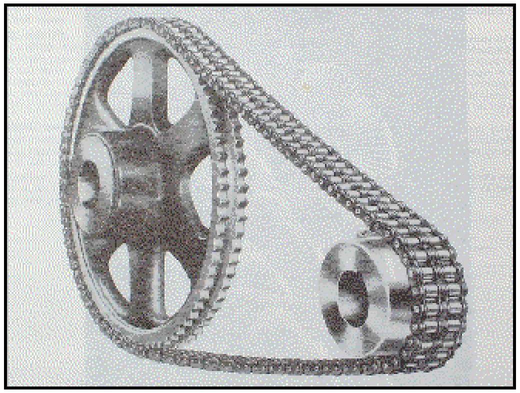
Flexible Mechanical Elements
Belt Drives

Introduction:

- When these elements are employed, they usually replace a group of gears, shafts, and bearings.
- They, thus greatly simplify a machine and consequently are a major cost-reducing elements.
- Flexible mechanical elements such as belts, ropes or chains are used for the transmission of power over comparatively long distances.



Belt drives:

- A belt drive is a method of transferring rotary motion between two parallel shafts. A belt drive includes one pulley on each shaft and one or more continuous belts over the two parallel pulleys. The motion of the driving pulley is transferred to the driven pulley via the friction between the belt and the pulley.

- Belts have the following characteristics:

1- Easy, flexible equipment design, as tolerances are not important.

2- Isolation from shock and vibration between driver and driven system.

3- Driven shaft speed conveniently changed by changing pulley sizes.

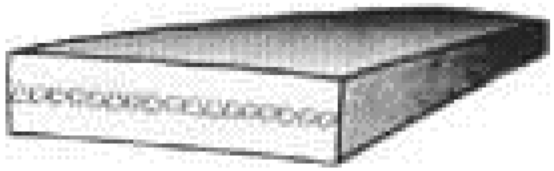
4- Belt drives require no lubrication.

5- Maintenance is relatively convenient.

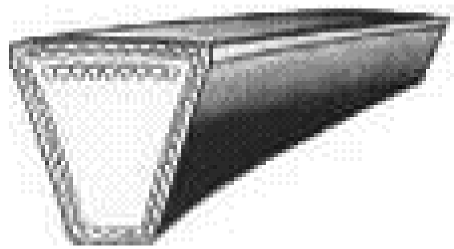
6- Very quiet compared to chain drives, and direct spur gear drives.

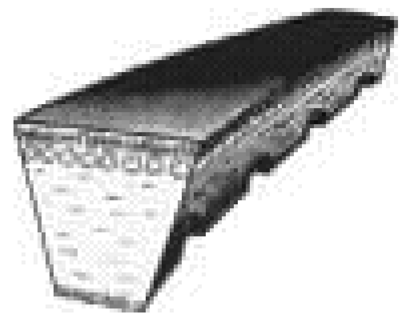
- Kind of Belts:

1- Flat belt drive:

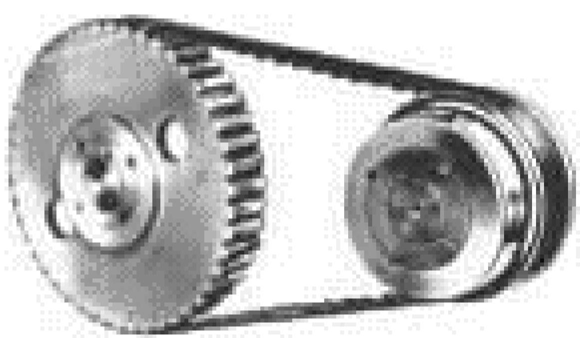


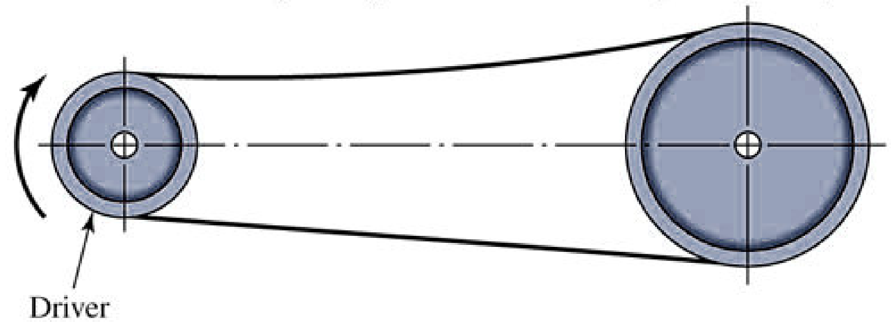
2- V- belt drive:



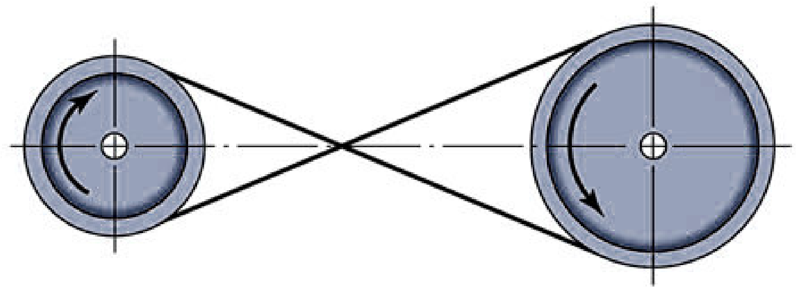


3- Timing belt drive:





(*a*)



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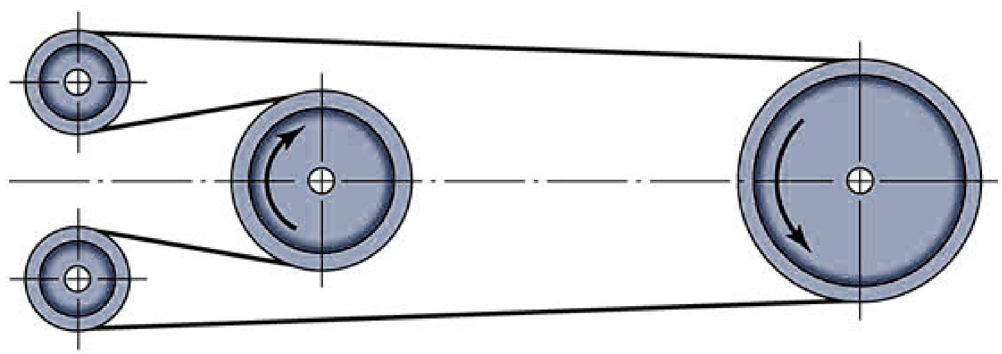

