Question 1: (50 points)

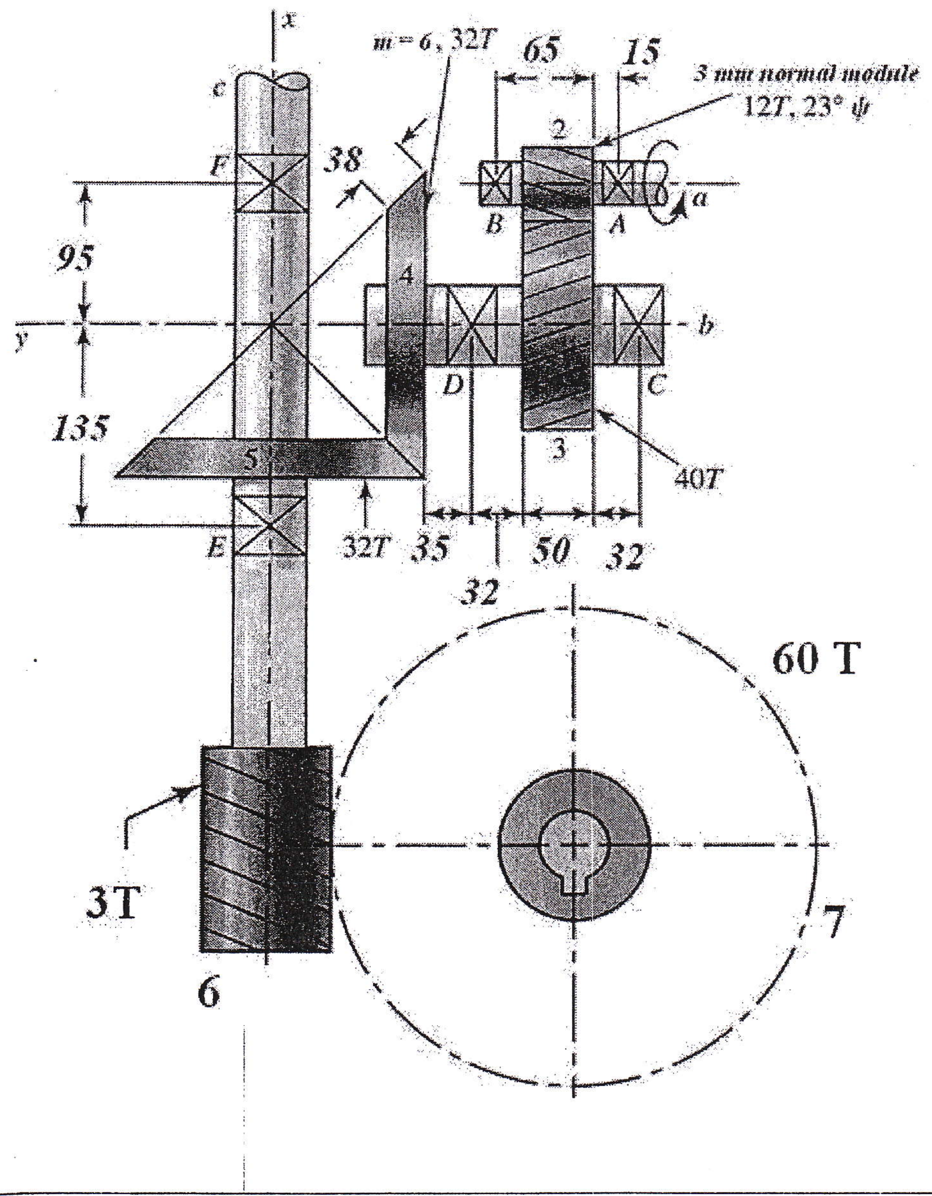
The gear train shown below consists of three stages. The first stage is a helical gear set, the second stage is a bevel gear set and the third stage is a worm and worm wheel. The pressure angle (φ and/or φ_n) for all gears is 20^o (other dimensions and data of the gears are shown on figure), **Determine:**

a) The magnitudes and directions of the speeds (rpm) of gears 3, 4, 5, 6 and 7 if the pinion 2 is rotating at 1750 rpm in the direction shown, (CCW).

b) The pitch circle diameter of the worm wheel 7 if the lead of the worm 6 is 18 mm.

c) The magnitudes and directions of the forces acting on gears 3 and 4 at the mesh points (the mid point along the face width) if the transmitted power is 5 kW.

d) The reactions at bearings C and D.



