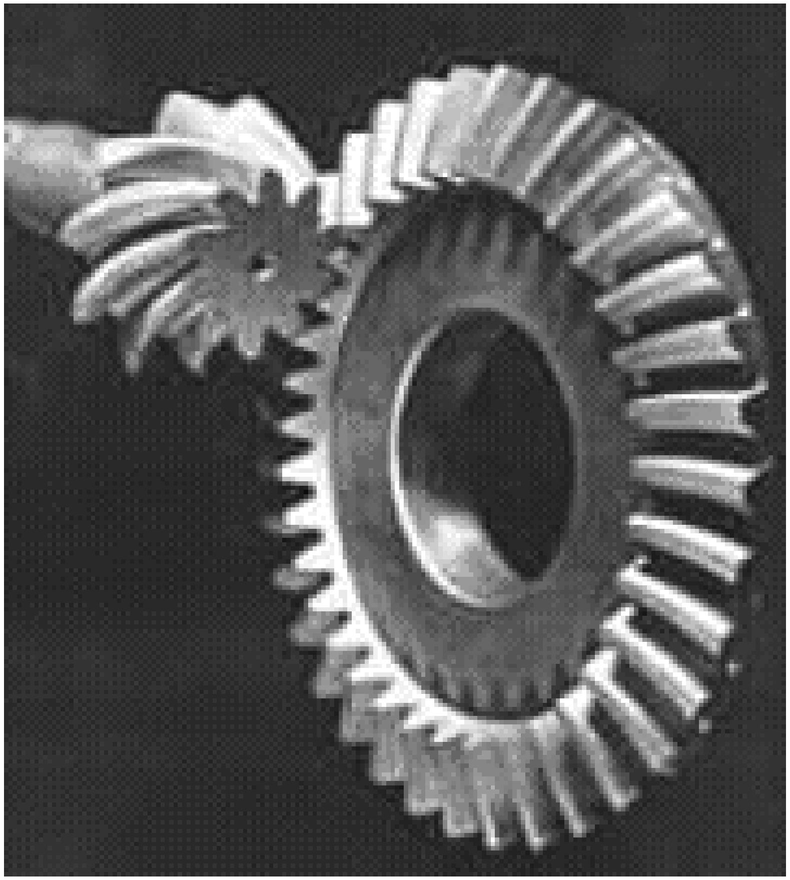
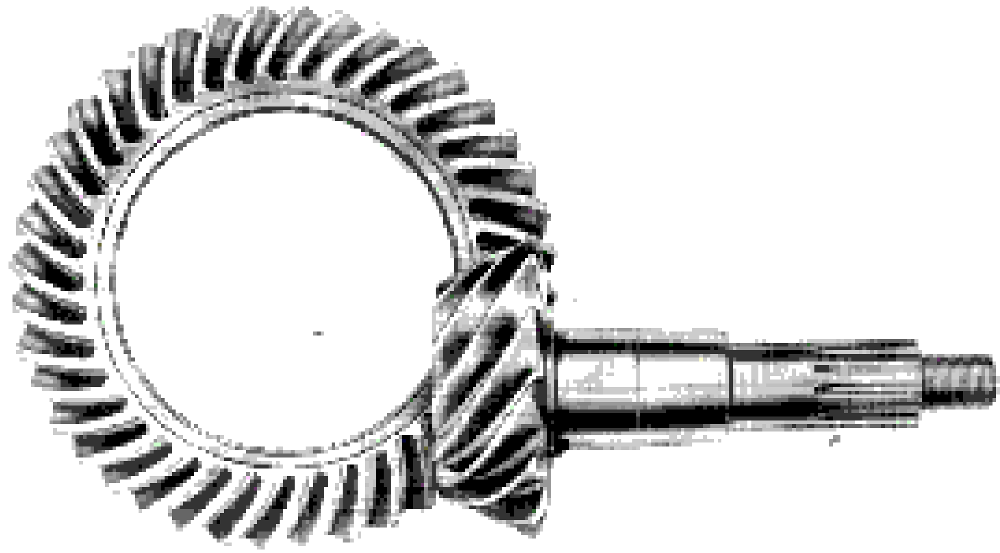
Bevel and Worm Gears

Bevel gears:

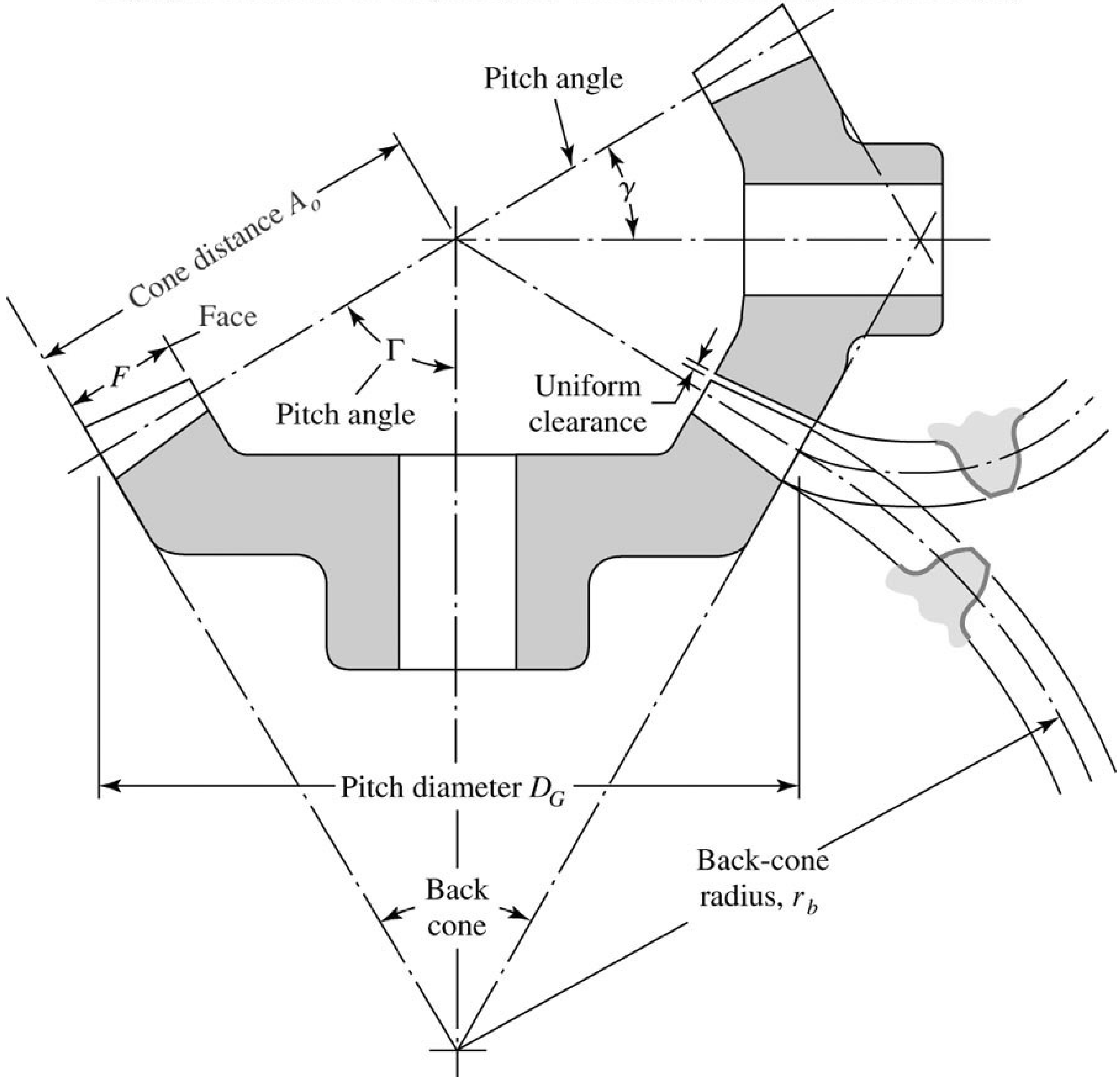
Bevel gears are usually used to transmit motion between two intersecting shafts (at any angle but usually 90^o). Bevel gear sets are shown in Fig. below.

