Tutorial # 6 - Solution

1- Figure 6-1 shows a double-reduction helical gearset. Pinion 2 is the driver, and it receives a torque of 135 N.m from its shaft in the direction shown. Pinion 2 has a normal module of 3 mm, 14 teeth, and a normal pressure angle of 20° and is cut left-handed with a helix angle of 30°. The mating gear 3 on shaft b has 36 teeth. Gear 4, which is the driver for the second pair of gears in the train, has a normal module of 5 mm, 15 teeth, and a normal pressure angle of 20° and is cut left-handed with a helix angle of 15°. Mating gear 5 has 45 teeth.
Find the magnitude and direction of the force exerted by the bearings C and D on shaft b if bearing C can take only radial load while bearing D is mounted to take both radial and thrust load.

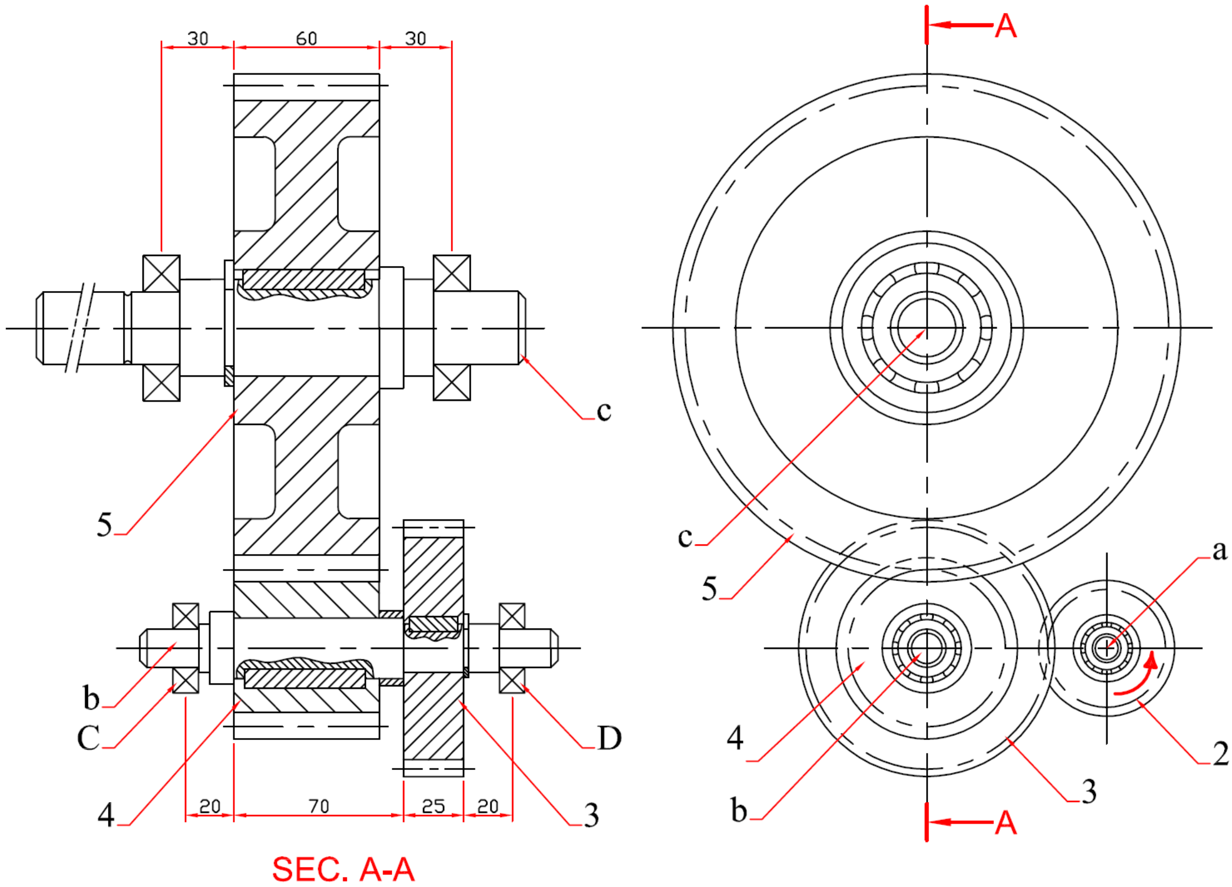


Figure 6-1

Solution:

Pinion 2 & Gear 3:

$T_a = 135$ N.m

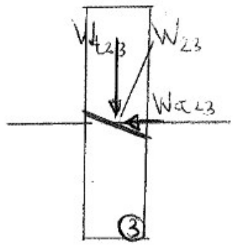
$m_{n2} = 3$ mm

$N_2 = 14$ Teeth

$\phi_{n2} = 20^\circ$

L.H., $\psi_2 = 30^\circ$

$N_3 = 36$ teeth



$$d_2 = \frac{m_{n2}}{\cos\psi_2} N_2 = 48.5 \text{ mm}$$

$$d_3 = \frac{m_{n2}}{\cos\psi_2} N_3 = 124.7 \text{ mm}$$

$$W_{t23} = \frac{T_a}{r_2} = 5.567 \text{ KN}$$

$$W_{r23} = W_{t23} \tan\phi_{t_2} = W_{t23}\frac{\tan\phi_{n_2}}{\cos\psi_2} = 2.34 \text{ KN} \quad \left(\tan\phi_t = \frac{\tan\phi_n}{\cos\psi}\right)$$

$$W_{a23} = W_{t23} \tan\psi_2 = 3.214 \text{ KN}$$

[directions are shown in the sketch)

Pinion 4 & Gear 5:

$m_{n4} = 5$ mm

$N_4 = 15$ teeth

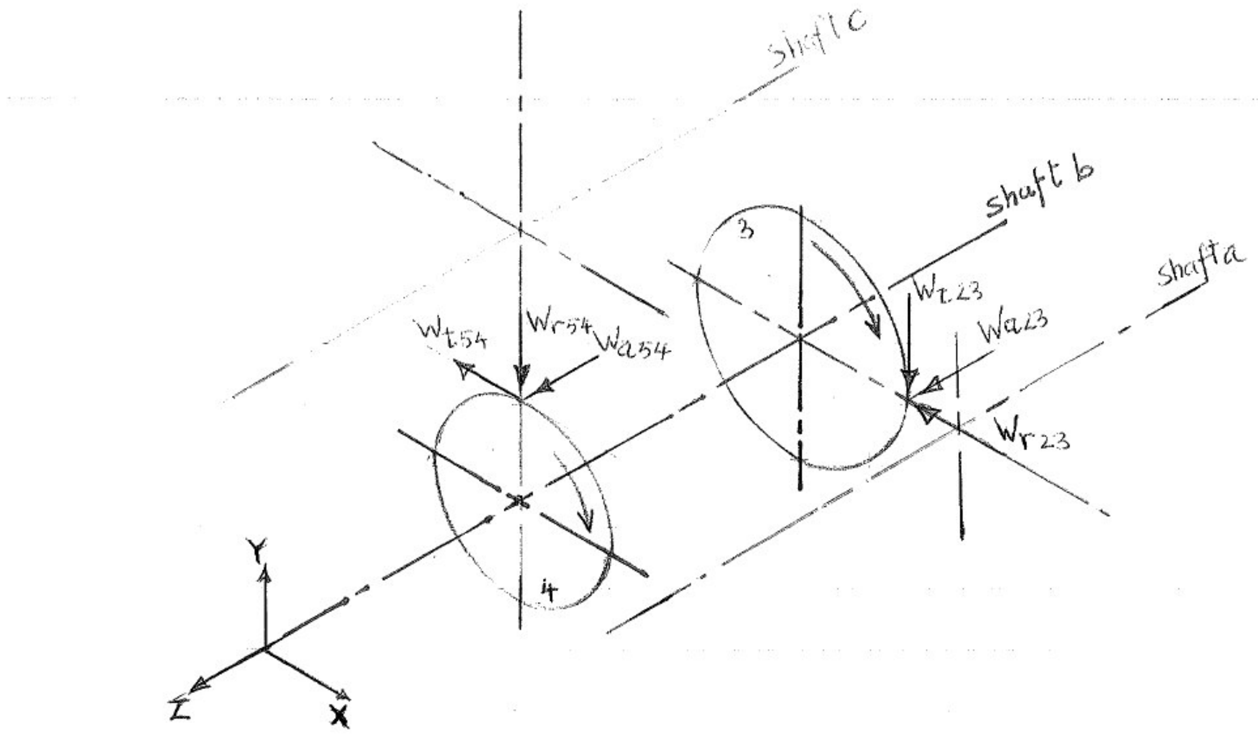
$\phi_{n_4} = 20^\circ$

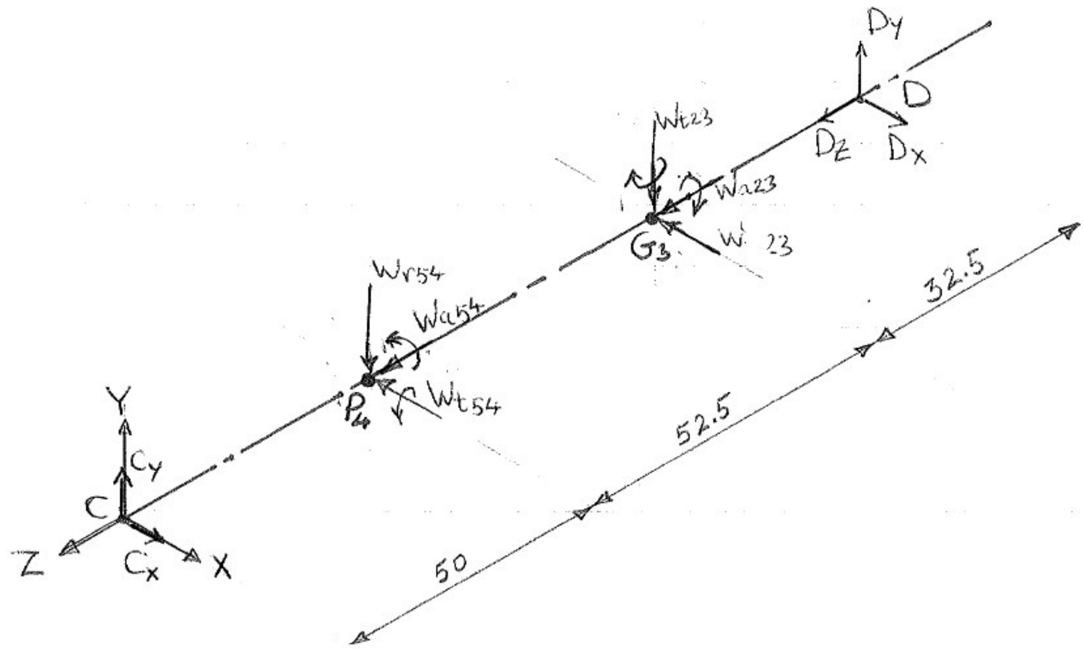
$\psi_4 = 15^\circ$ L.H.

$N_5 = 45$ teeth

$$d_4 = \frac{m_{n4}}{\cos\psi_4} N_4 = 77.64 \text{ mm}$$

$$d_5 = \frac{m_{n4}}{\cos\psi_4} N_5 = 232.9 \text{ mm}$$


shaft c
shaft b
shaft a
3
4
W_{t54}
W_{r54}
W_{a54}
W_{t23}
W_{a23}
W_{r23}
Y
Z
X


D_y
D
D_z
D_x
W_{t23}
W_{a23}
G_3
W_{r54}
W_{a54}
P_4
W_{t54}
Y
C_y
C
Z
C_x
X
50
52.5
32.5

