CS 273A Machine Learning (Fall 2017) Homework 3

October 31, 2017

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```
In [17]: import numpy as np
         np.random.seed(0)
         import mltools as ml
         import matplotlib.pyplot as plt   # use matplotlib for plotting with inli
         %matplotlib inline

         import os
         print os.getcwd()
```

```
/Users/sheilacwang/Documents/Study /17Fall/CS 273A/hw/hw3
```

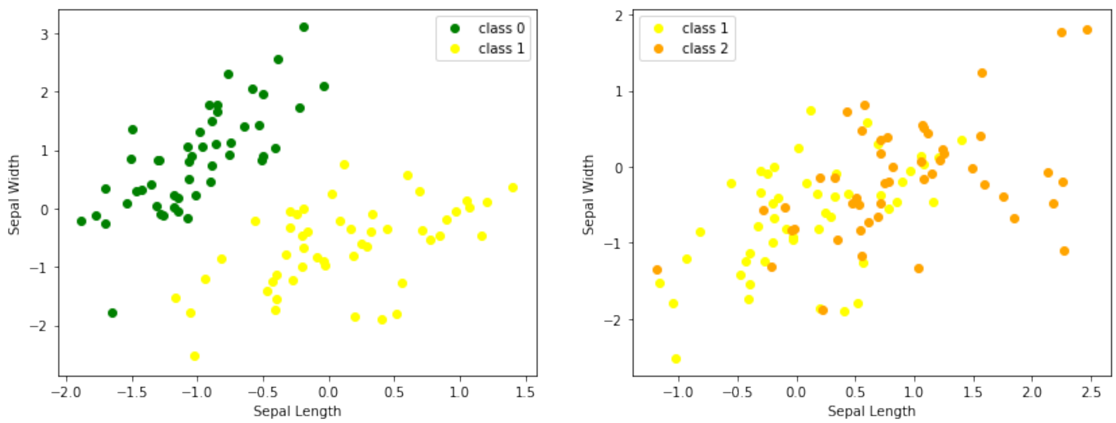
1 Problem 1: Logistic Regression

```
In [18]: iris = np.genfromtxt("data/iris.txt",delimiter=None)
         X, Y = iris[:,0:2], iris[:,-1] # get first two features & target
         X,Y = ml.shuffleData(X,Y) # reorder randomly (important later)
         X,_ = ml.transforms.rescale(X) # works much better on rescaled data

         XA, YA = X[Y<2,:], Y[Y<2] # get class 0 vs 1
         XB, YB = X[Y>0,:], Y[Y>0] # get class 1 vs 2
```

1.1 Question 1: Scatterplot

```
In [19]: fig, ax = plt.subplots(1, 2, figsize=(14, 5))
         ax = ax.flatten()
         crs = ['green', 'yellow', 'orange']

         for c in np.unique(YA):
             ax[0].scatter(XA[YA==c, 0], XA[YA==c, 1],\
                           color = crs[int(c)],\
                           label='class %d' % (int(c)))
             ax[0].legend()
             ax[0].set_xlabel('Sepal Length')
             ax[0].set_ylabel('Sepal Width')
```

```
for c in np.unique(YB):
    ax[1].scatter(XB[YB==c, 0], XB[YB==c, 1],\
                  color = crs[int(c)],\
                  label='class %d' % (int(c)))
    ax[1].legend()
    ax[1].set_xlabel('Sepal Length')
    ax[1].set_ylabel('Sepal Width')

plt.show()
```



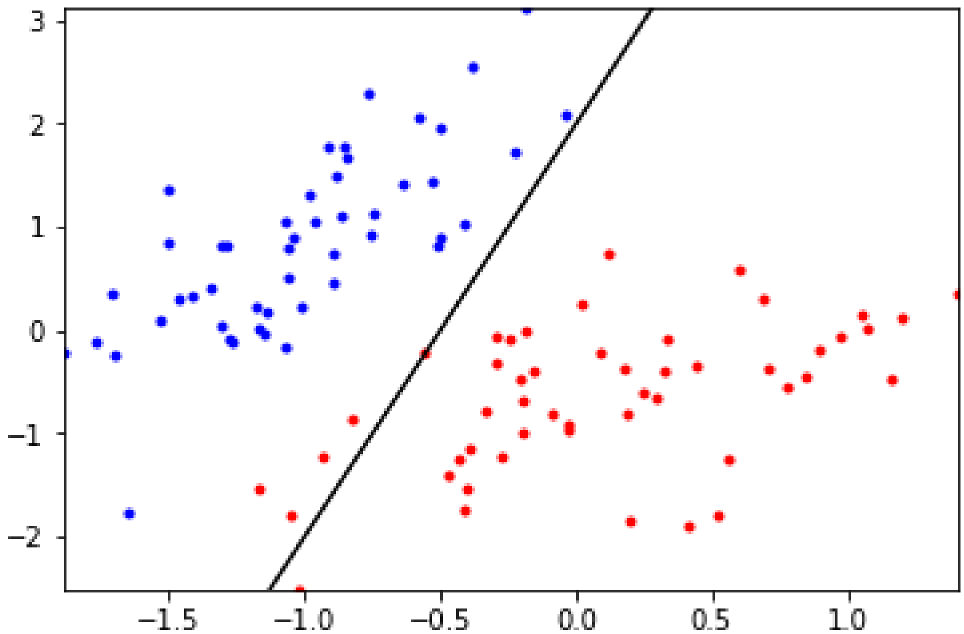
As are suggested by the scatterplots, class 0 and class 1 are linearly separable according the first two features, while class 1 and 2 are not.

1.2 Question 2

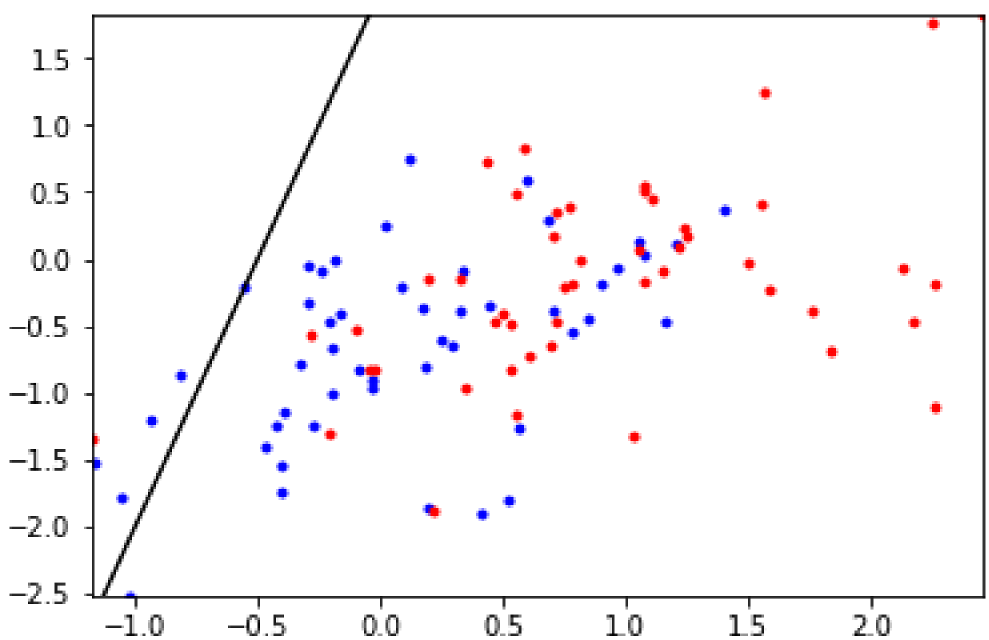
The lines of code I added to the function is:

x2b = - (self.theta[0] + self.theta[1] * x1b)/self.theta[2]

