



中国科学技术大学 计算机科学与技术系  
University of Science and Technology of China  
DEPARTMENT OF COMPUTER SCIENCE AND TECHNOLOGY

# 算法设计与分析

## Design and Analysis of Algorithms

主讲人 徐云

Fall 2018, USTC



## Part 1 Foundation

### 第1章 算法的重要性

#### 1.1 课程信息

#### 1.2 算法的学习意义、概念和重要性

### 第2章 算法初步

### 第3章 函数增长率

### 第3章 递归

### 第4章 概率分析和随机算法

## Part 2 Sorting and Order Statistics

## Part 3 Data Structure

## Part 4 Advanced Design and Analysis Techniques

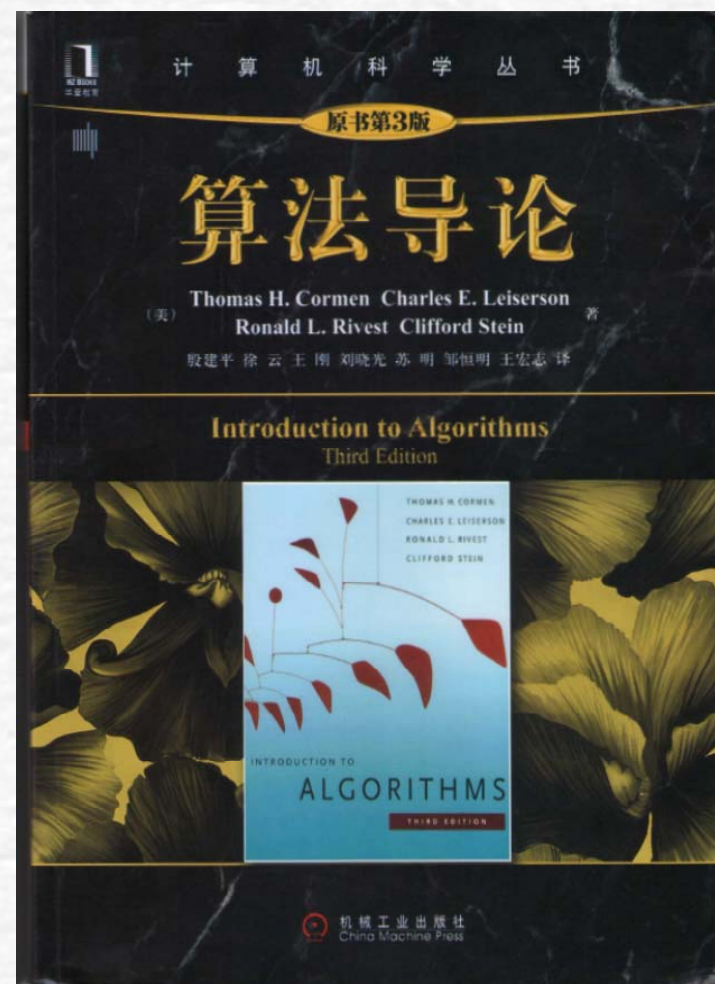
## Part 5 Advanced Data Structures

## Part 6 Graph Algorithms

## Part 7 Selected Topics

# 课程信息: Textbook

- 书名: 算法导论 (第三版)
- 著者: Thomas H. Cormen, Charles E. Leiserson, and Ronald L. Rivest
- ISBN: 978-7-111-40701-0
- 出版社: 机械工业出版社
- 出版日期: 2013-01



