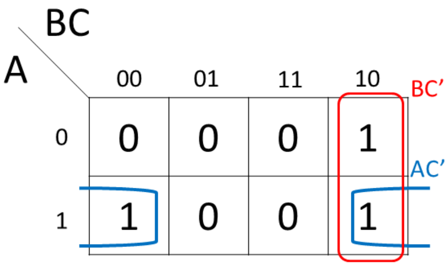
Cpr E 281 HW04 SOLUTION

ELECTRICAL AND COMPUTER ENGINEERING
IOWA STATE UNIVERSITY

Minimization and Karnaugh Maps
Assigned Date: Fourth Week
Due Date: Monday, Sep. 19, 2016

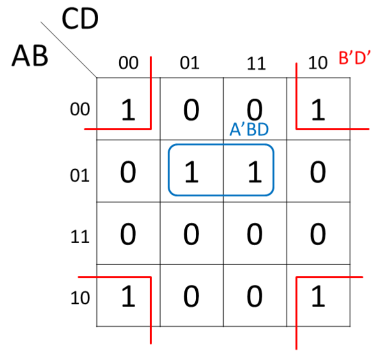
P1. (20 points) Use a K-map to find the minimal sum-of-products (SOP) expression for the following four problems. Show the terms that are grouped in each K-map.

a)



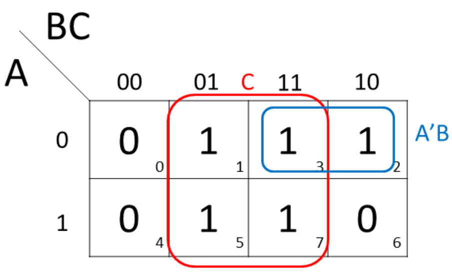
$$F(A,B,C) = AC' + BC'$$

b)



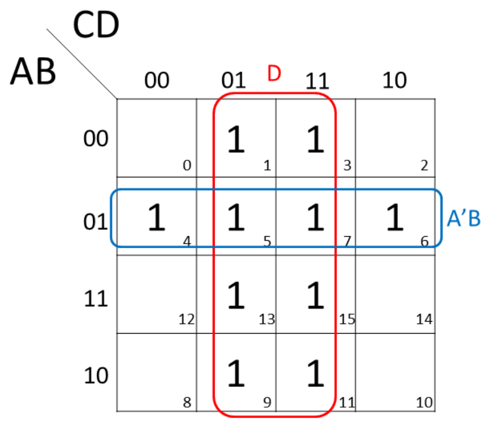
$$F(A,B,C,D) = B'D' + A'BD$$

c)



$$F(A,B,C) = C + A'B$$

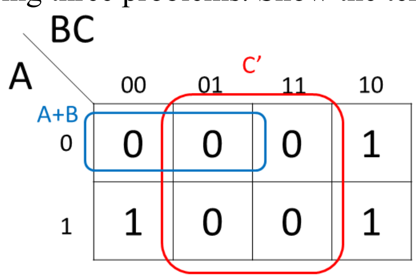
d)



$$F(A,B,C,D) = D + A'B$$

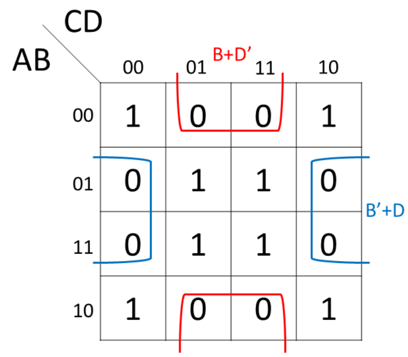
P2. (15 points) Use a K-map to find the minimal product-of-sums (POS) expression for the following three problems. Show the terms that are grouped in each K-map.

a)



