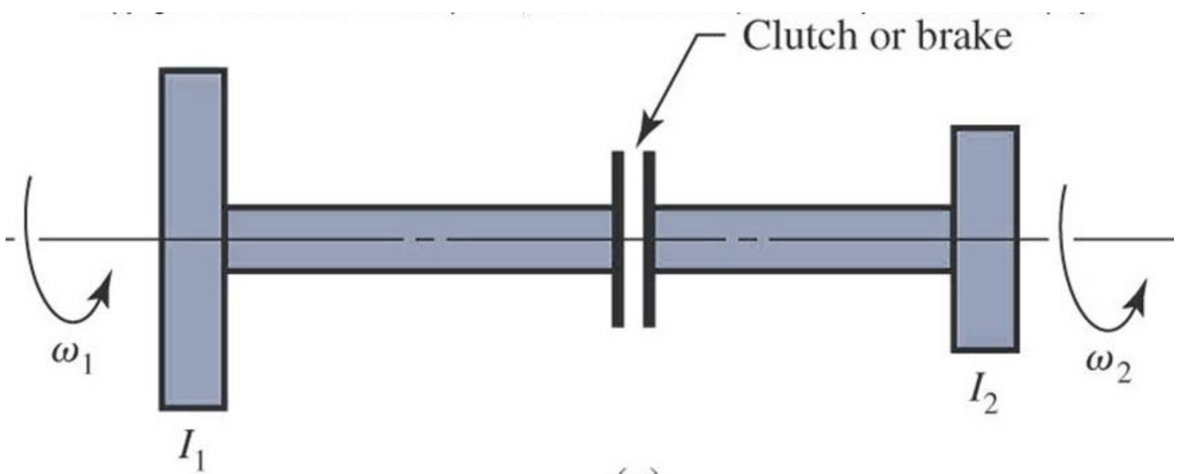
Clutches and Brakes

As shown in figure below, two inertias, I_1 and I_2 , rotating at the respective angular velocities ω_1 and ω_2 , are to be brought to the same speed by engaging the clutch or brake.

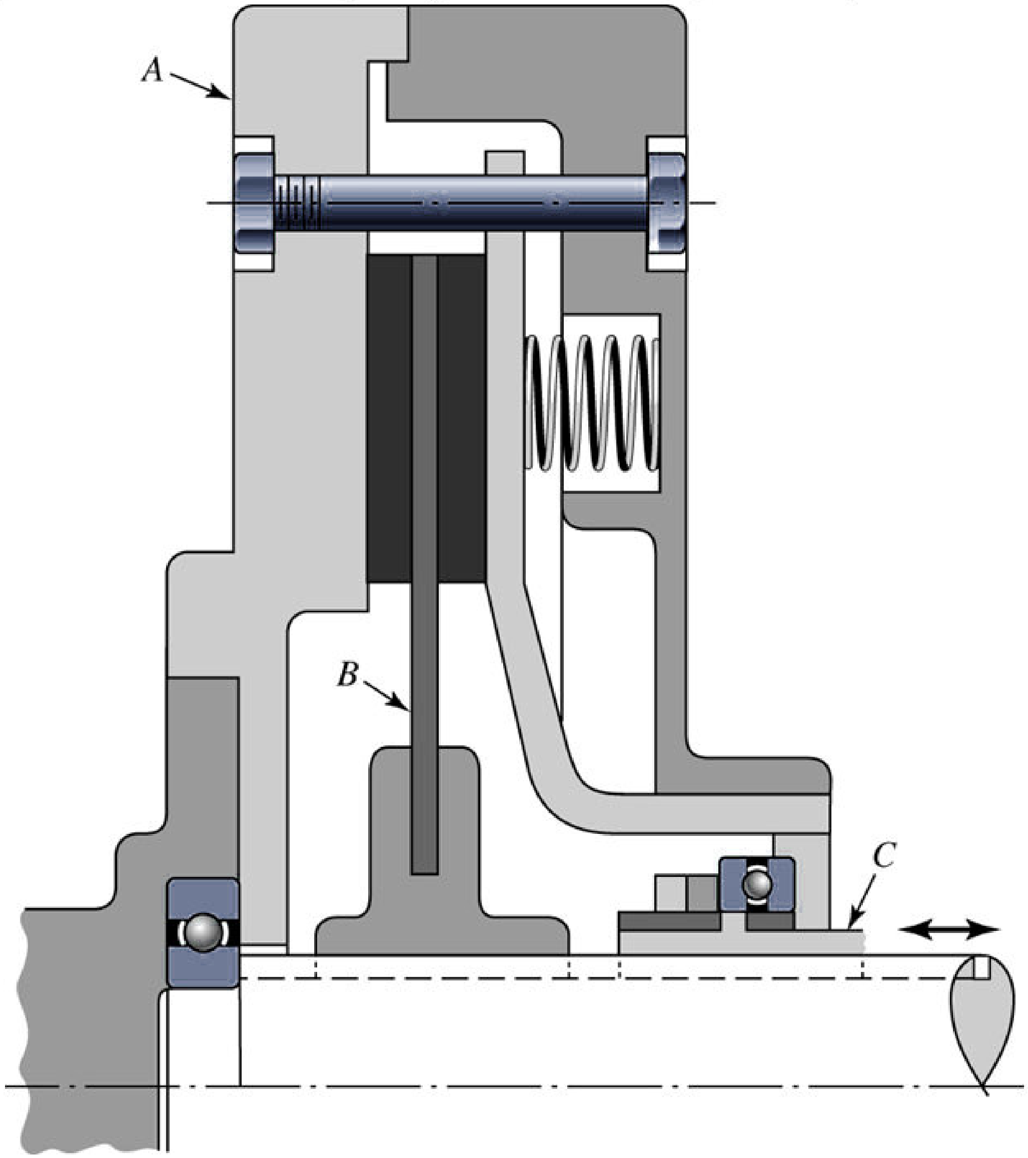
Note : In case of brake one of the two speeds is zero.

