

A Brief Introduction To Microwave Engineering and To EE 433

The microwave region is typically defined as those frequencies between 300 MHz and 300 GHz. (Recall 1 MHz = 1×10^6 Hz and 1 GHz = 1×10^9 Hz.) These frequencies include free-space wavelengths between 1 m and 1 mm. Here's a look at a few important regions in the electromagnetic spectrum...

Region	Frequency range	Wavelength range
Microwave	300 MHz – 300 GHz	1 m – 1 mm
Millimeter-Wave	30 GHz – 300 GHz	1 cm – 1 mm
Infrared	1000 GHz – 10000 GHz	0.3 mm – 30 μ m
Visible light	430000 GHz – 750000 GHz	700 nm – 400 nm

Note: 1000 GHz = 1 THz = 1×10^{12} Hz

Why are microwave frequencies of interest?

Perhaps the best way of answering this is to consider a primary application of microwaves -- wireless communication

The first application of microwaves that often comes to mind is wireless transmission of information. As we go higher in frequency, fractional bandwidth increases. For example, let's assume that we wish to transmit a number of 4 kHz wide voice signals through a wireless link. Further let's assume that we have two wireless systems to choose from, one operating at 500 MHz and the second at 4 GHz, each with a 10 % bandwidth around its center frequency.

In theory, the 500 MHz system could carry:

$$\text{Number of channels} = \frac{\text{Operating frequency} \times \text{percent BW}}{\text{BW per channel}} = \frac{0.5 \text{ GHz} \times 0.1}{4 \text{ kHz}} = 12,500$$

In theory, the 4 GHz system could carry:

$$\text{Number of channels} = \frac{4 \text{ GHz} \times 0.1}{4 \text{ kHz}} = 100,000$$

From the above, we see that as the system's operating frequency increases, ideally its capacity increases. Another advantage in going to higher frequency is antenna size. For a given aperture size, the gain of an antenna increases with frequency. To make portable wireless communications possible, we must operate at a frequency at which the required antenna size is reasonable! Another advantage of increased antenna gain with frequency is the potential for higher-resolution imaging systems.

While it may seem that one can simply increase the operating frequency of a microwave link to increase capacity, issues such as equipment cost, spectrum licensing, and

atmospheric attenuation must be considered. Other applications of microwaves include radar, navigation, remote sensing, and medical instrumentation.

The theoretical foundation for electromagnetics (and thus microwaves) was laid by James Clerk Maxwell in 1873. Oliver Heaviside cast Maxwell's equations into modern form and contributed to a topic particularly relevant to this class, namely, transmission line theory.

Recall:

$\vec{E} \equiv$ Electric Field Intensity [V/m]

$\vec{D} \equiv$ Electric Flux Density [C/m²]

$\vec{H} \equiv$ Magnetic Field Intensity [A/m]

$\vec{B} \equiv$ Magnetic Flux Density [Wb/m²]

$\epsilon \equiv$ Permittivity of the medium [F/m]

$\mu \equiv$ Permeability of the medium [H/m]

Maxwell's Equations:

Point Form

Integral Form

$$\nabla \cdot \vec{D} = \rho_v$$

$$\int_s \vec{D} \cdot d\vec{S} = \int_v \rho_v dv \quad (1)$$

$$\nabla \cdot \vec{B} = 0$$

$$\int_s \vec{B} \cdot d\vec{S} = 0 \quad (2)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\int_L \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_s \vec{B} \cdot d\vec{S} \quad (3)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\int_L \vec{H} \cdot d\vec{l} = \int_s \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S} \quad (4)$$

where:

$$\vec{D} = \epsilon \vec{E}$$

and

$$\vec{B} = \mu \vec{H}$$

- Equation 1 is Gauss' law and states that the volume charge density is identical to the divergence of the electric flux density (flux = charge enclosed).
- Equation 2 is in a similar form and implies that no isolated magnetic charge exists (magnetic flux lines close upon themselves).
- Equation 3 is Faraday's Law and states that the circulation of the electric field is equal to the negative rate of change of the flux of the magnetic field through a surface formed by the circulation contour.

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- Equation 4 is Ampere's Law (with Maxwell's displacement current) and states that the circulation of the magnetic field is equal to the surface integral of both the conduction current (\vec{J}) and the time varying displacement current ($\frac{\partial \vec{D}}{\partial t}$).

