Solutions to Homework 1

Radar Systems (Al Jouf University)

Problem 1.1 1.1

a) What should be the pulse repetition frequency of a radar in order to achieve a maximum unambiguous range of **60 nmi**?

$$R_{un} = \frac{c}{2f_P}$$

$$f_P = \frac{c}{2R_{un}} = \frac{3\times 10^8}{2\times 60\times 1852} = 1350\,Hz$$

b) How long does it take for the radar signal to travel out and back when the target is at the maximum unambiguous range?

$$T_P = \frac{1}{f_P} = \frac{1}{1350} = 0.74\,ms$$

c) If the radar has a pulse width of 1.5 µs, what is the extent (in meters) of the pulse energy in space in the range coordinate?

$$d = \tau\times c = \left(1.5\times 10^{-6}\right)\left(3\times 10^{8}\right) = 450\,m$$

d) How far apart in range (meters) must two equal-size targets be separated in order to be certain they are completely resolved by a pulse width of 1.5 µs (Resolution range)?

$$\Delta R = \frac{\tau c}{2} = \frac{\left(1.5\times 10^{-6}\right)\left(3\times 10^{8}\right)}{2} = 225\,m$$

e) If the radar has a peak power of 800 kW and a pulse width of 1.5 µs, what is its average power?

$$\because \frac{\tau}{T_P} = \frac{P_{av}}{P_{peak}} \qquad \therefore P_{ave} = P_{peak}\frac{\tau}{T_P} = 800\frac{1.5\times 10^{-6}}{0.74\times 10^{-3}} = 1620\,W$$

f) What is the duty cycle of this radar?

$$D_C = \frac{\tau}{T_P} = \frac{1.5\times 10^{-6}}{0.74\times 10^{-3}} \approx 0.002$$

Problem 1.2 1.2

A ground-based air-surveillance radar operates at a frequency of 1300 MHz (L band). Its maximum range is 200 nmi for the detection of a target with a radar cross section of one square meter ($\sigma = 1$ m^2). Its antenna is 12 m wide by 4 m high, and the antenna aperture efficiency is η_a = 0.65. The receiver minimum detectable signal is $S_{\min}$ = 10^{-13} W and overall radar loss of 0 dB.

Determine the following:

a) Antenna effective aperture A_e and antenna gain G [numerically and in dBi].

$$A_e = \eta_a A = 0.65\times 12\times 4 = 31.2\,m^2$$

$$\lambda = \frac{c}{f} = \frac{3\times 10^8}{1300\times 10^6} = \frac{3}{13}\ m$$

$$G = \frac{4\pi A_e}{\lambda^2} = 73622 = 38.67 dBi$$

b) Peak transmitter power.

$$P_R = \frac{P_T G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L}$$

$$P_T = \frac{P_R (4\pi)^3 R^4 L}{G^2 \lambda^2 \sigma} = \frac{S_{\min} (4\pi)^3 R^4 L}{G^2 \lambda^2 \sigma} = \frac{10^{-13} (4\pi)^3 \times (200 \times 1852)^4 (1)}{(73622)^2 \left(\frac{3}{13}\right)^2 (1)} = 12940\, W$$

c) Pulse repetition frequency to achieve a maximum unambiguous range of 200 nmi.

$$R_{un} = \frac{c}{2 f_P} \rightarrow \qquad f_P = \frac{c}{2 R_{un}} = \frac{3 \times 10^8}{2 \times 200 \times 1852} \approx 405\, Hz$$

d)

$$\therefore P_{av} = P_{peak} \frac{\tau}{T_P} = 12940 \left(2 \times 10 \quad \right)(405) = 10.48\, W$$

e) Duty cycle.

$$D_C = \frac{\tau}{T_P} = \left(2 \times 10^{-6}\right)(405) \approx 0.00081$$

f) Horizontal beamwidth (degrees).

