Heaven’s Light Is Our Guide

Rajshahi University of Engineering & Technology

Department of Computer Science & Engineering

Course Code: CSE 3209

Course Title: Digital Signal Processing

Assignment

Submitted By

Name : Md Habibur Rahman

Roll: **2003076**

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Submitted To

Md. Mazharul Islam

Lecturer

Department of CSE, RUET

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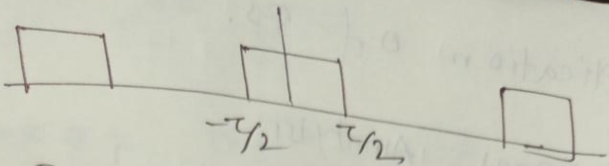
Problem



Determine F.S and power density spectrum

$$C_K = \frac{1}{T_P}\int_{-\tau/2}^{\tau/2} x(t)\, e^{-J2\pi K f_0 t}\, dt$$

$$= \frac{A}{T_P}\,\frac{1}{-J2\pi K f_0}\left[e^{-J\pi 2K f_0 t}\right]_{-\tau/2}^{\tau/2}$$

$$= \frac{A}{T_P} \times \frac{1}{-J2\pi K f_0} \times \left[e^{-J\pi 2K f_0 \cdot \tau/2} - e^{J\pi 2K f_0 \tau/2}\right]$$

$$= \frac{A}{T_P \times J2\pi K f_0} \times \left[e^{J\pi K f_0 \tau} - e^{-J\pi K f_0 \tau}\right]$$

$$= \frac{A}{T_P} \times \left(\frac{e^{J\pi K f_0 \tau} - e^{-J\pi K f_0 \tau}}{2J2\pi K f_0 \tau}\right)$$

$$= \frac{A\tau}{T_P} \times \sin(K\pi f_0 \tau)$$

$$C_K = \frac{A\tau}{T_P} \times \operatorname{sinc}(\pi K F_0 \tau)$$

power density spectrum $= P_K = |C_K|^2 = \frac{A\tau^2}{T_P^2} \sin$

(4) Convolution:-

$x_1(n) \xrightarrow{FT} x_1(\omega)$
$x_2(n) \xrightarrow{FT} x_2(\omega)$
$x_1(n)\, x_2(n) \xrightarrow{FT} X_1(\omega)\, X_2(\omega)$

(5) Wiener Khintchine

$r_{x_1 x_2} \xrightarrow{FT} S_{x_1 x_2}(\omega)$

Problem

$x(n) = a^{|n|} \quad -1 < a < 1$

$x(n) = x_1(n) + x_2(n)$ ——— (1)

$x_1(n) = \begin{cases} a^n & n \geq 0 \\ 0 & \text{Otherwise} \end{cases}$

$x_2(n) = \begin{cases} a^{-n} & n < 0 \\ 0 & \text{else} \end{cases}$

$X_1(\omega) = \sum_{0}^{\infty} x_1(n)\, e^{-J\omega n}$

$= \sum_{0}^{\infty} (a e^{-J\omega})^n$

$= \frac{1}{1 - a e^{-J\omega}}$

$X_2(\omega) = \sum_{-\infty}^{-1} a^{-n}\, e^{-J\omega n}$

$= \sum_{-\infty}^{-1} (a e^{J\omega})^{-n}$

$= \sum_{1}^{\infty} (a e^{J\omega})^k$

$= \frac{a e^{J\omega}}{1 - a e^{J\omega}}$

$X(\omega) = X_1(\omega) + X_2(\omega)$

$= \frac{1}{1 - a e^{-J\omega}} + \frac{a e^{J\omega}}{1 - a e^{J\omega}}$

$= \frac{1 - \cancel{a e^{J\omega}} + \cancel{a e^{J\omega}} - a^2}{(1 - a e^{-J\omega})(1 - a e^{J\omega})} = \frac{1 - a^2}{1 - 2a\cup + \cup^2}$

$=$

C	Magnitude	Phase
C_0	$1/2$	0
C_1	$\sqrt{(1/4)^2+(1/4)^2}$	$\pi/4$
C_2	0	undef.
C_3	$\sqrt{2}/4$	$-\pi/4$

FTDT aperiodic

$$\xrightarrow{\quad x \quad}$$

analysis

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

synthesis of DTFT:

$$x(n) = \frac{1}{2\pi} \int_{2\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

Problem $x(n) = a^n u(n)$ | $-1 < a < 1$

Since $|a| < 1$, $x(n)$ is absolutely summable we can apply FTDT

$$X(\omega) = \sum_{-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$= \sum_{-\infty}^{\infty} a^n\, e^{-j\omega n}$$

$$= \sum_{-\infty}^{\infty} \left(a e^{-j\omega}\right)^n$$

$$= \frac{1}{1 - a e^{-j\omega}}$$

Problem:- Given that

$x_1(n) = x_2(n) = \{1, 1, 1\}$

clearly $x_1(n) = x_2(n)$ real + even.

$$X_1(w) = X_2(w) = X(0) + 2\sum_{n=1}^{\infty} x(n)\cos(wn)$$

$$= 1 + 2\cos w$$

$$X(w) = X_1(w)\ X_2(w)$$

$$= (1+2\cos wn)(1+2\cos wn)$$

$$= (1+2\cos wn)^2$$

$$= 3 + 4\cos w + 2\cos 2w$$

$$= 3 + 4\cdot\tfrac{1}{2}(e^{jw} + e^{-jw}) + 2\cdot\tfrac{1}{2}(e^{2jw} + e^{-2jw})$$

$$= e^{2jw} + 2e^{jw} + 3 + 2e^{-jw} + e^{-2jw}$$

$x(n) = \{1, 2, 3, 2, 1\}$

every discrete signal

$x(n) = x_o(n) + x_e(n)$

Proof:- $x(n) = x_e(n) + x_o(n)$ — (i)

$x(-n) = x_e(-n) + x_o(-n) = x_e(n) - x_o(n)$ — (ii)

(i) + (ii) $x_e(n) = \dfrac{x(n) + x(-n)}{2}$

$x_e(-n) = \dfrac{x(-n) + x(n)}{2} = x_e(n)$

(i) − (ii) $x_o(n) = \dfrac{x(n) - x(-n)}{2}$

$\boxed{x_o(n) = -x_o(n)}$

DFT (শুধু মাত্র Periodic)

Synthesis:-

$$x(n) = \sum_{k=0}^{N-1} C_k \, e^{j2\pi k \frac{n}{N}}$$

analysis :-

$$C_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) \, e^{-j2\pi k \frac{n}{N}}$$

(a) **Problem:-**

$$x(n) = \cos(\sqrt{2}\pi n)$$

$$2\cancel{\pi} f = \sqrt{2}\cancel{\pi}$$

$$f = \frac{1}{\sqrt{2}}$$

$N = \sqrt{2}$ which is irrational. since N is not rational so this signal is aperiodic, that is why we cant determin DFT.

(b) **Problem:-** $x(n) = \cos\left(\frac{\pi n}{3}\right)$

$$2\cancel{\pi} f = \frac{\cancel{\pi}}{3}$$

$$\Rightarrow f = 1/6 \qquad \boxed{N = 6}$$