BAN Logic
A Logic of Authentication

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BAN Logic

The BAN logic was named after its inventors, Mike Burrows, Martín Abadí, and Roger Needham.

The logic is, as they stated, a *logic of belief and action*. It contains no logical inversions; therefore it cannot be used to prove a protocol flawed.

But when proof that a protocol is correct cannot be obtained, that protocol deserves to be treated with grave suspicion.

Notation

The logic reasons about beliefs. If Alice believes a proposition P, we write $A \mid\equiv\ P$ and say 'A *believes* P'.

Alice believes that K_{AT} is a good key for communicating with Trent. This is expressed as $A \mid\equiv\ A \overset{K_{AT}}{\longleftrightarrow} T$; we say A *believes* K_{AT} *is a good key for* A *and* T.

Trent acts as *authentication server* or *certification authority* in many of the protocols analyzed by BAN logic. If Alice believes that Trent can be trusted to create a 'good key' for communication with Bob, we write $A \mid\equiv\ T \Rightarrow A \overset{K}{\longleftrightarrow} B$; we say '$A$ *believes* T *has jurisdiction over* or *speaks for* *good keys for* A *and* B'.

Messages

When Alice receives a message (which, in our logic, always contains a proposition), we write $A \triangleleft P$ and say A *sees* P.

Seeing is not believing unless you know who said it. Note that, in this logic, nobody says anything he or she does not believe. If Alice sent a message containing the statement P, we may write $A \mid\!\sim P$ and say A *once said* P.

But, Alice may have said it so long ago that we can no longer trust the contents of her message. We need to know that Alice's statement is *fresh*. When a statement P is fresh we write $\#(P)$ and say P *is fresh*.

Notation Summary

$P \mid\equiv X$	P *believes* X
$P \triangleleft X$	P *sees* X
$P \mid\sim X$	P *once said* X
$\#(P)$	P is *fresh*
$P \Rightarrow X$	P has *jurisdiction over* X
$P \overset{K}{\longleftrightarrow} Q$	K is a *good key* for communicating between P and Q
$\overset{K}{\mapsto} P$	P has K as a *public key*
$P \overset{X}{\rightleftharpoons} Q$	X is a *secret* known only to P and Q
$\langle X\rangle_Y$	X *combined with* (secret) Y

Axioms

We adopt the notation in the BAN papers: $\frac{P}{Q}$ means *if* P *is true then* Q *is true*.

We assume that participants in protocols are good logicians: if Alice believes a proposition X and $\frac{X}{Y}$, then she believes Y too. This is true for the axioms also.

Message meaning rules

$$\frac{P \mid\equiv P \overset{K}{\longleftrightarrow} Q, P \triangleleft \{X\}_K}{P \mid\equiv Q \mid\sim X}$$

$$\frac{P \mid\equiv \overset{K}{\rightarrow} Q, P \triangleleft \{X\}_{K^{-1}}}{P \mid\equiv Q \mid\sim X}$$

$$\frac{P \mid\equiv P \overset{Y}{\rightleftharpoons} Q, P \triangleleft \langle X \rangle_Y}{P \mid\equiv Q \mid\sim X}$$

Nonce Verification

$$\frac{P \mid\equiv \#(X),\; P \mid\equiv Q \mid\sim X}{P \mid\equiv Q \mid\equiv X}$$

Jurisdiction Rule

$$\frac{P \mid\equiv Q \Rightarrow X, P \mid\equiv Q \mid\equiv X}{P \mid\equiv X}$$

More Rules

$$\frac{P \mid\equiv X, P \mid\equiv Y}{P \mid\equiv (X, Y)}$$

$$\frac{P \mid\equiv (X, Y)}{P \mid\equiv X}$$

$$\frac{P \mid\equiv Q \mid\equiv (X, Y)}{P \mid\equiv Q \mid\equiv X}$$

$$\frac{P \mid\equiv Q \mid\sim (X, Y)}{P \mid\equiv Q \mid\sim X}$$

And More

$$\frac{P \lhd (X, Y)}{P \lhd X}$$

$$\frac{P \lhd \langle X \rangle_Y)}{P \lhd X}$$

$$\frac{P \mid\equiv P \stackrel{K}{\longleftrightarrow} Q, P \lhd \{X\}_K}{P \lhd X}$$

