Theory of gearing
And
Basic definitions

Theory of gearing:

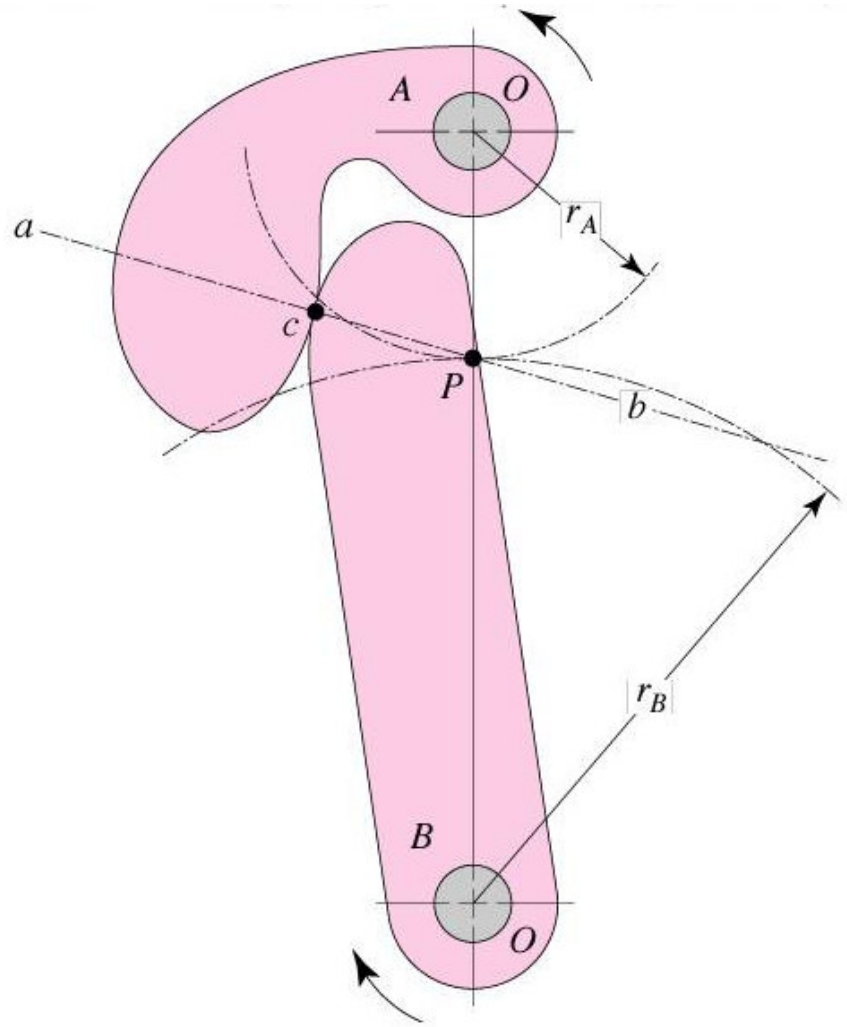
The fundamental law of gearing states that: the angular velocity ratio between the gears of a gear set must remain constant throughout the mesh.

Or

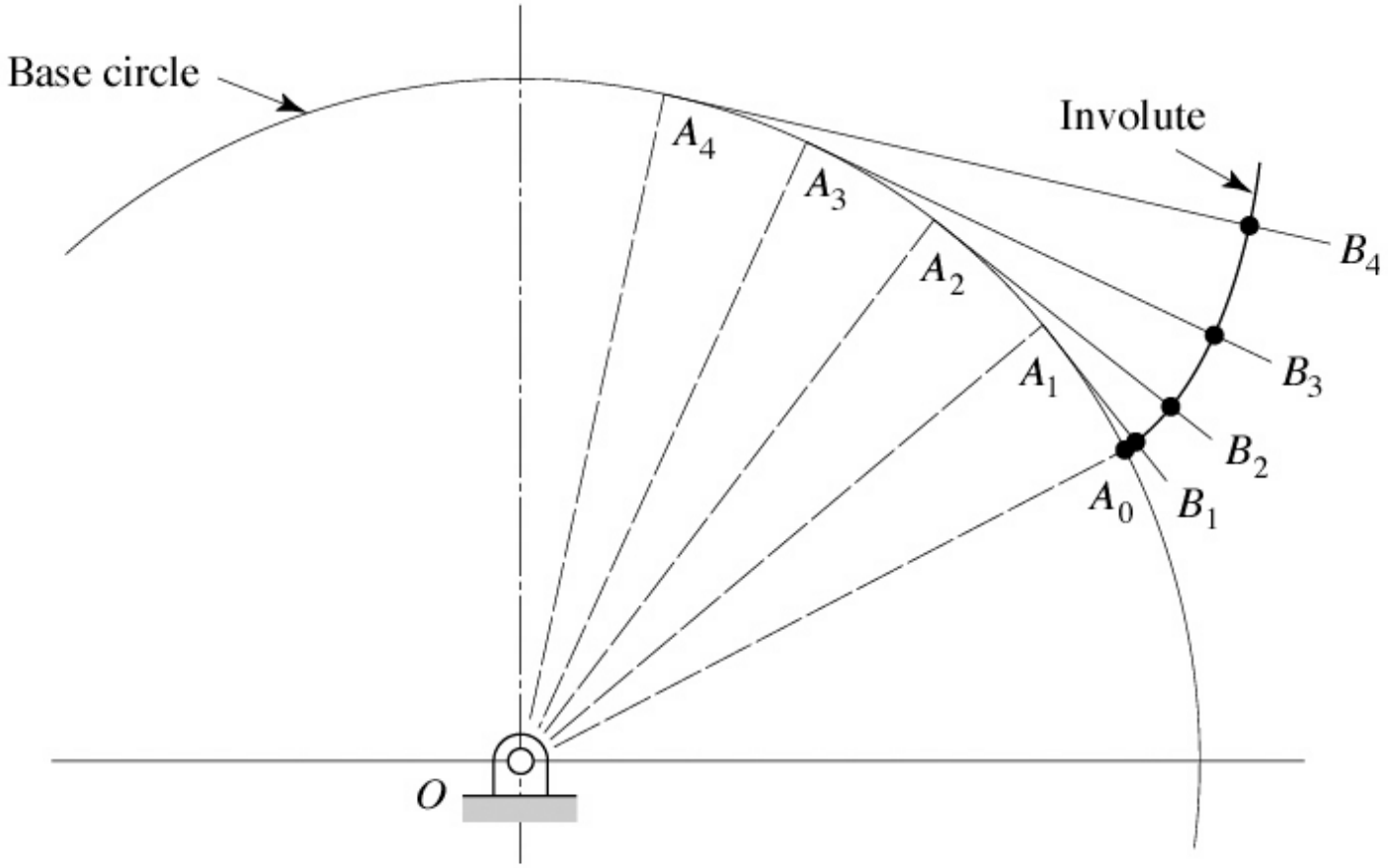
The common normal of the tooth profiles, at all contact points within the mesh, must always pass through a fixed point on the line of centers, called the pitch point

When the tooth profiles are designed so as to produce a constant angular velocity ratio during meshing, these are said to have a conjugate action.