```
In [20]: from logisticClassify2 import *

         learner = logisticClassify2(); # create "blank" learner
         learner.classes = np.unique(YA) # define class labels using YA or YB
         wts = np.array([0.5,1.,-0.25]); # TODO: fill in values
         learner.theta = wts; # set the learner's parameters
         learner.plotBoundary(XA,YA)
```




```
In [21]: learner = logisticClassify2(); # create "blank" learner
         learner.classes = np.unique(YB) # define class labels using YA or YB
         wts = np.array([0.5,1.,-0.25]); # TODO: fill in values
         learner.theta = wts; # set the learner's parameters
         learner.plotBoundary(XB,YB)
```



1.3 Question 3

The function definition is:

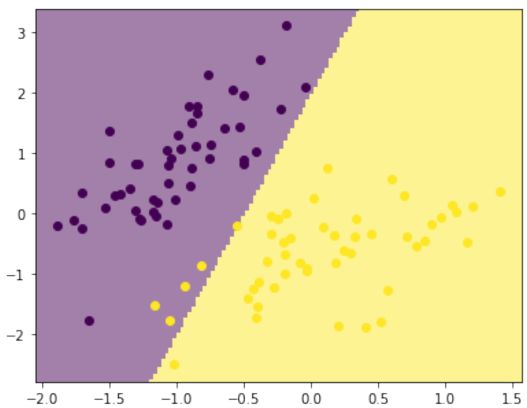
```
In [22]: # def predict(self, X):
         #     """ Return the predictied class of each data point in X"""
         #     r = np.zeros(X.shape[0]);
         #     Yhat = np.zeros(X.shape[0]);
         #     for i in range(X.shape[0]):
         #         r[i] = self.theta[0] + self.theta[1]*X[i,0]\
         #                + self.theta[2]*X[i,1]
         #         if r[i] > 0:
         #             Yhat[i] = self.classes[1]
         #         else:
         #             Yhat[i] = self.classes[0]

         #     return Yhat
```

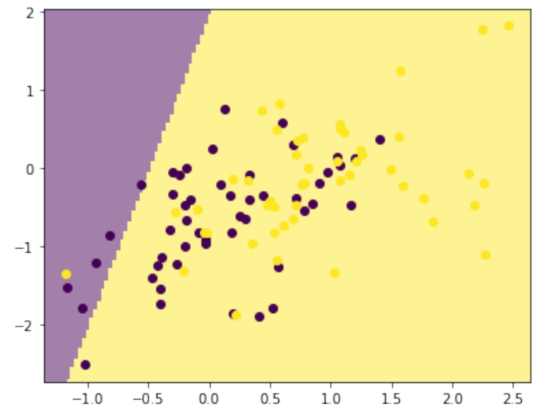
```
In [23]: wts = np.array([0.5,1.,-0.25])
         learner = logisticClassify2()
         learner.theta = wts; # set the learner's parameters
         YA_hat = learner.predict(XA)
         YB_hat = learner.predict(XB)

         print "classifier error rate on dataset A:",learner.err(XA, YA)
         print
         print "classifier error rate on dataset B:", learner.err(XB, YB-1)
         ## Since YB initial class values are 1 and 2
```

