

1. An oil company purchased an option on land in Alaska. Preliminary Geological studies assigned the following probabilities:

$$P(\text{high-quality oil}) = 0.5$$

$$P(\text{medium-quality oil}) = 0.2$$

$$P(\text{no oil}) = 0.3$$

a) What is the probability of finding oil?

b) After 200 feet of drilling on the first well, a soil test is made. The probabilities of finding the particular type of soil identified by the test are

$$P(\text{Soil} \mid \text{high quality oil}) = 0.2$$

$$P(\text{Soil} \mid \text{medium-quality oil}) = 0.8$$

$$P(\text{Soil} \mid \text{no oil}) = 0.2$$

How should the firm interpret the soil test? What are the revised probabilities, and what is the new probability of finding oil?

$$P(H) = 0.5 \quad P(M) = 0.2 \quad P(N) = 0.3$$

$$a) \quad P(H \cup M) = P(H) + P(M) - P(H \cap M) = 0.5 + 0.2 = 0.7$$

$$b) \quad P(S \mid H) = 0.2, \quad P(S \mid M) = 0.8, \quad P(S \mid N) = 0.2$$

$$P(H \mid S) = ?, \quad P(M \mid S) = ?, \quad P(N \mid S) = ?$$

$$\begin{aligned} P(H \mid S) &= \frac{P(S \mid H) \cdot P(H)}{P(S \mid H) \cdot P(H) + P(S \mid M) \cdot P(M) + P(S \mid N) \cdot P(N)} \\ &= \frac{0.2 \times 0.5}{0.2 \times 0.5 + 0.8 \times 0.2 + 0.2 \times 0.3} = \frac{5}{16} \end{aligned}$$

$$P(M \mid S) = \frac{0.8 \times 0.2}{0.2 \times 0.5 + 0.8 \times 0.2 + 0.2 \times 0.3} = \frac{8}{16}$$

$$P(N \mid S) = 1 - \left(\frac{5+8}{16} \right) = \frac{3}{16}$$

$$P(\text{oil} \mid S) = \frac{13}{16}$$