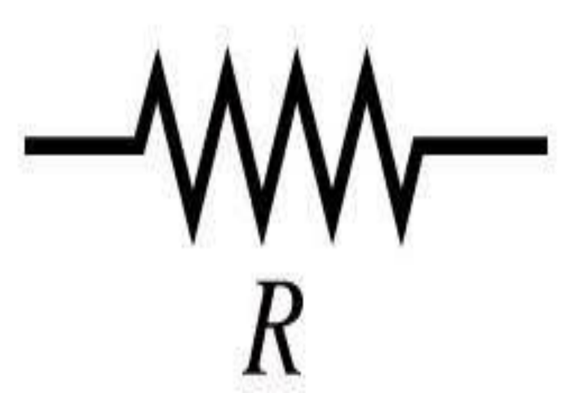
30-Resistors and Capacitors

Resistor

- A resistor is a passive two-terminal electrical component that implements electrical resistance as a circuit element.

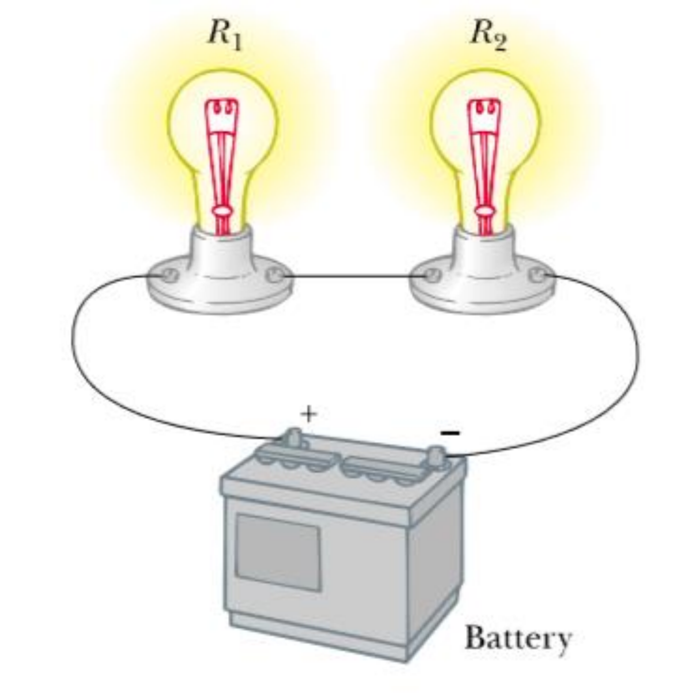
- SI unit of R is volts per ampere. One volt per ampere is defined to be 1 ohm (Ω)



$$1\ \Omega \equiv \frac{1\ \text{V}}{1\ \text{A}}$$

Resistors in Series

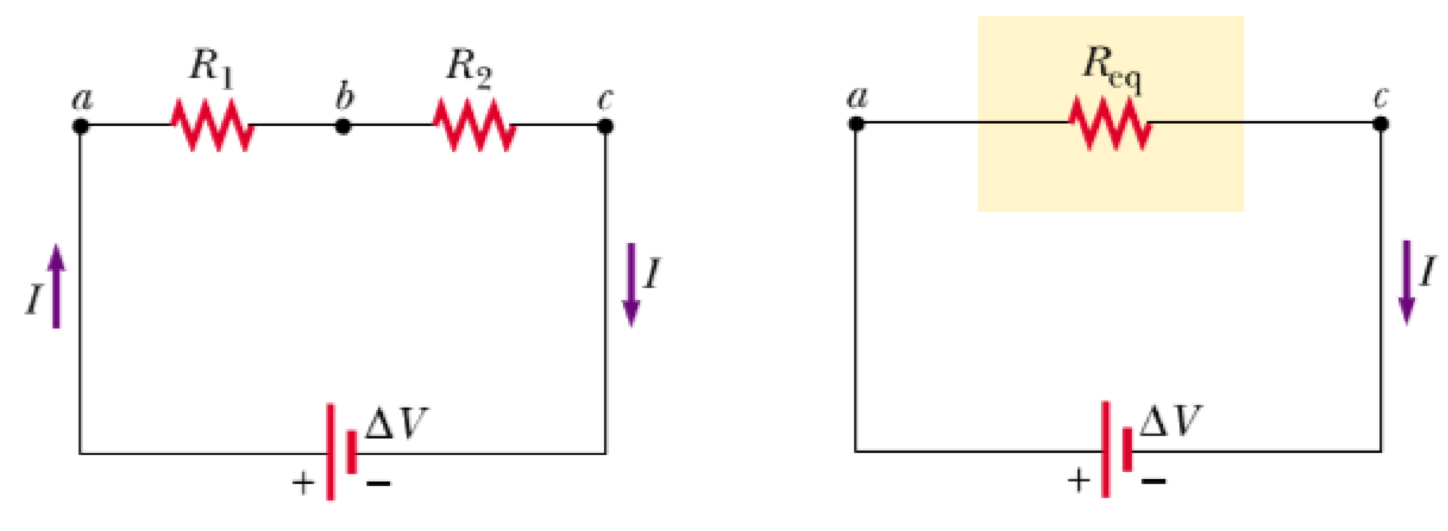
- For a series combination of resistors, the current in the two resistors are the same because any charge that passes through R_1 must also pass through R_2.
- The potential difference applied across the series combination of resistors will divide between the resistors.