```
classifier error rate on dataset A: 0.0505050505051

classifier error rate on dataset B: 0.464646464646
```

1.4 Question 4

```
In [24]: fig, ax = plt.subplots(1, 2, figsize=(14, 5));
         ax = ax.flatten();
         Y = [YA, YB-1];

         for i, X in enumerate([XA, XB]):
             ml.plotClassify2D( learner, X, Y[i], axis=ax[i] )

         plt.show()
```





The decision boundaries matches the plots I computed analytically.

1.5 Question 5

Since $\sigma(r) = (1 + exp(-r))^{-1}$, then $\sigma(x^{(j)}\theta^T) = (1 + exp(-x^{(j)}\theta^T))^{-1}$, and $J_j(\theta) = -y^{(j)}log(\sigma(x^{(j)}\theta^T)) - (1-y^{(j)})log(1-\sigma(x^{(j)}\theta^T))$.

By taking partial derivatives, we have:

$$\frac{\partial J^{(j)}}{\partial \theta_0} = (-\frac{y^{(j)}}{\sigma(x^{(j)}\theta^T)} + \frac{1-y^{(j)}}{1-\sigma(x^{(j)}\theta^T)})\frac{exp(-x^{(j)}\theta^T)}{(1+exp(-x^{(j)}\theta^T))^2} = -y^{(j)} + \sigma(x^{(j)}\theta^T)$$

$$\frac{\partial J^{(j)}}{\partial \theta_1} = (-\frac{y^{(j)}}{\sigma(x^{(j)}\theta^T)} + \frac{1-^{(j)}y}{1-\sigma(x^{(j)}\theta^T)})\frac{exp(-x^{(j)}\theta^T)}{(1+exp(-x^{(j)}\theta^T))^2}x_1 = (-y^{(j)} + \sigma(x^{(j)}\theta^T))x_1$$

$$\frac{\partial J^{(j)}}{\partial \theta_2} = (-\frac{y^{(j)}}{\sigma(x^{(j)}\theta^T)} + \frac{1-^{(j)}y}{1-\sigma(x^{(j)}\theta^T)})\frac{exp(-x^{(j)}\theta^T)}{(1+exp(-x^{(j)}\theta^T))^2}x_2 = (-y^{(j)} + \sigma(x^{(j)}\theta^T))x_2$$

1.6 Question 6

The complete implementation of train is:

```
In [9]: # def train(self, X, Y, initStep=0.2, stopTol=1e-4, \
        #              stopEpochs=5000, plot=None):
        #         """ Train the logistic regression using stochastic gradient desce
        #         M,N = X.shape;
        #         self.classes = np.unique(Y);
        #         XX = np.hstack((np.ones((M,1)),X))
        #         YY = ml.toIndex(Y,self.classes);
        #         if len(self.theta)!=N+1:
        #             self.theta=np.random.rand(N+1);
```

```
#           def sigma(r):
#               return 1/(1+np.exp(-r))
#           # init loop variables:
#           epoch=0; done=False; Jnll=[np.inf]; J01=[np.inf];
#           while not done:
#               stepsize, epoch = initStep*2.0/(2.0+epoch),\
#               epoch+1;
#               # Do an SGD pass through the entire data set:
#               for i in np.random.permutation(M):
#                   ri = np.dot(self.theta, XX[i,:]);
#                   # TODO: compute linear response r(x)
#                   gradi = (-YY[i] + sigma(ri))*XX[i,:];
#                   # TODO: compute gradient of NLL loss
#                   self.theta -= stepsize * gradi;

#               J01.append( self.err(X,Y) )

#               ## TODO: compute surrogate loss
#               jsur = 0;
#               for i in np.random.permutation(M):
#                   jsur += -YY[i]*np.log(sigma(np.dot(self.theta, XX[i,:])))
#                   (1-YY[i])*np.log(1-sigma(np.dot(self.theta, XX[i,:])))
#               Jsur = jsur/M
#               ##  Jsur = sum_i [ (log si) if yi==1 else (log(1-si)) ]
#               Jnll.append( Jsur ) # TODO evaluate the current NLL loss

#               plt.figure(1);
#               plt.plot(Jnll,'b-',J01,'r-');
#               plt.xlabel("epoch")
#               plt.title("Convergence of Surrogate Loss and Error Rate")
#               plt.draw();    # plot losses
#               if N==2:
#                   plt.figure(2);
#                   self.plotBoundary(X,Y);
#                   plt.title("Convergence of Classifier")
#                   plt.draw(); # & predictor if 2D
#               plt.pause(.01);

#               ## For debugging:
#               ## print self.theta, ' => ', Jnll[-1], ' / ', J01[-1]
#               ## raw_input()   # pause for keystroke

