C | SecureAES

- High-Performance AES Encryption in C -

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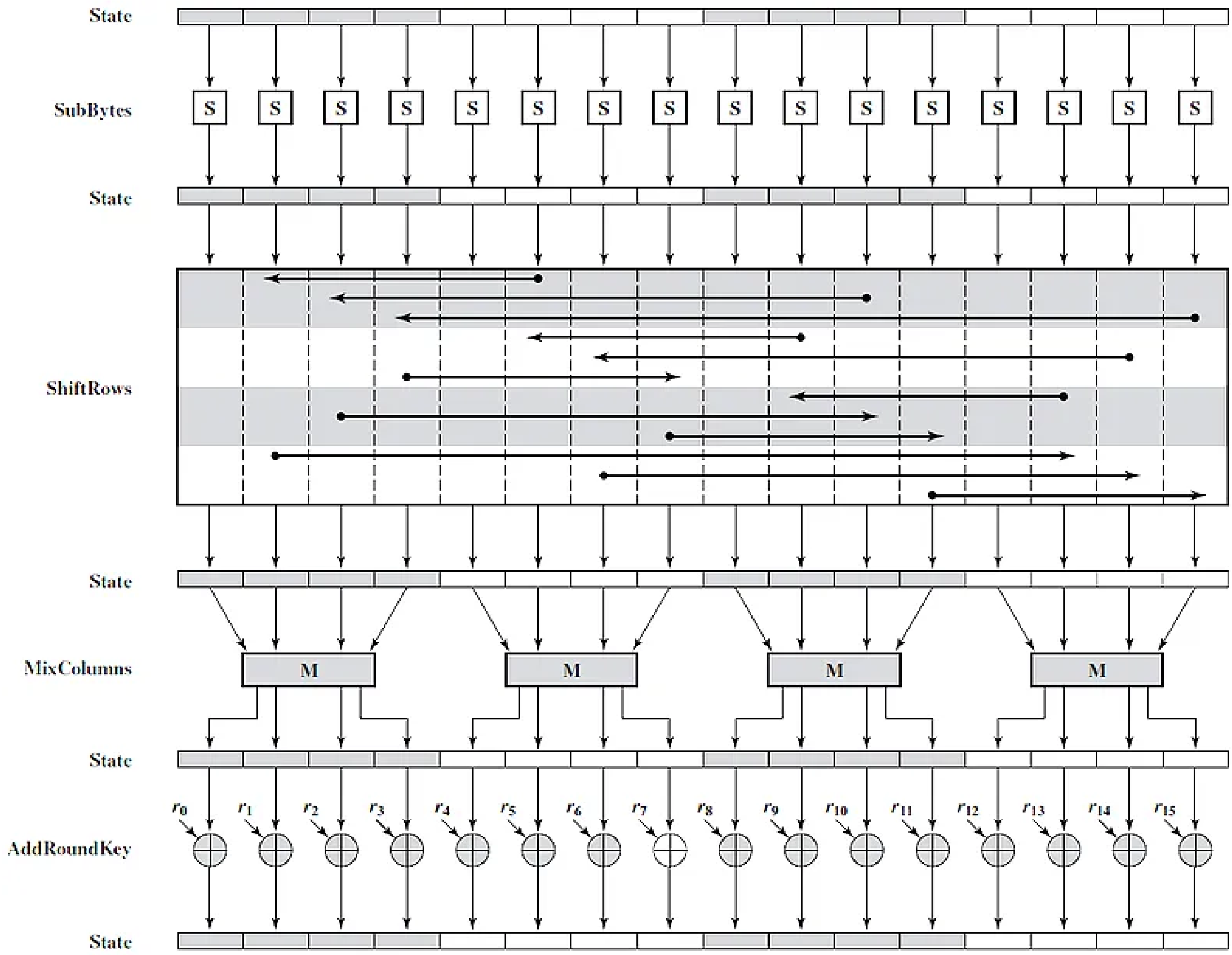

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Acknowledgements

Note (XOR Operation and Modular Reduction in $GF(2^n)$). In the context of Galois Field $GF(2^n)$, particularly in binary polynomial arithmetic, the XOR operation is equivalent to addition and also plays a crucial role in modular reduction. We explore this equivalence through the principles of field theory and polynomial arithmetic.

- **Field Properties:**

  A Galois Field, $GF(p^n)$, is a finite field that contains a finite number of elements, where

  - p is a prime number (base of the field) and
  - n is a positive integer (degree of the field).

  For the binary field $GF(2^n)$, $p = 2$, which implies that every element in this field is either 0 or 1.

- **Addition in $GF(2^n)$:**

  In $GF(2^n)$, the addition of two elements is performed modulo 2. For any two elements $a, b \in GF(2^n)$, the addition is defined as:

  $$a + b = a \oplus b$$

  Since 2 is the base of the field, the addition wraps around upon reaching 2, which is effectively what the XOR operation does.

- **Polynomial Representation:**

  Elements in $GF(2^n)$ can be represented as polynomials where each coefficient is in $GF(2) = \{0, 1\}$. A general element can be written as:

  $$a(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$$

  where $a_i \in \{0, 1\}$ for all i.

- **Modular Reduction:**

  Modular reduction in $GF(2^n)$ involves reducing a polynomial by a fixed irreducible polynomial of degree n, ensuring that the result remains within the field. Let $m(x)$ be the irreducible polynomial. The reduction of a polynomial $f(x)$ is given by: $f(x) \mod m(x)$

- **XOR as Modular Reduction:**

  During modular reduction, the subtraction used in polynomial division becomes XOR, because subtraction and addition are the same in $GF(2)$. Therefore, reducing a polynomial $f(x)$ by $m(x)$ is effectively performed using XOR on the coefficients of corresponding terms.

  For example, if $f(x)$ has a term x^k where $k \geq n$, and $m(x)$ has a term x^k, then reducing $f(x)$ by $m(x)$ involves XORing the coefficients of x^k in $f(x)$ and $m(x)$, effectively eliminating the x^k term in $f(x)$.

In summary, the XOR operation becomes equivalent to both addition and modular reduction in $GF(2^n)$ due to the binary nature of the field. This equivalence simplifies polynomial arithmetic in binary fields, making it a cornerstone of operations in cryptographic algorithms.

Contents

Chapter 1

Block Cipher

Block ciphers are a fundamental component in cryptographic systems. They transform fixed-size blocks of plaintext into ciphertext using a symmetric key. The transformation is designed to be reversible only with knowledge of the key.

1.1 Definition and Structure

- **Secure Pseudo-Random Permutation (PRP) and Substitution Groups:**
  - **Definition:** A block cipher is considered a secure PRP if it is indistinguishable from a random permutation of the input bits, making it resistant to cryptanalysis.
  - **Substitution Groups:** Block ciphers often use substitution-permutation networks (SPNs) that include substitution groups. These groups perform non-linear transformations, crucial for creating cryptographic strength.
- **Confidentiality for Fixed n-bit Data (Blocks):**
  - **Fixed Block Size:** Block ciphers encrypt and decrypt data in fixed-size blocks (commonly 64 or 128 bits). This fixed size is crucial for the algorithm's structure and security.
  - **Padding Schemes:** When the data doesn't fit perfectly into a block, padding schemes are used to fill the remaining space, ensuring consistent block sizes.
- **Block Cipher Operation Modes for Variable-Length Data:**
  - **Mode of Operation:** To handle variable-length data, block ciphers use different modes of operation like CBC (Cipher Block Chaining), CFB (Cipher Feedback), and GCM (Galois/Counter Mode).
  - **Ensuring Security:** Each mode offers distinct features for security and efficiency, often enhancing the cipher's resistance to various attack vectors.
- **Advantages Over Asymmetric Key Cryptography:**
  - **High-Speed Computation:** Block ciphers are generally faster and require less computational power compared to asymmetric key cryptography.
  - **Suitability:** This makes them suitable for encrypting large volumes of data and in environments with limited resources.

- **Deriving Other Cryptographic Functions:**
  - **Versatility:** Block ciphers can be used to design other cryptographic functions like hash functions, message authentication codes (MACs), and random number generators.
  - **Construction Techniques:** Techniques like Cipher Block Chaining-MAC (CBC-MAC) and Counter mode (CTR) are examples of how block ciphers can be adapted for these purposes.

Block ciphers are a critical element in the cryptographic landscape, providing a versatile and efficient means for securing digital data. Their adaptability and robustness make them an indispensable tool in the design of secure communication protocols and cryptographic systems.

1.2 Modes of Operations

Table 1.1: Comparison of Modes

Mode	Integrity	Authentication	EncryptBlk	DecryptBlk	Padding	IV	$\|P\| \stackrel{?}{=} \|C\|$
ECB	O	X	O	O	O	X	$\|P\| < \|C\|$
CBC	O	X	O	O	O	O	$\|P\| < \|C\|$
OFB	O	X	O	X	X	O	$\|P\| = \|C\|$
CFB	O	X	O	X	X	O	$\|P\| = \|C\|$
CTR	O	X	O	X	X	O	$\|P\| = \|C\|$
CBC – CS	O	X	O	O	X	O	$\|P\| = \|C\|$

1.2.1 Padding

Block ciphers require input lengths to be a multiple of the block size. Padding is used to extend the last block of plaintext to the required length. Without proper padding, the encryption process may be insecure or infeasible.

There are several padding schemes used in practice, such as:

Table 1.2: Padding Standards in Block Ciphers

Standard Name	Padding Method
PKCS#7	Pad with bytes all the same value as the number of padding bytes `...dd | dd dd dd dd dd dd dd dd dd dd dd dd 04 04 04 04 |`
ANSI X9.23	Pad with zeros, last byte is the number of padding bytes `...dd | dd dd dd dd dd dd dd dd dd dd dd 00 00 00 00 05 |`
ISO/IEC 7816-4	First byte is '80' (hex), followed by zeros `...dd | dd dd dd dd dd dd dd dd dd dd 80 00 00 00 00 00 |`
ISO 10126	Pad with random bytes, last byte is the number of padding bytes `...dd | dd dd dd dd dd dd dd dd dd dd 2e 49 1b c1 aa 06 |`

1.2.2 ECB (Electronic CodeBook)

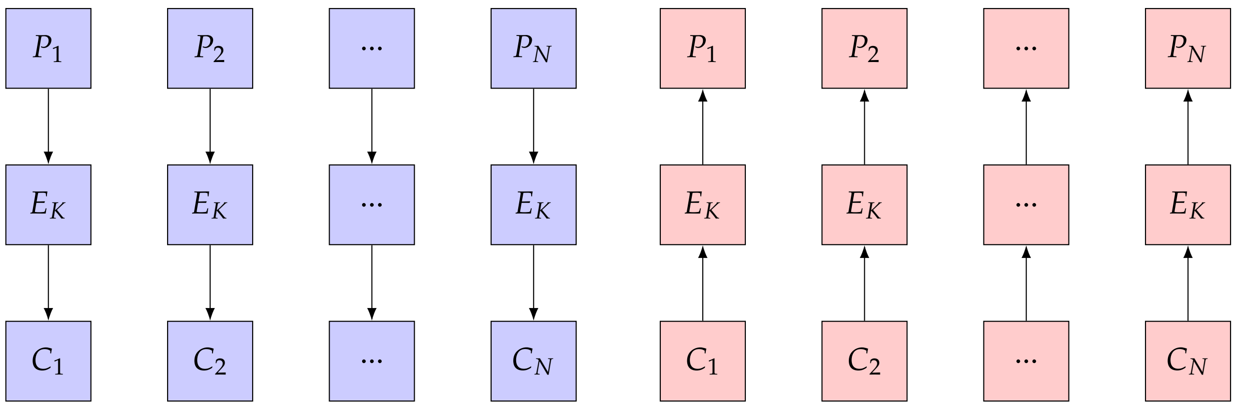
Algorithm 1: Electronic CodeBook

Input: K and $P = P_1 \parallel \cdots \parallel P_N$ $(P_i \in \{0, 1\}^n)$
Output: $C = C_1 \parallel \cdots \parallel C_N$ $(C_i \in \{0, 1\}^n)$

for $i \leftarrow 1$ **to** N **do**
  $C_i \leftarrow \mathsf{EncryptBlk}(K, P_i)$;
end
return $C = C_1 \parallel \cdots \parallel C_N$;

Input: K and $C = C_1 \parallel \cdots \parallel C_N$ $(C_i \in \{0, 1\}^n)$
Output: $P = P_1 \parallel \cdots \parallel P_N$ $(P_i \in \{0, 1\}^n)$

for $i \leftarrow 1$ **to** N **do**
  $P_i \leftarrow \mathsf{DecryptBlk}(K, C_i)$;
end
return $C = C_1 \parallel \cdots \parallel C_N$;



Remark 1.1.

(1) For a pair of plaintext/ciphertext (P, C) and (P', C'),

$$P = P' \implies C = C'.$$

(2) A single-bit error in the ciphertext affects only the corresponding block in the decryption.

(3) Block order changes, as well as additions/deletions, are possible. Therefore, to ensure the integrity of the ciphertext (i.e., detection or prevention of tampering), it should be used in conjunction with a checksum or a Message Authentication Code (MAC).

