

# Lambda Calculus

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```

λinput.output
function (input) {
    return output
}

λa.λb.

def add (a, b):
    return a + b

add(5,7)

def add (a):
    def adda(b):
        return a + b
    return adda

add(5)(7)

λa.(λb.(a + b))(5)(7)
T = λa.λb.a
F = λa.λb.b
λbool.bool(t)(f)
- returned if bool is true
- returned if bool is false

```

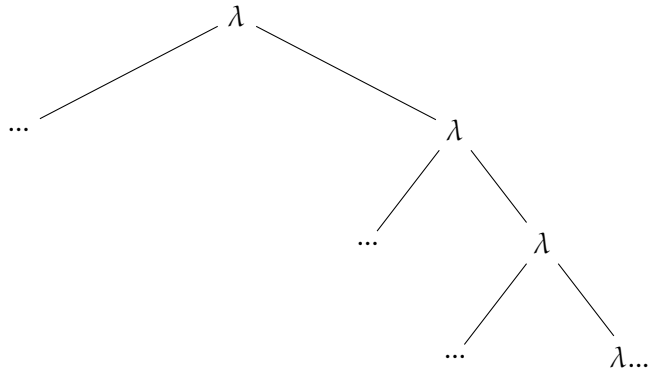
## Logic Gates

- $\neg = \lambda b.b(F)(T)$ 
  - returned if  $b$  is true
  - returned if  $b$  is false
- $\vee = \lambda a.\lambda b.a(T)(b)$ 
  - returned if  $a$  is true
  - returned if  $b$  is false

- $\wedge = \lambda a. \lambda b. a(b)(F)$

- returned if  $a$  is true

- returned if  $b$  is false



$$1 = \lambda f. \lambda a. f(a)$$

$$2 = \lambda f. \lambda a. f(f(a))$$

$$3 = \lambda f. \lambda a. f(f(f(a)))$$

$$4 = \lambda f. \lambda a. f(f(f(f(a))))$$

$$3(f)(a) = f(f(f(a)))$$

$$+1 = \lambda n. \lambda f. \lambda a. f(n(f)(a))$$

$$+ = \lambda x. \lambda y. x(+1)(y)$$

$$* = \lambda x. \lambda y. y(+ (x))(0)$$

$$** = \lambda x. \lambda y. y(* (x))(1)$$

## Introduction to Axiomatic Thinking

$$2 + 3 * 7$$

$$2 + 7 + 7 + 7$$

- Multiplication is defined as multiple addition
- Some rules are defined in terms of other rules
  - Multiplication is “redundant”

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## References

- [1] “Why functions are turing complete (Lambda Calculus)” YouTube, uploaded by A Byte of Code, 4 Sep 2022, <https://www.youtube.com/watch?v=m32kbFXBRR0>
- [2] “Lambda Calculus vs. Turing Machines (Theory of Computation)” YouTube, uploaded by Advait Shinde, 3 Mar 2020, [https://www.youtube.com/watch?v=ruOnPmI\\_40g](https://www.youtube.com/watch?v=ruOnPmI_40g)