Resistors in Series

- The voltage drop from *a* to *b* equals IR_1 and the voltage drop from *b* to *c* equals IR_2, the voltage drop from *a* to *c* is:

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$



Resistors in Series

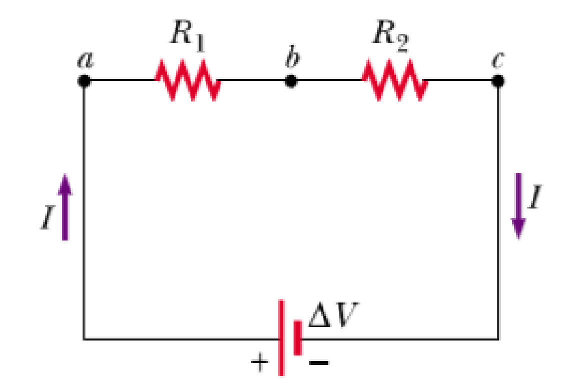
- We can replace the two resistors in series with a single resistor having an equivalent resistance R_{eq}, where

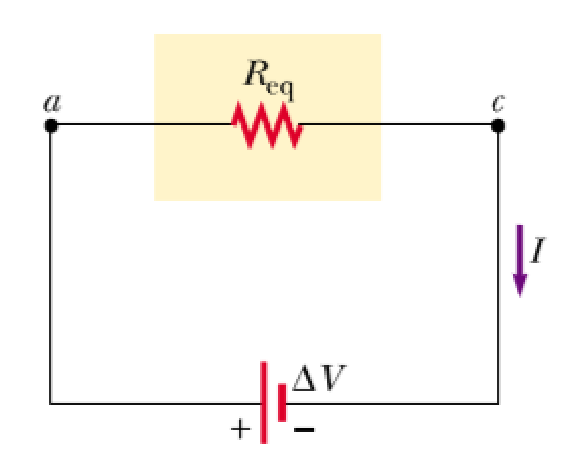
$$R_{eq} = R_1 + R_2$$

- The equivalent resistance of three or more resistors connected in series is:

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

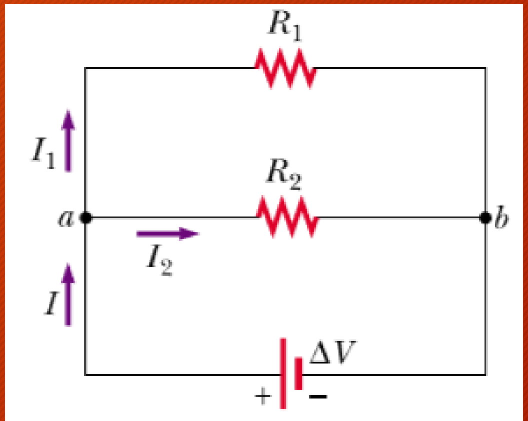
- The equivalent resistance of a series connection of resistors is always greater than any individual resistance.