Another important equation to include is the so-called "continuity equation" given below:

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$$

The continuity equation states that the current density that leaves a point is equal to the negative of the time rate of change of the charge at that point.

Maxwell's equations and the continuity equation are covered in standard EM courses. These courses often use the static form (non-time varying) of Maxwell's equations to calculate fields based on stationary electric charges or constant currents. Waves are then introduced through a manipulation of the time-varying equations. *We will make scant reference to Maxwell's equations during this course.* We will use the equations to investigate the dominant mode in a rectangular waveguide, but little else.

Why not dig deeply into Maxwell's equations?

We don't need to. Using Maxwell's equations to analytically solve problems can shed great light on many interesting EM problems. The number of problems that can be solved analytically using Maxwell's equations is rather limited however. Modern "full-wave" design tools utilize the equations, but make thousands of calculations behind our backs to generate a solution!

If we don't use Maxwell's equations, then what can we do?

Simplify the problems, and let simulators clean up the details. Let's take a step back to simple circuit theory and see how this might work...

Ohm's Law:

$$V = IZ$$

Voltage across element is proportional to the current through the element.

Kirchhoff's Voltage Law (KVL):

$$\sum_{\text{around a closed loop}} V = 0$$

The algebraic sum of all the voltages around any closed loop in a circuit equals zero.

Kirchhoff's Current Law (KCL):

$$\sum_{\text{into a node}} i = 0$$

The algebraic sum of all the currents at any node in a circuit equals zero.

It is interesting to note that the basic circuit laws given above are simplifications of the EM equations we have discussed previously. Let's see how.

Kirchhoff's Voltage Law

Let's start with the following from Maxwell's equations:

$$\int_L \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{S}$$

What happens if zero magnetic field leaves a surface (or if the time rate of change of the magnetic field is zero leaving the surface)?

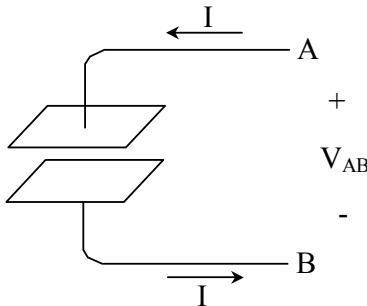
$$\int_L \vec{E} \cdot d\vec{l} = 0$$

But what is the line integral of the electric field across an element? It is the voltage across the element. Thus, if the surface (S) we are considering is in fact a loop in a circuit...

$$\int_L \vec{E} \cdot d\vec{l} = \sum_{\text{around a closed loop}} V = 0$$

Derivation of Ohm's Law

Let's derive Ohm's law in the case of a capacitor (and assume similar derivations for an inductor and resistor). Consider the illustration of a parallel plate capacitor given below.



Consider the circulation of the electric field assuming that the magnetic field is zero as before.

$$\int_L \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{S} = 0$$

Let's break up the integral as follows and integrate counterclockwise around the loop.

$$\int_L \vec{E} \cdot d\vec{l} = \int_{\text{Along wires}} \vec{E} \cdot d\vec{l} + \int_{\text{Between capacitor plates}} \vec{E} \cdot d\vec{l} + \int_{\text{Across AB gap}} \vec{E} \cdot d\vec{l}$$

Assuming that the wires are perfect electrical conductors (PEC), the first term is zero. We may say this as a PEC can maintain no voltage drop (think of what happens to the current in Ohm's law if the resistance drops to zero). Taking the charge on the top plate to be Q , the charge on the bottom plate is $-Q$ and the voltage across the plates is Q/C and thus

$$\int_{\text{Between capacitor plates}} \vec{E} \cdot d\vec{l} = \frac{Q}{C}$$

The final term is simply the voltage from B to A (counterclockwise rotation), that is,

$$\int_{\text{Across AB gap}} \vec{E} \cdot d\vec{l} = -V_{AB}$$

The voltage around the closed loop must be zero and thus we find,

$$\frac{Q}{C} = V_{AB}$$

Taking the time derivative of both sides,

$$\frac{1}{C} \frac{\partial Q}{\partial t} = \frac{\partial V_{AB}}{\partial t}$$

