Prolog Programming Assignment #2: State Space Problem Solving

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Abstract

In this assignment build up a program that will eventually create a valid solution to the Towers of Hanoi problem. This is an interesting program that uses Prolog's backend problem solving in very cool ways. We slowly place pieces of this code together until we have one that can solve level the problem with 5 disks. We do this by creating moves, valid states, goal and start states. The program does not resolve the problem with the perfect solution... so far.

Task 3: One Move Predicate and a Unit Test

State Space Operator Implementation Code:

```
m12([Tower1Before,Tower2Before,Tower3],[Tower1After,Tower2After,Tower3]) :-
Tower1Before = [H|T],
Tower1After = T,
Tower2Before = L,
Tower2After = [H|L].
```

Unit Test Code:

```
test__m12 :-
write('Testing: move_m12\n'),TowersBefore = [[t,s,m,l,h],[],[]],
trace('','TowersBefore',TowersBefore),
m12(TowersBefore,TowersAfter),
trace('','TowersAfter',TowersAfter).
```

Unit Test Demo:

```
3 ?- test__m12
.
Testing: move_m12
TowersBefore = [[t,s,m,l,h],[],[]]
TowersAfter = [[s,m,l,h],[t],[]]
true.
```

Task 4: The Remaining Five Move Predicates and a Unit Tests

State Space Operators Code:

```
% --- State Space operators
m12([Tower1Before,Tower2Before,Tower3],[Tower1After,Tower2After,Tower3]) :-
Tower1Before = [H|T],
Tower1After = T,
Tower2Before = L,
Tower2After = [H|L].
m13([Tower1Before,Tower2,Tower3Before],[Tower1After,Tower2,Tower3After]) :-
Tower1Before = [H|T],
Tower1After = T,
Tower3Before = L,
Tower3After = [H|L].
m21([Tower1Before,Tower2Before,Tower3],[Tower1After,Tower2After,Tower3]) :-
Tower2Before = [H|T],
Tower2After = T,
Tower1Before = L,
Tower1After = [H|L].
m23([Tower1,Tower2Before,Tower3Before],[Tower1,Tower2After,Tower3After]) :-
Tower2Before = [H|T],
Tower2After = T,
Tower3Before = L,
Tower3After = [H|L].
m31([Tower1Before,Tower2,Tower3Before],[Tower1After,Tower2,Tower3After]) :-
Tower3Before = [H|T],
Tower3After = T,
Tower1Before = L,
Tower1After = [H|L].
m32([Tower1,Tower2Before,Tower3Before],[Tower1,Tower2After,Tower3After]) :-
Tower3Before = [H|T],
Tower3After = T,
Tower2Before = L,
Tower2After = [H|L].
```

Unit Test Programs Code:

```
% --- State Space operator tests
test m12 :-
write('Testing: move_m12\n'),TowersBefore = [[t,s,m,l,h],[],[]],
trace('', 'TowersBefore', TowersBefore),
m12(TowersBefore, TowersAfter),
trace('', 'TowersAfter', TowersAfter).
test m13 :-
write('Testing: move_m13\n'),TowersBefore = [[t,s,m,l,h],[],[]],
trace('', 'TowersBefore', TowersBefore),
m13(TowersBefore, TowersAfter),
trace('','TowersAfter',TowersAfter).
test m21 :-
write('Testing: move m21\n'),TowersBefore = [[],[t,s,m,l,h],[]],
trace('', 'TowersBefore', TowersBefore),
m21(TowersBefore, TowersAfter),
trace('','TowersAfter',TowersAfter).
test m23 :-
write('Testing: move_m23\n'),TowersBefore = [[],[t,s,m,l,h],[]],
trace('', 'TowersBefore', TowersBefore),
m23(TowersBefore, TowersAfter),
trace('', 'TowersAfter', TowersAfter).
test m31 :-
write('Testing: move_m31\n'),TowersBefore = [[],[],[t,s,m,l,h]],
trace('', 'TowersBefore', TowersBefore),
m31(TowersBefore, TowersAfter),
trace('','TowersAfter',TowersAfter).
test__m32 :-
write('Testing: move m32\n'),TowersBefore = [[],[],[t,s,m,1,h]],
trace('', 'TowersBefore', TowersBefore),
m32(TowersBefore, TowersAfter),
trace('','TowersAfter',TowersAfter).
```