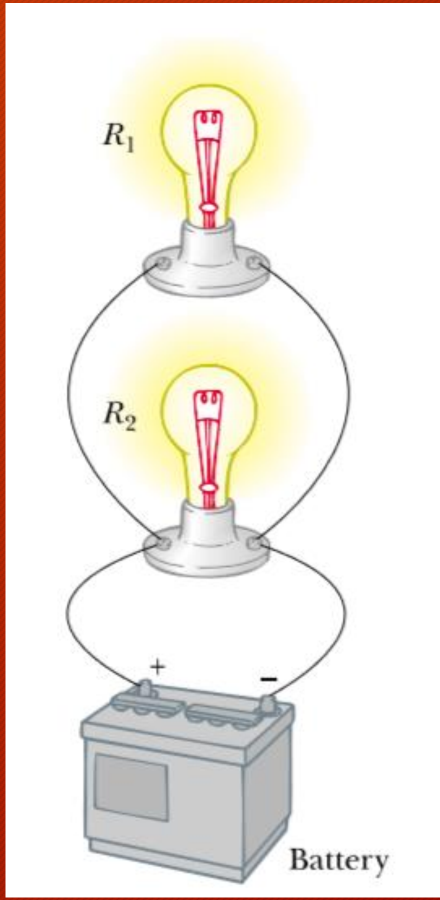




Resistors in Parallel

- Now consider two resistors connected in parallel. When the current ***I*** reach point ***a*** called a ***junction***, it splits into two parts, with I_1 going through R_1 and I_2 going through R_2. Because charge must be conserved, the current ***I*** that enter point a must equal the total current leaving that point: $I = I_1 + I_2$

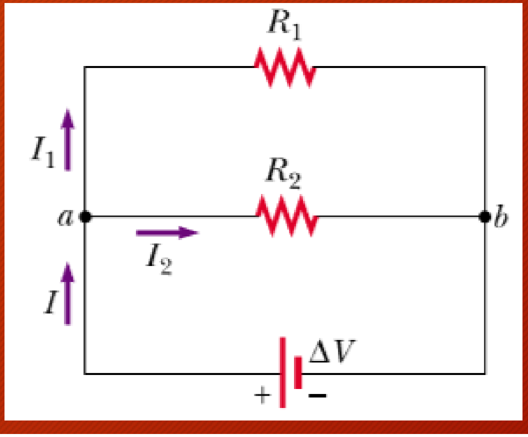




Resistors in Parallel

- When resistors are connected in parallel, the potential difference across them are the same.
  The expression ***ΔV = I R*** gives:

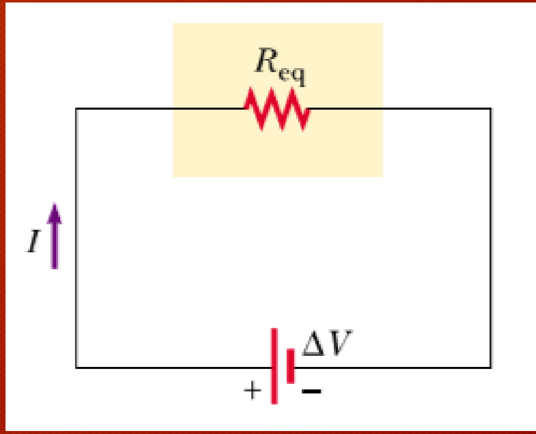
$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{eq}}$$



- The equivalent resistance of two resistors in parallel is given by:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$



Resistors in Parallel

- The equivalent resistance of several resistors in parallel is given by:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

- The equivalent resistance of two or more resistors connected in parallel is always less than the least resistance in the group

EXAMPLE 28.3 Find the Equivalent Resistance

Four resistors are connected as shown in Figure 28.6a. (a) Find the equivalent resistance between points *a* and *c*.

Solution The combination of resistors can be reduced in steps, as shown in Figure 28.6. The 8.0-Ω and 4.0-Ω resistors are in series; thus, the equivalent resistance between *a* and *b* is 12 Ω (see Eq. 28.5). The 6.0-Ω and 3.0-Ω resistors are in parallel, so from Equation 28.7 we find that the equivalent resistance from *b* to *c* is 2.0 Ω. Hence, the equivalent resistance from *a* to *c* is 14 Ω.

