Distributions: Gamma

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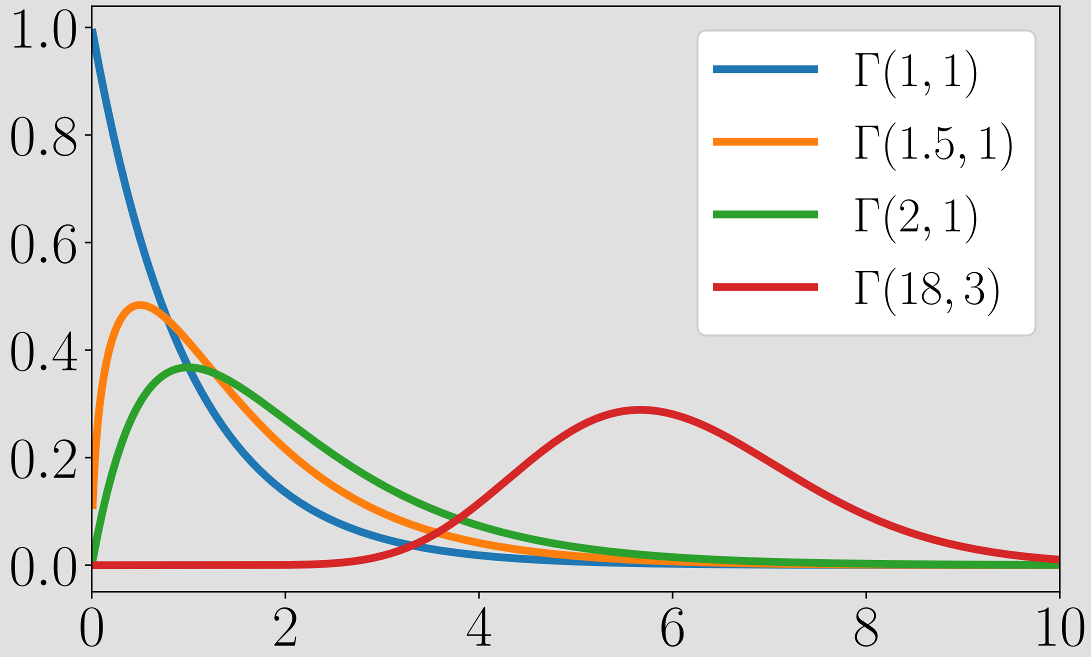

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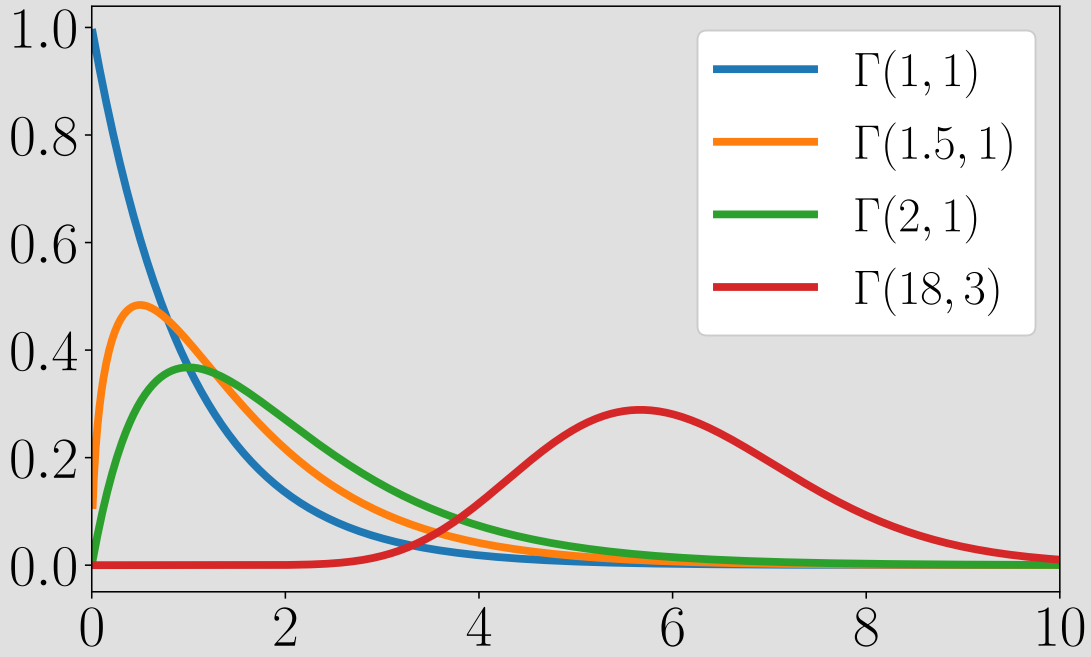

