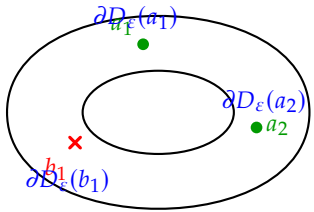
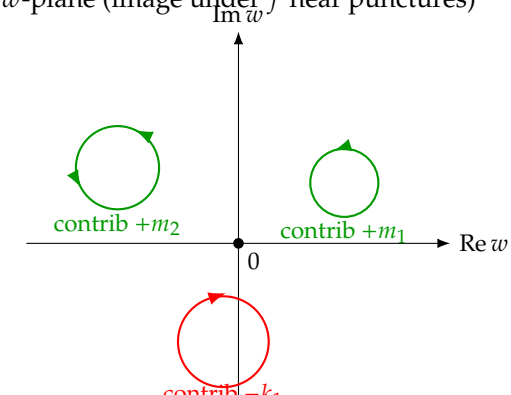
X (compact Riemann surface)



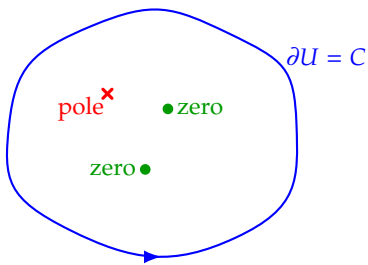
$$d\omega = 0 \text{ on } X \setminus \{a_i, b_j\}, \quad 0 = \int_{\partial(X \setminus \cup D_\varepsilon)} \omega = -\sum \int_{\partial D_\varepsilon(p)} \omega$$

w-plane (image under f near punctures)



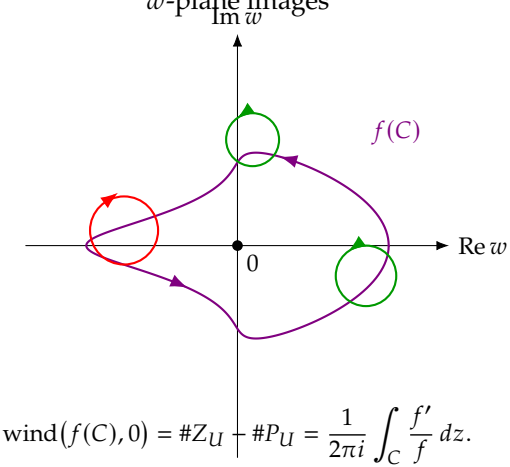
$$\sum_p \operatorname{Res}\left(\frac{df}{f}, p\right) = \sum_p \operatorname{ord}_p f = 0.$$

Planar domain $U \subset \mathbb{C}$



$$\int_{\partial U} \frac{df}{f} = \sum_{\text{punctures in } U} \int_{\partial D_\varepsilon(p)} \frac{df}{f} = 2\pi i \sum_{p \in U} \operatorname{ord}_p f.$$

w-plane images



$$\operatorname{wind}\big(f(C), 0\big) = \#Z_U - \#P_U = \frac{1}{2\pi i} \int_C \frac{f'}{f}\, dz.$$