Unit Test Programs Demo:

```
5 ?- test m12.
Testing: move m12
TowersBefore = [[t,s,m,l,h],[],[]]
TowersAfter = [[s,m,l,h],[t],[]]
true.
6 ?- test m13.
Testing: move m13
TowersBefore = [[t,s,m,l,h],[],[]]
TowersAfter = [[s,m,1,h],[],[t]]
true.
7 ?- test m21.
Testing: move m21
TowersBefore = [[],[t,s,m,1,h],[]]
TowersAfter = [[t],[s,m,1,h],[]]
true.
8 ?- test m23.
Testing: move m23
TowersBefore = [[],[t,s,m,1,h],[]]
TowersAfter = [[],[s,m,1,h],[t]]
true.
9 ?- test m31.
Testing: move_m31
TowersBefore = [[],[],[t,s,m,l,h]]
TowersAfter = [[t],[],[s,m,l,h]]
true.
10 ?- test m32.
Testing: move m32
TowersBefore = [[],[],[t,s,m,1,h]]
TowersAfter = [[],[t],[s,m,1,h]]
true.
```

Task 5: Valid State Predicate and Unit Test

valid_state Predicate Code:

```
% --- valid_state(S) :: S is a valid state
valid_state([A | [B | [C]]]) :- state(A), state(B), state(C).
state([]).
state([t]).
state([t,s]).
state([t,s,m]).
state([t,s,m,l]).
state([t,s,m,h]).
state([t,s,m,1,h]).
state([t,s,1]).
state([t,s,h]).
state([t,s,l,h]).
state([t,m]).
state([t,m,1]).
state([t,m,l,h]).
state([t,1]).
state([t,1,h]).
state([t,h]).
state([s]).
state([s,m]).
state([s,m,1]).
state([s,m,l,h]).
state([s,1]).
state([s,1,h]).
state([s,h]).
state([m]).
state([m,1]).
state([m,1,h]).
state([m,h]).
state([1]).
state([1,h]).
state([h]).
```

Unit Test Program Code:

```
% ----
% --- valid_state tests

test__valid_state :-
    write('Testing: vaid_state\n'),
    test__vs([[],t,s,m,h],[],[]),
    test__vs([[],[h,t,s,m],[1]]),
    test__vs([[],[h,t,s,m],[1]]),
    test__vs([[],[h],[1,m,s,t]]),
    test__vs([[],[h],[t,s,m,l]]).

test__vs(S) :-
    valid_state(S),
    write(S), write('i is valid.'), nl.

test__vs(S) :-
    write(S), write(' is invalid.'), nl.
```

Unit Test Program Demo:

```
11 ?- test_valid_state.
Testing: vaid_state
[[l,t,s,m,h],[],[]] is invalid.
[[t,s,m,l,h],[],[]]i is valid.
[[],[h,t,s,m],[l]] is invalid.
[[],[t,s,m,h],[l]]i is valid.
[[],[h],[l,m,s,t]] is invalid.
[[],[h],[t,s,m,l]]i is valid.
true
```

Task 6: Defining the write sequence predicate

write_sequence Predicate Code:

```
% --- write_sequence
write sequence([]).
write_sequence([H|T]) :-
    sequence(H,Explanation),
    write(Explanation), nl,
    write sequence(T).
sequence(m12, Explanation) :-
    Explanation = 'Transfer a disk from tower 1 to 2.'.
sequence(m13, Explanation) :-
    Explanation = 'Transfer a disk from tower 1 to 3.'.
sequence(m21, Explanation) :-
    Explanation = 'Transfer a disk from tower 2 to 1.'.
sequence(m23, Explanation) :-
    Explanation = 'Transfer a disk from tower 2 to 3.'.
sequence(m31, Explanation) :-
    Explanation = 'Transfer a disk from tower 3 to 1.'.
sequence(m32, Explanation) :-
    Explanation = 'Transfer a disk from tower 3 to 2.'.
```

Unit Tester Code:

```
test__write_sequence :-
   write('First test of write_sequence ...'), nl,
   write_sequence([m31,m12,m13,m21]),
   write('Second test of write_sequence ...'), nl,
   write_sequence([m13,m12,m32,m13,m21,m23,m13]).
```