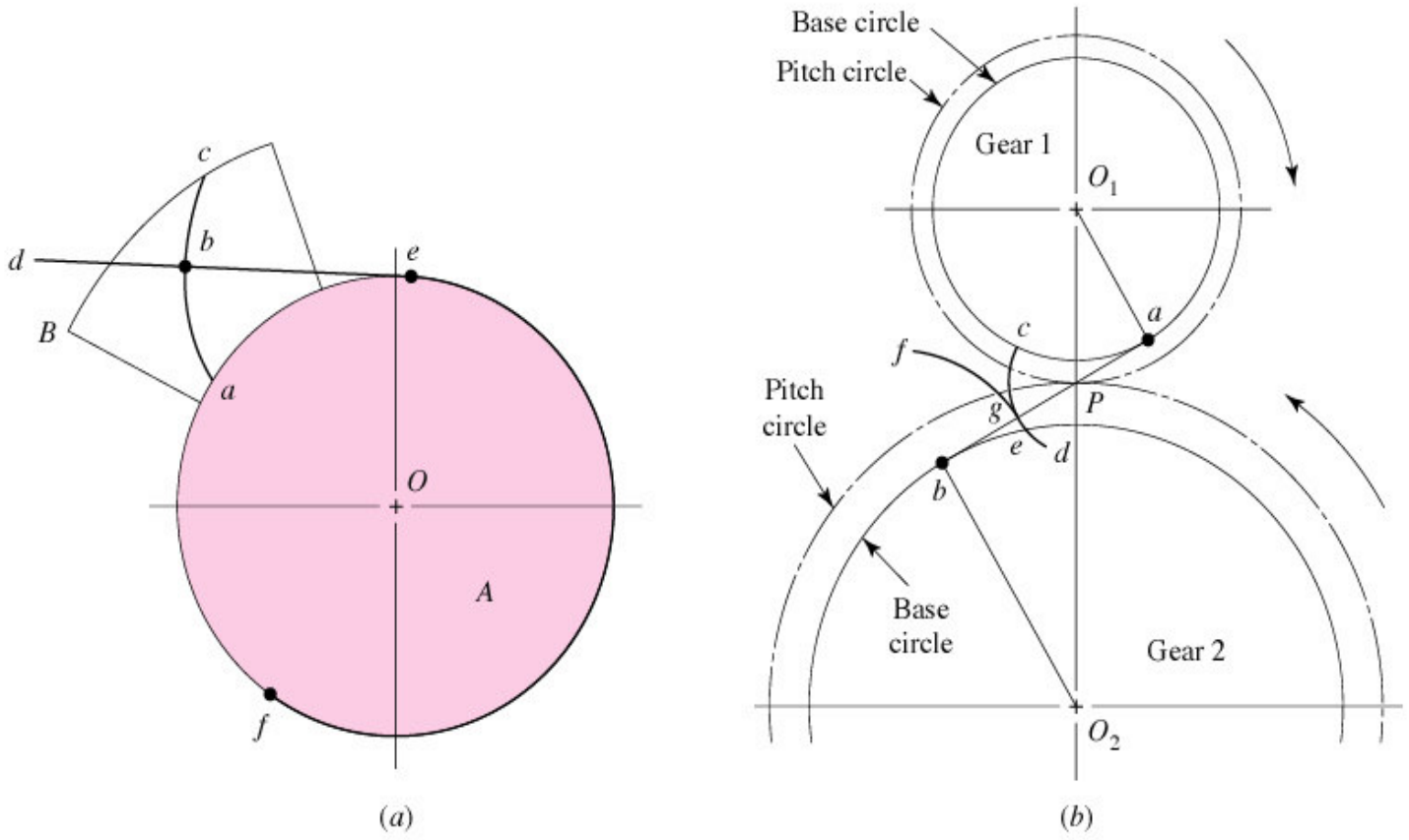
Example: involute profile



Construction of an involute curve

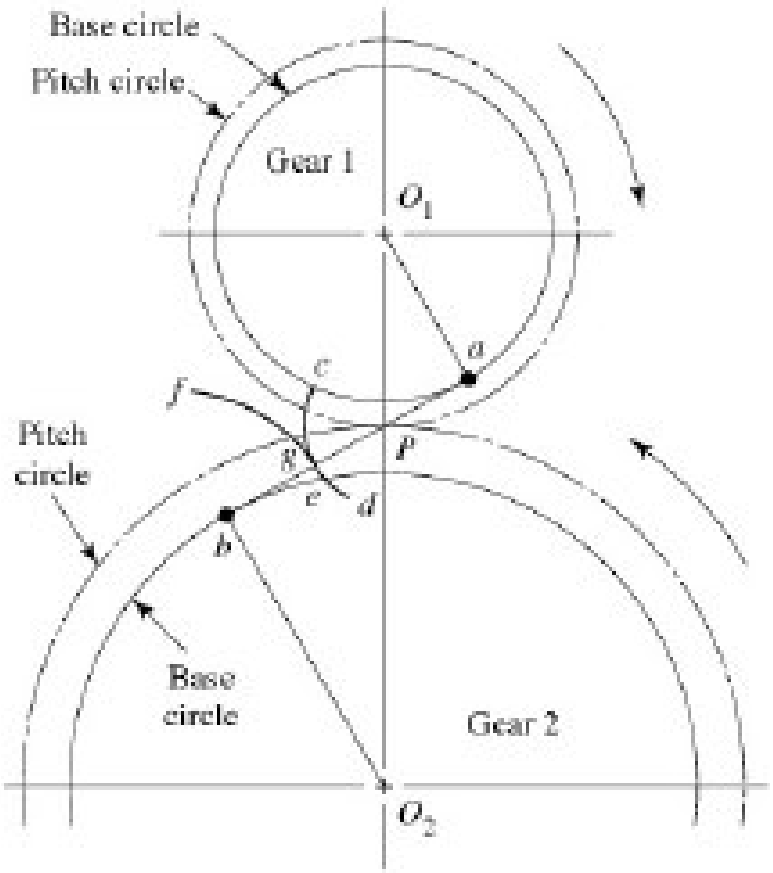


involute action:

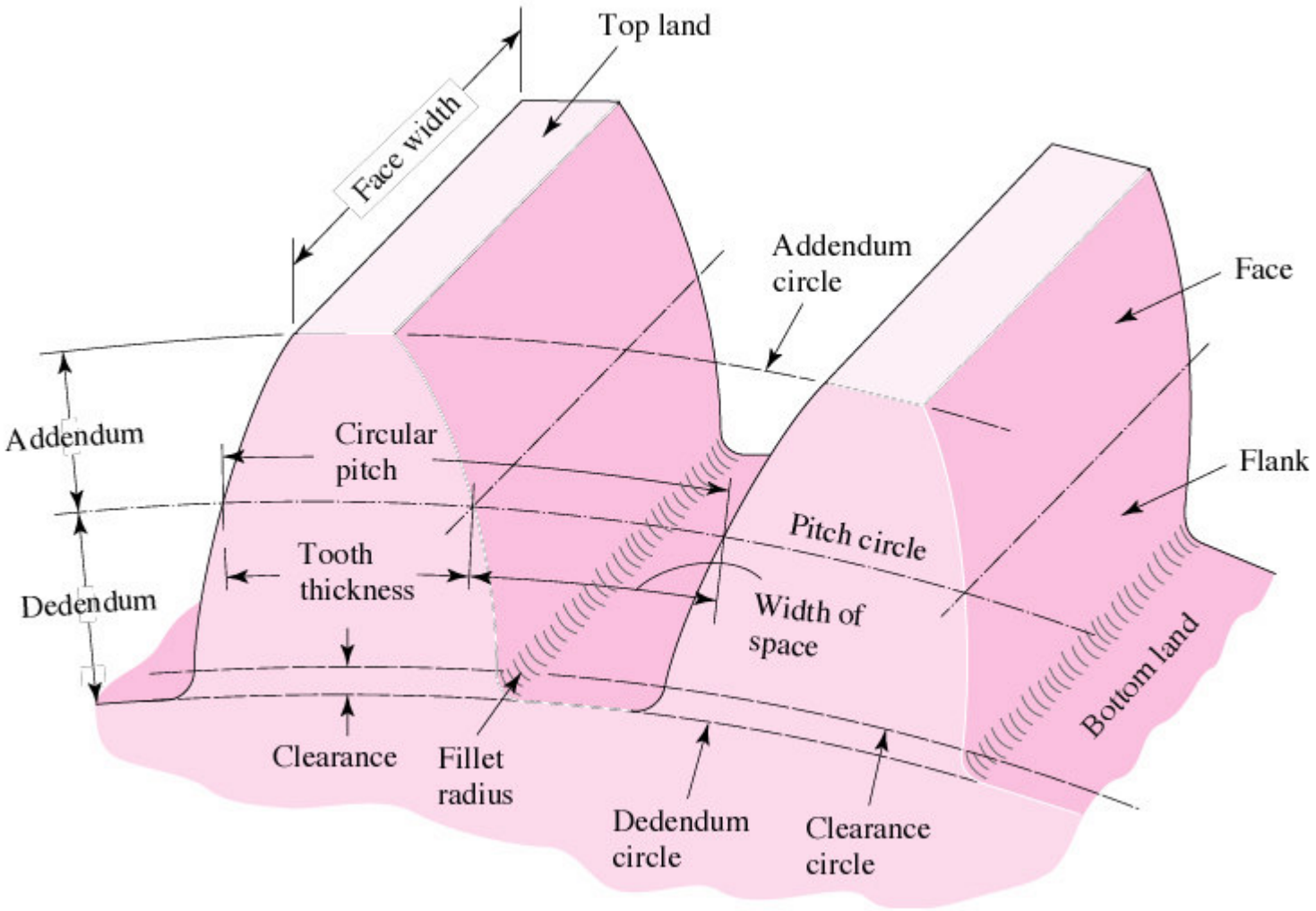


Involute Properties

- The circle on which the involute is generated is called the *base circle.*
- The point of contact (g) moves along the generating line (line of action); the generating line does not change position, because it is always tangent to the base circles.
- And since the generating line is always normal to the involutes at the point of contact, the requirement for uniform motion is satisfied.



Basic Definition:



As shown in figure , the following terms can be defined:

1- The pitch circle:

•Is the surface of the theoretical rolling cylinder (Tangent to each other).

(The pitch circle is a theoretical surface upon which all calculation are usually based.)

2- The module (m):

Is the ratio of the pitch circle diameter to the number of teeth

$$(m = d / N \text{ or } d = m N)$$

3- The circular pitch (P):

Is the distance measured on the pitch circle from a point on one tooth to a corresponding point on an adjacent tooth.

(P = tooth thickness + space width).

4- The addendum (a): (Added on):

Is the radial distance between the top land and the pitch circle.

