Cpr E 281 HW05 SOLUTION

ELECTRICAL AND COMPUTER ENGINEERING
IOWA STATE UNIVERSITY

Number Representation and Arithmetic Circuits

Assigned Date: Fifth Week
Due Date: Monday, Oct. 3, 2016

P1. (10 points) An expedition to Mars found the ruins of a civilization. The explorers were able to translate the mathematical equations:

$$5x^2 - 50x + 125 = 0$$

with the solutions: x=5 and x=8. The x=5 solution seemed okay, but x=8 was puzzling. The problem should be because Martians were using a non-decimal number system. Therefore, "50" is not fifty, but "50" in base b (50b=5×b+0×1=5b). The explorers reflected on the way in which Earth's number system developed. How many fingers would you say the Martians had? (*Hint*: What should be the value of the base b such that both 5 and 8 are solutions of the equation?)

Solution:

By transferring all coefficients from base b to decimal, we will get:

$$(5)x^2 - (5b+0)x + (1\times b^2 + 2\times b + 5) = 0$$

Substitute x=5 in the equation and we have:

$$b^2 - 23b + 130 = 0 \Rightarrow b = 13 \text{ or } b=10$$

Substitute x=8 in the equation and we have:

$$b^2 - 38b + 325 = 0 \Rightarrow b = 13 \text{ or } b=25$$

Therefore, b=13.

P2. (15 points) Complete the following table by converting the integers in decimal to **5-bit signed numbers** in binary. (1 point for each cell.)

	Decimal	Sign-and-Magnitude	1's Complement	2's Complement
Example	**-5**	**10101**	**11010**	**11011**
(a)	**-15**	11111	10000	10001
(b)	**-10**	11010	10101	10110
(c)	**-1**	10001	11110	11111
(d)	**0**	00000	00000	00000
(e)	**7**	00111	00111	00111

P3. (18 points) Perform the following conversions: (3 points each)

a) $(10011)_2$ in 5-bit sign-and-magnitude to 5-bit 1's complement
b) $(10011)_2$ in 5-bit sign-and-magnitude to 5-bit 2's complement
c) $(11000)_2$ in 5-bit 1's complement to 5-bit sign-and-magnitude
d) $(11000)_2$ in 5-bit 1's complement to 5-bit 2's complement
e) $(101110)_2$ in 6-bit 2's complement to 6-bit sign-and-magnitude
f) $(101110)_2$ in 6-bit 2's complement to 6-bit 1's complement

Solution:

a) 11100 — $(10011)_2$ S&M = $(-3)_{10}$
b) 11101
c) 10111 — $(11000)_2$ 1's = $(-7)_{10}$
d) 11001
e) 110010 — $(101110)_2$ 2's = $(-18)_{10}$
f) 101101

P4. (12 points) Negate the following 6-bit 2's complement binary numbers: (3 points each)

a) $(001010)_2$
b) $(110011)_2$
c) $(100100)_2$
d) $(010001)_2$

Solution:

a) 110110
b) 001101
c) 011100
d) 101111

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P5. (10 points) Consider 4-bit 2's complement representation for signed numbers.

a) (2 points) What is the largest integer in decimal that can be represented?
b) (2 points) What is the smallest integer in decimal that can be represented?
c) (6 points) For n-bit 2's complement representation, what are the largest and smallest integers in decimal that can be represented?

Solution:

a) +7
b) -8
c) $2^{n-1} - 1$
d) -2^{n-1}

P6. (10 points) Perform the following additions of **5-bit unsigned numbers** in binary and identify if overflow occurs. Check your answers by converting the numbers to decimal.

a) (5 points) 01111 + 01010
b) (5 points) 11000 + 01101

Solution:

a)

```
C: 0 1 1 1 0 0
     0 1 1 1 1   (+15)
 +   0 1 0 1 0   (+10)
 -------------
     1 1 0 0 1   (+25)
```

Overflow: NO

b)

```
C: 1 1 0 0 0 0
     1 1 0 0 0   (+24)
 +   0 1 1 0 1   (+13)
 -------------
     0 0 1 0 1   (+5)
```

Overflow: YES

P7. (15 points) Perform the following operations of **5-bit 2's complement numbers** in binary and identify if overflow occurs. Check your answers by converting the numbers to decimal.

a) (5 points) 01111 + 01010
b) (5 points) 11000 + 01101
c) (5 points) 01010 - 11101

Solution:

a)

$$
\begin{array}{rrl}
\text{C:} & \boxed{0\ 1}\,1\ 1\ 0\ 0 & \\
 & 0\ 1\ 1\ 1\ 1 & (+15) \\
+ & 0\ 1\ 0\ 1\ 0 & (+10) \\
\hline
 & 1\ 1\ 0\ 0\ 1 & (-7)
\end{array}
$$

Overflow: YES

b)

$$
\begin{array}{rrl}
\text{C:} & \boxed{1\ 1}\,0\ 0\ 0\ 0 & \\
 & 1\ 1\ 0\ 0\ 0 & (-8) \\
+ & 0\ 1\ 1\ 0\ 1 & (+13) \\
\hline
 & 0\ 0\ 1\ 0\ 1 & (+5)
\end{array}
$$

Overflow: NO

c)
To perform subtraction in 2's complement, you can negate the second number and then perform the regular addition.

Negation of $(11101)_2 = (00011)_2$ $(-3)_{10} \Rightarrow (+3)_{10}$

$$
\begin{array}{rrl}
\text{C:} & \boxed{0\ 0}\,0\ 1\ 0\ 0 & \\
 & 0\ 1\ 0\ 1\ 0 & (+10) \\
+ & 0\ 0\ 0\ 1\ 1 & (+3) \\
\hline
 & 0\ 1\ 1\ 0\ 1 & (+13)
\end{array}
$$

Overflow: NO

P8. (10 points) Read Section 3.2 from the textbook. A full-adder (FA) can be constructed with two half-adders (HAs). From Figure 3.4 on page 129 one can infer that the carry-out function for a FA is given by:

$$c_{i+1} = x_i y_i + c_i(x_i \oplus y_i)$$

On the other hand, the textbook states on page 126 that the carry-out function for a FA is:

$$c_{i+1} = x_i y_i + x_i c_i + y_i c_i$$

Prove that these two functions are the same using Boolean algebra.

<u>Solution:</u>

$$
\begin{aligned}
c_{i+1} &= x_i y_i + c_i(x_i \oplus y_i) \\
&= x_i y_i + c_i(x_i y_i' + x_i' y_i) \\
&= x_i y_i + c_i(x_i y_i' + x_i' y_i) \\
&= x_i y_i + c_i x_i y_i' + c_i x_i' y_i \\
&= x_i y_i + c_i x_i y_i' + x_i y_i + c_i x_i' y_i && \text{Add one more } x_i y_i \\
&= x_i(y_i + c_i y_i') + y_i(x_i + c_i x_i') && \text{Apply } A + \overline{A}B = A + B \\
&= x_i(y_i + c_i) + y_i(x_i + c_i) \\
&= x_i y_i + x_i c_i + y_i x_i + y_i c_i \\
&= x_i y_i + x_i c_i + y_i c_i
\end{aligned}
$$