Numerical Analysis - ENME 602
Engineering and Materials Science
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Practice Sheet 3A

Interpolation & Polynomial Approximation

Problem 1

1. For the given functions $f(x)$, let $x_0 = 1$, $x_1 = 1.25$ and $x_2 = 1.6$. Construct interpolation polynomials of degree at most one and two to approximate $f(1.4)$, and find the absolute error.

   a. $f(x) = \sin(\pi x)$
   b. $f(x) = \sqrt[3]{x-1}$
   c. $f(x) = \log_{10}(3x-1)$
   d. $f(x) = e^{2x} - x$

2. Use Theorem 3.3 to find an error bound for the approximations in Exercise **1**.

Solution:

Question	Polynomial	Approximation	Error
1.a	$p_1(x) = -0.697x + 0.1641$ $p_2(x) = 3.5525x^2 - 10.821x + 7.2691$	$p_1(1.4) = 0.8117$ $p_2(1.4) = -0.9174$	0.13935 0.03365
1.b	$p_1(x) = 0.6099x - 0.1325$ $p_2(x) = 3.1832x^2 + 9.682x + 6.4988$	$p_1(1.4) = 0.72136$ $p_2(1.4) = -0.81692$	0.01544 0.080
1.c	$p_1(x) = 0.4013 - 0.0622$ $p_2(x) = -0.2531x^2 + 1.1227x - 0.56852$	$p_1(1.4) = 0.49962$ $p_2(1.4) = 0.50718$	5.53×10^{-3} 2.03×10^{-3}
1.d	$p_1(x) = 34.289x - 31.929$ $p_2(x) = 26.854x^2 - 42.249x21.784$	$p_1(1.4) = 16.0756$ $p_2(1.4) = 15.269$	1.0306 0.22424

Question	Error Bound
2.a	for $P_1(x) : 0.15113$ for $P_2(x) : 0.06976$
2.b	for $P_1(x) : 0.0343$ for $P_2(x)$: There is no bound
2.c	for $P_1(x) : 7.9135 \times 10^{-3}$ for $P_2(x) : 6.6 \times 10^{-3}$
2.d	for $P_1(x) : 1.50262$ for $P_2(x) : 0.4416$

Problem 2

1. Use appropriate Lagrange interpolating polynomials of degrees one, two and three to approximate each of the following:

   a. $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$,$f(0.5) = 2.71828$, $f(0.75) = 4.48169$

   b. $f(0)$ if $f(-0.5) = 1.93750$, $f(-0.25) = 1.33203$,$f(0.25) = 0.800781$, $f(0.5) = 0.687500$

   c. $f(0.18)$ if $f(0.1) = -0.29004986$, $f(0.2) = -0.56079734$,$f(0.3) = -0.81401972$, $f(0.4) = -1.0526302$

   d. $f(0.25)$ if $f(-1) = 0.86199480$, $f(-0.5) = 0.95802009$,$f(0) = 1.0986123$, $f(0.5) = 1.2943767$

2. The data for Exercise **1** were generated using the following functions. Use the error formula to find a bound for the error, and compare the bound to the actual error for the cases $n = 1$ and $n = 2$.

   a. $f(x) = e^{2x}$

   b. $f(x) = x^4 - x^3 + x^2 - x + 1$

   c. $f(x) = x^2 \cos x - 3x$

   d. $f(x) = \ln(e^x + 2)$

Solution:

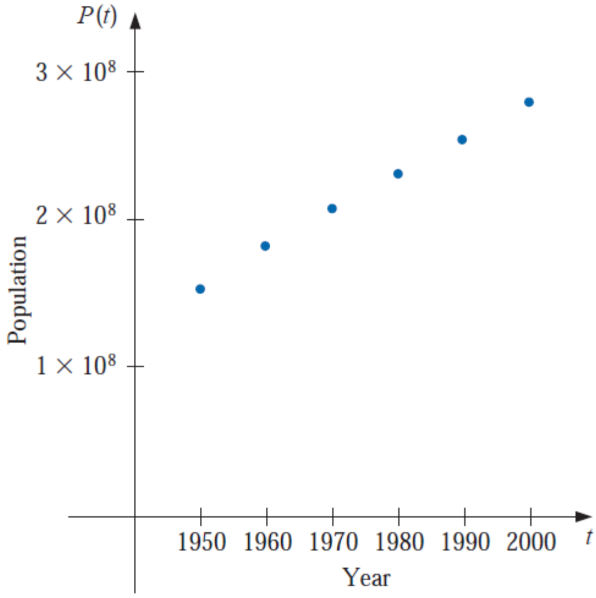
Q.	**Polynomial**	**Approximation**
1.a	$p_1(x) = 4.27824x + 0.57916$ $p_2(x) = 5.5508x^2 + 0.11514x + 1.2730$ $p_3(x) = 2.91211x^3 + 1.1826x^2 + 2.1172x + 1$	$p_1(0.43) = 2.4188$ $p_2(0.43) = 2.3488$ $p_3(0.43) = 2.3605$
1.b	$p_1(x) = -1.06249x + 1.0664055$ $p_2(x) = 0.282322x^2 - 0.6648405x + 0.94935$ $p_3(x) = -0.999944x^3 + 1.312504x^2 - x + 0.984374$	$p_1(0) = 1.0664$ $p_2(0) = 0.94935$ $p_3(0) = 0.984374$
1.c	$p_1(x) = -2.7074748x - 0.01930238$ $p_2(x) = 0.876255x^2 - 2.9703513x - 1.77728 \times 10^{-3}$ $p_3(x) = -0.48517x^3 + 1.1673x^2 - 3.0237x - 1.1338 \times 10^{-3}$	$p_1(0.18) = -0.50665$ $p_2(0.18) = -0.50805$ $p_3(0.18) = -0.50814$
1.d	$p_1(x) = 0.3915288x + 1.0986123$ $p_2(x) = 0.11034x^2 + 0.33636x + 1.0986$ $p_3(x) = -0.040888x^3 + 0.15046x^2 - 0.346196x + 1.09861$	$p_1(0.25) = 1.19649$ $p_2(0.25) = 1.1896$ $p_3(0.25) = 1.1952$

Question	**Error Bound**	**Absolute Error**
2.a	for $P_1(x) : 0.87313$ for $P_2(x) : 0.035938$	0.05564 0.01426
2.b	for $P_1(x) : 0.1328125$ for $P_2(x) : 0.06602$	0.0664 0.05065
2.c	for $P_1(x) : 2.425 \times 10^{-3}$ for $P_2(x) : 2.22 \times 10^{-4}$	1.47346×10^{-3} 7.3464×10^{-5}
2.d	for $P_1(x) : 7.74\times^{-3}$ for $P_2(x) : 7.6495 \times 10^{-3}$	7.42×10^{-3} 5.3006×10^{-4}

Problem 3

A census of the population of the United States is taken every 10 years. The following table lists the population, in thousands of people, from 1950 to 2000, and the data are also represented in the figure.

Year	1950	1960	1970	1980	1990	2000
Population (in thousands)	151,326	179,323	203,302	226,542	249,633	281,422



a. Use Lagrange interpolation to approximate the population in the years 1940, 1975, and 2020.

b. The population in 1940 was approximately 132,165,000. How accurate do you think your 1975 and 2020 figures are?

Solution:

a. $P_5(x) = 9.12166667 \times 10^{-7} x^5 - 0.00899604583 x^4 + 35.4884313333 x^3 - 69998.6501554 x^2 + 69033562.289 x - 27232574341.7$
$P_5(1940) = 102.396,\ P_5(1975) = 215.042,\ P_5(2020) = 513.442.$

b. The 1975 may not be very accurate, but the 2020 figure is likely to be extremely inaccurate

Problem 4

It is suspected that the high amounts of tannin in mature oak leaves inhibit the growth of the winter moth (Operophtera bromata L., Geometridae) larvae that extensively damage these trees in certain years. The following table lists the average weight of two samples of larvae at times in the first 28 days after birth. The first sample was reared on young oak leaves, whereas the second sample was reared on mature leaves from the same tree.

a. Use Lagrange interpolation to approximate the average weight curve for each sample.

b. Find an approximate maximum average weight for each sample by determining the maximum of the interpolating polynomial.

Day	0	6	10	13	17	20	28
Sample 1 average weight (mg)	6.67	17.33	42.67	37.33	30.10	29.31	28.74
Sample 2 average weight (mg)	6.67	16.11	18.89	15.00	10.56	9.44	8.89

Solution:

a. Sample 1: $P_6(x) = 0.0000409458x^6 - 0.00367168x^5 + 0.126902x^4 - 2.09464x^3 + 16.1427x^2 - 42.643x + 6.67$

Sample 2: $P_6(x) = 0.00000836160x^6 - 0.000752546x^5 + 0.0258413x^4 - 0.413799x^3 + 2.91281x^2 - 5.67821x + 6.67$

b. Sample 1 : 42.71 mg.
Sample 2: 19.42 mg.