a) $\dfrac{n_2}{n_3} = \dfrac{N_3}{N_2} \Rightarrow n_3 = \dfrac{N_2}{N_3} \times n_2$

10 $\therefore n_3 = \dfrac{12}{40} \times 1750 = 525$ rpm (CW) 2

$n_4 = n_3 = 525$ rpm (CW) 2

$\dfrac{n_4}{n_5} = \dfrac{N_5}{N_4} = \dfrac{32}{32} = 1$

$n_5 = n_4 = 525$ rpm (CCW) 2

$n_6 = n_5 = 525$ rpm (CCW) 2

$\dfrac{n_6}{n_7} = \dfrac{N_7}{N_6} = \dfrac{60}{3} = 20$

$N_7 = \dfrac{n_6}{20} = \dfrac{525}{20} = 26.25$ rpm (CCW) 2

b) $L = p_x N_W$

5 $p_x = \dfrac{L}{N_W} = \dfrac{18}{3} = 6$ mm 2

$\therefore p_t = p_x = 6$ mm 1

$d_G = \dfrac{N_G \cdot p_t}{\pi} = \dfrac{60 \times 6}{\pi} = 114.59$ mm 2

c) $W_t = \dfrac{H}{v_t}$

25

$v_{t_2} = \omega_2 \cdot \dfrac{d_2}{2}$

$\omega_2 = \dfrac{2\pi n_2}{60} = \dfrac{2\pi \times 1750}{60} = 183.259 \ s^{-1}$

$d_2 = m N_2 = \dfrac{m_n}{\cos\psi} \cdot N_2 = \dfrac{3}{\cos 23^\circ} \cdot 12 = 39.109 \text{ mm}$

$\therefore v_{t_2} = 183.259 \times \dfrac{39.109}{2} = 3583.54 \text{ mm/s}$

$= 3.5835 \text{ m/s}$ (2)

$W_{t_{2,3}} = \dfrac{5\times 10^3}{3.5835} = 1395.28 \text{ N}$ (2)

$W_{r_{2,3}} = W_{t_{2,3}} \tan\phi_t$

$\tan\phi_t = \dfrac{\tan\phi_n}{\cos\psi} = \dfrac{\tan 20^\circ}{\cos 23^\circ} = 0.3954$

$\therefore W_{r_{2,3}} = 1395.28 \times 0.3954 = 551.697 \text{ N}$ (2)

$W_{a_{2,3}} = W_t \tan\psi = 1395.28 \tan 23^\circ = 592.26 \text{ N}$ (2)

bevel gears:

$d_4 = m N_4 = 6 \times 32 = 192 \text{ mm}$

$r_{av} = \dfrac{d}{2} - \dfrac{F}{2} \sin\gamma$

$\tan\gamma_g = \dfrac{N_g}{N_p} = \dfrac{32}{32} = 1 = \tan\gamma_p \ , \gamma = 45^\circ$

$r_{av.} = \dfrac{192}{2} - \dfrac{38}{2} \sin 45^\circ = 96 - 13.435 = 82.565 \text{ mm}$ (2)

$v_{t\,av.} = \omega_4 \cdot r_{av.} = \dfrac{2\pi \times 525}{60} \times 82.565 = 4539.25 \text{ mm/s}$

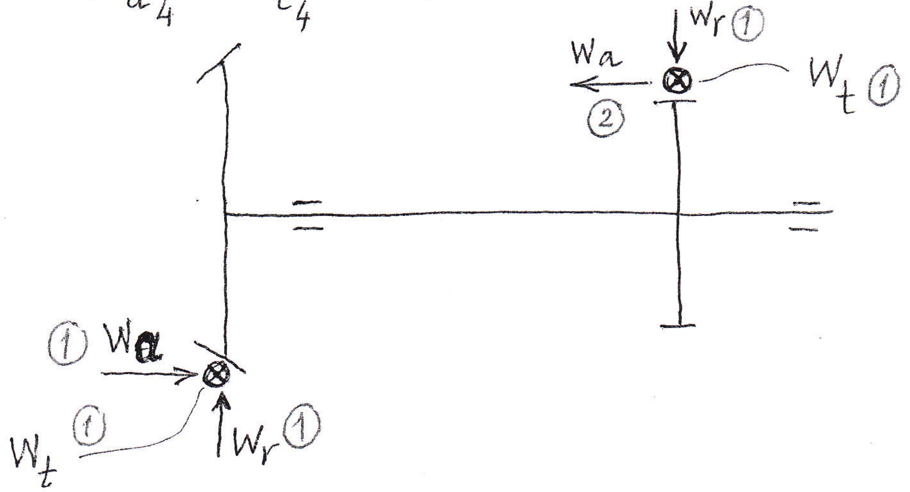
$= 4.5392 \text{ m/s}$ (2)

