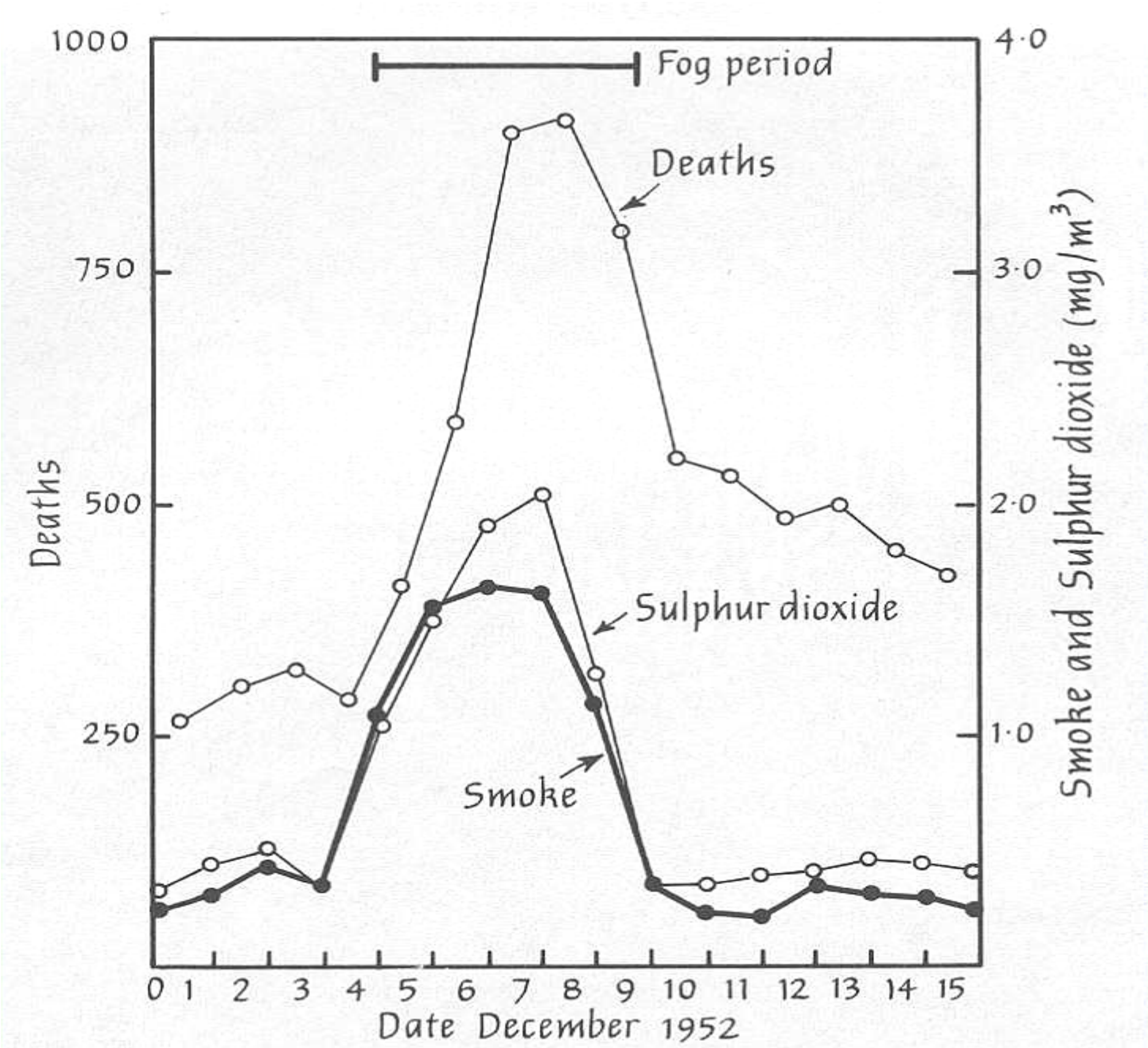
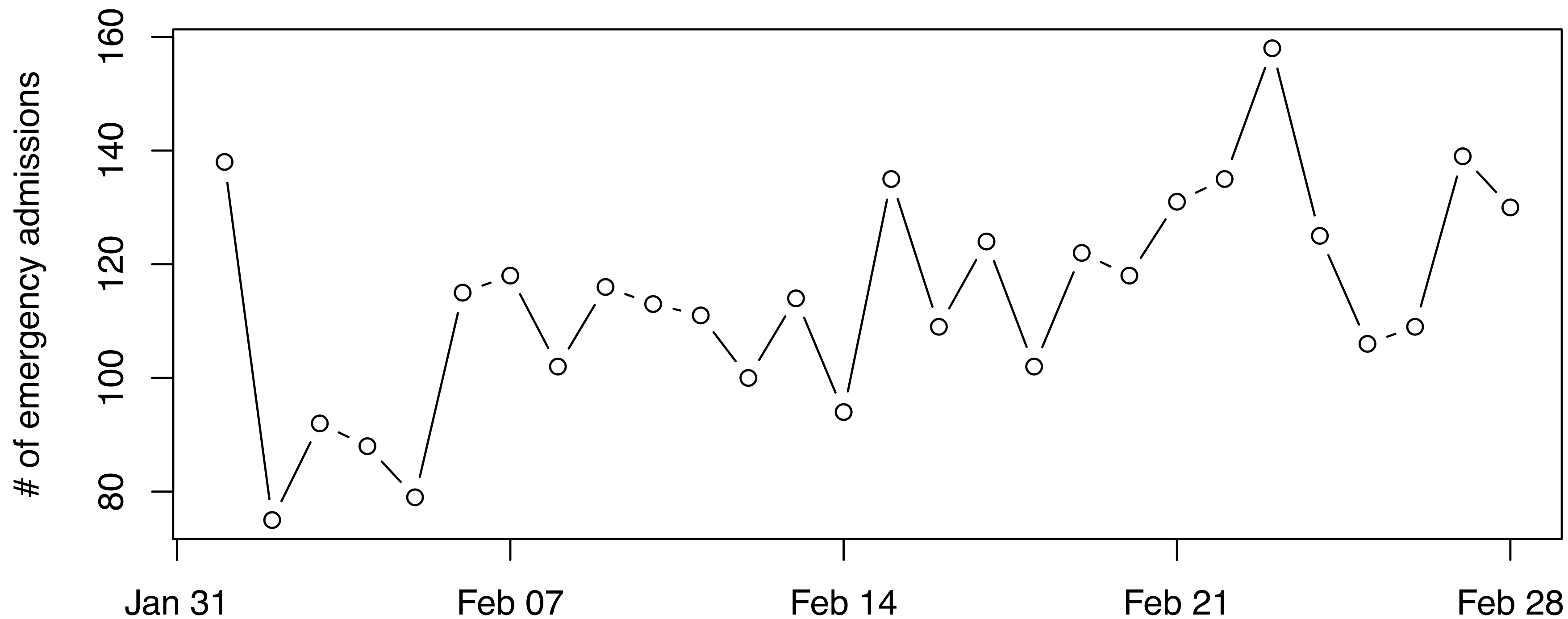
Time Series Analysis

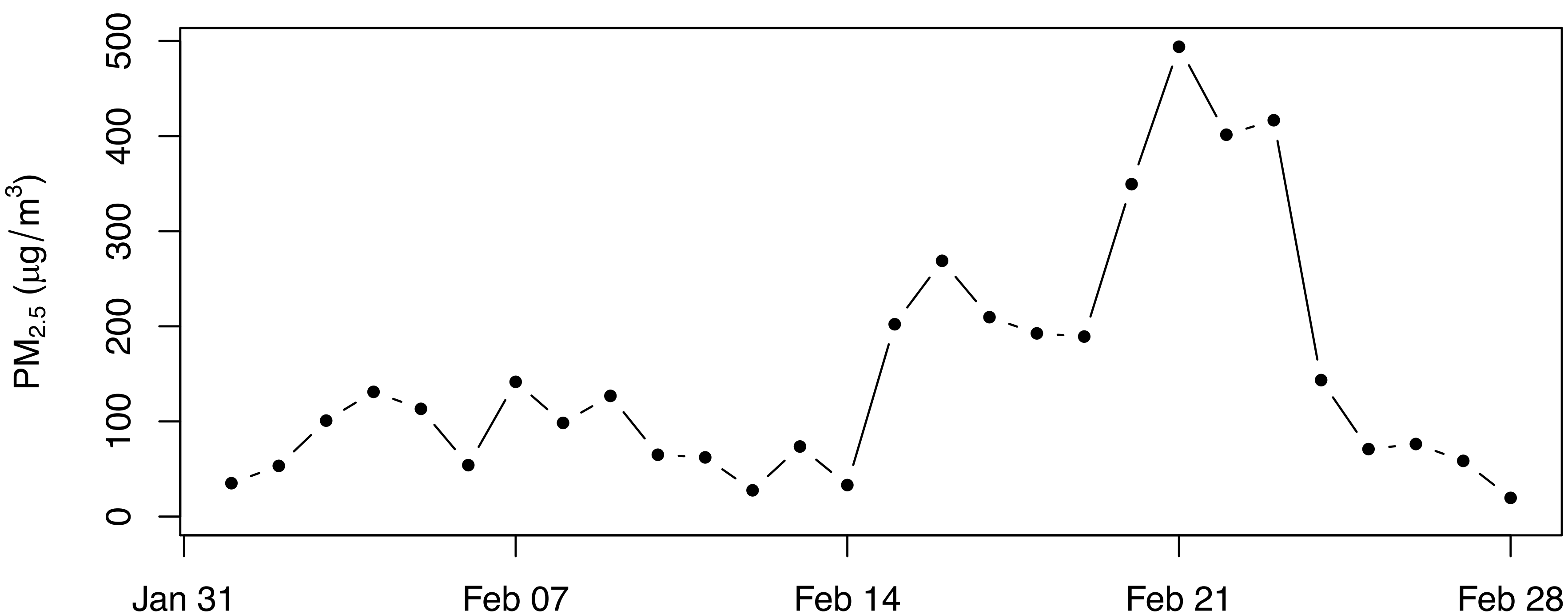
Biostatistics 140.712

London Fog (1952)

