

C | SecureAES

- High-Performance AES Encryption in C -

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Note (XOR Operation and Modular Reduction in $GF(2^n)$). In the context of Galois Field $GF(2^n)$, particularly in binary polynomial arithmetic, the XOR operation is equivalent to addition and also plays a crucial role in modular reduction. We explore this equivalence through the principles of field theory and polynomial arithmetic.

- **Field Properties:**

A Galois Field, $GF(p^n)$, is a finite field that contains a finite number of elements, where

- p is a prime number (base of the field) and
- n is a positive integer (degree of the field).

For the binary field $GF(2^n)$, $p = 2$, which implies that every element in this field is either 0 or 1.

- **Addition in $GF(2^n)$:**

In $GF(2^n)$, the addition of two elements is performed modulo 2. For any two elements $a, b \in GF(2^n)$, the addition is defined as:

$$a + b = a \oplus b$$

Since 2 is the base of the field, the addition wraps around upon reaching 2, which is effectively what the XOR operation does.

- **Polynomial Representation:**

Elements in $GF(2^n)$ can be represented as polynomials where each coefficient is in $GF(2) = \{0, 1\}$. A general element can be written as:

$$a(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1x + a_0$$

where $a_i \in \{0, 1\}$ for all i .

- **Modular Reduction:**

Modular reduction in $GF(2^n)$ involves reducing a polynomial by a fixed irreducible polynomial of degree n , ensuring that the result remains within the field. Let $m(x)$ be the irreducible polynomial. The reduction of a polynomial $f(x)$ is given by: $f(x) \bmod m(x)$

- **XOR as Modular Reduction:**

During modular reduction, the subtraction used in polynomial division becomes XOR, because subtraction and addition are the same in $GF(2)$. Therefore, reducing a polynomial $f(x)$ by $m(x)$ is effectively performed using XOR on the coefficients of corresponding terms.

For example, if $f(x)$ has a term x^k where $k \geq n$, and $m(x)$ has a term x^k , then reducing $f(x)$ by $m(x)$ involves XORing the coefficients of x^k in $f(x)$ and $m(x)$, effectively eliminating the x^k term in $f(x)$.

In summary, the XOR operation becomes equivalent to both addition and modular reduction in $GF(2^n)$ due to the binary nature of the field. This equivalence simplifies polynomial arithmetic in binary fields, making it a cornerstone of operations in cryptographic algorithms.

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Chapter 1

Block Cipher AES-128

1.1 Overview of AES-128

- KeyExpansion : $\{0, 1\}^{128} \rightarrow \{0, 1\}^{1408=4 \cdot (10+1) \cdot 32}$.
- AddRoundKey : $\{0, 1\}^{128} \times \{0, 1\}^{128} \rightarrow \{0, 1\}^{128}$.
- SubBytes/ShiftRows/MixColumns : $\{0, 1\}^{128} \rightarrow \{0, 1\}^{128}$.

Algorithm 1: Encryption of AES-128

Input: block $\text{src} \in \{0, 1\}^{128}$, round-keys $\{rk_i\}_{i=0}^{11}$ ($rk_i \in \{0, 1\}^{128}$)

Output: block $\text{dst} \in \{0, 1\}^{128}$

```
1  $t \leftarrow \text{AddRoundKey}(\text{src}, rk_0);$ 
2 for  $i \leftarrow 1$  to 9 do
3    $t \leftarrow (\text{MixColumns} \circ \text{ShiftRows} \circ \text{SubBytes})(t);$ 
4    $t \leftarrow \text{AddRoundKey}(t, rk_i);$ 
5 end
6  $t \leftarrow (\text{ShiftRows} \circ \text{SubBytes})(t);$ 
7  $t \leftarrow \text{AddRoundKey}(t, rk_{10});$ 
8  $\text{dst} \leftarrow t;$ 
9 return  $\text{dst};$ 
```

Algorithm 2: Decryption of AES-128

Input: block $\text{src} \in \{0, 1\}^{128}$, round-keys $\{rk_i\}_{i=0}^{11}$ ($rk_i \in \{0, 1\}^{128}$)

Output: block $\text{dst} \in \{0, 1\}^{128}$

```
1  $t \leftarrow \text{AddRoundKey}(\text{src}, rk_{10});$ 
2 for  $i \leftarrow 9$  to 1 do
3    $t \leftarrow (\text{InvSubBytes} \circ \text{InvShiftRows})(t);$ 
4    $t \leftarrow \text{AddRoundKey}(t, rk_i);$ 
5    $t \leftarrow \text{InvMixColumns}(t);$ 
6 end
7  $t \leftarrow (\text{InvShiftRows} \circ \text{InvSubBytes})(t);$ 
8  $t \leftarrow \text{AddRoundKey}(t, rk_0);$ 
9  $\text{dst} \leftarrow t;$ 
10 return  $\text{dst};$ 
```

1.2 Functions and Constants used in AES

1.2.1 Key Expansion

- **RotWord** : $\{0, 1\}^{32} \rightarrow \{0, 1\}^{32}$ is defined by

$$\text{RotWord}(X_0 \parallel X_1 \parallel X_2 \parallel X_3) := X_1 \parallel X_2 \parallel X_3 \parallel X_0 \quad \text{for } X_i \in \{0, 1\}^8.$$

Code 1.1: RotWord rotates the input word left by one byte

```
1 u32 RotWord(u32 word) {
2     return (word << 0x08) | (word >> 0x18);
3 }
```

- **SubWord** : $\{0, 1\}^{32} \rightarrow \{0, 1\}^{32}$ is defined by

$$\text{SubWord}(X_0 \parallel X_1 \parallel X_2 \parallel X_3) := s(X_0) \parallel s(X_1) \parallel s(X_2) \parallel s(X_3) \quad \text{for } X_i \in \{0, 1\}^8.$$

Here, $s : \{0, 1\}^8 \rightarrow \{0, 1\}^8$ is the **S-box**.

Code 1.2: SubWord applies the S-box to each byte of the input word

```
1 u32 SubWord(u32 word) {
2     return (u32)s_box[word >> 0x18] << 0x18 |
3         (u32)s_box[(word >> 0x10) & 0xFF] << 0x10 |
4         (u32)s_box[(word >> 0x08) & 0xFF] << 0x08 |
5         (u32)s_box[word & 0xFF];
6 }
```

- **Round Constant rCon**:

The constant $\text{rCon}_i \in \mathbb{F}_{2^8}$ used in generating the i -th round key corresponds to the value of x^{i-1} in the binary finite field \mathbb{F}_{2^8} and is as follows:

i	1	2	3	4	5	6	7	8	9	10
Rcon_i	0x01	0x02	0x04	0x08	0x10	0x20	0x40	0x80	0x1b	0x36

Code 1.3: rCon Array Declaration

```
1 static const u32 rCon[10] = {
2     0x01000000, 0x02000000, 0x04000000, 0x08000000,
3     0x10000000, 0x20000000, 0x40000000, 0x80000000,
4     0x1b000000, 0x36000000
5 };
```

Algorithm 3: Key Schedule (AES-128)

Input: User key $uk = (uk_0, \dots, uk_{15})$ ($uk_i \in \{0, 1\}^8$); // $uk \in \{0, 1\}^{128}$ is 16-byte

Output: round-keys $\{rk_i\}_{i=0}^{43}$ ($rk_i \in \{0, 1\}^{32}$); // $\{rk_i\}_{i=0}^{43} \in \{0, 1\}^{1408}$ is 176-byte

```

1  $rk_0 \leftarrow uk_0 \parallel uk_1 \parallel uk_2 \parallel uk_3$ ;
2  $rk_1 \leftarrow uk_4 \parallel uk_5 \parallel uk_6 \parallel uk_7$ ;
3  $rk_2 \leftarrow uk_8 \parallel uk_9 \parallel uk_{10} \parallel uk_{11}$ ;
4  $rk_3 \leftarrow uk_{12} \parallel uk_{13} \parallel uk_{14} \parallel uk_{15}$ ;
5 for  $i = 4$  to 43 do
6    $t \leftarrow rk_{i-1}$ ;
7   if  $i \bmod 4 = 0$  then
8     /* SubWord  $\circ$  RotWord :  $\{0, 1\}^{32} \rightarrow \{0, 1\}^{32}$  */
9      $t \leftarrow \text{RotWord}(t)$ ;
10     $t \leftarrow \text{SubWord}(t)$ ;
11     $t \leftarrow t \oplus (rCon_{i/4} \parallel 0x00 \parallel 0x00 \parallel 0x00)$ ;
12  end
13   $rk_i \leftarrow rk_{i-4} \oplus_{32} t$ ;
14 end

