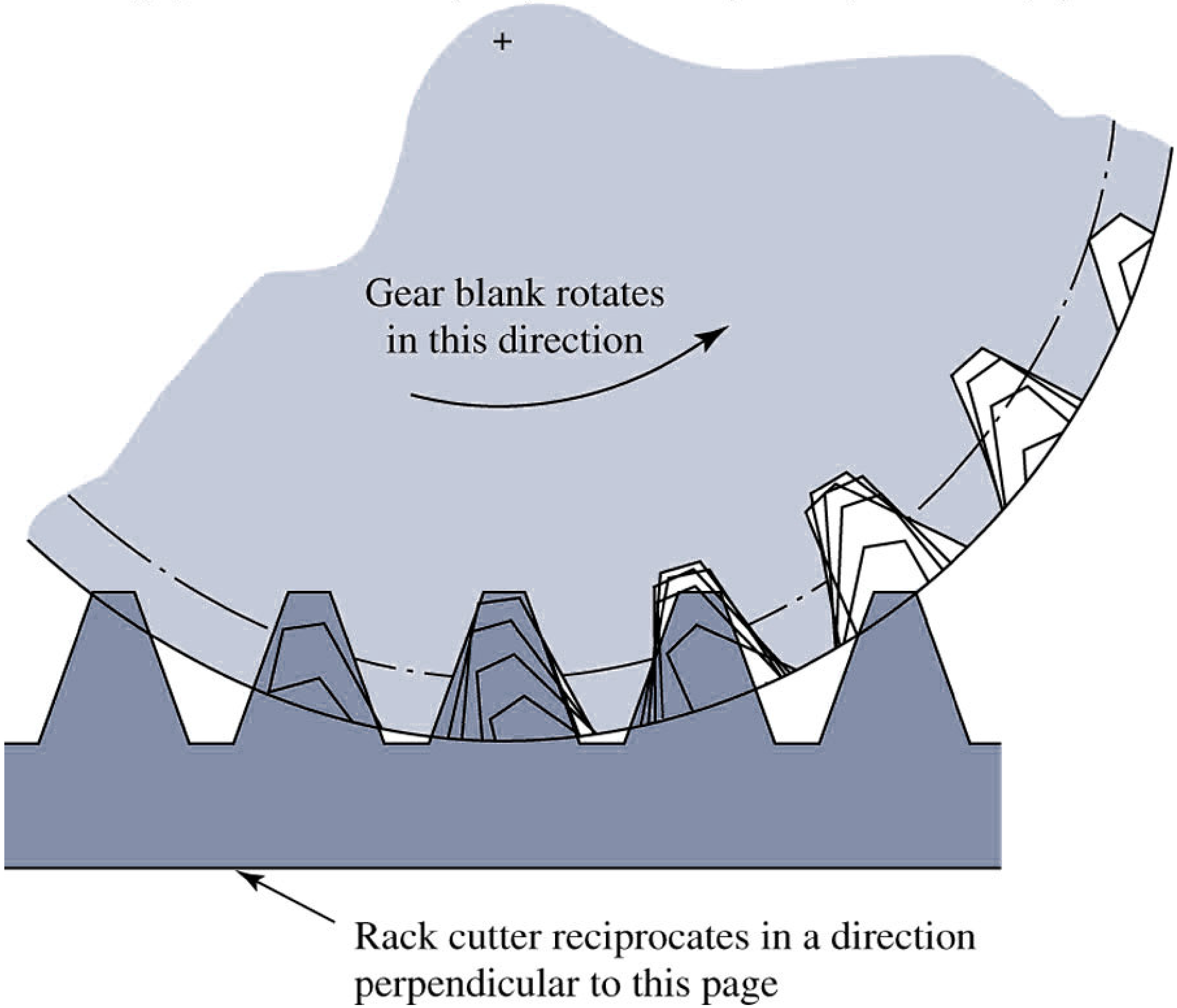
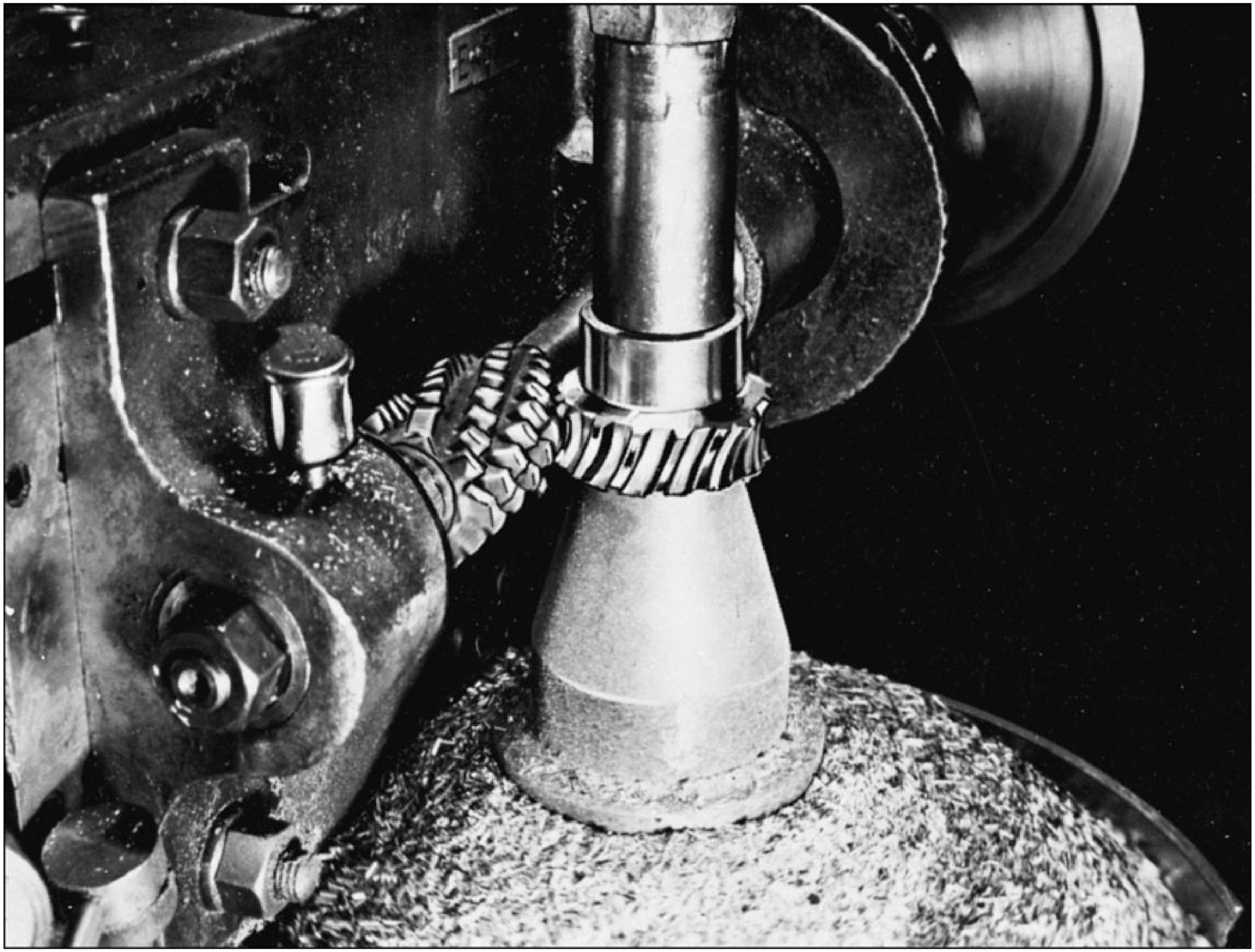
Force and Stress analysis of spur gears