1.2.3 CBC (Cipher Block Chaining)

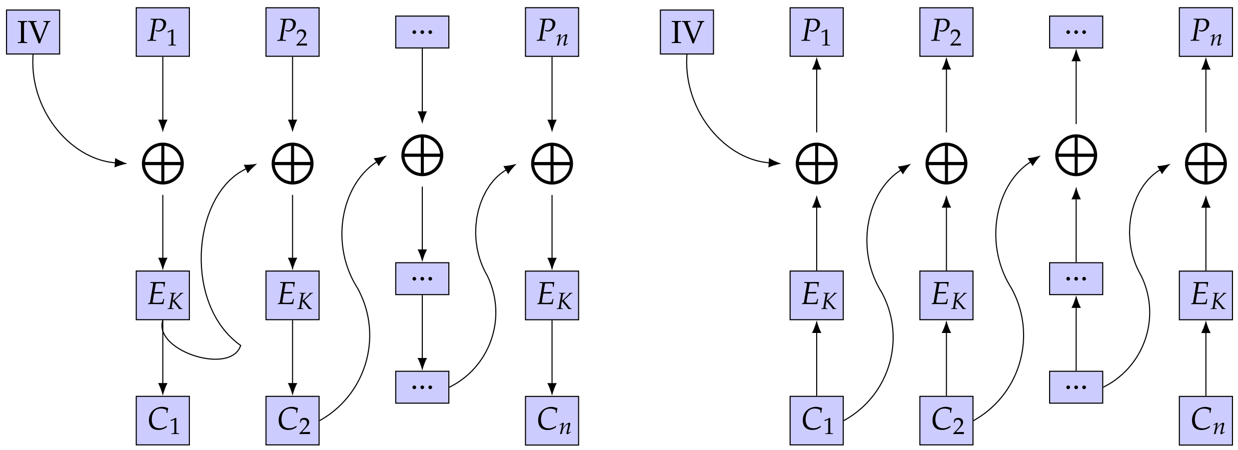
Algorithm 2: Cipher Block Chaining

Input: K, IV and $P = P_1 \parallel \cdots \parallel P_N$
Output: $C = C_1 \parallel \cdots \parallel C_N$

$C_0 \leftarrow IV$;
for $i \leftarrow 1$ **to** N **do**
  $C_i \leftarrow \mathsf{EncryptBlk}(K, P_i \oplus C_{i-1})$;
end
return $C = C_1 \parallel \cdots \parallel C_N$;

Input: K, IV and $C = C_1 \parallel \cdots \parallel C_N$
Output: $P = P_1 \parallel \cdots \parallel P_N$

$C_0 \leftarrow IV$;
for $i \leftarrow 1$ **to** N **do**
  $P_i \leftarrow C_{i-1} \oplus \mathsf{DecryptBlk}(K, C_i)$;
end
return $P = P_1 \parallel \cdots \parallel P_N$;



Remark 1.2.

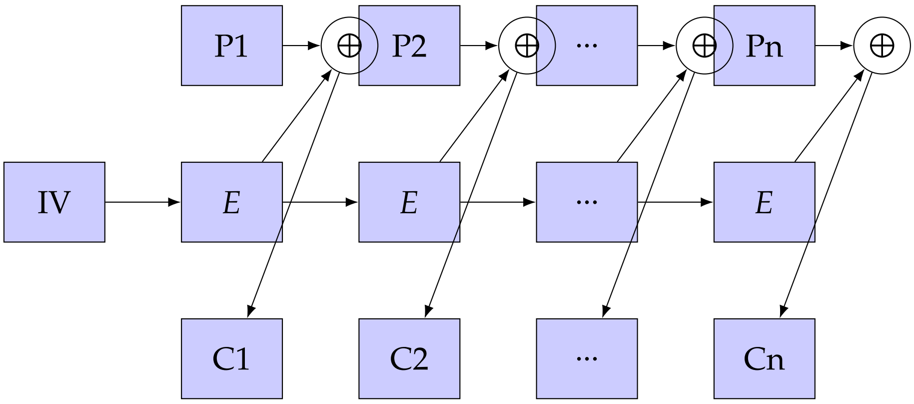
(1) For a set of plaintext/ciphertext pairs (IV, P, C) and (IV', P', C'),

$$IV = IV' \wedge P = P' \implies C = C'.$$

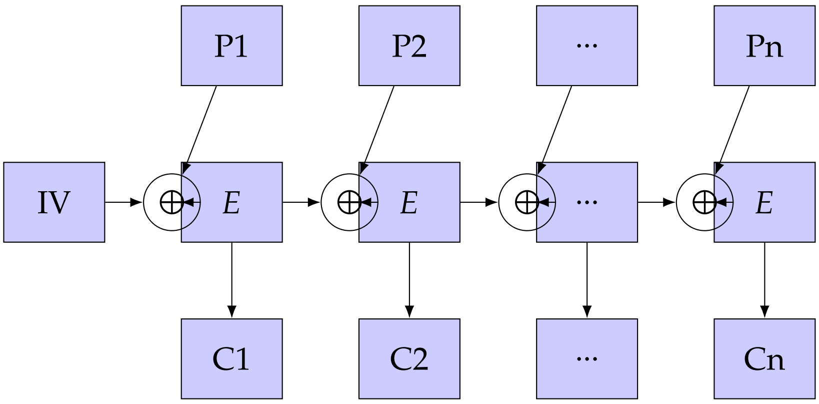
(2) A one-bit error in the ciphertext will affect the corresponding block and the subsequent block in the decryption process, thereby enabling the detection of such errors.

(3) Altering the order of blocks, as well as adding or deleting them, is not possible.

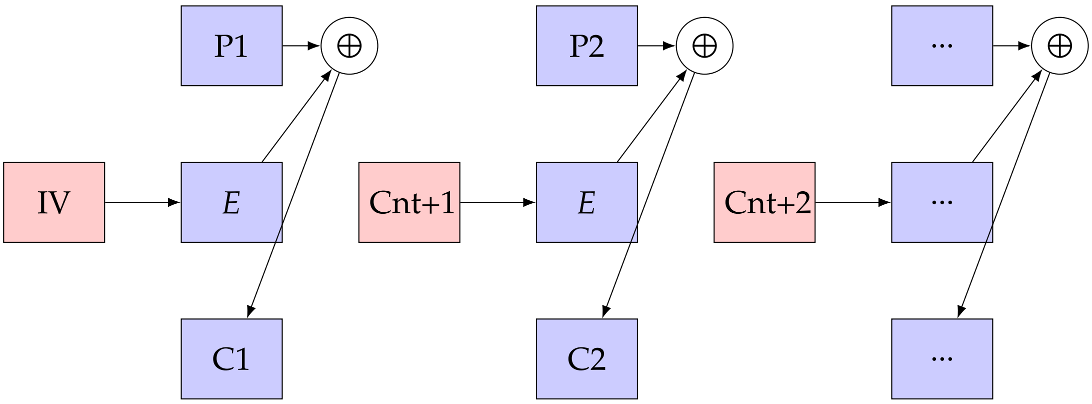
1.2.4 OFB (Output FeedBack)

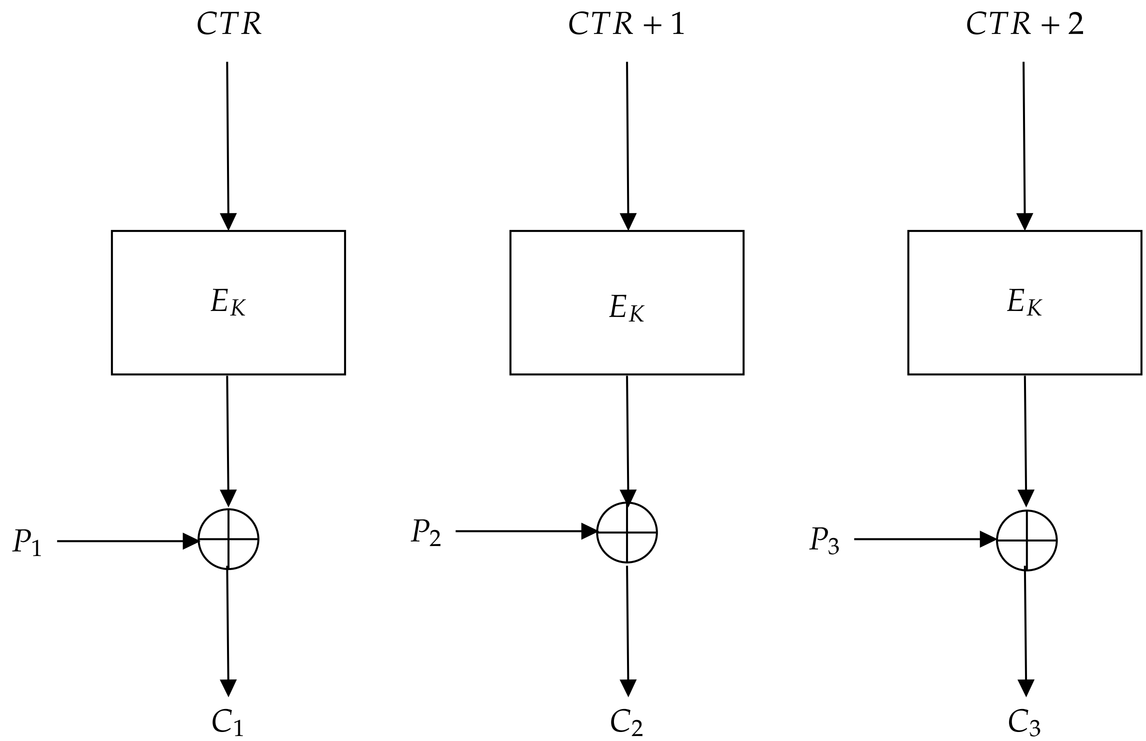


1.2.5 CFB (Ciphertext FeedBack)

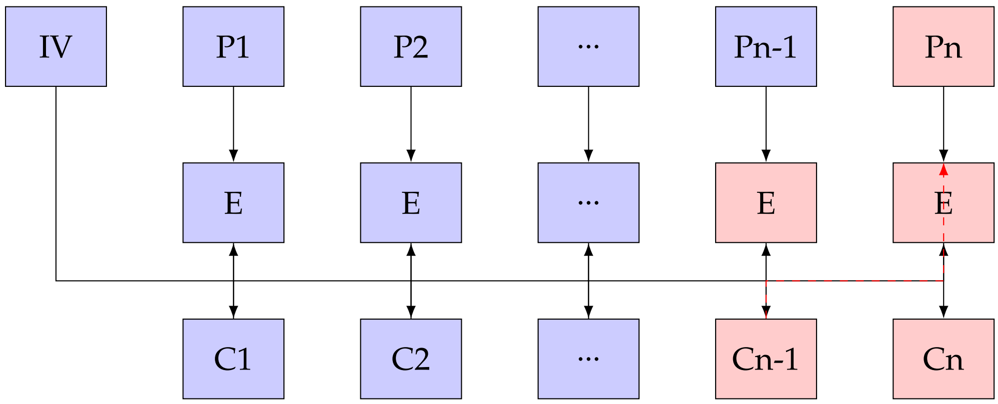


1.2.6 CTR (CounTeR)





1.2.7 CBC – CS **(Ciphertext Stealing)**



Chapter 2

AES Algorithm

2.1 S-Box ($GF(2^8)$)

Let $m(x) := x^8 + x^4 + x^3 + x + 1$.

$$\begin{aligned} GF(2^8) &= GF(2)[x]/\langle x^8 + x^4 + x^3 + x + 1\rangle \\ &= GF(2)[x]/\langle m(x)\rangle \\ &= \left\{b_0 + b_1 x + \cdots + b_6 x^6 + b_7 x^7 : b_0, b_1, \cdots, b_6, b_7 \in GF(2)\right\} \\ &= \left\{\sum_{i=0}^{7} b_i x^i : b_i \in \{0,1\}\right\}. \end{aligned}$$

Let $f(x), g(x) \in GF(2^8)$, say,

$$f(x) = \sum_{i=0}^{7} a_i x^i + \langle m(x)\rangle, \quad g(x) = \sum_{j=0}^{7} b_j x^j + \langle m(x)\rangle.$$

Then

$$f(x) + g(x) = \sum_{i=0}^{7} (a_i + b_i)x^i + \langle m(x)\rangle = \sum_{i=0}^{7} (a_i \oplus b_i)x^i + \langle m(x)\rangle.$$

Here, $\oplus : GF(2) \times GF(2) \longrightarrow GF(2)$. And

$$f(x) \cdot g(x) = m(x) \cdot q(x) + r(x).$$

Here $\deg r \le 7$.

