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Short-term electricity demand forecasting using double seasonal exponential smoothing

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This paper considers univariate online electricity demand forecasting for lead times from a half-hour-ahead to a day-ahead. A time series of demand recorded at half-hourly intervals contains more than one seasonal pattern. A within-day seasonal cycle is apparent from the similarity of the demand profile from one day to the next, and a within-week seasonal cycle is evident when one compares the demand on the corresponding day of adjacent weeks. There is strong appeal in using a forecasting method that is able to capture both seasonalities. The multiplicative seasonal ARIMA model has been adapted for this purpose. In this paper, we adapt the Holt–Winters exponential smoothing formulation so that it can accommodate two seasonalities. We correct for residual autocorrelation using a simple autoregressive model. The forecasts produced by the new double seasonal Holt–Winters method outperform those from traditional Holt–Winters and from a well-specified multiplicative double seasonal ARIMA model.

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Introduction

Online electricity demand prediction is required for the control and scheduling of power systems. The forecasts are required for lead times from a minute-ahead to a day-ahead. At National Grid, which is responsible for the transmission of electricity in England and Wales, online prediction is based on half-hourly data. A profiling heuristic is used to produce forecasts for each minute by interpolating between each half-hourly prediction. The National Grid 1 hour-ahead forecasts are a key input to the balancing market, which operates on a rolling 1 hour-ahead basis to balance supply and demand after the closure of bilateral trading between generators and suppliers.

Weather is a key influence on the variation in electricity demand (see Taylor and Buizza^{1,2}). However, in a real-time online forecasting environment, multivariate modelling is usually considered impractical. A multivariate online system would be very demanding in terms of weather forecast input and would require default procedures in order to ensure robustness.³ Univariate methods are considered to be sufficient for the short lead times involved because the weather variables tend to change in a smooth fashion, which will be captured in the demand series itself.

In this paper, we consider online, univariate forecasting of half-hourly data. A time series of electricity demand recorded at half-hourly intervals contains more than one seasonal pattern. Figure 1 shows half-hourly demand in

England and Wales for a fortnight in June 2000. A within-day seasonal cycle, of duration 48 half-hour periods, is apparent from the similarity of the demand profile from one day to the next, particularly on weekdays. A within-week seasonal cycle, of duration 336 half-hour periods, is evident when one compares the demand on the corresponding day of adjacent weeks. There is strong appeal in using a forecasting method that is able to capture information in both seasonalities.

Holt–Winters exponential smoothing is a popular approach to forecasting seasonal time series. The robustness and accuracy of exponential smoothing methods has led to their widespread use in applications where a large number of series necessitates an automated procedure, such as inventory control. This suggests that Holt–Winters might be a reasonable candidate for the automated application of online electricity demand forecasting. However, the method is only able to accommodate one seasonal pattern. The multiplicative seasonal ARIMA model has been extended in order to model the within-day and within-week seasonalities in electricity demand. In this paper, we adapt the Holt–Winters method so that it can accommodate two seasonalities. This involves the introduction of an additional seasonal index and an extra smoothing equation for the new seasonal index.

In the next section, we describe how ARIMA models have been adapted for online electricity demand forecasting, in order to capture multiple seasonalities in the demand series. We then show how the Holt–Winters method can be adapted for series with more than one seasonality. The section that follows presents an empirical forecast

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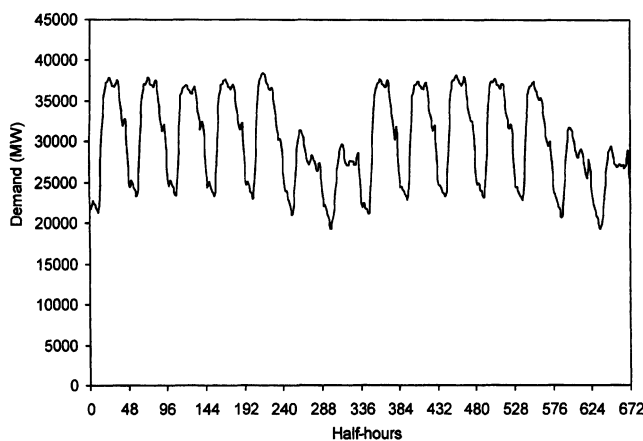


Figure 1 Half-hourly electricity demand in England and Wales from Monday 5 June 2000 to Sunday 18 June 2000.

comparison of the new formulation with the standard Holt–Winters method and with a multiplicative double seasonal ARIMA model. In the final section, we provide a summary and conclusion.