(b) What is the current in each resistor if a potential difference of 42 V is maintained between *a* and *c*?

Solution The currents in the 8.0-Ω and 4.0-Ω resistors are the same because they are in series. In addition, this is the same as the current that would exist in the 14-Ω equivalent resistor subject to the 42-V potential difference. Therefore, using Equation 27.8 ($R = \Delta V/I$) and the results from part (a), we obtain

$$I = \frac{\Delta V_{ac}}{R_{eq}} = \frac{42\text{ V}}{14\ \Omega} = 3.0\text{ A}$$

This is the current in the 8.0-Ω and 4.0-Ω resistors. When this 3.0-A current enters the junction at *b*, however, it splits, with part passing through the 6.0-Ω resistor (I_1) and part through the 3.0-Ω resistor (I_2). Because the potential difference is ΔV_{bc} across each of these resistors (since they are in parallel), we see that $(6.0\ \Omega)I_1 = (3.0\ \Omega)I_2$, or $I_2 = 2I_1$. Using this result and the fact that $I_1 + I_2 = 3.0$ A, we find that $I_1 = 1.0$ A and $I_2 = 2.0$ A. We could have guessed this at the start by noting that the current through the 3.0-Ω resistor has to be twice that through the 6.0-Ω resistor, in view of their relative resistances and the fact that the same voltage is applied to each of them.

As a final check of our results, note that $\Delta V_{bc} = (6.0\ \Omega)I_1 = (3.0\ \Omega)I_2 = 6.0$ V and $\Delta V_{ab} = (12\ \Omega)I = 36$ V; therefore, $\Delta V_{ac} = \Delta V_{ab} + \Delta V_{bc} = 42$ V, as it must.

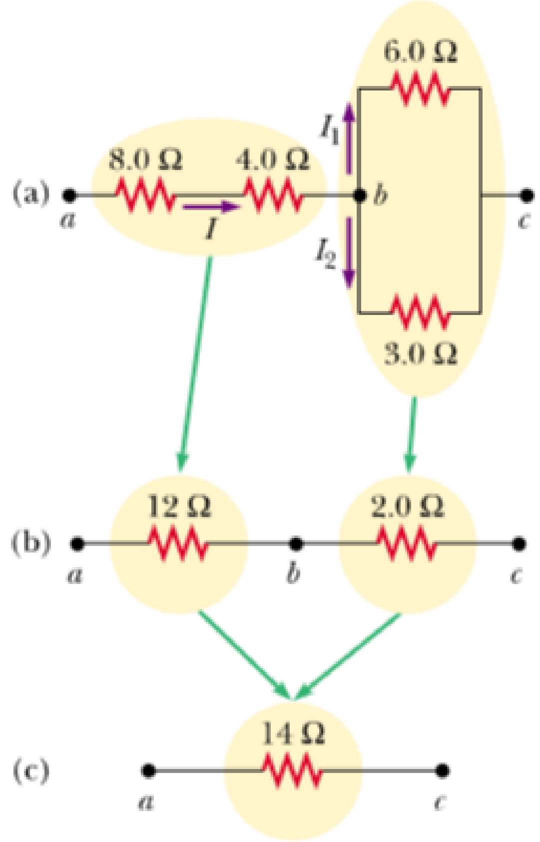


Figure 28.6

EXAMPLE 28.4 Three Resistors in Parallel

Three resistors are connected in parallel as shown in Figure 28.7. A potential difference of 18 V is maintained between points *a* and *b*. (a) Find the current in each resistor.

Solution The resistors are in parallel, and so the potential difference across each must be 18 V. Applying the relationship $\Delta V = IR$ to each resistor gives

$$I_1 = \frac{\Delta V}{R_1} = \frac{18\text{ V}}{3.0\ \Omega} = \boxed{6.0\text{ A}}$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18\text{ V}}{6.0\ \Omega} = \boxed{3.0\text{ A}}$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18\text{ V}}{9.0\ \Omega} = \boxed{2.0\text{ A}}$$

(b) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.