```

Code 1.4: AES-128 Key Expansion

```

1 void KeyExpansion(const u8* uKey, u32* rKey) {
2   u32 temp;
3   int i = 0;
4
5   // Copy the input key to the first round key
6   while (i < 4) {
7     rKey[i] = (u32)uKey[4*i] << 0x18 |
8     (u32)uKey[4*i+1] << 0x10 |
9     (u32)uKey[4*i+2] << 0x08 |
10    (u32)uKey[4*i+3];
11    i++;
12  }
13
14  i = 4;
15
16  // Generate the remaining round keys
17  while (i < 44) {
18    temp = rKey[i-1];
19    if (i % 4 == 0) {
20      temp = SubWord(RotWord(temp)) ^ rCon[i/4-1];
21    }
22    rKey[i] = rKey[i-4] ^ temp;
23    i++;
24  }
25 }

```

1.2.2 AddRoundKey

- $\text{AddRoundKey} : \{0, 1\}^{128} \times \{0, 1\}^{128} \rightarrow \{0, 1\}^{128}$ is defined by

$$\text{AddRoundKey}(\{X_i\}_{i=0}^{15}, \{rk_i\}_{i=0}^3) := \{X_i \oplus_8 uk_i\}_{i=0}^{15}.$$

Code 1.5: AES AddRoundKey

```

1 void AddRoundKey(u8* state, const u32* rKey) {
2     for (int i = 0; i < AES_BLOCK_SIZE; i++) {
3         // i = 0, 1, 2, 3 => wordIndex = 0
4         // i = 4, 5, 6, 7 => wordIndex = 1
5         // i = 8, 9, 10, 11 => wordIndex = 2
6         // i = 12, 13, 14, 15 => wordIndex = 3
7         int wordIndex = i / 4;
8
9         // i = 0, 1, 2, 3 => bytePosition = 0, 1, 2, 3
10        // i = 4, 5, 6, 7 => bytePosition = 0, 1, 2, 3
11        // i = 8, 9, 10, 11 => bytePosition = 0, 1, 2, 3
12        // i = 12, 13, 14, 15 => bytePosition = 0, 1, 2, 3
13        int bytePosition = i % 4;
14        /*
15         * +-----+-----+-----+-----+
16         * | i      | wordIndex | bytePosition | shiftedWord |
17         * +-----+-----+-----+-----+
18         * | 0-3    | 0        | 0          | rKey[0] >> 0x18 |
19         * |        |        | 1          | rKey[0] >> 0x10 |
20         * |        |        | 2          | rKey[0] >> 0x08 |
21         * |        |        | 3          | rKey[0]         |
22         * +-----+-----+-----+-----+
23         * | 4-7    | 1        | 0          | rKey[1] >> 24   |
24         * |        |        | 1          | rKey[1] >> 16   |
25         * |        |        | 2          | rKey[1] >> 8    |
26         * |        |        | 3          | rKey[1]         |
27         * +-----+-----+-----+-----+
28         * | ...    | ...      | ...        | ...             |
29         * +-----+-----+-----+-----+
30         * | 15     | 3        | 3          | rKey[3]         |
31         * +-----+-----+-----+-----+
32        */
33        u32 shiftedWord =
34            rKey[wordIndex] >> (8 * (3 - bytePosition));
35
36        u8 keyByte = shiftedWord & 0xFF;
37        state[i] ^= keyByte;
38
39        /* Extract the corresponding byte from the round key word */
40        // state[i] ^= (rKey[i / 4] >> (8 * (3 - (i % 4)))) & 0xFF;
41    }
42 }

```


1.2.3 SubBytes / InvSubBytes

- SubBytes : $\{0, 1\}^{128} \rightarrow \{0, 1\}^{128}$ is defined by

$$\text{SubBytes}(\{X_i\}_{i=0}^{15}) = \{s(X_i)\}_{i=0}^{15}.$$

- InvSubBytes : $\{0, 1\}^{128} \rightarrow \{0, 1\}^{128}$ is defined by

$$\text{SubBytes}(\{X_i\}_{i=0}^{15}) = \{s^{-1}(X_i)\}_{i=0}^{15}.$$

Table 1.1: Substitution Box

	00	01	02	03	04	05	06	07	08	09	0a	0b	0c	0d	0e	0f
00	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
10	ca	82
30
40
50
60
70
80
90
a0
b0
c0
d0	c1
e0	28	...
f0	16

Code 1.6: Byte Substitution

```

1 void SubBytes(u8* state) {
2     for (int i = 0; i < AES_BLOCK_SIZE; i++) {
3         state[i] = s_box[state[i]];
4     }
5 }

```

Code 1.7: Inverse Byte Substitution

```

1 void SubBytes(u8* state) {
2     for (int i = 0; i < AES_BLOCK_SIZE; i++) {
3         state[i] = inv_s_box[state[i]];
4     }
5 }

```

1.2.4 ShiftRows / InvShiftRows

- ShiftRows : $\{0, 1\}^{128} \rightarrow \{0, 1\}^{128}$ is defined by

X_0	X_4	X_8	X_{12}
X_1	X_5	X_9	X_{13}
X_2	X_6	X_{10}	X_{14}
X_3	X_7	X_{11}	X_{15}

 \Rightarrow

X_0	X_4	X_8	X_{12}
X_5	X_9	X_{13}	X_1
X_{10}	X_{14}	X_2	X_6
X_{15}	X_3	X_7	X_{11}

- InvShiftRows : $\{0, 1\}^{128} \rightarrow \{0, 1\}^{128}$ is defined by

X_0	X_4	X_8	X_{12}
X_1	X_5	X_9	X_{13}
X_2	X_6	X_{10}	X_{14}
X_3	X_7	X_{11}	X_{15}

 \Rightarrow

X_0	X_4	X_8	X_{12}
X_{13}	X_1	X_5	X_9
X_{10}	X_{14}	X_2	X_6
X_7	X_{11}	X_{15}	X_3

Code 1.8: ShiftRows

```

1 void ShiftRows(u8* state) {
2     u8 temp;
3
4     // Row 1: shift left by 1
5     temp = state[1];
6     state[1] = state[5];
7     state[5] = state[9];
8     state[9] = state[13];
9     state[13] = temp;
10
11    // Row 2: shift left by 2
12    temp = state[2];
13    state[2] = state[10];
14    state[10] = temp;
15    temp = state[6];
16    state[6] = state[14];
17    state[14] = temp;
18
19    // Row 3: shift left by 3 (or right by 1)
20    temp = state[15];
21    state[15] = state[11];
22    state[11] = state[7];
23    state[7] = state[3];
24    state[3] = temp;
25 }

```

Code 1.9: Inverse ShiftRows

```
1 void InvShiftRows(u8* state) {  
2     u8 temp;  
3  
4     // Row 1: shift left by 3 (or right by 1)  
5     temp = state[13];  
6     state[13] = state[9];  
7     state[9] = state[5];  
8     state[5] = state[1];  
9     state[1] = temp;  
10  
11    // Row 2: shift left by 2  
12    temp = state[2];  
13    state[2] = state[10];  
14    state[10] = temp;  
15    temp = state[6];  
16    state[6] = state[14];  
17    state[14] = temp;  
18  
19    // Row 3: shift left by 1  
20    temp = state[3];  
21    state[3] = state[7];  
22    state[7] = state[11];  
23    state[11] = state[15];  
24    state[15] = temp;  
25 }
```

1.2.5 MixColumns / InvMixColumns

- Multiplication in the finite field $GF(2^8)$.

$$\text{MUL}_{GF(2^8)} : \{0, 1\}^8 \times \{0, 1\}^8 \rightarrow \{0, 1\}^8.$$

Here,

$$\{0, 1\}^8 \simeq GF(2^8) = \mathbb{F}_{2^8} := \mathbb{F}_2[z]/(z^8 + z^4 + z^3 + z + 1) = \left\{ a_7z^7 + \dots + a_1z + a_0 : a_i \in \mathbb{F}_2 \right\}.$$

Note that

$$a(z) \times b(z) := a(z) \times b(z) \bmod (z^8 + z^4 + z^3 + z + 1)$$

Note. Given two polynomials $a(x)$ and $b(x)$ in $GF(2^8)$:

$$a(x) = a_7x^7 + a_6x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0,$$

$$b(x) = b_7x^7 + b_6x^6 + b_5x^5 + b_4x^4 + b_3x^3 + b_2x^2 + b_1x + b_0.$$

The algorithm performs polynomial multiplication in the finite field $GF(2^8)$. It uses a shift-and-add method, with an additional reduction step modulo an irreducible polynomial $m(x) = x^8 + x^4 + x^3 + x + 1$.

