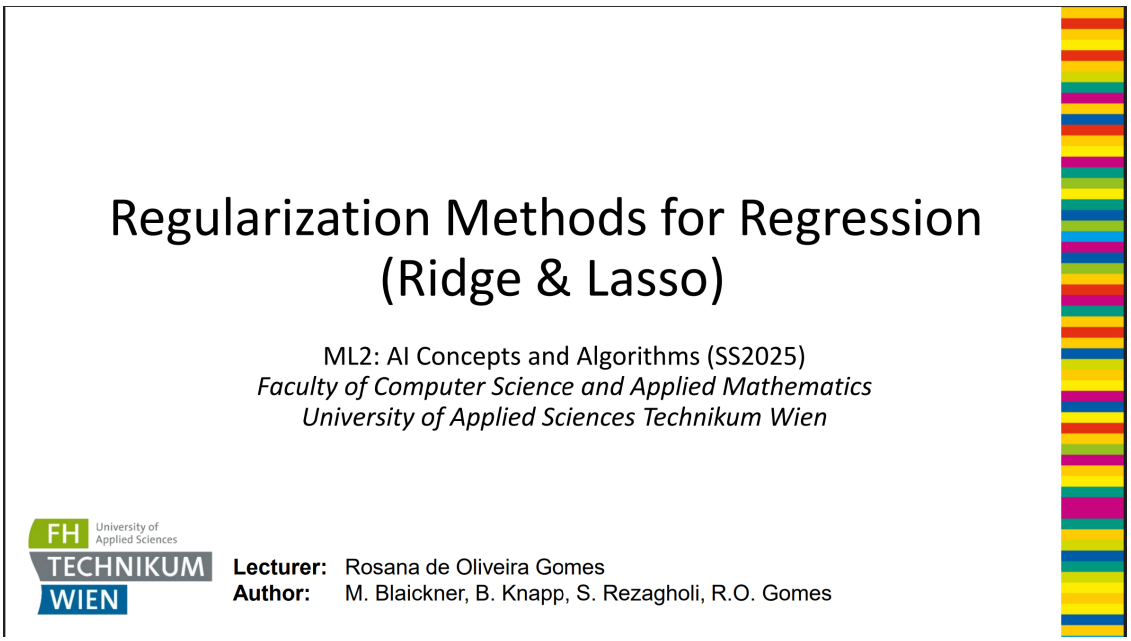
Regularization Methods for Regression

June 20, 2025



Welcome everyone.

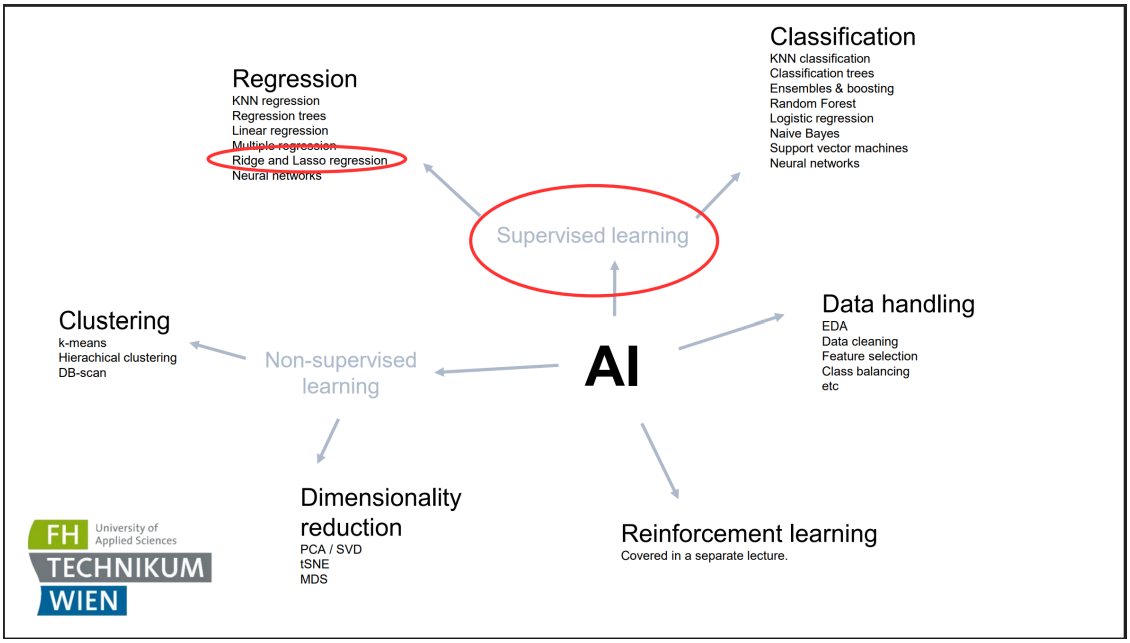
This presentation will cover **Regularization Methods for Regression**, focusing on two key techniques: **Ridge Regression** and **Lasso Regression**.

These methods are crucial when dealing with **overfitting** in regression models — especially in situations with **many predictors**, **collinear features**, or when we want to **select relevant variables** automatically.

Throughout this session, we'll:

- Understand the motivation behind regularization,
- See how Ridge and Lasso modify the loss function,
- Explore their mathematical formulations and use cases,
- And finally, compare them side-by-side and see how to tune them.

Let's begin with the first content slide. Please proceed.



This slide presents the **big picture of AI subfields**, with our current focus circled: **Ridge and Lasso Regression** under the **Regression** branch of **Supervised Learning**.

Let's break this down:

0.0.1 At the center: AI

Artificial Intelligence encompasses multiple learning paradigms, of which:

0.0.2 Supervised Learning

> Data includes input-output pairs (labels). The goal is to learn a mapping from inputs to outputs.

Two main families:

- **Classification** (e.g. logistic regression, SVMs)
- **Regression** (e.g. linear regression, Ridge & Lasso)

Our current focus — **Ridge and Lasso regression** — is **supervised** and **regression-based**, aiming to **predict continuous outcomes** while managing **model complexity**.

0.0.3 Non-supervised Learning

- **Clustering** (e.g. k-means)
- **Dimensionality reduction** (e.g. PCA, t-SNE)

These methods don't use output labels.

0.0.4 Data Handling

These are preprocessing techniques to clean, prepare, and balance datasets before feeding them into models.

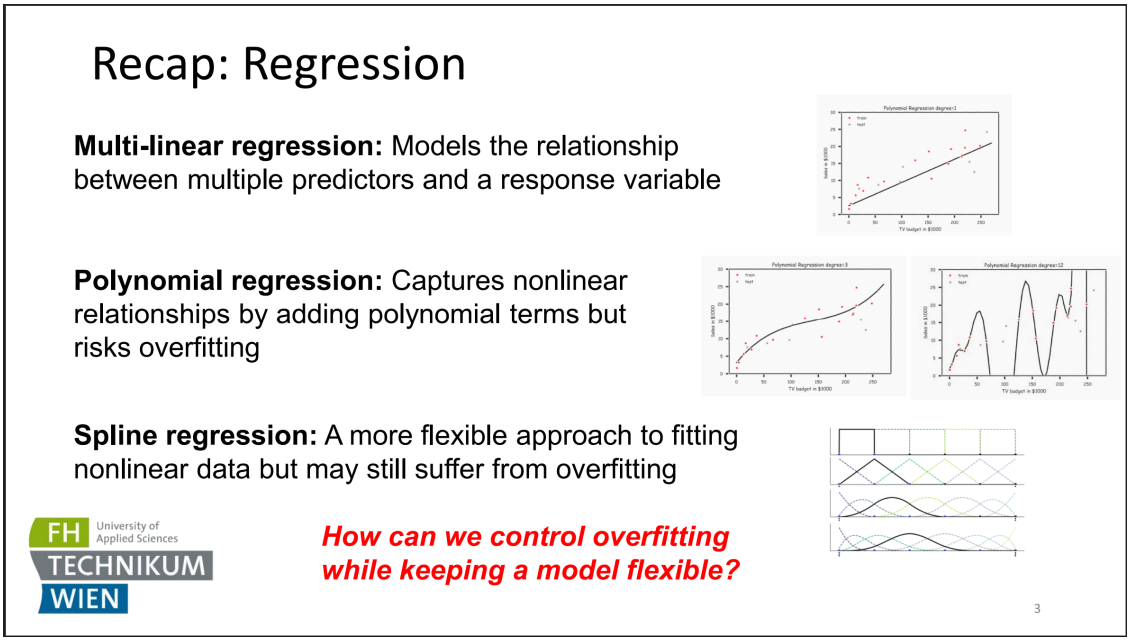
0.0.5 Reinforcement Learning

Mentioned but **covered elsewhere** — it's about learning policies through reward signals.

