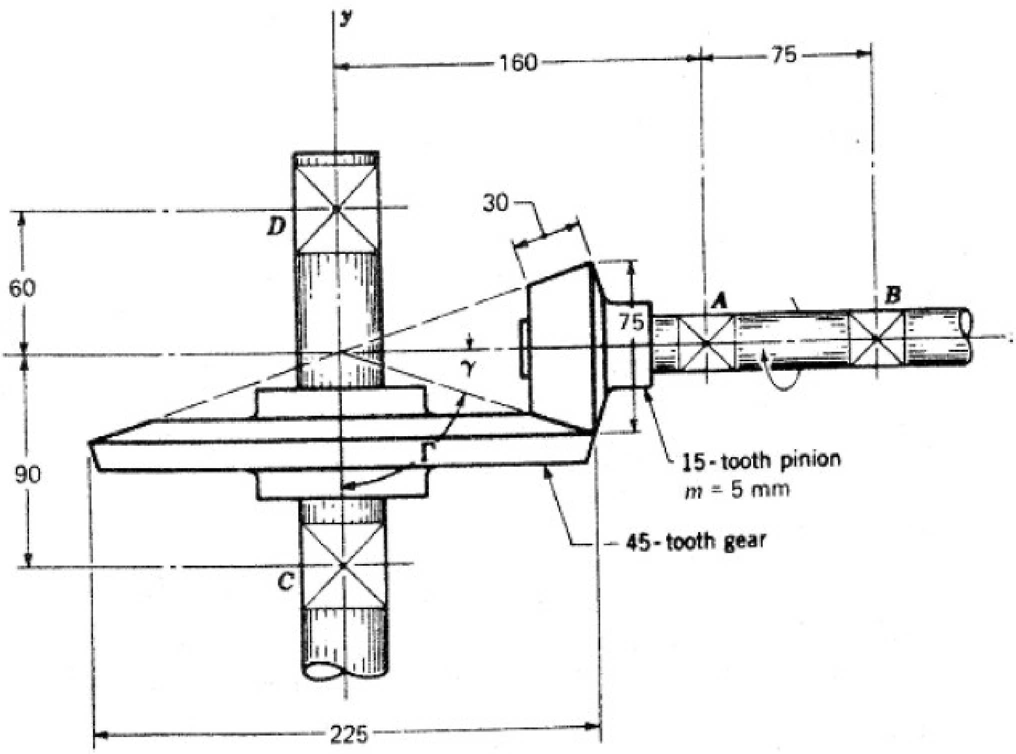
Question 1: (35 points)

The pinion of the bevel gear set shown below has 15 teeth, a module of 5 mm and a 20° pressure angle. The gear has 45 teeth and delivers 4 kW power to an oil pump at 330 rpm.

Determine: The reactions at bearings A & B of the pinion shaft.



Problem 1: $\boxed{35}$

$$i = \frac{N_g}{N_p} = \frac{45}{15} = 3$$

$$n_p = i \cdot n_g = 3 \times 330 = 990 \text{ rpm} \quad (2)$$

$$v = \frac{2\pi n}{60} \cdot \frac{d}{2}$$

$$r_{av.p} = r_p - \frac{F}{2}\sin\gamma$$

$$r_p = \frac{m N_p}{2} = \frac{5 \times 15}{2} = 37.5 \text{ mm (given)}$$

$$\tan\gamma = \frac{N_p}{N_g} = \frac{15}{45} = \frac{1}{3} \Rightarrow \gamma = 18.435^\circ \quad (2)$$

$$\therefore r_{avp} = 37.5 - \frac{30}{2}\sin(18.435) = 32.756 \text{ mm} \quad (2)$$

$$\therefore v^* = \frac{2\pi \times 990}{60} \times 32.756 = 3395.9 \text{ mm/s} = 3.396 \text{ m/s} \quad (5)$$

$$W_t^* = \frac{H}{v^*} = \frac{4 \times 1000}{3.396} = 1177.86 \text{ N} \quad (5)$$

