CS 273A Machine Learning (Fall 2017) Homework 1

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```
In [229]: import numpy as np
          import matplotlib.pyplot as plt
```

```
In [8]: import os
        os.getcwd()
```

```
Out[8]: '/Users/sheilacwang/Documents/Study /17Fall/CS 273A/hw/hw1'
```

1 Problem 1: Python & Data Exploration

```
In [217]: iris = np.genfromtxt("data/iris.txt",delimiter=None)
```

```
In [371]: Y = iris[:,-1] # target value is the last column
          X = iris[:,0:-1] # features are the other columns
```

1.1 Question 1

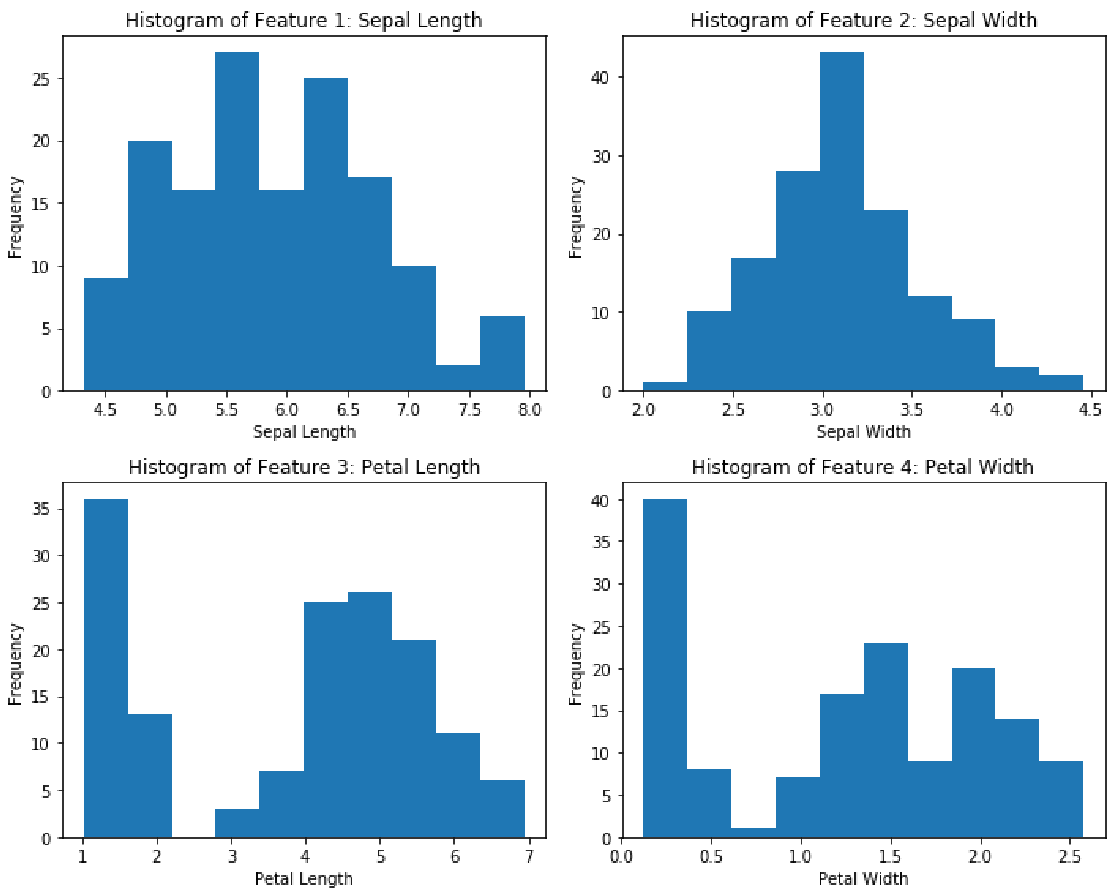
```
In [221]: print X.shape
```