5- The dedendum (b): (Deduced from):

Is the radial distance from the bottom land to the pitch circle.

6- The clearance (C):

Is the amount by which the dedendum in a given gear exceeded the addendum of its mating gear.

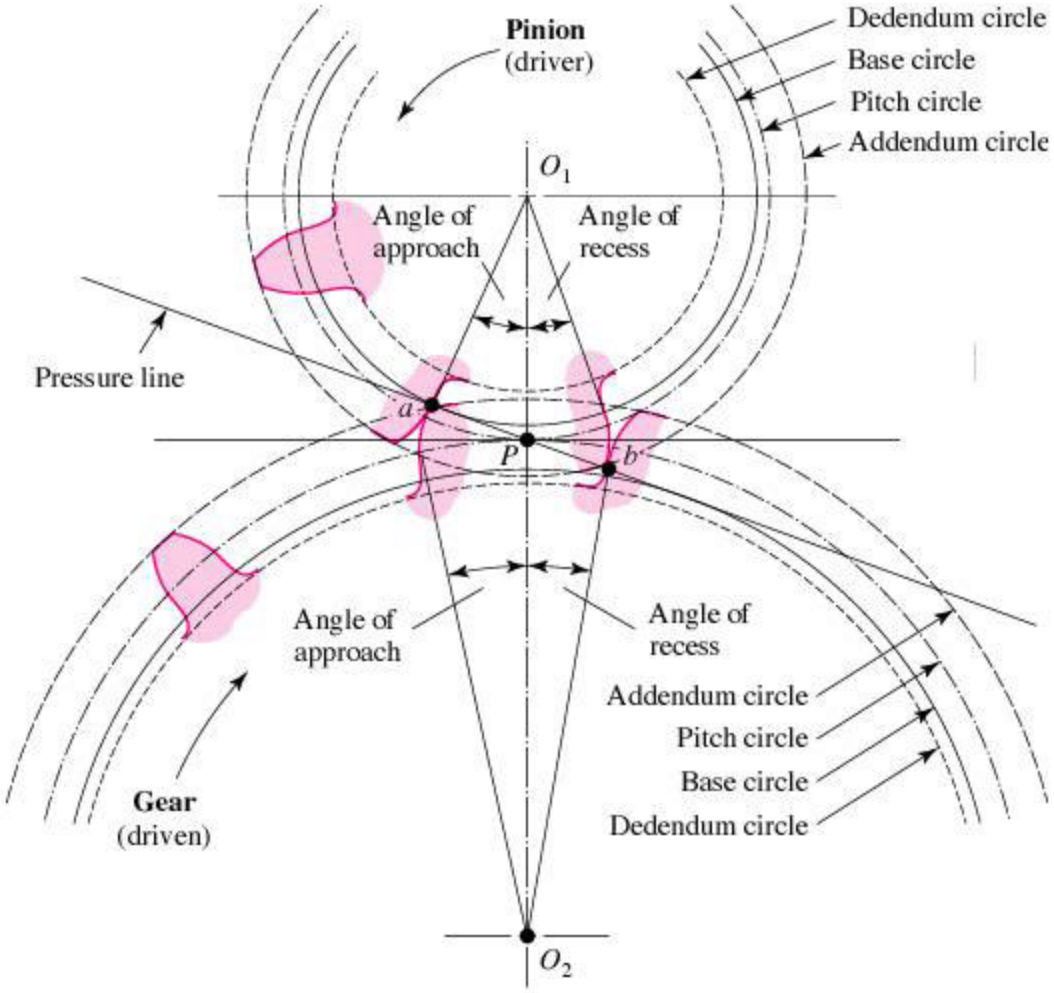
7- Addendum circle:

$$d_a = d + 2a$$
$$= d + 2m$$
$$= m\,N + 2m$$

8- Dedendum circle:

$$d_d = d - 2b$$
$$= d - 2\,(1.25\,m)$$
$$= m\,N - 2.5\,m$$

Mesh geometry:



Is the tangent to both base circles of the pinion and gear.

It represents the direction in which the resultant force acts between the gears.

- The pressure angle (φ):

  Is the angle between the pressure line and the pitch line.

  ($\varphi = 14.5^{o}, 20^{o}, 25^{o}$)

- Base circle diameter:

$$d_b = d \cos\varphi$$

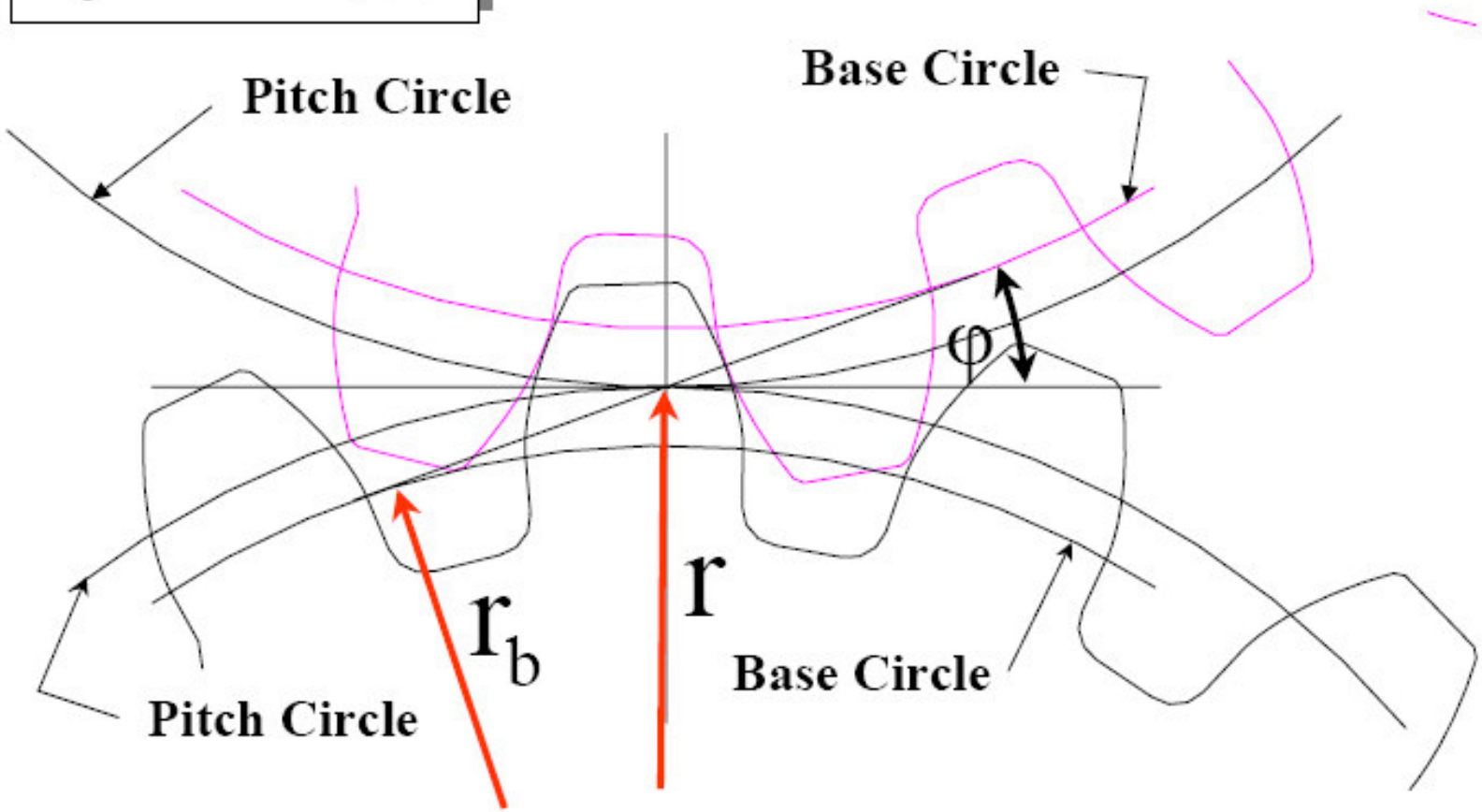
- Pressure line:

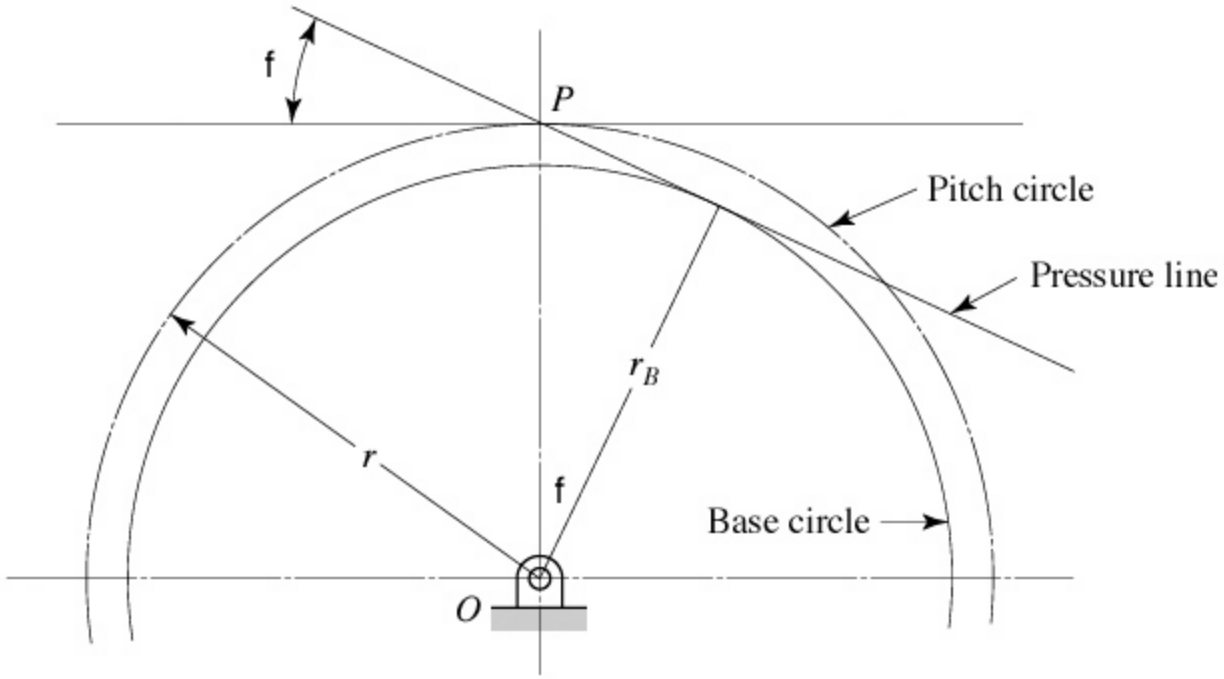
Note:

Involute profile enables changing the center distance between gears without changing the speed ratio. Only a new pitch circle should be considered.

Relationship Between Pitch and Base Circles

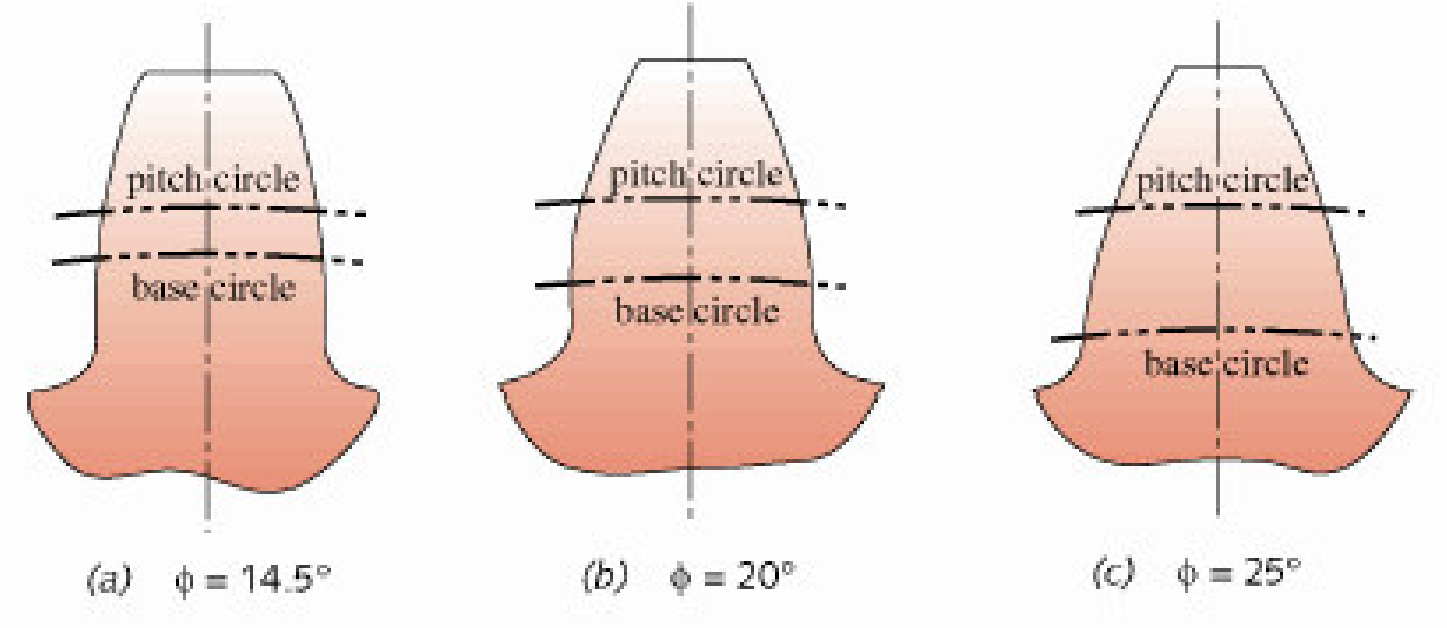
$$r_b = r\cos(\varphi)$$





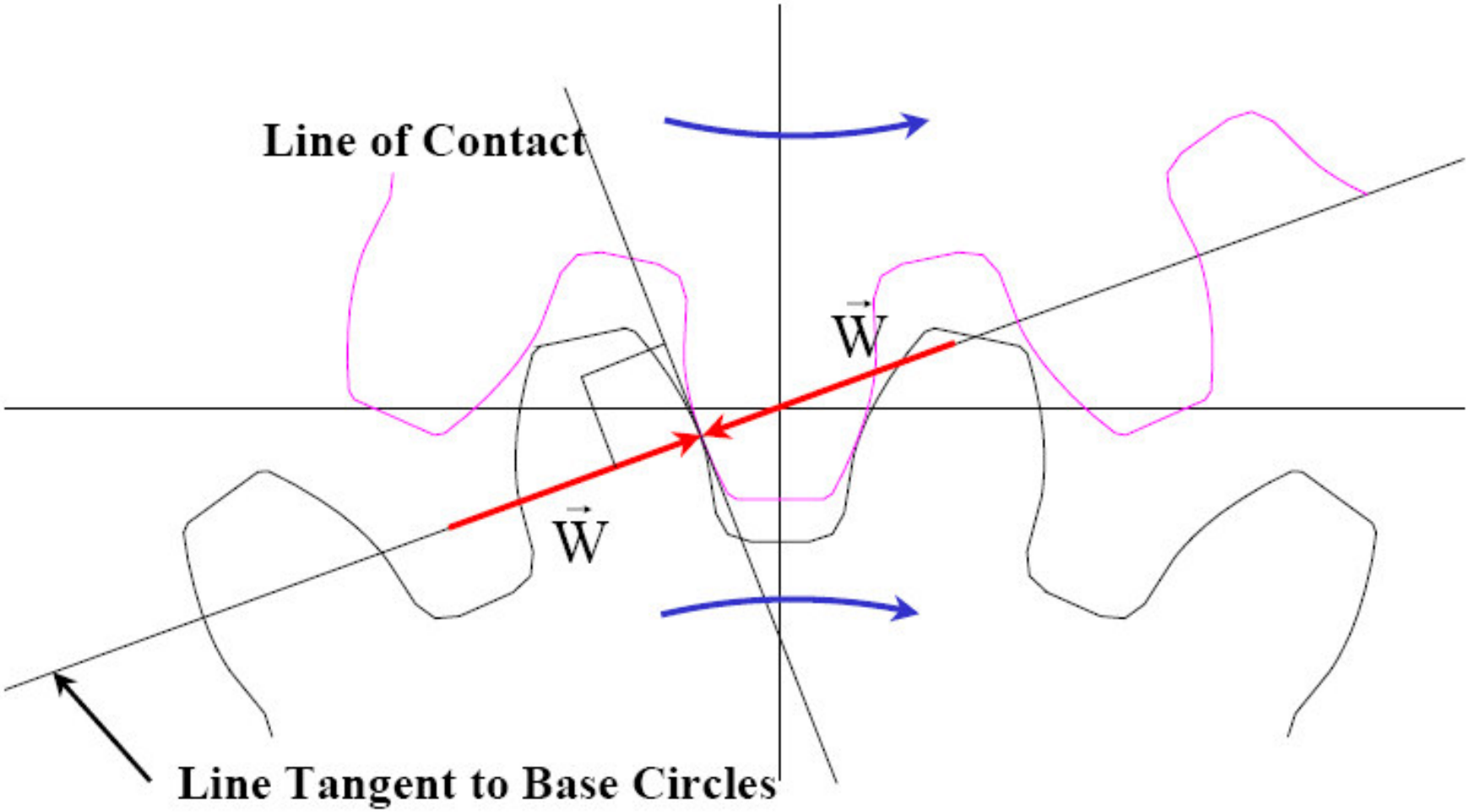
Tooth Profiles for Three Pressure Angles

- Larger the pressure angle, greater difference between pitch circle and base circle