Gear Manufacturing: Shaping

Gear Manufacturing:

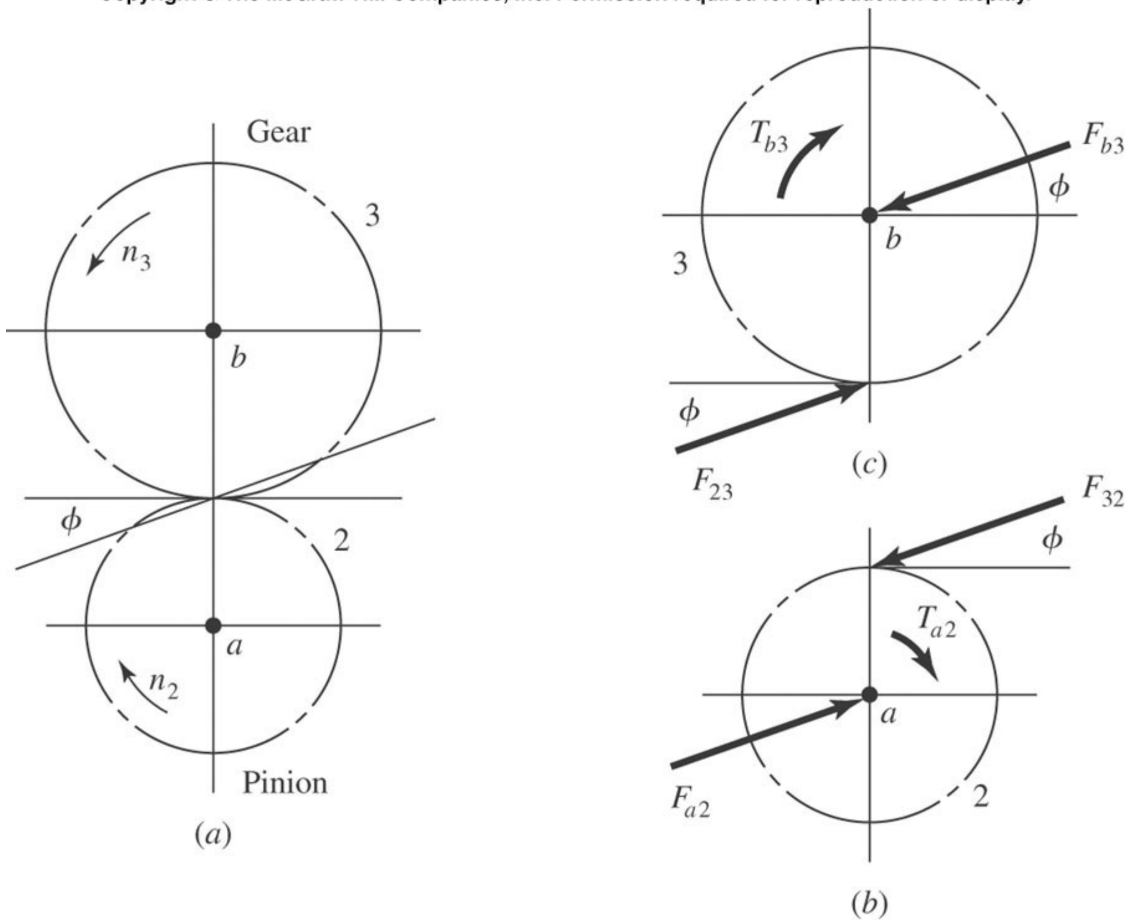


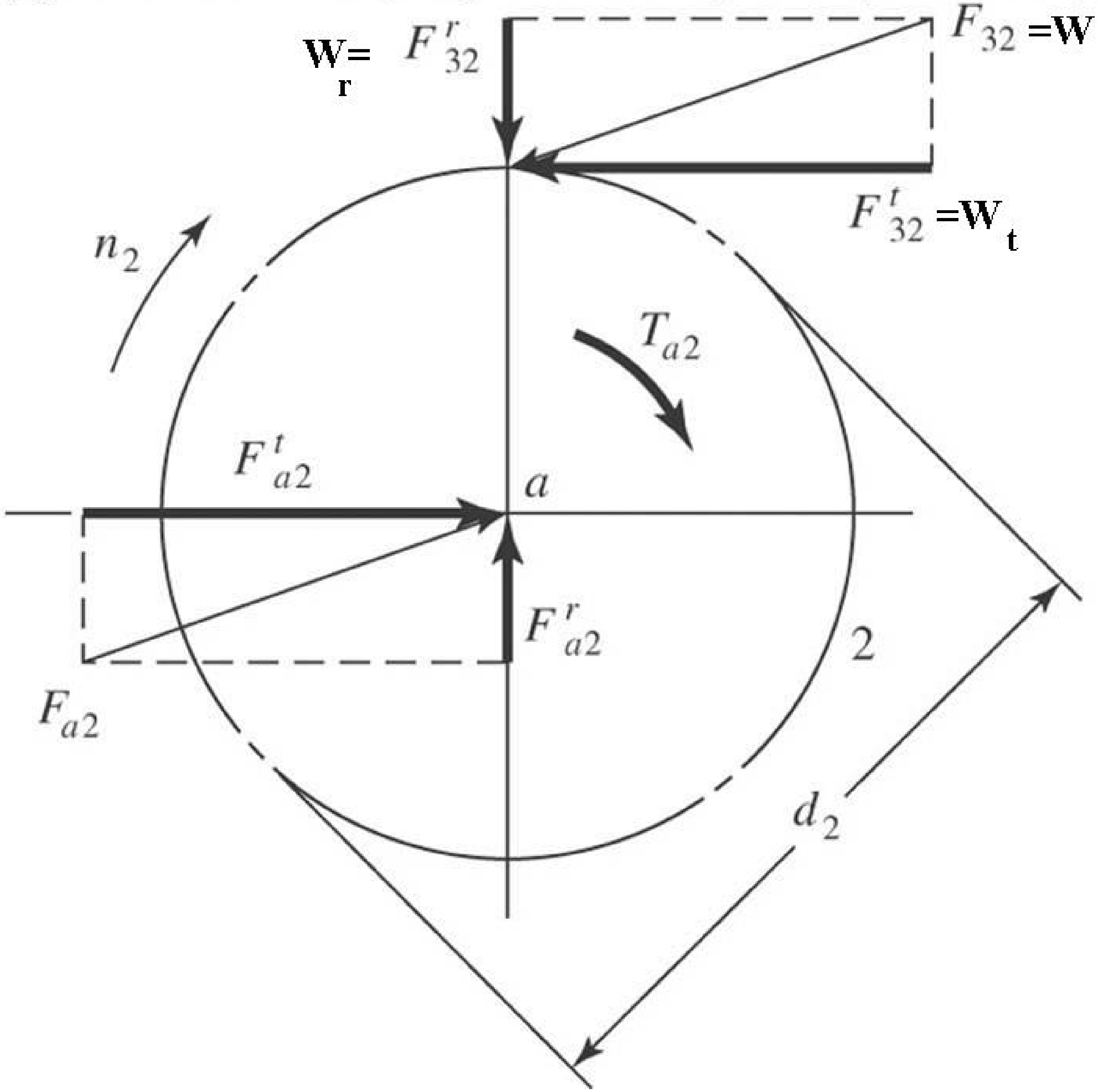
Gear Manufacturing: Hobbing



Force analysis of spur gears:

- Designate number 1 for the frame of the machine.
- Designate input gear (pinion) as gear 2.
- Designate successive gears as 3, 4,….. etc
- Designate shafts using lowercase letters a, b, c, ….etc

