Design of gears for Surface durability

Design of gears for Surface durability:

Surfaces of gear teeth are subjected to failure due to many repetitions of high contact stresses (pitting).

Hertz has shown that the contact stress between two cylinders can be computed from the equation:

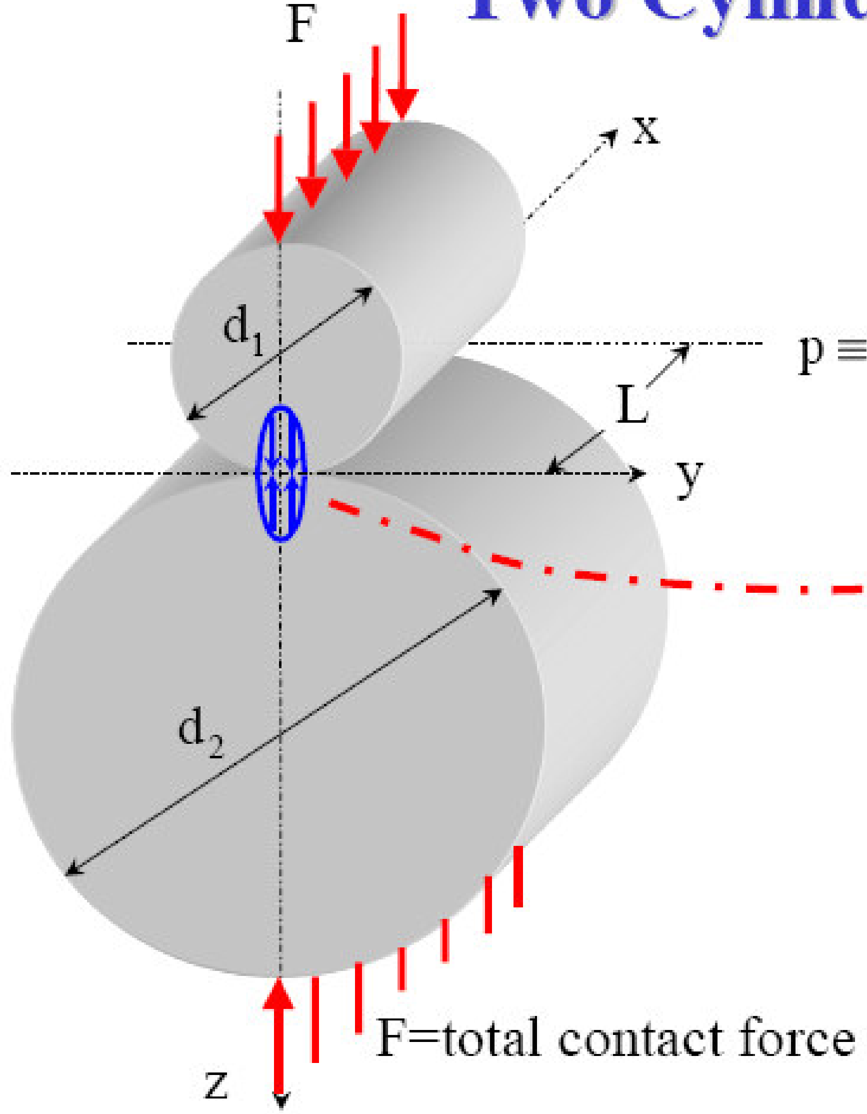
$$p_{max} = \frac{2F}{\pi b L}$$

Where p_{max} is the largest surface pressure

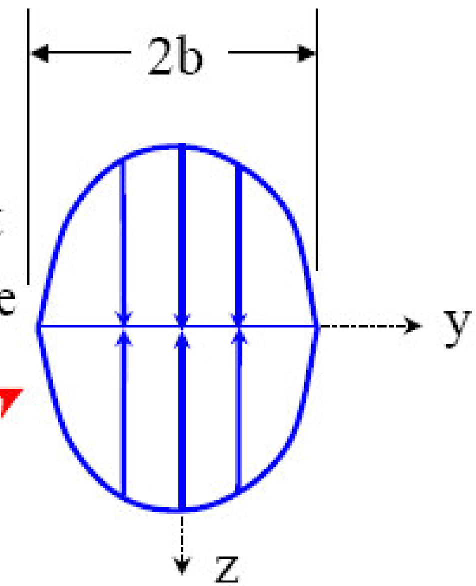
F is the force pressing the two cylinders together

