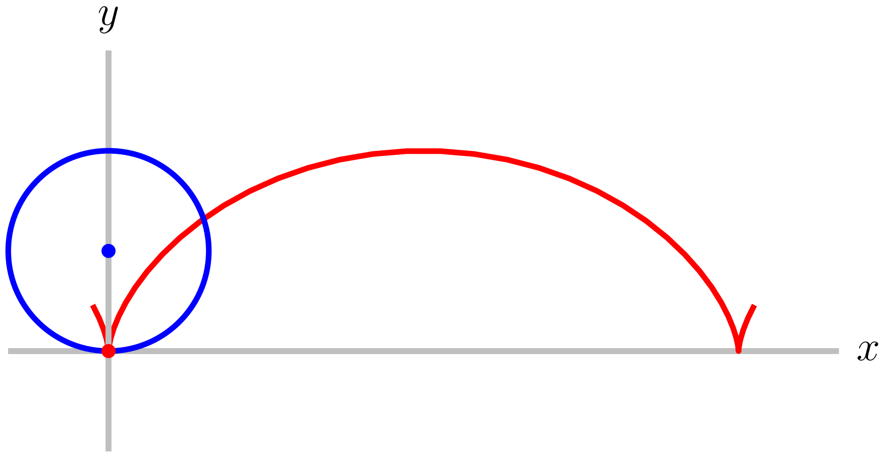

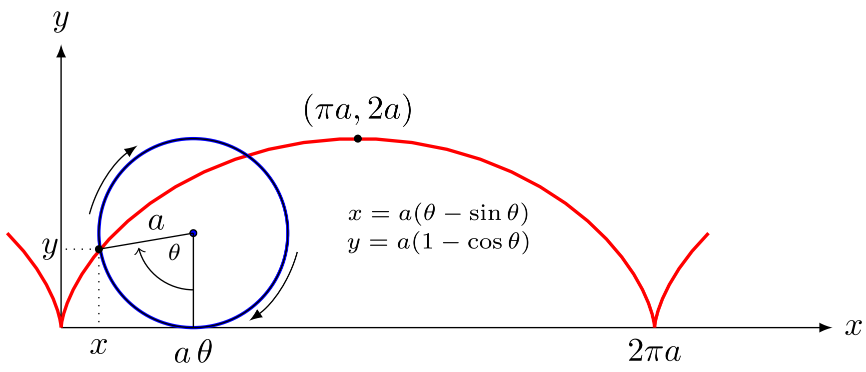

Let

$$\mathbf{r}(t) = \big\langle x(t),\ y(t)\big\rangle = \big\langle R\,t - R\sin t,\ R - R\cos t\big\rangle, \quad t \in [0, 2\pi].$$

Then

$$\mathbf{r}'(t) = \big\langle x'(t),\ y'(t)\big\rangle = \big\langle R(1-\cos t),\ R\sin t\big\rangle.$$

The differential arc–length is

$$\big\|\mathbf{r}'(t)\big\|\,dt = \sqrt{\big[R(1-\cos t)\big]^2 + \big[R\sin t\big]^2}\,dt.$$

Inside the square-root:

$$R^2(1-\cos t)^2 + R^2\sin^2 t = R^2\big[1 - 2\cos t + \cos^2 t + \sin^2 t\big] = R^2\big[2 - 2\cos t\big] = 2R^2\big(1-\cos t\big).$$

Hence

$$\|\mathbf{r}'(t)\| = \sqrt{2R^2(1-\cos t)} = R\sqrt{2(1-\cos t)} = 2R\big|\sin\tfrac{t}{2}\big|.$$

On $0 \le t \le 2\pi$, $\sin(t/2) \ge 0$, so

$$\|\mathbf{r}'(t)\| = 2R\sin\tfrac{t}{2}.$$

The total length of one arch is

$$L = \int_0^{2\pi} \|\mathbf{r}'(t)\|\,dt = \int_0^{2\pi} 2R\sin\tfrac{t}{2}\ dt.$$

Make the substitution $u = \frac{t}{2}$, so $dt = 2\,du$ and when t runs from 0 to 2π, u runs from 0 to π:

$$L = 2R\int_0^{2\pi}\sin\tfrac{t}{2}\,dt = 2R\int_0^{\pi}\sin u\,(2\,du) = 4R\int_0^{\pi}\sin u\,du = 4R\big[-\cos u\big]_0^{\pi} = 4R\big[-(-1)-(-1)\big] = 8R.$$

$$\boxed{L_{\text{one arch}} = 8R.}$$