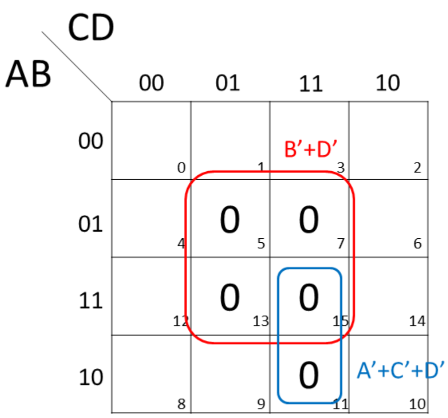
$$F(A,B,C) = (A+B)C'$$

b)



$$F(A,B,C,D) = (B'+D)(B+D')$$

c)



$$F(A,B,C,D) = (B'+D')(A'+C'+D')$$

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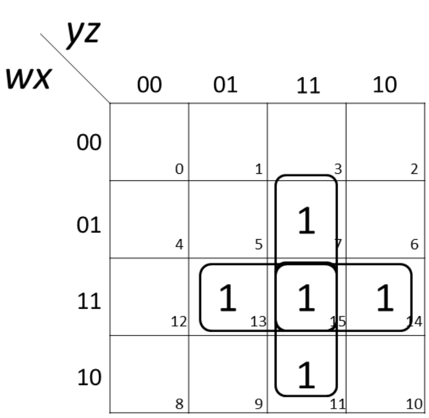
P3. (15 points) A four-variable function $F(w,x,y,z)$ is called a *majority* function if $F = 1$ when any three or all four of its input variables are equal to 1.

a) Truth table:

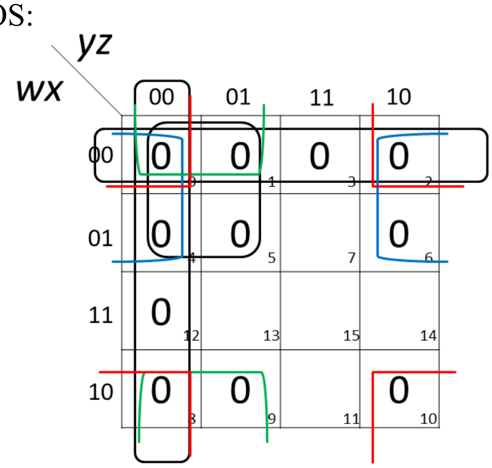
w	*x*	*y*	*z*	*F*
0	0	0	0	**0**
0	0	0	1	**0**
0	0	1	0	**0**
0	0	1	1	**0**
0	1	0	0	**0**
0	1	0	1	**0**
0	1	1	0	**0**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**0**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**1**
1	1	1	0	**1**
1	1	1	1	**1**

b) minimal SOP:



$F(w,x,y,z) = wxz + wxy + xyz + wyz$

c) minimal POS:



$F(w,x,y,z) = (w+x)(w+y)(w+z)(x+y)(x+z)(y+z)$

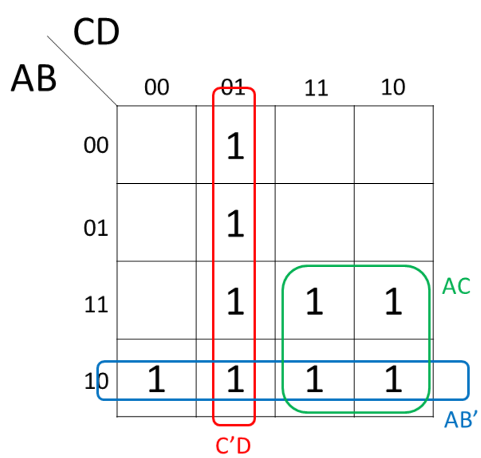
P4. (10 points) Use a K-map to derive the minimal SOP expressions for the following Boolean function:

$$F(A,B,C,D) = ACD' + C'D + AB' + ABCD$$

One way to solve this question is to build a truth table from the logic expression of F, and draw a K-map based on that truth table. Another way is to build the K-map by looking at each minterm in F:

$ACD' \Rightarrow$ 1□10 $\Rightarrow$ 1010,1110
$C'D \Rightarrow$ □□01 $\Rightarrow$ 0001,0101,1001,1101
$AB' \Rightarrow$ 10□□ $\Rightarrow$ 1000,1001,1010,1011
$ABCD \Rightarrow$ 1111

K-map:



The simplified SOP expression is:

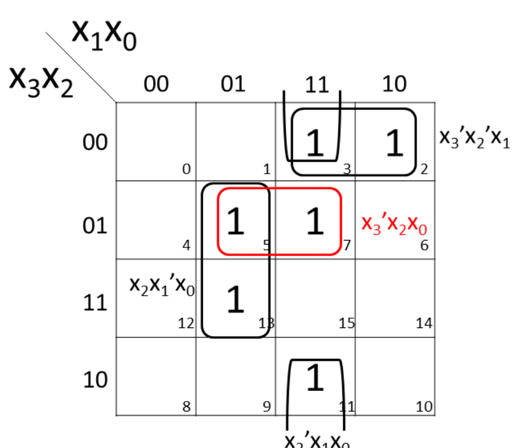
$$F(A,B,C,D) = AC + AB' + C'D.$$

P5. (20 points) Design a circuit that accepts a 4-bit number X= $x_3x_2x_1x_0$ as input and generates a 1-bit output *P* that is equal to 1 if the input number is a prime. (0 and 1 are not prime; 2, 3, 5, etc., are prime.)

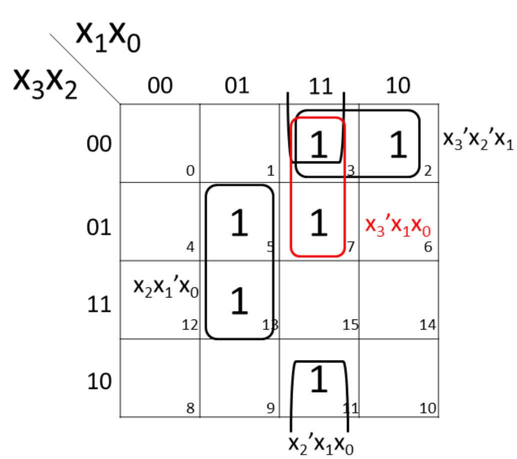
a) Truth table:

decimal	x_3	x_2	x_1	x_0	***P***
0	0	0	0	0	**0**
1	0	0	0	1	**0**
2	0	0	1	0	**1**
3	0	0	1	1	**1**
4	0	1	0	0	**0**
5	0	1	0	1	**1**
6	0	1	1	0	**0**
7	0	1	1	1	**1**
8	1	0	0	0	**0**
9	1	0	0	1	**0**
10	1	0	1	0	**0**
11	1	0	1	1	**1**
12	1	1	0	0	**0**
13	1	1	0	1	**1**
14	1	1	1	0	**0**
15	1	1	1	1	**0**

b) There exist two possible minimal SOP expressions for *P*. (students only need to have one of them in their answers)



$$F(x_3,x_2,x_1,x_0) = x_3'x_2'x_1 + x_2'x_1x_0 + x_2x_1'x_0 + \boxed{x_3'x_2x_0}$$



$$F(x_3,x_2,x_1,x_0) = x_3'x_2'x_1 + x_2'x_1x_0 + x_2x_1'x_0 + \boxed{x_3'x_1x_0}$$

P6. (20 points) Design a circuit that accepts a 3-bit number X= $x_2x_1x_0$ as input and generates a 6-bit number Y= $y_5y_4y_3y_2y_1y_0$ as output, which is equal to the square of the input number (i.e., Y=X^2).

a)

If the numbers are in decimal, we have the following table:

X	***Y***
0	**0**
1	**1**
2	**4**
3	**9**
4	**16**
5	**25**
6	**36**
7	**49**

Next, we convert the above table into binary numbers:

X			***Y***					
x_2	x_1	x_0	y_5	y_4	y_3	y_2	y_1	y_0
0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	1
0	1	0	0	0	0	1	0	0
0	1	1	0	0	1	0	0	1
1	0	0	0	1	0	0	0	0
1	0	1	0	1	1	0	0	1
1	1	0	1	0	0	1	0	0
1	1	1	1	1	0	0	0	1

b) The simplest SOP expressions of each output bit is as follows:

$$y_5 = x_2x_1$$

$$y_4 = x_2x_1' + x_2x_0$$

$$y_3 = x_2'x_1x_0 + x_2x_1'x_0$$

$$y_2 = x_2'x_1x_0' + x_2x_1x_0'$$

$$y_1 = 0$$

$$y_0 = x_0$$