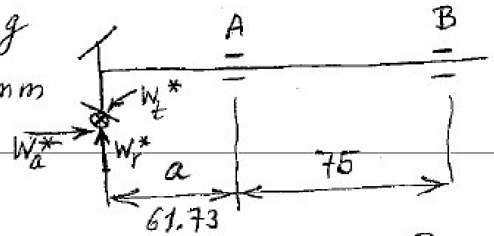
$$W_r^* = W_t^* \tan\phi \cos\gamma = 1177.86 \cdot \tan(20) \cos(18.435) = 406.7 \text{ N} \quad (2)$$

$$W_a^* = W_t^* \tan\phi \sin\gamma = 135.56 \text{ N} \quad (2)$$

$$r_{av.g} = \frac{225}{2} - \frac{30}{2}\sin(71.565) = 98.27 \text{ mm} \quad (2)$$

the distance $a = 160 - r_{av.g}$

$$\therefore a = 160 - 98.27 = 61.73 \text{ mm}$$



X-Y plane:

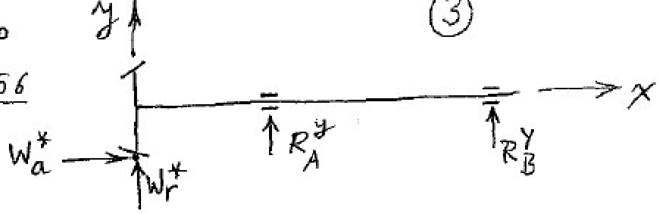
Moment about A:

$$W_r^* \times 61.73 - W_a^* \times 32.756 - R_B^Y \times 75 = 0$$

$$R_B^Y = \frac{406.7 \times 61.73 - 135.56 \times 32.756}{75}$$

$$= 275.535 \text{ N}$$

(3)

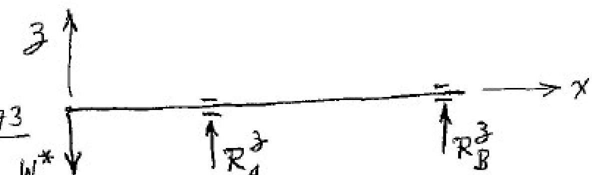


Moment about B:

$$W_r^* \times 136.73 - W_a \times 32.756 + R_A^Y \times 75 = 0$$

$$R_A^y = \frac{135.56 \times 32.756 - 406.7 \times 136.73}{75}$$

$$= -682.23 \text{ N}$$



x-z plane:

Moment about A:

$$W_t^* \times 61.73 + R_B^z \times 75 = 0$$

$$R_B^z = \frac{-1177.86 \times 61.73}{75} = -969.45 \text{ N}$$

Moment about B:

$$W_t^* \times 136.73 - R_A^z \times 75 = 0$$

$$R_A^z = \frac{1177.86 \times 136.73}{75} = 2147.31 \text{ N}$$

let Bearing B take the axial load then;

$$R_A = -682.23\, \underset{\sim}{j} + 2147.31\, \underset{\sim}{k} \qquad (4)$$

$$R_B = -135.56\, \underset{\sim}{i} + 275.535\, \underset{\sim}{j} - 969.45\, \underset{\sim}{k} \qquad (6)$$

Question 2: (50 points)

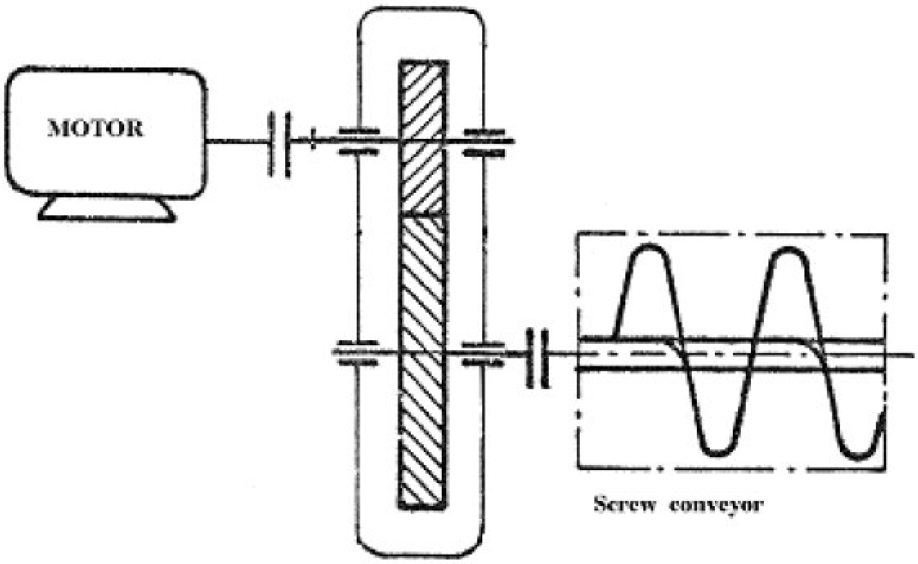
A single stage helical gear reducer is used to transmit the power of an electric motor to a screw conveyor as shown below. The number of teeth of the pinion is 18 teeth and the transmission ratio of the gear set is 3. The normal module is 4 mm and the normal pressure angle is 20°. The gears are made according to a quality number $Q_v = 6$.