$$Beamwidth = 65 \frac{\lambda}{D} = 65 \frac{3/13}{12} = 1.25^0$$

Problem 1.3 $\boxed{\text{1.3(a)(b)}}$

a) What is the peak power of a radar whose average transmitter power is 200 W, pulse width of 1 μs, and a pulse repetition frequency of 1000 Hz?

$$\therefore P_T = P_{peak} = P_{ave} \frac{T_P}{\tau} = P_{av} \frac{1}{\tau f_P} = 200 \frac{1}{\left(1 \times 10^{-6}\right)(1000)} = 200\ kW$$

b) What is the range (nmi) of this ground-based air-surveillance radar if it has to detect a target with a radar cross section of 2 m^2 when it operates at a frequency of 2.9 GHz (S band), with a rectangular-shaped antenna that is 5 m wide, 2.7 m high, antenna aperture efficiency of 0.6, overall radar loss of 0 dB and minimum detectable signal $S_{\min}$ equal to 10^{-12} W (based on P_T in the radar equation being the peak power)?

$$A_e = \eta_a A = 0.6 \times 5 \times 2.7 = 8.1\, m^2 \qquad \lambda = \frac{c}{f} = \frac{3 \times 10^8}{2.9 \times 10^9} = \frac{30}{29}\ m$$

$$G = \frac{4\pi A_e}{\lambda^2} = 95.11 = 19.78\, dBi$$

$$R_{\max} = \left(\frac{P_T G^2 \lambda^2 \sigma}{(4\pi)^3 L S_{\min}} \right)^{1/4} = \left(\frac{\left(200 \times 10^3\right)(95.11)^2 \left(\frac{30}{29}\right)^2 2}{(4\pi)^3 (1) 10^{-12}} \right)^{1/4} = 37375\, m$$

$$R_{\max} = \frac{37375}{1852} = 20.2\, nmi$$

Problem 1.4 $\boxed{1.5}$

A radar mounted on an automobile is to be used to determine the distance to a vehicle traveling directly in front of it. The radar operates at a frequency of 9375 MHz with a pulse width of 5ns. The maximum range is to be 150 m.

a) What is the pulse repetition frequency?

$$R_{un} = \frac{c}{2f_P} \rightarrow \qquad f_P = \frac{c}{2R_{un}} = \frac{3\times10^8}{2\times150} = 1\,MHz$$

b) What is the range resolution?

$$\Delta R = \frac{\tau c}{2} = \frac{\left(5\times10^{-9}\right)\left(3\times10^8\right)}{2} = 0.75\,m$$

c) If the antenna beamwidth were 3 deg, what would be the cross-range resolution at a range of 50 m and 100 m? do you think the resolution is sufficient?

$$\Delta x = R\times\theta_{3dB}$$

$$\theta_{3dB} = 3\times\frac{\pi}{180} = 0.0523\ \ rad$$

$$R = 50m \qquad \rightarrow\ \Delta x = 50\times0.0523 = 2.6\,m$$

$$R = 100m \qquad \rightarrow\ \Delta x = 100\times0.0523 = 5.23\,m$$

d) If the antenna dimensions were 0.5 ft by 0.5 ft and the antenna efficiency were 0.8, what would be the antenna gain?

$$A_e = \eta_a A = 0.8\times\tfrac{0.5}{3.28}\times\tfrac{0.5}{3.28} = 0.0186\,m^2$$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{9375\times10^6} = \frac{4}{125}\ m$$

$$G = \frac{4\pi A_e}{\lambda^2} = 228.25 = 23.58\,dBi$$

e) Find the average power required to detect a 10 m^2 radar cross section vehicle at a range of 150 m, if the minimum detectable signal is $5\text{x}10^{-12}$ W and overall radar loss of 0 dB?

$$P_T = \frac{P_R(4\pi)^3R^4L}{G^2\lambda^2\sigma} = \frac{S_{\min}(4\pi)^3R^4L}{G^2\lambda^2\sigma} = \frac{5\times10^{-12}(4\pi)^3\times(150)^4\,(1)}{(228.25)^2\left(\tfrac{4}{125}\right)^2(10)} = 9.41\ mW$$

$$\therefore P_{av} = P_{peak}\frac{\tau}{T_P} = 9.41\left(5\times10^{-9}\right)\left(1\times10^6\right) = 0.047\ mW$$