From equilibrium of Torques over shaft b:

$$W_{t23} \times \frac{d_3}{2} = W_{t54} \times \frac{d_4}{2}$$

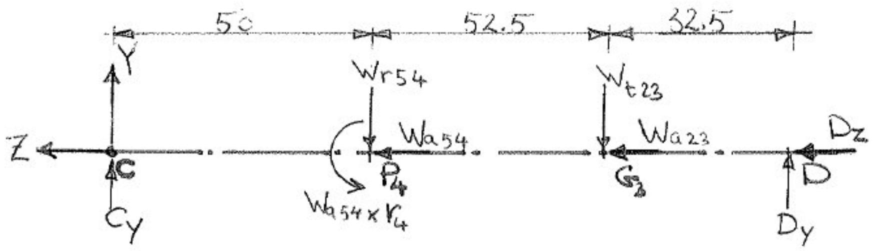
$$W_{t54} = 8.94 \text{ KN}$$

$$W_{r54} = W_{t54} \tan \phi_{t4} = W_{t54} \frac{\tan \phi_{n4}}{\cos \psi_4} = 3.37 \text{ KN.}$$

$$W_{a54} = W_{t54} \tan \psi_4 = 2.4 \text{ KN}$$ [direction are shown on the sketch

Shaft b:

Y-Z plane:



$$\Sigma F_y = 0$$

$$C_y - W_{r54} - W_{t23} + D_y = 0$$

$$C_y = 5.567 + 3.37 - D_y \qquad \text{—— (1)}$$

$$\Sigma M_x)_C = 0$$

$$-W_{r54} \times 50 \times 10^{-3} + W_{a54} \times r_4 - W_{t23} \times 102.5 \times 10^{-3} + 135 D_y = 0$$

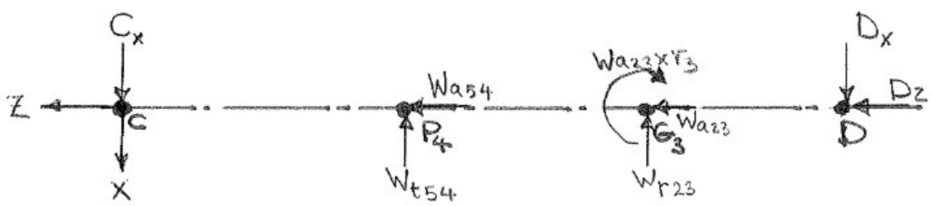
$$D_y = 4.785 \text{ KN}$$

Substituting in (1) $\qquad C_y = 4.15 \text{ KN}$

$$\Sigma F_Z = 0$$

$$-D_Z = W_{a54} + W_{a23} \qquad D_Z = -5.614 \text{ KN}$$

X-Z plane:



$$\Sigma F_x = 0$$

$$C_x - W_{t54} - W_{r23} + D_x = 0$$

$C_x = 8.94 + 2.34 - D_x \quad —— (2)$

$\Sigma M_y)_C = 0$

$W_{t54} \times 50 + W_{r23} \times 102.5 - W_{a23} \times r_3 - 135\, D_x = 0$

$D_x = 3.6$ KN

Sub in (2) . $C_x = 7.676$ kN

At Bearing C: $C_x = 7.676$ KN $\quad C_y = 4.15$ KN

At Bearing D: $D_x = 3.6$ kN $\quad D_y = 4.785$ kN $\quad D_z = 5.614$ KN

2- If the pinion described in problem 1 is running at 375 rpm, the gears are made of grade 1 steel, through-hardened at 200 Brinell, made to No. 6 quality standards, uncrowned, and are to be accurately and rigidly mounted. Assume a pinion life of 10^8 cycles, a reliability of 90% and a rim thickness of 20 mm (in gear 5).
Determine the AGMA bending and contact stresses acting on the gears and the corresponding factors of safety.

