

DS 116 Data Visualization

Multivariate numeric data

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Section 1

Bivariate data - scatterplots

Scatterplots

When you have two continuous variables, drawing the scatterplot is the first thing that comes to mind

The scatterplot is a useful exploratory method for providing a first look at bivariate data to see how they are distributed throughout the plane, for example, to see clusters of points, outliers, and so forth

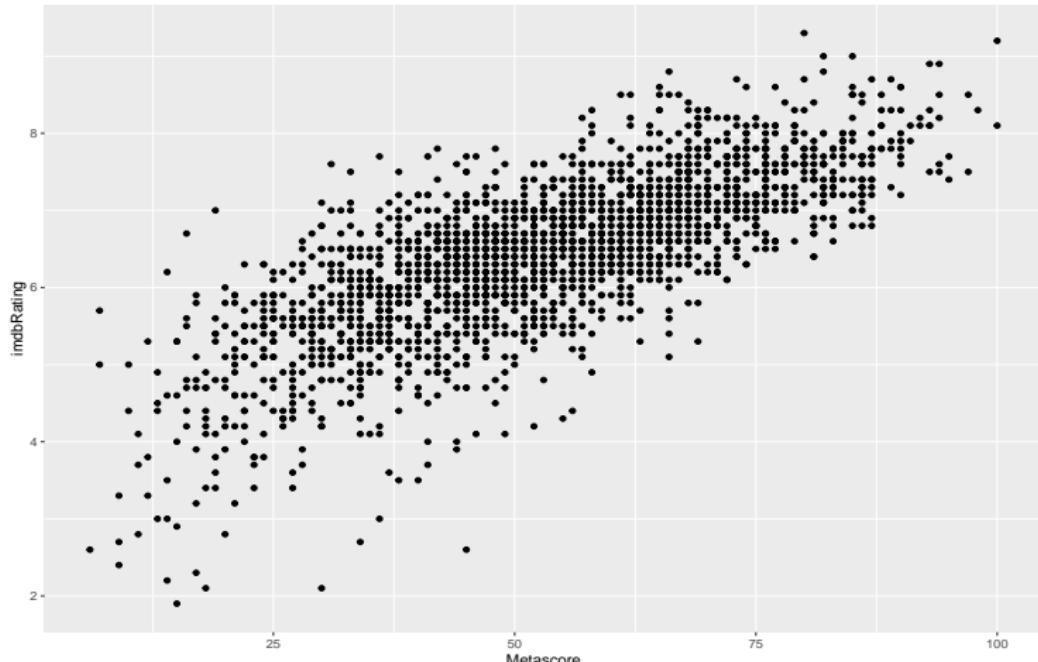
Scatterplots have the following usages:

- Reveal linear and non-linear relationship between two variables
- Detect the strength of the association (correlation) between the variables
- Detect outliers if there are any
- Detect the clusters of data

Scatterplots

in ggplot the scatterplot is created using `geom_point()`

```
movies <- read.csv('Data/movies_small.csv')
ggplot(movies, aes(x = Metascore, y = imdbRating)) + geom_point()
```



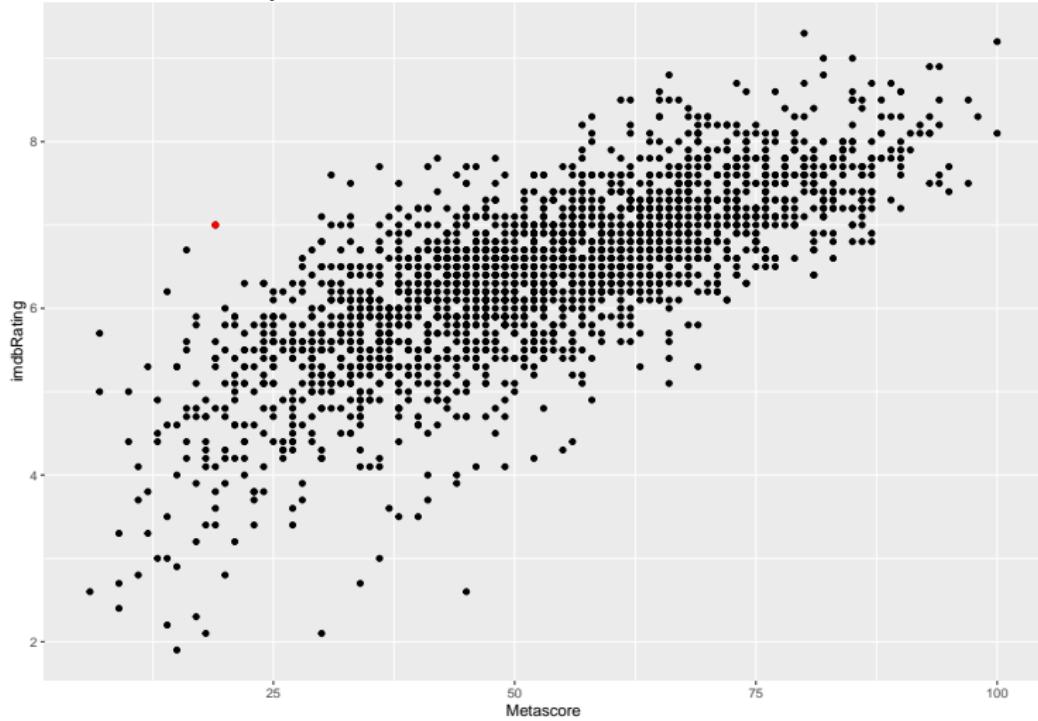
Scatterplots

Overplotting

- Overplotting is when the data or labels in a data visualization overlap, making it difficult to see individual data points and draw conclusion about the concentration of the data.
- Overplotting can result from very high sample sizes, or when one of the variables (say on x-axis) is discrete
- Fixes for overplotting include reducing the size of points, changing the shape of points, jittering, tiling, making points transparent, only showing a subset of points, and using algorithms to prevent labels from overlapping.

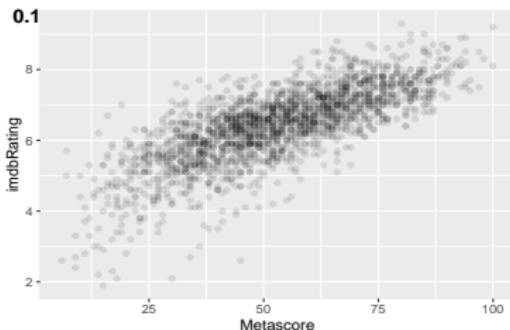
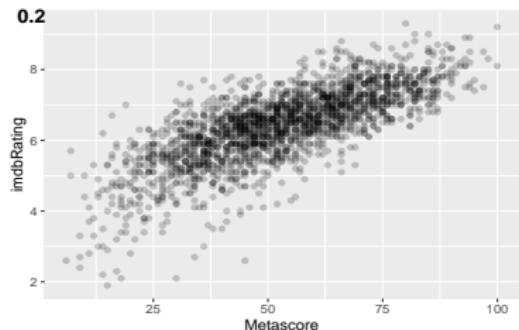
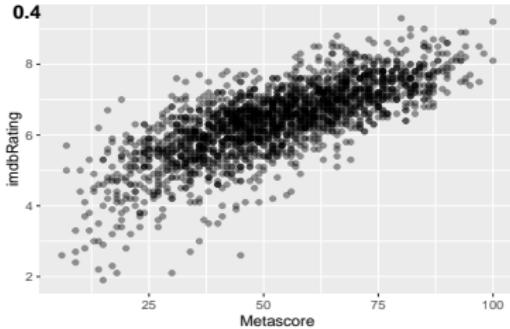
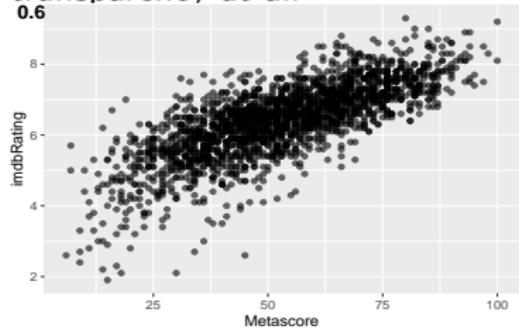
Scatterplots

Look at the red point



Scatterplots

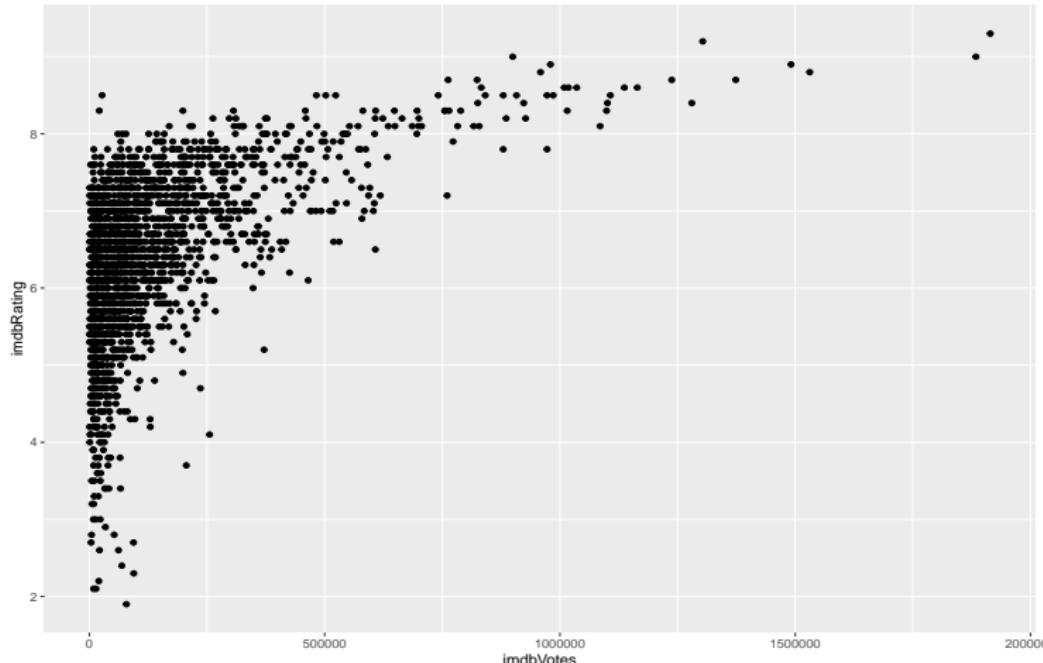
To understand where are the most of the points concentrated, use α transparency parameter alpha = 0 gives total transparency, while alpha = 1 gives no transparency at all



Scatterplots

imdbRating vs imdbVotes

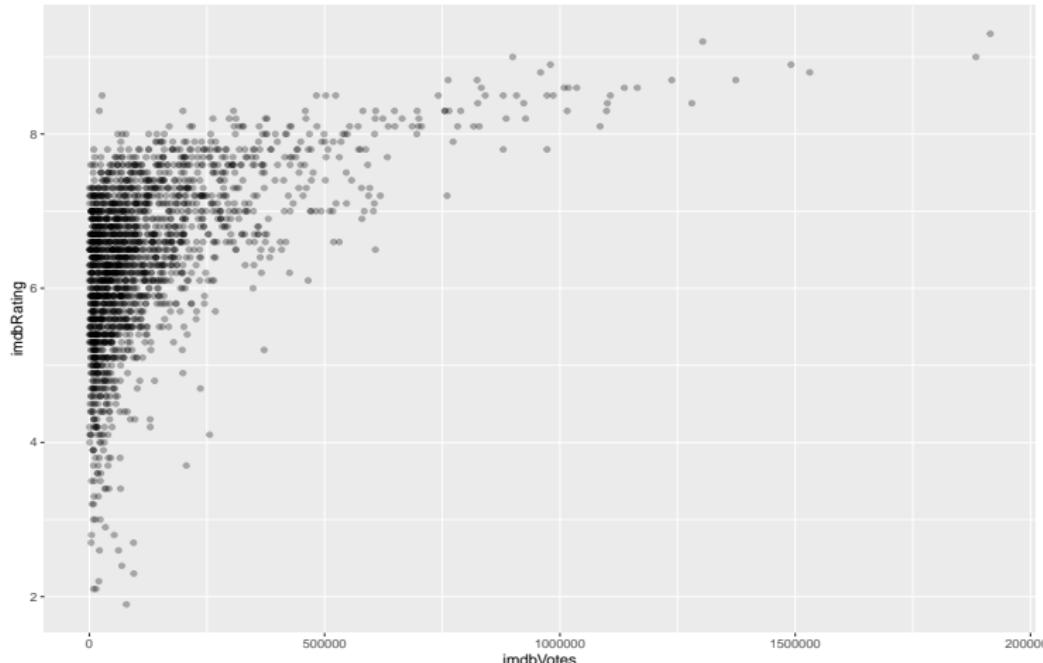
```
ggplot(movies, aes(x = imdbVotes, y = imdbRating)) +  
  geom_point()
```



Scatterplots

Add transparency

```
ggplot(movies, aes(x = imdbVotes, y = imdbRating)) +  
  geom_point(alpha = 0.3)
```



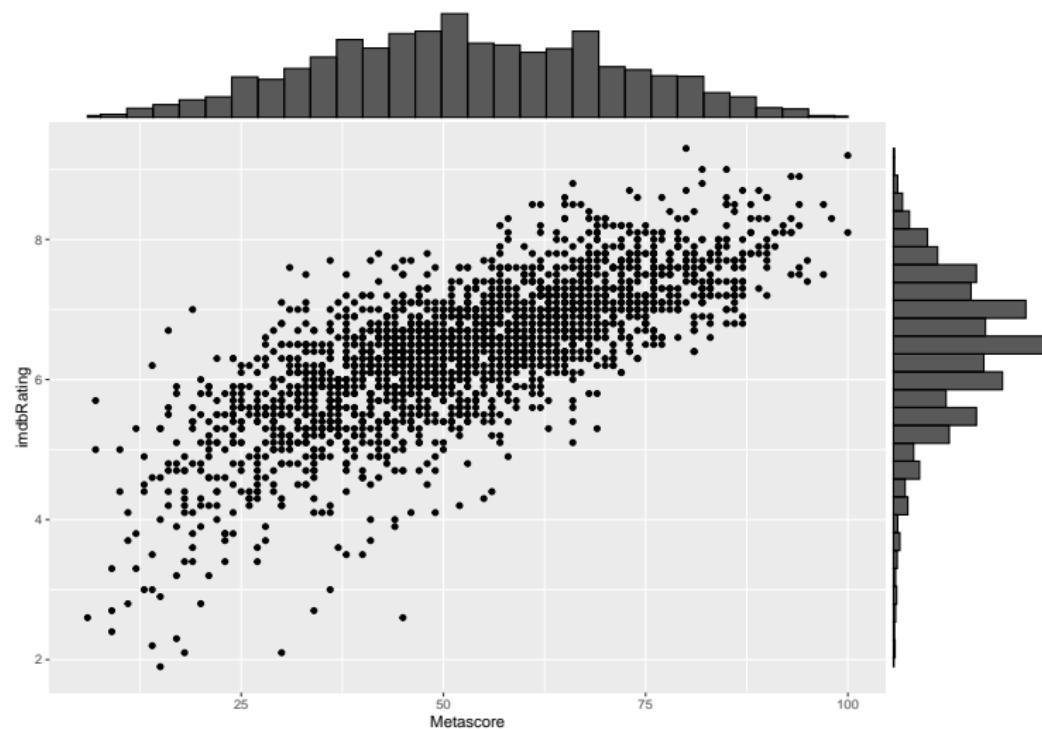
Scatterplots

If you need to see the concentration of the points, a good visual aid can be plotting the marginal distributions.
Use library ggExtra

Scatterplots

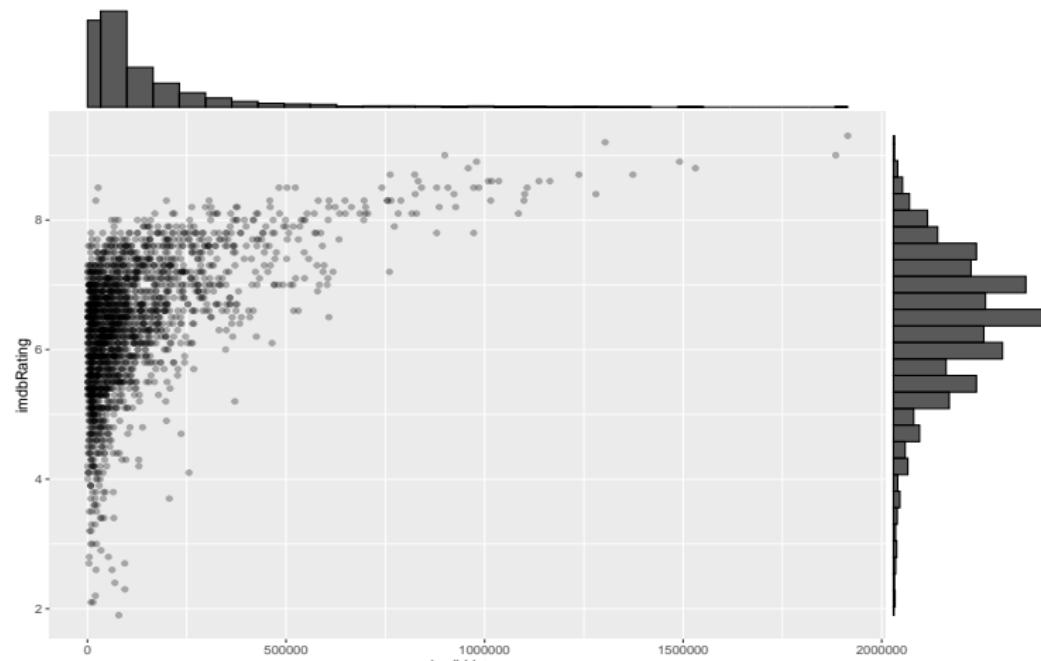
```
p1 <- ggplot(movies, aes(x = Metascore, y = imdbRating)) +  
  geom_point()  
ggMarginal(p1, type = 'histogram')
```

Scatterplots



Scatterplots

```
p1 <- ggplot(movies, aes(x = imdbVotes, y = imdbRating)) +  
  geom_point(alpha = 0.3)  
ggMarginal(p1, type = 'histogram')
```



Scatterplots

Jittering is an act of adding a bit of random noise to scatterplots, to better see the information contained in the data, usually when there is overplotting.
Seriea A football dataset

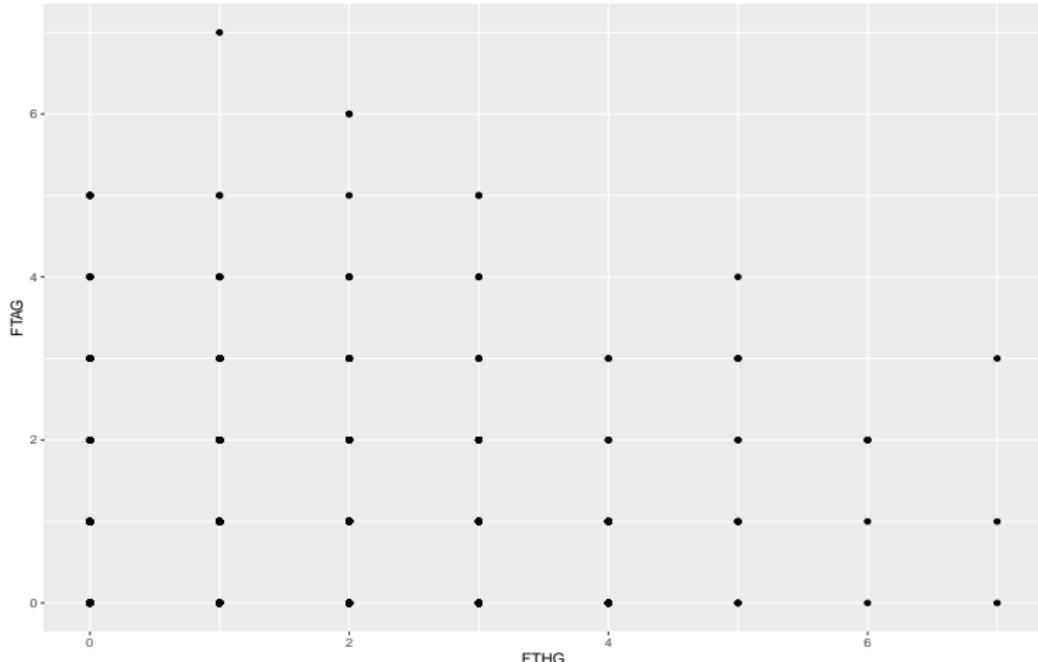
- FTHG - Full time goals by home team
- FTAG - Full time goals by away team
- Result = 1, if home team won, Result=0, if home team did not win

```
seriea <- read.csv('Data/seriea_games.csv')
head(seriea)
##           DATE HOMETEAM AWAYTEAM FTHG FTAG FTTG Result
## 1 8/20/2016 Juventus Fiorentina    2    1    3      1
## 2 8/20/2016      Roma Udinese     4    0    4      1
## 3 8/21/2016 Atalanta    Lazio     3    4    7      0
## 4 8/21/2016 Bologna   Crotone     1    0    1      1
## 5 8/21/2016 Chievo     Inter     2    0    2      1
## 6 8/21/2016 Empoli    Sampdoria    0    1    1      0
```

Scatterplots

plot FTHG against FTAG

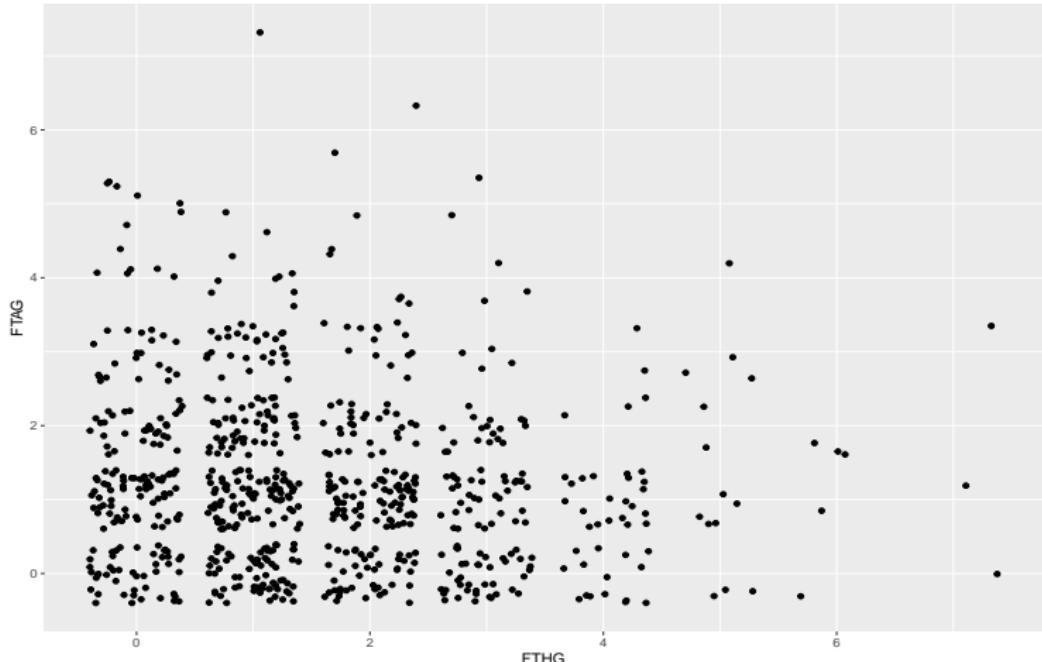
```
ggplot(seriea, aes(FTHG, FTAG)) + geom_point()
```



