

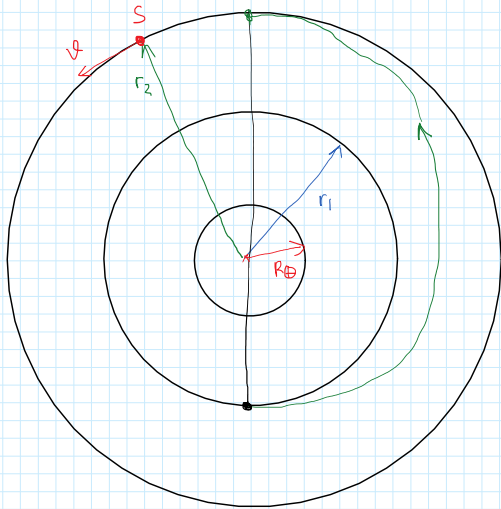
Tes 3

Friday, March 29, 2019 3:57 PM

1. Sebuah satelit telekomunikasi akan diletakkan pada sebuah orbit melingkar di atas ekuator, sedemikian sehingga dia berada tetap di satu titik di atas Brazil dengan deklinasi nol. Saat ini, satelit parkir pada ketinggian 320 km di atas permukaan Bumi, dan akan melakukan transfer orbit untuk naik ke orbit tetapnya.

- (a) Hitung radius orbit satelit akhir saat dia tepat di atas Brazil
(b) Hitung setengah sumbu mayor, eksentrisitas orbit transfer, dan lamanya transfer orbit.

Diketahui periode sideris rotasi Bumi adalah $23^h 56^m$ dan periode revolusi satelit di ketinggian parkir adalah 90 menit.



$$r_1 = R_E + 320$$

$$v_c = \sqrt{\frac{GM_E}{r_2}}$$

$$v_c = \frac{2\pi r_2}{P_2}$$

$$\frac{2\pi r_2}{P_2} = \sqrt{\frac{GM_E}{r_2}}$$

$$P_2 = 2\pi r_2 \cdot \sqrt{\frac{r_2}{GM_E}}$$

$$P_2 = 2\pi \sqrt{\frac{r_2^3}{GM_E}} = P_E$$

$$a) P_2 = 2\pi \sqrt{\frac{r_2^3}{GM_E}}$$

$$P_2^2 = 4\pi^2 \frac{r_2^3}{GM_E}$$

$$r_2^3 = \frac{P_2^2 \cdot GM_E}{4\pi^2}$$

$$r_2 = \sqrt[3]{\frac{P_2^2 \cdot GM_E}{4\pi^2}}$$

$$P_2 = P_E = 23^h 56^m \times 3600^s = 86160^s$$

$$= \sqrt[3]{\frac{(86160)^2 \cdot (6.673 \times 10^{-11}) \cdot (6 \times 10^{24})}{4\pi^2}} \quad m$$

$$r_2 = 42225.5 \text{ km}$$

b). a, e, T

$$a = \frac{r_1 + r_2}{2} = \frac{R_E + 320 + r_2}{2} = \frac{6378 + 320 + 42225.5}{2} \text{ km}$$

$$a = 24461.7 \text{ km}$$

$$e: \rightarrow r_p = r_1 = a(1-e)$$

$$R_E + 320 = a(1-e)$$

$$6378 + 320 = 24461.7 (1-e)$$

$$e = 0.726$$

$$T = \frac{1}{2}P \leftarrow \text{H. III Kepler: } \frac{a^3}{P^2} = \frac{GM_E}{4\pi^2} \rightarrow P^2 = \frac{4\pi^2 a^3}{GM_E}$$

$$= \frac{1}{2} \cdot 2\pi \sqrt{\frac{a^3}{GM_E}}$$

$$= \pi \sqrt{\frac{a^3}{GM_E}} = \pi \sqrt{\frac{(24461.7 \times 1000)^3}{(6.673 \times 10^{-11})(6 \times 10^{24})}} \text{ s}$$

$$T = 18995.19 \text{ s}$$

$$T = 5^h 16^m 35.19^s$$

2. Saat oposisi, sebuah planet yang merupakan planet superior memiliki jarak dari Matahari sebesar b AU, memiliki kecepatan sudut sideris $-\omega_1$ derajat per hari, dilihat dari pengamat di Bumi. Pada kuadratur berikutnya, kecepatan sudut siderisnya adalah $+\omega_2$ derajat per hari. Buktikan bahwa

Diagram illustrating the geometry of planetary motion around the Sun (S). The Earth (E₁) orbits the Sun at distance 1 AU. The planet (P₂) orbits the Sun at distance b AU. The angle between the Sun-Earth line and the Sun-Planet line is α . The Earth's orbital velocity is V_\oplus and the planet's orbital velocity is V_P . The relative velocity of the planet with respect to Earth is V_{P2} . The angle between the Sun-Planet line and the relative velocity vector is α . The angle between the Sun-Earth line and the relative velocity vector is α .

Equations derived from the diagram and geometry:

$$\frac{\omega_2}{\omega_1} = \frac{1}{b} \left(\frac{b-1}{b^{1/2}-1} \right)$$

$$E_2 P_2^2 = S P_2^2 - S E_2^2$$

$$= b^2 - 1$$

$$E_2 P_2 = \sqrt{b^2 - 1}$$

$$\omega_{P_2} = \omega_2 = \frac{V_P \sin \alpha}{E_2 P_2}$$

$$\omega_2 = \frac{V_P \sin \alpha}{\sqrt{b^2 - 1}}$$

$$\omega_{P_1} = -\omega_1 = \frac{V_P - V_\oplus}{E_1 P_1}$$

$$-\omega_1 = \frac{V_P - V_\oplus}{b-1}$$

$$\frac{\omega_2}{\omega_1} = - \left[\frac{V_P \sin \alpha}{\sqrt{b^2 - 1}} \cdot \frac{b-1}{V_P - V_\oplus} \right]$$

$$\sin \alpha = \frac{E_2 P_2}{S P_2} = \frac{\sqrt{b^2 - 1}}{b}$$

$$V = \sqrt{\frac{GM}{r}} \rightarrow V \propto \frac{1}{\sqrt{r}}$$

$$V_P \propto \frac{1}{\sqrt{b}}$$

$$V_\oplus \propto \frac{1}{\sqrt{1}} = 1$$

$$\frac{\omega_2}{\omega_1} = - \left[\frac{\frac{1}{\sqrt{b}} \cdot \frac{\sqrt{b^2 - 1}}{b}}{\frac{1}{\sqrt{b}} - 1} \cdot \frac{(b-1)}{\frac{1}{\sqrt{b}} - 1} \right]$$

$$= - \left[\frac{1}{\sqrt{b}} \cdot \frac{(b-1)}{\frac{1}{\sqrt{b}} - 1} \right]$$

$$= - \frac{1}{b} (b-1) \left[\frac{1}{\sqrt{b}} \cdot \left(\frac{1}{\frac{1}{\sqrt{b}} - 1} \right) \right]$$

$$= \frac{1}{b} (b-1) \left[\frac{1}{1 - \sqrt{b}} \right]$$

$$\frac{\omega_2}{\omega_1} = \frac{1}{b} (b-1) \left[\frac{1}{1 - \sqrt{b}} \right]$$