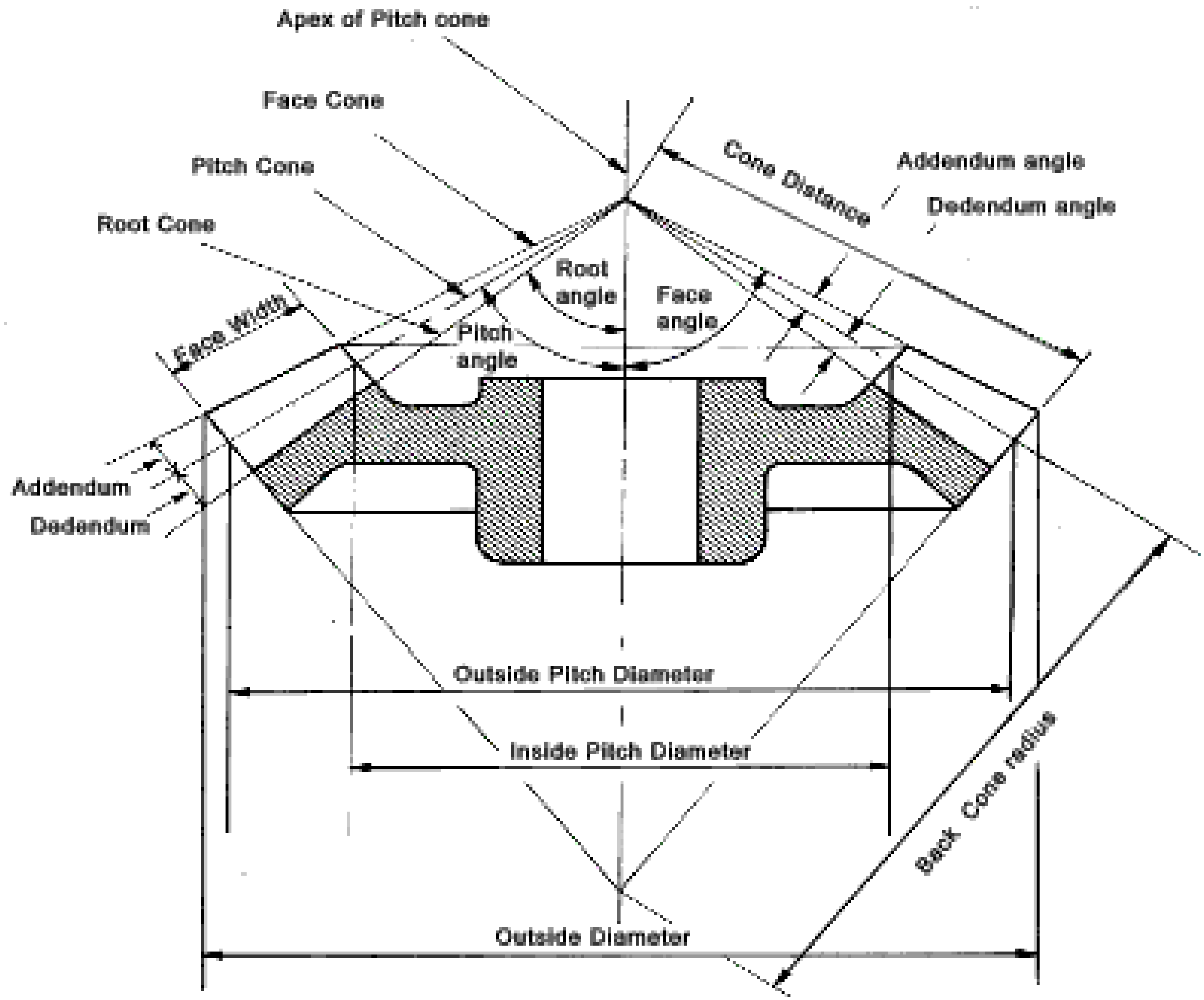




Hypoid bevel gears

Bevel gears kinematics:





Apex of Pitch cone
Face Cone
Pitch Cone
Root Cone
Cone Distance
Addendum angle
Dedendum angle
Root angle
Face angle
Pitch angle
Face Width
Addendum
Dedendum
Outside Pitch Diameter
Inside Pitch Diameter
Back Cone radius
Outside Diameter

1- The pitch circle of bevel gears is measured at the large end of the tooth (back cone). The pitch circle diameter and the circular pitch are calculated as in the spur gears:

$$d = mN$$

$$p_c = \pi m$$

2- The pitch angles γ, Γ are defined by the pitch cones meeting at the apex, as shown in Fig. They are related to the tooth numbers as follows:

$$\tan\gamma = \frac{N_p}{N_G}, \qquad \tan\Gamma = \frac{N_G}{N_p}$$

where γ, Γ are respectively the pitch angles of the pinion and gear.

3- The Face width is given as the shortest distance of:

$$F = \frac{A_o}{3} \quad \text{or} \quad F = (10-15)m$$

Force analysis of Straight bevel gears:

In determining shaft and bearing loads for bevel gear application, the usual practice is to use the tangential (transmitted) load which would occur if all the forces were concentrated at the midpoint of the tooth. While the actual resultant forces occur somewhere between the midpoint and the large end of the tooth.