Solution We apply the relationship $\mathcal{P} = (\Delta V)^2/R$ to each resistor and obtain

$$\mathcal{P}_1 = \frac{\Delta V^2}{R_1} = \frac{(18\text{ V})^2}{3.0\ \Omega} = \boxed{110\text{ W}}$$

$$\mathcal{P}_2 = \frac{\Delta V^2}{R_2} = \frac{(18\text{ V})^2}{6.0\ \Omega} = \boxed{54\text{ W}}$$

$$\mathcal{P}_3 = \frac{\Delta V^2}{R_3} = \frac{(18\text{ V})^2}{9.0\ \Omega} = \boxed{36\text{ W}}$$

This shows that the smallest resistor receives the most power. Summing the three quantities gives a total power of 200 W.

(c) Calculate the equivalent resistance of the circuit.

Solution We can use Equation 28.8 to find R_{eq}:

$$\frac{1}{R_{eq}} = \frac{1}{3.0\ \Omega} + \frac{1}{6.0\ \Omega} + \frac{1}{9.0\ \Omega}$$

$$= \frac{6}{18\ \Omega} + \frac{3}{18\ \Omega} + \frac{2}{18\ \Omega} = \frac{11}{18\ \Omega}$$

$$R_{eq} = \frac{18\ \Omega}{11} = \boxed{1.6\ \Omega}$$

Exercise Use R_{eq} to calculate the total power delivered by the battery.

Answer 200 W.

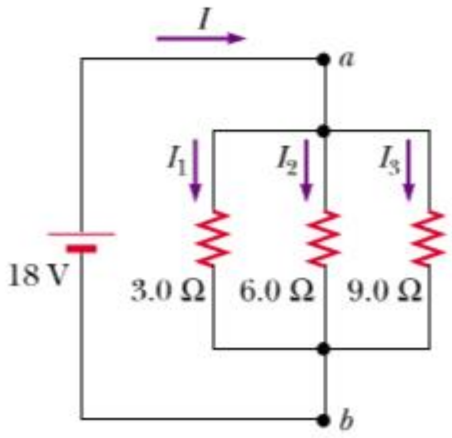
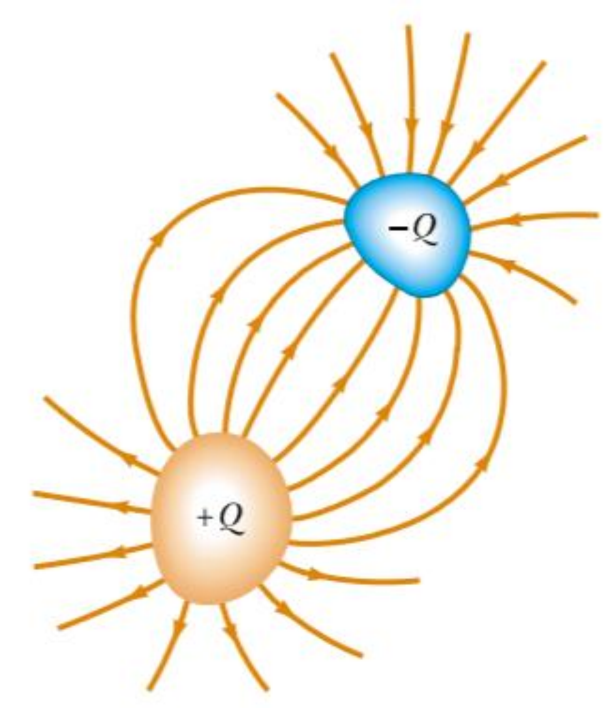


Figure 28.7 Three resistors connected in parallel. The voltage across each resistor is 18 V.

Capacitor

- Consider two conductors carrying charges of equal magnitude but of opposite sign, such a combination of two conductors is called capacitor. The conductors are called *plates*.



Definition of Capacitance

- The ***capacitance*** C of a capacitor is the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between them.

