Smith Chart

Philip Smith of Bell Laboratories developed the "Smith Chart" back in the 1930"s to expedite the tedious and repetative solution of certain rf design problems.

These include:

- Transmission line problems
- Rf amplifier design and analysis
- L-C impedance matching networks
- Plotting of antenna impedance
- Etc.

Construction of Smith Chart

- The Smith Chart is made up of a family of circles and a second family of arcs of circles.
- The circles are called “constant resistance circles”
- The arcs are “constant reactance circles”
- Impedances must be entered in rectangular form – broken down into a real and an imaginary component.
- The real part (resistance) determines the circle to use.
- The imaginary part (reactance) determines the arc to use.
- The intersection of an arc and a circle represents the plotted impedance.

Smith Chart Parametric Equations

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2. \qquad (2.116)$$

r_L circles

r_L circles are contained inside the unit circle

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2, \qquad (2.118)$$

x_L circles

Only parts of the x_L circles are contained within the unit circle

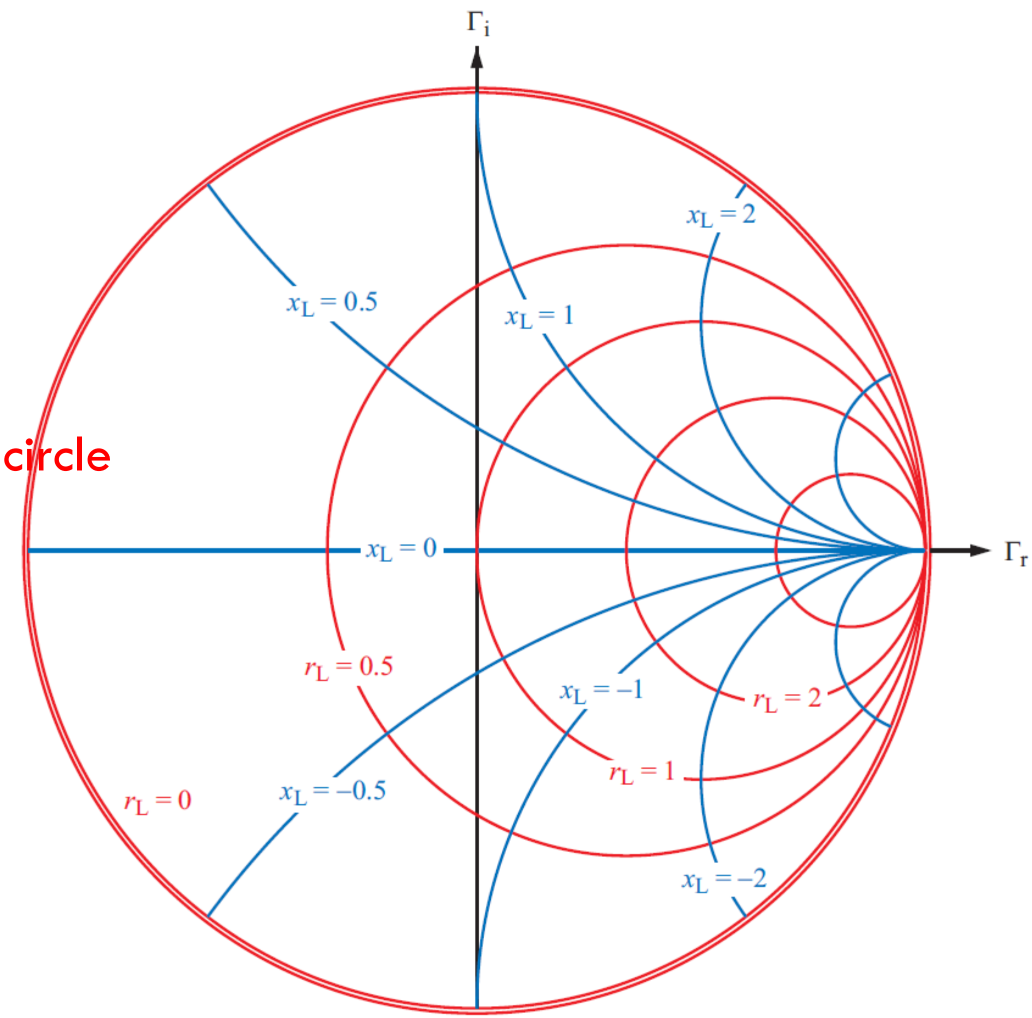
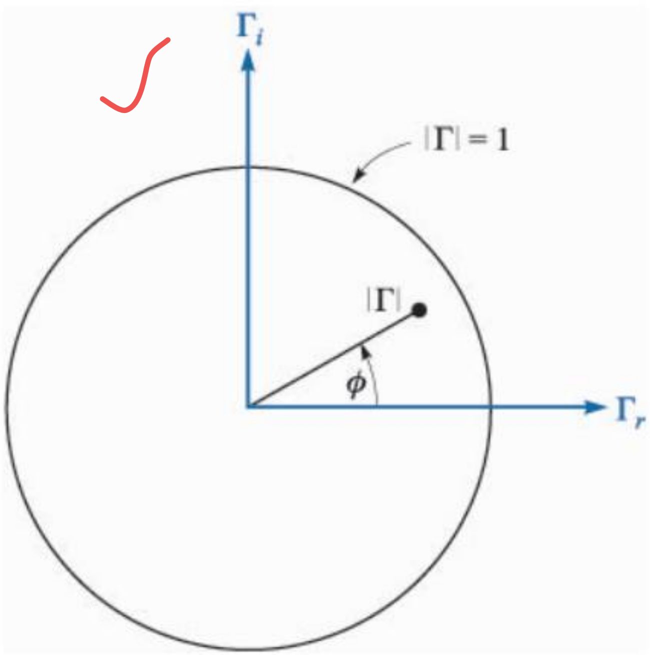