Scatterplots

add geom_jitter

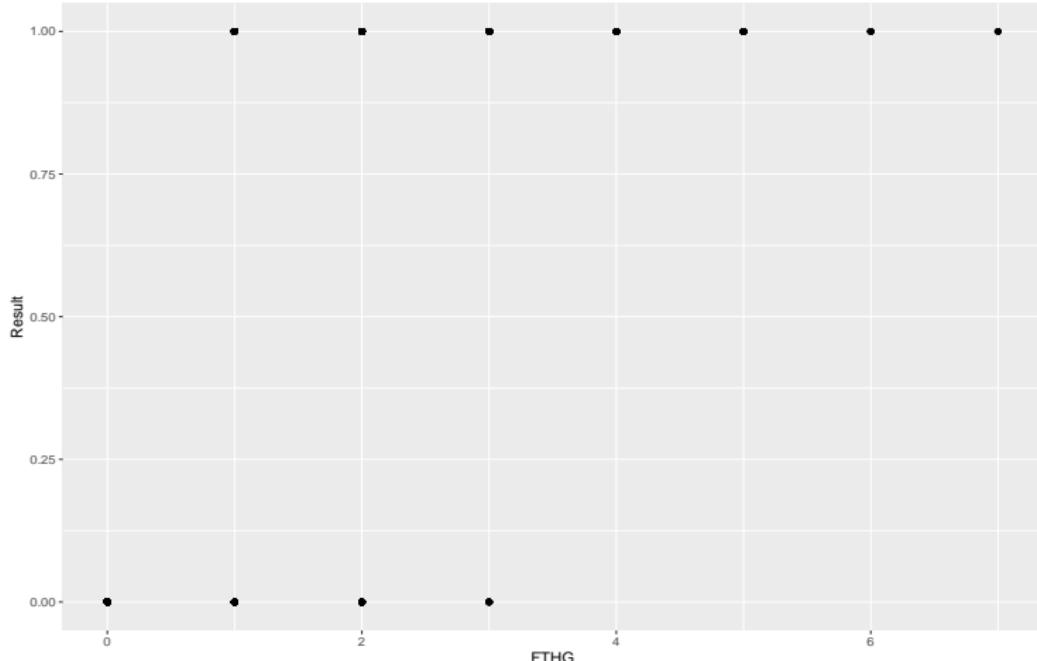
```
ggplot(seriea, aes(FTHG, FTAG)) + geom_jitter()
```



Scatterplots

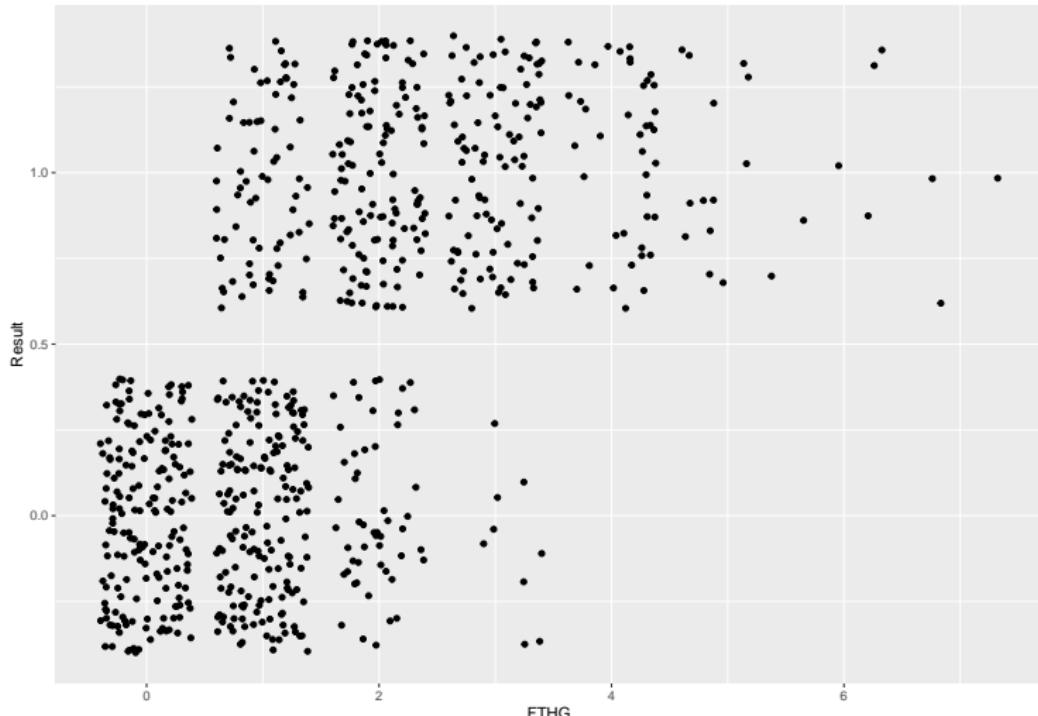
FTHG vs Result

```
ggplot(seriea, aes(FTHG, Result)) + geom_point()
```



Scatterplots

```
ggplot(seriea, aes(FTHG, Result)) + geom_jitter()
```



Scatterplots

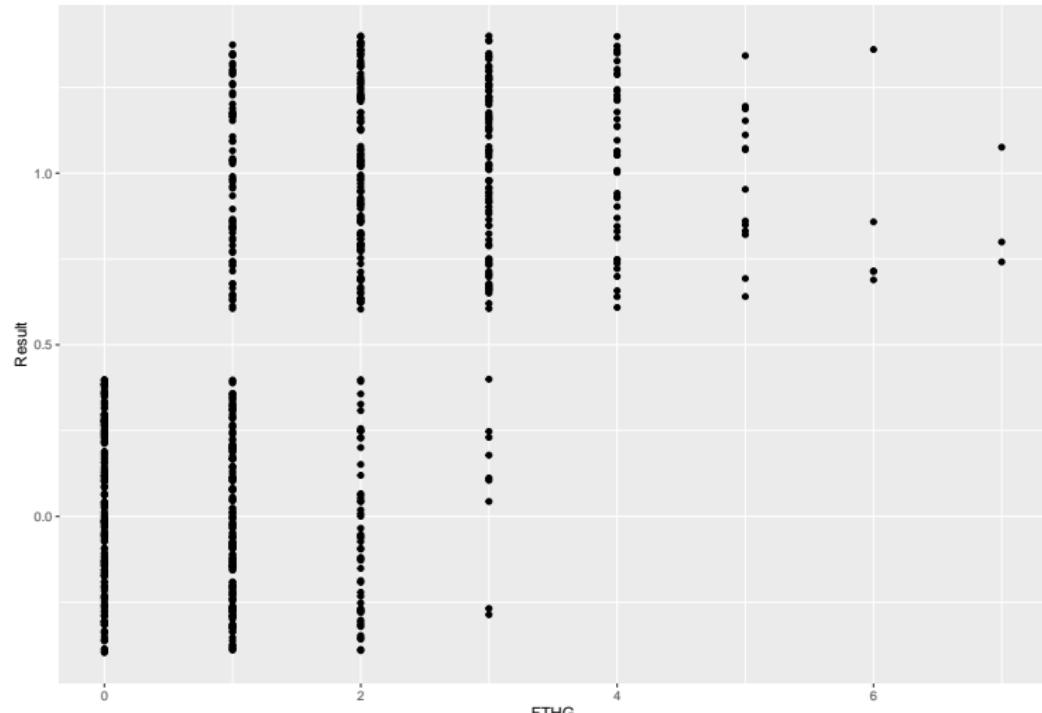
The amount of jittering is controlled with two parameters

- **height** Amount of vertical jitter.
- **width** Amount of horizontal jitter

The jitter is added in both positive and negative directions, so the total spread is twice the value specified here.

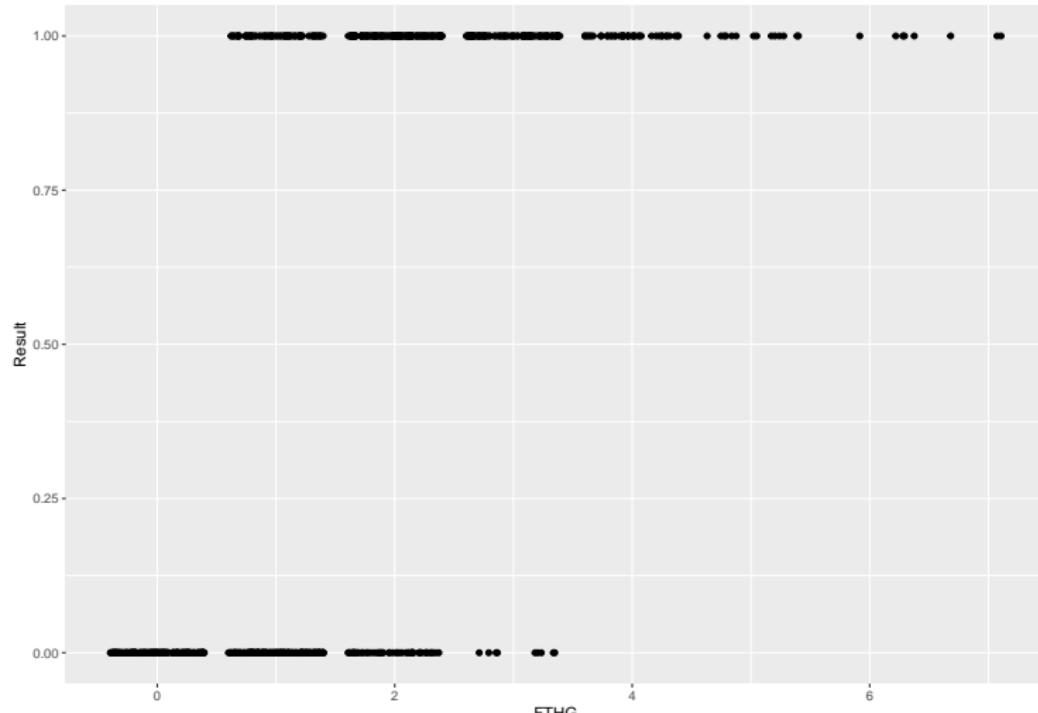
Scatterplots

```
ggplot(seriea, aes(FTHG, Result)) + geom_jitter(width=0)
```



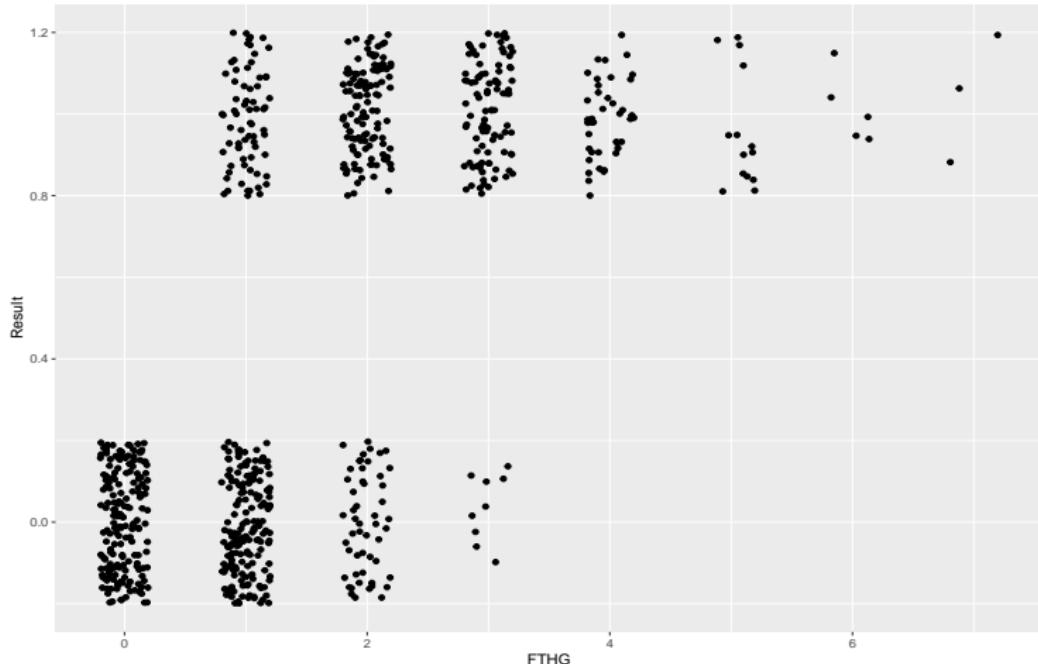
Scatterplots

```
ggplot(seriea, aes(FTHG, Result)) + geom_jitter(height=0)
```



Scatterplots

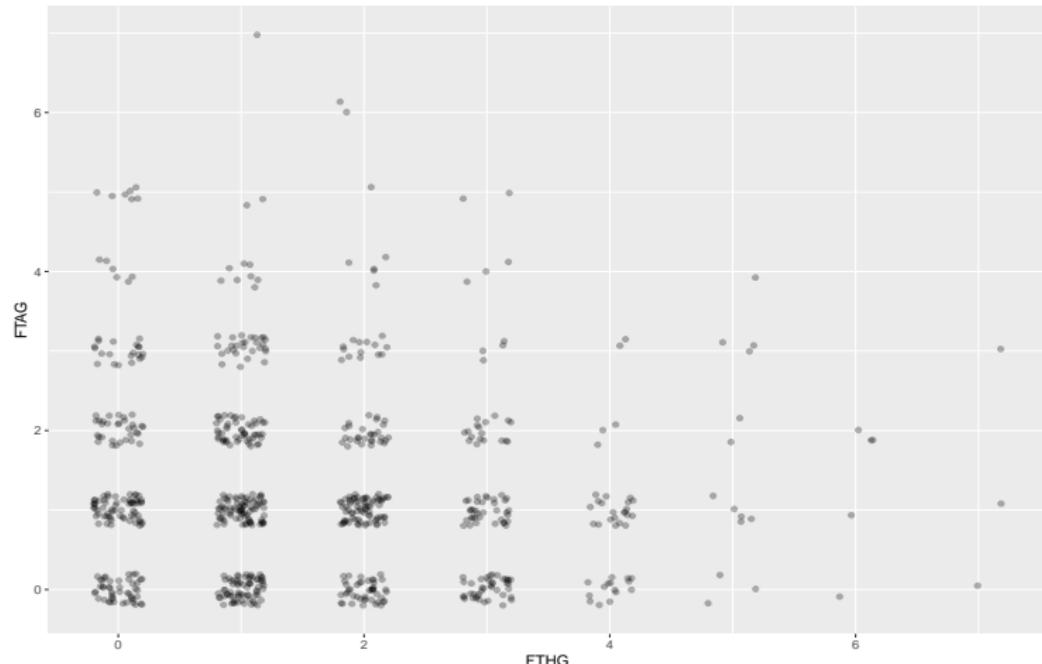
```
ggplot(seriea, aes(FTHG, Result)) +  
  geom_jitter(height=0.2, width = 0.2)
```



Scatterplots

jitter with alpha

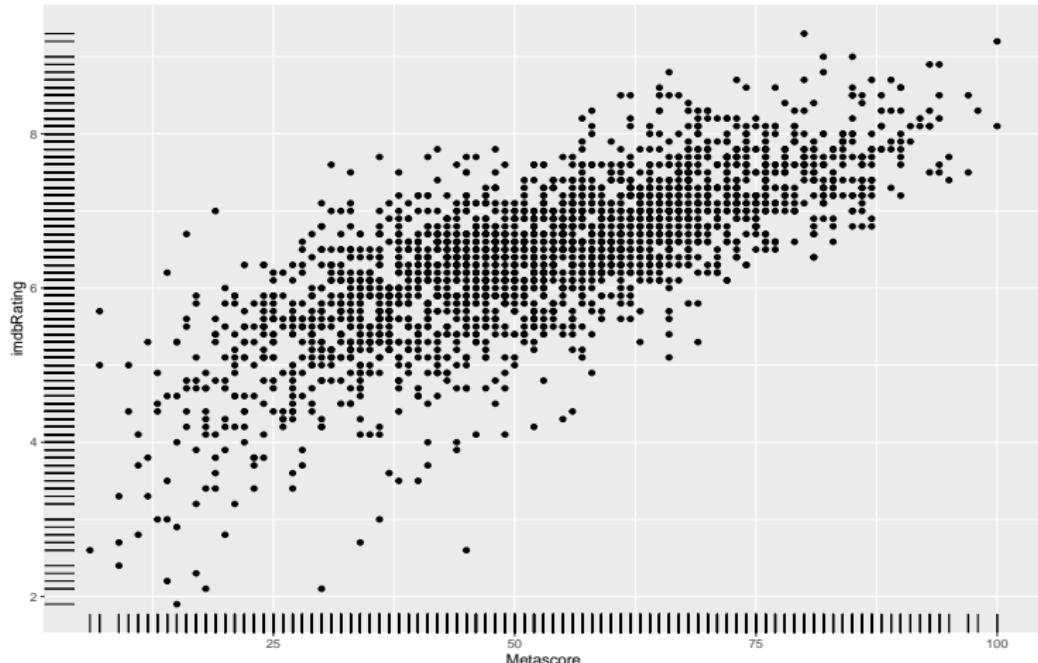
```
ggplot(seriea, aes(FTHG, FTAG)) +  
  geom_jitter(height=0.2, width = 0.2, alpha = 0.3)
```



Scatterplots

`geom_rug()` adds to scatterplot 1d marginal distribution, in a compact way

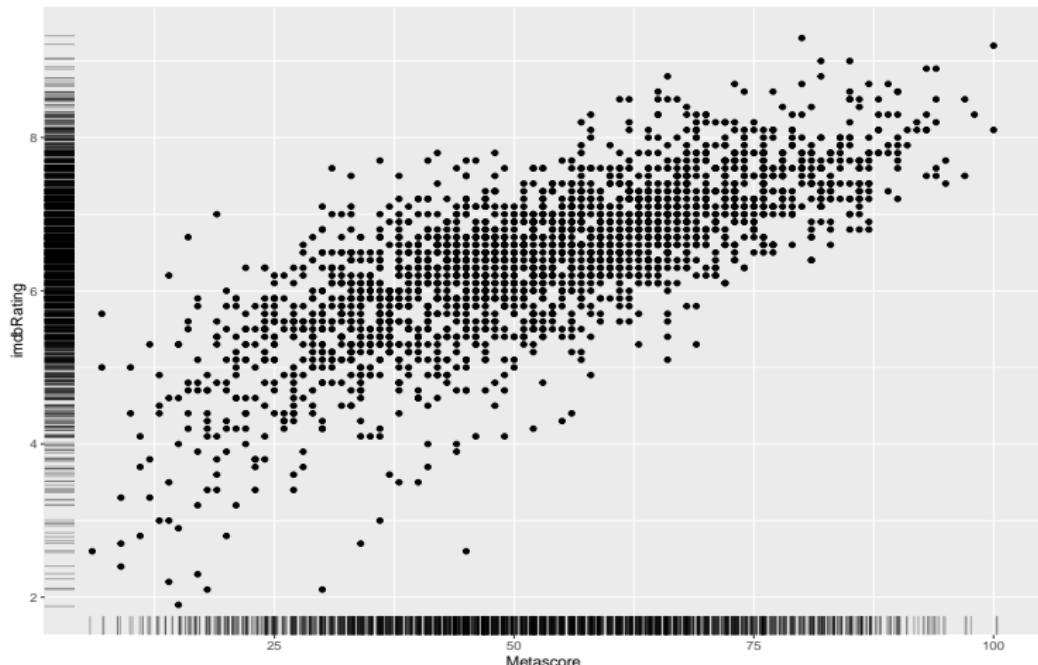
```
ggplot(movies, aes(x = Metascore, y = imdbRating)) +  
  geom_point() + geom_rug()
```



Scatterplots

Adding alpha and position = 'jitter'

```
ggplot(movies, aes(x = Metascore, y = imdbRating)) +  
  geom_point() + geom_rug(alpha = 0.3, position = 'jitter')
```



Scatterplots

Look at what are the aesthetics that can be used with `geom_rug()`
https://ggplot2.tidyverse.org/reference/geom_rug.html

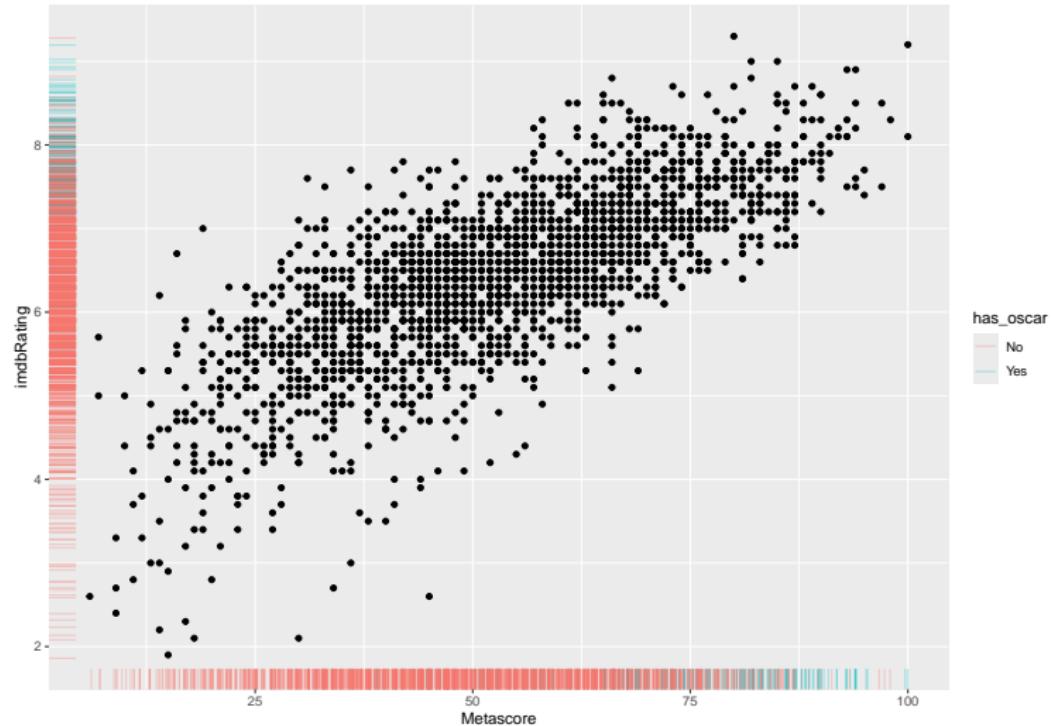
- alpha
- colour
- group
- linetype
- size
- x
- y

Scatterplots

Adding additional aesthetics

```
ggplot(movies, aes(x = Metascore, y = imdbRating)) +
  geom_rug(mapping = aes(color = has_oscar), alpha = 0.3,
  position = 'jitter')+
  geom_point()
```

Scatterplots



Section 2

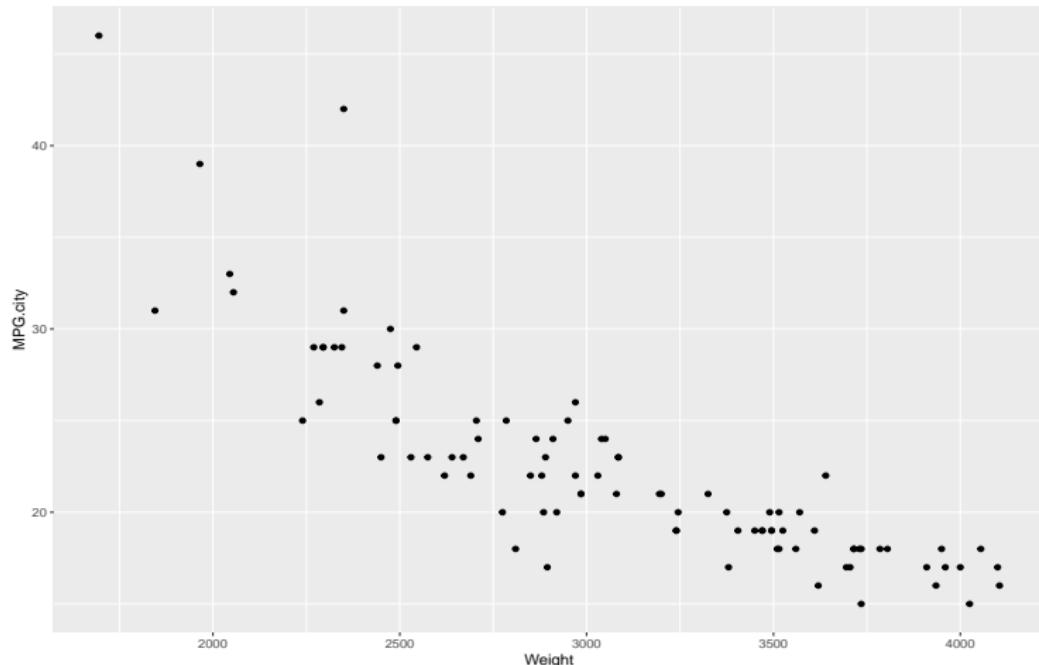
Bivariate data - smoothing

Smoothing

- It is always useful to add model fit line to the plot to understand what is the underlying relationship between two variables.
- In ggplot2 it is done using `geom_smooth` (`stat_smooth` if you want to use other geom)
- By default ggplot2 fits LOESS (locally estimated scatterplot smoothing) model for small data, and GAM (Generalized Additive Model) for large data

