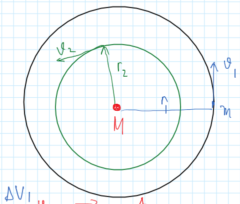
MBL, tasur, BG

Friday, March 29, 2019 8:03 AM

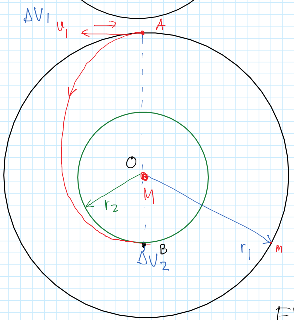
Transper orbit:
- circular (lingkaran)
- koplanar (sebidang)



$v_1 = \sqrt{\frac{GM}{r_1}}$

transper Hohmann

$v_2 = \sqrt{\frac{GM}{r_2}}$



$-\Delta V_1 = V_{elips} - V_{c,r_1}$

$-\Delta V_1 = V_{elips} - V = \boxed{\sqrt{GM\left[\frac{2}{r_1} - \frac{1}{a}\right]} - \sqrt{\frac{GM}{r_1}}}$

$\Delta V_2 = V_{c,r_2} - V_{elips} = \boxed{\sqrt{\frac{GM}{r_2}} - \sqrt{GM\left[\frac{2}{r_2} - \frac{1}{a}\right]}}$

Pers. energi Mekanik:

$Ek + Ep = h$

$\frac{1}{2}mV^2 - \frac{GMm}{r} = h$

$h = -\frac{GMm}{2a}$

$\frac{1}{2}\cancel{m}V^2 - \frac{GM\cancel{m}}{r} = -\frac{GM\cancel{m}}{2a}$

$\frac{1}{2}V^2 = \frac{GM}{r} - \frac{GM}{2a}$

$\cancel{\frac{1}{2}}V^2 = \cancel{\frac{1}{2}}GM\left[\frac{2}{r} - \frac{1}{a}\right]$

$V^2 = GM\left[\frac{2}{r} - \frac{1}{a}\right]$

$V = \sqrt{GM\left[\frac{2}{r} - \frac{1}{a}\right]}$

1) $a \rightarrow$ stgh sb. mayor orbit transfer

$a = \frac{AB}{2} = \frac{r_1 + r_2}{2}$

2). eksentrisitas, e

OB = jarak terdekat = periapsis $= a(1-e)$

OA = –//– terjauh = apoapsis $= a(1+e)$

$\frac{OB}{OA} = \frac{\cancel{a}(1-e)}{\cancel{a}(1+e)} = \frac{r_2}{r_1} \leftrightarrow r_1 - r_1 e = r_2 + r_2 e$

$r_1 - r_2 = r_1 e + r_2 e$

$r_1 - r_2 = e(r_1 + r_2)$

$\boxed{e = \frac{r_1 - r_2}{r_1 + r_2}}$

3) Durasi transper, τ

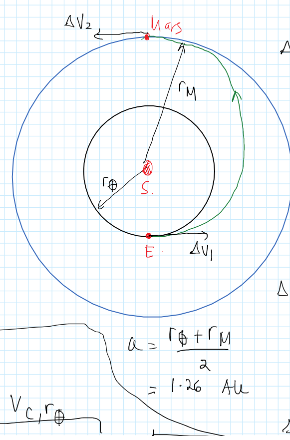
$\tau = \frac{1}{2}P$

H III kepler $\Rightarrow \frac{a^3}{P^2} = \frac{G}{4\pi^2}M \rightarrow P = \left[\frac{a^3}{\frac{G}{4\pi^2}M}\right]^{1/2}$

$\boxed{\tau = \frac{1}{2}\left[\frac{a^3}{\frac{G}{4\pi^2}M}\right]^{1/2}}$

4) perub. kecepatan, Δv.