Multiplicative double seasonal ARIMA models

The literature on short-term load forecasting contains a variety of univariate methods that could be implemented in an online prediction system. The range of different approaches includes state-space methods with the Kalman filter (eg Infield and Hill⁴), general exponential smoothing (eg Christiaanse⁵), artificial neural networks (eg Lamedica *et al*⁶), spectral methods (eg Laing and Smith⁷) and seasonal ARIMA models (eg Laing and Smith⁷, and Darbellay and Slama⁸). The most noticeable development in demand forecasting over the last decade has been the increasing interest shown by researchers and practitioners in artificial neural networks (see Hippert *et al*⁹). Although there is obvious appeal to using this modelling approach to find the nonlinear relationship between demand and weather variables, its appeal for univariate modelling is far less clear. The one short-term forecasting method that has remained popular over the years, and appears in many papers as a benchmark approach, is multiplicative seasonal ARIMA modelling.

The multiplicative seasonal ARIMA model, for a series, X_t , with just one seasonal pattern can be written as

$$\phi_p(L)\Phi_P(L^s)\nabla^d\nabla_s^D X_t = \theta_q(L)\Theta_Q(L^s)\varepsilon_t$$

where L is the lag operator, s is the number of periods in a seasonal cycle, ∇ is the difference operator, $(1-L)$, ∇_s is the seasonal difference operator, $(1-L^s)$, d and D are the orders of differencing, ε_t is a white noise error term, and ϕ_p , Φ_P , θ_q and Θ_Q are polynomial functions of orders p , P , q , and Q , respectively. The model is often expressed as $ARIMA(p,d,q) \times (P,D,Q)_s$. It is multiplicative in the sense that

the polynomial functions of L and L^s are multiplied on each side of the equation to give a rich function of the lag operator. Box *et al*¹⁰ (p 333) comment that the model can be extended for the case of multiple seasonalities. The multiplicative double seasonal ARIMA model can be written as

$$\begin{aligned} \phi_p(L)\Phi_{P_1}(L^{s_1})\Omega_{P_2}(L^{s_2})\nabla^d\nabla_{s_1}^{D_1}\nabla_{s_2}^{D_2}X_t \\ = \theta_q(L)\Theta_{Q_1}(L^{s_1})\Psi_{Q_2}(L^{s_2})\varepsilon_t \end{aligned} \quad (1)$$

where s_1 and s_2 are the number of periods in the different seasonal cycles, and Ω_{P_2} and Ψ_{Q_2} are polynomial functions of orders P_2 and Q_2 , respectively. This model can be expressed as $ARIMA(p,d,q) \times (P_1,D_1,Q_1)_{s_1} \times (P_2,D_2,Q_2)_{s_2}$. Applying the model to half-hourly electricity demand, Laing and Smith⁷ set $s_1 = 48$ to model the within-day seasonal cycle of 48 half-hours, and $s_2 = 336$ to model the within-week cycle of 336 half-hours. The forecasts from ARIMA models of this type are currently used at National Grid. In an application to hourly demand in the Czech Republic, Darbellay and Slama⁸ set $s_1 = 24$ to model the within-day seasonal cycle, and $s_2 = 168$ to model the within-week cycle.

The multiplicative seasonal ARIMA model can easily be extended to take care of three or more seasonalities by the introduction of additional polynomial functions of the lag operator and additional difference operators in expression (1). Therefore, the annual seasonal pattern in electricity demand could also be modelled. However, it is usual to assume that it is not significant in the context of lead times up to a day-ahead.⁷

In this section, we have shown how the multiplicative double seasonal ARIMA model is a straightforward extension of the standard multiplicative seasonal model. Motivated by this, and by the fact that exponential smoothing has been a competitive alternative to ARIMA models with a variety of different types of data,¹¹ in the next section, we adapt the standard Holt–Winters method for application to series with two seasonalities.