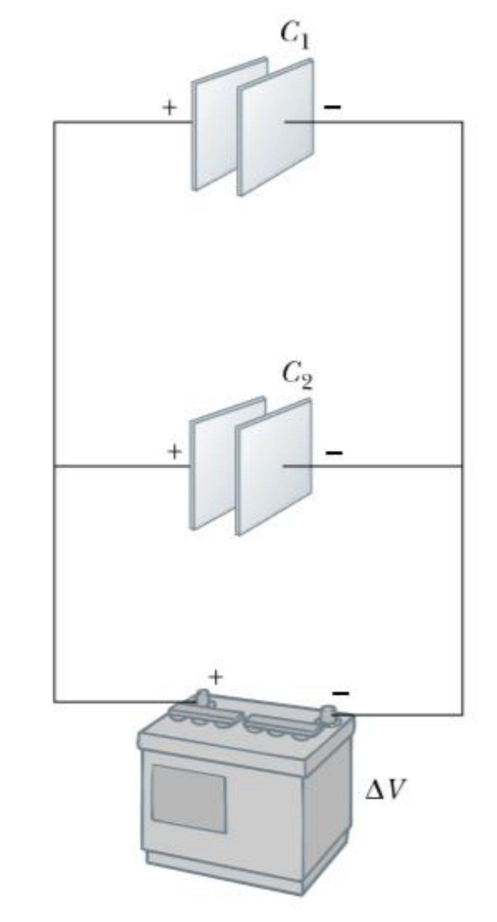
$$C \equiv \frac{Q}{\Delta V}$$

- The SI unit of capacitance is the farad (F)

$$1\ \mathrm{F} = 1\ \mathrm{C/V}$$

Parallel Combination of Capacitors

- The individual potential differences across capacitors connected in parallel are all the same and are equal to the potential difference applied across the combination.



Parallel Combination of Capacitors

- The total charge strored by two capacitor is

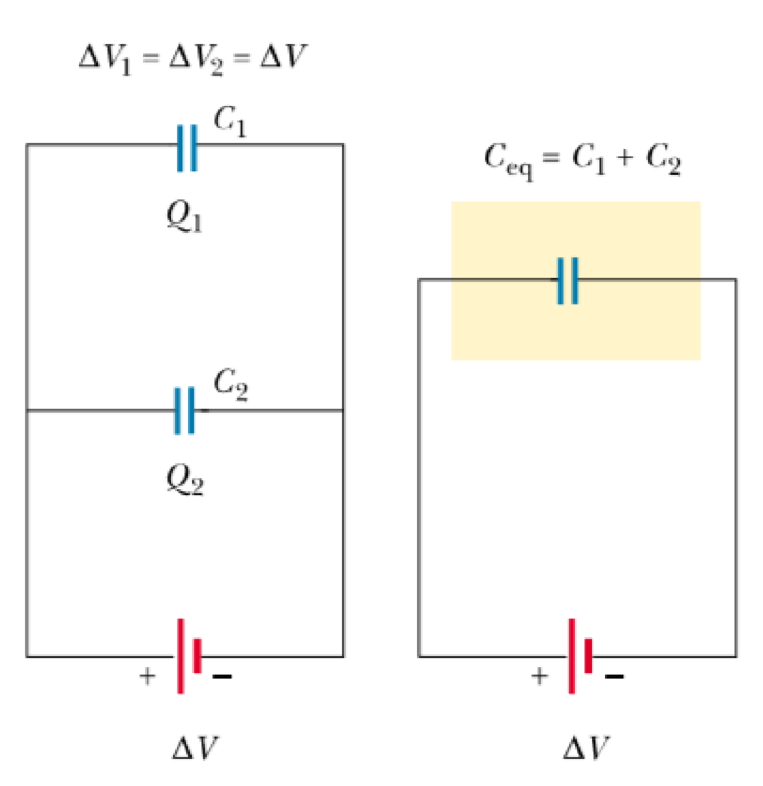
$$Q = Q_1 + Q_2$$

- The voltages across the capacitors are the same the charges that they carry are

$$Q_1 = C_1 \Delta V \qquad Q_2 = C_2 \Delta V$$

- For the equivalent capacitors:

$$Q = C_{eq} \Delta V$$



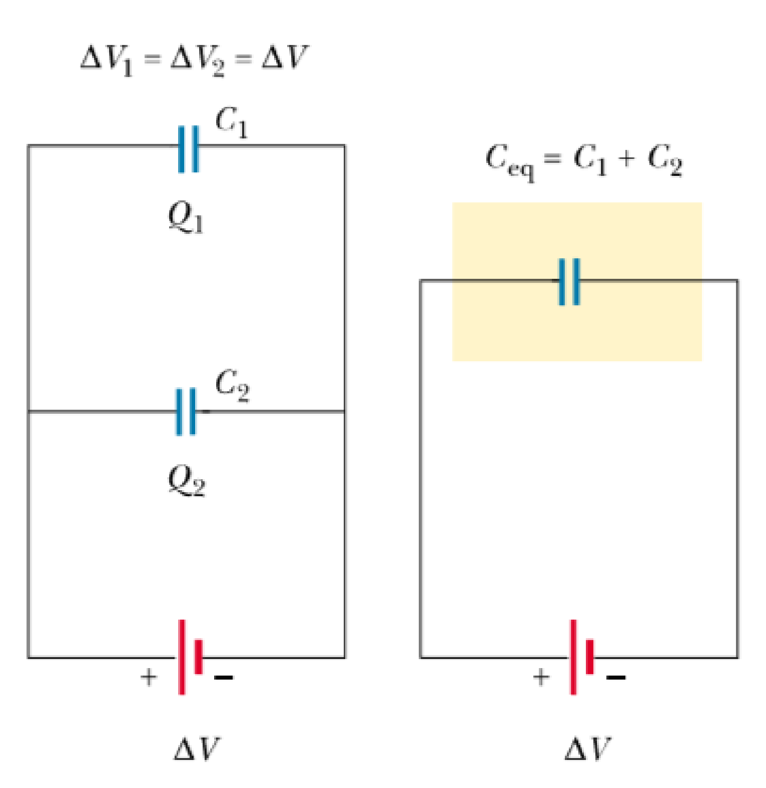
Parallel Combination of Capacitors

$$C_{eq}\,\Delta V = C_1\,\Delta V + C_2\,\Delta V$$

$$C_{eq} = C_1 + C_2 \qquad \begin{pmatrix}\text{parallel}\\ \text{combination}\end{pmatrix}$$

If we extend this treatment to three or more capacitors connected in parallel, we find the equivalent capacitance to be

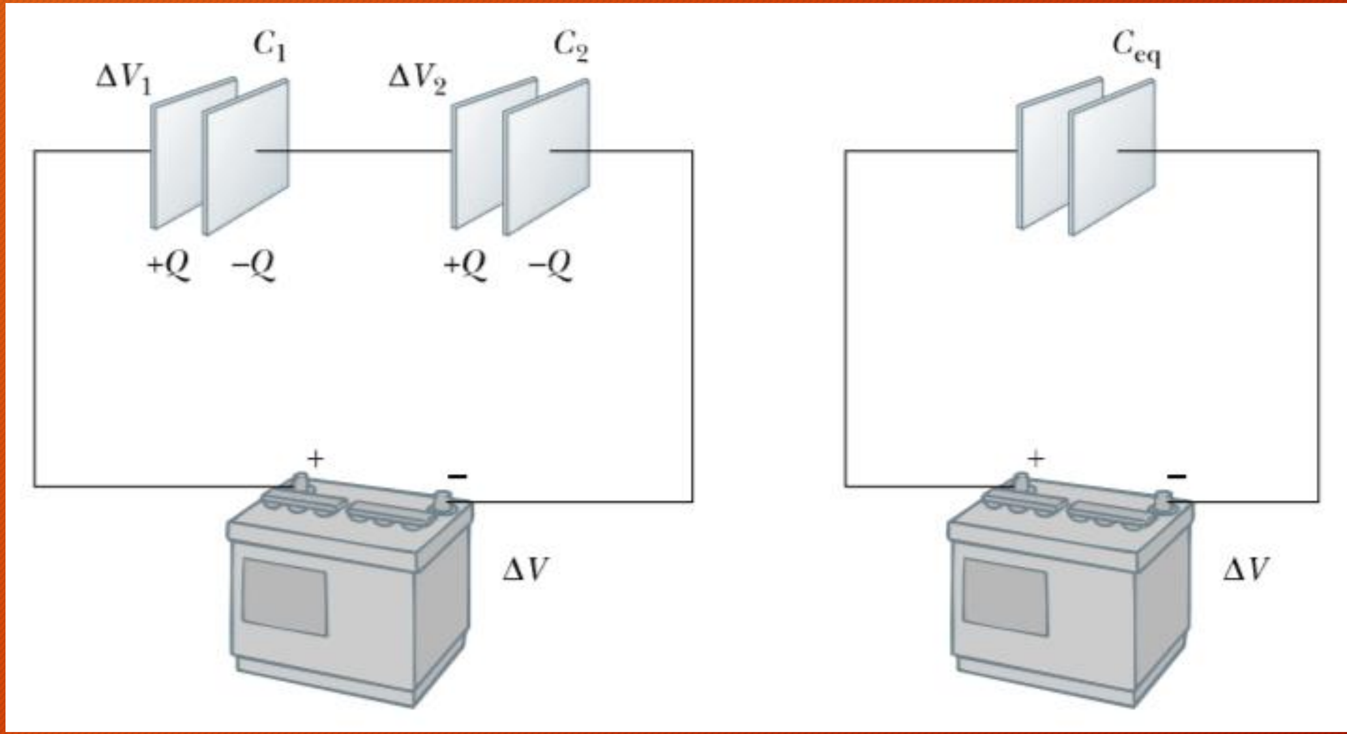
$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel combination)}$$



