chapter

1

An Introduction to Radar

1.1 BASIC RADAR

Radar is an electromagnetic system for the detection and location of reflecting objects such as aircraft, ships, spacecraft, vehicles, people, and the natural environment. It operates by radiating energy into space and detecting the echo signal reflected from an object, or target. The reflected energy that is returned to the radar not only indicates the presence of a target, but by comparing the received echo signal with the signal that was transmitted, its location can be determined along with other target-related information. Radar can perform its function at long or short distances and under conditions impervious to optical and infrared sensors. It can operate in darkness, haze, fog, rain, and snow. Its ability to measure distance with high accuracy and in all weather is one of its most important attributes.

The basic principle of radar is illustrated in Fig. 1.1. A transmitter (in the upper left portion of the figure) generates an electromagnetic signal (such as a short pulse of sinewave) that is radiated into space by an antenna. A portion of the transmitted energy is intercepted by the target and reradiated in many directions. The reradiation directed back towards the radar is collected by the radar antenna, which delivers it to a receiver. There it is processed to detect the presence of the target and determine its location. A single antenna is usually used on a time-shared basis for both transmitting and receiving when the radar waveform is a repetitive series of pulses. The range, or distance, to a target is found by measuring the time it takes for the radar signal to travel to the target and return back to the radar. (Radar engineers use the term *range* to mean *distance*, which is

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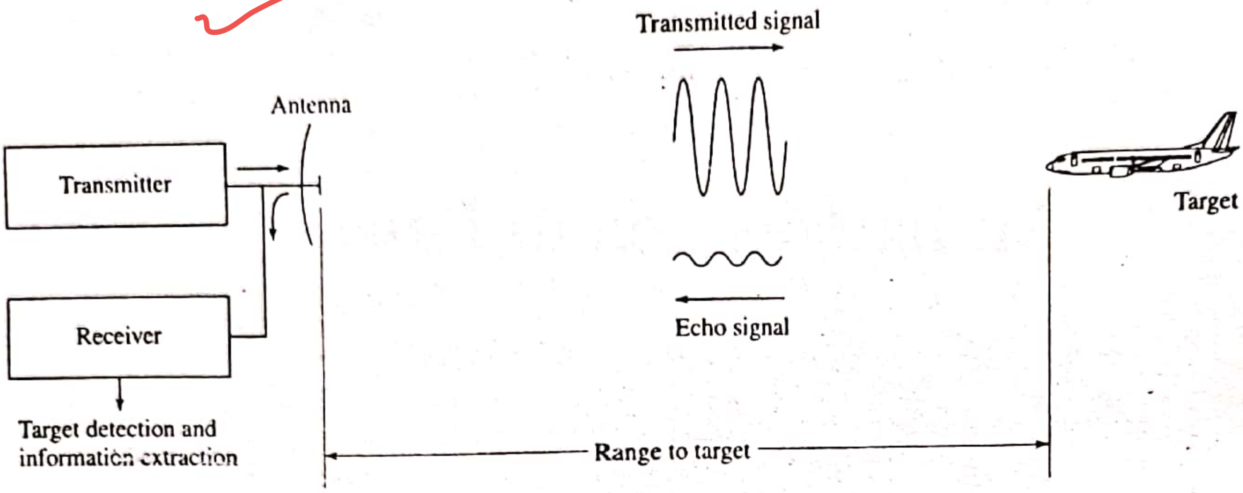

Figure 1.1 Basic principle of radar.

not the definition of range found in some dictionaries.*) The target's location in angle can be found from the direction the narrow-beamwidth radar antenna points when the received echo signal is of maximum amplitude. If the target is in motion, there is a shift in the frequency of the echo signal due to the doppler effect. This frequency shift is proportional to the velocity of the target relative to the radar (also called the radial velocity). The doppler frequency shift is widely used in radar as the basis for separating desired moving targets from fixed (unwanted) "clutter" echoes reflected from the natural environment such as land, sea, or rain. Radar can also provide information about the nature of the target being observed.

The term *radar* is a contraction of the words *ra*dio *d*etection *a*nd *r*anging. The name reflects the importance placed by the early workers in this field on the need for a device to detect the presence of a target and to measure its range. Although modern radar can extract more information from a target's echo signal than its range, the measurement of range is still one of its most important functions. There are no competitive techniques that can accurately measure long ranges in both clear and adverse weather as well as can radar.

Range to a Target The most common radar signal, or waveform, is a series of short-duration, somewhat rectangular-shaped pulses modulating a sinewave carrier. (This is sometimes called a *pulse train.*) The range to a target is determined by the time T_R it takes the radar signal to travel to the target and back. Electromagnetic energy in free space travels with the speed of light, which is $c = 3 \times 10^8$ m/s. Thus the time for the signal to travel to a target located at a range R and return back to the radar is $2R/c$. The range to a target is then

$$R = \frac{cT_R}{2} \tag{1.1}$$

*Webster's New Collegiate Dictionary defines *range* as "the horizontal distance to which a projectile can be propelled" or "the horizontal distance between a weapon and target." This is not how the term is used in radar. On the other hand, the dictionary defines *range finder* as "an instrument . . . to determine the distance to a target," which is its meaning in radar.

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With the range in kilometers or in nautical miles, and T in microseconds, Eq. (1.1) becomes

$$R(\text{km}) = 0.15\, T_R\,(\mu\text{s}) \qquad \text{or} \qquad R(\text{nmi}) = 0.081\, T_R\,(\mu\text{s})$$

Each microsecond of round-trip travel time corresponds to a distance of 150 meters, 164 yards, 492 feet, 0.081 nautical mile, or 0.093 statute mile. It takes 12.35 μs for a radar signal to travel a nautical mile and back.

Maximum Unambiguous Range Once a signal is radiated into space by a radar, sufficient time must elapse to allow all echo signals to return to the radar before the next pulse is transmitted. The rate at which pulses may be transmitted, therefore, is determined by the longest range at which targets are expected. If the time between pulses T_p is too short, an echo signal from a long-range target might arrive *after* the transmission of the next pulse and be mistakenly associated with that pulse rather than the actual pulse transmitted earlier. This can result in an incorrect or ambiguous measurement of the range. Echoes that arrive after the transmission of the next pulse are called *second-time-around echoes* (or *multiple-time-around echoes* if from even earlier pulses). Such an echo would appear to be at a closer range than actual and its range measurement could be misleading if it were not known to be a second-time-around echo. The range beyond which targets appear as second-time-around echoes is the *maximum unambiguous range*, R_{un}, and is given by

$$R_{un} = \frac{cT_p}{2} = \frac{c}{2f_p} \tag{1.2}$$

where T_p = pulse repetition period = $1/f_p$, and f_p = pulse repetition frequency (prf), usually given in hertz or pulses per second (pps). A plot of the maximum unambiguous range as a function of the pulse repetition frequency is shown in Fig. 1.2. The term *pulse repetition rate* is sometimes used interchangeably with *pulse repetition frequency*.

Radar Waveforms The typical radar utilizes a pulse waveform, an example of which is shown in Fig. 1.3. The peak power in this example is P_t = 1 MW, pulse width $\tau = 1\ \mu$s, and pulse repetition period T_p = 1 ms = 1000 μs. (The numbers shown were chosen for illustration and do not correspond to any particular radar, but they are similar to what might be expected for a medium-range air-surveillance radar.) The pulse repetition frequency f_p is 1000 Hz, which provides a maximum unambiguous range of 150 km, or 81 nmi. The average power (P_{av}) of a repetitive pulse-train waveform is equal to $P_t\tau/T_p = P_t\tau f_p$, so the average power in this case is $10^6 \times 10^{-6}/10^{-3}$ = 1 kW. The *duty cycle* of a radar waveform is defined as the ratio of the total time the radar is radiating to the total time it could have radiated, which is $\tau/T_p = \tau f_p$, or its equivalent P_{av}/P_t. In this case the duty cycle is 0.001. The energy of the pulse is equal to $P_t\tau$, which is 1 J (joule). If the radar could detect a signal of 10^{-12} W, the echo would be 180 dB below the level of the signal that was transmitted. A short-duration pulse waveform is attractive since the strong transmitter signal is not radiating when the weak echo signal is being received.

With a pulse width τ of 1 μs, the waveform extends in space over a distance $c\tau$ = 300 m. Two equal targets can be recognized as being resolved in range when they

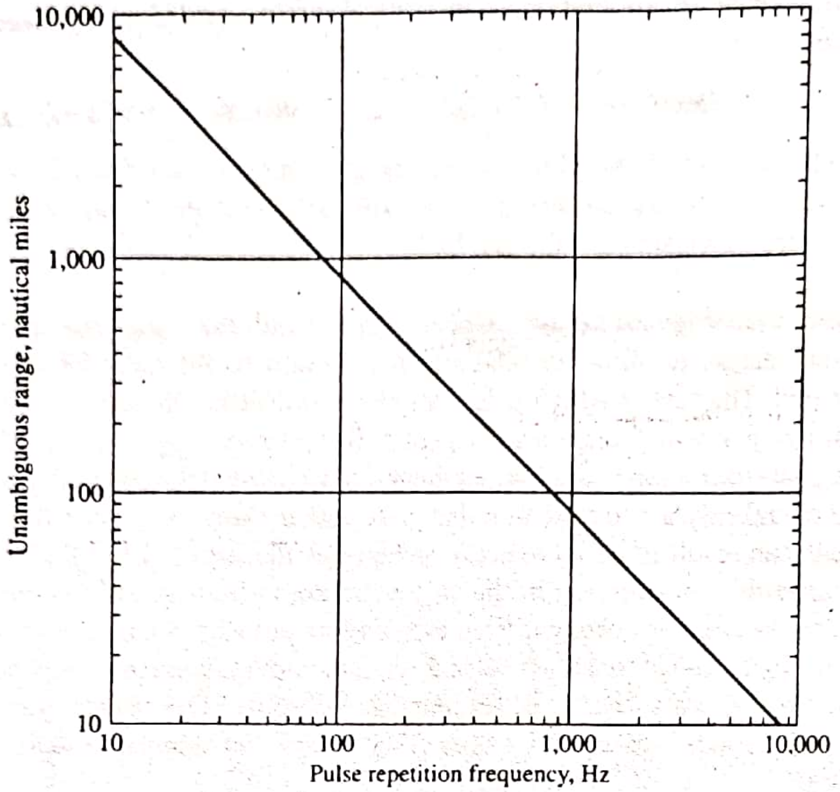

Figure ~~1.2~~ Plot of Eq. (1.2), the maximum unambiguous range R_{un} as a function of the pulse repetition frequency f_p.

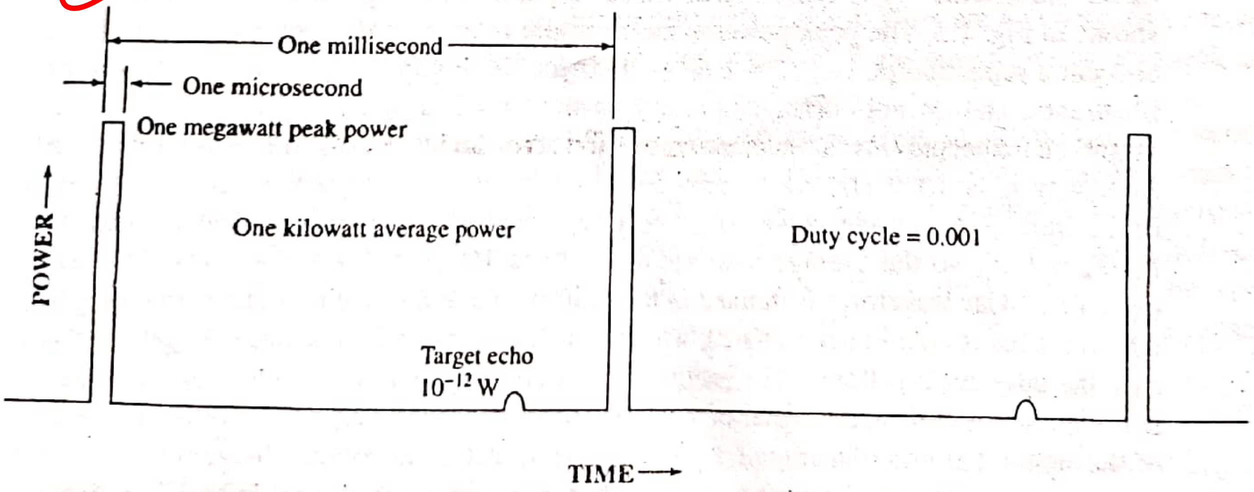

Figure ~~1.3~~ Example of a pulse waveform, with "typical" values for a medium-range air-surveillance radar. The rectangular pulses represent pulse-modulated sinewaves.

are separated a distance half this value, or $c\tau/2$. The factor of one-half results from the two-way travel of the radar wave. For example, when $\tau = 1\ \mu s$, two equal size targets can be resolved if they are separated by 150 m.

A very long pulse is needed for some long-range radars to achieve sufficient energy to detect small targets at long range. A long pulse, however, has poor resolution in the range dimension. Frequency or phase modulation can be used to increase the spectral width of a long pulse to obtain the resolution of a short pulse. This is called *pulse compression,* and is described in Sec. 6.5. Continuous wave (CW) waveforms have also been used in radar. Since they have to receive while transmitting, CW radars depend on the doppler frequency shift of the echo signal, caused by a moving target, to separate in the frequency domain the weak echo signal from the large transmitted signal and the echoes from fixed clutter (land, sea, or weather), as well as to measure the radial velocity of the target (Sec. 3.1). A simple CW radar does not measure range. It can obtain range, however, by modulating the carrier with frequency or phase modulation. An example is the frequency modulation (FM-CW) waveform used in the radar altimeter that measures the height (altitude) of an aircraft above the earth.

Pulse radars that extract the doppler frequency shift are called either *moving target indication* (MTI) or *pulse doppler* radars, depending on their particular values of pulse repetition frequency and duty cycle. An MTI radar has a low prf and a low duty cycle. A pulse doppler radar, on the other hand, has a high prf and a high duty cycle. Both types of doppler radars are discussed in Chap. 3. Almost all radars designed to detect aircraft use the doppler frequency shift to reject the large unwanted echoes from stationary clutter.

1.2 THE SIMPLE FORM OF THE RADAR EQUATION

The radar equation relates the range of a radar to the characteristics of the transmitter, receiver, antenna, target, and the environment. It is useful not only for determining the maximum range at which a particular radar can detect a target, but it can serve as a means for understanding the factors affecting radar performance. It is also an important tool to aid in radar system design. In this section, the simple form of the radar range equation is derived.

If the transmitter power P_t is radiated by an isotropic antenna (one that radiates uniformly in all directions), the *power density* at a distance R from the radar is equal to the radiated power divided by the surface area $4\pi R^2$ of an imaginary sphere of radius R, or

$$\text{Power density at range } R \text{ from an isotropic antenna} = \frac{P_t}{4\pi R^2} \qquad [1.3]$$

Power density is measured in units of watts per square meter. Radars, however, employ *directive* antennas (with narrow beamwidths) to concentrate the radiated power P_t in a particular direction. The *gain* of an antenna is a measure of the increased power density radiated in some direction as compared to the power density that would appear in that direction from an isotropic antenna. The maximum gain G of an antenna may be defined as

$$G = \frac{\text{maximum power density radiated by a directive antenna}}{\text{power density radiated by a lossless isotropic antenna with the same power input}}$$

The power density at the target from a directive antenna with a transmitting gain G is then

$$\text{Power density at range } R \text{ from a directive antenna} = \frac{P_t G}{4\pi R^2} \tag{1.4}$$

The target intercepts a portion of the incident energy and reradiates it in various directions. It is only the power density reradiated in the direction of the radar (the echo signal) that is of interest. The *radar cross section of the target* determines the power density returned to the radar for a particular power density incident on the target. It is denoted by σ and is often called, for short, *target cross section*, *radar cross section*, or simply *cross section*. The radar cross section is defined by the following equation:

$$\text{Reradiated power density back at the radar} = \frac{P_t G}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2} \tag{1.5}$$

The radar cross section has units of area, but it can be misleading to associate the radar cross section directly with the target's physical size. Radar cross section is more dependent on the target's shape than on its physical size, as discussed in Sec. 2.7.

The radar antenna captures a portion of the echo energy incident on it. The power received by the radar is given as the product of the incident power density [Eq. (1.5)] times the effective area A_e of the receiving antenna. The effective area is related to the physical area A by the relationship $A_e = \rho_a A$, where ρ_a = antenna aperture efficiency. The received signal power P_r (watts) is then

$$P_r = \frac{P_t G}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2} \cdot A_e = \frac{P_t G A_e \sigma}{(4\pi)^2 R^4} \tag{1.6}$$

The maximum range of a radar $R_{\max}$ is the distance beyond which the target cannot be detected. It occurs when the received signal power P_r just equals the minimum detectable signal $S_{\min}$. Substituting $S_{\min} = P_r$ in Eq. (1.6) and rearranging terms gives

$$R_{\max} = \left[\frac{P_t G A_e \sigma}{(4\pi)^2 S_{\min}}\right]^{1/4} \tag{1.7}$$

This is the fundamental form of the *radar range equation*. (It is also called, for simplicity, the *radar equation* or *range equation*.) The important antenna parameters are the transmitting gain and the receiving effective area. The transmitter power P_t has not been specified as either the average or the peak power. It depends on how $S_{\min}$ is defined. In this text, however, P_t denotes the peak power.