L is the contact length of cylinders

Contact Stress Between Two Cylinders



Contact Zone

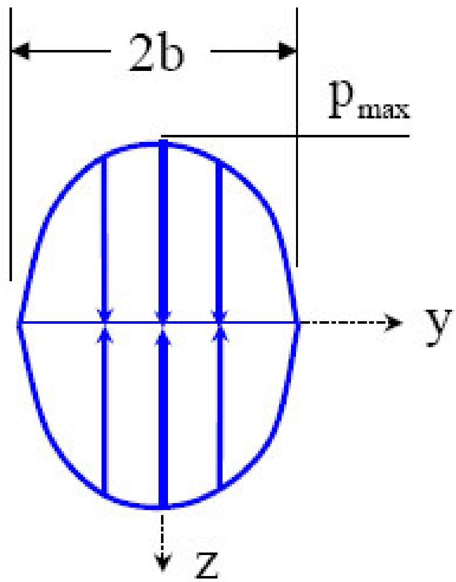


Contact Force

$$F = 2L\int_0^b p(y)\,dy$$

Hertz Contact Stress Equations

Contact Zone



Contact Width

$$b = \sqrt{\frac{2F}{\pi L} \frac{\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2}{1/d_1 + 1/d_2}}$$

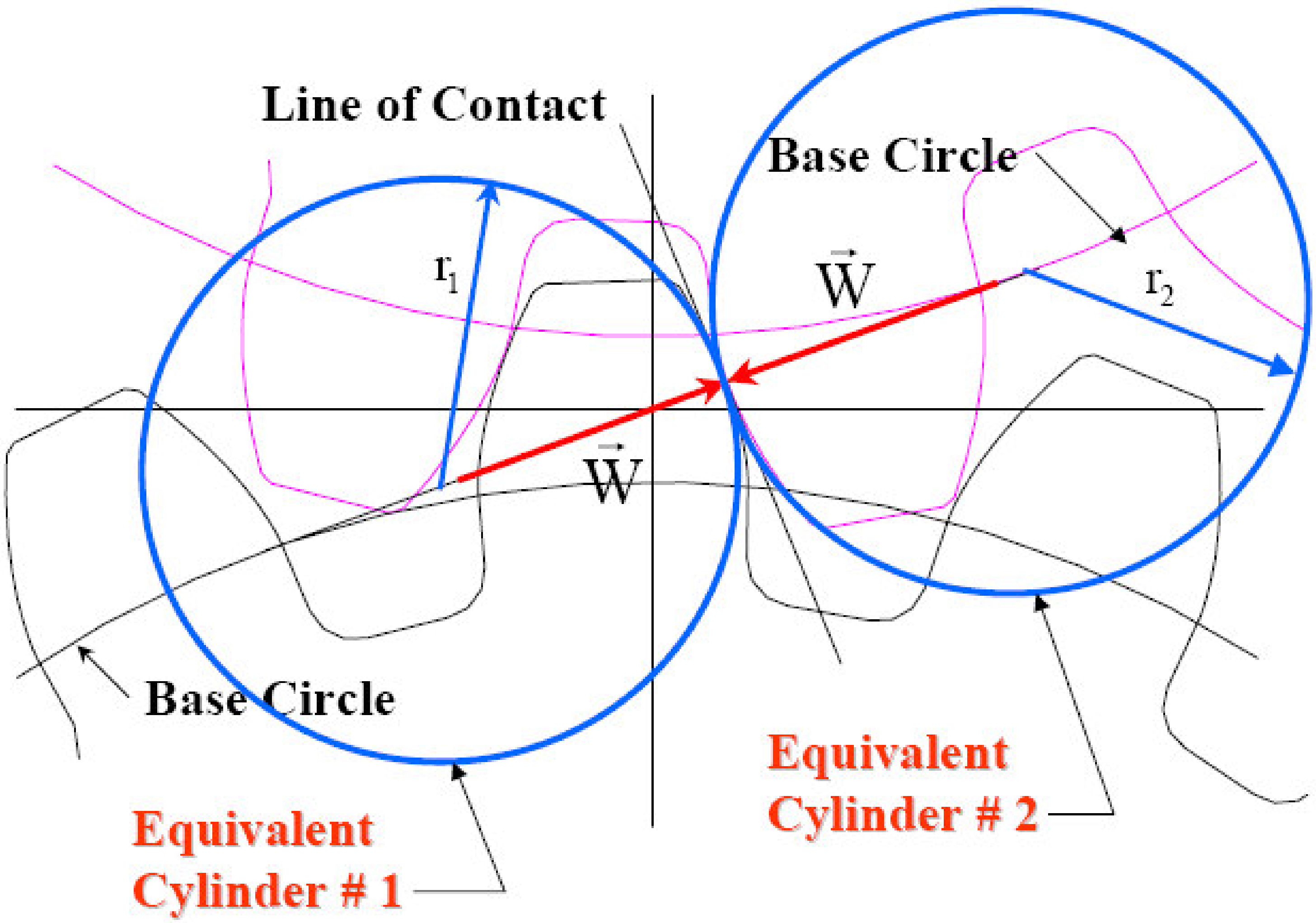
Maximum Contact Pressure

$$p_{max} = \frac{2F}{\pi b L}$$

Where υ_1, υ_2, E_1, E_2 are Poisson's ratios and the elastic constants

And d_1, d_2 are the diameters of the two contacting cylinders

For involute gears we may consider the teeth contact at every contact point as a contact between two cylinders with radii r_1 and r_2.



Now if we replace the force F by ($W = W_t / \cos\varphi$) , d by 2r and L by the face width of gears b and p_{max} by the contact stress σ_c. The contact stress between the two teeth can be expressed as :

$$\sigma_c = \sqrt{\frac{W_t}{\pi b \cos\varphi} \frac{(\frac{1}{r_1})+(\frac{1}{r_2})}{[(1-\nu_1^2)/E_1]+[(1-\nu_2^2)/E_2]}}$$

At the pitch point $r_1 = d_p\cos\varphi / 2$ and $r_2 = d_G\cos\varphi / 2$, where d_p and d_G are the pitch diameters of the pinion and gear.

Define an elastic coefficient :

$$C_p = \left[\frac{1}{\pi(\frac{1-\nu_p^2}{E_p}+\frac{1-\nu_G^2}{E_G})}\right]^{\frac{1}{2}}$$

The contact stress is: $$\sigma_c = -C_p \left[\frac{W_t}{b\cos\varphi}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\right]^{\frac{1}{2}}$$

The AGMA contact stress equation:

$$\sigma_c = C_p \sqrt{\frac{W_t}{bd_p I} K_v K_o K_s K_H C_f}$$