Smoothing

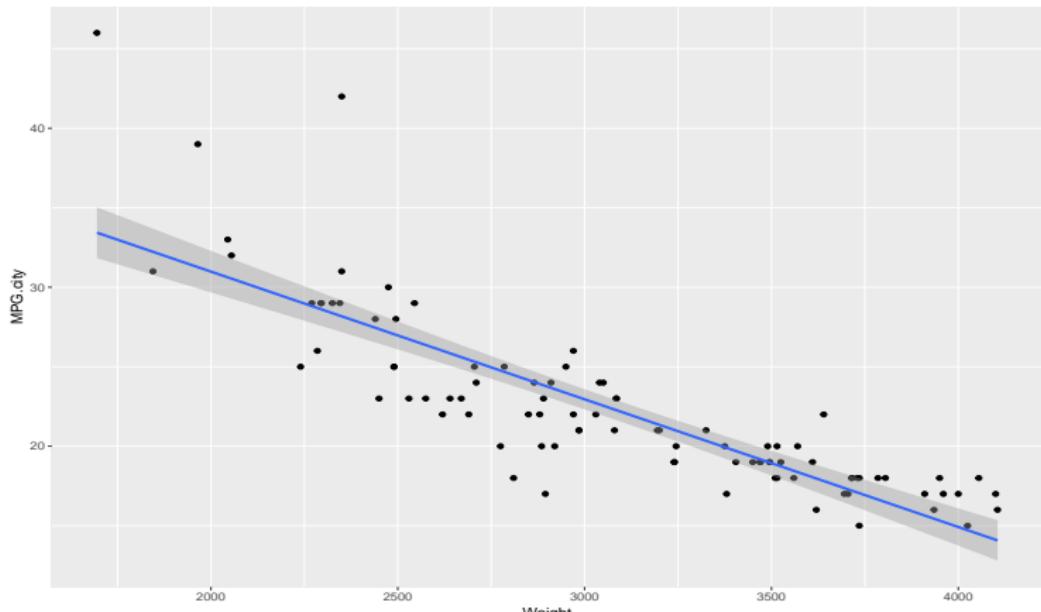
```
data(Cars93, package="MASS")
ggplot(Cars93, aes(Weight, MPG.city)) + geom_point()
```



Smoothing

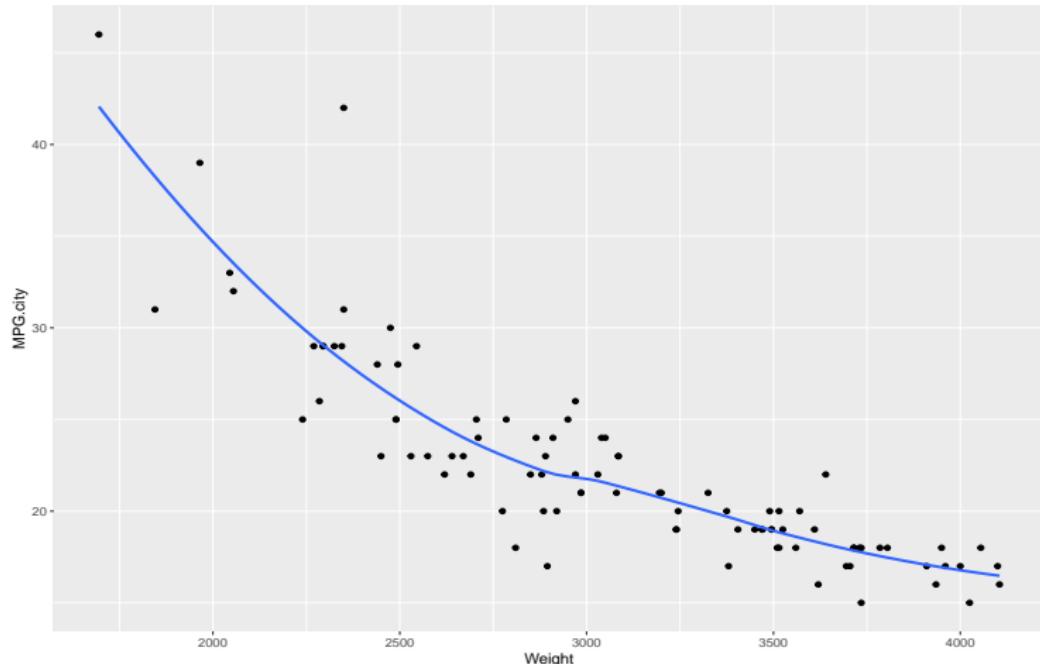
Fit regression line by overriding the method argument. The shaded area shows the confidence interval

```
ggplot(Cars93, aes(Weight, MPG.city)) + geom_point() +  
  geom_smooth(method = 'lm')
```



Smoothing

```
ggplot(Cars93, aes(Weight, MPG.city)) + geom_point() +  
  geom_smooth(se = F)
```



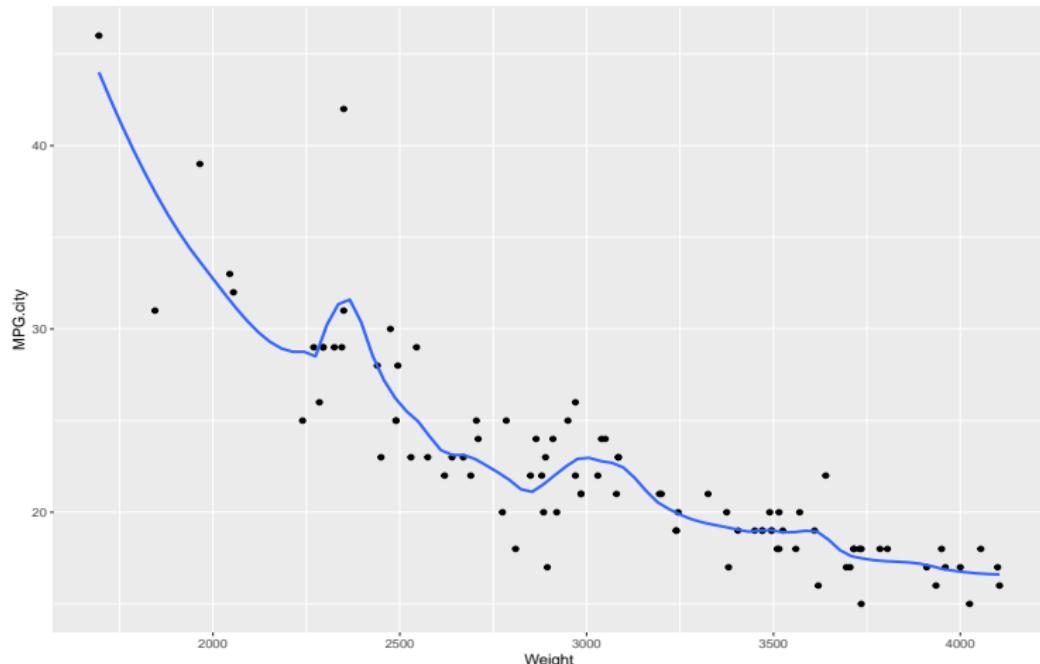
Smoothing

loess

- Fitting is done locally. That is, for the fit at point x , the fit is made using points in a neighborhood of x , weighted by their distance from x .
- The size of the neighborhood is controlled by α (the span parameter).
- higher value of α will result in smoother fit while smaller values will produce wigglier lines.

Smoothing

```
ggplot(Cars93, aes(Weight, MPG.city)) + geom_point() +  
  geom_smooth(se = F, span=0.2)
```

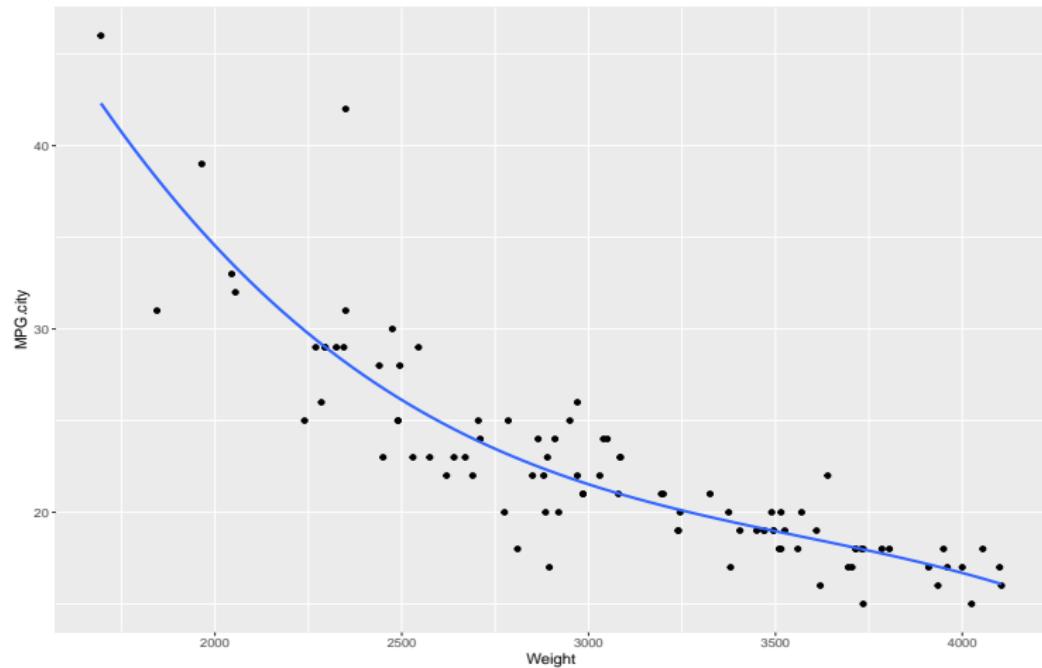


Smoothing

Actually you can define your own smoothing method and provide to geom_smooth poly() function in r fits polynomial regression with a given order

Smoothing

```
ggplot(Cars93, aes(Weight, MPG.city)) + geom_point() +  
  geom_smooth(method = 'lm', formula = y~poly(x,3), se=F)
```



Section 3

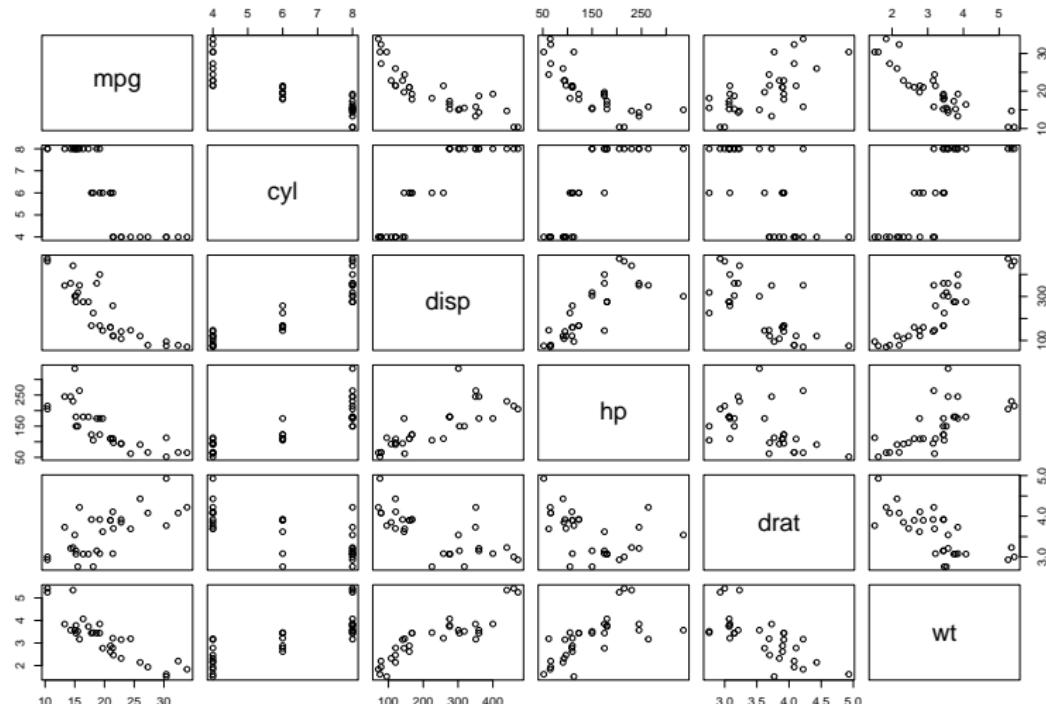
Scatterplot matrix

Scatterplot matrix

- A scatter plot matrix is a grid (or matrix) of scatter plots used to visualize bivariate relationships between combinations of variables.
- Each scatter plot in the matrix visualizes the relationship between a pair of variables, allowing many relationships to be explored in one chart.

Scatterplot matrix

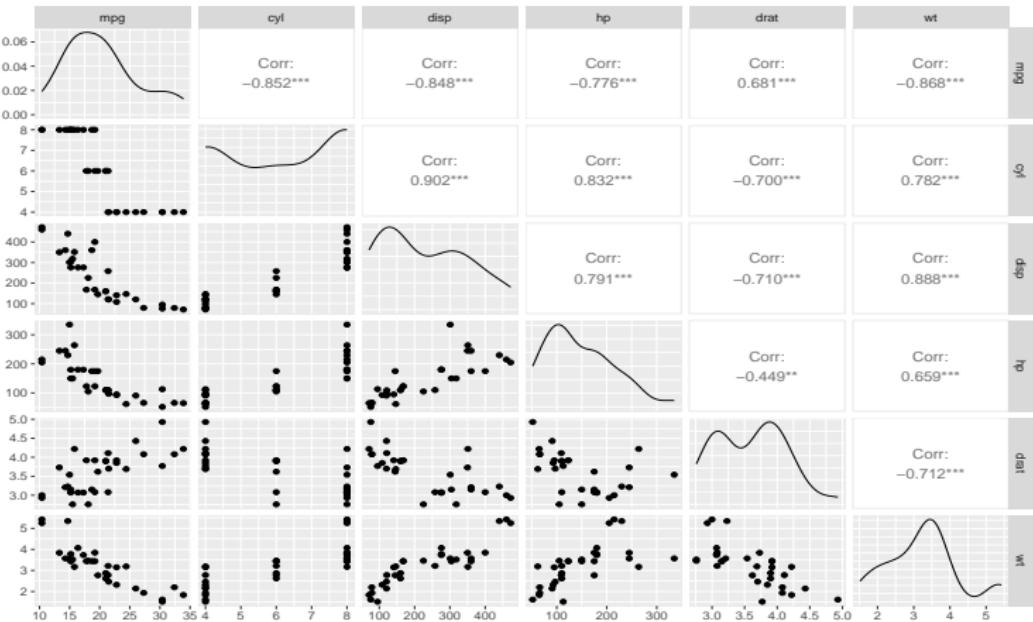
```
mtcars <- mtcars %>% dplyr::select(mpg:wt)  
pairs(mtcars)
```



Scatterplot matrix

ggplot style

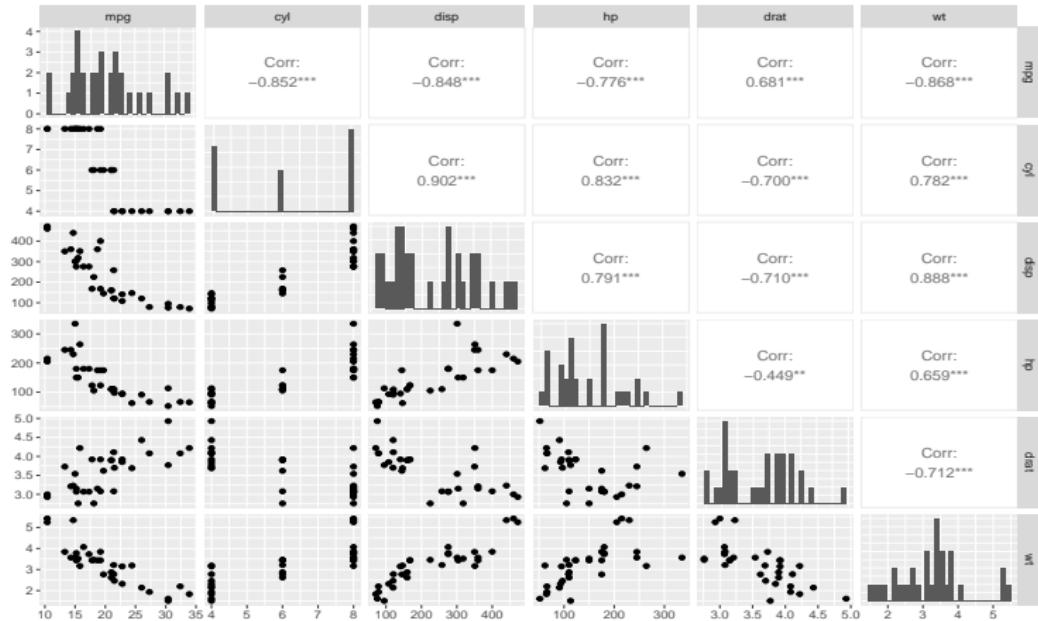
`ggpairs(mtcars)`



Scatterplot matrix

The scatterplot matrix can be customized see more

```
ggpairs(mtcars, diag = list(continuous = 'barDiag'))
```



Section 4

Parallel coordinates plot

Parallel coordinates plot

- Parallel plot or parallel coordinates plot allows to compare the feature of several individual observations (series) on a set of numeric variables.
- Each vertical bar represents a variable and often has its own scale.
- The units can even be different.
- Values are then plotted as series of lines connected across each axis.

Parallel coordinates plot

- With a scatterplot, the x and y axes are perpendicular to one another.
- In a parallel coordinate plot all axes are parallel to one another.
- Each variable has its own individual vertical (horizontal) axis.
- The axis is usually scaled from the minimum to the maximum case values for the variable, so that the full range of each axis is used.
- The values of each case on adjacent axes are joined by lines, so that a polygonal line across all axes defines a case.

Parallel coordinates plot

iris dataset

- create an id variable
- melt the data into long format with id.vars as the id variable and Species
- arrange by id

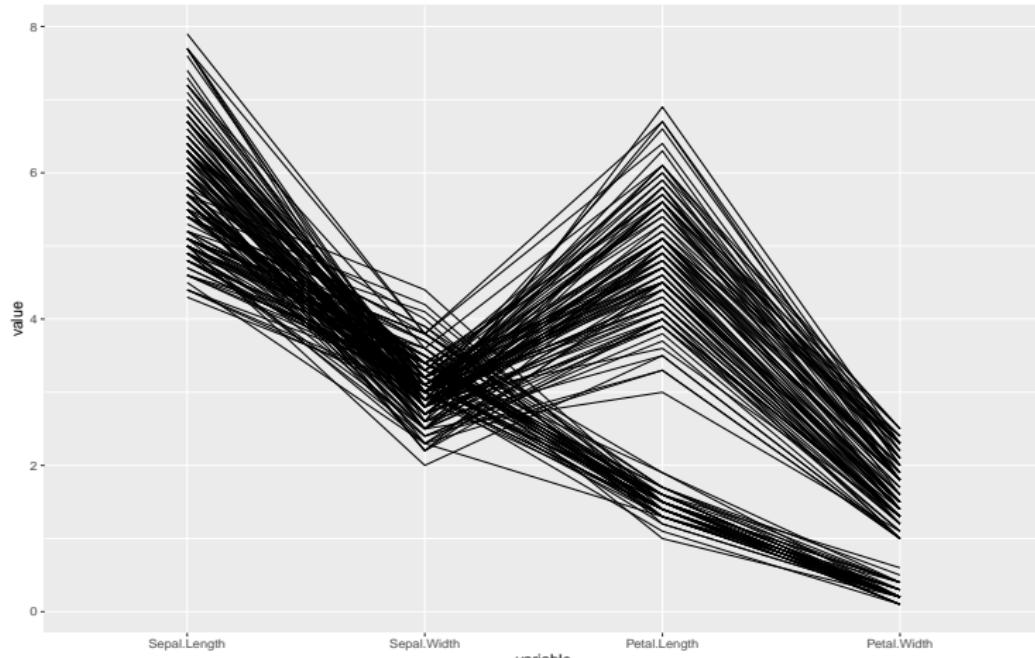
```
iris_melted <- iris %>% mutate(id = 1:150) %>%
  melt(id.vars = c('id', 'Species')) %>% arrange(id)
head(iris_melted)

##   id Species      variable value
## 1  1  setosa Sepal.Length  5.1
## 2  1  setosa Sepal.Width  3.5
## 3  1  setosa Petal.Length 1.4
## 4  1  setosa Petal.Width  0.2
## 5  2  setosa Sepal.Length  4.9
## 6  2  setosa Sepal.Width  3.0
```

Parallel coordinates plot

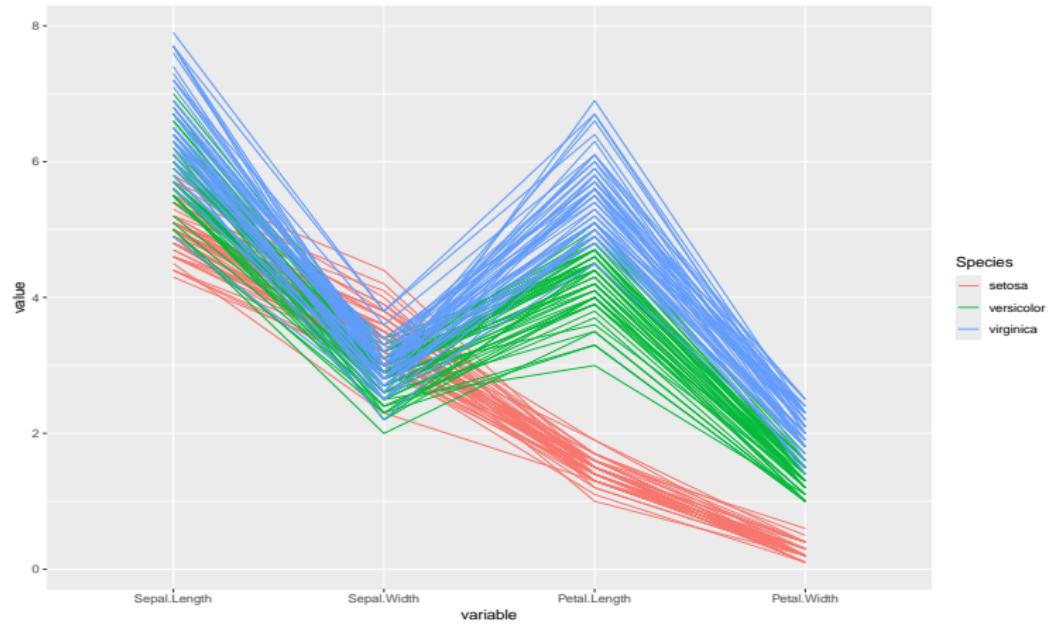
new aesthetics - group

```
ggplot(data = iris_melted, aes(x = variable, y = value)) +  
  geom_line(aes(group = id))
```



Parallel coordinates plot

```
ggplot(data = iris_melted, aes(x = variable, y = value)) +  
  geom_line(aes(group = id, color = Species))
```



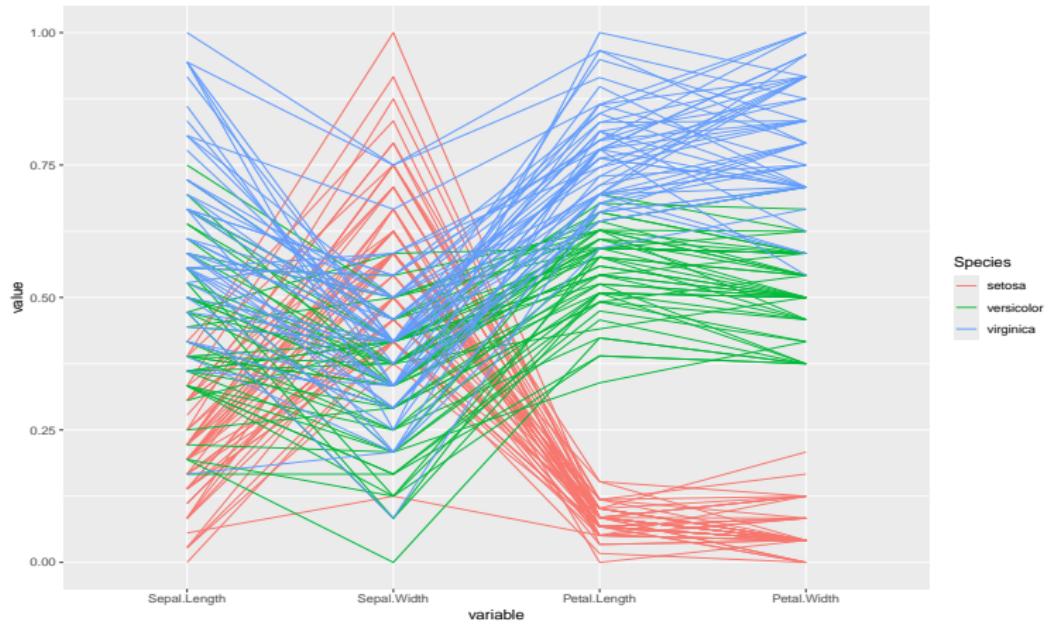
Parallel coordinates plot

- Usually the variables will come in different scales, thus we need to normalize them first
- MinMax normalization

```
min_max <- function(x){(x - min(x))/(max(x)-min(x))}  
iris_melted <- iris_melted %>% group_by(variable) %>%  
  mutate(value = min_max(value))
```