#               # TODO check stopping criteria:
#               if epoch > stopEpochs or \
#               np.abs(Jnll[-2] - Jnll[-1]) < stopTol:
#                   done = True;
#           plt.figure(3);
#           self.plotBoundary(X,Y);
```

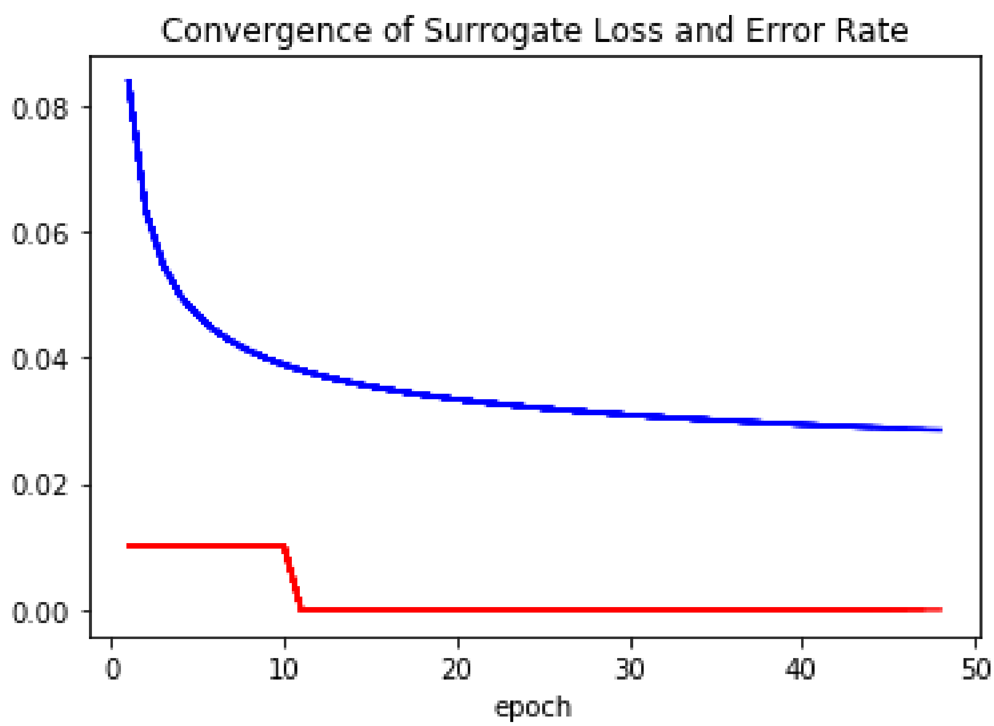
```
#          plt.title("Final Converged Classifier")
#          plt.draw();
```

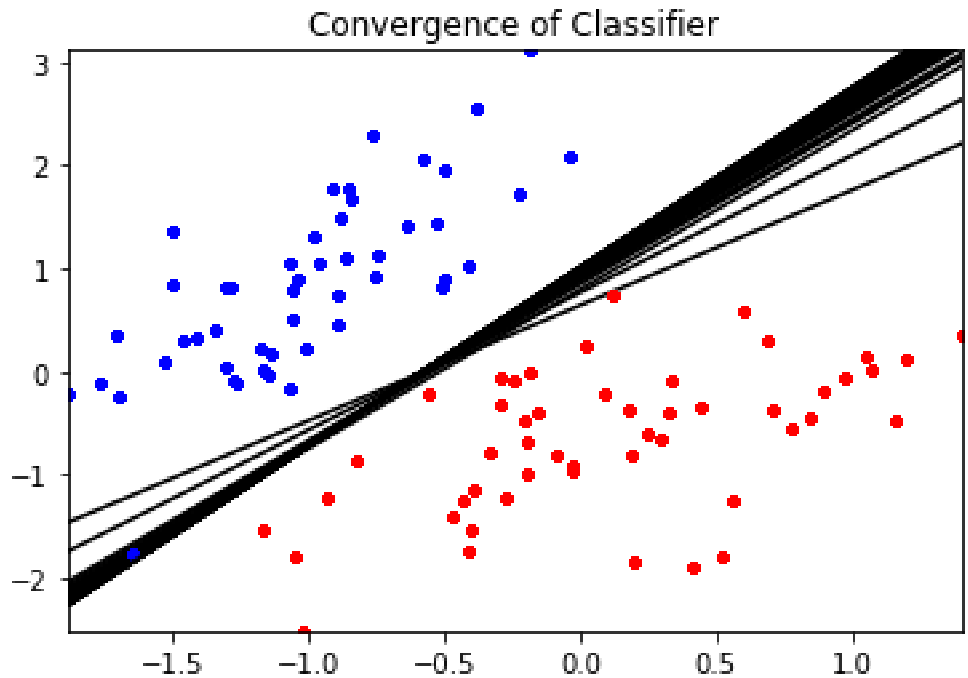
1.7 Question 7

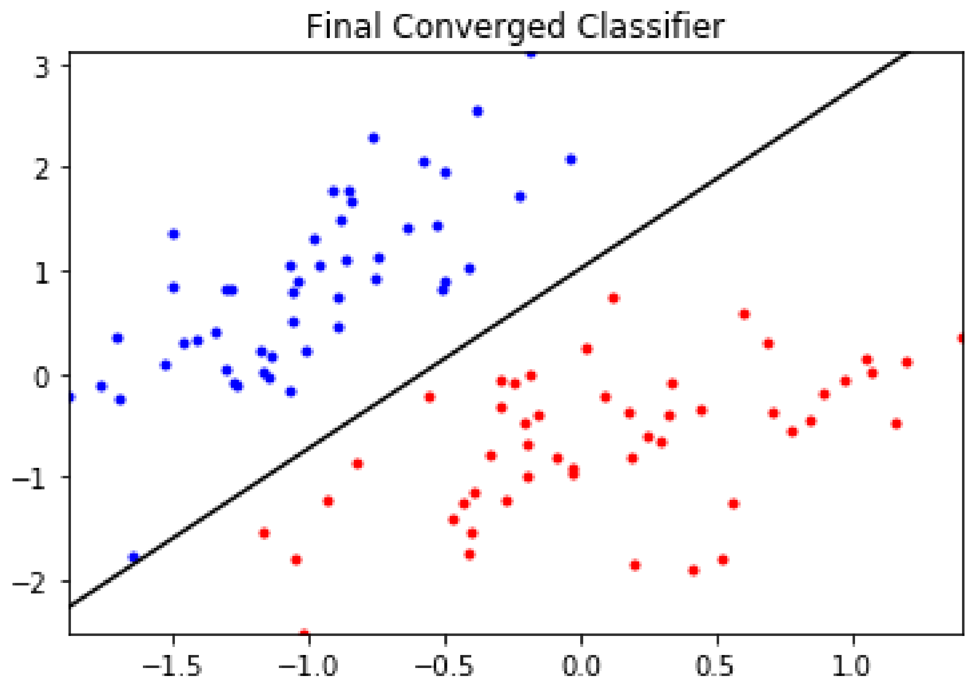
The plots of classifer on data set A are:

```
In [25]: wts = np.array([0.5,1.,-0.25])
         learner = logisticClassify2()
         learner.theta = wts; # set the learner's parameters

         learner.train(XA,YA,initStep=0.5)
         print "Final Theta:", learner.theta
```