0.0.6 Key Takeaway:

We are zooming into **Regression within Supervised Learning**, tackling the core challenge of **overfitting** using **regularization** techniques.

Ready for the next slide.



This slide is a **recap of regression methods** — reminding us why regularization is needed.

0.0.7 Multi-linear Regression

Models the linear relationship between **multiple independent variables** (predictors) and a single **dependent variable** (response). It's the baseline model — **simple**, but only captures **linear trends**.

0.0.8 Polynomial Regression

Captures **nonlinear** patterns by adding higher-degree terms (e.g. $x^2, x^3, ...$). It offers more **flexibility** but comes with a **high risk of overfitting** — as clearly shown in the right-hand plots:

- Degree 1 → underfit
- Degree 3 → decent fit
- Degree 12 → overfit chaos (model memorizes the noise)

0.0.9 Spline Regression

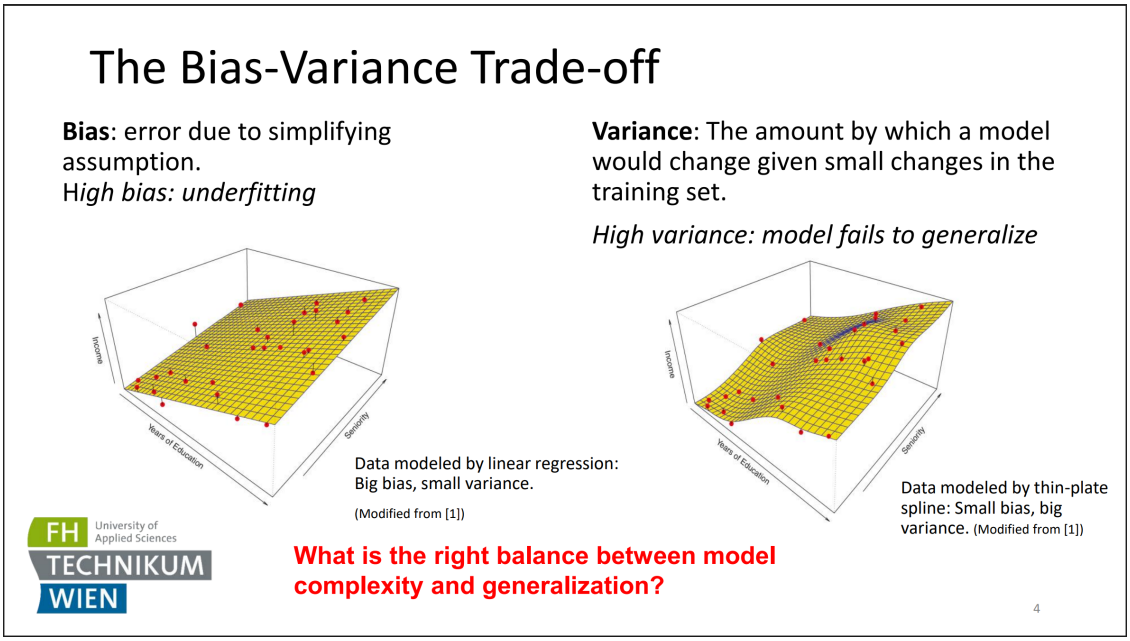
A compromise between flexibility and simplicity. Instead of one global polynomial, it stitches together **piecewise polynomials**, which can model local trends better. Still — without proper control — it **can also overfit**.

0.0.10 Key Problem Posed:

How can we control overfitting while keeping a model flexible?

This is where **regularization** techniques like Ridge and Lasso step in — providing a way to constrain models to avoid complexity without fully giving up flexibility.

Ready when you are for the next.



This slide introduces a foundational concept in statistical modeling and machine learning:

0.1 The Bias–Variance Trade-off

A critical challenge in supervised learning is **balancing two competing sources of error**:

0.1.1 Bias (left side)

- **Definition**: Error from overly simplistic assumptions in the model.
- **Effect**: The model **underfits** — it's too rigid to capture the underlying patterns.
- **Example**: A flat plane (linear regression) trying to model complex relationships.
- **Characteristics**:
  - Low model complexity
  - Small variance (doesn't change much across datasets)
  - High error due to systematic misrepresentation

0.1.2 Variance (right side)

- **Definition**: Error due to the model's sensitivity to small fluctuations in the training data.
- **Effect**: The model **overfits** — it captures noise and loses generalization.
- **Example**: A wavy spline bending to every data point.
- **Characteristics**:
  - High model complexity
  - Captures training data very well
  - But performance collapses on new data

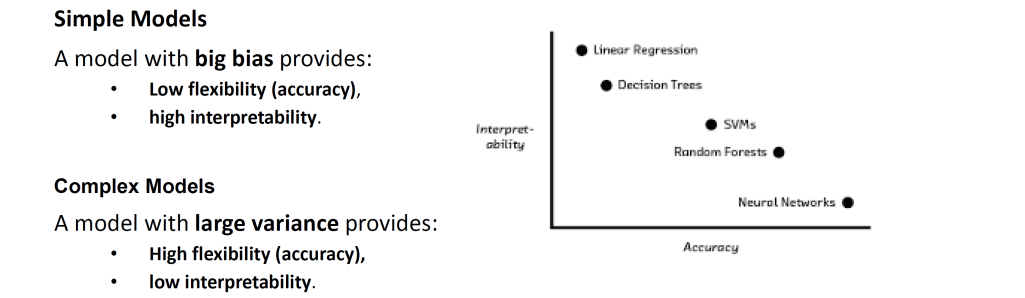
0.1.3 The Core Dilemma:

"What is the right balance between model complexity and generalization?"

This is exactly what **regularization** techniques like **Ridge** and **Lasso** aim to solve:

- By **penalizing model complexity**, they help reduce variance
- While accepting a **small increase in bias** for better overall performance

Next slide when you're ready.



This slide builds on the **bias-variance trade-off** by connecting it to **model interpretability and flexibility**.

0.2 Flexibility vs. Interpretability

There's a tension between having a model that is:

- **Simple and easy to understand**
- vs.
- **Flexible and able to capture complex patterns**

0.2.1 Simple Models (High Bias)

- **Examples**: Linear regression, shallow decision trees
- **Pros**:
  - Easy to **explain** and **interpret**
  - Low risk of overfitting
- **Cons**:
  - Miss complex patterns → low **accuracy** (underfitting)

0.2.2 Complex Models (High Variance)

- **Examples**: Neural networks, ensemble methods

- **Pros**:
  - High **flexibility** → can fit intricate data
- **Cons**:
  - Often **black boxes**
  - Risk of overfitting if not controlled

0.2.3 Diagram (top right)

Illustrates the trade-off:

- **Top-left**: Models like Linear Regression — very interpretable but not always accurate
- **Bottom-right**: Neural Networks — accurate but hard to interpret

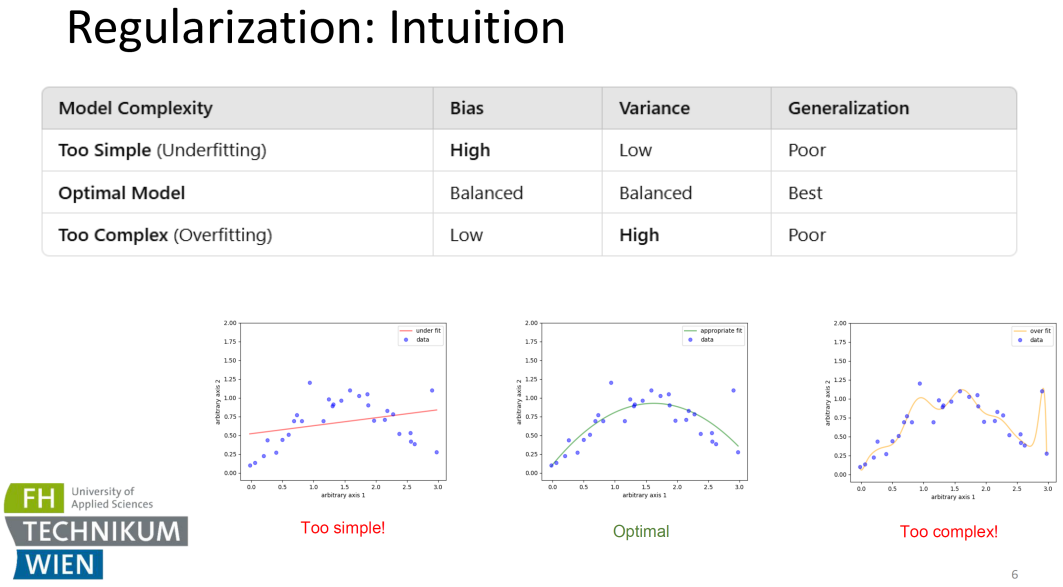
---

0.2.4 The Role of Regularization

> **Regularization** pushes complex models to be **simpler**, increasing **interpretability** without sacrificing too much predictive power.