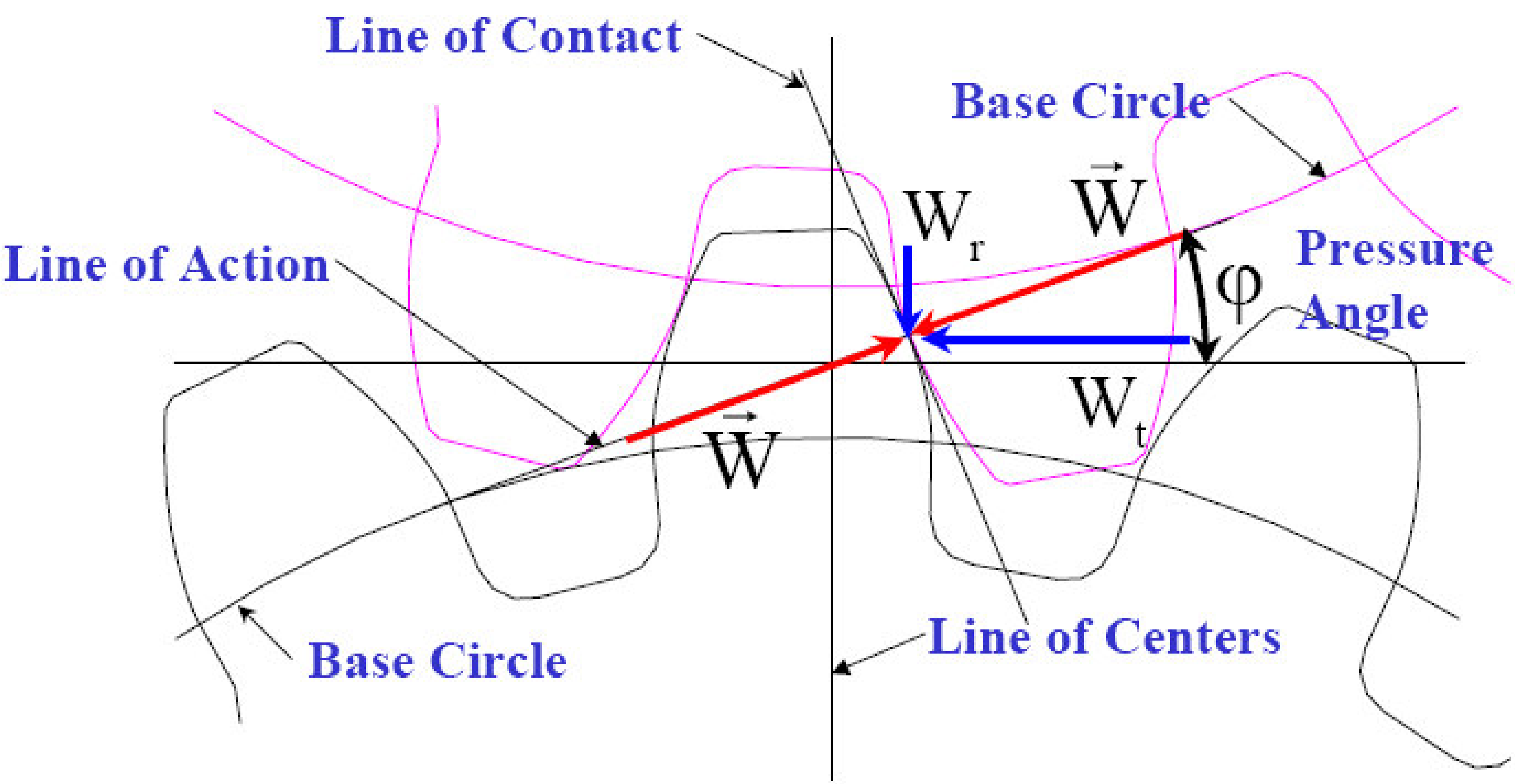




W$_r$= F^r_{32}
F_{32} =W
F^t_{32} =W$_t$
n_2
T_{a2}
F^t_{a2}
a
F^r_{a2}
2
F_{a2}
d_2

Tooth Load Equations

$$W_t = \frac{T}{d/2} \qquad W_r = W_t \cdot \tan\varphi \qquad \left|\vec{W}\right| = W_t / \cos\varphi$$



W : Total force between teeth

W_t : Tangential force (transmitted load)

W_r : Radial force

$$W = \sqrt{(W_t)^2 + (W_r)^2}$$

$$H = T.\omega$$

$$W_t = W\cos\varphi$$

$$T = \frac{H}{\omega}$$

$$W_r = W\sin\varphi$$

$$H = W_t.V$$

$$W_t = \frac{T}{r} = \frac{2T}{d}$$

$$V = \frac{\pi\ dn}{60}$$

Tooth stress analysis:

Limiting design factors in specifying the capacity of any gear drive:

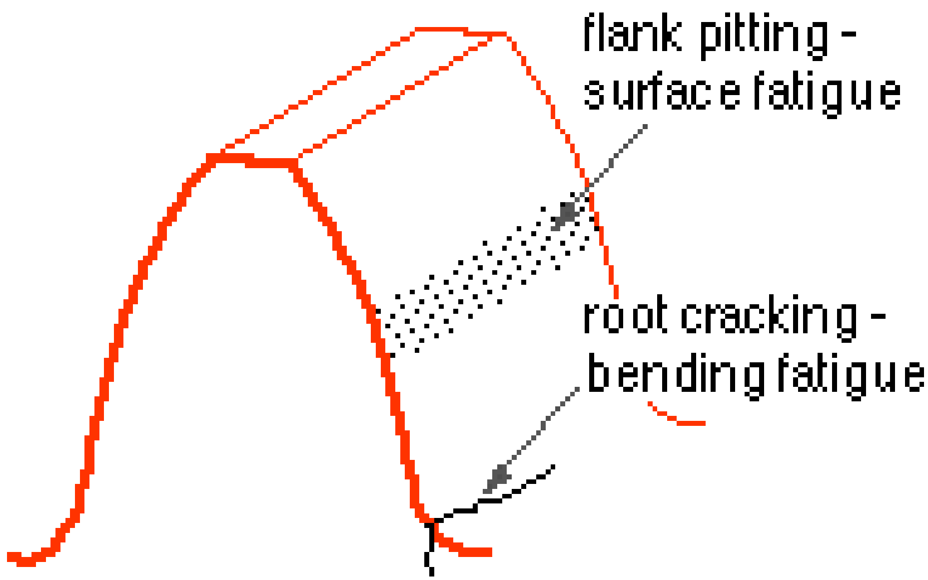
1- The heat generated during operation.

2- Failure of the teeth by breakage.

3- Fatigue failure of the tooth surfaces.

4- Abrasive wear of the tooth surface.

5- Noise as a result of high speeds, heavy loads, or mounting inaccuracies.



