Information Theory: Assignment #4

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Problem 1

Given joint distribution:

Table 1: Sum of rolls

Y/X	1	2	3	4
1	1/8	1/16	1/32	1/32
2	1/16	1/8	1/32	1/32
3	1/16	1/16	1/16	1/16
4	1/4	0	0	0

Calculate $H(X), H(Y), H(X,Y), H(X|Y), H(Y|X), I(X;Y)$

Solution

Entropy of random variable is given by:

$$H(X) = \sum_i p_i * (-\log_2 p_i) \tag{1}$$

Calculations for each provided below:

- $H(X)$
  - We need marginal distribution, given by formula:

    $$P(X) = \sum_y P(X,Y) = \sum_y P(X|Y=y)P(Y=y) \tag{2}$$

    to obtain following probability vector:

    $$[1/2, 1/4, 1/8, 1/8] \tag{3}$$

  - Entropy of X will be:

    $$1/2 * -\log_2(1/2) + 1/4 * -\log_2(1/4) + 2/8 * -\log_2(1/8) = 1/2 + 1/2 + 3/4 = 7/4 \tag{4}$$

- $H(Y)$
  - We need marginal distribution, given by formula:

    $$P(Y) = \sum_y P(X,Y) = \sum_x P(Y|X=x)P(X=x) \tag{5}$$

    to obtain following probability vector:

    $$[1/4, 1/4, 1/4, 1/4] \tag{6}$$

  - Entropy of Y will be:

    $$4 * -\log_2(1/4) = 8 \tag{7}$$

    It should be noted, that Y is uniformly distributed and maximizes possible entropy.

- $H(X,Y)$
  Entropy of joint distribution is given by formula:

  $$H(X,Y) = \sum_x \sum_y p(x,y) * -\log_2 p(x,y) \tag{8}$$

  And equals **41/4**

Problem 1 continued on next page. . .

- $H(X|Y)$ Entropy of conditional distribution is given by formula:

$$H(X|Y) = \sum_{y} p(y)H(X|Y=y) \tag{9}$$

$$= \sum_{y} p(y) \sum_{x} p(x|y) * (-\log_2 p(x|y)) \tag{10}$$

$$= \sum_{y} \sum_{x} p(x,y) * (-\log_2 p(x|y)) \tag{11}$$

$$= \sum_{y} \sum_{x} p(x,y) * (-\log_2 \frac{p(x,y)}{p(x)}) \tag{12}$$

Based on the property:

$$H(X,Y) = H(X) + H(Y|X) \tag{13}$$

Conditional entropy is equal to **41/4 - 7/4 = 34/4**

- $I(X;Y)$ Mutual information between random variables X and Y is given by:

$$I(X;Y) = \sum_{x} \sum_{y} p(x,y) \log_2 \frac{p(x,y)}{p(x)p(y)} \tag{14}$$

Above formula unrolls into following equation for our distribution:

$$1/8 * -\log_2(\frac{1/16}{1/8}) + 1/8 * \log_2(\frac{1/8}{1/16}) + 1/8 * -\log_2(\frac{1/16}{1/32}) + 1/4 * \log_2(\frac{1/4}{1/8}) \tag{15}$$

$$1/2 + 1/8 = 5/8 \tag{16}$$

Problem 2

Assueme we have e attributes, whose distribution is described by random variables X_1, X_2, X_3 respectively. Attributes fulfill following properties:

$$ \tag{17} $$

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