CS 228: Introduction to Data Structures
Lecture 28
Friday, November 4, 2016

Binary Trees

A **binary tree** is a tree in which every node has degree at most two, and every child is either a **left child** or a **right child**, even if it is the only child its parent has. The following equivalent recursive definition is useful for devising algorithms for binary trees.

Definition. A **binary tree** is a structure T defined on a finite set of nodes, such that either

- T is empty (i.e., contains no nodes), or
- T is composed of three disjoint sets of nodes:
  - a **root** node,
  - a binary tree called the **left subtree** of T, and
  - a binary tree called the **right subtree** of T.

Note that in the preceding definition, one or both of the left and right subtrees of T can be empty. In particular, T can consist of a single node (the root).

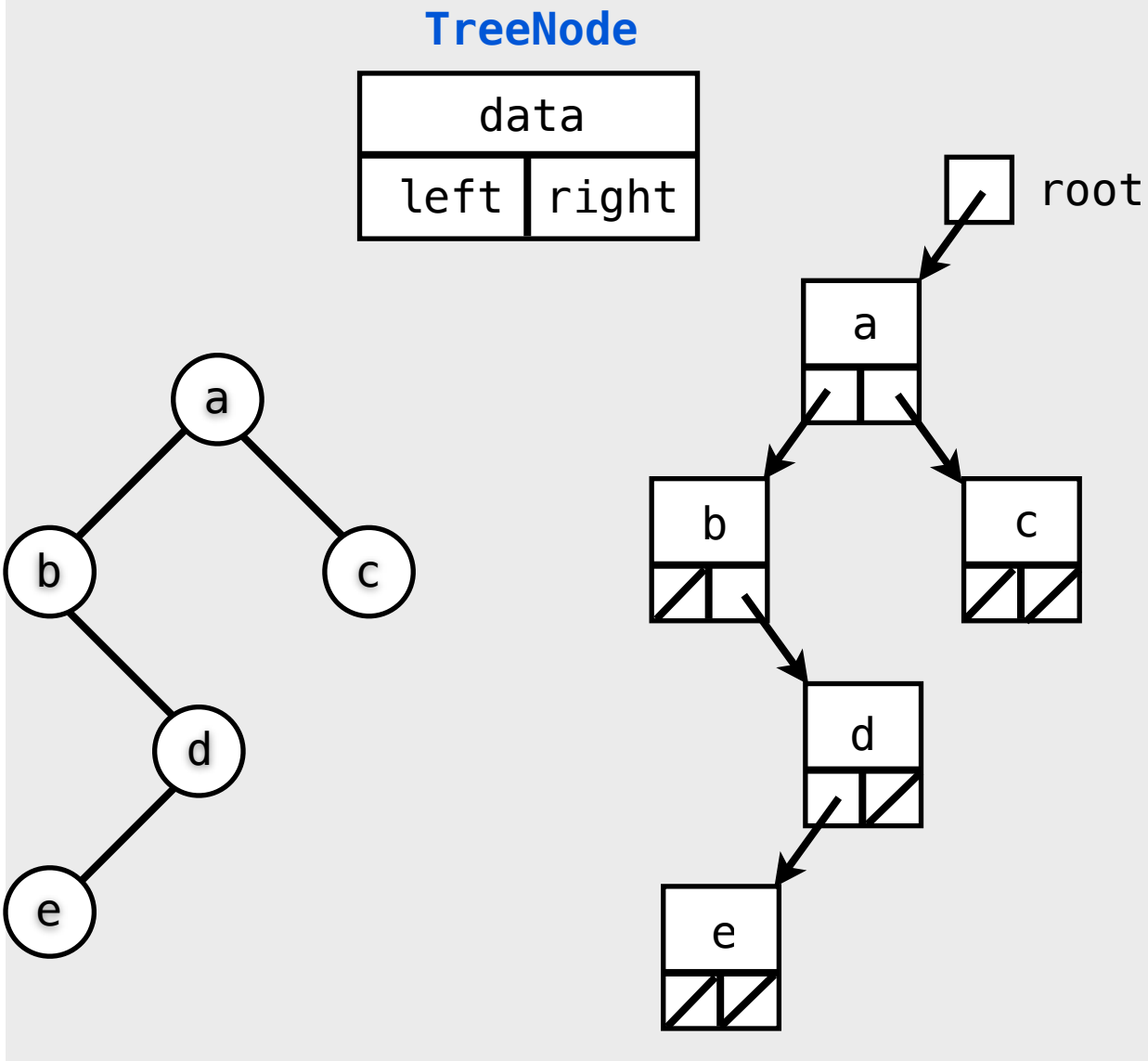
Representing Binary Trees

In the `TreeNode` class posted on Blackboard, each node has two successors, representing the left and right child. This particular implementation does not use child-to-parent links, but there are implementations that do.

```
public class TreeNode<E>
{
  protected TreeNode<E> left;
  protected TreeNode<E> right;
  protected E data;
```

The class includes various constructors; an `isLeaf()` method (whose purpose is obvious); getter methods `left()`, `right()`, and `data()`; and setter methods `setLeft()`, `setRight()`, and `setData()`.