If the same antenna is used for both transmitting and receiving, as it usually is in radar, antenna theory gives the relationship between the transmit gain G and the receive effective area A_e as[1,2]

$$G = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \rho_a A}{\lambda^2} \tag{1.8}$$

where λ = wavelength. (Wavelength $\lambda = c/f$, where c = velocity of propagation and f = frequency.) Equation (1.8) can be substituted in Eq. (1.7), first for A_e and then for G, to give two other forms of the radar equation

$$R_{max} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}}\right]^{1/4} \qquad [1.9]$$

$$R_{max} = \left[\frac{P_t A_e^2 \sigma}{4\pi \lambda^2 S_{min}}\right]^{1/4} \qquad [1.10]$$

These three forms of the radar equation [Eqs. (1.7), (1.9), and (1.10)] are basically the same; but there are differences in interpretation. For example, from Eq. (1.9) it might be concluded that the maximum range varies as $\lambda^{1/2}$, but Eq. (1.10) indicates the variation with range as $\lambda^{-1/2}$, which is just the opposite. On the other hand, Eq. (1.7) gives no explicit wavelength dependence for the range. The correct interpretation depends on whether the antenna gain is held constant with change in wavelength, or frequency, as implied by Eq. (1.9); or the effective area is held constant, as implied by Eq. (1.10). For Eq. (1.7) to be independent of frequency, two antennas have to be used. The transmitting antenna has to have a gain independent of wavelength and the receiving antenna has to have an effective aperture independent of wavelength. (This is seldom done, however.)

These simplified versions of the radar equation do not adequately describe the performance of actual radars. Many important factors are not explicitly included. The simple form of the radar range equation predicts too high a value of range, sometimes by a factor of two or more. In Chap. 2 the simple form of the radar equation is expanded to include other factors that allow the equation to be in better agreement with the observed range performance of actual radars.

1.3 RADAR BLOCK DIAGRAM

The operation of a pulse radar may be described with the aid of the simple block diagram of Fig. 1.4. The transmitter may be a *power amplifier*, such as the klystron, traveling wave tube, or transistor amplifier. It might also be a power oscillator, such as the magnetron. The magnetron oscillator has been widely used for pulse radars of modest capability; but the amplifier is preferred when high average power is necessary, when other than simple pulse waveforms are required (as in pulse compression), or when good performance is needed in detecting moving targets in the midst of much larger clutter echoes based on the doppler frequency shift (the subject of Chap. 3). A power amplifier is indicated in Fig. 1.4. The radar signal is produced at low power by a *waveform generator*, which is then the input to the power amplifier. In most power amplifiers, except for solid-state power sources, a modulator (Sec. 10.7) turns the transmitter on and off in synchronism with the input pulses. When a power oscillator is used, it is also turned on and off by a *pulse modulator* to generate a pulse waveform.

The output of the transmitter is delivered to the *antenna* by a waveguide or other form of transmission line, where it is radiated into space. Antennas can be mechanically steered parabolic reflectors, mechanically steered planar arrays, or electronically steered phased arrays (Chap. 9). On transmit the parabolic reflector focuses the energy into a narrow

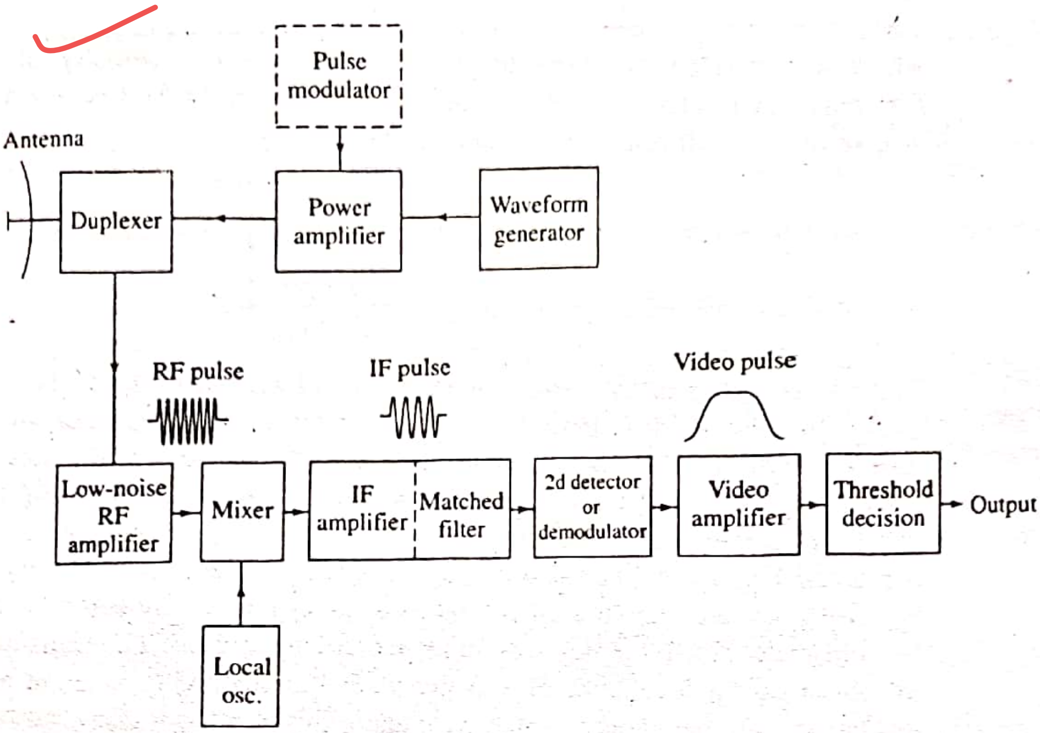

Figure 1.4 Block diagram of a conventional pulse radar with a superheterodyne receiver.

beam, just as does an automobile headlight or a searchlight. A phased array antenna is a collection of numerous small radiating elements whose signals combine in space to produce a radiating plane wave. Using phase shifters at each of the radiating elements, an electronically steered phased array can rapidly change the direction of the antenna beam in space without mechanically moving the antenna. When no other information is available about the antenna, the beamwidth (degrees) of a "typical" parabolic reflector is often approximated by the expression 65 λ/D, where D is the dimension of the antenna in the same plane as the beamwidth is measured, and λ is the radar wavelength. For example, an antenna with a horizontal dimension $D = 32.5$ wavelengths has an azimuth beamwidth of 2°. At a frequency of 3 GHz ($\lambda = 10$ cm), the antenna would be 3.25 m, or 10.7 ft, in extent. The rotation of a surveillance radar antenna through 360° in azimuth is called an antenna *scan*. A typical scan rate (or rotation rate) for a long-range civil air-traffic control air-surveillance radar might be 6 rpm. Military air-surveillance radars generally require a higher rotation rate.

The *duplexer* allows a single antenna to be used on a time-shared basis for both transmitting and receiving. The duplexer is generally a gaseous device that produces a short circuit (an arc discharge) at the input to the receiver when the transmitter is operating, so that high power flows to the antenna and not to the receiver. On reception, the duplexer directs the echo signal to the receiver and not to the transmitter. Solid-state ferrite circulators and receiver protector devices, usually solid-state diodes, can also be part of the duplexer.

The receiver is almost always a *superheterodyne*. The input, or RF,* stage can be a low-noise transistor amplifier. The mixer and local oscillator (LO) convert the RF signal

*In electrical engineering, RF is an abbreviation for *radio frequency*; but in radar practice it is understood to mean *radar frequency*. RF also is used to identify that portion of the radar that operates at RF frequencies, even though the inclusion of "frequencies" in this expression might seem redundant.

to an intermediate frequency (IF) where it is amplified by the IF amplifier. The signal bandwidth of a superheterodyne receiver is determined by the bandwidth of its IF stage. The IF frequency, for example, might be 30 or 60 MHz when the pulse width is of the order of 1 μs. (With a 1-μs pulse width, the IF bandwidth would be about 1 MHz.) The IF amplifier is designed as a *matched filter* (Sec. 5.2); that is, one which maximizes the output peak-signal-to-mean-noise ratio. Thus the matched filter maximizes the detectability of weak echo signals and attenuates unwanted signals. With the approximately rectangular pulse shapes commonly used in many radars, conventional radar receiver filters are close to that of a matched filter when the receiver bandwidth B is the inverse of the pulse width τ, or $B\tau \approx 1$.

Sometimes the low-noise input stage is omitted and the mixer becomes the first stage of the receiver. A receiver with a mixer as the input stage will be less sensitive because of the mixer's higher noise figure; but it will have greater dynamic range, less susceptibility to overload, and less vulnerability to electronic interference than a receiver with a low-noise first stage (Sec. 11.3). These attributes of a mixer input stage might be of interest for military radars subject to the noisy environment of hostile electronic countermeasures (ECM).

The IF amplifier is followed by a crystal diode, which is traditionally called the *second detector*, or *demodulator*. Its purpose is to assist in extracting the signal modulation from the carrier. The combination of IF amplifier, second detector, and video amplifier act as an *envelope detector* to pass the pulse modulation (envelope) and reject the carrier frequency. In radars that detect the doppler shift of the echo signal, the envelope detector is replaced by a *phase detector* (Sec. 3.1), which is different from the envelope detector shown here. The combination of IF amplifier and video amplifier is designed to provide sufficient amplification, or gain, to raise the level of the input signal to a magnitude where it can be seen on a display, such as a cathode-ray tube (CRT), or be the input to a digital computer for further processing.

At the output of the receiver a decision is made whether or not a target is present. The decision is based on the magnitude of the receiver output. If the output is large enough to exceed a predetermined threshold, the decision is that a target is present. If it does not cross the threshold, only noise is assumed to be present. The threshold level is set so that the rate at which false alarms occur due to noise crossing the threshold (in the absence of signal) is below some specified, tolerable value. This is fine if the noise remains constant, as when receiver noise dominates. If, on the other hand, the noise is external to the radar (as from unintentional interference or from deliberate noise jamming) or if clutter echoes (from the natural environment) are larger than the receiver noise, the threshold has to be varied adaptively in order to maintain the false alarm rate at a constant value. This is accomplished by a *constant false alarm rate* (CFAR) receiver (Sec. 5.7).

A radar usually receives many echo pulses from a target. The process of adding these pulses together to obtain a greater signal-to-noise ratio before the detection decision is made is called *integration*. The integrator is often found in the video portion of the receiver.

The *signal processor* is that part of the radar whose function is to pass the desired echo signal and reject unwanted signals, noise, or clutter. The signal processor is found in the receiver before the detection decision is made. The matched filter, mentioned previously, is an example of a signal processor. Another example is the doppler filter that

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separates desired moving targets (whose echoes are shifted in frequency due to the doppler effect) from undesired stationary clutter echoes.

Some radars process the detected target signal further, in the *data processor,* before displaying the information to an operator. An example is an *automatic tracker,* which uses the locations of the target measured over a period of time to establish the track (or path) of the target. Most modern air-surveillance radars and some surface-surveillance radars generate target tracks as their output rather than simply display detections. Following the data processor, or the decision function if there is no data processor, the radar output is displayed to an operator or used in a computer or other automatic device to provide some further action.

The signal processor and data processor are usually implemented with digital technology rather than with analog circuitry. The analog-to-digital (A/D) converter and digital memory are therefore important in modern radar systems. In some sophisticated radars in the past, the signal and data processors were larger and consumed more power than the transmitter and were a major factor in determining the overall radar system reliability; but this should not be taken as true in all cases.

A typical radar display for a surveillance radar is the PPI, or *plan position indicator* (the full term is seldom used). An example is shown in Fig. 1.5. The PPI is a presentation

Figure 1.5 Example of a PPI (plan position indicator) display. This is the output of an L-band radar with an antenna beam-width of 3°, without MTI processing. The range ring has a radius of 50 nmi. Clutter is seen in the near vicinity of the radar. Aircraft approaching an airport are seen in the northwest direction.
(Courtesy of George Linde of the Naval Research Laboratory.)

that maps in polar coordinates the location of the target in azimuth and range. The PPI in the past has been implemented with an intensity-modulated CRT. The amplitude of the receiver output modulates (turns on or off) the electron-beam intensity (called the z-axis of the CRT) as the electron beam is made to sweep outward (the range coordinate) from the center of the tube. The sweep of the electron beam rotates in angle in synchronism with the pointing of the antenna beam. A B-scope display is similar to a PPI except that it utilizes a rectangular format, rather than the polar format, to display range versus angle. Both the PPI and the B-scope CRT displays have limited dynamic range since they are intensity modulated. An A-scope is sometimes used for special purposes. It is an amplitude-modulated rectangular display that presents the receiver output on the y-axis and the range (or time delay) on the x-axis. (An example is shown in Fig. 7.21.) It is more suited for tracking radar or continuous staring applications than as a display for surveillance radar.

The early radars displayed to an operator *raw video,* which is the output of the radar receiver without further processing (with the exception of the matched filter). Modern radars usually present *processed video,* which is the output of the radar after signal processing and threshold detection or after automatic tracking. Only processed target detections or target tracks are presented. This relieves the burden on the operator, but processed video can also eliminate information about the environment and unusual operational situations that a trained and alert operator might be able to recognize and interpret.

Radars can operate in various modes by radiating different frequencies, with different polarizations. (The polarization of the radar wave is defined by the direction of the electric field vector.) The radar can also employ various waveforms with different pulse widths, pulse repetition frequencies, or other modulations; and different forms of processing for suppressing different types of clutter, interference, and jamming. The various waveforms and processing need to be selected wisely. A trained operator can fulfill this function, but an operator can become overloaded. When there are many available system options, the radar can be designed to automatically determine the proper mode of operation and execute what is required to implement it. The mode of radar operation is often changed as a function of the antenna look-direction and/or range, according to the nature of the environment.

1.4 RADAR FREQUENCIES

Conventional radars generally operate in what is called the *microwave* region (a term not rigidly defined). Operational radars in the past have been at frequencies ranging from about 100 MHz to 36 GHz, which covers more than eight octaves. These are not necessarily the limits. Operational HF over-the-horizon radars operate at frequencies as low as a few megahertz. At the other end of the spectrum, experimental millimeter wave radars have been at frequencies higher than 240 GHz.

During World War II, letter codes such as *S, X,* and *L* were used to designate the distinct frequency bands at which microwave radar was being developed. The original purpose was to maintain military secrecy; but the letter designations were continued after the war as a convenient shorthand means to readily denote the region of the spectrum at which a radar operated. Their usage is the accepted practice of radar engineers. Table 1.1 lists the radar-frequency letter-band designations approved as an IEEE Standard.[3] These are related to the specific frequency allocations assigned by the International Telecommunications Union (ITU) for radiolocation, or radar. For example, *L* band officially extends from 1000 MHz to 2000 MHz, but *L*-band radar is only allowed to operate within the region from 1215 to 1400 MHz since that is the band assigned by the ITU.

There have been other letter-band designations, but Table 1.1 is the only set of designations approved by the IEEE for radar. It has also been recognized by being listed in the U. S. Department of Defense Index of Specifications and Standards.[4] A different set of letter bands has been used by those working in electronic warfare. It was originally formulated by the U.S. Department of Defense for use only in conducting electronic countermeasure exercises.[5] Sometimes it is incorrectly extended to describe radar frequencies, but the use of the electronic warfare letter bands for radar can be confusing and it is not appropriate. (There may be *J*-band jammers, but according to the IEEE Letter-Band Standards there are no *J*-band radars.) Usually the context in which the nomenclature is employed can aid in distinguishing whether the letters refer to radar or to EW.

Table 1.1 IEEE standard radar-frequency letter-band nomenclature*

Band Designation	Nominal Frequency Range	Specific Frequency Ranges for Radar based on ITU Assignments in Region 2
HF	3–30 MHz	
VHF	30–300 MHz	138–144 MHz 216–225 MHz
UHF	300–1000 MHz	420–450 MHz 850–942 MHz
L	1–2 GHz	1215–1400 MHz
S	2–4 GHz	2300–2500 MHz 2700–3700 MHz
C	4–8 GHz	5250–5925 MHz
X	8–12 GHz	8500–10,680 MHz
K_u	12–18 GHz	13.4–14.0 GHz 15.7–17.7 GHz
K	18–27 GHz	24.05–24.25 GHz
K_a	27–40 GHz	33.4–36 GHz
V	40–75 GHz	59–64 GHz
W	75–110 GHz	76–81 GHz 92–100 GHz
mm	110–300 GHz	126–142 GHz 144–149 GHz 231–235 GHz 238–248 GHz

*From "IEEE Standard Letter Designations for Radar-Frequency Bands," IEEE Std 521–1984.

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Letter-band nomenclature is not a substitute for the actual numerical frequencies at which a radar operates. The specific numerical frequencies of a radar should be used whenever appropriate, but the letter designations of Table 1.1 should be used whenever a short notation is desired.

1.5 APPLICATIONS OF RADAR

Radar has been employed to detect targets on the ground, on the sea, in the air, in space, and even below ground. The major areas of radar application are briefly described below.

Military Radar is an important part of air-defense systems as well as the operation of offensive missiles and other weapons. In air defense it performs the functions of surveillance and weapon control. Surveillance includes target detection, target recognition, target tracking, and designation to a weapon system. Weapon-control radars track targets, direct the weapon to an intercept, and assess the effectiveness of the engagement (called *battle damage assessment*). A missile system might employ radar methods for guidance and fuzing of the weapon. High-resolution imaging radars, such as synthetic aperture radar, have been used for reconnaissance purposes and for detecting fixed and moving targets on the battlefield. Many of the civilian applications of radar are also used by the military. The military has been the major user of radar and the major means by which new radar technology has been developed.

Remote Sensing All radars are remote sensors; however, this term is used to imply the sensing of the environment. Four important examples of radar remote sensing are (1) weather observation, which is a regular part of TV weather reporting as well as a major input to national weather prediction; (2) planetary observation, such as the mapping of Venus beneath its visually opaque clouds; (3) short-range below-ground probing; and (4) mapping of sea ice to route shipping in an efficient manner.