Bending Stress: Lewis Bending equation

$$\sigma = \frac{M}{I} c$$

$$M = W_t \cdot l$$

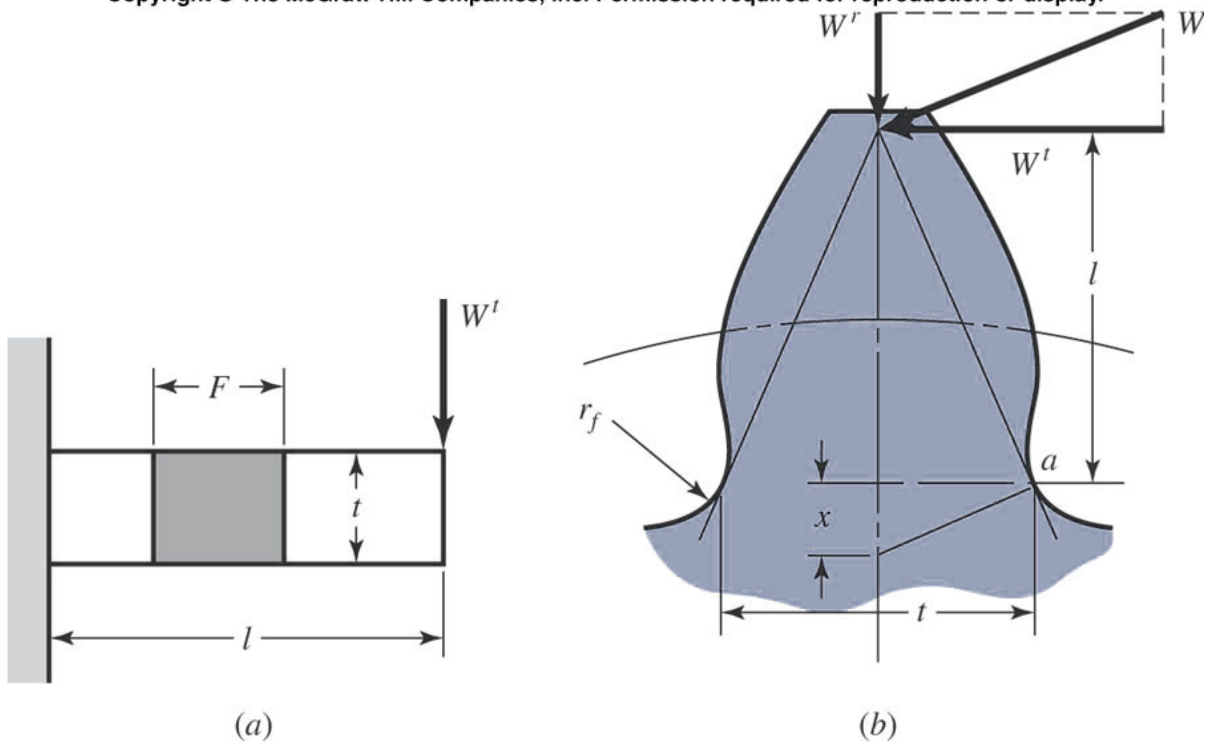
$$I = \frac{bt^3}{12}$$

$$c = \frac{t}{2}$$

$$\sigma = \frac{6W_t l}{bt^2}$$

$$\frac{t/2}{x} = \frac{l}{t/2}$$

$$x = \frac{t^2}{4l}$$



$$\sigma = \frac{6W_t l}{bt^2} = \frac{W_t}{b} \frac{1}{t^2/6l} = \frac{W_t}{b} \frac{1}{t^2/4l} \frac{1}{(\frac{4}{6})}$$

$$\sigma = \frac{W_t}{b(\frac{2}{3})x} \frac{p}{p},$$ b is the face width

$$\sigma = \frac{W_t p}{b(\frac{2}{3})xp}$$

let $y = \frac{2x}{3p}$, where y is Lewis form factor, then :

$$\sigma = \frac{W_t}{bpy}$$ which is the original Lewis equation

since $p = \pi m$

$$\sigma = \frac{W_t}{bm\pi y}$$ let $Y = \pi y$ then

$$\sigma = \frac{W_t}{bmY}$$ where Y can be calculated from tooth geometry

then they introduce a dynamic factor K_v in the equation to account for the dynamic effects of different pitch line velocity V_t.

$$\therefore \sigma = \frac{W_t}{bmY} K_v$$

Fatigue failure due to bending stress:

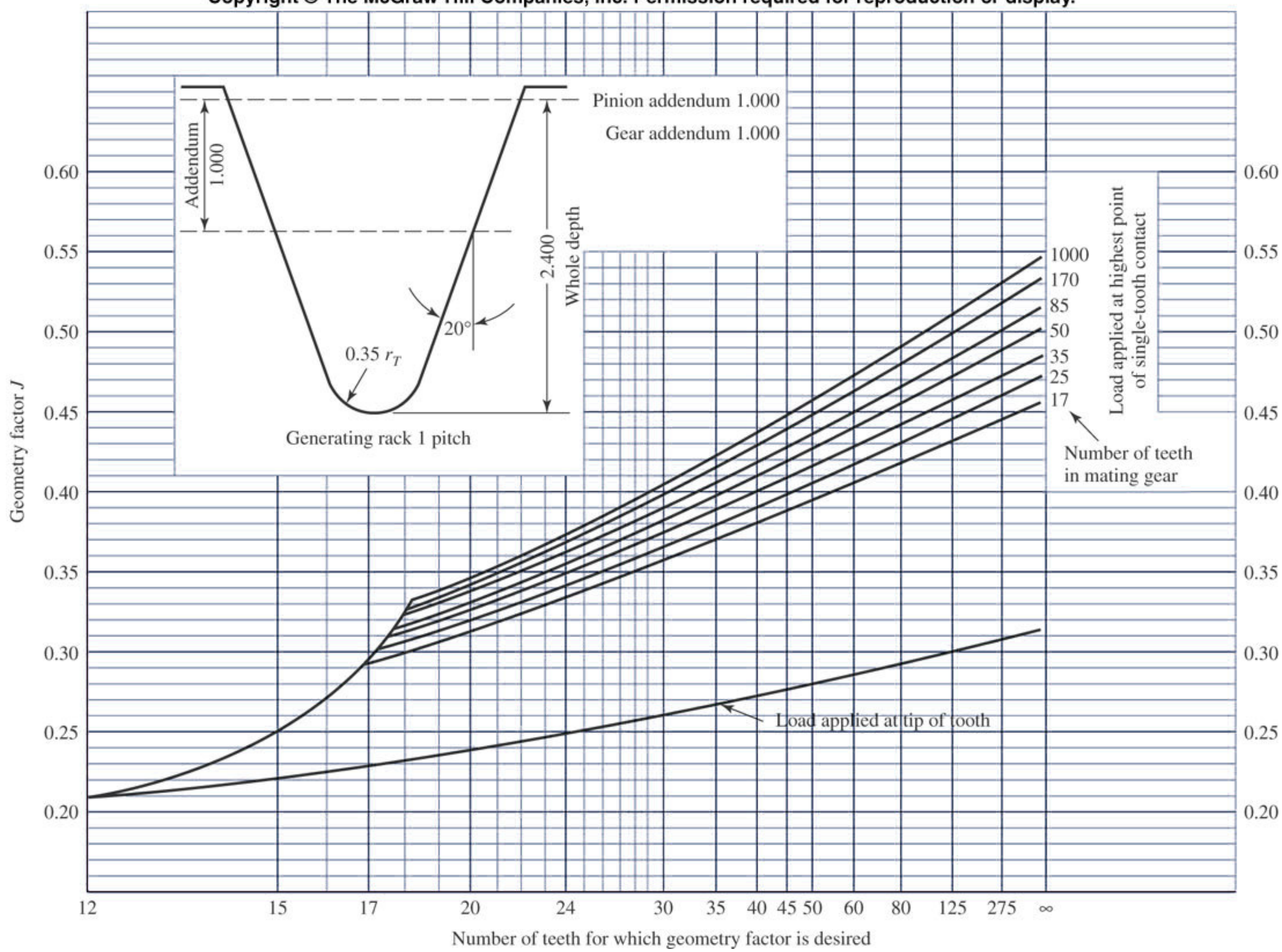
AGMA (American Gears Manufacturer Association) modified the original Lewis equation by introducing AGMA-form factor J instead of Y to include the root fillet stress concentration factor. Other factors were added later to the equation to be in the form:

$$\sigma = \frac{W_t}{bmJ} K_v K_0 K_s K_H K_B$$

σ	Tooth bending stress	K_v	Dynamic factor
W_t	Tangential force	K_0	Overload factor
b	Face width of tooth	K_s	Size factor
m	module	K_H	Load-distribution factor
J	AGMA-form factor	K_B	Rim-thickness factor