This is the motivation behind Ridge and Lasso: add a **penalty** that keeps models in a sweet spot between **simplicity** and **performance**.

Ready to continue.



This slide is a visual and intuitive summary of what **regularization** is meant to control.

0.3 Regularization: Intuition

It ties back directly to the **bias-variance trade-off** and provides **three concrete scenarios**:

0.3.1 Too Simple → Underfitting

- **Bias**: High (model misses patterns)
- **Variance**: Low (very stable)
- **Generalization**: Poor
- **Example**: Linear fit to curved data — it ignores structure

---

0.3.2 Optimal Model

- **Bias & Variance**: Balanced
- **Generalization**: Best
- **Example**: Just the right amount of curvature — fits the trend but not the noise

---

0.3.3 Too Complex → Overfitting

- **Bias**: Low
- **Variance**: High
- **Generalization**: Poor
- **Example**: A wiggly function that fits every point — including noise

---

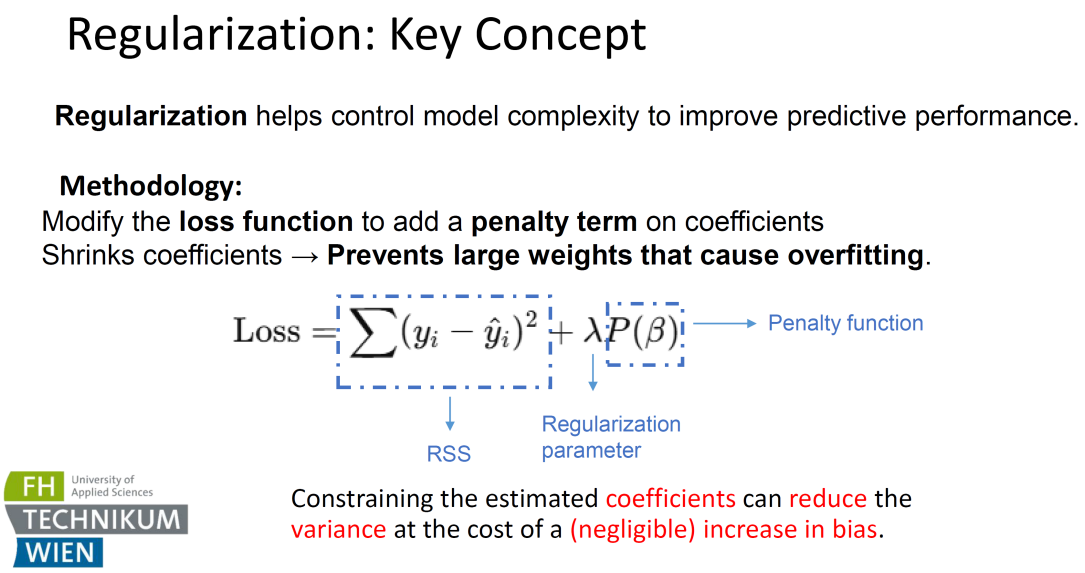
0.3.4 Regularization's Role

> Pushes the model back toward the **"optimal zone"** by **penalizing excessive complexity**.

Regularization doesn't force us to choose between underfitting and overfitting — it allows us to **tune the balance**.

This leads us into the next logical question: *how do we apply this idea mathematically?*

Ready when you are.



This slide presents the **core mathematical idea behind regularization**.

0.4 Regularization: Key Concept

The **goal** is to control model complexity — reducing the risk of overfitting — **by penalizing large coefficients** in the model.

0.4.1 Methodology: Modify the Loss Function

In plain linear regression, the loss function is the **Residual Sum of Squares (RSS)**:

$$\text{Loss} = \sum(y_i - \hat{y}_i)^2$$

With **regularization**, we **add a penalty term**:

$$\text{Loss} = \sum(y_i - \hat{y}_i)^2 + \lambda\, P(\beta)$$

- λ: the **regularization parameter** — controls how strong the penalty is
- $P(\beta)$: the **penalty function** — typically a norm of the coefficient vector

---

0.4.2 Intuition:

- Large weights are **penalized**
- The model prefers **smaller coefficients** unless they **strongly reduce error**
- This **shrinks** the model toward simpler forms

0.4.3 The Trade-Off:

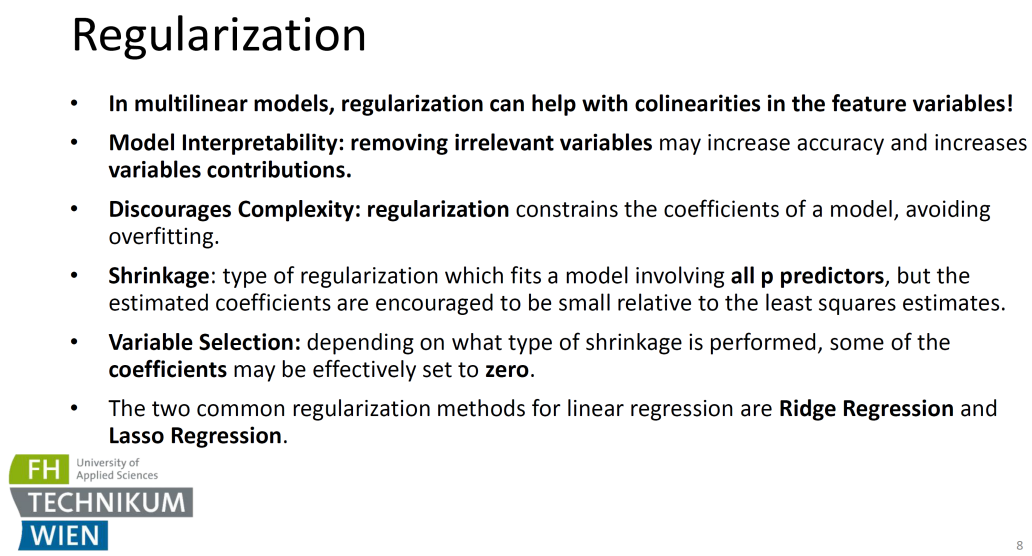
Variance decreases **Bias increases slightly** (but is usually acceptable)

Hence the line at the bottom:

"Constraining coefficients reduces variance at the cost of (negligible) bias."

This is the core mechanism behind **Ridge** (L2 norm) and **Lasso** (L1 norm) — which we'll now explore in detail.

Let's proceed.



This slide highlights the **practical motivations and benefits of regularization** in regression models.

0.5 Regularization: Why and When?

Let's go line by line:

0.5.1 Helps with Collinearity

- When predictor variables are **correlated**, standard regression coefficients become **unstable**.
- Regularization (especially Ridge) helps **stabilize the solution**.

0.5.2 Improves Interpretability

- By **shrinking or removing** irrelevant coefficients, models become **easier to interpret**.
- Especially true for **Lasso**, which can **zero out** variables completely.

0.5.3 Discourages Complexity

- Prevents the model from **fitting noise**.
- Keeps the model within a **controlled complexity range**.

0.5.4 Shrinkage

- A form of regularization where **all variables are included**, but coefficients are **shrunk toward zero**.
- **Ridge** does this — keeping all predictors but with small weights.

0.5.5 Variable Selection

- **Lasso** goes a step further: it can **set some coefficients to exactly zero**.
- This makes it useful for **automatic feature selection** in high-dimensional datasets.