Parallel coordinates plot

```
ggplot(data = iris_melted, aes(x = variable, y = value)) +  
  geom_line(aes(group = id, color = Species))
```

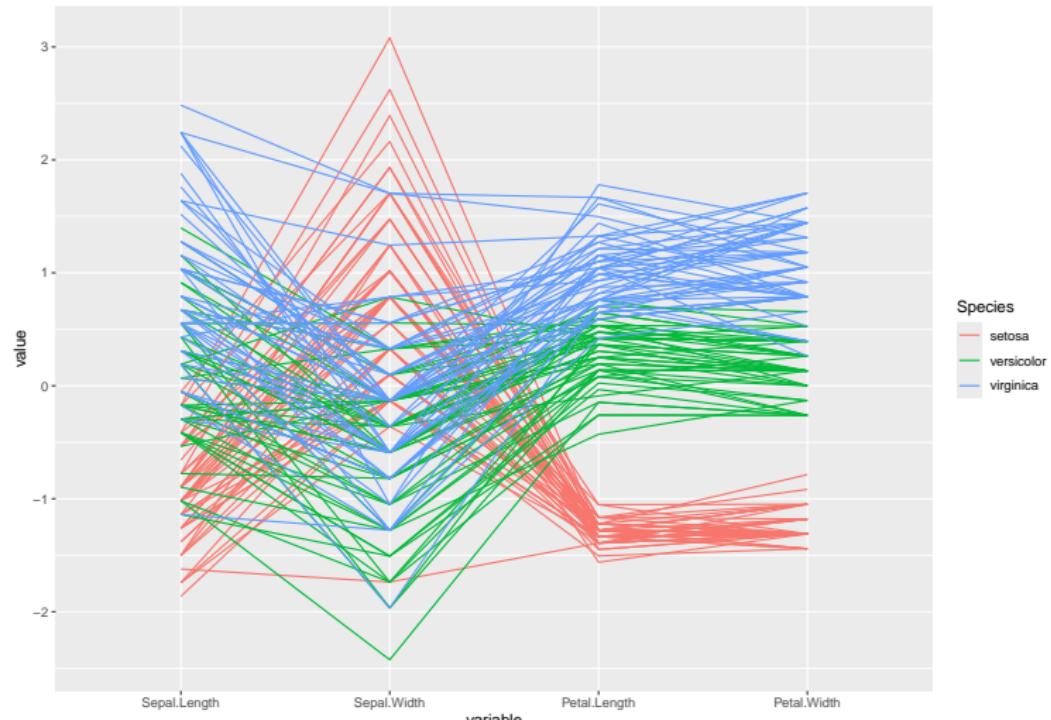


Parallel coordinates plot

- GGally is a great companion for ggplot and has a wrapper functions for different complex visualization
- The dataframe melting and normalization (z- score) is done by default

Parallel coordinates plot

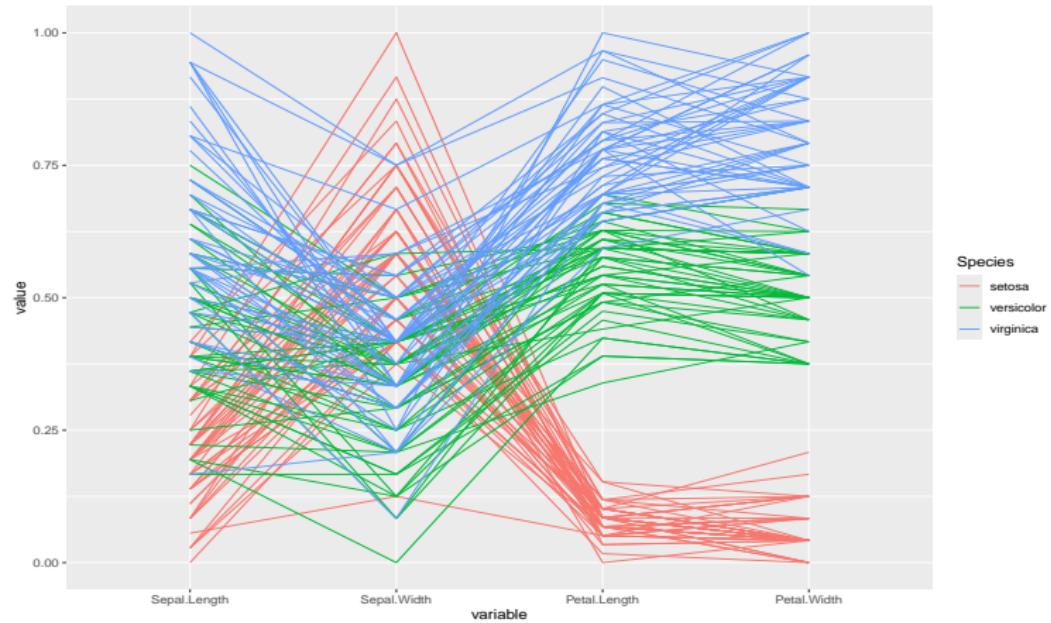
```
ggparcoord(data = iris, columns = 1:4, groupColumn = 'Species')
```



Parallel coordinates plot

minmax normalization

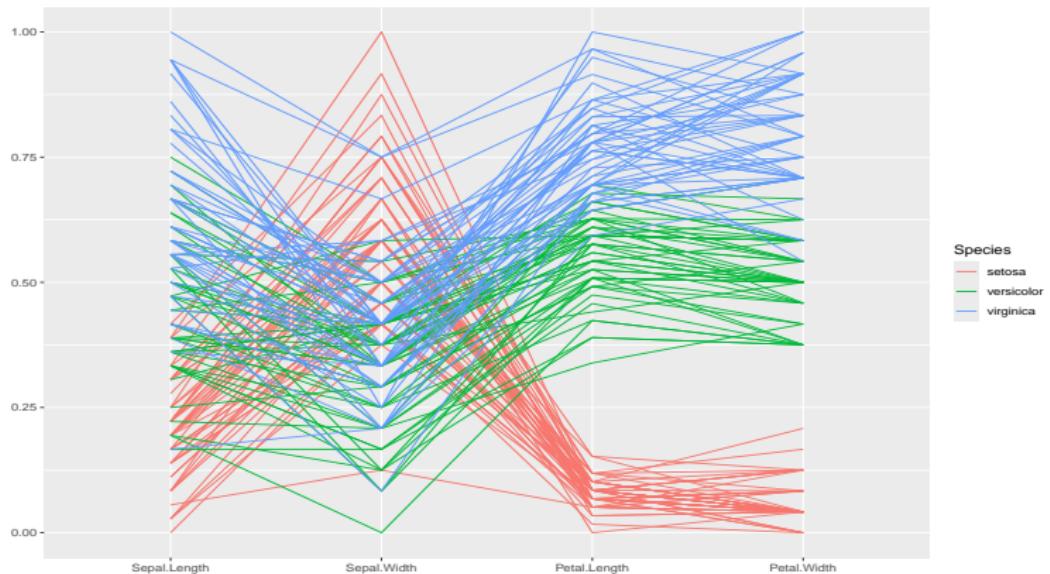
```
ggparcoord(data = iris, columns = 1:4, groupColumn = 'Species',  
           scale = 'uniminmax')
```



Parallel coordinates plot

result is a ggplot object, so can add layers as usual

```
ggparcoord(data = iris, columns = 1:4, groupColumn = 'Species',
            scale = 'uniminmax') +
  theme(axis.title = element_blank())
```

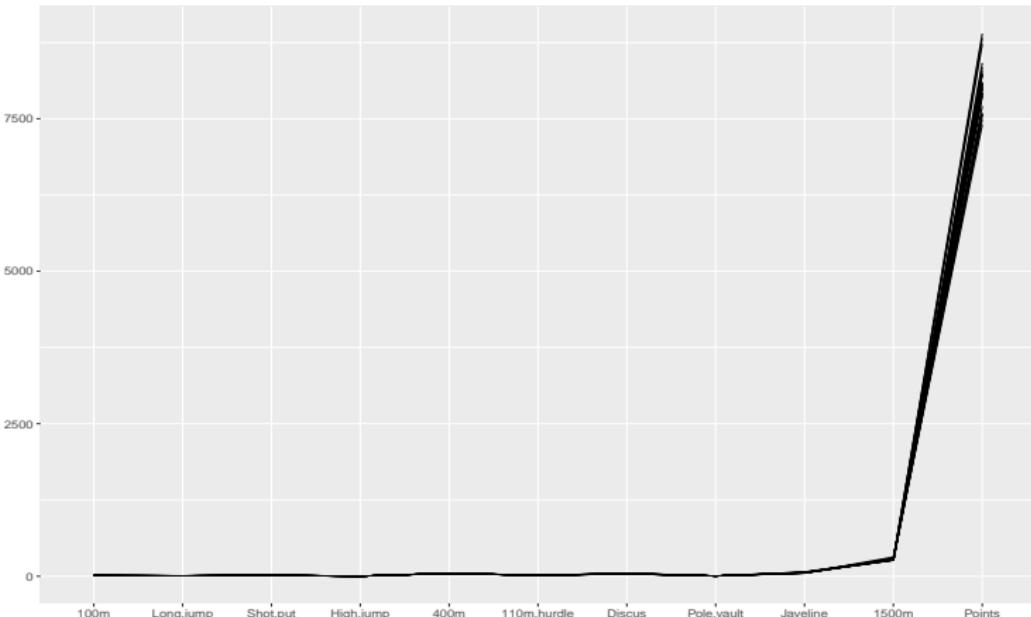


Parallel coordinates plot

Why to normalize ? Look at the decathlon data

Parallel coordinates plot

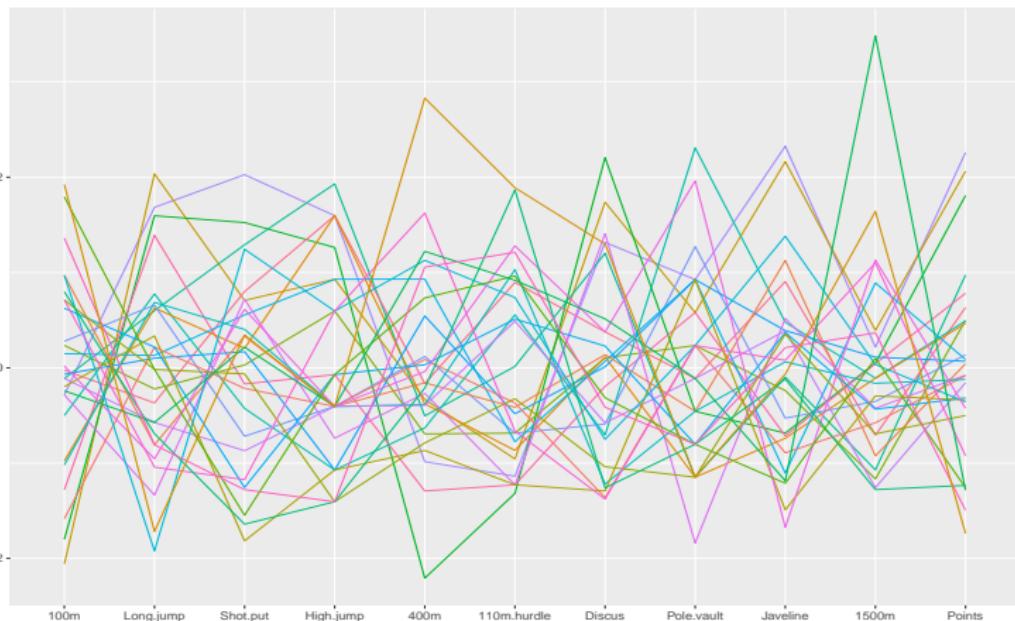
```
decathlon <- read_excel('Data/decathlon.xlsx')
dec_melted <- melt(decathlon)
ggplot(data = dec_melted, aes(x = variable, y = value)) +
  geom_line(aes(group = Sportsman)) + theme(axis.title = element_blank())
```



Parallel coordinates plot

The normalized version

```
ggparcoord(data = decathlon, columns = 2:12, groupColumn = 'Sportsman') +  
  theme(legend.position = "none") + theme(axis.title = element_blank())
```



Section 5

Correlation heatmaps

Correlation heatmaps

- Correlation heatmaps are used to visualize the correlation matrix.
- They are especially useful when you have multivariate data (more than two variables)

Decathlon data

```
decathlon <- read_excel('Data/decathlon.xlsx')
colnames(decathlon)
## [1] "Sportsman"      "100m"           "Long.jump"       "Shot.put"        "High.jump"
## [6] "400m"           "110m.hurdle"    "Discus"          "Pole.vault"     "Javeline"
## [11] "1500m"          "Points"
```

Correlation heatmaps

```
round(cor(decathlon[, -1]), 2)
##          100m Long.jump Shot.put High.jump 400m 110m.hurdle Discus
## 100m      1.00    -0.70   -0.37   -0.31   0.63     0.54   -0.23
## Long.jump -0.70     1.00    0.20    0.35  -0.67    -0.54    0.25
## Shot.put   -0.37    0.20     1.00    0.61  -0.20    -0.25    0.67
## High.jump  -0.31    0.35    0.61     1.00  -0.17    -0.33    0.52
## 400m       0.63   -0.67   -0.20   -0.17   1.00     0.52   -0.14
## 110m.hurdle 0.54   -0.54   -0.25   -0.33   0.52     1.00   -0.22
## Discus     -0.23    0.25    0.67    0.52  -0.14    -0.22    1.00
## Pole.vault -0.26    0.29    0.02   -0.04  -0.12    -0.15   -0.18
## Javeline    -0.01    0.09    0.38    0.20  -0.05    -0.08    0.25
## 1500m      0.06   -0.15    0.13    0.00   0.55     0.18    0.22
## Points     -0.71    0.77    0.62    0.63  -0.70    -0.65    0.51
##          Pole.vault Javeline 1500m Points
## 100m        -0.26   -0.01   0.06   -0.71
## Long.jump   0.29    0.09  -0.15    0.77
## Shot.put    0.02    0.38   0.13    0.62
## High.jump  -0.04    0.20   0.00    0.63
## 400m       -0.12   -0.05   0.55   -0.70
## 110m.hurdle -0.15   -0.08   0.18   -0.65
```

Correlation heatmaps: ggplot2

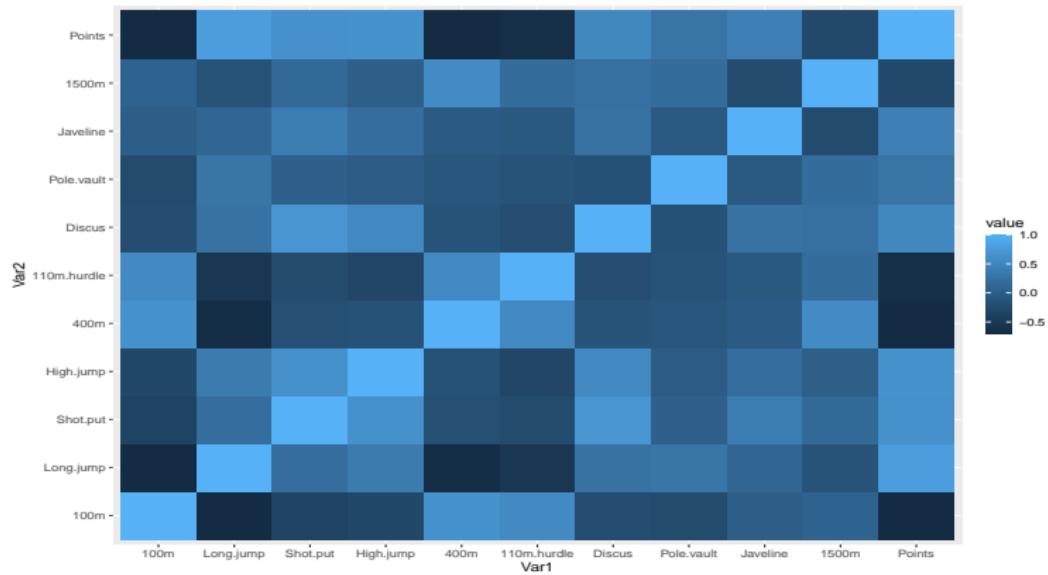
To create correlation heatmap with ggplot2, we first need to transform the data into long format.

```
cor_mat <- round(cor(decathlon[,-1]),2)
cor_mat <- melt(cor_mat)
head(cor_mat)
##           Var1 Var2 value
## 1      100m 100m  1.00
## 2 Long.jump 100m -0.70
## 3   Shot.put 100m -0.37
## 4 High.jump 100m -0.31
## 5      400m 100m  0.63
## 6 110m.hurdle 100m  0.54
```

Correlation heatmaps

We use `geom_tile()` to draw rectangles

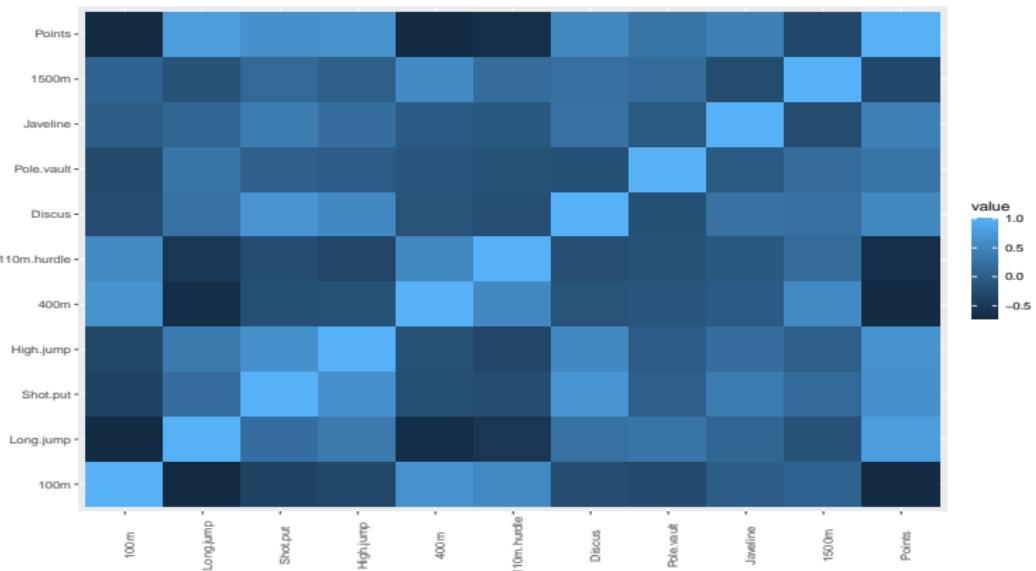
```
ggplot(data = cor_mat, aes(x=Var1, y=Var2, fill=value)) +  
  geom_tile()
```



Correlation heatmaps

Shape the plot a little

```
ggplot(data = cor_mat, aes(x=Var1, y=Var2, fill=value)) +  
  geom_tile() + theme(axis.title = element_blank(),  
                      axis.text.x = element_text(angle = 90))
```

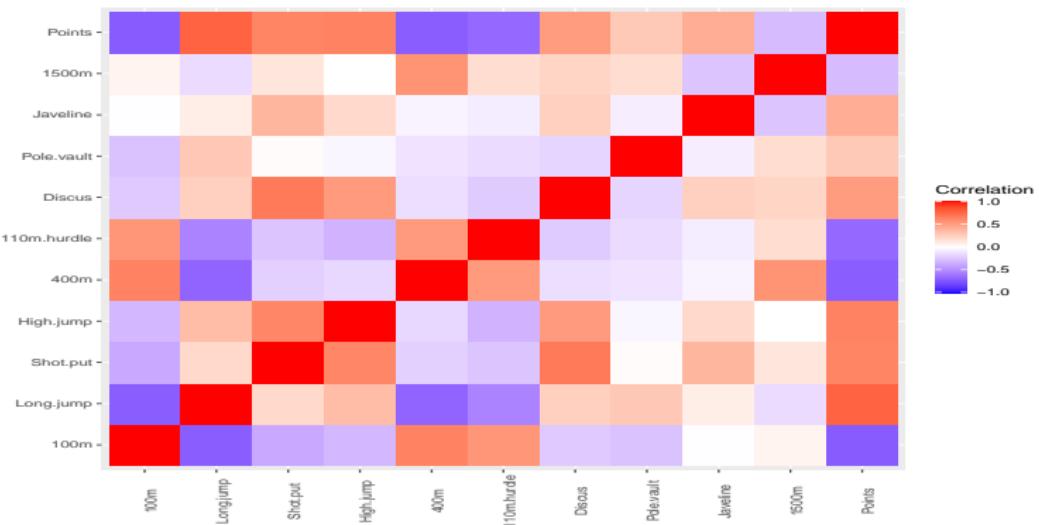


Correlation heatmaps

- Change the color scale to a diverging one
- `geom_tile()` is creating rectangles, if you want squares, use `coord_fixed()`

Correlation heatmaps

```
ggplot(data = cor_mat, aes(x=Var1, y=Var2, fill=value)) + geom_tile() +
  theme(axis.title = element_blank(),
  axis.text.x = element_text(angle = 90)) +
  scale_fill_gradient2(low = "blue", high = "red", mid = "white",
  midpoint = 0, limit = c(-1,1), name="Correlation") + coord_fixed()
```



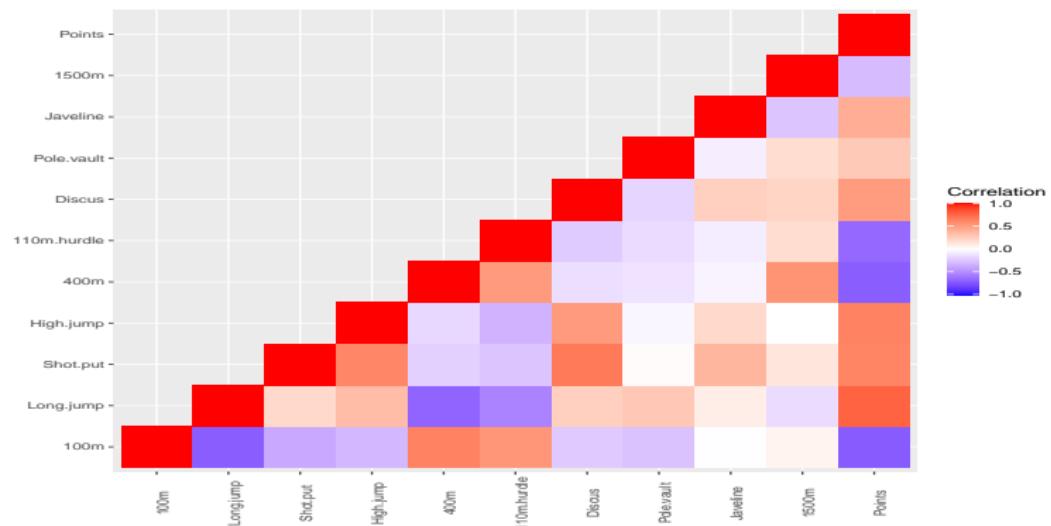
Correlation heatmaps

Only lower or upper triangle of the correlation matrix is really needed for the analysis.

```
cor_mat <- round(cor(decathlon[,-1]),2)
cor_mat[upper.tri(cor_mat)] <- NA
cor_mat <- melt(cor_mat, na.rm = T)
```

