# *BJT AC analysis*

→ for any Input → obtain DC analysis to get $I_C$ & make sure BJT is working as an Amplifier (in Active Region)

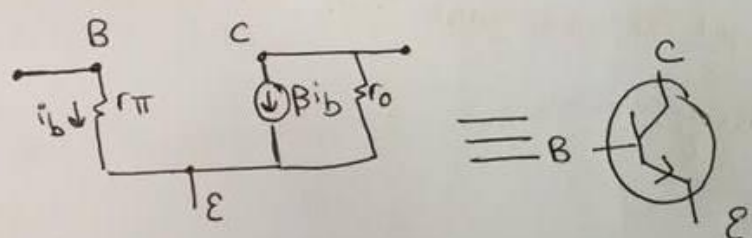
→ obtain AC analysis to get $R_{in}$, $R_{out}$, $A_v$

**Solution Steps :**

① obtain DC analysis to get $I_C$
- → cap. → O.C
- → AC voltage Source → S.C
- → AC current Source → O.C

② g from $I_C$
- → $g_m = \frac{I_C}{V_T}$
- → $r_\pi = \beta / g_m$
- → $r_o = \frac{V_A}{I_C}$ → Early voltage

③ AC analysis : Remove the transistor & put the Small Signal model instead.




in AC analysis → Cap → S.C
DC V.S → S.C
DC C.S → O.C

⑤ $A_v$ (gain) $= \frac{V_o}{V_i}$ $\qquad R_{out} = \frac{V_T}{I_T}\Big|_{V_{in}=0}$

$R_{in} = \frac{V_{in}}{I_{in}}$

* Important notes :-

1] If $V_A$ is not given → Assume $r_o = \infty$

2] for getting $R_{out}$, you apply Same Rule as thevenin ; where you switch off the independant AC source, Apply $V_T$ & $I_T$ @ $V_{out}$ & Solve to get $\boxed{\frac{V_T}{I_T}}$.

3] check whether $i_B = 0$ or not for $R_{out}$ especially, as if it's equal to zero the ($\beta i_b$) dependant circuit will become an O.C.

4] common Emmitter means that Vin is on B/C & $V_o$ is from B/C,
common Base means that Vin is on E/C & $V_o$ is taken from E/C
Same for common Collector, Vin → E/B
$V_o$ → E/B

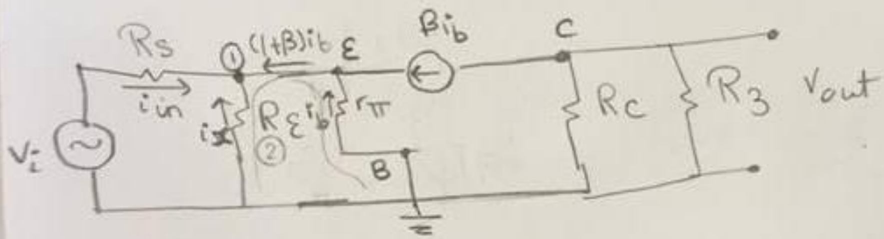
5] for solving, you can get the gain first then Rin & Rout or vice versa. Both are okay.

# *Sheet 5*

1) $R_S = 100\ \Omega$, $R_E = 4.3\,K$, $R_C = 2.2\,K$, $R_3 = 51\,K$ & $\beta = 100$

a) for $A_v$, $R_{in}$ & $R_{out}$ @ $I_C = 1\,mA$

→ Draw Small Signal Model :- (cap. SC, DC Voltage sources SC)




$$g_m = \frac{I_C}{V_T} = \frac{1mA}{25mV} = 40\ mA/V$$

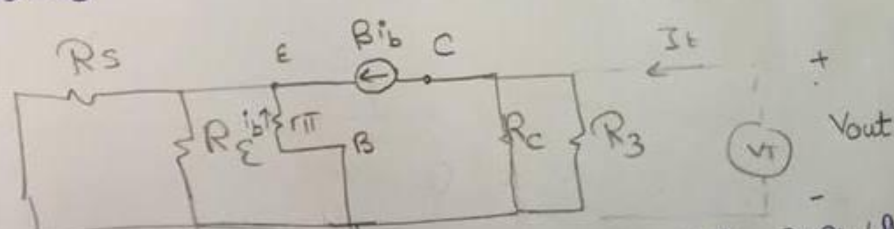
$$r_\pi = \beta / g_m = 100/40m = 2.5\,K\Omega$$

$$r_o = \frac{V_A}{I_C}, \quad \because V_A \text{ not given} \rightarrow r_o \simeq \infty$$

