z-plane

Im z

zero of order 2

$C : |z| = 1$

Re z

$p = 0.3 - 0.4i$

poles: none

$f(z) = c\,(z-p)^2, \quad c = 1 + 0.5\,i,$

$w = f(z)$-plane

Im w

$f(C)$

Re w

0

$$\operatorname{ord}_p f = \frac{1}{2\pi i} \oint_C \frac{df}{f} = \frac{1}{2\pi i} \oint_C \frac{2\,dz}{z - p} = 2$$
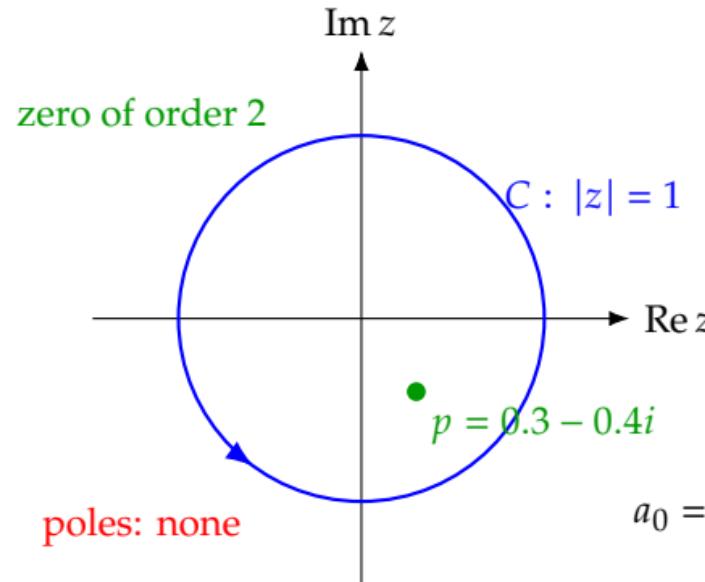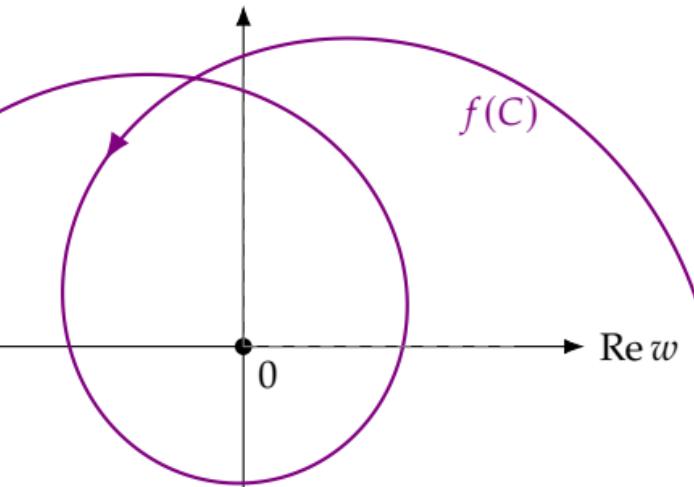
$$a_0 = f(p) = \frac{1}{2\pi i} \oint_C \frac{f(\zeta)}{\zeta - p}\,d\zeta = 0, \qquad a_1 = f'(p) = \frac{1}{2\pi i} \oint_C \frac{f(\zeta)}{(\zeta - p)^2}\,d\zeta = 0,$$

$$a_2 = \frac{f''(p)}{2!} = \frac{1}{2\pi i} \oint_C \frac{f(\zeta)}{(\zeta - p)^3}\,d\zeta = \frac{1}{2\pi i} \oint_C \frac{c}{\zeta - p}\,d\zeta = c.$$

$$\operatorname{wind}\big(f(C), 0\big) = 2 \;\Rightarrow\; \oint_C \frac{f'(z)}{f(z)}\,dz = 2\pi i \cdot 2.$$