Unit Tester Demo:

```
12 ?- test_write_sequence.
First test of write_sequence ...
Transfer a disk from tower 3 to 1.
Transfer a disk from tower 1 to 2.
Transfer a disk from tower 1 to 3.
Transfer a disk from tower 2 to 1.
Second test of write_sequence ...
Transfer a disk from tower 1 to 3.
Transfer a disk from tower 1 to 2.
Transfer a disk from tower 3 to 2.
Transfer a disk from tower 1 to 3.
Transfer a disk from tower 2 to 1.
Transfer a disk from tower 2 to 1.
Transfer a disk from tower 2 to 3.
Transfer a disk from tower 1 to 3.
```

Task 7: Run the Program to Solve the 3 Disk Problem

Intermediate output display demo:

```
14 ?- solve.
PathSoFar = [[[s,m,1],[],[]]]
Move = m12
NextState = [[m,1],[s],[]]
PathSoFar = [[[s,m,1],[],[]],[[m,1],[s],[]]]
Move = m12
NextState = [[1],[m,s],[]]
Move = m13
NextState = [[1],[s],[m]]
PathSoFar = [[[s,m,1],[],[]],[[m,1],[s],[]],[[1],[s],[m]]]
NextState = [[],[1,s],[m]]
Move = m13
NextState = [[],[s],[1,m]]
Move = m21
NextState = [[s,1],[],[m]]
PathSoFar = [[[s,m,1],[],[]],[[m,1],[s],[]],[[1],[s],[m]],[[s,1],[],[m]]]
Move = m12
NextState = [[1],[s],[m]]
Move = m13
NextState = [[1],[],[s,m]]
PathSoFar = [[[s,m,1],[],[]],[[m,1],[s],[]],[[1],[s],[m]],[[s,1],[],[m]],[[1],[],[s,m]]]
Move = m12
NextState = [[],[1],[s,m]]
PathSoFar = [[[s,m,1],[],[]],[[m,1],[s],[]],[[1],[s],[m]],[[s,1],[],[m]],[[1],[s,m]],[[],[1],[s,m]]]
NextState = [[1],[],[s,m]]
Move = m23
NextState = [[],[],[1,s,m]]
Move = m31
NextState = [[s],[1],[m]]
PathSoFar = [[[s,m,1],[],[]],[[m,1],[s],[]],[[1],[s],[m]],[[s,1],[],[m]],[[1],[],[s,m]],[[s],[]],[[s],[]],[m]]]
Move = m12
NextState = [[],[s,1],[m]]
PathSoFar = [[[s,m,1],[],[]],[[m,1],[s],[]],[[1],[s],[m]],[[s,1],[],[m]],[[1],[s,m]],[[],[s,m]],[[s],[1],[m]],[[],[s,1],[m]]]
```

```
Move = m21
NextState = [[s],[1],[m]]
Move = m23
NextState = [[],[1],[s,m]]
Move = m31
NextState = [[m],[s,1],[]]
Move = m12
NextState = [[],[m,s,1],[]]
Move = m13
NextState = [[],[s,1],[m]]
Move = m21
NextState = [[s,m],[1],[]]
PathSoFar = [[[s,m,1],[],[[],[[m,1],[s],[]],[[1],[s],[m]],[[1],[],[s,m]],[[],[s,m]],[[s],[n]],[[m],[s,1],[]],[[s,1],[]],[]]]
Move = m12
NextState = [[m],[s,1],[]]
Move = m13
NextState = [[m],[1],[s]]
,[s]]]
Move = m12
,[s]],[[],[m,1],[s]]]
Move = m21
NextState = [[m],[1],[s]]
Move = m23
NextState = [[],[1],[m,s]]
Move = m31
,[s]],[[],[m,1],[s]],[[s],[m,1],[]]]
Move = m12
NextState = [[],[s,m,1],[]]
Move = m23
 NextState = [[],[m,1],[s]]
 Move = m13
 NextState = [[],[m,1],[s]]
 Move = m21
 NextState = [[m,s],[1],[]]
 Move = m23
 NextState = [[s],[1],[m]]
 Move = m32
 NextState = [[],[s,m,1],[]]
 ,[s]],[[],[m,1],[s]],[[],[s,m,1],[]]]
 Move = m21
 NextState = [[s],[m,1],[]]
PathSoFar = [[[s,m,1],[],[[m,1],[s],[]],[[1],[s,m]],[[1],[],[s,m]],[[],[s,m]],[[s],[]],[m],[m],[m],[s,1],[]],[[s,m],[1],[]],[m],[1]
 ,[s]],[[],[m,1],[s]],[[],[s,m,1],[]],[[s],[m,1],[]]]
Move = m12
 NextState = [[],[s,m,1],[]]
 Move = m13
 NextState = [[],[m,1],[s]]
 Move = m21
 NextState = [[m,s],[1],[]]
 Move = m23
 NextState = [[s],[1],[m]]
 Move = m23
 NextState = [[],[m,1],[s]]
 Move = m13
 NextState = [[],[1],[m,s]]
 Move = m21
 NextState = [[1,m],[],[s]]
 Move = m23
 NextState = [[m],[],[1,s]]
 Move = m31
 NextState = [[s,m],[1],[]]
 Move = m32
```