1. Initialization: Set $p(x) = 0$ to initialize the product polynomial.
2. Iterate over each bit of $b(x)$, from LSB to MSB.
 - (i) If the current bit b_i of $b(x)$ is 1, update $p(x)$ as $p(x) \oplus a(x)$. In $GF(2^8)$, addition is equivalent to the XOR operation:

$$p(x) = p(x) \oplus a(x).$$

- (ii) Shift $a(x)$ left by 1 (multiply by x), increasing its degree by 1:

$$a(x) = a(x) \cdot x.$$

- (iii) If the coefficient of x^8 in $a(x)$ is 1, reduce $a(x)$ by $m(x)$ to keep the degree under 8:

$$a(x) = a(x) \oplus m(x).$$

- (iv) Shift $b(x)$ right by 1 (divide by x) for the next iteration:

$$b(x) = b(x) / x.$$

3. After all bits of $b(x)$ are processed, $p(x)$ be the product of $a(x)$ and $b(x)$ modulo $m(x)$.

Note (Modular Reduction in $GF(2^8)$ using XOR). In the context of multiplication in the binary finite field $GF(2^8)$, modular reduction ensures that results of operations remain within the field. The use of XOR for modular reduction is due to the properties of polynomial arithmetic over $GF(2)$ and the representation of elements in $GF(2^8)$.

– **Polynomial Representation in $GF(2^8)$:**

1. **Elements as Polynomials:** Each element in $GF(2^8)$ can be represented as a polynomial of degree less than 8, where each coefficient is either 0 or 1, i.e.,

$$GF(2^8) = \mathbb{F}_2[x]/(x^8 + x^4 + x^3 + x + 1) = \{a_7x^7 + \dots + a_1x + a_0 : a_i \in \mathbb{F}_2\}.$$

This corresponds to an 8-bit binary number, with each bit representing a coefficient of the polynomial, i.e.,

$$a_7x^7 + \dots + a_1x + a_0 \iff (a_7 \dots a_1a_0)_2.$$

2. **Binary Operations:** In $GF(2)$, addition and subtraction are equivalent to the XOR operation, since $1 + 1 = 0$ in this field, the same as $1 \oplus 1$.

– **Modular Reduction with an Irreducible Polynomial**

1. **Irreducible Polynomial:** In $GF(2^8)$, an irreducible polynomial of degree 8, typically $p(x) = x^8 + x^4 + x^3 + x + 1$ (represented as `0x11b` in binary), is used for modular reduction.
 2. **Modular Reduction Process:** After multiplying two polynomials, if the resulting polynomial's degree is 8 or higher, it must be reduced modulo the irreducible polynomial to ensure the result remains a polynomial of degree less than 8, thus staying within $GF(2^8)$.
 3. **XOR for Reduction:** XOR is used for modular reduction in $GF(2^8)$ because polynomial subtraction in $GF(2)$ is performed by XORing coefficients.
- Given two elements in $GF(2^8)$, $a(x)$ and $b(x)$, their product is $c(x) = a(x) \cdot b(x)$. If $\deg(c(x)) \geq 8$, then $c(x)$ must be reduced modulo the irreducible polynomial $p(x)$. This is achieved by XORing the coefficients of $c(x)$ and $p(x)$:

$$c(x) = a(x) \cdot b(x) \mod p(x)$$

If $c(x)$ has a term x^8 or higher, we subtract $p(x)$ from $c(x)$ to reduce its degree. In $GF(2)$, subtraction is equivalent to addition, performed by XORing coefficients:

$$c'(x) = c(x) \oplus p(x)$$

This operation effectively eliminates the term x^8 (or higher) in $c(x)$, ensuring that the result remains within $GF(2^8)$. Consider the product of two polynomials $a(x)$ and $b(x)$ in $GF(2^8)$:

$$a(x) = x^6 + x^4 + x^2 + x + 1 \quad \text{and} \quad b(x) = x^7 + x + 1$$

The product $c(x) = a(x) \cdot b(x)$ might yield a polynomial of degree 8 or higher. To reduce $c(x)$ modulo $p(x) = x^8 + x^4 + x^3 + x + 1$, we perform XOR between the coefficients of $c(x)$ and $p(x)$, ensuring the result stays within $GF(2^8)$.

Code 1.10: Multiplication in $GF(2^8)$

```

1  u8 MUL_GF256(u8 a, u8 b) {
2      u8 res = 0;
3      // Mask for detecting the MSB (0x80 = 0b10000000)
4      u8 MSB_mask = 0x80;
5      u8 MSB;
6      /*
7       * The reduction polynomial
8       * (x^8 + x^4 + x^3 + x + 1) = 0b100011011
9       * for AES, represented in hexadecimal
10     */
11     u8 modulo = 0x1B;
12
13     for (int i = 0; i < 8; i++) {
14         // Add a to result if LSB(b)=1
15         if (b & 1)
16             res ^= a;
17
18         MSB = a & MSB_mask; // Store the MSB of a
19         a <<= 1; // Multiplying it by x effectively
20
21         // Reduce the result modulo the reduction polynomial
22         if (MSB)
23             a ^= modulo;
24
25         b >>= 1; // Moving to the next bit
26     }
27
28     return res;
29 }
30
31 #define MUL_GF256(a, b) ({ \
32     u8 res = 0; \
33     u8 MSB_mask = 0x80; \
34     u8 MSB; \
35     u8 modulo = 0x1B; \
36     u8 temp_a = (a); \
37     u8 temp_b = (b); \
38     for (int i = 0; i < 8; i++) { \
39         if (temp_b & 1) \
40             res ^= temp_a; \
41         MSB = temp_a & MSB_mask; \
42         temp_a <<= 1; \
43         if (MSB) \
44             temp_a ^= modulo; \
45         temp_b >>= 1; \
46     } \
47     res; \
48 })

```

- MixColumns : $\{0, 1\}^{128} \rightarrow \{0, 1\}^{128}$ is defined by

$$\text{MixColumns} \begin{pmatrix} X_0 & X_4 & X_8 & X_{12} \\ X_1 & X_5 & X_9 & X_{13} \\ X_2 & X_6 & X_{10} & X_{14} \\ X_3 & X_7 & X_{11} & X_{15} \end{pmatrix} := \begin{pmatrix} 0x02 & 0x03 & 0x01 & 0x01 \\ 0x01 & 0x02 & 0x03 & 0x01 \\ 0x01 & 0x01 & 0x02 & 0x03 \\ 0x03 & 0x01 & 0x01 & 0x02 \end{pmatrix} \begin{pmatrix} X_0 & X_4 & X_8 & X_{12} \\ X_1 & X_5 & X_9 & X_{13} \\ X_2 & X_6 & X_{10} & X_{14} \\ X_3 & X_7 & X_{11} & X_{15} \end{pmatrix}.$$

- InvMixColumns : $\{0, 1\}^{128} \rightarrow \{0, 1\}^{128}$ is defined by

$$\text{MixColumns} \begin{pmatrix} X_0 & X_4 & X_8 & X_{12} \\ X_1 & X_5 & X_9 & X_{13} \\ X_2 & X_6 & X_{10} & X_{14} \\ X_3 & X_7 & X_{11} & X_{15} \end{pmatrix} := \begin{pmatrix} 0x0e & 0x0b & 0x0d & 0x09 \\ 0x09 & 0x0e & 0x0b & 0x0d \\ 0x0d & 0x09 & 0x0e & 0x0b \\ 0x0b & 0x0d & 0x09 & 0x0e \end{pmatrix} \begin{pmatrix} X_0 & X_4 & X_8 & X_{12} \\ X_1 & X_5 & X_9 & X_{13} \\ X_2 & X_6 & X_{10} & X_{14} \\ X_3 & X_7 & X_{11} & X_{15} \end{pmatrix}.$$

Code 1.11: MixColumns

```

1 void MixColumns(u8* state) {
2     u8 temp[4];
3     // Multiply and add the elements in the column
4     // by the fixed polynomial
5     for (int i = 0; i < 4; i++) {
6         temp[0] =
7             MUL_GF256(0x02, state[i * 4]) ^
8             MUL_GF256(0x03, state[i * 4 + 1]) ^
9             state[i * 4 + 2] ^
10            state[i * 4 + 3];
11
12        temp[1] =
13            state[i * 4] ^
14            MUL_GF256(0x02, state[i * 4 + 1]) ^
15            MUL_GF256(0x03, state[i * 4 + 2]) ^
16            state[i * 4 + 3];
17
18        temp[2] =
19            state[i * 4] ^
20            state[i * 4 + 1] ^
21            MUL_GF256(0x02, state[i * 4 + 2]) ^
22            MUL_GF256(0x03, state[i * 4 + 3]);
23
24        temp[3] =
25            MUL_GF256(0x03, state[i * 4]) ^
26            state[i * 4 + 1] ^
27            state[i * 4 + 2] ^
28            MUL_GF256(0x02, state[i * 4 + 3]);
29
30        // Copy the mixed column back to the state
31        for (int j = 0; j < 4; j++)
32            state[i * 4 + j] = temp[j];
33    }
34 }