Problem 1.5

Compute the single pulse SNR for a high PRF radar with the following parameters: Peak power 100 kW, antenna gain 20 dBi, operating frequency 5.6 GHz, losses 8 dB, noise figure 5 dB, dwell time 2s, duty cycle 0.3. The range of interest is 50 km. Assume target RCS 0.01 m^{2,} effective system noise temperature 290 k.

$$P_{peak} = 100\, kW$$

$$G = 20\, dB = 10^2 = 100$$

$$\lambda = \frac{c}{f} = \frac{3\times 10^8}{5.6\times 10^9} = \frac{3}{56}\ m$$

$$L = 8\, dB = 10^{0.8} = 6.31$$

$$F = 5\, dB = 10^{0.5} = 3.16$$

$$T_D = 2\, s, \quad D_C = 0.3 \qquad \rightarrow \tau = T_D D_C = 2\times 0.3 = 0.6\, s$$

$$R = 50\, km = 50\times 10^3\ m$$

$$\sigma = 0.01\, m^2$$

$$(SNR)_1 = \frac{P_{peak} G^2 \lambda^2 \sigma \tau}{(4\pi)^3 R^4 k T_{eff} F L} = \frac{\left(100\times 10^3\right)(100)^2\left(\frac{3}{56}\right)^2(0.01)(0.6)}{(4\pi)^3\left(50\times 10^3\right)^4\left(1.38\times 10^{-23}\right)(290)(3.16)(6.31)} = 17.39 = 12.4\, dB$$

Problem 1.6

An L-band radar operating at frequency 1.5 MHz with an antenna of gain 36 dBi is designed to obtain a single pulse minimum signal-to-noise ratio of 20 dB. Assume the receiver bandwidth of 4 MHz, RCS of 10 m^2, noise figure of 10 dB, and the maximum range of 120 km.

a) Find the minimum detectable signal,

$$G = 36 \text{ dB} \approx 4000, \quad \text{SNR} = 20 \text{ dB} = 100, \quad \lambda = \frac{3\times 10^8}{1.5\times 10^9} = 0.2 \text{ m}, \quad F = 10 \text{ dB} = 10$$

The minimum detectable signal is

$$S_{\min} = F k T_{eff} B (\text{SNR})_{\min} = (10)(1.38\times 10^{-23})(290)(4\times 10^6)(100) \approx 16\times 10^{-12}\text{ W}$$

b) Calculate the peak power

$$P_T = \frac{P_R (4\pi)^3 R^4 L}{G^2 \lambda^2 \sigma} = \frac{(4\pi)^3 R^4 S_{\min} L}{G^2 \lambda^2 \sigma} = \frac{(4\pi)^3 (120\times 10^3)^4 (16\times 10^{-12})(1)}{(4000)^2 (0.2)^2 (10)} \approx 1.03 \text{ MW}$$

c) Calculate the pulse width for this radar.

$$\tau = \frac{1}{B} = \frac{1}{4\times 10^6} = .25\ \mu\text{s}$$

Problem 1.7

A typical millimeter wave (MMW) search radar operating at a frequency of 94 GHz is used in a sector defined by $\pm 30^{\circ}$ azimuth and 4° elevation scan, and has the following specifications:

Antenna Gain	40 dB
Antenna diameter	0.25 m
Radar cross section	25 m^2
System losses	10 dB
Noise figure	3 dB
Transmit peak power	5 W
Pulse width	40 ns
Pulse repetition frequency	10 kHz

a) Find the detection range for a signal-to-noise ratio of 10 dB.