1. Hitung total perubahan kecepatan sebuah wahana dari saat dia mengorbit Bumi pada ketinggian 400 km di atas permukaan Bumi, sampai dia mengorbit planet Mars pada ketinggian 2000 km (diketahui jarak Matahari - Mars = 1.52 AU; Massa Mars = 6.39 x 10^23 kg). Jangan lupa ilustrasikan masalah ini dengan gambar
2. Jika wahana ini diluncurkan mendekati Matahari dari ketinggian 400 km di atas permukaan Bumi dan mengorbit Matahari pada jarak 1000000 km dari permukaan Matahari, berapakah total perubahan kecepatannya. Bandingkan dengan jawaban no. 1



$\Delta V_1 = V_{ellips, r_\oplus} - V_{c, r_\oplus}$

$= \sqrt{GM_\odot\left[\frac{2}{r_\oplus} - \frac{1}{a}\right]} - \sqrt{\frac{GM_\odot}{r_\oplus}}$

$\frac{a^3}{P^2} = \frac{G}{4\pi^2}M \rightarrow \frac{a^3}{P^2} = M$ (AU, tahun, $M_\odot$)

$\frac{G}{4\pi^2} = 1 \rightarrow G = 4\pi^2$

$\Delta V_1 = \sqrt{4\pi^2 \cdot M_\odot\left[\frac{2}{r_\oplus} - \frac{1}{a}\right]} - \sqrt{\frac{GM_\odot}{r_\oplus}}$

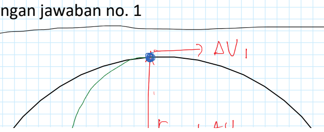
$= 2\pi\sqrt{(1)\left[\frac{2}{1} - \frac{1}{1.26}\right]} - 2\pi\sqrt{\frac{(1)}{(1)}}$ AU/tahun

$= 2\pi(1.1) - 2\pi$

$\Delta V_1 = 2\pi(0.1)$ AU/thn $\Rightarrow 0.63 \frac{AU}{tahun} = 0.63 \times 1.4959 \times 10^8$ km

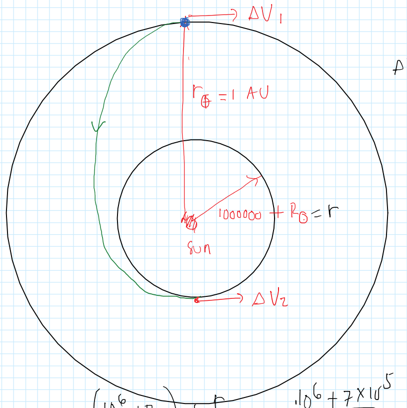
$a = \frac{r_\oplus + r_M}{2} = 1.26$ AU

2.



$\Delta V_1 = V_{e, r_\oplus} - V_{c, r_\oplus}$

2



$$\Delta V_1 = V_{e,r_\oplus} - V_{c,r_\oplus}$$
$$= \sqrt{GM_\odot\left[\frac{2}{r_\oplus} - \frac{1}{a}\right]} - \sqrt{\frac{GM_\odot}{r_\oplus}}$$
$$= \sqrt{4\pi^2(1)\left[\frac{2}{1} - \frac{1}{0.51}\right]} - \sqrt{\frac{4\pi^2(1)}{(1)}}$$
$$= 2\pi(0.2) - 2\pi$$
$$= -0.8 \cdot 2\pi$$
$$= 5.03\ \frac{AU}{tahun} = \frac{5.03 \times 1.4959 \times 10^8}{365.25 \times 24 \times 3600}\ \frac{km}{s}$$

$$a = \frac{(10^6 + R_\odot) + r_\oplus}{2} = \frac{\frac{10^6 + 7\times10^5}{1.4959\times10^8} + 1}{2}$$
$$= \frac{0.011 + 1}{2}\ AU$$
$$= 0.51\ AU$$