Correlation heatmaps

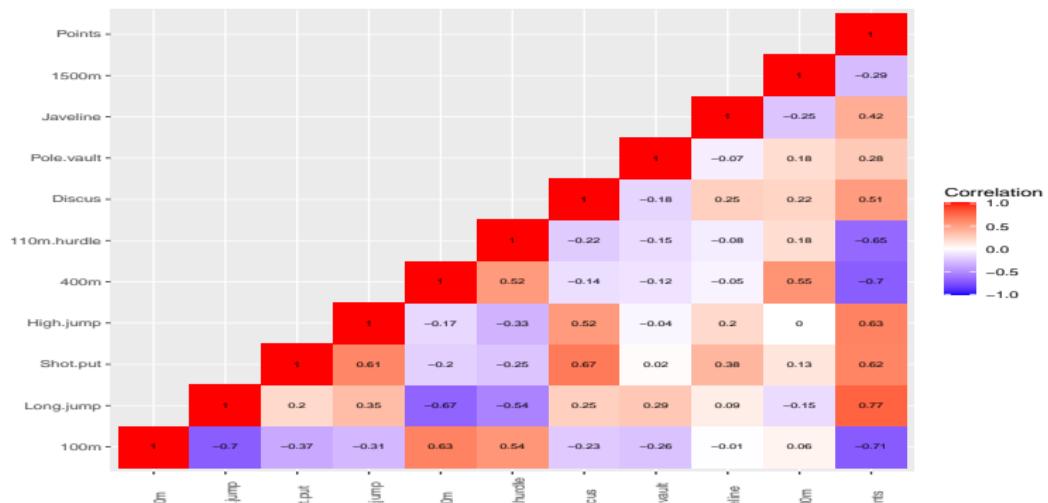
```
ggplot(data = cor_mat, aes(x=Var1, y=Var2, fill=value)) + geom_tile() +  
  theme(axis.title = element_blank(),  
  axis.text.x = element_text(angle = 90)) +  
  scale_fill_gradient2(low = "blue", high = "red", mid = "white",  
  midpoint = 0, limit = c(-1,1), name="Correlation") + coord_fixed()
```



Correlation heatmaps

Use `geom_text()` to add the correlation coefficients on the graph

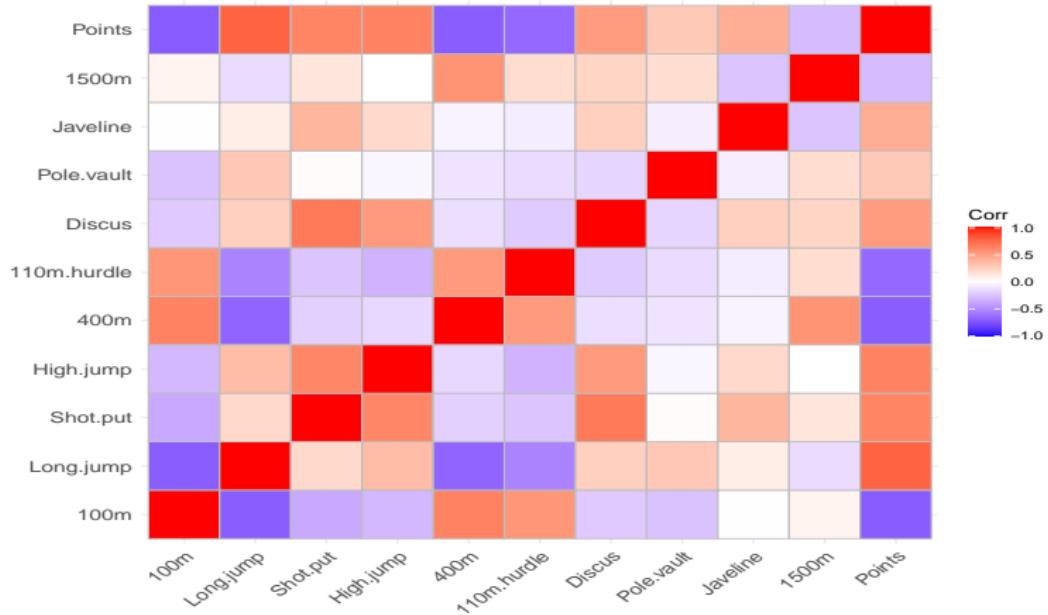
```
ggplot(data = cor_mat, aes(x=Var1, y=Var2, fill=value)) + geom_tile() +
  theme(axis.title = element_blank(),
  axis.text.x = element_text(angle = 90)) +
  scale_fill_gradient2(low = "blue", high = "red", mid = "white",
  midpoint = 0, limit = c(-1,1), name="Correlation") + coord_fixed() +
  geom_text(aes(label = value), color = "black", size = 2.5)
```



Correlation heatmaps

A ready to go solution, library `ggcorrplot` will do all the plotting with a single line

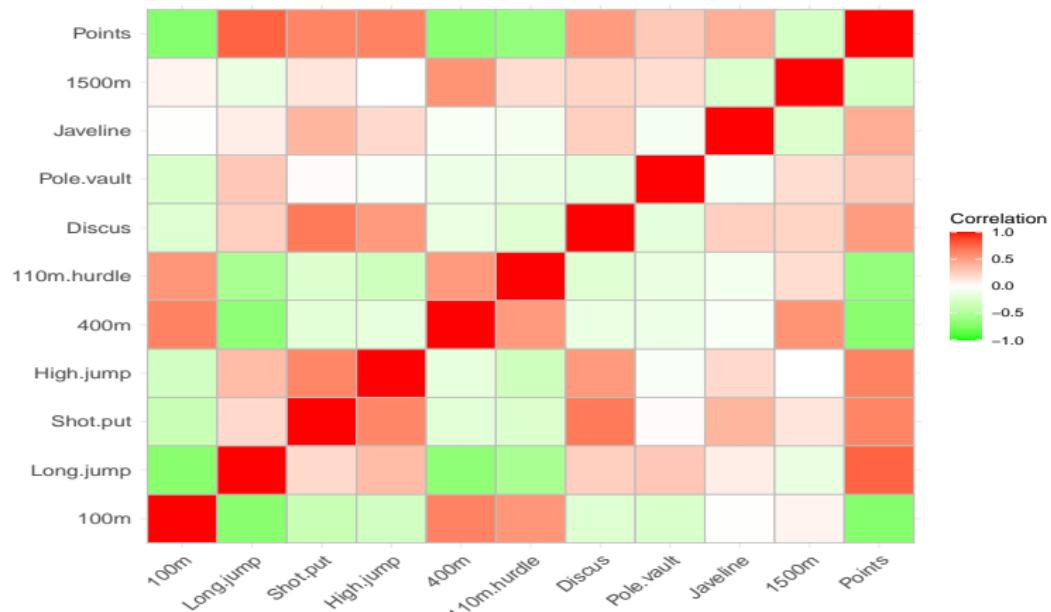
```
cor_mat <- round(cor(decathlon[,-1]),2)
ggcorrplot(cor_mat)
```



Correlation heatmaps

`ggcorrplot()` is creating a `ggplot` object, thus we can add standard `ggplot` layers to it

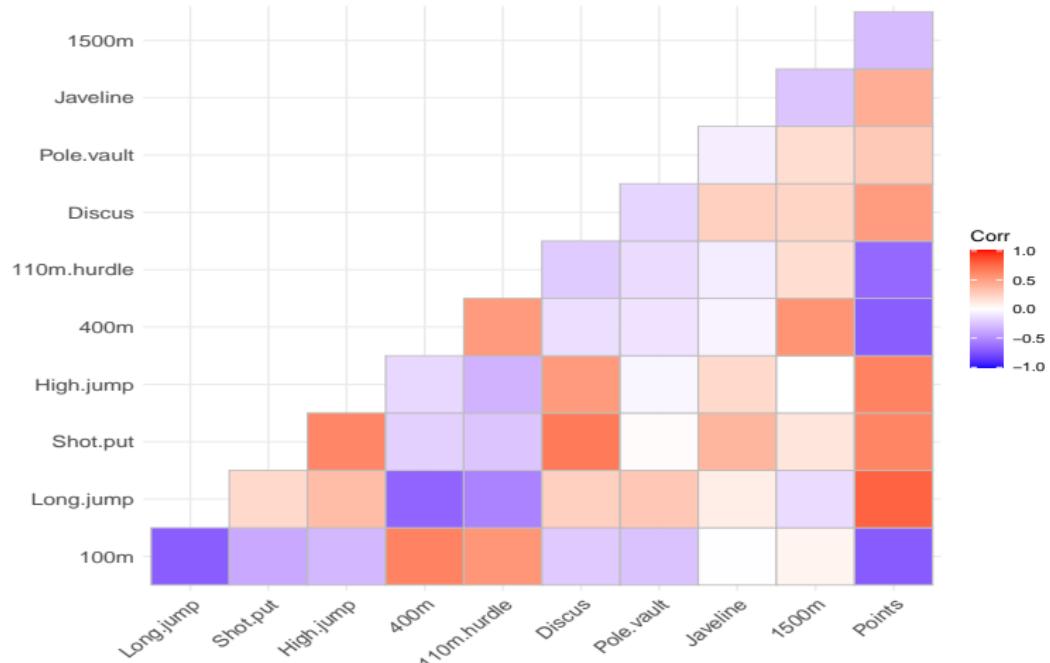
```
ggcorrplot(cor_mat) + scale_fill_gradient2(low = "green", high = "red",
    mid = "white", midpoint = 0, limit = c(-1,1), name="Correlation")
```



Correlation heatmaps

Keep only lower triangle

```
ggcorrplot(cor_mat, type = 'lower')
```

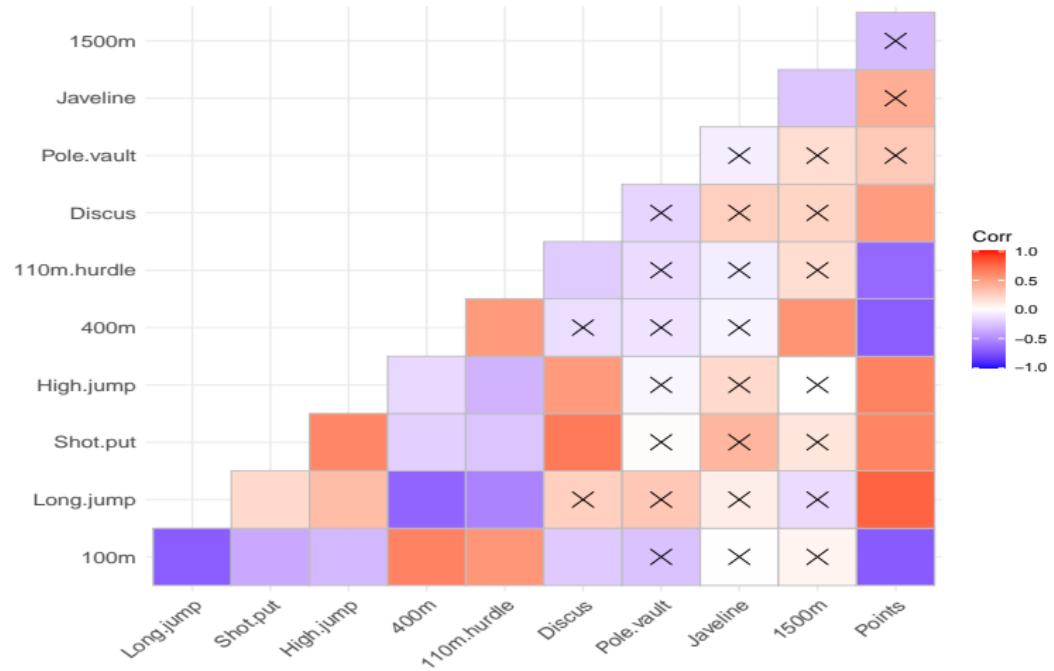


Correlation heatmaps

- function `cor_pmat()` will calculate p-values for correlation coefficients
- `p.mat` argument will mark non-significant coefficients

Correlation heatmaps

```
ggcorrplot(cor_mat, type = 'lower', p.mat = cor_pmat(cor_mat))
```



Section 6

Contour plots

Contour plots

- A contour plot is a graphical technique for representing a 3-dimensional surface by plotting constant z slices, called contours, on a 2-dimensional format.
- Thus given a value for z, lines are drawn for connecting the (x,y) coordinates where that z value occurs.
- In the other cases, you need to specify the z by yourself

Contour plots

The contour plot is formed by:

- Horizontal axis: Independent variable 1
- Vertical axis: Independent variable 2
- Lines: iso-response values

If the data (or function) do not form a regular grid, you typically need to perform a 2-D interpolation to form a regular grid.

Contour plots

fujitopo data contains three variables on Japan's mountain Fuji:

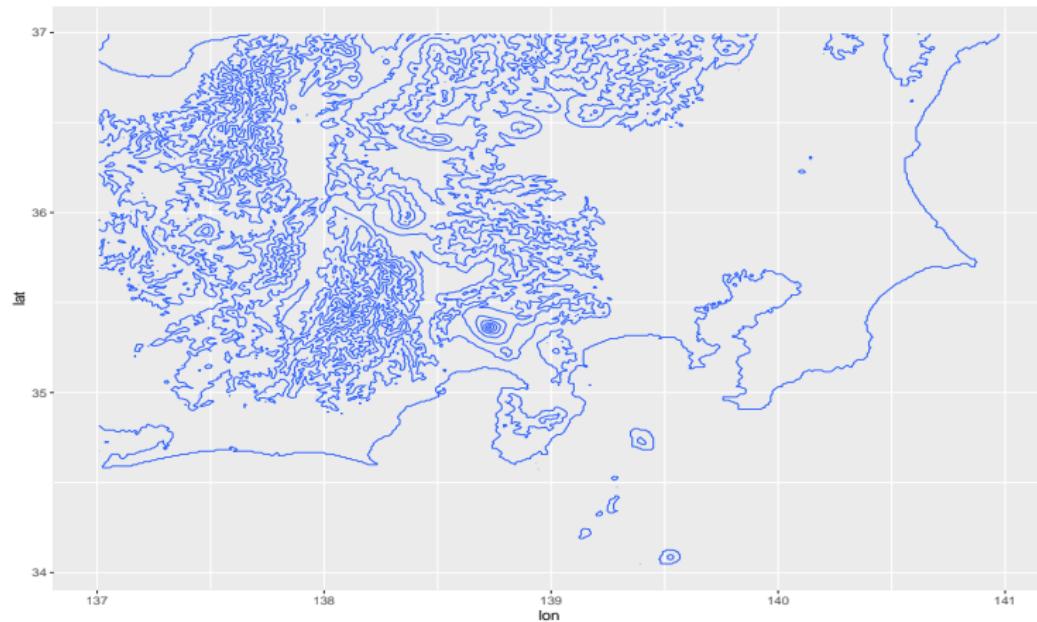
- Longitude,
- Latitude,
- Elevation

```
data("fujitopo")
fujitopo <- as.data.frame(fujitopo)
```

Contour plots

Additional aesthetics - z

```
ggplot(data = fujitopo, aes(x = lon, y = lat, z = z)) +  
  geom_contour()
```



Contour plots

Get the actual elevation map

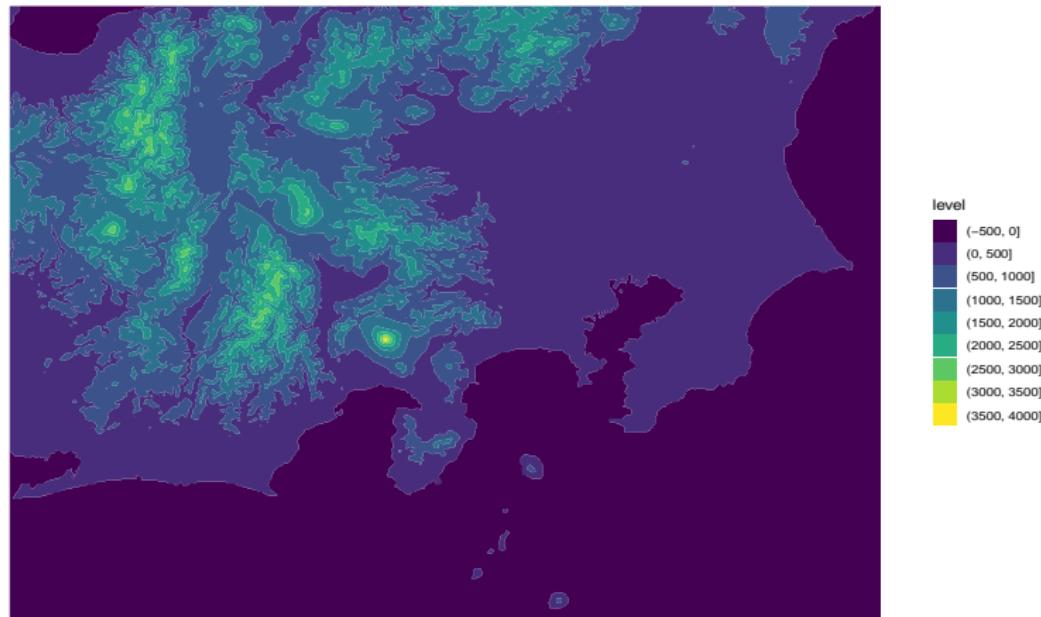
```
ggplot(data = fujitopo, aes(x = lon, y = lat, z = z)) +  
  geom_contour() + theme_void()
```



Contour plots

`geom_contour_filled()` fill geom with defined levels

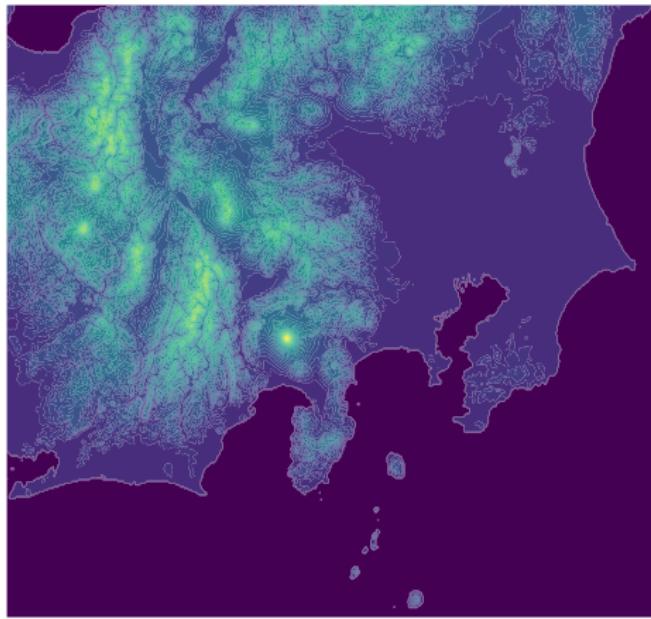
```
ggplot(data = fujitopo, aes(x = lon, y = lat, z = z)) +  
  geom_contour_filled() + theme_void()
```



Contour plots

You can specify your own bins, binwidth or breaks

```
ggplot(data = fujitopo, aes(x = lon, y = lat, z = z)) +  
  geom_contour_filled(binwidth = 100) + theme_void()
```



level
(-500, -400]
(-400, -300]
(-300, -200]
(-200, -100]
(-100, 0]
(0, 100]
(100, 200]
(200, 300]
(300, 400]
(400, 500]
(500, 600]
(600, 700]
(700, 800]
(800, 900]
(900, 1000]
(1000, 1100]
(1100, 1200]
(1200, 1300]
(1300, 1400]
(1400, 1500]
(1500, 1600]
(1600, 1700]
(1700, 1800]
(1800, 1900]
(1900, 2000]
(2000, 2100]
(2100, 2200]
(2200, 2300]
(2300, 2400]
(2400, 2500]
(2500, 2600]
(2600, 2700]
(2700, 2800]
(2800, 2900]
(2900, 3000]
(3000, 3100]
(3100, 3200]
(3200, 3300]
(3300, 3400]
(3400, 3500]
(3500, 3600]

Contour plots

Create contour plot for 3d data, where z is a function of x and y
Create a grid

```
x <- seq(-5,5, length.out = 100)
grid_df <- expand.grid(x = x, y = x)
head(grid_df)
##           x   y
## 1 -5.000000 -5
## 2 -4.898990 -5
## 3 -4.797980 -5
## 4 -4.696970 -5
## 5 -4.595960 -5
## 6 -4.494949 -5
```

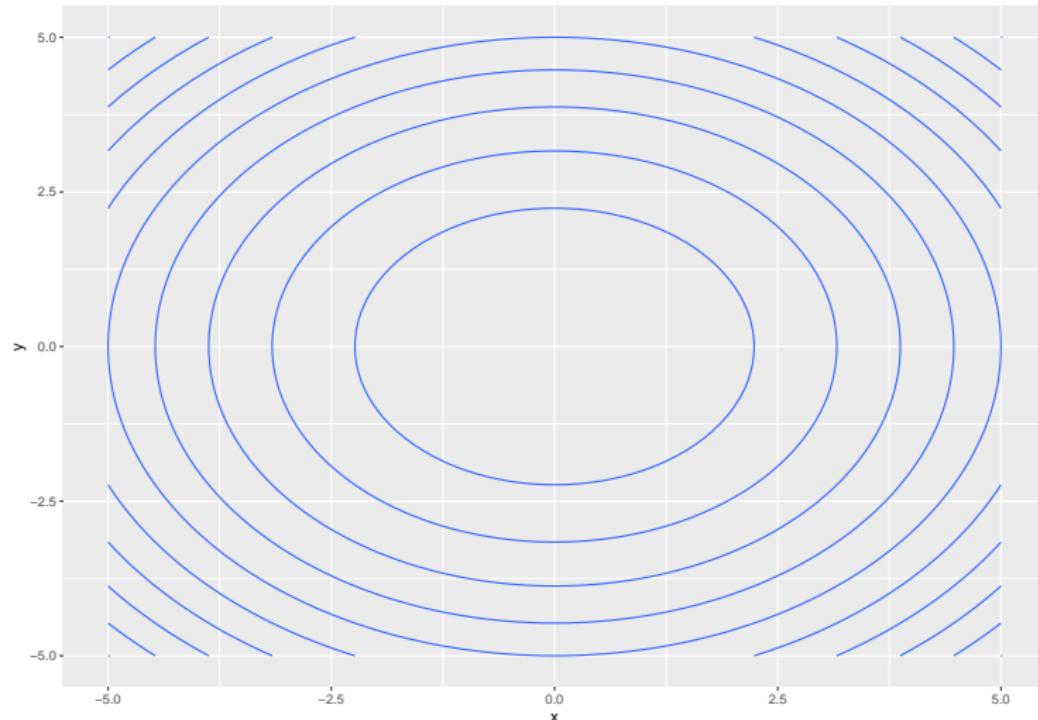
Contour plots

Lets make a function $z = x^2 + y^2$

```
grid_df$z <- grid_df$x^2 + grid_df$y^2
head(grid_df)
##           x     y      z
## 1 -5.000000 -5 50.00000
## 2 -4.898990 -5 49.00010
## 3 -4.797980 -5 48.02061
## 4 -4.696970 -5 47.06152
## 5 -4.595960 -5 46.12284
## 6 -4.494949 -5 45.20457
```

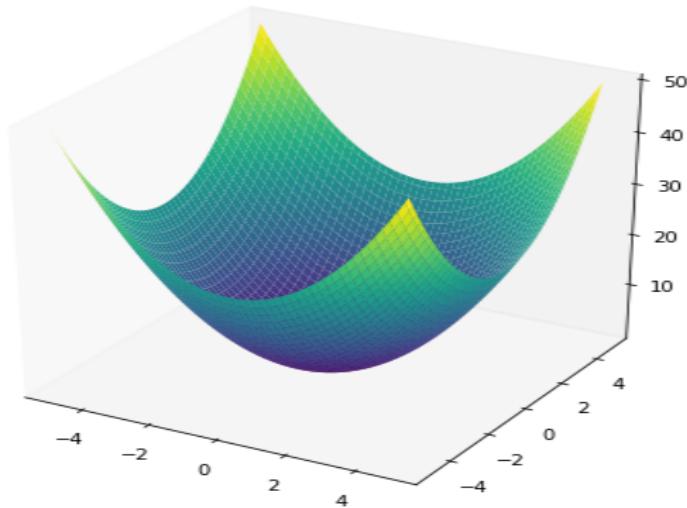
Contour plots

```
ggplot(grid_df, aes(x = x, y = y, z = z)) + geom_contour()
```