Double seasonal Holt–Winters exponential smoothing

Standard Holt–Winters

The standard Holt–Winters method was introduced by Winters¹² and is suitable for series with one seasonal pattern. The multiplicative seasonality version of the method is presented in expressions (2)–(5). It assumes an additive trend and estimates the local slope, T_t , by smoothing successive differences, $(S_t - S_{t-1})$, of the local level, S_t . The local s -period seasonal index, I_t , is estimated by smoothing the ratio of observed value, X_t , to local level, S_t .

$$\text{Level} \quad S_t = \alpha(X_t/I_{t-s}) + (1 - \alpha)(S_{t-1} + T_{t-1}) \quad (2)$$

$$\text{Trend} \quad T_t = \gamma(S_t - S_{t-1}) + (1 - \gamma)T_{t-1} \quad (3)$$

$$\text{Seasonality} \quad I_t = \delta(X_t/S_t) + (1 - \delta)I_{t-s} \quad (4)$$

$$\hat{X}_t(k) = (S_t + kT_t)I_{t-s+k} \quad (5)$$

where α , γ and δ are the smoothing parameters, and $\hat{X}_t(k)$ is the k -step-ahead forecast. The seasonality is multiplicative in the sense that the underlying level of the series is multiplied by the seasonal index. Holt–Winters for additive seasonality is an alternative formulation, which involves the addition of seasonal factors to the underlying trend. The multiplicative version is appropriate if the magnitude of the seasonal variation increases with an increase in the mean level of the series, while the additive version should be used if the seasonal effect does not depend on the current mean level. The multiplicative version is much more widely used and so for simplicity, in this paper, we provide only the multiplicative formulation.

It is worth noting that the use of the word “multiplicative” in the context of seasonal ARIMA models is quite different from its use in Holt–Winters exponential smoothing. In contrast with Holt–Winters for multiplicative seasonality, the seasonal effect for multiplicative seasonal ARIMA models does not depend on the mean level of the series. There is no equivalence between Holt–Winters for multiplicative seasonality and multiplicative seasonal ARIMA models. This point is, perhaps, emphasised by the fact that although there is an ARIMA model for which Holt–Winters for additive seasonality is optimal,¹³ there is no ARIMA model for which Holt–Winters for multiplicative seasonality is optimal.¹⁴

Double seasonal Holt–Winters

Although standard Holt–Winters is widely used for forecasting seasonal time series, the method is only able to accommodate one seasonal pattern. A formulation that can accommodate more than one seasonal pattern has not been considered in the exponential smoothing literature. This is evident from the recent taxonomies of Hyndman *et al.*¹⁵ and Taylor.¹⁶ The Holt–Winters method for double multiplicative seasonality is given in expressions (6)–(10). The method is suitable when there are two seasonal patterns in the time series. The formulation involves separate seasonal indices, D_t and W_t , for the two seasonalities. The local s_1 -period seasonal index, D_t , is estimated by smoothing the ratio of observed value, X_t , to the product of the local level, S_t , and local s_2 -period seasonal index, W_{t-s_2} . Similarly, the local s_2 -period seasonal index, W_t , is estimated by smoothing the ratio of observed value, X_t , to the product of the local level, S_t , and local s_1 -period seasonal index, D_{t-s_1} .

$$\text{Level} \quad S_t = \alpha(X_t / (D_{t-s_1} W_{t-s_2})) + (1 - \alpha)(S_{t-1} + T_{t-1}) \quad (6)$$

$$\text{Trend} \quad T_t = \gamma(S_t - S_{t-1}) + (1 - \gamma)T_{t-1} \quad (7)$$

$$\text{Seasonality 1} \quad D_t = \delta(X_t / (S_t W_{t-s_2})) + (1 - \delta)D_{t-s_1} \quad (8)$$

$$\text{Seasonality 2} \quad W_t = \omega(X_t / (S_t D_{t-s_1})) + (1 - \omega)W_{t-s_2} \quad (9)$$

$$\hat{X}_t(k) = (S_t + kT_t)D_{t-s_1+k}W_{t-s_2+k} \quad (10)$$

where α , γ , δ and ω are the smoothing parameters. Applying the method to a series of half-hourly demand, one would set $s_1 = 48$ and $s_2 = 336$, as in the multiplicative double seasonal ARIMA model of Laing and Smith⁷. D_t and W_t would then represent the within-day and within-week seasonalities, respectively. A double additive seasonality method can be developed in a similar way from the standard Holt–Winters method for additive seasonality. The formulation in expressions (6)–(10) can easily be extended for three or more seasonal patterns by introducing an extra seasonal index and smoothing equation for each additional seasonality.