a- Determine the number of teeth of the gear and the helix angle if the center distance between the input and output shafts is 154 mm.

b- The face width of the teeth is 45 mm and the gears are made of grade 1 steel through hardened to a hardness of 300 HB for the pinion and 250 HB for the gear. Determine, for a reliability of 90%, the max power that could be transmitted through this gear set if the motor speed is 1000 rpm and the gear reducer is designed to work for infinite life ($Y_N = Z_N = 0.96$), accurate mounting ($K_H = 1.3$) and uniform driving and driven conditions ($K_o = 1$).

Note: See the attached data sheets, assume any missing factor or coefficient to be unity.

Take $C_p = 191\ (MPa)^{0.5}$



Problem 2: $\boxed{50}$

a) $i = \dfrac{N_g}{N_p} \Rightarrow 3 = \dfrac{N_g}{18} \Rightarrow N_g = 54$ teeth. (2)

$$C = \frac{d_p + d_g}{2} = \frac{m_n}{\cos\psi}\left(\frac{N_g + N_p}{2}\right)$$

$$\therefore 154 = \frac{4}{\cos\psi}\left(\frac{18+54}{2}\right)$$

$$\therefore \cos\psi = 0.9351 \quad , \quad \psi = 20.76^\circ \quad (4)$$

b)

i) Bending Calculations

$$\sigma_b = \frac{W_t}{bmJ} \cdot K_v K_o K_s K_H K_B$$

$$J_p = 0.5 \times 0.982 = 0.491 \quad (2)$$

$$J_g = 0.58 \times 0.93 = 0.539 \quad (2)$$

$$Y_Z = 0.85 , \; Y_N = Z_N = 0.96 \; , \; K_o = K_H = 1 \; , K_s = 1$$

$$K_v = \left(\frac{A + \sqrt{200v}}{A}\right)^B$$

$$B = 0.25\,(12 - Q_v)^{2/3}$$

$$= 0.25\,(12-6)^{2/3} = 0.8255$$

$$A = 50 + 56\,(1-B)$$

$$= 50 + 56\,(1 - 0.8255) = 59.773$$

$$v = \frac{2\pi n_p}{60} \cdot \frac{d_p}{2} = \frac{2\pi n_p}{60} \cdot \frac{m N_p}{2}$$

$$m = \frac{m_n}{\cos\psi} = \frac{4}{\cos(20.76)} = 4.278 \quad (4)$$

$$d_p = 4.278 \times 18 = 77.004 \text{ mm}$$

$$v = \frac{2\pi \times 1000}{60} \times \frac{77.004}{2} = 4031.56 \text{ mm/s}$$

$$= 4.0316 \text{ m/s}$$

$$\therefore K_v = \left(\frac{59.773\sqrt{200 \times 4.0316}}{59.773}\right)^{0.8255}$$

$$K_v = 1.3783 \quad (4)$$

Bending Strength:

$$\sigma'_{FP} = 0.703\ HB + 113$$

$$\sigma'_{FP_p} = 0.703 \times 300 + 113 = 323.9 \text{ MPa} \quad 1$$

$$\sigma_{FP_p} = \sigma'_{FP_p}\left(\frac{Y_N}{Y_\theta Y_Z}\right) = 323.9\left(\frac{0.96}{1 \times 0.85}\right) = 365.82 \text{ MPa} \quad 2$$

$$\sigma'_{FP_g} = 0.703 \times 250 + 113 = 288.75 \text{ MPa} \quad 1$$

$$\sigma_{FP_g} = 288.75\left(\frac{0.96}{1 \times 0.85}\right) = 326.12 \text{ MPa}. \quad 2$$

let $\sigma_b = \sigma_{FP} = \frac{W_t}{b\,m\,J} \cdot K_v \cdot K_o \cdot K_S \cdot K_H \cdot K_B$

$$\therefore W^b_{t_p} = \frac{365.82 \times 45 \times 4.278 \times 0.491}{1.3783 \times 1 \times 1 \times 1.3 \times 1} = 19298.12 \text{ N} \quad 3$$