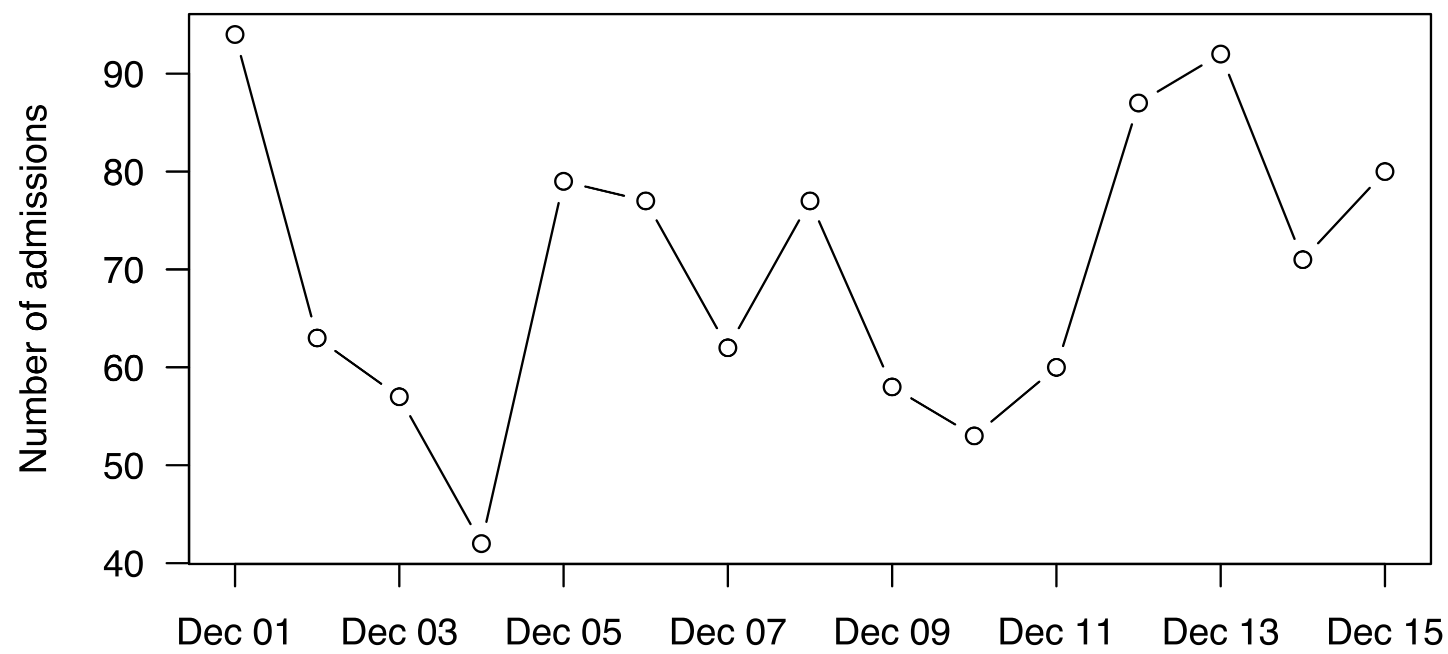


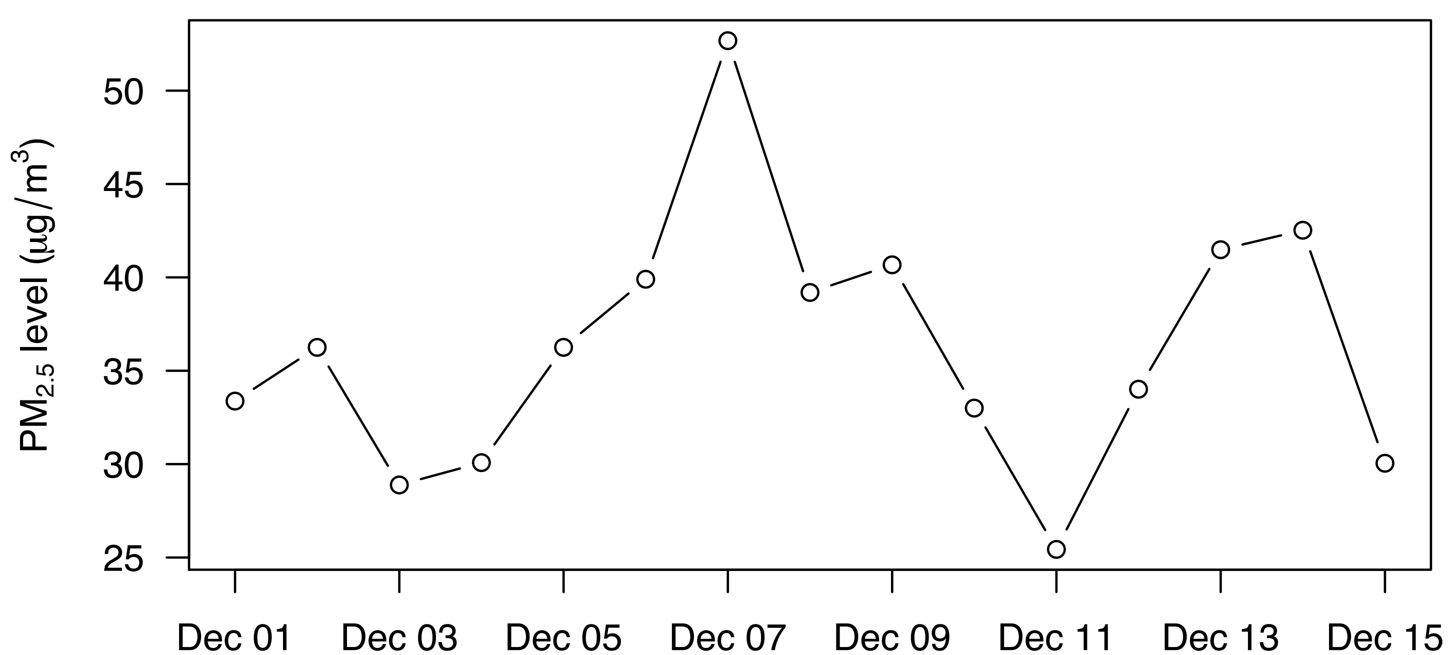
Beijing Fog (2011)





Chicago (2005)





Time Series Analysis

- Relating changes in an exposure X with changes in an outcome Y, adjusting for potential confounders Z
- The *units* of analysis are time points (seconds, minutes, days, months, etc.)
- Time series analysis is interesting because it comes with a “built-in covariate”: **Time itself**

Time Series or Longitudinal?

- Time series
  - focuses on a single series of data
  - only has “within-subject” variation
- Longitudinal data
  - usually focuses on replicates of very short series across subjects
  - within- and between-subject variation
  - Very long time series replicated across many subjects sometimes called *functional data*
- No clear rule!

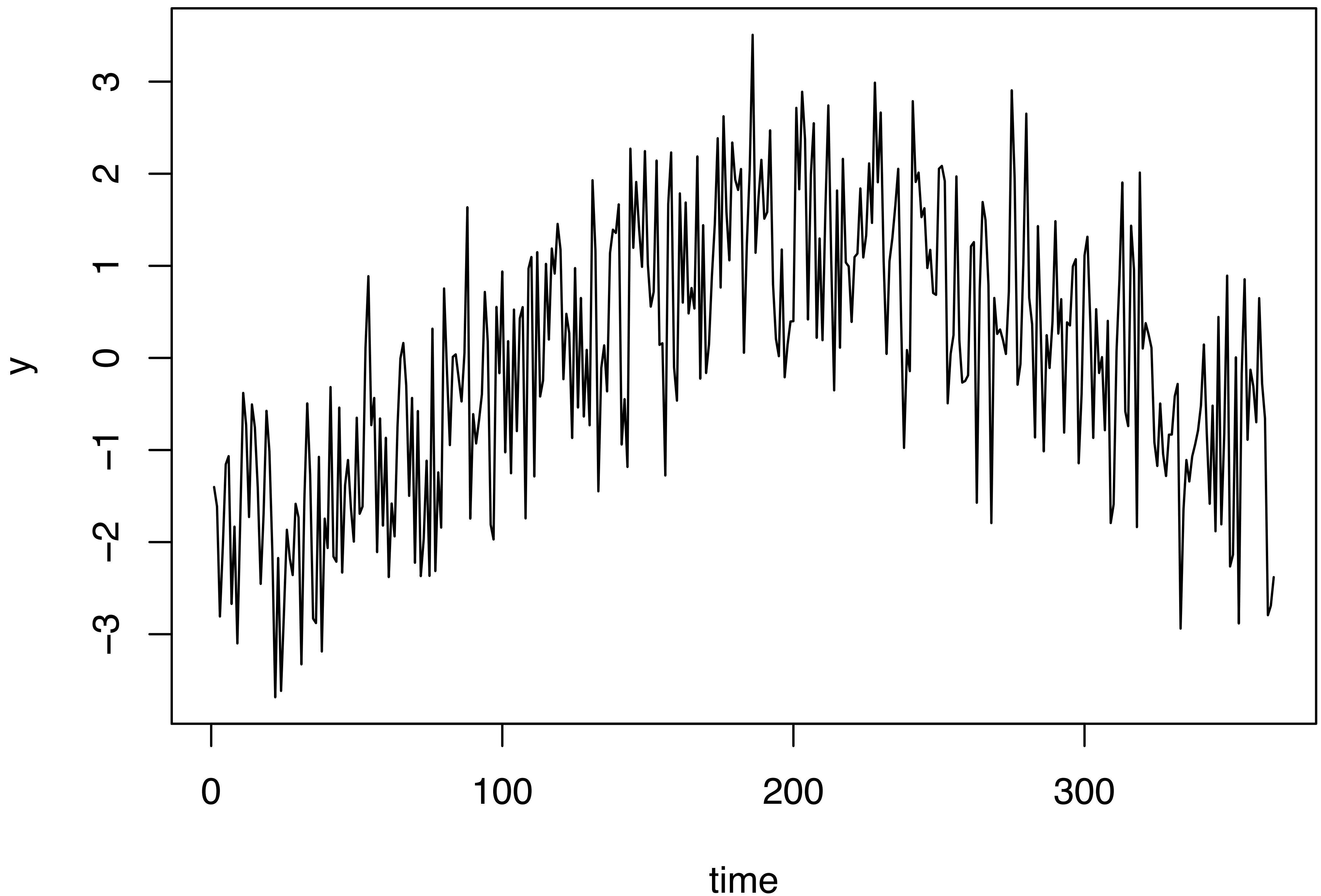
Time is Special

- Observations at neighboring time points are thought to be (positively) correlated with each other: **autocorrelation**
- Why are observations observed across time correlated?
- Time can be used as a “stand in” for other, **unobserved**, time-varying predictors
- Data can also be generated through a **dynamical process** whereby values at one time point may causally effect values at a future time point

Autocorrelation

- Correlation between elements of a time series is called autocorrelation
- Autocorrelation can occur between elements of a time series at different lag distances
- Autocorrelation can be estimated using the autocorrelation function (acf)
- The acf can be plotted as a function of the lag distance

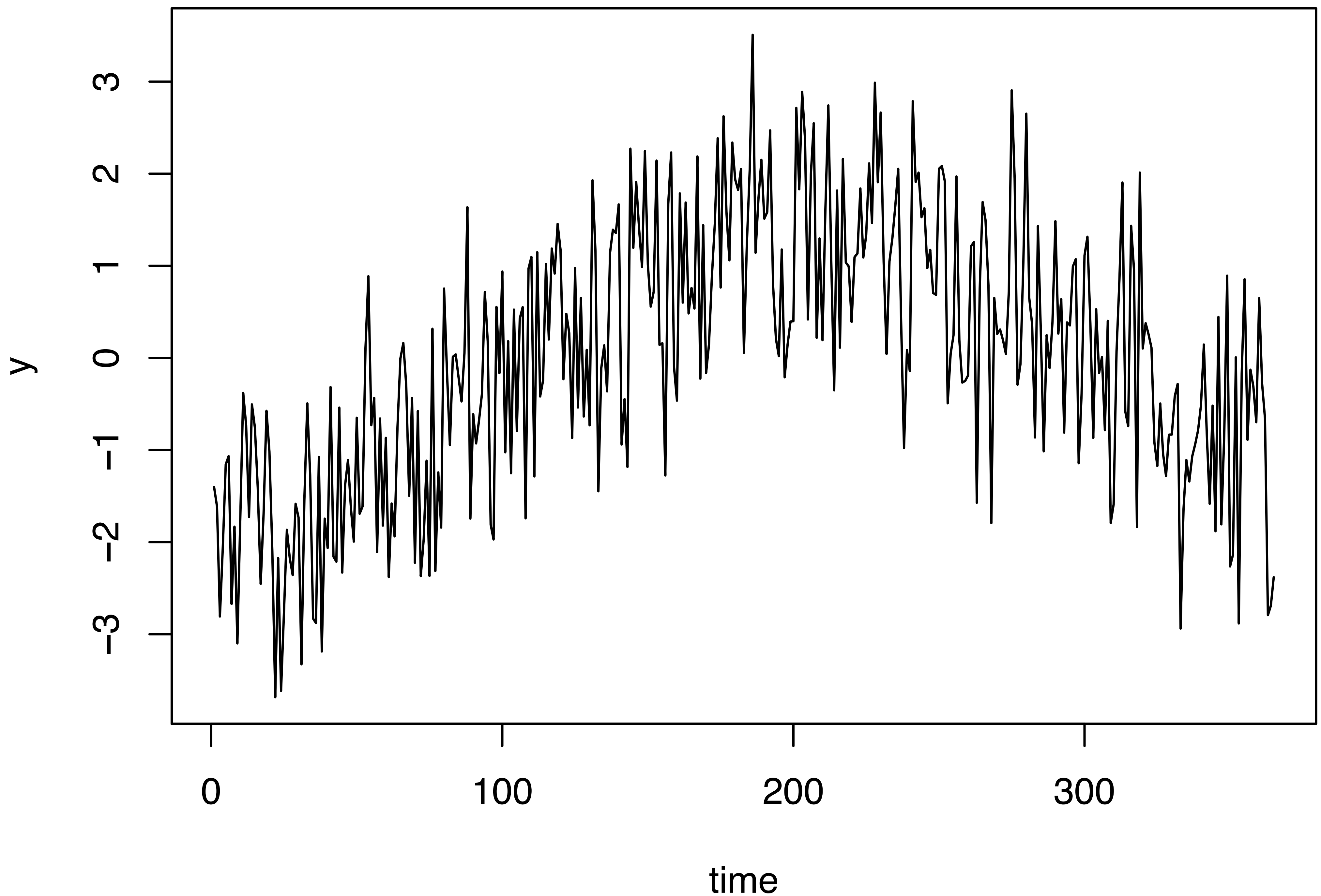
Autocorrelation?



