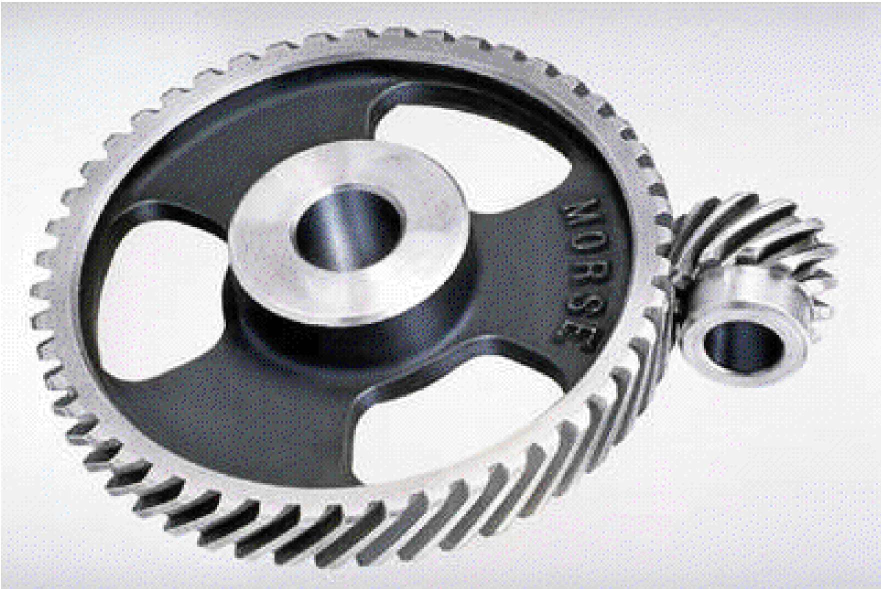
Helical Gears

Helical gears are used to transmit motion between parallel or skew shafts, as shown in Figure.



Crossed helical gear

Helical gear

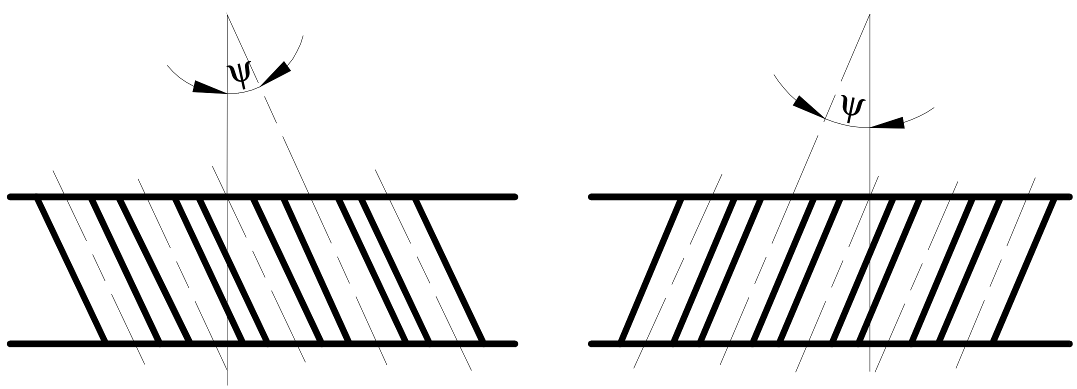
Parallel Helical gears kinematics:

1- Helical gears used to transmit motion between parallel shafts.

2- The initial contact of helical gear teeth is a point which changes into a line as the teeth come into more engagement. The line is diagonal across the face of the tooth. This gradual engagement of the teeth gives helical gears the ability to transmit heavy loads at high speeds.