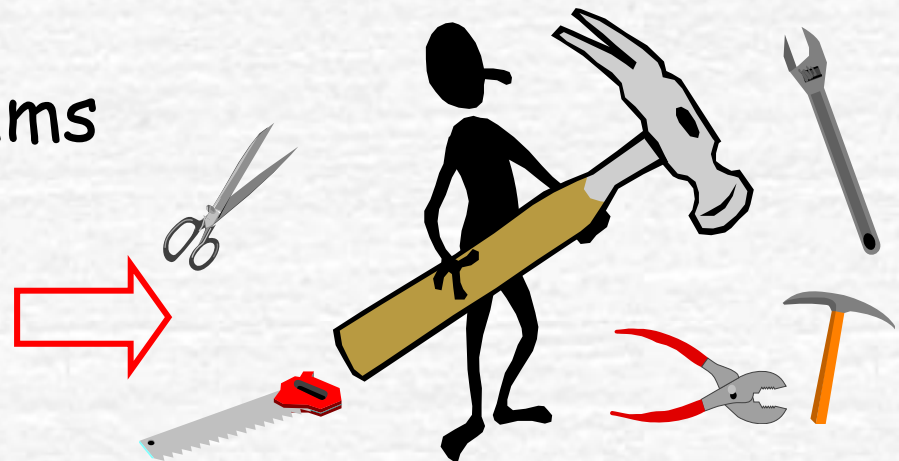
# 课程信息: Instructor and etc.

- Instructor: 徐云 教授
- Email: [xuyun@ustc.edu.cn](mailto:xuyun@ustc.edu.cn)
- Website: <http://staff.ustc.edu.cn/~xuyun/algssse.htm>
- Teaching Assistant: 程昊宇, 陈伟
- Grading Policy:
  - Homework & Projects : 30%
  - Tests in Class : 30%
  - Final Exam and Tests: 40%
  - Score: 5 excellent; 4 satisfactory; 3 some flaws;  
2 poor; 1 something turned in



# 课程信息: Goal

- A survey of algorithmic design techniques.
  - Abstract thinking.
  - How to develop new algorithms for any problem that may arise.
  - Be a great thinker and designer.
- 
- ~~Not: A list of algorithms~~
    - ~~Learn their code~~
    - ~~Trace them until work~~
    - ~~Implement them~~
    - ~~be a mundane programmer~~





## Part 1 Foundation

### 第1章 算法的重要性

#### 1.1 课程信息

#### 1.2 算法的学习意义、概念和重要性

### 第2章 算法初步

### 第3章 函数增长率

### 第3章 递归

### 第4章 概率分析和随机算法

## Part 2 Sorting and Order Statistics

## Part 3 Data Structure

## Part 4 Advanced Design and Analysis Techniques

## Part 5 Advanced Data Structures

## Part 6 Graph Algorithms

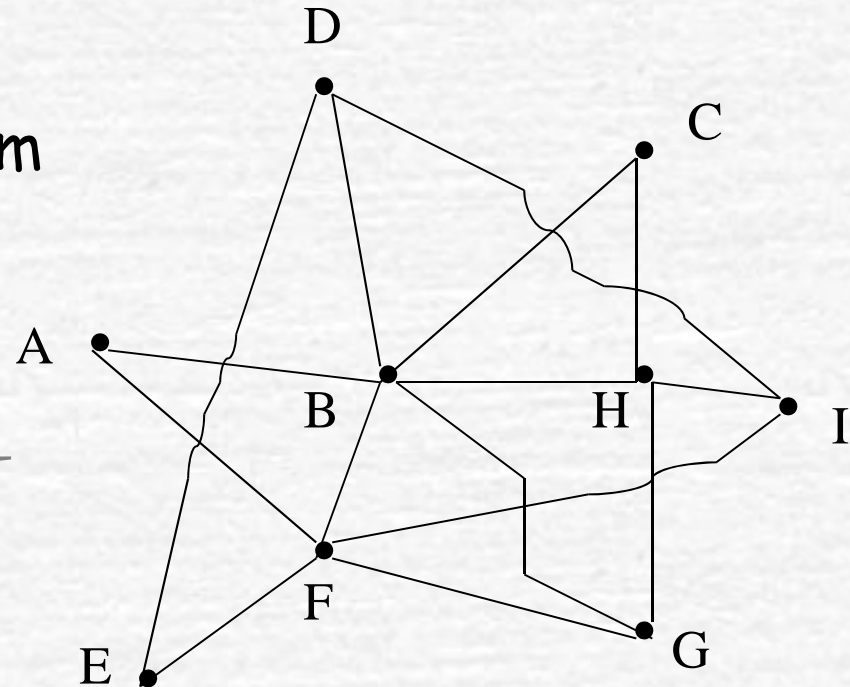
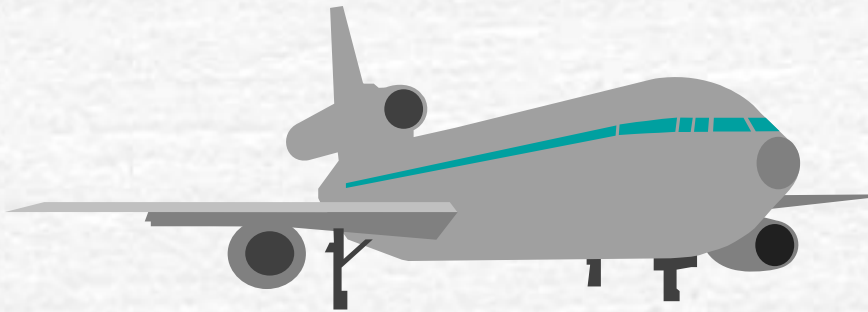
## Part 7 Selected Topics