```
(148, 4)
```

There are 148 data points and each one has 4 features.

1.2 Question 2

```
In [349]: fig, ax = plt.subplots(2,2, figsize=(10, 8))
          ax = ax.ravel()
          xaxes = ['Sepal Length','Sepal Width','Petal Length','Petal Width']

          for j,ax in enumerate(ax):
              ax.hist(X[:,j])
              ax.set_title('Histogram of Feature %s: %s'%(j+1, xaxes[j]))
              ax.set_xlabel(xaxes[j])
              ax.set_ylabel('Frequency')
          plt.tight_layout()
          plt.show()
```



1.3 Question 3

```
In [23]: np.set_printoptions(precision=4)
```

```
In [306]: print 'The mean of the data points for each feature:'
          print np.mean(X, axis=0)
```

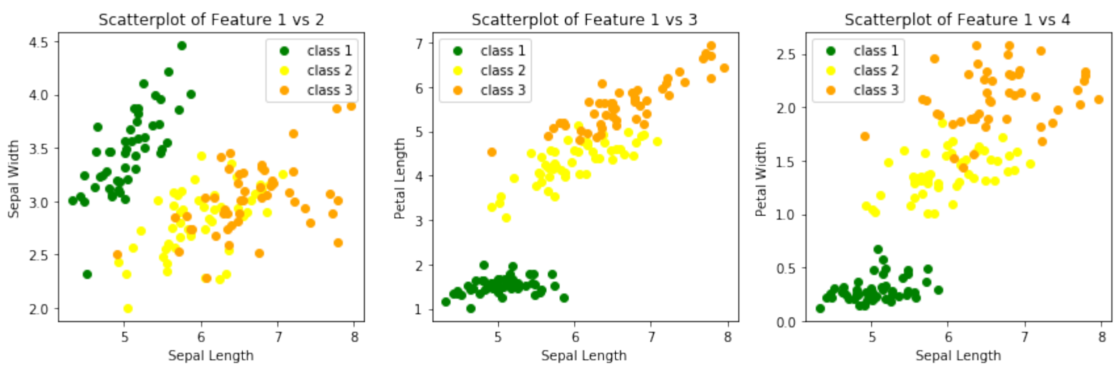
```
The mean of the data points for each feature:
[ 5.9001  3.0989  3.8196  1.2526]
```

```
In [307]: print 'The standard deviation of the data points for each feature:'
          print np.std(X, axis=0)
```

```
The standard deviation of the data points for each feature:
[ 0.8334  0.4363  1.7541  0.7588]
```

1.4 Question 4

In [381]:
```
fig, ax = plt.subplots(1,3, figsize=(12, 4))
ax = ax.ravel()
xaxes = ['Sepal Length','Sepal Width','Petal Length','Petal Width']
crs = ['green', 'yellow', 'orange']

for j,ax in enumerate(ax):

    for c in np.unique(Y):
        ax.plot(X[Y==c, 0], X[Y==c, j+1], 'o', color = crs[int(c)], \
                label='class %d' % (int(c)+1))