Figure 2-25: Families of r_L and x_L circles within the domain $|\Gamma| \leq 1$.

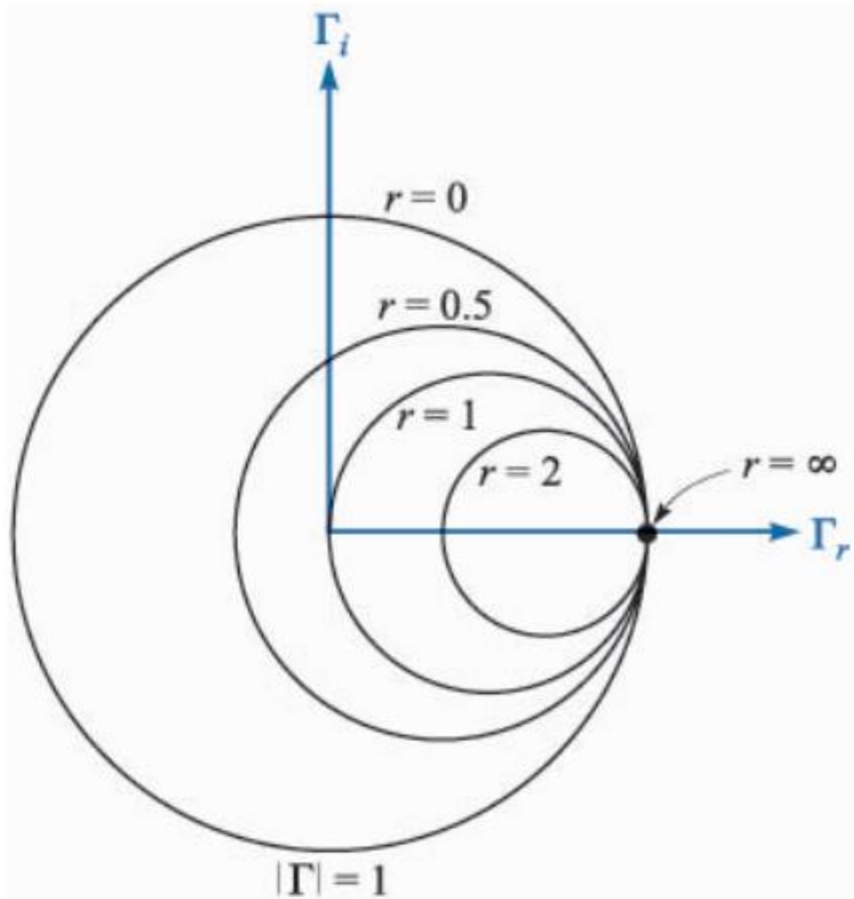
Smith Chart

The polar coordinates of the Smith chart are the magnitude and phase angle of the reflection coefficient; the cartesian coordinates are the real and imaginary parts of the reflection coefficient. The entire chart lies within the unit circle $|\Gamma| = 1$.

