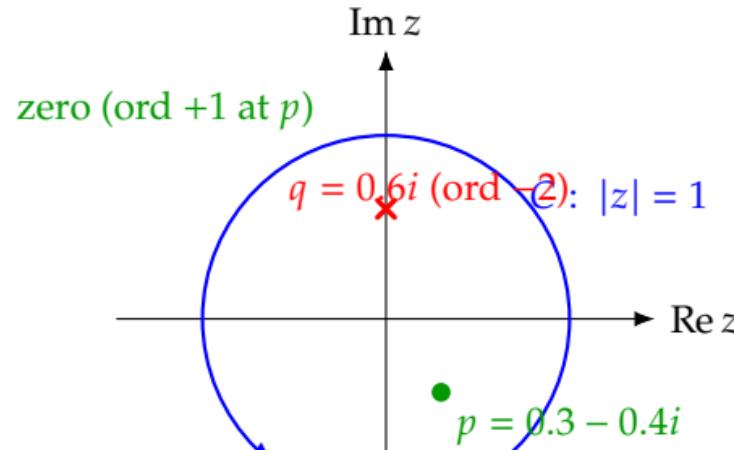


z -plane

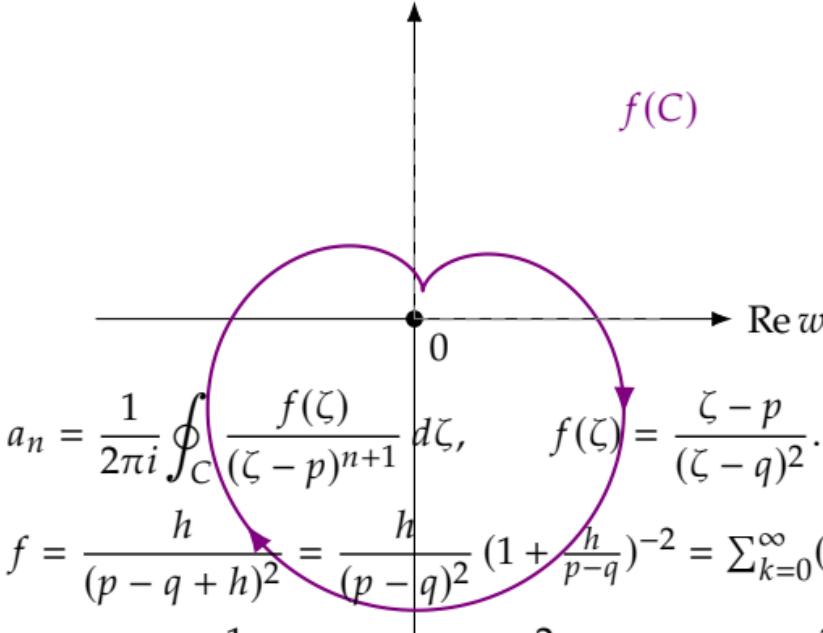


$$f(z) = \frac{z-p}{(z-q)^2}, \quad \text{poles} \quad \frac{df}{f} = \frac{dz}{z-p} + \frac{2\,dz}{z-q}. \quad \text{Let } h = \zeta - p. \quad \text{Then } f = \frac{h}{(p-q+h)^2}$$

$$\text{ord}_p f = \frac{1}{2\pi i} \oint_{\partial D_\varepsilon(p)} \frac{df}{f} = +1, \quad \text{ord}_q f = \frac{1}{2\pi i} \oint_{\partial D_\varepsilon(q)} \frac{df}{f} = -2. \quad a_0 = 1, \quad a_1 = \frac{1}{(p-q)^2}, \quad a_2 = -\frac{2}{(p-q)^3}, \quad a_3 = \frac{3}{(p-q)^4}, \dots$$

$$\Rightarrow \#Z - \#P = 1 - 2 = -1.$$

$w = f(z)$ -plane
Im w



$$a_n = \frac{1}{2\pi i} \oint_C \frac{f(\zeta)}{(\zeta-p)^{n+1}} d\zeta, \quad f(\zeta) = \frac{\zeta-p}{(\zeta-q)^2}.$$

$$\frac{h}{(p-q+h)^2} = \frac{h}{(p-q)^2} \left(1 + \frac{h}{p-q}\right)^{-2} = \sum_{k=0}^{\infty} (-1)^k (k+1) \frac{h^{k+1}}{(p-q)^{k+2}}.$$

$$\text{wind}(f(C), 0) = \#Z_C - \#P_C = 1 - 2 = -1 \Rightarrow \oint_C \frac{f'(z)}{f(z)} dz = -2\pi i.$$