Contour plots

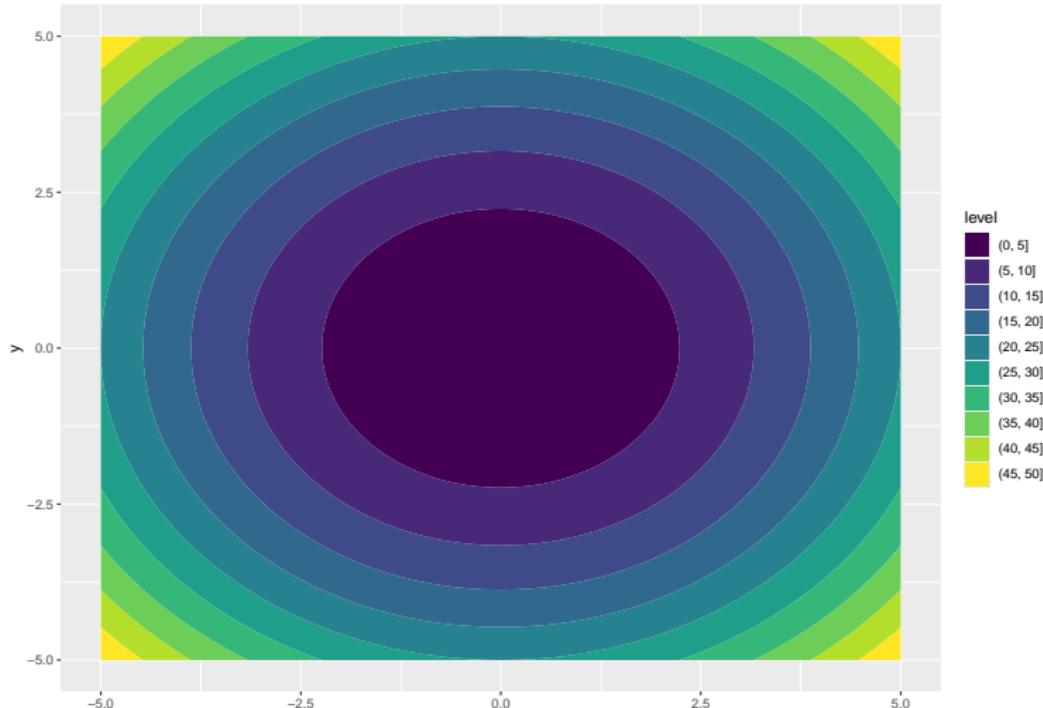
The 3d surface plot of the contour plot for the $z = x^2 + y^2$



Contour plots

With the filled layer, we are getting better picture

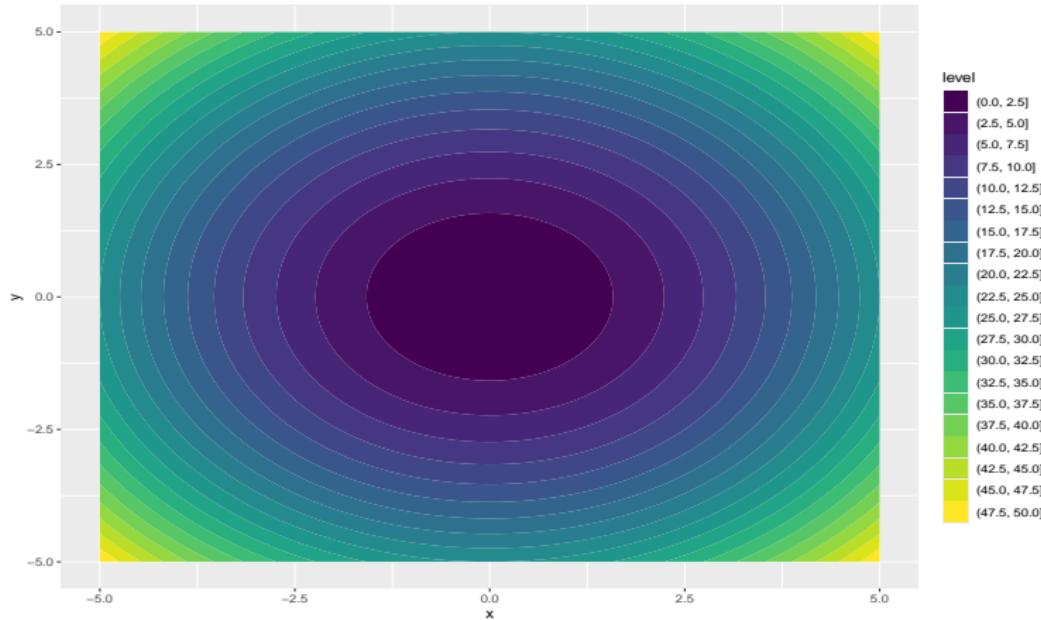
```
ggplot(grid_df, aes(x = x, y = y, z = z)) + geom_contour_filled()
```



Contour plots

Change the binwidth

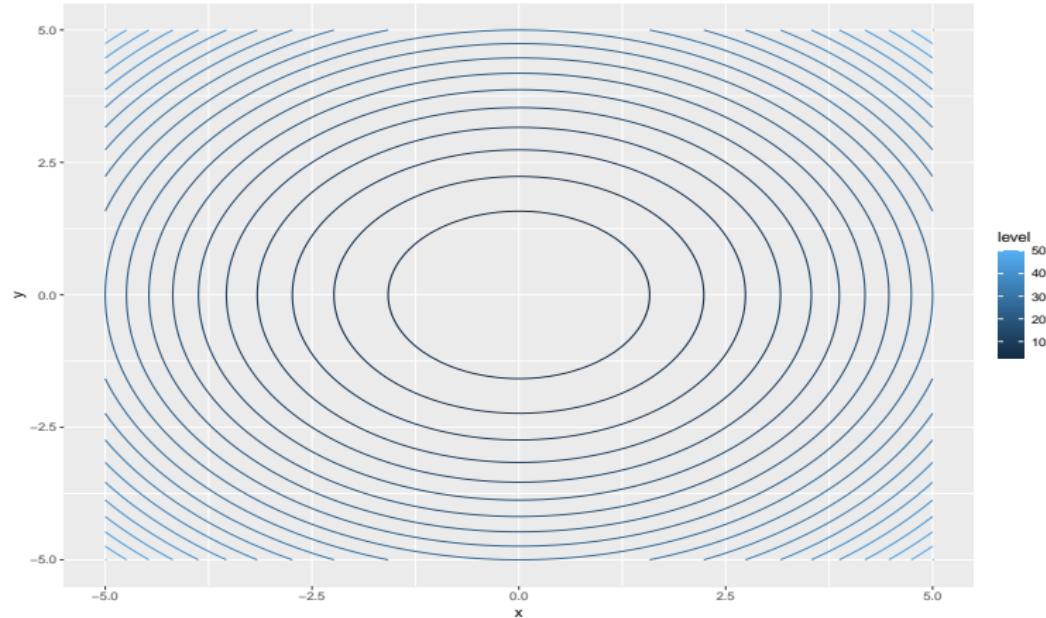
```
ggplot(grid_df, aes(x = x, y = y, z = z)) +  
  geom_contour_filled(binwidth = 2.5)
```



Contour plots

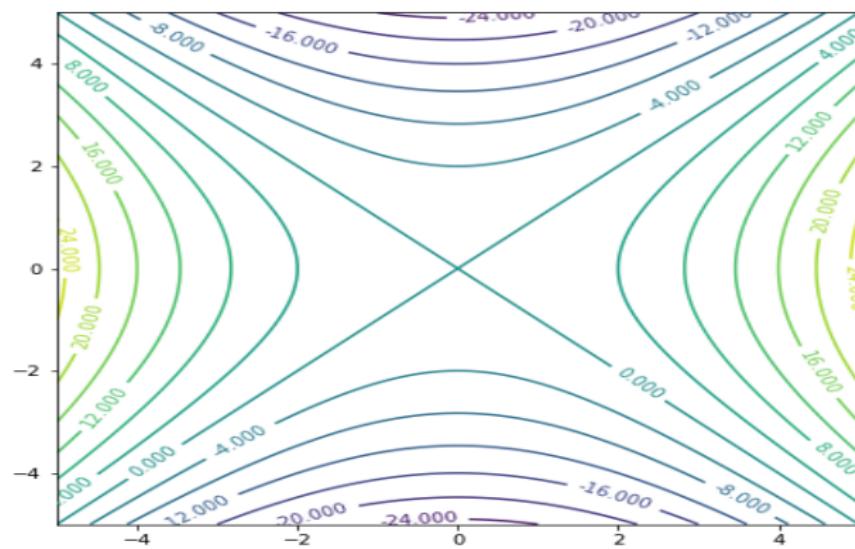
You can also color lines by levels

```
ggplot(grid_df, aes(x = x, y = y, z = z)) +  
  geom_contour(aes(color = ..level..), binwidth = 2.5)
```



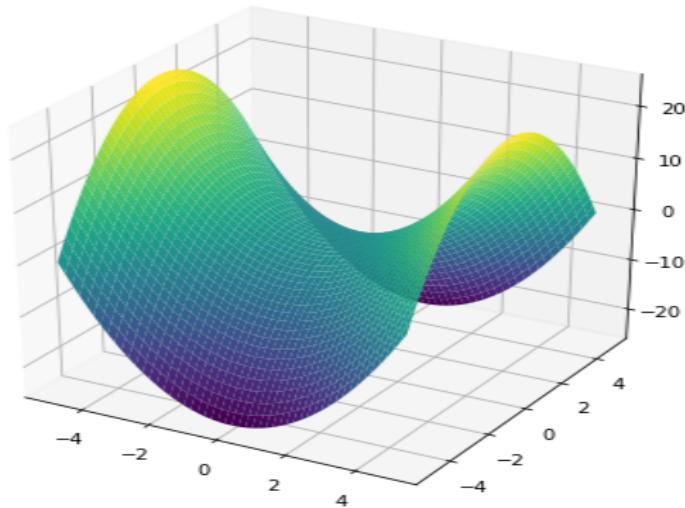
Contour plots

Can you guess the function from its contour plot ?



Contour plots

The 3d function plot for the same variable



Section 7

Visualizing joint distributions

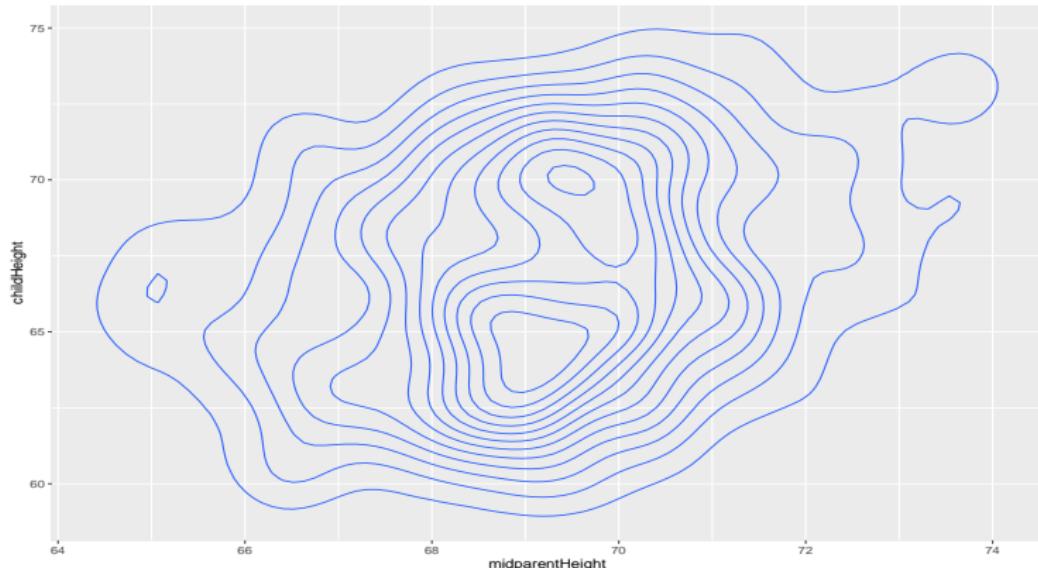
2d plots

- A 2D density plot or 2D histogram is an extension of the histogram.
- It shows the distribution of values in a data set across the range of two quantitative variables.
- It is really useful to avoid overplotting in a scatterplot, especially when you have too many points, the 2D density plot counts the number of observations within a particular area of the 2D space.
- This specific area can be a square (bin) or a hexagon (hexbin).
- We can also estimate a 2D kernel density estimation and represent it with contours.

2d plots

Visualize the relationship between midparent height and child height, add stat_density_2d as a layer.

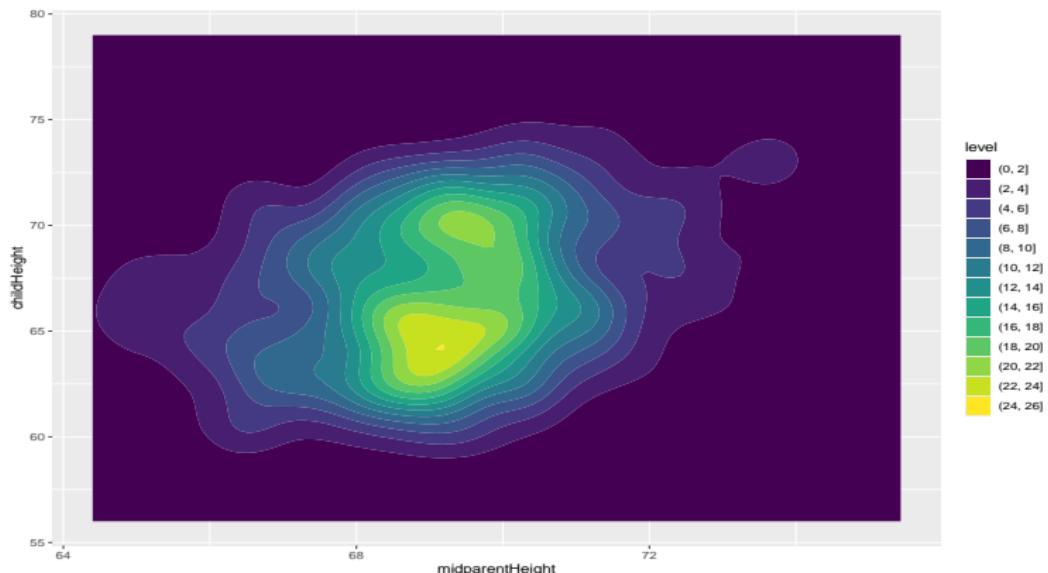
```
ggplot(GaltonFamilies, aes(x = midparentHeight, y = childHeight)) +  
  stat_density_2d()
```



2d plots

Better approach is to fill the contour plot with either the count of the points or the density count

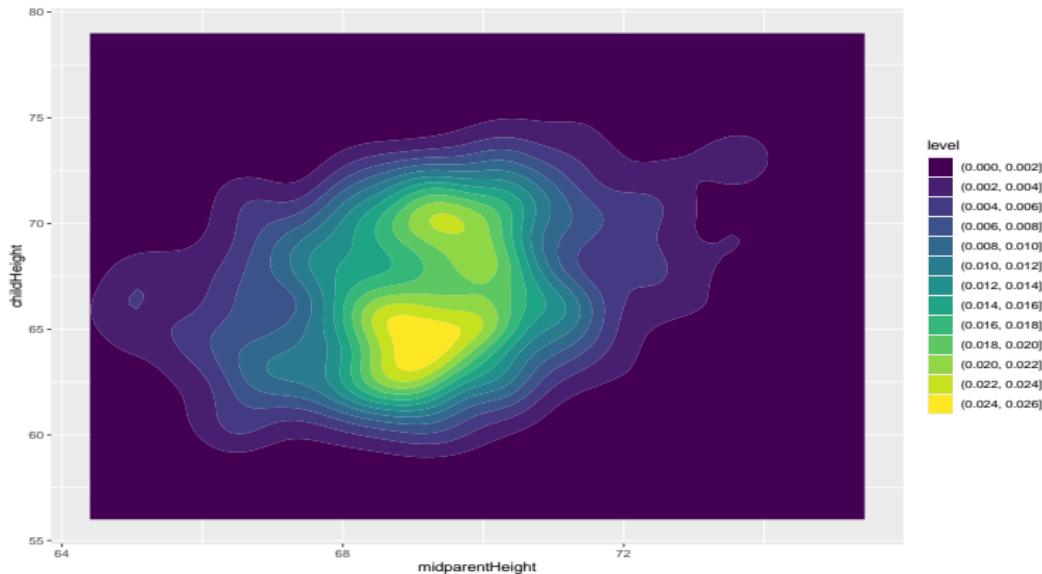
```
ggplot(GaltonFamilies, aes(x = midparentHeight, y = childHeight)) +  
  geom_density_2d_filled(contour_var = 'count')
```



2d plots

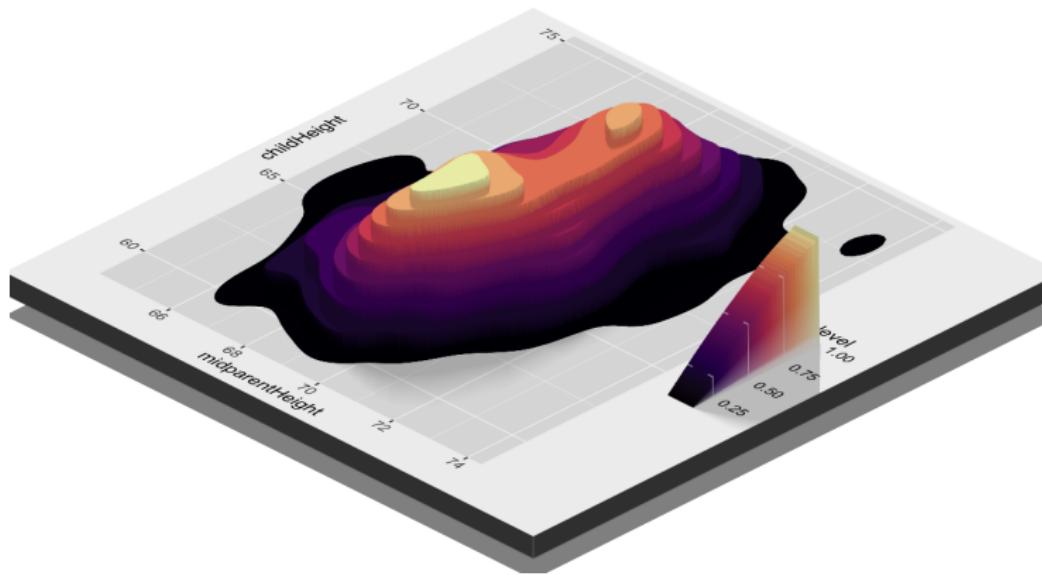
Density

```
ggplot(GaltonFamilies, aes(x = midparentHeight, y = childHeight)) +  
  geom_density_2d_filled(contour_var = 'density')
```



2d plots

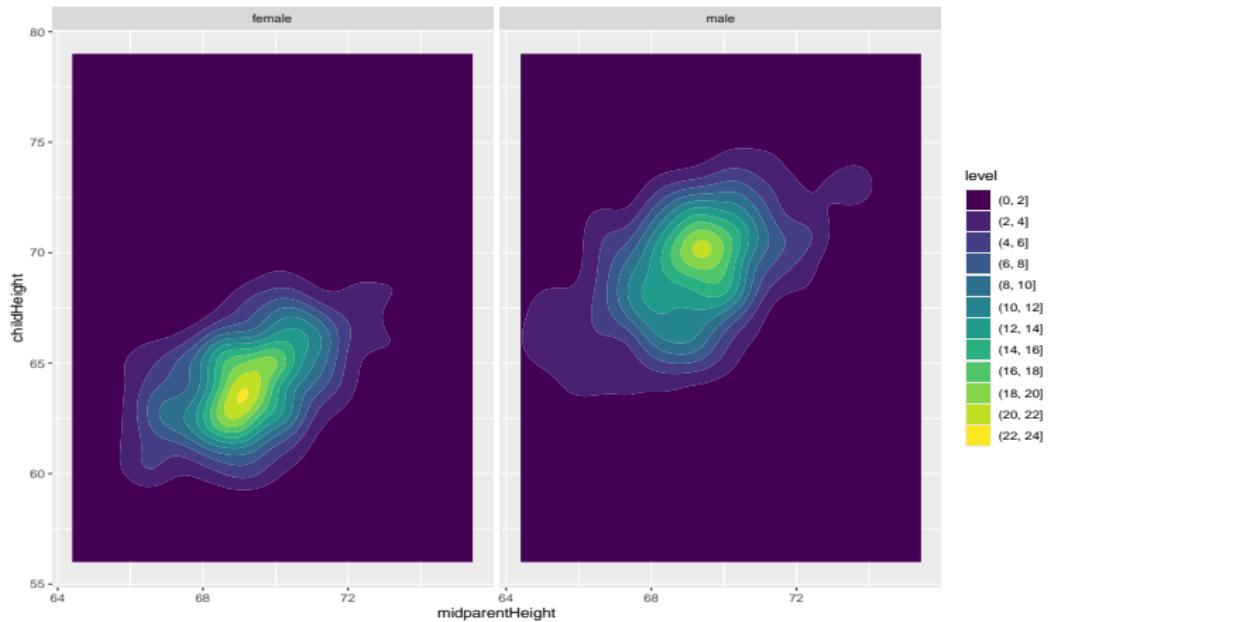
3d version



2d plots

Faceted by gender

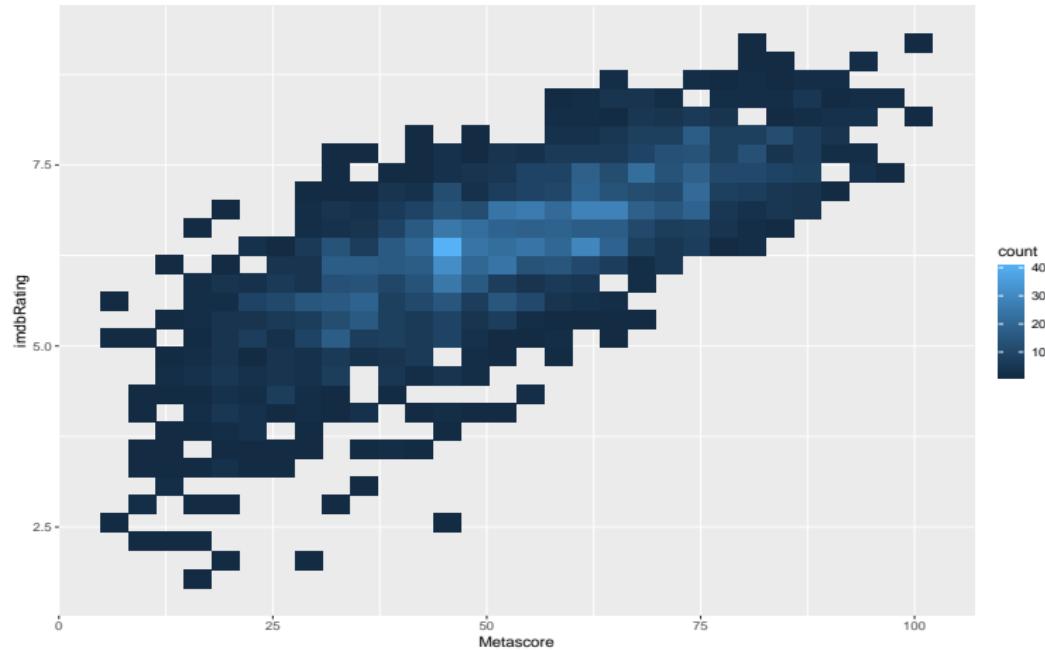
```
ggplot(GaltonFamilies, aes(x = midparentHeight, y = childHeight)) +  
  geom_density_2d_filled(contour_var = 'count') + facet_grid(.~gender)
```



2d plots

You can visualize with bins (stat = count)

