

Microwave Engineering David M. Pozar

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conductors, and the shunt conductance G is due to dielectric loss in the material between the conductors. R and G , therefore, represent loss. A finite length of transmission line can be viewed as a cascade of sections of the form shown in Figure 2.1b.

From the circuit of Figure 2.1b, Kirchhoff's voltage law can be applied to give

$$v(z, t) - R\Delta z i(z, t) - L\Delta z \frac{\partial i(z, t)}{\partial t} - v(z + \Delta z, t) = 0, \quad (2.1a)$$

and Kirchhoff's current law leads to

$$i(z, t) - G\Delta z v(z + \Delta z, t) - C\Delta z \frac{\partial v(z + \Delta z, t)}{\partial t} - i(z + \Delta z, t) = 0. \quad (2.1b)$$

Dividing (2.1a) and (2.1b) by Δz and taking the limit as $\Delta z \rightarrow 0$ gives the following differential equations:

$$\frac{\partial v(z, t)}{\partial z} = -Ri(z, t) - L \frac{\partial i(z, t)}{\partial t}, \quad (2.2a)$$
$$\frac{\partial i(z, t)}{\partial z} = -Gv(z, t) - C \frac{\partial v(z, t)}{\partial t}. \quad (2.2b)$$

These are the time domain form of the transmission line equations, also known as the *telegrapher equations*. I

For the sinusoidal steady-state condition, with cosine-based phasors, (2.2a) and (2.2b) simplify to

$$\frac{dV(z)}{dz} = -(R + j\omega L)I(z), \quad (2.3a)$$
$$\frac{dI(z)}{dz} = -(G + j\omega C)V(z). \quad (2.3b)$$

50 Chapter 2: Transmission Line Theory

Note the similarity in the form of (2.3a) and (2.3b) and Maxwell's curl equations of (1.41a) and (1.41b).

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Wave Propagation on a Transmission Line

The two equations (2.3a) and (2.3b) can be solved simultaneously to give wave equations for $V(z)$ and $I(z)$:

$$\frac{d^2 V(z)}{dz^2} - \gamma^2 V(z) = 0, \quad (2.4a)$$
$$\frac{d^2 I(z)}{dz^2} - \gamma^2 I(z) = 0, \quad (2.4b)$$

where

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where

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (2.5)$$

is the complex propagation constant, which is a function of frequency. Traveling wave solutions to (2.4) can be found as

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}, \quad (2.6a)$$
$$I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z}, \quad (2.6b)$$

where the $e^{-\gamma z}$ term represents wave propagation in the $+z$ direction, and the $e^{\gamma z}$ term represents wave propagation in the $-z$ direction. Applying (2.3a) to the voltage of (2.6a) gives the current on the line:

$$I(z) = \frac{\gamma}{R + j\omega L} (V_o^+ e^{-\gamma z} - V_o^- e^{\gamma z}).$$

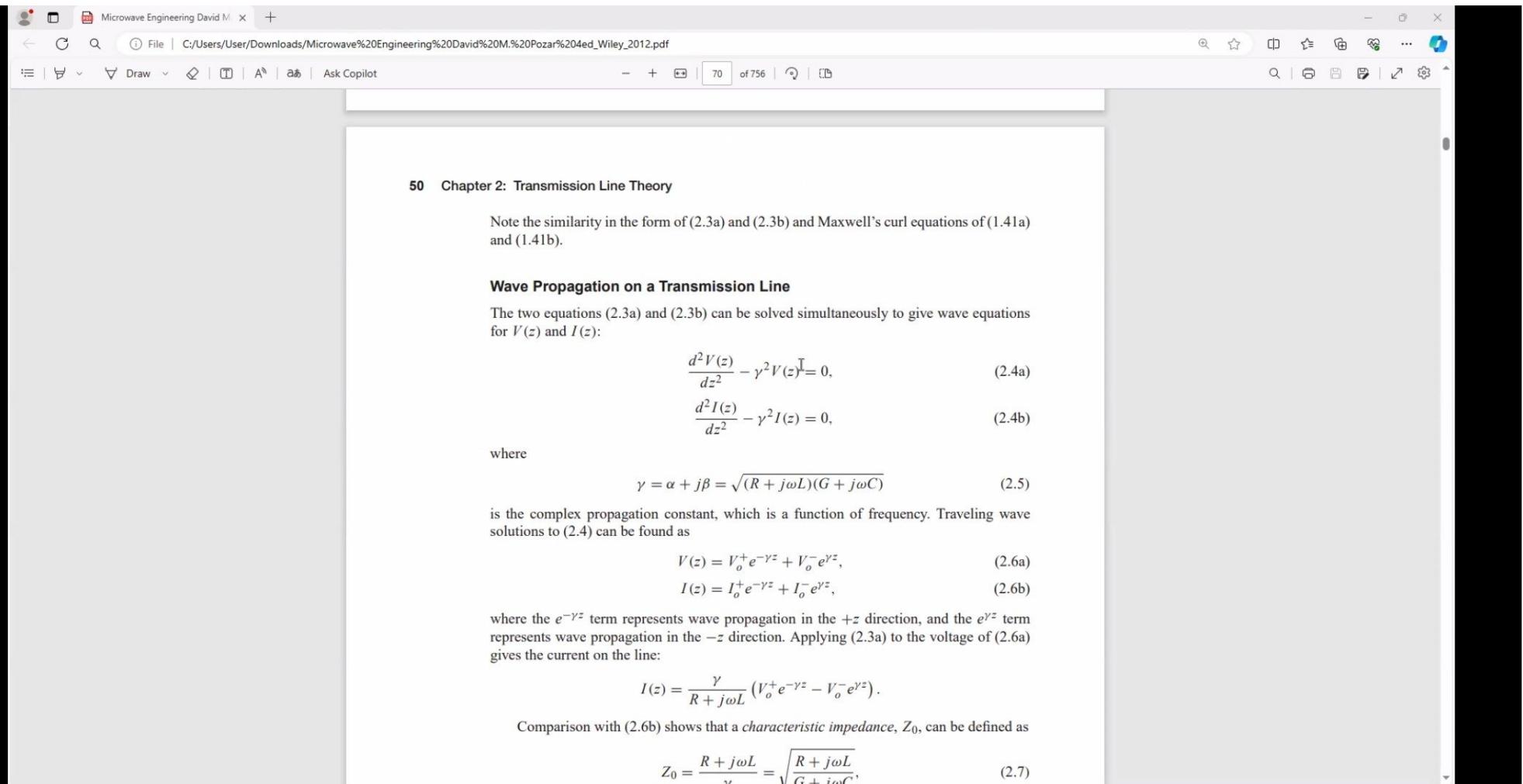
Comparison with (2.6b) shows that a *characteristic impedance*, Z_0 , can be defined as

$$Z_0 = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}, \quad (2.7)$$

to relate the voltage and current on the line as follows:

$$\frac{V_o^+}{I_o^+} = Z_0 = \frac{-V_o^-}{I_o^-}.$$

Then (2.6b) can be rewritten in the following form:



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THIS PROBLEM WILL ILLUSTRATE WAVE REFLECTION ON TRANSMISSION LINES, A FUNDAMENTAL PROPERTY OF DISTRIBUTED SYSTEMS.

Assume that an incident wave of the form $V_o^+ e^{-j\beta z}$ is generated from a source at $z < 0$. We have seen that the ratio of voltage to current for such a traveling wave is Z_0 , the characteristic impedance of the line. However, when the line is terminated in an arbitrary load $Z_L \neq Z_0$, the ratio of voltage to current at the load must be Z_L . Thus, a reflected wave

2.3 The Terminated Lossless Transmission Line 57

FIGURE 2.4 A transmission line terminated in a load impedance Z_L .

must be excited with the appropriate amplitude to satisfy this condition. The total voltage on the line can then be written as in (2.14a), as a sum of incident and reflected waves:

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}. \quad (2.34a)$$

Similarly, the total current on the line is described by (2.14b):

$$I(z) = \frac{V_o^+}{Z_0} e^{-j\beta z} - \frac{V_o^-}{Z_0} e^{j\beta z}. \quad (2.34b)$$

The total voltage and current at the load are related by the load impedance, so at $z = 0$ we must have

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_o^+ + V_o^-}{V_o^+ - V_o^-} Z_0.$$

Solving for V_o^- gives

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$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0, \quad (2.4b)$$

where

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (2.5)$$

is the complex propagation constant, which is a function of frequency. Traveling wave solutions to (2.4) can be found as

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}, \quad (2.6a)$$
$$I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z}, \quad (2.6b)$$

where the $e^{-\gamma z}$ term represents wave propagation in the $+z$ direction, and the $e^{\gamma z}$ term represents wave propagation in the $-z$ direction. Applying (2.3a) to the voltage of (2.6a) gives the current on the line:

$$I(z) = \frac{\gamma}{R + j\omega L} (V_o^+ e^{-\gamma z} - V_o^- e^{\gamma z}).$$

Comparison with (2.6b) shows that a *characteristic impedance*, Z_0 , can be defined as

$$Z_0 = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}, \quad (2.7)$$

to relate the voltage and current on the line as follows:

$$\frac{V_o^+}{I_o^+} = Z_0 = \frac{-V_o^-}{I_o^-}.$$

Then (2.6b) can be rewritten in the following form:

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$$\frac{d^2}{dz^2} - \gamma^2 I(z) = 0, \quad (2.40)$$

where

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (2.5)$$

is the complex propagation constant, which is a function of frequency. Traveling wave solutions to (2.4) can be found as