Example. Here is a tree represented using `TreeNode`.



Tree Traversals

A **traversal** is a manner of ***visiting*** each node in a tree once. What you do when visiting any particular node depends on the application; for instance, you might print the data stored in the node, or perform some calculation

upon it. We will study four traversal algorithms, which differ from each other in the order they visit the nodes. In what follows, T denotes a binary tree.

A **preorder traversal** of T first visits the root of T and then traverses the left and right subtrees of T.

```
PREORDER(T):

    if T is empty
        return

    let T_left and T_right be the left and right subtrees of T

    visit the root of T
    PREORDER(T_left)
    PREORDER(T_right)
```

Here's an implementation using `TreeNode`. This method just prints out the content of each node when it visits the node.

```
 public static void
 traversePreorder(TreeNode<?> node)
  {
    if (node == null) return;
    System.out.print(node.data().toString()
+ " ");
    traversePreorder(node.left());
    traversePreorder(node.right());
  }
```

A **postorder traversal** of T first traverses the left subtree of T, then the right subtree of T, and, finally, visits the root.

POSTORDER(T):

if T is empty
return

let T_{left} and T_{right} be the left and right subtrees of T

POSTORDER(T_{left})
POSTORDER(T_{right})
visit the root of T

An **inorder traversal** of T first traverses the left subtree of T, then visits the root, and, finally, traverses the right subtree of T.

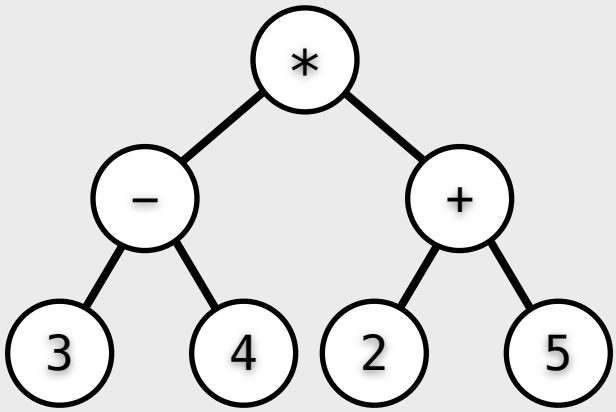
INORDER(T):

if T is empty
return

let T_{left} and T_{right} be the left and right subtrees of T

INORDER(T_{left})
visit the root of T
INORDER(T_{right})

Example. Consider the following expression tree.




```
Preorder:     * - 3 4 + 2 5
Inorder:      3 - 4 * 2 + 5
Postorder:    3 4 - 2 5 + *
```

Note that the preorder, inorder, and postorder traversals of an expression tree print a prefix, infix (without parentheses), and postfix expression, respectively.

A ***level order traversal*** first visits the root, then all the depth-1 nodes (from left to right), then all the depth-2 nodes, etc. It continues visiting nodes level by level, left-to-right within each level, until all nodes are visited.

Example. The level-order sequence for the expression tree of the preceding example is

```
* - + 3 4 2 5
```

Unlike the preorder, inorder, and postorder sequences,the level order sequence does not have a natural interpretation.

The easiest way to implement level order traversal is non-recursively, using a queue, as follows.

```
LevelOrder(T):
    Create an empty queue q
    if T is not empty
        q.enqueue(root)
    while !q.isEmpty()
        x = q.dequeue()
        let y_left be the left child of x
        let y_right be the right child of x
        if (y_left != null) q.enqueue(y_left)
        if (y_right != null) q.enqueue(y_right)
        visit x
```

We will not spend much time on the preceding algorithm now. You should study the above algorithm on your own, and try it out on a few examples, and convince yourselves that it is correct. We will return to this idea when we study non-binary trees and graphs, where level-order traversal — called ***breadth-first search*** in that context — is much more useful.

Application: Computing the Height of a Binary Tree

Tree traversals have several applications. Here we describe how to use post-order traversal to compute the ***height*** of a binary tree.

Recall that the height of a tree T is the length of the longest path from the root of T to a leaf. To devise an algorithm, it is easier to work with the following equivalent recursive definition.

1. If T is empty, then the height of T is –1.

2. Otherwise, the height of T is 1 + max {height left subtree, height right subtree}.

This recursive definition leads directly to the following implementation.

```
public static int height(TreeNode<?> node)
{
  if (node == null)
    return -1;
  int lHeight = height(node.left());
  int rHeight = height(node.right());
  if (lHeight > rHeight)
    return lHeight + 1;
  else
    return rHeight + 1;
}
```

Other examples of the applications of traversals are given in the class `ExampleTree`, posted on Blackboard.

Evaluating Expression Trees

We can use postorder traversal to evaluate an expression tree whose root is given. The algorithm is on the next page. The `evaluate()` method in the `ExpressionTree` posted on Blackboard implements this algorithm.

1. If the root is a leaf, then it must be a number, so just return it.

2. Otherwise, the root must be a binary operator op, so

   2.1. Let x_L be the result of evaluating the left subtree.

   2.2. Let x_R be the result of evaluating the right subtree.

   2.3. Return x_L op x_R.