ST2334 – Probability and Statistics
AY24/25, Y2S2
Tutorial

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December 3, 2025

Contents

1 Week 1

1.1 Exam Format Questions

1.1.1 Q1

Let A and B be two events of sample space S. Which of the following is INCORRECT:

(a) If $A \cup B = A$, then we must have $B \subseteq A$.

(b) If $A \cap B = A$, then we must have $A \subseteq B$.

(c) If $A \cup B \subseteq A$, then we must have $B \cup A \subseteq B$.

(d) All are correct.

(a): $\forall b \in B, b \in A \cup B . A \cup B = A \implies B \subseteq A$

(b): $A \cap B = A \implies \forall a \in A, a \subseteq A \cap B \implies \forall a \in A, A \subseteq B$

(c): Let $A = \{1, 2\}$, $B = \{1\}$, Proof by counter-example

1.1.2 Q2

$((A \cup B) \cap C)' = ?$

(a) $(A' \cap C') \cap (B' \cap C')$

(b) $(A' \cap C') \cup (B' \cap C')$

(c) $(A' \cup C') \cap (B' \cup C')$

(d) $(A' \cup C') \cup (B' \cup C')$

$((A \cup B) \cap C)' = ((A \cup B)' \cup C') = ((A' \cap B') \cup C') = (A' \cup C') \cap (B' \cup C')$

1.1.3 Q3

There are 5 vowels and 21 consonants among 26 alphabets. If a sample of 3 alphabets are selected without replacement, how many samples have at least 1 vowels?

$nCr(26, 3) - nCr(21, 3) = 2600 - 1330 = 1270$

1.1.4 Q4

How many ways can 4 men and 3 women sit in a row if no two women are allowed to sit together?

$nPr(4, 4) \times nCr(5, 3) \times nPr(3, 3) = 1440$

1.1.5 Q5

A contractor wishes to build 9 houses, each of different in design in 9 plots of land. In how many ways can be placed these houses on a street if 6 lots are on South side of the street and 3 lots are on the North side? (Note: The 9 lots are fixed.)

$nPr(9, 9) = 362880$

1.2 Analytical Questions

1.2.1 Q1

The NUS library has five copies of a certain text on reserve. Two copies (1 and 2) are first editions, and the other three (3, 4 and 5) are second editions. A student examines these books in random order, stopping only when a second edition has been selected. One possible outcome is 5, and another is 213.

i. List the outcomes in the sample space S.

$S = \{3, 4, 5, 13, 14, 15, 23, 24, 25, 123, 124, 125, 213, 214, 215\}$

ii. Let A denote the event that exactly one book must be examined. List the outcomes in A.

$A = \{3, 4, 5\}$

iii. Let B be the event that book 5 is the one selected. List the outcomes in B.

$B = \{5, 15, 25, 125, 215\}$

iv. Let C be the event that book 1 is not examined. List the outcomes in C.

$C = \{3, 4, 5, 23, 24, 25\}$

v. List the outcomes in $A \cap B$, $A \cup B$, and $A \cap B \cap C$ respectively. Are A and B mutually exclusive?

$A \cap B = \{5\}$
$A \cup B = \{3, 4, 5, 15, 25, 125, 215\}$
$A \cup B \cup C = \{5\}$
No, they are not mutually exclusive.

1.2.2 Q2

Consider the digits 0, 2, 4, 6, 8 and 9. If each digit can be used only once,

i. how many three-digit numbers can be formed?

$5 \times 5 \times 4 = 100$

ii. how many of these numbers in (i) are odd numbers?

$4 \times 4 \times 1 = 16$

iii. how many of these odd numbers in (ii) are greater than or equal to 620?
$1 \times 4 \times 1 + 1 \times 3 \times 1 = 7$

1.2.3 Q3

An exam paper consists of seven questions. Candidates are asked to answer five questions. Find the number of ways to select five questions (in each of the following cases) if

i. there are no restrictions;

$nCr(7, 5) = 21$

ii. the first two questions must be answered;

$nCr(5, 3) = 10$

iii. at least one of the first two questions must be answered; and

$nCr(5, 3) + 2 \times nCr(5, 4) = 20$

iv. exactly two from the first three questions must be answered.

