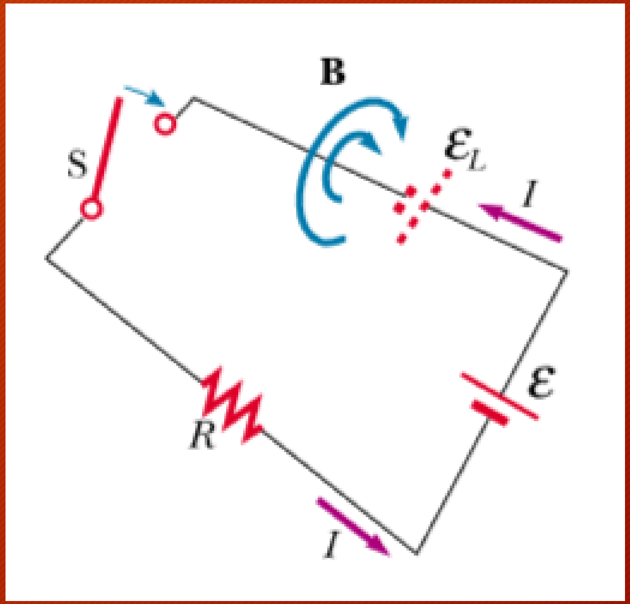
29-Inductors and Inductance

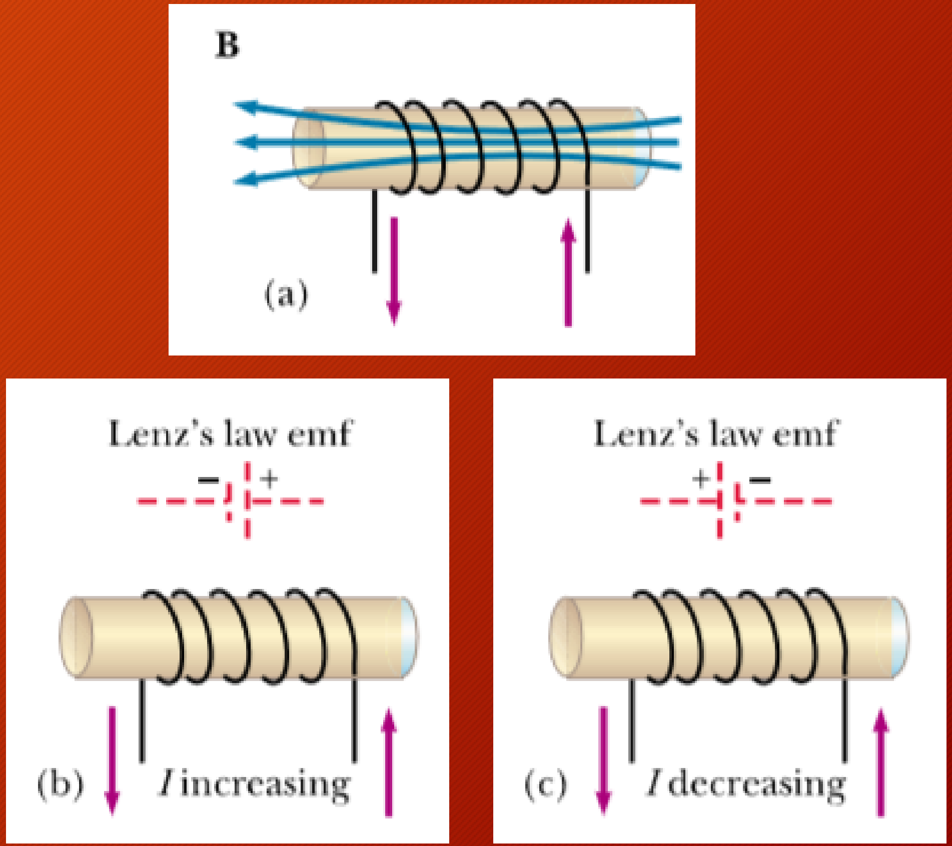
Self Inductance

- Consider a circuit consisting of a switch, a resistor and a source of emf.
- When the switch is thrown closed, the current produce a magnetic flux through the area enclosed by the loop.
- As the current increases towards its equilibrium value, this magnetic flux changes in time and induces an emf in the loop.
- The battery symbol drawn with dashed lines represents the self induced emf.



Self Inductance

- Consider a coil wound on a cylindrical iron core.
- A current in the coil produces a magnetic field directed to the left.
- If the current increases, the increasing magnetic flux creates an induced emf having polarity shown by the dashed battery.
- The polarity of the induced emf reverses if the current decreases.



Self Inductance

- The magnetic flux is proportional to the magnetic field due to the source current, which in turn is proportional to the source current.
- A self induced emf $\mathcal{E}_L$ is always proportional to the time rate of change of the source current.

$$\mathcal{E}_L = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

- Where L is a proportionality constant ----- called the inductance of the coil.