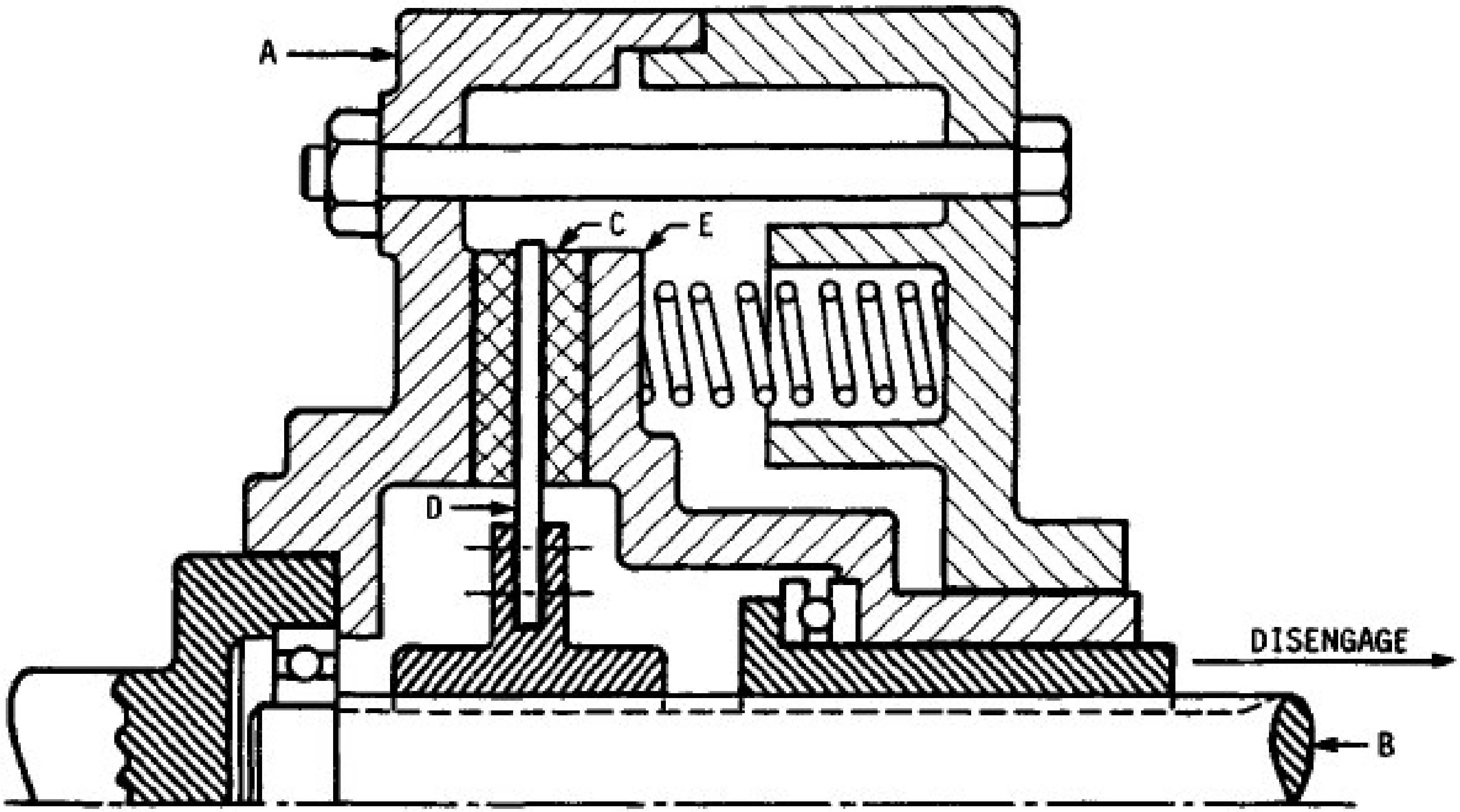
Slippage occurs because the two elements are rotating at different speeds and energy is dissipated during actuation in the form of heat, resulting in a temperature rise.



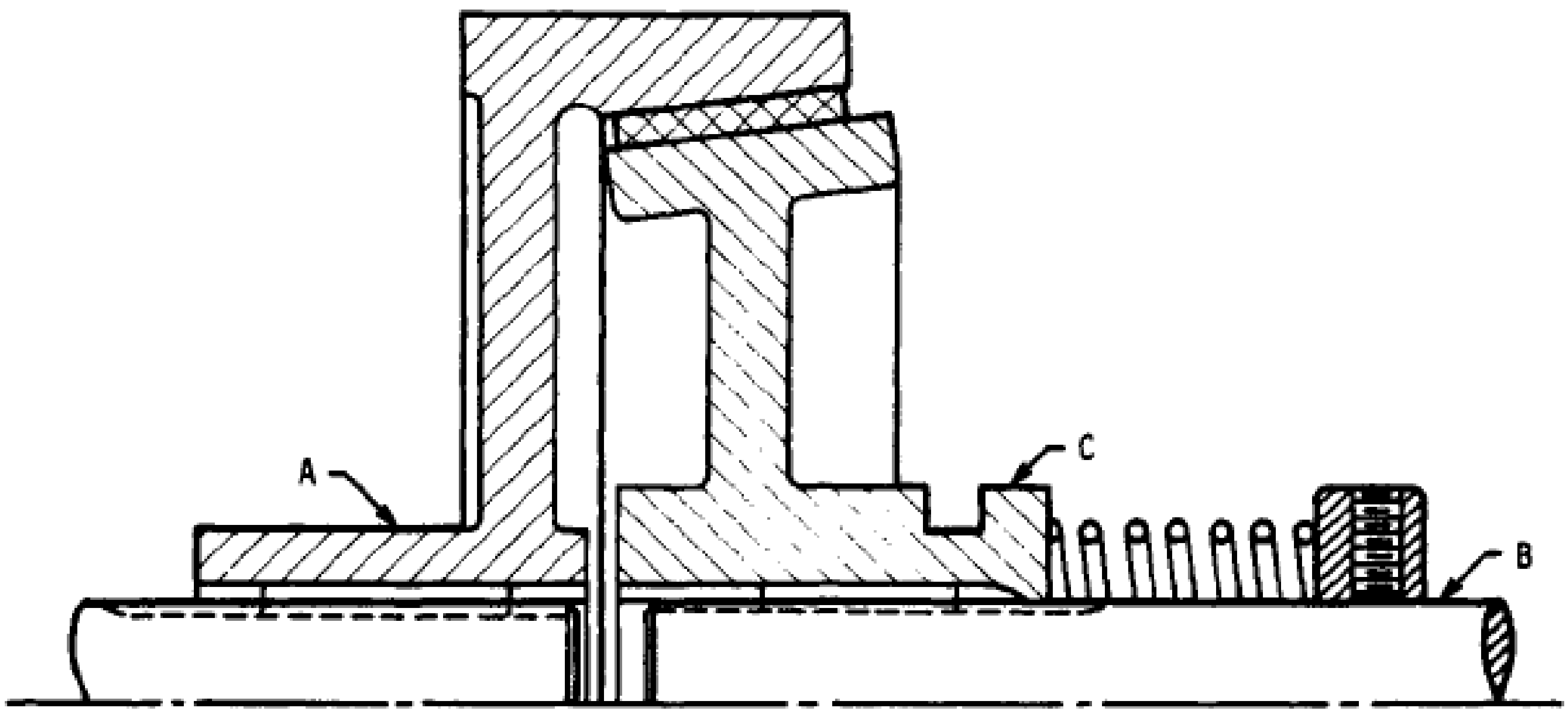
(a)