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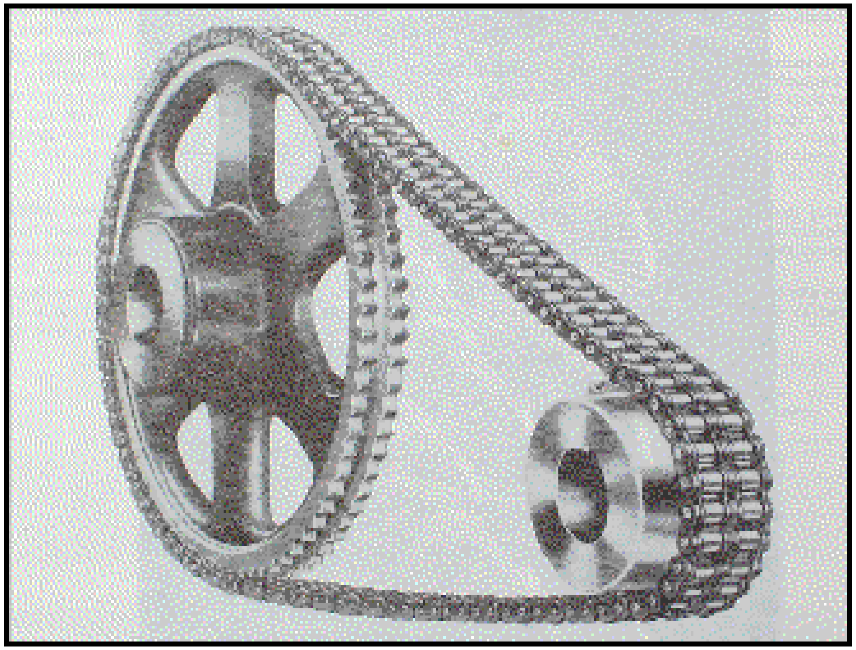
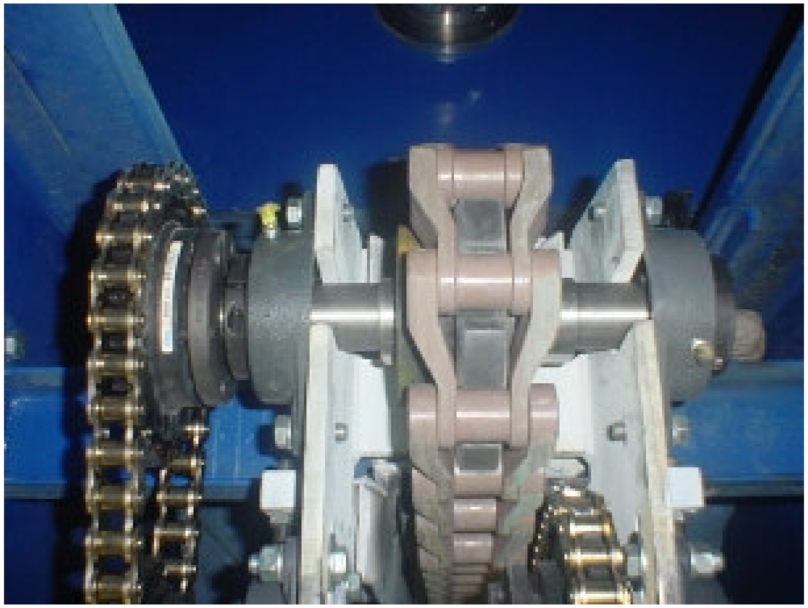
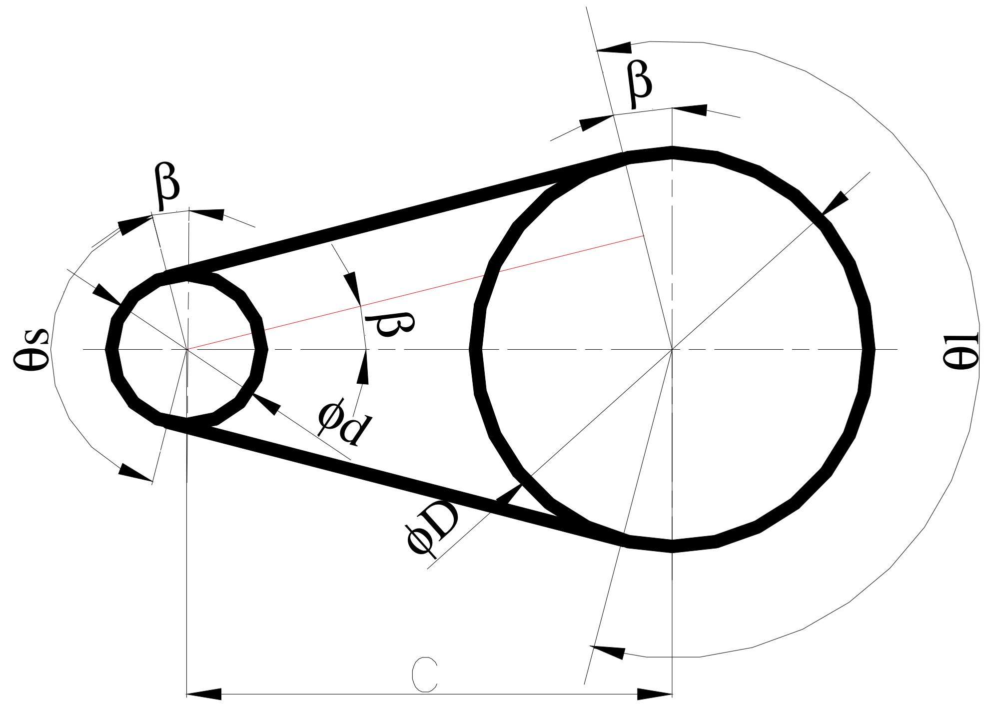
<u>- Chain drive:</u>





Flat Belt drives:

Geometrical relations:



- θ : wrap angle (contact angle), s for small & l for large pulleys.
- d : diameter of small pulley.
- D : diameter of large pulley.
- C : center distance.