$W^*_{t_{4,5}} = \dfrac{H}{v_{t\,av.}} = \dfrac{5\times 10^3}{4.5392} = 1101.52 \text{ N}$ (2)

$$W_{r_4}^{*} = W_{t_4}^{*} \tan\phi \cos\gamma$$

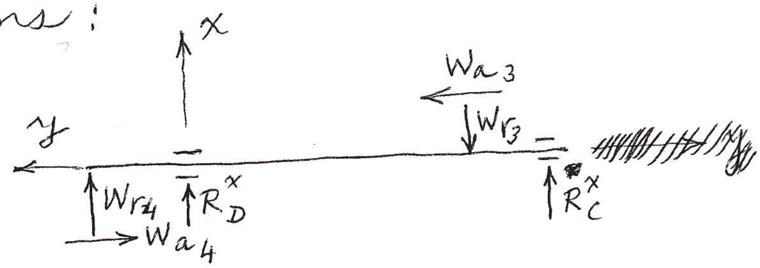
$$= 1101.52 \times \tan 20^\circ \cos 45^\circ = 283.493\ \text{N} \quad 2$$

$$W_{a_4}^{*} = W_{t_4}^{*} \tan\phi \sin\gamma = 283.493\ \text{N} \quad 2$$



d) Bearings Reactions:

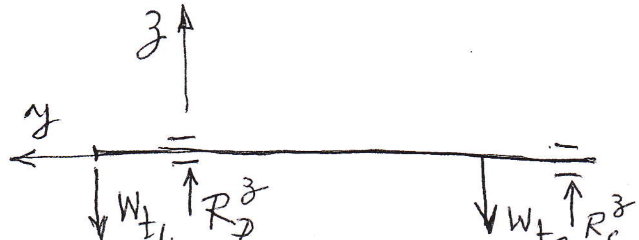
10 i) x-y plane



$R_C^x = -147.66$ N

$R_D^x = 415.87$ N

ii) y-z plane



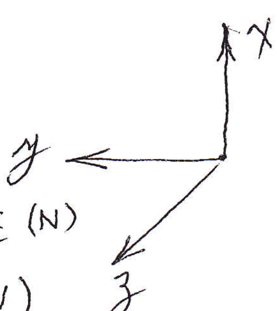
$R_C^z = 229.64$ N

$R_D^z = 2267.16$ N

iii) $R_C^y = W_{a_3} - W_{a_4}$

$= 592.26 - 283.493$

$= 308.77$ N



$R_C = -147.66\,\underset{\sim}{i} + 308.77\,\underset{\sim}{j} + 229.64\,\underset{\sim}{k}$ (N) 2 2 2

$R_D = 415.87\,\underset{\sim}{i} + 2267.16\,\underset{\sim}{k}$ (N) 2 2

Question 2: (50 points)
Determine, for a reliability of 90%, the safety factors guarding against bending and surface fatigue of the two helical gears described in question (1) if the pinion and gear are made of grade 1 steel through hardened to a hardness of 240 HB for the pinion and 200 HB for the gear. The gears are made according to a quality number $Q_v = 6$. The face width of the teeth is 45 mm and the gears are designed to work for infinite life ($Y_N = Z_N = 0.96$), accurate mounting ($K_H = 1.3$) and uniform driving and driven conditions ($K_o = 1$).
Note: See the attached data sheets, assume any missing factor or coefficient to be unity, take $C_p = 191\ (MPa)^{0.5}$

Bending stresses:

$$\left.\begin{aligned} W_t &= 1395.28 \text{ N} \\ v_t &= 3.5835 \text{ m/s} \end{aligned}\right\} *$$

$$\sigma_b = \frac{W_t}{b\,m\,J} \cdot K_v \cdot K_o \cdot K_s \cdot K_H \cdot K_B$$

$$K_v = \left(\frac{A + \sqrt{200 v}}{A}\right)^B$$

$$B = 0.25(12-6)^{2/3} = 0.825$$

$$A = 50 + 56(1-B) = 59.8$$

$$K_v = \left(\frac{59.8 + \sqrt{200 \times 3.5835}}{59.8}\right)^{0.825} = 1.3569 \quad (4)$$