We define

$$\begin{array}{rccc} \texttt{xtime} & : & GF(2^8) & \longrightarrow & GF(2^8) \\ & & f(x) & \longmapsto & x \cdot f(x) \end{array}.$$

Then

1. $f(x) = a_0 + a_1 x + \cdots + a_6 x^6 + a_7 x^7$

2.

$$\begin{aligned} x \cdot f(x) &= a_0 x + a_1 x^2 + \cdots + a_6 x^7 + a_7 x^8 \\ &= a_0 x + a_1 x^2 + \cdots + a_6 x^7 + a_7 (x^4 + x^3 + x + 1) \quad \because x^8 \equiv x^4 + x^3 + x + 1 \pmod{m(x)} \\ &= \begin{cases} \sum_{i=0}^{6} a_i x^{i+1} + 0 & : a_7 = 0 \\ \sum_{i=0}^{6} a_i x^{i+1} + (x^4 + x^3 + x + 1) & : a_7 = 1 \end{cases} \\ &= \begin{cases} (f(x) \ll 1) & : a_7 = 0 \\ (f(x) \ll 1) \oplus \texttt{0x1b} & : a_7 = 1 \end{cases} \end{aligned}$$

```
u8 GF256_xtime(u8 f) {
    return (f & 0x80) ? (f << 1) ^ 0x1b : f << 1;
}
```

$$\begin{aligned} g(x) \cdot f(x) &= \sum_{i=0}^{7} g(x) a_i x^i \\ &= g(x) a_0 + \sum_{i=1}^{7} g(x) a_i x^i \\ &= g(x) a_0 + x \sum_{i=1}^{7} g(x) a_i x^{i-1} \\ &= g(x) a_0 + x \left(g(x) a_1 + \sum_{i=2}^{7} g(x) a_i x^{i-2} \right) \\ &= g(x) a_0 + x \left(g(x) a_1 + x \left(g(x) a_2 + \sum_{i=3}^{7} g(x) a_i x^{i-3} \right) \right) \\ &= \cdots \\ &= g(x) a_0 + x(g(x) a_1 + x(g(x) a_2 + x(g(x) a_3 + x(g(x) a_4 + x(g(x) a_5 + x(g(x) a_6 + x(g(x) a_7))))))) \end{aligned}$$

That is,

$$\begin{array}{lll} x & \cdot 0 & +(g(x) \cdot a_7) \\ x & \cdot (g(x) \cdot a_7) & +(g(x) \cdot a_6) \\ x & \cdot (x \cdot (g(x) \cdot a_7) + (g(x) \cdot a_6)) & +(g(x) \cdot a_5) \end{array}$$

Step 1. $x \cdot 0 + g(x) \cdot a_7$

Step 2. $x \cdot (g(x) \cdot a_7) + (g(x) \cdot a_6)$

Step 3. $x \cdot (x \cdot (g(x) \cdot a_7) + (g(x) \cdot a_6)) + (g(x) \cdot a_5)$

$\ldots$

Final. $g \cdot a_0 + x(g \cdot a_1 + \cdots + x(g \cdot a_5 + x(g \cdot a_6 + x(g \cdot a_7))))$

```
u8 GF256_mul(u8 f, u8 g) {
    u8 h = 0x00; // h = f * g
    u8 coef;

    for (i8 i = 7; i >= 0; i--) {
        coef = (f >> i) & 0x01; // Get the coefficient f_i
        h = GF256_xtime(h);     // Multiply h by x
        if (coef == 1) h ^= g;  // If f_i = 1, add g(x) to h(x)
    }

    return h;
}
```

```
u8 GF256_MUL(u8 f, u8 g) {
    u8 h = 0;
    for (i8 = 0; i < 8; i++) {
        h ^= (g & 1) * f;
        g >>= 1;     // g = g / 2.

        u8 msb = f & 0x80;
        f <<= 1;     // f = f * 2.
        // Conditional XOR without branching.
        f ^= (0 - ((f & 0x80) >> 7)) & 0x1b; // (f & 0x80) is the
           msb
    }

    return h;
}
```

2.2 MixColumns ($GF(2^8)[x]/\langle x^4+1\rangle$)

$$\begin{array}{lcccc} \textsf{MixColumns} & : & \{0,1\}^{128} & \longrightarrow & \{0,1\}^{128} \\ & & \sum_{i=0}^{127} a_i x^i & \longmapsto & \sum_{j=0}^{127} b_j x^j \end{array}$$

X_0	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}

$$\Downarrow$$

X'_0	X'_1	X'_2	X'_3	X'_4	X'_5	X'_6	X'_7	X'_8	X'_9	X'_{10}	X'_{11}	X'_{12}	X'_{13}	X'_{14}	X'_{15}

$$\begin{array}{|c|c|c|c|}\hline X_0 & X_4 & X_8 & X_{12} \\ \hline X_1 & X_5 & X_9 & X_{13} \\ \hline X_2 & X_6 & X_{10} & X_{14} \\ \hline X_3 & X_7 & X_{11} & X_{15} \\ \hline \end{array} \quad \Longrightarrow \quad \begin{array}{|c|c|c|c|}\hline X'_0 & X'_4 & X'_8 & X'_{12} \\ \hline X'_1 & X'_5 & X'_9 & X'_{13} \\ \hline X'_2 & X'_6 & X'_{10} & X'_{14} \\ \hline X'_3 & X'_7 & X'_{11} & X'_{15} \\ \hline \end{array}$$

Consider

$$GF(2^8)[x]/\langle x^4+1\rangle = \left\{a_0 + a_1x + a_2x^3 + a_3x^3 : a_i \in GF(2^8)\right\}.$$

Note that $x^5 = (x^4+1)x + x \equiv x \pmod{\langle x^4+1\rangle}$. We choose

$$a(x) = (\texttt{0x03})\cdot x^3 + (\texttt{0x01})\cdot x^2 + (\texttt{0x01})\cdot x + (\texttt{0x02}) \in GF(2^8)[x]/\langle x^4+1\rangle,$$

where

$$\begin{aligned}
\texttt{0x01} &= \texttt{0b 0000 0001} & &= 1,\\
\texttt{0x02} &= \texttt{0b 0000 0010} & &= x,\\
\texttt{0x03} &= \texttt{0b 0000 0011} & &= x+1.
\end{aligned}$$

Let $b(x) = b_0 + b_1x + b_2x^2 + b_3x^3$, $c(x) = c_0 + c_1x + c_2x^2 + c_3x^3$, and let

$$\begin{pmatrix} b_0\\ b_1\\ b_2\\ b_3 \end{pmatrix} \mapsto \begin{pmatrix} c_0\\ c_1\\ c_2\\ c_3 \end{pmatrix} = \begin{pmatrix} a_0\cdot b_0 \bmod x^4+1\\ a_1\cdot b_1 \bmod x^4+1\\ a_2\cdot b_2 \bmod x^4+1\\ a_3\cdot b_3 \bmod x^4+1 \end{pmatrix}.$$

Then

$$\begin{aligned}
a(x)\cdot b(x) = (a_3\cdot b_3)\,&x^6 + (a_3\cdot b_2 + a_2\cdot b_3)\,x^5\\
&+ (a_3\cdot b_1 + a_2\cdot b_2 + a_1\cdot b_3)\,x^4\\
&+ (a_3\cdot b_0 + a_2\cdot b_1 + a_1\cdot b_2 + a_0\cdot b_3)x^3\\
&+ (a_2\cdot b_0 + a_1\cdot b_1 + a_0\cdot b_2)x^2\\
&+ (a_1\cdot b_0 + a_0\cdot b_1)x\\
&+ (a_0\cdot b_0)\\
\\
= (a_3\cdot b_3)\,&x^2 + (a_3\cdot b_2 + a_2\cdot b_3)\,x\\
&+ (a_3\cdot b_1 + a_2\cdot b_2 + a_1\cdot b_3)\,1\\
&+ (a_3\cdot b_0 + a_2\cdot b_1 + a_1\cdot b_2 + a_0\cdot b_3)x^3\\
&+ (a_2\cdot b_0 + a_1\cdot b_1 + a_0\cdot b_2)x^2\\
&+ (a_1\cdot b_0 + a_0\cdot b_1)x\\
&+ (a_0\cdot b_0) \quad \text{over} \quad GF(2^8)[x]/\langle x^4+1\rangle\\
\\
&= (a_3\cdot b_0 + a_2\cdot b_1 + a_1\cdot b_2 + a_0\cdot b_3)x^3\\
&= (a_2\cdot b_0 + a_1\cdot b_1 + a_0\cdot b_2 + a_3\cdot b_3)x^2\\
&= (a_1\cdot b_0 + a_0\cdot b_1 + a_3\cdot b_2 + a_2\cdot b_3)x\\
&= (a_0\cdot b_0 + a_3\cdot b_1 + a_2\cdot b_2 + a_1\cdot b_3)\\
\\
&= c(x).
\end{aligned}$$

Thus, we have

$$
\begin{aligned}
T \quad : \quad [GF(2^8)]^4 &\longrightarrow [GF(2^8)]^4 \\
\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} &\longmapsto \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} a_3 & a_2 & a_1 & a_0 \\ a_2 & a_1 & a_0 & a_3 \\ a_1 & a_0 & a_3 & a_2 \\ a_0 & a_3 & a_2 & a_1 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix},
\end{aligned}
$$

where

$$
\begin{pmatrix} a_3 & a_2 & a_1 & a_0 \\ a_2 & a_1 & a_0 & a_3 \\ a_1 & a_0 & a_3 & a_2 \\ a_0 & a_3 & a_2 & a_1 \end{pmatrix} = \begin{pmatrix} \texttt{0x02} & \texttt{0x03} & \texttt{0x01} & \texttt{0x01} \\ \texttt{0x01} & \texttt{0x02} & \texttt{0x03} & \texttt{0x01} \\ \texttt{0x01} & \texttt{0x01} & \texttt{0x02} & \texttt{0x03} \\ \texttt{0x03} & \texttt{0x01} & \texttt{0x01} & \texttt{0x02} \end{pmatrix}.
$$

Here, T has GF-MUL $\times$ 16.

Let $a \in \{\texttt{0}, \texttt{1}\}^8$. Then

$$
\begin{aligned}
\texttt{02} \otimes a &= \texttt{xtime}\,(a)\,, \\
\texttt{03} \otimes a &= (\texttt{02} \oplus \texttt{01}) \otimes a \\
&= (\texttt{02} \otimes a) \oplus (\texttt{01} \otimes a) \\
&= \texttt{xtime}\,(a) \oplus a.
\end{aligned}
$$

Thus,

$$
\begin{aligned}
c_0 &= (\texttt{02} \otimes b_0) \oplus (\texttt{03} \otimes b_1) \oplus b_2 \oplus b_3 \\
&= \texttt{xtime}\,(b_0) \oplus \texttt{xtime}\,(b_1) \oplus b_1 \oplus b_2 \oplus b_3 \\
&= \texttt{xtime}\,(b_0 \oplus b_1) \oplus b_1 \oplus b_2 \oplus b_3 \qquad \because\ ax + ab = x(a+b) \\
&= \texttt{temp0} \oplus b_1 \oplus b_2 \oplus b_3.
\end{aligned}
$$

and so

$$
\begin{aligned}
c_0 &= (b_0 \oplus b_1 \oplus b_2 \oplus b_3) \oplus \texttt{temp0} \oplus b_0 && \text{with } \texttt{temp0} = \texttt{xtime}\,(b_0 \oplus b_1), \\
c_1 &= (b_0 \oplus b_1 \oplus b_2 \oplus b_3) \oplus \texttt{temp1} \oplus b_1 && \text{with } \texttt{temp1} = \texttt{xtime}\,(b_1 \oplus b_2), \\
c_2 &= (b_0 \oplus b_1 \oplus b_2 \oplus b_3) \oplus \texttt{temp2} \oplus b_2 && \text{with } \texttt{temp2} = \texttt{xtime}\,(b_2 \oplus b_3), \\
c_3 &= (b_0 \oplus b_1 \oplus b_2 \oplus b_3) \oplus \texttt{temp3} \oplus b_3 && \text{with } \texttt{temp3} = \texttt{xtime}\,(b_3 \oplus b_0).
\end{aligned}
$$

That is,

$\texttt{sum} \leftarrow b_0 \oplus b_1 \oplus b_2 \oplus b_3$;
$c_0 \leftarrow \texttt{sum} \oplus \texttt{temp0} \oplus b_0$;
$c_0 \leftarrow \texttt{sum} \oplus \texttt{temp1} \oplus b_1$;
$c_0 \leftarrow \texttt{sum} \oplus \texttt{temp2} \oplus b_2$;
$c_0 \leftarrow \texttt{sum} \oplus \texttt{temp3} \oplus b_3$;

It has $\texttt{xtime}\,() \times 4$.

Consider

$$\begin{aligned}
[a(x)]^1 &= a_3x^3 + a_2x^2 + a_1x + a_0,\\
[a(x)]^2 &= (a_3^2 + a_1^2)x^2 + (a_2^2 + a_0^2),\\
[a(x)]^4 &= a_3^4 + a_2^4 + a_1^4 + a_0^4.
\end{aligned}$$

Let $a(x) = (\texttt{03})x^3 + x^2 + x + (\texttt{02})$. Then

$$\begin{aligned}
[a(x)]^2 &= (\texttt{04})x^2 + (\texttt{05}),\\
[a(x)]^3 &= (\texttt{0b})x^3 + (\texttt{0d})x^2 + (\texttt{09})x + (\texttt{0e}),
\end{aligned}$$

and so $[a(x)]^{-1} = [a(x)]^3 = (\texttt{0b})x^3 + (\texttt{0d})x^2 + (\texttt{09})x + (\texttt{0e})$.

