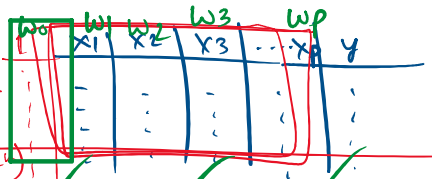
07 July 2024 20:55

OLS in Vector form



Linear Model (Scaler Not.)

$$\Rightarrow \hat{y} = \omega_0 + \omega_1 x_1 + \omega_2 x_2 + \omega_3 x_3 + \cdots + \omega_p x_p$$ Scalar Notation.

$X = N \times p \Rightarrow X = N \times (p+1)$

$y = N \times 1$, $\hat{y} = N \times 1$

$$w = \begin{bmatrix} \omega_0 \\ \omega_1 \\ \omega_2 \\ \vdots \\ \omega_p \end{bmatrix} \quad (p+1) \times 1$$

N = No. of rows

p = No. of features.

$\hat{y} = Xw$ ⇒ Linear Reg. Model in Vector Notn.

$N \times (p+1)$ $\quad (p+1) \times 1$

$X = N \times (p+1)$, $\quad y$ & $\hat{y} = N \times 1$

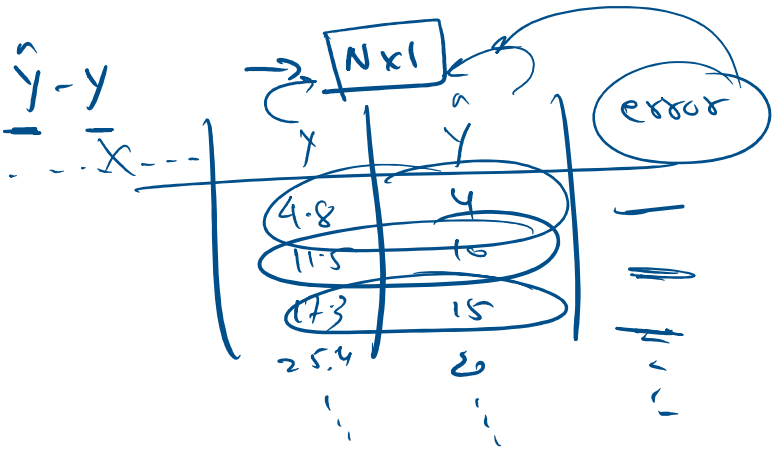
$w = (p+1) \times 1$, $\quad \hat{y} = Xw$

$$v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad v^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$v^T v = ? \begin{bmatrix} 1\times1 + 2\times2 + 3\times3 \end{bmatrix} = \begin{bmatrix} 1^2 + 2^2 + 3^2 \end{bmatrix}$$

v^T: 1×3, v: 3×1, result: 1×1

$$\text{error} = \hat{y} - y$$



$$L = \frac{1}{N}(\hat{y}-y)^T(\hat{y}-y)$$ Loss Fⁿ in vector form.

$\hat{y} = xw$

$$L = \frac{1}{N}(xw-y)^T(xw-y)$$

$$L = \frac{1}{N}\left[(xw)^T \cdot xw - (xw)^T y - y^T(xw) + y^T y\right]$$

$$= \frac{1}{N}\left[(w^T x^T)(xw) - (w^T x^T \cdot y) - (y^T xw) + y^T y\right]$$

$$L = \frac{1}{N}\left[w^T w \cdot (x^T x) - 2w^T x^T y + y^T y\right]$$

$$\frac{\partial L}{\partial w} = \frac{1}{N}\left[2w(x^T x) - 2x^T y\right] = 0$$

$$\frac{dL}{dw} = 0 \Rightarrow (x^T x)w = x^T y$$

$$\frac{\partial w^T}{\partial w} = 1$$

Find OLS soln

$$w = (x^T x)^{-1} x^T y$$