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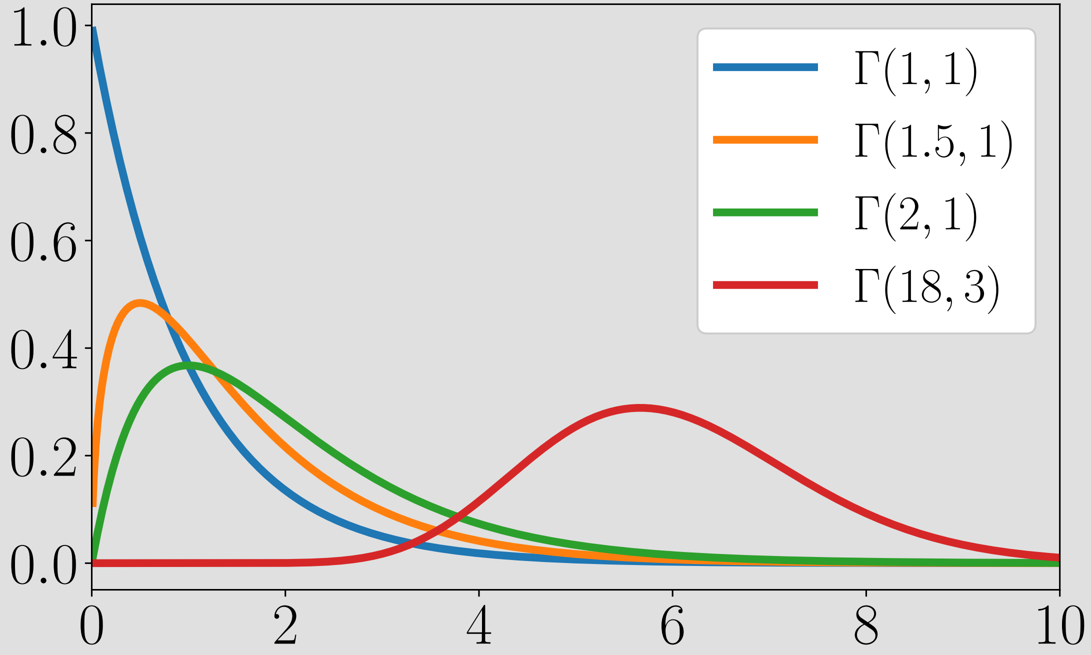