Geometry Factor J:

It is obtained from the following figure for spur gears having a 20º pressure angle and full-depth teeth



Dynamic factor K_v:

$$K_v = \left(\frac{A+\sqrt{200V}}{A}\right)^B, \quad \text{V is the tangential velocity in m/s}$$

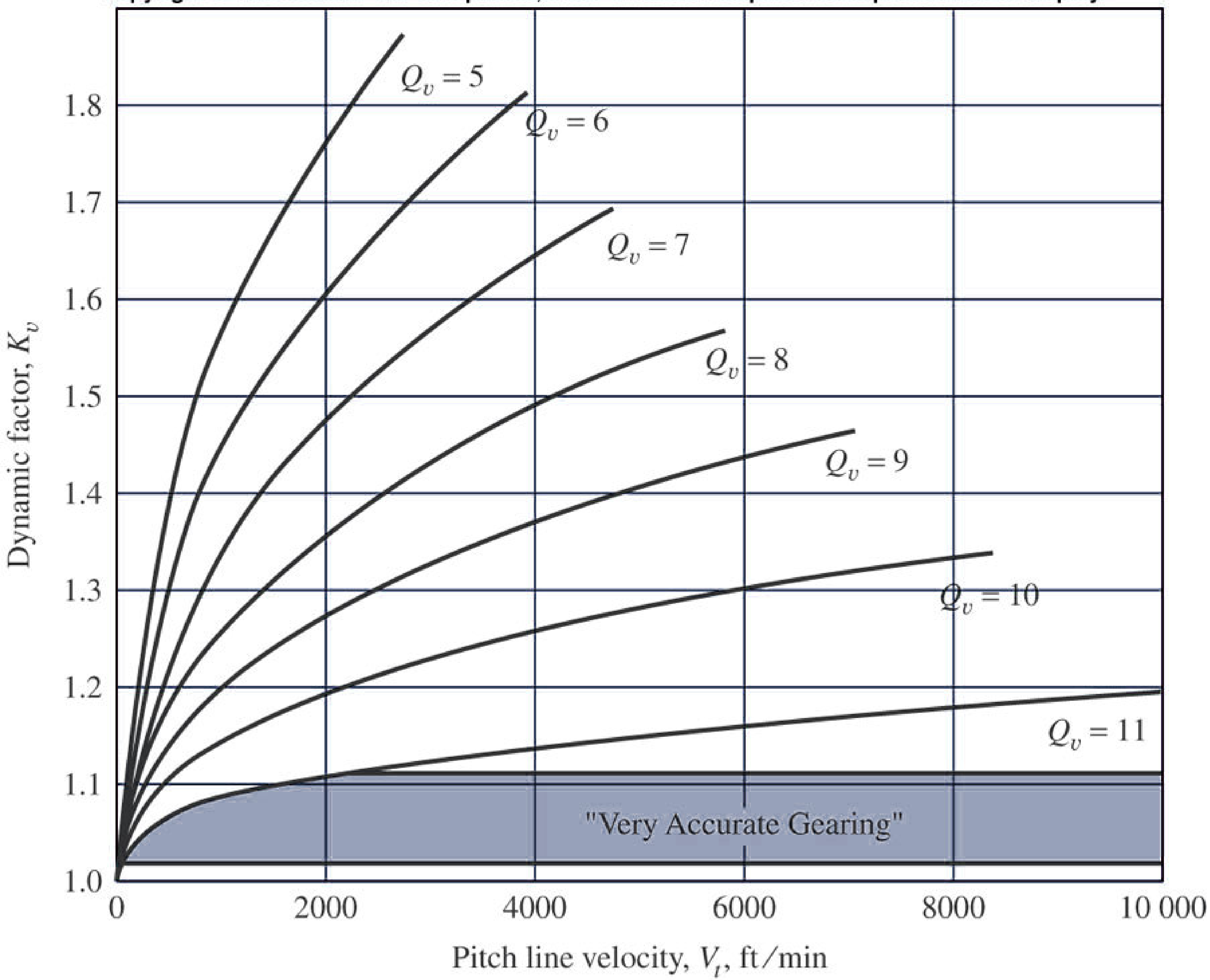
$$A = 50 + 56(1-B)$$

$$B = 0.25(12-Q_v)^{\frac{2}{3}}$$

where Q_v is the AGMA transmission accuracy-level number (quality number)

The maximum velocity, for a certain quality number, representing the end of a certain Q_v curve is:

$$(V_t)_{\max} = \frac{[A+(Q_v-3)]^2}{200} \quad \text{m/s}$$



Dynamic factor, K_v
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
0
2000
4000
6000
8000
10 000
Pitch line velocity, V_t, ft/min
$Q_v = 5$
$Q_v = 6$
$Q_v = 7$
$Q_v = 8$
$Q_v = 9$
$Q_v = 10$
$Q_v = 11$
"Very Accurate Gearing"

Overload factor, K_o

Is intended to make allowances for all externally applied loads in excess of The nominal tangential (transmitted) load W_t . It may be selected from the following table according to the type of loading:

Source of power	Driven machinery		
	Uniform	Moderate shock	Heavy shock
Uniform	1	1.25	1.75
Light shock	1.25	1.5	2.0
Medium shock	1.5	1.75	2.25

Size factor, K_s

The AGMA has not yet established standards for size factors and recommends that K_s be set to 1 unless very large teeth are used.

In this case K_s could be treated as the reciprocal of the fatigue size factor k_b.

i.e. $K_s = 1 / k_b$

where k_b can be selected from tables according to the module

Module *m*	Factor k_b	Module *m*	Factor k_b
1-2	1.000	11	0.843
2.25	0.984	12	0.836
2.5	0.974	14	0.824
2.75	0.965	16	0.813
3	0.956	18	0.804
3.5	0.942	20	0.796
4	0.930	22	0.788
4.5	0.920	25	0.779
5	0.910	28	0.770
5.5	0.902	32	0.760
6	0.894	36	0.752
7	0.881	40	0.744
8	0.870	45	0.736
9	0.860	50	0.728
10	0.851		

Load distribution factor, K_H

It modifies the stress equations to reflect non-uniform distribution of load Across the line of contact. AGMA has a detailed procedure to calculate this factor.

$$K_H = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_{mc} = 1 \qquad \text{for uncrowned teeth}$$

$$= 0.8 \qquad \text{for crowned teeth}$$

$$C_{pf} = \frac{b}{10d} - 0.025 \qquad \text{for b} <= 25\text{ mm}$$

$$= \frac{b}{10d} - 0.0375 + 0.0125b \qquad \text{for } 25 < \text{B} <= 430\text{ mm}$$