Frictional-Contact axial Clutches



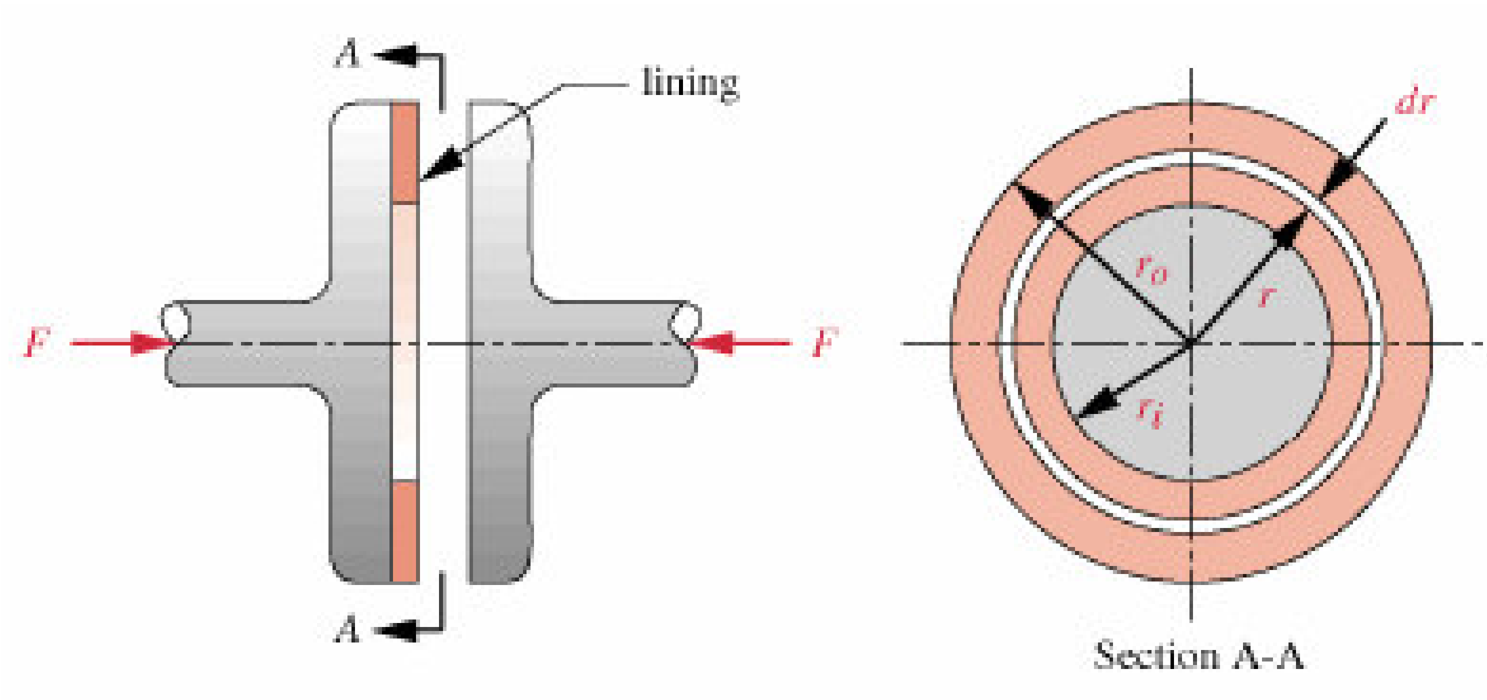


A
C
E
D
DISENGAGE
B


A
C
B

Frictional-Contact axial Clutches
Disk Clutches

The simplest disk clutch may consist of two disks (plates), pressed together axially with a normal force to generate the friction force required to transmit the torque. (one disk could be lined with a high friction material as shown)



The normal force can be supplied mechanically, hydraulically, pneumatically or electromagnetically.
The pressure between the contacting surfaces may approach a uniform distribution over the surface if the disks are flexible enough. In such cases, the Wear will be greater at larger diameters because wear is proportional to the pressure times the peripheral velocity (pv) where the velocity increases linearly with radius.
As the wear increases towards the outer diameters of the disks, the loss of material will change the pressure distribution to a non uniform one, and the disks will approach a uniform wear condition where pv = constant.
A new clutch may be close to the uniform-pressure condition and approaches the uniform wear conditions with use.
The uniform wear assumption gives a more conservative clutch rating, so it is favored by some designers.

Plate (Disk) Clutches:

The figure shows a friction disk having an outside diameter D and an inside diameter d. The objective is to obtain the axial force F necessary to produce a friction force to transmit a torque T.