$$W^b_{t_g} = \frac{326.12 \times 45 \times 4.278 \times 0.539}{1.3783 \times 1 \times 1 \times 1.3 \times 1} = 18885.66 \text{ N} \quad 3$$

ii) Contact Stress Calculations:

$$\sigma_c = C_p \sqrt{\frac{W_t}{b\,d_p\,I} K_v K_o K_S K_H C_f} \quad , \quad C_f = 1$$

$$I = \frac{\cos\phi_t \sin\phi_t}{2\,m_N} \cdot \frac{i}{i+1}$$

$$m_N = \frac{p_N}{0.95\,Z}$$

$$p_N = p_m \cos\phi_m = \pi\,m_m \cos\phi_m = 4\pi\cos(20^\circ) = 11.808 \text{ mm}$$

$$\tan\phi_m = \tan\phi_t \cos\psi$$

$$\therefore \phi_t = \tan^{-1}\left(\frac{\tan\phi_m}{\cos\psi}\right) = 21.25^\circ$$

$$Z = \sqrt{\left(\frac{4.278 \times 18}{2} + 4\right)^2 - \left(\frac{4.278 \times 18}{2}\cos(21.25^\circ)\right)^2} + \sqrt{\left(\frac{4.278 \times 54}{2} + 4\right)^2 - \left(\frac{4.278 \times 54}{2}\cos(21.25^\circ)\right)^2} - 154\sin(21.25^\circ)$$

$$Z = 18.858 \text{ mm} \quad 2$$

$$\therefore m_N = \frac{11.808}{0.95 \times 18.858} = 0.6591 \quad (1)$$

$$\therefore I = \frac{0.932 \times 0.3624}{2 \times 0.6591} \cdot \frac{3}{(3+1)} = 0.192 \quad (2)$$

Contact Strength:

$$\sigma_{HP}' = 2.22\, HB + 200$$

$$\sigma_{HP_p}' = 2.22 \times 300 + 200 = 866 \text{ MPa} \quad (1)$$

$$\sigma_{HP_p} = \sigma_{HP_p}' \frac{Z_N C_H}{Y_\theta Y_Z} = 866 \frac{0.96 \times 1}{1 \times 0.85} = 978.07 \text{ MPa.} \quad (2)$$

$$\sigma_{HP_g}' = 2.22(250) + 200 = 755 \text{ MPa} \quad (1)$$

$$\sigma_{HP_g} = 755 \left(\frac{0.96 \times 1}{1 \times 0.85} \right) = 852.706 \text{ MPa.} \quad (2)$$

let $\sigma_c = \sigma_{HP}$

$$\therefore W_{t_p}^c = \frac{45 \times (4.278 \times 18) \times 0.192 \times (978.07)^2}{(191)^2 \times 1.3783 \times 1 \times 1 \times 1.3 \times 1} = 9736.7 \text{ N} \quad (3)$$

$$W_{t_g}^c = \frac{45 \times 77.004 \times 0.192 \times (852.706)^2}{(191)^2 \times 1.3783 \times 1 \times 1 \times 1.3 \times 1} = 7400.34 \text{ N} \quad (3)$$

The max power that can be transmitted:

$$H_{max} = \min.\left(W_{t_p}^b, W_{t_g}^b, W_{t_p}^c, W_{t_g}^c\right) \times v$$

$$= 7400.34 \times 4.0316$$

$$= 29836.58 \text{ W}$$

$$= 29.836 \text{ kW} \quad (3)$$

Question 3: (points)

The deep groove ball bearing # 6316 was selected for a certain application where the shaft was rotating at 2000 rpm and the equivalent radial load on this bearing was 11900 N. The dynamic loading capacity of the bearing was found to be 94300 N.

a- Find the expected service life of this bearing.

b- What would be the maximum equivalent load that this bearing could support for a service life of 12000 hours if the shaft is rotating at 1500 rpm.

Problem 3 : $\boxed{15}$

a) $$C_R = F_{eq}\left[\frac{L_D}{L_R}\cdot\frac{n_D}{n_R}\right]^{1/a}$$

$L_R = 500$ hrs

$n_R = 100/3$ rpm

$a = 3$

$$\therefore \quad 94300 = 11900\left[\frac{L_D}{500}\cdot\frac{2000}{(100/3)}\right]^{1/3}$$

$\therefore \; L_D = 4146.79$ hrs. (8)

b) $$94300 = F_{eq}\left[\frac{12000}{500}\cdot\frac{1500}{(100/3)}\right]^{1/3}$$

$\therefore \; F_{eq} = 9191.16$ N (7)