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}, \quad (2.6a)$$

$$I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z}, \quad (2.6b)$$

where the $e^{-\gamma z}$ term represents wave propagation in the $+z$ direction, and the $e^{\gamma z}$ term represents wave propagation in the $-z$ direction. Applying (2.3a) to the voltage of (2.6a) gives the current on the line:

$$I(z) = \frac{\gamma}{R + j\omega L} (V_o^+ e^{-\gamma z} - V_o^- e^{\gamma z}).$$

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$$\frac{V_o^+}{I_o^+} = Z_0 \frac{-V_o^-}{I_o^-}.$$

Then (2.6b) can be rewritten in the following form:

$$I(z) = \frac{V_o^+}{Z_0} e^{-\gamma z} - \frac{V_o^-}{Z_0} e^{\gamma z}. \quad (2.8)$$

Converting back to the time domain, we can express the voltage waveform as

$$v(z, t) = |V_o^+| \cos(\omega t - \beta z + \phi^+) e^{-\alpha z} + |V_o^-| \cos(\omega t + \beta z + \phi^-) e^{\alpha z}, \quad (2.9)$$

where ϕ^\pm is the phase angle of the complex voltage V_o^\pm . Using arguments similar to those in Section 1.4, we find that the wavelength on the line is

$$\lambda = \frac{2\pi}{\gamma} \quad (2.10)$$

solutions to (2.4) can be found as

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}, \quad (2.6a)$$

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where the $e^{-\gamma z}$ term represents wave propagation in the $+z$ direction, and the $e^{\gamma z}$ term represents wave propagation in the $-z$ direction. Applying (2.3a) to the voltage of (2.6a) gives the current on the line:

$$I(z) = \frac{\gamma}{R + j\omega L} (V_o^+ e^{-\gamma z} - V_o^- e^{\gamma z}).$$

Comparison with (2.6b) shows that a *characteristic impedance*, Z_0 , can be defined as

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$$\lambda = \frac{2\pi}{\beta}, \quad (2.10)$$

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and the phase velocity is

$$v_p = \frac{\omega}{\beta} = \lambda f. \quad (2.11)$$

The Lossless Line

The above solution is for a general transmission line, including loss effects, and it was seen that the propagation constant and characteristic impedance were complex. In many practical cases, however, the loss of the line is very small and so can be neglected, resulting in a simplification of the results. Setting $R = G = 0$ in (2.5) gives the propagation constant as

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC},$$

or

$$\beta = \omega\sqrt{LC}, \quad (2.12a)$$
$$\alpha = 0. \quad (2.12b)$$

As expected for a lossless line, the attenuation constant α is zero. The characteristic impedance of (2.7) reduces to

$$Z_0 = \sqrt{\frac{L}{C}}, \quad (2.13)$$

which is now a real number. The general solutions for voltage and current on a lossless transmission line can then be written as

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}, \quad (2.14a)$$
$$I(z) = \frac{V_o^+}{Z_0} e^{-j\beta z} - \frac{V_o^-}{Z_0} e^{j\beta z}. \quad (2.14b)$$

The wavelength is λ

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}}, \quad (2.15)$$

and the phase velocity is

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}. \quad (2.16)$$

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$\alpha = \frac{1}{2} \left[\frac{R_s}{\eta \ln b/a} \left(\frac{1}{a} + \frac{1}{b} \right) + \omega \epsilon'' \eta \right],$

where $\eta = \sqrt{\mu/\epsilon'}$ is the intrinsic impedance of the dielectric material filling the coaxial line. In addition, $\beta = \omega\sqrt{LC} = \omega\sqrt{\mu\epsilon'}$ and $Z_0 = \sqrt{L/C} = (\eta/2\pi) \ln b/a$. ■

This method for the calculation of attenuation requires that the line parameters L , C , R , and G be known. These can sometimes be derived using the formulas of (2.17)–(2.20), but a more direct and versatile procedure is to use the perturbation method, to be discussed shortly.

The Distortionless Line

As can be seen from the exact equations (2.82)–(2.83) for the propagation constant of a lossy line, the phase term β is generally a complicated function of frequency ω when loss is present. In particular, we note that β is generally not exactly a linear function of frequency, as in (2.85b), unless the line is lossless. If β is not a linear function of frequency (of the form $\beta = a\omega$), then the phase velocity $v_p = \omega/\beta$ will vary with frequency. The implication of this is that the various frequency components of a wideband signal will travel with different phase velocities and so arrive at the receiver end of the transmission line at slightly different times. This will lead to *dispersion*, a distortion of the signal, and is generally an undesirable effect. Granted, as we have argued, the departure of β from a linear function may be quite small, but the effect can be significant if the line is very long. This effect leads to the concept of group velocity, which we will address in detail in Section 3.10.

There is a special case, however, of a lossy line that has a linear phase factor as a function of frequency. Such a line is called a *distortionless* line, and it is characterized by line parameters that satisfy the relation

$$\frac{R}{L} = \frac{G}{C}. \quad (2.87)$$

From (2.83) the exact complex propagation constant, under the condition specified by (2.87), reduces to

$$\begin{aligned} \gamma &= j\omega\sqrt{LC} \sqrt{1 - 2j\frac{R}{\omega L} - \frac{R^2}{\omega^2 L^2}} \\ &= j\omega\sqrt{LC} \left(1 - j\frac{R}{\omega L} \right) \\ &= R\sqrt{\frac{C}{L}} + j\omega\sqrt{LC} = \alpha + j\beta, \end{aligned} \quad (2.88)$$

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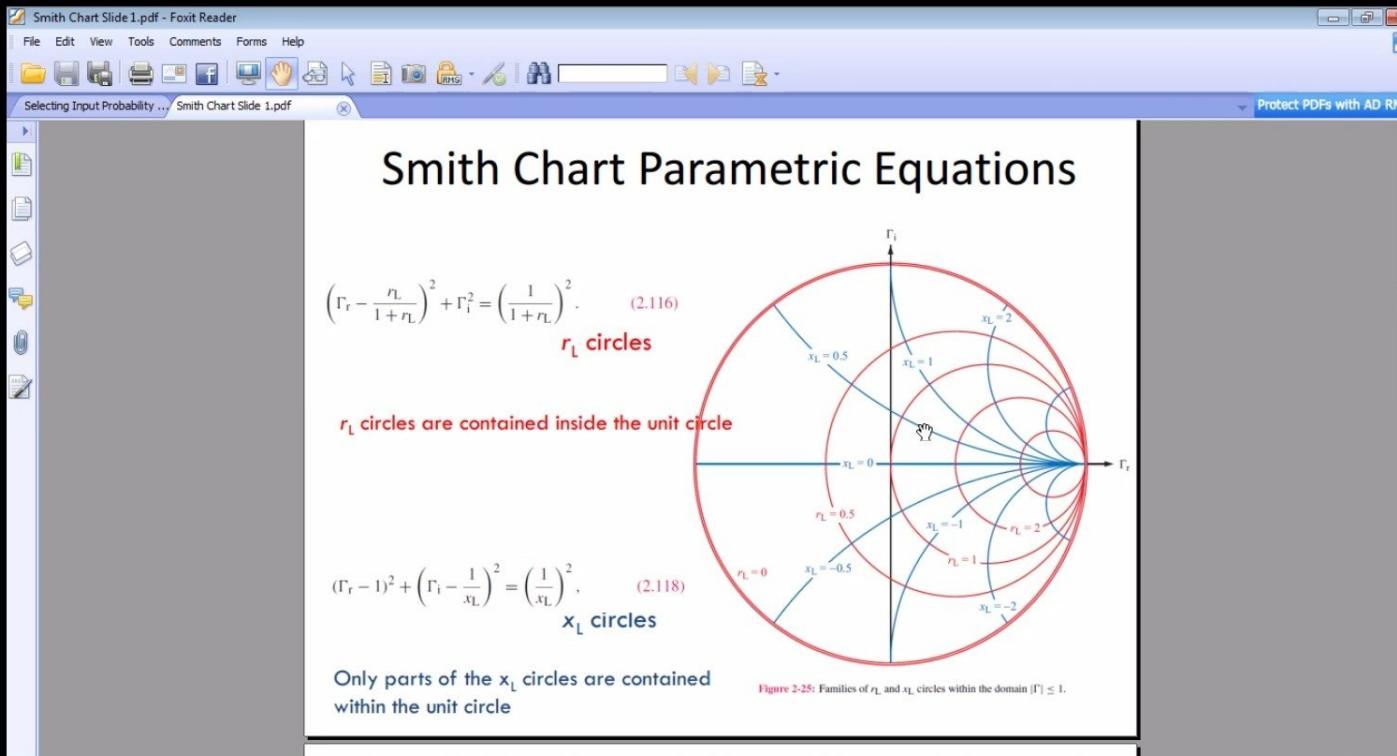
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Smith Chart

Philip Smith of Bell Laboratories developed the “Smith Chart” back in the 1930’s to expedite the tedious and repetitive solution of certain rf design problems.

These include:

- Transmission line problems
- Rf amplifier design and analysis
- L-C impedance matching networks
- Plotting of antenna impedance
- Etc.