0.5.6 Summary

The two key tools:

- **Ridge Regression**: shrinks but keeps all variables
- **Lasso Regression**: shrinks and may eliminate some variables

These methods are both based on the same principle: **penalize coefficient size**, but they differ in **how** they do it — and that's exactly what we'll explore next.

Proceed when ready.

Ridge Regression

- **Ridge regression is related to L2-regularization**. In ridge regression one minimizes the following expression.

$$\text{RSS} + \lambda \|\beta\|_2^2 = \text{RSS} + \lambda \sum_{j=1}^{p} \beta_j^2$$

where $\lambda \geq 0$ is a **hyperparameter** to be determined.

Shrinkage penalty $\lambda \|\beta\|_2^2$: shrinks the parameters towards zero.

Small for $\beta_1, \ldots, \beta_p$ close to zero.

L2 Penalty

$$P(\beta) = \sum_{j=1}^{p} \beta_j^2$$

FH University of Applied Sciences TECHNIKUM WIEN

9

Now we enter the specifics of the **first regularization method**:

0.6 Ridge Regression

Ridge regression applies what's called **L2 regularization** — meaning it **penalizes the sum of squared coefficients**.

0.6.1 The Loss Function

$$\text{Loss} = \text{RSS} + \lambda \|\beta\|_2^2 = \sum (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^{p} \beta_j^2$$

- The first part is the **usual least squares error** (RSS).
- The second part is the **L2 penalty**:
  - $\sum \beta_j^2 \rightarrow$ sum of squared model weights.
  - Multiplied by (lambda), the regularization strength.

0.6.2 Role of (Lambda)

- is a **hyperparameter** — it controls how much weight we give to the penalty.
  - $= \mathbf{0} \rightarrow$ Ridge regression becomes **ordinary least squares** (no regularization).
  - $> \mathbf{0} \rightarrow$ Penalty starts to take effect.
  - $\rightarrow \boldsymbol{\infty} \rightarrow$ All weights are pushed toward **zero**.

0.6.3 Why Use Ridge?

To **shrink** large weights, especially in cases of:

- **Multicollinearity**
- **Many features**
- **High variance models**

0.6.4 Takeaway:

Ridge doesn't eliminate features — it **keeps all predictors** but **shrinks** their influence proportionally.

Coming up: How affects the solution — and then the comparison to Lasso. Ready when you are.

Ridge Regression

$$\text{RSS} + \lambda \|\beta\|_2^2 = \text{RSS} + \lambda \sum_{j=1}^{p} \beta_j^2$$

- If **λ=0**, the penalty term has **no effect** and ridge regression will reproduce the least squares estimates.
- As **λ →∞** the impact of the shrinkage penalty grows and the absolute values of the estimated parameters **approach zero**.
- Ridge regression produces a **different set** of parameters for each value of λ

What is a **good value** for λ?

FH University of Applied Sciences TECHNIKUM WIEN

10

This slide explains how the **hyperparameter** affects **Ridge Regression behavior**.

0.7 Ridge Regression — The Role of

Let's revisit the formula:

$$\text{Loss} = \text{RSS} + \lambda \sum_{j=1}^{p} \beta_j^2$$

Now we look at **what happens as changes**:

0.7.1 $= 0$

- No regularization.
- The model behaves exactly like **Ordinary Least Squares**.
- No shrinkage at all — **pure variance minimization** on the training set.

0.7.2 $\to \infty$

- The penalty term **dominates** the loss.
- All coefficients β_j are driven **closer to zero**.
- Eventually, the model becomes the **null model** (predicts a constant for all inputs).

0.7.3 Each yields a different solution

- Ridge regression gives a **continuum of models**, one for each .
- Choosing the **right** is crucial — and leads to the red question at the bottom:

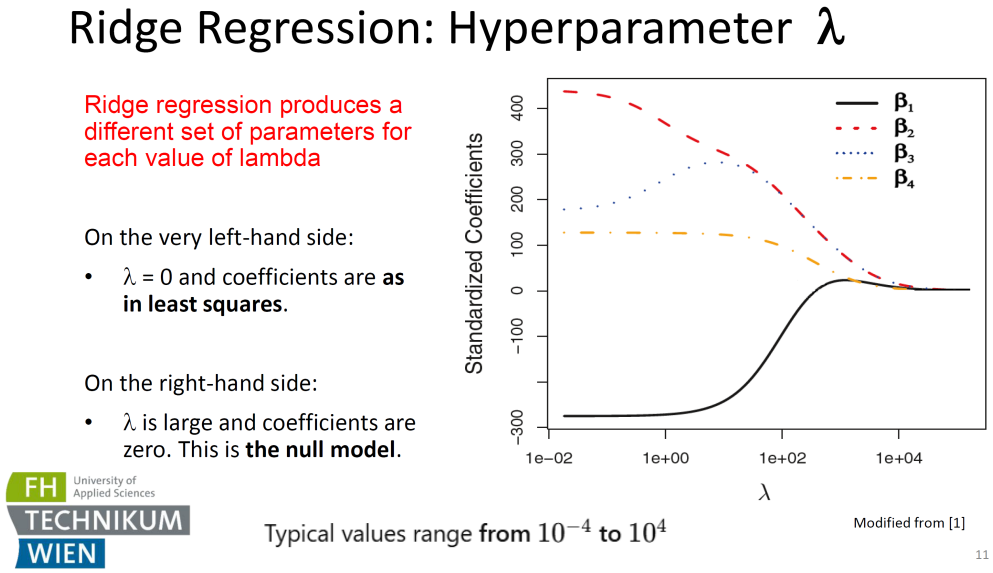
What is a good value for ?

We typically **tune** using:

- **Cross-validation**: train on subsets, test on held-out data
- Select that minimizes **validation error**

The next slides will illustrate this visually.

Ready?



This slide **visualizes** the effect of on the **Ridge regression coefficients**.

0.8 Ridge Regression: Hyperparameter

As mentioned before, **each gives a different set of coefficients**. This graph illustrates that.

0.8.1 Plot Explanation (Right Side)

- **X-axis**: (on a logarithmic scale, from 10^{-2} to 10^{4})
- **Y-axis**: Standardized coefficient values for , , ,

0.8.2 Observations:

- On the **left-hand side** (small 0):
  - All coefficients match **least squares estimates** — no penalty.
- Moving to the **right**:
  - As increases, **all coefficients shrink** gradually.
  - Eventually, all approach **zero** — the **null model**.

This shrinking is **smooth and continuous** — a key property of **L2 regularization**.

0.8.3 Note:

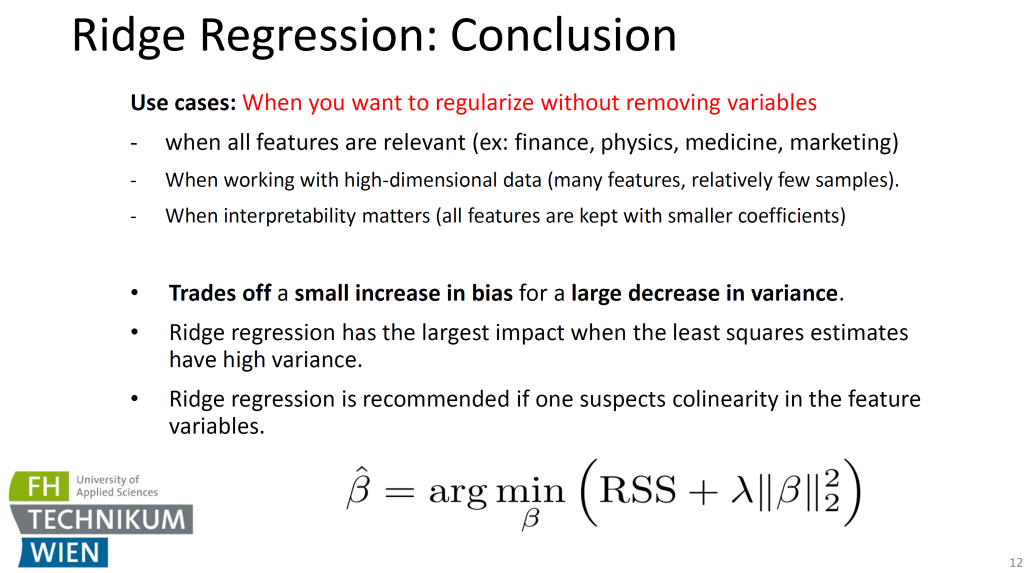
- Ridge **never eliminates** coefficients completely — it **dampens them**.
- Contrast this with **Lasso**, which can actually **set some exactly to zero**.