ТЕХНИЧЕСКИЙ СЛАЙД

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$\Gamma(5) = 24$

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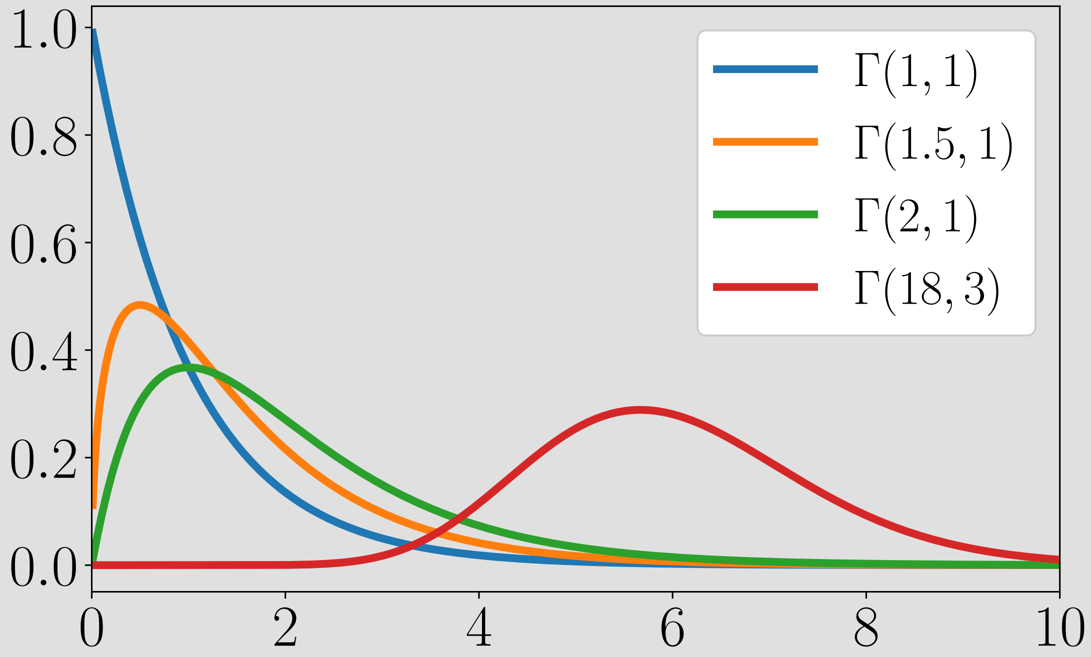

Statistics

$$\Gamma(\gamma|a,b) = \frac{b^a}{\Gamma(a)}\gamma^{a-1}e^{-b\gamma}$$

$$\mathbb{E}[\gamma] = a/b$$

$$\mathrm{Mode}[\gamma] = \frac{a-1}{b}$$

$$\mathrm{Var}[\gamma] = {}^{a}\!/\!{}_{b^2}$$

Example

You run $5\mathrm{km} \pm 100\mathrm{m}$ a day

ТЕХНИЧЕСКИЙ СЛАЙД

You run $5\text{km} \pm 100\text{m}$ a day

So is a random variable

We could model it with a normal

Example

Std.