Chapter 3

AES-128

3.1 Overview of AES-128

- $\mathsf{KeyExpansion} : \{\mathtt{0}, \mathtt{1}\}^{128} \to \{\mathtt{0}, \mathtt{1}\}^{1408=4\cdot(10+1)\cdot 32}$.
- $\mathsf{AddRoundKey} : \{\mathtt{0}, \mathtt{1}\}^{128} \times \{\mathtt{0}, \mathtt{1}\}^{128} \to \{\mathtt{0}, \mathtt{1}\}^{128}$.
- $\mathsf{SubBytes}/\mathsf{ShiftRows}/\mathsf{MixColumns} : \{\mathtt{0}, \mathtt{1}\}^{128} \to \{\mathtt{0}, \mathtt{1}\}^{128}$.

Algorithm 3: Encryption of AES-128

Input: block src $\in \{\mathtt{0}, \mathtt{1}\}^{128}$, round-keys $\{rk_i\}_{i=0}^{11}$ $(rk_i \in \{\mathtt{0}, \mathtt{1}\}^{128})$
Output: block dst $\in \{\mathtt{0}, \mathtt{1}\}^{128}$

$t \leftarrow \mathsf{AddRoundKey}(\texttt{src}, rk_0)$;
for $i \leftarrow 1$ **to** 9 **do**
  $t \leftarrow (\mathsf{MixColumns} \circ \mathsf{ShiftRows} \circ \mathsf{SubBytes})(t)$;
  $t \leftarrow \mathsf{AddRoundKey}(t, rk_i)$;
end
$t \leftarrow (\mathsf{ShiftRows} \circ \mathsf{SubBytes})(t)$;
$t \leftarrow \mathsf{AddRoundKey}(t, rk_{10})$;
dst $\leftarrow t$;
return dst;

Algorithm 4: Decryption of AES-128

Input: block src $\in \{\mathtt{0}, \mathtt{1}\}^{128}$, round-keys $\{rk_i\}_{i=0}^{11}$ $(rk_i \in \{\mathtt{0}, \mathtt{1}\}^{128})$
Output: block dst $\in \{\mathtt{0}, \mathtt{1}\}^{128}$

$t \leftarrow \mathsf{AddRoundKey}(\texttt{src}, rk_{10})$;
for $i \leftarrow 9$ **to** 1 **do**
  $t \leftarrow (\mathsf{InvSubBytes} \circ \mathsf{InvShiftRows})(t)$;
  $t \leftarrow \mathsf{AddRoundKey}(t, rk_i)$;
  $t \leftarrow \mathsf{InvMixColumns}(t)$;
end
$t \leftarrow (\mathsf{InvShiftRows} \circ \mathsf{InvSubBytes})(t)$;
$t \leftarrow \mathsf{AddRoundKey}(t, rk_0)$;
dst $\leftarrow t$;
return dst;

3.2 Functions and Constants used in AES

3.2.1 Key Expansion

- RotWord : $\{0,1\}^{32} \to \{0,1\}^{32}$ is defined by

$$\text{RotWord}\left(X_0 \parallel X_1 \parallel X_2 \parallel X_3\right) := X_1 \parallel X_2 \parallel X_3 \parallel X_0 \quad \text{for} \quad X_i \in \{0,1\}^8 .$$

Code 3.1: RotWord rotates the input word left by one byte

```
u32 RotWord(u32 word) {
    return (word << 0x08) | (word >> 0x18);
}
```

- SubWord : $\{0,1\}^{32} \to \{0,1\}^{32}$ is defined by

$$\text{SubWord}(X_0 \parallel X_1 \parallel X_2 \parallel X_3) := s(X_0) \parallel s(X_1) \parallel s(X_2) \parallel s(X_3) \quad \text{for} \quad X_i \in \{0,1\}^8 .$$

Here, $s : \{0,1\}^8 \to \{0,1\}^8$ is the S-box.

Code 3.2: SubWord applies the S-box to each byte of the input word

```
u32 SubWord(u32 word) {
    return (u32)s_box[word >> 0x18] << 0x18 |
        (u32)s_box[(word >> 0x10) & 0xFF] << 0x10 |
        (u32)s_box[(word >> 0x08) & 0xFF] << 0x08 |
        (u32)s_box[word & 0xFF];
}
```

- Round Constant rCon:

The constant $\texttt{rCon}_i \in \mathbb{F}_{2^8}$ used in generating the i-th round key corresponds to the value of x^{i-1} in the binary finite field $\mathbb{F}_{2^8}$ and is as follows:

i	1	2	3	4	5	6	7	8	9	10
$\texttt{rCon}_i$	0x01	0x02	0x04	0x08	0x10	0x20	0x40	0x80	0x1b	0x36

Code 3.3: rCon Array Declaration

```
static const u32 rCon[10] = {
    0x01000000, 0x02000000, 0x04000000, 0x08000000,
    0x10000000, 0x20000000, 0x40000000, 0x80000000,
    0x1b000000, 0x36000000
};
```

Algorithm 5: Key Schedule (AES-128)

Input: User key $uk = (uk_0, \ldots, uk_{15})$ ($uk_i \in \{0,1\}^8$); // $uk \in \{0,1\}^{128}$ is 16-byte
Output: round-keys $\{rk_i\}_{i=0}^{43}$ ($rk_i \in \{0,1\}^{32}$); // $\{rk_i\}_{i=0}^{43} \in \{0,1\}^{1408}$ is 176-byte

```
rk_0 ← uk_0 || uk_1 || uk_2 || uk_3;
rk_1 ← uk_4 || uk_5 || uk_6 || uk_7;
rk_2 ← uk_8 || uk_9 || uk_10 || uk_11;
rk_3 ← uk_12 || uk_13 || uk_14 || uk_15;
for i = 4 to 43 do
    t ← rk_{i-1};
    if i mod 4 = 0 then
        /* SubWord ∘ RotWord : {0,1}^32 → {0,1}^32 */
        t ← RotWord(t);
        t ← SubWord(t);
        t ← t ⊕ (rCon_{i/4} || 0x00 || 0x00 || 0x00);
    end
    rk_i ← rk_{i-4} ⊕_32 t;
end
```

Code 3.4: AES-128 Key Expansion

```
void KeyExpansion(const u8* uKey, u32* rKey) {
    u32 temp;
    int i = 0;

    // Copy the input key to the first round key
    while (i < 4) {
        rKey[i] = (u32)uKey[4*i] << 0x18 |
        (u32)uKey[4*i+1] << 0x10 |
        (u32)uKey[4*i+2] << 0x08 |
        (u32)uKey[4*i+3];
        i++;
    }

    i = 4;

    // Generate the remaining round keys
    while (i < 44) {
        temp = rKey[i-1];
        if (i % 4 == 0) {
            temp = SubWord(RotWord(temp)) ^ rCon[i/4-1];
        }
        rKey[i] = rKey[i-4] ^ temp;
        i++;
    }
}
```

3.2.2 AddRoundKey

- AddRoundKey : $\{0,1\}^{128} \times \{0,1\}^{128} \longrightarrow \{0,1\}^{128}$ is defined by

$$\text{AddRoundKey}\left(\{X_i\}_{i=0}^{15}, \{rk_i\}_{i=0}^{3}\right) := \{X_i \oplus_8 uk_i\}_{i=0}^{15}.$$

Code 3.5: AES AddRoundKey

```c
void AddRoundKey(u8* state, const u32* rKey) {
    for (int i = 0; i < AES_BLOCK_SIZE; i++) {
        // i =  0,  1,  2,  3 => wordIndex = 0
        // i =  4,  5,  6,  7 => wordIndex = 1
        // i =  8,  9, 10, 11 => wordIndex = 2
        // i = 12, 13, 14, 15 => wordIndex = 3
        int wordIndex = i / 4;

        // i =  0,  1,  2,  3 => bytePosition = 0,  1,  2,  3
        // i =  4,  5,  6,  7 => bytePosition = 0,  1,  2,  3
        // i =  8,  9, 10, 11 => bytePosition = 0,  1,  2,  3
        // i = 12, 13, 14, 15 => bytePosition = 0,  1,  2,  3
        int bytePosition = i % 4;
/*
 * +-------+------------+--------------+---------------------+
 * | i     | wordIndex  | bytePosition | shiftedWord         |
 * +-------+------------+--------------+---------------------+
 * | 0-3   | 0          | 0            | rKey[0] >> 0x18     |
 * |       |            | 1            | rKey[0] >> 0x10     |
 * |       |            | 2            | rKey[0] >> 0x08     |
 * |       |            | 3            | rKey[0]             |
 * -----------------------------------------------------------
 * | 4-7   | 1          | 0            | rKey[1] >> 24       |
 * |       |            | 1            | rKey[1] >> 16       |
 * |       |            | 2            | rKey[1] >> 8        |
 * |       |            | 3            | rKey[1]             |
 * -----------------------------------------------------------
 * | ...   | ...        | ...          | ...                 |
 * -----------------------------------------------------------
 * | 15    | 3          | 3            | rKey[3]             |
 * +-------+------------+--------------+---------------------+
*/
        u32 shiftedWord =
            rKey[wordIndex] >> (8 * (3 - bytePosition));

        u8 keyByte = shiftedWord & 0xFF;
        state[i] ^= keyByte;

/* Extract the corresponding byte from the round key word */
// state[i] ^= (rKey[i / 4] >> (8 * (3 - (i % 4)))) & 0xFF;
    }
}
```

3.2.3 SubBytes / InvSubBytes

- SubBytes : $\{0, 1\}^{128} \to \{0, 1\}^{128}$ is defined by

$$\mathrm{SubBytes}(\{X_i\}_{i=0}^{15}) = \left\{s(X_i)\right\}_{i=0}^{15}.$$

- InvSubBytes : $\{0, 1\}^{128} \to \{0, 1\}^{128}$ is defined by

$$\mathrm{SubBytes}(\{X_i\}_{i=0}^{15}) = \left\{s^{-1}(X_i)\right\}_{i=0}^{15}.$$

Table 3.1: Substitution Box

	00	01	02	03	04	05	06	07	08	09	0a	0b	0c	0d	0e	0f
00	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
10	ca	82	...	...	...	...	...	...	...	...	...	...	...	...	...	...
30	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
40	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
50	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
60	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
70	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
80	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
90	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
a0	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
b0	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
c0	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
d0	...	...	...	...	...	...	...	...	...	...	...	...	...	c1	...	...
e0	...	...	...	...	...	...	...	...	...	...	...	...	...	...	28	...
f0	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	16

Code 3.6: Byte Substitution

```c
void SubBytes(u8* state) {
    for (int i = 0; i < AES_BLOCK_SIZE; i++) {
        state[i] = s_box[state[i]];
    }
}
```

Code 3.7: Inverse Byte Substitution

```c
void InvSubBytes(u8* state) {
    for (int i = 0; i < AES_BLOCK_SIZE; i++) {
        state[i] = inv_s_box[state[i]];
    }
}
```

3.2.4 ShiftRows / InvShiftRows

- ShiftRows : $\{0, 1\}^{128} \longrightarrow \{0, 1\}^{128}$ is defined by

X_0	X_4	X_8	X_{12}
X_1	X_5	X_9	X_{13}
X_2	X_6	X_{10}	X_{14}
X_3	X_7	X_{11}	X_{15}

$\Longrightarrow$

X_0	X_4	X_8	X_{12}
X_5	X_9	X_{13}	X_1
X_{10}	X_{14}	X_2	X_6
X_{15}	X_3	X_7	X_{11}

- InvShiftRows : $\{0, 1\}^{128} \longrightarrow \{0, 1\}^{128}$ is defined by

X_0	X_4	X_8	X_{12}
X_1	X_5	X_9	X_{13}
X_2	X_6	X_{10}	X_{14}
X_3	X_7	X_{11}	X_{15}

$\Longrightarrow$

X_0	X_4	X_8	X_{12}
X_{13}	X_1	X_5	X_9
X_{10}	X_{14}	X_2	X_6
X_7	X_{11}	X_{15}	X_3

Code 3.8: ShiftRows

```c
void ShiftRows(u8* state) {
    u8 temp;

    // Row 1: shift left by 1
    temp = state[1];
    state[1] = state[5];
    state[5] = state[9];
    state[9] = state[13];
    state[13] = temp;

    // Row 2: shift left by 2
    temp = state[2];
    state[2] = state[10];
    state[10] = temp;
    temp = state[6];
    state[6] = state[14];
    state[14] = temp;

    // Row 3: shift left by 3 (or right by 1)
    temp = state[15];
    state[15] = state[11];
    state[11] = state[7];
    state[7] = state[3];
    state[3] = temp;
}
```

Code 3.9: Inverse ShiftRows

```
void InvShiftRows(u8* state) {
    u8 temp;

    // Row 1: shift left by 3 (or right by 1)
    temp = state[13];
    state[13] = state[9];
    state[9] = state[5];
    state[5] = state[1];
    state[1] = temp;

    // Row 2: shift left by 2
    temp = state[2];
    state[2] = state[10];
    state[10] = temp;
    temp = state[6];
    state[6] = state[14];
    state[14] = temp;

    // Row 3: shift left by 1
    temp = state[3];
    state[3] = state[7];
    state[7] = state[11];
    state[11] = state[15];
    state[15] = temp;
}
```

3.2.5 MixColumns / InvMixColumns

- Multiplication in the finite filed GF(2^8).

$$\mathrm{MUL}_{GF256} : \{0, 1\}^8 \times \{0, 1\}^8 \to \{0, 1\}^8 .$$

Here,

$$\{0, 1\}^8 \simeq GF(2^8) = \mathbb{F}_{2^8} := \mathbb{F}_2[z]/(z^8 + z^4 + z^3 + z + 1) = \left\{a_7 z^7 + \cdots + a_1 z + a_0 : a_i \in \mathbb{F}_2\right\} .$$

Note that

$$a(z) \times b(z) := a(z) \times b(z) \bmod (z^8 + z^4 + z^3 + z + 1)$$

Note. Given two polynomials $a(x)$ and $b(x)$ in $GF(2^8)$:

$$a(x) = a_7x^7 + a_6x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0,$$
$$b(x) = b_7x^7 + b_6x^6 + b_5x^5 + b_4x^4 + b_3x^3 + b_2x^2 + b_1x + b_0.$$

The algorithm performs polynomial multiplication in the finite field $GF(2^8)$. It uses a shift-and-add method, with an additional reduction step modulo an irreducible polynomial $m(x) = x^8 + x^4 + x^3 + x + 1$.

