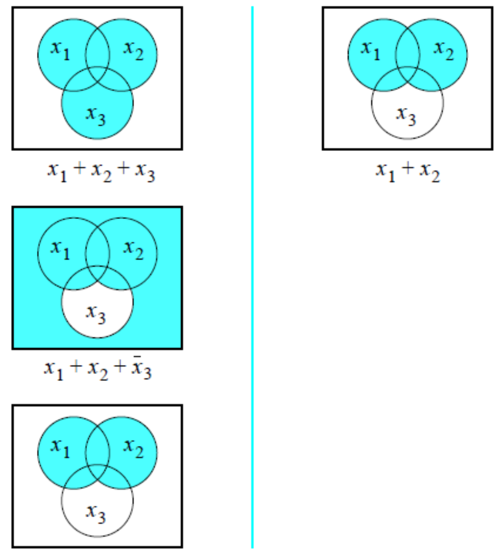
Initial Stuff and Basics

Assigned Date: Second Week
Due Date: First class of 3rd week

P1. Proof using Venn diagrams:

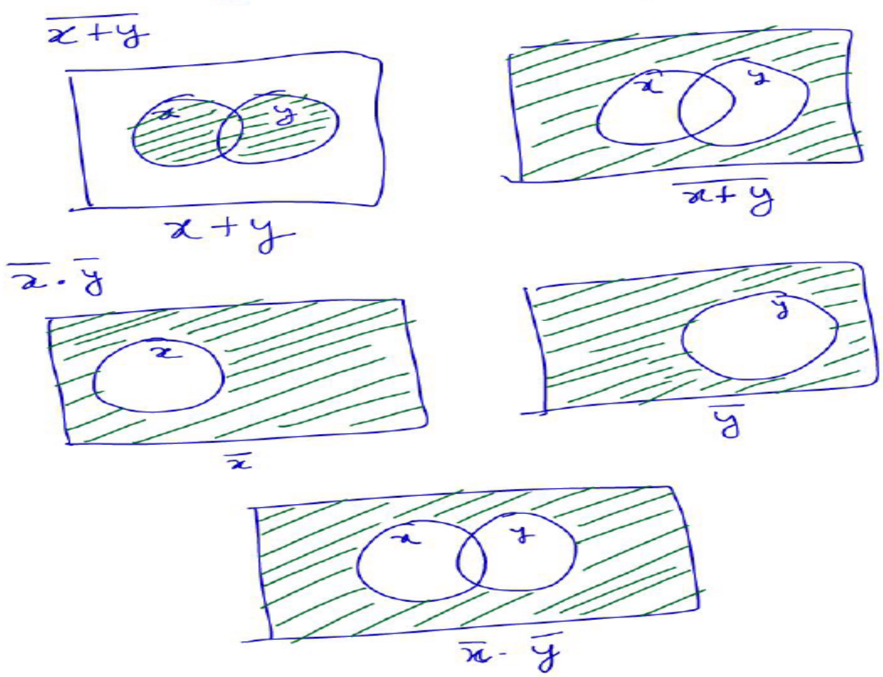


$x_1 + x_2 + x_3$; $x_1 + x_2$

$x_1 + x_2 + \bar{x}_3$

$(x_1 + x_2 + x_3) \cdot (x_1 + x_2 + \bar{x}_3)$

(b) $\overline{x+y} = \bar{x} \cdot \bar{y}$ DeMorgan's theorem



P2. (a) The answer is:
LHS
(x1+x2+x3) . (x1+x2+x3') = x1x1+x1x2+x1x3'+x2x1+x2x2+x2x3'+x3x1+x3x2+x3x3'

Using the Boolean algebra properties in the book section 2.5 page 34 and 35 we can simplify the last equation as follow:

x1x1=x1, x2x2=x2, x1x2+x2x1+x1x2+x1x2+x1x2, x3x3'=0,

Then the equation will be reduced to:
x1+x2+x1x2+x2x3+x2x3'+x1x3+x1x3'

