Numerical Analysis - ENME 602
Engineering and Materials Science
Assoc.Prof.Dr. Hesham H. Ibrahim
Spring 2021

GUC German University in Cairo الجامعة الألمانية بالقاهرة

Practice Sheet 3B

Interpolation & Polynomial Approximation

Problem 1

Use the Newton forward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

a. $f(8.4)$ if $f(8.1) = 16.94410$, $f(8.3) = 17.56492$, $f(8.6) = 18.50515$, $f(8.7) = 18.82091$

b. $f(0.9)$ if $f(0.6) = -0.17694460$, $f(0.7) = 0.01375227$, $f(0.8) = 0.22363362$, $f(1.0) = 0.65809197$

Solution:

Question	Polynomial	Approximation
1.a	$p_1(x) = 16.9441 + 3.1041(x - 8.1)$ $p_2(x) = p_1(x) + 0.06(x - 8.1)(x - 8.3)$ $p_3(x) = p_2(x) - 0.00208333(x - 8.1)(x - 8.3)(x - 8.6)$	$p_1(8.4) = 17.87533$ $p_2(8.4) = 17.87713$ $p_3(8.4) = 17.87714$
1.b	$p_1(x) = -0.1769446 + 1.9069687(x - 0.6)$ $p_2(x) = p_1(x) + 0.959224(x - 0.6)(x - 0.7)$ $p_3(x) = p_2(x) - 1.785741(x - 0.6)(x - 0.7)(x - 0.8)$	$p_1(0.9) = 0.395146$ $p_2(0.9) = 0.4526995$ $p_3(0.9) = 0.4419850$

Problem 2

Use the Newton forward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

a. $f(0.43)$ if $f(0) = 1,\ f(0.25) = 1.64872,\ f(0.5) = 2.71828,\ f(0.75) = 4.48169$

b. $f(0.18)$ if $f(0.1) = -0.29004986,\ f(0.2) = -0.56079734,\ f(0.3) = -0.81401972,\ f(0.4) = -1.0526302$

Solution:

$$s = \frac{x - x_0}{h}$$

Question	Polynomial	Approximation
1.a	$s = 1.72$ $p_1(s) = 1.0 + 0.64872s$ $p_2(s) = p_1(s) + 0.21042s(s-1)$ $p_3(s) = p_2(s) + 0.04550167s(s-1)(s-2)$	$p_1(s) = 2.115798400$ $p_2(s) = 2.376382528$ $p_3(s) = 2.360604734$
1.b	$s = 0.8$ $p_1(s) = -0.29004986 - 0.2707474800s$ $p_2(s) = p_1(s) + 0.00876255s(s-1)$ $p_3(s) = p_2(s) - 0.0004855333s(s-1)(s-2)$	$p_1(s) = -0.5066478440$ $p_2(s) = -0.5080498520$ $p_3(s) = -0.5081430744$

Problem 3

Use the Newton backward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

a. $f(-\frac{1}{3})$ if $f(-0.75) = -0.07181250,\ f(-0.5) = -0.024750,\ f(-0.25) = 0.33493750,\ f(0) = 1.1010$

b. $f(0.25)$ if $f(0.1) = -0.62049958,\ f(0.2) = -0.28398668,\ f(0.3) = 0.00660095,\ f(0.4) = 0.24842440$

Solution:

$$s = \frac{x - x_n}{h}$$

Question	Polynomial	Approximation
1.a	$s = -4/3$ $p_1(s) = 1.101 + 0.7660625s$ $p_2(s) = p_1(s) + 0.406375s(s-1)$ $p_3(s) = p_2(s) + 0.09375s(s-1)(s-2)$	$p_1(s) = 0.0795833$ $p_2(s) = 0.1698889$ $p_3(s) = 0.1745185$
1.b	$s = -1.5$ $p_1(s) = 0.2484244 + 0.2418235s$ $p_2(s) = p_1(s) - 0.04876419s(s-1)/2$ $p_3(s) = p_2(s) - 0.00283891s(s-1)(s-2)/6$	$p_1(s) = -0.1143108$ $p_2(s) = -0.1325973$ $p_3(s) = -0.1327748$

Problem 4

The following data are given for a polynomial $P(x)$ of unknown degree.

x	0	1	2	3
$P(x)$	4	9	15	18

Determine the coefficient of x^3 in $P(x)$ if all fourth-order forward differences are 1.