The derivative of charge with respect to time is simply the current I . Thus,

$$\frac{1}{C} I = \frac{\partial V_{AB}}{\partial t} = j\omega V_{AB}$$

$$V_{AB} = \frac{1}{j\omega C} I \rightarrow V_{AB} = IZ$$

Derivation of Kirchhoff's Current Law

Let's start with the continuity equation.

$$\nabla \bullet \vec{J} = -\frac{\partial \rho_v}{\partial t}$$

For steady state currents,

$$\frac{\partial \rho_v}{\partial t} = 0$$

and thus,

$$\nabla \bullet \vec{J} = 0 \rightarrow \sum_{\text{into a node}} i = 0$$

This states that the current leaving a point (node) under steady-state conditions is zero.

The upshot of all this: *The circuit laws (Ohm's Law, KCL, KVL) may be derived from Maxwell's equations.*

Clearly, the circuit laws are easier to understand and apply when compared with Maxwell's equations. Can we use the simple circuit laws, in favor of Maxwell's equations, in designing and describing our microwave circuits?

The answer: sometimes.

Whether the simple circuit laws may be used depends on the size of our circuit in relation to the wavelength corresponding to the operating frequency.

The free-space wavelength is related to the operating frequency by

$$\lambda = \frac{c}{f}$$

where λ is the wavelength, c is the speed of light in vacuum ($\sim 3 \times 10^8$ m/s), and f is the frequency in Hz.

If the size of the circuit (or element) in question is much smaller than the operating wavelength ($\sim \lambda/100$ or smaller), the simple circuit laws apply. In such a case, we say that the elements of the circuit are "lumped" elements.

If the size of the circuit in question is comparable to that of the operating wavelength ($\sim \lambda/10$ to $\sim \lambda$), the simple circuit laws do not apply. In such a case, we say that the elements of the circuit, are "distributed" elements. In situations in which the largest dimensions fall between the $\lambda/10$ and $\lambda/100$ guidelines, it is safest to assume a distributed character, though the 5% guideline is often invoked. The 5% guideline maintains that when the size of the element in question is smaller than 5% of the guided wavelength, it can be treated as a lumped element.

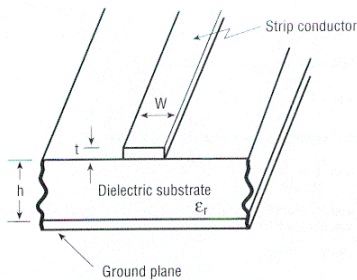
Phase variations of currents and voltages across elements are insignificant when dealing with lumped elements, whereas accounting for the phase variations across elements is critically important when dealing with distributed circuits.

It is of interest to note what happens when elements are much greater than the operating wavelength. Such a case often arises in optics and the simplified rules of geometric optics (refection, refraction laws) apply.

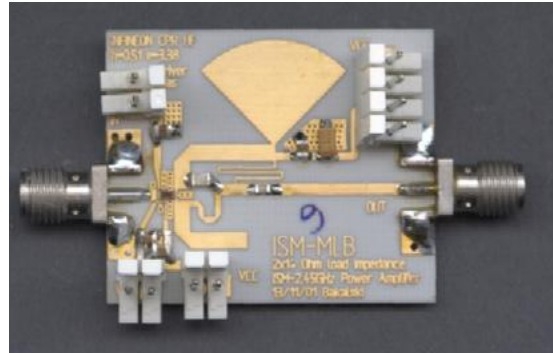
Since the operating wavelength is quite small at microwave frequencies, we often will not use circuit theory. In such cases, we may always fall back on Maxwell's equations as they are universally valid. That being said, Maxwell's equations are too cumbersome to handle in all but the most simple of situations. Thus we seek tools to describe microwave circuits that, while perhaps not as simple as Ohm's Law, KCL and KVL, are much simpler than direct application of Maxwell's Equations.

Enter Transmission Line Theory...