→ AC analysis :-

$$V_{out} = -\beta i_b \times R_{out}$$

→ to get $R_{out}$ → $V_{in}$ SC, add $V_T$ & $I_T$




→ ~~ib is off~~ zero ~~since there is no current in the left half of the circuit, thus ib is off~~. $I_b$ is zero as there's no source conducting $i_B$ in the left half of the circuit.

$$\therefore \frac{V_T}{I_T} = R_{out} = R_C \,//\, R_3 = 2.2\,K\Omega \,//\, 51\,K\Omega$$

$$\boxed{= 2.11\,K\Omega}$$

to get $R_{in}$ :-

$$\frac{V_{in}}{i_{n}} = R_{in}$$

$i_{in} \rightarrow$ by KCL @ node 1 :-

$$i_{in} = -i_x - (1+\beta) i_b \longrightarrow 1$$

KVL @ loop 2

$$R_E i_x = -i_b r_\pi \rightarrow i_x = \frac{-i_b r_\pi}{R_E} \rightarrow 2$$

$\rightarrow$ Substitute in 1 :-

$$\boxed{i_{in} = -\left(1+\beta+\frac{r_\pi}{R_E}\right) \times i_b} \longrightarrow 3$$

$$\cancel{\therefore \frac{V_{in}}{\phantom{x}} = R_S i_{in} + - r_\pi i_b}$$
$$\cancel{= R_S i_{in} +}$$

$$V_{in} = R_S i_{in} + (-R_E \times i_x)$$
$$= R_S i_{in} - i_b r_\pi = R_S i_{in} + \frac{i_{in} r_\pi}{\left(1+\beta+\frac{r_\pi}{R_E}\right)}$$

$$\boxed{\frac{V_{in}}{i_{in}} = R_S + \frac{r_\pi}{1+\beta+\frac{r_\pi}{R_E}} = 124.6\,\Omega} = R_{in}$$

$$\frac{V_o}{V_{in}} = \frac{\cancel{-}\beta \cancel{i_b} \times R_{out}}{R_S \times (1+\beta+\frac{r_\pi}{R_E})\cancel{i_b} + \cancel{i_b} r_\pi}$$

gain $\Rightarrow$ $$\boxed{= \frac{\beta \times R_{out}}{R_S \times (1+\beta+\frac{r_\pi}{R_E}) + r_\pi}}$$

$V_{in}$ in terms of $i_b$ :-

$R_S i_{in} - i_b r_\pi$

$$\boxed{V_{in} = R_S \times (-1+\beta+\frac{r_\pi}{R_E}) i_b - i_b r_\pi}$$

b) $I_C = 10\ \mu A$, $R_E = 430\ K$, $R_C = 220\ K$, $R_3 = 510\ K$ ③

→ Same steps as ⓐ

$$g_m = \frac{I_C}{V_T} = \frac{10\ \mu A}{25\ mV} = 0.4\ mA/V$$

$$r_\pi = \beta / g_m = \frac{100}{0.4m} = 250\ K\Omega$$

$$r_o = \infty$$

$$R_{out} = R_C \,//\, R_3 = 220\ K \,//\, 510\ K$$
$$\boxed{= 153.7\ K\Omega}$$

$$R_{in} = R_S + \frac{r_\pi}{1+\beta+\frac{r_\pi}{R_E}} = 2.561\ K\Omega$$

$$\boxed{A_v = \frac{V_{out}}{v_{in}} = \frac{\beta R_{out}}{R_S * \left(1+\beta+\frac{r_\pi}{R_E}\right) + r_\pi}}$$

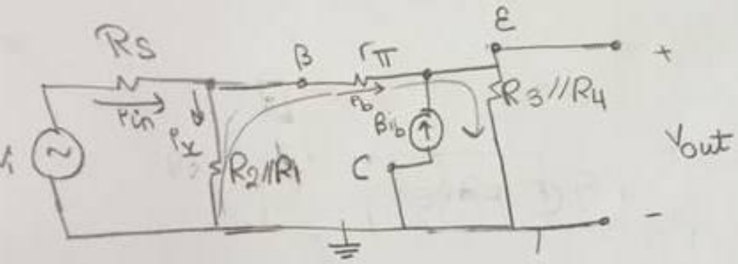
Problem ② :-



$R_S = 2K$ , $R_1 = 100K$, $R_2 = 300K$, $R_3 = 13K$, $R_4 = 100K$, $\beta = 100$ &