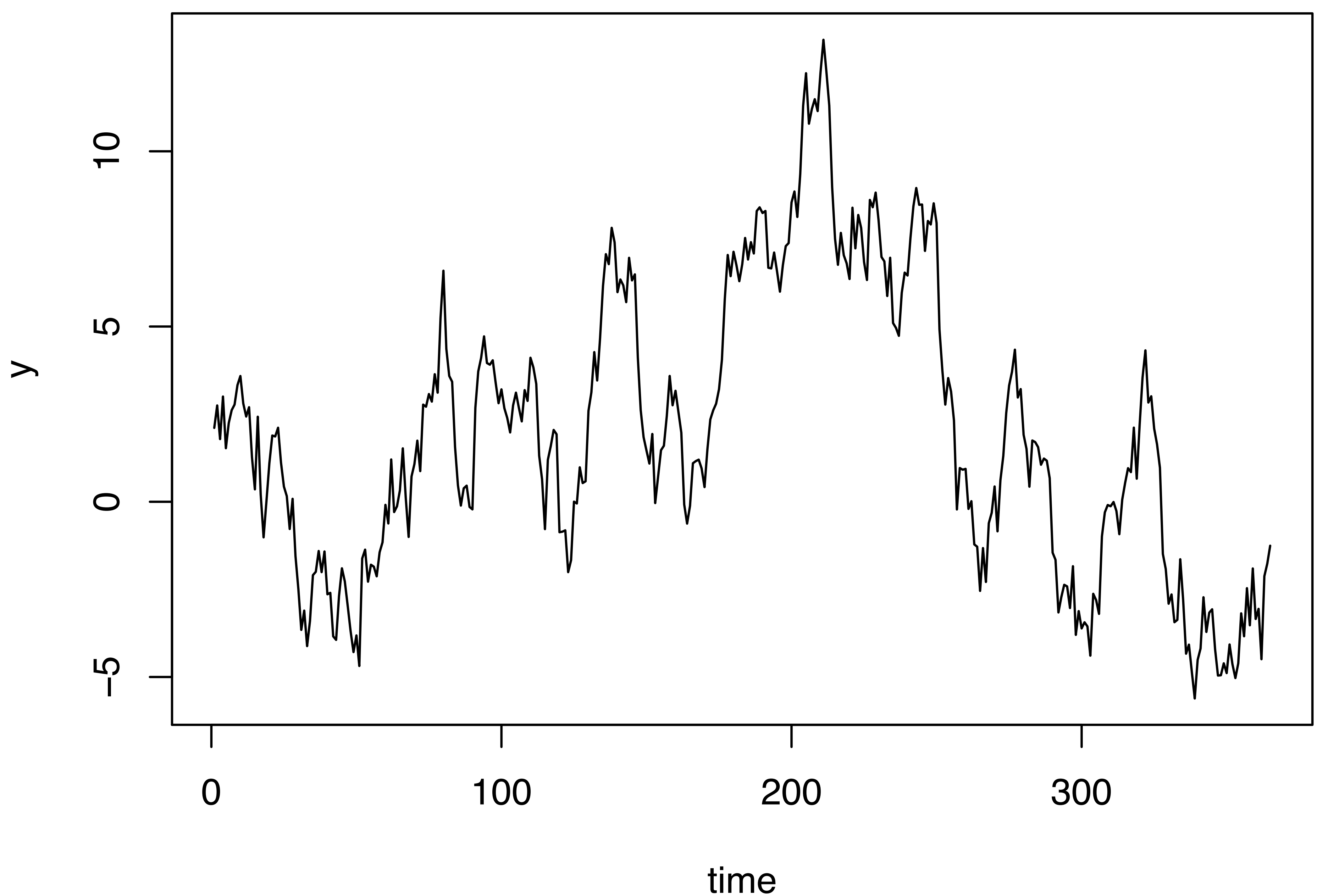
Random vs. Fixed Variation

- Like with any statistical modeling it is important to separate out what is **random** and what is **fixed**
- Traditional time series modeling methods focus on modeling the random aspects (often no mention of fixed aspects)
- In environmental health applications, often many things are fixed
  - season, day-of-week, temperature, etc.
  - identifying those fixed effects is key part of science
- Residual variation may still be autocorrelated
- Because you often only observe a single time series, it is not possible for a model to determine what is fixed and what is random

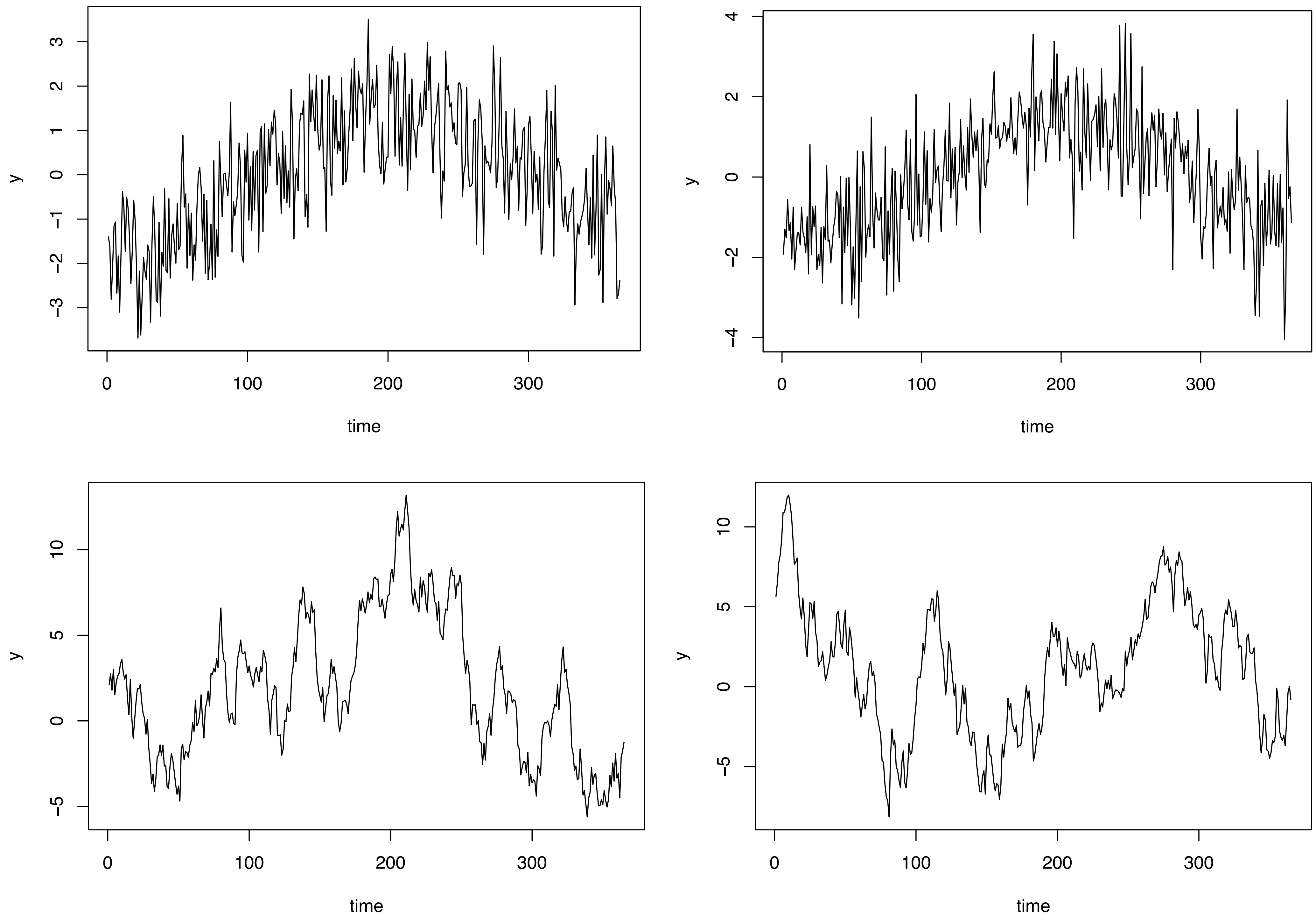
Random or Fixed?



Random or Fixed?



Random or Fixed?



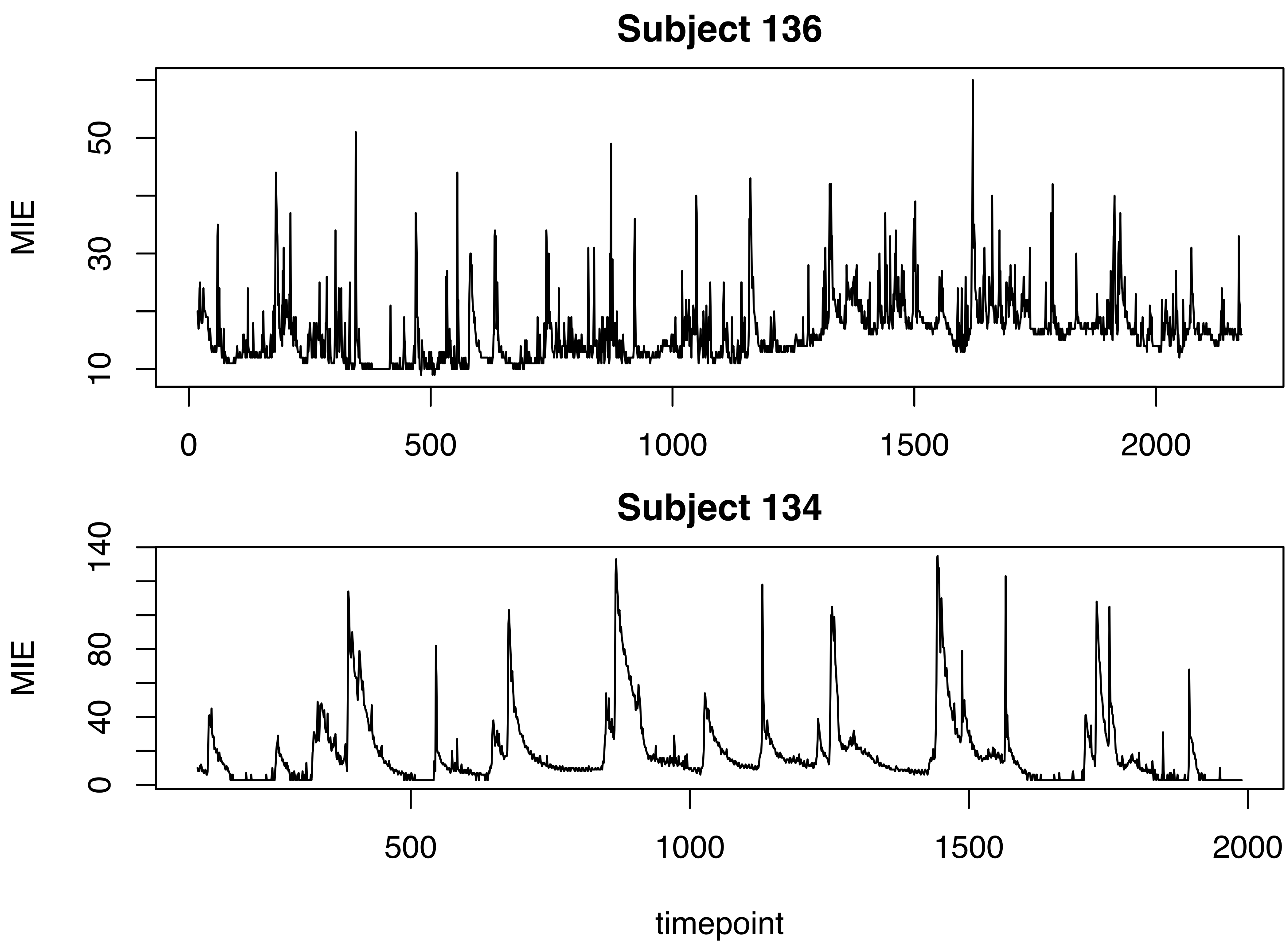
Time Series Analysis in Environmental Health