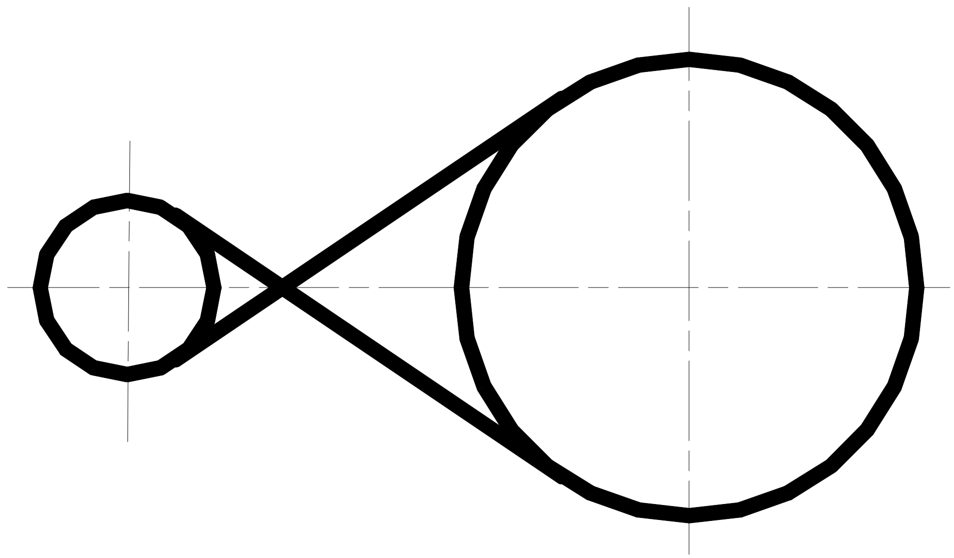
- From geometry, the length of the belt:.

$$L = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}$$

- 0r from Shigley:

$$L = \sqrt{4C^2 - (D-d)^2} + \frac{1}{2}(D\theta_l + d\theta_s) \quad (\ \theta \text{ in radians})$$

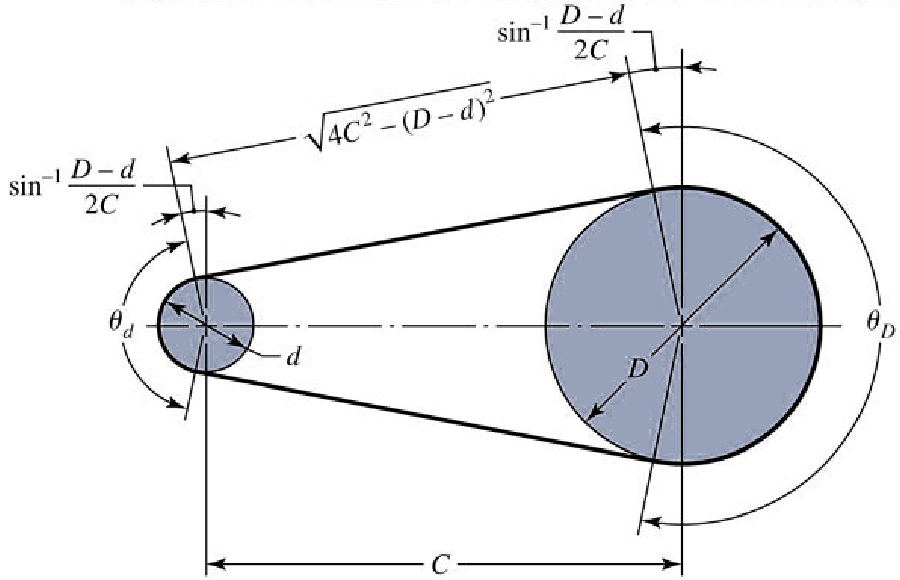
- For crossed belt:.



$$L_c = 2C + \frac{\pi(D+d)}{2} + \frac{(D+d)^2}{4C}$$

- 0r from Shigley:

$$L_c = \sqrt{4C^2 - (D+d)^2} + \frac{\theta}{2}(D+d) \qquad (\ \theta \text{ in radians})$$