0.8.4 Final Remark:

Typical values for tuning lie between 10^{-4} and 10^{4}, selected via **cross-validation**.

We'll now wrap up Ridge and transition into **Lasso Regression**, where things behave differently.

Ready for the next.



This slide summarizes the key takeaways for **Ridge Regression**.

0.9 Ridge Regression: Conclusion

0.9.1 Use Cases — When to Use Ridge

"When you want to regularize **without removing** any variables"

Ideal when:

- **All features are believed to be useful**
  - Examples: **finance**, **physics**, **medicine**, **marketing**
- You're dealing with **high-dimensional data**
  - e.g., many features, but few samples
- **Interpretability matters**
  - Ridge keeps all features — just **with smaller, stabilized coefficients**

0.9.2 What Ridge Offers

- **Shrinks all coefficients**, but never sets them exactly to zero
- **Greatly reduces variance**, at the cost of a **small increase in bias**
- Very effective in the presence of **multicollinearity**

0.9.3 Mathematical Form:

$$\hat{\beta} = \arg\min_{\beta} \left(\text{RSS} + \lambda \|\beta\|_2^2 \right)$$

This highlights:

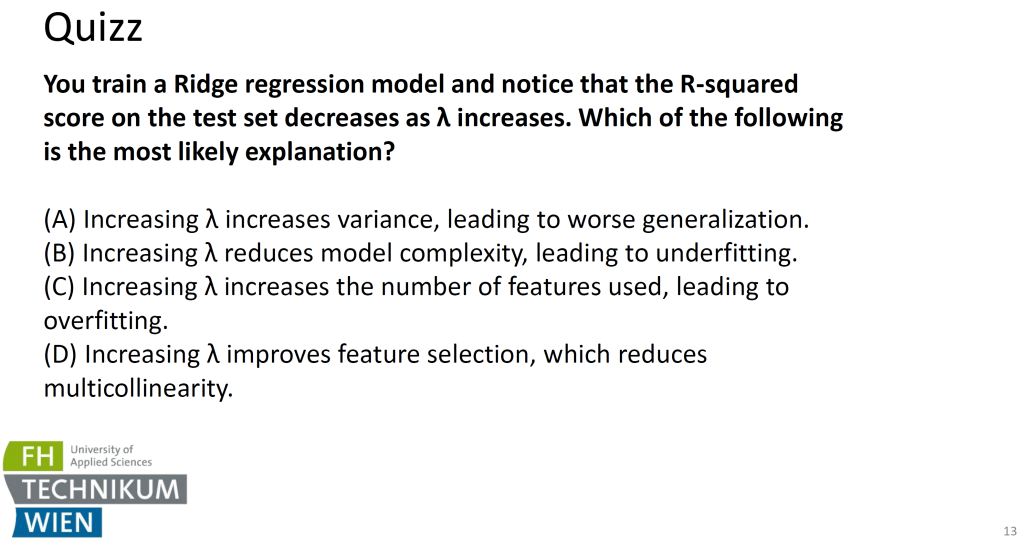
- The **minimization goal**
- The **balance between fit and penalty**

0.9.4 Bottom Line:

Use Ridge when you want a **stable**, **generalizable**, and **interpretable** model without discarding features.

Now let's switch gears and explore **Lasso Regression**, where variable selection comes into play.

Ready when you are.



This is a quick quiz slide to **test your understanding** of Ridge regression and the role of .

0.9.5 Question Recap:

> *You train a Ridge regression model and notice that the R^2 score on the test set* ***decreases*** *as* ***increases****. What's the most likely explanation?*

Let's go through the choices:

(A) Increasing increases variance, leading to worse generalization Incorrect — Increasing **reduces variance** (not increases). It's one of the key purposes of regularization.

(B) Increasing reduces model complexity, leading to underfitting Correct — As becomes too large, the model becomes too simple (shrinks coefficients too much), leading to **underfitting** and worse performance.

(C) Increasing increases the number of features used, leading to overfitting Incorrect — Ridge **never adds** features or causes overfitting as increases. It **shrinks all coefficients** regardless of feature count.

(D) Increasing improves feature selection, which reduces multicollinearity Incorrect — Ridge doesn't perform feature selection (that's **Lasso's job**). It addresses multicollinearity, yes — but **not via feature elimination**.

0.9.6 Final Answer: (B)

> Increasing reduces model complexity, and too much reduction leads to **underfitting**, hence lower R^2.

Shall we continue with Lasso?

Lasso Regression

- In Lasso regression one **minimizes** the following expression:

$$\text{RSS} + \lambda \|\beta\|_1 = \text{RSS} + \lambda \sum_{j=1}^{n} |\beta_j|$$

where $\lambda \geq 0$ is a **hyperparameter** to be determined.

This is Tikhonov regularization using the **L1-norm**.

Shrinkage penalty: more **pronounced tendency to shrink single parameters strongly** (often to practically **zero)** when λ is sufficiently large **(Variable Selection)**.

Leads to more interpretable models!

FH University of Applied Sciences TECHNIKUM WIEN

L1 Penalty

$$P(\beta) = \sum_{j=1}^{n} |\beta_j|$$

14

Now we move on to the second major regularization method:

0.10 Lasso Regression — L1 Regularization

Lasso stands for **Least Absolute Shrinkage and Selection Operator**. It uses the **L1 norm** to penalize model complexity.

0.10.1 Loss Function:

$$\text{Loss} = \text{RSS} + \lambda\|\beta\|_1 = \sum(y_i - \hat{y}_i)^2 + \lambda\sum_{j=1}^{n}|\beta_j|$$

- λ: regularization strength (as in Ridge)
- $\|\beta\|_1$: sum of absolute values of coefficients — the **L1 penalty**

0.10.2 Key Difference from Ridge:

- Ridge (**L2**) → sum of squares of β_j
- **Lasso (**L1**)** → sum of **absolute values** of β_j

0.10.3 Shrinkage Behavior:

- Lasso **strongly penalizes** large individual coefficients
- It has the **unique ability to set some coefficients exactly to zero**
- This results in **automatic variable selection** (only the most relevant features stay)

0.10.4 Benefits:

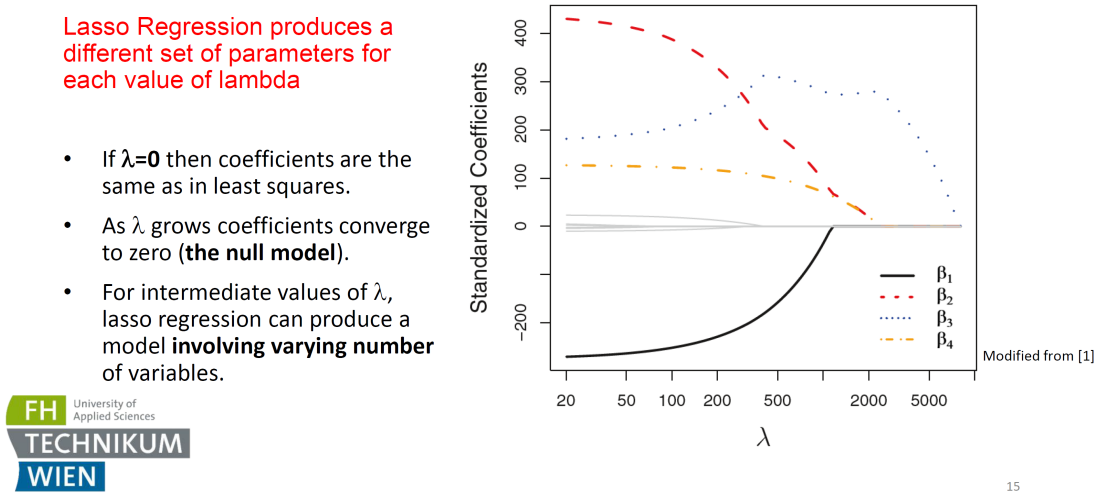
- Results in **sparser models**
- **More interpretable**
- Especially useful when we believe **only a subset of features are actually relevant**

0.10.5 Summary:

Ridge = shrinkage Lasso = shrinkage + **selection**

More on how affects Lasso in the next slides.

