

# Lightricks Take-Home Project

Hadar Sharvit

April 30, 2022

## 1 Question I

let  $|H| = n$  and  $|B| = m$ . Looking at the algorithms definition, for every  $u \in H$  we perform two summations, each one over  $|B| = m$  elements. In other words, for every  $u \in H$  we perform  $O(1) \times 2m = O(m)$  operations, where the  $O(1)$  indicates the complexity of the multiplication  $w(u, v)I(v)$  for some  $u$  and some  $v$  (in the numerator), or the "multiplication" of  $w(u, v)$  with 1 (in the denominator). This results in an  $O(nm)$  complexity operation overall.

The above result could also be understood in terms of vector and matrices multiplication, as was implemented in the code. More specifically, our weight function  $w : H \times B \rightarrow \mathbb{R}^{>0}$  was represented as a matrix  $W$  with dimensions  $n \times m$ , and the sum over all  $v \in B$  could be represented as a dot product of  $W$  with the vector that represents all points in  $B$  (in the numerator) or the dot product with  $W$  and a vector of ones with size  $|B| = m$  (for the denominator).

Furthermore, as the boundary is generated using at most 8 neighbours of a pixel in the hole, it could contain at most  $8n = O(n)$  points, therefore  $O(nm) = O(n^2)$ . (Do notice though that only if  $n = 1$  we have 8 neighbours, and in any other case that considers only one hole, the number of neighbours is usually smaller)

## 2 Question II

A plausible solution to this would be as followed: divide our Hole  $H$  into  $\ell$  disjoint sections  $\{H_i\}_{i=1}^{\ell}$  (notice that  $\ell$  is a constant, predefined number which is not a function of  $n$ ). Next, calculate the center of every  $H_i$  (Center of mass, if you will)  $\{\mu_i\}_{i=1}^{\ell}$ . From here, in an iterative process, set  $I(H_j)$  using the same algorithm as before, using  $\mu_j$  instead of  $u$ . In other words, for every  $u_j \in H_j$  we set  $I(u_j) = \frac{\sum_{v \in B} w(\mu_j, v)I(v)}{\sum_{v \in B} w(\mu_j, v)}$ . Notice that in this scenario, the weight function could be represented as a different vector for every  $\ell$  (or a matrix  $\ell \times m$ , rather than a matrix of size  $O(n^2)$ )

In terms of time complexity, generating  $\{H_i\}_{i=1}^{\ell}$  takes  $O(n)$  as we go over all points in  $H$  and split. From here we loop  $\ell$  times, and in each iteration we calculate the center of mass (which is an  $O(n)$  operation), the corresponding weight vector that represents distances between the current center of mass to all points in  $B$  (which is an  $O(n)$  operation as well), and finally the summation which adds up  $|B| = O(n)$  elements as well (this was represented as vector multiplication), All together we are dealing with  $\ell \times (O(n) + O(n) + O(n)) = O(n)$ .

## 3 Question III