There are two different criteria :

Uniform Wear:

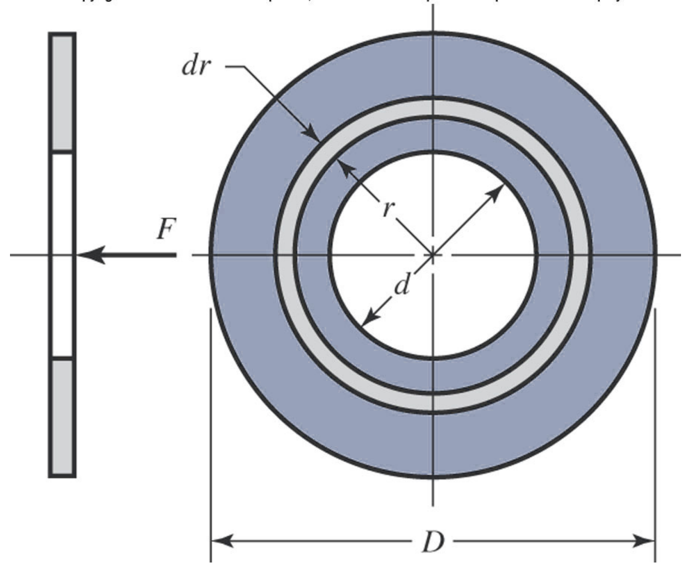
Since uniform pressure is assumed constant allover the plate face,

Then $pv = \text{constant}$ where p is the pressure, $v = \omega r$ is the speed

$p\omega r = \text{constant}$, but since ω is constant for the plate

$pr = \text{constant}$

$p_{max}\, r_i = p_{min}\, r_o$



For an element of the disk of radius r and thickness dr, the normal force acting upon this element is

$$dF = 2\pi r\, p\, dr$$

The total normal force on the plate

$$F = \int_{d/2}^{D/2} 2\pi p r dr = \pi p_{\max} d \int_{d/2}^{D/2} dr = \frac{\pi p_{\max} d}{2}(D - d)$$

The torque is found by integrating the product of the frictional force And the radius:

$$T = \int_{d/2}^{D/2} 2\pi f p r^2 dr = \pi f p_{\max} d \int_{d/2}^{D/2} r dr = \frac{\pi f p_{\max} d}{8}(D^2 - d^2)$$

$where\ f$ is the coefficient of friction

by substituting the value of the force F, then

$$T = \frac{fF}{4}(D + d)$$

For a multi-disk clutch with m acting friction surfaces
The torque is:

$$T = m.\frac{fF}{4}(D + d)$$

Uniform Pressure:

When uniform pressure is assumed over the area of the disk, then

$$F = \frac{\pi p}{4}(D^2 - d^2)$$

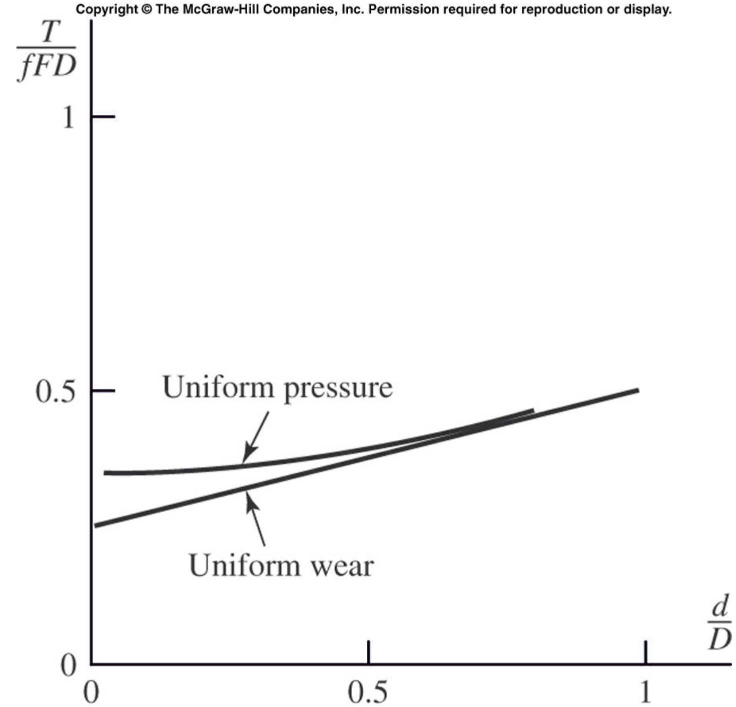
the torque is

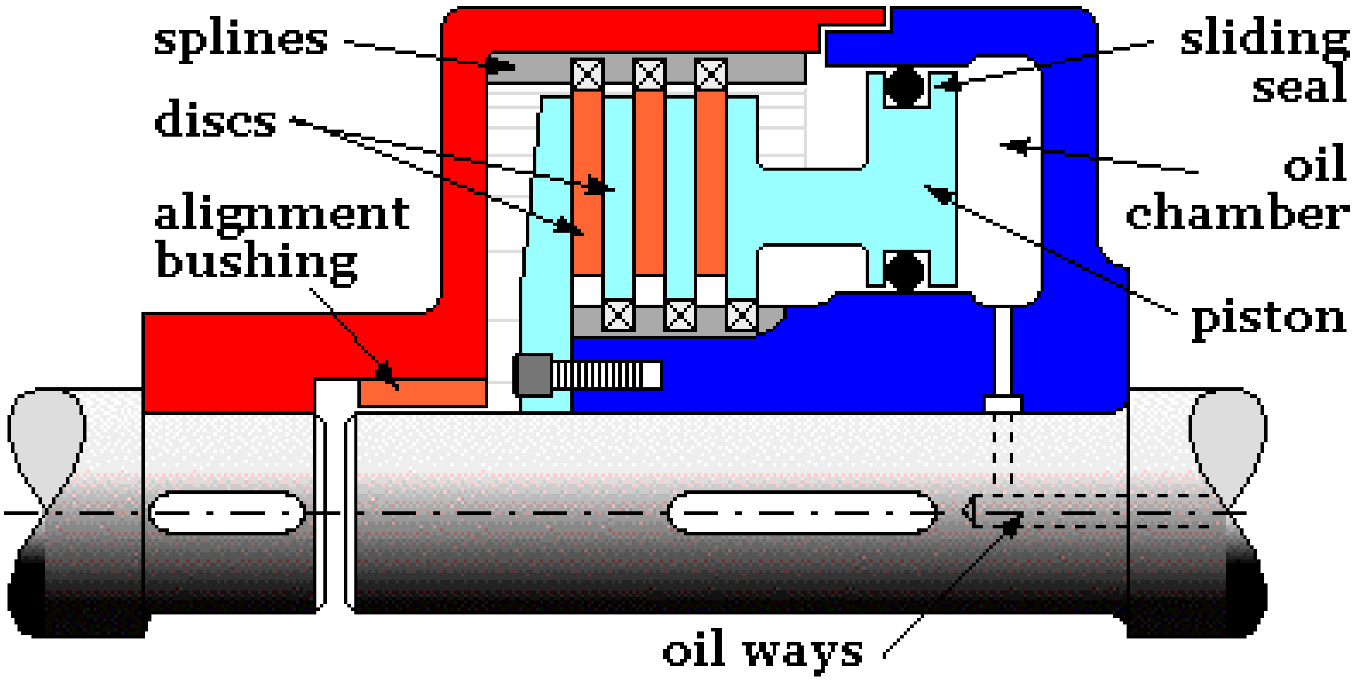
$$T = 2\pi f p \int_{d/2}^{D/2} r^2 dr = \frac{\pi f p}{12}(D^3 - d^3)$$

