Group 5: Euler's Method

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Description of Euler's Method

Euler's Method is a numerical method used in numerical analysis for solving ordinary differential equations (ODEs) with a given initial value. It is named after the mathematician Leonhard Euler who developed the method in the 18th century.
The basic idea of Euler's method is to approximate the solution to the ODE by using the tangent line at the initial point to make a local linear approximation of the solution. Then, the solution at a small time increment from the initial point is estimated by following the tangent line for that increment.
The points are computed iteratively as follows:

$$y_{n+1} = y_n + h * f(x_n, y_n)$$

where h is the time step size and $x_{n+1} = x_n + h$.

Step by Step procedure

1. Given an initial value problem of the form $y' = f(x, y), y(x_0) = y_0,$ choose a time step size h.
2. Set $x = x_0$ and $y = y_0 as$ the initial values.
3. Use Euler's method to compute the values y_{n+1} for $n = 0, 1, \dots, n - 1$:

$$\begin{aligned} x_{n+1} &= x_n + h \\ y_{n+1} &= y_n + h * f'(x_n, y_n) \end{aligned}$$

4. Repeat step 3 until you reach the desired endpoint or number of time steps.
5. Plot the graph to visualize the solution

Solved example

Question: Given that $y = x^2$ and the initial points are p(1, 1) with step-size $h = 0.2$. Obtain $y(2.0)$

Solution:
$n = (y_n - y_0)$divide by$h$
$n = (2.0 - 1)/5$ therefore: $n = 5$.The values of x_n and y_n will be:

n	x_n	y_n	Accurate
0	1	1	1
1	1.2	1.48	1.44
2	1.4	2.04	1.96
3	1.6	2.68	2.56
4	1.8	3.40	3.24
5	2.0	4.16	4.00

Table: Values of x_n and y_n

Python Code I

Here's an example implementation of Euler's method in Python:

```
import matplotlib.pyplot as plt


def f(x):
    return x ** 2


def df(x):
    return 2 * x


def euler_method(f_prime, x0, y0, h, num_steps):
    x = [x0]
    y = [y0]
```

Continuation ...

```
for i in range(num_steps):
    x_i = x[i]  # Get the current time
    y_i = y[i]  # Get the current solution value
    y_next = y_i + h * f_prime(x_i)

    x_next = x_i + h

    x.append(x_next)
    y.append(y_next)

return x, y
```

Continuation ...

```
x_initial = 1
y_initial = 1
size_step = 0.2
n = 5

x_values, y_values = euler_method(df, x_initial, y_initial,
size_step, n)
y_euler = y_values[-1]
y_accurate = f(x_values[-1])

for pt in range(len(x_values)):
    print(f"Value at y({x_values[pt]}) = {y_values[pt]}")

print()
print(f"Error = {y_accurate-y_euler}")
```

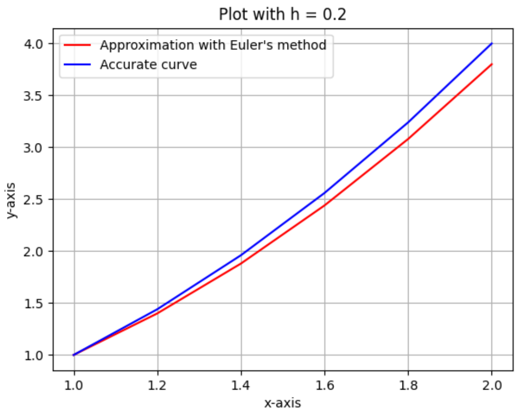
Continuation ...

```
x_real = [1, 1.1, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9]
y_real = [num ** 2 for num in x_values]

plt.plot(x_values, y_values, color='red',
label="Approximation with Euler's method")

plt.plot(x_values, y_real, color='blue',
label='Accurate curve')

plt.legend()
plt.xlabel('x-axis')
plt.ylabel('y-axis')
plt.grid()
plt.show()
```

Output of the code with $h = 0.2$

```
Value at y(1) = 1
Value at y(1.2) = 1.4
Value at y(1.4) = 1.88
Value at y(1.5999999999999999) = 2.44
Value at y(1.7999999999999998) = 3.08
Value at y(1.9999999999999998) = 3.8

Error = 0.1999999999999993
```

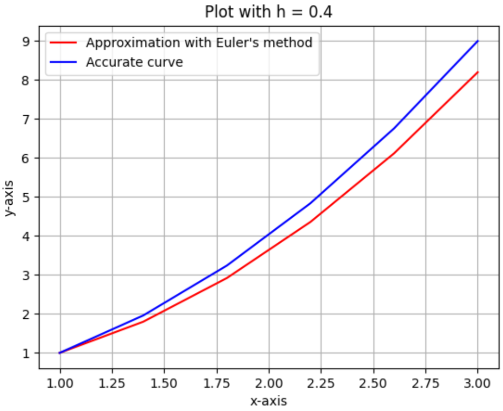
Euler's Plot with $h = 0.2$



Output of the code when $h = 0.4$

```
Value at y(1) = 1
Value at y(1.4) = 1.8
Value at y(1.7999999999999998) = 2.92
Value at y(2.1999999999999997) = 4.359999999999999
Value at y(2.5999999999999996) = 6.119999999999999
Value at y(2.9999999999999996) = 8.2

Error = 0.7999999999999989
```

Euler's Plot with $h = 0.4$



Conclusion

Euler's method is a **simple** and **straightforward** algorithm for approximating solutions to ordinary differential equations (ODEs). However:

Euler's method is a **first-order method**, meaning that the error in the approximation is **proportional to the step size h used in the algorithm.** As a result, small step sizes are required to achieve high accuracy, which can make the computation time-consuming and impractical for large-scale problems, also it can **only be applied directly to first-order ODEs.** For higher-order ODEs, the method must be modified by transforming the higher order ODE into a system of first-order ODEs which can make the computation more complex and less efficient.Therefore, more advanced numerical methods such as the **Runge-Kutta methods** or the **Adams-Bashforth methods** are typically used