    ax.set_title('Scatterplot of Feature 1 vs %s' %(j+2))
    ax.set_xlabel(xaxes[0])
    ax.set_ylabel(xaxes[j+1])
    ax.legend()
plt.tight_layout()
plt.show()
```



2 Problem 2: KNN Predictions

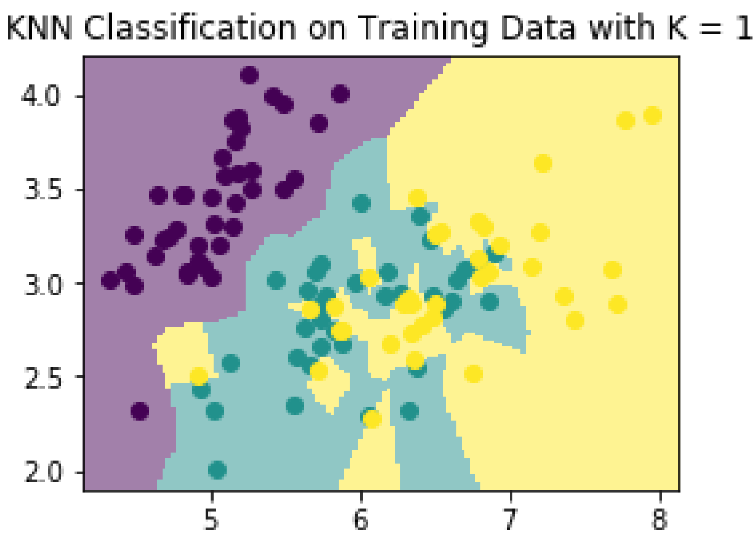
In [389]:
```
import mltools as ml

Y = iris[:,-1] # target value is the last column
X = iris[:,0:2] # features are the first 2 columns
```

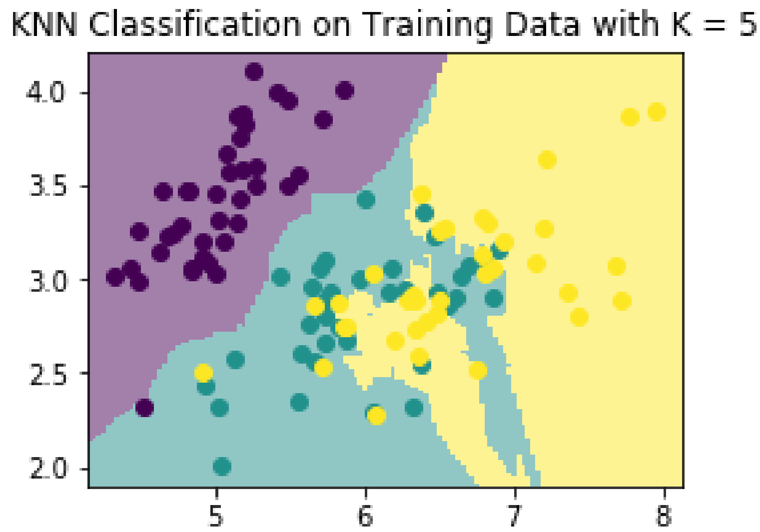
In [390]:
```
np.random.seed(0)