NextState = [[m],[s,1],[]]

NextState = [[1,s,m],[],[]]

Move = m21

Move = m23

```
NextState = [[s,m],[],[1]]
],[1]]]
Move = m12
NextState = [[m],[s],[1]]
PathSoFar = [[[s,m,1],[],[]],[[m,1],[s],[]],[[1],[s],[m]],[[s],[]],[[1],[],[s,m]],[[s],[]],[[]],[[]],[[],[s],[
],[1]],[[m],[s],[1]]]
Move = m12
NextState = [[],[m,s],[1]]
Move = m13
NextState = [[],[s],[m,1]]
PathSoFar = [[[s,m,1],[],[]],[[m,1],[s],[]],[[1],[s],[m]],[[s,1],[],[m]],[[s,m],[[s,m],[[s,m],[]],[]],[[s,m],[
],[1]],[[m],[s],[1]],[[],[s],[m,1]]]
Move = m21
NextState = [[s],[],[m,1]]
],[1]],[[m],[s],[1]],[[],[s],[m,1]],[[s],[],[m,1]]]
Move = m12
NextState = [[],[s],[m,1]]
Move = m13
NextState = [[],[],[s,m,1]]
],[1]],[[m],[s],[1]],[[],[s],[m,1]],[[s],[],[m,1]],[[],[],[s,m,1]]]
SolutionSoFar = [m12,m13,m21,m13,m12,m31,m12,m31,m21,m23,m12,m13,m21,m13]
```

English only solution display demo:

```
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 3 to tower 1.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 3 to tower 1.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 2 to tower 3.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 1 to tower 3.
```

Task 7 Q&A:

- 1. What was the length of your program's solution to the three disk problem?
 - My program's solution was 14 moves
- 2. What is the length of the shortest solution to the three disk problem?
 - -The shortest solution to the three disk problem is 7.
- 3. How do you account for the discrepancy?
- -The program is constrained by not being able to think ahead. It makes moves mindlessly, constrained by the rules of its code, until it finds a valid solution. It is unaware of the most efficient path to the goal state, but it will always find a valid one.

Task 8: Run the Program to Solve the 4 Disk Problem

4 Disk English Solution Demo:

```
Solution ...
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 3 to tower 1.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 3 to tower 1.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 3 to tower 1.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 3 to tower 1.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 3 to tower 1.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 3 to tower 1.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 3 to tower 1.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 3 to tower 1.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 1 to tower 3.
```

Task 8 Q&A:

- 1. What was the length of your program's solution to the four disk problem?
 - My program's solution length was 40 moves.
- 2. What is the length of the shortest solution to the four disk problem?
 - -The shortest 4 disk problem solution is 15 moves.