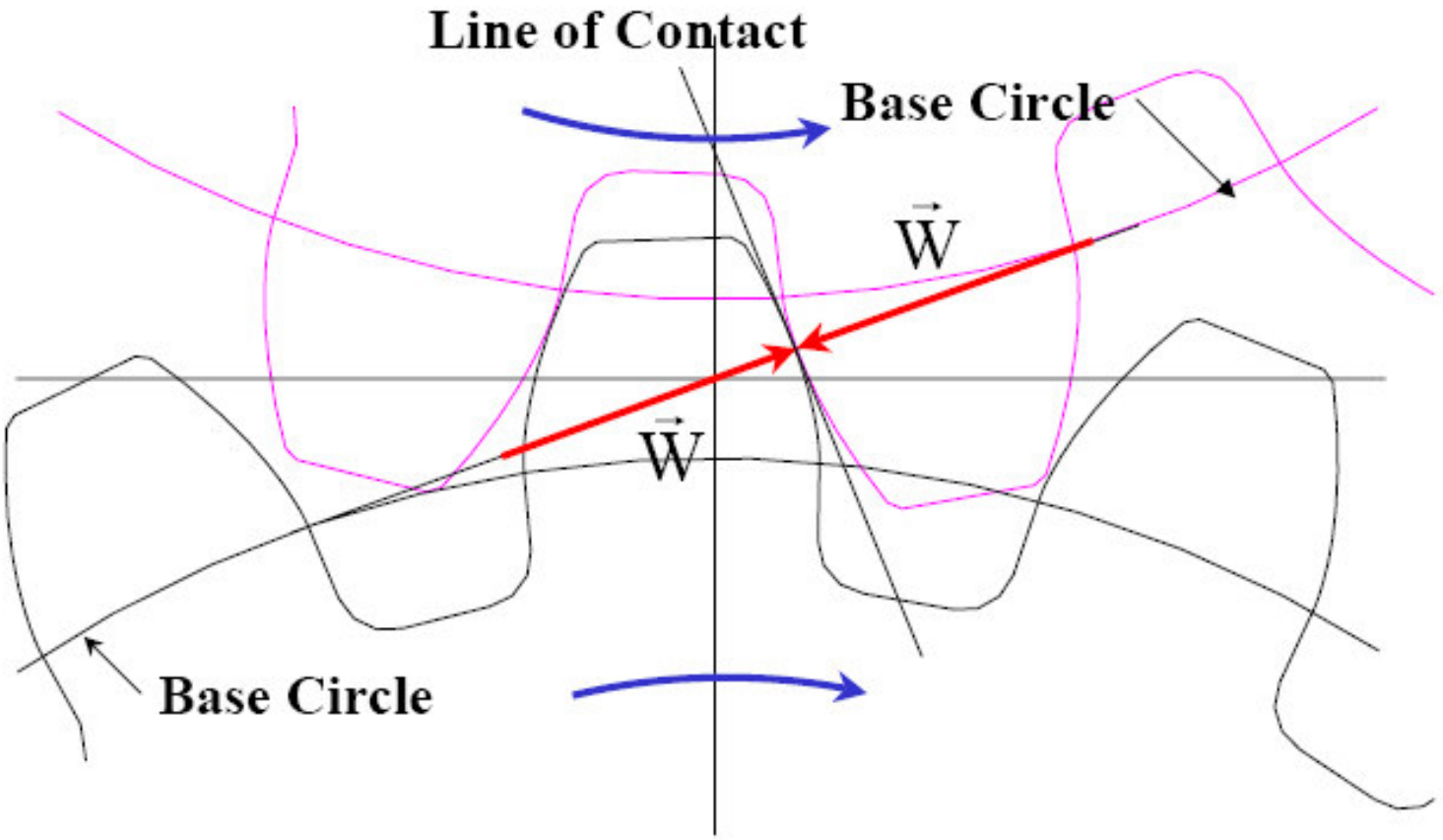


(a) $\phi = 14.5°$ (b) $\phi = 20°$ (c) $\phi = 25°$

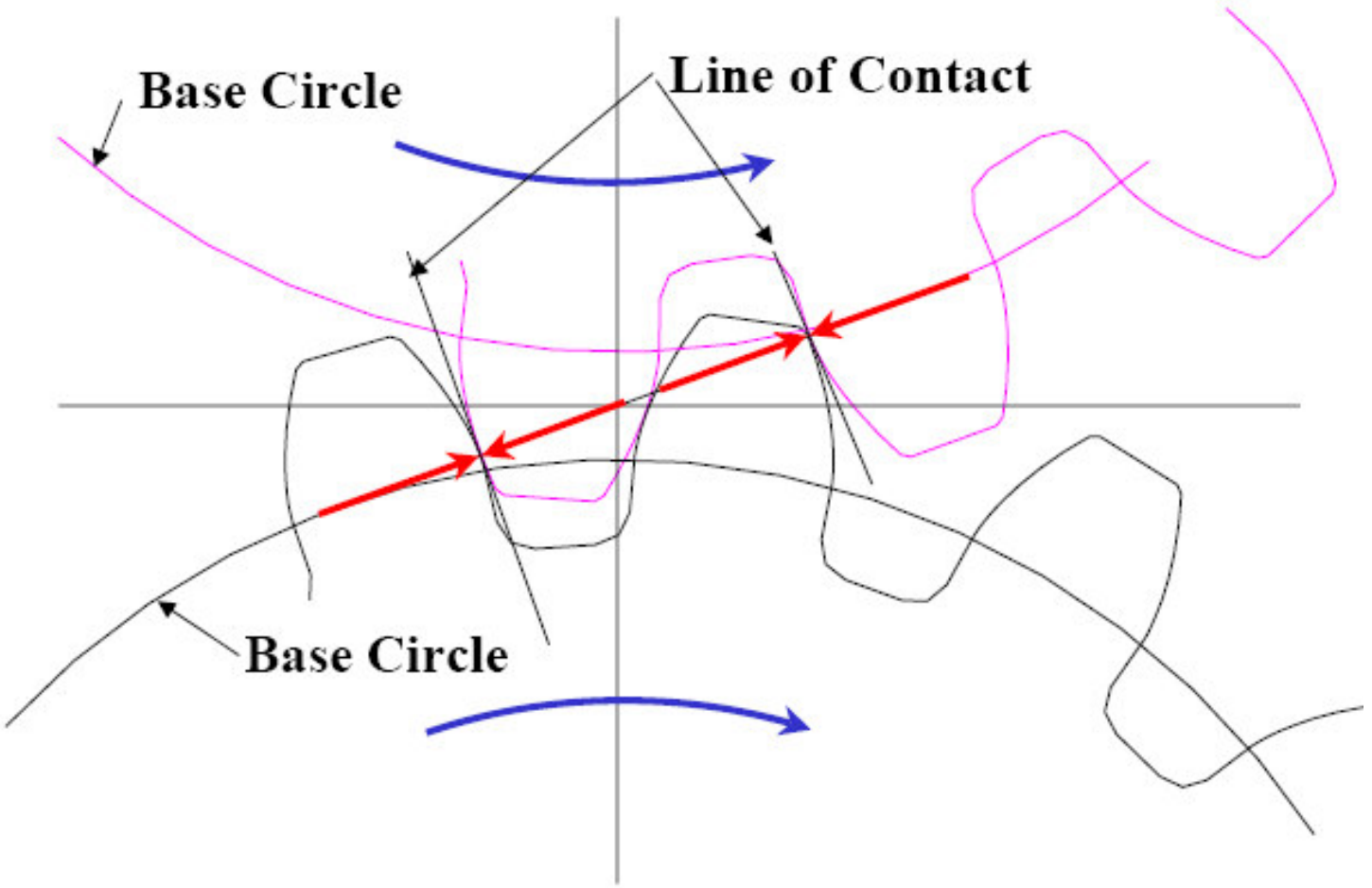
1st Close Up of Meshed Teeth



2nd Close Up of Meshed Teeth



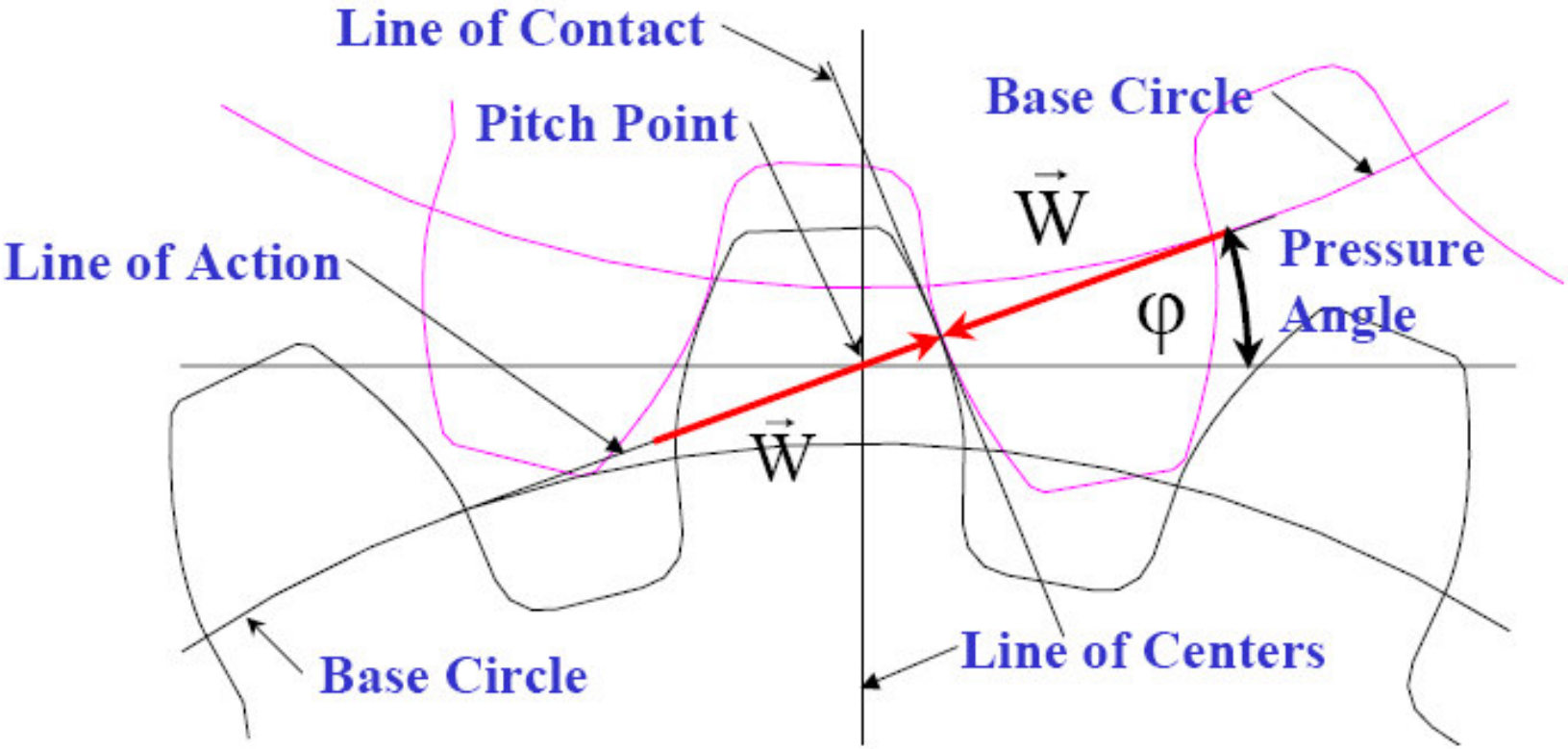