$$\Delta V_1 = 23.83\ \frac{km}{s}$$

$$a = \frac{r_\oplus + r_M}{2} = 1.26\ AU$$

$$= 2\pi\sqrt{(1)\left[\frac{2}{1} - \frac{1}{1.26}\right]} - 2\pi\sqrt{\frac{(1)}{(1)}}\ AU/tahun$$
$$= 2\pi(1.1) - 2\pi$$
$$\Delta V_1 = 2\pi(0.1)\ AU/thn \Rightarrow 0.63\ \frac{AU}{tahun} = \frac{0.63 \times 1.4959 \times 10^8}{365.25 \times 24 \times 3600}\ \frac{km}{s}$$
$$\Delta V_1 = 2.98\ \frac{km}{s}$$

$$\Delta V_2 = V_{c,r_M} - V_{e,r_M}$$
$$= \sqrt{\frac{GM_\odot}{r_M}} - \sqrt{GM_\odot\left[\frac{2}{r_M} - \frac{1}{a}\right]}$$
$$= \sqrt{\frac{4\pi^2 \cdot (1)}{(1.52)}} - \sqrt{4\pi^2(1)\left[\frac{2}{1.52} - \frac{1}{1.26}\right]}$$
$$= 2\pi \cdot (0.81) - 2\pi \cdot (0.72)$$
$$\Delta V_2 = 2\pi(0.087)\ \frac{AU}{tahun} \Rightarrow 0.55\ \frac{AU}{tahun}$$
$$\Delta V_2 = \frac{0.55 \times 1.4959 \times 10^8}{365.25 \times 24 \times 3600}\ \frac{km}{s}$$
$$\Delta V_2 = 2.6\ \frac{km}{s}$$

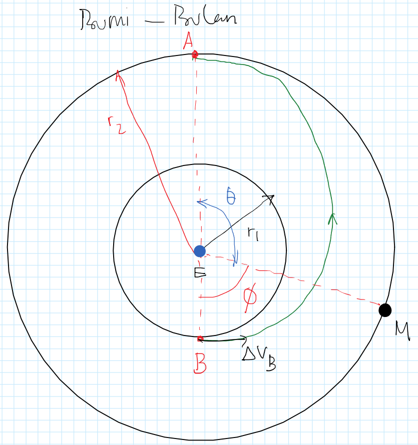
$$\Delta V_{TOT} = \Delta V_1 + \Delta V_2$$
$$= 2.98 + 2.6 = 5.58\ \frac{km}{s}$$

$$\Delta V_2 = V_{c,r} - V_{e,r}$$
$$= \sqrt{\frac{GM_\odot}{r}} - \sqrt{GM_\odot\left[\frac{2}{r} - \frac{1}{a}\right]}$$
$$= \sqrt{\frac{4\pi^2(1)}{0.011}} - \sqrt{4\pi^2(1)\left[\frac{2}{0.011} - \frac{1}{0.51}\right]}$$
$$= 2\pi \cdot 9.53 - 2\pi(13.41)$$
$$\Delta V_2 = -24.39\ \frac{AU}{tahun} = \frac{-24.39 \times 1.4959 \times 10^8}{365.25 \times 24 \times 3600}\ \frac{km}{s}$$
$$\Delta V_2 = 115.59\ \frac{km}{s}$$

$$\Delta V_{TOT} = |\Delta V_1| + |\Delta V_2|$$
$$= 23.83 + 115.59$$
$$= 139.42\ \frac{km}{s}$$