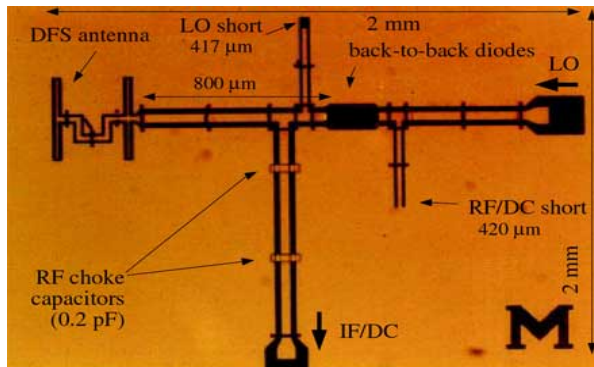
Transmission Line Examples:



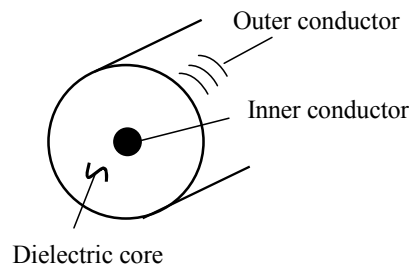
Microstrip



Microstrip Power Amplifier



154 GHz receiver in Coplanar Waveguide (CPW)



Coaxial line

Definitions For The Common Microwave Impedances

In discussing microwave propagation, one may hear a variety of uses of the word impedance. A few follow.

Impedance

The “traditional” definition of impedance for ac circuits is simply the complex ratio of the voltage to current.

$$Z = \frac{V}{I}$$

Intrinsic Impedance

The intrinsic impedance of a medium *depends only on the dielectric properties of the medium*. The intrinsic impedance is identical to the impedance of a *plane wave* in that medium.

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

ϵ is the permittivity of the medium (F/m)
 μ is the permeability of the medium (H/m)

Wave Impedance

The wave impedance is a characteristic of the particular type of wave (i.e. its field configuration) and depends on the material properties in which the wave propagates and on frequency.

Characteristic Impedance

The characteristic impedance is given by the ratio of voltage and current waves. The characteristic impedance is predominantly a transmission line concept and is unique if the propagating mode is transverse electromagnetic (TEM). If the transmission line is lossless,

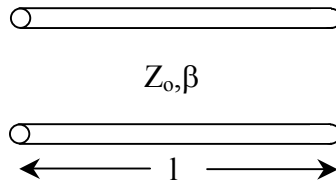
$$Z_o = \sqrt{\frac{L}{C}}$$

where L and C depend on transmission line geometry and material properties.

Transmission Line Summary Sheet

This short note is meant to condense the lecture material into a few equations and a few concepts that you should understand and be able to apply.

Familiar depiction of a general transmission line...



Transmission line theory demonstrated that the voltage and current on a transmission line are in the form of *traveling waves* and may be expressed as shown below.

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}$$

$$I(z) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z}$$

From these equations, we see that a transmission line is characterized by two key parameters, *characteristic impedance* (Z_o) and *propagation constant* (γ). Both parameters are defined by the transmission line geometry and the properties of the material(s) upon which the transmission line is built.

The **characteristic impedance** is the ratio of the forward voltage wave to the forward current wave traveling along the transmission line. From transmission line theory we found the characteristic impedance to be given by:

$$Z_o = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad [\Omega]$$

where R, L, G and C represent resistance, inductance, conductance and capacitance per unit length along the TL.

In the case of a lossless transmission line, this reduces to:

$$Z_o = \sqrt{\frac{L}{C}}$$

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The **propagation constant** is the sum of two terms, the attenuation constant (α) and the phase constant (β) and describes both the change in amplitude (via α) and phase (via β) of the wave as it travels along the TL. The propagation constant can be related to the distributed parameters as follows:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad [1/m].$$

In the case of a lossless transmission line, the propagation constant reduces to

$$\gamma = j\beta = j\omega\sqrt{LC},$$

and thus phase constant alone dictates the wave's propagation constant. In this case,

$$\boxed{\beta = \omega\sqrt{LC}} \quad [\text{radians/m}]$$

The velocity of a fixed phase point on a wave, its **phase velocity**, is given by:

$$\boxed{v_p = \frac{\omega}{\beta}} \quad [\text{m/s}]$$

A lossless transmission line is thus dispersionless. (Do you remember why? What is the significance to signal transfer along the TL?)