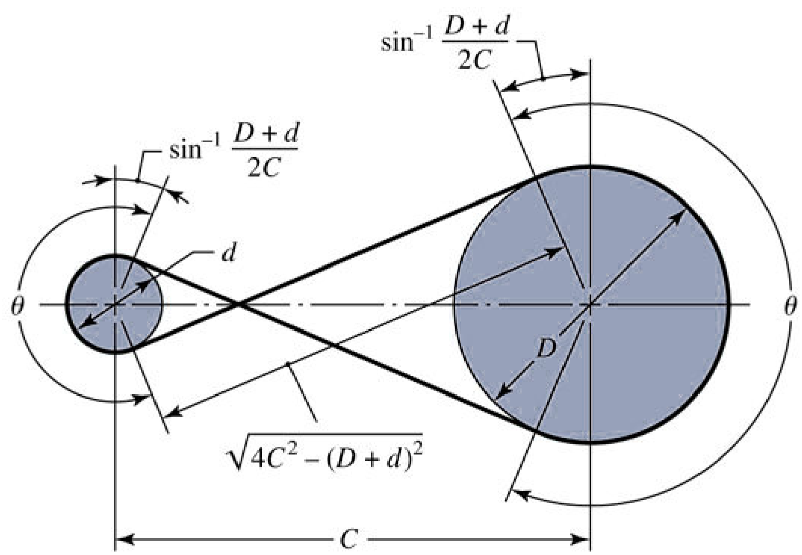


$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C}$$

$$L = \sqrt{4C^2-(D-d)^2} + \tfrac{1}{2}(D\theta_D + d\theta_d)$$

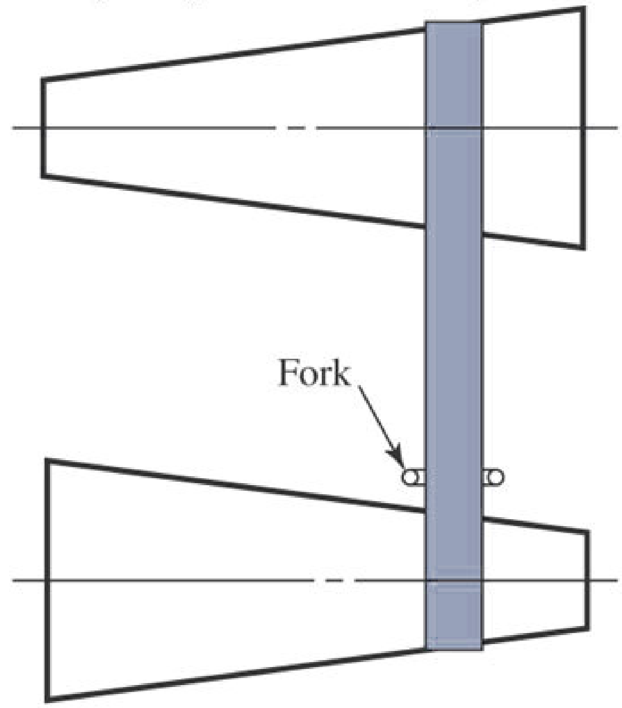
(*a*)



$$\theta = \pi + 2\sin^{-1}\frac{D+d}{2C}$$

$$L = \sqrt{4C^2-(D+d)^2} + \tfrac{1}{2}(D+d)\theta$$

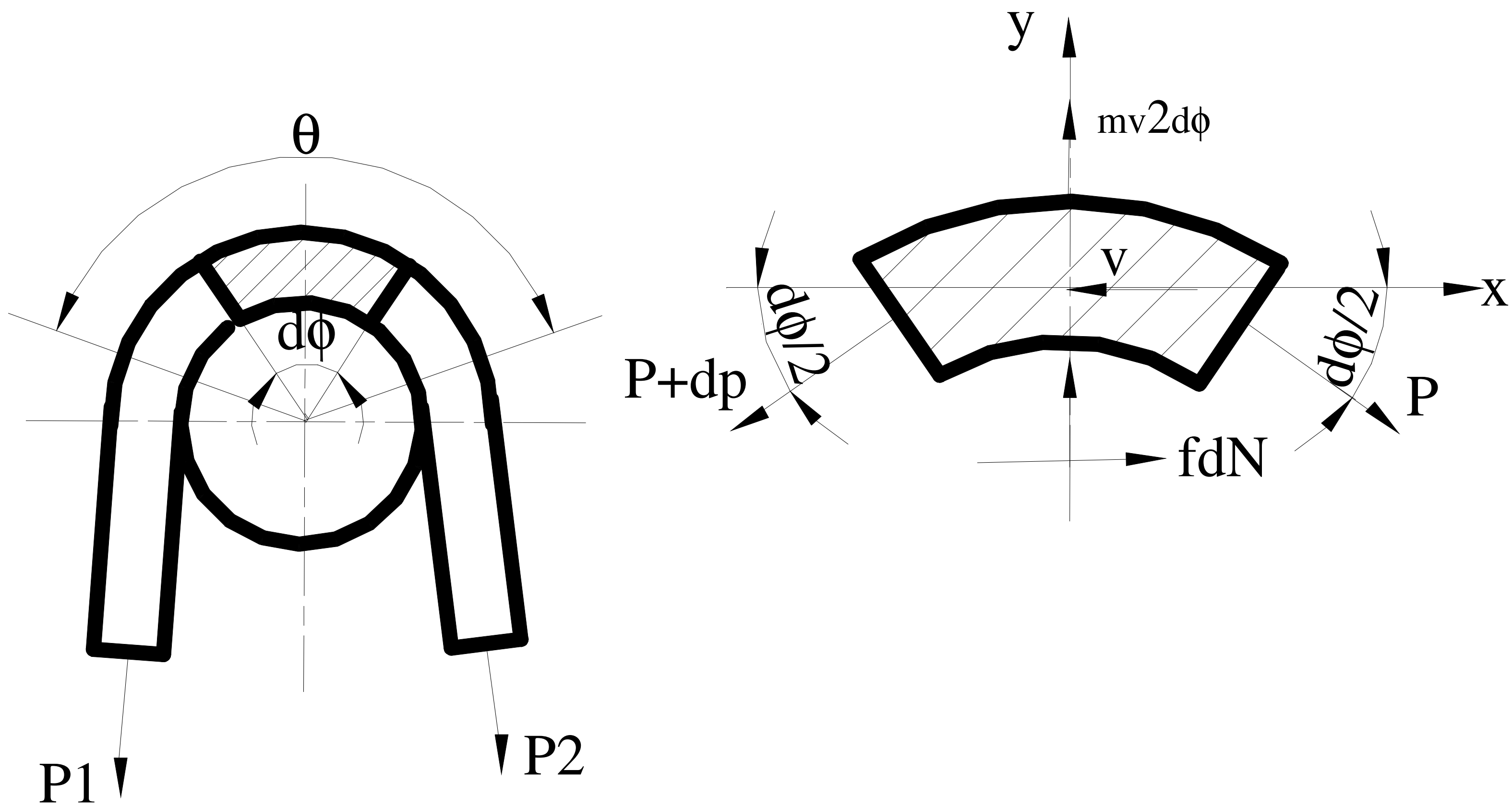
(*b*)