Air Traffic Control (ATC) Radars have been employed around the world to safely control air traffic in the vicinity of airports (Air Surveillance Radar, or ASR), and en route from one airport to another (Air Route Surveillance Radar, or ARSR) as well as ground-vehicular traffic and taxiing aircraft on the ground (Airport Surface Detection Equipment, or ASDE). The ASR also maps regions of rain so that aircraft can be directed around them. There are also radars specifically dedicated to observing weather (including the hazardous downburst) in the vicinity of airports, which are called Terminal Doppler Weather Radar, or TDWR. The Air Traffic Control Radar Beacon System (ATCRBS and Mode-S) widely used for the control of air traffic, although not a radar, originated from military IFF (Identification Friend or Foe) and uses radar-like technology.

Law Enforcement and Highway Safety The radar speed meter, familiar to many, is used by police for enforcing speed limits. (A variation is used in sports to measure the speed of a pitched baseball.) Radar has been considered for making vehicles safer by warning

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of pending collision, actuating the air bag, or warning of obstructions or people behind a vehicle or in the side blind zone. It is also employed for the detection of intruders.

Aircraft Safety and Navigation The airborne weather-avoidance radar outlines regions of precipitation and dangerous wind shear to allow the pilot to avoid hazardous conditions. Low-flying military aircraft rely on terrain avoidance and terrain following radars to avoid colliding with obstructions or high terrain. Military aircraft employ ground-mapping radars to image a scene. The radio altimeter is also a radar used to indicate the height of an aircraft above the terrain and as a part of self-contained guidance systems over land.

Ship Safety Radar is found on ships and boats for collision avoidance and to observe navigation buoys, especially when the visibility is poor. Similar shore-based radars are used for surveillance of harbors and river traffic.

Space Space vehicles have used radar for rendezvous and docking, and for landing on the moon. As mentioned, they have been employed for planetary exploration, especially the planet Earth. Large ground-based radars are used for the detection and tracking of satellites and other space objects. The field of radar astronomy using Earth-based systems helped in understanding the nature of meteors, establishing an accurate measurement of the Astronomical Unit (the basic yardstick for measuring distances in the solar system), and observing the moon and nearby planets before adequate space vehicles were available to explore them at close distances.

Other Radar has also found application in industry for the noncontact measurement of speed and distance. It has been used for oil and gas exploration. Entomologists and ornithologists have applied radar to study the movements of insects and birds, which cannot be easily achieved by other means.

Some radar systems are small enough to be held in one's hand. Others are so large that they could occupy several football fields. They have been used at ranges close enough to almost touch the target and at ranges that reach to the planets.

1.6 THE ORIGINS OF RADAR[6,7,8]

The basic concept of radar was first demonstrated by the classical experiments conducted by the German physicist Heinrich Hertz from 1885 to 1888.[9] Hertz experimentally verified the predictions of James Clerk Maxwell's theory of the electromagnetic field published in 1864. Hertz used an apparatus that was similar in principle to a pulse radar at frequencies in the vicinity of 455 MHz. He demonstrated that radio waves behaved the same as light except for the considerable difference in frequency between the two. He showed that radio waves could be reflected from metallic objects and refracted by a dielectric prism.

Hertz received quick and widespread recognition for his work, but he did not pursue its practical applications. This was left to others. The potential of Hertz's work for the

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detection and location of reflecting objects—which is what radar does—was advanced by another German, Christian Hulsmeyer. In the early 1900s he assembled an instrument that would today be known as a monostatic (single site) pulse radar. It was much improved over the apparatus used by Hertz. In 1904 he obtained a patent in England[10] and other countries. Hulsmeyer's radar detected ships, and he extensively marketed it for preventing collisions at sea. He demonstrated his apparatus to shipping companies and to the German Navy. Although it was a success and much publicized, there apparently was no interest for a ship collision-avoidance device. His invention and his demonstrations faded from memory and were all but forgotten. Radar would have to be rediscovered a few more times before it eventually became an operational reality.

During the 1920s other evidence of the radar method appeared. S. G. Marconi, the well-known pioneer of wireless radio, observed the radio detection of targets in his experiments and strongly urged its use in a speech delivered in 1922 before the Institute of Radio Engineers (now the IEEE).[11] Apparently unaware of Marconi's speech, A. Hoyt Taylor and Leo C. Young of the U.S. Naval Research Laboratory in Washington, D.C. accidentally observed, in the autumn of 1922, a fluctuating signal at their receiver when a ship passed between the receiver and transmitter located on opposite sides of a river. This was called a *CW wave-interference system*, but today it is known as *bistatic CW radar*. (*Bistatic* means the radar requires two widely separated sites for the transmitter and receiver.) In 1925, the pulse radar technique was used by Breit and Tuve of the Carnegie Institution in Washington, D.C. to measure the height of the ionosphere. The Breit and Tuve apparatus was indeed a radar, but it was not recognized at the time that the same principle might be applied for the detection of ships and aircraft.[12] There were additional reported detections of aircraft and other targets by the CW wave-interference (bistatic radar) method in several countries of the world; but this type of radar did not have, and still doesn't have, significant utility for most applications.

It was the appearance of the heavy military bomber aircraft in the late 1920s and early 1930s that eventually gave rise to operational military radar. After World War I, the bomber was transformed from a fabric-coated biplane with open cockpit to an all-metal, single-wing aircraft with enclosed cockpit, which flew at high altitude over long distance with a heavy bomb load. Long-range warning of the approach of the heavy bomber became an important military need. In most of the countries that responded to this threat, the possible detection methods examined were similar even though the developments were covered by secrecy. Sound locators were the first of the sensors to be examined. They were deployed in many armies up to the start of World War II even though they were recognized much earlier to be inadequate for the task. Attempts were made to detect the spark-plug ignition noise radiated at radio frequencies by the aircraft engine; but they were abandoned once it was realized that the radiated noise could be suppressed by proper shielding. Infrared was examined but it did not have adequate range capability; it was not all-weather; and it did not determine the range of the target. The bistatic CW radar then followed from the accidental detection of aircraft, ships, or other targets as they passed between the transmitter and receiver of a radio system. This two-site configuration was cumbersome and merely acted as a trip wire to detect the passage of a target as it crossed the line connecting transmitter and receiver. The radar method did not become truly useful until the transmitter and receiver were colocated at a single site and pulse waveforms were used.

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PROBLEMS

1.1 *a.* What should be the pulse repetition frequency of a radar in order to achieve a maximum unambiguous range of 60 nmi?

b. How long does it take for the radar signal to travel out and back when the target is at the maximum unambiguous range?

c. If the radar has a pulse width of 1.5 μs, what is the extent (in meters) of the pulse energy in space in the range coordinate?

d. How far apart in range (meters) must two equal-size targets be separated in order to be certain they are completely resolved by a pulse width of 1.5 μs?

e. If the radar has a peak power of 800 kW, what is its average power?

f. What is the duty cycle of this radar?

1.2 A ground-based air-surveillance radar operates at a frequency of 1300 MHz (*L* band). Its maximum range is 200 nmi for the detection of a target with a radar cross section of one square meter ($\sigma = 1$ m^2). Its antenna is 12 m wide by 4 m high, and the antenna aperture efficiency is $\rho_a = 0.65$. The receiver minimum detectable signal is $S_{min} = 10^{-13}$ W. Determine the following:

a. Antenna effective aperture A_e (square meters) and antenna gain G [numerically and in dB, where G (in dB) $= 10 \log_{10} G$ (as a numeric)].

b. Peak transmitter power.

c. Pulse repetition frequency to achieve a maximum unambiguous range of 200 nmi.

d. Average transmitter power, if the pulse width is 2 μs.

e. Duty cycle.

f. Horizontal beamwidth (degrees).

1.3 *a.* What is the peak power of a radar whose average transmitter power is 200 W, pulse width of 1 μs, and a pulse repetition frequency of 1000 Hz?

b. What is the range (nmi) of this ground-based air-surveillance radar if it has to detect a target with a radar cross section of 2 m^2 when it operates at a frequency of 2.9 GHz (*S* band), with a rectangular-shaped antenna that is 5 m wide, 2.7 m high, antenna aperture efficiency ρ_a of 0.6, and minimum detectable signal S_{min} equal to 10^{-12} W (based on P_t in the radar equation being the peak power)?

c. Sketch the received echo-signal power as a function of range from 10 to 80 nmi.

1.4 The moon as a radar target may be described as follows: average distance to the moon is 3.844×10^8 m (about 208,000 nmi); experimentally measured radar cross section is 6.64×10^{11} m^2 (mean value over a range of radar frequencies); and its radius is 1.738×10^6 m.

a. What is the round-trip time (seconds) of a radar pulse to the moon and back?

b. What should the pulse repetition frequency (prf) be in order to have no range ambiguities?

c. For the purpose of probing the nature of the moon's surface, a much higher prf could be used than that found in (b). How high could the prf be if the purpose is to observe the echoes from the moon's front half?

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d. If an antenna with a diameter of 60 ft and aperture efficiency of 0.6 were used at a frequency of 430 MHz with a receiver having a minimum detectable signal of 1.5×10^{-16} W, what peak power is required? Does your answer surprise you; and if so, why?

e. The radar cross section of a smooth perfectly conducting sphere of radius a is πa^2. What would be the radar cross section of the moon if it were a sphere with a perfectly smooth, conducting surface? Why might the measured cross section of the moon (given above) be different from this value?

1.5 A radar mounted on an automobile is to be used to determine the distance to a vehicle traveling directly in front of it. The radar operates at a frequency of 9375 MHz (*X* band) with a pulse width of 10 ns (10^{-8} s). The maximum range is to be 500 ft.

a. What is the pulse repetition frequency that corresponds to a range of 500 ft?

b. What is the range resolution (meters)?

c. If the antenna beamwidth were 6°, what would be the cross-range resolution (meters) at a range of 500 ft? Do you think this value of cross-range resolution is sufficient?

d. If the antenna dimensions were 1 ft by 1 ft and the antenna efficiency were 0.6, what would be the antenna gain (dB)?

e. Find the average power required to detect a 10 m^2 radar cross section vehicle at a range of 500 ft, if the minimum detectable signal is 5×10^{-13} W.

1.6 Determine (a) the peak power (watts) and (b) the antenna physical area (m^2) which make the cost of the following radar a minimum:

Frequency: 1230 MHz (*L* band)

Antenna aperture efficiency: 0.6

Receiver minimum detectable signal: 3×10^{-13} W

Unit cost of transmitter: $2.20 per watt of peak power

Unit cost of antenna: $1400 per square meter of physical area

Cost of receiver and other items: $1,000,000

The radar must detect a target of 2 m^2 cross section at a range of 200 nmi.

(You will have to use one of the simple forms of the radar range equation.)

c. What is the cost of the antenna and the cost of the transmitter?

d. In a new radar design, how would you try, as a first attempt, to allocate the costs between the antenna and the transmitter (based only on the answers to the above problem)?

1.7 Who invented radar? (Please explain your answer.)

1.8 Three variants of the simple form of the radar equation have been given. In Eq. (1.9) the wavelength is in the numerator, in Eq. (1.10) the wavelength is in the denominator, and in Eq. (1.7), there is no explicit indication of wavelength. How would you respond to the question: "How does the radar range vary with the radar wavelength, everything else being the same?"

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1.9 If the weight of a transmitter is proportional to the transmitter power (i.e., $W_T = k_T P_t$) and if the weight of an antenna is proportional to its volume (so that we can say its weight is proportional to the 3/2 power of the antenna aperture area A, or $W_A = k_A A^{3/2}$), what is the relationship between the weight of the antenna and the weight of the transmitter that makes the total weight $W = W_T + W_A$ a minimum, assuming a fixed range? (You will need the simple form of the radar equation to obtain a relationship between P_t and A.)

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chapter

2

The Radar Equation

2.1 INTRODUCTION

The simple form of the radar equation, derived in Sec. 1.2, expressed the maximum radar range R_{max} in terms of the key radar parameters and the target's radar cross section when the radar sensitivity was limited by receiver noise. It was written:*

$$R_{max} = \left[\frac{P_t G A_e \sigma}{(4\pi)^2 S_{min}} \right]^{1/4} \qquad [2.1]$$

where

P_t = transmitted power, W
G = Antenna gain
A_e = Antenna effective aperture, m^2
σ = Radar cross section of the target, m^2
S_{min} = Minimum detectable signal, W

Except for the target's radar cross section, the parameters of this simple form of the radar equation are under the control of the radar designer. It states that if long ranges are desired, the transmitted power should be large, the radiated energy should be concentrated into a narrow beam (large transmitting gain), the echo energy should be received by a large antenna aperture (also synonymous with large gain), and the receiver should be sensitive to weak signals.

*This can also be written in terms of gain or effective aperture by using the relationship $G = 4\pi A_e/\lambda^2$.

In practice, however, this simple form of the radar equation does not adequately predict the range performance of actual radars. It is not unusual to find that when Eq. (2.1) is used, the actual range might be only half that predicted.[1] The failure of the simple form of the radar equation is due to (1) the statistical nature of the minimum detectable signal (usually determined by receiver noise), (2) fluctuations and uncertainties in the target's radar cross section, (3) the losses experienced throughout a radar system, and (4) propagation effects caused by the earth's surface and atmosphere. The statistical nature of receiver noise and the target cross section requires that the maximum radar range be described probabilistically rather than by a single number. Thus the specification of range must include the probability that the radar will detect a specified target at a particular range, and with a specified probability of making a false detection when no target echo is present. The range of a radar, therefore, will be a function of the *probability of detection*, P_d, and the *probability of false alarm*, P_{fa}.

The prediction of the radar range cannot be performed with arbitrarily high accuracy because of uncertainties in many of the parameters that determine the range. Even if the factors affecting the range could be predicted with high accuracy, the statistical nature of radar detection and the variability of the target's radar cross section and other effects make it difficult to accurately verify the predicted range. In spite of it not being as precise as one might wish, the radar equation is an important tool for (1) assessing the performance of a radar, (2) determining the system trade-offs that must be considered when designing a new radar system, and (3) aiding in generating the technical requirements for a new radar procurement.

In this chapter, the simple radar equation will be extended to include many of the important factors that influence the range of a radar when its performance is limited by receiver noise. A pulse waveform will be assumed, unless otherwise noted. In addition to providing a more complete representation of the radar range, this chapter introduces a number of basic radar concepts.

A thorough discussion of all the factors that influence the prediction of radar range is beyond the scope of a single chapter. For this reason, many subjects may appear to be treated lightly. More detailed information can be found in some of the subsequent chapters and in the references listed, especially those by Lamont Blake.[2,3]

2.2 DETECTION OF SIGNALS IN NOISE

The ability of a radar receiver to detect a weak echo signal is limited by the ever-present noise that occupies the same part of the frequency spectrum as the signal. The weakest signal that can just be detected by a receiver is the *minimum detectable signal*. In the radar equation of Eq. (2.1) it was denoted as S_{min}. Use of the minimum detectable signal, however, is not common in radar and is not the preferred method for describing the ability of a radar receiver to detect echo signals from targets, as shall be seen in Sec. 2.3.

Detection of a radar signal is based on establishing a threshold at the output of the receiver. If the receiver output is large enough to exceed the threshold, a target is said to be present. If the receiver output is not of sufficient amplitude to cross the threshold, only noise is said to be present. This is called *threshold detection.* Figure 2.1 represents the output of a radar receiver as a function of time. It can be thought of as the video output displayed on an A-scope (amplitude versus time, or range). The fluctuating appearance of the output is due to the random nature of receiver noise.

When a large echo signal from a target is present, as at *A* in Fig. 2.1, it can be recognized on the basis of its amplitude relative to the rms noise level. If the threshold level is set properly, the receiver output should not normally exceed the threshold if noise alone were present, but the output would exceed the threshold if a strong target echo signal were present along with the noise. If the threshold level were set too low, noise might exceed it and be mistaken for a target. This is called a *false alarm.* If the threshold were set too high, noise might not be large enough to cause false alarms, but weak target echoes might not exceed the threshold and would not be detected. When this occurs, it is called a *missed detection.* In early radars, the threshold level was set based on the judgment of the radar operator viewing the radar output on a cathode-ray tube display. In radars with automatic detection (electronic decision making), the threshold is set according to classical detection theory described later in this chapter.

The output that is shown in Fig. 2.1 is assumed to be from a *matched-filter* receiver. A matched filter, as was mentioned in Sec. 1.3, is one that maximizes the output signal-to-noise ratio. (This is discussed in detail in Sec. 6.2). Almost all radars employ a matched filter or a close approximation. A matched filter does not preserve the shape of the input waveform. For example, a rectangular-like pulse will be somewhat triangular in shape at the output of the matched filter. For this reason the receiver output drawn in this figure is more a series of triangular-like pulses rather than rectangular. The fact that the matched filter changes the shape of the received signal is of little consequence. The filter is not designed to preserve the signal shape, but to maximize detectability.