$I_c = 0.25\ mA$

Small signal Model :-




$$g_m = \frac{I_c}{V_T} = \frac{0.25\ mA}{25\ mV} = 10\ mA/V$$

$$r_\pi = \beta / g_m = 100/10m = 10\ K$$

$$r_o = \infty$$

→ to get $R_{in}$ :-

$V_{in} = i_{in} \times R_{in}$

$i_{in} = i_b + i_x$

KVL @ loop ① :

$$-i_x (R_2 // R_1) + i_b r_\pi + (1+\beta) i_b \times R_3 // R_4 = 0$$

$$i_x = i_b \frac{r_\pi + (1+\beta) \times R_3 // R_4}{R_2 // R_1}$$

$$\therefore i_{in} = i_b + i_b \times \frac{r_\pi + (1+\beta) \times R_3 // R_4}{R_2 // R_1} \rightarrow i_b = \frac{i_{in}}{1 + \frac{r_\pi + (1+\beta) \times R_3 // R_4}{R_2 // R_1}}$$

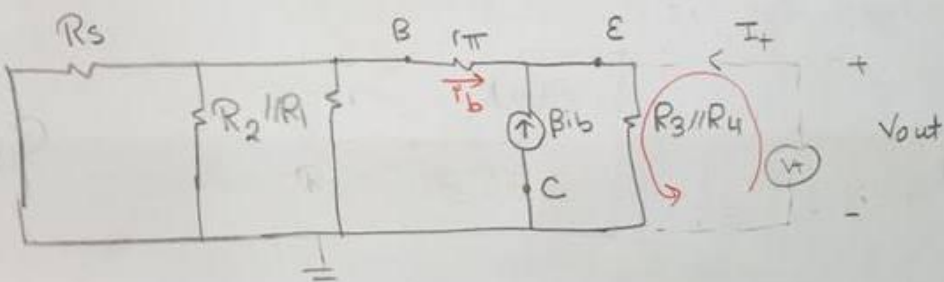
$$R_{in} = \frac{V_{in}}{i_{in}}$$

$$V_{in} = R_S \times i_{in} + i_b\left(r_\pi + (1+\beta) R_3 // R_4\right)$$

$$= R_S \times i_{in} + \frac{i_{in}}{1 + \frac{r_\pi + (1+\beta) \times R_3 // R_4}{R_1 // R_2}} \times r_\pi + (1+\beta) R_3 // R_4$$

$$\boxed{\frac{V_i}{i_{in}} = R_S + R_1 // R_2 \times \frac{r_\pi + (1+\beta) R_3 // R_4}{1 + r_\pi + (1+\beta) \times R_3 // R_4}} \Rightarrow R_{in}$$

→ to get $R_{out}$ :-




$$V_T = (+I_t + \beta i_b + i_b) * R_3 // R_4$$

$$\because V_T = V_E \ \& \ i_b = \frac{-V_T}{(R_S // R_2 // R_1) + r_\pi}$$

$$V_t = \left(I_t + \frac{(1+\beta) * - V_T}{(R_S // R_2 // R_1) + r_\pi}\right) * R_3 // R_4$$

$$V_t\left(\frac{1}{R_3 // R_4} + \frac{1+\beta}{(R_S // R_2 // R_1) + r_\pi}\right) = I_t$$

$$\boxed{\frac{V_t}{I_t} = \frac{1}{\frac{1}{R_3 // R_4} + \frac{1+\beta}{(R_S // R_2 // R_1) + r_\pi}}} \rightarrow R_{out}$$

$$V_{out} = (1+\beta)i_b \times (R_3 // R_4)$$

$$\boxed{\frac{V_o}{V_i} = \frac{(1+\beta)\times(R_3//R_4)}{R_S\left(1+\dfrac{r_\pi + (1+\beta)R_3//R_4}{R_1//R_2}\right) + (r_\pi + (1+\beta)R_3//R_4)}} \rightarrow \text{gain}$$