Ready to proceed?



This slide visualizes how **affects Lasso regression coefficients**, and how **feature selection emerges**.

0.11 Lasso Regression: Hyperparameter

As with Ridge, each gives a different solution — but here the behavior is quite different:

0.11.1 Plot Overview (Right Side)

- **X-axis**: (log scale, from 20 to 5000)
- **Y-axis**: Standardized coefficients for several features (, , …)
- As increases, **some coefficients abruptly drop to zero**.

0.11.2 Key Properties of Lasso

- **= 0**: Ordinary Least Squares (no penalty), all coefficients included
- **→ ∞**: All coefficients go to 0 — the **null model**
- **Intermediate** : Some coefficients remain; others are **set to zero** → This is **automatic feature selection**

0.11.3 Important Difference from Ridge:

- Ridge: All coefficients shrink gradually, none go exactly to zero
- Lasso: Some coefficients shrink **to zero abruptly** — effectively **removing features**

This gives Lasso its **sparsity property**, which is critical in high-dimensional datasets where many features may be irrelevant.

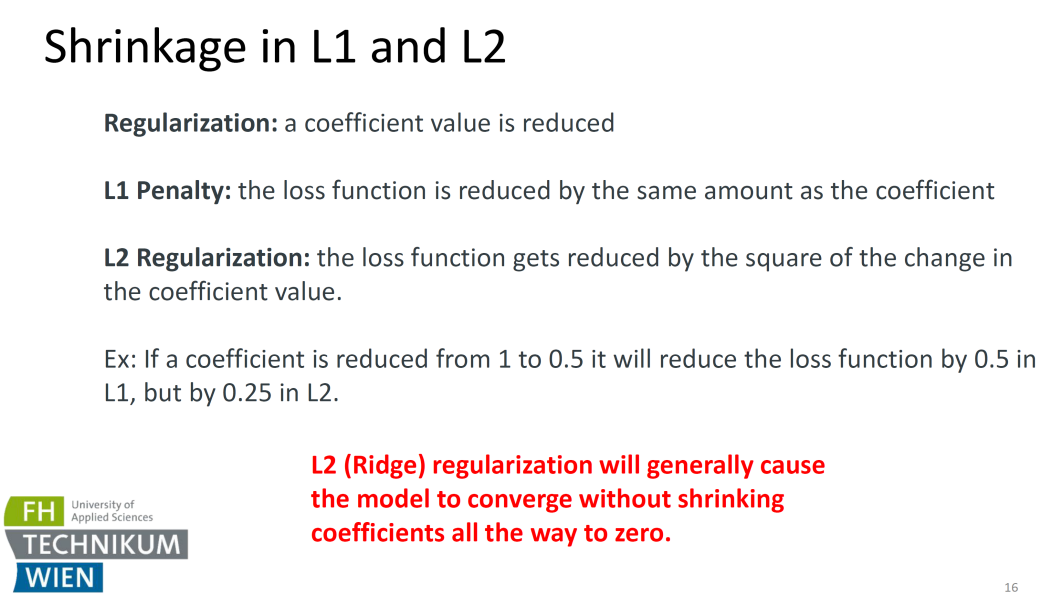
0.11.4 Summary Line:

Lasso regression can create **simpler models with fewer variables** as increases.

More → more sparsity → fewer features → higher interpretability (but risk of underfitting)

We'll now go deeper into how shrinkage differs between L1 and L2.

Shall we continue?



This slide offers a **technical comparison of how shrinkage behaves under L1 and L2 penalties** — explaining why Lasso leads to sparsity, and Ridge does not.

0.12 Shrinkage in L1 vs. L2

0.12.1 Regularization, in general:

Means **reducing the size of the coefficients** to prevent overfitting.

0.12.2 L1 Penalty (Lasso):

- The penalty is **proportional to the absolute value** of the coefficient:

$$\sum |\beta_j|$$

- This creates a **constant gradient**, leading to a strong pull towards zero.
- Coefficients are **shrunk equally**, and often **set exactly to zero** — enabling **feature selection**.

0.12.3 L2 Penalty (Ridge):

- The penalty is **proportional to the square** of the coefficient:

$$\sum \beta_j^2$$

- This results in **smaller gradients** as coefficients get smaller.
- Coefficients are shrunk **progressively less** as they get closer to zero.
- Ridge thus **rarely sets coefficients to exactly zero** — it **shrinks**, but doesn't **eliminate**.

0.12.4 Numerical Example:

- If a coefficient drops from $1 \rightarrow 0.5$:
  - **L1 penalty** drops by **0.5**
  - **L2 penalty** drops by **0.25**

This is why **L1 is more aggressive** in eliminating features.

0.12.5 Key Conclusion:

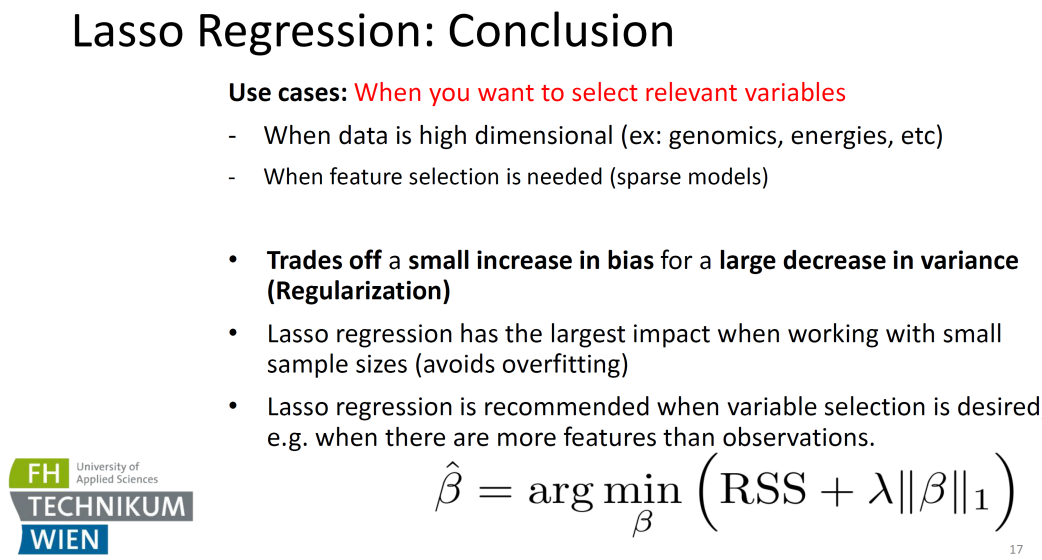
> "**L2 (Ridge) regularization will generally cause the model to converge without shrinking coefficients all the way to zero.**"

Which is why:

- **Ridge** is great for **stabilization**
- **Lasso** is great for **simplification**

We'll soon see use cases and a summary next.

Ready?



This slide wraps up **Lasso Regression** with its **ideal use cases**, trade-offs, and optimization formula.

0.13 Lasso Regression: Conclusion

0.13.1 Use Cases — When Lasso Is a Good Choice:

"**When you want to select relevant variables**"

- **High-dimensional data**: more features than samples e.g., genomics, energy data, text analysis
- **Feature selection** is important → Lasso can **drop irrelevant variables**, creating **sparse models**

0.13.2 Regularization Trade-Off

- Accepts a **small increase in bias**
- Gains a **large reduction in variance**
- Improves **generalization** and prevents **overfitting**

0.13.3 Greatest Impact When:

- The dataset has **few observations** but **many features**
- Example: You have 50 samples and 1,000 predictors → Lasso can shrink 950+ of them to zero

---

0.13.4 Optimization Objective:

$$\hat{\beta} = \arg\min_{\beta} \left(\text{RSS} + \lambda \|\beta\|_1\right)$$

- The L1 norm enables **automatic variable selection**
- controls **how many variables are kept**

0.13.5 Summary:

> Use Lasso when you want a **simpler, interpretable, and sparse model**, and you suspect **only a few features truly matter**.

Shall we continue with the Lasso quiz?

Quizz

You are using Lasso regression on a dataset with 100 features, but only 10 are truly useful for predicting the target variable. However, you observe that the selected features change slightly each time you retrain the model. What is the most likely explanation?