- Primary task:
    - Model Y vs. X controlling for Z
    - We usually do *not* want to predict Y from a set of predictors X
    - We usually do *not* want to predict a future value of Y from a set of predictors X
- Key problems:
    - Temporal misalignment
    - Missing data is often a problem
- Most concerned with *residual* autocorrelation after removing fixed effects

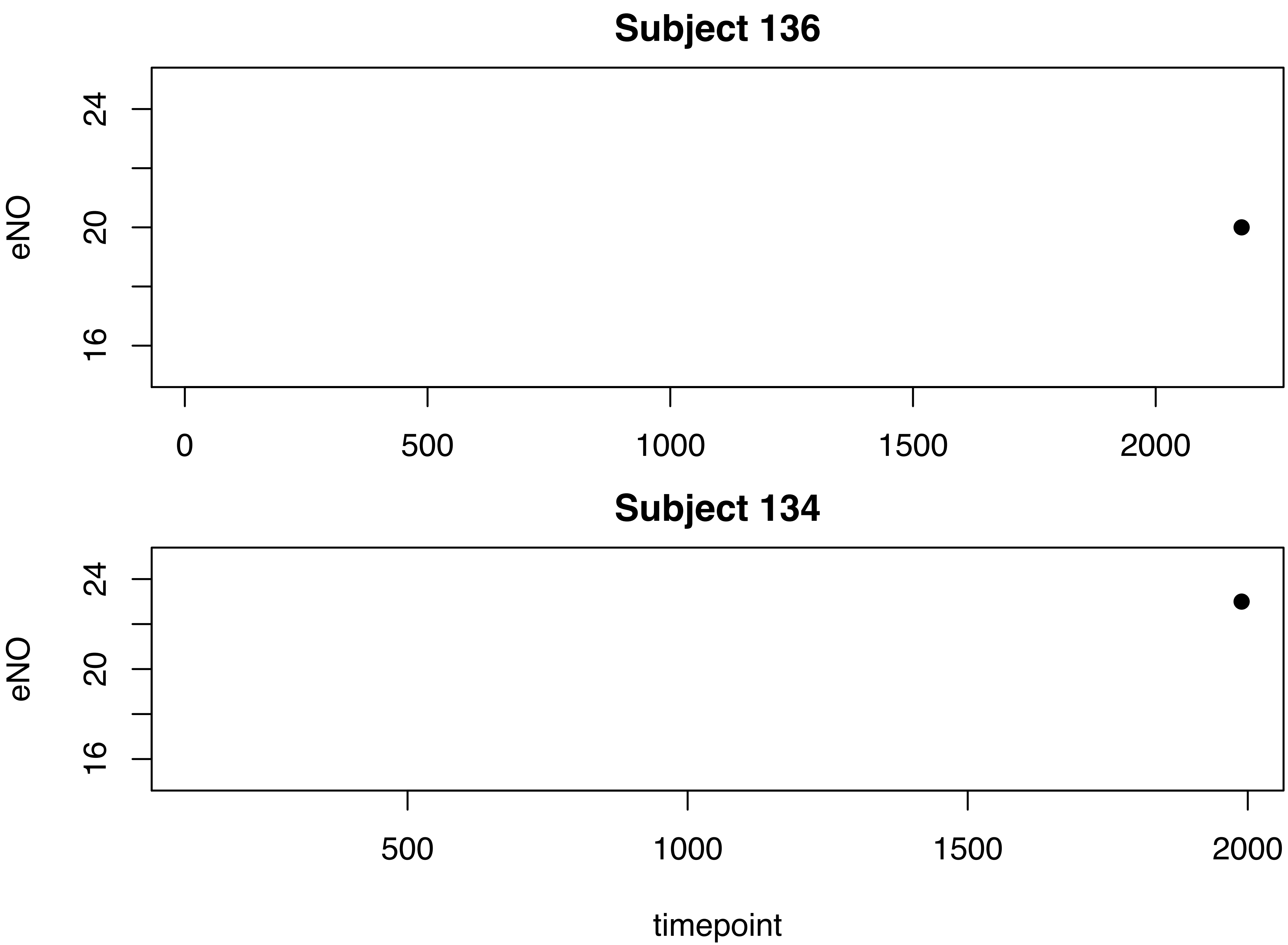
Example: pDRs and COPD

- A study of elderly adults with COPD living in Baltimore, MD; total of 85 subjects
- Longitudinal followup with 1 visit every 3 months (total 3 visits)
- Personal DataRAM (pDR) placed in home to measure PM2.5 at 5-minute intervals over 7-days
- Are indoor levels of PM2.5 associated with COPD?

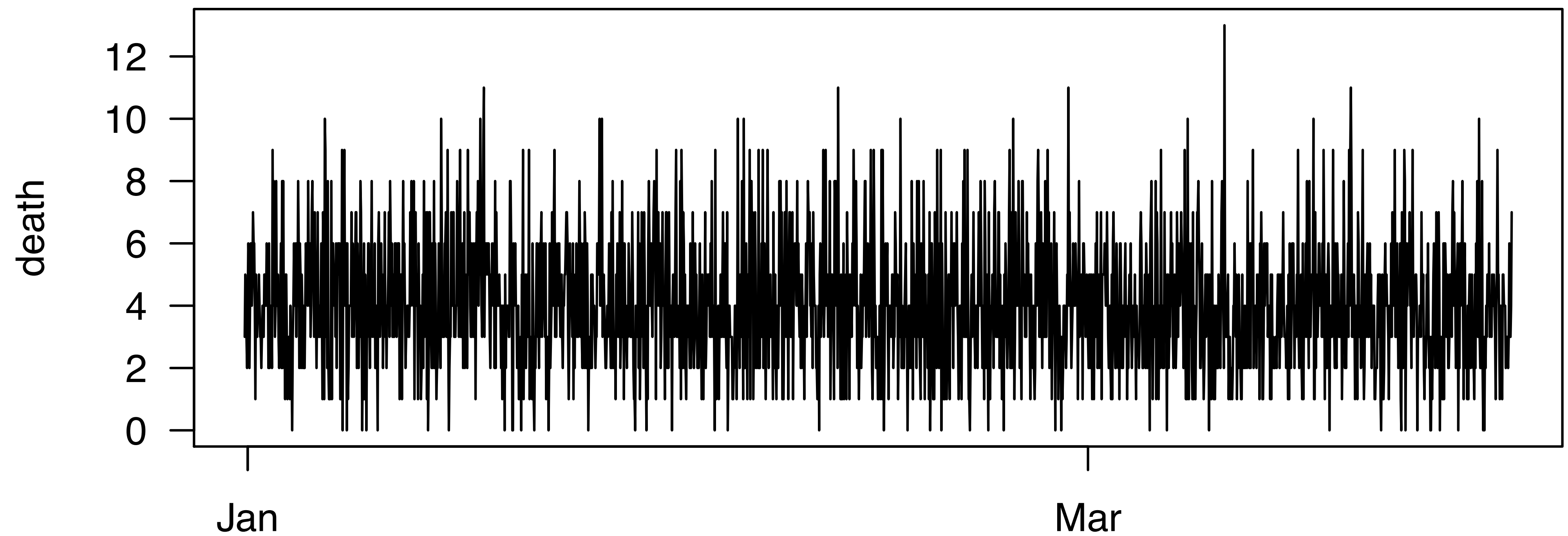
Real-Time $PM_{2.5}$

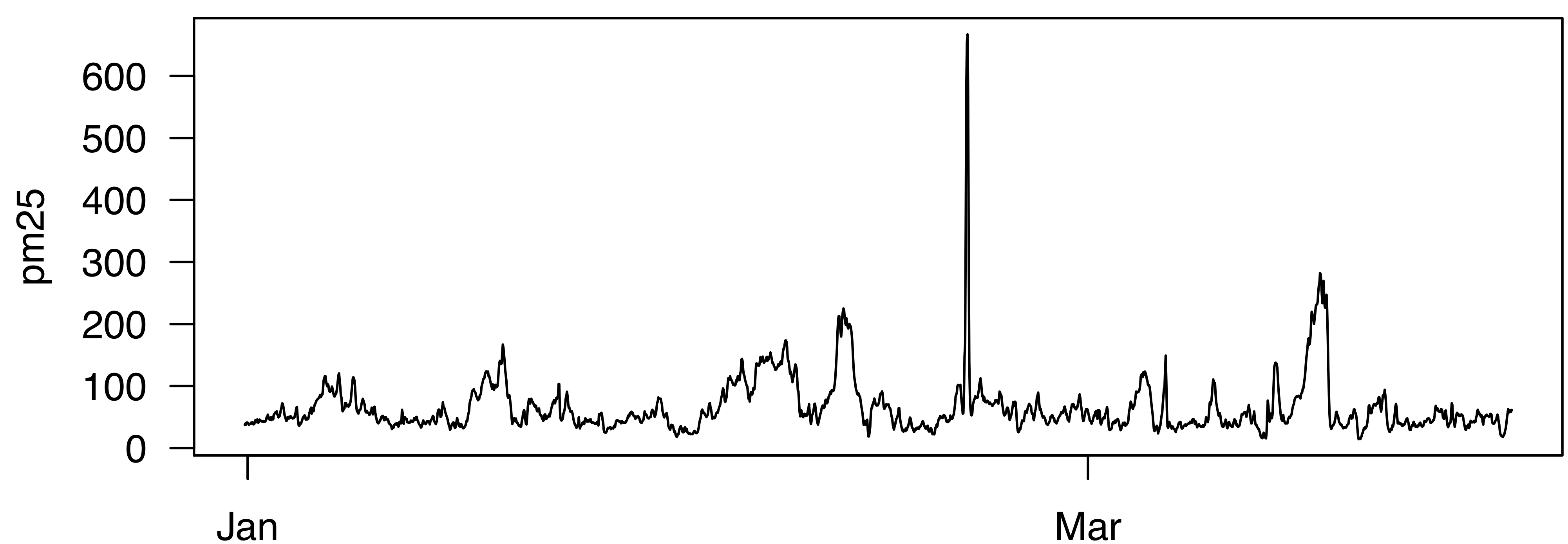


Less-Than-Real-Time Outcome



Hourly PM2.5 and Mortality (Seoul, Korea)





Estimation or Prediction?

- One approach is to design models to optimally **predict the outcome**
- Another approach is to develop models that **estimate an association** and adequately control for confounding
- These two approaches do not necessarily lead to the same model!
- If X and Y have an inherently weak relationship, then it makes little sense to include X in a model for optimally predicting Y
  - e.g. step-wise model selection methods will usually remove X from the model; then what?
  - Potential confounders weakly correlated with Y (but perhaps strongly correlated with X) may not be included in a prediction model

Confounding in Time Series

- A confounder is something that is associated with both the outcome and the risk factor
- In time series studies, the association between a risk factor and outcome is potentially confounded by things that vary in time (day to day, week to week)
    - e.g. weather, season, temperature, pollutants, long-term trends
- Things that “do not vary over time” are not confounders

Timescale of Variation

- In time series analysis the *timescale of variation* is an important aspect to consider
- Do we care about year-to-year, month-to-month, or day-to-day variation in X and Y?
- On what timescales do potential confounders vary?
- If a potential confounder does not vary on your timescale of interest, then it is not a confounder.