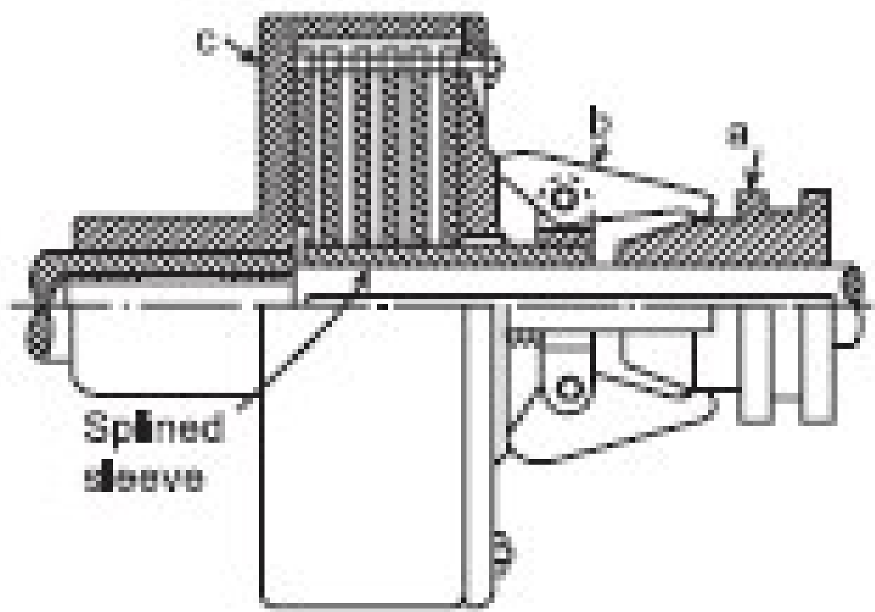
substituti ng for the force F

$$T = \frac{fF}{3}\frac{(D^3 - d^3)}{(D^2 - d^2)}$$

generally $$T = m.\frac{fF}{3}\frac{(D^3 - d^3)}{(D^2 - d^2)}$$



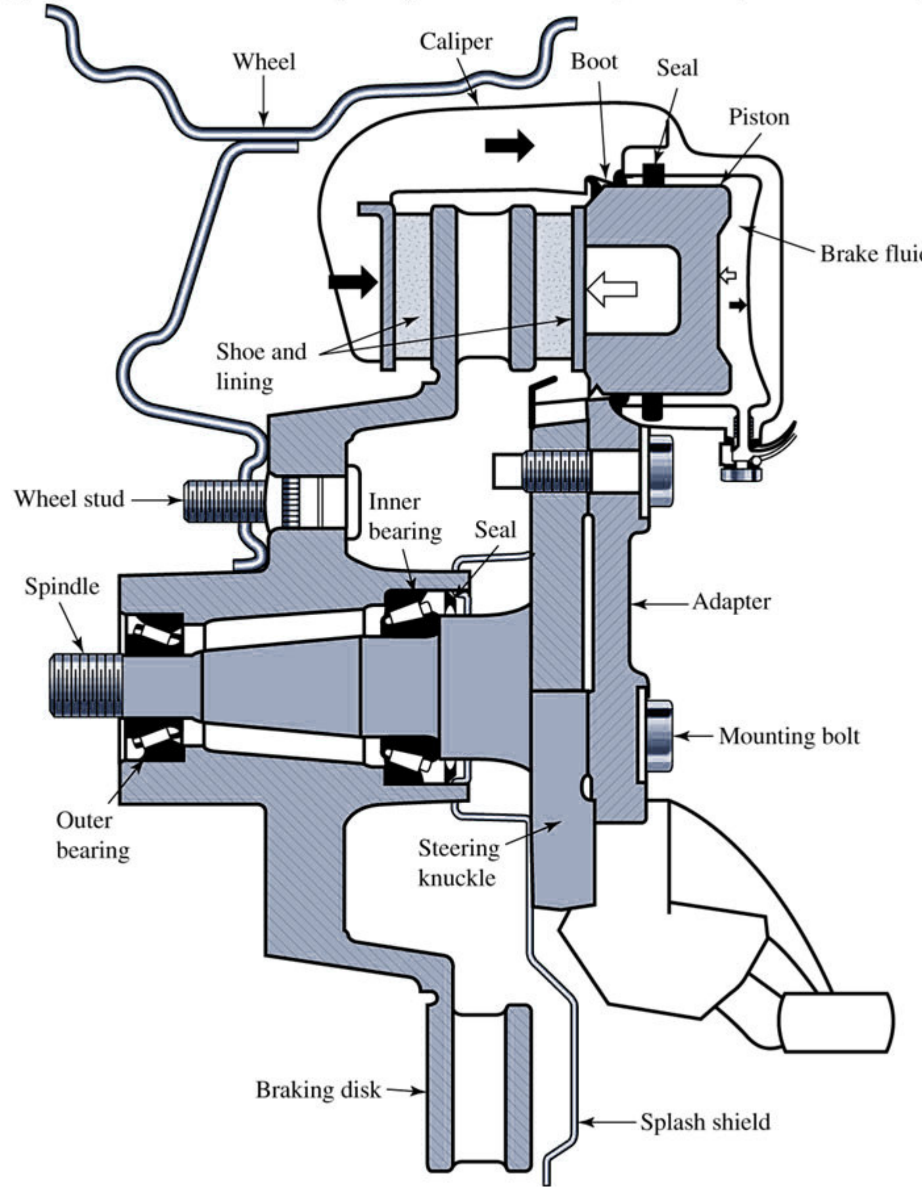

splines
discs
alignment
bushing
sliding
seal
oil
chamber
piston
oil ways

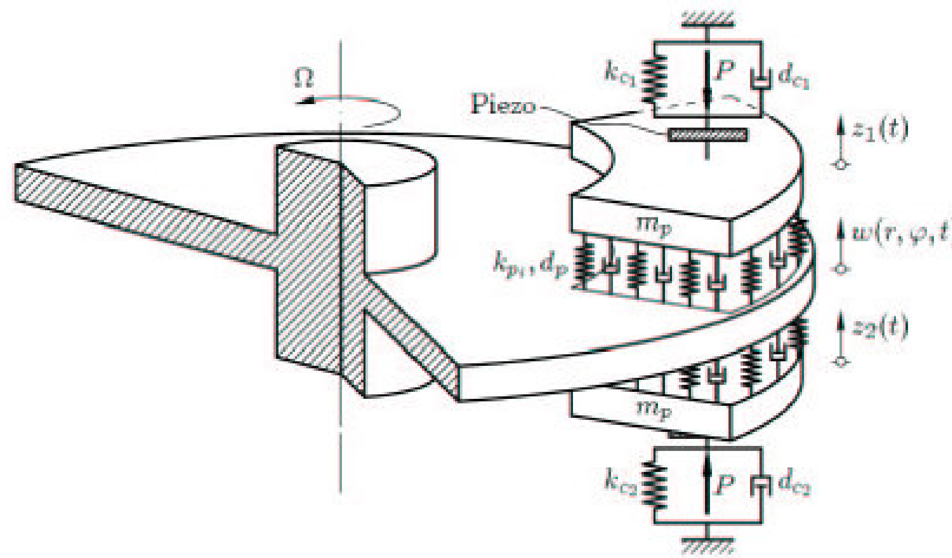

c
b
a
Splined
sleeve

Typical lining pressures (in N/mm^2) and dry coefficients of friction are shown below:

Material	Working Pressure	Coefficient of friction
Molded materials and sintered metals	1 to 2	
Cast iron on cast iron	1 to 1.7	0.15 - 0.2