```
Final Theta: [ 3.51209268  6.01069038 -3.45764397]
```



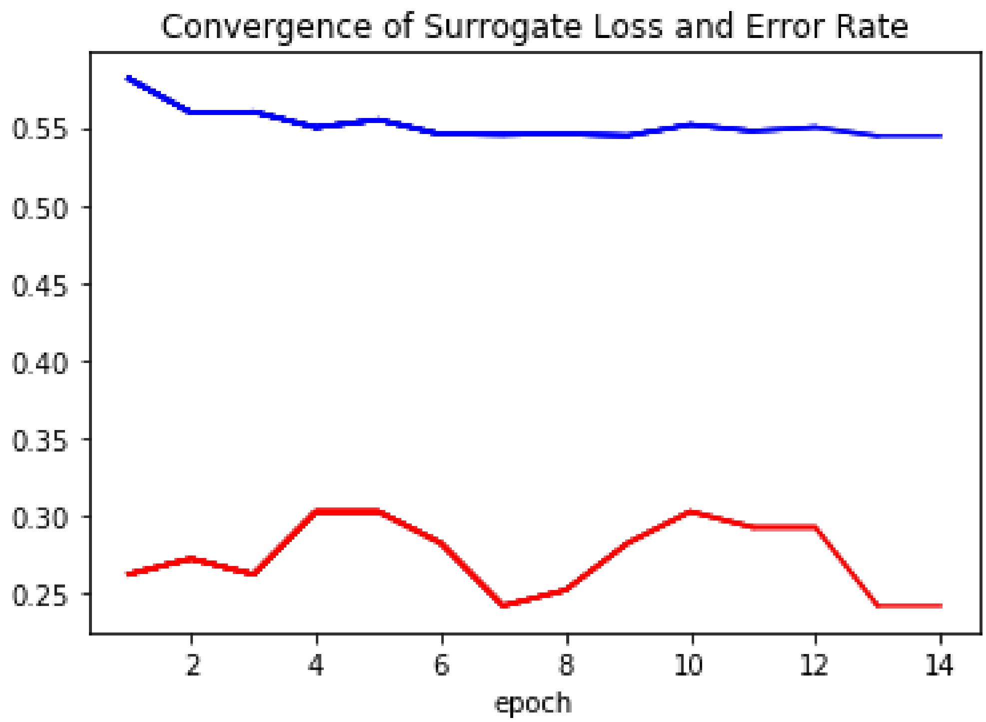


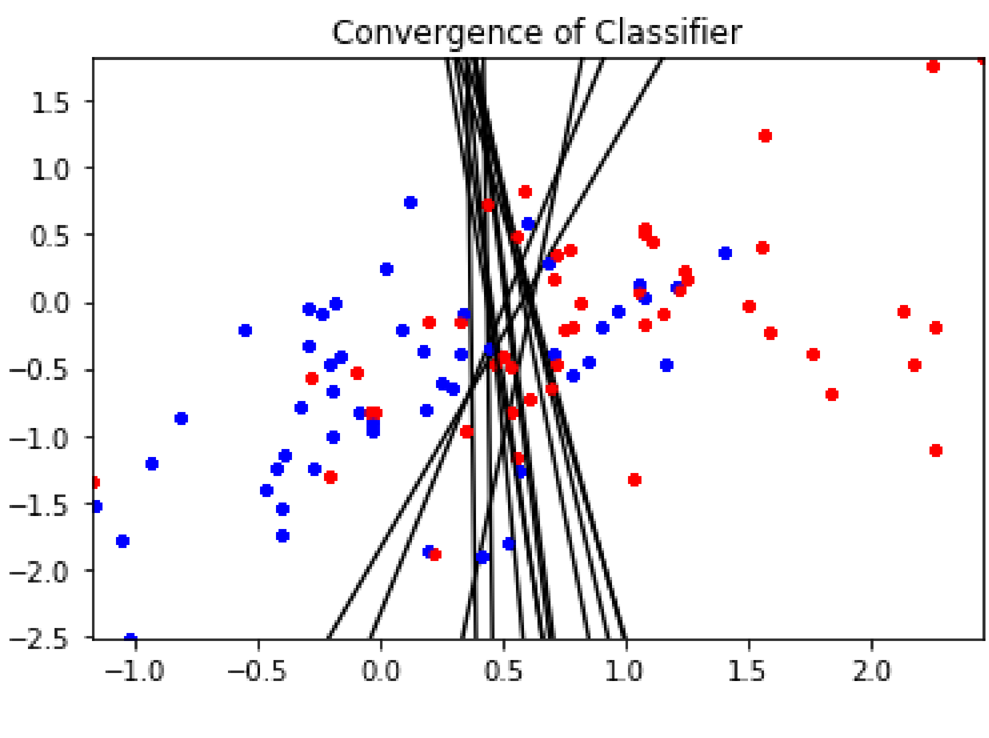


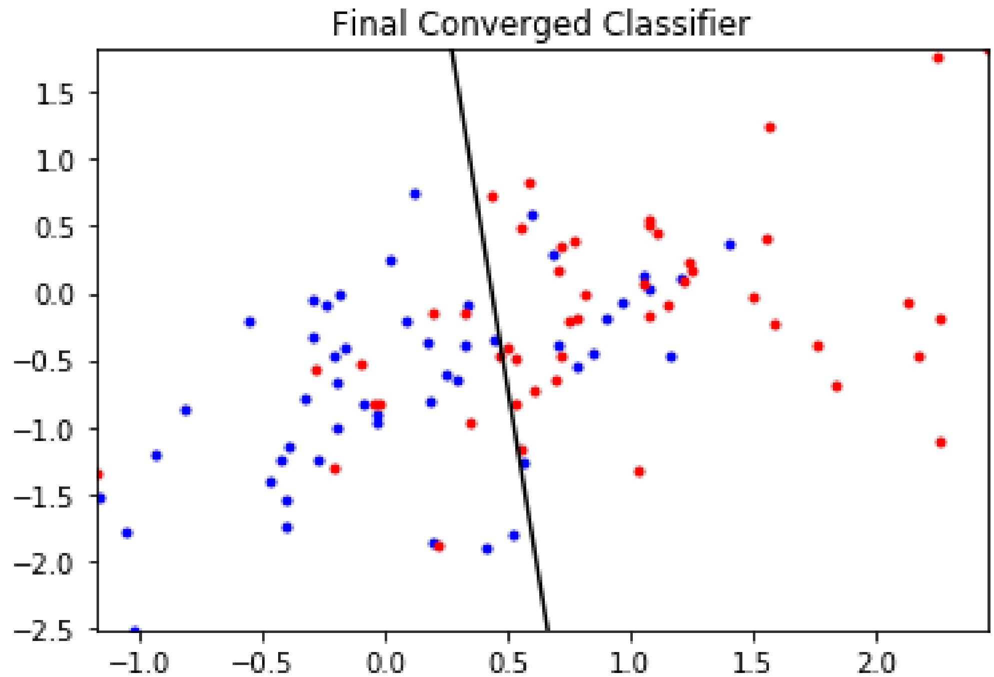
The plots of classifer on data set B are:

```
In [26]: learner.train(XB,YB,initStep=0.5)
         print "Final Theta:", learner.theta
```

```
Final Theta: [-0.75075015  1.71808938  0.15528256]
```







1.8 Question 8

The new gradient equations after adding the regularization term are:

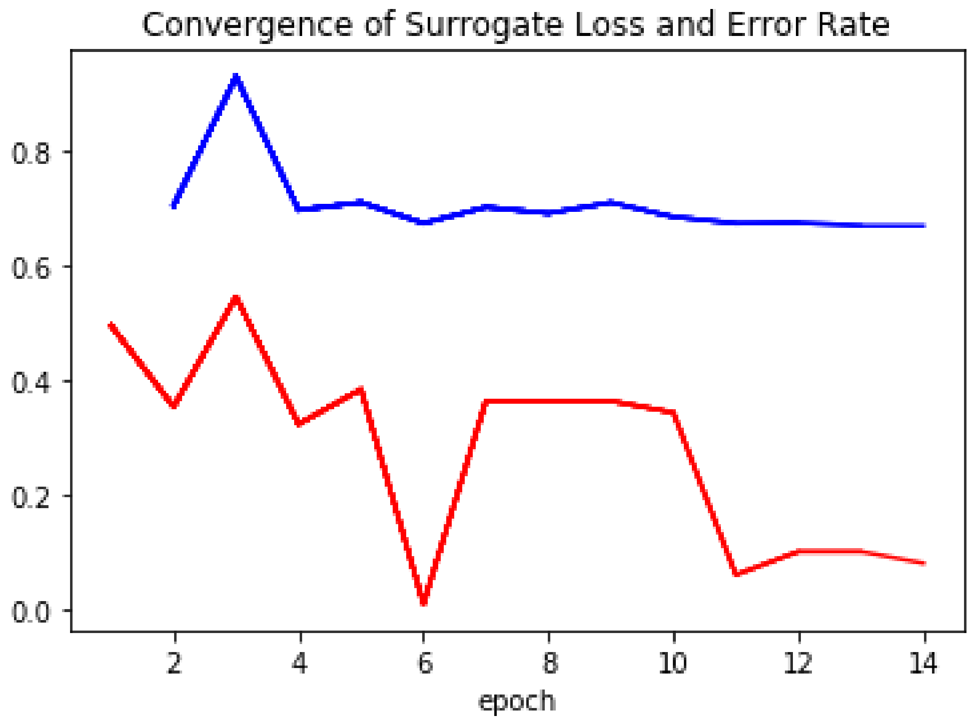
$$\frac{\partial J^{(j)}}{\partial \theta_0} = -y^{(j)} + \sigma(x^{(j)}\theta^T) + 2\alpha\theta_0$$

$$\frac{\partial J^{(j)}}{\partial \theta_1} = (-y^{(j)} + \sigma(x^{(j)}\theta^T))x_1 + 2\alpha\theta_1$$

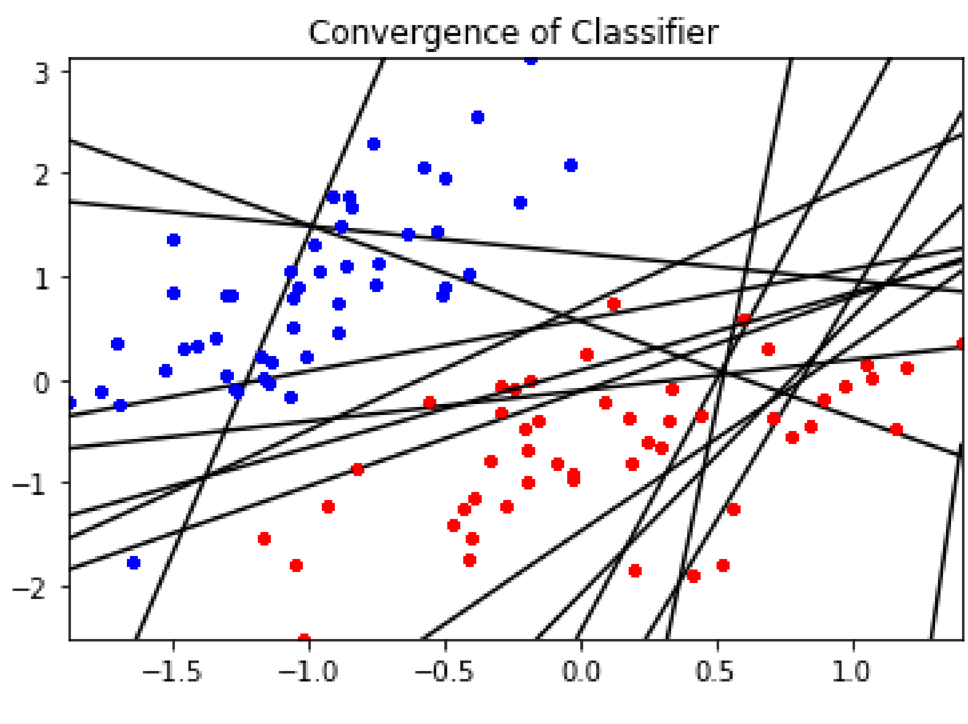
$$\frac{\partial J^{(j)}}{\partial \theta_2} = (-y^{(j)} + \sigma(x^{(j)}\theta^T))x_2 + 2\alpha\theta_2$$