You run $5\text{km} \pm 100\text{m}$ a day

Expectation

Example

You run 5km $\pm$ 100m a day

$$\mathbb{E}[x] = {}^{a}/_{b} = 5,\ Var[x] = {}^{a}/_{b^2} = 0.1^2$$

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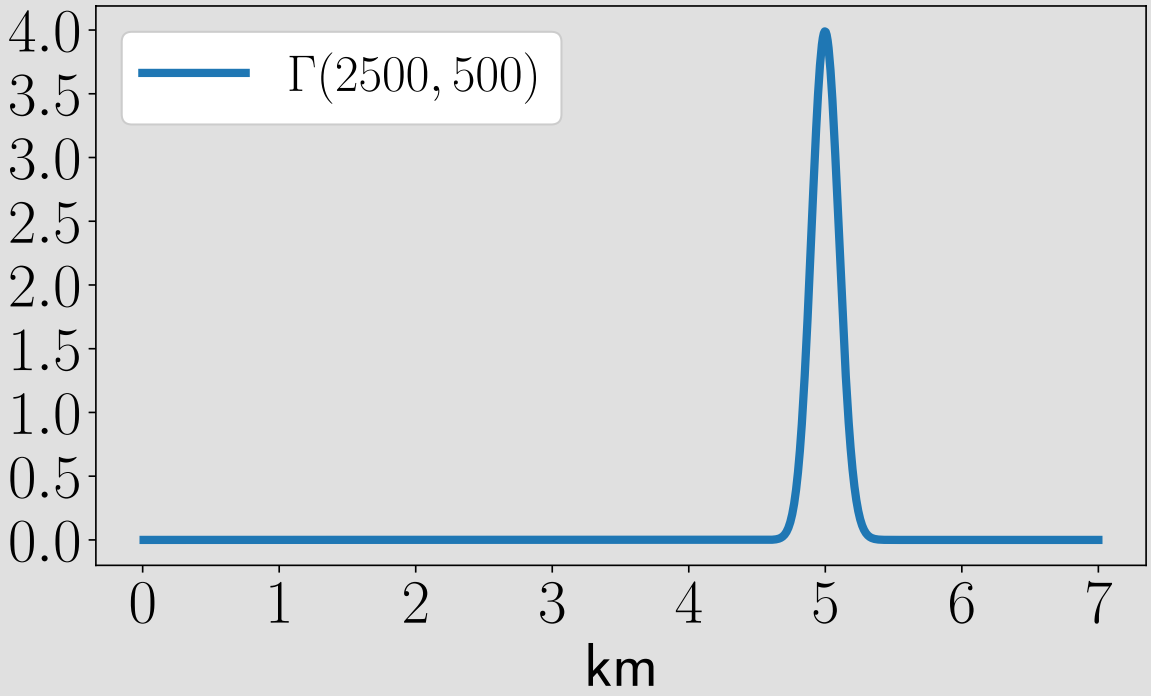
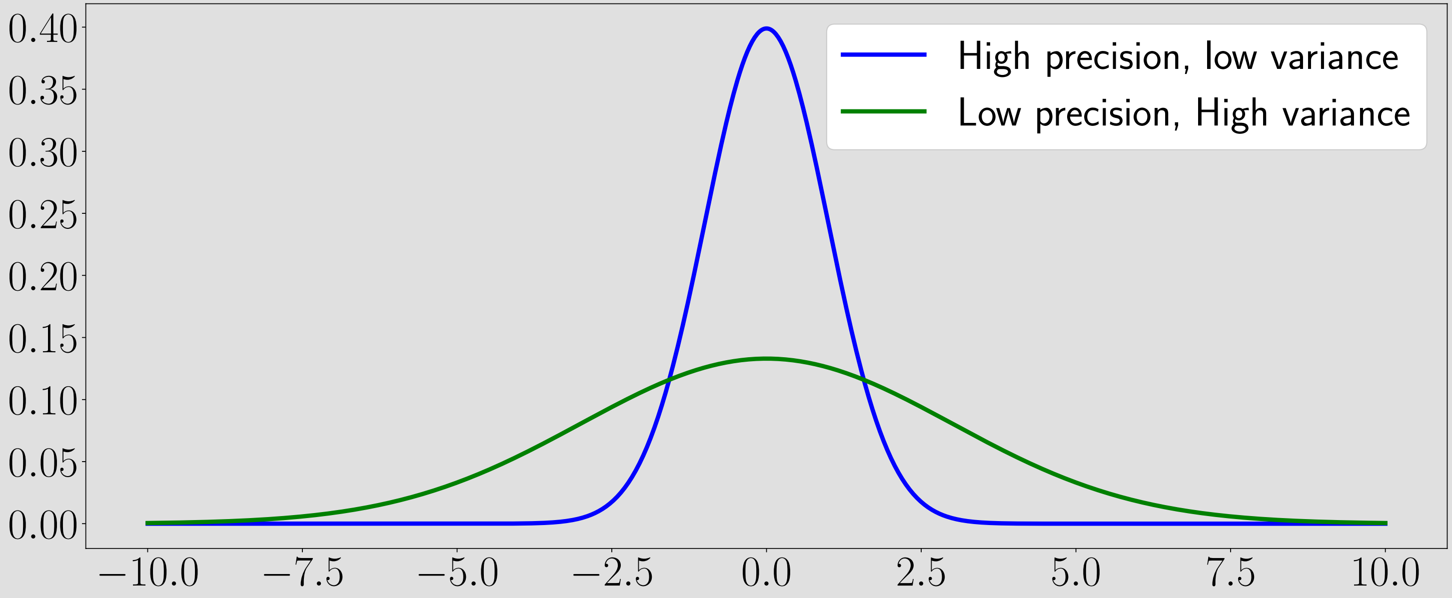


Example: Normal, precision

Precision

Precision $\gamma = \frac{1}{\sigma^2}$ Variance



Precision

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$$p(\gamma|x) = \Gamma(a+\tfrac{1}{2}, b+\tfrac{(x-\mu)^2}{2})$$