(A) Lasso struggles with feature selection when features are highly correlated.
(B) The dataset size is too small, making Ridge regression a better choice.
(C) The regularization parameter (λ\lambdaλ) is too small, causing overfitting.
(D) Lasso is deterministic, so this should not happen unless the dataset changes.

FH University of Applied Sciences TECHNIKUM WIEN

18

This quiz tests your understanding of **Lasso's feature selection behavior**, especially in high-dimensional settings.

0.14 Question Recap:

> *You're using Lasso on a dataset with 100 features, only 10 of which are truly relevant. Each retraining gives a slightly different subset of selected features. Why is that happening?*

Let's evaluate the options:

0.14.1 (A) Lasso struggles with feature selection when features are highly correlated.

Correct — Lasso has difficulty deciding **which feature to keep** when multiple features are **strongly correlated**. It may **arbitrarily drop one** and keep the other, leading to **instability** across runs. This is a well-known limitation of Lasso.

0.14.2 (B) The dataset size is too small, making Ridge regression a better choice.

Incorrect — Lasso is often **preferred** for small datasets with many features, since it can eliminate irrelevant ones. Ridge doesn't do variable selection.

0.14.3 (C) The regularization parameter is too small, causing overfitting.

Unlikely — If were too small, Lasso would act like plain linear regression, yes — but the instability described is more typical of **correlated predictors**, not issues per se.

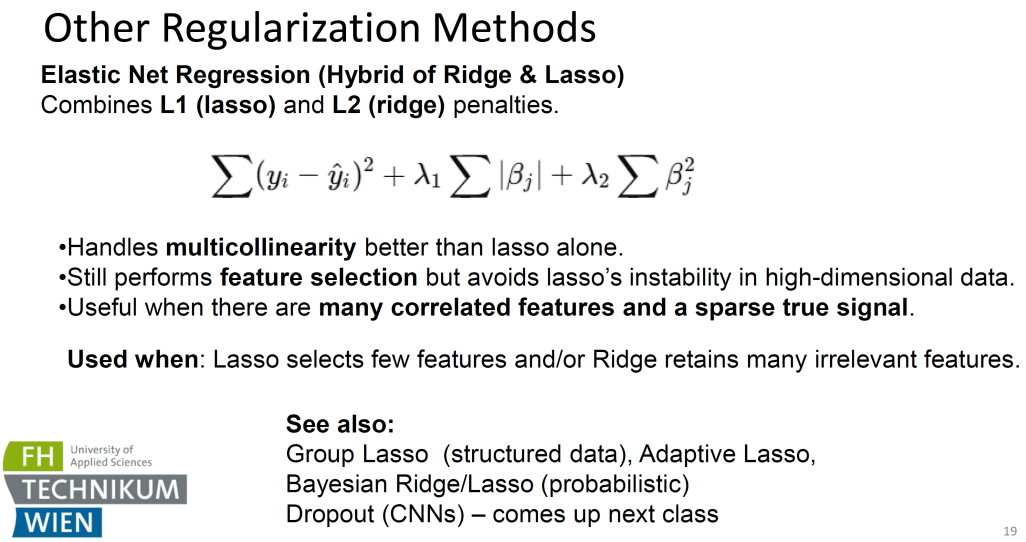
0.14.4 (D) Lasso is deterministic, so this should not happen unless the dataset changes.

Incorrect — Even with the same dataset, randomness (e.g. initialization, solver tolerances) or **correlated variables** can make Lasso select **different subsets**.

0.15 Final Answer: (A)

> Lasso struggles with feature selection when features are **highly correlated**, causing **instability in the selected subset**.

Let me know when to proceed.



This slide introduces **other regularization methods**, with a focus on **Elastic Net Regression**, a powerful hybrid approach.

0.16 Elastic Net Regression

A **blend of Ridge (L2)** and **Lasso (L1)** penalties:

$$\sum(y_i - \hat{y}_i)^2 + \lambda_1 \sum |\beta_j| + \lambda_2 \sum \beta_j^2$$

It combines:

- **L1 penalty** → drives coefficients to **zero** (like Lasso)
- **L2 penalty** → handles **multicollinearity** and stabilizes coefficients (like Ridge)

0.16.1 Why Use Elastic Net?

Handles correlated predictors better than Lasso

- Lasso can drop features **arbitrarily** when predictors are highly correlated.
- Elastic Net **shares weight** among them — stabilizing the selection.

Still supports feature selection

- Because of the L1 component

Ideal for:

- **High-dimensional data**
- Where you expect:
  - Many predictors
  - Only a few are truly relevant (**sparse true signal**)

0.16.2 When to Prefer It:

When:

- Lasso drops **too many** features
- Ridge retains **too many irrelevant** features

Elastic Net provides a **middle ground**.

0.16.3 Also Worth Exploring:

- **Group Lasso**: For structured feature groups
- **Adaptive Lasso**: Weighted penalties
- **Bayesian Ridge/Lasso**: Probabilistic approaches
- **Dropout (CNNs)**: A kind of regularization in deep learning — coming up next session

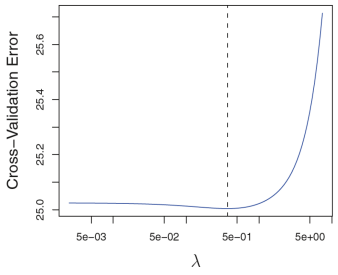
Shall we continue with selection and model comparison?

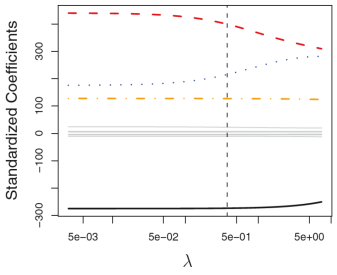
Selecting λ

- Choice of a grid of λ values and computation of cross-validation error for each value of λ.
- Selection of λ for which the cross-validation error is smallest.
- Refit using all of the available observations and the selected λ.

Left:

Cross-validation errors from applying ridge regression.





Right:

Coefficient estimates as a function of λ. The vertical dashed lines indicate the value of λ selected by cross-validation.
From [1].

FH University of Applied Sciences TECHNIKUM WIEN

20

This slide explains **how to select the optimal** — the key hyperparameter in regularization.

0.17 Selecting (Lambda)

0.17.1 Step-by-Step Procedure:

1. **Choose a grid of values**
   - e.g. logarithmic scale: $\lambda \in [10^{-4}, 10^{4}]$
2. **Perform cross-validation**
   - For each : train on training folds, validate on the hold-out fold
3. **Pick the with the lowest validation error**
4. **Refit** the model using **all the data** and the selected

---

0.17.2 Left Plot:

- **X-axis**: values (log scale)
- **Y-axis**: Cross-validation error
- **U-shaped curve**:
  - Small : overfitting → high variance
  - Large : underfitting → high bias
  - **Minimum** → ideal
- **Dashed vertical line** shows the selected

0.17.3 Right Plot:

- Tracks **coefficient paths** as changes
- Vertical dashed line = selected
- You can see which coefficients remain active and how much shrinkage occurs

---

0.17.4 Summary:

> Use **cross-validation** to find the that balances bias and variance for best generalization.

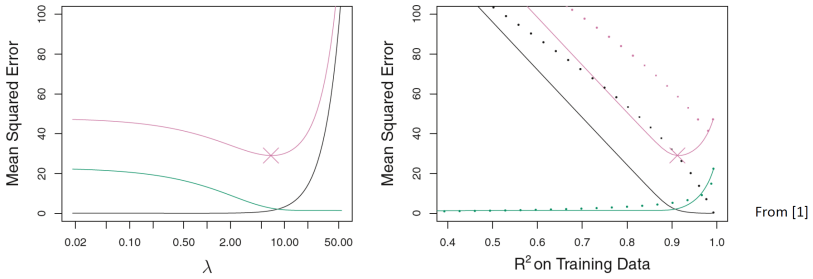
This approach applies to **Ridge**, **Lasso**, and **Elastic Net** alike.

Shall we go to the comparison summary?

Comparing Lasso and Ridge Regression

- Left: Plots of **in-sample error (black)**, **variance (green)**, and **test MSE (pink)** for lasso regression.
- Right: Comparison of squared bias, variance and test MSE between **lasso (solid)** and **ridge regression (dashed)** plotted against R^2 on training data.

The crosses indicate the lasso models for which MSE is smallest.