The wavelength on a given transmission line is:

$$\boxed{\lambda = \frac{2\pi}{\beta}} \quad [\text{m}]$$

The wavelength on a given transmission line is often referred to as the “**guided wavelength**” and is equally well represented by:

$$\boxed{\lambda_g = \lambda = \frac{\lambda_o}{\sqrt{\epsilon_{\text{eff}}}}} \quad [\text{m}]$$

where λ_o is the free-space wavelength and ϵ_{eff} is the effective dielectric constant of the transmission line (a parameter determined by line geometry and material properties).

The **electrical length** of a section of transmission line of length l , is given by:

$$\boxed{\text{Electrical Length} = \beta l} \quad [\text{degrees or radians}]$$

Bringing back the time factor

$$V(z,t) = |V_0^+| \cos(\omega t - \beta z + \phi^+) e^{-\alpha z} + |V_0^-| \cos(\omega t + \beta z + \phi^-) e^{\alpha z}$$

$$I(z,t) = \frac{|V_0^+|}{Z_0} \cos(\omega t - \beta z + \phi^+) e^{-\alpha z} - \frac{|V_0^-|}{Z_0} \cos(\omega t + \beta z + \phi^-) e^{\alpha z}$$

where $V_0^\pm = |V_0^\pm| \angle \phi^\pm$

▷ What is the phase velocity? (i.e. velocity of a fixed phase point)

$$\text{phase} = \omega t - \beta z + \phi^+ = \text{constant}$$

\uparrow phase changes with time \nwarrow phase change with distance \nearrow constant

$$v_p \equiv \text{phase velocity} = \frac{dz}{dt} = \boxed{\frac{\omega}{\beta} = v_p}$$

That is $\frac{d}{dt} (\omega t - \beta z + \phi^+) = \omega - \beta \frac{dz}{dt} = \omega - \beta \frac{\omega}{\beta} =$

▷ What is the wavelength? phase is constant \nearrow

$$(\omega t - \beta z + \phi^+) - (\omega t - \beta(z + \lambda) + \phi^+) = 2\pi$$

$$\beta \lambda = 2\pi$$

$$\boxed{\lambda = 2\pi / \beta}$$

⑨

Recall that $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$

In general then, α and β are functions of R, L, G, C and ω . Practical transmission lines are low loss and so to simplify matters they are often assumed to be lossless.

Lossless TL $\rightarrow R = G = 0$

$\rightarrow \gamma = \alpha + j\beta = \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC}$

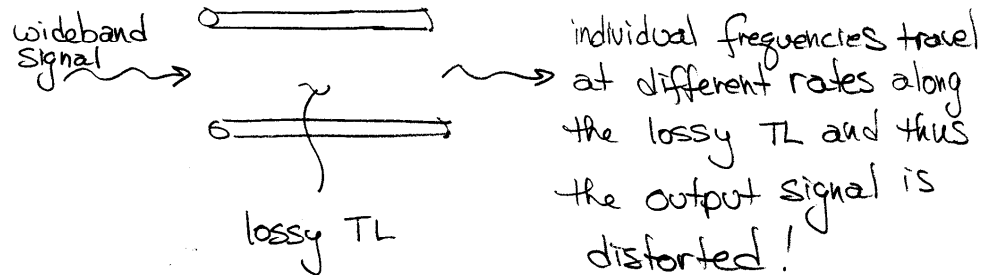
$\rightarrow \alpha = 0 \quad \beta = \omega\sqrt{LC}$

Thus $\boxed{v_p = \frac{1}{\sqrt{LC}}}$

This is an important result as it states that the phase velocity on a lossless transmission line is independent of frequency.

\rightarrow Zero dispersion

Why is zero dispersion important?

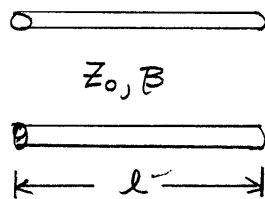


Recall that in the general case

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

If the TL is lossless $Z_0 = \sqrt{L/C}$

Electrical Length



$Z_0 \equiv$ characteristic impedance [Ω]
 $\beta \equiv$ phase constant [rad/m]
 $l \equiv$ physical length

Electrical length $\equiv \beta l$ [radians or degrees]

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<end lecture 2 >

Microwave Transmission Lines

There are a variety of TL types currently used at microwave frequencies. Some examples are: Coaxial line ("coax"), microstrip, stripline, coplanar waveguide, and coplanar strip line. Important considerations in choosing a TL include size, compatibility with other components, and loss/dispersion performance.