X,Y = ml.shuffleData(X,Y);  # shuffle data randomly
Xtr,Xva,Ytr,Yva = ml.splitData(X,Y, 0.75);
# split data into 75/25 train/validation
```

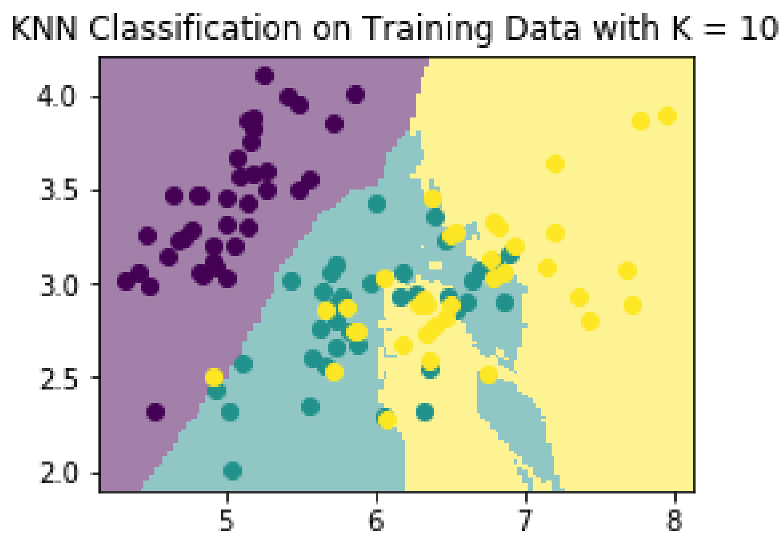
2.1 Question 1

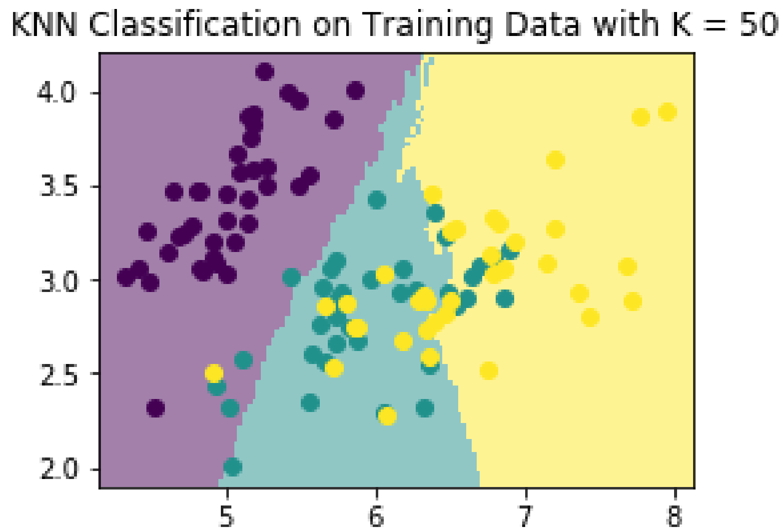
The visualization plots of the training data classification are given below:

```
In [409]: K = [1,5,10,50];

          for i, k in enumerate(K):
              knn = ml.knn.knnClassify()
              knn.train(Xtr, Ytr, k)
              YvaHat = knn.predict(Xva)
              fig, ax = plt.subplots(1, 1, figsize=(4, 3))
              ml.plotClassify2D( knn, Xtr, Ytr )
              ax.set_title(
                  'KNN Classification on Training Data with K = %s' %k
              )