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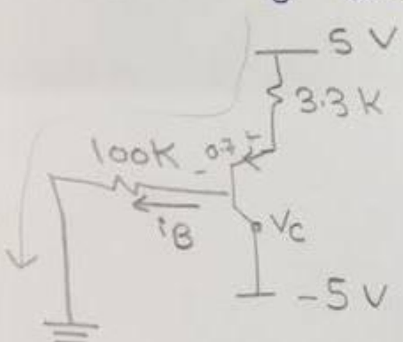
Problem 3 :

→ first we need to get $I_c$ as it's not given, thus we have to solve with DC-analysis first. $\boxed{\beta = 120}$




$$-5 + 3.3K\,I_E + 0.7 + 100K\,I_B = 0$$

$$-5 + 3.3K(1+\beta)I_B + 0.7 + 100K\,I_B = 0$$

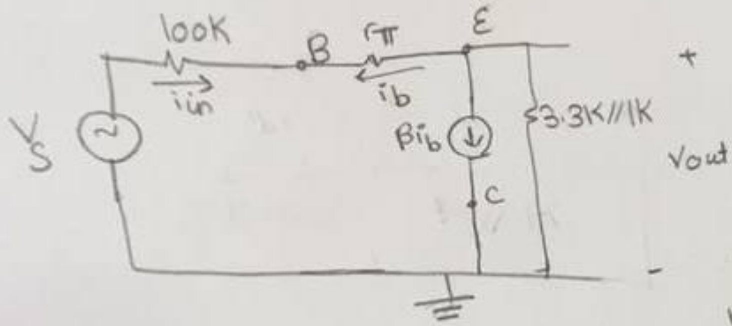
$$-4.3 + 499.3K\,I_B = 0$$

$$\boxed{I_B = 8.61\,\mu A}$$

$$\boxed{I_C = 1.03\text{ mA}}$$

$V_{CE} > 0.2$ ✓ Active

→ Second we need to Solve using the Small Signal model.




Note: AC Model for PNP is the Same as NPN, Just notice the currents direction.

$$g_m = \frac{I_C}{V_T}$$

$$r_\pi = \beta / g_m$$

$$r_o = \infty$$

to get $R_{in}$ :-

$$\frac{V_{in}}{i_{in}} = R_{in}$$

$\longrightarrow V_{in} = 100K \times i_{in} - r_\pi i_b - (3.3K // 1K) * (1+\beta) i_b$

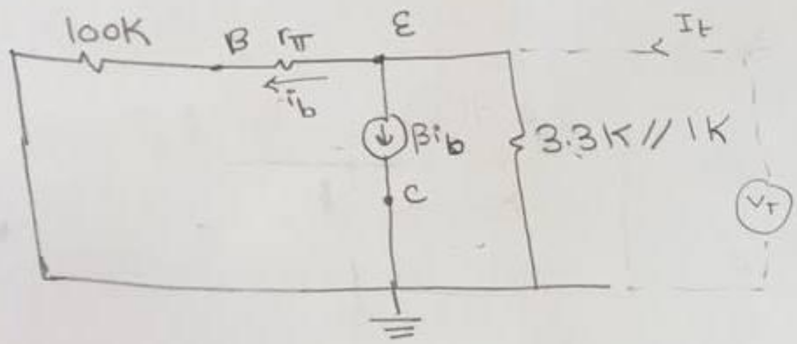
$\because i_{in} = -i_b$

$$\therefore \boxed{\frac{V_{in}}{i_{in}} = 100K + r_\pi + (3.3K // 1K) * (1+\beta)} \Rightarrow R_{in}$$

$V_{in} = i_{in} \times R_{in}$

$V_{out} = (1+\beta) i_b \times (3.3K // 1K)$

for Rout




$V_T = 1K//3.3K * (I_t - \beta i_b - i_b)$

$\because V_T = V_E$ & $i_b = \dfrac{V_T}{R_\pi + 100K}$

$$V_T = 1K//3.3K * \left(I_t - \frac{\beta V_T}{r_\pi + 100K} - \frac{V_T}{r_\pi + 100K}\right)$$

$$\frac{V_T}{1K//3.3K} + \frac{(1+\beta) V_T}{r_\pi + 100K} = I_t$$

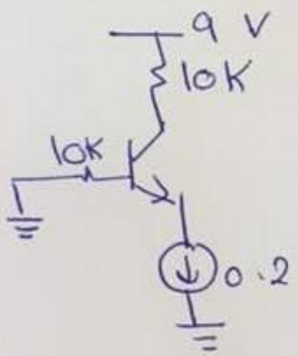
$$V_T\left(\frac{1}{1K//3.3K} + \frac{1+\beta}{r_\pi + 100K}\right) = I_t$$

$$\boxed{\frac{V_T}{I_t} = \frac{1}{\dfrac{1}{1K//3.3K} + \dfrac{1+\beta}{r_\pi + 100K}}} \rightarrow \text{Rout}$$

$$\boxed{\frac{V_o}{v_i} = \frac{(1+\beta) \times (3.3K//1K)}{100K + r_\pi + [(1+\beta) * (3.3K//1K)]}} \rightarrow \text{gain}$$

