Numerical Analysis - ENME 602
Engineering and Materials Science
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Spring 2021

Practice Sheet 2a

Algorithms & Solutions of Non-Linear Equations

Bisection Method

Problem 1

1. Use the **Bisection** method to find p_3 for $f(x) = \sqrt{x} - \cos x$ on $[0, 1]$.
2. Let $f(x) = 3(x+1)(x-\frac{1}{2})(x-1)$. Use the **Bisection** method on the following intervals to find p_3.

   a. $[-2, 1.5]$ **b.** $[-1.25, 2.5]$

3. Use the **Bisection** method to find solutions accurate to within 10^{-2} for $x^3 - 7x^2 + 14x - 6 = 0$ on each interval.

   a. $[0, 1]$ **b.** $[1, 3.2]$ **c.** $[3.2, 4]$

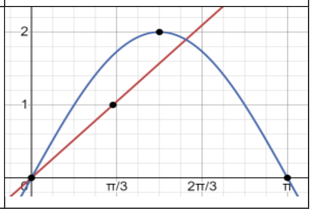
Solution:

Question	p_n
1	0.625
2.a	-0.6875
2.b	1.09376
3.a	0.5859375
3.b	3.00234375
3.c	3.41875

Problem 2

1. Sketch the graphs of $y = x$ and $y = 2\sin x$.
2. Use the **Bisection** method to find an approximation to within 10^{-5} to the first positive value of x with $x = 2\sin x$.

Solution:

<table>
<tr><th>Question</th><th>Solution</th></tr>
<tr><td>1</td><td>
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Problem 3

1. Find an approximation to $\sqrt[3]{25}$ correct to within 10^{-4} using the **Bisection** Algorithm on $[2, 3]$.
2. Use Theorem **2.1** to find a bound for the number of iterations needed to achieve an approximation with accuracy 10^{-3} to the solution of $x^3 + x - 4 = 0$ lying in the interval $[1, 4]$. Find an approximation to the root with this degree of accuracy.

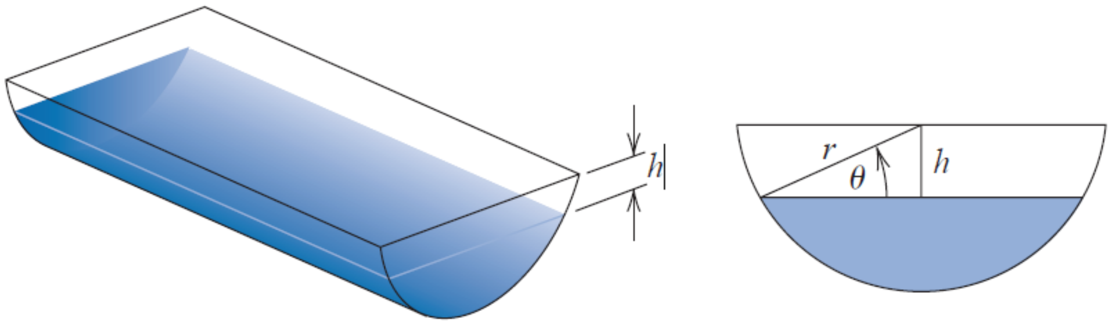
Solution:

Question	Solution
1.$\mathbf{p_n}$	2.92401
2.NoOfIter	12
2.$\mathbf{p_n}$	1.378662109375

Problem 4

A trough of length L has a cross section in the shape of a semicircle with radius r (see the accompanying figure). When filled with water to within a distance h of the top, the volume V of the water is

$$V = L\left[0.5\pi r^2 - r^2 \arcsin\left(\frac{h}{r}\right) - h(r^2 - h^2)^{1/2}\right]$$



Suppose $L = 10\,\text{ft}, r = 1\,\text{ft}$ and $V = 10.4\,\text{ft}^3$. Find the depth of water in the trough to within 0.01 ft.

Solution:

Question	Solution
1.Interval	[1 4]
2.$\mathbf{p_n}$	0.838

Algorithms

Problem 5

Write an algorithm to sum the finite series $\sum_{i=1}^{N} x_i$ in reverse order.