A threshold level is shown in Fig. 2.1 by the long dash line. If the signal is large enough, as at *A*, it is not difficult to decide that a target echo signal is present. But consider the two weaker signals at *B* and *C* representing two target echoes of equal amplitude. The noise accompanying the signal at *B* is assumed to be of positive amplitude and adds to the target signal so that the combination of signal plus noise crosses the threshold and is declared a target. At *C* the noise is assumed to subtract from the target signal, so that the resultant of signal and noise does not cross the threshold and is a missed

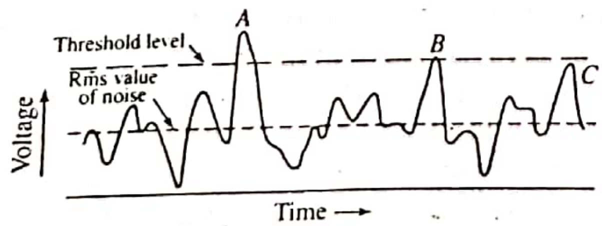

Figure 2.1 Envelope of the radar receiver output as a function of time (or range). *A*, *B*, and *C* represent signal plus noise. *A* and *B* would be valid detections, but *C* is a missed detection.

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detection. The ever-present noise, therefore, will sometimes enhance the detection of marginal signals, but it may also cause loss of detection.

The signal at C would have been detected if the threshold were lower. But too low a threshold increases the likelihood that noise alone will exceed the threshold and be improperly called a detection. The selection of the proper threshold is therefore a compromise that depends upon how important it is to avoid the mistake of (1) failing to recognize a target signal that is present (missed detection) or (2) falsely indicating the presence of a target signal when none exists (false alarm).

The signal-to-noise ratio, as has been mentioned, is a better measure of a radar's detection performance than is the minimum detectable signal. The relationship between the two is developed next.

2.3 RECEIVER NOISE AND THE SIGNAL-TO-NOISE RATIO

At microwave frequencies, the noise with which the target echo signal competes is usually generated within the receiver itself. If the radar were to operate in a perfectly noise-free environment so that no external sources of noise accompany the target signal, and if the receiver itself were so perfect that it did not generate any excess noise, there would still be noise generated by the thermal agitation of the conduction electrons in the ohmic portion of the receiver input stages. This is called *thermal noise* or *Johnson noise*. Its magnitude is directly proportional to the bandwidth and the absolute temperature of the ohmic portions of the input circuit. The available thermal-noise power (watts) generated at the input of a receiver of bandwidth B_n (hertz) at a temperature T (degrees Kelvin) is[4]

$$\text{available thermal-noise power} = kTB_n \qquad [2.2]$$

where k = Boltzmann's constant = 1.38×10^{-23} J/deg. (The term *available* means that the device is operated with a matched input and a matched load.) The bandwidth of a superheterodyne receiver (and almost all radar receivers are of this type) is taken to be that of the IF amplifier (or matched filter).

In Eq. (2.2) the bandwidth B_n is called the *noise bandwidth*, defined as

$$B_n = \frac{\int_0^\infty |H(f)|^2 \, df}{\boxed{|H(f0)|^2}} \qquad [2.3]$$

where $H(f)$ = frequency-response function of the IF amplifier (filter) and f_0 = frequency of the maximum response (usually occurs at midband). Noise bandwidth is not the same as the more familiar half-power, or 3-dB, bandwidth. Equation (2.3) states that the noise bandwidth is the bandwidth of the equivalent rectangular filter whose noise-power output is the same as the filter with frequency response function $H(f)$. The *half-power bandwidth*, a term widely used in electronic engineering, is defined by the separation between the points of the frequency response function $H(f)$ where the response is reduced 0.707 (3 dB in power) from its maximum value. Although it is not the same as the noise bandwidth, the half-power bandwidth is a reasonable approximation for many practical radar receivers.[5,6] Thus the half-power bandwidth B is usually used to approximate the noise bandwidth B_n, which will be the practice in the remainder of the chapter.

The noise power in practical receivers is greater than that from thermal noise alone. The measure of the noise out of a real receiver (or network) to that from the ideal receiver with only thermal noise is called the *noise figure* and is defined as

$$F_n = \frac{\text{noise out of practical receiver}}{\text{noise out of ideal receiver at std temp } T_0} = \frac{N_{out}}{kT_0BG_a} \qquad [2.4]$$

where N_{out} = noise out of the receiver, and G_a = available gain. The noise figure is defined in terms of a standard temperature T_0, which the IEEE defines as 290 K (62°F). This is close to room temperature. (A standard temperature assures uniformity in measurements that might be made at different temperatures.) With this definition, the factor kT_0 in the definition of noise figure is 4×10^{-21} W/Hz, a quantity easier to remember than Boltzmann's constant. The available gain G_a is the ratio of the signal out, S_{out}, to the signal in, S_{in}, with both the output and input matched to deliver maximum output power. The input noise, N_{in}, in an ideal receiver is equal to kT_0B_n. The definition of noise figure given by Eq. (2.4) therefore can be rewritten as

$$F_n = \frac{S_{in}/N_{in}}{S_{out}/N_{out}} \qquad [2.5]$$

This equation shows that the noise figure may be interpreted as a measure of the degradation of the signal-to-noise ratio as the signal passes through the receiver.

Rearranging Eq. (2.5), the input signal is

$$S_{in} = \frac{kT_0BF_nS_{out}}{N_{out}} \qquad [2.6]$$

If the minimum detectable signal S_{min} is that value of S_{in} which corresponds to the minimum detectable signal-to-noise ratio at the output of the IF, $(S_{out}/N_{out})_{min}$, then

$$S_{min} = kT_0BF_n\left(\frac{S_{out}}{N_{out}}\right)_{min} \qquad [2.7]$$

Substituting the above into Eq. (2.1), and omitting the subscripts on S and N, results in the following form of the radar equation:

$$R_{max}^4 = \frac{P_tGA_e\sigma}{(4\pi)^2kT_0BF_n(S/N)_{min}} \qquad [2.8]$$

For convenience, R_{max} on the left-hand side is usually written as the fourth power rather than take the fourth root of the right-hand side of the equation.

The minimum detectable signal is replaced in the radar equation by the *minimum detectable signal-to-noise ratio* $(S/N)_{min}$. The advantage is that $(S/N)_{min}$ is independent of the receiver bandwidth and noise figure; and, as we shall see in Sec. 2.5, it can be expressed in terms of the probability of detection and the probability of false alarm, two parameters that can be related to the radar user's needs.

The signal-to-noise ratio in the above is that at the output of the IF amplifier, since maximizing the signal-to-noise ratio at the output of the IF is equivalent to maximizing the video output where the threshold decision is made.[7]

Before continuing the development of the radar equation, it is necessary to digress and briefly review the concept of the *probability density function* in order to describe the signal-to-noise ratio in statistical terms. Those familiar with this subject can omit the next section.

2.4 PROBABILITY DENSITY FUNCTIONS

In this section, we introduce the concept of the probability density function and give some examples that are important in the detection of radar signals.

Noise is a random phenomenon; hence, the detection of signals in the presence of noise is also a random phenomenon and should be described in probabilistic terms. *Probability* is a measure of the likelihood of the occurrence of an event. The scale of probability ranges from 0 to 1. (Sometimes probabilities are expressed in percent—from 0 to 100 percent—rather than 0 to 1.) An event that is certain has a probability of 1. An impossible event has a probability 0. The intermediate probabilities are assigned so that the more likely an event, the greater is its probability. Probabilities represent discrete events. Continuous functions, such as random noise, are represented by the *probability density function*, abbreviated *pdf*.

Consider the variable x as representing the value of a random process such as a noise voltage or current. Imagine each x to define a point on a straight vertical line corresponding to the distance from a fixed reference point. The distance x from the reference point might represent the value of the noise voltage or noise current. Divide the line into small segments of length Δx and count the number of times that x falls within each interval. The probability density function is then defined as

$$p(x) = \lim_{\substack{\Delta x \to 0 \\ N \to \infty}} \frac{(\text{number of values within } \Delta x \text{ at } x)/\Delta x}{\text{total number of values} = N} \qquad [2.9]$$

Thus $p(x)$ expresses probability as a density rather than discrete values, and is more appropriate for continuous functions of time as is noise in a radar receiver.

The probability that a particular value of x lies within the infinitesimal interval dx centered at x is simply $p(x)dx$. The probability that the value of x lies within the finite range from x_1 to x_2 is found by integrating $p(x)$ over the range of interest, or

$$\text{probability } (x_1 < x < x_2) = \int_{x_1}^{x_2} p(x)\, dx \qquad [2.10]$$

The probability density function, by definition, is always positive. Since every measurement must yield some value, the integral of the probability density function over all values of x must equal unity; that is,

$$\int_{-\infty}^{\infty} p(x)\, dx = 1 \qquad [2.11]$$

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This condition is used to normalize the pdf. The average value of a variable function $\phi(x)$ that is described by the probability density function $p(x)$ is

$$\langle \phi(x) \rangle_{av} = \int_{-\infty}^{\infty} \phi(x) p(x)\, dx \tag{2.12}$$

This follows from the definitions of an average value and the probability density function. From the above, the mean, or average, value of x is

$$\langle x \rangle_{av} = m_1 = \int_{-\infty}^{\infty} x p(x)\, dx \tag{2.13}$$

and the mean square value of x is

$$\langle x^2 \rangle_{av} = m_2 = \int_{-\infty}^{\infty} x^2 p(x)\, dx \tag{2.14}$$

The quantities m_1 and m_2 are called the *first* and *second moments* of the random variable x. If x represents an electric voltage or current, m_1 is the d-c component. It is the value read by a direct-current voltmeter or ammeter. The mean square value m_2 of the current, when multiplied by the resistance, gives the average power. (In detection theory, it is customary to take the resistance as 1 ohm, so that m_2 is often stated to be the average power.) The *variance* σ^2 is the mean square deviation of x about its mean m_1 and can be expressed as

$$\sigma^2 = \langle (x - m_1)^2 \rangle_{av} = \int_{-\infty}^{\infty} (x - m_1)^2 p(x)\, dx = m_2 - m_1^2 \tag{2.15}$$

It is sometimes called the *second central moment.* If the random variable x is a noise current, the product of the variance and resistance is the mean power of the a-c component. The square root of the variance is the *standard deviation* and is the root mean square (rms) of the a-c component. It is usually designated as σ. We next consider four examples of probability density functions.

Uniform pdf This is shown in Fig. 2.2a and is defined as

$$\begin{aligned} p(x) &= k && \text{for } a < x < a + b \\ &= 0 && \text{for } x < a \text{ and } x > a + b \end{aligned}$$

where k is a constant. It describes the phase of a random sinewave relative to a particular origin of time, where the phase of the sinewave is, with equal probability, anywhere from 0 to 2π radians. The uniform pdf also describes the distribution of the round-off (quantizing) error in numerical computations and in analog-to-digital converters.

The constant k is found to be equal to $1/b$ by requiring that the integral of the probability density function over all values of x equal unity [Eq. (2.11)]. From Eq. (2.13) the average value of the uniform distribution is $a + (b/2)$, which could have been found from inspection in this simple example. The variance from Eq. (2.15) equals $b^2/12$.

Gaussian pdf The gaussian pdf is important in detection theory since it describes many sources of noise, including receiver thermal noise. Also, it is more convenient to

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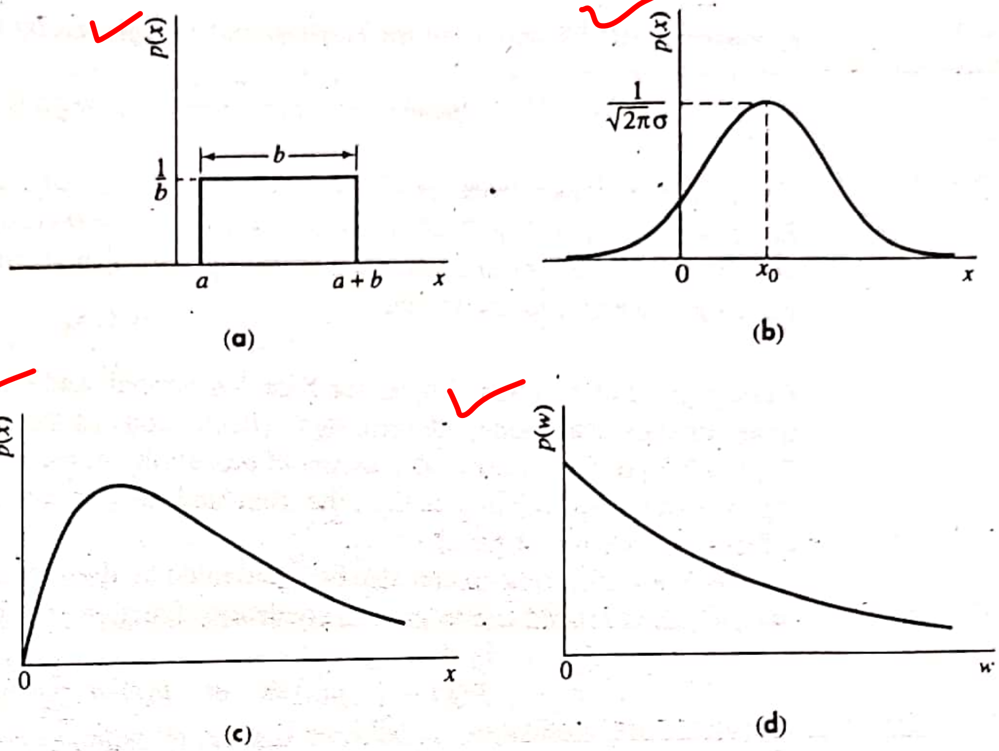
Figure 2.2 Examples of probability density functions. (a) uniform; (b) gaussian; (c) Rayleigh (voltage); (d) Rayleigh (power), or exponential.



manipulate mathematically than many other pdfs. The gaussian probability density function has a bell-shaped appearance, Fig. 2.2b, and is given by

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-x_0)^2}{2\sigma^2}\right] \quad [2.16]$$

where exp [·] is the exponential function, σ^2 is the variance of x, and x_0 is the mean value of x.

The ability of the gaussian pdf to represent many statistical phenomena is a consequence of the *central limit theorem*. It states that, under very general assumptions, the probability density function of the sum of a large number of independently distributed quantities approaches the gaussian pdf no matter what the individual pdfs might be.

HW

Rayleigh pdf This is of interest for radar since the envelope of a narrowband filter (such as the IF filter of a radar receiver) is described by the Rayleigh pdf when the input noise voltage is gaussian. The statistical behavior of the radar cross section of some types of targets and some types of clutter also fit this pdf. It is given as

$$p(x) = \frac{2x}{m_2} \exp\left(-\frac{x^2}{m_2}\right) \qquad x \geq 0 \quad [2.17]$$

where $m_2 = \langle x^2 \rangle_{av}$ is the mean square value of x. The Rayleigh pdf is shown in Fig. 2.2c. It is a one-parameter pdf (the mean square value), and has the property that its standard deviation is equal to $\sqrt{(4/\pi) - 1} = 0.523$ times the mean value.

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HW

Exponential pdf When x^2 in the Rayleigh pdf is replaced by w, the pdf becomes

$$p(w) = \frac{1}{w_0} \exp\left(-\frac{w}{w_0}\right) \qquad w \geq 0 \tag{2.18}$$

where w_0 is the mean value of w. This is the *exponential* pdf, but is sometimes called the *Rayleigh-power* pdf, Fig. 2.2d. If the parameter x in the Rayleigh pdf is the voltage, then w represents the power and w_0 is the average power. The standard deviation of the exponential pdf is equal to the mean.

Other pdfs Later in this chapter the Rice, log normal, and chi-square pdfs will be mentioned as statistical models describing the fluctuations of the target's radar cross section. Section 7.5 provides further discussion of probability density functions as applied to the statistics of clutter. (Clutter is the echo from land, sea, or weather that interferes with the detection of desired targets.)

Statistical phenomena can also be represented by the probability distribution function $P(x)$, which is related to the probability density function $p(x)$ by

$$P(x) = \int_{-\infty}^{x} p(x)\,dx \quad \text{or} \quad p(x) = \frac{d}{dx}P(x) \tag{2.19}$$

Example of the Use of Probability Density Functions The following is a simple example of the use of probability density functions. It involves the calculation of the mean value (d-c component) of the voltage output of a half-wave linear rectifier when the input is gaussian noise voltage of zero mean (thermal noise).[8] The answer itself is of little consequence for our interest in radar detection, but the method used is similar to the more complicated procedures for finding the statistical outputs of a radar receiver mentioned in the next section.

The probability density function (pdf) of the zero-mean gaussian noise voltage x at the input is

$$p(x)\,dx = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx \qquad -\infty < x < \infty$$

The output y of a half-wave rectifier for an input x is given as

$$y = ax \qquad x \geq 0,$$

and

$$y = 0 \qquad x < 0$$

where a = constant. To find the mean value of the output, we have to find the pdf at the output. There are three components to the output pdf. The first component is the probability that the rectifier output, $y > 0$, will lie between y and $y + dy$. It is the same as the probability that x lies between x and $x + dx$ when $x > 0$. Thus,

$$p(y)\,dy = p(x)\,dx = \frac{1}{\sqrt{2\pi}a\sigma} e^{-\frac{y^2}{2a^2\sigma^2}} dy \qquad y > 0$$

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The second component is the probability that $y = 0$, which is the same as the probability that $x < 0$, which is 1/2 (since half the time the noise voltage is negative and is not passed by the half-wave rectifier). This is represented by $(1/2)\delta(y)$, where $\delta(y)$ is the delta function which has the value 1 when $y = 0$, and is 0 otherwise. The third component is when $y < 0$. There is no output from a rectifier when $y < 0$, thus the probability is 0 that $y < 0$. Combining the three components gives

$$p(y)\,dy = \frac{1}{\sqrt{2\pi}a\sigma} e^{-\frac{y^2}{2a^2\sigma^2}}\,dy + \frac{1}{2}\delta(y)\,dy + 0 \qquad y \geq 0$$

The d-c component is $m_1 = \int_{-\infty}^{\infty} yp(y)\,dy$, or

$$m_1 = \frac{1}{\sqrt{2\pi}a\sigma}\int_0^{\infty} ye^{-\frac{y^2}{2a^2\sigma^2}}\,dy + \frac{1}{2}\int_{-\infty}^{\infty} y\delta(y)\,dy$$

The second integral containing the delta function is zero, since $\delta(y)$ has a value only when $y = 0$. The first integral is easily evaluated. The result is the d-c component, which is $a\sigma/\sqrt{2\pi}$.