Timescale Decomposition

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$$

$$x_t = \bar{x}_t^Y + (x_t - \bar{x}_t^Y)$$

If $\beta_1 = \beta_2$, then we have the first model

$$y_t = \beta_0 + \beta_1 \bar{x}_t^Y + \beta_2 (x_t - \bar{x}_t^Y) + \varepsilon_t$$

Timescale Decomposition

$$y_t = \beta_0 + \beta_1 \bar{x}_t^Y + \beta_2 (x_t - \bar{x}_t^Y) + \varepsilon_t$$

$$z_t = \bar{z}_t^S + (z_t - \bar{z}_t^S)$$

$$y_t = \beta_0 + \beta_1 \bar{x}_t^Y + \beta_2 \bar{z}_t^S + \beta_3 (z_t - \bar{z}_t^S) + \varepsilon_t$$

$$u_t = \bar{u}_t^W + (u_t - \bar{u}_t^W)$$

Timescale Decomposition

$$u_t = \bar{u}_t^W + (u_t - \bar{u}_t^W)$$

$$x_t = \bar{x}_t^Y + \bar{z}_t^S + \bar{u}_t^W + r_t$$

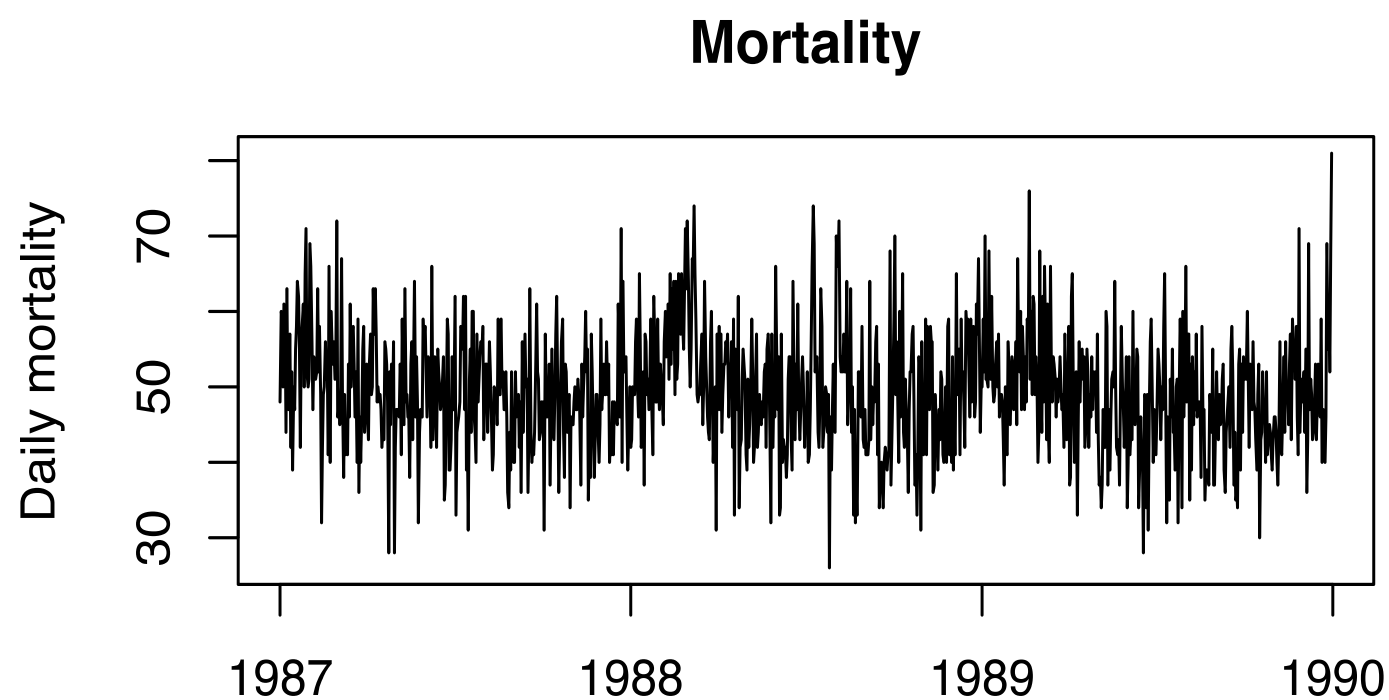
$$y_t = \beta_0 + \beta_1 \bar{x}_t^Y + \beta_2 \bar{z}_t^S + \beta_3 \bar{u}_t^W + \beta_4 r_t + \varepsilon_t$$

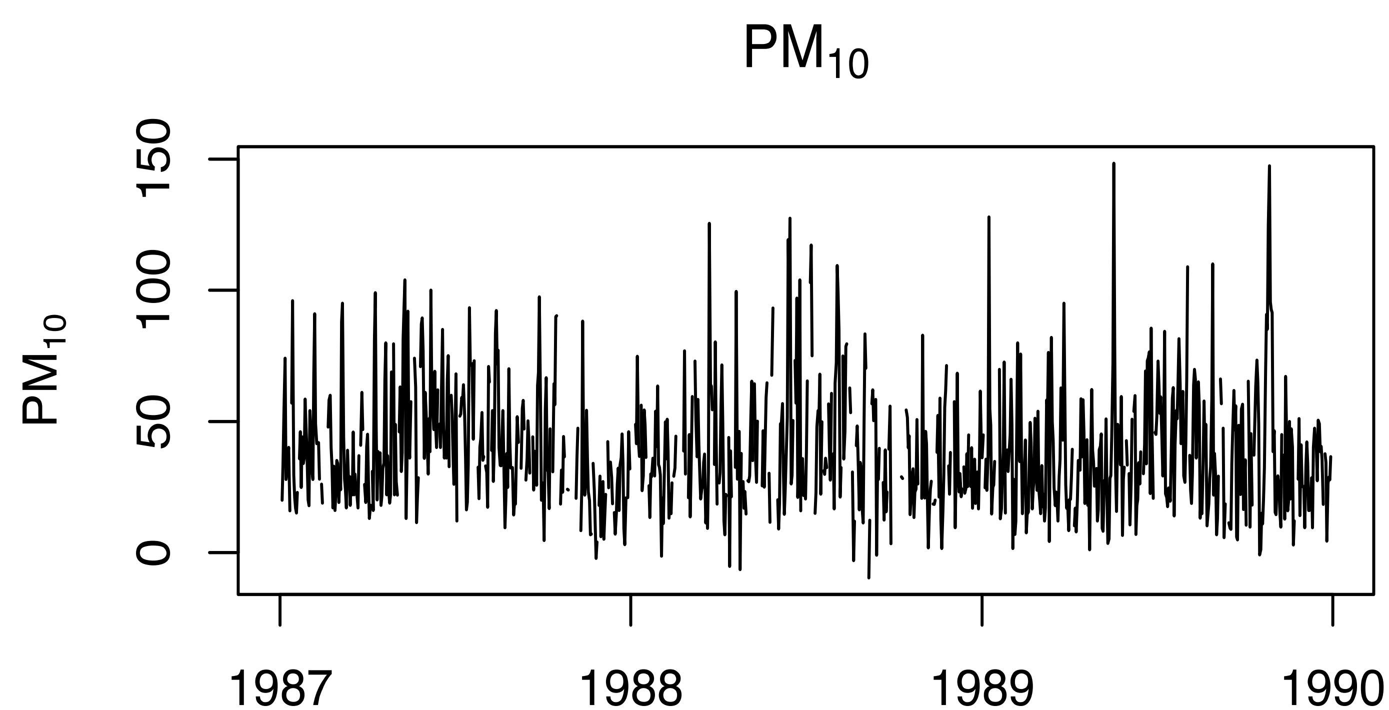
365-day MA

90-day MA

7-day MA

Detroit Mortality and PM_{10}



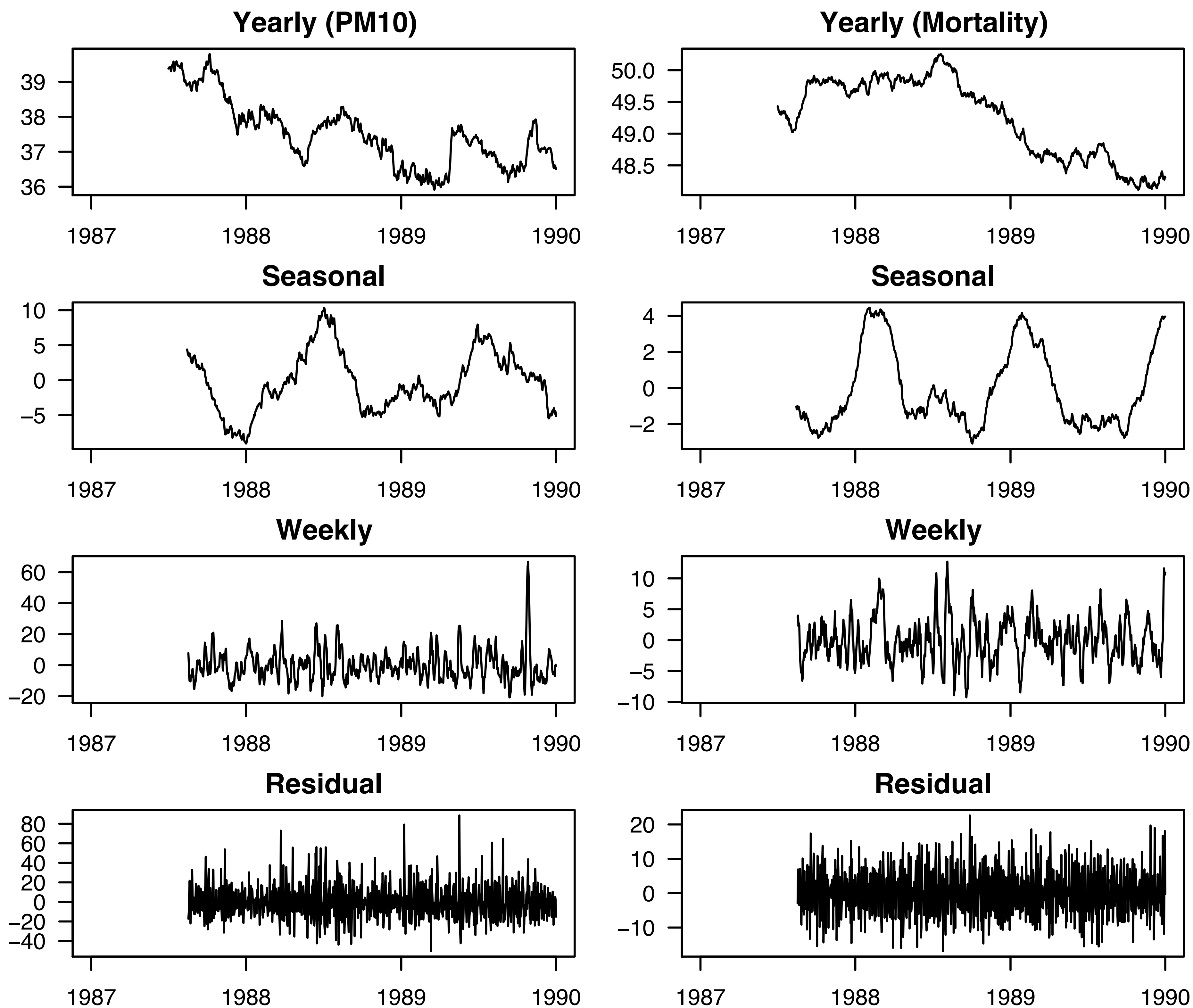


Detroit PM_{10} and Mortality

$$Y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$$

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	46.1798	0.2263	204.11	0.0000
x	0.0232	0.0057	4.06	0.0000

Timescales of Variation



Detroit PM_{10} and Mortality

$$Y_t = \beta_0 + \beta_1 \textbf{yearly}_t + \beta_2 \textbf{seasonal}_t + \beta_3 \textbf{weekly}_t + \beta_4 \textbf{residual}_t + \varepsilon_t$$

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	34.1031	1.3098	26.04	0.0000
x.yearly	0.3783	0.0383	9.88	0.0000
z.seasonal	−0.4354	0.0295	−14.76	0.0000
u.weekly	0.0532	0.0123	4.33	0.0000
r	0.0215	0.0070	3.07	0.0022

Which Estimate Do We Use?