C_p is the elastic coefficient ($\sqrt{\text{MPa}}$)

C_f is the surface condition factor

d_p is the pitch diameter of the pinion

I is the geometry factor for pitting resistance

K_v, K_o, K_s, K_H are the dynamic, overload, size and the load-distribution factors as explained before.

Surface-strength geometry factor I:

In the original equation of the contact stress:

$$\sigma_c = -C_p \left[\frac{W_t}{b \cos \varphi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{\frac{1}{2}}$$

At the pitch point $r_1 = d_p \sin \varphi$ and $r_2 = d_G \sin\varphi$ where d_p and d_G are the pitch diameters of the pinion and gear.

$$\therefore \frac{1}{r_1} + \frac{1}{r_2} = \frac{2}{\sin \varphi} \left(\frac{1}{d_p} + \frac{1}{d_G} \right)$$

Define the speed ratio $m_G = (N_G / N_p) = (d_G / d_p)$

$$\therefore \frac{1}{r_1} + \frac{1}{r_2} = \frac{2}{d_p \sin \varphi} \left(\frac{m_G + 1}{m_G} \right)$$

$$\therefore \sigma_c = -C_p \left[\frac{W_t}{b \cos \varphi} \left(\frac{2}{d_p \sin \varphi} \frac{m_G + 1}{m_G} \right) \right]^{\frac{1}{2}}$$

$$\therefore \sigma_c = -C_p \left[\frac{W_t}{bd_p} \left(\frac{2}{\cos\varphi \sin\varphi} \frac{m_G + 1}{m_G} \right) \right]^{\frac{1}{2}}$$

define $\quad I = \dfrac{\cos\varphi \sin\varphi}{2} \dfrac{m_G}{m_G + 1}$

$$\therefore \sigma_c = -C_p \left[\frac{W_t}{bd_p I} \right]^{\frac{1}{2}}$$

The surface-strength geometry factor :

$$I = \frac{\cos\varphi \sin\varphi}{2} \frac{m_G}{m_G + 1} \qquad \text{for rxternal gears}$$

$$I = \frac{\cos\varphi \sin\varphi}{2} \frac{m_G}{m_G - 1} \qquad \text{for internal gears}$$

