1. $G = \forall x(\neg(\exists y\, P(x,y)) \vee Q(x))$ (Push negation in)
$= \forall x(\neg \exists y\, P(x,y) \vee Q(x))$ (Skolemize)
$= \forall x(\neg P(x, f(x)) \vee Q(x))$ (Drop Universal Quantifier)
$= \neg P(x, f(x)) \vee Q(x)$

$F = \exists x \exists y (P(x,y) \Rightarrow Q(x))$ (Replace $\Rightarrow$)
$= \exists x \exists y (\neg P(x,y) \vee Q(x))$ (Skolemize $\exists x$)
$= \exists y (\neg P(a,y) \vee Q(a))$ (Skolemize $\exists y$)
$= \neg P(a,b) \vee Q(a)$

Since G & F are in the same form $(a = x, f(x) = b)$
$G \Rightarrow F$ is valid.

2. $H = \forall x \forall y (P(x,y) \Rightarrow \neg Q(x))$ (Replace $\Rightarrow$)
$= \forall x \forall y (\neg P(x,y) \vee \neg Q(x))$ (Drop Universal Quantifier $\forall x$)
$= \forall y (\neg P(x,y) \vee \neg Q(x))$ (Drop Universal Quantifier $\forall y$)
$= \neg P(x,y) \vee \neg Q(x)$

$F \wedge H = (\neg P(x,y) \vee Q(x)) \wedge (\neg P(x,y) \vee \neg Q(x))$

To prove this is satisfiable, we just need to find an intepretation to make this true.

Let P be true if $x > y$ and it takes integers.
Let Q be true if x is positive and it takes Integers.

if $x = 1$, $y = 2$

$F \wedge H = (\neg P(1,2) \vee Q(1)) \wedge (\neg P(1,2) \vee \neg Q(1))$
$= (t \vee t) \wedge (t \vee f)$
$= t \wedge t$
$= t$

Because there is an intepretation that makes $F \wedge H$ true, it is satisfiable.

3. To show $F \wedge H \Rightarrow \neg J$ is valid, we are going to make a resolution-based inference with $F \wedge H$.

$$
\begin{aligned}
J &= \forall x \forall y P(x,y) && \text{(Drop Universal Quantifer } \forall x\text{)} \\
&= \forall y P(x,y) && \text{(Drop Universal Quantifier } \forall y\text{)} \\
&= P(x,y)
\end{aligned}
$$

$\neg J = \neg P(x,y)$

Resolution-based inference

F

$\neg P(x,y), Q(x)$

H

$\neg P(x,y), \neg Q(x)$

$\neg P(x,y) \wedge \neg P(x,y)$ (Simplify duplicate clauses)

$= \neg P(x,y)$

$= \neg J$

$\therefore (F \wedge H) \Rightarrow \neg J$ is valid.