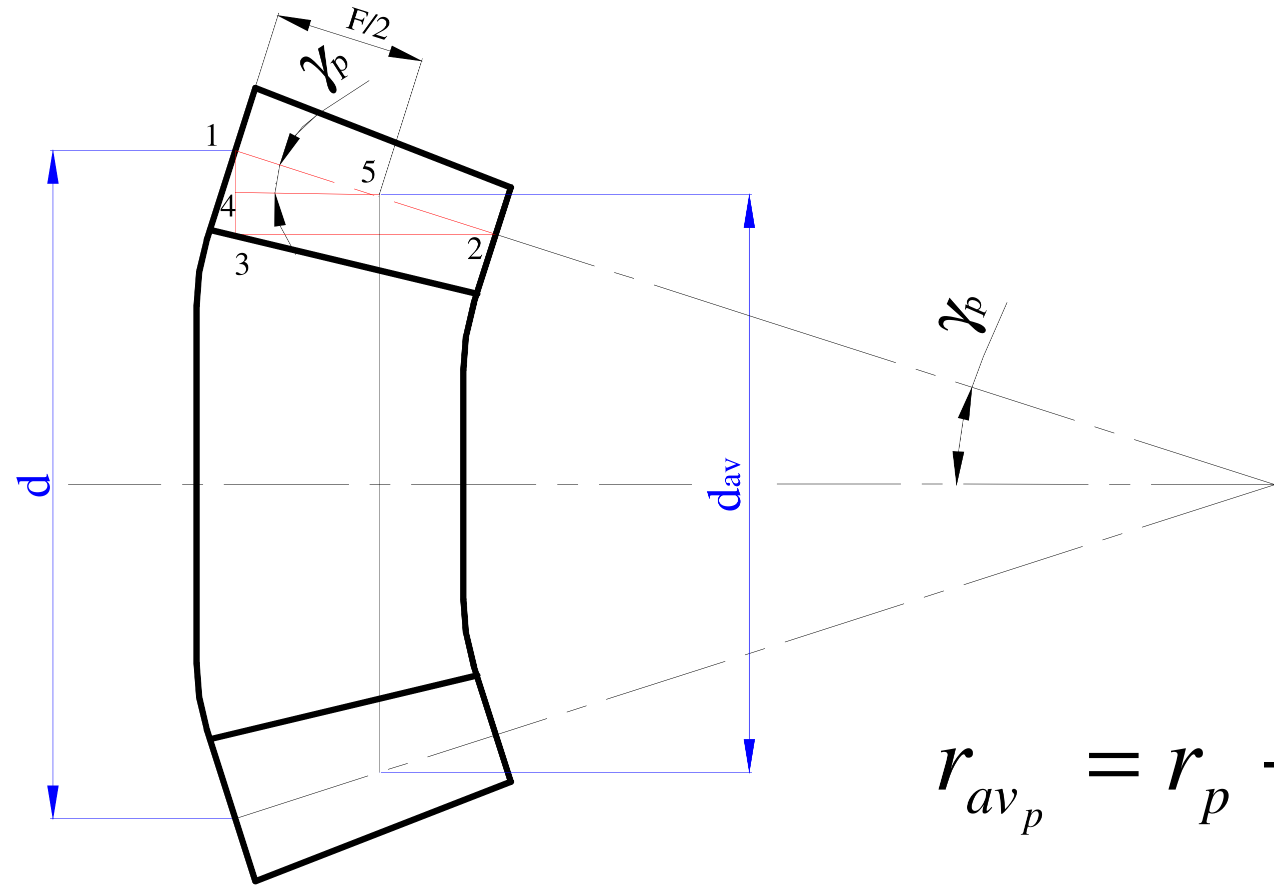
$$W_t^* = \frac{T}{r_{av}} = \frac{H}{V_{av}}$$

(at the midpoint of the tooth)

Where r_{av} is the pitch radius of the gear under consideration at the midpoint of the tooth.

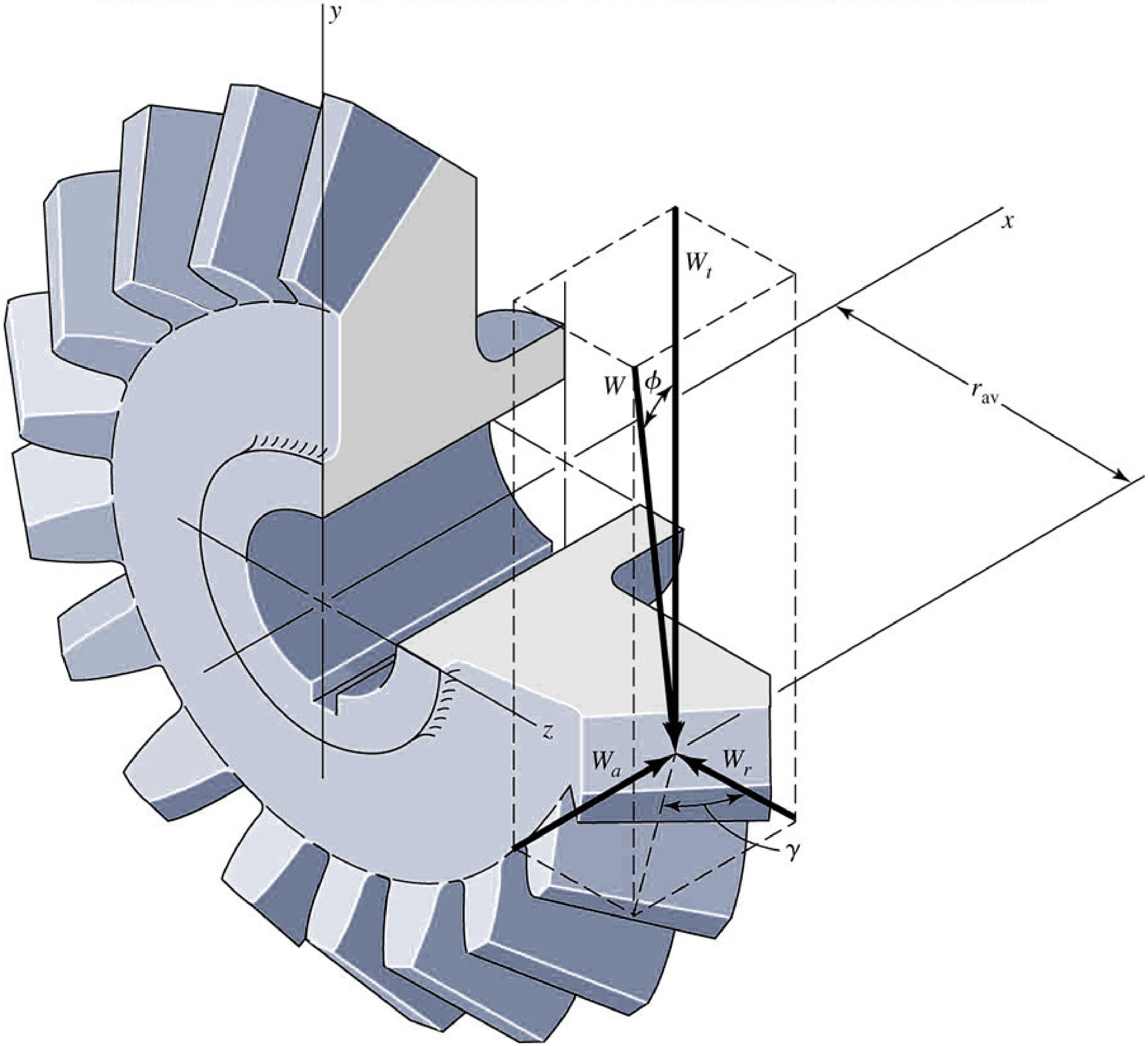
The average velocity is determined from: $V_{av} = \omega r_{av}$