Also x2x3'+x3x2=x2x3+x2x3'=x2 and x1x3+x1x3'=x1, x1+x1x2=x1
x1+x2+x1+x2+x1=x1+x2= RHS ###

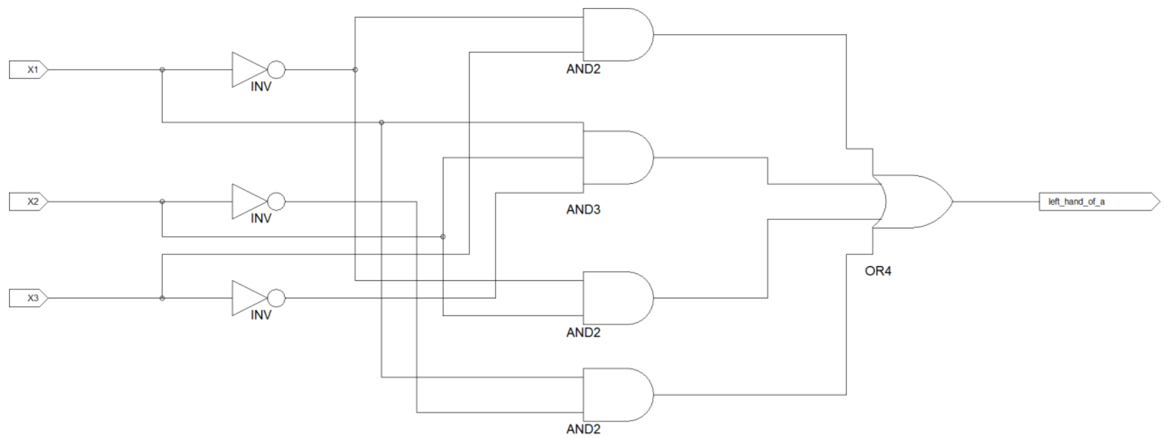
P3. Here, we will substitute the value of x1, x2 and x3 in the following equation to validate that LHS=RHS

$$(x_1 + x_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + x_2) = (x_1 + x_2)(x_2 + x_3)(\bar{x}_1 + \bar{x}_3)$$

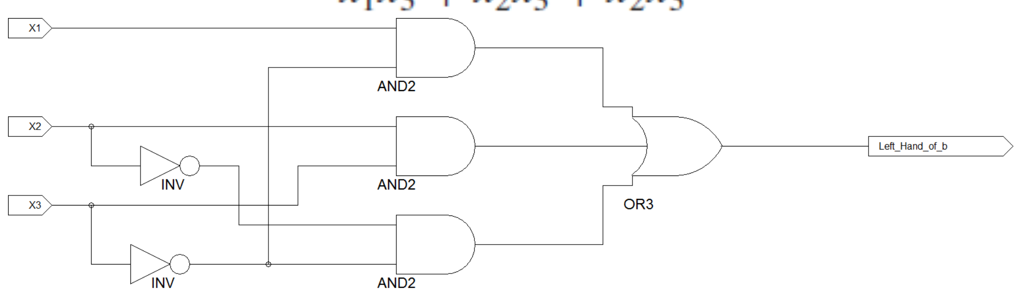
X1	X2	X3	Left Hand	Right Hand	Valid?
0	0	0	0*1*1=0	0*0*1=0	Y
0	0	1	1*1*1=1	0*1*1=0	N
0	1	0	0*1*1=0	1*1*1=1	N
0	1	1	1*1*1=1	1*1*1=1	Y
1	0	0	1*1*0=0	1*0*1=0	Y
1	0	1	1*1*0=0	1*1*0=0	Y
1	1	0	1*1*1=1	1*1*1=1	Y
1	1	1	1*0*1=0	1*1*0=0	Y

P4. Please see the three figures below:

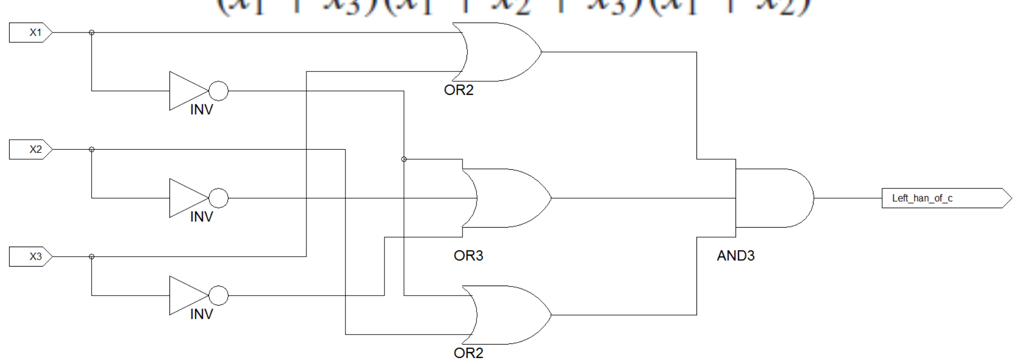
$$\bar{x}_1 x_3 + x_1 x_2 \bar{x}_3 + \bar{x}_1 x_2 + x_1 \bar{x}_2$$



$$x_1\overline{x}_3 + x_2x_3 + \overline{x}_2\overline{x}_3$$



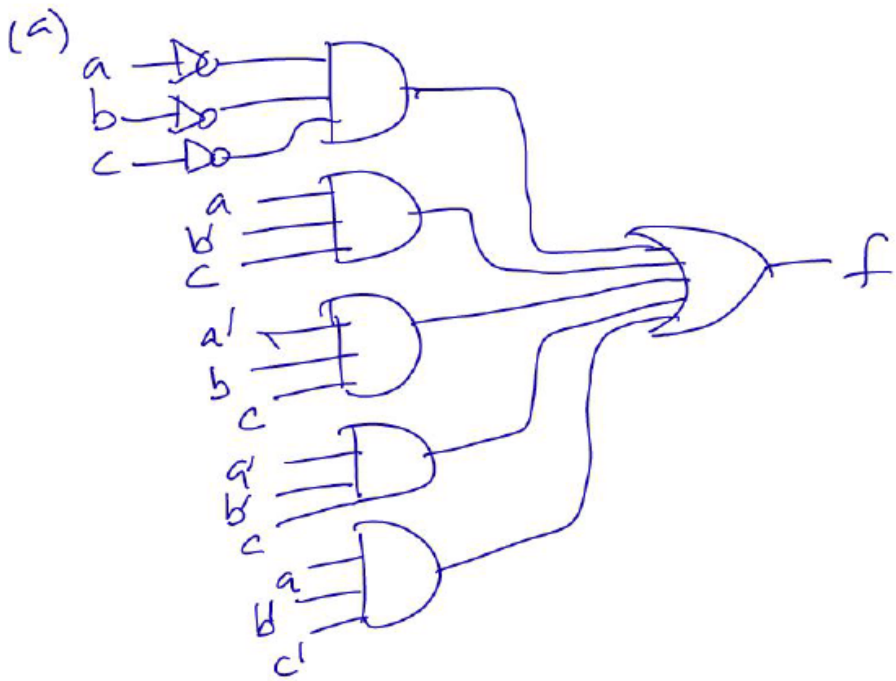
$$(x_1 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3)(\overline{x}_1 + x_2)$$



P5. (a) The diagram is shown below

P5. $f(a,b,c) = a'b'c' + ab'c + a'bc + a'b'c + ab'c'$

(a)



(b) Cost = 9 + 23 = 32

(c) $f = \underline{a'b'c'} + \underline{ab'c} + a'bc + \underline{a'b'c} + \underline{ab'c'}$