In this example we started with the pdf describing the input and found the pdf describing the output. In the next section we will follow a similar procedure to find the probabilities of detection and false alarm, but will only provide the answers rather than go through the more elaborate mathematical derivation.

2.5 PROBABILITIES OF DETECTION AND FALSE ALARM

Next it is shown how to find the minimum signal-to-noise ratio required to achieve a specified probability of detection and probability of false alarm. The signal-to-noise ratio is needed in order to calculate the maximum range of a radar using the radar range equation as was given by Eq. (2.8). The basic concepts for the detection of signals in noise may be found in a classical review paper by Rice[9] or one of several texts on detection theory.[10]

Envelope Detector Figure 2.3 shows a portion of a superheterodyne radar receiver with IF amplifier of bandwidth B_{IF}, second detector,* video amplifier with bandwidth B_v, and a threshold where the detection decision is made. The IF filter, second detector, and video filter form an *envelope detector* in that the output of the video amplifier is the envelope, or modulation, of the IF signal. (An envelope detector requires that the video bandwidth $B_v \geq B_{IF}/2$ and the IF center frequency $f_{IF} >> B_{IF}$. These conditions are usually met in

*The diode stage in the envelope detector of a superheterodyne receiver has traditionally been called the *second detector* since the mixer stage, which also employs a diode, was originally called the first detector. The mixer stage is no longer known as the first detector, but the name second detector has been retained in radar practice to distinguish it from other forms of detectors used in radar receivers (such as phase detectors and phase-sensitive detectors).

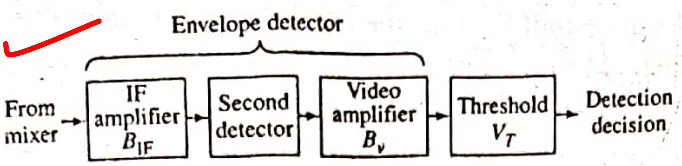

Figure 2.3 Portion of the radar receiver where the echo signal is detected and the detection decision is made.

radar.) The envelope detector passes the modulation and rejects the carrier. The second detector is a nonlinear device (such as a diode). Either a linear or a square-law detector characteristic may be assumed since the effect on the detection probability is relatively insensitive to the choice. (A square-law characteristic is usually easier to handle mathematically, but a linear law is preferred in practice since it allows a larger dynamic range than the square law.) The bandwidth of the radar receiver is the bandwidth of the IF amplifier. The envelope of the IF amplifier output is the signal applied to the threshold detector. When the receiver output crosses the threshold, a signal is declared to be present.

Probability of False Alarm The receiver noise at the input to the IF filter (the terms *filter* and *amplifier* are used interchangeably here) is described by the gaussian probability density function of Eq. (2.16) with mean value of zero, or

$$p(v) = \frac{1}{\sqrt{2\pi\Psi_0}} \exp\left(-\frac{v^2}{2\Psi_0}\right) \tag{2.20}$$

where $p(v)\,dv$ is the probability of finding the noise voltage v between the values of v and $v + dv$ and Ψ_0 is the mean square value of the noise voltage (mean noise power). S. O. Rice has shown in a *Bell System Technical Journal* paper[9] that when gaussian noise is passed through the IF filter, the probability density function of the envelope R is given by a form of the Rayleigh pdf:

$$p(R) = \frac{R}{\Psi_0} \exp\left(-\frac{R^2}{2\Psi_0}\right) \tag{2.21}$$

The probability that the envelope of the noise voltage will exceed the voltage threshold V_T is the integral of $p(R)$ evaluated from V_T to ∞, or

$$\text{Probability } (V_T < R < \infty) = \int_{V_T}^{\infty} \frac{R}{\Psi_0} \exp\left(-\frac{R^2}{2\Psi_0}\right) dR = \exp\left(\frac{-V_T^2}{2\Psi_0}\right) \tag{2.22}$$

This is the *probability of a false alarm* since it represents the probability that noise will cross the threshold and be called a target when only noise is present. Thus, the probability of a false alarm, denoted P_{fa}, is

$$P_{fa} = \exp\left(\frac{-V_T^2}{2\Psi_0}\right) \tag{2.23}$$

By itself, the probability of false alarm as given by Eq. (2.23) does not indicate whether or not a radar will be troubled by excessive false indications of targets. The time between false alarms is a better measure of the effect of noise on radar performance.

Figure 2.4 illustrates the occurrence of false alarms. The average time between crossings of the decision threshold when noise alone is present is called the *false-alarm time*, T_{fa}, and is given by

$$T_{fa} = \lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} T_k \qquad [2.24]$$

where T_k is the time between crossings of the threshold V_T by the noise envelope. The false-alarm time is something a radar customer or operator can better relate to than the probability of false alarm. The false-alarm probability can be expressed in terms of false-alarm time by noting that the false-alarm probability P_{fa} is the ratio of the time the envelope is actually above the threshold to the total time it could have been above the threshold, or

$$P_{fa} = \frac{\sum_{k=1}^{N} t_k}{\quad} = \frac{\langle t_k \rangle_{av}}{\quad} = \frac{1}{T_{fa}B} \qquad [2.25]$$

where t_k and T_k are shown in Fig. 2.4, and B is the bandwidth of the IF amplifier of the radar receiver. The average duration of a threshold crossing by noise $\langle t_k \rangle_{av}$ is approximately the reciprocal of the IF bandwidth B. The average of T_k is the *false-alarm time*, T_{fa}. Equating Eqs. (2.23) and (2.25) yields

$$T_{fa} = \frac{1}{B}\exp\left(\frac{V_T^2}{2\psi_0}\right) \qquad [2.26]$$

A plot of T_{fa} as a function of $V_T^2/2\Psi_0$ is shown in Fig. 2.5. If, for example, the bandwidth of the IF amplifier were 1 MHz and the average time between false alarms were specified to be 15 min, the probability of a false alarm is 1.11×10^{-9}. The threshold voltage,

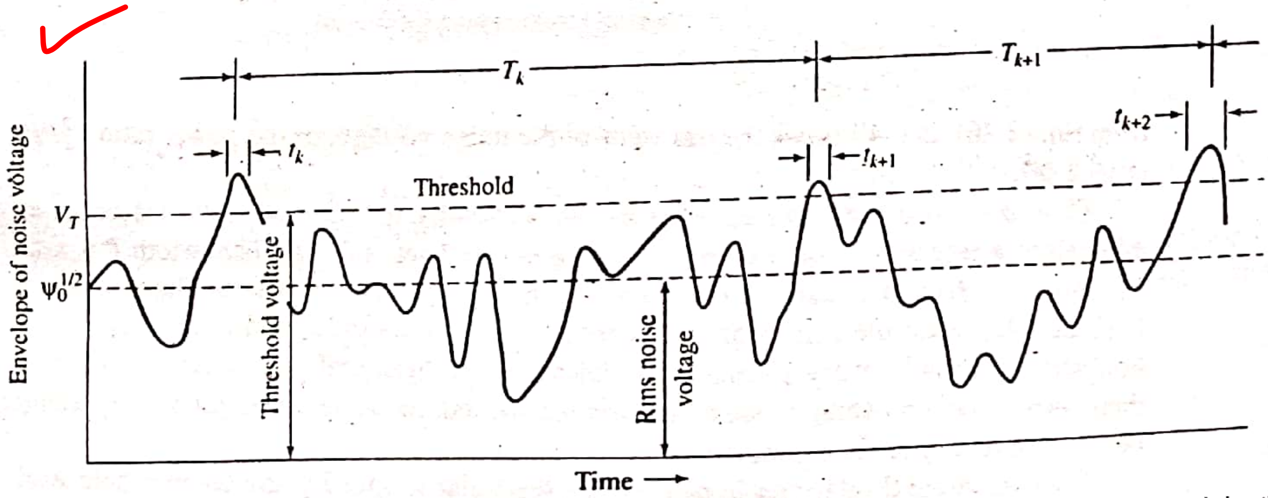

Figure 2.4 Envelope of the receiver output with noise alone, illustrating the duration of false alarms and the time between false alarms.

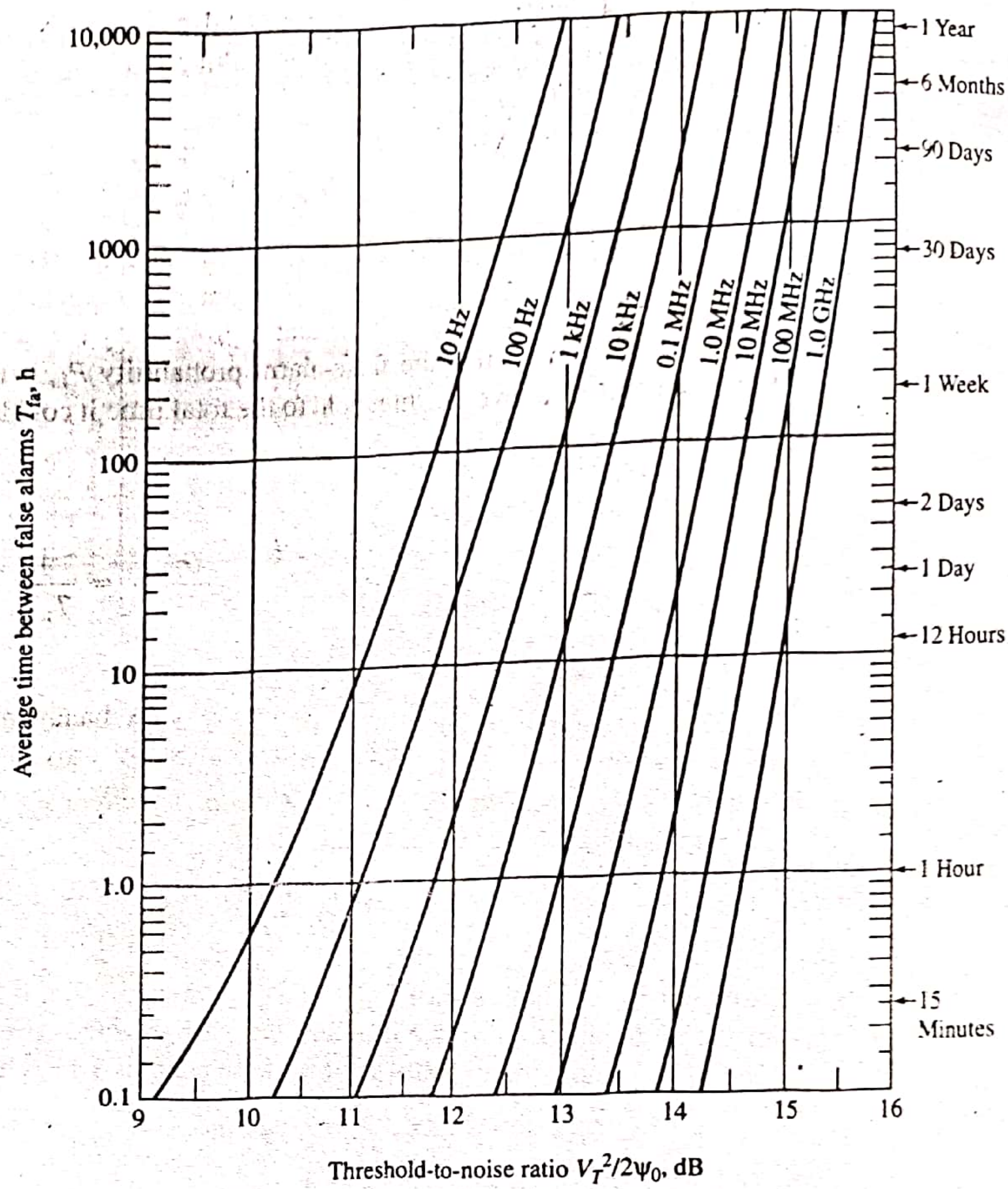

Figure 2.5 Average time between false alarms as a function of the threshold level V_T and the receiver bandwidth B; Ψ_0 is the mean square noise voltage.

from Eq. (2.26), is 6.42 times the rms value of the noise voltage, or the power ratio V_T^2/Ψ_0 is 16.2 dB.

The false-alarm probabilities of radars are generally quite small since a decision as to whether a target is present or not is made every $1/B$ second. The bandwidth B is usually large, so there are many opportunities during one second for a false alarm to occur. For example, when the bandwidth is 1 MHz (as with a 1-μs pulse width) there are 1 million decisions made every second as to whether noise or signal plus noise is present. If there were to be, on average, one false alarm per second, the false-alarm probability would be 10^{-6} in this specific example.

The exponential relationship between the false-alarm time T_{fa} and the threshold level V_T [Eq. (2.26)] results in the false-alarm time being sensitive to small variations in the threshold. For example, if the bandwidth were 1 MHz, a value of $10\log(V_T^2/2\Psi_0) = 13.2$

dB results in a false-alarm time of about 20 min. A 0.5-dB decrease in the threshold to 12.7 dB decreases the false-alarm time by an order of magnitude, to about 2 min.

If the threshold is set slightly higher than required and maintained stable, there is little likelihood of false alarms due to thermal noise. In practice, false alarms are more likely to occur from clutter echoes (ground, sea, weather, birds, and insects) that enter the radar and are large enough to cross the threshold. In the specification of the radar's false-alarm time, however, clutter is almost never included, only receiver noise.

Although the crossing of the threshold by noise is called a false alarm, it is not necessarily a *false-target report.* Declaration of a target generally requires more than one detection made on multiple observations by the radar (Sec. 2.13). In many cases, establishing the track of a target is required before a target is declared as being present. Such criteria can allow a higher probability of false alarm for each detection; hence, the threshold can be lowered to improve detection without obtaining excessive false-target reports. In the present chapter, however, most of the discussion relating to the radar equation concerns a detection decision based on a single crossing of the threshold.

If the receiver were turned off (or gated) for a small fraction of the time, as it normally would be during the transmission of the radiated pulse, the false-alarm probability will be *increased* by the fraction of time the receiver is not operative. This assumes the false-alarm time remains constant. The effect of gating the receiver off for a short time seldom needs to be taken into account since the resulting change in the probability of false alarm and the change in the threshold level are small.

HW

Probability of Detection So far, we have discussed only the noise input at the radar receiver. Next, consider an echo signal represented as a sinewave of amplitude A along with gaussian noise at the input of the envelope detector. The probability density function of the envelope R at the video output is given by[9]

$$p_s(R) = \frac{R}{\Psi_0} \exp\left(-\frac{R^2 + A^2}{2\Psi_0}\right) I_0\left(\frac{RA}{\Psi_0}\right) \qquad [2.27]$$

where $I_0(Z)$ is the modified Bessel function of zero order and argument Z. For large Z, an asymptotic expansion for $I_0(Z)$ is

$$I_0(Z) = \frac{e^Z}{\sqrt{2\pi Z}}\left(1 + \frac{1}{8Z} + \cdots\right) \qquad [2.28]$$

When the signal is absent, $A = 0$ and Eq. (2.27) reduces to Eq. (2.21), the pdf for noise alone. Equation (2.27) is called the *Rice probability density function.*

The probability of detecting the signal is the probability that the envelope R will exceed the threshold V_T (set by the need to achieve some specified false-alarm time). Thus the probability of detection is

$$P_d = \int_{V_T}^{\infty} p_s(R)\, dR \qquad [2.29]$$

When the probability density function $p_s(R)$ of Eq. (2.27) is substituted in the above, the probability of detection P_d cannot be evaluated by simple means. Rice[9] used a series approximation to solve for P_d. Numerical and empirical methods have also been used.

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The expression for P_d, Eq. (2.29), along with Eq. (2.27), is a function of the signal amplitude A, threshold V_T, and mean noise power Ψ_0. In radar systems analysis it is more convenient to use signal-to-noise power ratio S/N than $A^2/2\Psi_0$. These are related by

$$\frac{A}{\Psi_0^{1/2}} = \frac{\text{signal amplitude}}{\text{rms noise voltage}} = \frac{\sqrt{2}\,(\text{rms signal voltage})}{\text{rms noise voltage}}$$

$$= \left(2\,\frac{\text{signal power}}{\text{noise power}}\right)^{1/2} = \left(\frac{2S}{N}\right)^{1/2}$$

The probability of detection P_d can then be expressed in terms of S/N and the ratio of the threshold-to-noise ratio $V_T^2/2\Psi_0$. The probability of false alarm, Eq. (2.23) is also a function of $V_T^2/2\Psi_0$. The two expressions for P_d and P_{fa} can be combined, by eliminating the threshold-to-noise ratio that is common to each, so as to provide a single expression relating the probability of detection P_d, probability of false alarm P_{fa}, and the signal-to-noise ratio S/N. The result is plotted in Fig. 2.6.