```

Code 1.12: Inverse MixColumns

```

1  void InvMixColumns(u8* state) {
2      u8 temp[4];
3
4      for (int i = 0; i < 4; i++) {
5          temp[0] =
6              MUL_GF256(0x0e, state[i * 4]) ^
7              MUL_GF256(0x0b, state[i * 4 + 1]) ^
8              MUL_GF256(0x0d, state[i * 4 + 2]) ^
9              MUL_GF256(0x09, state[i * 4 + 3]);
10
11         temp[1] =
12             MUL_GF256(0x09, state[i * 4]) ^
13             MUL_GF256(0x0e, state[i * 4 + 1]) ^
14             MUL_GF256(0x0b, state[i * 4 + 2]) ^
15             MUL_GF256(0x0d, state[i * 4 + 3]);
16
17         temp[2] =
18             MUL_GF256(0x0d, state[i * 4]) ^
19             MUL_GF256(0x09, state[i * 4 + 1]) ^
20             MUL_GF256(0x0e, state[i * 4 + 2]) ^
21             MUL_GF256(0x0b, state[i * 4 + 3]);
22
23         temp[3] =
24             MUL_GF256(0x0b, state[i * 4]) ^
25             MUL_GF256(0x0d, state[i * 4 + 1]) ^
26             MUL_GF256(0x09, state[i * 4 + 2]) ^
27             MUL_GF256(0x0e, state[i * 4 + 3]);
28
29         for (int j = 0; j < 4; j++)
30             state[i * 4 + j] = temp[j];
31     }
32 }

```


Chapter 2

AES - 128 / 192 / 256 (Byte Version)

2.1 Specification

Table 2.1: Parameters of the Block Cipher AES

Algorithms	Block Size (N_b -byte)	Key Length (N_k -byte)	Number of Rounds (N_r)	Round-Key Length (byte)	Number of Round-Keys ($N_r + 1$)	Total Size of Round-Keys ($N_b(N_r + 1)$)
AES-128	16	16 (4·32-bit)	10	16	11	176
AES-192	16	24 (6·32-bit)	12	16	13	208
AES-256	16	32 (8·32-bit)	14	16	15	240

Code 2.1: Configuration

```
1 // Define macros for AES key length
2 #define AES_VERSION 128 // Can be 128, 192, or 256
3 // Define macro for AES block size
4 #define AES_BLOCK_SIZE 16
5
6 // Define Nk and Nr based on AES key length
7 #if AES_VERSION == 128
8     #define Nk 4
9     #define Nr (Nk + 6) // 10
10    #define ROUND_KEYS_SIZE (16 * (Nr + 1)) // 176
11 #elif AES_VERSION == 192
12     #define Nk 6
13     #define Nr (Nk + 6) // 12
14     #define ROUND_KEYS_SIZE (16 * (Nr + 1)) // 208
15 #elif AES_VERSION == 256
16     #define Nk 8
17     #define Nr (Nk + 6) // 14
18     #define ROUND_KEYS_SIZE (16 * (Nr + 1)) // 240
19 #else
20     #error "Invalid AES ky length"
21 #endif
```

2.2 Key Expansion (General Version)

Algorithm 4: Key Schedule (General Version)

Input: User-key $uk = (uk_0, \dots, uk_{N_k-1})$ ($uk_i \in \{0, 1\}^8$); // uk is 16/24/32-byte

Output: Round-key $\{rk_i\}_{i=0}^{4(N_r+1)-1}$ ($rk_i \in \{0, 1\}^{32}$)

/* $\{rk_i\}_{i=0}^{4(N_r+1)-1}$ is 176/208/240-byte */

```

1  $l \leftarrow N_k/4;$  //  $l = 4, 6, 8$ 
2 for  $i = 0$  to  $l - 1$  do
3    $rk_i \leftarrow uk_{4i} \parallel uk_{4i+1} \parallel uk_{4i+2} \parallel uk_{4i+3};$ 
4 end
5 for  $i = l$  to  $4(N_r + 1) - 1$  do
6    $t \leftarrow rk_{i-1};$ 
7   if  $i \bmod l = 0$  then
8      $t \leftarrow (\text{SubWord} \circ \text{RotWord})(t);$ 
9      $t \leftarrow t \oplus_{32} (\text{rCon}_{i/l} \parallel 0x00 \parallel 0x00 \parallel 0x00);$ 
10  else if  $l > 6 \ \&\& \ i \bmod l = 4$  then
11     $t \leftarrow \text{SubWord}(t);$ 
12  end
13   $rk_i \leftarrow rk_{i-l} \oplus_{32} t;$ 
14 end

```

Code 2.2: Key Expansion (General ver.)

```

1 void KeyExpansion(const u8* uKey, u32* rKey) {
2   u32 temp;
3
4   for (int i = 0; i < Nk; i++) {
5     rKey[i] = (u32)uKey[4*i] << 0x18 |
6              (u32)uKey[4*i+1] << 0x10 |
7              (u32)uKey[4*i+2] << 0x08 |
8              (u32)uKey[4*i+3];
9   }
10
11  for (int i = Nk; i < (Nr + 1) * 4; i++) {
12    temp = rKey[i - 1];
13    if (i % Nk == 0) {
14      temp = SubWord(RotWord(temp)) ^ rCon[i / Nk - 1];
15    } else if (Nk > 6 && i % Nk == 4) {
16      // Additional S-box transformation for AES-256
17      temp = SubWord(temp);
18    }
19    rKey[i] = rKey[i - Nk] ^ temp;
20  }
21 }

```

2.3 8-bit AES - 128 / 192 / 256

Algorithm 5: Encryption of 8-bit AES

Input: block $\text{src} \in \{0, 1\}^{128}$, round-keys $\{rk_i\}_{i=0}^{N_r+1}$ ($rk_i \in \{0, 1\}^{32 \times 4}$)

Output: block $\text{dst} \in \{0, 1\}^{128}$

```

1  $t \leftarrow \text{AddRoundKey}(\text{src}, rk_0);$            //  $\text{AddRoundKey}: \{0, 1\}^{8 \times 16} \times \{0, 1\}^{32 \times 4} \rightarrow \{0, 1\}^{8 \times 16}$ 
2 for  $i \leftarrow 1$  to  $N_r - 1$  do
3    $t \leftarrow \text{SubBytes}(t);$                      //  $\text{SubBytes}: \{0, 1\}^{8 \times 16} \rightarrow \{0, 1\}^{8 \times 16}$ 
4    $t \leftarrow \text{ShiftRows}(t);$                    //  $\text{ShiftRows}: \{0, 1\}^{8 \times 16} \rightarrow \{0, 1\}^{8 \times 16}$ 
5    $t \leftarrow \text{MixColumns}(t);$                    //  $\text{MixColumns}: \{0, 1\}^{8 \times 16} \rightarrow \{0, 1\}^{8 \times 16}$ 
6    $t \leftarrow \text{AddRoundKey}(t, rk_i);$ 
7 end
8  $t \leftarrow \text{SubBytes}(t);$ 
9  $t \leftarrow \text{ShiftRows}(t);$ 
10  $t \leftarrow \text{AddRoundKey}(t, rk_{N_r});$ 
11  $\text{dst} \leftarrow t;$ 
12 return  $\text{dst};$ 

```

Code 2.3: 8-bit AES Encryption

```

1 void AES_Encrypt(const u8* plaintext, const u8* key,
2   u8* ciphertext) {
3   // AES-128/192/256: roundKey[44]/roundKey[52]/roundKey[60]
4   u32 roundKey[ROUND_KEYS_SIZE / sizeof(u32)];
5   u8 state[AES_BLOCK_SIZE]; // state[16]
6
7   // Copy plaintext to state
8   for (int i = 0; i < AES_BLOCK_SIZE; i++)
9     state[i] = plaintext[i];
10
11   KeyExpansion(key, roundKey);
12
13   // 0: roundKey[ 0] | roundKey[1] | roundKey[ 2] | roundKey[ 3]
14   AddRoundKey(state, roundKey); // Initial round
15
16   for (int round = 1; round <= Nr; round++) { // Main rounds
17     SubBytes(state); ShiftRows(state);
18     if (round != Nr) MixColumns(state);
19     // 1: roundKey[ 4] | roundKey[5] | roundKey[ 6] | roundKey[ 7]
20     // 2: roundKey[ 8] | roundKey[9] | roundKey[10] | roundKey[11]
21     // i: roundKey[4*i] | ... | ... | roundKey[4*i+3]
22     AddRoundKey(state, roundKey + 4 * round);
23   }
24
25   // Copy state to ciphertext
26   for (int i = 0; i < AES_BLOCK_SIZE; i++)
27     ciphertext[i] = state[i];
28 }