In polar form, we have used $|\Gamma|$ and ϕ as the magnitude and angle of Γ; let us now select Γ_r and Γ_i as the real and imaginary parts of Γ,

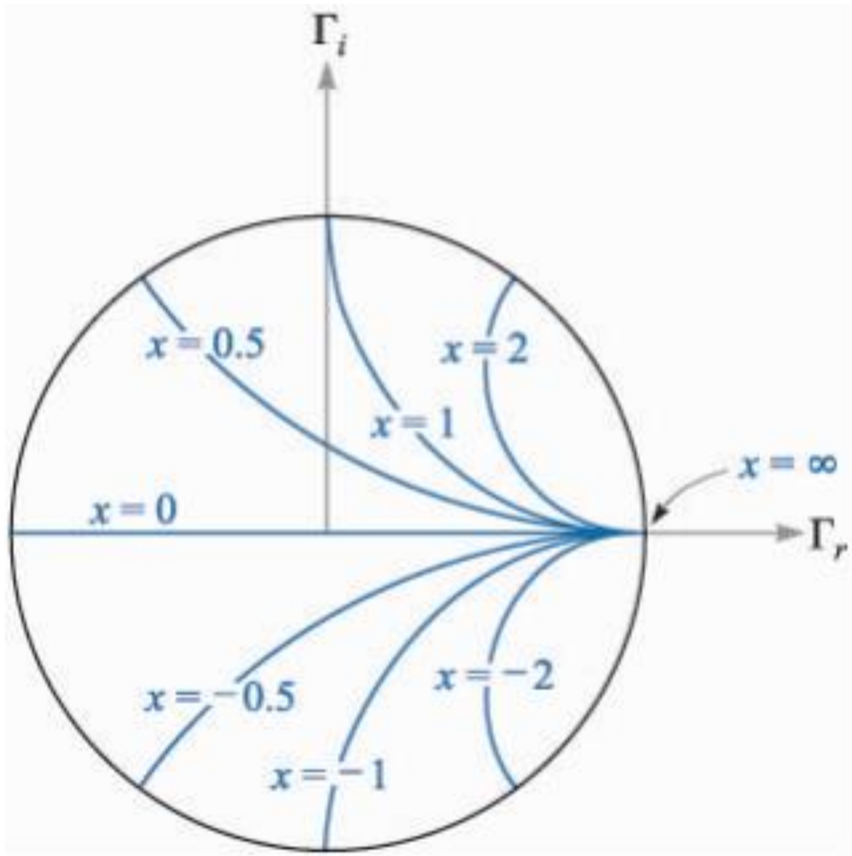


Smith Chart



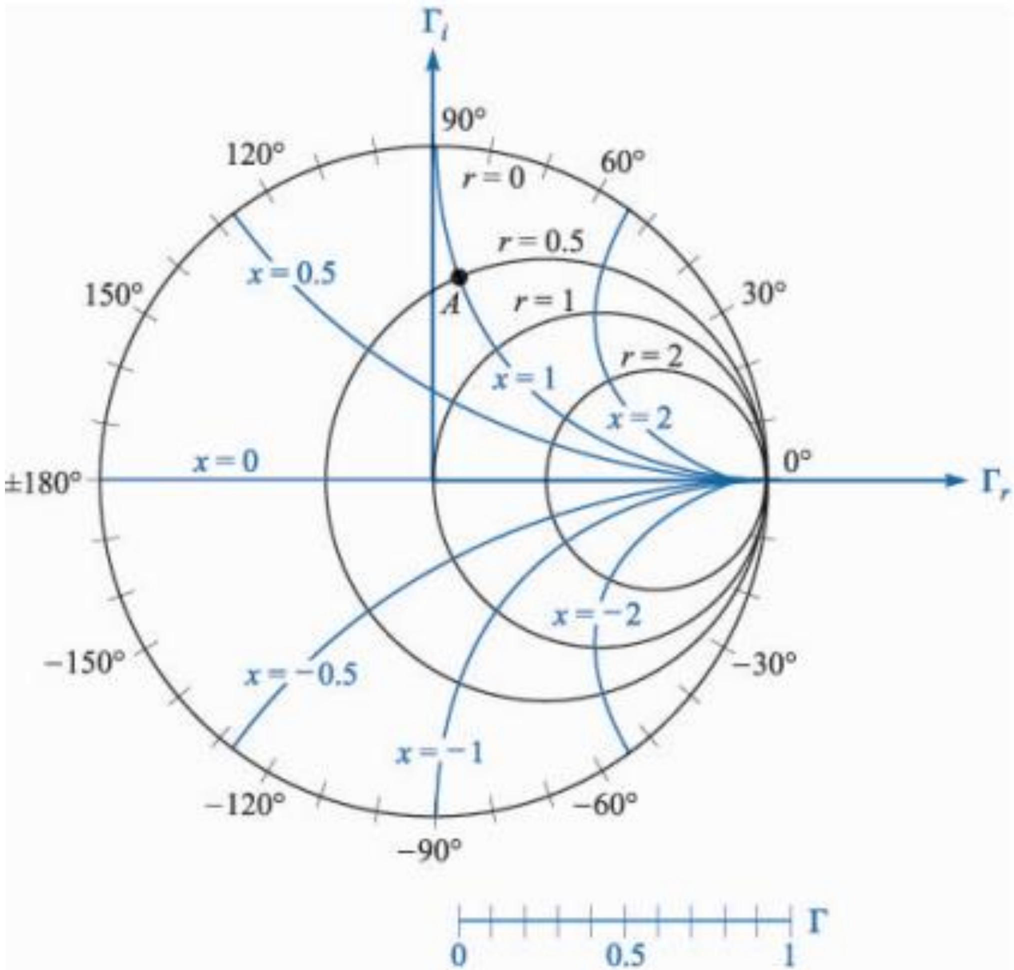
Constant-r circles are shown on the Γ_r, Γ_i plane. The radius of any circle is $1/(1 + r)$.