```
In [27]: learner.train(XA,YA,alpha=2)
         print "Final Theta:", learner.theta
```

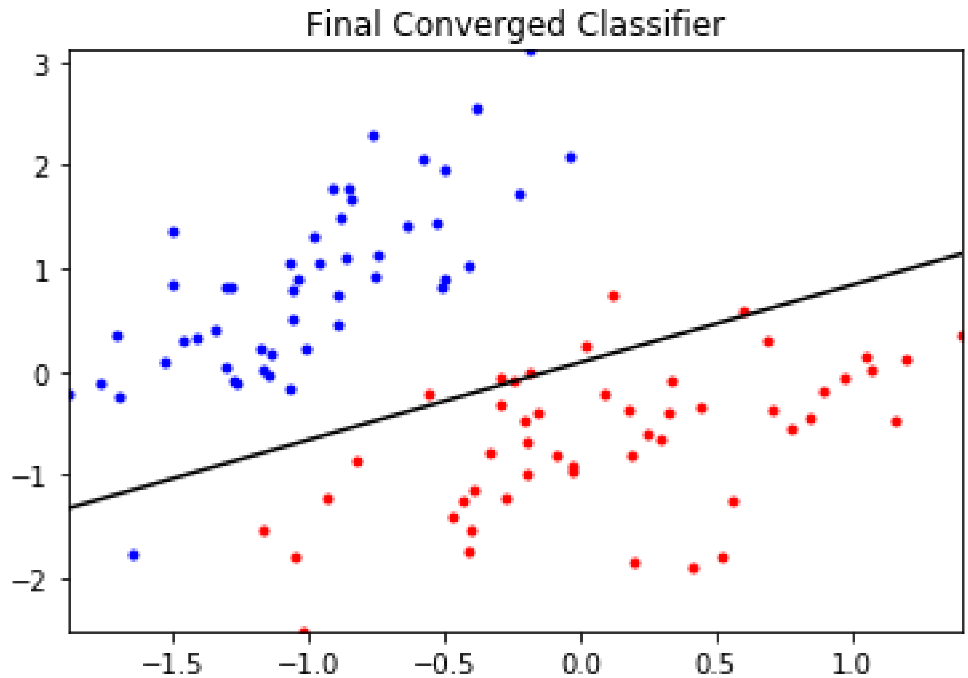
```
Final Theta: [ 0.00930968  0.07858302 -0.10484828]
```



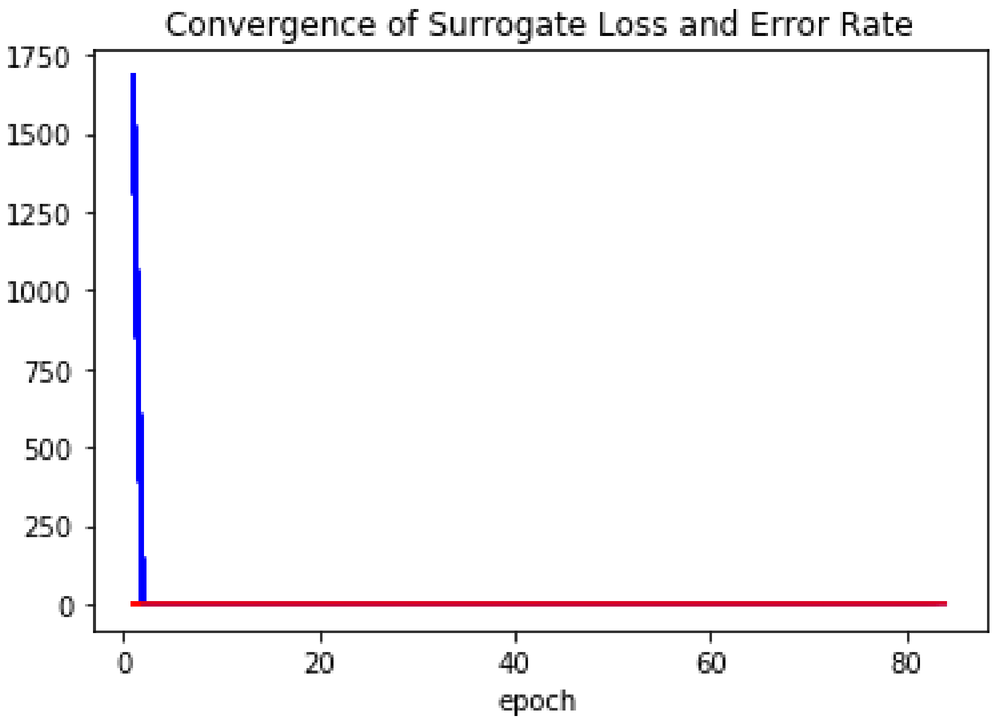
Convergence of Surrogate Loss and Error Rate
0.8
0.6
0.4
0.2
0.0
2
4
6
8
10
12
14
epoch

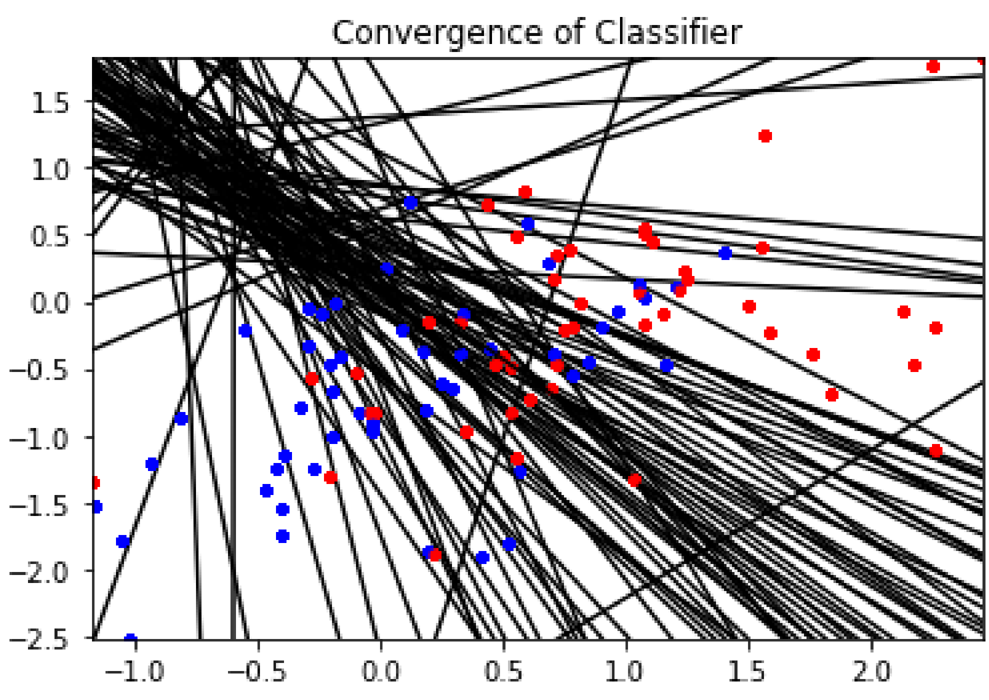


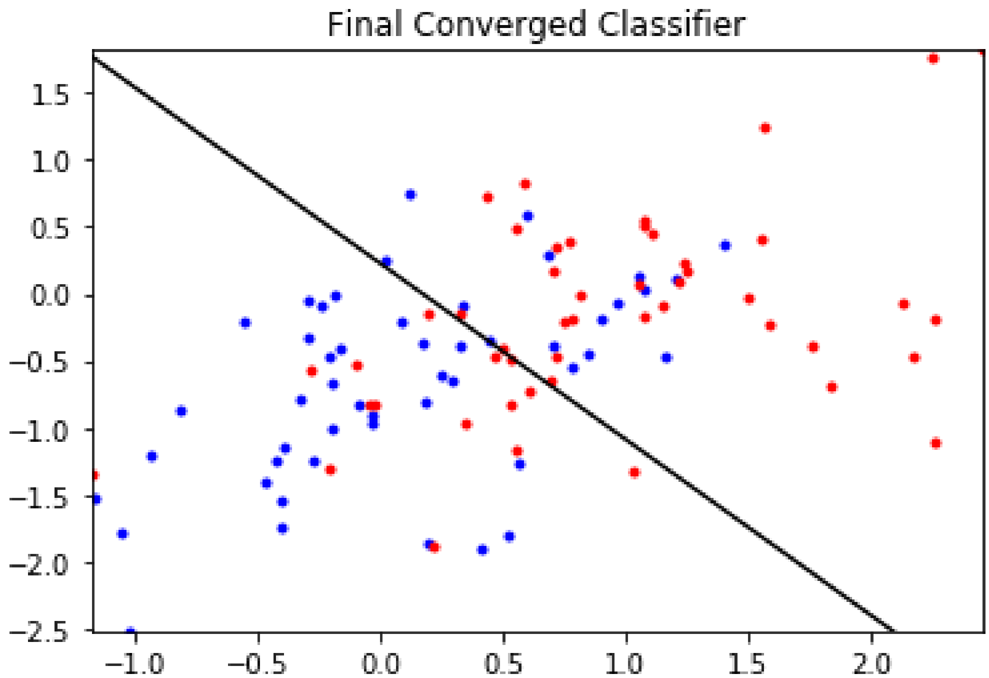
Convergence of Classifier
3
2
1
0
−1
−2
−1.5
−1.0
−0.5
0.0
0.5
1.0