Albersheim[11,12] developed a simple empirical formula for the relationship between S/N, P_d, and P_{fa}, which is

$$S/N = A + 0.12AB + 1.7B \qquad [2.30]$$

where

$$A = \ln\,[0.62/P_{fa}] \quad \text{and} \quad B = \ln\,[P_d/(1 - P_d)]$$

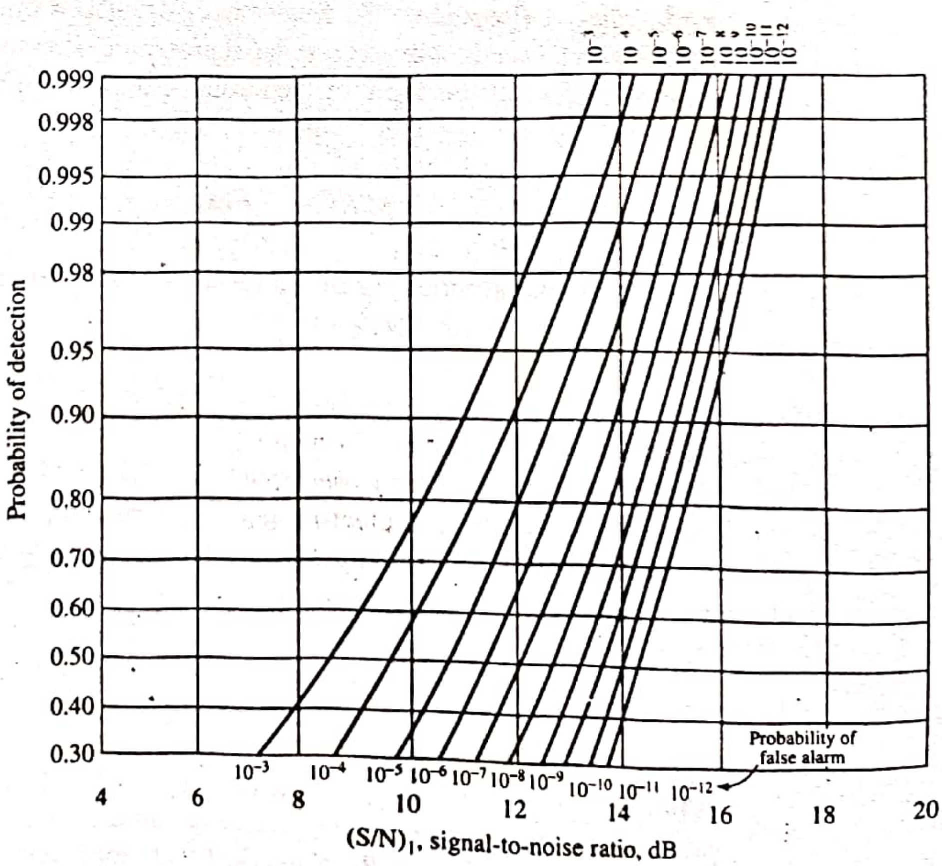

Figure 2.6 Probability of detection for a sinewave in noise as a function of the signal-to-noise (power) ratio and the probability of false alarm.

The signal-to-noise ratio in the above is a numeric, and not in dB; and ln is the natural logarithm. Equation (2.30) is said to be accurate to within 0.2 dB for P_{fa} between 10^{-3} and 10^{-7}, and P_d between 0.1 and 0.9. (It is probably suitable for rough calculations for even greater values of P_d and lower values of P_{fa}.) From such an expression or from a graph such as Fig. 2.6, the minimum signal-to-noise ratio required for a particular probability of detection and a specified probability of false alarm can be found and entered into the radar range equation. The above applies for a single pulse. The case for multiple pulses is given later.

The detection probability and the false-alarm probability are specified by the system requirements as derived from the customer's needs. From the specified detection and false-alarm probabilities, the minimum signal-to-noise ratio is found. For example, suppose that the average time between false alarms is required to be 15 min. If the bandwidth were 1 MHz, Eq. (2.25) gives a false-alarm probability of 1.11×10^{-9}. Figure 2.6 indicates that a signal-to-noise ratio of 13.05 dB is required for a probability of detection of 0.50, 14.7 dB for $P_d = 0.90$, and 15.75 dB for $P_d = 0.99$. Thus, a change of less than 3 dB can mean the difference between highly reliable detection (0.99) and marginal detection (0.50).

At first glance one might be inclined to think that the signal-to-noise ratios required for reliable detection in the above example are relatively high. They are of this magnitude because in the above example it is required that, on average, there be no more than one false alarm every 900 million possible detection decisions. Compared to telecommunications services, however, the signal-to-noise ratios for radar are relatively low. For TV the signal-to-noise ratio is said[13] to be 40 dB; there is some snow with 35 dB; objectionable interference at 30 dB; and the picture is all snow with 25 dB. Telephone service is said to require a signal-to-noise ratio of 50 dB. By comparison, radar detection is highly efficient.

The material in this section assumed only a single pulse was being used for detection. Most radars, however, utilize more than one pulse to make the detection decision, as will be discussed next.

2.6 INTEGRATION OF RADAR PULSES

The number of pulses returned from a point target by a scanning radar with a pulse repetition rate of f_p Hz, an antenna beamwidth θ_B degrees, and which scans at a rate of $\dot{\theta}_s$ degrees per second is

$$n = \frac{\theta_B f_p}{\dot{\theta}_s} = \frac{\theta_B f_p}{6\omega_r} \qquad [2.31]$$

where ω_r = revolutions per minute (rpm) if a 360° rotating antenna. The number of pulses received n is usually called *hits per scan* or *pulses per scan*. It is the number of pulses within the one-way beamwidth θ_B. Example values for a long-range ground-based air-surveillance radar might be 340-Hz pulse repetition rate, 1.5° beamwidth, and an antenna rotation rate of 5 rpm (30°/s). These numbers, when substituted into Eq. (2.31), yield $n = 17$ pulses per scan. (If n is not a whole number it can be either rounded off or the number can be used as is. It will make little difference in the calculation of radar range whichever you choose to do, unless n is small.)

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The process of summing all the radar echoes available from a target is called *integration* (even though an *addition* is actually performed). Many techniques have been considered in the past to provide integration of pulses A common integration method in early radars was to take advantage of the persistence of the phosphor of the cathode-ray-tube display combined with the integrating properties of the eye and brain of the radar operator. Analog storage devices, such as narrowband filters, can act as integrators; but they have been replaced with digital methods.

Integration that is performed in the radar receiver before the second detector (in the IF) is called *predetection integration* or *coherent integration.* Predetection integration is theoretically lossless, but it requires the phase of the echo signal pulses to be known and preserved in order to combine the sinewave pulses in phase without loss. Integration after the second detector is known as *postdetection integration* or *noncoherent integration.* It is easier to accomplish than predetection integration since the phases of the echoes are not preserved and only the envelopes of the pulses need be aligned to perform addition. There is a theoretical integration loss, however, with the use of postdetection integration.

If n pulses, all of the same signal-to-noise ratio, were perfectly integrated by an ideal lossless predetection integrator, the integrated signal-to-noise (power) ratio would be exactly n times that of a single pulse. Therefore, in this case, we can replace the single-pulse signal-to-noise ratio $(S/N)_1$ in the radar equation with $(S/N)_n = (S/N)_1/n$, where $(S/N)_n$ is the required signal-to-noise ratio per pulse when there are n pulses integrated predetection without loss. If the same n pulses were integrated by an ideal postdetection device, the resultant signal-to-noise ratio would be less than n times that of a single pulse. This loss in integration efficiency is caused by the nonlinear action of the second detector, which converts some of the signal energy to noise energy in the rectification processes. An integration efficiency for postdetection integration may be defined as

$$E_i(n) = \frac{(S/N)_1}{n(S/N)_n} \tag{2.32}$$

where the symbols have been defined in the above. The improvement in signal-to-noise ratio when n pulses are integrated is called the *integration improvement factor* $I_i(n) = nE_i(n)$. It can also be thought of as the *equivalent number of pulses integrated* $n_{eq} = nE_i(n)$. For postdetection integration n_{eq} is less than n; for ideal predetection integration $n_{eq} = n$. Thus for the same integrated signal-to-noise ratio, postdetection integration requires more pulses than predetection, assuming the signal-to-noise ratio per pulse in the two cases is the same.

The postdetection integration efficiency and the required signal-to-noise ratio per pulse $(S/N)_n$ may be found by use of statistical detection theory, similar to that outlined in the previous section. This was originally undertaken for radar application in the classic work of J. I. Marcum.[14] (His work originally appeared in 1954 as a highly regarded, widely distributed, but not generally available, Rand Corporation report.) Marcum defined an integration loss in dB as $L_i(n) = 10 \log [1/E_i(n)]$. The integration loss and the integration improvement factor are plotted in Fig. 2.7. They vary only slightly with probability of detection or probability of false alarm.

Marcum used the *false-alarm number* n_f in his calculations rather the probability of false alarm. His false-alarm number is the reciprocal of our false-alarm probability

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defined by Eqs. (2.23) and (2.25). On average, there will be one false-alarm decision out of n_f possible decisions within the false-alarm time T_{fa}. In other words, the average number of possible decisions between false alarms is n_f. If τ is the pulse width and T_p is the pulse repetition period $= 1/f_p$, then the number of possible decisions n_f in the time T_{fa} is equal to the number of range intervals per pulse period (T_p/τ) times the number of pulse periods per second (f_p) times the false-alarm time (T_{fa}). Combining the above, we get $n_f = (T_p/\tau) \times f_p \times T_{fa} = T_{fa}/\tau$. Since $\tau \approx 1/B$, where B = bandwidth, the false-alarm number is $n_f = T_{fa}B = 1/P_{fa}$.

The above assumed that the radar made decisions at a rate equal to the bandwidth B. If the radar integrates n pulses per scan before making a target detection decision, then the rate at which decisions are made is B/n. This results in a false-alarm probability n times as great as when decisions are made at a rate B times per second. This does not mean there will be more false alarms when n pulses are integrated since we have assumed that the average time between false alarms remains the same when pulses are integrated. The rate at which detection decisions are made is lower. The above is another reason why false-alarm probability is not as good a descriptor of false alarms as is the average false-alarm time. A probability by itself has little meaning unless the rate at which events occur is known.

Following the practice of Marcum, P_{fa} will be taken in this text as the reciprocal of $T_{fa}B = n_f$, even when n pulses are integrated. Some authors, on the other hand, prefer to define a false-alarm number $n'_f = n_f/n$ that accounts for the number of pulses integrated. Therefore, caution should be exercised when using different authors' computations or different computer programs for finding the signal-to-noise ratio as a function of probability of detection and probability of false alarm (or false-alarm number). There is no standardization of definitions. Correct values of signal-to-noise ratio for use in the radar equation can be obtained from most sources provided the particular assumptions used by the sources are understood.

The solid straight line in Fig. 2.7a represents a perfect lossless predetection integrator. When only a few pulses are integrated (implying large signal-to-noise ratio per pulse), Fig. 2.7a shows that the performance of the postdetection integrator is not much different from the predetection integrator. When a large number of pulses are integrated (small signal-to-noise ratio per pulse), the difference between predetection and postdetection integration is more pronounced.

The dash straight line in this figure is proportional to $n^{1/2}$. In the early days of radar it was thought that the integration improvement factor of a radar operator viewing a cathode-ray-tube display, such as a PPI, was equal to $n^{1/2}$. This is not necessarily correct. The $n^{1/2}$ relation was based on an incorrect theory and poor displays. When individual pulses are displayed so that they do not overlap or saturate the phosphor screen, the integration improvement achieved by an operator can be equivalent to that predicted by the Marcum theory for signal integration outlined in this chapter.[15]

The radar equation when n pulses are integrated is

$$R^4_{max} = \frac{P_t G A_e \sigma}{(4\pi)^2 k T_0 B F_n (S/N)_n} \qquad [2.33]$$

where the parameters are the same as previously used, except that $(S/N)_n$ is the signal-to-noise ratio of each of the n equal pulses that are integrated. Also, the half-power bandwidth B is used instead of the noise bandwidth B_n, as mentioned in Sec. 2.3. To employ this form of the equation it is necessary to have, for each value of n, a set of curves for $(S/N)_n$ similar to those of Fig. 2.6 for $n = 1$. Such curves are available[16] but are not necessary since only Figs. 2.6 and 2.7 are needed. Substituting Eq. (2.32) for $(S/N)_n$ into (2.33) gives

$$R^4_{\max} = \frac{P_t G A_e \sigma n E_i(n)}{(4\pi)^2 k T_0 B F_n (S/N)_1} \tag{2.34}$$

The value of $(S/N)_1$ is found from Fig. 2.6, and the integration improvement factor $nE_i(n)$ is found from Fig. 2.7a.

An approximation for the signal-to-noise ratio per pulse is given by an empirical formula due to Albersheim,[11,12] which is an extension of Eq. (2.30).

$$(S/N)_n = -5 \log_{10} n + \left(6.2 + \frac{4.54}{\sqrt{n - 0.44}}\right) \times \log_{10} (A + 0.12AB + 1.7B) \tag{2.35}$$

where the signal-to-noise ratio per pulse $(S/N)_n$ is in dB, n is the number of independent (pulse) samples integrated, and A and B are the same as defined for Eq. (2.30). This equation is said to have an error of less than 0.2 dB over the range of $n = 1$ to $n = 8096$, $P_d = 0.1$ to 0.9, and $P_{fa} = 10^{-3}$ to 10^{-7}. (As noted with Eq. (2.30), Eq. (2.35) is probably a good approximation for rough calculations when P_d is even greater and P_{fa} is even lower than the above.)

The discussion of integration loss or efficiency in this section is theoretical loss. In addition, there can be loss due to the actual method used for implementing the integration process in a radar.

~~2.7~~ RADAR CROSS SECTION OF TARGETS

The radar cross section σ is the property of a scattering object, or target, that is included in the radar equation to represent the magnitude of the echo signal returned to the radar by the target. In the derivation of the simple form of the radar equation in Sec. 1.2 the radar cross section was defined in terms of Eq. (1.5), which was

$$\text{Reradiated power density back at the radar} = \frac{P_t G}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2} \tag{1.5}$$

A definition of the radar cross section found in some texts on electromagnetic scattering is

$$\sigma = \frac{\text{power reflected toward source/unit solid angle}}{\text{incident power density}/4\pi} = 4\pi R^2 \frac{|E_r|^2}{|E_i|^2} \tag{2.36}$$

where R is the range to the target, E_r is the electric field strength of the echo signal back at the radar, and E_i is the electric field strength incident on the target. It is assumed in the above that the target is far enough from the radar that the incident wave can be considered to be planar rather than spherical. Equation (2.36) is equivalent to the simple form of the radar equation derived in Sec. 1.2. Sometimes the radar cross section σ is said to be a (fictional) area that intercepts a part of the power incident at the target which, if scattered uniformly in all directions, produces an echo power at the radar equal to that produced at the radar by the real target. Real targets, of course, do not scatter the incident energy uniformly in all directions.

The power scattered from a target in the direction of the radar receiver, and hence the radar cross section, can be calculated by solving Maxwell's equations with the proper boundary conditions applied or by computer modeling. The radar cross section can also be measured, based on the radar equation, using either full-size or scale models of targets.

Radar cross section depends on the characteristic dimensions of the object compared to the radar wavelength. When the wavelength is large compared to the object's dimensions, scattering is said to be in the *Rayleigh region.* It is named after Lord Rayleigh who first observed this type of scattering in 1871, long before the existence of radar, when investigating the scattering of light by microscopic particles. The radar cross section in the Rayleigh region is proportional to the fourth power of the frequency, and is determined more by the volume of the scatterer than by its shape. At radar frequencies, the echo from rain is usually described by Rayleigh scattering.

At the other extreme, where the wavelength is small compared to the object's dimensions, is the *optical region.* Here radar scattering from a complex object such as an aircraft is characterized by significant changes in the cross section when there is a change in frequency or aspect angle at which the object is viewed. Scattering from aircraft or ships at microwave frequencies generally is in the optical region. In the optical region, the radar cross section is affected more by the shape of the object than by its projected area.

In between the Rayleigh and the optical regions is the *resonance region* where the radar wavelength is comparable to the object's dimensions. For many objects, the radar cross section is larger in the resonance region than in the other two regions. These three distinct scattering regions are illustrated by scattering from the sphere described next.

Simple Targets The sphere, cylinder, flat plate, rod, ogive, and cone are examples of simple targets. Analytical expressions exist for the radar cross sections of some of these objects. Sometimes the radar cross sections of complex targets can be calculated by describing the target as a collection of simple shapes whose cross sections are known. The total cross section is obtained by summing vectorially the contributions from the individual simple shapes. A few examples will be presented to illustrate the nature of radar cross section behavior.

Sphere The sphere is the simplest object for illustrating radar scattering since it has the same shape no matter from what aspect it is viewed. Its calculated radar cross section is shown in Fig. 2.8 as a function of $2\pi a/\lambda$, the circumference measured in wavelengths, where a is the radius of the sphere and λ is the radar wavelength. The cross section in this figure is normalized by the projected physical area of the sphere, πa^2. The three different scattering regions that characterize the sphere are labeled in the figure. In the Rayleigh region where $2\pi a/\lambda << 1$, the radar cross section is proportional to f^4, where f = frequency = c/λ, and c = velocity of propagation.

