

$$\frac{df}{dt} = -\lambda f + w \quad | \mathcal{F}(\cdot)$$

$$(i\omega)\hat{f} = -\lambda\hat{f} + \hat{w}$$

$$\hat{f} = \frac{\hat{w}}{i\omega + \lambda}$$

$$S(\omega) = \frac{E[|\hat{w}|^2]}{\omega^2 + \lambda^2} = \frac{q}{\omega^2 + \lambda^2}$$

$$\begin{aligned} h(\tau) &= \frac{1}{2\pi} \int \frac{q}{\omega^2 + \lambda^2} \exp(i\omega\tau) d\omega \\ &= \frac{q}{2\lambda} \exp(-\lambda|\tau|) \end{aligned}$$

$$\begin{aligned} \frac{df}{dt} &= -\lambda f + w \quad | e^{\lambda t} \\ e^{\lambda t} \frac{df}{dt} + \lambda e^{\lambda t} f &= w e^{\lambda t} \\ \frac{d}{dt} (e^{\lambda t} f) &= w e^{\lambda t} \quad | \int \end{aligned}$$

$$e^{\lambda t} f(t) - e^{\lambda t_0} f(t_0) = \int_{t_0}^t e^{\lambda s} w(s) ds$$

$$\begin{aligned} f(t) &= e^{-\lambda(t-t_0)} f(t_0) \\ &+ \int_{t_0}^t e^{-\lambda(t-s)} w(s) ds \end{aligned}$$

$$\begin{aligned}
 & \overbrace{p(\phi(t) | \phi(t_0))}^{\tau_0} \quad \mathbb{Q} \\
 &= \mathcal{N}(\phi(t) | A\phi(t_0), \mathbb{Q}) \\
 & A = e^{-\lambda(t-t_0)} \\
 & \mathbb{Q} = \int_{t_0}^t e^{-2\lambda(t-s)} g \, ds \\
 &= \frac{g}{2\lambda} (1 - e^{-2\lambda(t-t_0)})
 \end{aligned}$$

$$\begin{cases}
 f_{k+1} = A \cdot \phi_k + g, \quad g \sim \mathcal{N}(0, \mathbb{Q}) \\
 y_k = H \cdot \phi_k + \epsilon
 \end{cases}$$