# 算法学习意义

- Story 1 and Story 2
- ACM's Turing Awards
- CS2013(Computer Science Curricula 2013)

# 算法学习意义: Story -1

## Traveling Salesman Problem



TSP :  $(n-1)!$  TOURS

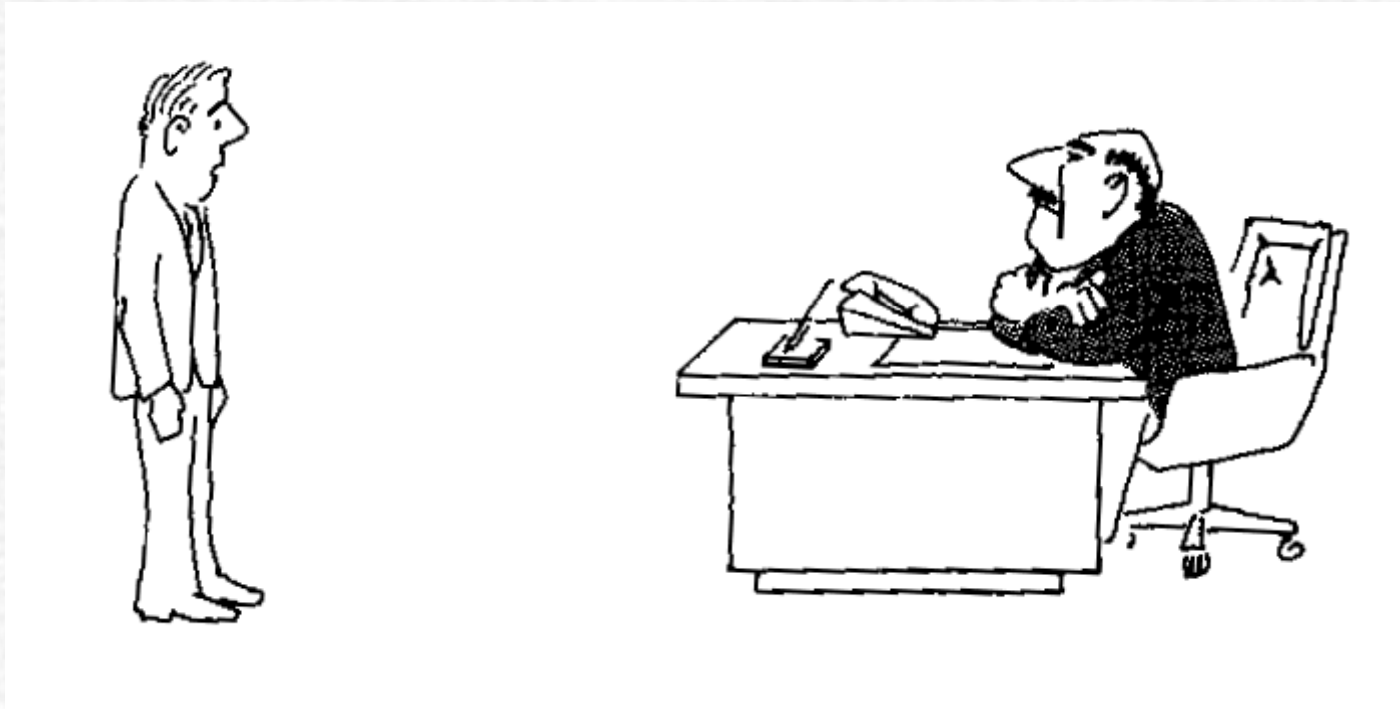
Computer speed:  $10^{10}$  enumerations per second