3rd Close Up of Meshed Teeth

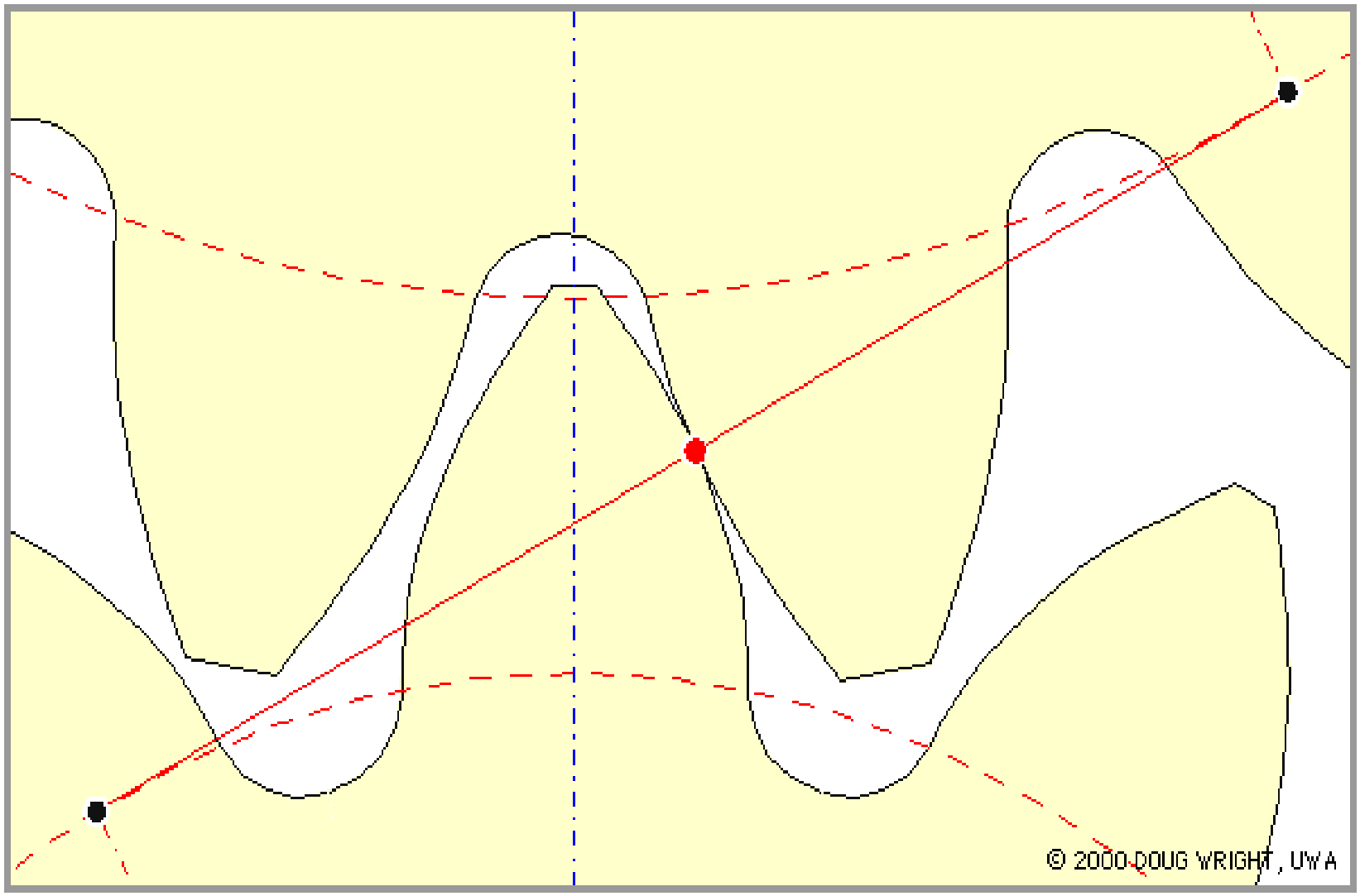


Line of Action – Line tangent to both base circles

Pressure Angle – Angle between the line normal to the line of centers and the line of action.