Smith Chart Parametric Equations

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2. \quad (2.116)$$

r_L circles

r_L circles are contained inside the unit circle

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2, \quad (2.118)$$

x_L circles

Only parts of the x_L circles are contained within the unit circle

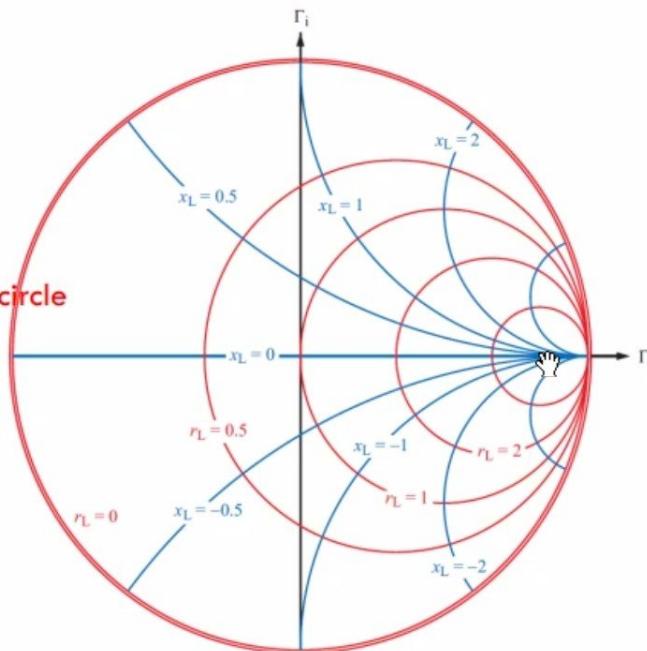
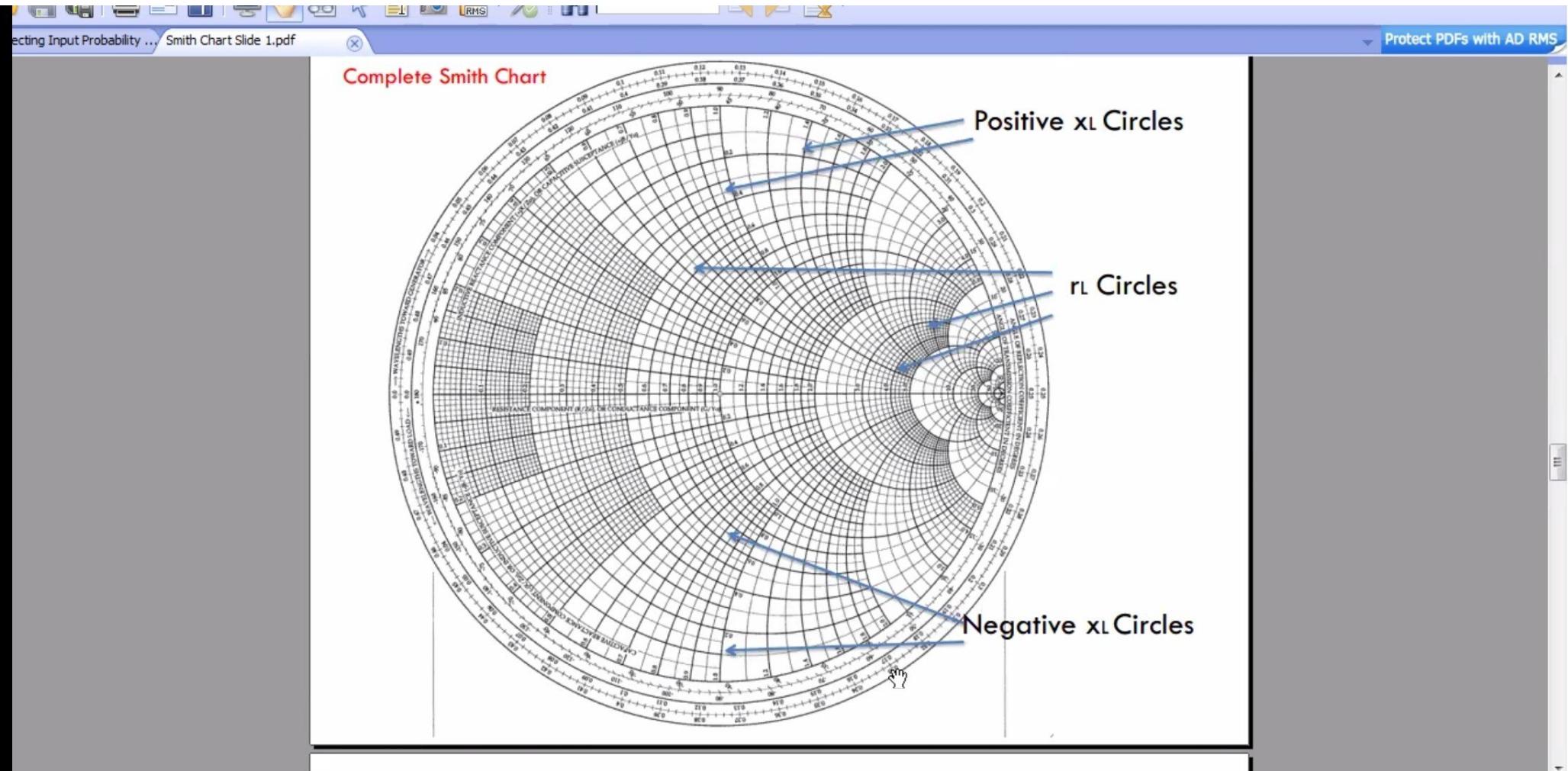
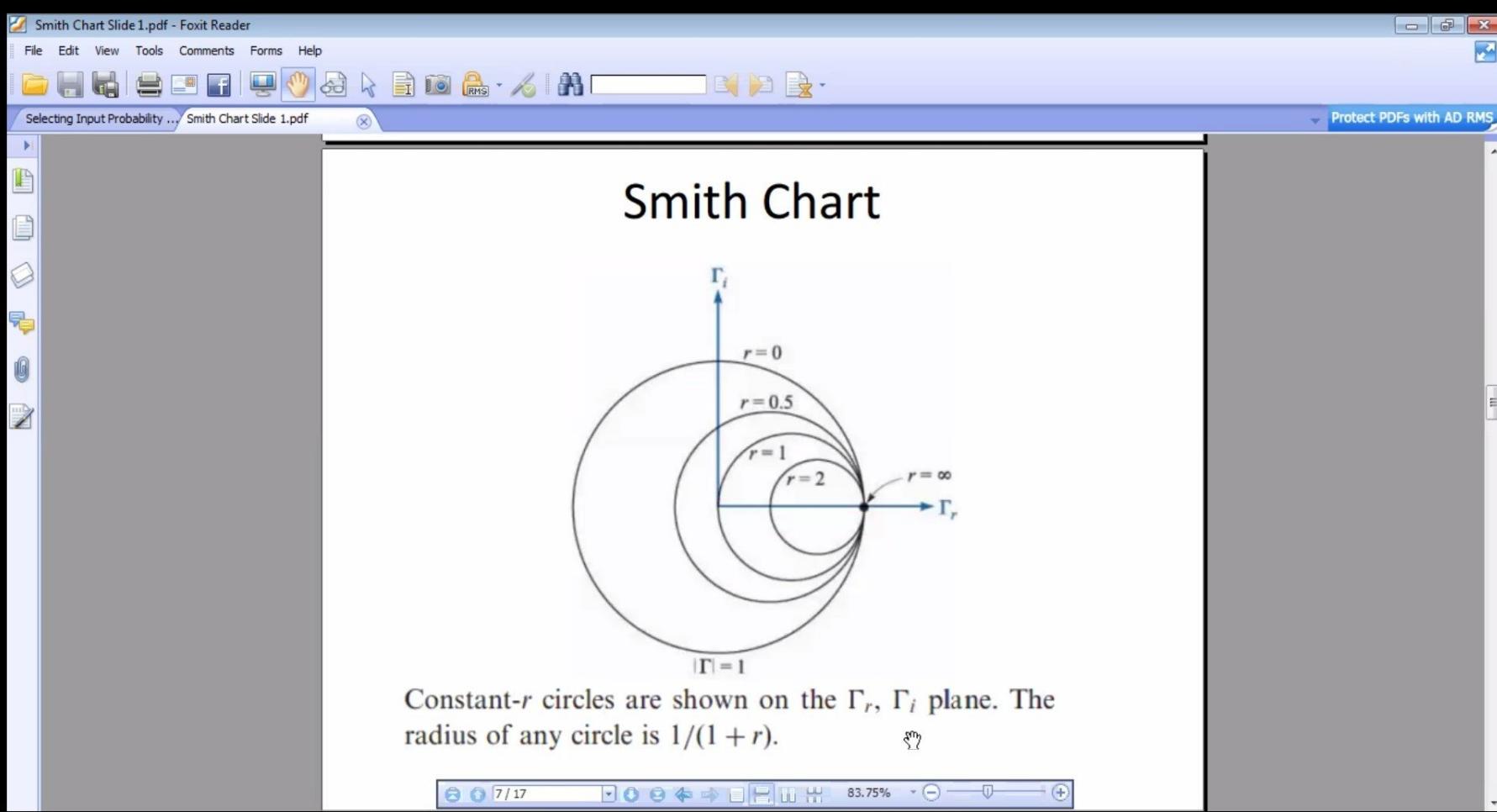


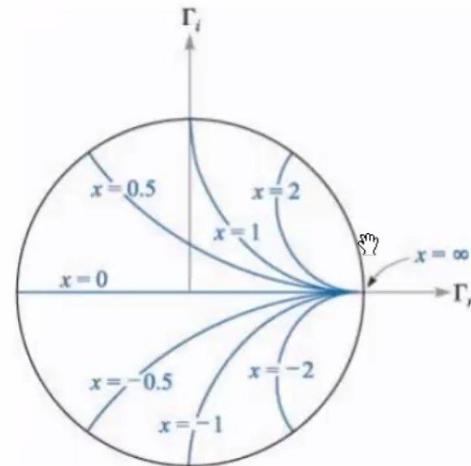
Figure 2-25: Families of r_L and x_L circles within the domain $|\Gamma| \leq 1$.