Empirical comparison of methods

Empirical analysis in order to address two main issues is carried out. Firstly, whether the new double seasonal Holt–Winters method offers an improvement on the standard Holt–Winters method in terms of forecast accuracy is investigated. Secondly, forecasting performance of the new formulation is compared with a well-specified multiplicative double seasonal ARIMA model.

The data used were 12 weeks of half-hourly electricity demand in England and Wales from Monday 5 June 2000 to Sunday 27 August 2000. It is shown in Figure 2. We used the first 8 weeks of data to estimate method parameters and the remaining 4 weeks to evaluate post-sample forecasting performance. This amounts to 2688 half-hourly observations for estimation and 1344 for evaluation. To simplify our comparison of methods, we chose a period that did not contain any ‘special’ days, such as national holidays. Demand on these days is so very unlike the rest of the year that online univariate methods are generally unable to produce reasonable forecasts. In practice, interactive facilities tend to be used for special days, which allow operator experience to supplement or override the system offline. If a forecasting method is unable to tolerate gaps in the historical series, the special days can be smoothed over, leaving the natural periodicities of the data intact.⁷

Multiplicative double seasonal ARIMA

The process of model identification is impractical in an online demand forecasting system, and so the model is chosen offline. We used the Box–Jenkins modelling methodology to identify the most suitable ARIMA model based on the 2688 observations in the estimation sample. The autocorrelation function and partial autocorrelation function were used to select the order of the model, which was then estimated by maximum likelihood. The residuals were inspected for any remaining autocorrelation. Laing and Smith⁷ explain that, in the multiplicative double seasonal ARIMA formulation in expression (1), polynomials of order greater than two are rarely necessary when fitting a model to half-hourly data for England and Wales. In view of this, we considered polynomials up to order two, but we also

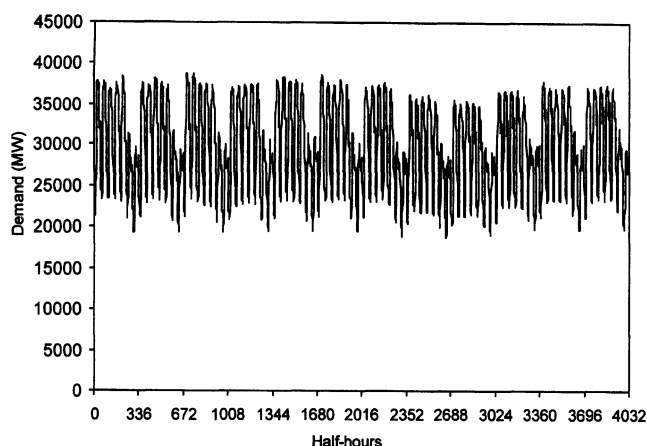


Figure 2 Half-hourly electricity demand in England and Wales from Monday 5 June 2000 to Sunday 27 August 2000.

checked the autocorrelation function of the residuals for any remaining higher order autocorrelation. We compared the Schwarz Bayesian criterion (SBC) for an extensive range of different ARIMA models. We investigated differencing and a logarithmic transformation for demand but found neither to improve the SBC. The model with lowest SBC and satisfactory residuals was the following $ARIMA(2,0,0) \times (2,0,2)_{48} \times (2,0,2)_{336}$ model, which we shall refer to as the *Double seasonal ARIMA* model:

$$\begin{aligned} & (1 - 1.02L + 0.08L^2)(1 - 0.32L^{48} - 0.31L^{96}) \\ & \times (1 - 0.52L^{336} - 0.44L^{672})(X_t - 21625) \\ & = (1 - 0.16L^{96})(1 - 0.15L^{336} - 0.19L^{672})e_t \end{aligned}$$

Holt–Winters exponential smoothing

We produced forecasts using the following three Holt–Winters methods:

Holt–Winters for within-day seasonality — This is standard Holt–Winters for multiplicative seasonality, given in expressions (2)–(5), using only a 48-period seasonal cycle.