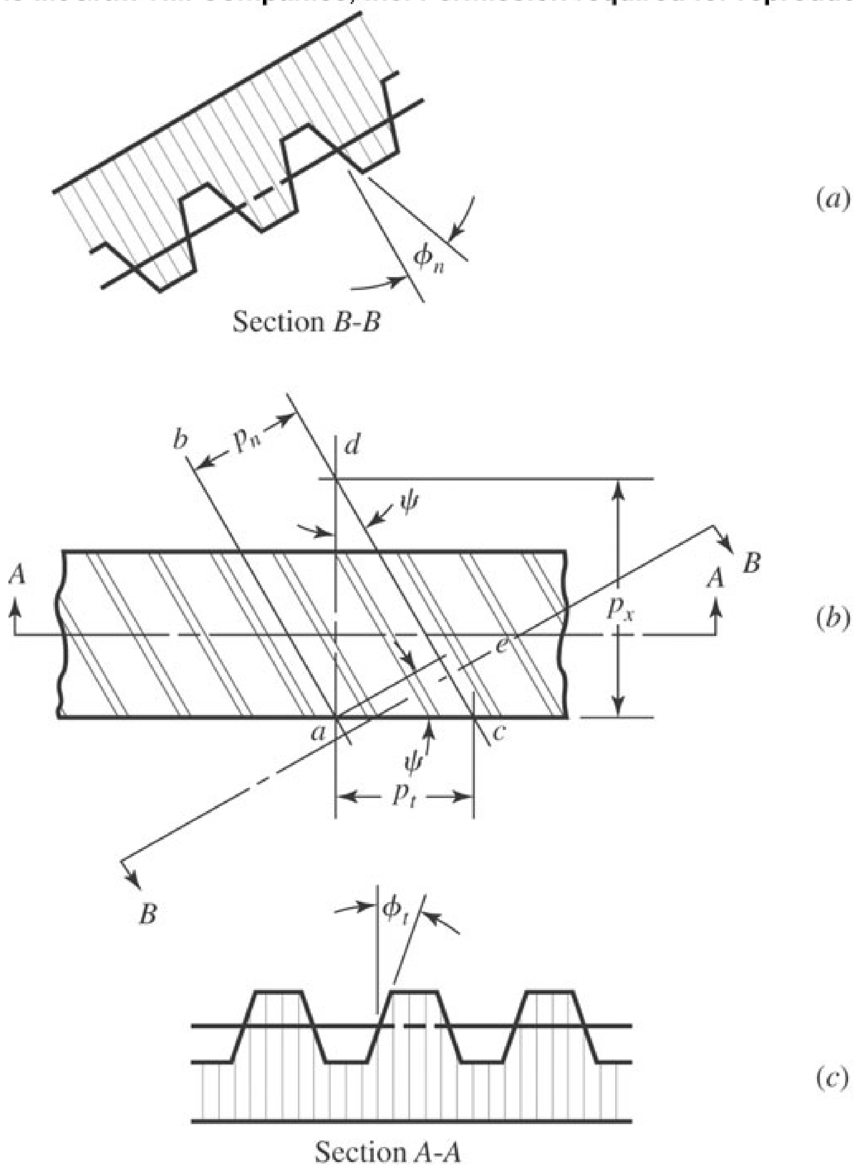
3- Helical gears subject the shaft bearings to both radial and thrust loads.

4- The helix angle ψ is the same on each gear, but one gear must have a right hand (R.H) helix and the other a left hand (L.H) helix, see Figure below.



Left hand helix　　　Right hand helix

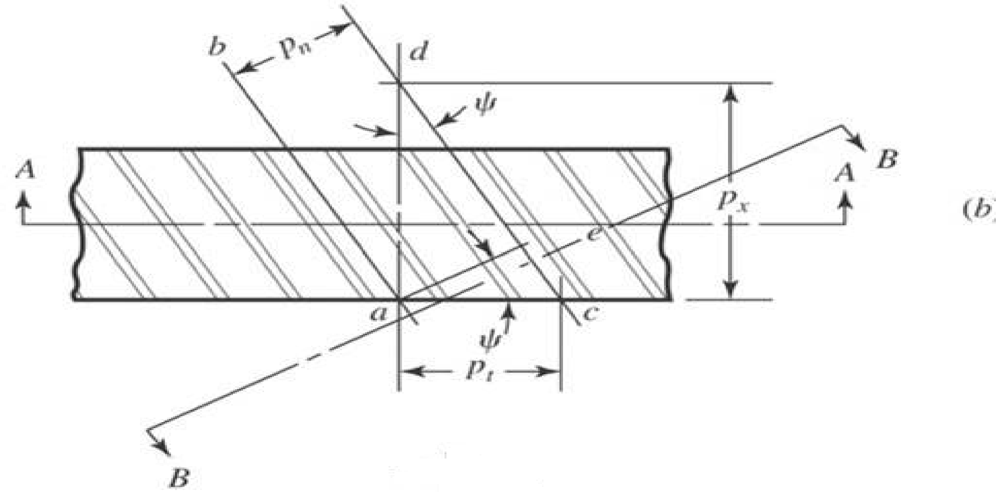
5- The Figure represents a portion of the top view of a helical rack. Lines *ab* and *cd* are the centerlines of two adjacent teeth taken on the pitch plane.