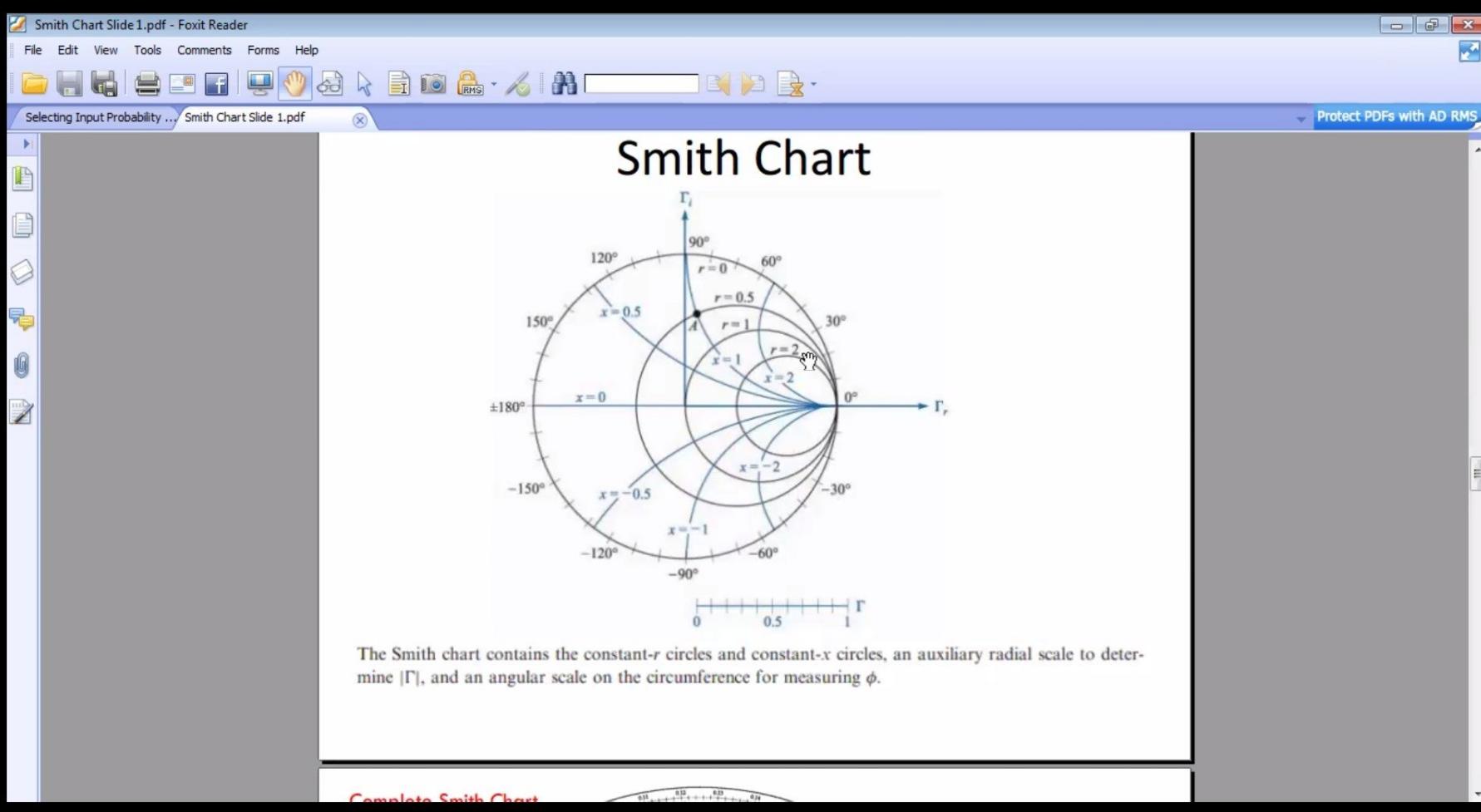


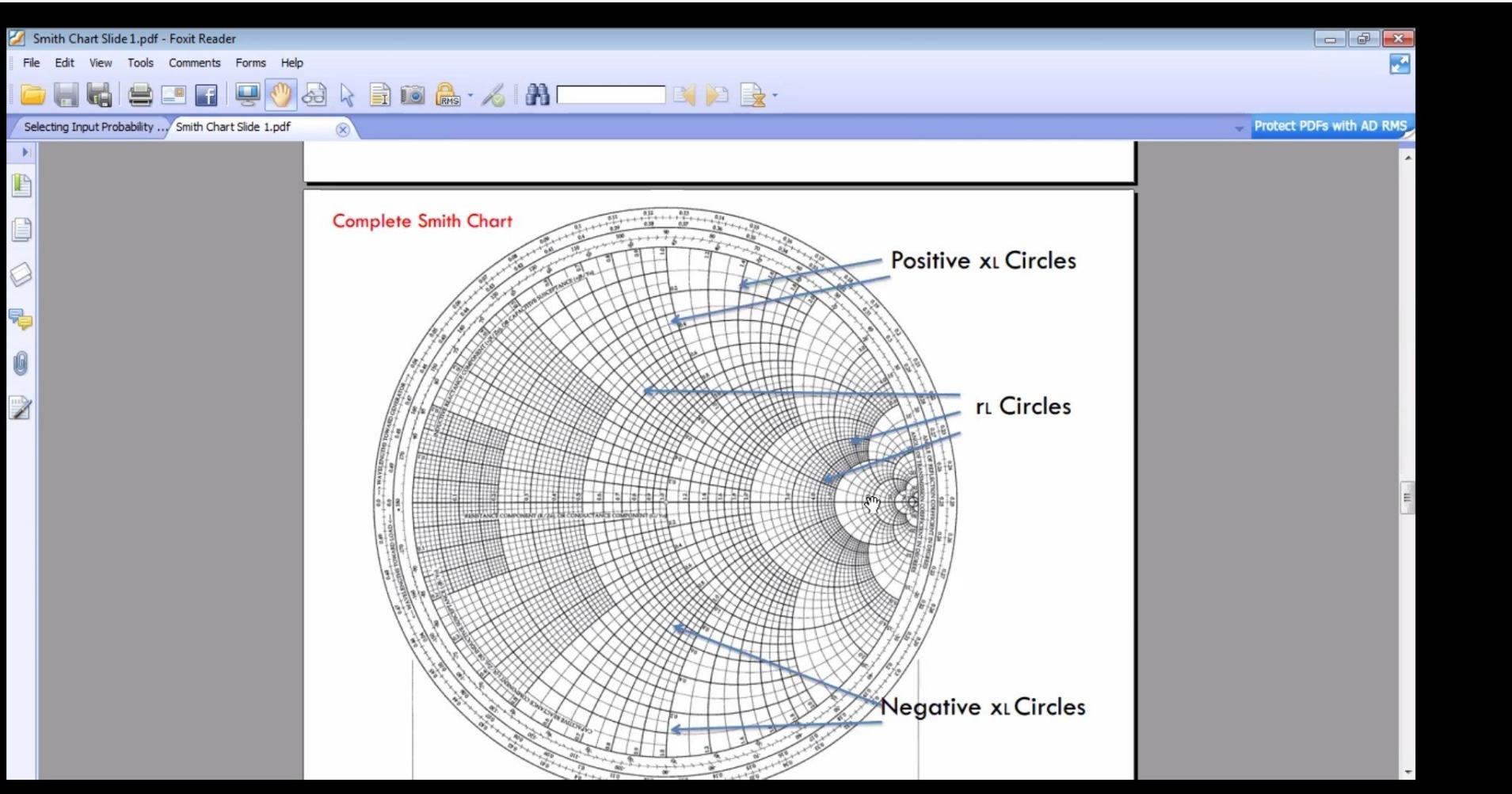


Smith Chart



The portions of the circles of constant x lying within $|\Gamma| = 1$ are shown on the Γ_r , Γ_i axes. The radius of a given circle is $1/|x|$.



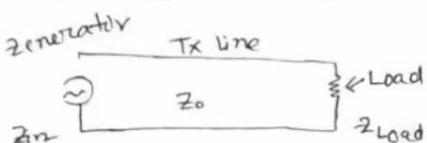


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Finding Load impedance on Smith Chart



Impedance is a complex number, $Z = R + jX$

We can plot Z_{load} on a SMITH CHART.
Then we can easily find VSWR and Γ

Plotting Z_{load} on SMITH CHART

- ① NORMALIZE (Divide Z_L by Z_0) ^{real part}
- ② Find R' on chart (Resistive/ real) part.
- ③ Find X' on chart (Reactive/ imaginary) part.
- ④ Plot A point where they meet.

Example $z_L = (50 + j50) \Omega$

$$z_0 = 50 \Omega$$

$$\therefore z_L' = \left(\frac{50}{50} + j \frac{50}{50} \right) \Omega = (1 + j1) \Omega$$

$\uparrow \quad \uparrow$
 $R' \quad X'$

geinz Ex. $z_L = (300 - j25) \Omega$

$$z_0 = 50 \Omega$$

$$\therefore z_L' = \left(\frac{300}{50} - j \frac{25}{50} \right) \Omega = (6 - j0.5) \Omega$$