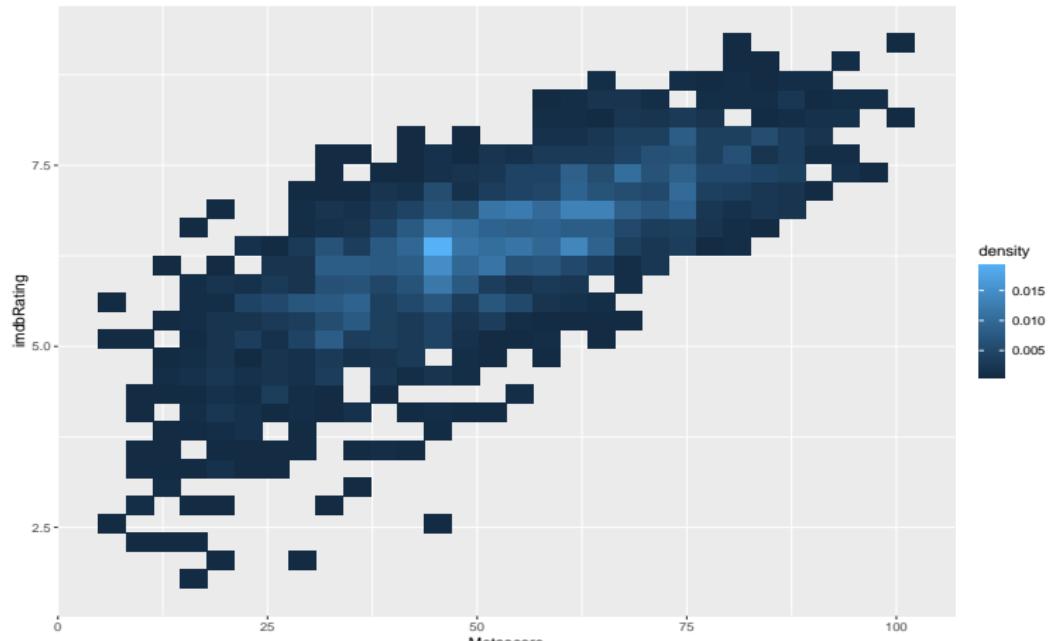
```
ggplot(movies, aes(x = Metascore, y = imdbRating)) + stat_bin_2d()
```



2d plots

Stat = density

```
ggplot(movies, aes(x = Metascore, y = imdbRating)) +  
  stat_bin_2d(aes(fill = ..density..))
```



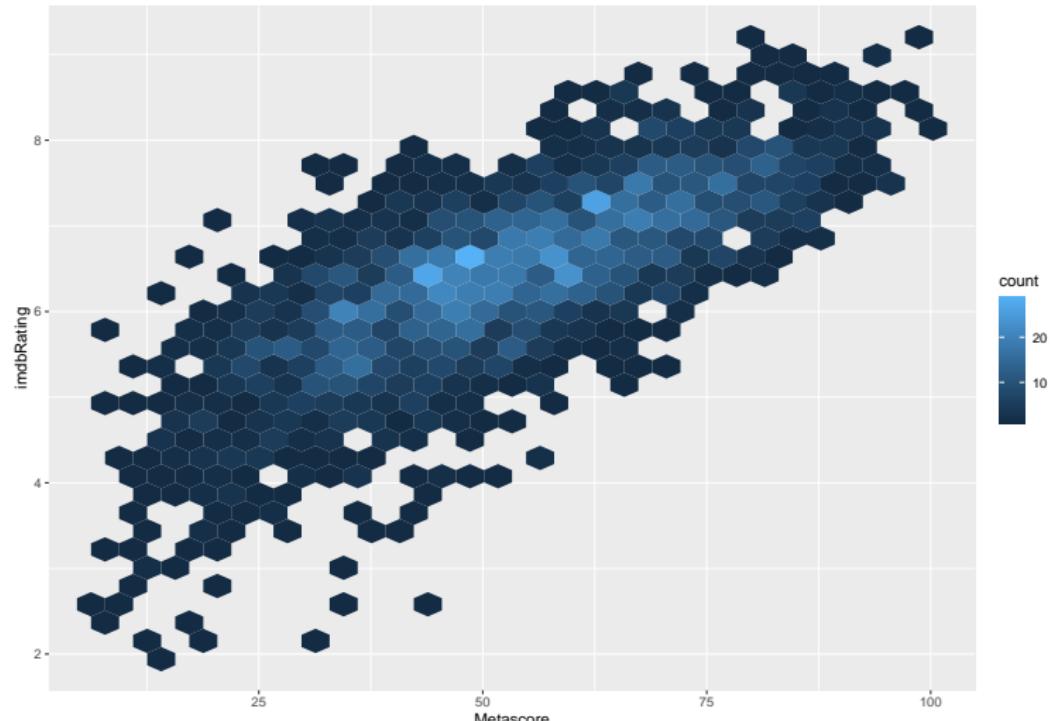
2d plots

Hexbins

- A Hexbin plot is useful to represent the relationship of 2 numerical variables when you have a lot of data points.
- Instead of overlapping, the plotting window is split in several hexbins, and the number of points per hexbin is counted. - The color denotes this number of points or the density.

2d plots

```
ggplot(movies, aes(x = Metascore, y = imdbRating)) + geom_hex()
```



2d plots

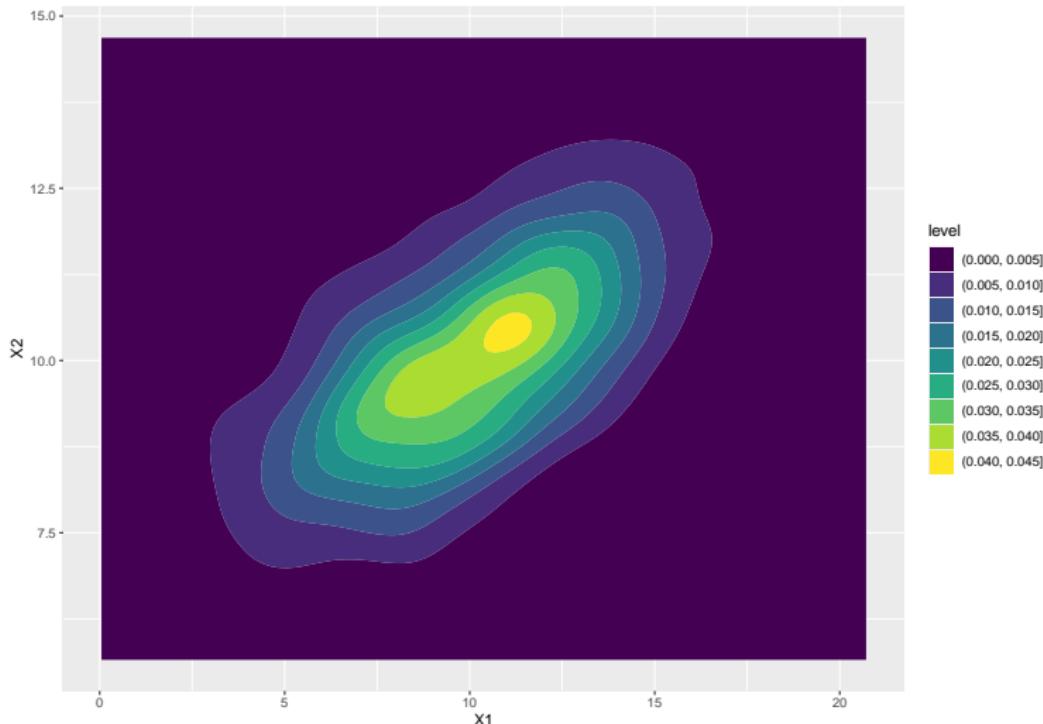
First simulate multivariate normal distribution

```
library(MASS)
sigma_m <- matrix(c(10,3,3,2),2,2)
sigma_m
##      [,1] [,2]
## [1,]    10     3
## [2,]     3     2
```

```
df_norm <- mvrnorm(n = 1000, mu = c(10,10), Sigma = sigma_m) %>% data.frame()
```

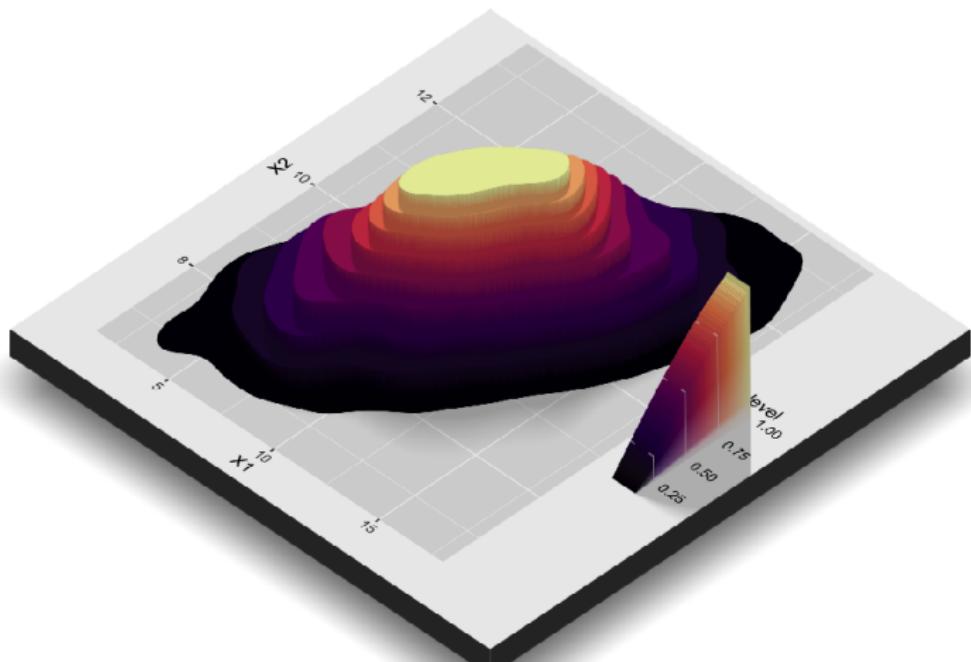
2d plots

```
ggplot(df_norm, aes(x = X1, y = X2)) + stat_density_2d_filled()
```



2d plots

The 3d version

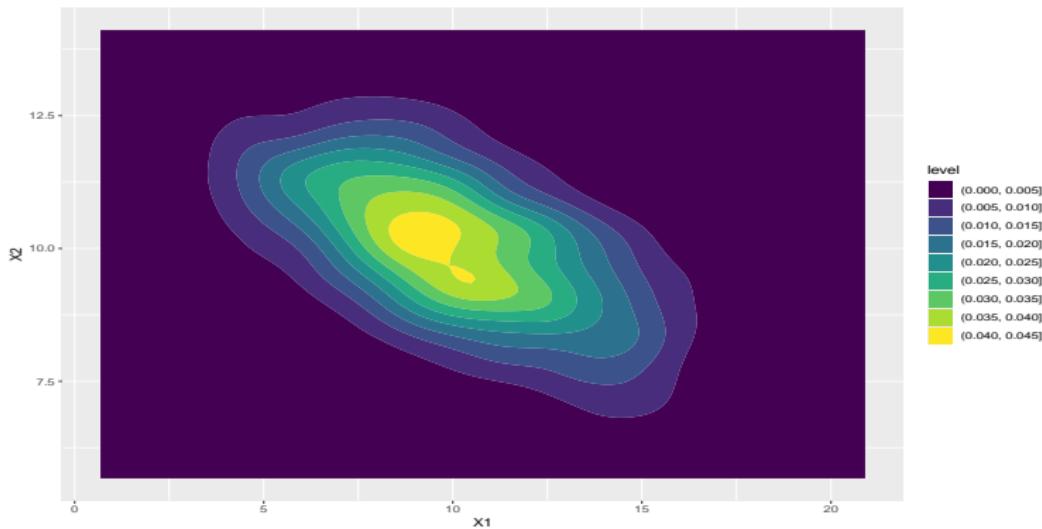


2d plots

What will happen if you change covariance from positive to negative?

```
sigma_m <- matrix(c(10,-3,-3,2),2,2)
df_norm <- mvrnorm(n = 1000, mu = c(10,10), Sigma = sigma_m)
df_norm <- data.frame(df_norm)

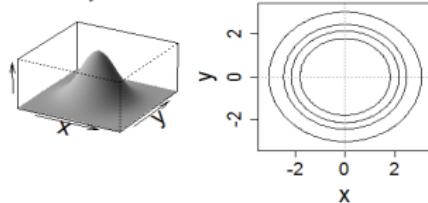
ggplot(df_norm, aes(x = X1, y = X2)) + stat_density_2d_filled()
```



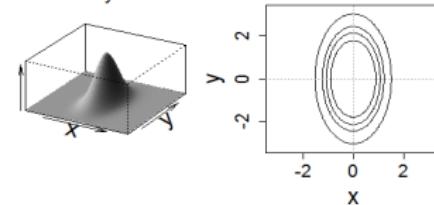
2d plots

Plots for different covariance matrices

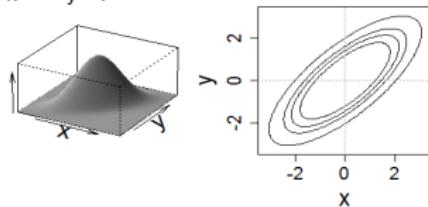
$$\sigma_x = \sigma_y, \rho = 0$$



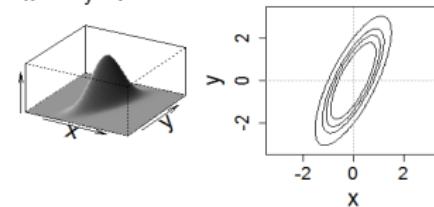
$$2\sigma_x = \sigma_y, \rho = 0$$



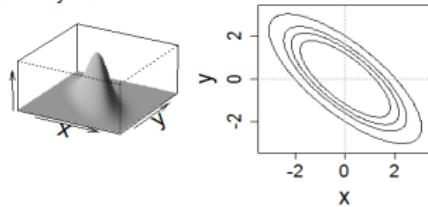
$$\sigma_x = \sigma_y, \rho = 0.75$$



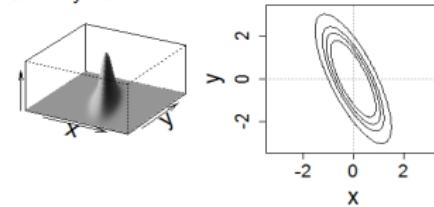
$$2\sigma_x = \sigma_y, \rho = 0.75$$



$$\sigma_x = \sigma_y, \rho = -0.75$$



$$2\sigma_x = \sigma_y, \rho = -0.75$$



Section 8

Case study

Case study: fertility and religion

The file `fertility.csv` contains the following variables

- Country - Country name
- Code - 3 letter ISO code of the country
- Life Expectancy at birth
- GDP_PPP GDP per capita (purchasing power parity)
- Population in millions
- Fertility rate - babies born per woman
- Region
- Main religion of the country (Christianity, Islam, Other)

Year 2015, source: <https://ourworldindata.org>

Case study: fertility and religion

```
fertility <- read.csv('Data/fertility.csv', stringsAsFactors = T)
head(fertility)

##           Country Code Year Life_expectancy GDP_PPP Population Fertility_rate
## 1 Afghanistan AFG 2015      63.377     1.9      34.4        4.80
## 2    Albania ALB 2015      78.025    10.9      2.9        1.71
## 3   Algeria DZA 2015      76.090    13.0      39.7        2.83
## 4    Angola AGO 2015      59.398     8.6      27.9        5.76
## 5 Argentina ARG 2015      76.068    19.3      43.1        2.30
## 6 Armenia ARM 2015      74.467     9.6      2.9        1.62

##           Region Religion
## 1          Asia    Islam
## 2        Europe    Islam
## 3       Africa    Islam
## 4       Africa Christianity
## 5 South America Christianity
## 6        Asia Christianity
```

Case study: fertility and religion

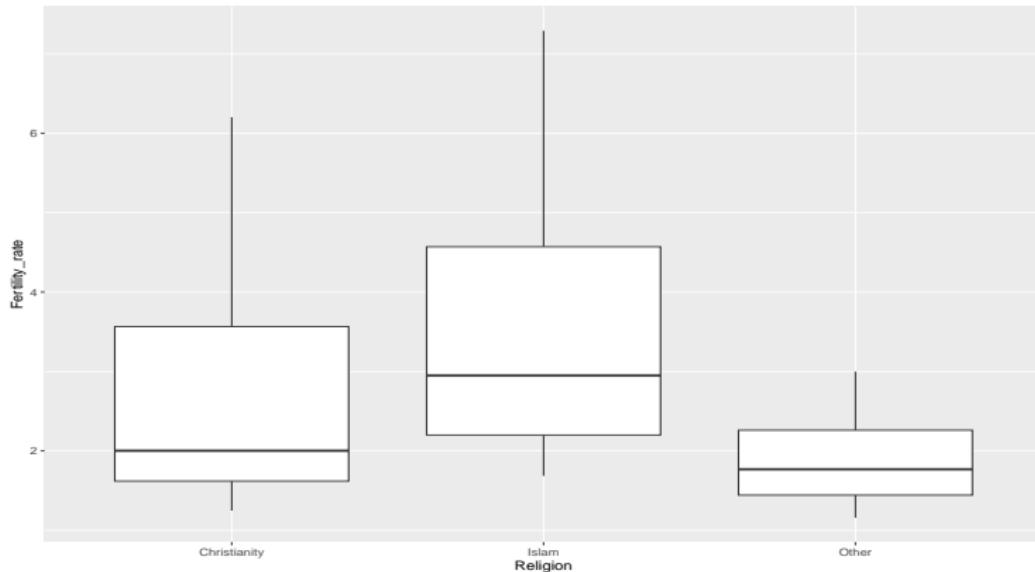
There is a long going myth that Islamic countries have a higher fertility rate than Christian countries.

Is the religion reason why some nations are having higher population growth?

Case study: fertility and religion

Fertility rate is higher for islamic nations

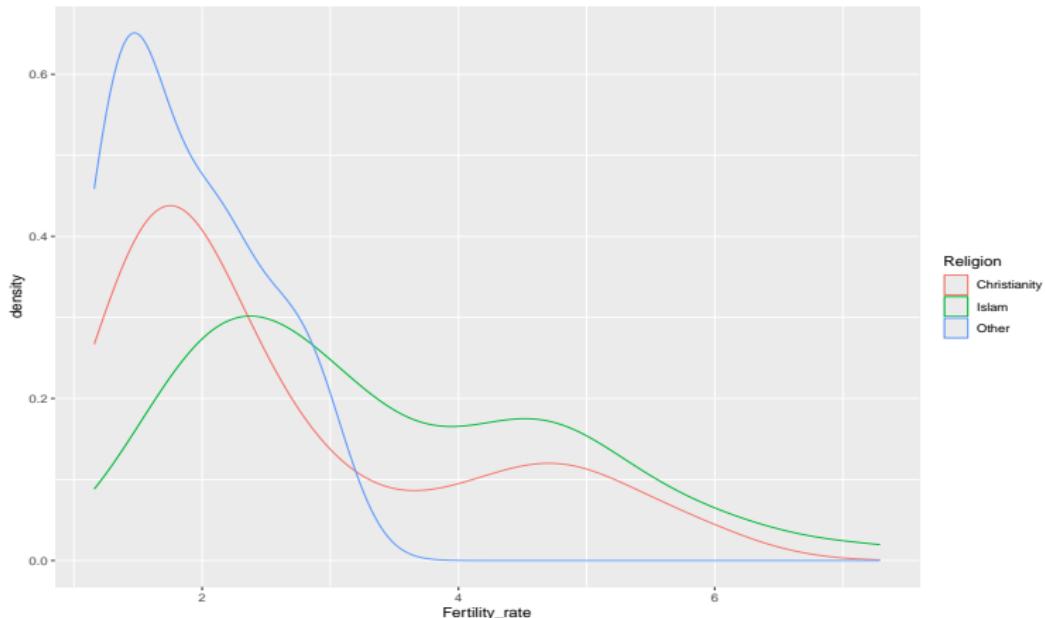
```
ggplot(fertility, aes(x = Religion, y = Fertility_rate)) +  
  geom_boxplot()
```



Case study: fertility and religion

Density plot shows the same picture

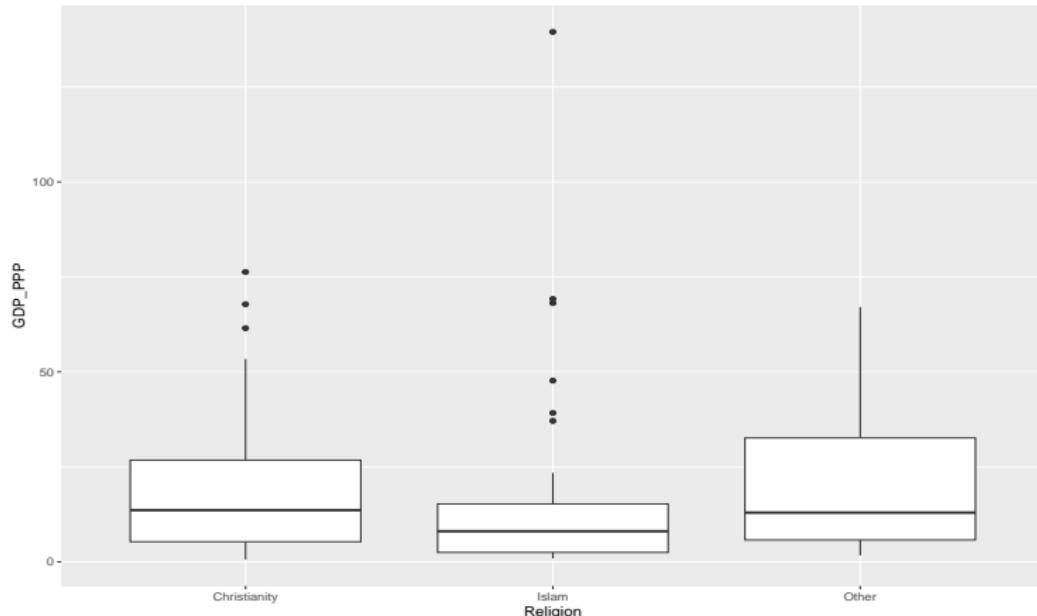
```
ggplot(fertility, aes(x = Fertility_rate, color = Religion)) +  
  geom_density()
```



Case study: fertility and religion

However, christian nations have higher per capita income

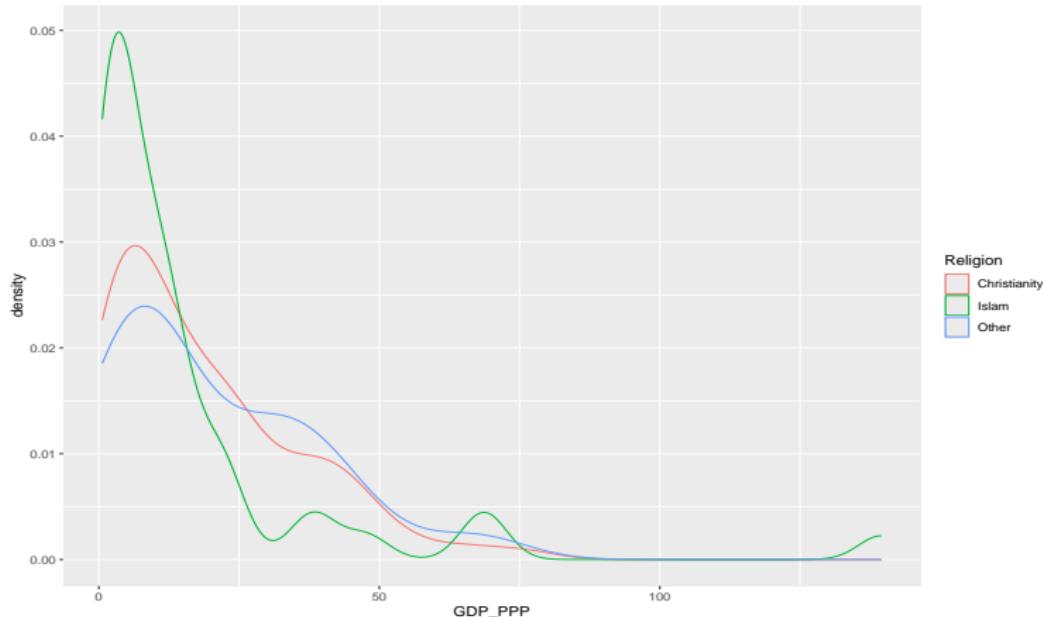
```
ggplot(fertility, aes(x = Religion, y = GDP_PPP)) +  
  geom_boxplot()
```