```

Algorithm 6: Decryption of 8-bit AES

Input: block $\text{src} \in \{0, 1\}^{128}$, round-keys $\{rk_i\}_{i=0}^{N_r+1}$ ($rk_i \in \{0, 1\}^{32 \times 4}$)

Output: block $\text{dst} \in \{0, 1\}^{128}$

```

1  $t \leftarrow \text{AddRoundKey}(\text{src}, rk_{N_r});$ 
2 for  $i \leftarrow N_r - 1$  to 1 do
3    $t \leftarrow \text{InvShiftRows}(t);$ 
4    $t \leftarrow \text{InvSubBytes}(t);$ 
5    $t \leftarrow \text{AddRoundKey}(t, rk_i);$ 
6    $t \leftarrow \text{InvMixColumns}(t);$ 
7 end
8  $t \leftarrow \text{InvShiftRows}(t);$ 
9  $t \leftarrow \text{InvSubBytes}(t);$ 
10  $t \leftarrow \text{AddRoundKey}(t, rk_0);$ 
11  $\text{dst} \leftarrow t;$ 
12 return  $\text{dst};$ 

```

Code 2.4: 8-bit AES Decryption

```

1 void AES_Decrypt(const u8* ciphertext, const u8* key,
2   u8* plaintext) {
3   u32 roundKey[ROUND_KEYS_SIZE / sizeof(u32)];
4   u8 state[AES_BLOCK_SIZE];
5
6   KeyExpansion(key, roundKey);
7
8   for (int i = 0; i < AES_BLOCK_SIZE; i++)
9     state[i] = ciphertext[i];
10
11   // Initial round with the last round key
12   AddRoundKey(state, roundKey + 4 * Nr);
13
14   // Main rounds in reverse order
15   for (int round = Nr - 1; round >= 0; round--) {
16     InvShiftRows(state);
17     InvSubBytes(state);
18     // i: roundKey[4*i] | ... | roundKey[4*i+3]
19     // 1: roundKey[ 4] | roundKey[5] | roundKey[ 6] | roundKey[ 7]
20     // 0: roundKey[ 0] | roundKey[1] | roundKey[ 2] | roundKey[ 3]
21     AddRoundKey(state, roundKey + 4 * round);
22     if (round != 0)
23       InvMixColumns(state);
24   }
25
26   for (int i = 0; i < AES_BLOCK_SIZE; i++)
27     plaintext[i] = state[i];
28 }

```

Chapter 3

AES - 128 / 192 / 256 (32-bit Version)

3.1 SubMix and InvSubInvMix

- $\text{SubMix} : \{0, 1\}^{128} \rightarrow \{0, 1\}^{128}$ and $\text{InvSubInvMix} : \{0, 1\}^{128} \rightarrow \{0, 1\}^{128}$ are defined by

$$\begin{aligned}\text{SubMix} &:= \text{MixColumns} \circ \text{SubBytes} \\ \text{InvSubInvMix} &:= \text{InvMixColumns} \circ \text{InvSubBytes}.\end{aligned}$$

Algorithm 7: Encryption and Decryption of AES using SubMix and InvSubInvMix

Input: block $\text{src} \in \{0, 1\}^{128}$, round-keys $\{rk_i\}_{i=0}^{N_r+1}$ ($rk_i \in \{0, 1\}^{128}$)

Output: block $\text{dst} \in \{0, 1\}^{128}$

1 **Function** AES_ENC:

```
2    $t \leftarrow \text{src};$ 
3    $t \leftarrow \text{AddRoundKey}(t, rk_0);$ 
4   for  $i \leftarrow 1$  to  $N_r - 1$  do
5        $t \leftarrow \text{ShiftRows}(t);$ 
6        $t \leftarrow \text{SubMix}(t);$ 
7        $t \leftarrow \text{AddRoundKey}(t, rk_i);$ 
8   end
9    $t \leftarrow \text{SubBytes}(t);$ 
10   $t \leftarrow \text{ShiftRows}(t);$ 
11   $t \leftarrow \text{AddRoundKey}(t, rk_{N_r});$ 
12   $\text{dst} \leftarrow t;$ 
13  return  $\text{dst};$ 
14 end
```

1 **Function** AES_DEC:

```
2    $t \leftarrow \text{src};$ 
3    $t \leftarrow \text{AddRoundKey}(t, rk_{N_r});$ 
4   for  $i \leftarrow N_r - 1$  downto 1 do
5        $t \leftarrow \text{InvShiftRows}(t);$ 
6        $t \leftarrow \text{InvSubInvMix}(t);$ 
7        $t' \leftarrow \text{InvMixColumns}(rk_i);$ 
8        $t \leftarrow \text{AddRoundKey}(t, t');$ 
9   end
10   $t \leftarrow \text{InvShiftRows}(t);$ 
11   $t \leftarrow \text{InvSubBytes}(t);$ 
12   $t \leftarrow \text{AddRoundKey}(t, rk_0);$ 
13   $\text{dst} \leftarrow t;$ 
14  return  $\text{dst};$ 
15 end
```

3.2 8×32 Table Look Up

Convert the 8-bit string $\{X_i\}_{i=0}^{15}$ ($X_i \in \{0, 1\}^8$) into a 32-bit string $\{W_i\}_{i=0}^3$ ($W_i \in \{0, 1\}^{32}$) as follows:

$$W_i := (X_{4i} \ll 24) \parallel (X_{4i+1} \ll 16) \parallel (X_{4i+2} \ll 8) \parallel (X_{4i+3}) \in \{0, 1\}^{32}$$

for $i = 0, 1, 2, 3$. In other words,

X_0	$(W_0 \gg 0x18) \& 0xff$	X_4	$(W_1 \gg 0x18) \& 0xff$
X_1	$(W_0 \gg 0x10) \& 0xff$	X_5	$(W_1 \gg 0x10) \& 0xff$
X_2	$(W_0 \gg 0x08) \& 0xff$	X_6	$(W_1 \gg 0x08) \& 0xff$
X_3	$(W_0) \& 0xff$	X_7	$(W_1) \& 0xff$
X_8	$(W_2 \gg 0x18) \& 0xff$	X_{12}	$(W_3 \gg 0x18) \& 0xff$
X_9	$(W_2 \gg 0x10) \& 0xff$	X_{13}	$(W_3 \gg 0x10) \& 0xff$
X_{10}	$(W_2 \gg 0x08) \& 0xff$	X_{14}	$(W_3 \gg 0x08) \& 0xff$
X_{11}	$(W_2) \& 0xff$	X_{15}	$(W_3) \& 0xff$

Note that

X_0	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}
W_0				W_1				W_2				W_3			

Algorithm 8: SubMix and InvSubInvMix

Input: $(W_0, W_1, W_2, W_3) \in \{0, 1\}^{128=32 \times 4}$

Output: $(V_0, V_1, V_2, V_3) \in \{0, 1\}^{128=32 \times 4}$

1 **Function** SubMix:

2 **for** $i \leftarrow 0$ **to** 3 **do**

3 $V_i \leftarrow Te_0((W_i \gg 0x18) \& 0xff)$
4 $\oplus_{32} Te_1((W_i \gg 0x10) \& 0xff)$
5 $\oplus_{32} Te_2((W_i \gg 0x08) \& 0xff)$
6 $\oplus_{32} Te_3(W_i \& 0xff);$

7 **end**

8 **return** $(V_0, V_1, V_2, V_3);$

9 **end**

Input: $(W_0, W_1, W_2, W_3) \in \{0, 1\}^{128=32 \times 4}$

Output: $(U_0, U_1, U_2, U_3) \in \{0, 1\}^{128=32 \times 4}$

1 **Function** InvSubInvMix:

2 **for** $i \leftarrow 0$ **to** 3 **do**

3 $U_i \leftarrow Td_0((W_i \gg 0x18) \& 0xff)$
4 $\oplus_{32} Td_1((W_i \gg 0x10) \& 0xff)$
5 $\oplus_{32} Td_2((W_i \gg 0x08) \& 0xff)$
6 $\oplus_{32} Td_3(W_i \& 0xff);$