              plt.show()
```



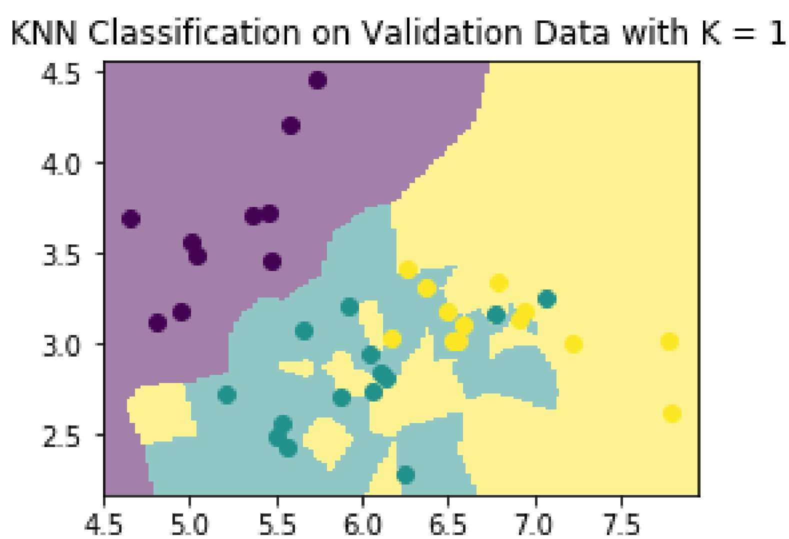

KNN Classification on Training Data with K = 5
4.0
3.5
3.0
2.5
2.0
5
6
7
8


KNN Classification on Training Data with K = 10
4.0
3.5
3.0
2.5
2.0
5
6
7
8

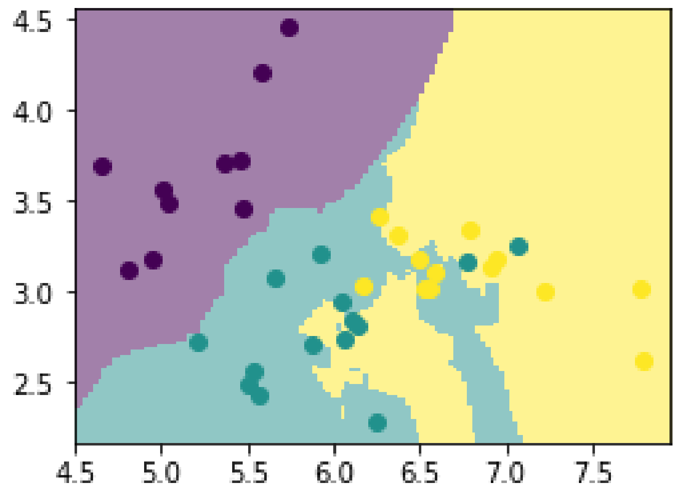


Next I'll visualize the classification boundries using the validation data:

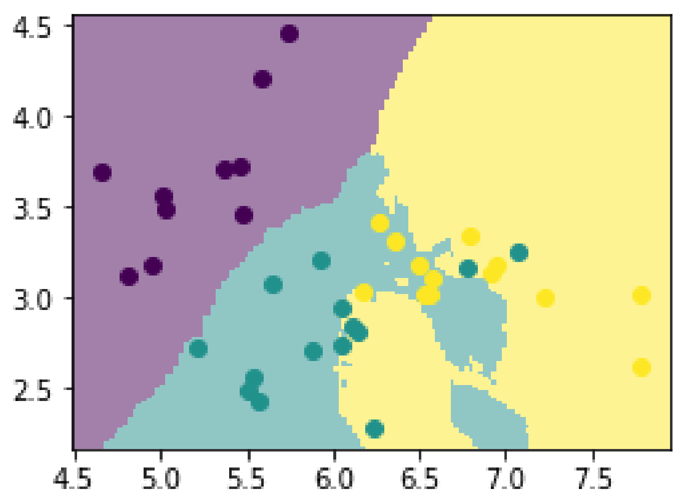
```
In [410]: K = [1,5,10,50];

          for i, k in enumerate(K):
              knn = ml.knn.knnClassify()
              knn.train(Xtr, Ytr, k)
              YvaHat = knn.predict(Xva)
              fig, ax = plt.subplots(1, 1, figsize=(4, 3))
              ml.plotClassify2D( knn, Xva, Yva )
              ax.set_title(
                  'KNN Classification on Validation Data with K = %s' %k
              )
              plt.show()
```

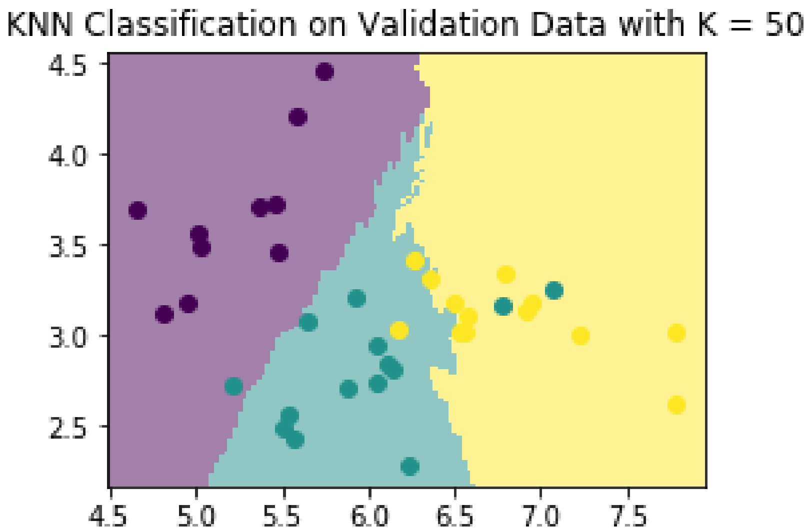


KNN Classification on Validation Data with K = 5



KNN Classification on Validation Data with K = 10





2.2 Question 2

Using only the first two features, error rates on both the training data and validation data for each value of k are computed, and a semi-log plot is shown as following:

```
In [351]: Y = iris[:,-1] # target value is the last column
          X = iris[:,0:2] # features are the first 2 columns

          np.random.seed(0)
          X,Y = ml.shuffleData(X,Y);  # shuffle data randomly
          Xtr,Xva,Ytr,Yva = ml.splitData(X,Y, 0.75);


          K = [1,2,5,10,50,100,200];
          errTrain = [];
          errVa = [];


          fig, ax = plt.subplots(1, 1, figsize=(5, 4))

          for i,k in enumerate(K):
              learner = ml.knn.knnClassify()
              k = K[i]
              print 'k =', k
              learner.train(Xtr, Ytr, k)
              Yhat = learner.predict(Xtr)
              errTrain.append(np.mean(Yhat != Ytr))
              YvaHat = learner.predict(Xva)
              errVa.append(np.mean(YvaHat != Yva))
```

```
        print "Training Error Rate:", errTrain[i]
        print "Validation Error Rate:", errVa[i]
        print

    ax.semilogx(K, errTrain, "r", label = "Training Error Rate")
    ax.semilogx(K, errVa, "g", label = "Validation Error Rate" )
    ax.legend()
    plt.show()
```

```
k = 1
Training Error Rate: 0.0
Validation Error Rate: 0.297297297297

k = 2
Training Error Rate: 0.117117117117
Validation Error Rate: 0.297297297297

k = 5
Training Error Rate: 0.135135135135
Validation Error Rate: 0.27027027027

k = 10
Training Error Rate: 0.18018018018
Validation Error Rate: 0.378378378378

k = 50
Training Error Rate: 0.261261261261
Validation Error Rate: 0.135135135135

k = 100
Training Error Rate: 0.414414414414
Validation Error Rate: 0.324324324324

k = 200
Training Error Rate: 0.648648648649
Validation Error Rate: 0.72972972973
```

Based on these results, the value of k I recommend is 50 because it has the least error rate on the validation data.

2.3 Question 3

```
In [352]: ## With all the features:

          Y = iris[:,-1] # target value is the last column
          X = iris[:,0:-1] # features are the first 4 columns

          np.random.seed(0)
          X,Y = ml.shuffleData(X,Y);  # shuffle data randomly
          Xtr,Xva,Ytr,Yva = ml.splitData(X,Y, 0.75);


          K = [1,2,5,10,50,100,200];
          errTrain = [];
          errVa = [];

          fig, ax = plt.subplots(1, 1, figsize=(5, 4))

          for i,k in enumerate(K):
              learner = ml.knn.knnClassify()
              k = K[i]
              print 'k =', k
              learner.train(Xtr, Ytr, k)
```

```
            Yhat = learner.predict(Xtr)
            errTrain.append(np.mean(Yhat != Ytr))
            YvaHat = learner.predict(Xva)
            errVa.append(np.mean(YvaHat != Yva))
            print "Training Error Rate:", errTrain[i]
            print "Validation Error Rate:", errVa[i]
            print

        ax.semilogx(K, errTrain, "r", label = "Training Error Rate")
        ax.semilogx(K, errVa, "g", label = "Validation Error Rate" )
        ax.legend()
        plt.show()
```