- Which timescale estimate is most appropriate or useful (i.e. tells the best or interesting “story”)?
- Which potential confounders affect which timescale estimate and how?
- At which timescale do we have the best data, measurements, knowledge/hypotheses?

Regression Model Assumptions

- Errors are independent of covariates
- Errors have mean zero
- Errors are independent of each other (autocorrelation?)
- Variance of errors is constant and > 0
- [Errors are Normally distributed with mean 0 and variance σ^2]

Time Series Regression Model Assumptions

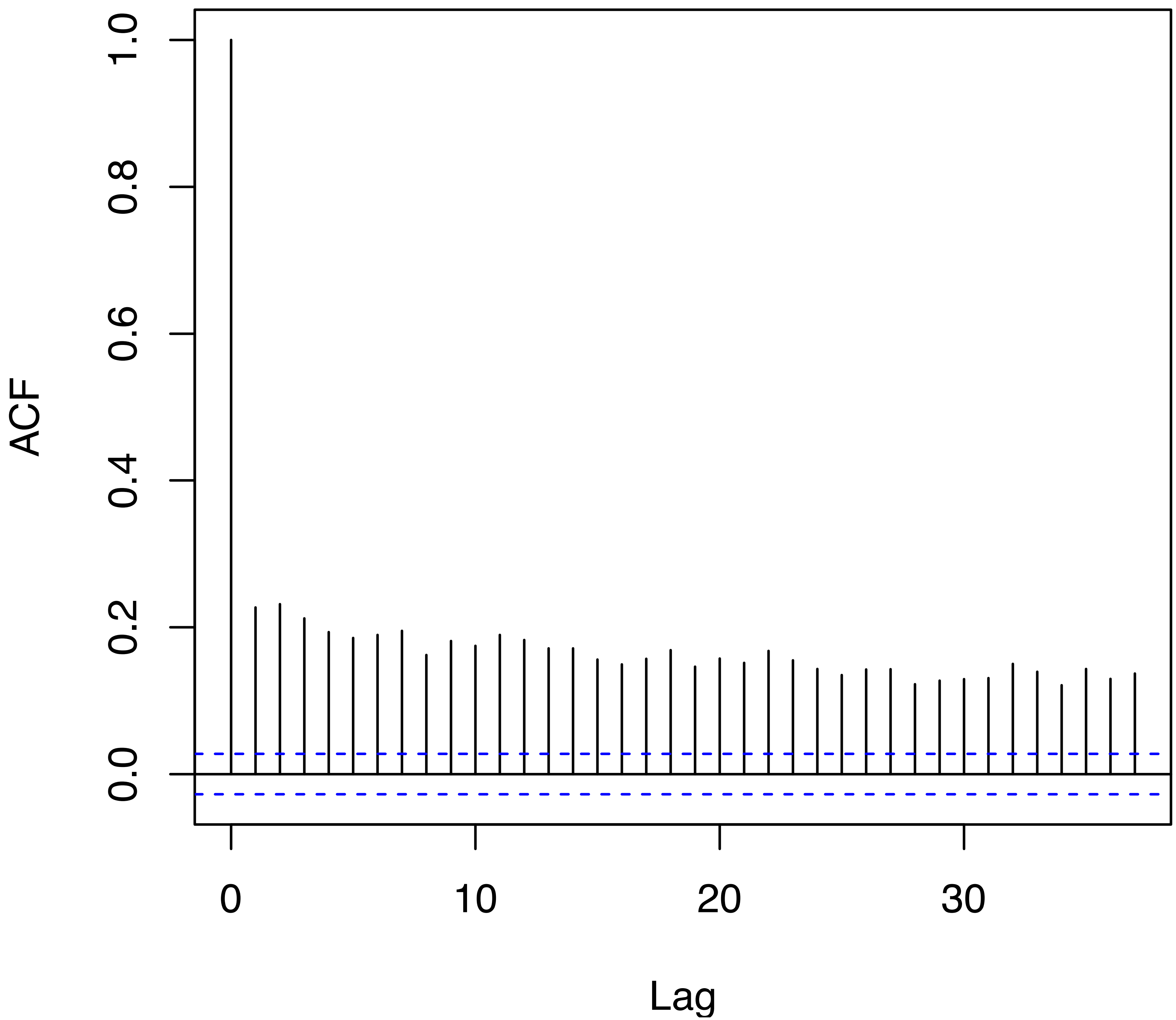
- In a time series regression model, errors may be correlated
  - Why are errors correlated? Can we explain it?
- Correlated errors (if ignored) can affect estimation of parameters
  - Is that important? Do we need to model it?
- What is random? What is fixed?

Detroit PM_{10} and Mortality Analysis

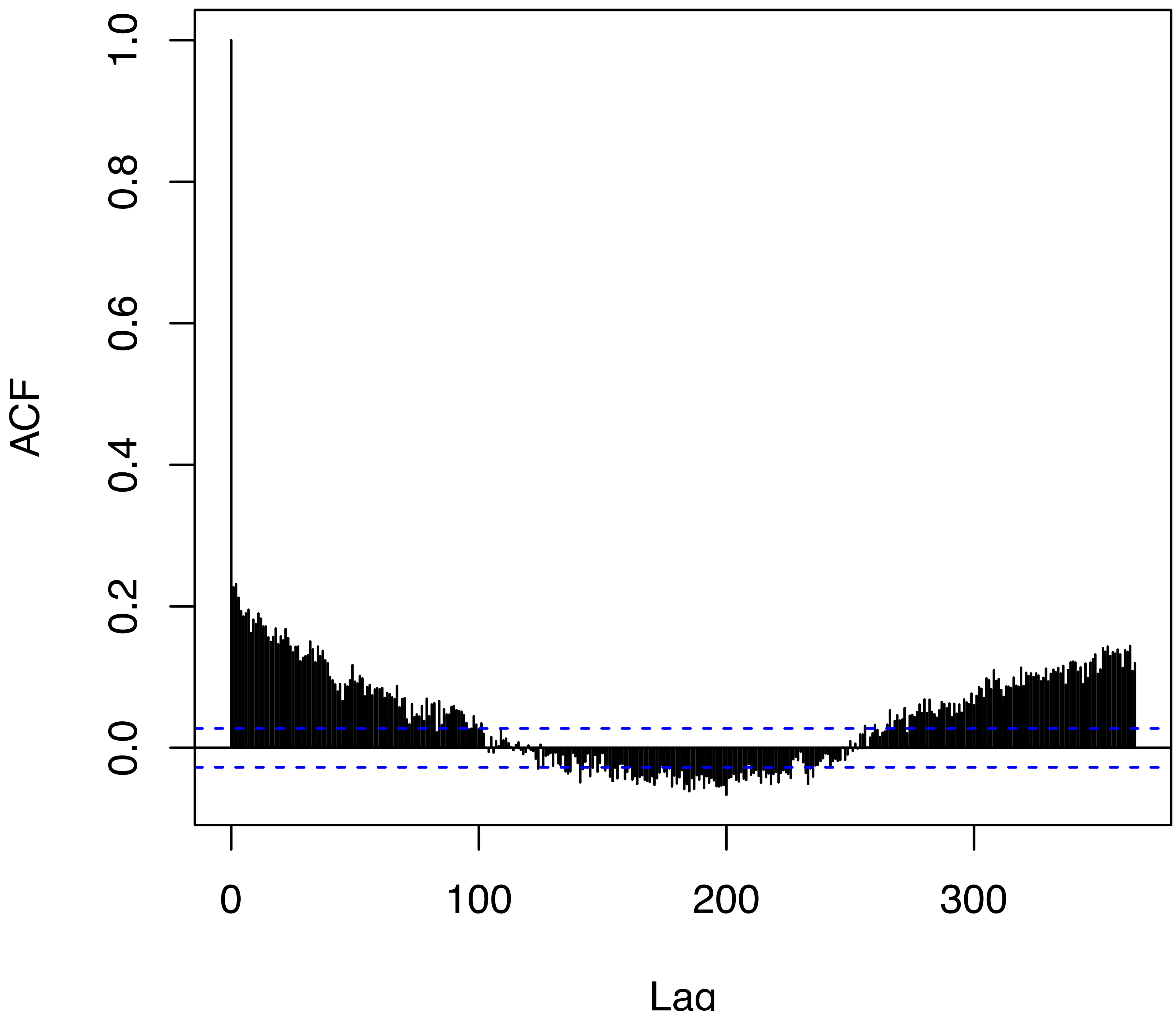
$$Y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$$

```
Call:
lm(formula = y ~ x)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.83914 -0.11227  0.00688  0.11807  0.55958 

Coefficients:
             Estimate Std. Error  t value Pr(>|t|)
(Intercept) 3.8348985  0.0024221 1583.311   <2e-16
x           0.0003014  0.0001253    2.406   0.0162

Residual standard error: 0.1732 on 5112 degrees of freedom
Multiple R-squared: 0.001131,   Adjusted R-squared: 
  0.0009357 
F-statistic: 5.788 on 1 and 5112 DF,  p-value: 0.01617
```

Residual autocorrelation?



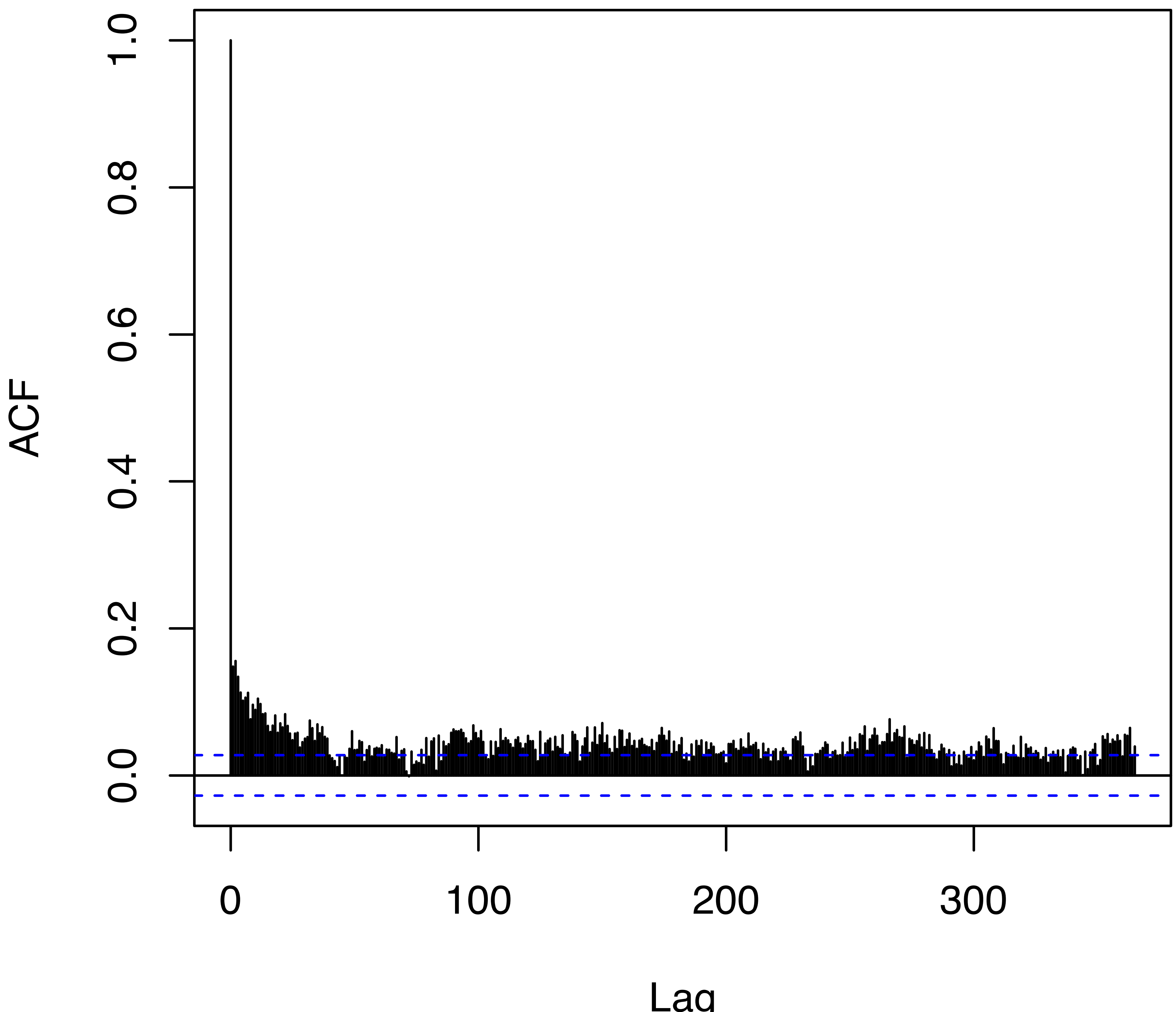
Residual autocorrelation?