Holt–Winters for within-week seasonality — This is standard Holt–Winters for multiplicative seasonality, given in expressions (2)–(5), using only a 336-period seasonal cycle.

Double seasonal Holt–Winters — This is the new Holt–Winters for double multiplicative seasonality, given in expressions (6)–(10), using both a 48-period cycle for the within-day seasonality and a 336-period seasonal cycle for the within-week seasonality.

Williams and Miller¹⁷ use simple averages of the first few data observations to calculate initial smoothed values for the level, trend and seasonal components in the standard Holt–Winters method. We implemented their procedure for Holt–Winters for within-day seasonality and Holt–Winters for within-week seasonality. We adapted the procedure for double seasonal Holt–Winters. The initial trend, T_0 , was chosen as the average of (1) $1/336$ of the difference between

the mean of the first 336 and second 336 observations and (2) the average of the first differences for the first 336 observations. The initial level, S_0 , was chosen as the mean of the first 672 observations minus 336.5 times the initial trend. The initial values for the within-day seasonal index, D_t , were set as the average of the ratios of actual observation to 48-point centred moving average, taken from the corresponding half-hour period in each of the first 7 days of the time series. The initial values for the within-week seasonal index, W_t , were set as the average of the ratios of actual observation to 336-point centred moving average, taken from the corresponding half-hour period on the same day of the week in each of the first 2 weeks of the demand series, divided by the corresponding initial value of the smoothed within-day seasonal index, D_t .

We derived parameter values by the common procedure of minimising the sum of squared 1-step-ahead forecast errors using a nonlinear optimisation routine. The estimated parameters are shown in Table 1. The zero values for γ in Holt–Winters for within-week seasonality and double seasonal Holt–Winters were accompanied by a constant small value for the smoothed trend, T_t . This seems reasonable since variation in the 8-week estimation period is dominated by seasonality. The high value for γ in Holt–Winters for within-day seasonality was accompanied by highly varying, and quite sizeable, values for the smoothed trend. Since the method is unable to pick up the weekly seasonality in the data, it has incorporated this variability in its estimate of the trend.

Figure 3 compares the post-sample forecasting accuracy of the three Holt–Winters methods and the ARIMA model for lead times up to a day-ahead. The figure shows the mean absolute percentage error (MAPE), which is the most widely used error summary measure in electricity demand forecasting. Following the recommendation of Hippert *et al.*,⁹ we also calculated the mean absolute error, root-mean-square error, and root-mean-square percentage error, but we do not report these results here because the relative performances of the methods for these measures were very similar to those for the MAPE. The results for Holt–Winters for within-day seasonality were so poor that it was impractical to plot the MAPE values beyond 2 steps ahead on the same graph as the MAPE values for the other methods. This is due to the method failing to accommodate within-week seasonality. This might have been anticipated from Figure 1, which shows how very different the demand on Saturdays and Sundays is from that on weekdays. Holt–Winters for within-week seasonality is far more competitive, suggesting that the within-week seasonality accounts for a large proportion of the variation in the data. However, double seasonal Holt–Winters outperforms Holt–Winters for within-week seasonality for 38 of the 48 lead times, indicating that there is benefit in using a method that is able to pick up both seasonalities. Beyond 12 hours ahead, the accuracy of these two methods tends to improve with the lead time. This is due

Table 1 Holt–Winters parameters calculated from the 8-week estimation sample

| | Level α | Trend γ | Within-day seasonality δ | Within-week seasonality ω |
|------------------------------------------|----------------|----------------|---------------------------------|----------------------------------|
| Holt–Winters for within-day seasonality | 0.98 | 0.90 | 1.00 | — |
| Holt–Winters for within-week seasonality | 0.82 | 0.00 | — | 1.00 |
| Double seasonal Holt–Winters | 0.90 | 0.00 | 1.00 | 1.00 |

No adjustment for autocorrelation in the residuals of the methods.