Pitch Point – Intersection of the line of centers with the line of action

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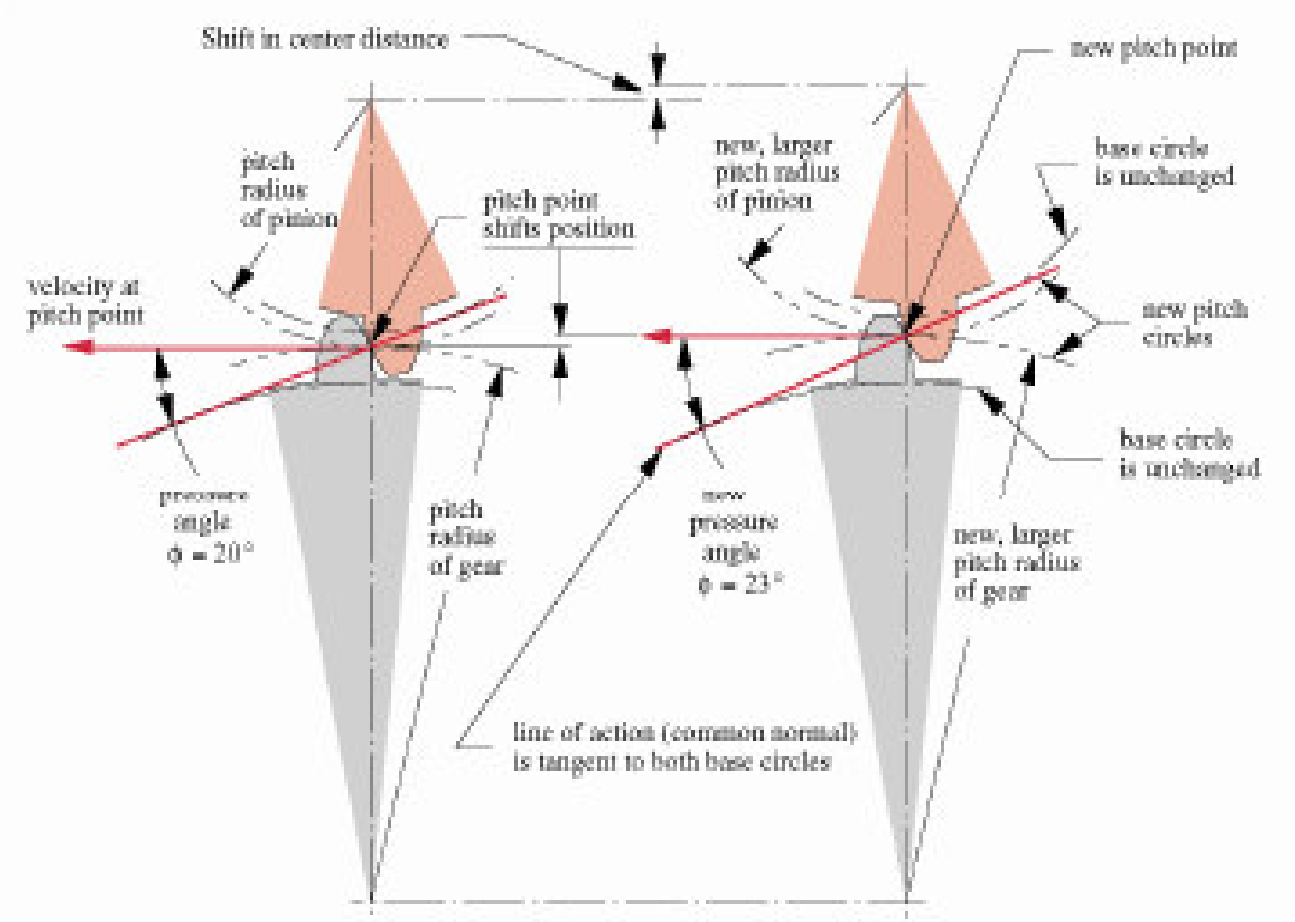
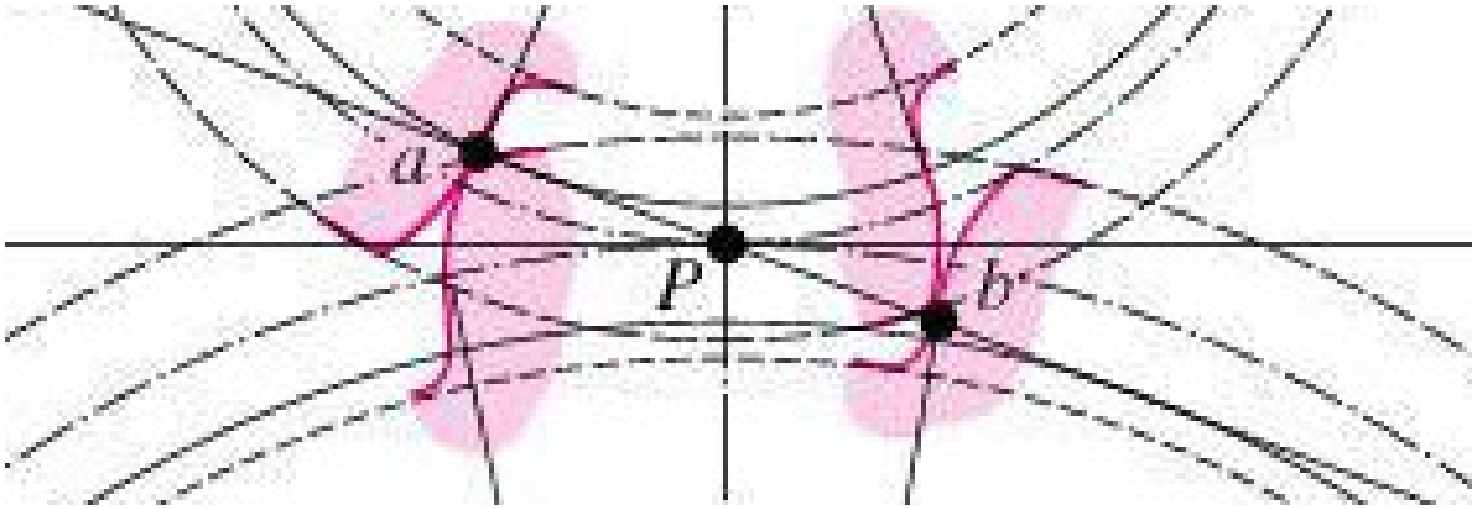
Changing Center Distance

- With the involute tooth form, the fundamental law of gearing is followed, even if the center distance changes
- Pressure angle increases



Contact ratio:



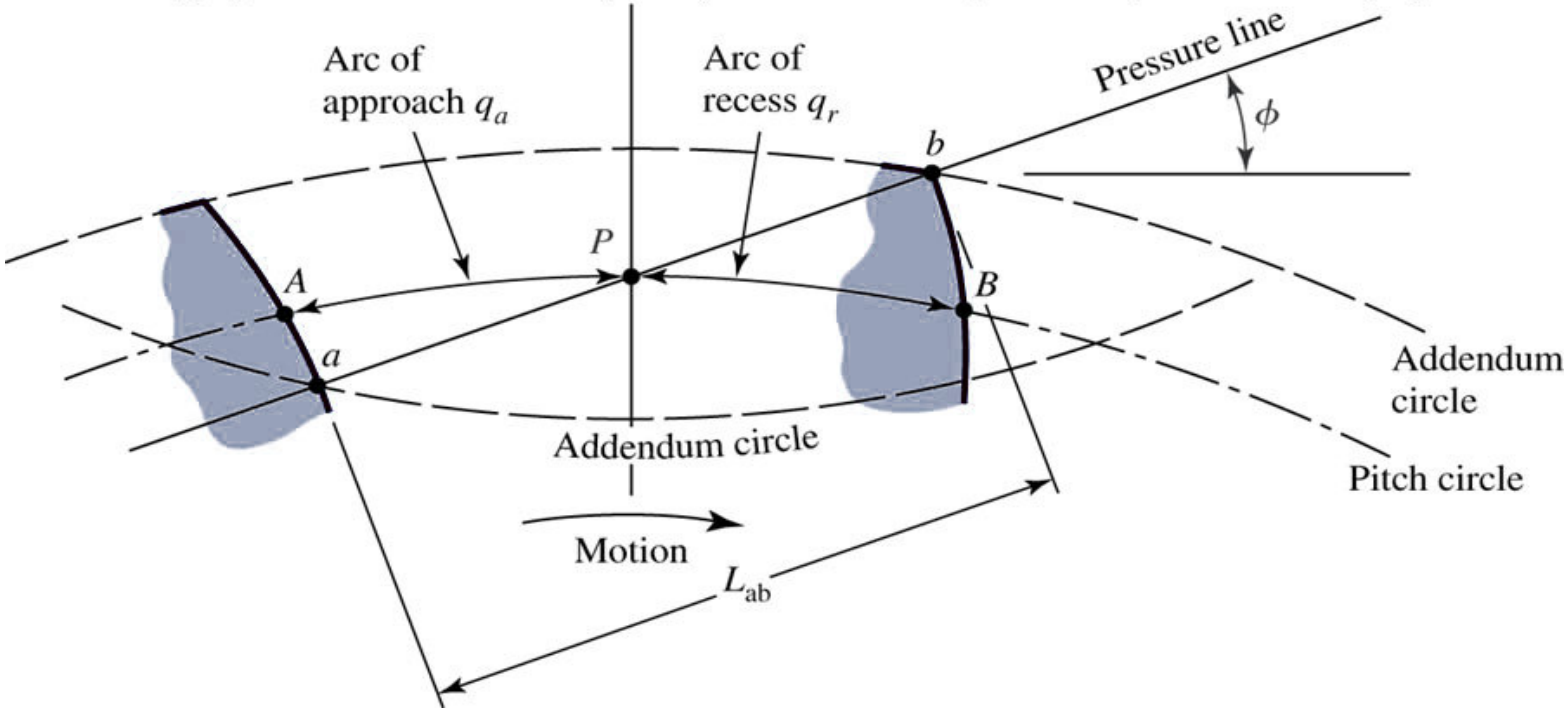
The points (a, b) of beginning and leaving contact define the mesh of the pinion and gear. The distance between these points along the line of action (pressure line) is called the length of action, Z, where:

$$Z = \sqrt{(r_p + a_p)^2 - (r_p \cos\varphi)^2} + \sqrt{(r_g + a_g)^2 - (r_g \cos\varphi)^2} - C \sin\varphi$$

r_p, r_g : are the pitch circle radii of pinion and gear.

a_p, a_g : are the addenda of pinion and gear

C : is the center distance



The length of action (L_{ab}) or parameter, Z, my be defined by the intersection of the respective addendum circles with the line of action (pressure line).

If the arc of contact AB (along the pitch circle) is greater than the circular pitch. This means that more than one pair of teeth can be in contact during a part of mesh duration

The contact ratio defines the average number of teeth in contact at any one time. It is calculated from:

$$m_c = \frac{q_t}{P}$$

where q_t is the arc of action and p is the circular pitch.