Solution:

$W_{t23} = 5.567$ kN $\qquad W_{t54} = 8.94$ kN

$b_2 = b_3 = 25$ mm $\qquad b_4 = b_5 = 60$ mm

a) bending:

	(2)	(3)	(4)	(5)
J	$J_2 = 0.44*0.965 = 0.425$	$J_3 = 0.5*0.93 = 0.465$	$J_4 = 0.48*0.98 = 0.47$	$J_5 = 0.58*0.92 = 0.53$
K_v	$V_2 = \omega_2 r_2 = \frac{2\pi n_2}{60} \times r_2 = 0.95$ m/s		$V_4 = \frac{2\pi n_2}{60} \times \frac{N_2}{N_3} \times r_4 = 0.59$ m/s	
	$B = 0.8255 \quad A = 59.77$		→ $B = 0.8255 \quad A = 59.77$ (same Q_v)	
	$K_{v2} = K_{v3} = 1.186$		$K_{v4} = K_{v5} = 1.1478$	
K_o	consider as uniform/uniform $K_o = 1$		$K_o = 1$	
K_s	$K_b = 0.956 \quad K_{s2} = K_{s3} = 1.046$		$K_b = 0.91 \quad K_{s4} = K_{s5} = 1.099$	
K_H	$C_{mc} = 1$		$C_{mc} = 1$	
	$C_{pf} = 0.0265$		$C_{pf} = 0.04$	
	$C_{pm} = 1.1$		$C_{pm} = 1.1$	
	$C_e = 1$		$C_e = 1$	
	$C_{ma} = 0.14$ (fig)		$C_{ma} = 0.16$ (fig)	
	$K_{H2} = 1.17 = K_{H3}$		$K_{H4} = K_{H5} = 1.204$	

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Gears	(2)	(3)	(4)	(5)
K_B	$K_{B2} = K_{B3} = 1$		$K_{B4} = K_{B5} = 1$	
σ'_{FP}	$\sigma'_{FP} = 253.6$ MPa		$\sigma'_{FP} = 253.6$ MPa	
Y_θ	$Y_\theta = 1$		$Y_\theta = 1$	
Y_Z	$Y_Z = 0.85$		$Y_Z = 0.85$	
Y_N	$L_2 = 10^8$ cycles	$L_3 = \frac{14}{36} \times 10^8 = 3.8 \times 10^7$	$L_4 = L_3$	$L_5 = \frac{15}{45} \times 3.8 \times 10^7 = 1.26 \times 10^7$
	$Y_{N2} = 0.928$	$Y_{N3} = 0.9578$	$Y_{N4} = 0.9578$	$Y_{N5} = 0.9926$
σ_b	$\sigma_{b2} = 219.4$ MPa	$\sigma_{b3} = 200.6$ MPa	$\sigma_{b4} = 93$ MPa	$\sigma_{b5} = 82.5$ MPa
σ_{FP}	$\sigma_{FP2} = 276.84$ MPa	$\sigma_{FP3} = 285.76$ MPa	$\sigma_{FP4} = 285.76$ MPa	$\sigma_{FP5} = 296.14$ MPa
S_F	$S_{F2} = 1.26$	$S_{F3} = 1.42$	$S_{F4} = 3.07$	$S_{F5} = 3.59$

b) Contact:

	(2)	(3)	(4)	(5)
C_P	$C_{P2} = C_{P3} = 191$ (Table)		$C_{P4} = C_{P5} = 191$ (Table)	
C_f	take $C_f = 1$		$C_f = 1$	
I	$m_N = \frac{\pi m_n \cos\phi_n}{0.95 Z} = 0.6766$		$m_N = 0.68$	
	$I = 0.19$		$I = 0.18$	
σ_C	$\sigma_C = 1081$ MPa		$\sigma_C = 768.59$ MPA	
σ'_{HP}	$\sigma'_{HP} = 644$ MPa		$\sigma'_{HP} = 644$	
Z_N	$L_2 = 10^8$			
	$Z_{N2} = 0.948$	$Z_{N3} = 0.97$	$Z_{N4} = 0.97$	$Z_{N5} = 0.99$
C_H	$C_H = 1$		$C_H = 1$	
σ_{HP}	$\sigma_{HP2} = 718.25$ MPa	$\sigma_{HP3} = 735$ MPa	$\sigma_{HP4} = 735$ MPa	$\sigma_{HP5} = 750$ MPa
n_C	$n_{C2} = 0.44$ (failure!)	$n_{C3} = 0.46$ (failure)	$n_{C4} = 0.95$ (failure!)	$n_{C5} = 0.97$ (failure!)