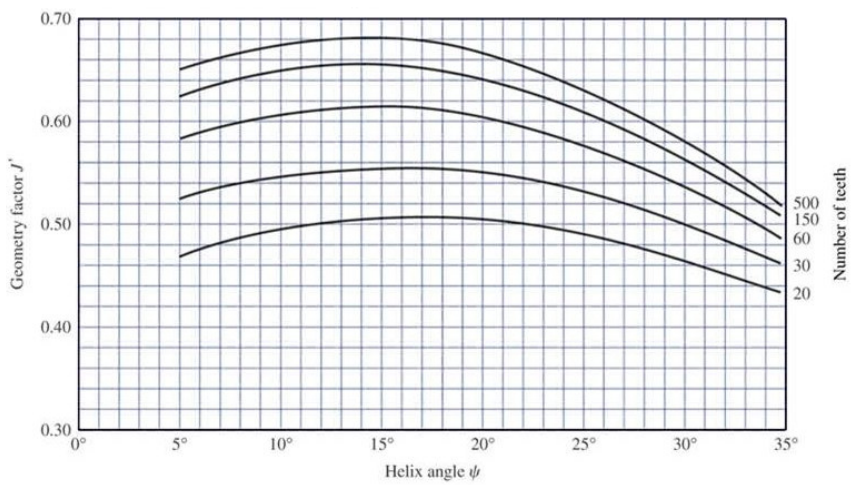
Stress analysis on helical gear tooth

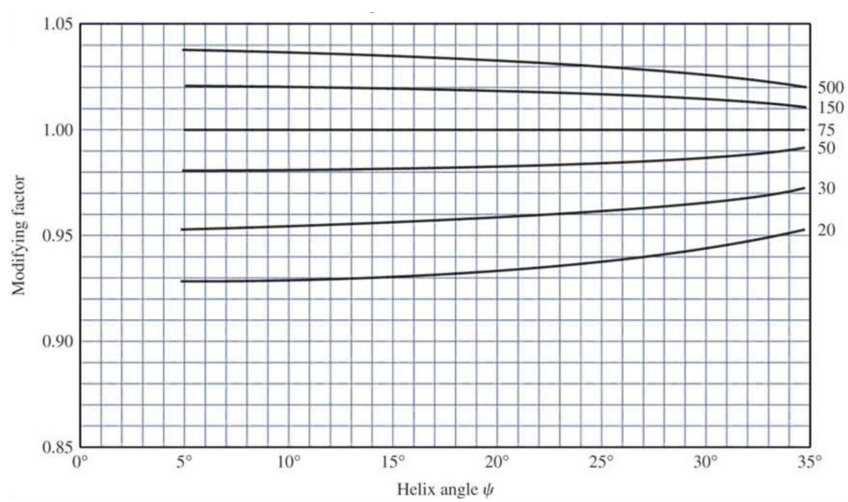
The same AGMA equations that were used for spur gears are used for helical gears to calculate the bending and contact stresses.

<u>Bending Stress:</u>

$$\sigma = \frac{W_t}{bmJ} K_v K_0 K_s K_H K_B$$

Where m is the transversal module and the AGMA form factor J is obtained using the following figures:





Where the modifying factor can be applied to the J\ factor when other than 75 teeth are used in the mating gear.

Bending Strength:

the allowable stress number (AGMA bending strength), for through Hardening steels may be determined from a relation of the form:

$$\sigma_{FP}^{\backslash} = 0.703\ HB + 113 \qquad MPa$$

where HB is the Brinell hardness number

Values of stress numbers are given in Tables and figures for different Hardening conditions. These stresses are modified by several factors that produce a limiting value of the bending stress:

$$\sigma_{FP} = \sigma_{FP}^{\backslash}\ (Y_N / Y_\theta\ Y_Z)$$

When the load cycles are reversed not repeated as in two-way bending of Gear tooth (Idler or intermediate gears), the strength is reduced to 0.7 of its corrected value.

The safety factor guarding against bending fatigue of gear tooth is given by:

$$n = \frac{\sigma_{FP}}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}}$$

Contact Stress:

$$\sigma_c = C_p \sqrt{\frac{W_t}{b d_p I} K_v K_o K_s K_H C_f}$$

$$I = \frac{\cos\varphi_t \sin\varphi_t}{2m_N} \frac{m_G}{m_G + 1} \qquad \text{for rxternal gears}$$

$$I = \frac{\cos\varphi_t \sin\varphi_t}{2m_N} \frac{m_G}{m_G - 1} \qquad \text{for internal gears}$$

where m_N is the load sharing ratio which is equal to the face width divided by the minimum total length of the lines of contact. conservative approximation of m_N is given by :

$$m_N = \frac{p_N}{0.95\,Z}$$

where Z is the length of line of action in the transverse plane.

p_N is the normal base pitch, $p_N = p_n \cos\varphi_n$

$p_n = \pi m_n$ is the normal circular pitch.