Determination of average diameter



$$r_{av_p} = r_p - \frac{F}{2}\sin\gamma$$

$$r_{av_g} = r_g - \frac{F}{2}\sin\Gamma$$



y
x
W_t
W
ϕ
r_{av}
z
W_a
W_r
γ

a- The force W is first resolved into two components:

$$W_t^* = W \cos\varphi$$
$$W_1^* = W \sin\varphi$$

b- The force W_1^* is resolved into two components:

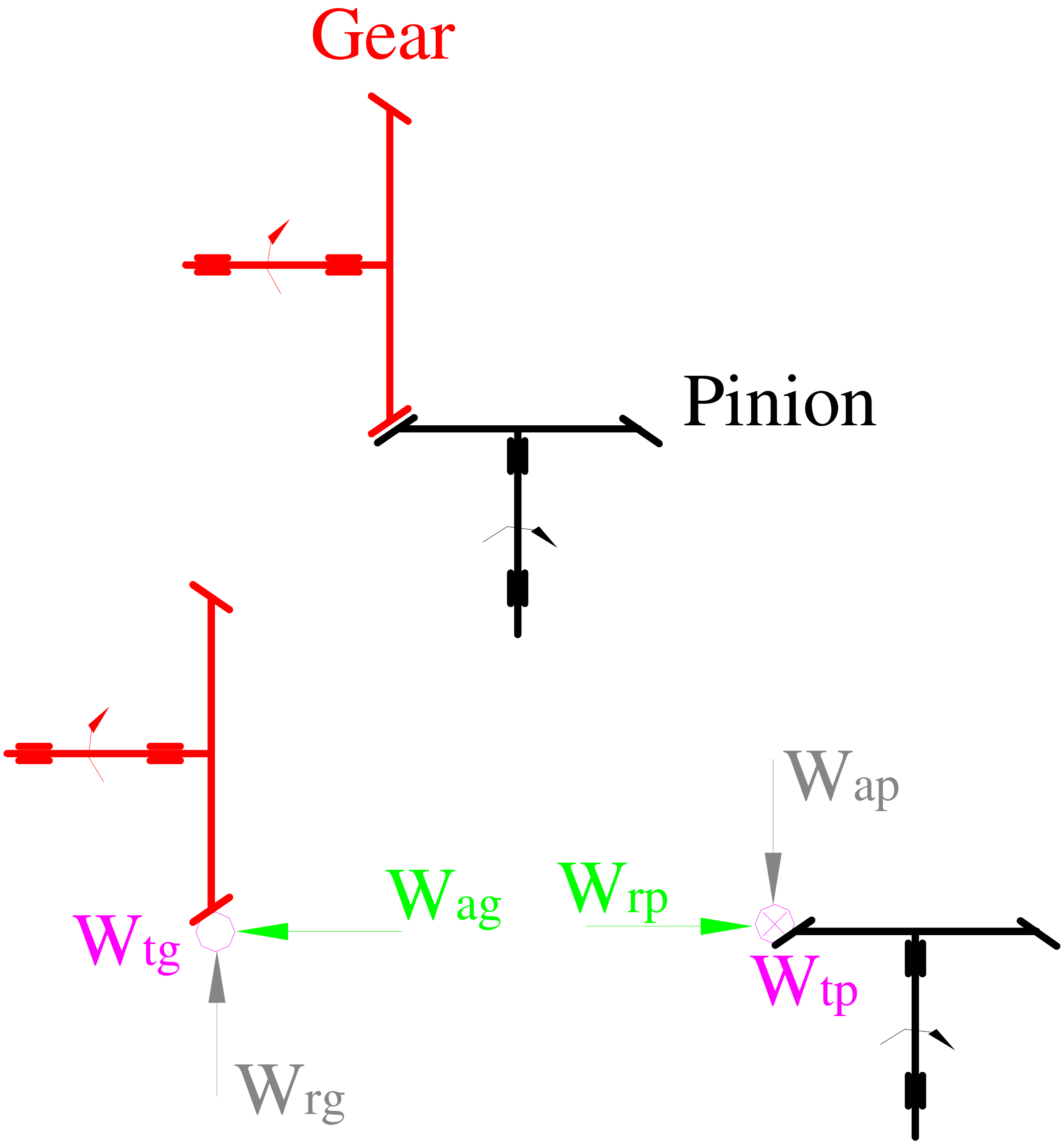
$$W_r^* = W \sin\varphi \cos\gamma$$
$$W_a^* = W \sin\varphi \sin\gamma$$

Usually W_t^* is given, and the other forces are required, therefore:

$$W = \frac{W_t^*}{\cos\varphi} \qquad W_r^* = W_t^* \tan\varphi \cos\gamma$$
$$W_a^* = W_t^* \tan\varphi \sin\gamma \tag{14-36}$$

Where: φ :is the pressure angle

Note: W_t^*, W_r^*, W_a^* are used for shaft and bearing design (reactions and bending moment diagrams)



Gear
Pinion
Wap
Wag
Wrp
Wtg
Wtp
Wrg

Stress analysis of Straight bevel gears:

Bending Stress:

$$\sigma = \frac{W_t}{bmJ} K_v K_0 K_s K_H$$

Bending Strength:

For through-hardened steel gears

$$\sigma'_{FP} = 0.3H_B + 14.48 \quad \text{MPa} \qquad \text{for grade 1}$$

$$= 0.33H_B + 41.24 \quad \text{MPa} \qquad \text{for grade 2}$$

The corrected strength :

$$\sigma_{FP} = \sigma'_{FP} \frac{Y_N}{Y_\theta Y_Z}$$

Contact Stress:

$$\sigma_c = C_p \sqrt{\frac{W_t}{b d_p I} K_v K_o K_s K_H C_{xc}}$$

Where C_{xc} is the crowning factor for pitting

Contact Strength:

for through hardened steel gears:

$$\sigma^{\backslash}_{HP} = 2.35 H_B + 162.89 \text{ MPa} \qquad \text{for grade 1}$$

$$= 2.51 H_B + 203.86 \text{ MPA} \qquad \text{for grade 2}$$

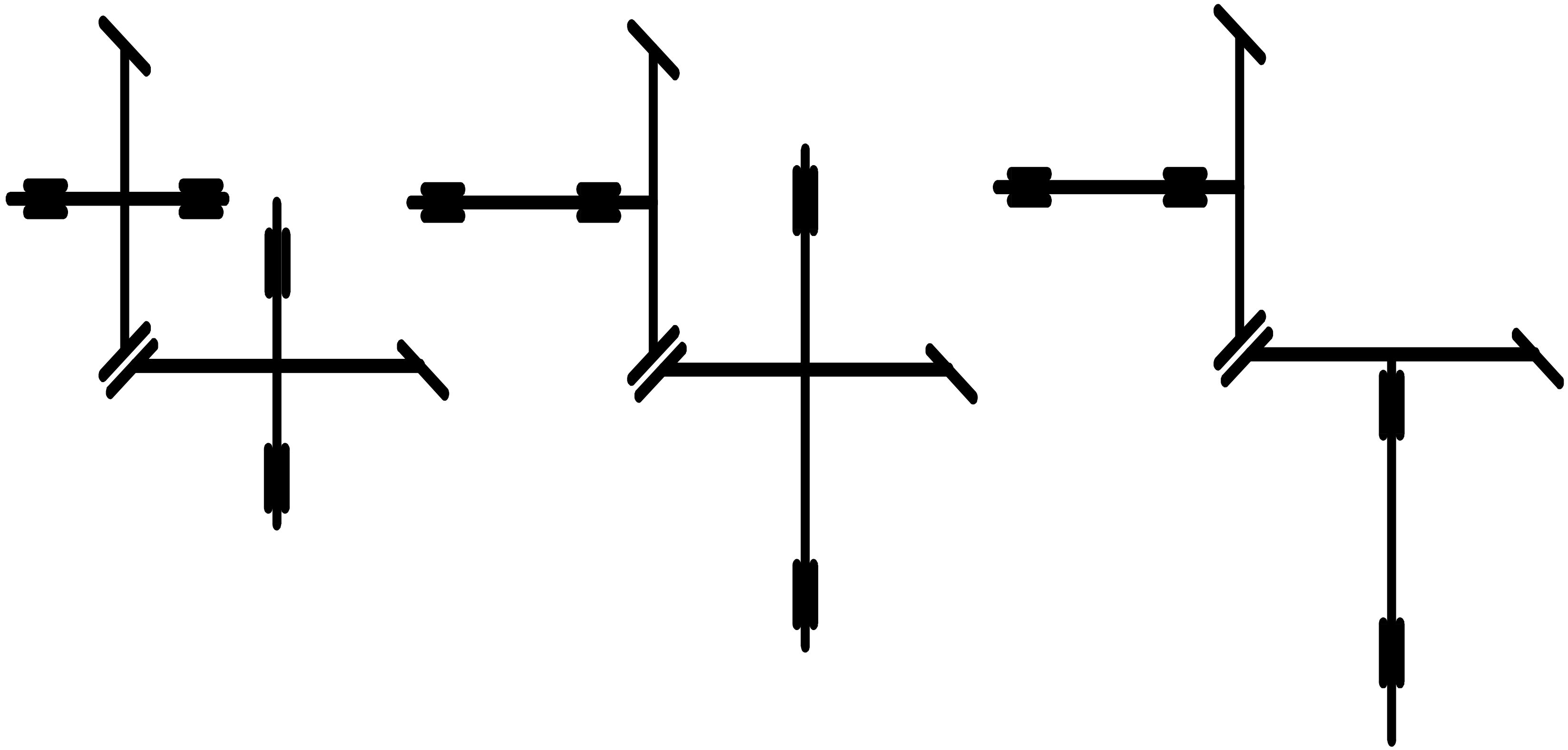
The corrected contact strength

$$\sigma_{\mathrm{HP}} = \sigma^{\backslash}_{HP} \frac{Z_N C_H}{Y_\theta Y_Z}$$

Notes:

1- All factors are given in graphical form or as equations in the text book

2- For all the above relations the module and diameter of the gear at the back cone is considered.