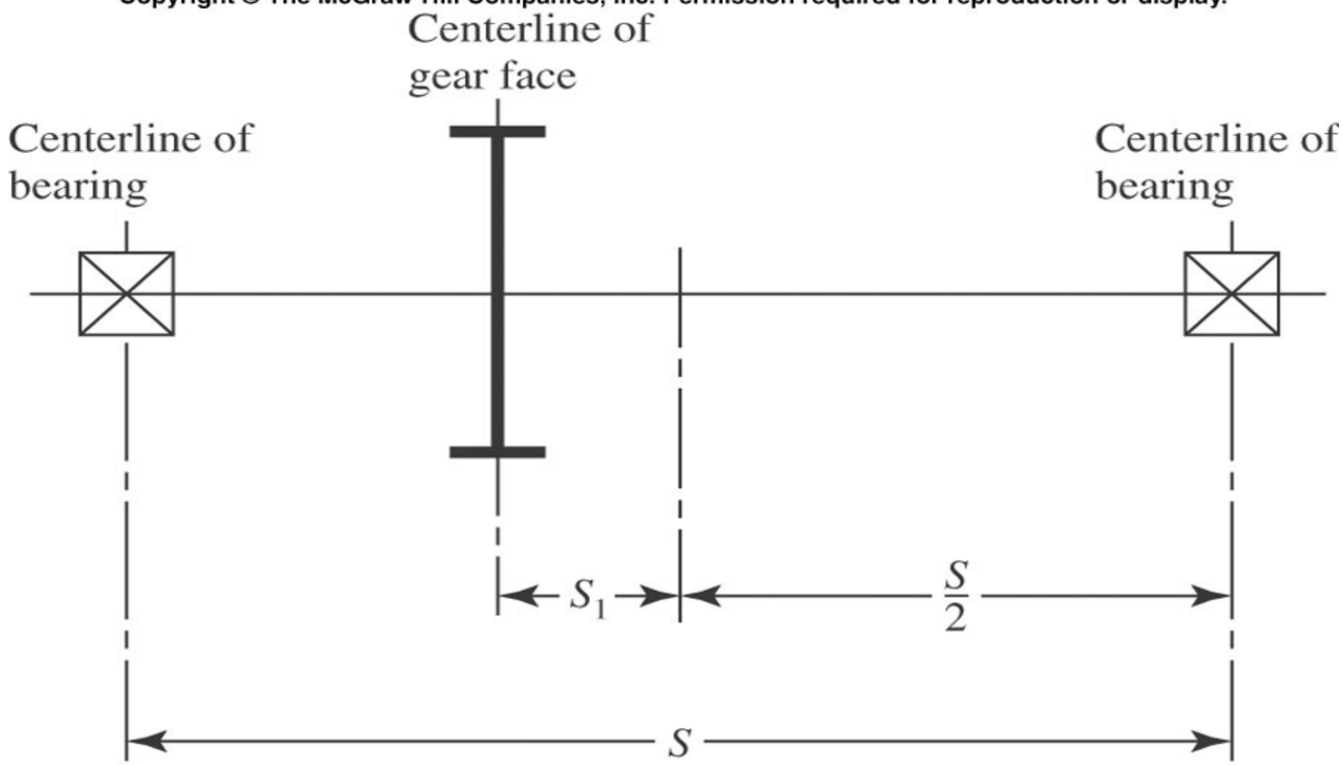
$$= \frac{b}{10d} - 0.1109 + 0.0207b - 0.000228b^2 \qquad \text{for } 430 < \text{b} < 1000\text{ mm}$$

$$\text{for values of } \frac{\text{b}}{\text{10d}} < 0.05 \text{ use } \frac{\text{b}}{\text{10d}} = 0.05$$

where b is the face width of the gear

$$C_{pm} = 1 \quad \text{for straddle-mounted pinion with } S_1/S < 0.175$$

$$= 1.1 \quad \text{for straddle-mounted pinion with } S_1/S >= 0.175$$



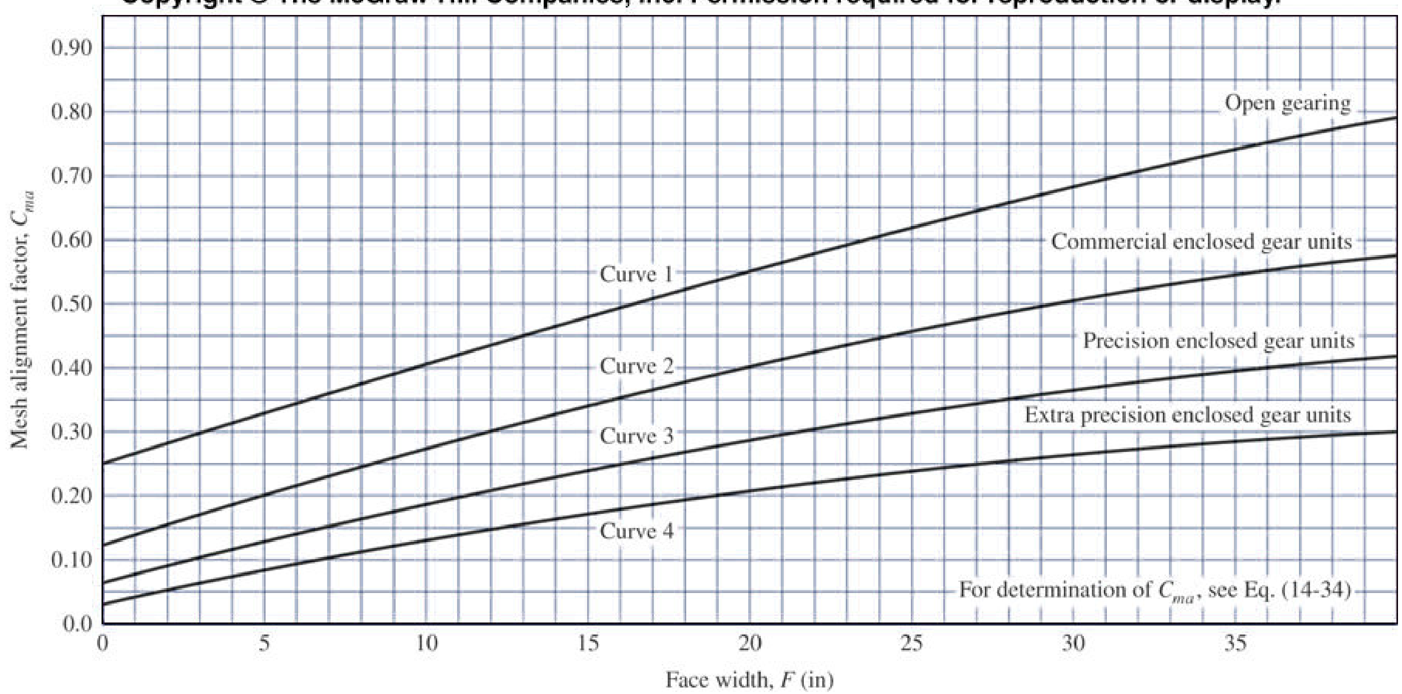
$$C_e = 0.8 \quad \text{for gearing adjusted at assembly or compatibility is improved by lapping, or both.}$$

Across

$$= 1 \quad \text{for all other conditions}$$

$$C_{ma} = A + Bb + Cb^2$$

Condition	A	B	C
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial Enclosed unit	0.127	0.0158	$-0.930(10^{-4})$
Precision Enclosed unit	0.0675	0.0128	$-0.926(10^{-4})$
Extra precision Enclosed unit	0.0036	0.0102	$-0.822(10^{-4})$



Instead of this rather complex procedure the following table can be used to estimate the value of the load distribution factor K_H.