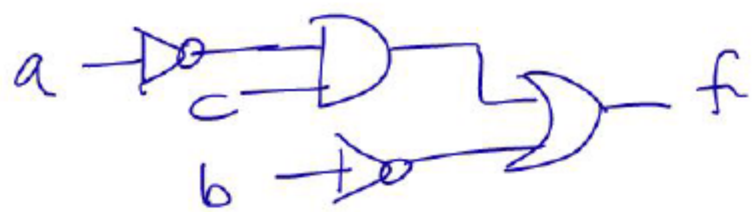
$= b'c'(a+a') + b'c(a+a') + a'bc$

$= \underline{b'c' + b'c} + a'bc$

$= b' + a'bc$

$= (b'+b) \cdot (b'+a'c)$

$f = b' + a'c$

(d) 

(e) cost $= 4 + 6 = 10$

P6. a) The Truth table:

X	y	z	w	f
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

b) SOP:
f= x'.y'.z'.w + x'.y'.z.w' + x'.y.z'.w' + x.y'.z'.w' + x'.y.z.w + x.y'.z.w + x.y.z'.w + x.y.z.w'

c) SOP in shorthand: f=$\sum m(1,2,4,7,8,11,13,14)$

d) SOP: $f' = \sum m(0,3,5,6,9,10,12,15)$

e) POS:
f = (x+y+z+w'). (x+y+z'+w). (x+y'+z+w). (x'+y+z+w). (x+y'+z'+w'). (x'+y+z'+w'). (x'+y'+z+w'). (x'+y'+z'+w)

f) $f = \Pi M(0,3,5,6,9,10,12,15)$

g) $f' = \Pi M(1,2,4,7,8,11,13,14)$

P7. Starting with the canonical product-of-sums for f can derive:

$$
\begin{aligned}
f &= (x_1+x_2+x_3)(x_1+x_2+\overline{x}_3)(x_1+\overline{x}_2+x_3)(x_1+\overline{x}_2+\overline{x}_3)\cdot \\
&\quad (\overline{x}_1+x_2+x_3)(\overline{x}_1+x_2+\overline{x}_3)(\overline{x}_1+\overline{x}_2+x_3) \\
&= ((x_1+x_2+x_3)(x_1+x_2+\overline{x}_3))((x_1+\overline{x}_2+x_3)(x_1+\overline{x}_2+\overline{x}_3))\cdot \\
&\quad ((\overline{x}_1+x_2+x_3)(\overline{x}_1+x_2+\overline{x}_3))((\overline{x}_1+\overline{x}_2+x_3)(\overline{x}_1+x_2+x_3)) \\
&= (x_1+x_2+x_3\overline{x}_3)(x_1+\overline{x}_2+x_3\overline{x}_3)\cdot \\
&\quad (\overline{x}_1+x_2+x_3\overline{x}_3)(\overline{x}_1+\overline{x}_2x_2+x_3) \\
&= (x_1+x_2)(x_1+\overline{x}_2)(\overline{x}_1+x_2)(\overline{x}_1+x_3) \\
&= (x_1+x_2\overline{x}_2)(\overline{x}_1+x_2x_3) \\
&= x_1(\overline{x}_1+x_2x_3) \\
&= x_1\overline{x}_1+x_1x_2x_3 \\
&= x_1x_2x_3
\end{aligned}
$$

P8. Derivation of the minimum sum-of-products expression:

$$
\begin{aligned}
f &= x_1\overline{x}_2\overline{x}_3+x_1x_2x_4+x_1\overline{x}_2x_3\overline{x}_4 \\
&= x_1\overline{x}_2\overline{x}_3(\overline{x}_4+x_4)+x_1x_2x_4+x_1\overline{x}_2x_3\overline{x}_4 \\
&= x_1\overline{x}_2\overline{x}_3\overline{x}_4+x_1\overline{x}_2\overline{x}_3x_4+x_1x_2x_4+x_1\overline{x}_2x_3\overline{x}_4 \\
&= x_1\overline{x}_2\overline{x}_3+x_1\overline{x}_2(\overline{x}_3+x_3)\overline{x}_4+x_1x_2x_4 \\
&= x_1\overline{x}_2\overline{x}_3+x_1\overline{x}_2\overline{x}_4+x_1x_2x_4
\end{aligned}
$$