7 **end**

8 **return** $(U_0, U_1, U_2, U_3);$

9 **end**

3.3 Generation of 8×32 Tables

Note that

$$\begin{aligned}
 \begin{pmatrix} V_0 & V_4 & V_8 & V_{12} \\ V_1 & V_5 & V_9 & V_{13} \\ V_2 & V_6 & V_{10} & V_{14} \\ V_3 & V_7 & V_{11} & V_{15} \end{pmatrix} &= \text{SubMix} \left(\begin{pmatrix} X_0 & X_4 & X_8 & X_{12} \\ X_1 & X_5 & X_9 & X_{13} \\ X_2 & X_6 & X_{10} & X_{14} \\ X_3 & X_7 & X_{11} & X_{15} \end{pmatrix} \right) := \\
 &\begin{pmatrix} \mathbf{0x02} & \mathbf{0x03} & \mathbf{0x01} & \mathbf{0x01} \\ \mathbf{0x01} & \mathbf{0x02} & \mathbf{0x03} & \mathbf{0x01} \\ \mathbf{0x01} & \mathbf{0x01} & \mathbf{0x02} & \mathbf{0x03} \\ \mathbf{0x03} & \mathbf{0x01} & \mathbf{0x01} & \mathbf{0x02} \end{pmatrix} \begin{pmatrix} s(X_0) & s(X_4) & s(X_8) & s(X_{12}) \\ s(X_1) & s(X_5) & s(X_9) & s(X_{13}) \\ s(X_2) & s(X_6) & s(X_{10}) & s(X_{14}) \\ s(X_3) & s(X_7) & s(X_{11}) & s(X_{15}) \end{pmatrix}, \\
 \begin{pmatrix} U_0 & V_4 & U_8 & U_{12} \\ U_1 & V_5 & U_9 & U_{13} \\ U_2 & V_6 & U_{10} & U_{14} \\ U_3 & V_7 & U_{11} & U_{15} \end{pmatrix} &= \text{InvSubInvMix} \left(\begin{pmatrix} X_0 & X_4 & X_8 & X_{12} \\ X_1 & X_5 & X_9 & X_{13} \\ X_2 & X_6 & X_{10} & X_{14} \\ X_3 & X_7 & X_{11} & X_{15} \end{pmatrix} \right) := \\
 &\begin{pmatrix} \mathbf{0x0e} & \mathbf{0x0b} & \mathbf{0x0d} & \mathbf{0x09} \\ \mathbf{0x09} & \mathbf{0x0e} & \mathbf{0x0b} & \mathbf{0x0d} \\ \mathbf{0x0d} & \mathbf{0x09} & \mathbf{0x0e} & \mathbf{0x0b} \\ \mathbf{0x0b} & \mathbf{0x0d} & \mathbf{0x09} & \mathbf{0x0e} \end{pmatrix} \begin{pmatrix} s^{-1}(X_0) & s^{-1}(X_4) & s^{-1}(X_8) & s^{-1}(X_{12}) \\ s^{-1}(X_1) & s^{-1}(X_5) & s^{-1}(X_9) & s^{-1}(X_{13}) \\ s^{-1}(X_2) & s^{-1}(X_6) & s^{-1}(X_{10}) & s^{-1}(X_{14}) \\ s^{-1}(X_3) & s^{-1}(X_7) & s^{-1}(X_{11}) & s^{-1}(X_{15}) \end{pmatrix}.
 \end{aligned}$$

Then

$$\begin{aligned}
 \begin{pmatrix} V_{4i} \\ V_{4i+1} \\ V_{4i+2} \\ V_{4i+3} \end{pmatrix} &= \begin{pmatrix} \mathbf{0x02} & \mathbf{0x03} & \mathbf{0x01} & \mathbf{0x01} \\ \mathbf{0x01} & \mathbf{0x02} & \mathbf{0x03} & \mathbf{0x01} \\ \mathbf{0x01} & \mathbf{0x01} & \mathbf{0x02} & \mathbf{0x03} \\ \mathbf{0x03} & \mathbf{0x01} & \mathbf{0x01} & \mathbf{0x02} \end{pmatrix} \begin{pmatrix} s(X_{4i}) \\ s(X_{4i+1}) \\ s(X_{4i+2}) \\ s(X_{4i+3}) \end{pmatrix} \\
 &= s(X_{4i}) \begin{pmatrix} \mathbf{0x02} \\ \mathbf{0x01} \\ \mathbf{0x01} \\ \mathbf{0x03} \end{pmatrix} \oplus_{32} s(X_{4i+1}) \begin{pmatrix} \mathbf{0x03} \\ \mathbf{0x02} \\ \mathbf{0x01} \\ \mathbf{0x01} \end{pmatrix} \oplus_{32} s(X_{4i+2}) \begin{pmatrix} \mathbf{0x01} \\ \mathbf{0x03} \\ \mathbf{0x02} \\ \mathbf{0x01} \end{pmatrix} \oplus_{32} s(X_{4i+3}) \begin{pmatrix} \mathbf{0x01} \\ \mathbf{0x01} \\ \mathbf{0x03} \\ \mathbf{0x02} \end{pmatrix}, \\
 \begin{pmatrix} U_{4i} \\ U_{4i+1} \\ U_{4i+2} \\ U_{4i+3} \end{pmatrix} &= \begin{pmatrix} \mathbf{0x0e} & \mathbf{0x0b} & \mathbf{0x0d} & \mathbf{0x09} \\ \mathbf{0x09} & \mathbf{0x0e} & \mathbf{0x0b} & \mathbf{0x0d} \\ \mathbf{0x0d} & \mathbf{0x09} & \mathbf{0x0e} & \mathbf{0x0b} \\ \mathbf{0x0b} & \mathbf{0x0d} & \mathbf{0x09} & \mathbf{0x0e} \end{pmatrix} \begin{pmatrix} s^{-1}(X_{4i}) \\ s^{-1}(X_{4i+1}) \\ s^{-1}(X_{4i+2}) \\ s^{-1}(X_{4i+3}) \end{pmatrix} \\
 &= s^{-1}(X_{4i}) \begin{pmatrix} \mathbf{0x0e} \\ \mathbf{0x09} \\ \mathbf{0x0d} \\ \mathbf{0x0b} \end{pmatrix} \oplus s^{-1}(X_{4i+1}) \begin{pmatrix} \mathbf{0x0b} \\ \mathbf{0x0e} \\ \mathbf{0x09} \\ \mathbf{0x0d} \end{pmatrix} \oplus s^{-1}(X_{4i+2}) \begin{pmatrix} \mathbf{0x0d} \\ \mathbf{0x0b} \\ \mathbf{0x0e} \\ \mathbf{0x09} \end{pmatrix} \oplus s^{-1}(X_{4i+3}) \begin{pmatrix} \mathbf{0x09} \\ \mathbf{0x0d} \\ \mathbf{0x0b} \\ \mathbf{0x0e} \end{pmatrix}.
 \end{aligned}$$

We define $Te_i/Td_i : \{\mathbf{0}, \mathbf{1}\}^8 \rightarrow \{\mathbf{0}, \mathbf{1}\}^{32}$ ($i \in \{0, 1, 2, 3\}$) as follows:

$$\begin{aligned} Te_0(X) &:= \left(\mathbf{0x02} \otimes_8 s(X), s(X), s(X), \mathbf{0x03} \otimes_8 s(X) \right), \\ Te_1(X) &:= \left(\mathbf{0x03} \otimes_8 s(X), \mathbf{0x02} \otimes_8 s(X), s(X), s(X) \right), \\ Te_2(X) &:= \left(s(X), \mathbf{0x03} \otimes_8 s(X), \mathbf{0x02} \otimes_8 s(X), s(X) \right), \\ Te_3(X) &:= \left(s(X), s(X), \mathbf{0x03} \otimes_8 s(X), \mathbf{0x02} \otimes_8 s(X) \right), \end{aligned}$$

and

$$\begin{aligned} Td_0(X) &:= \left(\mathbf{0x0e} \otimes_8 s^{-1}(X), \mathbf{0x09} \otimes_8 s^{-1}(X), \mathbf{0x0d} \otimes_8 s^{-1}(X), \mathbf{0x0b} \otimes_8 s^{-1}(X) \right), \\ Td_1(X) &:= \left(\mathbf{0x0b} \otimes_8 s^{-1}(X), \mathbf{0x0e} \otimes_8 s^{-1}(X), \mathbf{0x09} \otimes_8 s^{-1}(X), \mathbf{0x0d} \otimes_8 s^{-1}(X) \right), \\ Td_2(X) &:= \left(\mathbf{0x0d} \otimes_8 s^{-1}(X), \mathbf{0x0b} \otimes_8 s^{-1}(X), \mathbf{0x0e} \otimes_8 s^{-1}(X), \mathbf{0x09} \otimes_8 s^{-1}(X) \right), \\ Td_3(X) &:= \left(\mathbf{0x09} \otimes_8 s^{-1}(X), \mathbf{0x0d} \otimes_8 s^{-1}(X), \mathbf{0x0b} \otimes_8 s^{-1}(X), \mathbf{0x0e} \otimes_8 s^{-1}(X) \right). \end{aligned}$$

