$$\frac{df}{dt} = -\lambda f + w \qquad | \mathcal{F}(\cdot)$$

$$(i\omega)\hat{f} = -\lambda \hat{f} + \hat{w}$$

$$\hat{f} = \frac{\hat{w}}{i\omega + \lambda}$$

$$S(\omega) = \frac{E[|\hat{w}|^2]}{\omega^2 + \lambda^2} = \frac{q}{\omega^2 + \lambda^2}$$

$$h(\tau) = \frac{1}{2\pi}\int \frac{q}{\omega^2 + \lambda^2} \exp(i\omega\tau)\, d\omega$$

$$= \frac{q}{2\lambda} \exp(-\lambda|\tau|)$$

$$\frac{df}{dt} = -\lambda f + w \qquad | e^{\lambda t}$$

$$e^{\lambda t}\frac{df}{dt} + \lambda e^{\lambda t} f = e^{\lambda t} w$$

$$\frac{d}{dt}\left(e^{\lambda t} f\right) = e^{\lambda t} w \qquad | \int_{t_0}^{t}$$

$$e^{\lambda t} f(t) - e^{\lambda t_0} f(t_0) = \int_{t_0}^{t} e^{\lambda s} w(s)\, ds$$

$$f(t) = \overbrace{e^{-\lambda(t-t_0)}}^{A} f(t_0) + \int^{t} e^{-\lambda(t-t_0)} w(s)\, ds$$

T_0 Q

$$p(f(t) \mid f(t_0))$$
$$= \mathcal{N}(f(t) \mid A f(t_0), Q)$$
$$A = e^{-\lambda(t - t_0)}$$
$$Q = \int_{t_0}^{t} e^{-2\lambda(t-s)} q \, ds$$
$$= \frac{q}{2\lambda}\left(1 - e^{-2\lambda(t - t_0)}\right)$$

$$\begin{cases} f_{k+1} = A \cdot f_k + q, \quad q \sim \mathcal{N}(0, Q) \\ y_k = H \cdot f_k + \varepsilon \end{cases}$$