```
k = 1
Training Error Rate: 0.0
Validation Error Rate: 0.0540540540541

k = 2
Training Error Rate: 0.027027027027
Validation Error Rate: 0.027027027027

k = 5
Training Error Rate: 0.018018018018
Validation Error Rate: 0.027027027027

k = 10
Training Error Rate: 0.018018018018
Validation Error Rate: 0.027027027027

k = 50
Training Error Rate: 0.117117117117
Validation Error Rate: 0.0540540540541

k = 100
Training Error Rate: 0.378378378378
Validation Error Rate: 0.378378378378

k = 200
Training Error Rate: 0.648648648649
Validation Error Rate: 0.72972972973
```

Now with all the features in the data set, the semi-log plot is very different from the previous one, and this time the k value of 5 and 10 brings the least error rate on both training data and validation data.

3 Problem 3: Naive Bayes Classifiers

3.1 Question 1

The class probability:

$$P(y=1) = \frac{4}{10} = \frac{2}{5}; P(y=-1) = \frac{6}{10} = \frac{3}{5}$$

The individual feature probabilities for each class y and feature X_i:

$$P(X_1=0|y=1) = \frac{1}{4}; P(X_1=1|y=1) = \frac{3}{4}$$
$$P(X_1=0|y=-1) = \frac{1}{2}; P(X_1=1|y=-1) = \frac{1}{2}$$

$$P(X_2=0|y=1) = 1; P(X_2=1|y=1) = 0$$
$$P(X_2=0|y=-1) = \frac{1}{6}; P(X_2=1|y=-1) = \frac{5}{6}$$

$$P(X_3=0|y=1) = \frac{1}{4}; P(X_3=1|y=1) = \frac{3}{4}$$
$$P(X_3=0|y=-1) = \frac{1}{3}; P(X_3=1|y=-1) = \frac{2}{3}$$

$$P(X_4 = 0|y = 1) = \frac{1}{2}; P(X_4 = 1|y = 1) = \frac{1}{2}$$
$$P(X_4 = 0|y = -1) = \frac{1}{6}; P(X_4 = 1|y = -1) = \frac{5}{6}$$
$$P(X_5 = 0|y = 1) = \frac{3}{4}; P(X_5 = 1|y = 1) = \frac{1}{4}$$
$$P(X_5 = 0|y = -1) = \frac{2}{3}; P(X_5 = 1|y = -1) = \frac{1}{3}$$

3.2 Question 2

For $X = [0, 0, 0, 0, 0]$, the predicted class is y = +1; For $X = [1, 1, 0, 1, 0]$, the predicted class is y = -1.

3.3 Question 3

$P(y = 1|X = [1, 1, 0, 1, 0]) = \frac{P(X = [1, 1, 0, 1, 0]|y = 1)P(y = 1)}{P(X = [1, 1, 0, 1, 0]|y = 1)P(y = 1) + P(X = [1, 1, 0, 1, 0]|y = -1)P(y = -1)}$

Since $P(X_2 = 1|y = 1) = 0$, therefore the posterior probability is 0.

3.4 Question 4

Becasuse our data samples are less the number of parameters 2^5. For some feature combinations that we don't observe in the training data, the "joint" Bayes classifier would predict 0 probability in the validation data, which could result in misclassification in this case since for ties we prefer to predict class +1. And using more complex models would cause overfitting.

3.5 Question 5

The naive Bayes model over features x_2, x_3, x_4, x_5 is:

$$P(y = 1|X = [x_2, x_3, x_4, x_5]) =$$

$$\frac{P(X = [x_2, x_3, x_4, x_5]|y = 1)P(y = 1)}{P(X = [x_2, x_3, x_4, x_5]|y = 1)P(y = 1) + P(X = [x_2, x_3, x_4, x_5]|y = -1)P(y = -1)},$$

where

$$P(X = [x_2, x_3, x_4, x_5]|y = 1) = P(x_2|y = 1)P(x_3|y = 1)P(x_4|y = 1)P(x_5|y = 1)$$

4 Statement of Collaboration

I have abided by the rules of conduct and academic honesty adoped by UC Irvine. I did not discuss the specific solutions to this homework with any person.

Chenxi Wang 10/8/2017