We know that TLs support EM waves. The EM wave may take on various configurations within a TL depending on its geometry, size and the operating frequency. The field configurations are termed "modes." Such modes include

▷ TEM - "Transverse Electromagnetic"

- $\vec{E} \perp \vec{H} \perp$ direction of propagation
- Plane waves are TEM
- Coax supports TEM
- cutoff frequency is 0Hz (ie. will propagate to zero frequency)

(13)

▷ TE - "Transverse Electric"

- $\vec{E} \perp$ direction of propagation
- \vec{H} has at least some component \parallel to direction of propagation
- The dominant mode of a standard rectangular waveguide is TE.

▷ TM - "Transverse Magnetic"

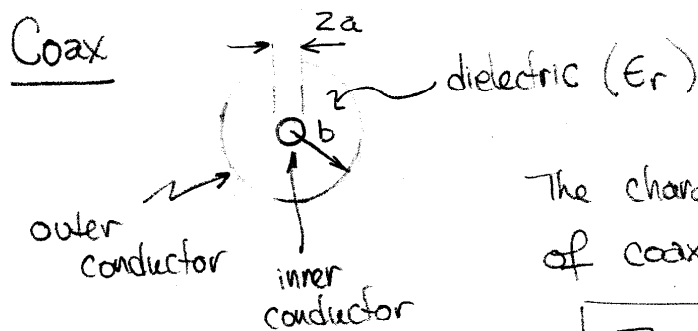
- $\vec{H} \perp$ direction of propagation
- \vec{E} has at least some component \parallel to direction of propagation
- Dielectric waveguides support TM modes

▷ Hybrid

- Combination of TE and TM modes

For more on modes see text.

Typically it is desirable to have only a single mode propagate along a TL. The first mode that is capable of propagating (determined by TL geometry, size and operating frequency) is termed the "dominant mode." "Higher order modes" are field configurations that are supported by a given TL but can only propagate at frequencies beyond the onset of that of the dominant mode.



The characteristic impedance of coax is given by:

$$Z_0 = \frac{\eta}{2\pi} \ln \frac{b}{a}$$

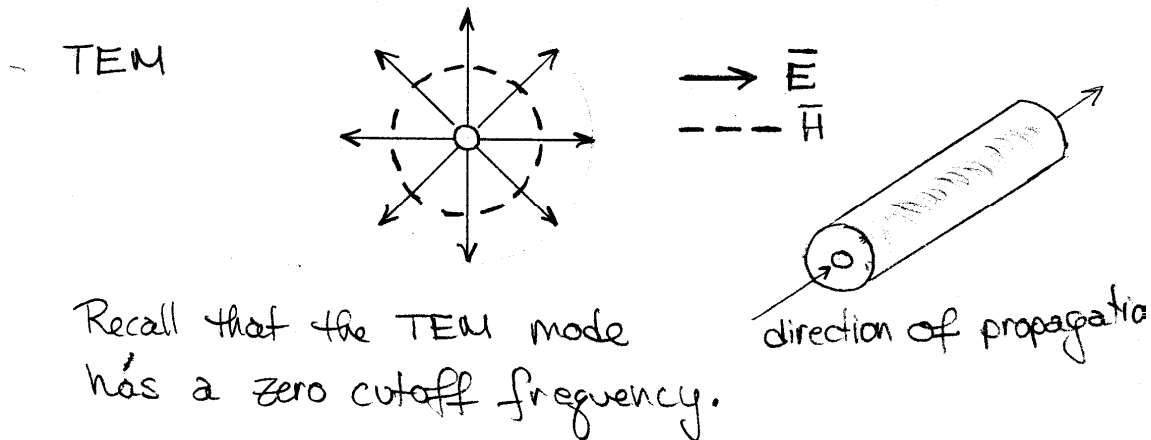
$\eta \equiv$ intrinsic impedance of

medium $\eta = \sqrt{\mu/\epsilon} = \eta_0 \sqrt{\frac{\mu_r}{\epsilon_r}}$

$\eta_0 \equiv$ intrinsic impedance of free-space

$$\eta_0 = \sqrt{\mu_0/\epsilon_0} = 120\pi \approx 377 \Omega$$

The dominant mode in a COAX is TEM



The second mode to propagate in the coax is a TE_{11} mode. Its cutoff frequency is given by

$$f_{c_{TE_{11}}} = \frac{c}{\pi \sqrt{\epsilon_r} (a+b)}$$

As an example consider RG-142 coax.