Contact Strength:

The contact fatigue strength at 107 cycles and 0.99 reliability for through Hardened steel gears may be obtained from a relation in the form:

$$\sigma^{\backslash}_{HP} = 2.22\ HB + 200 \qquad MPa$$

The corrected value of the contact strength:

$$\sigma_{HP} = \sigma^{\backslash}_{HP} \frac{Z_N C_H}{Y_\theta Y_Z}$$

Safety Factors for Surface Fatigue

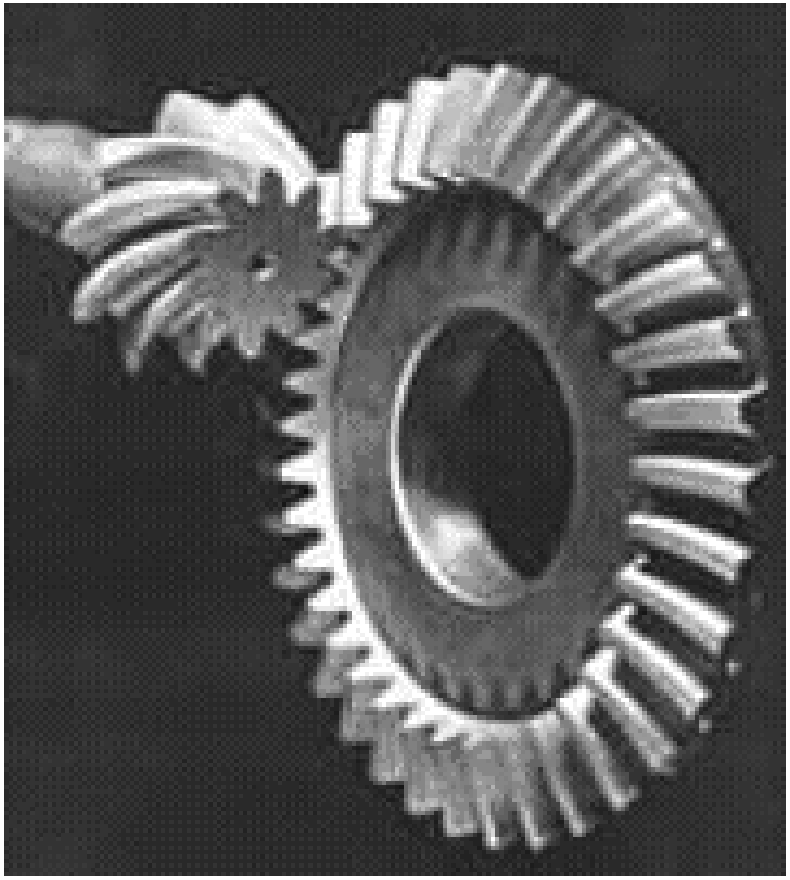
$$n^{\backslash}_c = \frac{\sigma_{HP}}{\sigma_c} = \frac{\text{the fully corrected contact strength}}{\text{contact stress}}$$

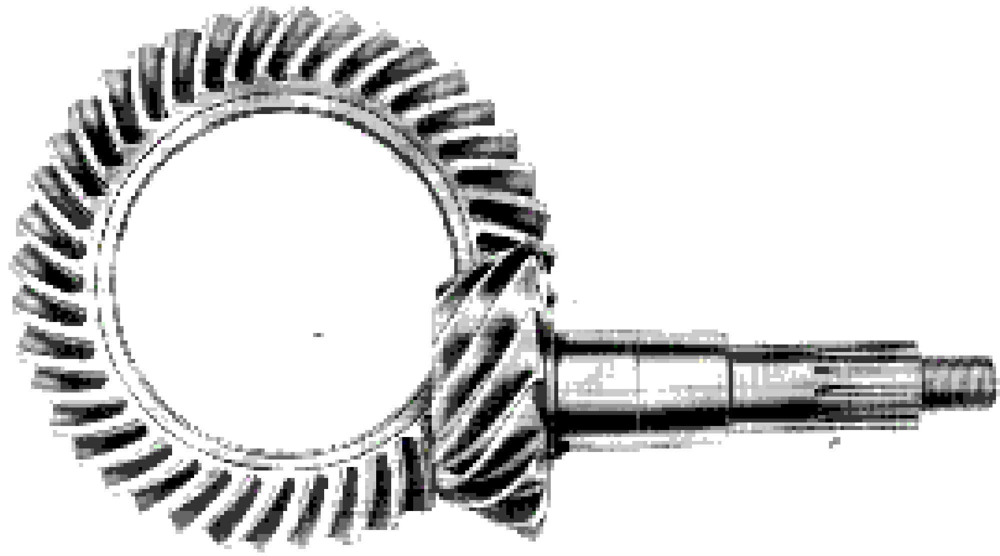
$$n_c = \left(\frac{\sigma_{HP}}{\sigma_c}\right)^2 = \left(\frac{\text{the fully corrected contact strength}}{\text{contact stress}}\right)^2$$

Bevel Gears

Bevel gears:

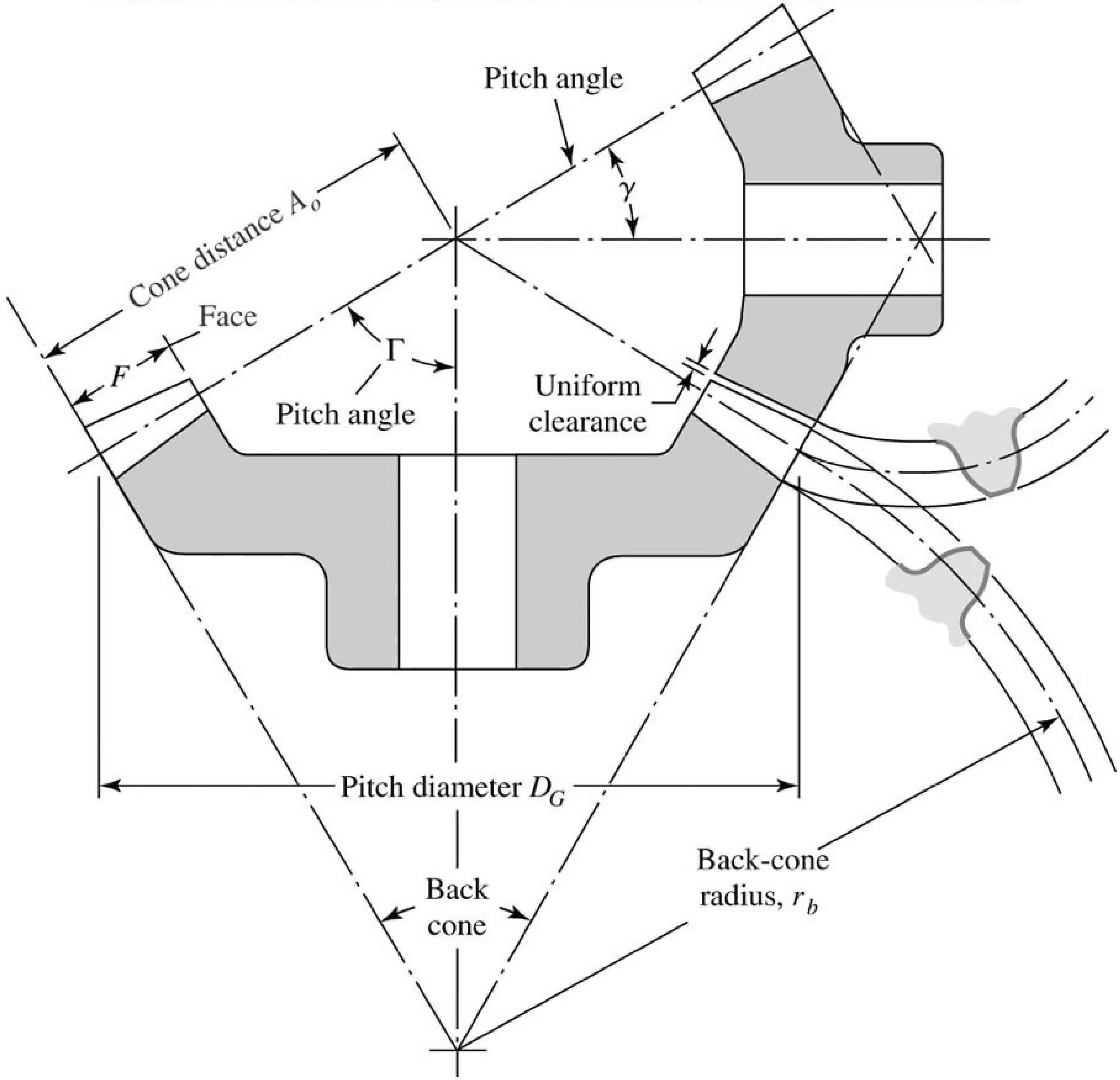
Bevel gears are usually used to transmit motion between two intersecting shafts (at any angle but usually 90^o). Bevel gear sets are shown in Fig. below.

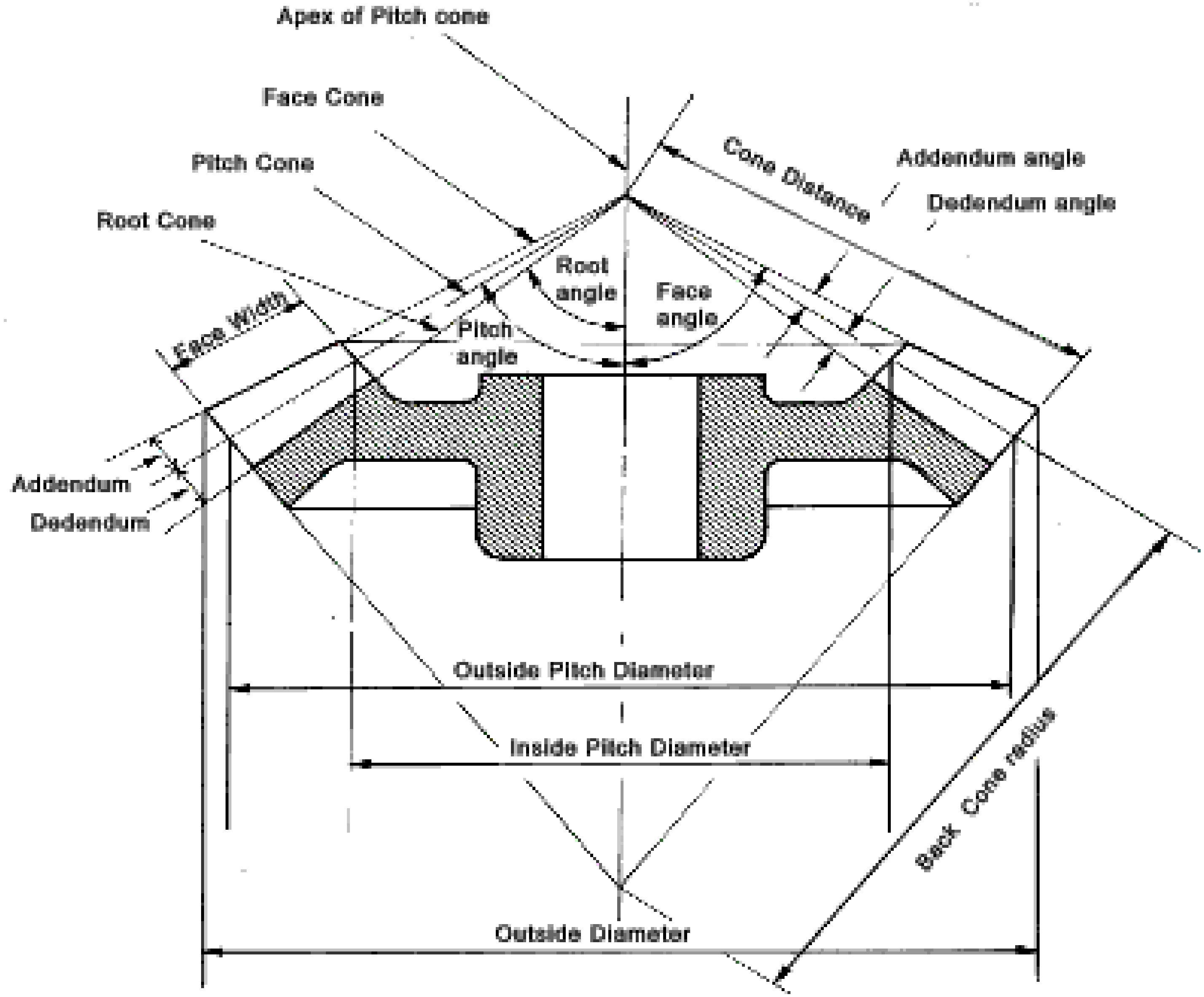




Hypoid bevel gears

Bevel gears kinematics:





Apex of Pitch cone
Face Cone
Pitch Cone
Root Cone
Cone Distance
Addendum angle
Dedendum angle
Root angle
Face angle
Pitch angle
Face Width
Addendum
Dedendum
Outside Pitch Diameter
Inside Pitch Diameter
Back Cone radius
Outside Diameter

1- The pitch circle of bevel gears is measured at the large end of the tooth (back cone). The pitch circle diameter and the circular pitch are calculated as in the spur gears:

$$d = mN$$

$$p_c = \pi m$$

2- The pitch angles γ, Γ are defined by the pitch cones meeting at the apex, as shown in Fig. They are related to the tooth numbers as follows:

$$\tan\gamma = \frac{N_p}{N_G}, \qquad \tan\Gamma = \frac{N_G}{N_p}$$

where γ, Γ are respectively the pitch angles of the pinion and gear.

3- The Face width is given as the shortest distance of:

$$F = \frac{A_o}{3} \quad \text{or} \quad F = (10-15)m$$

Force analysis of Straight bevel gears:

In determining shaft and bearing loads for bevel gear application, the usual practice is to use the tangential (transmitted) load which would occur if all the forces were concentrated at the midpoint of the tooth. While the actual resultant forces occur somewhere between the midpoint and the large end of the tooth.

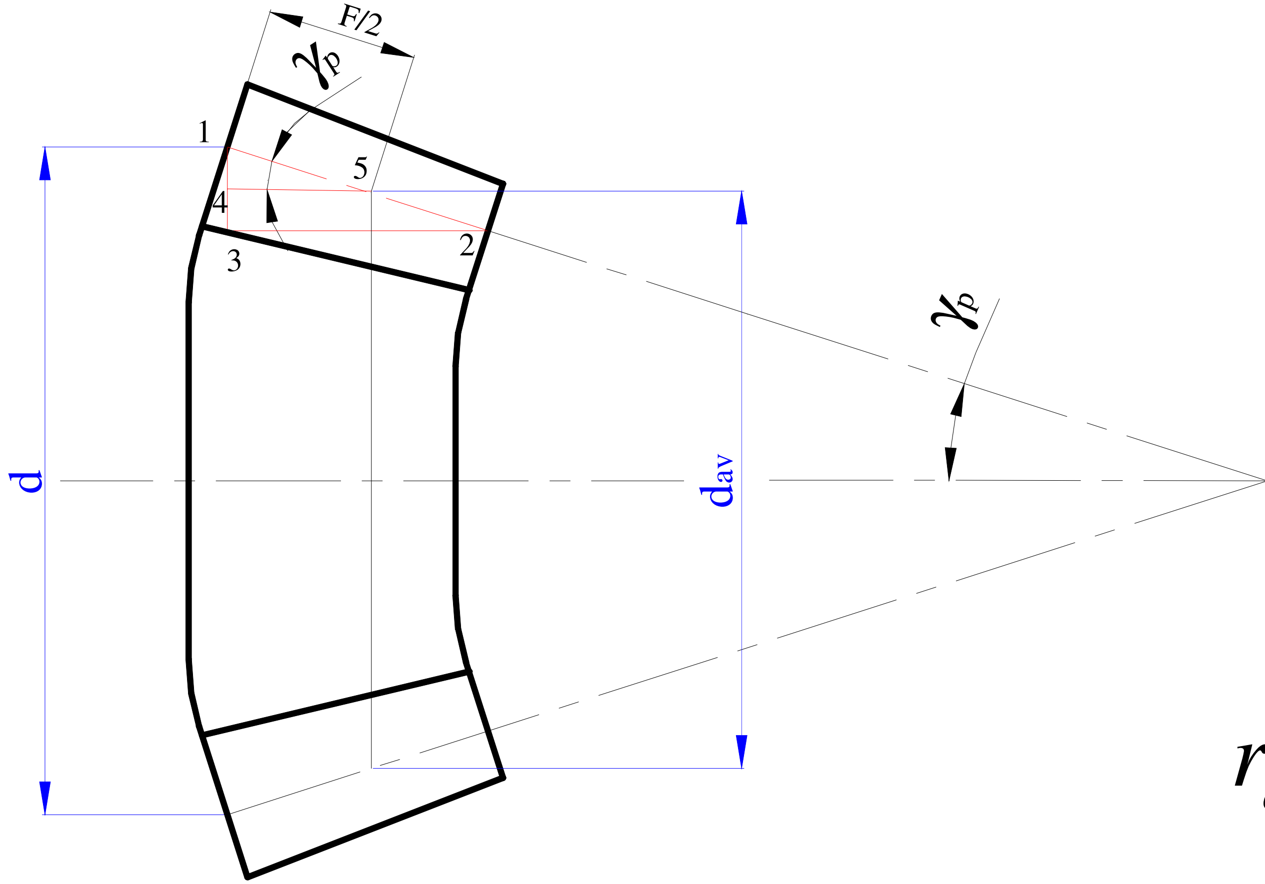
$$W_t^* = \frac{T}{r_{av}} = \frac{H}{V_{av}}$$

(at the midpoint of the tooth)

Where r_{av} is the pitch radius of the gear under consideration at the midpoint of the tooth.