Steel on cast iron	0.8 to 1.4	0.2 - 0.3
Bronze on cast iron	0.5 to 0.8	
Wood on cast iron	0.4 to 0.6	0.2 - 0.25
Cork on metal	0.05 to 0.1	0.35
Asbestos blocks on metal	0.25 to 1.1	0.4 - 0.48

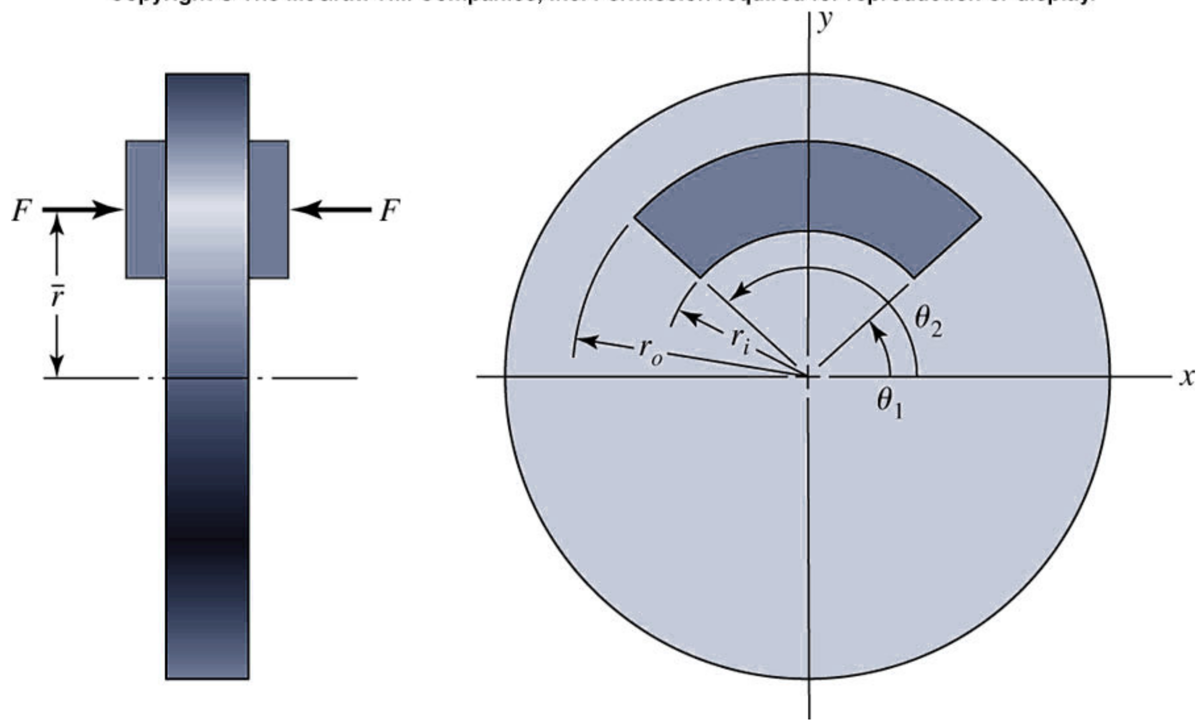
Disk Brakes:


Wheel
Caliper
Boot
Seal
Piston
Brake fluid
Shoe and
lining
Wheel stud
Inner
bearing
Seal
Spindle
Adapter
Mounting bolt
Outer
bearing
Steering
knuckle
Braking disk
Splash shield



Ω
Piezo
k_{c_1}
P
d_{c_1}
$z_1(t)$
m_p
$w(r,\varphi,t)$
k_{p_1}, d_p
$z_2(t)$
m_p
k_{c_2}
P
d_{c_2}

Disk Brake is a type of brakes where a friction material is pressed Against the faces of a rotating disk.