SubMix/InvSubInvMix : $\{\mathbf{0}, \mathbf{1}\}^{128} \rightarrow \{\mathbf{0}, \mathbf{1}\}^{128}$ are computed as follows:

$$\begin{aligned} \text{SubMix}(W_0, W_1, W_2, W_3) &:= (V_0, V_1, V_2, V_3), \\ \text{InvSubInvMix}(W_0, W_1, W_2, W_3) &:= (U_0, U_1, U_2, U_3) \end{aligned}$$

$$\text{where } \begin{cases} W_0 := X_0 \parallel X_1 \parallel X_2 \parallel X_3 \\ W_1 := X_4 \parallel X_5 \parallel X_6 \parallel X_7 \\ W_2 := X_8 \parallel X_9 \parallel X_{10} \parallel X_{11} \\ W_3 := X_{12} \parallel X_{13} \parallel X_{14} \parallel X_{15} \end{cases} \quad (W_i \in \{\mathbf{0}, \mathbf{1}\}^{32}) \text{ and}$$

$$\begin{aligned} V_0 &:= Te_0(X_0) \oplus_{32} Te_1(X_1) \oplus_{32} Te_2(X_2) \oplus_{32} Te_3(X_3), \\ V_1 &:= Te_0(X_4) \oplus_{32} Te_1(X_5) \oplus_{32} Te_2(X_6) \oplus_{32} Te_3(X_7), \\ V_2 &:= Te_0(X_8) \oplus_{32} Te_1(X_9) \oplus_{32} Te_2(X_{10}) \oplus_{32} Te_3(X_{11}), \\ V_3 &:= Te_0(X_{12}) \oplus_{32} Te_1(X_{13}) \oplus_{32} Te_2(X_{14}) \oplus_{32} Te_3(X_{15}), \end{aligned}$$

$$\begin{aligned} U_0 &:= Td_0(X_0) \oplus_{32} Td_1(X_1) \oplus_{32} Td_2(X_2) \oplus_{32} Td_3(X_3), \\ U_1 &:= Td_0(X_4) \oplus_{32} Td_1(X_5) \oplus_{32} Td_2(X_6) \oplus_{32} Td_3(X_7), \\ U_2 &:= Td_0(X_8) \oplus_{32} Td_1(X_9) \oplus_{32} Td_2(X_{10}) \oplus_{32} Td_3(X_{11}), \\ U_3 &:= Td_0(X_{12}) \oplus_{32} Td_1(X_{13}) \oplus_{32} Td_2(X_{14}) \oplus_{32} Td_3(X_{15}). \end{aligned}$$

Chapter 4

Cryptanalysis

4.1 Linear Cryptanalysis

Linear cryptanalysis is an advanced cryptanalytic technique primarily used to attack symmetric key ciphers. It exploits the linear relations (approximations) between input bits, output bits, and key bits of a cipher. The technique was first introduced by Mitsuru Matsui and has since been instrumental in the analysis of various cryptographic algorithms.

4.1.1 Linear Approximation Example

Consider a block cipher with an input block X and an output block Y . The essence of linear cryptanalysis lies in finding a linear approximation that correlates certain bits of X , Y , and the key K . A typical linear approximation can be expressed as follows:

$$P(X) \oplus Q(Y) = 0$$

where $P(X)$ and $Q(Y)$ are linear functions of the input and output blocks, respectively, \oplus denotes the bitwise XOR operation, and K' is a subkey or a combination of bits from the key K . Ideally, this equation holds with a probability significantly different from 0.5.

4.1.2 Linear Approximation Table (LAT)

A crucial tool in linear cryptanalysis is the Linear Approximation Table (LAT), particularly for analyzing S-boxes in block ciphers. An S-box is a fundamental component in many block ciphers that performs substitution, and LAT is used to measure the correlation between input and output bits of this S-box.

The LAT is a matrix where each cell corresponds to a particular input-output bit combination and contains a value representing the correlation between these bits. These values are critical in identifying which linear approximations are strong (i.e., have a high correlation) and can be exploited in an attack.

4.1.3 Attack Process

The general process of conducting a linear cryptanalytic attack involves several steps:

1. **Identify Strong Linear Approximations:** Using the LAT, identify the linear approximations with the highest bias. The bias is the deviation of the approximation's probability from 0.5.
 - *Mathematical Foundation:* Analyze the Linear Approximation Table (LAT) for strong linear correlations. LAT is a 2D matrix with biases of linear approximations as entries.
 - *Calculation:* For an S-box with n input and output bits, LAT is a $2^n \times 2^n$ matrix. Bias is calculated as $\frac{\#matches}{2^n} - \frac{1}{2}$.
 - *Selection Criterion:* Choose approximations with the highest absolute bias values.
2. **Estimate Bias:** Accurately estimate the bias of the chosen approximation. This estimation is crucial for the success of the attack.
 - *Theory:* Bias is the deviation of the approximation's probability from 0.5.
 - *Mathematical Approach:* Use statistical methods like maximum likelihood estimation or Bayesian inference for bias refinement.
 - *Accuracy Considerations:* More data leads to more accurate bias estimation.
3. **Collect Plaintext-Ciphertext Pairs:** Amass a large number of plaintext-ciphertext pairs that correspond to the chosen approximation. The number of required pairs depends on the bias; weaker biases require more pairs.
 - *Data Requirement Analysis:* Required pairs $N \approx \frac{1}{\epsilon^2}$, where ϵ is the bias.
 - *Collection Methodology:* Data collection can vary based on the cipher and scenario.
 - *Computational Aspects:* Consider the complexity of storing and processing large amounts of data.
4. **Key Recovery:** Apply statistical methods to analyze the collected data. Exploit the bias to make informed guesses about the key bits or subkey, thereby incrementally recovering parts of the encryption key.
 - *Statistical Analysis:* Use statistical tests like chi-squared tests to analyze data.
 - *Exploiting Bias:* Use bias to infer key bits probabilistically.
 - *Incremental Key Recovery:* Recover different segments of the key gradually.
 - *Complexity Considerations:* Analyze the time complexity and employ optimization techniques if necessary.

In practice, the effectiveness of linear cryptanalysis depends on the cipher's structure, the quality of the linear approximations, and the amount of available data. It has been notably applied to attack DES (Data Encryption Standard) and has influenced the design of new cryptographic algorithms to be resistant to this form of analysis.

4.2 Differential Cryptanalysis

Differential Cryptanalysis is a method used in cryptography to analyze the effect of specific differences in input pairs on the differences in the resulting output pairs. This technique is especially powerful in the analysis of block ciphers.

4.2.1 Differential Example

Given a pair of inputs (X, X') and their corresponding outputs (Y, Y') under a cryptographic transformation. The differentials are defined as follows:

$$\Delta X = X \oplus X', \quad \Delta Y = Y \oplus Y'$$

where \oplus denotes the bitwise XOR operation. This operation highlights the changes from X to X' and from Y to Y' at the bit level.

4.2.2 Differential Characteristic

A differential characteristic is a crucial concept in this analysis. It is a sequence of expected input and output differentials for each round or stage of the cipher. These characteristics are probabilistic in nature and provide insights into how certain input differentials propagate to output differentials through the cipher's structure.

Probability of a Differential Characteristic

The probability of a differential characteristic is a key metric in differential cryptanalysis. It measures the likelihood that a specific input differential will result in the expected output differential after passing through the rounds of the cipher.

4.2.3 Attack Process

1. **Select Pairs of Plaintexts with a Certain Difference:** Choose plaintext pairs (P, P') such that the difference $\Delta P = P \oplus P'$ is specific and strategically chosen based on the cipher's structure.
 - *Concept:* Choose plaintext pairs (P_1, P_2) such that $\Delta P = P_1 \oplus P_2$, where \oplus is the bitwise XOR operation.
 - *Method:* Selection of ΔP is based on cipher's structure analysis. Aim to identify a ΔP that can cause predictable effects in ciphertexts.
 - *Mathematical Formulation:* This involves a heuristic approach to identify exploitable differences.
2. **Analyze How This Difference Propagates Through the Cipher:** Study the propagation of ΔP through the cipher. This involves analyzing the impact of the differential on the cipher's components like S-boxes, permutation layers, etc.
 - *Theory:* Study the propagation of ΔP through the cipher's rounds.
 - *Mathematical Tools:* Use differential characteristics and probabilistic analysis of the cipher components.