$30! / 10^{10} > 2.65 \times 10^{21}$  (s)

i.e.  $2.65 \times 10^{21} / (365 \times 24 \times 60 \times 60) > 8.4 \times 10^{13}$  (years)



# 算法学习意义: Story-1



“I can't find an efficient algorithm, I guess I'm just too dumb.”

Serious damage to your  
position within the company !!!

# 算法学习意义: Story-1



“I can't find an efficient algorithm, because no such algorithm is possible!”

Unfortunately, proving intractability can be just as hard as finding efficient algorithms !!!

∴ No hope !!!

**$P \neq NP$**

# 算法学习意义: Story-1



“I can't find an efficient algorithm, but neither can all these famous people”

# 算法学习意义: Story-1



“Anyway, we have to solve this problem.  
Can we satisfy with a good solution ? ”

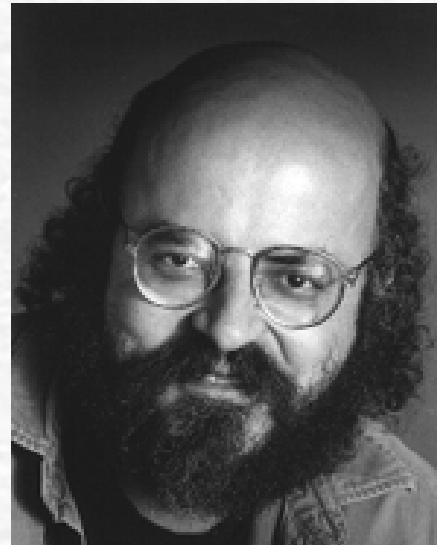


# 算法学习意义: Story-2

- Gates, William H. and Christos H. Papadimitriou.  
Bounds for sorting by prefix reversal.  
*Discrete Mathematics* 27 (1979), 47--57.



Harvard University(1973)  
Microsoft (1975)



Princeton University  
(MS 1974 and PhD 1976)

# 算法学习意义: ACM's Turing Awards

- 1966年至2017年67位图灵奖获得者中有大约20人直接或间接地与算法相关:  
D.E. Knuth, M.O. Rabin and D.S. Scott,  
R.W. Floyd, C.A.R. Hoare, S.A. Cook,  
N. Wirth, R.M. Karp,  
J.E. Hopcroft and R.E. Steams, M. Blum,  
Andrew Chi-Chih Yao, L.M. Adleman, *et al.*
- 上个世纪算法界: 三大成就
  - 快速傅里叶变换
  - 两个大整数乘法
  - 矩阵乘法

# 算法学习意义:中国计算机科学与技术学科

- 算法在CC2013(Computer Science Curricula 2013)中的地位:
  - 7种发展年代: 算法年代(第3代)
  - 知识体系的演变: 强调算法
  - 学科的根本问题: 能被有效地自动化
  - 学科3形态(理论、抽象、设计)的基础
  - 学科方法(数学和系统科学)的基础
  - 18大知识体系的核心
  - 重要的核心课程