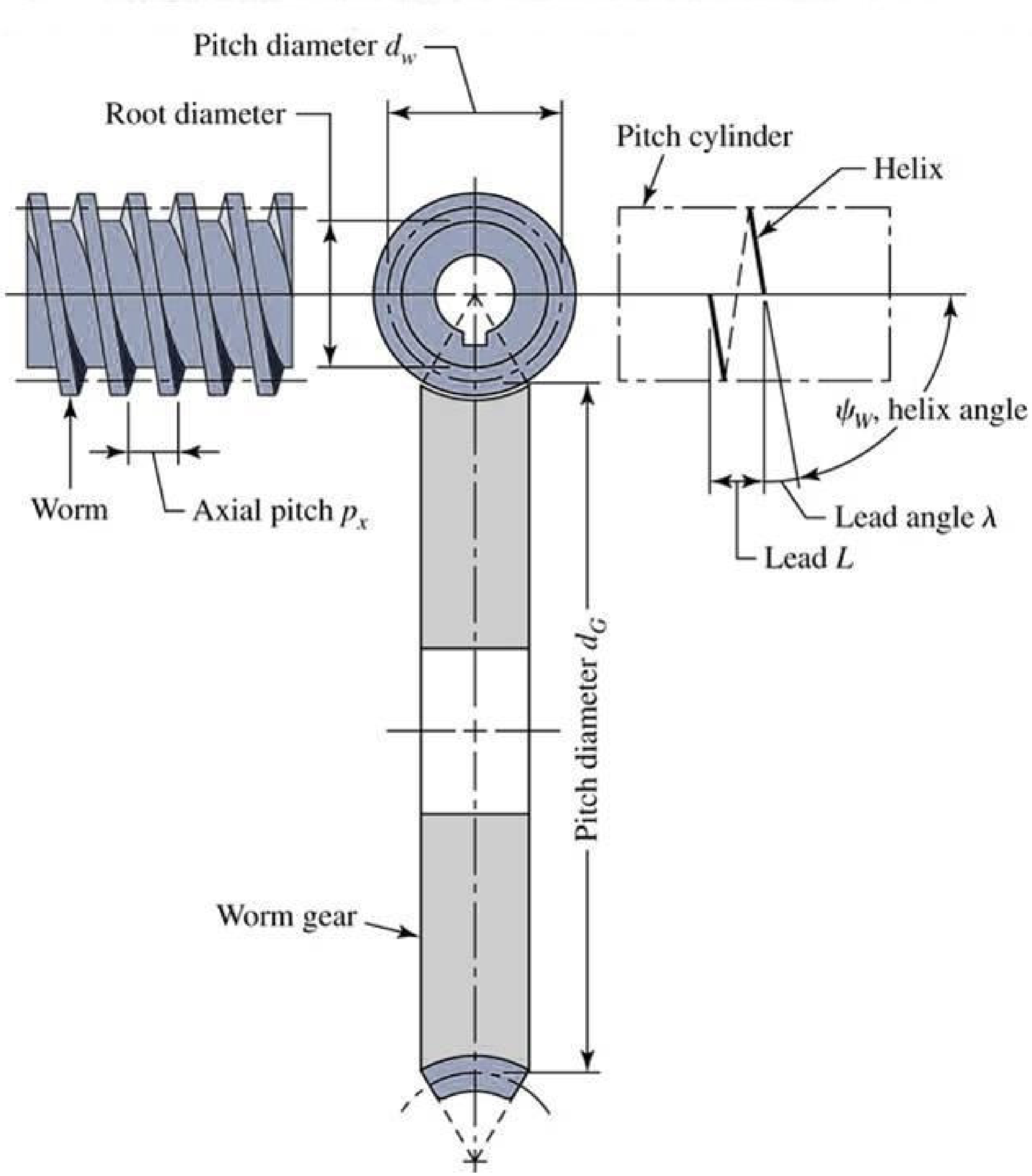


Both gears inboard   One gear outboard   Both gears outboard

Mounting of bevel gears

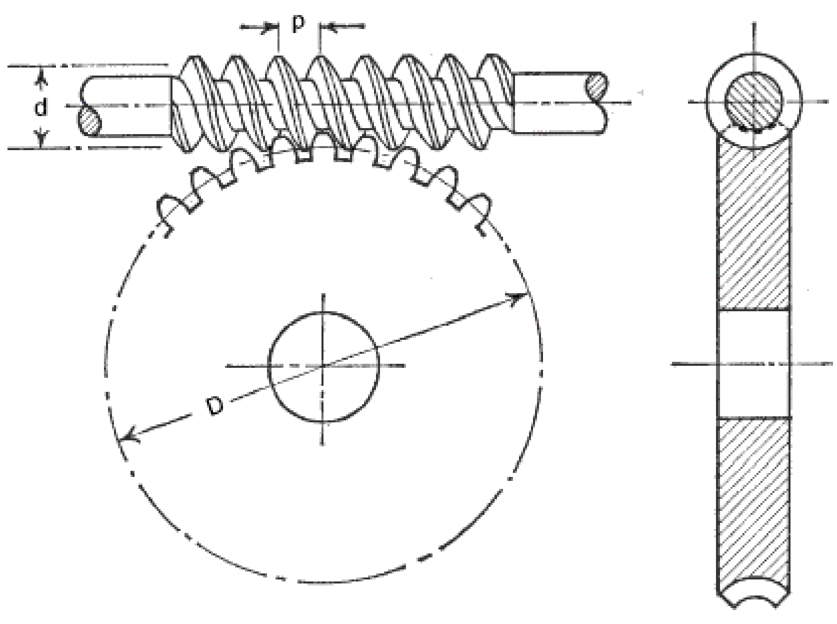
Worm Gears

A worm gear is used when a large speed reduction ratio is required between crossed axis shafts which do not intersect.



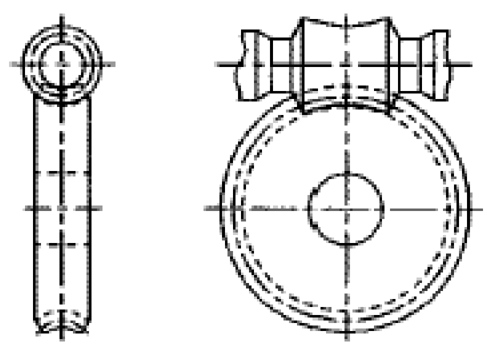
The worm is similar to a screw and the worm wheel is similar to a section of a nut.

- Worm gearset are either single or double enveloping.
- In single enveloping set, the worm wheel has its width cut into a concave surface, thus partially enclosing the worm when in mesh.
- In double enveloping set, in addition to having the worm wheel width cut concavely, this type has the worm length cut concavely. The result is that both the worm and gear partially enclose each. A double enveloping set will have more teeth in contact and will have area rather than line contact, thus permitting greater load transmission.



Single enveloping gearset

Fig. 4



Double enveloping gearset

- Worm gearing kinematics:

- Helix angles are quite different.
- Worm and gear have same hand of helix.
- Worm has large helix angle, gear has small helix angle.
- Specify lead angle λ on worm and helix angle ψ_g on the gear. These are the same for a 90° shaft angle.

$$(\lambda = \psi_g \quad \text{for } 90^o \text{ shafts})$$

- The axial pitch (p_x) of the worm and the transverse circular pitch (p_t) of the gear are equal for 90^o shafts.

$$(p_x = p_t \quad \text{for } 90^o \text{ shafts})$$

- The pitch diameter of the worm is not related to the number of teeth. It is chosen such that:

$$\frac{C^{0.875}}{3} \leq d_W \leq \frac{C^{0.875}}{1.7}$$

Where:

C: is the center distance.

- The pitch diameter of the gear is the diameter measured on a plane containing the worm axis.

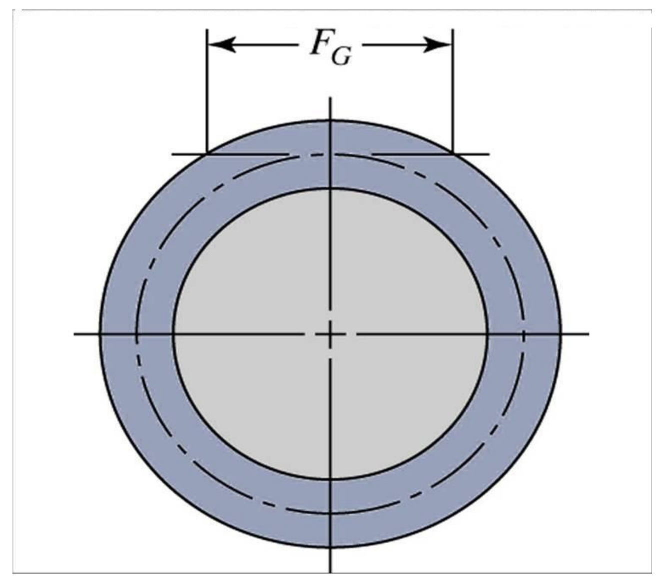
$$d_G = \frac{N_G p_t}{\pi}$$

- The lead (L) and the lead angle (λ) of the worm have the following relations:

$$L = p_x N_W \qquad \tan\lambda = \frac{L}{\pi d_W}$$

Where N_w is the Number of teeth of worm (number of starts)

- The face width F_g of the worm gear should
- be made equal to the length of a tangent to
- the worm pitch circle between its points of
- intersection with the addendum circle.