4. Rendezvous problem

Bumi – Bulan



$$\phi = ?$$
$$\phi = \pi - \theta \quad \text{radian}$$
$$= \pi - \omega_M \cdot t_M$$

$$t_M = \tau = \frac{1}{2}P$$

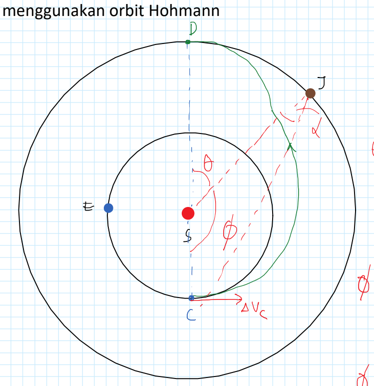
$$\frac{a^3}{P^2} = \frac{G}{4\pi^2}M_\oplus$$
$$P^2 = \frac{a^3 \cdot 4\pi^2}{GM_\oplus}$$
$$P = 2\pi\sqrt{\frac{a^3}{GM_\oplus}}$$

$$t_M = \frac{1}{\cancel{2}}\left[\cancel{2}\pi\sqrt{\frac{a^3}{GM_\oplus}}\right]$$

$$\phi = \pi - \omega_M \cdot \left[\pi\sqrt{\frac{a^3}{GM_\oplus}}\right]$$
$$= \pi - \frac{2\pi}{T_M}\left[\pi\sqrt{\frac{a^3}{GM_\oplus}}\right]$$

$$a = \frac{r_1 + r_2}{2}$$

1. Tentukan besar sudut fase planet Jupiter saat wahana luar angkasa dari Bumi berangkat supaya bisa bertemu pada satu titik di orbitnya.
2. Hitung lamanya perjalanan dari Bumi ke Jupiter dengan menggunakan orbit Hohmann

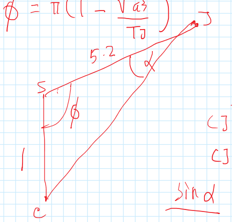


$\phi = \pi - \theta_J$
$= \pi - \omega_J \cdot t_J$

$t_J = \tau = \frac{1}{2} P$
$= \frac{1}{2} \left[\sqrt{\frac{4\pi^2 a^3}{G \cdot M_\odot}} \right]$
$= \frac{1}{2} \left[2\pi \sqrt{\frac{a^3}{G\, M_\odot}} \right]$ ✓

$\phi = \pi - \omega_J \cdot \frac{1}{2} \left[2\pi \sqrt{\frac{a^3}{G\, M_\odot}} \right]$
$= \pi - \frac{2\pi}{T_J} \left[\pi \sqrt{\frac{a^3}{G\, M_\odot}} \right]$
$\phi = \pi - \frac{2\pi}{T_J} \left[\pi \sqrt{\frac{a^3}{4\pi^2}} \right]$
$= \pi - \frac{2\pi}{T_J} \left[\frac{\pi}{2\pi} \sqrt{a^3} \right]$
$\phi = \pi - \frac{2\pi}{T_J} \cdot \frac{1}{2} \sqrt{a^3}$

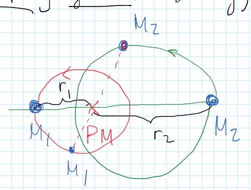
$\phi = \pi - \frac{\pi}{T_J} \sqrt{a^3}$
$\phi = \pi \left(1 - \frac{\sqrt{a^3}}{T_J} \right)$



$CJ^2 = 1^2 + 5.2^2 - 2 \cdot 5.2 \cos \phi$
$CJ = \dots$
$\frac{\sin \alpha}{1} = \frac{\sin \phi}{CJ} \Rightarrow \alpha = \dots$