6- The distance *ac* is the traverse circular pitch, p_t or p in the plane of rotation (usually called the circular pitch).

7- The distance *ae* is the normal circular pitch, p_n and is related to the traverse circular pitch by the relation:

$$p_n = p_t \cos \psi$$



8- The distance *ad* is the axial pitch, p_x and is related to the traverse circular pitch by the relation:

$$p_x = \frac{p_t}{\tan \psi}$$

9- The module in the normal direction, m_n is related to the module in the transversal direction, m_t or m by the relation:

$$m_n = m \cos \psi$$

10- The pitch circle diameter of the helical gear is given by:

$$d = mN$$

$$= \frac{m_n}{\cos \psi} N$$

11- The pressure angle in the normal direction, φ_n is related to the pressure angle in the direction of rotation (transversal), φ_t by the relation:

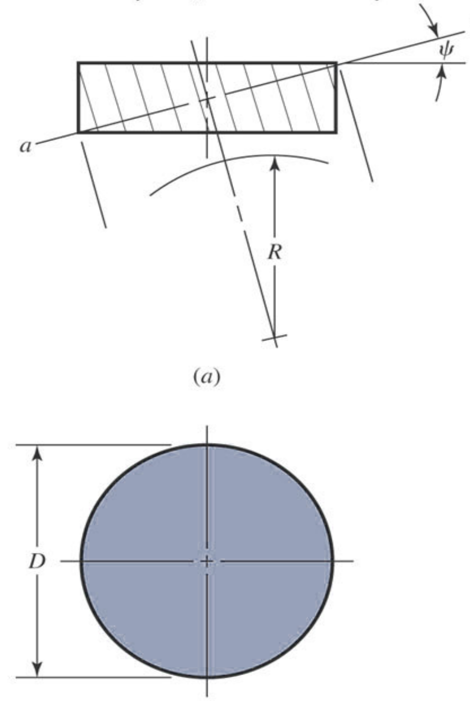
$$\cos\psi = \frac{\tan\varphi_n}{\tan\varphi_t} \quad \text{or} \quad \tan\varphi_n = \tan\varphi_t \cos\psi$$

12- Addendum circle: $d_a = mN + 2m_n$

Dedendum circle: $d_d = mN - 2.5m_n$

13- Virtual number of teeth of a helical gear, $N^\backslash$ is related to the actual number, N by the relation:

$$N^{\backslash} = \frac{N}{\cos^{3}\psi}$$



(a)

(b)

This figure shows a cylinder cut by an oblique plane at an angle ψ to a right section. The oblique plane cuts out an arc having a radius of curvature of *R*. This radius represents the apparent radius of a helical gear tooth when viewed in the direction of the tooth elements. Therefore, a gear of the same pitch and with the radius *R* will have a greater number of teeth because of the increased radius. (which means that we can use less number of teeth in helical gears than in spur gears without interference)

In the helical gear design this is called the virtual number of teeth.