Case study: fertility and religion

The same picture with density plots

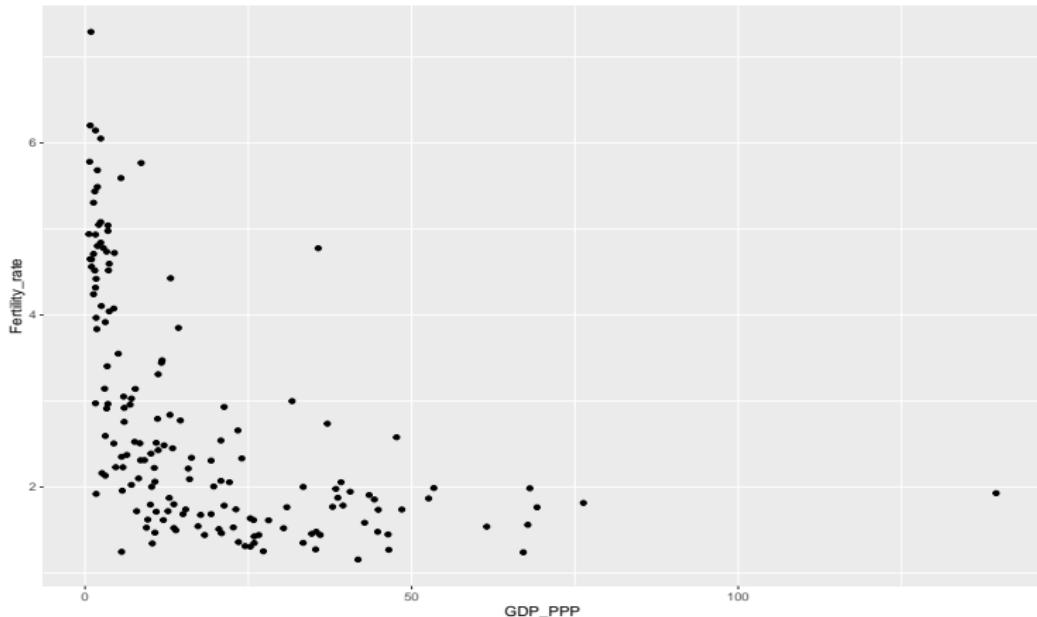
```
ggplot(fertility, aes(x = GDP_PPP, color = Religion)) +  
  geom_density()
```



Case study: fertility and religion

Relationship between GDP per capita and fertility rates

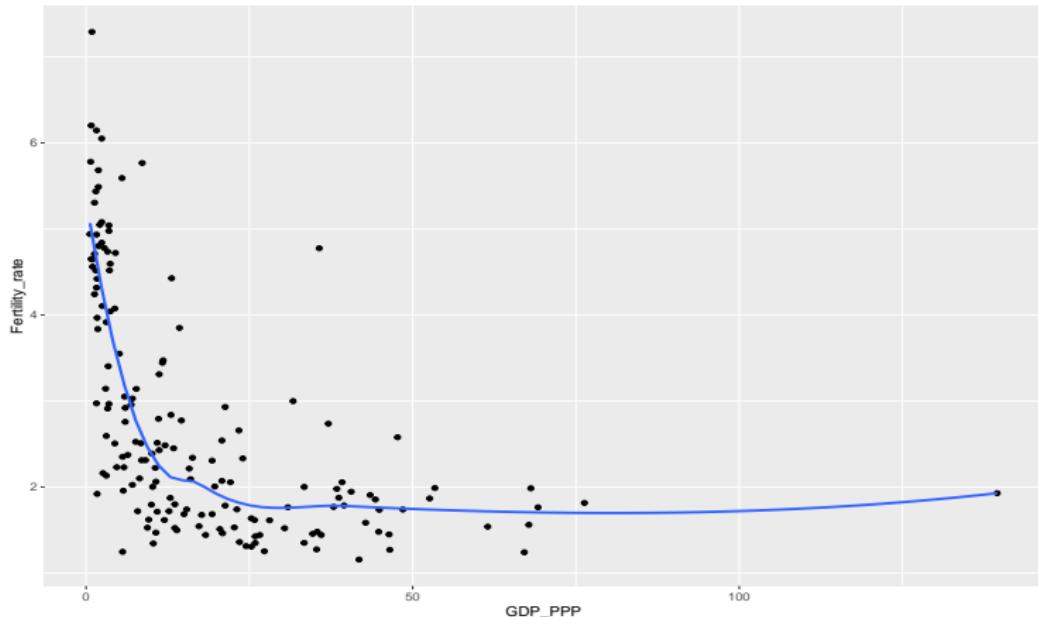
```
ggplot(fertility, aes(GDP_PPP, Fertility_rate)) +  
  geom_point()
```



Case study: fertility and religion

Adding loess smooth line

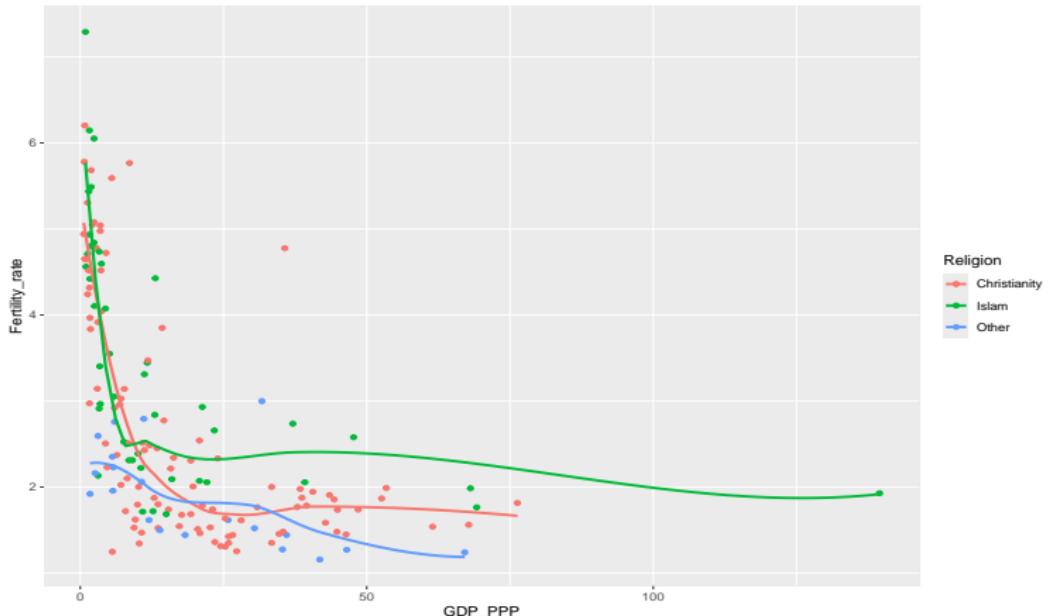
```
ggplot(fertility, aes(GDP_PPP, Fertility_rate)) +  
  geom_point() + geom_smooth(se=F)
```



Case study: fertility and religion

Relationship between fertility and GDP per capita for different religions

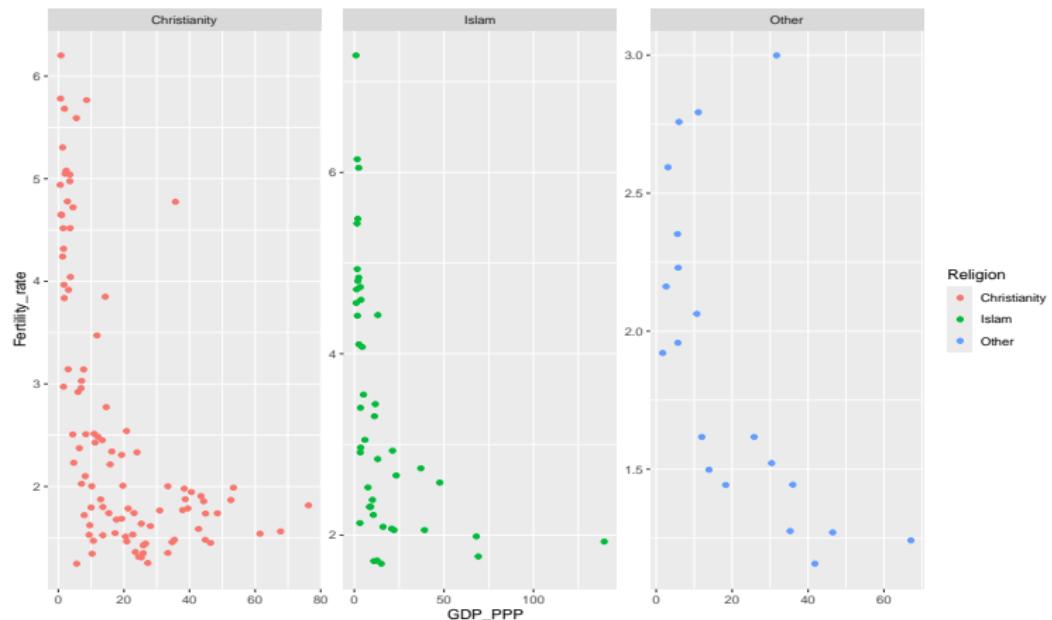
```
ggplot(fertility, aes(GDP_PPP, Fertility_rate, color = Religion)) +  
  geom_point() + geom_smooth(se=F)
```



Case study: fertility and religion

The same picture with facet_wrap()

```
ggplot(fertility, aes(GDP_PPP, Fertility_rate, color = Religion)) +  
  geom_point() + facet_wrap(~Religion, scales = 'free')
```

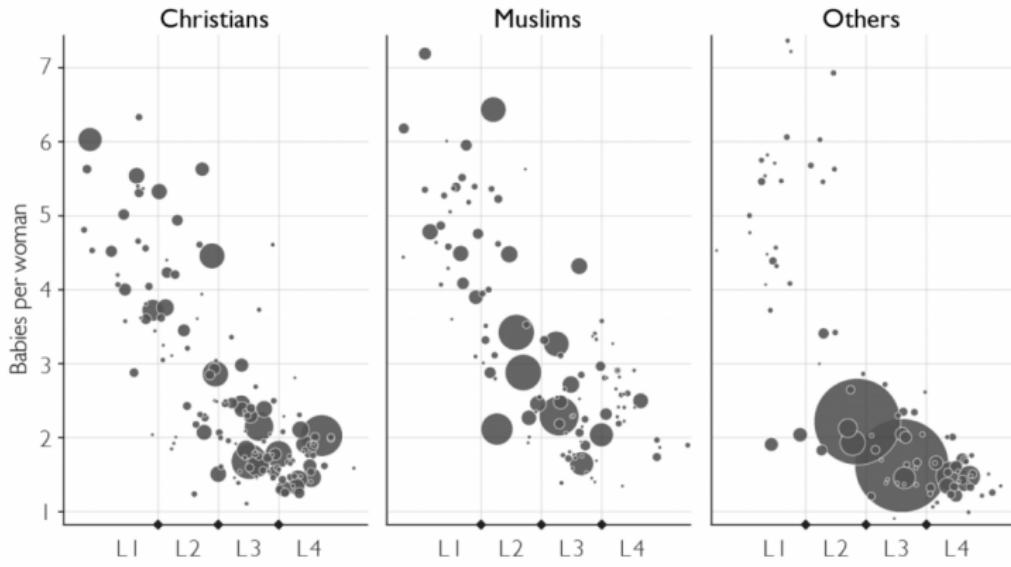


Case study: fertility and religion

- This is a famous example from Hans Rosling's book "**Factfulness: Ten Reasons We're Wrong About the World—and Why Things Are Better Than You Think**". Watch out his famous TED talk

WITH HIGHER INCOME COMES FEWER BABIES

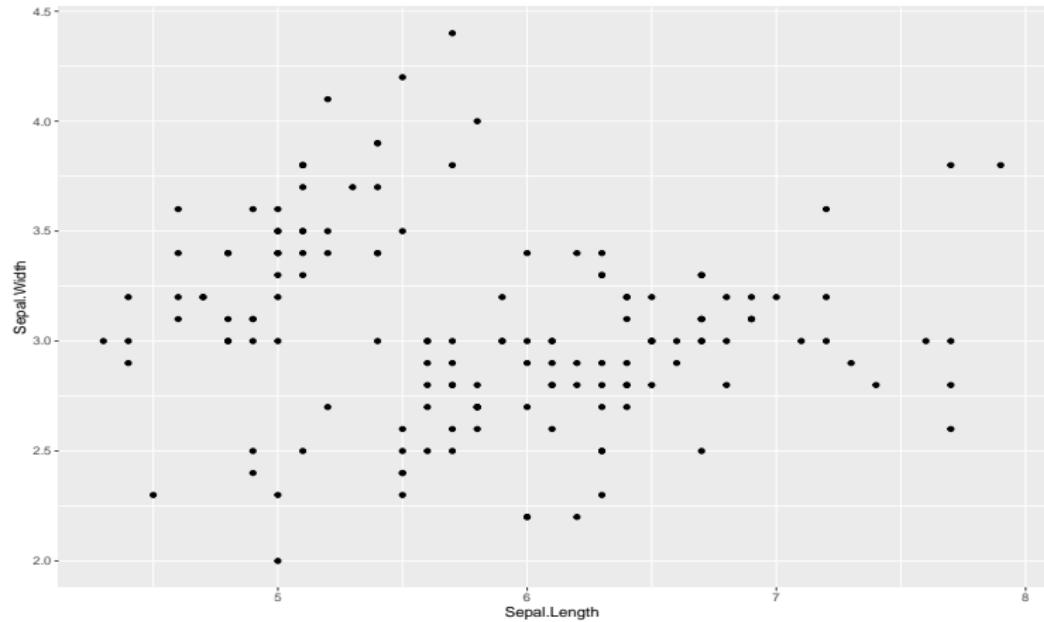
All countries split into religious groups, 2017. Bubble size shows number of people.



Case study: Group or not ?

What kind of relationship do you see?

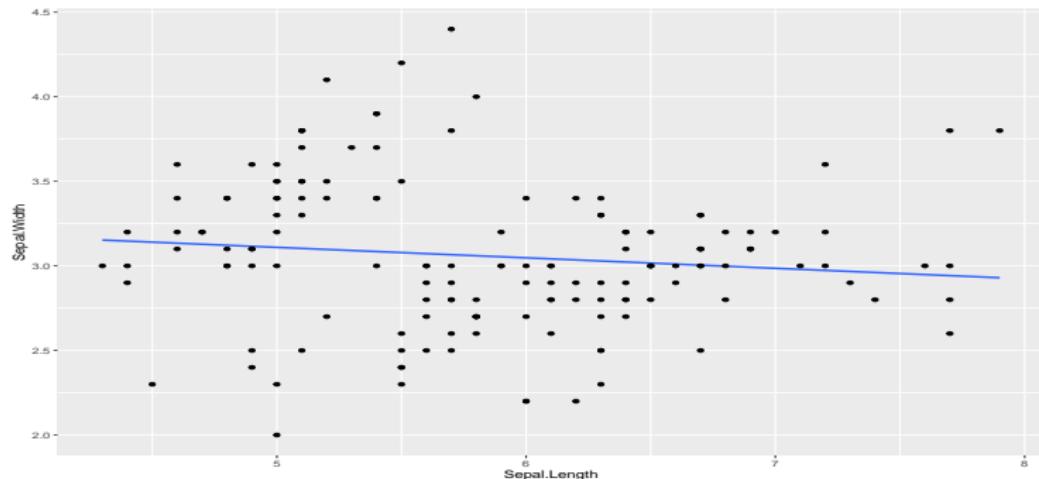
```
ggplot(iris, aes(x = Sepal.Length, y = Sepal.Width)) + geom_point()
```



Case study: Group or not ?

Weak negative relationship

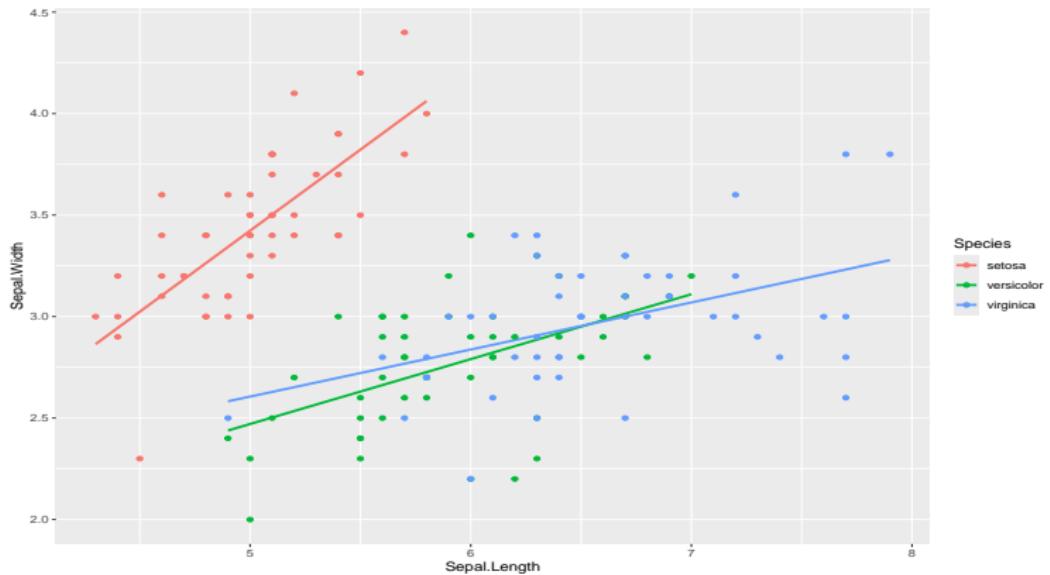
```
ggplot(iris, aes(x = Sepal.Length, y = Sepal.Width)) +  
  geom_smooth(method = "lm", se = FALSE) +  
  geom_point()
```



Case study: Group or not ?

Strong positive relationship

```
ggplot(iris, aes(x = Sepal.Length, y = Sepal.Width, color = Species)) +  
  geom_smooth(method = "lm", se = FALSE) +  
  geom_point()
```



Case study: Group or not ?

```
cor(iris[, c("Sepal.Length", "Sepal.Width")])  
## Sepal.Length Sepal.Width  
## Sepal.Length  1.0000000 -0.1175698  
## Sepal.Width   -0.1175698  1.0000000
```

Correlation by groups

```
iris %>%  
  group_by(Species) %>%  
  summarise(cor_sepal_length_width = cor(Sepal.Length, Sepal.Width))  
## # A tibble: 3 x 2  
##   Species      cor_sepal_length_width  
##   <fct>          <dbl>  
## 1 setosa         0.743  
## 2 versicolor     0.526  
## 3 virginica      0.457
```

Case study: Group or not ?

- Karl Pearson, one of the founders of modern statistics, had a thing for data collection.
- Once he collected 806 male skulls and 340 female skulls from the Paris Catacombs.
- He computed the correlation between skull length and skull breadth.



Case study: Group or not ?

We will replicate the data

- We will create two multivariate normal pair of variables.
- Define the means (for Length and Breadth) and covariance matrices

For males

```
mean <- c(188, 135)
cov <- matrix(c(6.33, 0.2, 0.2, 6.33), nrow = 2)
xy <- mvrnorm(800, mu = mean, Sigma = cov)
cor(xy[,1], xy[,2])
## [1] 0.07946103
```

Case study: Group or not ?

Create dataframe

```
skull <- data.frame(Length = xy[,1], Breadth = xy[,2])
skull$Sex <- 'Male'
head(skull)

##      Length Breadth Sex
## 1 190.3918 134.3107 Male
## 2 187.2291 138.2938 Male
## 3 191.7968 137.7464 Male
## 4 188.4713 132.0543 Male
## 5 186.4344 134.0172 Male
## 6 190.8282 131.9258 Male
```

Case study: Group or not ?

For females

```
mean <- c(177, 132)
cov <- matrix(c(8.44, 0.2, 0.2, 6.13), nrow = 2)
xy <- MASS::mvrnorm(300, mu = mean, Sigma = cov)
cor(xy[,1], xy[,2])
## [1] -0.02882048
```

Case study: Group or not ?

Combine into a dataframe

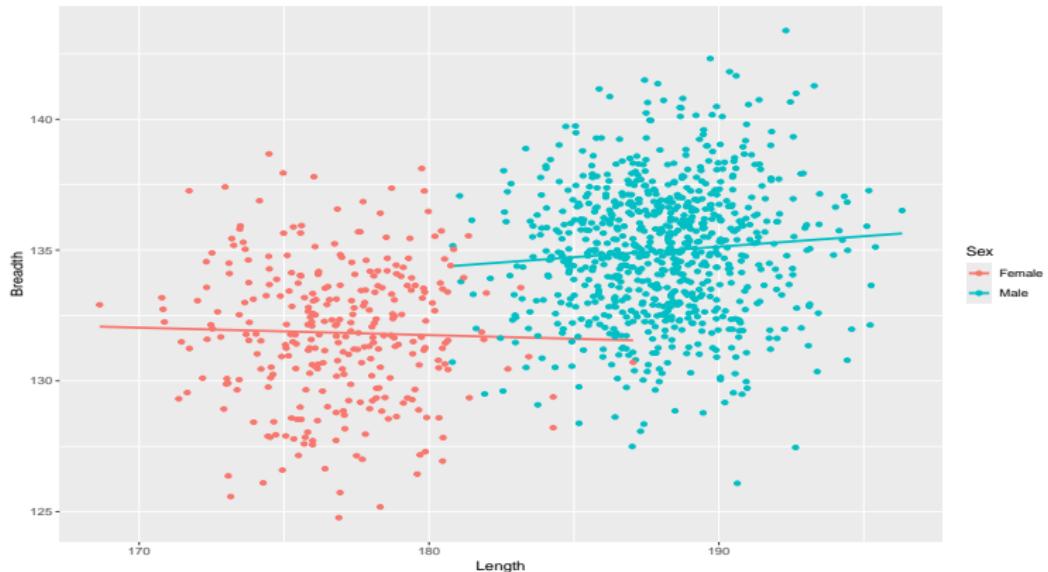
```
skull_female <- data.frame(Length = xy[,1], Breadth = xy[,2])
skull_female$Sex <- 'Female'

skull = rbind(skull, skull_female)
```

Case study: Group or not ?

Weak relationship when looked separately

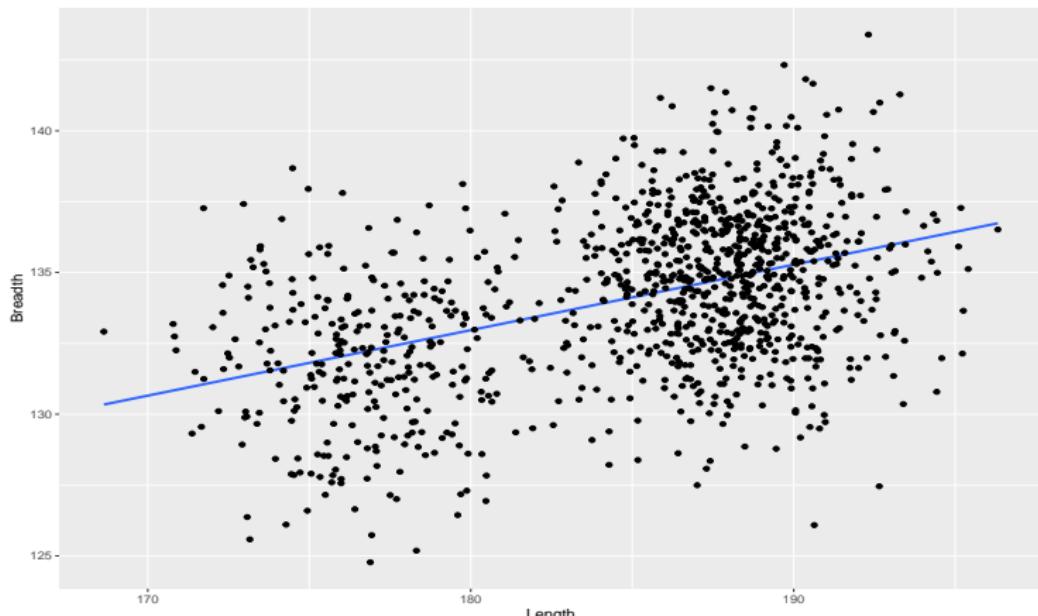
```
ggplot(skull, aes(x = Length, y = Breadth, color = Sex)) +  
  geom_smooth(method = "lm", se = FALSE) +  
  geom_point()
```



Case study: Group or not ?

Strong relationship when pulled together

```
ggplot(skull, aes(x = Length, y = Breadth)) +  
  geom_smooth(method = "lm", se = FALSE) +  
  geom_point()
```



Case study: Group or not ?

If you want to know more about when to pool the data and when not, look here -
Judea Pearl - The Book of Why: The New Science of Cause and Effect