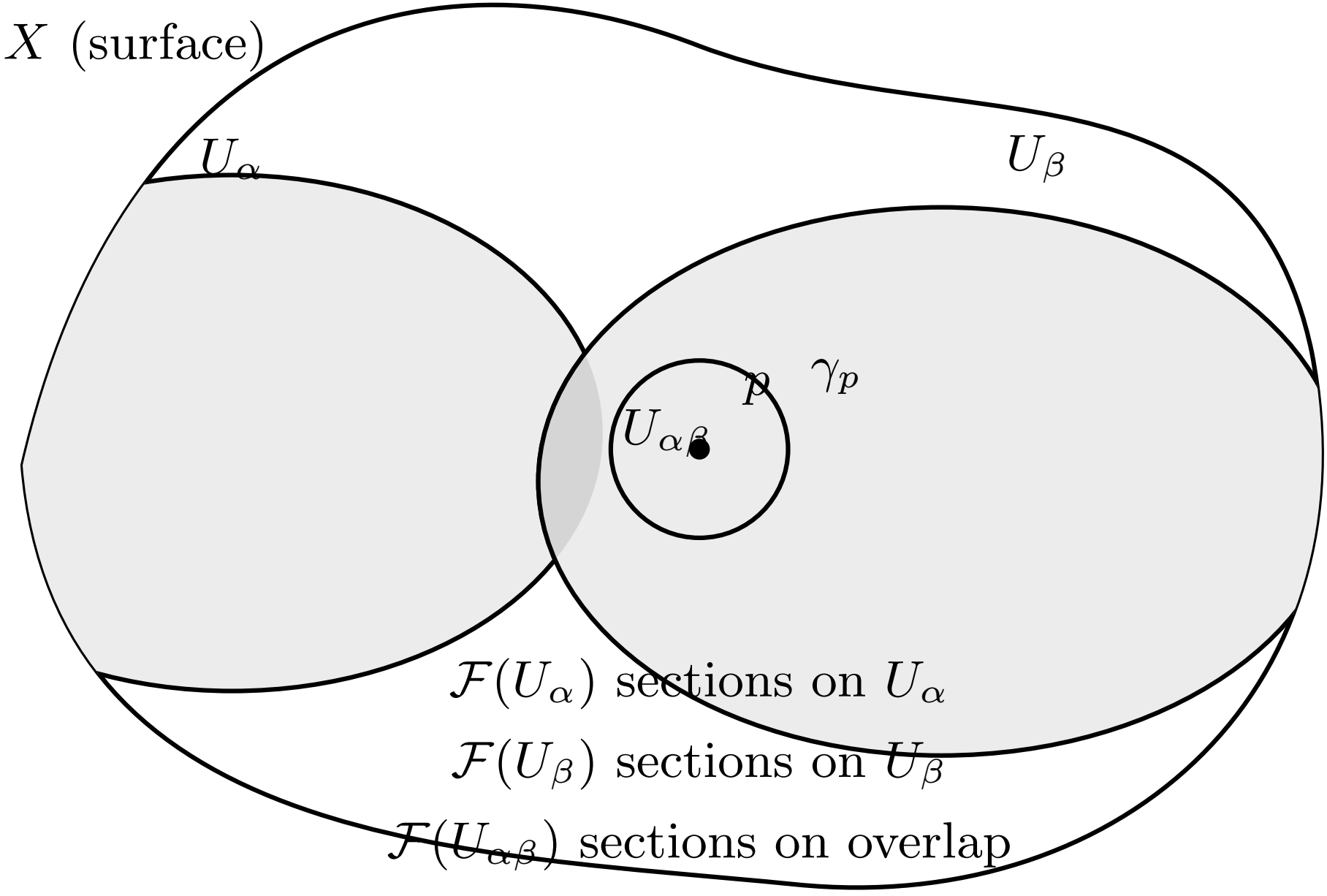
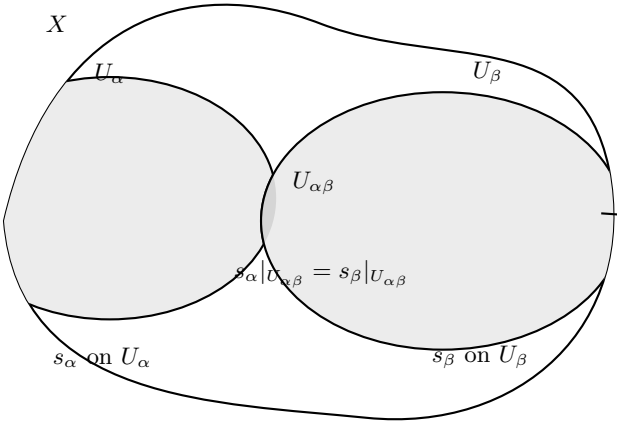
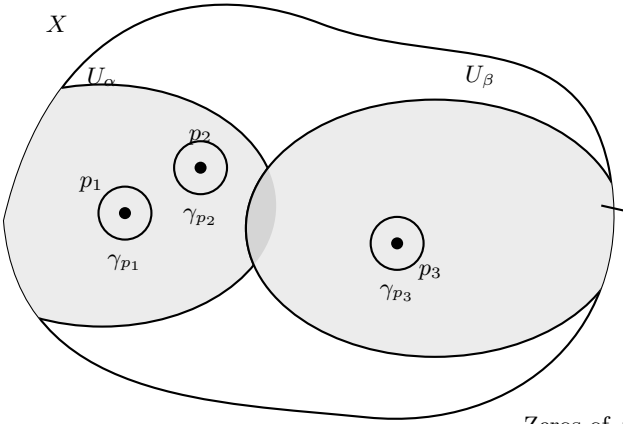

X (surface)
U_α
U_β
p
γ_p
$U_{\alpha\beta}$
$\mathcal{F}(U_\alpha)$ sections on U_α
$\mathcal{F}(U_\beta)$ sections on U_β
$\mathcal{F}(U_{\alpha\beta})$ sections on overlap



$\exists!\ s$ on $U_\alpha \cup U_\beta$
with $s|_{U_\alpha} = s_\alpha,\ s|_{U_\beta} = s_\beta$

Sheaf gluing axiom (S2)



Visual rule for $\mathcal{O}_X(D)$:

$s \in \mathcal{O}_X(D)(V)$ means $f_\alpha s$ is holomorphic on $V \cap U_\alpha$.

Equivalently, at each $p \in V$: $\operatorname{ord}_p(s) \geq -\operatorname{ord}_p(f_\alpha)$.

Winding form:

$$\operatorname{ord}_p(s) = \frac{1}{2\pi i} \int_{\gamma_p} \frac{ds}{s}.$$

Zeros of f_α indicate where s may have poles (bounded order).