Removing season

```
Call:
lm(formula = y ~ season + x)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.81887 -0.10184  0.00779  0.11293  0.55844 

Coefficients:
              Estimate Std. Error t value    Pr(>|t|)
(Intercept)  3.9155533  0.0046581 840.587     < 2e-16
seasonQ2    -0.1008966  0.0065837 -15.325     < 2e-16
seasonQ3    -0.1448564  0.0065731 -22.038     < 2e-16
seasonQ4    -0.0754779  0.0065410 -11.539     < 2e-16
x            0.0005982  0.0001205   4.965 0.000000711

Residual standard error: 0.1652 on 5109 degrees of freedom
Multiple R-squared: 0.09179,	Adjusted R-squared: 0.09108
F-statistic: 129.1 on 4 and 5109 DF,  p-value: < 2.2e-16
```

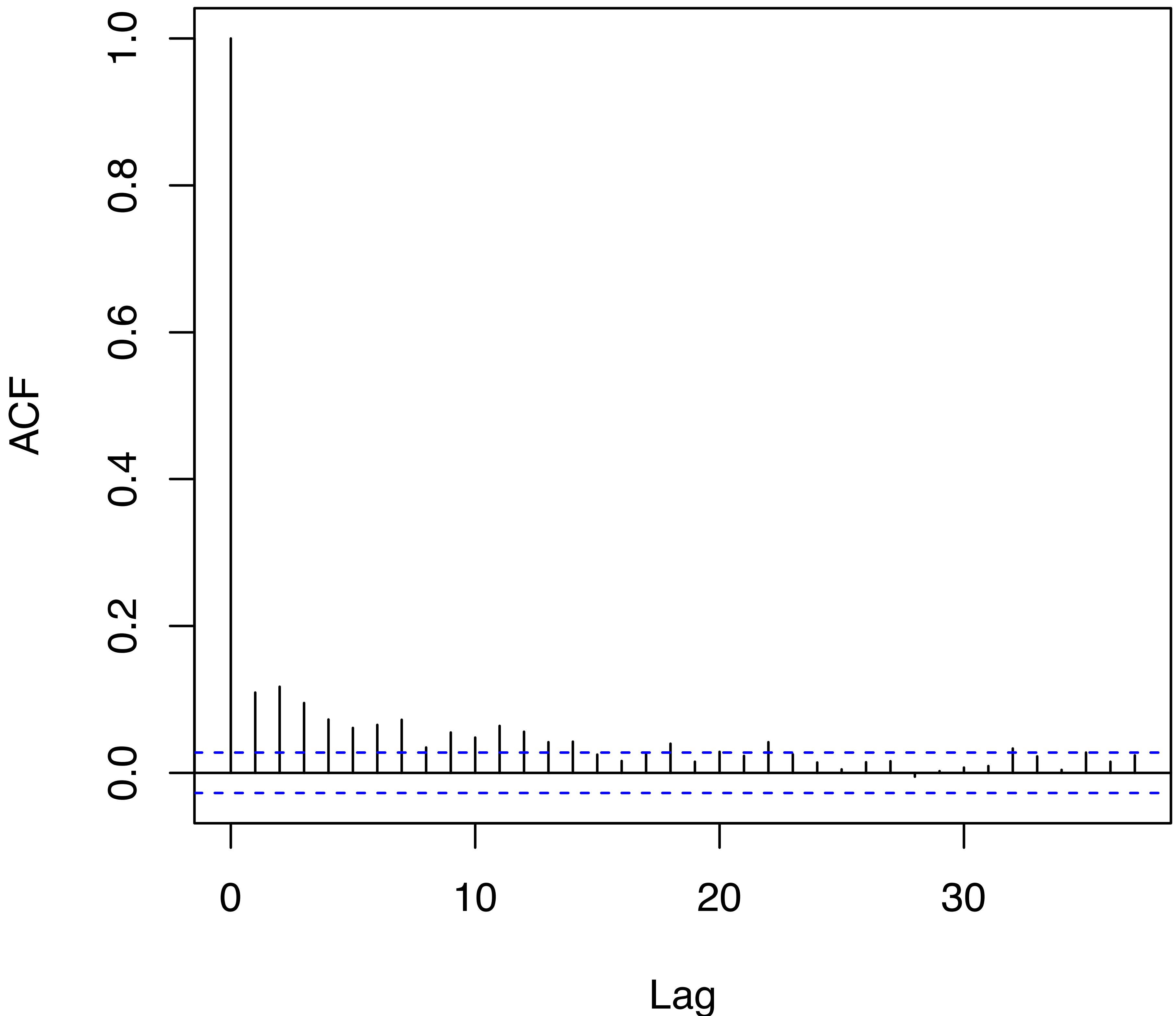
Residual autocorrelation?



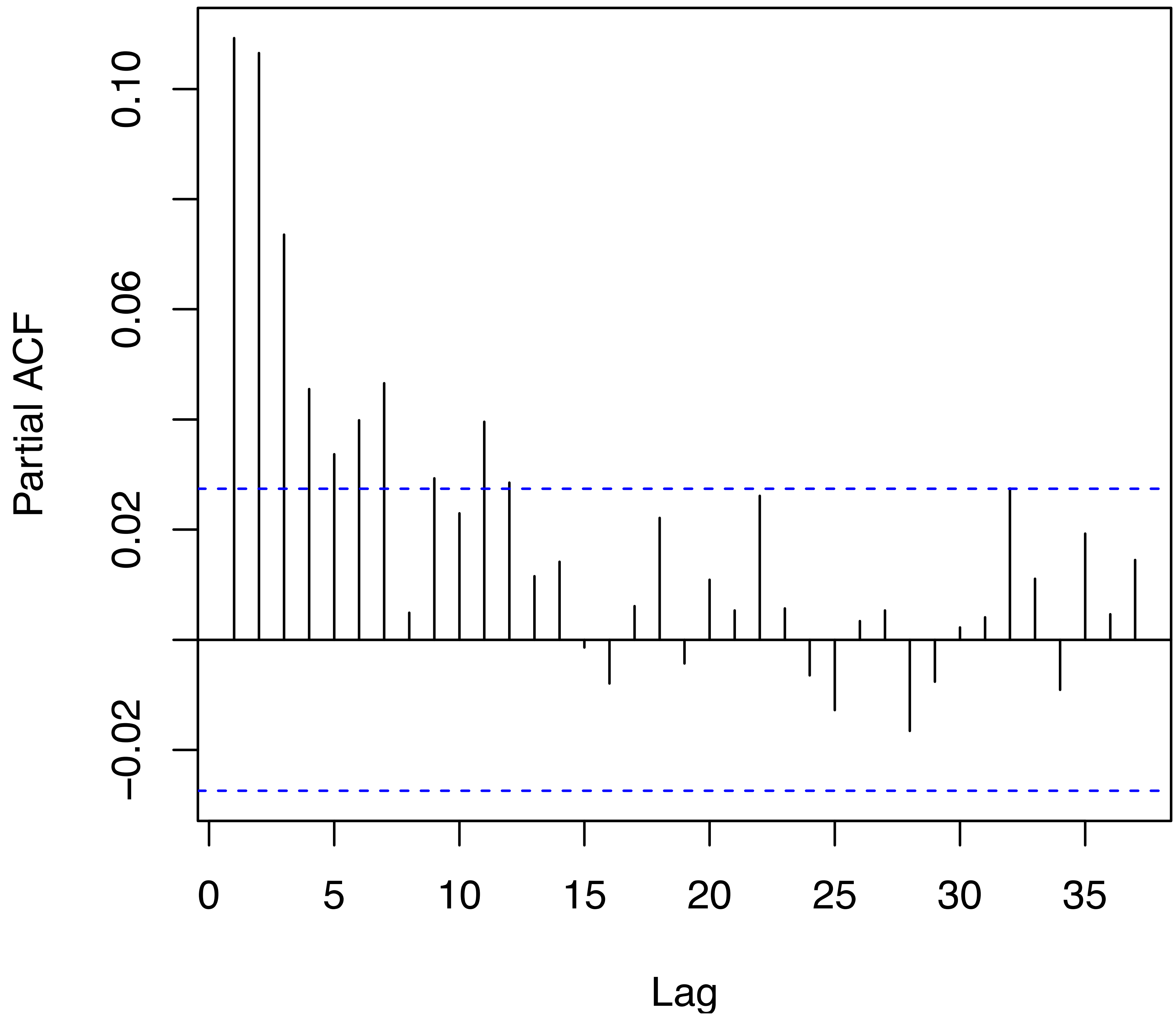
Remove season + trend

```
Call:
lm(formula = y ~ season + day + x)

Residuals:
     Min       1Q   Median       3Q      Max
-0.76282 -0.10167  0.00566  0.10936  0.55349

Coefficients:
                Estimate   Std. Error t value    Pr(>|t|)
(Intercept)  4.117496865  0.013998439 294.140     < 2e-16
seasonQ2    -0.098744775  0.006440817 -15.331     < 2e-16
seasonQ3    -0.140557052  0.006435061 -21.842     < 2e-16
seasonQ4    -0.069072422  0.006411230 -10.774     < 2e-16
day         -0.000023407  0.000001534 -15.257     < 2e-16
x            0.000589037  0.000117849   4.998 0.000000598

Residual standard error: 0.1616 on 5108 degrees of freedom
Multiple R-squared: 0.1314,      Adjusted R-squared: 0.1305
F-statistic: 154.5 on 5 and 5108 DF,  p-value: < 2.2e-16
```

Residual autocorrelation?