Solution:

The coefficient of x^3 is $-\frac{11}{12}$.

Problem 5

For a function f , the forward-divided differences are given by

$x_0 = 0.0$	$f[x_0]$		
		$f[x_0, x_1]$	
$x_1 = 0.4$	$f[x_1]$		$f[x_0, x_1, x_2] = \frac{50}{7}$
		$f[x_1, x_2] = 10$	
$x_1 = 0.7$	$f[x_2] = 6$		

Determine the missing entries in the table

Solution:

$f[x_0] = 1,\ f[x_1] = 3,\ f[x_0, x_1] = 5.$

Problem 6

a. Approximate $f(0.05)$ using the following data and the Newton forward-difference formula:

x	0.0	0.2	0.4	0.6	0.8
$P(x)$	1.00000	1.22140	1.49182	1.82212	2.22554

b. Use the Newton backward-difference formula to approximate $f(0.65)$.

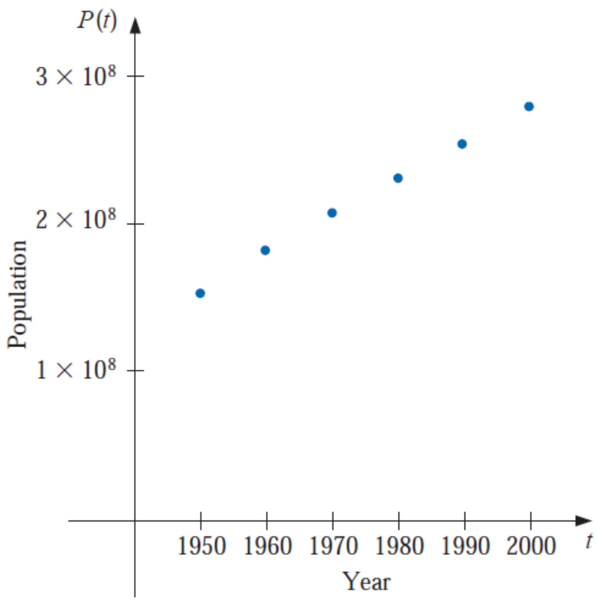
Solution:

Question	Polynomial	Approximation
a	$p_4(s) = 1 + 0.2214s + 0.02451s(s-1) + 1.81 * 10^{-3}s(s-1)(s-2) + 9.91667 * 10^{-5}s(s-1)(s-2)(s-3)$	$p_4(0.25) \approx f(0.05) = 1.051259$
b	$p_4(s) = 2.22554 + 0.40342s + 0.03656s(s+1) + 2.206667 * 10^{-3}s(s+1)(s+2) + 9.9166667 * 10^{-5}s(s+1)(s+2)(s+3)$	$p_4(-0.75) \approx f(0.65) = 1.91555$

Problem 7

A census of the population of the United States is taken every 10 years. The following table lists the population, in thousands of people, from 1950 to 2000, and the data are also represented in the figure.

Year	1950	1960	1970	1980	1990	2000
Population (in thousands)	151,326	179,323	203,302	226,542	249,633	281,422



a. Use appropriate divided differences to approximate the population in the years 1940, 1975, and 2020.

b. The population in 1940 was approximately 132,165,000. How accurate do you think your 1975 and 2020 figures are?

Solution:

a. $P_5(1940) = 102.396$, $P_5(1975) = 215.0428$, $P_5(2020) = 513.443$.

b. The 1975 may not be very accurate, but the 2020 figure is likely to be extremely inaccurate