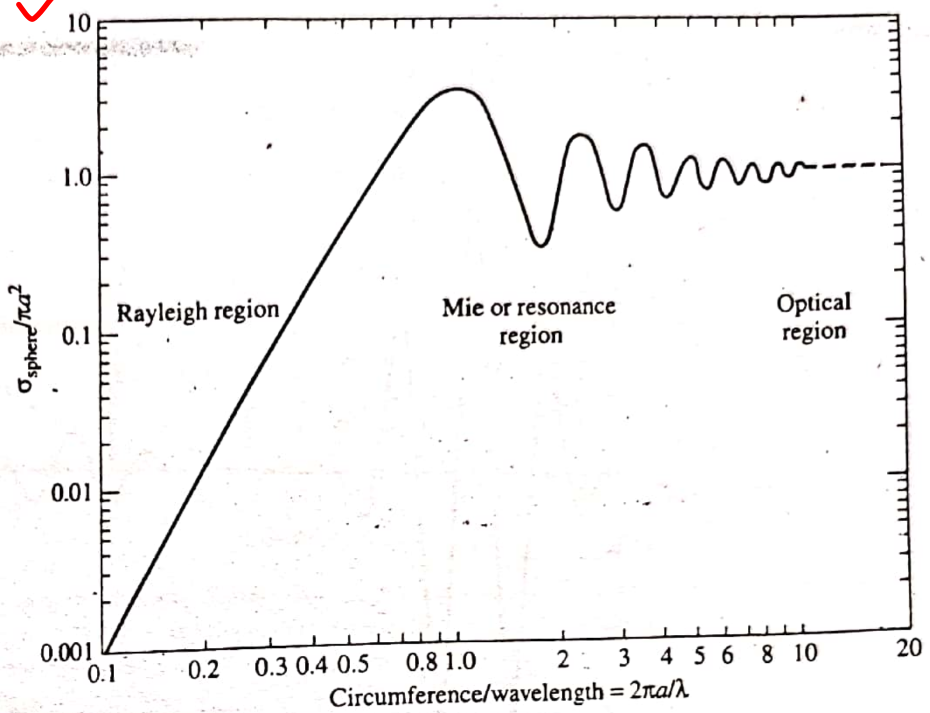

Figure 2.8 Normalized radar cross section of a sphere as a function of its circumference ($2\pi a$) measured in wavelengths. a = radius; λ = wavelength.

In the optical region, $2\pi a/\lambda >> 1$, the radar cross section approaches the physical area of the sphere as the frequency is increased. This unique circumstance can mislead one into thinking that the geometrical area of a target is a measure of its radar cross section. It applies to the sphere, but not to other targets. In the optical region, scattering does not take place over the entire hemisphere that faces the radar, but only from a small bright spot at the tip of the smooth sphere. It is more like what would be seen if a polished metallic sphere, such as a large ball bearing, were photographed with a camera equipped with a flash. The only illumination is at the tip, rather than from the entire hemispherical surface. A diffuse sphere or rough-surface sphere, such as a white billiard ball, would reflect from its entire surface, as does the full moon when viewed visually.

The radar cross section of the sphere in the *resonance region* oscillates as a function of frequency, or $2\pi a/\lambda$. Its maximum occurs at $2\pi a/\lambda = 1$, and is 5.6 dB greater than its value in the optical region. The first null is 5.5 dB below the optical region value. Changes in cross section occur with changing frequency because there are two waves that interfere constructively and destructively. One is the direct reflection from the front face of the sphere. The other is the *creeping wave* that travels around the back of the sphere and returns to the radar where it interferes with the reflection from the front of the sphere. The longer the electrical path around the sphere, the greater the loss, so the smaller will be the magnitude of the fluctuation with increasing frequency.

Figure 2.9 illustrates the backscatter that would be produced by a very short pulse radar that can resolve the specular echo reflected from the forward part of the sphere from the creeping wave which travels around the back of the sphere. The incident waveform in this figure is a shaped pulse of sinewave of the form $0.5[1 + \cos(\pi t/t_0)]$, where the pulse extends from $-t_0$ to $+t_0$.[17] The radius of the sphere in this example is equal to the

2.13 OTHER RADAR EQUATION CONSIDERATIONS

Prediction of Radar Range This chapter discussed many, but not all, of the factors that might enter into the radar equation for the prediction of range, when limited by receiver noise. The simple form of the radar equation we started with as Eq. (2.1), with the modifications indicated in this chapter, now becomes

$$R_{max}^4 = \frac{P_{av} G A \rho_a \sigma n E_i(n) F^4 e^{-2\alpha R_{max}}}{(4\pi)^2 k T_0 F_n (B\tau) f_p (S/N)_1 L_f L_s} \qquad [2.61]$$

where

R_{max} = Maximum radar range, m
P_{av} = Average transmitter power, W
G = Antenna gain
A = Antenna area, m^2
ρ_a = Antenna aperture efficiency
σ = Radar cross section of the target, m^2
n = Number of pulses integrated
$E_i(n)$ = Integration efficiency
F^4 = Propagation factor
α = Attenuation coefficient, nepers per unit distance
k = Boltzmann's constant = 1.38×10^{-23} J/deg
T_0 = Standard temperature = 290 K
F_n = Receiver noise figure
B = Receiver bandwidth, Hz
τ = Pulse width, s
f_p = Pulse repetition frequency, Hz
$(S/N)_1$ = Signal-to-noise ratio required as if detection were based on only a single pulse
L_f = Fluctuation loss (for a Swerling target model)
L_s = System loss

The product $kT_0 = 4 \times 10^{-21}$ W/Hz. In most radar designs the product $B\tau \approx 1$. The average power can be expressed as $P_{av} = P_t \tau f_p = E_p f_p$, where E_p is the energy in a transmitted pulse. The total energy transmitted in n pulses is $E_T = nE_p$. The signal-to-noise ratio for a rectangular pulse can be expressed as an energy ratio since $S/N = (E/\tau)/N_0 B = E/(N_0 B\tau)$, where E is the energy of the received pulse, and N_0 is the receiver noise power per unit bandwidth. When $B\tau = 1$, then $(S/N)_1 = (E/N_0)_1$. Omitting the propagation factors, atmospheric attenuation, and the fluctuation loss, the radar equation can be written

$$R_{max}^4 = \frac{E_T G A \rho_a \sigma E_i(n)}{(4\pi)^2 k T_0 (E/N_0)_1 L_s} \qquad [2.62]$$

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This radar equation can be applied to any waveform, not just a rectangular pulse, so long as a matched filter is used on reception and the energy parameters are properly defined.

The radar equation, Eq. (2.61), developed in this chapter for a pulse waveform can be modified for other radars, such as CW, FM-CW, pulse doppler, and MTI. It also can be adapted to specialized radar applications such as the surveillance-radar equation, derived next. Tracking radars, synthetic aperture radars, HF over-the-horizon radars, and other specialized radars require modification of the classical radar equation to account for the special attributes of different radar systems.

When radar performance is limited by clutter echoes rather than receiver noise, the radar equation takes on a completely different form from the equations presented here, as discussed in Chap. 7. When assessing radar performance when hostile ECM noise jamming dominates, receiver noise in the denominator of the radar equation is replaced by the jamming noise that enters the radar receiver.

Surveillance-Radar Range Equation The radar equation described so far applies to a radar that dwells on the target for n pulses. The radar equation for a surveillance radar, however, is slightly different since it must account for the defining characteristic of a surveillance radar which is that it search a specified angular region in a given time. The *scan time,* or *revisit time,* is t_s, in seconds. The angular region to be searched is denoted by Ω, in steradians. (A steradian is the area subtended by a solid angle Ω on the surface of a sphere of unit radius. The total solid angle about a point is therefore 4π steradians. If the region Ω, for instance, represents 360° in azimuth and 30° in elevation, the solid angle in steradians is $2\pi \sin 30° = \pi$ steradians.)

The scan time, t_s, is equal to $t_0\Omega/\Omega_0$, where $t_0 = n/f_p$ is the time that the radar beam dwells on the target, n is the number of pulses received as the antenna scans past the target, f_p = prf, and Ω_0 is the solid angular beamwidth that is approximately equal, for small beamwidths, to the product of the azimuth beamwidth θ_a times the elevation beamwidth θ_e in radians. (This assumes θ_A/θ_a and θ_E/θ_e are integers, where θ_A is the total azimuth coverage and θ_E the total elevation coverage.) The antenna gain is approximately $G = 4\pi/\Omega_0$. With the above substitutions into a slightly simplified Eq. (2.61), the surveillance-radar equation becomes

$$R^4_{max} = \frac{P_{av} A_e \sigma E_i(n)}{4\pi k T_0 F_n (S/N)_1 L_s} \frac{t_s}{\Omega} \qquad [2.63]$$

This equation shows that the important parameters of a surveillance radar under the control of the radar designer are the *average power* and the *effective aperture.* The *power-aperture product,* therefore, is an important measure of the capability of a radar to perform long-range surveillance. The frequency does not appear explicitly. In practice, however, it is easier to achieve high power and large antennas at lower rather than higher frequencies. Furthermore, weather effects are less of a bother at the lower frequencies, which is something not indicated by this form of the surveillance-radar equation.

Although Eq. (2.63) illustrates the basic radar characteristics that affect the range of a surveillance radar, it is not a good equation on which to base a radar design. Too many factors are not explicitly stated. It is better to use Eq. (2.61) and the several auxiliary equations that relate to the surveillance application, such as the number of pulses received

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per scan [Eq. (2.31)], and the relationship between the scan time and the coverage volume.

The surveillance radar equation does not explicitly contain the number of pulses per dwell. There should, of course, be at least one pulse; but in most cases there need to be more than one pulse. If only one or two pulses are obtained from a target, the beam-shape loss is large. In an MTI or pulse doppler radar for the detection of moving targets in clutter, the greater the time on target, the larger the number of pulses processed, and the greater will be the reduction in clutter (as discussed in Sec. 3.7 on antenna scanning modulation in MTI radar.) The surveillance radar equation given above, therefore, might need to be modified when doppler processing requires a fixed dwell time or minimum number of pulses.

M-out-of-N Criterion What has been discussed thus far is the probability of detection based on a single scan, or single observation, as the radar antenna scans by the target. A surveillance radar, however, seldom makes a detection decision that a target is present based on only a single observation. One criterion for announcing that a target is present is based on requiring M detections on N scans, where $1 < M \leq N$. For instance, the criterion for detection might be to require a detection (threshold crossing) on each of 2 successive scans, or 2 detections over 3 scans, or 3 out of 4, or 3 out of 5. Denoting the probability of detection on a single trial (scan) by P, the probability of detecting a target on M out of N trials, is given by the classical expression

$$\text{prob}\,[M \text{ out of } N] = \sum_{k=M}^{N} \frac{N!}{k!(N-k)!} P^k (1-P)^{N-k} \qquad [2.64]$$

From this expression the probability of detecting a target on $M = 2$ out of $N = 3$ scans is $3P^2 - 2P^3$. With a 2-out-of-3 criterion, the detection probability is larger than that of a single scan when the single-scan probability is greater than 0.5.

The probability of false alarm with the M-out-of-N criterion also can be found from Eq. (2.64). It will be much lower than the single-scan probability of false alarm. This means that a higher false-alarm probability per scan can be tolerated in order to achieve a specified overall false-alarm probability. For example, if the single-scan false-alarm probability were 10^{-8}, Eq. (2.64) shows that the probability of obtaining a false report of a target, when the detection criterion is 2 out of 3, would be 3×10^{-16}, which is a very low number. If the false-report probability is to be 10^{-8} when the detection criterion is 2 out of 3, the single-scan false-alarm probability can be set equal to 0.6×10^{-4}, which results in a lowering of the required detection threshold with a concurrent savings in transmitter power (or its equivalent).

Track Establishment as a Detection Criterion Many modern air-surveillance radars declare that a target is present when a track is established rather than when a single detection decision is made. As discussed in Sec. 4.9, establishment of a track requires multiple observations of a target. Since the likelihood is very small that noise alone can establish a logical track, the single-observation probability of false alarm can be relaxed. Thus the false-alarm probability of a threshold-crossing with noise alone might be as high as 10^{-3} without excessive false track-reports. One criterion used to establish a track is that there

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be target echoes detected on at least 3 out of 5 scans. This is similar to the M-out-of-N criterion, except for the added constraint that the track should be within an expected speed range and not exhibit unusual changes in its trajectory. When the establishment of a valid track is taken as the criterion for the report of a target's presence, a false alarm can be made an exceedingly rare event with a well-designed radar and well-designed tracking algorithms.

Cumulative Probability of Detection If a target is observed over multiple scans, the cumulative probability of detection can be large even though the single-scan probability is small. Cumulative probability is the probability that the target is detected *at least* once on N scans. Consider a radar that can observe a target on N successive scans of the rotating antenna. It is assumed, for convenience of discussion, that the range does not change significantly over the N scans so that the change in received signal power with range need not be taken into account. The probability of detecting a target at least once during the N scans is called the *cumulative probability of detection*, P_c, and is written

$$P_c = 1 - (1 - P_d)^N \quad [2.65]$$

where P_d = single-scan probability of detection. (The maximum radar range based on the cumulative probability of detection has been said to vary as the third power rather than the more usual fourth power variation based on the single-scan probability.[66,67]) The cumulative probability of detection, however, is *not* a good measure of radar performance since a target-detection decision can seldom be made on the basis of a single threshold crossing. Generally, more than one observation of a detection is needed before a reliable report of the presence of a target can be announced.

Verification of Predicted Range This chapter has shown that there are many factors affecting the range of a radar, and they are not always known accurately. The prediction of the maximum range is not something that can always be done as well as might be desired. Even if one could make an accurate prediction of range, there is the problem of trying to verify the prediction experimentally. Suppose, for example, an air-surveillance radar is required to have a 0.90 probability of detecting a one square meter target at a range of 200 nmi, with a false-alarm probability of 10^{-8}. A large number of observations are needed to insure that the probability of detection is actually 0.90 and not 0.80 or 0.95. It is often difficult to account for the varying target cross section not being exactly one square meter (or whatever other value the radar is designed to detect), especially when the cross section varies with viewing aspect. The effect of atmospheric refraction and multipath lobing of the elevation pattern must be known. One can, and should, experimentally determine the range performance of a radar, but one should not expect highly precise measurements.

Radar range performance of a ground-based air-surveillance radar is sometimes determined experimentally by measuring the *blip-scan ratio* as a function of range. The blip-scan ratio is an experimental approximation to the single-scan probability of detection. It is typically found by having a radar fly back and forth at constant altitude on a radial course relative to the radar, and on each scan of the antenna it is recorded whether or not a target blip is detected. This process is repeated many times until sufficient data are

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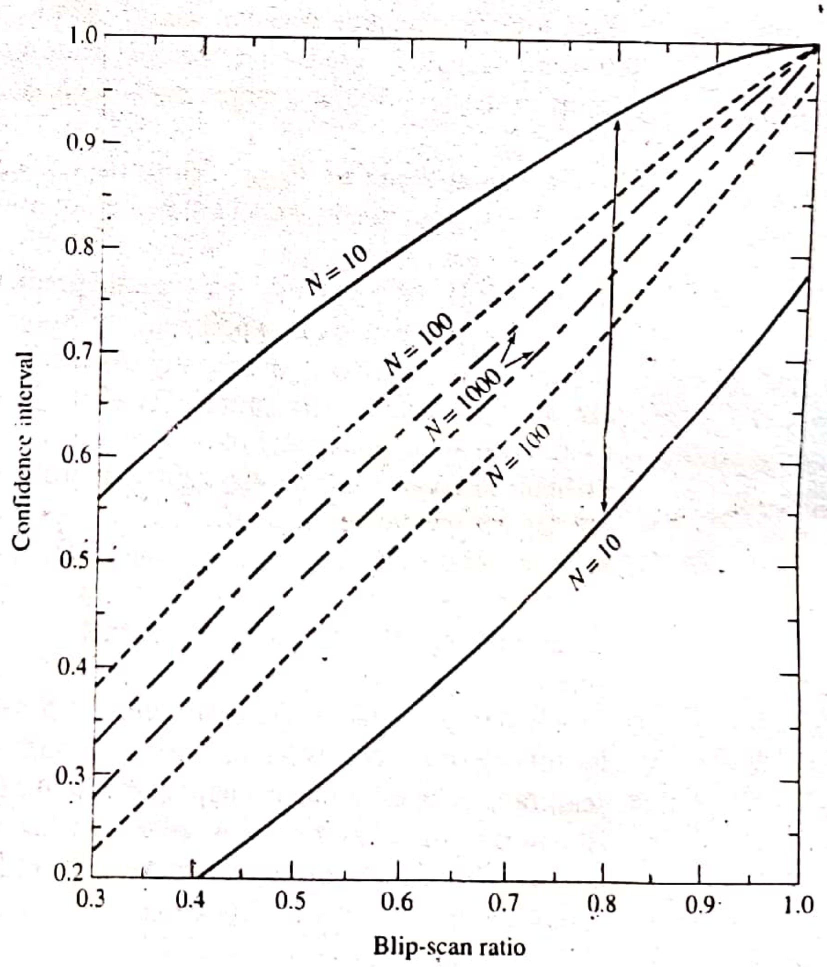
obtained to compute, as a function of range, the number of scans (blips) on which the target was detected, divided by the number of times it could have been detected (scans). It provides a measure of performance for a particular aircraft flying at a particular altitude when viewed from the head-on and tail-on aspects. Ducting and other anomalous propagation effects that might occur during testing can make for difficulty, and ought to be avoided if possible.

Just as in tossing a coin many times to determine the fraction of events that it is heads, the blip-scan ratio is a statistical quantity whose accuracy depends on how many times the measurement is attempted. This is a classical statistical problem in Bernoulli trials. It is related to Eq. (2.64), which describes the probability of obtaining at least M successes out of N trials. In the experimental verification of a radar's performance, however, the problem is: given M successes out of N trials, what is the underlying probability of detection. When the number of trials is large, the probability of detection approaches M/N. Figure 2.27, provided to the author by Fred Staudaher, shows pairs of curves that statistically bound the correct values, with a confidence level of 90 percent, when the number of trials N equals 10, 100, and 1000. The abscissa is the experimentally measured blip-scan ratio of M/N. The ordinate is the range of values within which the true value of blip-scan ratio might be, with a specified probability of confidence. For example, assume that

Figure 2.27 The ordinate of each pair of similarly labeled curves gives the range of confidence that the abscissa (measured blip-scan ratio) represents the true value of the blip-scan ratio (or single-scan probability of detection) with a confidence coefficient of 90 percent. N is the number of trials. See text for example. (Courtesy of Fred Staudaher.)



the measured blip-scan ratio M/N is 0.80 after 10 trials. Figure 2.27 states that there is 90 percent confidence that the true value lies between 0.54 and 0.93 (see the vertical line in Fig. 2.27). If there were only 10 trials, one would not have a good idea of the true value of the blip-scan ratio. If the measured blip-scan ratio were again 0.80 after 100 trials, its true value has 90 percent confidence of lying between 0.73 and 0.86. With 1000 trials, the true value lies within 0.78 and 0.82 with 90 percent confidence. Thus a relatively large number of trials might be required to be sure the radar meets its performance specifications.