The actuating force F And the friction torque T are given by:

$$F = \int_{\theta_1}^{\theta_2} \int_{r_i}^{r_o} prdrd\theta$$

$$= (\theta_1 - \theta_2) \int_{r_i}^{r_o} prdr$$

$$T = \int_{\theta_1}^{\theta_2} \int_{r_i}^{r_o} fpr^2 drd\theta$$

$$= (\theta_2 - \theta_1) f \int_{r_i}^{r_0} pr^2 dr$$

The equivalent radius r_e (of the application of the friction force) can be found from the relation:

$$fFr_e = T \qquad \therefore r_e = \frac{T}{fF} = \frac{\int_{r_i}^{r_o} pr^2 dr}{\int_{r_i}^{r_o} pr dr}$$

The location r_F of the activation force F is found by equating the moments about the x-axis:

$$M_x = \int_{\theta_1}^{\theta_2}\int_{r_i}^{r_o} pr(\sin\theta) dr d\theta = (\cos\theta_1 - \cos\theta_2)\int_{r_i}^{r_o} pr^2 dr$$

$$M_x = Fr_F$$

$$\therefore r_F = \frac{M_x}{F} = \frac{\cos\theta_1 - \cos\theta_2}{\theta_1 - \theta_2} r_e$$

Uniform Wear: $pr = \text{constant} = p_{\max}.r_i$

$$p = \frac{r_i}{r} p_{\max}$$

$$\therefore F = (\theta_2 - \theta_1) p_{\max}.r_i (r_o - r_i)$$

$$T = (\theta_2 - \theta_1) f \int_{r_i}^{r_o} p_{\max}.r_i r dr$$

$$T = \frac{1}{2}(\theta_2 - \theta_1) f p_{\max}.r_i (r_o^2 - r_i^2)$$

$$r_e = \frac{p_{\max} r_i \int_{r_i}^{r_o} r dr}{p_{\max} r_i \int_{r_i}^{r_o} dr} = \frac{r_o^2 - r_i^2}{2} \frac{1}{r_o - r_i} = \frac{r_i + r_o}{2}$$

$$r_F = \frac{\cos\theta_1 - \cos\theta_2}{\theta_2 - \theta_1} \frac{r_i + r_o}{2}$$

Uniform Pressure:

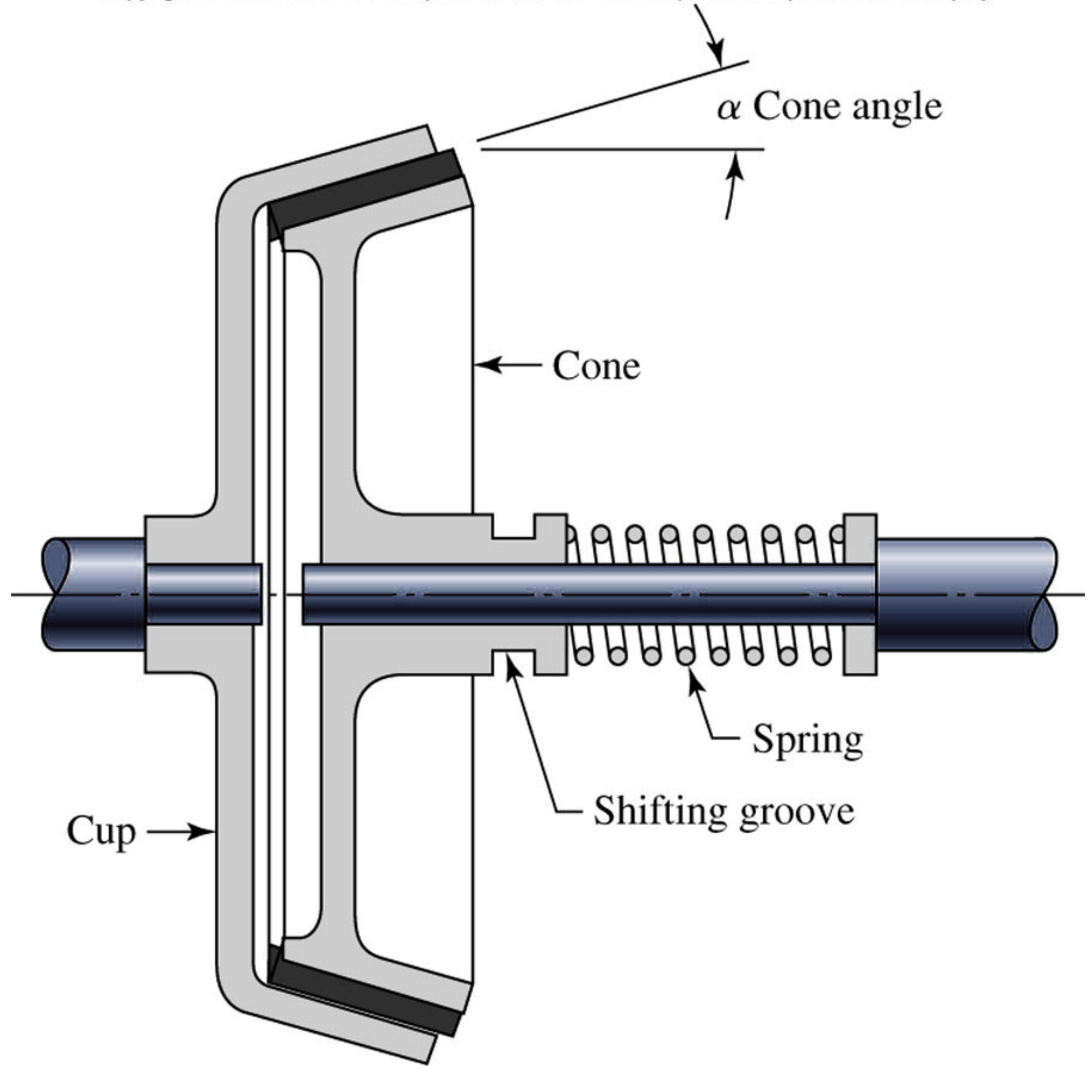
$$F = (\theta_2 - \theta_1)\, p \int_{r_i}^{r_o} r\,dr = \tfrac{1}{2}(\theta_2 - \theta_1)\, p(r_o^2 - r_i^2)$$