And More

$$\frac{P \mid\equiv \stackrel{K}{\rightarrow} P,\ P \lhd \{X\}_K}{P \lhd X}$$

$$\frac{P \mid\equiv \stackrel{K}{\rightarrow} Q,\ P \lhd \{X\}_{K^{-1}}}{P \lhd X}$$

$$\frac{P \mid\equiv \#(X)}{A \mid\equiv \#((X, Y))}$$

Most analyses start with assumptions such as

$$A \mid\equiv A \stackrel{K_{AT}}{\longleftrightarrow} T \qquad B \mid\equiv B \stackrel{K_{BT}}{\longleftrightarrow} T$$

$$A \mid\equiv T \Rightarrow A \stackrel{K_{AB}}{\longleftrightarrow} B \qquad B \mid\equiv T \Rightarrow B \stackrel{K_{AB}}{\longleftrightarrow} B$$

$$A \mid\equiv \#(N_A) \qquad B \mid\equiv \#(N_B)$$

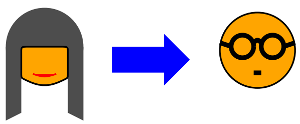
$$T \mid\equiv A \stackrel{K_{AB}}{\longleftrightarrow} B$$

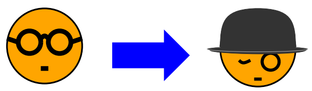
and need to conclude with

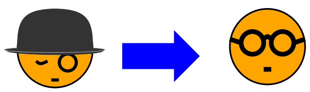
$$A \mid\equiv A \stackrel{K_{AB}}{\longleftrightarrow} B \qquad B \mid\equiv A \stackrel{K_{AB}}{\longleftrightarrow} B$$

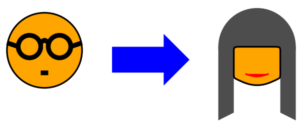
$$A \mid\equiv B \mid\equiv A \stackrel{K_{AB}}{\longleftrightarrow} B \qquad B \mid\equiv A \mid\equiv A \stackrel{K_{AB}}{\longleftrightarrow} B$$

The Ottway-Rees Authentication Protocol

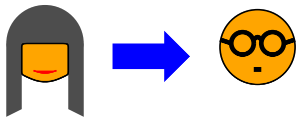
1.  $M, A, B, \{N_A, M, A, B\}_{K_{AT}}$

2.  $M, A, B, \{N_A, M, A, B\}_{K_{AT}}, \{N_B, M, A, B\}_{K_{BT}}$

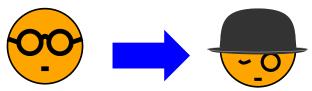
3.  $M, \{N_A, K_{AB}\}_{K_{AT}}, \{N_B, K_{AB}\}_{K_{BT}}$

4.  $M, \{N_A, K_{AB}\}_{K_{AT}}$

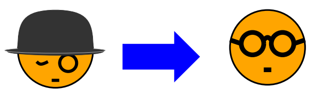
Analysis — The Protocol

1.  $M, A, B, \{N_A, M, A, B\}_{K_{AT}}$

   $\{N_A, N_C\}_{K_{AS}}$

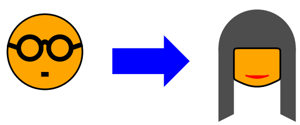
2.  $M, A, B, \{N_A, M, A, B\}_{K_{AT}}, \{N_B, M, A, B\}_{K_{BT}}$

   $\{N_A, N_C\}_{K_{AS}}, \{N_B, N_C\}_{K_{BS}}$

3.  $M, \{N_A, K_{AB}\}_{K_{AT}}, \{N_B, K_{AB}\}_{K_{BT}}$

   $\{N_A, A \overset{K_{AB}}{\longleftrightarrow} B, B \mid\!\sim\ N_C\}_{K_{AT}},$