## problem 4

### DC- analysis :-




$I_E = 0.2\,mA$

$(1+\beta) I_b = 0.2\,mA$

$\boxed{I_b = 3.92\,\mu A}$

$\boxed{I_C = 0.196\,mA}$

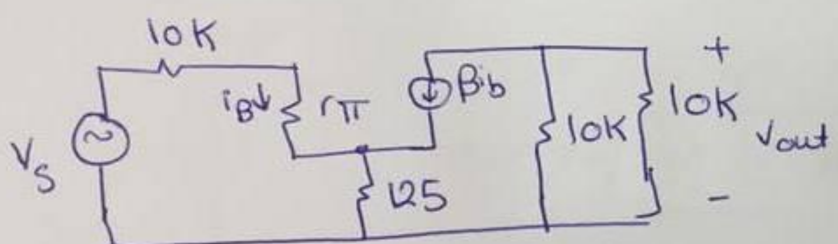
$V_{CE} > 0.2$ ✓ → Active

$$g_m = \frac{I_C}{V_T} = \frac{0.196\,mA}{25\,mV} = 7.84\,mA/V$$

$$r_\pi = \beta / g_m = \frac{50}{7.84\,m} = 6.38\,K\Omega$$

$r_o = \infty$

### → AC-analysis :-




$\boxed{V_{out} = -\beta i_b \times R_{out}}$

$V_{in} = i_b \times R_{in}$

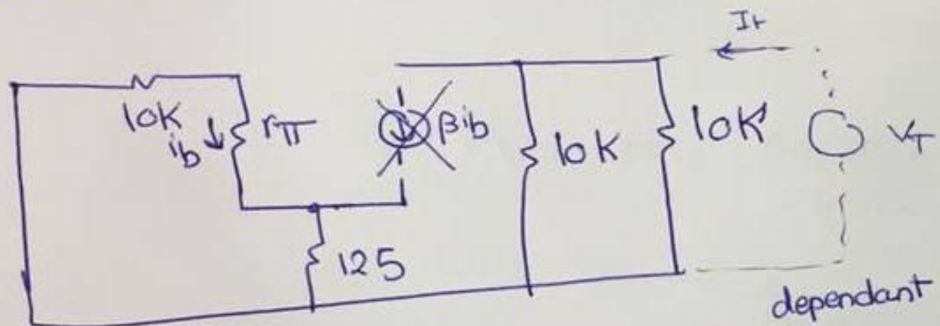
$$\boxed{\frac{V_o}{V_i} = \frac{-\beta R_{out}}{R_{in}}} \rightarrow \text{gain}$$

$V_S = 10K i_B + r_\pi i_b + (1+\beta) i_b \times 125$

$\because i_b = i_{in}$

$\boxed{\frac{V_S}{i_b} = 10K + r_\pi + (1+\beta) \times 125} \rightarrow R_{in}$

for $R_{out}$




dependant

→ To check whether $i_B = 0$ or not, we close the dependant current source & check whether there's any source driving the current needed ($I_b$). In this case, $I_b = 0$ as there's no source to produce the current, thus $\beta I_b$ is an O.C.

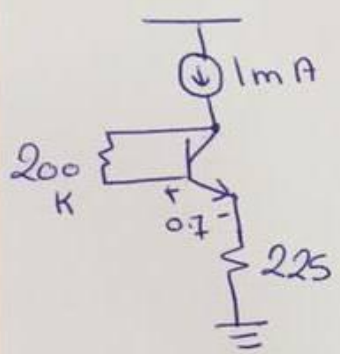
$\boxed{\therefore \frac{V_T}{I_T} = 10K \parallel 10K = 5K} \rightarrow R_{out}$

$\boxed{\frac{V_o}{V_i} = \frac{-\beta \times 5K}{10K + r_\pi + (1+\beta) \times 125}} \rightarrow gain$