Characteristic of support	Face Width, mm			
	0-50	150	225	400 up
Accurate mounting, small bearing clearance, minimum deflection, precision gears	1.3	1.4	1.5	1.8
Less rigid mountings, less accurate gears, contact across full face	1.6	1.7	1.8	2.2
Accuracy and mounting such that less than full face contact exists	Over 2.2			

Rim-thickness factor, K_B

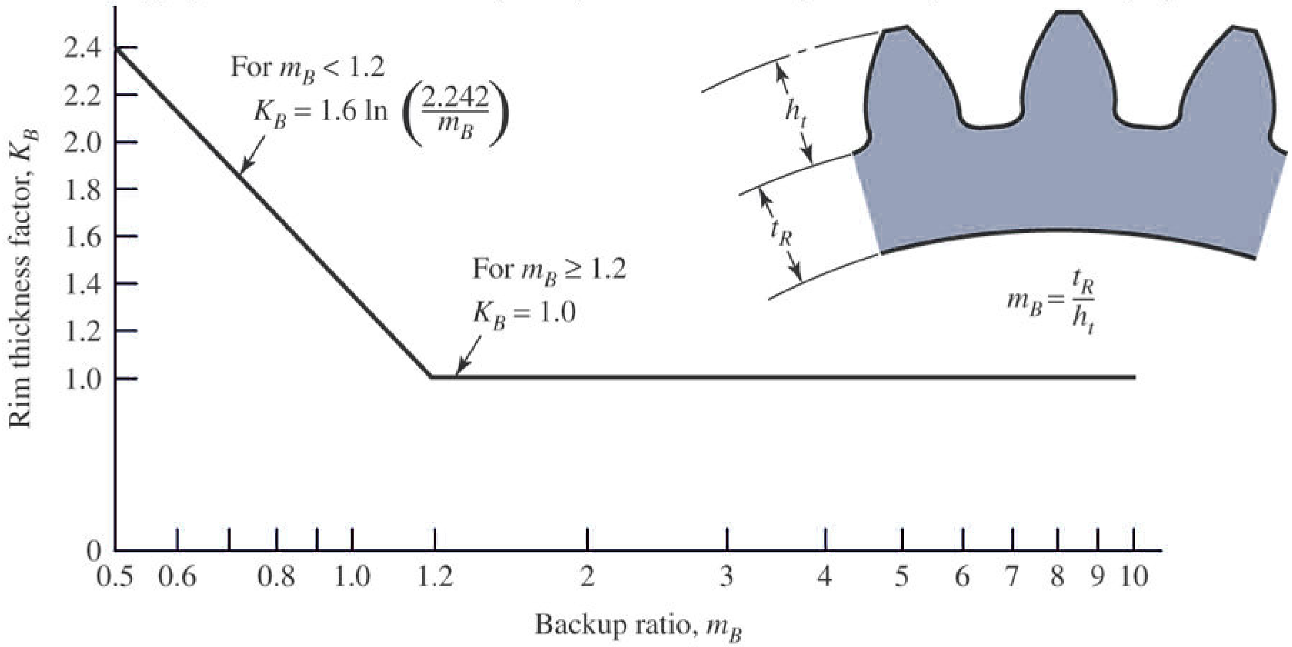
When the rim thickness is not sufficient to provide full support for the tooth root, The location of bending fatigue failure may be through the gear rather than at the tooth fillet. This factor adjusts the estimated bending stress for the Thin-rimmed gear .

$$K_B = 1.6 \ln (2.242 / m_B) \quad \text{if} \quad m_B < 1.2$$
$$= 1 \qquad m_B >= 1.2$$

Where $m_B = t_R / h_t$

t_R **rim thickness below the tooth**

h_t **the tooth height**



AGMA Strength Equations

AGMA uses allowable stress numbers instead of the strength for gear materials. These stress numbers are derived from the hardness and hardening conditions of the gear.

For example , the allowable stress number (AGMA bending strength), for through Hardening steels may be determined from the relation:

$$\sigma'_{FP} = 0.703\ H_B + 113 \qquad \text{MPa}$$

Where H_B is the Brinell hardness number

Values of stress numbers are given in Tables and figures for different Hardening conditions.

These stresses are modified by several factors that produce a limiting Value of the bending stress:

$$\sigma_{FP} = \sigma'_{FP}\ (Y_N / Y_\theta Y_Z)$$

Where Y_θ is the temp. factor

Y_Z is the reliability factor

Y_N is the stress cycle life factor, which is used to modify the strength for lives other than 10^7 cycles.

Temp. factor:

For temperatures up to 120, use $Y_\theta = 1$. For higher temperatures, the factor should be greater than unity.

Reliability factor:

Reliability	Y_Z
0.9999	1.5
0.999	1.25
0.99	1
0.9	0.85
0.5	0.7

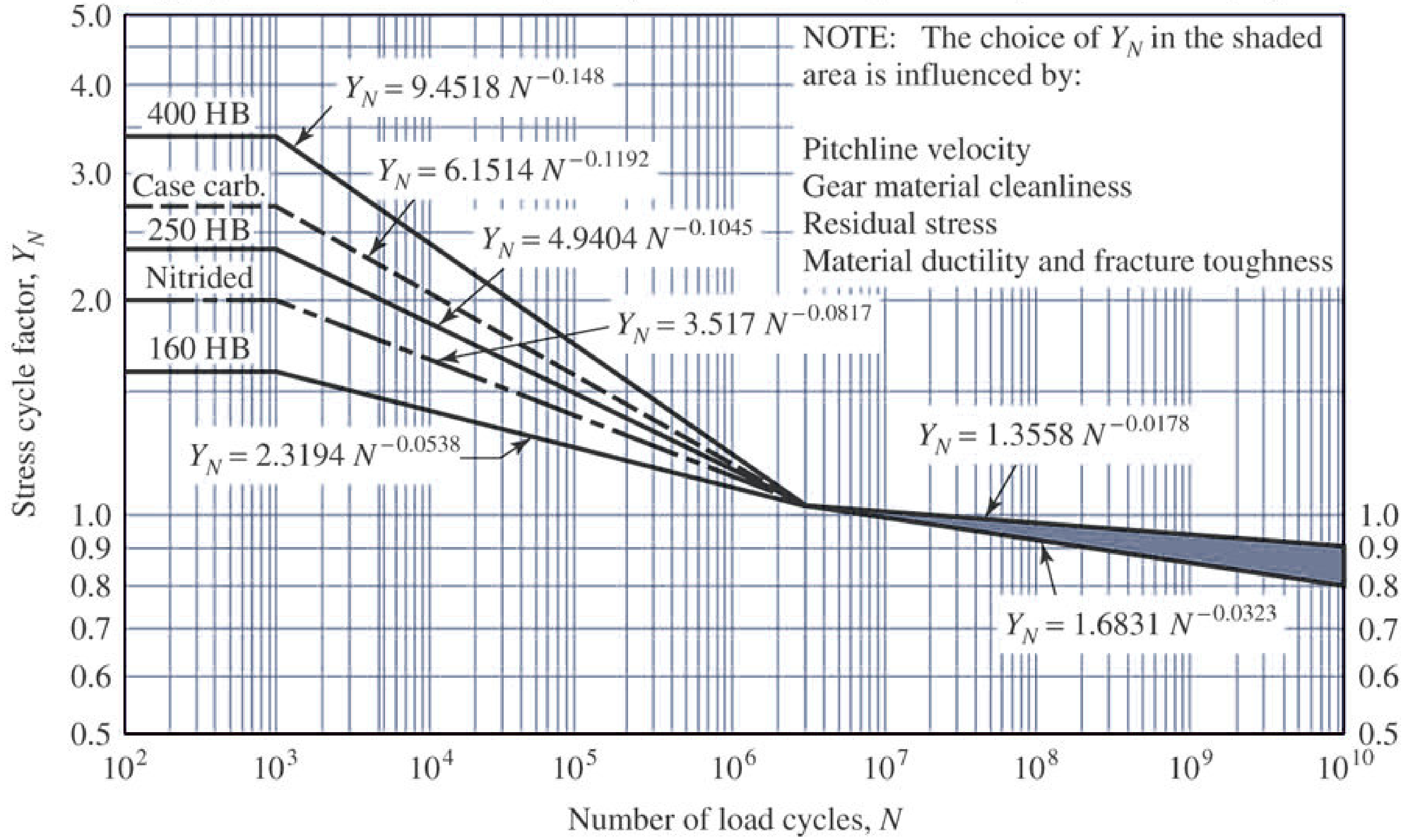
For other values of reliability, Y_Z may be interpolated by the relations:

$$Y_Z = 0.658 - 0.0759 \ln (1 - R) \qquad 0.5 < R < 0.99$$

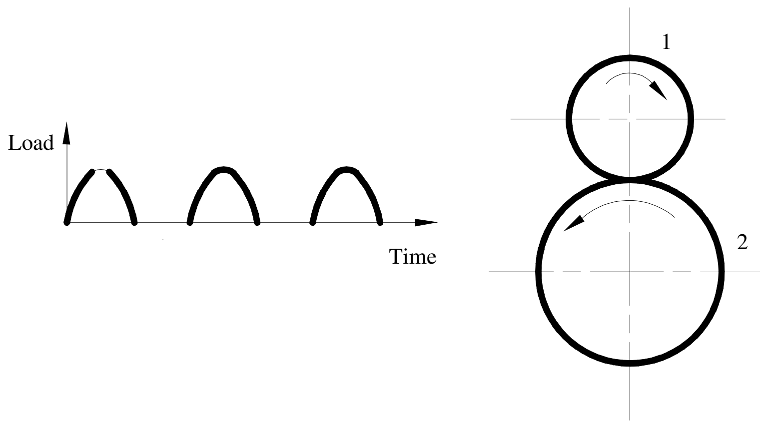
$$Y_Z = 0.5 - 0.109 \ln (1 - R) \qquad 0.99 < R < 0.9999$$

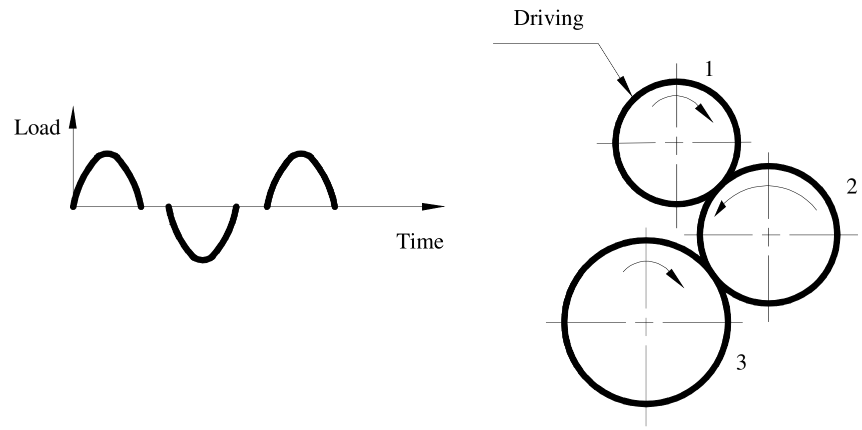
Stress cycle life factor:

The AGMA strength for bending fatigue of gear materials are based on 10^7 load cycles repeatedly applied. For lives other than 10^7 cycles, the load cycle factor is used as a modification factor. The values of this factor are Given in the following figure:



When the load cycles are reversed not repeated as in two-way bending of Gear tooth (Idler or intermediate gears), the strength is reduced to <u>0.7</u> of its corrected value.
(from 0.6 for Gerber failure criterion to 0.75 for Goodman line criterion)





<u>Repeated (one-way bending) cycles</u> **<u>Two-way bending</u>**

The safety factor guarding against bending fatigue of gear tooth is given by:

$$n = \frac{\sigma_{FP}}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}}$$

Example: A 17-tooth 20^o pressure angle spur pinion rotates at 1800 rpm and transmits 3 kW to a 52-tooth disk gear. The module is 2.5 mm, the face width is 38 mm and the quality standard is No. 6. the gears are straddle-mounted with bearings immediately adjacent. The pinion is grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is grade 1 steel, through-hardened, with a Brinell hardness of 200, tooth surface and core. Poisson's ratio is 0.3 and Young's modulus is 207 GPa. $J_p = 0.3$ and $J_G = 0.4$. The loading is smooth because of motor and load. It is a commercial enclosed gear unit and the tooth profile is uncrowned.

For a reliability of 90% and a pinion life of 10^8 cycles (use

$Y_N = 1.3558\ N^{-0.0178}$ and $Z_N = 1.4488\ N^{-0.023}$)

Find: The factor of safety of gears against bending fatigue

Solution :

$$d_p = mN_p = 2.5 \times 17 = 42.5 \text{ mm}$$

$$d_G = mN_G = 2.5 \times 52 = 130 \text{ mm}$$

$$V_t = \omega \times r = \frac{2\pi n_p}{60}\frac{d_p}{2} = \frac{2\pi \times 1800}{60}\frac{42.5}{2} = 4005.5 \text{ mm/s} = 4.005 \text{ m/s}$$

$$W_t = \frac{H}{V_t} = \frac{3 \times 1000}{4.005} = 749.06 \text{ N}$$

the dynamic factor $\mathrm{K_v}$:

$$K_v = \left(\frac{A + \sqrt{200V}}{A}\right)^B$$

$$B = 0.25(12 - Q_v)^{\frac{2}{3}} = 0.25(12 - 6)^{\frac{2}{3}} = 0.8255$$

$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.8255) = 59.772$$

$$K_v = \left(\frac{59.772 + \sqrt{200 \times 4.005}}{59.772}\right)^{0.8255} = 1.377$$

since the load is uniform $K_o = 1$

Assuming constant thickness gears $K_B = 1$

The size factor $K_s = \frac{1}{k_b} = \frac{1}{0.974} = 1.03$

The load distribution factor:

$$K_H = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$K_H = 1 + 1(0.0695 \times 1 + 0.15 \times 1) = 1.22$$

NOTE: K_H may be taken from the to be 1.3

The load cycle factor:

the speed ratio $\mathrm{m_G} = \frac{N_G}{N_p} = \frac{52}{17} = 3.059$

$$(Y_N)_p = 1.3558(10^8)^{-0.0178} = 0.977$$

$$(Y_N)_G = 1.3558\left(\frac{10^8}{3.059}\right)^{-0.0178} = 0.996$$

The bending stress :

$$\sigma = \frac{W_t}{bmJ} K_v K_0 K_s K_H K_B$$

$$\sigma_p = \frac{749.06}{38 \times 2.5 \times 0.3} 1.377 \times 1 \times 1.03 \times 1.3 \times 1$$

$$= 48.46 \quad \text{MPa}$$

$$\sigma_G = \frac{749.06}{38 \times 2.5 \times 0.4} 1.377 \times 1 \times 1.03 \times 1.3 \times 1$$

$$= 36.345 \quad \text{MPa}$$

The bending strength :

$$\sigma'_{\text{FP}} = 0.533 H_B + 88.3 \quad \text{MPa}$$

$$(\sigma'_{\text{FP}})_p = 0.533 \times 240 + 88.3 = 216.22 \quad \text{MPa}$$

$$(\sigma'_{\text{FP}})_G = 0.533 \times 200 + 88.3 = 194.9 \quad \text{MPa}$$

The corrected bending strength :

$$\sigma_{FP} = \sigma_{\text{FP}}^{\backslash} \frac{Y_N}{Y_\theta Y_Z}$$

$$(\sigma_{FP})_p = 216.22 \times \frac{0.977}{1 \times 0.85} = 248.52 \quad \text{MPa}$$

$$(\sigma_{FP})_G = 194.9 \times \frac{0.996}{1 \times 0.85} = 228.38 \quad \text{MPa}$$

The safety factor against bending fatigue :

$$n = \frac{\sigma_{FP}}{\sigma}$$

$$n_p = \frac{248.52}{48.46} = 5.12$$

$$n_G = \frac{228.38}{36.345} = 6.28$$