   $\{N_B, A \overset{K_{AB}}{\longleftrightarrow} B, A \mid\!\sim\ N_C\}_{K_{BT}}$

4.  $M, \{N_A, K_{AB}\}_{K_{AT}}$

   $\{N_A, A \overset{K_{AB}}{\longleftrightarrow} B, B \mid\!\sim\ N_C\}_{K_{AT}}$

Analysis — Assumptions

$$A \mid\equiv A \stackrel{K_{AT}}{\longleftrightarrow} T \qquad B \mid\equiv B \stackrel{K_{BT}}{\longleftrightarrow} T$$

$$T \mid\equiv A \stackrel{K_{AT}}{\longleftrightarrow} T \qquad T \mid\equiv B \stackrel{K_{BT}}{\longleftrightarrow} T$$

$$T \mid\equiv A \stackrel{K_{AB}}{\longleftrightarrow} B$$

$$A \mid\equiv T \Rightarrow A \stackrel{K_{AB}}{\longleftrightarrow} B \qquad B \mid\equiv T \Rightarrow B \stackrel{K_{AB}}{\longleftrightarrow} B$$

$$A \mid\equiv T \Rightarrow B \mid\sim X \qquad B \mid\equiv T \Rightarrow A \mid\sim X$$

$$A \mid\equiv \#(N_A) \qquad B \mid\equiv \#(N_B)$$

$$A \mid\equiv \#(N_C)$$

$$\frac{B \triangleleft \{N_B, A \stackrel{K_{AB}}{\longleftrightarrow} B\}_{K_{BT}}, B \mid\equiv B \stackrel{K_{BT}}{\longleftrightarrow} T}{B \mid\equiv T \mid\sim N_B, A \stackrel{K_{AB}}{\longleftrightarrow} B}$$

$$\frac{B \triangleleft \{N_B, A \stackrel{K_{AB}}{\longleftrightarrow} B\}_{K_{BT}}, B \mid\equiv \#(N_B)}{B \mid\equiv \#(A \stackrel{K_{AB}}{\longleftrightarrow} B)}$$

$$\frac{B \mid\equiv T \mid\sim A \stackrel{K_{AB}}{\longleftrightarrow} B, B \mid\equiv \#(A \stackrel{K_{AB}}{\longleftrightarrow} B)}{B \mid\equiv T \mid\equiv A \stackrel{K_{AB}}{\longleftrightarrow} B}$$

$$\frac{B \mid\equiv T \mid\equiv A \stackrel{K_{AB}}{\longleftrightarrow} B, B \mid\equiv T \Rightarrow A \stackrel{K_{AB}}{\longleftrightarrow} B}{B \mid\equiv A \stackrel{K_{AB}}{\longleftrightarrow} B}$$

Analysis — Conclusion

Similarly, we derive $A \mid\equiv A \overset{K_{AB}}{\longleftrightarrow} B$.

We can also derive

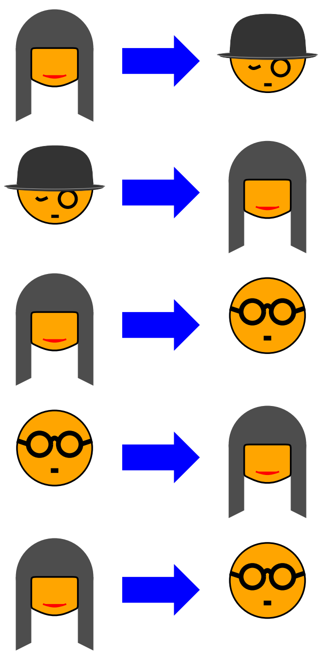
$$A \mid\equiv B \mid\equiv N_C,$$

but we can only prove

$$B \mid\equiv A \mid\sim N_C.$$

N_A is not needed, N_C can be used instead.