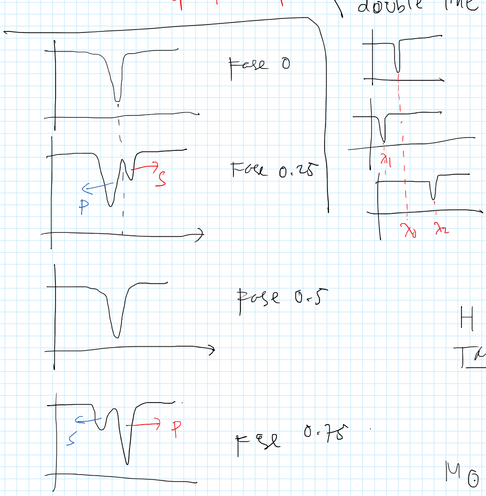
2) $\tau = \frac{1}{2} P$

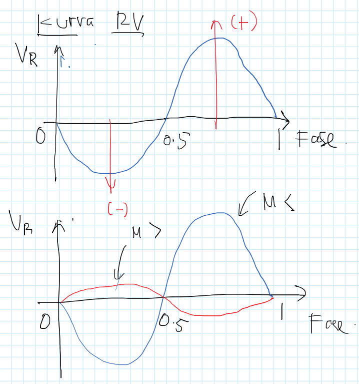
- Bintang ganda (Binary)



$\boxed{M_1 r_1 = M_2 r_2}$

BS — Visual / spectroscopic — single line / double line





H. III Kepler
Tatasurya Matahari - J

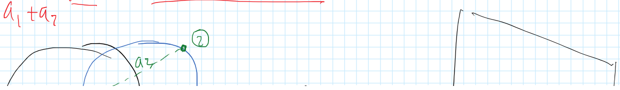
$\frac{a^3}{P^2} = \frac{G}{4\pi^2} (M_\odot + M_J)$
$M_\odot >> M_J \longrightarrow M_\odot + M_J \approx M_\odot$

$\frac{a^3}{P^2} = \frac{G}{4\pi^2} (M_\odot)$

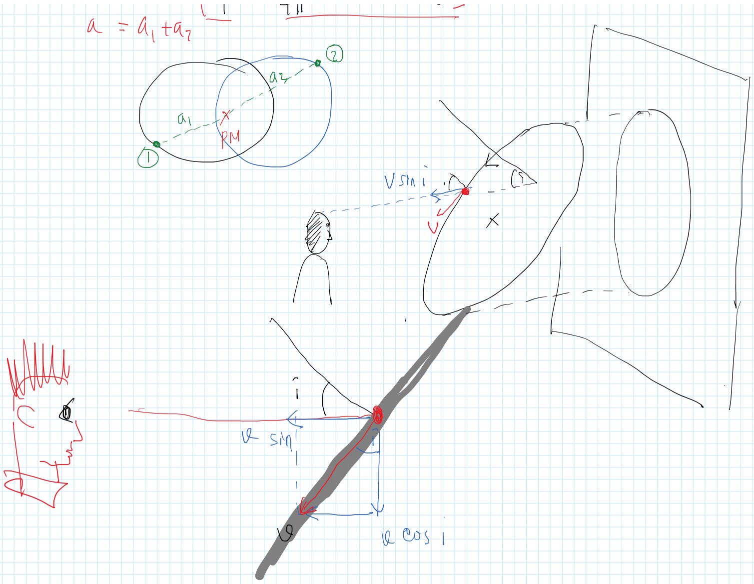
u/ Bintang Ganda

$\boxed{\frac{a^3}{P^2} = \frac{G}{4\pi^2} (M_1 + M_2)}$

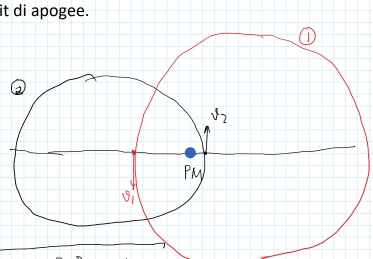
$a = a_1 + a_2$



$a = a_1 + a_2$



Dua buah satelit memiliki orbit elips di sekitar Bumi dan keduanya memiliki setengah sumbu mayor yang besarnya sama. Perbandingan kecepatan kedua satelit saat di perigee adalah 3/2 dan eksentrisitas orbit satelit yang memiliki perigee lebih besar diketahui sebesar 0.5. Tentukan eksentrisitas orbit satelit yang lain dan tentukan perbandingan kecepatan kedua satelit di apogee.