Force analysis of worm gearing:

1- Neglecting friction

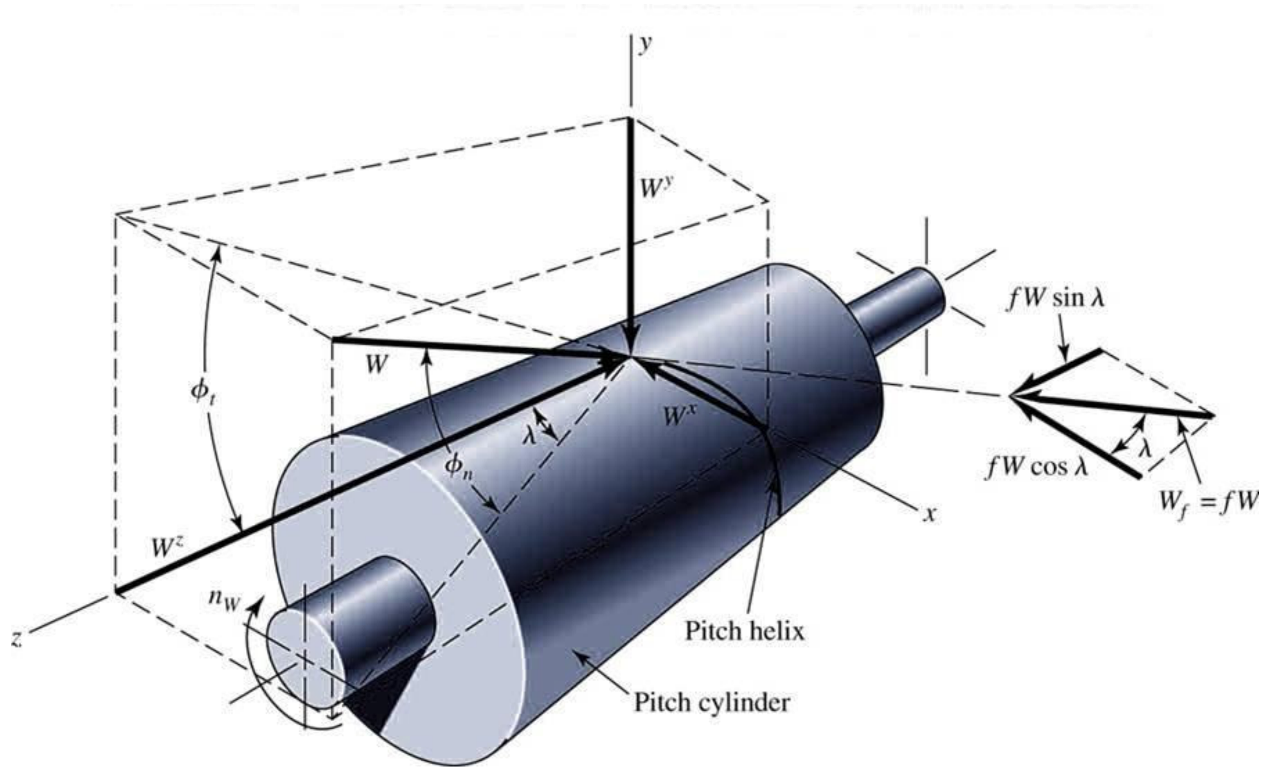
Force exerted by gear onto the worm is **W.**

$$\mathbf{W} = W_x\mathbf{i} + W_y\mathbf{j} + W_z\mathbf{k}$$

$$W_x = W\cos\phi_n \sin\lambda$$

$$W_y = W\sin\phi_n$$

$$W_z = W\cos\phi_n \cos\lambda$$



Drawing of the pitch cylinder of a worm, showing the forces exerted upon it by the worm gear.

$$W_{Wt} = -W_{Ga} = W_x$$

$$W_{Wr} = -W_{Gr} = W_y$$

$$W_{Wa} = -W_{Gt} = W_z$$

Subscript *W* and *G* represent forces acting on Worm and Gear respectively.
Gear axis is parallel to *x*, worm axis is parallel to *z*, right handed coordinate system

2- Including friction:

- Relative motion between worm and gear is sliding
  - Friction is important.
  - Need to introduce coefficient of friction μ.

- Given a load ***W*** acting normal to the tooth profile, $W_f = \mu W$
- with a component $\mu W \cos\lambda$ in negative *x* direction
- and $\mu W \sin\lambda$ in positive *z* direction

$$W_x = W\left(\cos\phi_n \sin\lambda + \mu\cos\lambda\right)$$

$$W_y = W\sin\phi_n$$

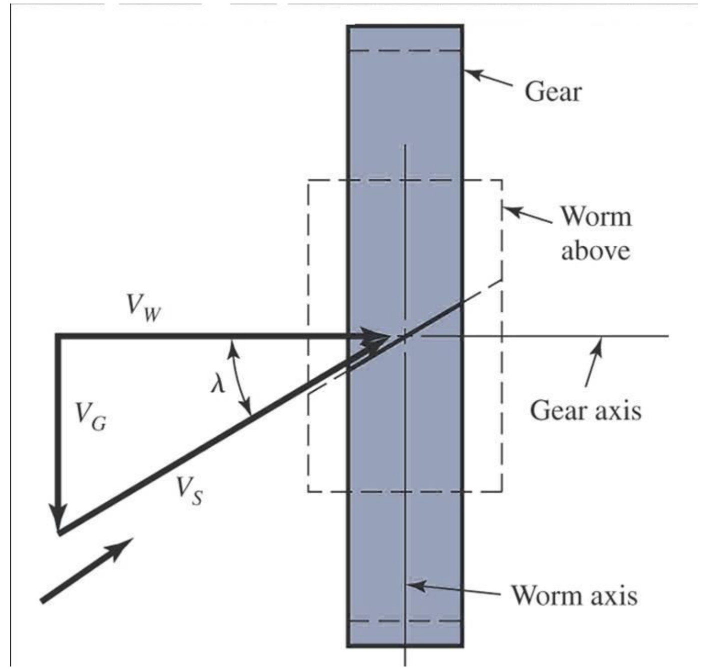
$$W_z = W\left(\cos\phi_n \cos\lambda - \mu\sin\lambda\right)$$

Where: μ: is the coefficient of friction between teeth.