The prediction of the range of a radar is not as exact as might be desired, and the accurate experimental measurement of its range performance is not easy. For this reason, the acceptance of a new radar system by a buyer is not usually based on the costly experimental measurement of performance. One might make a limited number of trials to insure that radar performance is not far out of line. However, for contracting purposes when buying a radar, the performance of the individual subsystems is specified (such as transmitter power, antenna gain, receiver noise figure, receiver dynamic range, and so forth) since these can be measured and used in calculations to predict what the actual performance might be when the radar is operated as a system. If each of the subsystem specifications are met, and if the specifications are properly devised, the buyer can be confident that the radar will perform as predicted.

Accuracy of the Radar Range Computation There are those who believe that each parameter that enters into the computation of range should be determined with the highest accuracy possible. The author, however, is of the opinion that the limited accuracy of many of the values that enter the radar equation, as well as the difficulty in experimentally verifying the predicted radar performance, do not justify a precision approach to radar prediction. One can't be sloppy, of course, but in engineering one cannot always be overly precise. In spite of difficulties, the engineer has to be able to guarantee that the radar can perform as required.

Calculation of the Radar Equation The range of a radar can be obtained with nothing more complicated than a simple calculator and a set of tables or graphs similar to what has been presented in this chapter. There exist, however, computer programs on the market for calculation of the radar equation.[68] They make the calculation of range easy; but they are not necessary except when it is required to plot coverage diagrams that take into account the effect of the earth's surface and other propagation factors mentioned in Chap. 8.

This chapter has been concerned with radar detection when receiver noise is the dominant factor hindering detection. The prediction of radar range when clutter echoes from the land, sea, or rain are larger than receiver noise requires a different formulation of the radar range equation, as well as a different design of the radar. Detection of targets in clutter is the subject of Chap. 7. It has been suggested[69] that computer simulation on a computer can be employed when clutter echoes, jamming noise, and receiver noise have to be considered and cannot be faithfully represented by gaussian statistics. The success of simulation depends on how well the clutter and other interfering effects can be modeled. They need to be accurate, which is not always easy to achieve in the "real world."

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Radar Equation in Design In this chapter the radar equation has been discussed mainly as a means for predicting the range of a radar. It also serves the important role of being the basis for radar system design. Some parameters that enter into the radar equation are given by the customer based on the nature of the task the radar is to perform. Examples are the required range, coverage, and target characteristics. Generally there might be trade-offs and compromises required in selecting values of the other parameters that are under the control of the radar designer. It is the radar equation that is used to examine the effect of the trade-offs among the various parameters, such as the trade between a large antenna or a large transmitter power. The decision as to the radar frequency is also something that should come from examining the radar equation. Thus almost all radar design starts with the radar equation.

Conservative Design Because of the lack of precision in knowing many of the parameters that enter into the radar equation, it is advisable to design a radar as conservatively as possible. This means taking full account of all the factors affecting performance that are knowable—and then adding a safety factor to increase the signal-to-noise ratio. (One method for doing this is to specify the Swerling Case 1 target model with a probability of detection of 0.9 or greater.) Such practice has produced excellent radars that do the job required. In a few cases the radar signal-to-noise ratio was 20 dB greater than actually needed. This may be high, however, by today's standards. Only a few radars have been built with a large safety factor, but they have verified the validity of this approach. Unfortunately, the procurement practices of most agencies that buy radars, as well as the competitive nature of the marketplace, usually do not permit this degree of conservative design to occur very often.

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PROBLEMS

2.1 If the noise figure of a receiver is 2.5 dB, what reduction (measured in dB) occurs in the signal-to-noise ratio at the output compared to the signal-to-noise ratio at the input?

2.2 What is the noise bandwidth B_n of a low-pass RC filter whose frequency-response function is $H(f) = \dfrac{1}{1 + j(f/B_v)}$, where B_v is the half-power bandwidth? That is, find the ratio B_n/B_v.

2.3 The random variable x has an exponential probability density function:

$$p(x) = a \exp[-bx] \qquad x > 0$$

where a and b are constants.

a. Determine the relation between a and b required for normalization.
b. Determine the mean m_1 and the variance σ^2 for the normalized $p(x)$.
c. Sketch $p(x)$ for $a = 1$.
d. Find the probability distribution function $P(x)$, and sketch the result for a = 1.

2.4 Show that the standard deviation of the Rayleigh probability density function [Eq. (2.17)] is proportional to the mean value. You should use integral tables when integration cannot be performed in a simple manner. (This result is used in deriving the form of the log-FTC clutter suppression circuit described in Sec. 7.8.)

2.5 The average time between false alarms is specified as 30 min and the receiver bandwidth is 0.4 MHz.

a. What is the probability of false alarm?
b. What is the threshold-to-noise power ratio (V_T^2/Ψ_0)?
c. Repeat (*a*) and (*b*) for an average false alarm time of one year (8760 h).
d. Assume the threshold-to-noise power ratio is to be set to achieve a 30-min false-alarm time [value as in part (*b*)]; but, for some reason, the threshold is actually set lower by 0.3 dB than the value found in part (*b*). What is the resulting average time between false alarms with the lower threshold?
e. What would be the average time between false alarms if the threshold were to increase by 0.3 dB?
f. Examine the two values of threshold-to-noise ratio you have calculated in (*d*) and (*e*) and comment on the practicability of precisely achieving a specified value of false-alarm time.

2.6 A radar has a bandwidth $B = 50$ kHz and an average time between false alarms of 10 min.

a. What is the probability of false alarm?
b. If the pulse repetition frequency (prf) were 1000 Hz and if the first 15 nmi of range were gated out (receiver is turned off) because of the use of a long pulse, what would be the new probability of false alarm? (Assume the false-alarm time has to remain constant.
c. Is the difference between (*a*) and (*b*) significant?
d. What is the pulse width that results in a minimum range of 15 nmi?

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2.7 A transmission line with loss L is connected to the input of a receiver whose noise figure is F_r. What is the overall noise figure of the combination?

2.8 A radar at a frequency of 1.35 GHz has an antenna of width $D = 32$ ft, a maximum unambiguous range of 220 nmi, and an antenna scan time (time to make one rotation of the antenna) of 10 s.

a. What is the number of echo pulses per scan received by the radar from a point target? [Use the relationship that the antenna half-power beamwidth in radians is $\theta_B = 1.2\lambda/D$ (λ = wavelength).]

b. What is the integration loss and the integration-improvement factor when the probability of detection is 0.9 and the probability of false alarm is 10^{-4}?

2.9 Show that the far right-hand side of Eq. (2.36), a definition of the radar cross section, is the same as the simple radar equation, Eq. (1.6). [It easier to start with Eq. (2.36) and obtain Eq. (1.6), than vice versa.]

2.10 *a.* What frequency will result in the maximum radar cross section of a metallic sphere whose diameter is 1 m?

b. At what frequency will the radar cross section of a ball bearing one millimeter in diameter be maximum?

2.11 *a.* What is the maximum radar cross section (square meters) of an automobile license plate that is 12 inches wide by 6 inches high, at a frequency of 10.525 GHz (the frequency of an X-band speed radar)?

b. How many degrees in the vertical plane should the plate be tilted in order to reduce its cross section by 10 dB? For purposes of this problem you may assume the license plate is perfectly flat. The radar cross section of a flat plate as a function of the incidence angle ϕ may be written, for ϕ not too large, as:

$$\sigma(\phi) \approx \sigma_{max} \frac{\sin^2 [2\pi(H/\lambda) \sin \phi]}{[2\pi(H/\lambda) \sin \phi]^2}$$

where σ_{max} = maximum radar cross section of a flat plate = $4\pi A^2/\lambda^2$, A = area of plate, λ = radar wavelength, and H = height of the plate. (Be careful of units. You will have to sketch a portion of the cross section pattern as a function of ϕ to find the value of ϕ corresponding to -10 dB.)

c. What other parts of an automobile might contribute to its radar cross section when viewed directly from the front?

2.12 Describe briefly the behavior of the radar cross section (in the microwave region) of a raindrop and a large aircraft with respect to its dependence on (*a*) frequency and (*b*) viewing aspect.

2.13 Describe the chief characteristic of the radar echo from a target when its radar cross section is in the (*a*) Rayleigh region, (*b*) resonance region, and (*c*) optical region.

2.14 A typical value of an individual "sea spike" echo at X band (wavelength = 3.2 cm), as discussed in Sec. 7.4, might be 1 m^2. What is the dimension of the side of a square flat plate that produces the same radar echo when the plate is viewed at normal incidence?

2.15 A radar noncoherently integrates 18 pulses, each of uniform amplitude (the nonfluctuating case). The IF bandwidth is 100 kHz.

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a. If the average time between false alarms is 20 min, what must be the signal-to-noise ratio per pulse $(S/N)_n$ in order to achieve a probability of detection of 0.80? (Suggest the use of Albersheim's equation.)

b. What is the corresponding value of $(S/N)_1$?

c. What would $(S/N)_1$ be if the target cross section fluctuated according to a Swerling Case 1 model?

2.16 Why does the cross section of a complex target, such as that in Fig. 2.15, fluctuate so rapidly with a small change in aspect angle when the radar wavelength is small compared to the target's dimensions?

2.17 Show that the probability density function for the Swerling Case 1 model is the same as the chi-square of degree 2 [Eq. (2.47)].

2.18 *a.* What signal-to-noise ratio is required for a radar that makes a detection on the basis of a single pulse, when the probability of detection is 0.50 and the probability of false alarm is 10^{-6}? Assume a nonfluctuating target echo.

b. Repeat for a 0.99 probability of detection and the same probability of false alarm.

c. Repeat parts (*a*) and (*b*), but for a Swerling Case 1 fluctuating target.

d. Compare your results in a table. What conclusions can you obtain from this?

2.19 A radar measures an apparent range of 7 nmi when the prf is 4000 Hz, but it measures an apparent range of about 18.6 nmi when the prf is 3500 Hz. What is the true range (nmi)?

2.20 *a.* Show that the echo signal power P_r received from an aircraft flying at a constant height h over a perfectly conducting flat earth is independent of the range R, when the antenna elevation gain varies as the cosecant-squared of the elevation angle ϕ (that is: $G = G_0 \csc^2 \phi$).

b. In addition to having a received signal that is independent of the range (requiring less dynamic range in the receiver), what is another reason for employing an antenna with a $\csc^2 \phi$ elevation pattern for an air-surveillance radar when compared to a conventional unshaped fan-beam elevation pattern?

c. What are the limitations in applying the simple result of (*a*) to a radar in the real world?

2.21 A surface-based air-surveillance radar with a fan-beam antenna that rotates 360° in azimuth has a maximum range of 150 nmi and height coverage to 60,000 ft. Its maximum elevation-angle coverage is 30°. What percentage of the total available volume coverage is lost because of the overhead "hole" compared to a radar with complete angular coverage up to 90° (no hole in the coverage)? Assume, for simplicity, a flat earth.

2.22 A radar receives five pulses within its half-power (3 dB) beamwidth as the antenna beam scans past a point target. The middle of the five pulses is transmitted when the maximum of the antenna pattern points in the direction of the target. The first and the fifth pulses are transmitted when the leading and trailing half-power points are, respectively, directed at the target. What is the two-way beam-shape loss (dB) in this case?

2.23 Five identical radars, each with a receiver having a square-law detector, have partial overlap in their radar coverages so that not all radars are guaranteed to see each target. The outputs of all five radars are combined before a detection decision is made. If a target is

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seen on only one of the five radars and the other four radars see only receiver noise, what is the collapsing loss when the detection probability is 0.5 and the false-alarm probability is 10^{-4}?

2.24 A civil marine radar is employed on boats and ships for observing navigation buoys, detecting land-sea boundaries, piloting, and avoiding collisions. Consider the following civil-marine radar:

frequency: 9400 MHz (*X* band)
antenna: horizontal beamwidth = 0.8°
vertical beamwidth = 15°
gain = 33 dB
azimuth rotation rate = 20 rpm
peak power: 25 kW
pulse width: 0.15 μs
pulse repetition rate: 4000 Hz
receiver noise figure: 5 dB
receiver bandwidth: 15 MHz
system losses: 12 dB
average time between false alarms: 4 hours

a. Plot the single-scan probability of detection as function of range (nmi), assuming a constant cross-section target of 10 m^2 (a navigation buoy) and free-space propagation. [You will find it easier to select the probability of detection and find the corresponding signal-to-noise ratio, rather than the reverse. You need only consider probabilities of detection from 0.30 to 0.99. You may, for purposes of this problem, select a single (average) value of the integration improvement factor rather than try to find it as a function of P_d (since the curve in the text does not permit otherwise).]
b. Repeat (*a*) for a Swerling Case 1 target fluctuation model with average cross section of 10 m^2. Plot on the same diagram as (*a*).
c. Comment on whether the average power of this radar is too low, just right, or too high for the job it has to perform here.
d. Why do you think this ship-mounted radar antenna has a 15° elevation beamwidth when all the targets are located on the surface of the sea?

2.25 Consider the following air-surveillance radar:

frequency: 2.8 GHz (*S* band)
peak power: 1.4 MW
pulse width: 0.6 μs
pulse repetition frequency: 1040 Hz
receiver noise figure: 4 dB
antenna rotation rate: 12.8 rpm

antenna gain: 33 dB

antenna azimuth beamwidth: 1.35 deg

system losses : 12 dB

average false-alarm time: 20 min

target cross section: 2 m^2

Plot each of the following on the same coordinates (with range as the abscissa):

a. The free-space single-scan probability of detection as a function of range (in nautical miles) for a constant cross-section target. [You will find it easier to select the probability of detection and find the corresponding signal-to-noise ratio, rather than the reverse. You need only consider probabilities of detection from 0.30 to 0.99. You may, for purposes of this problem, select a single (average) value of the integration improvement factor rather than try to find it as a function of P_d (since the curve in the text does not permit otherwise).]

b. The probability of detection as a function of range for the same situation as part (*a*) but with the detection criterion that the target must be found on at least 2 out of 3 scans of the rotating antenna. [You may assume that the range and the received signal power do not change appreciably over the three scans. For convenience of this calculation, you may assume that the single-scan false-alarm probability is the same as used in part (*a*).]

c. Repeat (*a*) for a Swerling Case 1 with average target cross section of 2 m^2.

d. Repeat (*b*) for a Swerling Case 1 with average target cross section of 2 m^2.

e. Is the prf adequate for avoiding range ambiguities?

(The radar in this problem is similar to the airport surveillance radar known as the ASR.)

2.26 Starting with Eq. (2.51) derive the surveillance radar equation [Eq. (2.63)]. You may omit the propagation factor, attenuation, and fluctuation loss.

2.27 What is the effect of receiver bandwidth on the maximum range of a well-designed radar, assuming the average power remains constant? Explain your answer.

2.28 *a.* What is the probability of detecting a target on at least 2 out of 4 scans when the single-scan probability of detection is 0.8?

b. What is the corresponding probability of false alarm in this case when the single-scan false-alarm probability is 10^{-8}?

c. What should be the single-scan false-alarm probability if the overall false-alarm probability with a detection criterion of 2 out of 4 scans is 10^{-8}?

d. When the higher single-scan probability of false alarm of part (*c*) is employed rather than a 10^{-8} single-scan probability of false alarm, what reduction in the signal-to-noise ratio can be obtained?

2.29 In this problem, it is assumed that the targets for an air-surveillance radar are characterized by a Swerling Case 1 model. There are *n* pulses received from a target. Half of the *n* pulses are at one frequency and the other half are at a second frequency that is far enough removed from the first to completely decorrelate the second set of *n*/2 pulses relative to

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the first set of $n/2$ pulses. What is the improvement in signal-to-noise ratio obtained from this use of frequency diversity when (*a*) $P_d = 0.95$ and (*b*) $P_d = 0.6$? (*c*) If the radial extent (in range) of a target is 30 m, what must the difference in the two frequencies be to decorrelate the target echo?

2.30 *a*. Make a list of the system losses that might occur in a long-range air-surveillance radar, and estimate an approximate value for the loss due to each factor. You need not include losses due to doppler processing. (There is, of course, no unique answer for this question.)

b. Using the total system loss you have estimated, what is the reduction in radar range that occurs due to the system losses if the radar range without losses is 200 nmi?

2.31 Question 1.8 of Chapter 1 asked "How does radar range depend on the wavelength?" Based on Chapter 2, how would you now answer this question for an air-surveillance radar? (Please justify your answer.)

2.32 An experimental measurement of the blip-scan ratio (single-scan probability of detection) of a particular target at a particular range gives a value of 0.5 after 10 trails (antenna scans).

a. What is the confidence interval of this measurement if the confidence level has to be 90 percent?

b. What is the confidence interval (with the same confidence level of 90 percent) after 100 scans, assuming the measured blip-scan ratio is still 0.5?

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