$nCr(3, 2) + nCr(4, 3) = 12$

1.2.4 Q4

Little Red Riding Hood lives at point A : (0,0), and wants to visit her grandmother at point B : (13,8). At each step, she can only go East (Right) or North (Up) along the grid as shown below. The Big Bad Wolf lives at Y : (10,6).

i. How many ways can Little Red Riding Hood go to visit her grandmother regardless of whether she will pass by the Big Bad Wolf?

$nCr(13+8,8) = 203490$

ii. How many ways can she go to visit her grandmother avoiding the Big Bad Wolf?

$n203490 - nCr(10+6,6) - nCr(5,2) = 123410$

iii. Little Red Riding Hood wants to buy a gift for her grandmother at X : (2,2). How many ways can she go to visit her grandmother stopping by X but avoiding Y?

$nCr(4,2) \times (nCr(17,6) - nCr(12,4) \times nCr(5,2)) = 44556$

2 Week 2

2.1 Exam Format Questions

2.1.1 Q1

Let A and B be events satisfying $P(A \cup B) = P(A) + P(B)$. Which of the following statement is NOT true?

(a) If A and B are independent, then P(A) = 0 or P(B) = 0.

(b) If $A \neq B$, then A and B are mutually exclusive.

(c) If $P(A) > P(B) > 0$, then A and B are not independent.

(d) If A = S, the sample space, then P(B) = 0.

(a) False

(b) 1,2,6,3,4,6

(c) ?

(d) True

2.1.2 Q2

Draw 4 balls randomly without replacement from a basket containing 4 blue balls, 4 green balls, and 2 red balls. What is the probability to get 2 blue balls, 1 green ball, and 1 red ball?

(a) 7/105

(b) 8/105

(c) 9/35

(d) 8/35

(a)

(b)

(c)

(d)

2.1.3 Q3

The probability that a Singaporean company will set up a factory in City A is 0.7. The probability that it will set up a factory in City B is 0.4, and the probability that it will set up in either City A or City B or both is 0.8. Which of the following statements is INCORRECT?

(a) The probability that the company will set up a factory in both cities is 0.3.

(b) The probability that the company will set up a factory in neither city is 0.2.

(c) Whether the company will set up a factory in City A will not affect whether it will set up a factory in City B.

(d) Whether the company will set up a factory in City B will affect whether it will set up a factory in City A.

(e) None of the given options.

(a) $P(A \cap B) = 0.3$

(b) $P(A' \cap B') = 1 - P(A \cup B) = 0.2$

(c) $P(A \cap B) \neq P(A)P(B)$

(d) Refer (c)

(e) Refer (a,b,c,d)

2.1.4 Q4

Consider 5-card poker hands dealt from a standard 52 card deck. Two important events are A = {You draw a flush}, B = {You draw a straight}. If you are dealt a 5-card hand, find the following probabilities:

- P(A) = $\frac{12}{51} \times \frac{11}{50} \times \frac{10}{49} \times \frac{9}{48} = 0.001981$
- P(B) = $\frac{4}{51} \times 4 = 10 * 1020/nCr(52, 5)c = 0.003925$

2.2 Analytical Questions

2.2.1 Q1

Suppose there are 500 applicants for five equivalent positions at a factory. The company is able to narrow the field to 30 equally qualified applicants. Seven of the finalists are minority candidates. Assume that the five who are chosen are selected at random from this final group of thirty.

i. In how many ways can the selection be made?

$nCr(30, 5) = 142506$

ii. What is the probability that none of the minority candidates are hired?

$\frac{nCr(23,5)}{nCr(30,5)} = 0.2361$

iii. What is the probability that no more than one minority candidates is hired?