$\epsilon_r = 2.08$ (teflon)

$a = 0.036''$
 $b = 0.117''$
 $c = 0.141''$

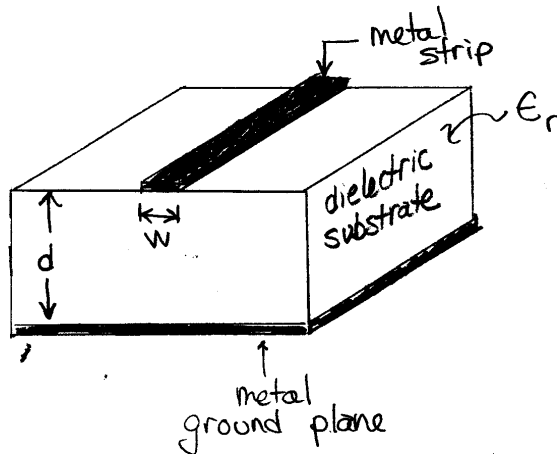
} these are typical values

$$Z_0 = \frac{120\pi}{2\pi\sqrt{2.08}} \ln\left(\frac{0.117}{0.036}\right) = 49.52$$

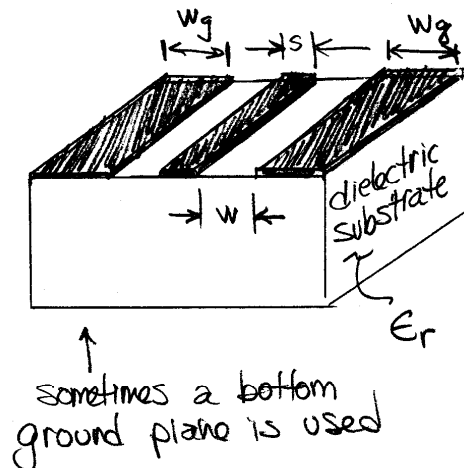
$$f_{c_{TE_{11}}} = \frac{3 \times 10^8 \text{ m/s}}{\pi \sqrt{2.08} (0.117'' + 0.036'') \left(\frac{2.54 \text{ cm}}{1 \text{ inch}}\right) \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)} \approx 17 \text{ GHz}$$

→ Operate below 17 GHz to ensure single-mode propagation

The two most popular planar transmission lines are microstrip and coplanar waveguide (CPW)



MICROSTRIP



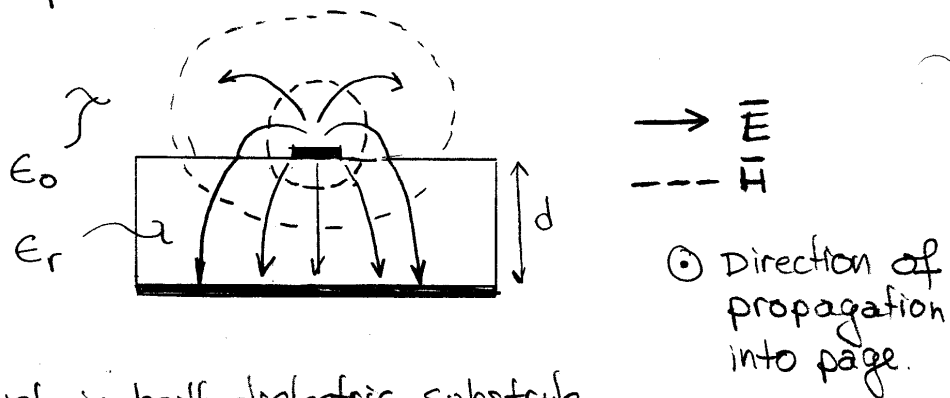
CPW

The following are often-stated advantages & disadvantages of these two TLs

Microstrip: lower loss, higher power handling capability, characteristics of line and discontinuities well known with accurate models widely available. But characteristics sensitive to substrate thickness which may become a problem at $f > 20 \text{ GHz}$ and mounting of devices may require vias.