Task 9: Review Your Code and Archive it

Towers of Hanoi Code

```
% ------
    % --- File: towers of hanoi.pro
  % --- Line: Program to solve the Towers of Hanoi problem
    :- consult('inspectors.pro').
    % --- make_move(S,T,SSO) :: Make a move from state S to state T by SSO
    make move(TowersBeforeMove,TowersAfterMove,m12) :-
    m12(TowersBeforeMove, TowersAfterMove).
11
12
    make_move(TowersBeforeMove,TowersAfterMove,m13) :-
    m13(TowersBeforeMove, TowersAfterMove).
    make move(TowersBeforeMove,TowersAfterMove,m21) :-
    m21(TowersBeforeMove, TowersAfterMove).
18
    make move(TowersBeforeMove, TowersAfterMove, m23) :-
    m23(TowersBeforeMove, TowersAfterMove).
    make move(TowersBeforeMove,TowersAfterMove,m31) :-
    m31(TowersBeforeMove, TowersAfterMove).
    make move(TowersBeforeMove,TowersAfterMove,m32) :-
    m32(TowersBeforeMove, TowersAfterMove).
```

```
% --- valid_state(S) :: S is a valid state
valid_state([A | [B | [C]]]) :- state(A), state(B), state(C).
state([]).
state([t]).
state([t,s]).
state([t,s,m]).
state([t,s,m,1]).
state([t,s,m,h]).
state([t,s,m,l,h]).
state([t,s,1]).
state([t,s,h]).
state([t,s,l,h]).
state([t,m]).
state([t,m,1]).
state([t,m,1,h]).
state([t,1]).
state([t,1,h]).
state([t,h]).
state([s]).
state([s,m]).
state([s,m,1]).
state([s,m,l,h]).
state([s,1]).
state([s,1,h]).
state([s,h]).
state([m]).
state([m,1]).
state([m,1,h]).
state([m,h]).
state([1]).
state([1,h]).
state([h]).
```

```
% --- valid_state tests
      test__valid_state :-
          write('Testing: vaid_state\n'),
          test__vs([[1,t,s,m,h],[],[]]),
          test__vs([[t,s,m,l,h],[],[]]),
          test__vs([[],[h,t,s,m],[1]]),
          test__vs([[],[t,s,m,h],[1]]),
          test__vs([[],[h],[1,m,s,t]]),
          test__vs([[],[h],[t,s,m,1]]).
      test vs(S):-
          valid state(S),
          write(S), write('i is valid.'), nl.
      test_vs(S) :-
          write(S), write(' is invalid.'), nl.
      % --- solve(Start,Solution) :: succeeds if Solution represents a path
      % --- from the start state to the goal state.
      solve :-
      extend_path([[[t,s,m,1],[],[]]],[],Solution),
      write_solution(Solution).
      extend_path(PathSoFar,SolutionSoFar,Solution) :-
      PathSoFar = [[[],[],[t,s,m,1]]|_],
      showr('PathSoFar',PathSoFar),
      showr('SolutionSoFar',SolutionSoFar),
      Solution = SolutionSoFar.
      extend_path(PathSoFar,SolutionSoFar,Solution) :-
      PathSoFar = [CurrentState|_],
      showr('PathSoFar',PathSoFar),
      make move(CurrentState, NextState, Move),
      show('Move',Move),
      show('NextState',NextState),
      not(member(NextState,PathSoFar)),
      valid_state(NextState),
      Path = [NextState|PathSoFar],
      Soln = [Move|SolutionSoFar],extend_path(Path,Soln,Solution).
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```

```
% ------
     % --- write_sequence
     write_sequence([]).
     write_sequence([H|T]) :-
         sequence(H,Explanation),
         write(Explanation),nl,
         write_sequence(T).
     sequence(m12, Explanation) :-
         Explanation = 'Transfer a disk from tower 1 to tower 2.'.
      sequence(m13, Explanation) :-
         Explanation = 'Transfer a disk from tower 1 to tower 3.'.
     sequence(m21, Explanation) :-
         Explanation = 'Transfer a disk from tower 2 to tower 1.'.
     sequence(m23, Explanation) :-
         Explanation = 'Transfer a disk from tower 2 to tower 3.'.
     sequence(m31, Explanation) :-
128
         Explanation = 'Transfer a disk from tower 3 to tower 1.'.
      sequence(m32, Explanation) :-
         Explanation = 'Transfer a disk from tower 3 to tower 2.'.
     test write sequence :-
         write('First test of write_sequence ...'), nl,
         write_sequence([m31,m12,m13,m21]),
         write('Second test of write sequence ...'), nl,
         write_sequence([m13,m12,m32,m13,m21,m23,m13]).
     % --- write sequence reversed(S) :: Write the sequence, given by S,
     % --- expanding the tokens into meaningful strings.
     write_solution(S) :-
     nl, write('Solution ...'), nl, nl,
     reverse(S,R),
     write_sequence(R),nl.
```