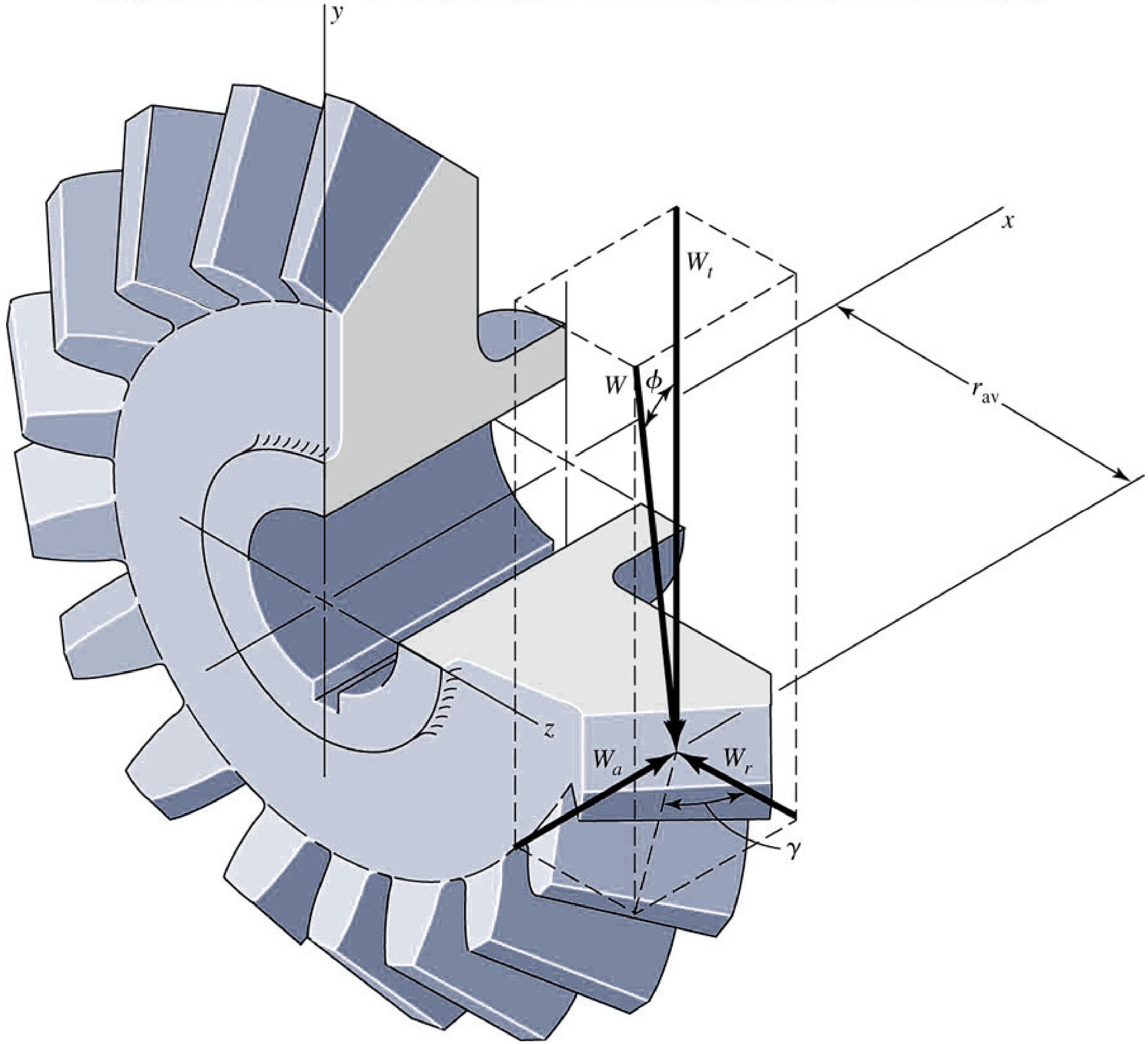
The average velocity is determined from: $$V_{av} = \omega r_{av}$$

Determination of average diameter



$$r_{av_p} = r_p - \frac{F}{2}\sin\gamma$$

$$r_{av_g} = r_g - \frac{F}{2}\sin\Gamma$$


y
x
z
W_t
W
ϕ
r_{av}
W_a
W_r
γ

a- The force W is first resolved into two components:

$$W_t^* = W \cos\varphi$$
$$W_1^* = W \sin\varphi$$

b- The force W_1^* is resolved into two components:

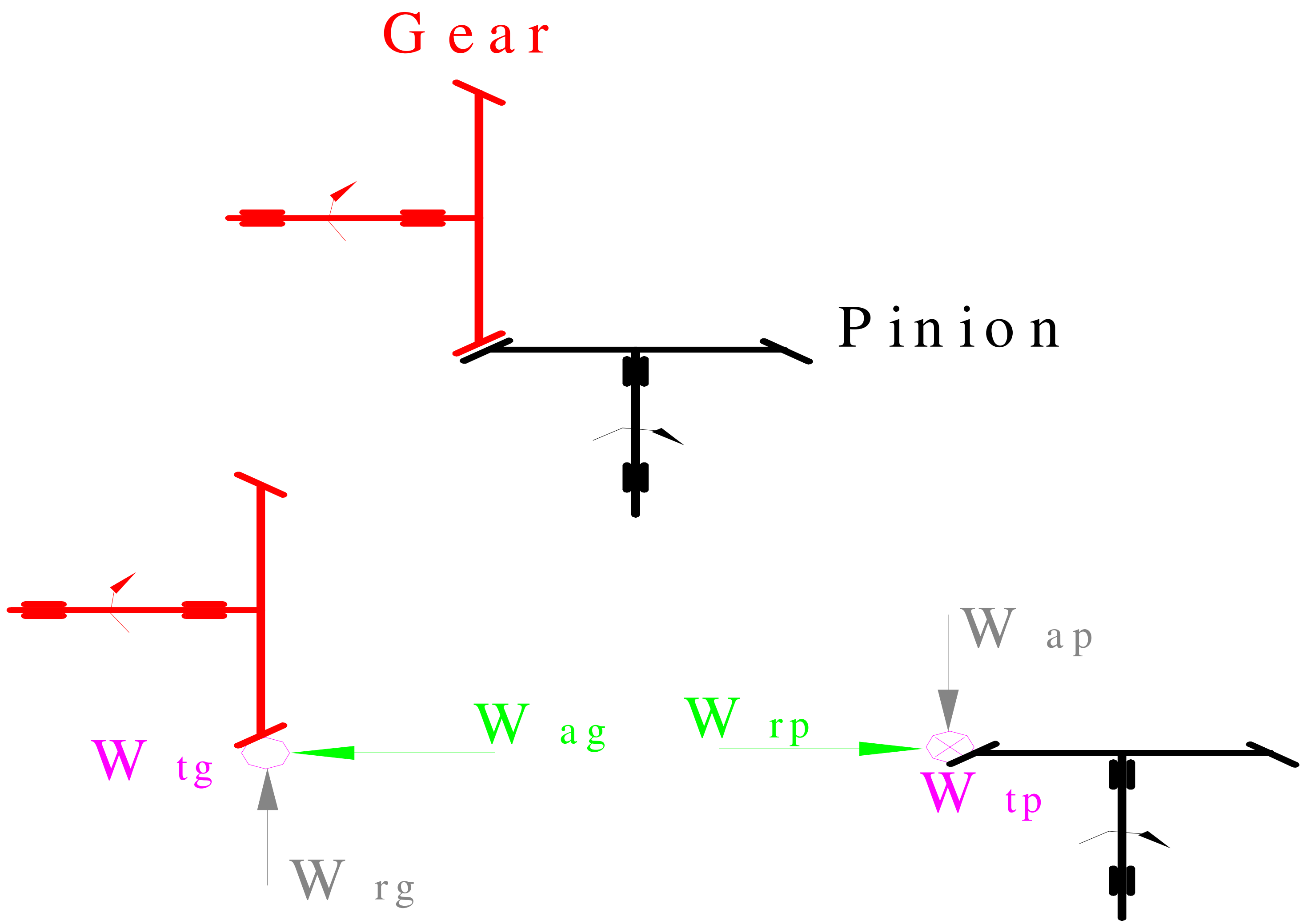
$$W_r^* = W \sin\varphi \cos\gamma$$
$$W_a^* = W \sin\varphi \sin\gamma$$

Usually W_t^* is given, and the other forces are required, therefore:

$$W = \frac{W_t^*}{\cos\varphi} \qquad W_r^* = W_t^* \tan\varphi \cos\gamma \qquad W_a^* = W_t^* \tan\varphi \sin\gamma \tag{14-36}$$

Where: φ :is the pressure angle

<u>Note:</u> W_t^*, W_r^*, W_a^* are used for shaft and bearing design (reactions and bending moment diagrams)


Gear
Pinion
W tg
W ag
W rg
W rp
W ap
W tp

Stress analysis of Straight bevel gears:

Bending Stress:

$$\sigma = \frac{W_t}{bmJ} K_v K_0 K_s K_H$$

Bending Strength:

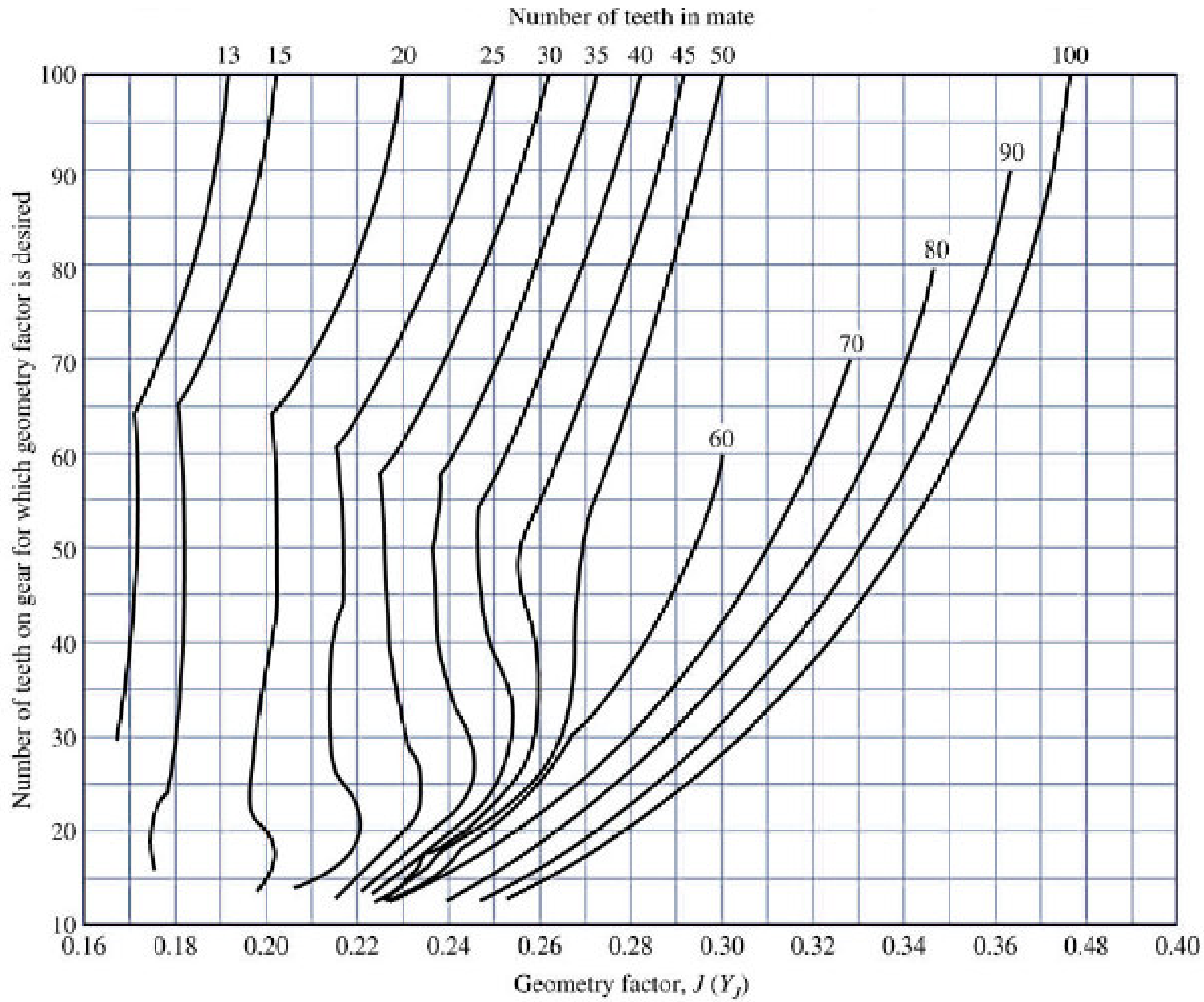
For through-hardened steel gears

$$\sigma^{\backslash}_{FP} = 0.3 H_B + 14.48 \quad \text{MPa} \qquad \text{for grade 1}$$

$$= 0.33 H_B + 41.24 \quad \text{MPa} \qquad \text{for grade 2}$$

The corrected strength :

$$\sigma_{FP} = \sigma^{\backslash}_{FP} \frac{Y_N}{Y_\theta Y_Z}$$



Number of teeth in mate
13
15
20
25
30
35
40
45
50
60
70
80
90
100
Number of teeth on gear for which geometry factor is desired
10
20
30
40
50
60
70
80
90
100
0.16
0.18
0.20
0.22
0.24
0.26
0.28
0.30
0.32
0.34
0.36
0.48
0.40
Geometry factor, J (Y_J)

Contact Stress:

$$\sigma_c = C_p \sqrt{\frac{W_t}{b d_p I} K_v K_o K_s K_H C_{xc}}$$

Where C_{xc} is the crowning factor for pitting

Contact Strength:

for through hardened steel gears:

$$\sigma^{\backslash}_{HP} = 2.35 H_B + 162.89 \text{ MPa} \qquad \text{for grade 1}$$