CPW: Uniplanar and thus series & shunt elements may be easily added w/o need for vias, substrate thickness largely irrelevant. But loss may be somewhat larger and ground plane equalization often necessary. (17)

Microstrip



Field exists in both dielectric substrate and in air.

→ inhomogeneous dielectric and thus microstrip does not support a pure TEM mode

In most practical cases, the substrate is electrically thin ($d \ll \lambda$), encouraging most of the field to reside in the dielectric. In such cases, the microstrip is considered to be a "quasi-TEM" structure

How is it then modeled?

CASE I: Air substrate

$$\epsilon_r = 1$$



$$v_p = c, \beta = \beta_0$$

CASE II: single dielectric fills all space

$$\epsilon_r \neq 1$$



$$v_p = c/\sqrt{\epsilon_r}, \beta = \beta_0 \sqrt{\epsilon_r}$$

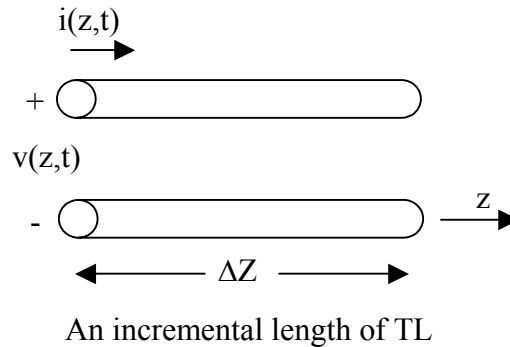
The actual microstrip is modeled somewhere between

those two extreme cases. → $v_p = c/\sqrt{\epsilon_{eff}}, \beta = \beta_0 \sqrt{\epsilon_{eff}}$
 $\epsilon_{eff} \equiv$ effective dielectric constant $1 < \epsilon_{eff} < \epsilon_r$ (18)

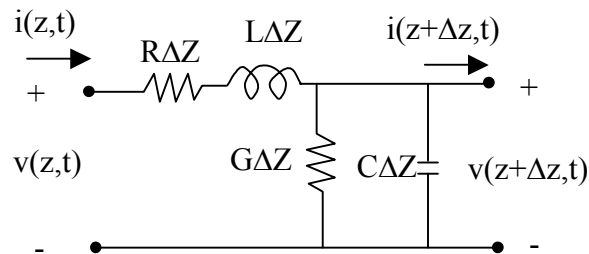
SEE SECTION 3.8 IN OUR TEXT FOR DETAILED THEORY ON MICROSTRIP LINES. WE WILL BE USING A PROGRAM CALLED 'MLIN' THAT IS PART OF THE ADS SOFTWARE. A FREE PROGRAM FROM AGILENT CALLED "APPCAD" ALSO GIVE CLOSE RESULTS FOR MICROSTRIP AND STRIPLINE CALCULATIONS. A LINK WILL BE PROVIDED ON THE WEBSITE TO GET APPCAD (avo)

A Brief Overview Of Transmission Line Theory

Transmission lines are often depicted as two wire lines:



Think about a coaxial line for a moment. It seems reasonable to consider that the conductors contribute both a series resistance and a series inductance. The resistance would come from the finite conductivity of the metal and the inductance from magnetic flux linkage. In the coax, a dielectric separates the two conductors. A dielectric separating two conductors sounds like a capacitance. Naturally, the dielectric will have some loss associated with it. Since the capacitance and this dielectric loss go from one conductor to another, we should include both shunt capacitance and a shunt conductance to our model. We will now create a lumped element model of an incremental section of TL based on our reasoning.



Equivalent circuit of an incremental (length Δz) section of TL

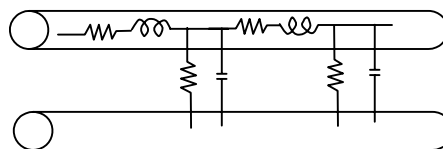
$R \equiv$ Series resistance per unit length [Ω/m] (represents conductive loss)

$L \equiv$ Series inductance per unit length [H/m]

$C \equiv$ Shunt capacitance per unit length [F/m]

$G \equiv$ Shunt conductance per unit length [S/m] (represents dielectric loss)

These elements are actually distributed along the length of the TL. That is,



<end lecture 3>