* problem 5



## -DC analysis:-




$I_C + I_B = 1\,mA$

$\beta I_b + I_b = 1\,mA$

$I_b = 9.9\,\mu A$

$\boxed{I_C = 0.99\,mA}$

check for $V_{CE} > 0.2$ ✓✓

$$g_m = \frac{0.99\,mA}{25\,mV} = 0.0396\,mA/V$$

$r_\pi = \beta / g_m = 2.5\,K\Omega$

$r_o = \infty$

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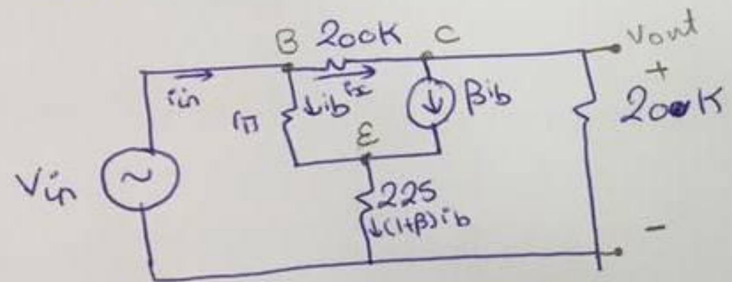
## AC-analysis:-




## To get Rin :-

$V_{in} = i_b(r_\pi + (1+\beta)\times 225) \rightarrow (1)$

$V_{in} = 200K\, i_x + (i_x - \beta i_b) * 20K \rightarrow (2)$

$V_{in} = 200K(i_{in} - i_b) + (i_{in} - (\beta+1) i_b) * 20K$

$V_{in} = i_{in}(200K + 20K) - i_b(200K + (1+\beta)20K)$

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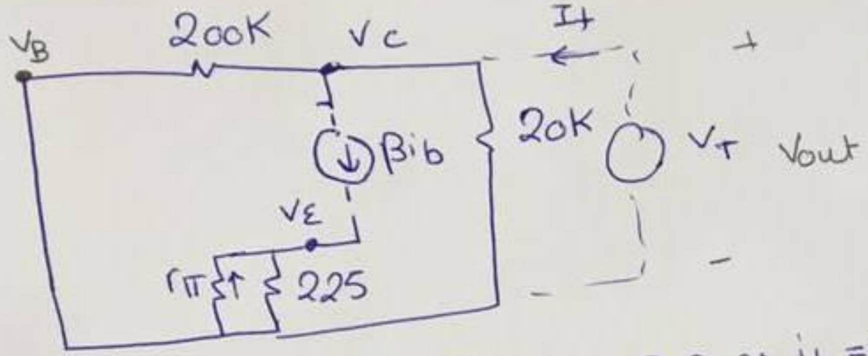
$$V_{in} = i_n(20K + 200K) - V_{in}\frac{(225 + (\beta+1)20K)}{r_\pi + (\beta+1)\times 225}$$

$$V_{in}\left[1 + \frac{225 + 20K(1+\beta)}{r_\pi + (1+\beta)\times 225}\right] = i_{in}(20K + 200K)$$

$$\boxed{\frac{V_{in}}{I_{in}} = \frac{20K + 200K}{1 + \frac{225 + 20K(1+\beta)}{r_\pi + (1+\beta)\times 225}}} \rightarrow R_{in}$$

→ To get Rout :-




→ Same case as problem 4, $\beta i_b$ is an O.C as $i_b = 0$.

$$\therefore \frac{V_T}{I_T} = R_{out} = 20K \,//\, 200K$$

$$\boxed{= 18.18\,K}$$

to get gain :-

$$\frac{V_o}{20K} + \beta i_b + \frac{V_o - V_i}{200K} = 0$$

$$V_o\left(\frac{1}{20K} + \frac{1}{200K}\right) = \frac{V_i}{200K} + \frac{-\beta V_i}{r_\pi + (1+\beta)\times 225}$$

$$\frac{V_o}{V_i} = \frac{\frac{1}{200K} + \frac{-\beta}{r_\pi + (1+\beta)\times 225}}{\frac{1}{20K} + \frac{1}{200K}}$$

$$\boxed{= \frac{200K \,//\, [(r_\pi + (1+\beta)\times 225) \times -\beta]}{20K \,//\, 200K}} \rightarrow \text{gain}$$

Best of Luck