Topology I

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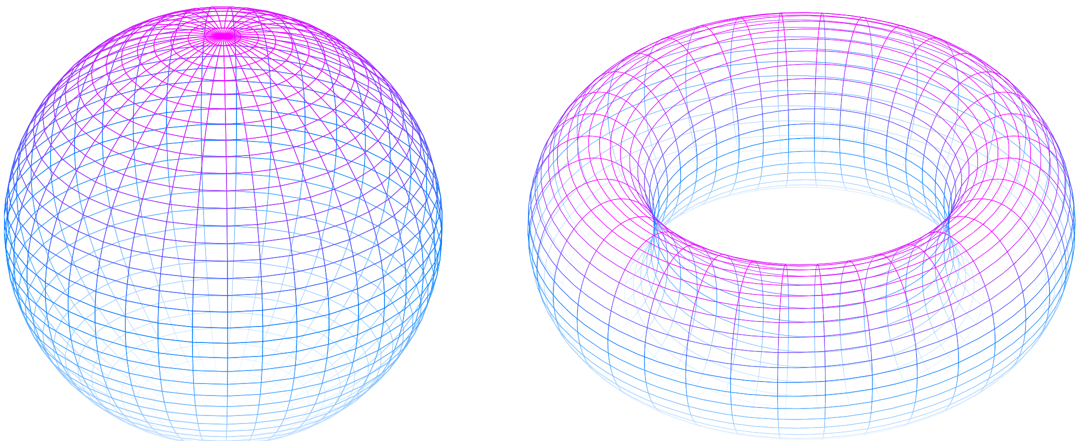
We cover the following topics in this note.

- Topology



Topology

Definition. Let $S = \varnothing$ be a non-empty set. A **topology** on S is a subset $\mathscr{T} = \{H : H \subseteq S\} \subseteq 2^S$ that satisfies the open set axioms:

(O1)[a] $\varnothing \in \mathscr{T}$ and $S \in \mathscr{T}$;

(O2)[b] $\langle H_\alpha \rangle_{\alpha\in\Lambda} \subseteq \mathscr{T} \implies \bigcup_{\alpha\in\Lambda} H_\alpha \in \mathscr{T}$;

(O3)[c] $\langle H_i \rangle_{i=1}^{n} \subseteq \mathscr{T} \implies \bigcap_{i=1}^{n} H_i \in \mathscr{T}$.

A topological spaces $(S, \mathscr{T})$ is a non-empty set $S \neq \varnothing$ with a topology $\mathscr{T}$.

[a] Complete Set is Element of Topology: (O1) follows from (O2) and (O3), but (O1) is usually included for clarity.

[b] Union of Open Sets: $\mathscr{T}$ is closed under *arbitrary* unions

[c] Intersection of Open Sets: $\mathscr{T}$ is closed under *finite* intersection

Example 1 (Cofinite Topology). Let X be a set. Define $\mathscr{T}_C \subseteq 2^X$ by

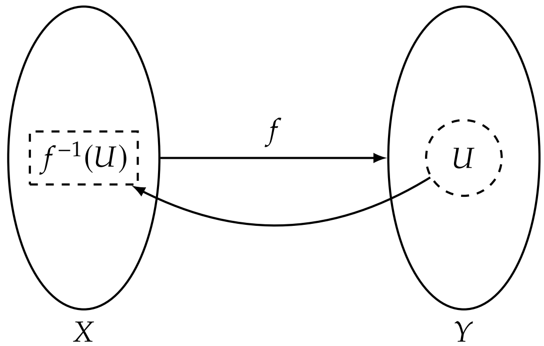
$$\mathscr{T}_C := \left\{ T \subseteq S : T^C \subseteq S \text{ is a finite set} \right\} \cup \{\varnothing\}$$

We claim that $\mathscr{T}_C$ be a topology on X:

(i)

(ii)

(iii)



References

[1] 수학의 즐거움, Enjoying Math. "수학 공부, 기초부터 대학원 수학까지, 8. 위상수학 (a) 위상공간의 정의." YouTube Video, 41:25. Published September 27, 2019. URL: `https://www.youtube.com/watch?v=q8BtXIFzo2Q`.

A Complement of Family

Note.

$$\left(\bigcup_{i\in\Lambda} E_i\right)^C = \bigcap_{i\in\Lambda} (E_i)^C$$

Proof. content... □