Smith Chart



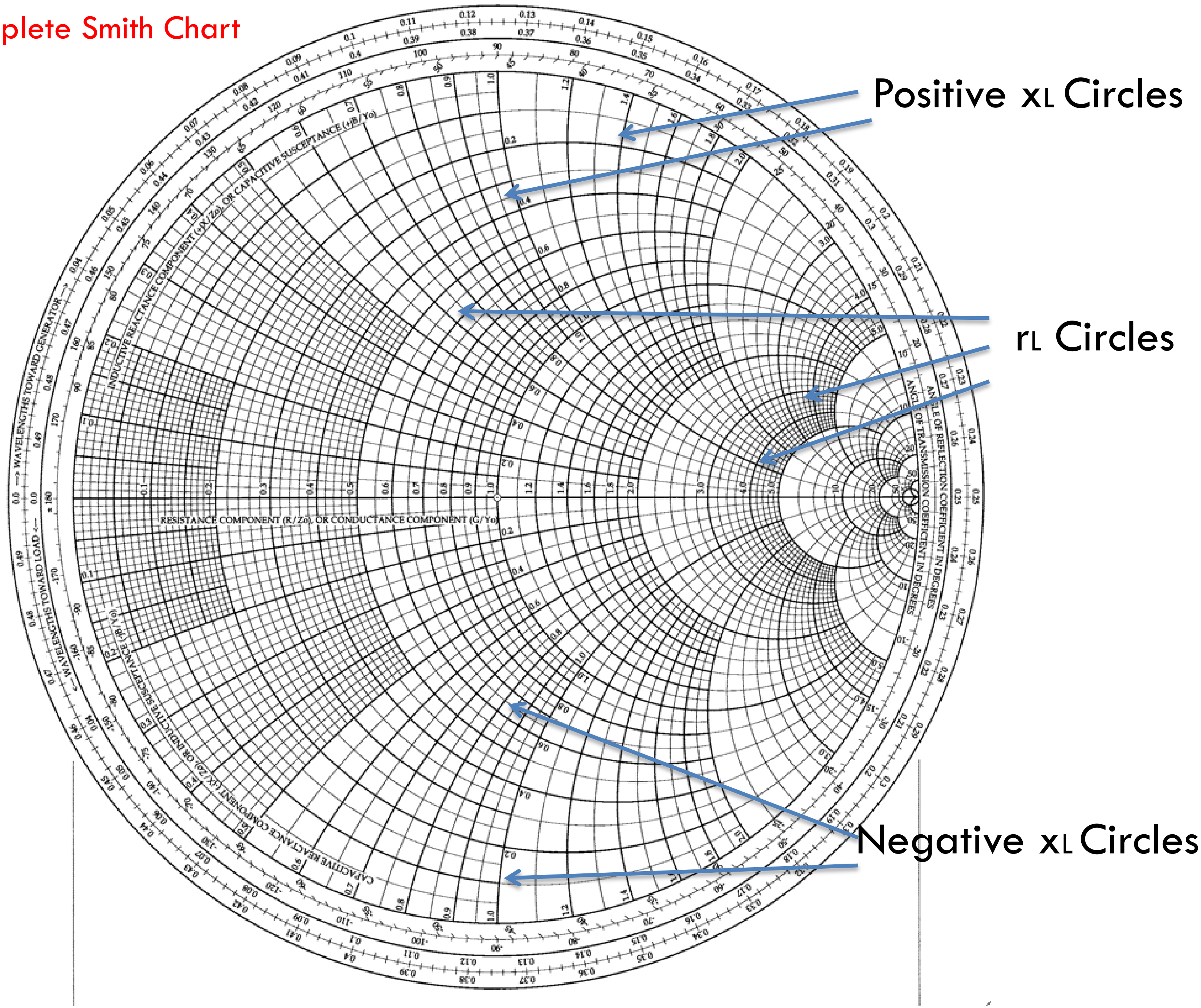
The portions of the circles of constant x lying within $|\Gamma| = 1$ are shown on the Γ_r, Γ_i axes. The radius of a given circle is $1/|x|$.

Smith Chart

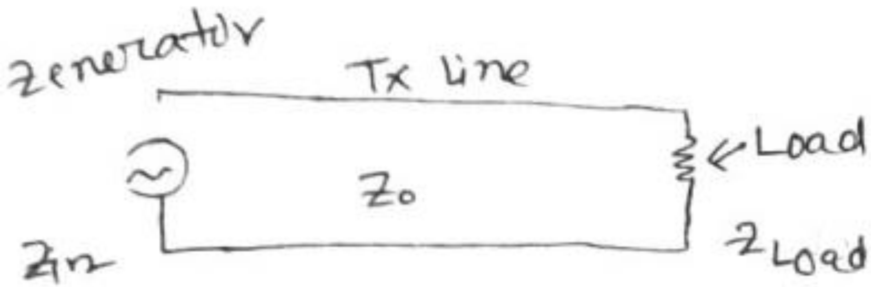


The Smith chart contains the constant-r circles and constant-x circles, an auxiliary radial scale to determine $|\Gamma|$, and an angular scale on the circumference for measuring ϕ.

Complete Smith Chart



Finding Load impedance on Smith Chart



Impedance is a complex number, $Z = R + jx$

R ↑ Real

jx ↑ imaginary

We can plot Z_{Load} on a SMITH CHART.

Then we can easily find VSWR and Γ

Plotting Z_{Load} on SMITH CHART

1. NORMALIZE (Divide Z_L by Z_o)
2. Find R' on Chart (Resistive/Real) part
3. Find x' on chart (Reactive/imaginary) Part.
4. Plot A Point where they meet.

Finding Load impedance on Smith Chart

Example $Z_L = (50 + J50)\Omega$

$Z_0 = 50\Omega$

$\therefore Z_L' = \left(\frac{50}{50} + J\frac{50}{50}\right)\Omega = (\underbrace{1}_{R'} + J\underbrace{1}_{X'})\Omega$

Again Ex. $Z_L = (300 - j25)\Omega$

$Z_0 = 50\Omega$

$\therefore Z_L' = \left(\frac{300}{50} - j\frac{25}{50}\right)\Omega = (\underbrace{6}_{R'} - J\underbrace{0.5}_{X'})\Omega$

Finding VSWR on Smith Chart

Steps:

1. PLOT Z_{Load}
2. Draw VSWR circle
3. Look!

Ex: 3 Loads,

(A) $Z_L = (100 - j100)\,\Omega = (z_L' = 2 - j2)$ VSWR = 4.