1. Initialization: Set $p(x) = 0$ to initialize the product polynomial.
2. Iterate over each bit of $b(x)$, from LSB to MSB.
   (i) If the current bit b_i of $b(x)$ is 1, update $p(x)$ as $p(x) \oplus a(x)$. In $GF(2^8)$, addition is equivalent to the XOR operation:
   $$p(x) = p(x) \oplus a(x).$$
   (ii) Shift $a(x)$ left by 1 (multiply by x), increasing its degree by 1:
   $$a(x) = a(x) \cdot x.$$
   (iii) If the coefficient of x^8 in $a(x)$ is 1, reduce $a(x)$ by $m(x)$ to keep the degree under 8:
   $$a(x) = a(x) \oplus m(x).$$
   (iv) Shift $b(x)$ right by 1 (divide by x) for the next iteration:
   $$b(x) = b(x) \;/\; x.$$
3. After all bits of $b(x)$ are processed, $p(x)$ be the product of $a(x)$ and $b(x)$ modulo $m(x)$.

Note (Modular Reduction in $GF(2^8)$ using XOR). In the context of multiplication in the binary finite field $GF(2^8)$, modular reduction ensures that results of operations remain within the field. The use of XOR for modular reduction is due to the properties of polynomial arithmetic over GF(2) and the representation of elements in $GF(2^8)$.

– **Polynomial Representation in $GF(2^8)$:**

1. **Elements as Polynomials**: Each element in $GF(2^8)$ can be represented as a polynomial of degree less than 8, where each coefficient is either 0 or 1, i.e.,

$$GF(2^8) = \mathbb{F}_2[x]/(x^8 + x^4 + x^3 + x + 1) = \left\{a_7x^7 + \cdots + a_1x + a_0 : a_i \in \mathbb{F}_2\right\}.$$

This corresponds to an 8-bit binary number, with each bit representing a coefficient of the polynomial, i.e.,

$$a_7x^7 + \cdots + a_1x + a_0 \iff (a_7 \dots a_1a_0)_2.$$

2. **Binary Operations**: In $GF(2)$, addition and subtraction are equivalent to the XOR operation, since $1 + 1 = 0$ in this field, the same as $1 \oplus 1$.

– **Modular Reduction with an Irreducible Polynomial**

1. **Irreducible Polynomial**: In $GF(2^8)$, an irreducible polynomial of degree 8, typically $p(x) = x^8 + x^4 + x^3 + x + 1$ (represented as `0x11b` in binary), is used for modular reduction.
2. **Modular Reduction Process**: After multiplying two polynomials, if the resulting polynomial's degree is 8 or higher, it must be reduced modulo the irreducible polynomial to ensure the result remains a polynomial of degree less than 8, thus staying within $GF(2^8)$.
3. **XOR for Reduction**: XOR is used for modular reduction in $GF(2^8)$ because polynomial subtraction in GF(2) is performed by XORing coefficients.

– Given two elements in $GF(2^8)$, $a(x)$ and $b(x)$, their product is $c(x) = a(x) \cdot b(x)$. If $\deg(c(x)) \geq 8$, then $c(x)$ must be reduced modulo the irreducible polynomial $p(x)$. This is achieved by XORing the coefficients of $c(x)$ and $p(x)$:

$$c(x) = a(x) \cdot b(x) \mod p(x)$$

If $c(x)$ has a term x^8 or higher, we subtract $p(x)$ from $c(x)$ to reduce its degree. In GF(2), subtraction is equivalent to addition, performed by XORing coefficients:

$$c'(x) = c(x) \oplus p(x)$$

This operation effectively eliminates the term x^8 (or higher) in $c(x)$, ensuring that the result remains within $GF(2^8)$. Consider the product of two polynomials $a(x)$ and $b(x)$ in $GF(2^8)$:

$$a(x) = x^6 + x^4 + x^2 + x + 1 \quad \text{and} \quad b(x) = x^7 + x + 1$$

The product $c(x) = a(x) \cdot b(x)$ might yield a polynomial of degree 8 or higher. To reduce $c(x)$ modulo $p(x) = x^8 + x^4 + x^3 + x + 1$, we perform XOR between the coefficients of $c(x)$ and $p(x)$, ensuring the result stays within $GF(2^8)$.

Code 3.10: Multiplication in GF(2^8)

```
u8 MUL_GF256(u8 a, u8 b) {
    u8 res = 0;
    // Mask for detecting the MSB (0x80 = 0b10000000)
    u8 MSB_mask = 0x80;
    u8 MSB;
    /*
     * The reduction polynomial
     * (x^8 + x^4 + x^3 + x + 1) = 0b100011011
     * for AES, represented in hexadecimal
    */
    u8 modulo = 0x1B;

    for (int i = 0; i < 8; i++) {
        // Add a to result if LSB(b)=1
        if (b & 1)
            res ^= a;

        MSB = a & MSB_mask; // Store the MSB of a
        a <<= 1; // Multiplying it by x effectively

        // Reduce the result modulo the reduction polynomial
        if (MSB)
            a ^= modulo;

        b >>= 1; // Moving to the next bit
    }

    return res;
}

#define MUL_GF256(a, b) ({ \
    u8 res = 0; \
    u8 MSB_mask = 0x80; \
    u8 MSB; \
    u8 modulo = 0x1B; \
    u8 temp_a = (a); \
    u8 temp_b = (b); \
    for (int i = 0; i < 8; i++) { \
        if (temp_b & 1) \
        res ^= temp_a; \
        MSB = temp_a & MSB_mask; \
        temp_a <<= 1; \
        if (MSB) \
        temp_a ^= modulo; \
        temp_b >>= 1; \
    } \
    res; \
})
```

- MixColumns : $\{0,1\}^{128} \to \{0,1\}^{128}$ is defined by

$$\text{MixColumns}\left(\begin{pmatrix} X_0 & X_4 & X_8 & X_{12} \\ X_1 & X_5 & X_9 & X_{13} \\ X_2 & X_6 & X_{10} & X_{14} \\ X_3 & X_7 & X_{11} & X_{15} \end{pmatrix}\right) := \begin{pmatrix} \texttt{0x02} & \texttt{0x03} & \texttt{0x01} & \texttt{0x01} \\ \texttt{0x01} & \texttt{0x02} & \texttt{0x03} & \texttt{0x01} \\ \texttt{0x01} & \texttt{0x01} & \texttt{0x02} & \texttt{0x03} \\ \texttt{0x03} & \texttt{0x01} & \texttt{0x01} & \texttt{0x02} \end{pmatrix} \begin{pmatrix} X_0 & X_4 & X_8 & X_{12} \\ X_1 & X_5 & X_9 & X_{13} \\ X_2 & X_6 & X_{10} & X_{14} \\ X_3 & X_7 & X_{11} & X_{15} \end{pmatrix}.$$

- InvMixColums : $\{0,1\}^{128} \to \{0,1\}^{128}$ is defined by

$$\text{MixColums}\left(\begin{pmatrix} X_0 & X_4 & X_8 & X_{12} \\ X_1 & X_5 & X_9 & X_{13} \\ X_2 & X_6 & X_{10} & X_{14} \\ X_3 & X_7 & X_{11} & X_{15} \end{pmatrix}\right) := \begin{pmatrix} \texttt{0x0e} & \texttt{0x0b} & \texttt{0x0d} & \texttt{0x09} \\ \texttt{0x09} & \texttt{0x0e} & \texttt{0x0b} & \texttt{0x0d} \\ \texttt{0x0d} & \texttt{0x09} & \texttt{0x0e} & \texttt{0x0b} \\ \texttt{0x0b} & \texttt{0x0d} & \texttt{0x09} & \texttt{0x0e} \end{pmatrix} \begin{pmatrix} X_0 & X_4 & X_8 & X_{12} \\ X_1 & X_5 & X_9 & X_{13} \\ X_2 & X_6 & X_{10} & X_{14} \\ X_3 & X_7 & X_{11} & X_{15} \end{pmatrix}.$$

Code 3.11: MixColumns

```
void MixColumns(u8* state) {
    u8 temp[4];
    // Multiply and add the elements in the column
    // by the fixed polynomial
    for (int i = 0; i < 4; i++) {
        temp[0] =
            MUL_GF256(0x02, state[i * 4]) ^
            MUL_GF256(0x03, state[i * 4 + 1]) ^
            state[i * 4 + 2] ^
            state[i * 4 + 3];

        temp[1] =
            state[i * 4] ^
            MUL_GF256(0x02, state[i * 4 + 1]) ^
            MUL_GF256(0x03, state[i * 4 + 2]) ^
            state[i * 4 + 3];

        temp[2] =
            state[i * 4] ^
            state[i * 4 + 1] ^
            MUL_GF256(0x02, state[i * 4 + 2]) ^
            MUL_GF256(0x03, state[i * 4 + 3]);

        temp[3] =
            MUL_GF256(0x03, state[i * 4]) ^
            state[i * 4 + 1] ^
            state[i * 4 + 2] ^
            MUL_GF256(0x02, state[i * 4 + 3]);

        // Copy the mixed column back to the state
        for (int j = 0; j < 4; j++)
            state[i * 4 + j] = temp[j];
    }
}
```

Code 3.12: Inverse MixColumns

```
void InvMixColumns(u8* state) {
    u8 temp[4];

    for (int i = 0; i < 4; i++) {
        temp[0] =
            MUL_GF256(0x0e, state[i * 4]) ^
            MUL_GF256(0x0b, state[i * 4 + 1]) ^
            MUL_GF256(0x0d, state[i * 4 + 2]) ^
            MUL_GF256(0x09, state[i * 4 + 3]);

        temp[1] =
            MUL_GF256(0x09, state[i * 4]) ^
            MUL_GF256(0x0e, state[i * 4 + 1]) ^
            MUL_GF256(0x0b, state[i * 4 + 2]) ^
            MUL_GF256(0x0d, state[i * 4 + 3]);

        temp[2] =
            MUL_GF256(0x0d, state[i * 4]) ^
            MUL_GF256(0x09, state[i * 4 + 1]) ^
            MUL_GF256(0x0e, state[i * 4 + 2]) ^
            MUL_GF256(0x0b, state[i * 4 + 3]);

        temp[3] =
            MUL_GF256(0x0b, state[i * 4]) ^
            MUL_GF256(0x0d, state[i * 4 + 1]) ^
            MUL_GF256(0x09, state[i * 4 + 2]) ^
            MUL_GF256(0x0e, state[i * 4 + 3]);

        for (int j = 0; j < 4; j++)
            state[i * 4 + j] = temp[j];
    }
}
```

Chapter 4

AES - 128 / 192 / 256 (Byte Version)

4.1 Specification

Table 4.1: Parameters of the Block Cipher AES (1-word = 32-bit)

Algorithms	Block Size (N_b-word)	Key Length (N_k-word)	Number of Rounds (N_r)	Round-Key Length (word)	Number of Round-Keys ($N_r + 1$)	Total Size of Round-Keys ($N_b(N_r + 1)$)
AES-128	4	4 (4·32-bit)	10	4	11	44 (176-byte)
AES-192	4	6 (6·32-bit)	12	4	13	52 (208-byte)
AES-256	4	8 (8·32-bit)	14	4	15	60 (240-byte)

