

**1. Convert each of the following into a 32-bit two's complement binary number:**

(a) 512<sub>ten</sub>

$$512_{\text{ten}} = 0000000000000000000000001000000000_{\text{two}}$$

(b)  $-1,023_{\text{ten}}$

- To do this: first I convert  $1,023_{\text{ten}}$  to binary, then get the 1's complement of that number, and add 1 to the 1's complement to get the 2's complement:
  - $1,023_{\text{ten}} = 0000\ 0000\ 0000\ 0000\ 0000\ 0011\ 1111\ 1111_{\text{two}}$
  - 1's complement =  $1111\ 1111\ 1111\ 1111\ 1111\ 1100\ 0000\ 0000_{\text{two}}$
  - 2's (adding 1) =  $1111\ 1111\ 1111\ 1111\ 1111\ 1100\ 0000\ 0001_{\text{two}}$

(c)  $-4,000,000_{\text{ten}}$

- $4,000,000_{\text{ten}} = 0000\ 0000\ 0011\ 1101\ 0000\ 1001\ 0000\ 0000_{\text{two}}$
- 1's complement =  $1111\ 1111\ 1100\ 0010\ 1111\ 0110\ 1111\ 1111_{\text{two}}$
- 2's complement =  $1111\ 1111\ 1100\ 0010\ 1111\ 0111\ 0000\ 0000_{\text{two}}$

2. Write the decimal number represented by each of the following two's complement binary numbers:

(a) (1111 1111 1111 1111 1111 1110 0000 1100)<sub>two</sub>

- The sign is negative, change to normal decimal then add the sign.
- 0000 0000 0000 0000 0000 0001 1111 0100<sub>two</sub> = 500<sub>ten</sub>
- 1111 1111 1111 1111 1111 1110 0000 1100<sub>two</sub> = -500<sub>ten</sub>

(b) (1111 1111 1111 1111 1111 1111 1111 1111) two

- Sign is negative.
- 0000 0000 0000 0000 0000 0000 0000 0001<sub>two</sub> = 1<sub>ten</sub>
- 1111 1111 1111 1111 1111 1111 1111 1111<sub>two</sub> = -1<sub>ten</sub>

(c) (0111 1111 1111 1111 1111 1111 1111 1111)<sub>two</sub>

- The sign is positive.
- $0111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111_{\text{two}} = 2147483647_{\text{ten}}$

**3.**

(a) For the hexadecimal number 0x 7FFF FFFA, what binary number does it represent?

- $0x7FFF\ FFFA = 0111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1010_{two} = 2147483642_{ten}$

(b) Write the hexadecimal number represented by the following binary number:

$$(1100\ 1010\ 1111\ 1110\ 1111\ 1010\ 1100\ 1110)_{\text{two}}$$

- The sign is negative
- $1100\ 1010\ 1111\ 1110\ 1111\ 1010\ 1100\ 1110_{\text{two}} = 0x\text{CAFE FACE}$

4. Two friends, Harry and David, are arguing. Harry says, “All integers greater than zero and exactly divisible by six have exactly two 1s in their binary representation.” David disagrees. He says, “No, but all such numbers have an even number of 1s in their binary representation.” Do you agree with Harry or with David, or with neither? Justify your answer with a proof or counterexamples.

- Both are wrong.
- For example  $42 = 101010$
- Which proves both of them are wrong since 42 is divisible by 6 and has three 1's in its binary representation.

5. Bits in the memory of a computer have no inherent meaning. They mean something only under an interpretation.

Given the bit pattern:

1000 1111 1110 1111 1100 0000 0000 0000

What does it represent under each of the following interpretations?

(a) A two's complement integer?

- The sign is negative.
- $0111\ 0000\ 0001\ 0000\ 0100\ 0000\ 0000\ 0000_{\text{two}} = 1880113152_{\text{ten}}$
- $1000\ 1111\ 1110\ 1111\ 1100\ 0000\ 0000\ 0000_{\text{two}} = -1880113152_{\text{ten}}$

(b) An unsigned integer?

- $1000\ 1111\ 1110\ 1111\ 1100\ 0000\ 0000\ 0000_{\text{two}} = 2414854144_{\text{ten}}$

(c) A MIPS instruction?

- Load Word (LW)