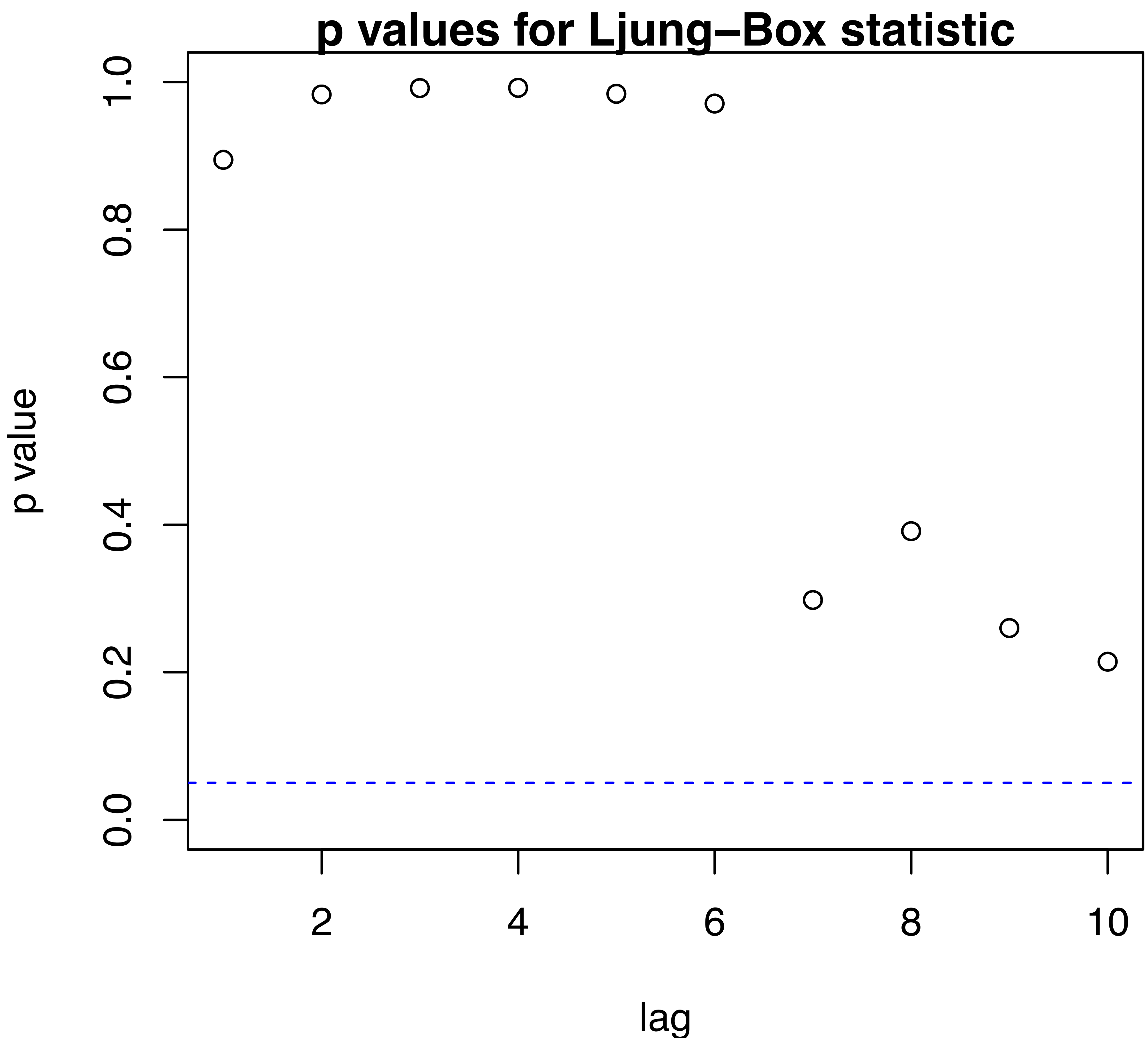
PACF Plot



AR(6) model results

```
Call:
arima(x = y, order = c(6, 0, 0), xreg = mm, include.mean = F)


Coefficients:
         ar1     ar2     ar3     ar4     ar5     ar6  (Intercept)  seasonQ2
      0.0834  0.0909  0.0639  0.0392  0.0307  0.0403       4.1137   -0.0938
s.e.  0.0140  0.0140  0.0141  0.0141  0.0140  0.0140       0.0211    0.0094
      seasonQ3  seasonQ4  day       x
       -0.1356   -0.0670    0  0.0006
s.e.    0.0095    0.0094    0  0.0001


sigma^2 estimated as 0.02521:  log likelihood = 2154.22,  aic = -4282.44
```

Test for residual autocorrelation



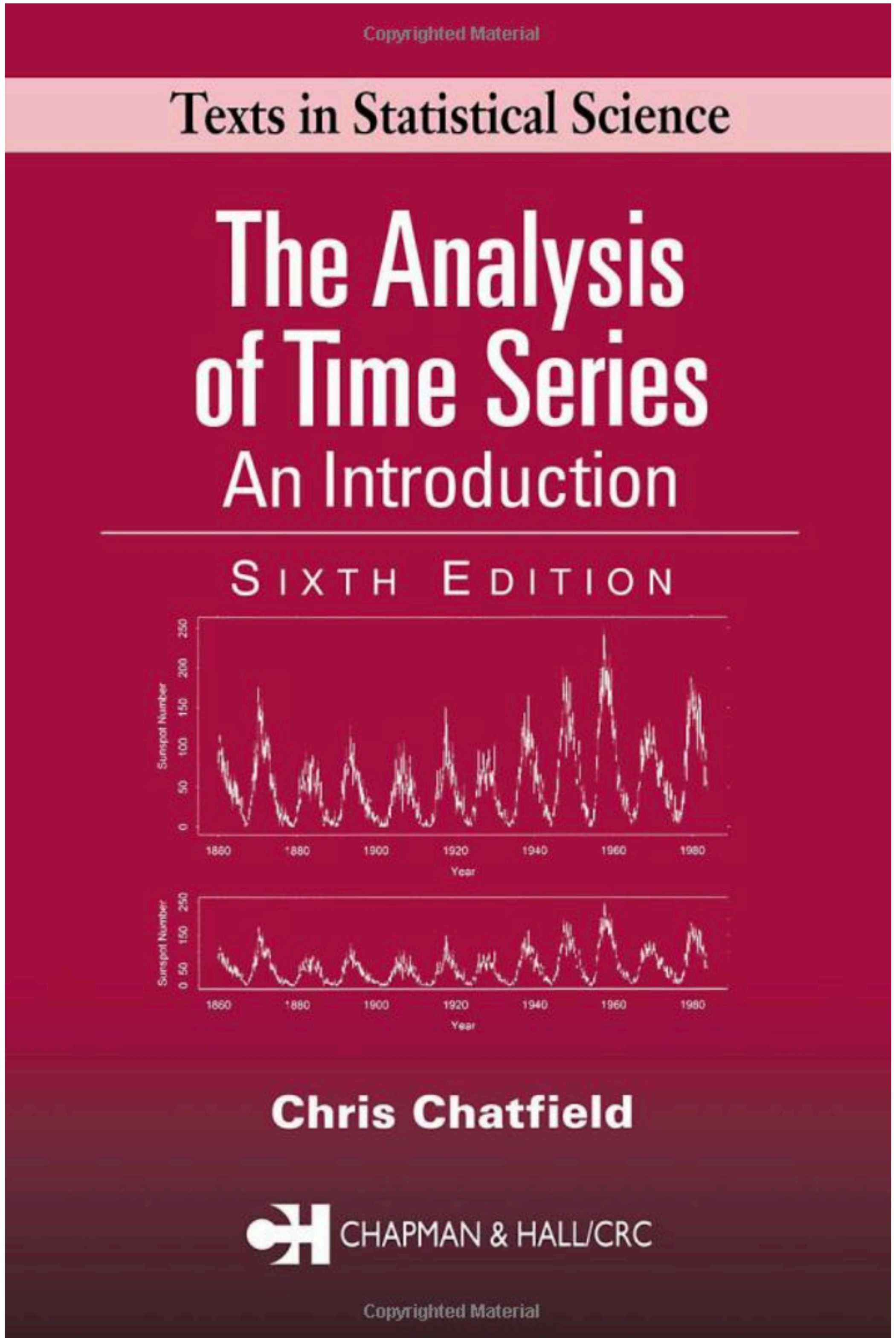
Robust variance or AR model?

	Coef	Naive	Robust	AR(6)
(Intercept)	4.117497	0.013998	0.017070	0.020225
seasonQ2	-0.098745	0.006441	0.007945	0.009088
seasonQ3	-0.140557	0.006435	0.008498	0.009153
seasonQ4	-0.069072	0.006411	0.007999	0.009058
day	-0.000023	0.000002	0.000002	0.000014
x	0.000589	0.000118	0.000131	0.000120

Summary

- Time series data relate changes over time of an exposure and outcome
- Different models can be used for estimation/explanation and prediction
- Understanding timescales of variation for exposure and outcome is important in time series analysis
- Autocorrelation in residuals can indicate unexplained variability (but often can be explained!)

Literature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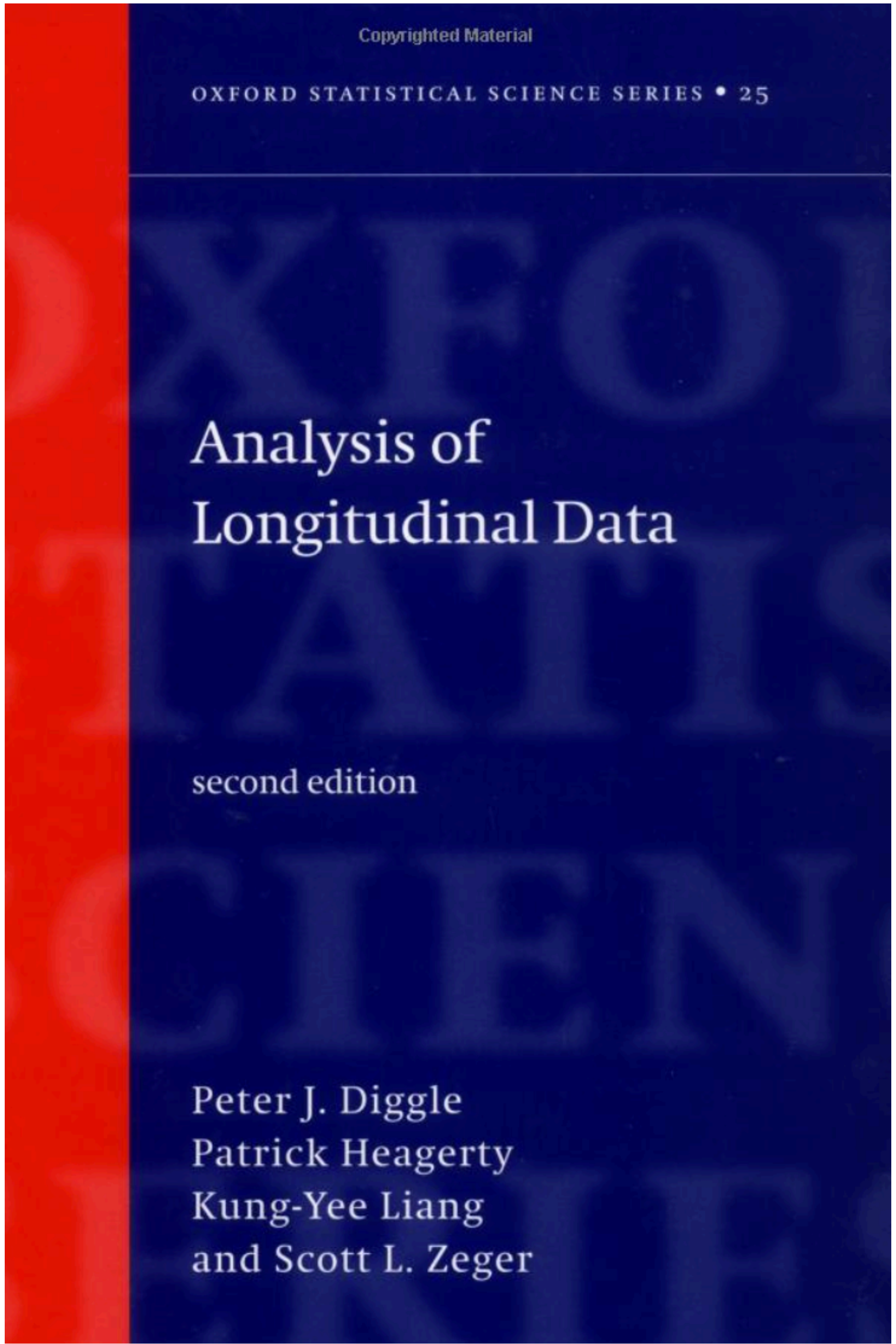
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