$\uparrow \quad \uparrow$
 $R' \quad X'$

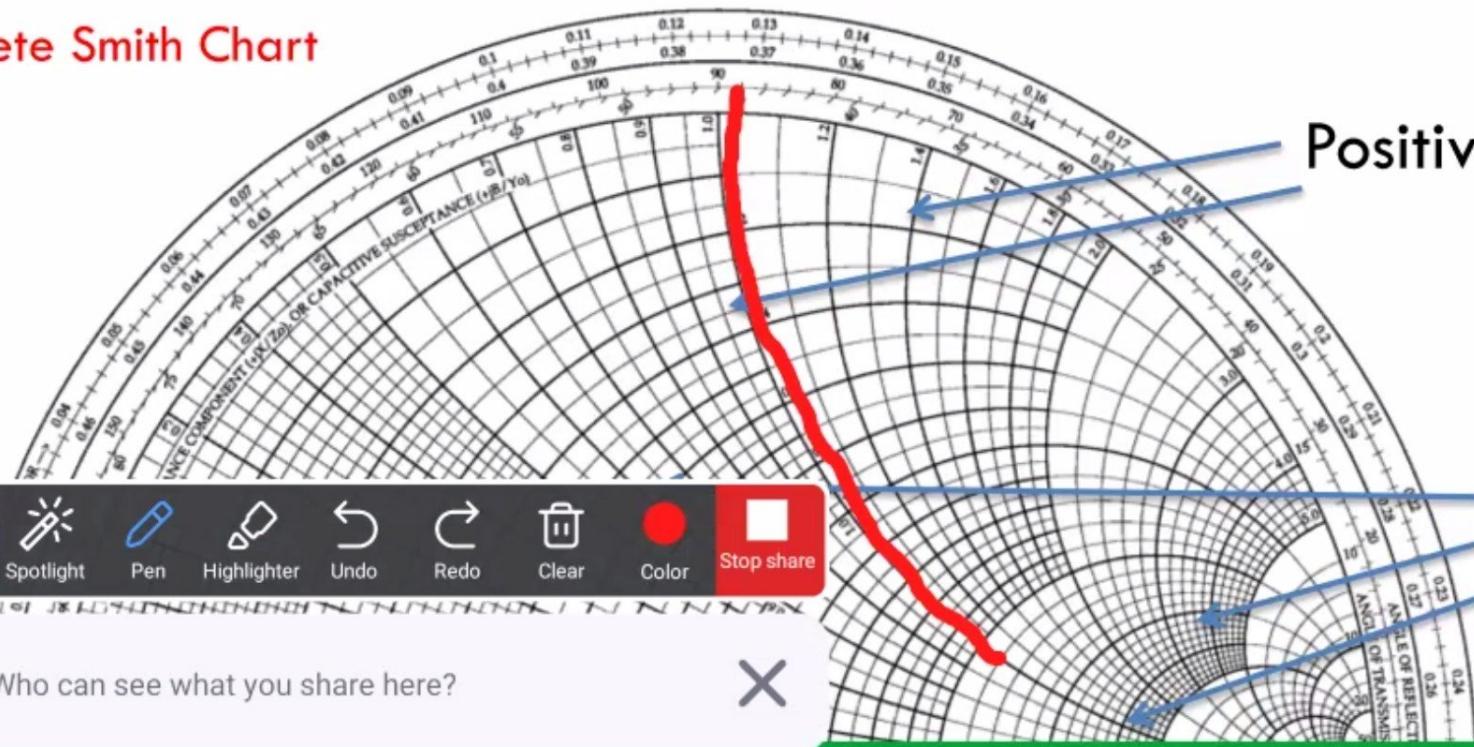
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← Introduction to Smith Chart 1.pdf

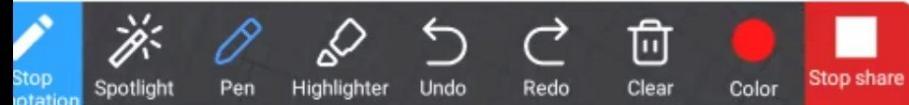


Incomplete Smith Chart



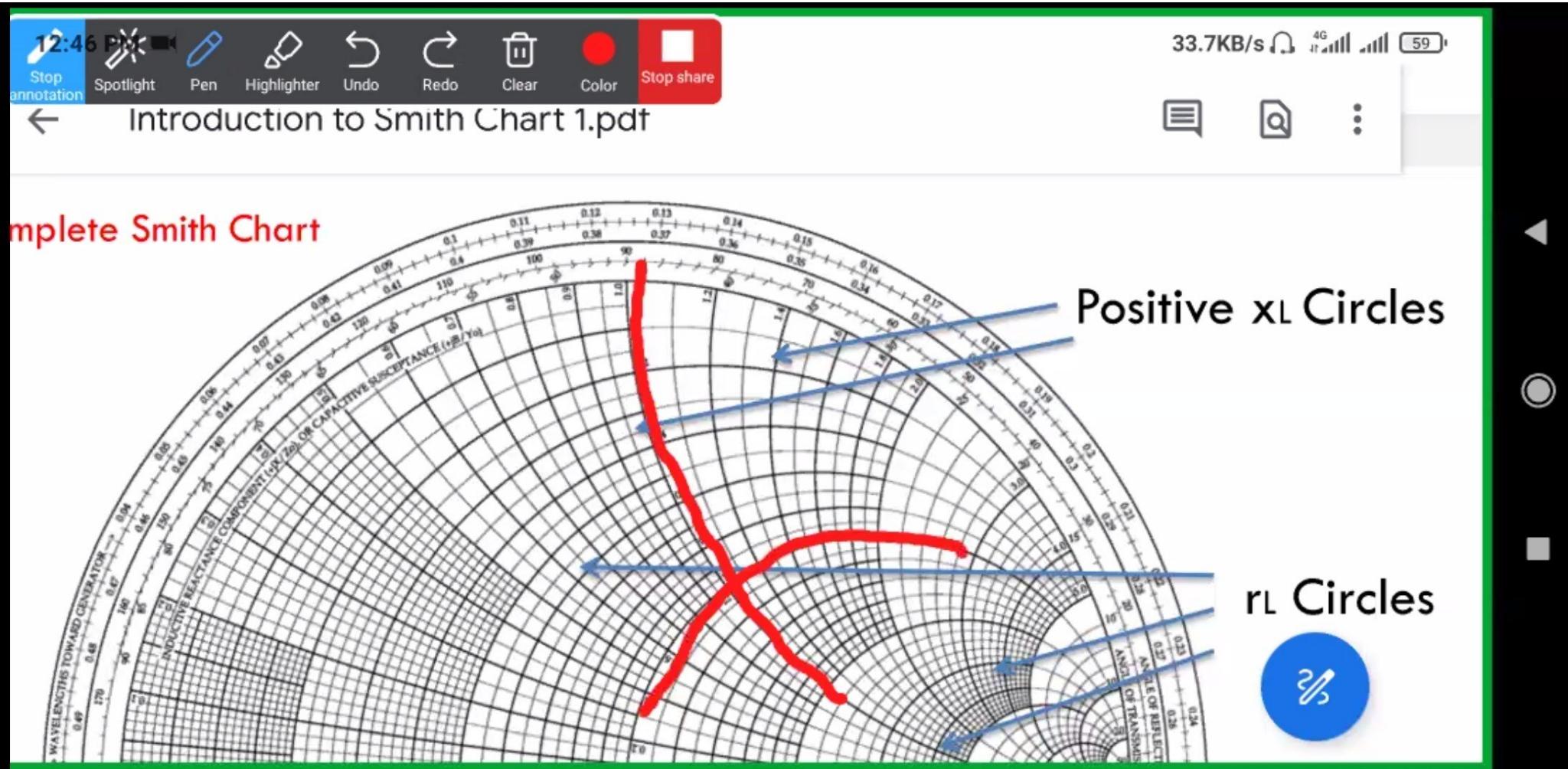
Positive x_L Circles

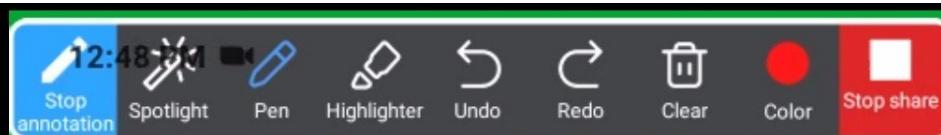
Positive r_L Circles



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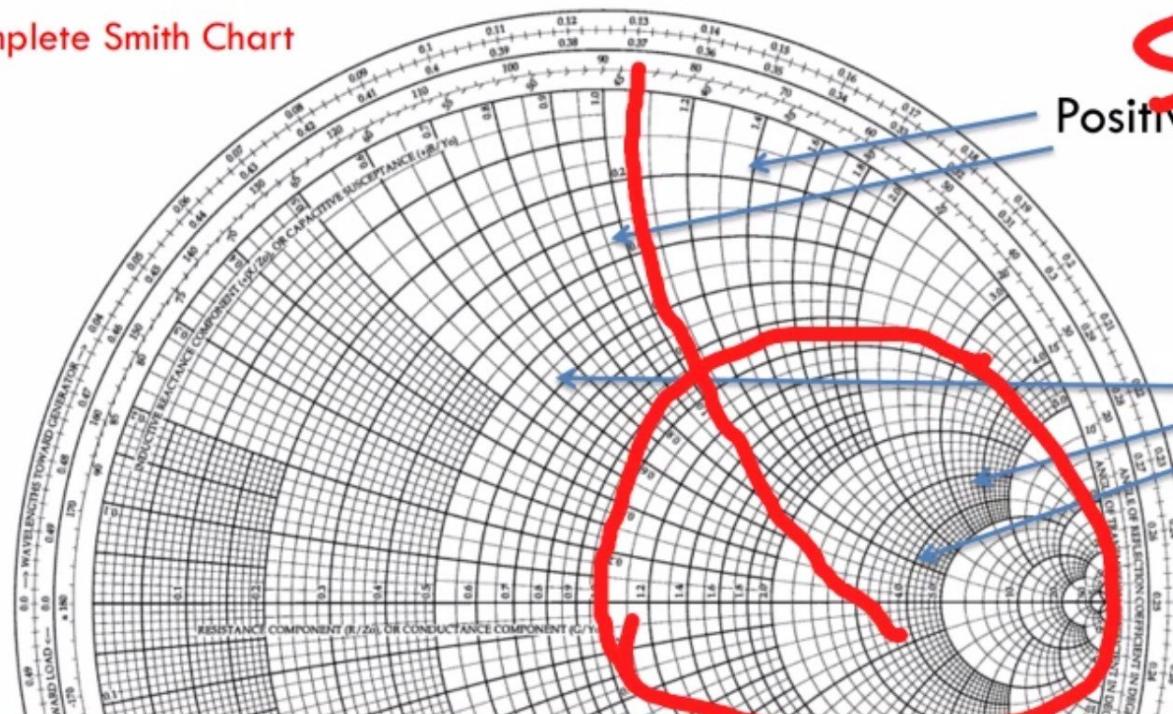