In terms of the length of action, the relation is :

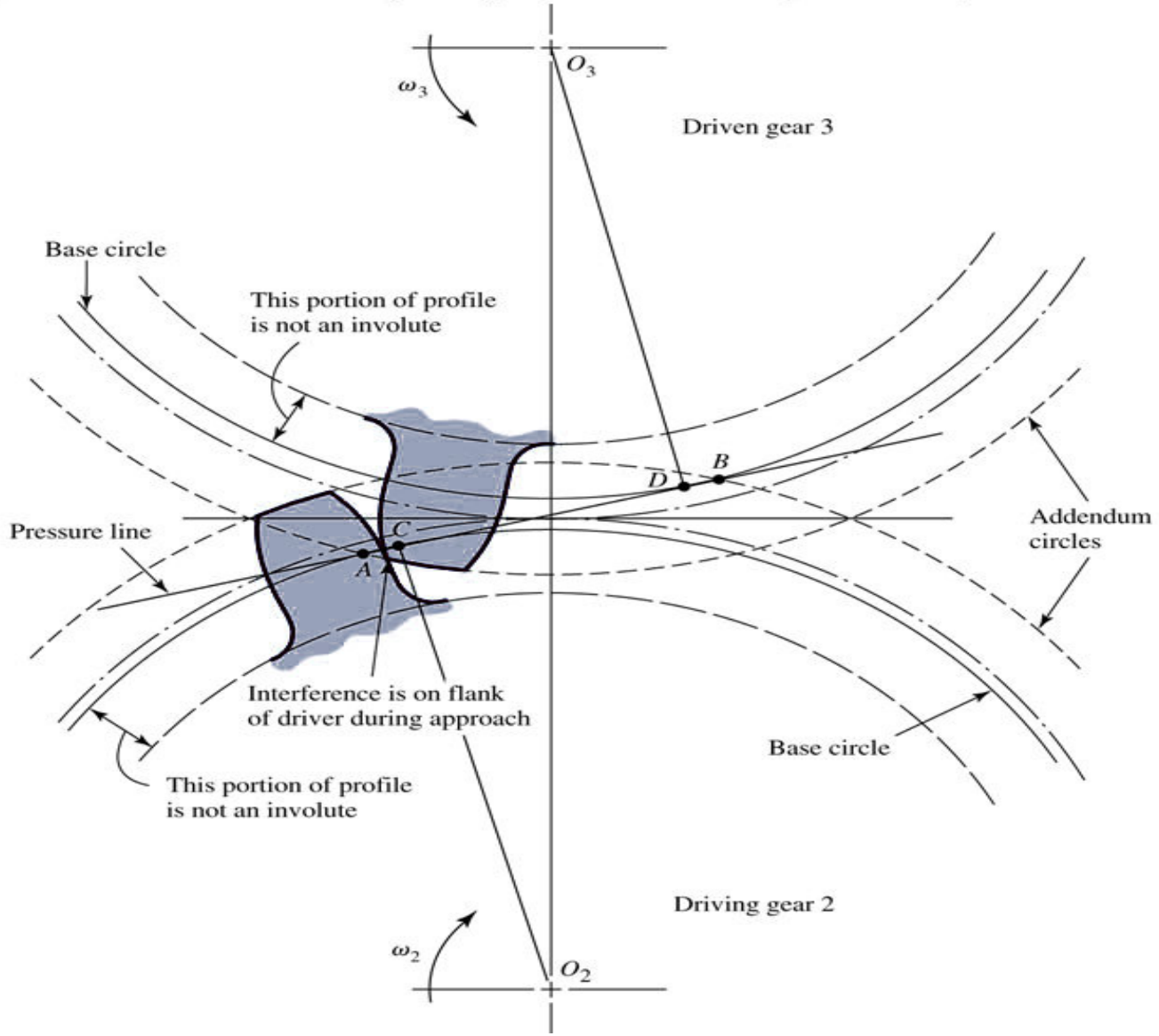
$$m_c = \frac{L_{ab}}{p_b} = \frac{Z}{p\cos\phi}$$

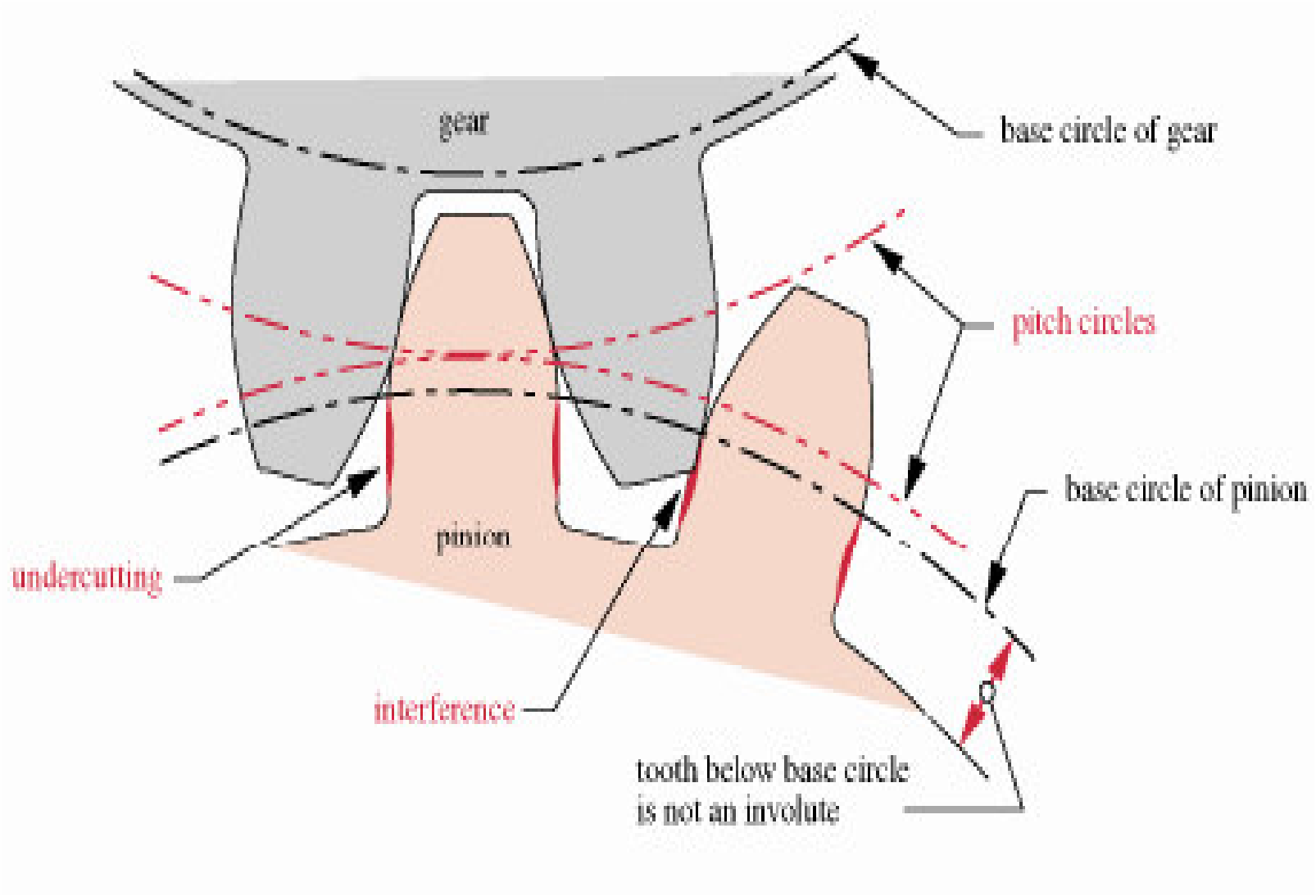
where p_b is the base circle pitch

The contact ratio is usually between 1.2 and 1.6

The magnitude of Z can be calculated from the equation or can be measured from a scaled drawing of the two addendum circles and the pressure line.

Interference:




gear
base circle of gear
pitch circles
base circle of pinion
pinion
undercutting
interference
tooth below base circle
is not an involute

Interference:

- The involute tooth form is only defined outside of the base circle.
- If the base circle is larger than the dedendum circle, then the portion of the tooth below the base circle will not be an involute.
- This part (portion) will interfere with the tip of the tooth on the mating gear, which is an involute. The actual effect is that the involute tip of the tooth tends to dig out the non-involute part of the flank of the mating gear.
-To avoid interference, the minimum number of teeth of a spur pinion that can be used is:

$$N_p = \frac{2k}{(1+2m_G)\sin^2\phi}\left(m_G + \sqrt{m_G^2 + (1+2m_G)\sin^2\phi}\right)$$

Where $m_G = N_G / N_p$ is the transmission ratio, $m_G > 1$

k = 1 for full-depth teeth

= 0.8 for stub teeth

φ = the pressure angle

The largest gear which can mesh with a specified pinion without interference:

$$N_G = \frac{N_P^2 \sin^2 \phi - 4k^2}{4k - 2N_P \sin^2 \phi}$$

Full depth pinion and full depth gears of various sizes for 20° pressure angle

Minimum Pinion Teeth	Maximum Gear Teeth
17	1 309
16	101
15	45
14	26
13	16