Assuming $Z_0 = 50\,\Omega$

(B) $Z_L = (50 + j100)\,\Omega$ $(z_L' = 1 - j2)$ VSWR = 5.8

(C) $Z_L = 50\,\Omega$ $(z_L' = 1 + j0)$

VSWR circle is a perfect match

Radius = 0

VSWR = 1 (Perfect impedance match)

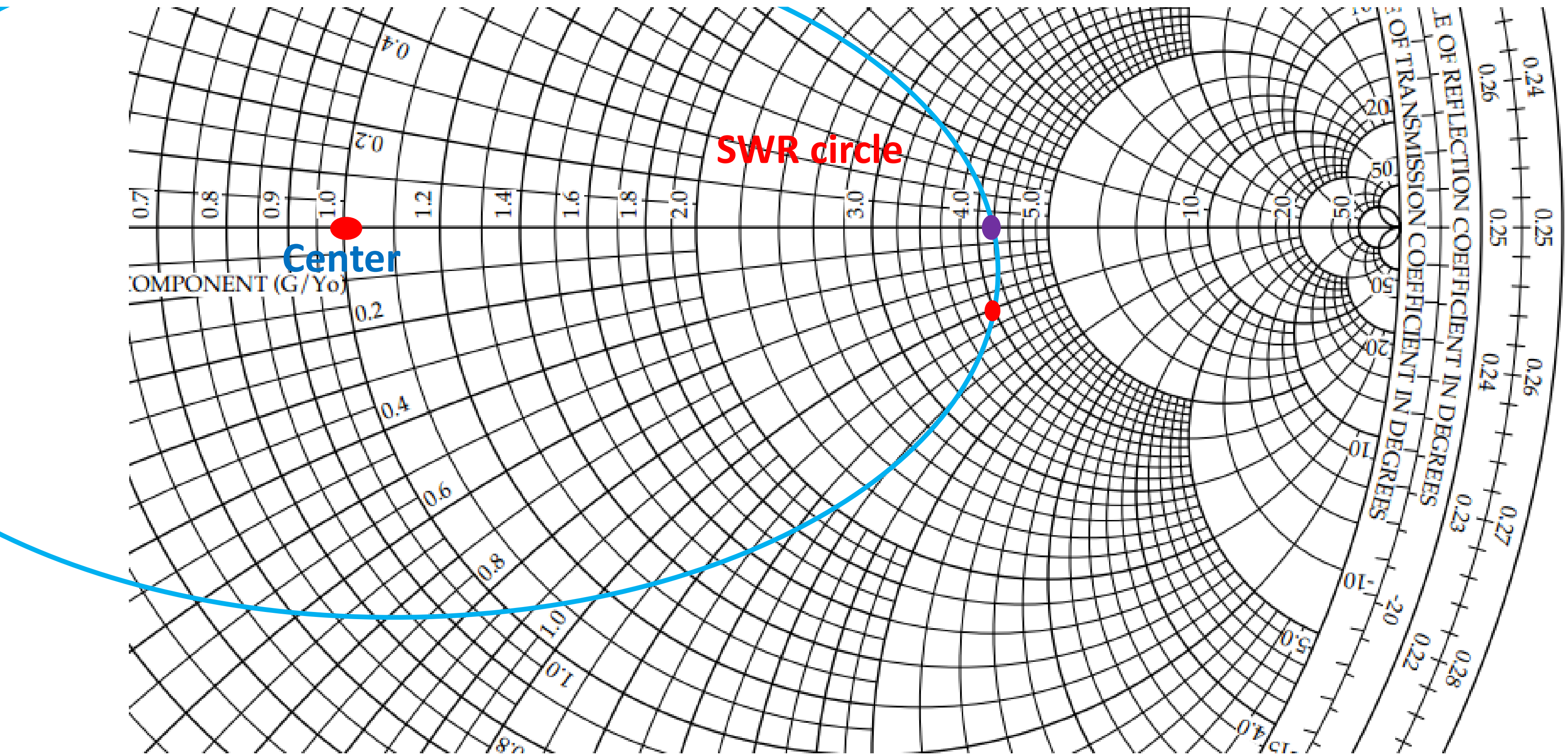
No Reflected power.