<u>Force analysis of helical gears:</u>

The figure shown is a three dimensional view of the forces acting against a helical gear tooth. The point of application of the forces is in the pitch plane and in the center of the gear face. From the geometry of the figure, it is clear that:

the radial force:

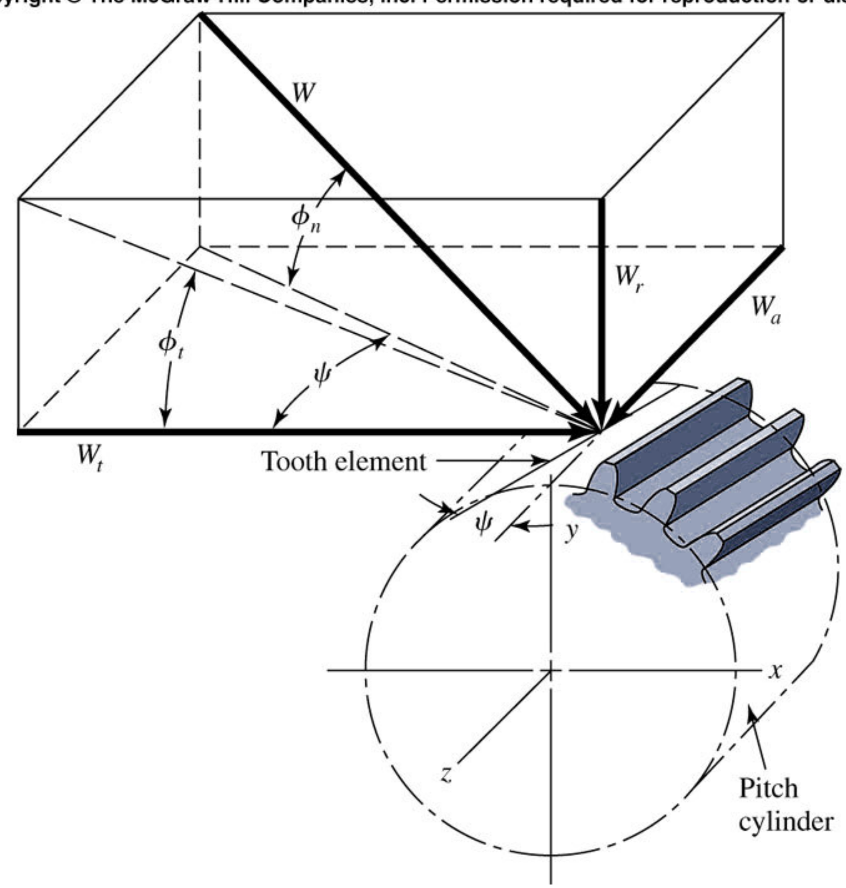
$$W_r = W \sin \varphi_n$$

the tangential force:

$$W_t = W \cos \varphi_n \cos \psi$$

the axial force:

$$W_a = W \cos \varphi_n \sin \psi$$



Usually W_t is given (or Calculated), and the other forces are required, therefore:

$$W = \frac{W_t}{\cos\varphi_n \cos\psi}$$

$$W_r = \frac{W_t}{\cos\varphi_n \cos\psi}\sin\varphi_n = W_t \frac{\tan\varphi_n}{\cos\psi}$$

$$\because \tan\varphi_t = \frac{\tan\varphi_n}{\cos\psi} \quad \text{or} \quad \tan\varphi_n = \tan\varphi_t \cos\psi$$