# 算法的概念

- 什么是算法？
- 算法的特征
- 问题及问题实例
- 算法求解的问题示例
- 输入实例和问题规模
- 算法与程序的区别
- The poetry of computation



# 算法的概念：什么是算法？

- 算法 (Algorithm) 是通过一个有限的指令序列集合对特定问题进行求解的一种计算执行描述。
- An algorithm is a finite set of precise instructions for performing a computation or for solving a problem, NOT necessarily tied to computers.
- Historical note: The word *algorithm* comes from the name of a Persian mathematician and author (al-Khuwarizmi) who lived around 825 a.d.



# 算法的概念：什么是算法？

- Algorithm 一本书的算法概念

Any well-defined computational procedure that takes some value as *input* and produces some value as *output*. That means **an algorithm is a sequence of computational steps that transforms the input into the output**, achieving the desired input-output relationship.

# 算法的概念：算法的特征

- 一个算法通常应具有下列特征：
  - (1) 输入：一个算法具有零个或多个取自指定集合的输入值；
  - (2) 输出：对算法的每一次输入，算法具有一个或多个与输入值相联系的输出值；
  - (3) 确定性：算法的每一个指令步骤都是明确的；
  - (4) 有限性：对算法的每一次输入，算法都必须在有限步骤（即有限时间）内结束；
  - (5) 正确性：对每一次输入，算法应产生出正确的输出值；
  - (6) 通用性：算法的执行过程可应用于所有同类求解问题，而不是仅适用于特殊的输入。



# 算法的概念：问题及问题实例

- Problem — 问题

The general term the desired input/output relationship. A computational Problem is a specification of the desired input-output relationship.

- Instance of a problem — 问题实例

An Instance of a problem is all the inputs needed to compute a solution to the problem.

- Example 1.1 Sorting Problem and Instance

Input — A sequence of  $n$  numbers  $(a_1, a_2, \dots, a_n)$ ;

Output — A permutation  $(a'_1, a'_2, \dots, a'_n)$  of the input sequence such that  $a'_1 \leq a'_2 \leq \dots \leq a'_n$ .

i.e.,  $(31, 41, 59, 26, 41, 58) \rightarrow (26, 31, 41, 41, 58, 59)$

input

output



# 算法的概念：算法求解的问题示例

- What kinds of problems are solved by Algorithms?
  - (1)Sorting
  - (2)Biological problems, such as gene identification
  - (3)Information retrieval
  - (4)Privacy and security
  - (5)Allocation of scarce resources
  - (6)Shortest route from one point to one other point/all other points
  - (7)Optimal order of matrix multiplication
  - (8)Factoring large numbers, especially when looking for prime numbers
- Remark: Type of Problems: Decision Problem, Optimization Problem, etc,

# 算法的概念：输入实例和问题规模

- 输入实例

问题的具体计算例子。

如，排序问题的3个输入实例：

① 3, 4, 2, 6, 5;

② 5, 4, 1, 6, 3;

③ 7, 3, 1, 6, 4, 8, 5, 9;

- 问题规模

算法的输入实例大小。

如上面排序问题的3个输入实例的规模大小分别为5, 5, 8

# 算法的概念：算法与程序的区别

- 算法的概念与程序十分相似，但实际上有很大不同。程序并不都满足算法所要求的上述特征，例如有限性特征。算法代表了对特定问题的求解，而程序则是算法在计算机上的实现。因此算法也常常称为是一个能行过程。一个函数如果可以用一个算法来计算，那么我们称该函数是能行可计算的。

如，操作系统是一种程序而不是算法



# Great algorithms are the poetry of computation

- Fast arithmetic: Cryptography.
- QuickSort: Databases.
- FFT: Signal processing.
- Huffman codes: Data compression.
- Network flow: Routing Internet packets.
- Linear programming: Planning, decision-making.





# End of Chap1