- The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitances.

Series Combination of Capacitors

- Two capacitors connected as shown in figure are known as series combination of capacitors.



Series Combination of Capacitors

- The charges on capacitors connected in series are the same.
- The voltage ΔV across the battery terminals is split between the two capacitors.

$$\Delta V = \Delta V_1 + \Delta V_2$$

- Applying the definition of capacitance

$$\Delta V = \frac{Q}{C_{eq}}$$

Series Combination of Capacitors

- The potential difference across each is

$$\Delta V_1 = \frac{Q}{C_1} \qquad \Delta V_2 = \frac{Q}{C_2}$$

- Noting that $\Delta V = Q/C_{eq}$ we have

$$\frac{Q}{C_{\mathrm{eq}}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

- Cancelling Q, we arrive at the relationship

$$\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Series Combination of Capacitors

- When this analysis is applied to three or more capacitors connected in series, the relationship for equivalent capacitance is

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

- The equivalent capacitance of a series combination is always less than the individual capacitance in the combination.

Example 26.4 Equivalent Capacitance

Find the equivalent capacitance between a and b for the combination of capacitors shown in Figure 26.10a. All capacitances are in microfarads.

Solution Using Equations 26.8 and 26.10, we reduce the combination step by step as indicated in the figure. The 1.0-μF and 3.0-μF capacitors are in parallel and combine according to the expression $C_{eq} = C_1 + C_2 = 4.0\ \mu$F. The 2.0-$\mu$F and 6.0-$\mu$F capacitors also are in parallel and have an equivalent capacitance of 8.0 μF. Thus, the upper branch in Figure 26.10b consists of two 4.0-μF capacitors in series, which combine as follows:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0\ \mu\text{F}} + \frac{1}{4.0\ \mu\text{F}} = \frac{1}{2.0\ \mu\text{F}}$$

$$C_{eq} = \frac{1}{1/2.0\ \mu\text{F}} = 2.0\ \mu\text{F}$$

The lower branch in Figure 26.10b consists of two 8.0-μF capacitors in series, which combine to yield an equivalent capacitance of 4.0 μF. Finally, the 2.0-μF and 4.0-μF capacitors in Figure 26.10c are in parallel and thus have an equivalent capacitance of 6.0 μF.

Exercise Consider three capacitors having capacitances of 3.0 μF, 6.0 μF, and 12 μF. Find their equivalent capacitance when they are connected (a) in parallel and (b) in series.

Answer (a) 21 μF; (b) 1.7 μF.