

Category Theory

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Category

A **category** C consists of

- a class $C_0 = \text{Obj}(C)$ of **objects** X, Y, Z, \dots
- a class $C_1 = \text{Mor}(C)$ of **morphisms** (or **arrow**) f, g, h, \dots between its objects
 - for every morphism f , an object $\text{Dom}(f)$ (called its **source** or **domain**), and an object $\text{Cdm}(f)$ (called its **target** or **codomain**)
 - for every pair of morphisms f and g , where $\text{Cdm}(f) = \text{Dom}(g)$, a morphism $g \circ f$, called **composite** (also written gf or some times $f;g$)
 - for every object X , a morphism id_X (or 1_X), called the **identity morphism** on X ;

The morphisms of C satisfy the following property:

(C1) (Composition)

$$\begin{aligned} X, Y, Z \in \text{Obj}(C) &\implies [f \in \text{Hom}(X, Y), g \in \text{Hom}(Y, Z), \text{Cdm}(f) = \text{Dom}(g)] \\ &\implies \exists (g \circ f) \in \text{Hom}(X, Z) \end{aligned}$$

(C2) (Identity)

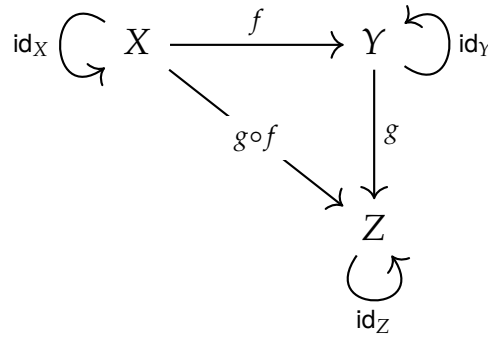
$$\begin{aligned} X \in \text{Obj}(C) &\implies \exists \text{id}_X \in \text{Hom}(X, X) : \\ &[Y \in \text{Obj}(C), f \in \text{Hom}(X, Y), g \in \text{Hom}(Y, Z)] \\ &\implies f \circ \text{id}_X = f, \text{id}_X \circ g = g \end{aligned}$$

(C3) (Associativity)

$$f, g, h \in \text{Mor}(C) \implies f \circ (g \circ h) = (f \circ g) \circ h.$$

Remark 1. To describe a metacategory it is necessary to specify:

- The collection $\text{Obj}(C)$ of objects;
- The collection $\text{Mor}(C)$ of morphisms;
- For each object X , an identity morphism $\text{id}_X : X \rightarrow X$;
- For every appropriate pair of morphisms f, g , the composite morphism $g \circ f$.



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References

- [1] ProofWiki. "Definition:Metacategory" Accessed on [May 05, 2024]. <https://proofwiki.org/wiki/Definition:Metacategory>.
- [2] nLab. "category" Accessed on [May 05, 2024]. <https://ncatlab.org/nlab/show/category#Grothendieck61>.