Finding VSWR on Smith Chart

Example: Find out VSWR of $Z_L = (200 - j50)\Omega$ where $Z_0 = 50\Omega$

Solution:

1. The red dot shows the normalized load impedance $Z_L{}' = \frac{200}{50} - \mathrm{j}\frac{50}{50} = (4 - j1)$
2. SWR circle intersects the center line at value of 4.3



Finding Reflection coefficient on Smith Chart

Finding the reflection coefficient:

Steps:

1. Plot Z_L
2. Draw VSWR circle
3. Use compass to find $|\Gamma|$
4. Draw line to find angle

Example:

1. Find Γ of Z_L= (200-j50)Ω where Z_0= 50Ω

Solution:

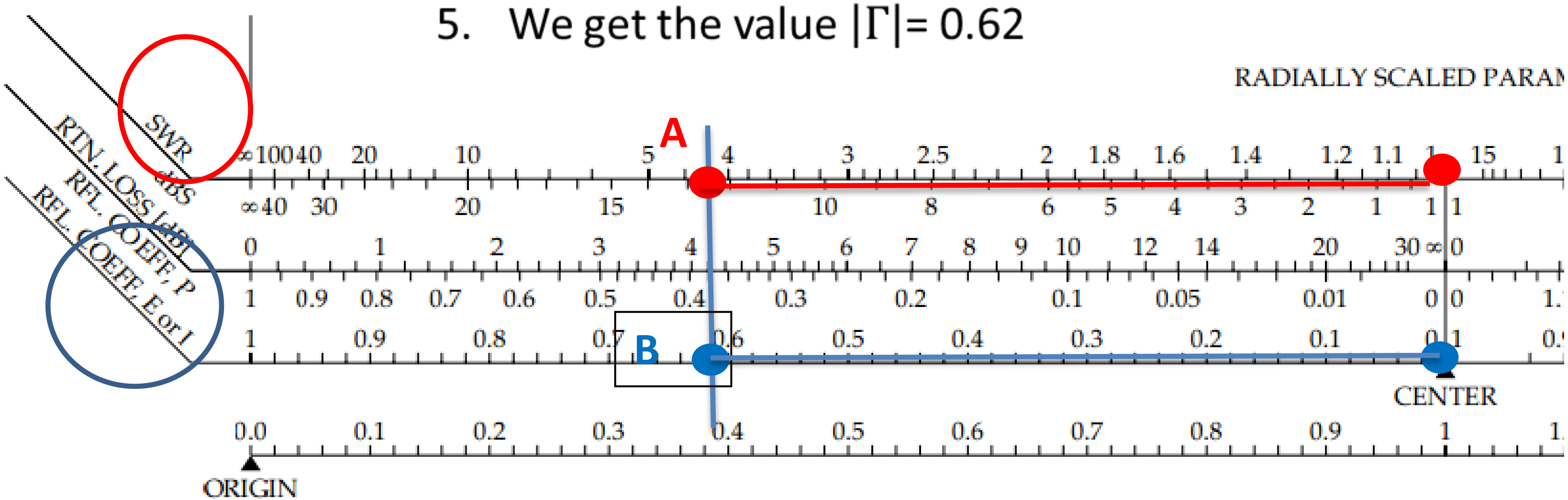
Γ is a complex number . We have to find the magnitude of Γ i;e $|\Gamma|$

And also the angle of Γ in degrees.

Finding the magnitude of reflection coefficient

Solution:

1. Now we have to find $|\Gamma|$
2. The following picture is the bottom side of smith chart.
3. Point out the value of SWR on the SWR scale i;e point A
4. Use compass to find point B on Reflection coefficient scale from center O.
5. We get the value $|\Gamma|$= 0.62



Finding the angle of reflection coefficient

Solution:

1. The angle shows the Point D; value= -7^o
2. So the value of reflection coefficient Γ = $0.62 \angle -7^o$