$\frac{nCr(23,5)+nCr(7,1)\times nCr(23,4)}{nCr(30,5)} = 0.6711$

2.2.2 Q2

There are two intersections with traffic lights along the route taken by a motorist driving to work. The probability that he must stop at the first light is 0.4, the probability that he must stop at the second light is 0.5, and the probability that he must stop at at least one of the two lights is 0.6. What is the probability that he must stop

i. at both lights?

$P(A) = 0.4 \qquad P(B) = 0.5 \qquad P(A \cup B) = 0.6$
$P(A \cap B) = P(A) - (P(A \cup B) - P(B)) = 0.4 - (0.6 - 0.5) = 0.3$

ii. at exactly one light?

$P(A \oplus B) = P(A \cup B) - P(A \cap B) = 0.6 - 0.3 = 0.3$

iii. at neither light?

$P(A' \cap B') = P(A \cup B)' = 1 - 0.6 = 0.4$

iv. at the second light given that he has stopped at the first light?

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.3}{0.4} = 0.75$

v. Is the event "stopping at the first traffic light" independent of the event "stopping at the second traffic light"?

No

2.2.3 Q3

A soft-drink bottling company maintains records concerning the number of unacceptable bottles of soft drink obtained from the filling and capping machines. Based on the past data, the probability that a bottle came from machine I and was nonconforming is 0.01, and the probability that a bottle came from machine II and was nonconforming is 0.025. Half the bottles are filled on machine I and the other half are filled on machine II. If a filled bottle of soft drink is selected at random, what is the probability that

i. it is a nonconforming bottle?

$P(I \cap X) = 0.01 \qquad P(II \cap X) = 0.025 \qquad P(I) = P(II) = 0.5$
$P(X) = 0.01 + 0.025 = 0.035$

ii. it was filled on machine II?

$P(II) = 0.5$

iii. it was filled on machine II and is a conforming bottle?

$P(II \cap X') = 0.5 - 0.025 = 0.475$

iv. It was filled on machine I or is a conforming bottle?

$P(I \cup X') = P(I) + P(X') - P(I \cap X') = 0.5 + (1 - 0.035) - 0.49 = 0.975$

v. Suppose you know that the bottle was produced on machine I. What is the probability that it is nonconforming?

$P(X|I) = \frac{P(X \cap I)}{P(I)} = \frac{0.01}{0.5} = 0.02$

vi. Suppose you know that the bottle is nonconforming. What is the probability that it was produced on machine I?

$P(I|X) = \frac{P(I \cap X)}{P(X)} = \frac{0.01}{0.035} = 0.2875$

vii. Explain the difference in the answers to (v) and (vi).

They are supposed to be different things. $P(X) < P(I)$ so it will be much more likely to get a bottle that is produced on machine I that is nonconforming.

3 Week 3

3.1 Exam Format Questions

3.1.1 Q1

Which of the following can define probability distributions?

(a) $f(x) = \frac{x}{14}$ for $x = 0, 1, 2, 3, 4$

(b) $f(x) = \frac{3-x^2}{4}$ for $x = 0, 1, 2$

(c) $f(x) = \frac{1}{5}$ for $x = 5, 6, 7, 8, 9$

(d) $f(x) = \frac{2x+1}{50}$ for $x = 0, 1, 2, 3, 4$

(a) No

(b) No, negative

(c) Yes

(d) No

3.1.2 Q2

Which of the following figures draw the cdf for random variable X, where

$$f(x) = \begin{cases} 0.1 & 10 \leq x \leq 20 \\ 0 & \text{elsewhere} \end{cases}$$

(a)

(b) Yes

(c)

(d)

3.1.3 Q3

A worker needs to drive to work from his home daily. There is only one route available, on which there are two speeding cameras working independently. The speeding cameras at each of these locations operates 50% and 75% of the time respectively. Based on the worker's driving habit, he will speed 40% of the time; and whether he will speed at different time points are independent. What is the probability that the worker will not receive a speeding ticket for each day? Note: whether the camera is working at any time is also independent with whether a driver is speeding when s/he drives through that camera.