Code 4.1: Configuration in C

```c
// Define macros for AES key length
#define AES_VERSION 128 // Can be 128, 192, or 256
// Define macro for AES block size
#define AES_BLOCK_SIZE 16

// Define Nk and Nr based on AES key length
#if AES_VERSION == 128
    #define Nk 4
#elif AES_VERSION == 192
    #define Nk 6
#elif AES_VERSION == 256
    #define Nk 8
#else
    #error "Invalid AES ky length"
#endif

#define Nr (Nk + 6) // 10 / 12 / 14
#define ROUND_KEYS_SIZE (16 * (Nr + 1)) // 176 / 208 / 240
```

Code 4.2: Configuration in Rust

```
// Define a constant for the AES key length.
pub const AES_VERSION: u32 = 128; // Can be 128, 192, or 256

// Define constant for AES block size
pub const AES_BLOCK_SIZE: usize = 16;

// Define constants Nk and Nr based on AES key length
pub const NK: usize = match AES_VERSION {
    128 => 4,
    192 => 6,
    256 => 8,
    _ => panic!("Invalid AES key length"),
};

pub const NR: usize = NK + 6;
pub const ROUND_KEYS_SIZE: usize = 16 * (NR + 1);
```

4.2 Key Expansion (General Version)

Algorithm 6: Key Schedule (General Version)

Input: User-key $uk = (uk_0, \ldots, uk_{N_k-1})$ $(uk_i \in \{0,1\}^8)$; // uk is 16/24/32-byte

Output: Round-key $\{rk_i\}_{i=0}^{4(N_r+1)-1}$ $(rk_i \in \{0,1\}^{32})$

/* $\{rk_i\}_{i=0}^{4(N_r+1)-1}$ is 176/208/240-byte */

$l \leftarrow N_k/4$; // $l = 4, 6, 8$
for $i = 0$ **to** $l - 1$ **do**
  $rk_i \leftarrow uk_{4i} \parallel uk_{4i+1} \parallel uk_{4i+2} \parallel uk_{4i+3}$;
end
for $i = l$ **to** $4(N_r + 1) - 1$ **do**
  $t \leftarrow rk_{i-1}$;
  if $i \bmod l = 0$ **then**
    $t \leftarrow (\mathrm{SubWord} \circ \mathrm{RotWord})(t)$;
    $t \leftarrow t \oplus_{32} (\mathrm{rCon}_{i/l} \parallel \texttt{0x00} \parallel \texttt{0x00} \parallel \texttt{0x00})$;
  else if $l > 6$ && $i \bmod l = 4$ **then**
    $t \leftarrow \mathrm{SubWord}(t)$;
  end
  $rk_i \leftarrow rk_{i-l} \oplus_{32} t$;
end

Code 4.3: Key Expansion in C (General ver.)

```c
void KeyExpansion(const u8* uKey, u32* rKey) {
    u32 temp;

    for (int i = 0; i < Nk; i++) {
        rKey[i] = (u32)uKey[4*i] << 0x18 |
                  (u32)uKey[4*i+1] << 0x10 |
                  (u32)uKey[4*i+2] << 0x08 |
                  (u32)uKey[4*i+3];
    }

    for (int i = Nk; i < (Nr + 1) * 4; i++) {
        temp = rKey[i - 1];
        if (i % Nk == 0) {
            temp = SubWord(RotWord(temp)) ^ rCon[i / Nk - 1];
        } else if (Nk > 6 && i % Nk == 4) {
            // Additional S-box transformation for AES-256
            temp = SubWord(temp);
        }
        rKey[i] = rKey[i - Nk] ^ temp;
    }
}
```

Code 4.4: Key Expansion Test

```
void RANDOM_KEY_GENERATION(u8* key) {
    srand((u32)time(NULL));

    // Initialize pointer to the start of the key array
    u8* p = key;

    // Set the counter to 16 bytes
    int cnt = 0;

    // Loop until all 16 bytes are filled
    while (cnt < AES_BLOCK_SIZE) {
        *p = rand() & 0xff; // Assign a random byte (0 to 255)
        p++;                // Move to the next byte
        cnt++;              // Decrement the byte count
    }
}

void KeyExpansionTest() {
    u8 uKey[AES_BLOCK_SIZE] = { 0x00, };
    RANDOM_KEY_GENERATION(uKey);
    // u8 uKey[AES_BLOCK_SIZE] = {
    //   0x2b, 0x7e, 0x15, 0x16, 0x28, 0xae, 0xd2, 0xa6,
    //   0xab, 0xf7, 0x15, 0x88, 0x09, 0xcf, 0x4f, 0x3c
    // };
    for (int i = 0; i < AES_BLOCK_SIZE; i++) {
        printf("%02x", uKey[i]);
    } printf("\n");

    u32 rKeys[ROUND_KEYS_SIZE / sizeof(u32)];
    KeyExpansion(uKey, rKeys);
    for (int i = 0; i < ROUND_KEYS_SIZE / sizeof(u32); i++) {
        printf("%08x\n", rKeys[i]);
    }
}

int main() {
    KeyExpansionTest();
    return 0;
}
```

4.3 8-bit AES - 128 / 192 / 256

Algorithm 7: Encryption of 8-bit AES

Input: block src $\in \{0,1\}^{128}$, round-keys $\{rk_i\}_{i=0}^{N_r+1}$ ($rk_i \in \{0,1\}^{32*4}$)

Output: block dst $\in \{0,1\}^{128}$

$t \leftarrow$ AddRoundKey(src, rk_0); // AddRoundKey: $\{0,1\}^{8*16} \times \{0,1\}^{32*4} \rightarrow \{0,1\}^{8*16}$

for $i \leftarrow 1$ **to** $N_r - 1$ **do**

  $t \leftarrow$ SubBytes(t); // SubBytes: $\{0,1\}^{8*16} \rightarrow \{0,1\}^{8*16}$

  $t \leftarrow$ ShiftRows(t); // ShiftRows: $\{0,1\}^{8*16} \rightarrow \{0,1\}^{8*16}$

  $t \leftarrow$ MixColumns(t); // MixColumns: $\{0,1\}^{8*16} \rightarrow \{0,1\}^{8*16}$

  $t \leftarrow$ AddRoundKey(t, rk_i);

end

$t \leftarrow$ SubBytes(t);

$t \leftarrow$ ShiftRows(t);

$t \leftarrow$ AddRoundKey(t, rk_{N_r});

dst $\leftarrow t$;

return dst;

Code 4.5: 8-bit AES Encryption

```c
void AES_Encrypt(const u8* plaintext, const u8* key,
    u8* ciphertext) {
    // AES-128/192/256: roundKey[44]/roundKey[52]/roundKey[60]
    u32 roundKey[ROUND_KEYS_SIZE / sizeof(u32)];
    u8 state[AES_BLOCK_SIZE]; // state[16]

    // Copy plaintext to state
    for (int i = 0; i < AES_BLOCK_SIZE; i++)
        state[i] = plaintext[i];

    KeyExpansion(key, roundKey);

// 0: roundKey[  0] | roundKey[1] | roundKey[ 2] | roundKey[    3]
    AddRoundKey(state, roundKey); // Initial round

    for (int round = 1; round <= Nr; round++) { // Main rounds
        SubBytes(state); ShiftRows(state);
        if (round != Nr) MixColumns(state);
// 1: roundKey[  4] | roundKey[5] | roundKey[ 6] | roundKey[    7]
// 2: roundKey[  8] | roundKey[9] | roundKey[10] | roundKey[   11]
// i: roundKey[4*i] |    ...      |    ...       | roundKey[4*i+3]
        AddRoundKey(state, roundKey + 4 * round);
    }

    // Copy state to ciphertext
    for (int i = 0; i < AES_BLOCK_SIZE; i++)
        ciphertext[i] = state[i];
}
```

Algorithm 8: Decryption of 8-bit AES

Input: block src $\in \{0, 1\}^{128}$, round-keys $\{rk_i\}_{i=0}^{N_r+1}$ $(rk_i \in \{0, 1\}^{32*4})$

Output: block dst $\in \{0, 1\}^{128}$

$t \leftarrow$ AddRoundKey(src, rk_{N_r});
for $i \leftarrow N_r - 1$ **to** 1 **do**
    $t \leftarrow$ InvShiftRows(t);
    $t \leftarrow$ InvSubBytes(t);
    $t \leftarrow$ AddRoundKey(t, rk_i);
    $t \leftarrow$ InvMixColumns(t);
end
$t \leftarrow$ InvShiftRows(t);
$t \leftarrow$ InvSubBytes(t);
$t \leftarrow$ AddRoundKey(t, rk_0);
dst $\leftarrow t$;
return dst;

Code 4.6: 8-bit AES Decryption

```c
void AES_Decrypt(const u8* ciphertext, const u8* key,
    u8* plaintext) {
    u32 roundKey[ROUND_KEYS_SIZE / sizeof(u32)];
    u8 state[AES_BLOCK_SIZE];

    KeyExpansion(key, roundKey);

    for (int i = 0; i < AES_BLOCK_SIZE; i++)
        state[i] = ciphertext[i];

    // Initial round with the last round key
    AddRoundKey(state, roundKey + 4 * Nr);

    // Main rounds in reverse order
    for (int round = Nr - 1; round >= 0; round--) {
        InvShiftRows(state);
        InvSubBytes(state);
// i: roundKey[4*i] |     ...     |     ...     | roundKey[4*i+3]
// 1: roundKey[  4] | roundKey[5] | roundKey[ 6] | roundKey[     7]
// 0: roundKey[  0] | roundKey[1] | roundKey[ 2] | roundKey[     3]
        AddRoundKey(state, roundKey + 4 * round);
        if (round != 0)
            InvMixColumns(state);
    }

    for (int i = 0; i < AES_BLOCK_SIZE; i++)
        plaintext[i] = state[i];
}
```

Chapter 5

Pre-Computation using SubMix

5.1 SubMix and InvSubInvMix

- $\mathrm{SubMix} : \{0, 1\}^{128} \to \{0, 1\}^{128}$ and $\mathrm{InvSubInvMix} : \{0, 1\}^{128} \to \{0, 1\}^{128}$ are defined by

$$\mathrm{SubMix} := \mathrm{MixColumns} \circ \mathrm{SubBytes}$$
$$\mathrm{InvSubInvMix} := \mathrm{InvMixColumns} \circ \mathrm{InvSubBytes}.$$

Algorithm 9: Encryption and Decryption of AES using SubMix and InvSubInvMix

Input: block src $\in \{0, 1\}^{128}$, round-keys $\{rk_i\}_{i=0}^{N_r+1}$ $(rk_i \in \{0, 1\}^{128})$

Output: block dst $\in \{0, 1\}^{128}$

Function AES_ENC**:**
    $t \leftarrow$ src;
    $t \leftarrow \mathrm{AddRoundKey}(t, rk_0)$;
    for $i \leftarrow 1$ **to** $N_r - 1$ **do**
        $t \leftarrow \mathrm{ShiftRows}(t)$;
        $t \leftarrow$ **SubMix**(t);
        $t \leftarrow \mathrm{AddRoundKey}(t, rk_i)$;
    end
    $t \leftarrow \mathrm{SubBytes}(t)$;
    $t \leftarrow \mathrm{ShiftRows}(t)$;
    $t \leftarrow \mathrm{AddRoundKey}(t, rk_{N_r})$;
    dst $\leftarrow t$;
    return dst;
end

Function AES_DEC**:**
    $t \leftarrow$ src;
    $t \leftarrow \mathrm{AddRoundKey}(t, rk_{N_r})$;
    for $i \leftarrow N_r - 1$ **downto** 1 **do**
        $t \leftarrow \mathrm{InvShiftRows}(t)$;
        $t \leftarrow$ **InvSubInvMix**(t);
        $t' \leftarrow \mathrm{InvMixColumns}(rk_i)$;
        $t \leftarrow \mathrm{AddRoundKey}(t, t')$;
    end
    $t \leftarrow \mathrm{InvShiftRows}(t)$;
    $t \leftarrow \mathrm{InvSubBytes}(t)$;
    $t \leftarrow \mathrm{AddRoundKey}(t, rk_0)$;
    dst $\leftarrow t$;
    return dst;
end