$$\lambda = \frac{3\times10^{8}}{94\times10^{9}} = 0.0031 \text{ m} \quad \text{G} = 40 \text{ dBi} = 10^{4},\ L = 10 \text{ dB} = 10,\ \ F = 3 \text{ dB} = 2,$$

$$SNR = 10\ dB = 10^{1} \qquad f_P = 10\,kHz$$

$$\theta_{3dB} \approx \frac{\lambda}{D} = \frac{(0.0031)}{.25} = 0.0124\ rad = 0.014\times\frac{180}{\pi} = 0.71^{\circ},\ \ \Omega = (60)\times 5 = 300^{\circ}$$

$$SNR = \frac{P_T G^2 \lambda^2 \tau\sigma}{(4\pi)^3 R^4 k T_{eff} F L} \quad \to R = \left(\frac{P_T G^2 \lambda^2 \tau\sigma}{(4\pi)^3 k T_{eff} F L (SNR)}\right)^{1/4}$$

$$R = \left(\frac{P_T G^2 \lambda^2 \tau\sigma}{(4\pi)^3 k T_{eff} F L (SNR)}\right)^{1/4} = \left[\frac{(5)(10^4)^2(0.0031)^2\left(40\times10^{-9}\right)(25)}{(4\pi)^3(1.38\times10^{-23})(290)(2)(10)(10)}\right]^{1/4} = 1.32\ km$$

b) Find the number of antenna beam positions needed to cover the search volume, the antenna coverage rate, and the time on-target (dwell time) if the coverage is obtained in a radar frame time of 6 seconds.

$$T_{scan} = 6\,s$$

$$n_B = \frac{\Omega}{\theta_{3dB}^2} = \frac{300}{0.71} = 422$$

the antenna coverage rate $= \dfrac{\Omega}{T_{scan}} = \dfrac{300}{6} = 50\,degree/s$

$$T_D = \frac{T_{scan}}{n_B} = \frac{6}{422} = 14.22\,ms$$

c) Find the number of integrated pulses.

$$n = T_D f_P = (0.0142)(10^4) = 142$$

d) Find the detection range when an integration loss$((L_I)$ of 3 dB is included.

$$(L_I) = 3\,dB = 10^{0.3} = 2$$

For the case of n pulses integrated and integration loss included

$$R_{142} = \left(\frac{P_T G^2 \lambda^2 \tau \sigma n}{(4\pi)^3 k T_{eff} F L L_I (SNR)}\right)^{1/4} = R\left(\frac{n}{L_I}\right)^{1/4} = 1.32\left(\frac{142}{2}\right)^{1/4} = 3.83\,km$$

which indicates a significant improvement in the detection range

e) Find the maximum unambiguous range

$$R_{un} = \frac{T_P c}{2} = \frac{c}{2f_P} = \frac{3\times 10^8}{(2)(10\times 10^3)} = 15\ km$$

which justifies that the detection range even with integration is below the maximum unambiguous range.

Problem 1.8

A radar is subject to interference by a self-protection jammer. The radar and jammer specifications are:

Radar transmit power	60 kW
Radar antenna gain	50 dB
Radar pulse width	2.5 μs
Radar losses	10 dB
Jammer power	180 W
Jammer antenna gain	10 dB
Jammer bandwidth	45 MHz
Jammer losses	0 dB

a) Find the cross over range for a target of RCS σ = 5 m^2.

$$G = 50\text{ dB} = 10^5 \quad L = 10\text{ dB} = 10 \quad G_J = 10\text{ dB} = 10 \quad L_J = 0\text{ dB} = 1 \quad B = \frac{1}{2.5\times 10^{-6}} = 4\times 10^5$$

$$R_{CO} = \left(\frac{P_T G B_J L_J \sigma}{4\pi P_J G_J B_r L}\right)^{1/2} = \left[\frac{(60\times 10^3)(10^5)(45\times 10^6)(1)(5)}{(4\pi)(180)(10)(4\times 10^5)(10)}\right]^{1/2} = 3.86\text{ km}$$

b) Find the detection range if the required SJR for detection is 10 dB.

$$R_D = \frac{R_{CO}}{\sqrt{(S/J)_{\min}}} = \frac{3,862.74}{\sqrt{10}} = 1.22\text{ km}$$