(a) 0.56

(b) 0.48

(c) 0.36

(d) 0.72

$P(S \cap A) \cup P(S \cap B) = 0.2 + 0.3$.
Minus $P(A \cap B)$, then 1 minus to get 0.56.

3.1.4 Q4

Which of the following figures draw the CDF for a random variable X?

(a)

(b) Yes

(c) Yes

(d) Yes

3.1.5 Q5

Let X be a continuous random variable; and let Y be a discrete random variable. It is possible to find a real number a, such that P(X = a) ¿ P(Y = a).
TRUE.

3.2 Analytical Questions

3.2.1 Q1

A manufacturer of printed circuit boards exposes all finished boards to an online automated verification test. During one period, 900 boards were completed and 890 passed the test. The test is not infallible. Of 30 boards intentionally made to have noticeable defects, 25 were detected by the test.

a Approximate P(Board passes Test $\|BoardhasDefects$)

b Approximate P(Board has Defects)

c Approximate P(Board has Defects $\|BoardPassesTest$)

3.2.2 Q2

For customers purchasing a full set of tires at a particular tire store, consider the events A = tires purchased were made in the United States, B = purchaser has tires balanced immediately, C = purchaser requests for front-end alignment. Assume the following unconditional and conditional probabilities: $P(A) = 0.75, P(B|A) = 0.9, P(B|A') = 0.8, P(C|A \cap B) = 0.8, P(C|A' \cap B) = 0.7$

a $P(A \cap B \cap C) = 0.75 \times 0.9 \times 0.8 = 0.5265$

b $P(B) = 0.9 \times 0.75 + 0.8 \times 0.25 = 0.875$

c $P(A|B) = \frac{0.75 \times 0.9}{0.875} = 0.771429$

d $P(B \cap C) = 0.5265 + 0.25 \times 0.8 \times 0.7 = 0.6665$

e $P(A|B \cap C) = \frac{0.5265}{0.6665} = 0.789947$

3.2.3 Q3

Total quality management (TQM) is a management philosophy and system of management techniques to improve product and service quality and worker productivity. TQM involves such techniques as teamwork, empowerment of workers, improved communication with customers, evaluation of work processes, and statistical analysis of processes and their output. One hundred Singapore companies were surveyed and it was found that 30 had implemented TQM. Among the 100 companies surveyed, 60 reported an increase in sales last year. Of those 60, 20 had implemented TQM. Suppose one of the 100 surveyed companies is to be selected randomly for additional analysis.

a What is the probability that a firm that implemented TQM is selected? That a firm whose sales increased is selected?

0.3 and 0.6

b Are the two events {TQM implemented} and {Sales increased} independent or dependent? Explain.

Dependent. 0.6 times 0.3 is not 0.2

c Suppose that among the 60 firms reporting sales increases, there were only 18 TQM-implementers (instead of 20). Now are the events {TQM implemented} and {Sales increased} independent or dependent? Explain.

Independent, 0.6 times 0.3 is 0.18

3.2.4 Q4

A company uses three different assembly lines, A1,A2, and A3, to manufacture a particular component. Of those manufactured by line A1, 5% need rework to remedy a defect, whereas 8% of A2's components need rework, and 10% of A3's components need rework. Suppose that 50% of all components are produced by line A1, while 30% are produced by line A2, and 20% come from line A3. If a randomly selected component needs rework, what is the probability that it came

a from line A_1: 0.362319

b from line A_2: 0.347826

c from line A_3: 0.289855

4 Week 3

4.1 Exam Format Questions

4.1.1 Q1

(a)

(b)

(c)

(d)

4.1.2 Q2

4.1.3 Q3

4.1.4 Q4

4.2 Analytical Questions

4.2.1 Q1

i.

ii.

iii.

iv.

v.

4.2.2 Q2

a

b

c

d

e

4.2.3 Q3

4.2.4 Q4