Table 2 Holt–Winters parameters calculated from the 8-week estimation sample

| | Level α | Trend γ | Within-day seasonality δ | Within-week seasonality ω | AR λ |
|------------------------------------------|----------------|----------------|---------------------------------|----------------------------------|--------------|
| Holt–Winters for within-day seasonality | 0.92 | 0.00 | 1.00 | — | 0.71 |
| Holt–Winters for within-week seasonality | 0.02 | 0.00 | — | 0.41 | 0.91 |
| Double seasonal Holt–Winters | 0.01 | 0.00 | 0.21 | 0.24 | 0.92 |

The methods included an AR(1) model for the residuals.

to the within-day seasonality, and it implies that a forecast for 12h ahead would be better made from a forecast origin 12h prior to the current period. In his analysis of online methods, Smith¹⁸ also concludes that the choice of forecast origin should depend on the forecast horizon. Comparing the two double seasonal methods, we see that double seasonal ARIMA outperforms double seasonal Holt–Winters for all but the last 5 lead times.

Adjusting for error autocorrelation in the Holt–Winters methods

Inspection of the 1-step-ahead errors, in the estimation sample of 2688 periods, revealed sizeable first-order autocorrelation for all three Holt–Winters methods, indicating that the forecasts were suboptimal. Gardner¹⁹ reports how the forecasts from exponential smoothing methods can sometimes be improved by using a simple adjustment, initially proposed by Reid²⁰ and Gilchrist²¹ (pp 202–203). The adjustment involves an AR(1) model, $e_t = \lambda e_{t-1} + \xi_t$, being fitted to the 1-step-ahead errors, e_t . The k -step-ahead forecasts from forecast origin τ are then modified by adding the term $\lambda^k e_\tau$. Chatfield²² found that the modification resulted in improvements in accuracy when applied to the autocorrelated errors from Holt–Winters for multiplicative seasonality. Using just the estimation sample, we fitted AR(1) models to the residuals from each of the three Holt–Winters methods described in the previous section. This led to improved post-sample results for all three methods at the very early lead times.

Estimating the parameters of a Holt–Winters method and then fitting a model to the residuals is a two-stage estimation approach. Chatfield²² suggests that it may be more efficient to estimate all of the parameters for a method in a single stage. We did this for each of the three Holt–Winters

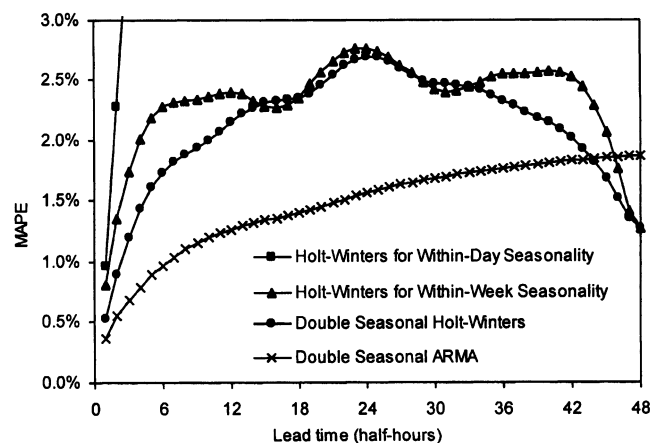


Figure 3 Comparison of MAPE results for the 4-week post-sample period. No adjustment for autocorrelation in the residuals of the Holt–Winters methods.

methods by minimising the sum of squared 1-step-ahead errors from the estimation sample. This led to far greater improvements in post-sample accuracy than were found using the two-stage estimation approach. Before presenting the post-sample MAPE results, let us first consider the estimated parameters resulting from the single-stage estimation approach. These are shown in Table 2. The parameters are noticeably different from those shown in Table 1 for the same methods without residual autocorrelation adjustment. The introduction of the AR(1) model for the residuals has caused a sizeable reduction in the smoothing parameter for the level in Holt–Winters for within-week seasonality and double seasonal Holt–Winters. It would seem that the introduction of the model for the residuals has, to a large degree, replaced the smoothing equation for the level. Incidentally, we did not fit a model to the residuals of the