5.2 8 × 32 Table Look Up

Convert the 8-bit string $\{X_i\}_{i=0}^{15}$ ($X_i \in \{0,1\}^8$) into a 32-bit string $\{W_i\}_{i=0}^{3}$ ($W_i \in \{0,1\}^{32}$) as follows:

$$W_i := (X_{4i} \ll 24) \parallel (X_{4i+1} \ll 16) \parallel (X_{4i+2} \ll 8) \parallel (X_{4i+3}) \in \{0,1\}^{32}$$

for $i = 0, 1, 2, 3$. In other words,

X_0	$(W_0 \gg \texttt{0x18})\ \&\ \texttt{0xff}$	X_4	$(W_1 \gg \texttt{0x18})\ \&\ \texttt{0xff}$
X_1	$(W_0 \gg \texttt{0x10})\ \&\ \texttt{0xff}$	X_5	$(W_1 \gg \texttt{0x10})\ \&\ \texttt{0xff}$
X_2	$(W_0 \gg \texttt{0x08})\ \&\ \texttt{0xff}$	X_6	$(W_1 \gg \texttt{0x08})\ \&\ \texttt{0xff}$
X_3	$(W_0)\ \&\ \texttt{0xff}$	X_7	$(W_1)\ \&\ \texttt{0xff}$
X_8	$(W_2 \gg \texttt{0x18})\ \&\ \texttt{0xff}$	X_{12}	$(W_3 \gg \texttt{0x18})\ \&\ \texttt{0xff}$
X_9	$(W_2 \gg \texttt{0x10})\ \&\ \texttt{0xff}$	X_{13}	$(W_3 \gg \texttt{0x10})\ \&\ \texttt{0xff}$
X_{10}	$(W_2 \gg \texttt{0x08})\ \&\ \texttt{0xff}$	X_{14}	$(W_3 \gg \texttt{0x08})\ \&\ \texttt{0xff}$
X_{11}	$(W_2)\ \&\ \texttt{0xff}$	X_{15}	$(W_3)\ \&\ \texttt{0xff}$

Note that

<table>
<tr><td>X_0</td><td>X_1</td><td>X_2</td><td>X_3</td><td>X_4</td><td>X_5</td><td>X_6</td><td>X_7</td><td>X_8</td><td>X_9</td><td>X_{10}</td><td>X_{11}</td><td>X_{12}</td><td>X_{13}</td><td>X_{14}</td><td>X_{15}</td></tr>
<tr><td colspan="4">W_0</td><td colspan="4">W_1</td><td colspan="4">W_2</td><td colspan="4">W_3</td></tr>
</table>

Algorithm 10: SubMix and InvSubInvMix

Input: $(W_0, W_1, W_2, W_3) \in \{0,1\}^{128=32\times4}$
Output: $(V_0, V_1, V_2, V_3) \in \{0,1\}^{128=32\times4}$

```
Function SubMix:
    for i ← 0 to 3 do
        V_i ← Te_0((W_i ≫ 0x18) & 0xff)
              ⊕_32 Te_1((W_i ≫ 0x10) & 0xff)
              ⊕_32 Te_2((W_i ≫ 0x08) & 0xff)
              ⊕_32 Te_3(W_i & 0xff);
    end
    return (V_0, V_1, V_2, V_3);
end
```

Input: $(W_0, W_1, W_2, W_3) \in \{0,1\}^{128=32\times4}$
Output: $(U_0, U_1, U_2, U_3) \in \{0,1\}^{128=32\times4}$

```
Function InvSubInvMix:
    for i ← 0 to 3 do
        U_i ← Td_0((W_i ≫ 0x18) & 0xff)
              ⊕_32 Td_1((W_i ≫ 0x10) & 0xff)
              ⊕_32 Td_2((W_i ≫ 0x08) & 0xff)
              ⊕_32 Td_3(W_i & 0xff);
    end
    return (U_0, U_1, U_2, U_3);
end
```

5.3 Generation of 8×32 Tables

Note that

$$\begin{pmatrix} V_0 & V_4 & V_8 & V_{12} \\ V_1 & V_5 & V_9 & V_{13} \\ V_2 & V_6 & V_{10} & V_{14} \\ V_3 & V_7 & V_{11} & V_{15} \end{pmatrix} = \mathrm{SubMix}\left(\begin{pmatrix} X_0 & X_4 & X_8 & X_{12} \\ X_1 & X_5 & X_9 & X_{13} \\ X_2 & X_6 & X_{10} & X_{14} \\ X_3 & X_7 & X_{11} & X_{15} \end{pmatrix} \right) :=$$
$$\begin{pmatrix} \mathtt{0x02} & \mathtt{0x03} & \mathtt{0x01} & \mathtt{0x01} \\ \mathtt{0x01} & \mathtt{0x02} & \mathtt{0x03} & \mathtt{0x01} \\ \mathtt{0x01} & \mathtt{0x01} & \mathtt{0x02} & \mathtt{0x03} \\ \mathtt{0x03} & \mathtt{0x01} & \mathtt{0x01} & \mathtt{0x02} \end{pmatrix} \begin{pmatrix} s(X_0) & s(X_4) & s(X_8) & s(X_{12}) \\ s(X_1) & s(X_5) & s(X_9) & s(X_{13}) \\ s(X_2) & s(X_6) & s(X_{10}) & s(X_{14}) \\ s(X_3) & s(X_7) & s(X_{11}) & s(X_{15}) \end{pmatrix},$$

$$\begin{pmatrix} U_0 & V_4 & U_8 & U_{12} \\ U_1 & V_5 & U_9 & U_{13} \\ U_2 & V_6 & U_{10} & U_{14} \\ U_3 & V_7 & U_{11} & U_{15} \end{pmatrix} = \mathrm{InvSubInvMix}\left(\begin{pmatrix} X_0 & X_4 & X_8 & X_{12} \\ X_1 & X_5 & X_9 & X_{13} \\ X_2 & X_6 & X_{10} & X_{14} \\ X_3 & X_7 & X_{11} & X_{15} \end{pmatrix} \right) :=$$
$$\begin{pmatrix} \mathtt{0x0e} & \mathtt{0x0b} & \mathtt{0x0d} & \mathtt{0x09} \\ \mathtt{0x09} & \mathtt{0x0e} & \mathtt{0x0b} & \mathtt{0x0d} \\ \mathtt{0x0d} & \mathtt{0x09} & \mathtt{0x0e} & \mathtt{0x0b} \\ \mathtt{0x0b} & \mathtt{0x0d} & \mathtt{0x09} & \mathtt{0x0e} \end{pmatrix} \begin{pmatrix} s^{-1}(X_0) & s^{-1}(X_4) & s^{-1}(X_8) & s^{-1}(X_{12}) \\ s^{-1}(X_1) & s^{-1}(X_5) & s^{-1}(X_9) & s^{-1}(X_{13}) \\ s^{-1}(X_2) & s^{-1}(X_6) & s^{-1}(X_{10}) & s^{-1}(X_{14}) \\ s^{-1}(X_3) & s^{-1}(X_7) & s^{-1}(X_{11}) & s^{-1}(X_{15}) \end{pmatrix}.$$

Then

$$\begin{pmatrix} V_{4i} \\ V_{4i+1} \\ V_{4i+2} \\ V_{4i+3} \end{pmatrix} = \begin{pmatrix} \mathtt{0x02} & \mathtt{0x03} & \mathtt{0x01} & \mathtt{0x01} \\ \mathtt{0x01} & \mathtt{0x02} & \mathtt{0x03} & \mathtt{0x01} \\ \mathtt{0x01} & \mathtt{0x01} & \mathtt{0x02} & \mathtt{0x03} \\ \mathtt{0x03} & \mathtt{0x01} & \mathtt{0x01} & \mathtt{0x02} \end{pmatrix} \begin{pmatrix} s(X_{4i}) \\ s(X_{4i+1}) \\ s(X_{4i+2}) \\ s(X_{4i+3}) \end{pmatrix}$$
$$= s(X_{4i}) \begin{pmatrix} \mathtt{0x02} \\ \mathtt{0x01} \\ \mathtt{0x01} \\ \mathtt{0x03} \end{pmatrix} \oplus_{32} s(X_{4i+1}) \begin{pmatrix} \mathtt{0x03} \\ \mathtt{0x02} \\ \mathtt{0x01} \\ \mathtt{0x01} \end{pmatrix} \oplus_{32} s(X_{4i+2}) \begin{pmatrix} \mathtt{0x01} \\ \mathtt{0x03} \\ \mathtt{0x02} \\ \mathtt{0x01} \end{pmatrix} \oplus_{32} s(X_{4i+3}) \begin{pmatrix} \mathtt{0x01} \\ \mathtt{0x01} \\ \mathtt{0x03} \\ \mathtt{0x02} \end{pmatrix},$$

$$\begin{pmatrix} U_{4i} \\ U_{4i+1} \\ U_{4i+2} \\ U_{4i+3} \end{pmatrix} = \begin{pmatrix} \mathtt{0x0e} & \mathtt{0x0b} & \mathtt{0x0d} & \mathtt{0x09} \\ \mathtt{0x09} & \mathtt{0x0e} & \mathtt{0x0b} & \mathtt{0x0d} \\ \mathtt{0x0d} & \mathtt{0x09} & \mathtt{0x0e} & \mathtt{0x0b} \\ \mathtt{0x0b} & \mathtt{0x0d} & \mathtt{0x09} & \mathtt{0x0e} \end{pmatrix} \begin{pmatrix} s^{-1}(X_{4i}) \\ s^{-1}(X_{4i+1}) \\ s^{-1}(X_{4i+2}) \\ s^{-1}(X_{4i+3}) \end{pmatrix}$$
$$= s^{-1}(X_{4i}) \begin{pmatrix} \mathtt{0x0e} \\ \mathtt{0x09} \\ \mathtt{0x0d} \\ \mathtt{0x0b} \end{pmatrix} \oplus s^{-1}(X_{4i+1}) \begin{pmatrix} \mathtt{0x0b} \\ \mathtt{0x0e} \\ \mathtt{0x09} \\ \mathtt{0x0d} \end{pmatrix} \oplus s^{-1}(X_{4i+2}) \begin{pmatrix} \mathtt{0x0d} \\ \mathtt{0x0b} \\ \mathtt{0x0e} \\ \mathtt{0x09} \end{pmatrix} \oplus s^{-1}(X_{4i+3}) \begin{pmatrix} \mathtt{0x09} \\ \mathtt{0x0d} \\ \mathtt{0x0b} \\ \mathtt{0x0e} \end{pmatrix}.$$

We define $Te_i/Td_i : \{\mathtt{0}, \mathtt{1}\}^8 \to \{\mathtt{0}, \mathtt{1}\}^{32}$ $(i \in \{0, 1, 2, 3\})$ as follows:

$$Te_0(X) := \left(\mathtt{0x02} \otimes_8 s(X), s(X), s(X), \mathtt{0x03} \otimes_8 s(X) \right),$$
$$Te_1(X) := \left(\mathtt{0x03} \otimes_8 s(X), \mathtt{0x02} \otimes_8 s(X), s(X), s(X) \right),$$
$$Te_2(X) := \left(s(X), \mathtt{0x03} \otimes_8 s(X), \mathtt{0x02} \otimes_8 s(X), s(X) \right),$$
$$Te_3(X) := \left(s(X), s(X), \mathtt{0x03} \otimes_8 s(X), \mathtt{0x02} \otimes_8 s(X) \right),$$

and

$$Td_0(X) := \left(\mathtt{0x0e} \otimes_8 s^{-1}(X), \mathtt{0x09} \otimes_8 s^{-1}(X), \mathtt{0x0d} \otimes_8 s^{-1}(X), \mathtt{0x0b} \otimes_8 s^{-1}(X) \right),$$
$$Td_1(X) := \left(\mathtt{0x0b} \otimes_8 s^{-1}(X), \mathtt{0x0e} \otimes_8 s^{-1}(X), \mathtt{0x09} \otimes_8 s^{-1}(X), \mathtt{0x0d} \otimes_8 s^{-1}(X) \right),$$
$$Td_2(X) := \left(\mathtt{0x0d} \otimes_8 s^{-1}(X), \mathtt{0x0b} \otimes_8 s^{-1}(X), \mathtt{0x0e} \otimes_8 s^{-1}(X), \mathtt{0x09} \otimes_8 s^{-1}(X) \right),$$
$$Td_3(X) := \left(\mathtt{0x09} \otimes_8 s^{-1}(X), \mathtt{0x0d} \otimes_8 s^{-1}(X), \mathtt{0x0b} \otimes_8 s^{-1}(X), \mathtt{0x0e} \otimes_8 s^{-1}(X) \right).$$

SubMix/InvSubInvMix : $\{\mathtt{0}, \mathtt{1}\}^{128} \longrightarrow \{\mathtt{0}, \mathtt{1}\}^{128}$ are computed as follows:

$$\mathrm{SubMix}(W_0, W_1, W_2, W_3) := (V_0, V_1, V_2, V_3),$$
$$\mathrm{InvSubInvMix}(W_0, W_1, W_2, W_3) := (U_0, U_1, U_2, U_3)$$

where $\begin{cases} W_0 := X_0 \parallel X_1 \parallel X_2 \parallel X_3 \\ W_1 := X_4 \parallel X_5 \parallel X_6 \parallel X_7 \\ W_2 := X_8 \parallel X_9 \parallel X_{10} \parallel X_{11} \\ W_3 := X_{12} \parallel X_{13} \parallel X_{14} \parallel X_{15} \end{cases}$ $\quad (W_i \in \{\mathtt{0}, \mathtt{1}\}^{32})$ and

$$\begin{array}{lcccccccc}
V_0 & := & Te_0(X_0) & \oplus_{32} & Te_1(X_1) & \oplus_{32} & Te_2(X_2) & \oplus_{32} & Te_3(X_3), \\
V_1 & := & Te_0(X_4) & \oplus_{32} & Te_1(X_5) & \oplus_{32} & Te_2(X_6) & \oplus_{32} & Te_3(X_7), \\
V_2 & := & Te_0(X_8) & \oplus_{32} & Te_1(X_9) & \oplus_{32} & Te_2(X_{10}) & \oplus_{32} & Te_3(X_{11}), \\
V_3 & := & Te_0(X_{12}) & \oplus_{32} & Te_1(X_{13}) & \oplus_{32} & Te_2(X_{14}) & \oplus_{32} & Te_3(X_{15}), \\
\\
U_0 & := & Td_0(X_0) & \oplus_{32} & Td_1(X_1) & \oplus_{32} & Td_2(X_2) & \oplus_{32} & Td_3(X_3), \\
U_1 & := & Td_0(X_4) & \oplus_{32} & Td_1(X_5) & \oplus_{32} & Td_2(X_6) & \oplus_{32} & Td_3(X_7), \\
U_2 & := & Td_0(X_8) & \oplus_{32} & Td_1(X_9) & \oplus_{32} & Td_2(X_{10}) & \oplus_{32} & Td_3(X_{11}), \\
U_3 & := & Td_0(X_{12}) & \oplus_{32} & Td_1(X_{13}) & \oplus_{32} & Td_2(X_{14}) & \oplus_{32} & Td_3(X_{15}).
\end{array}$$