$$\left.\frac{V_2}{V_1}\right|_{pe} = \frac{3}{2}$$

$$e_1 = 0.5$$

a) e_2

b) $\left.\frac{V_2}{V_1}\right|_{ap} = ?$

$$V = \sqrt{GM_\oplus\left[\frac{2}{r} - \frac{1}{a}\right]}$$

w/ perigee $\rightarrow r = r_p$ sehingga $V = \sqrt{GM_\oplus\left[\frac{2}{r_p} - \frac{1}{a}\right]}$

$$\left.\frac{V_2}{V_1}\right|_{pe} = \sqrt{\frac{\cancel{GM_\oplus}\left[\frac{2}{r_{p_2}} - \frac{1}{a}\right]}{\cancel{GM_\oplus}\left[\frac{2}{r_{p_1}} - \frac{1}{a}\right]}} = \frac{3}{2}$$

$$\frac{\frac{2}{r_{p_2}} - \frac{1}{a}}{\frac{2}{r_{p_1}} - \frac{1}{a}} = \left(\frac{3}{2}\right)^2$$

$$\frac{2}{r_{p_2}} - \frac{1}{a} = \frac{9}{4}\left[\frac{2}{r_{p_1}} - \frac{1}{a}\right]$$

$$\frac{2}{r_{p_2}} - \frac{1}{a} = \frac{9}{2}\frac{1}{r_{p_1}} - \frac{9}{4a}$$

$$\frac{2}{r_{p_2}} - \frac{9}{2r_{p_1}} = \frac{1}{a} - \frac{9}{4a}$$

$$= \frac{1}{a}\left[1 - \frac{9}{4}\right]$$

$$r_{p_1} = a(1-e_1)$$
$$r_{p_2} = a(1-e_2)$$

$$\frac{2}{\cancel{a}(1-e_2)} - \frac{9}{2\cancel{a}(1-e_1)} = \frac{1}{\cancel{a}}\left(-\frac{5}{4}\right)$$

$$\frac{2}{1-e_2} - \frac{4.5}{1-e_1} = -\frac{5}{4}$$

$$e_1 = 0.5 \rightarrow \frac{2}{1-e_2} - \frac{4.5}{1-0.5} = -\frac{5}{4}$$

$$\frac{2}{1-e_2} = 9 - \frac{5}{4}$$

$$\frac{2}{1-e_2} = \frac{31}{4}$$

$$8 = 31 - 31e_2$$

$$e_2 = \frac{31-8}{31}$$

$$e_2 = 0.74$$

a)

b) $$\left.\frac{V_2}{V_1}\right|_{ap} = \sqrt{\frac{\cancel{GM_\oplus}\left[\frac{2}{r_{A_2}} - \frac{1}{a}\right]}{\cancel{GM_\oplus}\left[\frac{2}{r_{A_1}} - \frac{1}{a}\right]}}$$

$$r_{A_1} = a(1+e_1) = a(1+0.5) = 1.5a$$
$$r_{A_2} = a(1+e_2) = a(1+0.74) = 1.74a$$

$$\left.\frac{V_2}{V_1}\right|_{Ap} = \sqrt{\frac{\frac{2}{1.74a} - \frac{1}{a}}{\frac{2}{1.5a} - \frac{1}{a}}} = \sqrt{\frac{\frac{2-1.74}{1.74\cancel{a}}}{\frac{2-1.5}{1.5\cancel{a}}}}$$

$$\left.\frac{V_2}{V_1}\right|_{Ap} = \sqrt{\frac{\frac{2}{1.74a} - \frac{1}{a}}{\frac{2}{1.5a} - \frac{1}{a}}} = \sqrt{\frac{\frac{\;}{1.74\cancel{a}}}{\frac{2-1.5}{1.5\cancel{a}}}}$$

$$= \sqrt{\frac{0.15}{0.3}}$$

$$\boxed{\left.\frac{V_2}{V_1}\right|_{Ap} = 0.71}$$