Practice Sheet 4A

Linear Systems: Iterative Methods

Problem 1

1. Find the first two iterations of the **Jacobi method** for the following linear systems, using $X^{(0)} = 0$:

   a. $\begin{aligned} 3x_1 &- x_2 &+ x_3 &= 1, \\ 3x_1 &+ 6x_2 &+ 2x_3 &= 0, \\ 3x_1 &+ 3x_2 &+ 7x_3 &= 4. \end{aligned}$

   b. $\begin{aligned} 10x_1 &- x_2 & &= 9, \\ -x_1 &+ 10x_2 &- 2x_3 &= 7, \\ &- 2x_2 &+ 10x_3 &= 6. \end{aligned}$

   c. $\begin{aligned} 10x_1 &+ 5x_2 & & &= 6, \\ 5x_1 &+ 10x_2 &- 4x_3 & &= 25, \\ &- 4x_2 &+ 8x_3 &- x_4 &= -11, \\ & &- x_3 &+ 5x_4 &= -11. \end{aligned}$

   d. $\begin{aligned} 4x_1 &+ x_2 &+ x_3 & & &+ x_5 &= 6, \\ -x_1 &- 3x_2 &+ x_3 &+ x_4 & & &= 6, \\ 2x_1 &+ x_2 &+ 5x_3 &- x_4 &- x_5 & &= 6, \\ -x_1 &- x_2 &- x_3 &+ x_4 & & &= 6, \\ &2x_2 &- x_3 &+ x_4 &+ 4x_5 & &= 6. \end{aligned}$

2. Use the **Jacobi method** to solve the linear systems in **Exercise 1**, with $TOL = 10^{-3}$ in the l_∞ norm.

3. Use the **Gauss-Seidel method** to solve the linear systems in **Exercise 1**, with $TOL = 10^{-3}$ in the l_∞ norm.

Solution:

Question	Approximation	Error
1.a	$X^{(2)} = (0.14286, -0.35714, 0.42857)$	0.35714
1.b	$X^{(2)} = (0.97000, 0.87000, 0.54000)$	0.17
1.c	$X^{(2)} = (-0.65000, 1.65000, -0.40000, -2.47500)$	1.25
1.d	$X^{(2)} = (1.32500, -0.10000, 2.50000, 6.70000, 1.30000)$	1.9
2.a	$X^{(9)} = (0.03510, -0.23664, 0.65813)$	0.00075
2.b	$X^{(6)} = (0.99572, 0.95777, 0.79145)$	0.00052
2.c	$X^{(21)} = (-0.79711, 2.79517, -0.25940, -2.25179)$	0.00096
2.d	$X^{(47)} = (0.83072, 2.30758, 2.30753, 11.44654, -1.93836)$	0.00085
3.a	$X^{(6)} = (0.03535, -0.23679, 0.65776)$	0.00080
3.b	$X^{(4)} = (0.99575, 0.95787, 0.79157)$	00080
3.c	$X^{(9)} = (-0.79691, 2.79462, -0.25918, -2.25184)$	0.00085
3.d	$X^{(8)} = (0.83075, 2.30773, 2.30775, 11.44624, -1.93849)$	0.00078