5.4 Implementation of SubMix and InvSubInvMix

Code 5.1: SubMix

```
void SubMix(u8* state) {
    u32 temp[4];

    // temp[0] = Te0[state[ 0]] ^ Te1[state[ 1]] ^
    //           Te2[state[ 2]] ^ Te3[state[ 3]];
    // temp[1] = Te0[state[ 4]] ^ Te1[state[ 5]] ^
    //           Te2[state[ 6]] ^ Te3[state[ 7]];
    // temp[2] = Te0[state[ 8]] ^ Te1[state[ 9]] ^
    //           Te2[state[10]] ^ Te3[state[11]];
    // temp[3] = Te0[state[11]] ^ Te1[state[12]] ^
    //           Te2[state[13]] ^ Te3[state[14]];

    for (int i = 0; i < 4; i++) {
        temp[i] = Te0[state[4*i + 0]] ^
                  Te1[state[4*i + 1]] ^
                  Te2[state[4*i + 2]] ^
                  Te3[state[4*i + 3]];
    }

    for (int i = 0; i < 4; i++) {
        state[4*i + 0] = (temp[i] >> 0x18) & 0xff;
        state[4*i + 1] = (temp[i] >> 0x10) & 0xff;
        state[4*i + 2] = (temp[i] >> 0x08) & 0xff;
        state[4*i + 3] = temp[i] & 0xff;
    }
}
```

Code 5.2: InvSubInvMix

```
void InvSubInvMix(u8* state) {
    u32 temp[4];

    for (int i = 0; i < 4; i++) {
        temp[i] = Td0[state[4*i + 0]] ^
                  Td1[state[4*i + 1]] ^
                  Td2[state[4*i + 2]] ^
                  Td3[state[4*i + 3]];
    }

    for (int i = 0; i < 4; i++) {
        state[4*i + 0] = (temp[i] >> 0x18) & 0xff;
        state[4*i + 1] = (temp[i] >> 0x10) & 0xff;
        state[4*i + 2] = (temp[i] >> 0x08) & 0xff;
        state[4*i + 3] = temp[i] & 0xff;
    }
}
```

Chapter 6

32-bit AES

Code 6.1: SubWord applies the S-box to each byte of the input word

```
/* Table Annotation:
| i | Shift | Mask          | Description                           |
|---|-------|-------------|-----------------------------------|
| 0 | 0x18  | 0xFF << 0x18| Process most significant byte (1st)|
| 1 | 0x10  | 0xFF << 0x10| Process 2nd byte                      |
| 2 | 0x08  | 0xFF << 0x08| Process 3rd byte                      |
| 3 | 0x00  | 0xFF << 0x00| Process least significant byte (4th)|
*/

u32 SubWord(u32* word) {
    u8 temp;
    for (int i = 0; i < 4; i++) {
        temp = Te3[(*word >> (0x18 - 0x08 * i)) & 0x000000ff];
        *word &= ~(0xFF << (0x18 - 0x08 * i)); // Clear the byte
        *word |= (temp & 0x000000ff) << (0x18 - 0x08 * i); // Set
           the new value
    }
}
```

Code 6.2: SubWord applies the S-box to each byte of the input word

```
u32 SubWord(u32 word) {
    u8 temp0, temp1, temp2, temp3;

    // Process the most significant byte (1st byte)
    temp0 = Te3[(word >> 24) & 0x000000ff];
    word &= 0x00ffffff; // Clear the most significant byte
    word |= (temp0 & 0x000000ff) << 24; // Set the new value

    // Process the 2nd byte
    temp1 = Te3[(word >> 16) & 0x000000ff];
    word &= 0xff00ffff; // Clear the 2nd byte
    word |= (temp1 & 0x000000ff) << 16; // Set the new value

    // Process the 3rd byte
    temp2 = Te3[(word >> 8) & 0x000000ff];
    word &= 0xffff00ff; // Clear the 3rd byte
    word |= (temp2 & 0x000000ff) << 8; // Set the new value

    // Process the least significant byte (4th byte)
    temp3 = Te3[word & 0x000000ff];
    word &= 0xffffff00; // Clear the least significant byte
    word |= (temp3 & 0x000000ff); // Set the new value

    return word;
}
```

Code 6.3: SubWord applies the S-box to each byte of the input word

```
u32 SubWord(u32 word) {
    // Apply the transformation to each byte and combine them
    return (Te3[(word >> 0x18) & 0xff] & 0xff000000) |
    (Te3[(word >> 0x10) & 0xff] & 0x00ff0000) |
    (Te1[(word >> 0x08) & 0xff] & 0x0000ff00) |
    (Te1[ word        & 0xff] & 0x000000ff );
}
```

Appendix A

Additional Data A

A.1 Substitution-BOX

```
static const u8 s_box[256] = {
    0x63, 0x7c, 0x77, 0x7b, 0xf2, 0x6b, 0x6f, 0xc5,
    0x30, 0x01, 0x67, 0x2b, 0xfe, 0xd7, 0xab, 0x76,
    0xca, 0x82, 0xc9, 0x7d, 0xfa, 0x59, 0x47, 0xf0,
    0xad, 0xd4, 0xa2, 0xaf, 0x9c, 0xa4, 0x72, 0xc0,
    0xb7, 0xfd, 0x93, 0x26, 0x36, 0x3f, 0xf7, 0xcc,
    0x34, 0xa5, 0xe5, 0xf1, 0x71, 0xd8, 0x31, 0x15,
    0x04, 0xc7, 0x23, 0xc3, 0x18, 0x96, 0x05, 0x9a,
    0x07, 0x12, 0x80, 0xe2, 0xeb, 0x27, 0xb2, 0x75,
    0x09, 0x83, 0x2c, 0x1a, 0x1b, 0x6e, 0x5a, 0xa0,
    0x52, 0x3b, 0xd6, 0xb3, 0x29, 0xe3, 0x2f, 0x84,
    0x53, 0xd1, 0x00, 0xed, 0x20, 0xfc, 0xb1, 0x5b,
    0x6a, 0xcb, 0xbe, 0x39, 0x4a, 0x4c, 0x58, 0xcf,
    0xd0, 0xef, 0xaa, 0xfb, 0x43, 0x4d, 0x33, 0x85,
    0x45, 0xf9, 0x02, 0x7f, 0x50, 0x3c, 0x9f, 0xa8,
    0x51, 0xa3, 0x40, 0x8f, 0x92, 0x9d, 0x38, 0xf5,
    0xbc, 0xb6, 0xda, 0x21, 0x10, 0xff, 0xf3, 0xd2,
    0xcd, 0x0c, 0x13, 0xec, 0x5f, 0x97, 0x44, 0x17,
    0xc4, 0xa7, 0x7e, 0x3d, 0x64, 0x5d, 0x19, 0x73,
    0x60, 0x81, 0x4f, 0xdc, 0x22, 0x2a, 0x90, 0x88,
    0x46, 0xee, 0xb8, 0x14, 0xde, 0x5e, 0x0b, 0xdb,
    0xe0, 0x32, 0x3a, 0x0a, 0x49, 0x06, 0x24, 0x5c,
    0xc2, 0xd3, 0xac, 0x62, 0x91, 0x95, 0xe4, 0x79,
    0xe7, 0xc8, 0x37, 0x6d, 0x8d, 0xd5, 0x4e, 0xa9,
    0x6c, 0x56, 0xf4, 0xea, 0x65, 0x7a, 0xae, 0x08,
    0xba, 0x78, 0x25, 0x2e, 0x1c, 0xa6, 0xb4, 0xc6,
    0xe8, 0xdd, 0x74, 0x1f, 0x4b, 0xbd, 0x8b, 0x8a,
    0x70, 0x3e, 0xb5, 0x66, 0x48, 0x03, 0xf6, 0x0e,
    0x61, 0x35, 0x57, 0xb9, 0x86, 0xc1, 0x1d, 0x9e,
    0xe1, 0xf8, 0x98, 0x11, 0x69, 0xd9, 0x8e, 0x94,
    0x9b, 0x1e, 0x87, 0xe9, 0xce, 0x55, 0x28, 0xdf,
    0x8c, 0xa1, 0x89, 0x0d, 0xbf, 0xe6, 0x42, 0x68,
    0x41, 0x99, 0x2d, 0x0f, 0xb0, 0x54, 0xbb, 0x16
};
```

```
static const u8 inv_s_box[256] = {
    0x52, 0x09, 0x6a, 0xd5, 0x30, 0x36, 0xa5, 0x38,
    0xbf, 0x40, 0xa3, 0x9e, 0x81, 0xf3, 0xd7, 0xfb,
    0x7c, 0xe3, 0x39, 0x82, 0x9b, 0x2f, 0xff, 0x87,
    0x34, 0x8e, 0x43, 0x44, 0xc4, 0xde, 0xe9, 0xcb,
    0x54, 0x7b, 0x94, 0x32, 0xa6, 0xc2, 0x23, 0x3d,
    0xee, 0x4c, 0x95, 0x0b, 0x42, 0xfa, 0xc3, 0x4e,
    0x08, 0x2e, 0xa1, 0x66, 0x28, 0xd9, 0x24, 0xb2,
    0x76, 0x5b, 0xa2, 0x49, 0x6d, 0x8b, 0xd1, 0x25,
    0x72, 0xf8, 0xf6, 0x64, 0x86, 0x68, 0x98, 0x16,
    0xd4, 0xa4, 0x5c, 0xcc, 0x5d, 0x65, 0xb6, 0x92,
    0x6c, 0x70, 0x48, 0x50, 0xfd, 0xed, 0xb9, 0xda,
    0x5e, 0x15, 0x46, 0x57, 0xa7, 0x8d, 0x9d, 0x84,
    0x90, 0xd8, 0xab, 0x00, 0x8c, 0xbc, 0xd3, 0x0a,
    0xf7, 0xe4, 0x58, 0x05, 0xb8, 0xb3, 0x45, 0x06,
    0xd0, 0x2c, 0x1e, 0x8f, 0xca, 0x3f, 0x0f, 0x02,
    0xc1, 0xaf, 0xbd, 0x03, 0x01, 0x13, 0x8a, 0x6b,
    0x3a, 0x91, 0x11, 0x41, 0x4f, 0x67, 0xdc, 0xea,
    0x97, 0xf2, 0xcf, 0xce, 0xf0, 0xb4, 0xe6, 0x73,
    0x96, 0xac, 0x74, 0x22, 0xe7, 0xad, 0x35, 0x85,
    0xe2, 0xf9, 0x37, 0xe8, 0x1c, 0x75, 0xdf, 0x6e,
    0x47, 0xf1, 0x1a, 0x71, 0x1d, 0x29, 0xc5, 0x89,
    0x6f, 0xb7, 0x62, 0x0e, 0xaa, 0x18, 0xbe, 0x1b,
    0xfc, 0x56, 0x3e, 0x4b, 0xc6, 0xd2, 0x79, 0x20,
    0x9a, 0xdb, 0xc0, 0xfe, 0x78, 0xcd, 0x5a, 0xf4,
    0x1f, 0xdd, 0xa8, 0x33, 0x88, 0x07, 0xc7, 0x31,
    0xb1, 0x12, 0x10, 0x59, 0x27, 0x80, 0xec, 0x5f,
    0x60, 0x51, 0x7f, 0xa9, 0x19, 0xb5, 0x4a, 0x0d,
    0x2d, 0xe5, 0x7a, 0x9f, 0x93, 0xc9, 0x9c, 0xef,
    0xa0, 0xe0, 0x3b, 0x4d, 0xae, 0x2a, 0xf5, 0xb0,
    0xc8, 0xeb, 0xbb, 0x3c, 0x83, 0x53, 0x99, 0x61,
    0x17, 0x2b, 0x04, 0x7e, 0xba, 0x77, 0xd6, 0x26,
    0xe1, 0x69, 0x14, 0x63, 0x55, 0x21, 0x0c, 0x7d
};
```