TUTORIAL 1

COMPUTER ARITHMETIC & ERROR ANALYSIS

Numerical Analysis (ENME 602)

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Computer Arithmetic

Floating point form:

- Normalized decimal floating point form: Machine numbers are represented in the normalized decimal floating-point form

$$\pm 0.d_1 d_2 \cdots d_{k-1} d_k \times 10^n, \quad 1 \leq d_1 \leq 9, \quad 0 \leq d_i \leq 9$$

Computer Arithmetic

- **Floating point form**:

$$y = 0.d_1 d_2 \cdots d_{k-1} d_k d_{k+1} \cdots \times 10^n$$

  - k-**digit Chopping**: all digits $d_{k+1} d_{k+2}$ ... are chopped off.

$$fl(y) = 0.d_1 d_2 \cdots d_{k-1} d_k \times 10^n$$

  - k-**digit Rounding**: is given by:

$$fl(y) = \begin{cases} 0.d_1 d_2 \cdots d_{k-1} d_k \times 10^n & \text{if } 0 < d_{k+1} < 5 \\ 0.d_1 d_2 \cdots d_{k-1} (d_k + 1) \times 10^n & \text{if } 5 \leq d_{k+1} \leq 9 \end{cases}$$

Note that :
Chop or **round** at each step of a computation not only done on the final output.

Error Analysis

Types of Error:

- suppose p^* is the approximation of p
  - **Actual error**: $(p - p^*)$
  - **Absolute error**: $|p - p^*|$
  - **Relative error**: $\frac{|p - p^*|}{|p|}$, where $p \neq 0$

Problem 1

Compute the absolute error and relative error in approximations of p by p^*.

a) $p = \pi,\ p^* = 22/7$

Problem 2

Suppose p^* must approximate p with relative error at most 10^{-3}. Find the largest interval in which p^* must lie for each value of p.

a) 150

Problem 3

2. Use four-digit rounding arithmetic to perform the following calculations. Compute the absolute error and relative error with the exact value determined to at least five digits.

c) (121-0.327)-119

d) (121-119)-0.327

g) $\left(\frac{2}{9}\right) \cdot \left(\frac{9}{7}\right)$

h) $\frac{\pi-\frac{22}{7}}{\frac{1}{17}}$

Problem 3

3. Use three-digit chopping arithmetic to perform the following calculations. Compute the absolute error and relative error with the exact value determined to at least five digits.

c) (121-0.327)-119

d) (121-119)-0.327

g) $\left(\frac{2}{9}\right) \cdot \left(\frac{9}{7}\right)$

h) $\dfrac{\pi - \frac{22}{7}}{\frac{1}{17}}$

Problem 5

The number e is defined by $e = \sum_{n=0}^{\infty}(1/n!)$. Use four-digit chopping arithmetic to compute the following approximations to e, and determine the absolute and relative errors.

$$e \approx \sum_{n=0}^{5} \frac{1}{n!}$$