Inductance of an N-turn coil

- Here L is a proportionality constant –called the inductance of the coil---that depends on the geometry of the circuit and other physical characteristics.

$$L = \frac{N\Phi_B}{I}$$

- We can also write the inductance as the ratio

$$L = -\frac{\mathcal{E}_L}{dI/dt}$$

- Just as resistance is a measure of the opposition to current (R=ΔV/I), inductance is a measure of the opposition to a change in a current.

Unit of Inductance

- The SI unit of inductance is the henry(H), which is 1 volt-second per ampere.

$$1\ \mathrm{H} = 1\ \frac{\mathrm{V}\cdot\mathrm{s}}{\mathrm{A}}$$

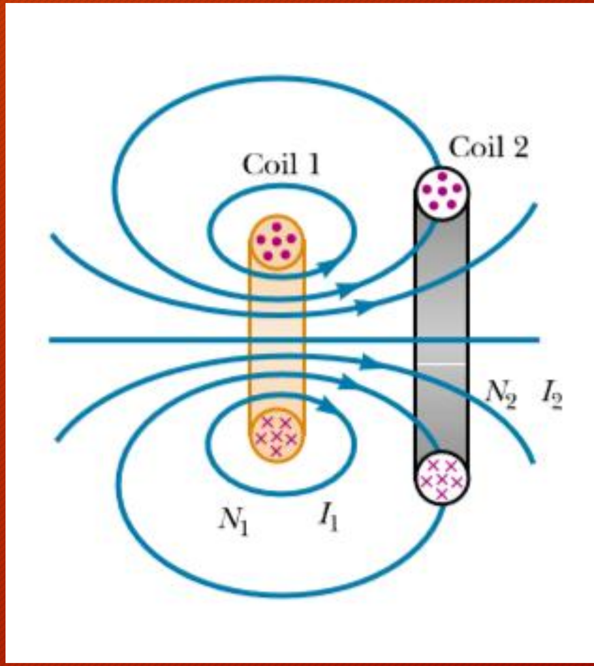
Mutual Inductance

- Very often, The magnetic flux through the area enclosed by a circuit varies with time because of time-varying currents in nearby circuits. This condition induces an emf through a process known as mutual induction, because it depends on the interaction of two circuits.

Definition of Mutual Induction

- Consider the two closely wound coils of wire shown in cross-sectional view. The current I1 in coil 1, which has N1 turns, creates magnetic field lines, some of which pass through coil 2, which has N2 turns. The magnetic flux caused by the current in coil 1 and passing through coil 2 is represented by Φ12.
- We define the mutual inductance M12 of coil 2 with respect to coil 1:

$$M_{12} \equiv \frac{N_2 \Phi_{12}}{I_1}$$



EXAMPLE 32.1 Inductance of a Solenoid

Find the inductance of a uniformly wound solenoid having N turns and length ℓ. Assume that ℓ is much longer than the radius of the windings and that the core of the solenoid is air.

$$B = \mu_0 n I = \mu_0 \frac{N}{\ell} I$$

where $n = N/\ell$ is the number of turns per unit length. The magnetic flux through each turn is

$$\Phi_B = BA = \mu_0 \frac{NA}{\ell} I$$

where A is the cross-sectional area of the solenoid. Using this expression and Equation 32.2, we find that

$$L = \frac{N\Phi_B}{I} = \boxed{\frac{\mu_0 N^2 A}{\ell}} \tag{32.4}$$

This result shows that L depends on geometry and is proportional to the square of the number of turns. Because $N = n\ell$, we can also express the result in the form

$$L = \mu_0 \frac{(n\ell)^2}{\ell} A = \mu_0 n^2 A\ell = \mu_0 n^2 V \tag{32.5}$$

where $V = A\ell$ is the volume of the solenoid.

EXAMPLE 32.2 Calculating Inductance and emf

(a) Calculate the inductance of an air-core solenoid containing 300 turns if the length of the solenoid is 25.0 cm and its cross-sectional area is 4.00 cm^2.

Solution Using Equation 32.4, we obtain

$$L = \frac{\mu_0 N^2 A}{\ell}$$

$$= (4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})\,\frac{(300)^2(4.00 \times 10^{-4}\ \mathrm{m}^2)}{25.0 \times 10^{-2}\ \mathrm{m}}$$

$$= 1.81 \times 10^{-4}\ \mathrm{T\cdot m^2/A} = \boxed{0.181\ \mathrm{mH}}$$

(b) Calculate the self-induced emf in the solenoid if the current through it is decreasing at the rate of 50.0 A/s.

Solution Using Equation 32.1 and given that $dI/dt = -50.0$ A/s, we obtain

$$\mathcal{E}_L = -L\frac{dI}{dt} = -(1.81 \times 10^{-4}\ \mathrm{H})(-50.0\ \mathrm{A/s})$$

$$= \boxed{9.05\ \mathrm{mV}}$$