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Introduction to Smith Chart 1.pdt



Complete Smith Chart



$50 + j50$

$- + j \cdot 1$

r_L Circles



Finding VSWR on Smith Chart

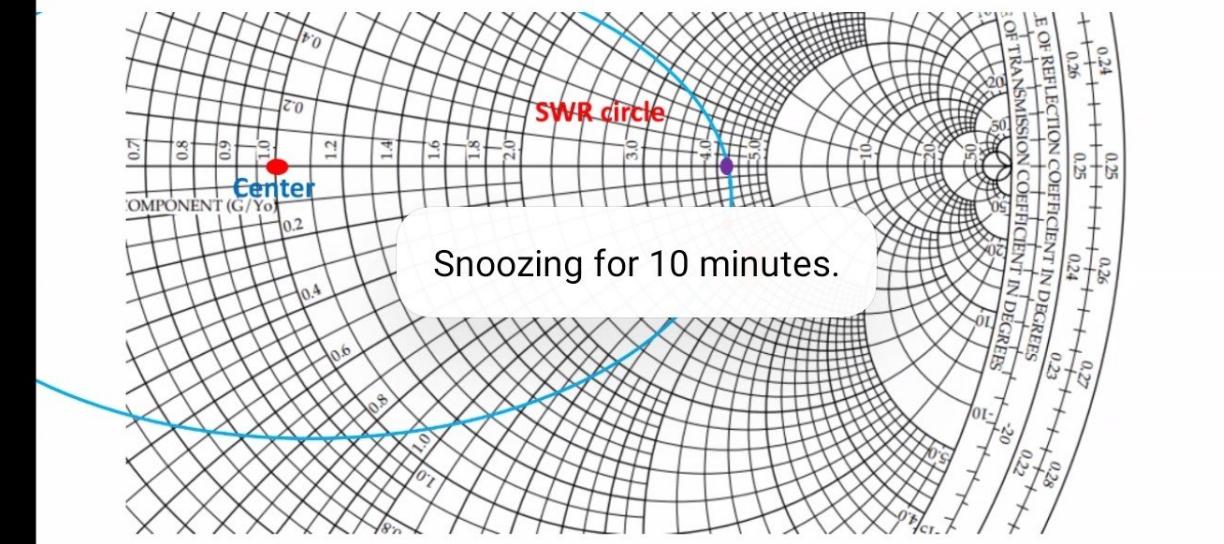
Example: Find out VSWR of $Z_L = (200 - j50)\Omega$ where $Z_0 = 50\Omega$

Solution:

1. The red dot shows the normalized load impedance

$$Z_L' = \frac{200}{50} - j \frac{50}{50} = (4 - j1)$$

2. SWR circle intersects the center line at value of 4.3



Finding VSWR on Smith Chart

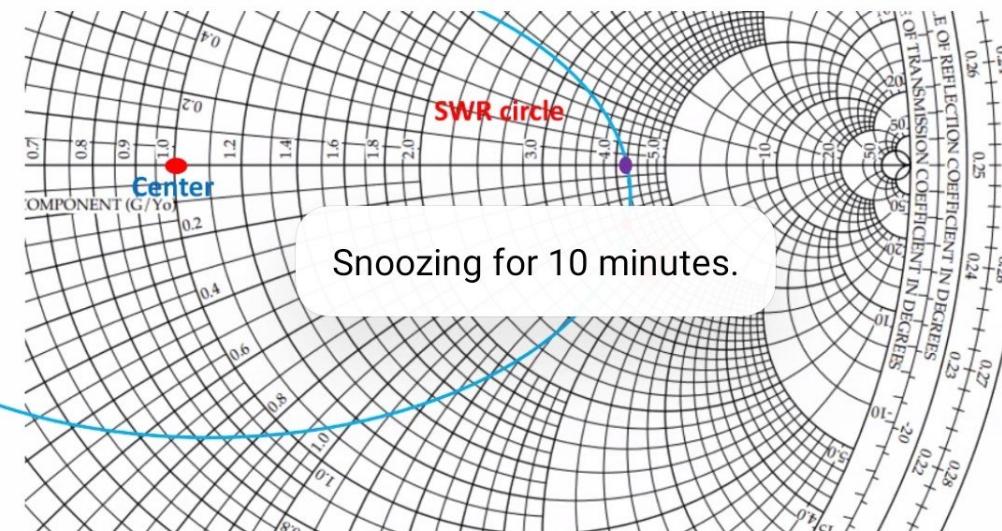
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Solution:

1. The red dot shows the normalized load impedance

$$Z_L' = \frac{200}{50} - j \frac{50}{50} = (4 - j1)$$

2. SWR circle intersects the center line at value of 4.3



Finding Reflection coefficient on Smith Chart

Finding the reflection coefficient:

Steps:

1. Plot Z_L
2. Draw VSWR circle
3. Use compass to find $|\Gamma|$
4. Draw line to find angle

Example:

1. Find Γ of $Z_L = (200 + j500)$ when $Z_0 = 500$
Snoozing for 10 minutes.

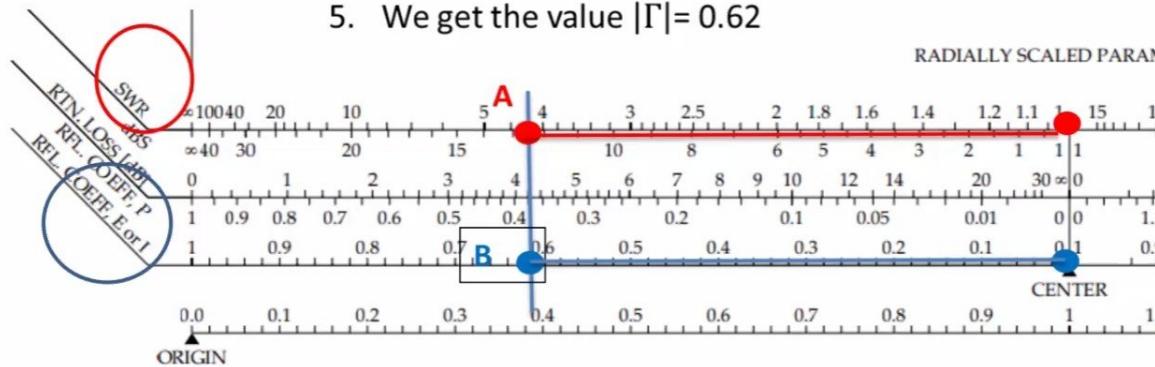
Solution:

Γ is a complex number. We have to find the magnitude of Γ i.e $|\Gamma|$
And also the angle of Γ in degrees.

Finding the magnitude of reflection coefficient

Solution:

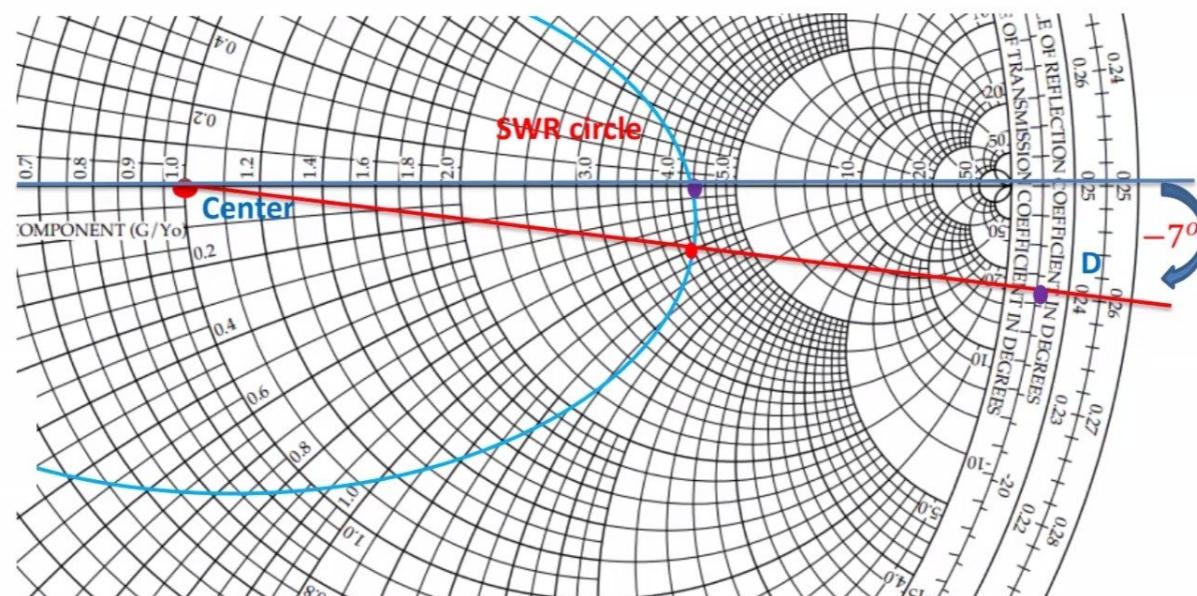
1. Now we have to find $|\Gamma|$
2. The following picture is the bottom side of smith chart.
3. Point out the value of SWR on the SWR scale i.e point A
4. Use compass to find point B on Reflection coefficient scale from center O.
5. We get the value $|\Gamma|=0.62$

