Choose a Positive Integer at Random

Theorem. It is impossible to choose a uniform positive integer at random

Proof. Suppose that we have a uniform distribution over the positive integers and that

$$0 < \varepsilon < 1$$

is a candidate for the probability of a specific positive integer occurring. Then

$$\begin{aligned}
\Pr\left[\left\{1, 2, \cdots, \left\lceil \frac{1}{\varepsilon} \right\rceil + 1\right\}\right] &= \varepsilon \cdot \left(\left\lceil \frac{1}{\varepsilon} \right\rceil + 1\right) \\
&\geq \varepsilon \cdot \left(\frac{1}{\varepsilon} + 1\right) \\
&= 1 + \varepsilon \\
&> 1.
\end{aligned}$$

This is a contradiction as *no* probability may be strictly greater than 1. □

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