```
% --- State Space operators
      m12([Tower1Before,Tower2Before,Tower3],[Tower1After,Tower2After,Tower3]) :-
      Tower1Before = [H|T],
      Tower1After = T,
      Tower2Before = L,
      Tower2After = [H|L].
      m13([Tower1Before,Tower2,Tower3Before],[Tower1After,Tower2,Tower3After]) :-
      Tower1Before = [H|T],
      Tower1After = T,
      Tower3Before = L,
      Tower3After = [H|L].
      m21([Tower1Before,Tower2Before,Tower3],[Tower1After,Tower2After,Tower3]) :-
      Tower2Before = [H|T],
      Tower2After = T,
      Tower1Before = L,
      Tower1After = [H|L].
170
      m23([Tower1,Tower2Before,Tower3Before],[Tower1,Tower2After,Tower3After]) :-
171
      Tower2Before = [H|T],
      Tower2After = T,
173
      Tower3Before = L,
      Tower3After = [H|L].
176
      m31([Tower1Before,Tower2,Tower3Before],[Tower1After,Tower2,Tower3After]) :-
      Tower3Before = [H|T],
178
      Tower3After = T,
      Tower1Before = L,
      Tower1After = [H|L].
      m32([Tower1,Tower2Before,Tower3Before],[Tower1,Tower2After,Tower3After]) :-
      Tower3Before = [H|T],
      Tower3After = T,
      Tower2Before = L,
      Tower2After = [H|L].
```

```
% --- State Space operator tests
      test m12 :-
      write('Testing: move_m12\n'),TowersBefore = [[t,s,m,l,h],[],[]],
      trace('', 'TowersBefore', TowersBefore),
      m12(TowersBefore, TowersAfter),
      trace('', 'TowersAfter', TowersAfter).
      test m13 :-
      write('Testing: move m13\n'),TowersBefore = [[t,s,m,l,h],[],[]],
      trace('', 'TowersBefore', TowersBefore),
      m13(TowersBefore, TowersAfter),
      trace('','TowersAfter',TowersAfter).
      test m21 :-
      write('Testing: move m21\n'),TowersBefore = [[],[t,s,m,l,h],[]],
      trace('', 'TowersBefore', TowersBefore),
      m21(TowersBefore, TowersAfter),
      trace('', 'TowersAfter', TowersAfter).
      test m23 :-
210
      write('Testing: move_m23\n'),TowersBefore = [[],[t,s,m,l,h],[]],
      trace('', 'TowersBefore', TowersBefore),
211
212
      m23(TowersBefore, TowersAfter),
213
      trace('', 'TowersAfter', TowersAfter).
214
215
      test m31 :-
      write('Testing: move m31\n'),TowersBefore = [[],[],[t,s,m,l,h]],
216
      trace('', 'TowersBefore', TowersBefore),
217
218
      m31(TowersBefore, TowersAfter),
219
      trace('', 'TowersAfter', TowersAfter).
220
221
      test m32 :-
222
      write('Testing: move_m32\n'),TowersBefore = [[],[],[t,s,m,l,h]],
      trace('', 'TowersBefore', TowersBefore),
224
      m32(TowersBefore, TowersAfter),
225
      trace('','TowersAfter',TowersAfter).
```

Inspectors.pro Code

```
% ------
% ------
% --- File: inspectors.pro
% --- Line: Utilities for inspecting memory during program execution
% -----
% --- These two can be used to print the value of a variable, labelled
% --- in two ways, and pause for the programmer to check out the
% --- situation. The firstone is generally useful. The second one is
% --- applicable only whenthe value of the veriable is a list, and it
% --- will print the valuein reverse order which is sometimes just
% --- what is desired. The first label generally pertains to a location
% --- in the program. The second label is just the name of the variable
% --- to which the value is bound.
check(Label,Name,Value) :-
write(Label),
write(Name), write(' = '),
write(Value),nl,
read(_).
checkr(Label,Name,Value) :-
write(Label),
write(Name), write(' = '),
reverse(Value, RValue),
write(RValue), nl,
read(_).
% --- These two are like the previously described checking predicates,
% --- except that they do not do the pause.
trace(Label,Name,Value) :-
write(Label),
write(Name), write(' = '),
write(Value), nl.
tracer(Label, Name, Value) :-
write(Label),
write(Name),write(' = '),
reverse(Value, RValue),
write(RValue), nl.
% -----
% --- Like trace, but without the extra labelling functionality.
show(Name, Value) :-
write(Name),write(' = '),
write(Value), nl.
showr(Name, Value) :-
write(Name),write(' = '),
reverse(Value, RValue),
write(RValue), nl.
```