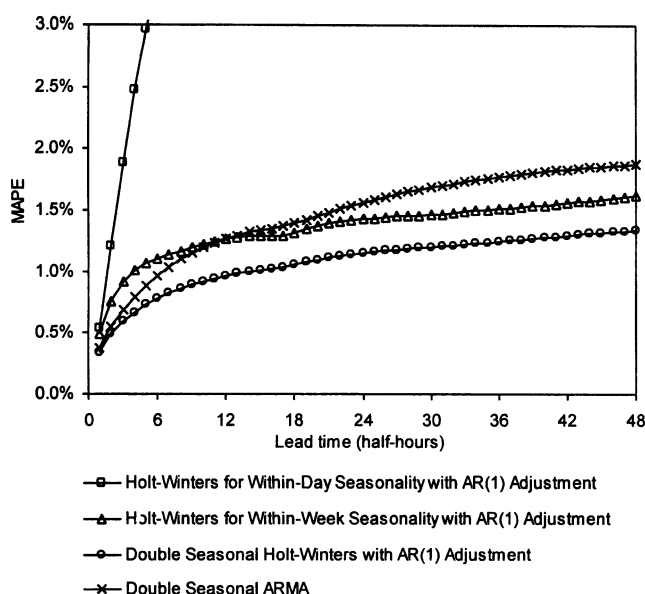


Figure 4 Comparison of *MAPE* results for the 4-week post-sample period. The Holt–Winters methods included an AR(1) model for the residuals.

double seasonal ARIMA model because it was estimated after careful diagnostic evaluation, and so there was no autocorrelation in its residuals.

Figure 4 shows the post-sample forecasting performance for the three Holt–Winters methods with residual autocorrelation adjustment. The *MAPE* results for double seasonal ARIMA, which were plotted in Figure 3, are also shown in Figure 4. The new results for all three Holt–Winters methods have improved substantially from Figure 3. The relative performance of the three methods has not changed, but double seasonal Holt–Winters is now the best of the three for all 48 lead times. Interestingly, the method now also outperforms double seasonal ARIMA for all the lead times. Beyond 12-periods ahead, the ARIMA model is also outperformed by Holt–Winters for within-week seasonality.

Intuitively, it is not surprising that, for the electricity demand data, the new double seasonal Holt–Winters method was more accurate than the two implementations of the standard Holt–Winters method. Application of the standard method was relatively naïve because it is unable to accommodate more than one seasonality. Nevertheless, it is pleasing to find that the empirical results support intuition. It is less clear why the new method with residual autocorrelation adjustment outperforms the ARIMA model. It cannot simply be due to the multiplicative nature of the double seasonal Holt–Winters method because an additive version of the method performed similarly. A possible explanation is provided by the comments of Chatfield²³ and Chatfield and Yar²⁴ in their consideration of the choice between ARIMA modelling and exponential smoothing. They explain that ARIMA modelling is worth considering if the series is dominated by short-term correlation but not

when it is dominated by trend and seasonal variation. Since electricity demand, recorded at half-hourly or hourly intervals, is dominated by seasonal variation, it follows that Holt–Winters formulations should perform well in comparison with ARIMA models.

Summary and conclusions

Online short-term electricity demand forecasting requires a robust, univariate procedure. Inspection of a time series of half-hourly demand reveals a within-day seasonality and a within-week seasonality. A popular approach is to use a multiplicative double seasonal ARIMA model. The robustness of exponential smoothing methods suggests that Holt–Winters would be a reasonable candidate for online short-term demand forecasting. However, the method is only able to accommodate one seasonal pattern. In this paper, we have shown how the method can be adapted for time series with two seasonalities. This involves the introduction of an additional seasonal index and an extra smoothing equation for this new seasonal index.

Using a series of half-hourly electricity demand, the new formulation outperformed standard Holt–Winters for forecast lead times from a half-hour-ahead to a day-ahead. The Holt–Winters methods were improved by the inclusion of an AR(1) model for the residuals. The best results were achieved by estimating the AR(1) model parameter in the same estimation procedure as the exponential smoothing parameters. The resulting forecasts for the new double seasonal Holt–Winters method outperformed those from standard Holt–Winters and also those from a well-specified multiplicative double seasonal ARIMA model. We, therefore, conclude that there is strong potential for the use of the new double seasonal Holt–Winters formulation in online short-term electricity demand forecasting. However, rather than recommending the new method in preference to all others, we feel that a more useful approach would be to use several different methods. Smith¹⁸ discusses combining online electricity demand forecasts from different methods with weights varying according to the particular period of the week and the forecast origin. Taylor and Majithia²⁵ describe how smooth transition combining methods can be used to enable a smooth form of switching between different demand forecasting methods.

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