Numerical Analysis - ENME 602
Engineering and Materials Science
Assoc.Prof.Dr. Hesham H. Ibrahim
Spring 2021

GUC
German University in Cairo
الجامعة الألمانية بالقاهرة

Practice Sheet 1

Computer Arithmetic & Error Analysis

Problem 1

Compute the absolute error and relative error in approximations of p by p^*.

a. $p = \pi,\ p^* = 22/7$

b. $p = \pi,\ p^* = 3.1416$

c. $p = e,\ p^* = 2.718$

d. $p = \sqrt{2},\ p^* = 1.414$

e. $p = e^{10},\ p^* = 22000$

f. $p = 10^{\pi},\ p^* = 1400$

g. $p = 8!,\ p^* = 39900$

h. $p = 9!,\ p^* = \sqrt{18\pi}(9/e)^9$

Solution:

Question	Absolute Error	Relative Error
a	1.264×10^{-3}	4.025×10^{-4}
b	7.346×10^{-6}	2.338×10^{-6}
c	2.818×10^{-4}	1.037×10^{-4}
d	2.136×10^{-4}	1.51×10^{-4}
e	26.466	1.202×10^{-3}
f	14.544	1.05×10^{-2}
g	420	1.042×10^{-2}
h	3343.127	9.213×10^{-3}

Problem 2

Suppose p^* must approximate p with relative error at most 10^{-3}. Find the largest interval in which p^* must lie for each value of p.

a. 150 **b.** 900 **c.** 1500 **d.** 90

Solution:

Question	**Interval**
a	$p^* \in [149.85, 150.15]$
b	$p^* \in [899.1, 900.9]$
c	$p^* \in [1498.5, 1501.5]$
d	$p^* \in [89.91, 90.9]$

Problem 3

1. Use three-digit rounding arithmetic to perform the following calculations. Compute the absolute error and relative error with the exact value determined to at least five digits.

a. $133 + 0.921$

b. $133 - 0.499$

c. $(121 - 0.327) - 119$

d. $(121 - 119) - 0.327$

e. $\dfrac{\frac{13}{14} - \frac{6}{7}}{2e - 5.4}$

f. $-10\pi + 6e - \dfrac{3}{62}$

g. $(\frac{2}{9}).(\frac{9}{7})$

h. $\dfrac{\pi - \frac{22}{7}}{\frac{1}{17}}$

2. Repeat part **1** using four-digit rounding arithmetic.

3. Repeat part **1** using three-digit chopping arithmetic.

4. Repeat part **1** using four-digit chopping arithmetic.

Solution:

1.

Question	Absolute Error	Relative Error
a	0.079	5.9×10^{-4}
b	0.499	3.77×10^{-3}
c	0.327	0.195
d	3×10^{-3}	1.79×10^{-3}
e	0.154	0.786
f	0.0546	3.6×10^{-3}
g	2.86×10^{-4}	10^{-3}
h	0.0215	1

2.

Question	**Absolute Error**	**Relative Error**
a	0.021	1.568×10^{-4}
b	0.001	7.55×10^{-6}
c	0.027	0.01614
d	0	0
e	0.03246	0.01662
f	0.005377	3.548×10^{-4}
g	1.429×10^{-5}	5×10^{-5}
h	0.0045	0.2092

3.

Question	**Absolute Error**	**Relative Error**
a	0.921	6.88×10^{-3}
b	0.501	3.78×10^{-3}
c	0.673	0.402
d	0.003	1.79×10^{-3}
e	1.60	0.817
f	0.0454	0.00299
g	0.00171	0.00600
h	0.02150	1

4.

Question	**Absolute Error**	**Relative Error**
a	0.021	1.568×10^{-4}
b	0.001	7.55×10^{-6}
c	0.073	0.04363
d	0	0
e	0.02945	0.01508
f	0.004622	3.050×10^{-4}
g	2.143×10^{-4}	7.5×10^{-4}
h	0.0045	0.2092

Problem 4

Use three-digit chopping arithmetic to compute the sum $\sum_{i=1}^{10}(1/i^2)$ first by $\frac{1}{1}+\frac{1}{4}+\cdots+\frac{1}{100}$ and then by $\frac{1}{100}+\frac{1}{81}+\cdots+\frac{1}{1}$. Which method is more accurate, and why?

Solution:

$$\frac{1}{1}+\frac{1}{4}+\cdots+\frac{1}{100}=1.53;\ \frac{1}{100}+\frac{1}{81}+\cdots+\frac{1}{1}=15.4.$$

The actual value is 1.549; Thus Method 2 is more accurate.

Problem 5

The number e is defined by $e = \sum_{n=0}^{\infty}(1/n!)$. Use four-digit chopping arithmetic to compute the following approximations to e, and determine the absolute and relative errors.

a. $e \approx \sum_{n=0}^{5} \frac{1}{n!}$

b. $e \approx \sum_{j=0}^{5} \frac{1}{(5-j)!}$

c. $e \approx \sum_{n=0}^{10} \frac{1}{n!}$

d. $e \approx \sum_{j=0}^{10} \frac{1}{(10-j)!}$

Solution:

Question	Approximation	Absolute Error	Relative Error
a	2.715	3.282×10^{-3}	1.207×10^{-3}
b	2.716	2.282×10^{-3}	8.394×10^{-4}
c	2.716	2.282×10^{-3}	8.394×10^{-4}
d	2.718	2.818×10^{-4}	1.037×10^{-4}