- *Computational Analysis:* Combine analytical techniques and computational experiments to understand difference propagation.
3. **Gather Plaintext-Ciphertext Pairs Following the Differential:** Collect a significant number of plaintext-ciphertext pairs that conform to the chosen differential. This data is used to analyze the behavior of the cipher under the differential attack.
- *Data Collection:* Accumulate pairs (P, C) that conform to ΔP .
 - *Statistical Analysis:* Analyze the distribution of output difference $\Delta C = C_1 \oplus C_2$.
 - *Data Requirements:* The number of pairs depends on the differential characteristic's probability and cipher's strength.
4. **Make Key Hypotheses Based on Output Differences:** Based on the differences observed in the ciphertext pairs, hypothesize about the possible key or subkey values. These hypotheses are then tested and refined to recover the key or reduce the key space.
- *Hypothesis Generation:* Formulate hypotheses about the key based on ΔC .
 - *Mathematical Techniques:* Use linear algebra, probability theory, and optimization algorithms for hypothesis refinement.
 - *Verification:* Test hypotheses against multiple plaintext-ciphertext pairs.

Appendix A

Additional Data A

A.1 Substitution-BOX

```
1 static const u8 s_box[256] = {
2     0x63, 0x7c, 0x77, 0x7b, 0xf2, 0x6b, 0x6f, 0xc5,
3     0x30, 0x01, 0x67, 0x2b, 0xfe, 0xd7, 0xab, 0x76,
4     0xca, 0x82, 0xc9, 0x7d, 0xfa, 0x59, 0x47, 0xf0,
5     0xad, 0xd4, 0xa2, 0xaf, 0x9c, 0xa4, 0x72, 0xc0,
6     0xb7, 0xfd, 0x93, 0x26, 0x36, 0x3f, 0xf7, 0xcc,
7     0x34, 0xa5, 0xe5, 0xf1, 0x71, 0xd8, 0x31, 0x15,
8     0x04, 0xc7, 0x23, 0xc3, 0x18, 0x96, 0x05, 0x9a,
9     0x07, 0x12, 0x80, 0xe2, 0xeb, 0x27, 0xb2, 0x75,
10    0x09, 0x83, 0x2c, 0x1a, 0x1b, 0x6e, 0x5a, 0xa0,
11    0x52, 0x3b, 0xd6, 0xb3, 0x29, 0xe3, 0x2f, 0x84,
12    0x53, 0xd1, 0x00, 0xed, 0x20, 0xfc, 0xb1, 0x5b,
13    0x6a, 0xcb, 0xbe, 0x39, 0x4a, 0x4c, 0x58, 0xcf,
14    0xd0, 0xef, 0xaa, 0xfb, 0x43, 0x4d, 0x33, 0x85,
15    0x45, 0xf9, 0x02, 0x7f, 0x50, 0x3c, 0x9f, 0xa8,
16    0x51, 0xa3, 0x40, 0x8f, 0x92, 0x9d, 0x38, 0xf5,
17    0xbc, 0xb6, 0xda, 0x21, 0x10, 0xff, 0xf3, 0xd2,
18    0xcd, 0x0c, 0x13, 0xec, 0x5f, 0x97, 0x44, 0x17,
19    0xc4, 0xa7, 0x7e, 0x3d, 0x64, 0x5d, 0x19, 0x73,
20    0x60, 0x81, 0x4f, 0xdc, 0x22, 0x2a, 0x90, 0x88,
21    0x46, 0xee, 0xb8, 0x14, 0xde, 0x5e, 0x0b, 0xdb,
22    0xe0, 0x32, 0x3a, 0x0a, 0x49, 0x06, 0x24, 0x5c,
23    0xc2, 0xd3, 0xac, 0x62, 0x91, 0x95, 0xe4, 0x79,
24    0xe7, 0xc8, 0x37, 0x6d, 0x8d, 0xd5, 0x4e, 0xa9,
25    0x6c, 0x56, 0xf4, 0xea, 0x65, 0x7a, 0xae, 0x08,
26    0xba, 0x78, 0x25, 0x2e, 0x1c, 0xa6, 0xb4, 0xc6,
27    0xe8, 0xdd, 0x74, 0x1f, 0x4b, 0xbd, 0x8b, 0x8a,
28    0x70, 0x3e, 0xb5, 0x66, 0x48, 0x03, 0xf6, 0x0e,
29    0x61, 0x35, 0x57, 0xb9, 0x86, 0xc1, 0x1d, 0x9e,
30    0xe1, 0xf8, 0x98, 0x11, 0x69, 0xd9, 0x8e, 0x94,
31    0x9b, 0x1e, 0x87, 0xe9, 0xce, 0x55, 0x28, 0xdf,
32    0x8c, 0xa1, 0x89, 0x0d, 0xbf, 0xe6, 0x42, 0x68,
33    0x41, 0x99, 0x2d, 0x0f, 0xb0, 0x54, 0xbb, 0x16
34 };
```

```

1  static const u8 inv_s_box[256] = {
2      0x52, 0x09, 0x6a, 0xd5, 0x30, 0x36, 0xa5, 0x38,
3      0xbf, 0x40, 0xa3, 0x9e, 0x81, 0xf3, 0xd7, 0xfb,
4      0x7c, 0xe3, 0x39, 0x82, 0x9b, 0x2f, 0xff, 0x87,
5      0x34, 0x8e, 0x43, 0x44, 0xc4, 0xde, 0xe9, 0xcb,
6      0x54, 0x7b, 0x94, 0x32, 0xa6, 0xc2, 0x23, 0x3d,
7      0xee, 0x4c, 0x95, 0x0b, 0x42, 0xfa, 0xc3, 0x4e,
8      0x08, 0x2e, 0xa1, 0x66, 0x28, 0xd9, 0x24, 0xb2,
9      0x76, 0x5b, 0xa2, 0x49, 0x6d, 0x8b, 0xd1, 0x25,
10     0x72, 0xf8, 0xf6, 0x64, 0x86, 0x68, 0x98, 0x16,
11     0xd4, 0xa4, 0x5c, 0xcc, 0x5d, 0x65, 0xb6, 0x92,
12     0x6c, 0x70, 0x48, 0x50, 0xfd, 0xed, 0xb9, 0xda,
13     0x5e, 0x15, 0x46, 0x57, 0xa7, 0x8d, 0x9d, 0x84,
14     0x90, 0xd8, 0xab, 0x00, 0x8c, 0xbc, 0xd3, 0x0a,
15     0xf7, 0xe4, 0x58, 0x05, 0xb8, 0xb3, 0x45, 0x06,
16     0xd0, 0x2c, 0x1e, 0x8f, 0xca, 0x3f, 0x0f, 0x02,
17     0xc1, 0xaf, 0xbd, 0x03, 0x01, 0x13, 0x8a, 0x6b,
18     0x3a, 0x91, 0x11, 0x41, 0x4f, 0x67, 0xdc, 0xea,
19     0x97, 0xf2, 0xcf, 0xce, 0xf0, 0xb4, 0xe6, 0x73,
20     0x96, 0xac, 0x74, 0x22, 0xe7, 0xad, 0x35, 0x85,
21     0xe2, 0xf9, 0x37, 0xe8, 0x1c, 0x75, 0xdf, 0x6e,
22     0x47, 0xf1, 0x1a, 0x71, 0x1d, 0x29, 0xc5, 0x89,
23     0x6f, 0xb7, 0x62, 0x0e, 0xaa, 0x18, 0xbe, 0x1b,
24     0xfc, 0x56, 0x3e, 0x4b, 0xc6, 0xd2, 0x79, 0x20,
25     0x9a, 0xdb, 0xc0, 0xfe, 0x78, 0xcd, 0x5a, 0xf4,
26     0x1f, 0xdd, 0xa8, 0x33, 0x88, 0x07, 0xc7, 0x31,
27     0xb1, 0x12, 0x10, 0x59, 0x27, 0x80, 0xec, 0x5f,
28     0x60, 0x51, 0x7f, 0xa9, 0x19, 0xb5, 0x4a, 0x0d,
29     0x2d, 0xe5, 0x7a, 0x9f, 0x93, 0xc9, 0x9c, 0xef,
30     0xa0, 0xe0, 0x3b, 0x4d, 0xae, 0x2a, 0xf5, 0xb0,
31     0xc8, 0xeb, 0xbb, 0x3c, 0x83, 0x53, 0x99, 0x61,
32     0x17, 0x2b, 0x04, 0x7e, 0xba, 0x77, 0xd6, 0x26,
33     0xe1, 0x69, 0x14, 0x63, 0x55, 0x21, 0x0c, 0x7d
34 };

```