- Experimentally, coefficient of friction is dependent on sliding velocity

$$\mathbf{V}_W = \mathbf{V}_G + \mathbf{V}_S$$

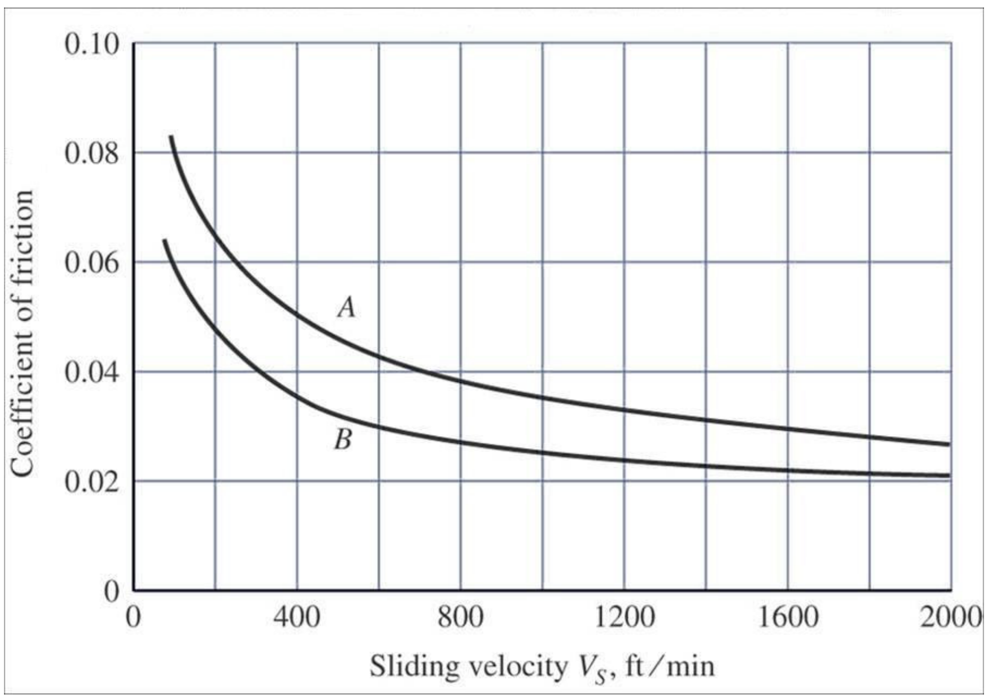
$$V_S = \frac{V_W}{\cos\lambda}$$



Where:

V_G = pitch line velocity of gear

V_w = pitch line velocity of worm



The Figure on the left represents values of the coefficient of friction for worm gears.

Curve A when more friction is expected (C.I).

Curve B for high quality materials (case hardened worm mating with a phosphor-bronze gear)

Efficiency:

$$\eta = \frac{\text{required power without friction}}{\text{required power with friction}}$$

$$= \frac{W_{wt}.\omega \quad (\text{without friction})}{W_{wt}.\omega \qquad (\text{with friction})}$$

$$= \frac{\cos\phi_n - \mu\tan\lambda}{\cos\phi_n + \mu\cot\lambda}$$

Power rating of worm gearing:

$$H = H_o + H_l$$

Where:

- H_l: is the losses power, and is given as:

$$H_l = \mathrm{W_f} \text{ x } V_s$$

Where:

W_f : is the friction force (= μW)

V_s : is the sliding speed

$\therefore$
$$H_l = \mu \mathrm{W} \text{ x } V_s$$

- H_o: is the output power, and is given as:

$$H_o = \mathrm{W_{tg}} \text{ x } V_{tg}$$

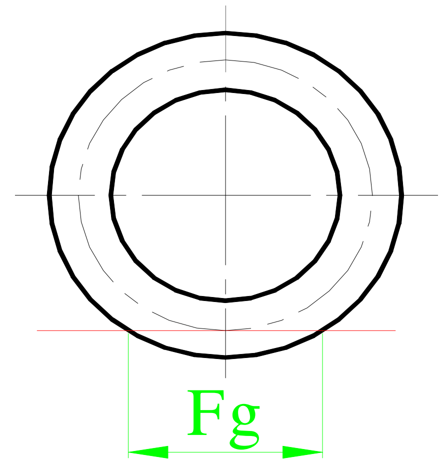
Where:

$$V_{tg} = \omega_g r_g = \left(\frac{2\pi n_g}{60}\right)\left(\frac{d_g}{2}\right)$$

The maximum (permissible) tangential force on the gear is given as:

$$W_{tg} = \beta\left(K_s d_g^{0.8} F_e K_m K_v\right)$$

Where



β : Conversion factor = 0.0131

K_s : Material factor (from table 1)

d_g : Pitch diameter of the gear

F_e : Effective face width

= smaller of ($\frac{2}{3} d_w$ or F_g (where F_g is shown in figure))

K_m : Ratio correction factor (from table2)

K_v : Velocity factor (from table 3)

Face width, mm	Sand-Cast Bronze	Static-Chill Cast Bronze	Centrifugal Cast Bronze
Up to 75	700	800	1000
100	665	780	975
125	640	760	940
150	600	720	900
175	570	680	850
200	530	640	800
225	500	600	750

Table 1: Material factor (K_s) for cylindrical worm gears

Tr. Ratio	K_m	Tr. Ratio	K_m	Tr. Ratio	K_m
3.0	0.500	8.0	0.724	30	0.825
3.5	0.554	9.0	0.744	40	0.815
4.0	0.593	10	0.760	50	0.785
4.5	0.620	12	0.783	60	0.745
5.0	0.645	14	0.799	70	0.687
6.0	0.679	16	0.809	80	0.622
7.0	0.706	20	0.820	100	0.490

Table 2: Ratio correction factor (K_m)

Table 3: Velocity factor (K_v)

V_s(m/s)	K_v	V_s(m/s)	K_v	V_s(m/s)	K_v
0.005	0.649	1.50	0.47	7.2	0.216
0.008	0.646	1.80	0.45	8	0.200
0.050	0.644	2.00	0.42	9	0.187
0.100	0.638	2.25	0.395	10	0.175
0.150	0.631	2.50	0.375	11	0.168
0.200	0.625	2.80	0.360	12	0.156
0.300	0.615	3.00	0.340	13	0.148
0.400	0.600	3.60	0.310	14	0.140
0.500	0.590	4.00	0.285	16	0.134
0.750	0.560	4.50	0.265	20	0.106
1.000	0.530	5.00	0.258	25	0.089
1.250	0.500	6.00	0.235	30	0.079