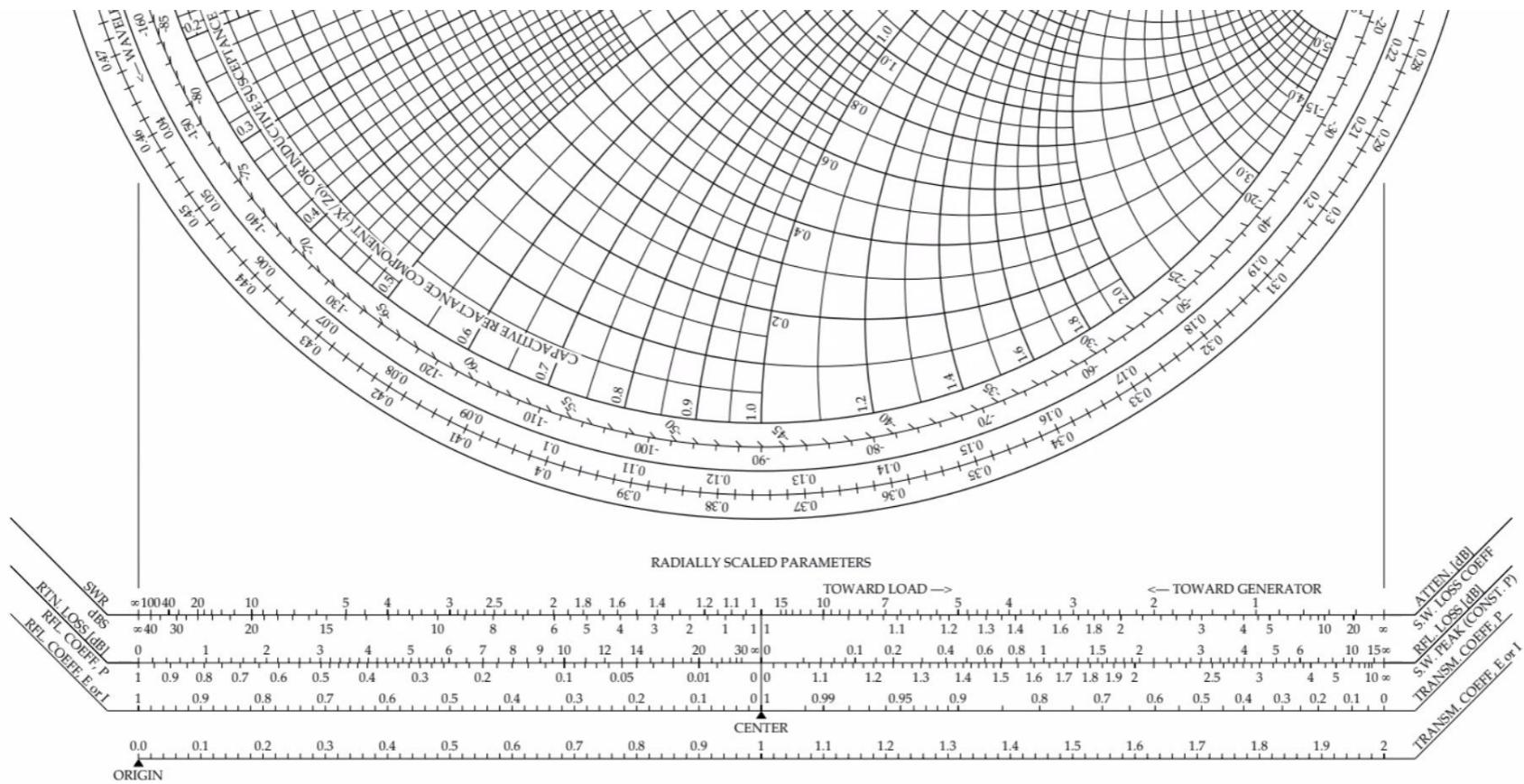


Finding the angle of reflection coefficient

Solution:

1. The angle shows the Point D; value= -7°
2. So the value of reflection coefficient $\Gamma = 0.62 \angle -7^{\circ}$

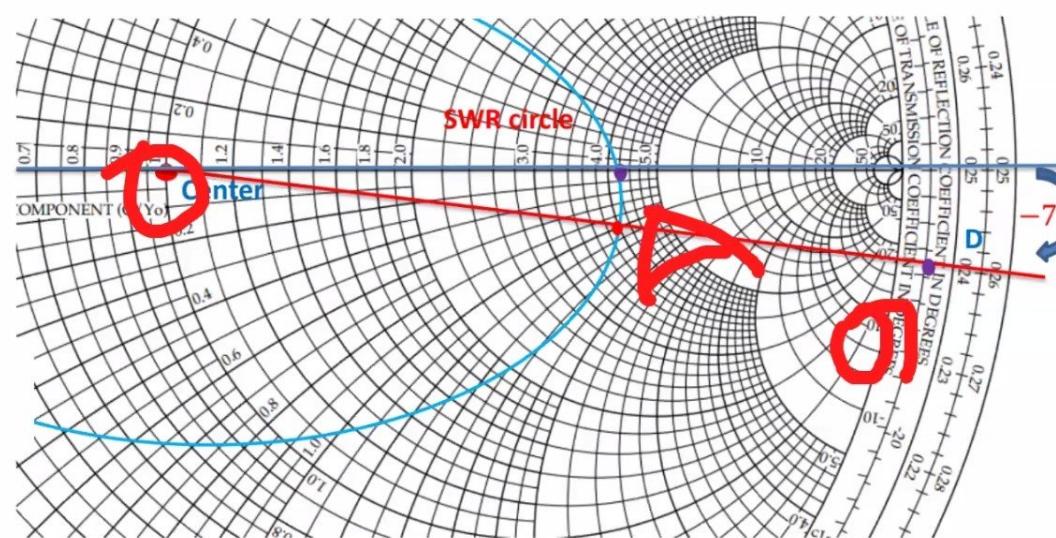




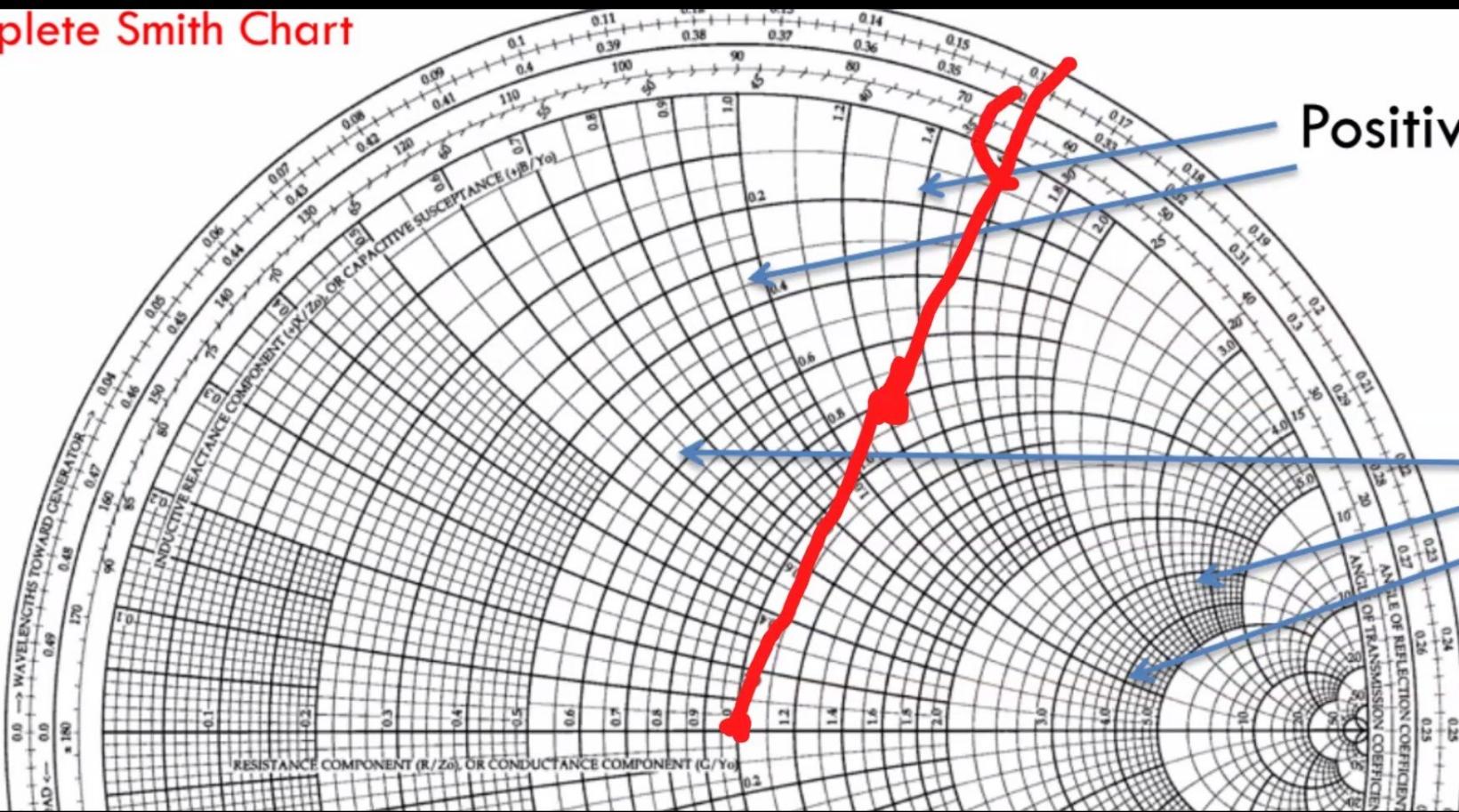
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Complete Smith Chart

