Fast Reliable Algorithms for Matrices with Structure

Fast Reliable Algorithms for Matrices with Structure

Edited by

T. Kailath
Stanford University
Stanford, California

A. H. Sayed
University of California
Los Angeles, California

siam

Society for Industrial and Applied Mathematics
Philadelphia

10 9 8 7 6 5 4 3 2 1

Library of Congress Cataloging-in-Publication Data

Fast reliable algorithms for matrices with structure / edited by
T. Kailath, A.H. Sayed

p. cm.

Includes bibliographical references (p. -) and index.

ISBN 0-89871-431-1 (pbk.)

1. Matrices -- Data processing. 2. Algorithms. I. Kailath, Thomas. II. Sayed, Ali H.

QA188 .F38 1999

512.9'434 -- dc21 99-26368

CIP

rev.

CONTRIBUTORS

Dario A. BINI
Dipartimento di Matematica
Università di Pisa
Pisa, Italy

Sheryl BRANHAM
Dept. Math. and Computer Science
Lehman College
City University of New York
New York, NY 10468, USA

Richard P. BRENT
Oxford University Computing Laboratory
Wolfson Building, Parks Road
Oxford OX1 3QD, England

Raymond H. CHAN
Department of Mathematics
The Chinese University of Hong Kong
Shatin, Hong Kong

Shivkumar CHANDRASEKARAN
Dept. Electrical and Computer Engineering
University of California
Santa Barbara, CA 93106, USA

Patrick DEWILDE
DIMES, POB 5031, 2600GA Delft
Delft University of Technology
Delft, The Netherlands

Victor S. GRIGORASCU
Facultatea de Electronica and Telecomunicatii
Universitatea Politehnica Bucuresti
Bucharest, Romania

Thomas KAILATH
Department of Electrical Engineering
Stanford University
Stanford, CA 94305, USA

Beatrice MEINI
Dipartimento di Matematica
Università di Pisa
Pisa, Italy

Victor Y. PAN
Dept. Math. and Computer Science
Lehman College
City University of New York
New York, NY 10468, USA

Michael K. NG
Department of Mathematics
The University of Hong Kong
Pokfulam Road, Hong Kong

Phillip A. REGALIA
Signal and Image Processing Dept.
Inst. National des Télécommunications
F-91011 Evry cedex, France

Rhys E. ROSHOLT
Dept. Math. and Computer Science
City University of New York
Lehman College
New York, NY 10468, USA

Ali H. SAYED
Electrical Engineering Department
University of California
Los Angeles, CA 90024, USA

Paolo TILLI
Scuola Normale Superiore
Piazza Cavalieri 7
56100 Pisa, Italy

Ai-Long ZHENG
Deptartment of Mathematics
City University of New York
New York, NY 10468, USA

CONTENTS

PREFACE

The design of fast and numerically reliable algorithms for large-scale matrix problems with structure has become an increasingly important activity, especially in recent years, driven by the ever-increasing complexity of applications arising in control, communications, computation, and signal processing.

The major challenge in this area is to develop algorithms that blend speed and numerical accuracy. These two requirements often have been regarded as competitive, so much so that the design of fast and numerically reliable algorithms for large-scale structured linear matrix equations has remained a significant open issue in many instances.

This problem, however, has been receiving increasing attention recently, as witnessed by a series of international meetings held in the last three years in Santa Barbara (USA, Aug. 1996), Cortona (Italy, Sept. 1996), and St. Emilion (France, Aug. 1997). These meetings provided a forum for the exchange of ideas on current developments, trends, and issues in fast and reliable computing among peer research groups. The idea of this book project grew out of these meetings, and the chapters are selections from works presented at the meetings. In the process, several difficult decisions had to be made; the editors beg the indulgence of participants whose contributions could not be included here.

Browsing through the chapters, the reader soon will realize that this project is unlike most edited volumes. The book is not merely a collection of submitted articles; considerable effort went into blending the several chapters into a reasonably consistent presentation. We asked each author to provide a contribution with a significant tutorial value. In this way, the chapters not only provide the reader with an opportunity to review some of the most recent advances in a particular area of research, but they do so with enough background material to put the work into proper context. Next, we carefully revised and revised again each submission to try to improve both clarity and uniformity of presentation. This was a substantial undertaking since we often needed to change symbols across chapters, to add cross-references to other chapters and sections, to reorganize sections, to reduce redundancy, and to try to state theorems, lemmas, and algorithms uniformly across the chapters. We did our best to ensure a uniformity of presentation and notation but, of course, errors and omissions may still exist and we apologize in advance for any of these. We also take this opportunity to thank the authors for their patience and for their collaboration during this time-consuming process. In all we believe the book includes a valuable collection of chapters that cover in some detail different aspects of the most recent trends in the theory of fast algorithms, with emphasis on implementation and application issues.