$$J_p = J_p' \cdot C_p = 0.47 \times 0.97 = 0.46 \quad (2)$$

$$J_g = 0.56 \times 0.92 = 0.5152 \quad (2)$$

$$\sigma_{bp} = \frac{1395.28}{45 \times 3.259 \times 0.46} \times 1.3569 \times 1 \times 1 \times 1.3 \times 1$$

$$= 36.48 \text{ MPa.} \quad (4)$$

$$\sigma_{bg} = 32.575 \text{ MPa.} \quad (4)$$

Bending strength:

$$\sigma'_{FP} = 0.703\ HB + 113$$

$$\sigma'_{FP_p} = 0.703 \times 240 + 113$$

$$= 281.72\ \text{MPa}$$

$$\sigma_{FP_p} = \sigma'_{FP_p} \cdot \frac{Y_N}{Y_\theta Y_Z} = 281.72\ \frac{0.96}{1 \times 0.85} = 318.18\ \text{MPa}\quad 3$$

$$\sigma'_{FP_g} = 0.703 \times 200 + 113 = 253.6\ \text{MPa.}\quad 3$$

$$\sigma_{FP_g} = 253.6\ \frac{0.96}{1 \times 0.85} = 286.42\ \text{MPa.}$$

Bending Fatigue Safety factors:

$$n_p = \frac{\sigma_{FP_p}}{\sigma_{b_p}} = \frac{318.18}{36.48} = 8.72\quad 2$$

$$n_g = \frac{286.42}{32.575} = 8.79\quad 2$$

Contact Stresses :-

$$\sigma_c = C_p \sqrt{\frac{W_t}{b \cdot d_p \cdot I} \cdot K_v K_o K_s K_H C_f}$$

$$d_p = \frac{m_n}{\cos\psi} \cdot N_p = \frac{3}{\cos 23^\circ} \cdot 12 = 39.109\ \text{mm},\ d_g = 130.36\ \text{mm}$$

$$I = \frac{\cos\phi_t \sin\phi_t}{2 m_N} \cdot \frac{m_G}{m_G + 1}$$

$$\tan\phi_t = \frac{\tan\phi_n}{\cos\psi} = 0.3954 \longrightarrow \phi_t = 21.574^\circ$$

$$m_G = i = \frac{N_g}{N_p} = \frac{40}{12} = 3.3333 = \frac{10}{3}$$

$$m_N = \frac{p_N}{0.95\ Z}$$

$$p_N = p_n \cos\phi_n = 3 \times \pi \times \cos 20^\circ = 8.8564$$

$$Z = \sqrt{\left(\left(\frac{39.109}{2}\right)+3\right)^2 - \left(\frac{39.109}{2}\cos 21.574\right)^2}$$

$$+ \sqrt{\left(\left(\frac{130.36}{2}\right)+3\right)^2 - \left(\frac{130.36}{2}\cos 21.574\right)^2}$$

$$- \left(\frac{39.109+130.36}{2}\right)\sin 21.574$$

$$= 13.3426 + 31.2168 - 31.1571 = 13.402 \text{ mm} \quad 3$$

$$\therefore m_N = \frac{8.8564}{0.95 \times 13.402} = 0.6956 \quad 2$$

$$I = \frac{\cos(21.574)\sin(21.574)}{2 \times 0.6956} \times \frac{\frac{10}{3}}{\frac{10}{3}+1} = 0.18907 \quad 4$$

$$\therefore \sigma_c = 191\sqrt{\frac{1395.28}{45 \times 39.109 \times 0.18907} \times 1.3569 \times 1 \times 1 \times 1.3}$$

$$= 519.46 \text{ MPa}. \quad 5$$

strength :-

$$\sigma'_{HP_p} = 2.22\,(240) + 200 = 732.8 \text{ MPa}.$$

$$\sigma_{HP_p} = \sigma'_{HP_p} \cdot \frac{Z_n C_H}{Y_\theta Y_Z} = 732.8\,\frac{0.96 \times 1}{1 \times 0.85} = 827.63 \text{ MPa}. \quad 3$$

$$\sigma'_{HP_g} = 2.22(200) + 200 = 644 \text{ MPa}$$

$$\sigma_{HP_g} = 644\,\frac{0.96 \times 1}{1 \times 0.85} = 727.34 \text{ MPa} \quad 3$$

$$n_{c_p} = \left(\frac{\sigma_{HP_p}}{\sigma_c}\right)^2 = \left(\frac{827.63}{519.46}\right)^2 = (1.5932)^2 = 2.538 \quad 2$$

$$n_{c_g} = \left(\frac{727.34}{519.46}\right)^2 = (1.4)^2 = 1.96 \quad 2$$