(a)

(b)

Belt tensions:



- P_1 : Tension of tight side.
- P_2 : Tension of loose side.
- m : Mass of unit length (of one meter) of the belt (kg).
- f : Coefficient of friction between belt & pulley..

- An element of the belt is subjected to:

  i) Tension P & $P+dP$

  ii) Normal & frictional forces dN & fdN

  iii) Centrifugal forces = $mrd\varphi \times \dfrac{v^2}{r} = mv^2 d\varphi$

- An equilibrium in x & y directions leads to:

$$(P+dP)\cos\frac{d\varphi}{2} - P\cos\frac{d\varphi}{2} - fdN = 0 \qquad (1)$$

$$(P+dP)\sin\frac{d\varphi}{2} + P\sin\frac{d\varphi}{2} - mv^2 d\varphi - dN = 0 \qquad (2)$$

- For small value of $d\varphi$:

$$\cos\left(\frac{d\varphi}{2}\right) = 1, \quad \sin\left(\frac{d\varphi}{2}\right) = \frac{d\varphi}{2}$$

- Substitute in equation (1):

$$dP - f\,dN = 0 \Rightarrow dN = \frac{dP}{f} \qquad (3)$$

- Substitute from equation (3) in equation (2) and neglecting second order terms:

$$\therefore \qquad \frac{dP}{\left(P - mv^2\right)} = f\,d\varphi \qquad (4)$$

- Integrating equation (4): $\int_{P_2}^{P_1} \frac{dP}{P - mv^2} = f \int_0^{\theta} d\varphi$

$$\therefore \qquad \frac{P_1 - mv^2}{P_2 - mv^2} = e^{f\theta}$$

- Neglecting the centrifugal forces:

$$\frac{P_1}{P_2} = e^{f\theta} \quad \text{Euler equation}$$

- For V-belts:

$$\frac{P_1 - mv^2}{P_2 - mv^2} = e^{\frac{f\theta}{\sin\left(\psi/2\right)}}$$

- Transmitted torque:

$$T_2 = \left(P_1 - P_2\right) \times \frac{D}{2}$$

$$T_1 = \left(P_1 - P_2\right) \times \frac{d}{2}$$

- Transmission ratio:

$$i = \frac{D}{d}$$

- Power:

$$H = \left(P_1 - P_2\right)v$$

- Condition of maximum power transmission:

- P_i : initial tightening force.

$$\therefore \quad \begin{aligned} P_1 &= P_i + \Delta P \\ P_2 &= P_i - \Delta P \end{aligned}$$

$$\therefore \quad P_i = \frac{P_1 + P_2}{2}$$