The book may be divided into four distinct parts:

1. The first four chapters deal with fast *direct* methods for the triangular factorization

of structured matrices, as well as the solution of structured linear systems of equations. The emphasis here is mostly on the generalized Schur algorithm, its numerical properties, and modifications to ensure numerical stability.

2. Chapters 5, 6, and 7 deal with fast *iterative* methods for the solution of structured linear systems of equations. The emphasis here is on the preconditioned conjugate gradient method and on Newton's method.

3. Chapters 8 to 10 deal with extensions of the notion of structure to the block case, the tensor case, and to the input-output framework. Chapter 8 presents fast algorithms for block Toeplitz systems of equations and considers applications in Markov chains and queueing theory. Chapter 9 studies tensor displacement structure and applications in polyspectral interpolation. Chapter 10 discusses realization theory and computational models for structured problems.

4. We have included two appendices that collect several useful matrix results that are used in several places in the book.

Acknowledgments. We gratefully acknowledge the support of the Army Research Office and the National Science Foundation in funding the organization of the Santa Barbara Workshop. Other grants from these agencies, as well as from the Defense Advanced Research Projects Agency and the Air Force Office of Scientific Research, supported the efforts of the editors on this project. We are also grateful to Professors Alan Laub of University of California Davis and Shivkumar Chandrasekaran of University of California Santa Barbara for their support and joint organization with the editors of the 1996 Santa Barbara Workshop. It is also a pleasure to thank Professors M. Najim of the University of Bordeaux and P. Dewilde of Delft University, for their leading role in the St. Emilion Workshop, and Professor D. Bini of the University of Pisa and several of his Italian colleagues, for the fine 1996 Toeplitz Workshop in Cortona.

October 1998

T. Kailath
Stanford, CA

A. H. Sayed
Westwood, CA

NOTATION

$\mathbf{N}$	The set of natural numbers.
$\mathbf{Z}$	The set of integer numbers.
$\mathbb{R}$	The set of real numbers.
$\mathbb{C}$	The set of complex numbers.
$\emptyset$	The empty set.
$\mathcal{C}_{2\pi}$	The set of 2π-periodic complex-valued continuous functions defined on $[-\pi, \pi]$.
$\mathcal{C}_0(\mathbb{R})$	The set of complex-valued continuous functions with bounded support in $\mathbb{R}$.
$\mathcal{C}_b(\mathbb{R})$	The set of bounded and uniformly continuous complex-valued functions over $\mathbb{R}$.
$.^T$	Matrix transposition.
$.^*$	Complex conjugation for scalars and conjugate transposition for matrices.
$a \triangleq b$	The quantity a is defined as b.
$\text{col}\{a, b\}$	A column vector with entries a and b.
$\text{diag}\{a, b\}$	A diagonal matrix with diagonal entries a and b.
$\text{tridiag}\{a, b, c\}$	A tridiagonal Toeplitz matrix with b along its diagonal, a along its lower diagonal, and c along its upper diagonal.
$a \oplus b$	The same as $\text{diag}\{a, b\}$.
$\hat{\imath}$	$\sqrt{-1}$
$\lceil x \rceil$	The smallest integer $m \geq x$.
$\lfloor x \rfloor$	The largest integer $m \leq x$.
0	A zero scalar, vector, or matrix.
I_n	The identify matrix of size $n \times n$.
$\mathcal{L}(x)$	A lower triangular Toeplitz matrix whose first column is x.
$\diamond$	The end of a proof, an example, or a remark.

$\|\cdot\|_2$	The Euclidean norm of a vector or the maximum singular value of a matrix.
$\|\cdot\|_1$	The sum of the absolute values of the entries of a vector or the maximum absolute column sum of a matrix.
$\|\cdot\|_\infty$	The largest absolute entry of a vector or the maximum absolute row sum of a matrix.
$\|\cdot\|_F$	The Frobenius norm of a matrix.
$\|\cdot\|$	Some vector or matrix norm.
$\|A\|$	A matrix with elements $\|a_{ij}\|$.
$\lambda_i(A)$	ith eigenvalue of A.
$\sigma_i(A)$	ith singular value of A.
$\kappa(A)$	Condition number of a matrix A, given by $\|A\|_2\|A^{-1}\|_2$.
$\mathrm{cond}_k(A)$	Equal to $\|A\|_k\|A^{-1}\|_k$.
ε	Machine precision.
$O(n)$	A constant multiple of n, or of the order of n.
$O_n(\varepsilon)$	$O(\varepsilon c(n))$, where $c(n)$ is some polynomial in n.
$\hat{\cdot}$	A computed quantity in a finite precision algorithm.
$\bar{\cdot}$	An intermediate exact quantity in a finite precision algorithm.
CG	The conjugate gradient method.
LDU	The lower-diagonal-upper triangular factorization of a matrix.
PCG	The preconditioned conjugate gradient method.
QR	The QR factorization of a matrix.