From [1]

FH University of Applied Sciences TECHNIKUM WIEN

21

This slide provides a **side-by-side performance comparison** between **Lasso** and **Ridge Regression**, using error metrics and R^2 values.

0.18 Comparing Lasso and Ridge

0.18.1 Left Plot: Lasso Regression Performance vs.

- **X-axis**: (log scale)
- **Y-axis**: Mean Squared Error (MSE)

Curves:

- **Black** → In-sample error
- **Green** → Variance
- **Pink** → Test MSE

Cross mark: where **Lasso** achieves its **lowest test MSE**

Interpretation:

- Low → high variance (overfit)
- High → high bias (underfit)
- Sweet spot: balanced bias-variance tradeoff = lowest test MSE

0.18.2 Right Plot: Lasso vs. Ridge Comparison

- **X-axis**: R^2 on training data (i.e., goodness of fit)

- **Y-axis**: Mean Squared Error (MSE)

Line styles:

- **Solid** → Lasso
- **Dashed** → Ridge

You can compare:

- **Bias (black)**,
- **Variance (green)**,
- **Test error (pink)**

Key Takeaways:

- **Lasso** reaches minimum test error at higher R^2 than Ridge
- **Ridge** may generalize better in some cases (especially with correlated features)
- **Lasso** can have more variance in selection, but achieves sparsity

---

0.18.3 Pitfall:

- If over-tuned, both can **underfit or overfit**, as seen in the curve edges

---

0.19 Bottom Line:

Use cross-validation to decide not just , but also **which method (Lasso or Ridge)** performs better on your data.

Final summary slide coming next?

Takeaway: Ridge and Lasso Regularization

Aspect	Ridge Regression (L2)	Lasso Regression (L1)
Effect on Coefficients	Shrinks all coefficients but keeps them	Shrinks some coefficients to exactly **zero**
Feature Selection?	❌ No	✅ Yes
Handles Collinearity?	✅ Distributes weights among correlated features	❌ Picks only one feature and ignores others
Computational Complexity	✅ Faster (solves closed-form solution)	❌ Slower (uses iterative optimization)
Best Used When	Features are **all relevant**, collinear data	**Sparse models**, many irrelevant features

The # of predictors that is related to the response is never known a priori.

Cross-validation can be used in order to determine which approach is better.

FH University of Applied Sciences TECHNIKUM WIEN

22

This final content slide offers a **comprehensive summary** comparing **Ridge** and **Lasso** — and when to use which.

0.20 Takeaway: Ridge vs. Lasso Regularization

Aspect	Ridge (L2)	Lasso (L1)
Effect on Coefficients	Shrinks all, **none go to zero**	Shrinks **some to exactly zero**
Feature Selection?	No	Yes (automatic variable selection)
Handles Collinearity?	Yes (distributes weight among correlated features)	Picks one, ignores others
Computational Complexity	Faster (closed-form solution)	Slower (requires iterative optimization)
Best Used When	All features are likely **relevant**, or collinear	Many features, but most are **irrelevant**

0.20.1 Final Notes:

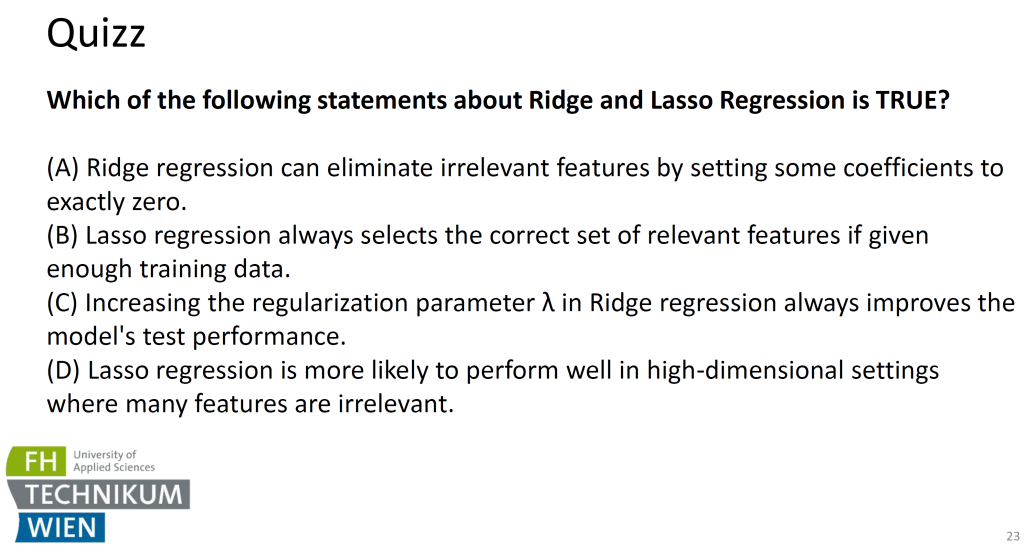
- **You never know in advance** how many features are truly relevant.
- So you should always use **cross-validation** to choose:
  - the **method** (Ridge or Lasso)
  - the **value of** (the regularization strength)

0.20.2 Bottom Line:

Ridge = Keep all, stabilize Lasso = Select few, simplify

Use data-driven validation to decide which works best for your case.

We're done with the core slides — would you like to continue with the quiz and assignment slide?



This final quiz checks if you've internalized the core differences between **Ridge** and **Lasso** regression.

0.21 Question Recap:

*Which of the following statements about Ridge and Lasso Regression is **TRUE**?*

0.21.1 (A) Ridge regression can eliminate irrelevant features by setting some coefficients to exactly zero.

False — Ridge (L2) **shrinks**, but **never zeroes out** coefficients.

0.21.2 (B) Lasso regression always selects the correct set of relevant features if given enough training data.

False — Lasso **can fail** in presence of **highly correlated features** or **small signal-to-noise ratio**. It's not guaranteed to recover the true model even with more data.

0.21.3 (C) Increasing the regularization parameter in Ridge regression always improves the model's test performance.

False — Increasing **too much** causes **underfitting**, which hurts performance. There's an **optimal** , not "the higher the better."

0.21.4 (D) Lasso regression is more likely to perform well in high-dimensional settings where many features are irrelevant.

True — Lasso is **designed for sparsity**. It does **feature selection**, and works best when only **a few features matter** — which is common in **high-dimensional** problems.

0.22 Correct Answer: (D)

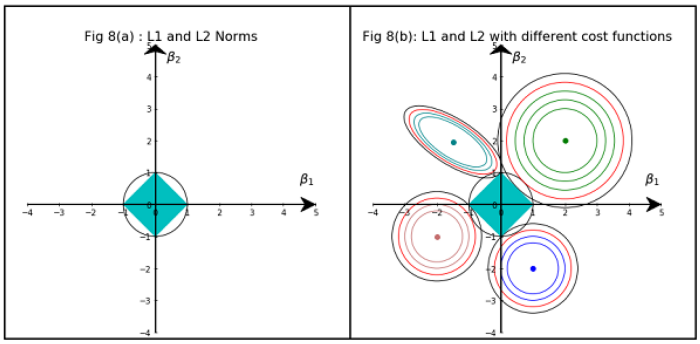
Let me know if you'd like to continue with the assignment slide or wrap up.

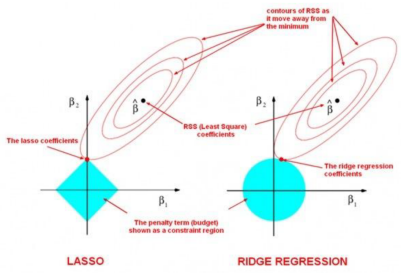
References

[1] Gareth James, Daniela Witten, Trevor Hastie, Robert Tibshirani. An Introduction to Statistical Learning: with Applications in R. New York: Springer, 2013.

FH University of Applied Sciences TECHNIKUM WIEN

25





L1 geometry (Lasso): Unit ball is square.
L2 geometry ("Ridge"): Unit ball is disc.

Look at the intersection where the gradient descent contours intersect with the lasso and ridge areas. Lasso has a greater propensity for intersection at the axes, that is to set coefficients to zero.

https://www.quora.com/Why-is-it-that-the-lasso-unlike-ridge-regression-results-in-coefficient-estimates-that-are-exactly-equal-to-zero
https://www.r-bloggers.com/2020/05/quick-tutorial-on-lasso-regression-with-example/

FH University of Applied Sciences TECHNIKUM WIEN