the Needham-Schroeder Protocol



1. A, B, N_A

2. $\{N_A, B, K_{AB}, \{A, K_{AB}\}_{K_{BT}}\}_{K_{AT}}$

3. $\{A, K_{AB}\}_{K_{BT}}$

4. $\{N_B\}_{K_{AB}}$

5. $\{N_B - 1\}_{K_{AB}}$

Needham-Schroeder Analysed

Assumptions:

$$T \mid\equiv A \overset{K_{AB}}{\longleftrightarrow} B$$

$$A \mid\equiv A \overset{K_{AT}}{\longleftrightarrow} T \qquad B \mid\equiv B \overset{K_{BT}}{\longleftrightarrow} T$$

$$T \mid\equiv A \overset{K_{AT}}{\longleftrightarrow} T \qquad T \mid\equiv B \overset{K_{BT}}{\longleftrightarrow} T$$

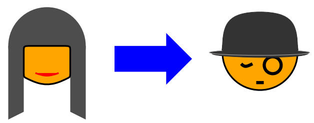
$$A \mid\equiv T \Rightarrow A \overset{K_{AB}}{\longleftrightarrow} B \qquad B \mid\equiv T \Rightarrow B \overset{K_{AB}}{\longleftrightarrow} B$$

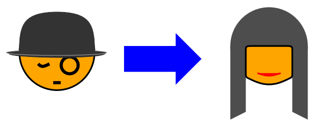
$$A \mid\equiv T \Rightarrow \#(K_{AB})$$

$$A \mid\equiv \#(N_A) \qquad B \mid\equiv \#(N_B)$$

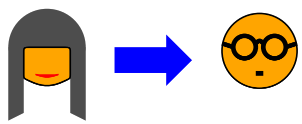
$$T \mid\equiv \#(K_{AB}) \qquad B \mid\equiv \#(A \overset{K}{\longleftrightarrow} B)$$

Messages

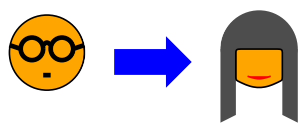
1.  A, B, N_A

2.  $\{N_A, B, K_{AB}, \{A, K_{AB}\}_{K_{BT}}\}_{K_{AT}}$

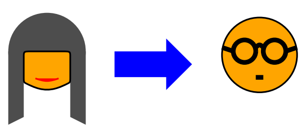
$$\{N_A, A \overset{K_{AB}}{\longleftrightarrow} B,\ \#(A \overset{K_{AB}}{\longleftrightarrow} B)\ , \{A \overset{K_{AB}}{\longleftrightarrow} B\}_{K_{BT}}\}_{K_{AT}}$$

3.  $\{A, K_{AB}\}_{K_{BT}}$

$$\{A \overset{K_{AB}}{\longleftrightarrow} B\}_{K_{BT}}$$

4.  $\{N_B\}_{K_{AB}}$

$$\{N_B, A \overset{K_{AB}}{\longleftrightarrow} B\}_{K_{AB}} from B$$

5.  $\{N_B - 1\}_{K_{AB}}$

$$\{N_B, A \overset{K_{AB}}{\longleftrightarrow} B\}_{K_{AB}} from A$$

Analysis

As in the Ottway-Rees protocol, we have the message for Alice saying $\{N_A, A \overset{K_{AB}}{\longleftrightarrow} B\}_{K_{AT}}$, so we can prove in an identical manner $A \mid\equiv A \overset{K_{AB}}{\longleftrightarrow} B$.

But Bob gets no message linking Trent's statement $A \overset{K_{AB}}{\longleftrightarrow} B$ to something he knows to be fresh. We cannot get beyond $B \mid\equiv T \mid\sim A \overset{K_{AB}}{\longleftrightarrow} B$.

Needham-Schroeder Exposed

This exactly puts the finger on the weakness in the Needham-Schroeder protocol: Mallory could record Alice and Bob's key exchange and trick Bob into accepting the key in Message 3 any time he chooses.

Mallory will be thwarted, however, when he cannot respond to Bob's Message 4.

But if he can afford to spend a year of CPU time cracking K_{AB}, he can get Bob to accept *and use* a year-old key.