$$T = (\theta_2 - \theta_1)\, fp \int_{r_i}^{r_o} r^2\,dr = \tfrac{1}{3}(\theta_2 - \theta_1)\, fp(r_0^3 - r_i^3)$$

$$r_e = \frac{p \int_{r_i}^{r_o} r^2\,dr}{p \int_{r_i}^{r_o} r\,dr} = \frac{2}{3}\,\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}$$

$$r_F = \frac{\cos\theta_1 - \cos\theta_2}{\theta_2 - \theta_1}\, r_e$$

Cone clutches & Brakes



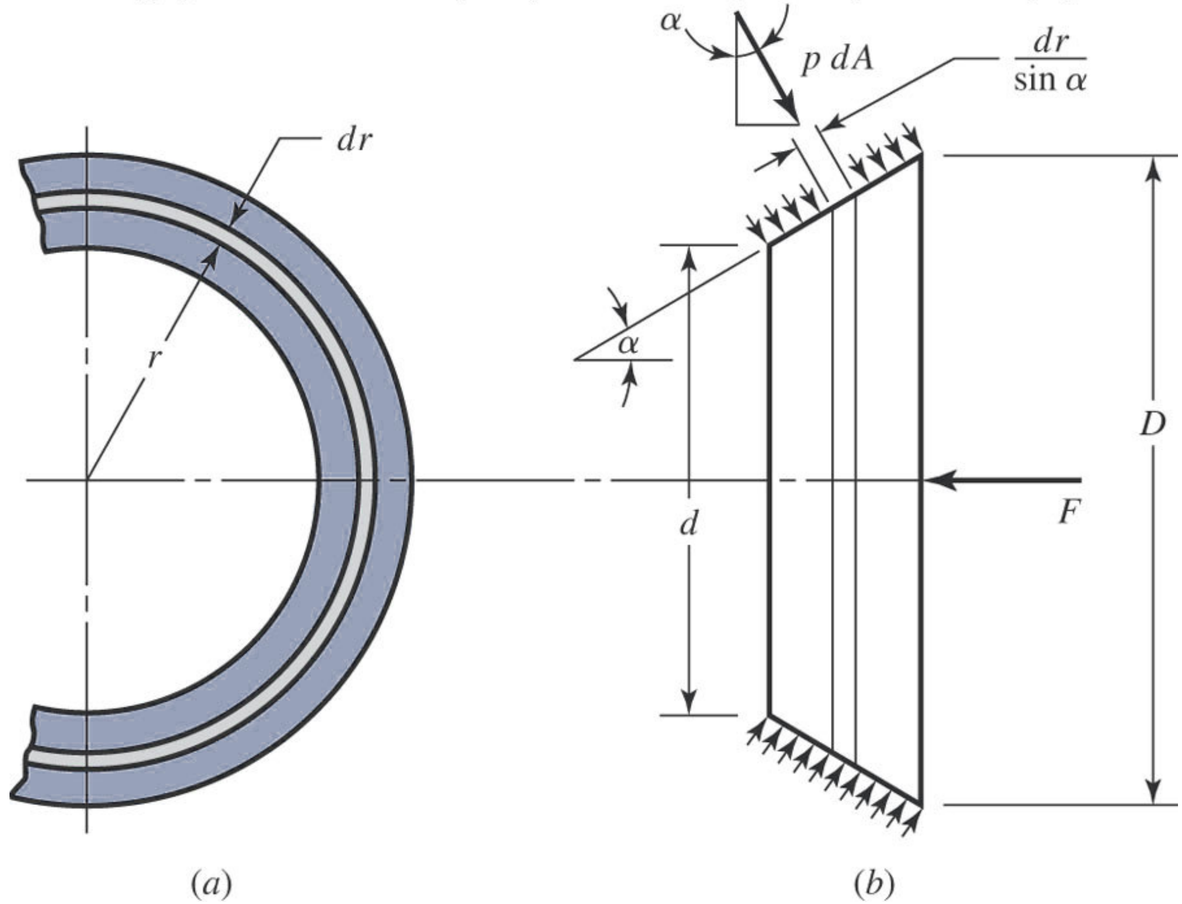
Uniform Wear:

For an element of area dA

$$dA = \frac{2\pi r dr}{\sin\alpha}$$

$$F = \int p dA \sin\alpha$$

$$p = p_{\max}\frac{d}{2r}$$



$$\therefore F = \int_{\frac{d}{2}}^{\frac{D}{2}} \left(p_{\max} \frac{d}{2r}\right)\left(\frac{2\pi r dr}{\sin\alpha}\right)\sin\alpha$$

$$= \pi p_{\max} d \int_{\frac{d}{2}}^{\frac{D}{2}} dr = \frac{\pi p_{\max} d}{2}(D - d)$$

$$T = \int rfp\,dA = \int_{\frac{d}{2}}^{\frac{D}{2}} (rf)\left(p_{\max} \frac{d}{2r}\right)\left(\frac{2\pi r dr}{\sin\alpha}\right)$$

$$= \frac{\pi f p_{\max} d}{\sin\alpha} \int_{\frac{d}{2}}^{\frac{D}{2}} r\,dr = \frac{\pi f p_{\max} d}{8\sin\alpha}(D^2 - d^2)$$

in terms of F: $$T = \frac{fF}{\sin\alpha}\left(\frac{D+d}{4}\right)$$

Uniform Pressure:

$$F = \int p\,dA \sin\alpha = \int_{\frac{d}{2}}^{\frac{D}{2}} p\left(\frac{2\pi r\,dr}{\sin\alpha}\right)\sin\alpha$$

$$= \frac{\pi p}{4}\left(D^2 - d^2\right)$$

$$T = \int r f p\,dA = \int_{\frac{d}{2}}^{\frac{D}{2}} (r f p)\,\frac{2\pi r\,dr}{\sin\alpha}$$

$$= \frac{\pi f p}{12\sin\alpha}\left(D^3 - d^3\right)$$

in terms of F

$$T = \frac{fF}{3\sin\alpha}\,\frac{D^3 - d^3}{D^2 - d^2}$$