$$= 2.51 H_B + 203.86 \text{ MPA} \qquad \text{for grade 2}$$

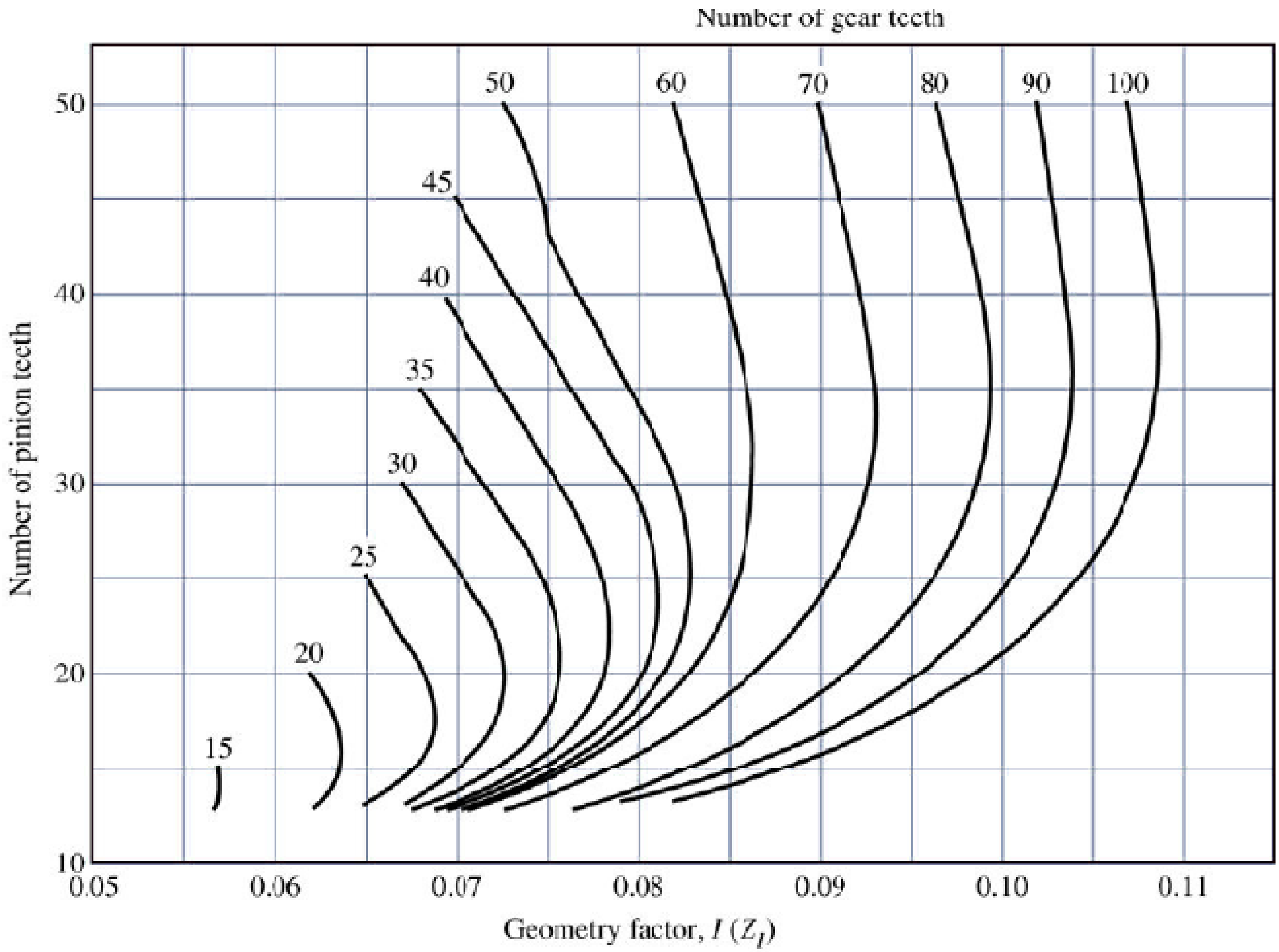
The corrected contact strength

$$\sigma_{\mathrm{HP}} = \sigma^{\backslash}_{HP} \frac{Z_N C_H}{Y_\theta Y_Z}$$

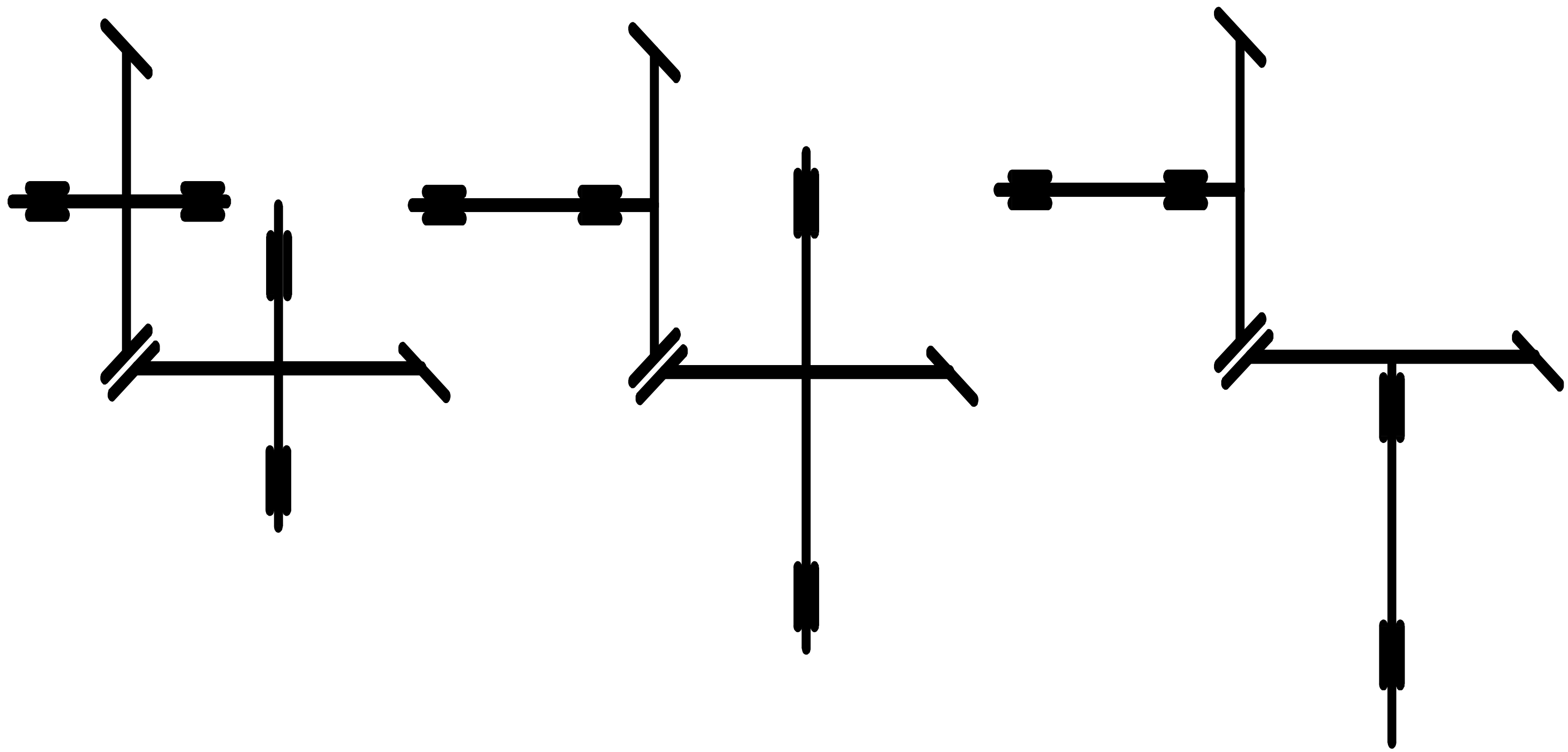
Notes:

1- All factors are given in graphical form or as equations in the text book

2- For all the above relations the module and diameter of the gear at the back cone is considered.



Number of gear teeth
50
60
70
80
90
100
45
40
35
30
25
20
15
Number of pinion teeth
10
20
30
40
50
0.05
0.06
0.07
0.08
0.09
0.10
0.11
Geometry factor, I (Z_I)



Both gears inboard One gear outboard Both gears outboard

Mounting of bevel gears

GMA load distribution factor for bevel gears.

Application	Both gears inboard	One gear outboard	Both gears outboard
General industrial	1-1.1	1.1-1.25	1.25-1.4
Automotive	1-1.1	1.1-1.25	
Aircraft	1-1.25	1.1-1.40	1.25-1.5