$$\boxed{\therefore W_r = W_t \tan\varphi_t}$$

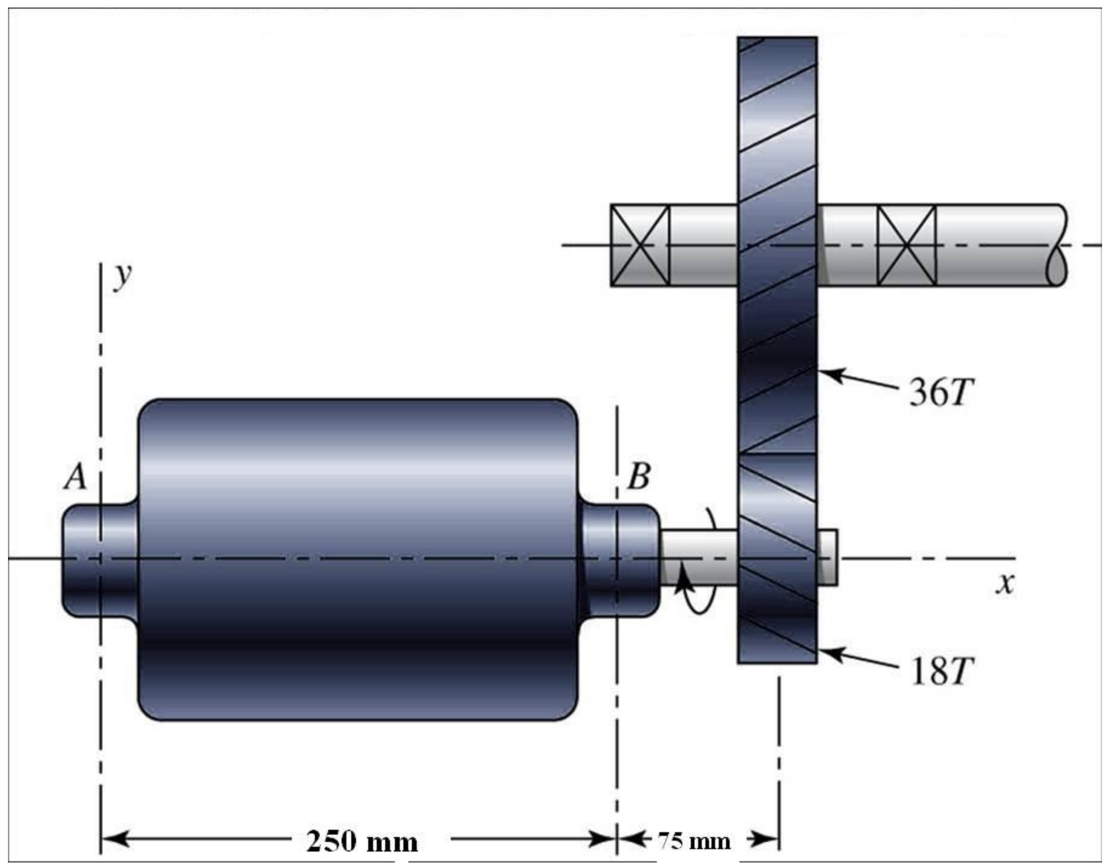
$$W_a = \frac{W_t}{\cos\varphi_n \cos\psi}\cos\varphi_n \sin\psi$$

$$\boxed{\therefore W_a = W_t \tan\psi}$$

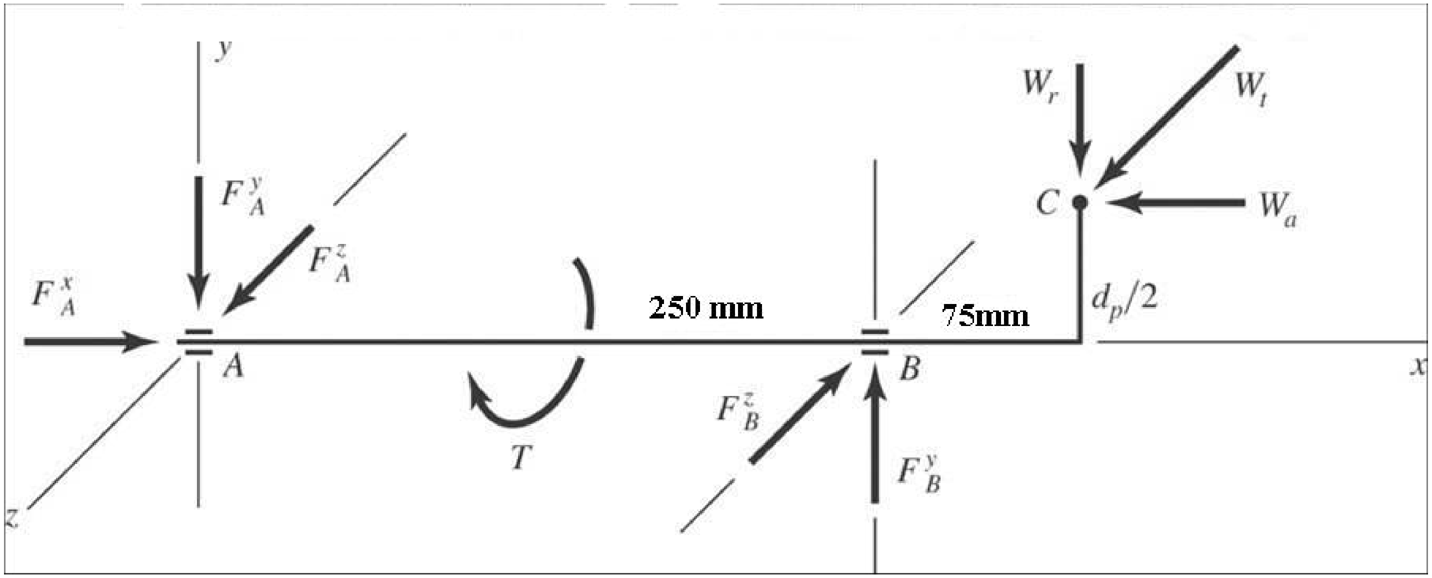
Example:

The figure shows an electric motor driving a helical gear set. The power of the motor is 2 kW and it is running at 100 rpm. The number of teeth of the pinion and gear are 18 and 36 respectively. The normal module is 2 mm, the normal pressure angle is 20°, the helix angle is 30°.

Find the magnitude and direction of the forces acting on the pinion and the reactions on the motor shaft at bearings A & B.



Solution:



$$d_p = m \cdot N = \frac{m_n}{\cos\psi} \cdot N_p = \frac{2}{\cos 30} \cdot 18 = 41.57 \quad \text{mm}$$

$$\mathrm{v} = \frac{2 \cdot \pi \cdot \mathrm{n_p}}{60} \cdot \frac{d_p}{2} = \frac{2 \times \pi \times 1000}{60} \cdot \frac{41.57}{2} = 2177 \quad \text{mm / s}$$

$$= 2.177 \quad \text{m / s}$$

$$\mathrm{W_t} = \frac{H}{\mathrm{v}} = \frac{2 \times 1000}{2.177} = 918.88 \quad \text{N}$$

$$\mathrm{W}_r = W_t \tan\varphi_t = W_t \frac{\tan\varphi_n}{\cos\psi} = 918.88 \frac{\tan 20}{\cos 30} = 386.18 \quad \text{N}$$

$$\mathrm{W_a} = W_t \tan\psi = 918.88 \times \tan 30 = 530.516 \quad \text{N}$$

Reactions at A & B:

Taking moments about the z - axis at A :

$$W_r \times (250 + 75) - W_a \times (\frac{d_p}{2}) - F_B^y \times 250 = 0$$

$$386.18 \times 325 - 530.516 \times \frac{41.57}{2} - F_B^y \times 250 = 0$$

$$F_B^y = \frac{386.18 \times 325 - 530.516 \times (\frac{41.57}{2})}{250} = 457.93 \quad \text{N}$$

Taking moments about the z - axis at B :

$$W_r \times 75 - W_a \times \frac{d_p}{2} - F_A^y \times 250 = 0$$

$$F_A^y = \frac{386.18 \times 75 - 530.516 \times (\frac{41.57}{2})}{250} = 71.75 \quad \text{N}$$

Taking moments about the y - axis at A :

$$W_t \times (250 + 75) - F_B^z \times 250 = 0$$

$$F_B^z = \frac{918.88 \times 325}{250} = 1194.54 \quad \text{N}$$

Taking moments about the y - axis at B :

$$W_t \times 75 - F_A^z \times 250 = 0$$

$$F_A^z = \frac{918.88 \times 75}{250} = 275.66 \quad \text{N}$$

$$F_A^x = W_a = 530.516 \quad \text{N}$$

The reactions at the bearings are :

$$R_A = 530.51 i - 71.75 \text{ j} + 275.66 \text{ k}$$

$$R_B = 457.93 \text{ j} - 1194.54 \text{ k}$$