```
In [28]: learner.train(XB,YB,alpha=2)
         print "Final Theta:", learner.theta

Final Theta: [-0.00978878  0.05655184  0.04317285]
```


Convergence of Surrogate Loss and Error Rate
1750
1500
1250
1000
750
500
250
0
0
20
40
60
80
epoch


Convergence of Classifier
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0



2 Problem 2: Shattering and VC Dimension

(1). $T(a + bx_1)$

The classifer would be a straight line perpendicular to the x1 axis, which is a one-dimensional line. So the VC dim = 2, only (a)(b) can be shattered by the learner.

(2). $T((a * b)x_1 + (c/a)x_2)$

This is a 2-dimensional line across the original point, so it does not have a constant term. In this case, the VC dim =2, only (a)(b) can be shattered by the learner.

(3). $T((x_1 - a)^2 + (x_2 - b)^2 + c)$

The classifer is a circle with changing center and radius. VC dim = 3, (a)(b)(c) can be shattered by the learner.

(4). $T(a + bx_1 + cx_2) \times T(d + bx_1 + cx_2)$

The classifer is two parallel lines distinguishing what's in between from outside. VC dim $\geqslant 4$, (a)(b)(c)(d) can be shattered by the learner.

3 Statement of Collaboration

I have abided by the rules of conduct and academic honesty adoped by UC Irvine. I did not discuss the specific solutions to this homework with any person.

Chenxi Wang

10/31/2017