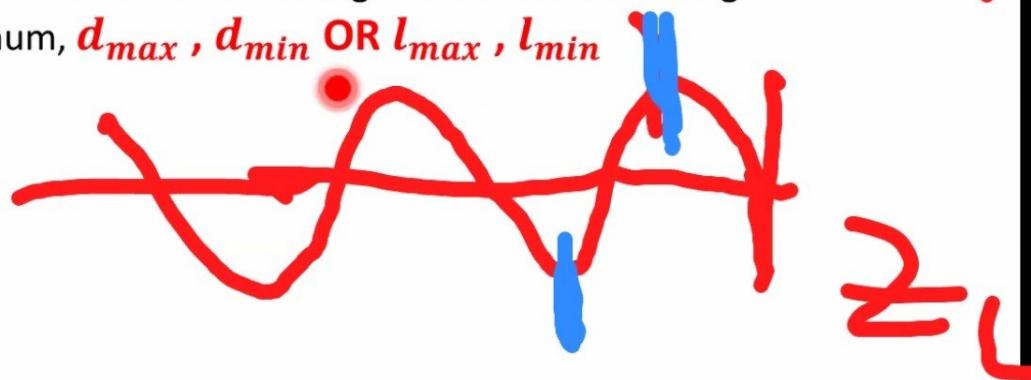


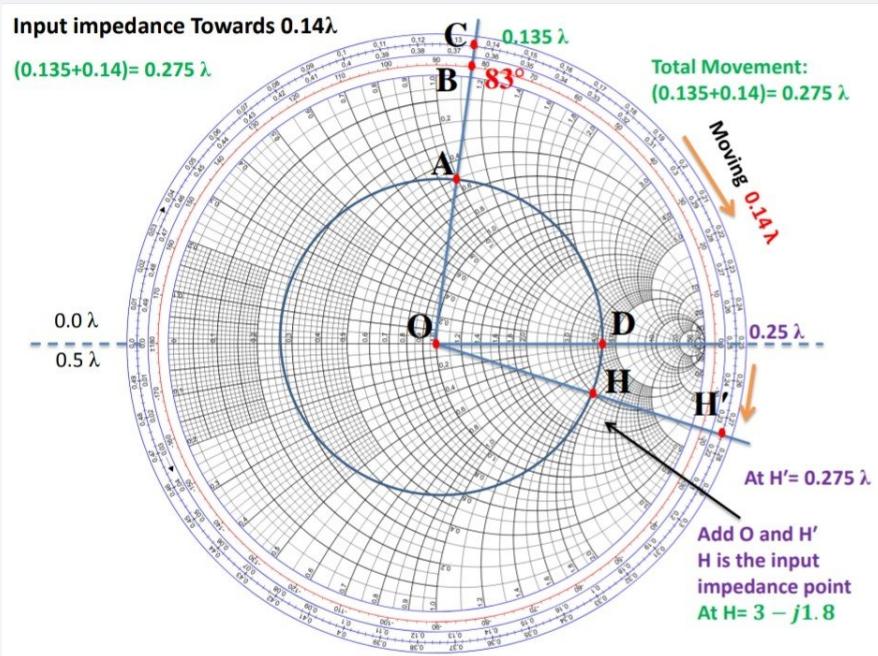
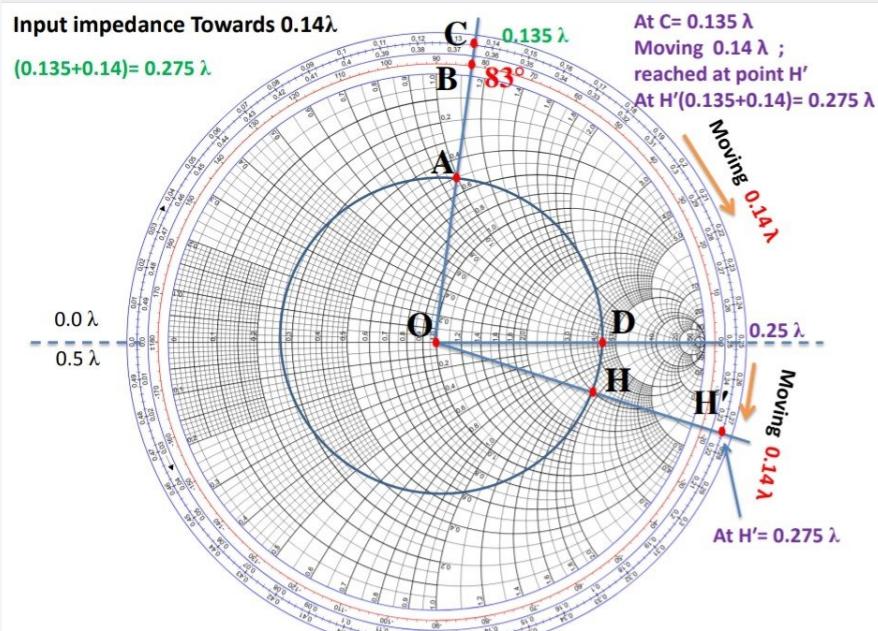
Positive x_L Circles

rl Circles

We have to find out using Smith chart

- a) Voltage reflection coefficient, Γ
- b) Voltage standing-wave ratio, **VSWR**
- c) Finding input impedance, Z_{in}
- d) Finding load impedance, Z_L
- e) the distances of the voltage maximum and voltage minimum, d_{max}, d_{min} OR l_{max}, l_{min}





S is numerically equal to the value of r_0 at P_{max} , the point at which the SWR circle intersects the real Γ axis to the right of the chart's center.

S = 4.2

At $H = 3 - j1.8$

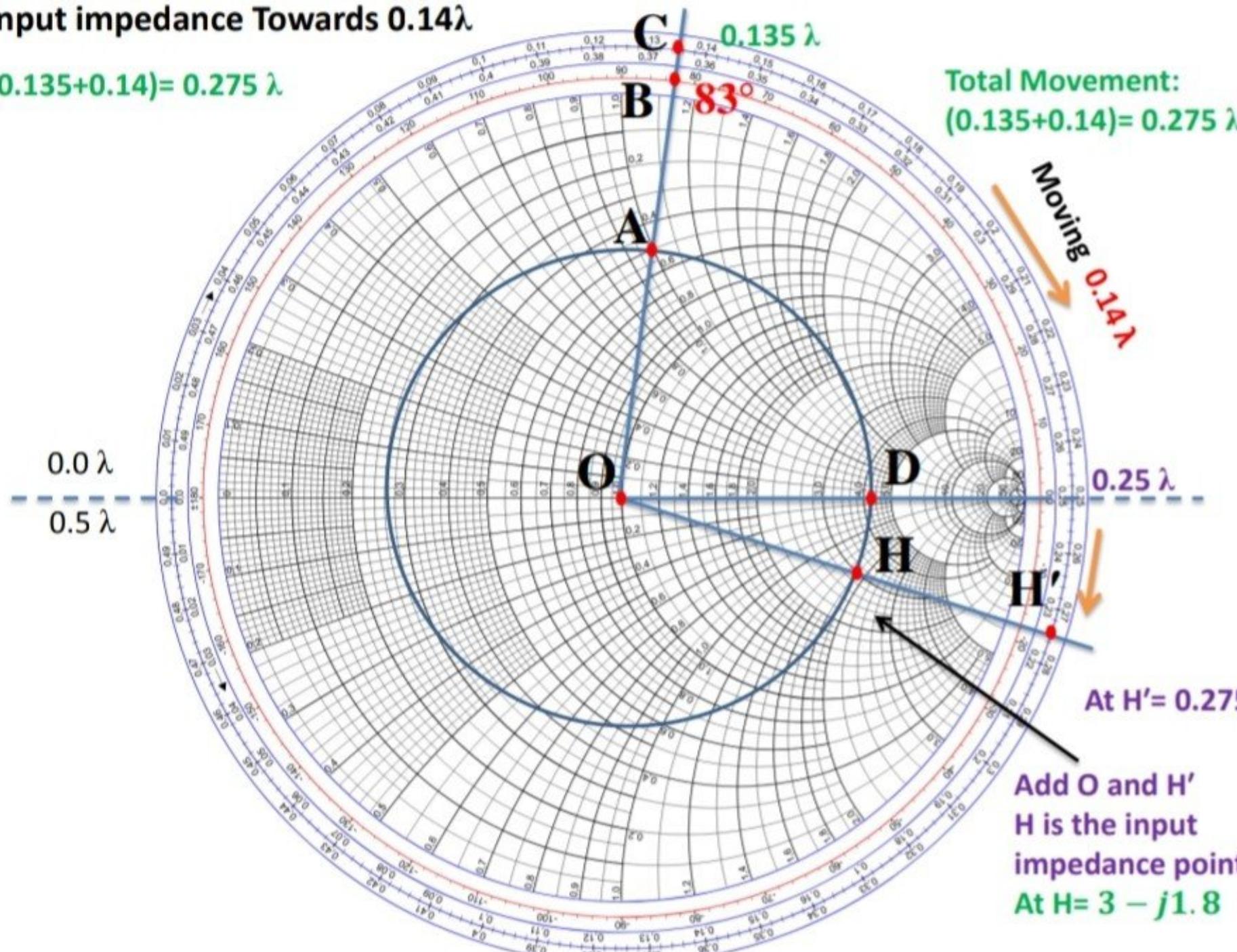
↓
R X

So the input impedance, Z_{in} at $H = (3-j1.8) Z_0 \Omega$
 $= (3-j1.8) 50 \Omega$
 $= (150 - j90) \Omega$



Input impedance Towards 0.14λ

$$(0.135+0.14) = 0.275 \lambda$$



Smith Chart slotted line example.

$$d_{\min} = \frac{5}{40} = 0.125\lambda$$

$$z_L = 0.6 - j0.8$$

$$Z_L = 50(0.6 - j0.8) = (30 - j40) \Omega$$