The elastic constant C_p :

May be obtained from the tables or calculated directly from the relation

$$C_p = \sqrt{\frac{1}{\pi\left(\frac{1-\nu_p^2}{E_p}+\frac{1-\nu_G^2}{E_G}\right)}}$$

Pinion	Modulus of elasticity *E*, GPa	Gear					
		Ste el	Malleabl e iron	Nodular iron	Cast iron	Aluminum bronze	Tin bronze
Steel	200	191	181	179	174	162	158
Malleable Iron	170	181	174	172	168	158	154
Nodular iron	170	179	172	170	166	156	152
Cast iron	150	174	168	166	163	154	149
Aluminum bronze	120	162	158	156	154	145	141
Tin bronze	110	158	154	152	149	141	137

Surface-condition factor C_f :

It depends on surface finish, residual stresses, and work hardening. It is not yet standardized by AGMA but when the surface conditions are not of high quality we may use $C_f > 1$.

AGMA Surface-Strength :

The contact fatigue strength at 10^7 cycles and 0.99 reliability for through Hardened steel gears may be obtained from a relation in the form:

$$\sigma^{\backslash}_{HP} = 2.22\, H_B + 200 \quad \text{MPa}$$

The corrected value of the contact strength:

$$\sigma_{HP} = \sigma^{\backslash}_{HP} \frac{Z_N C_H}{Y_\theta Y_Z}$$

Where Z_N , Y_θ and Y_Z are the stress-cycle life factor, the temperature factor, and the reliability factor respectively.

The Hardness Ratio Factor C_H :

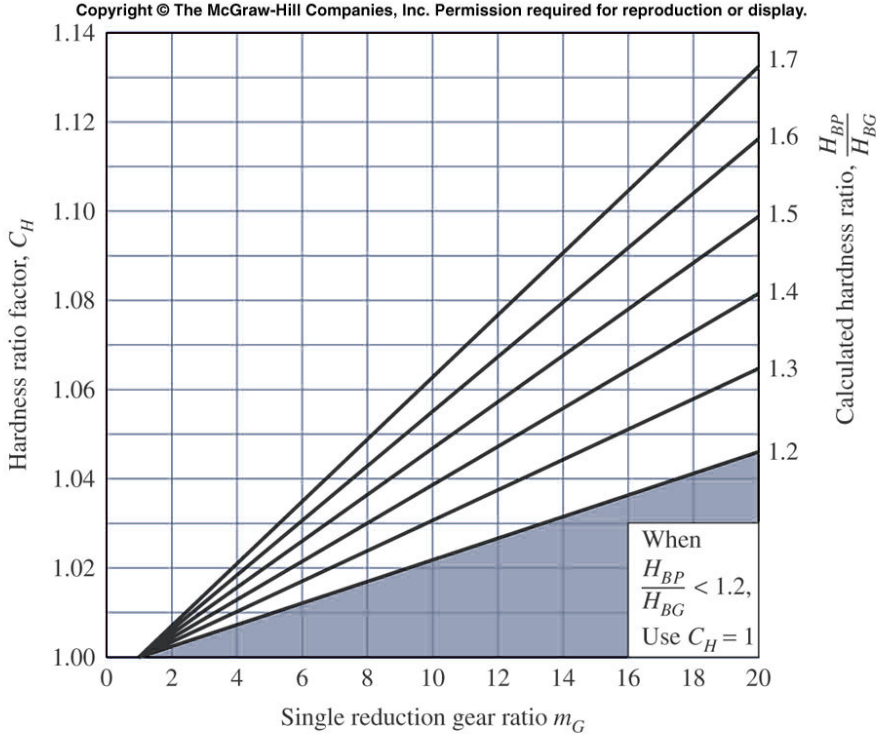
The pinion generally has a smaller number of teeth than the gear and Consequently is subjected to more cycles of contact stress.
The hardness ratio factor <u>C_H is used only for the gear</u> to adjust the surface Strength for this effect.

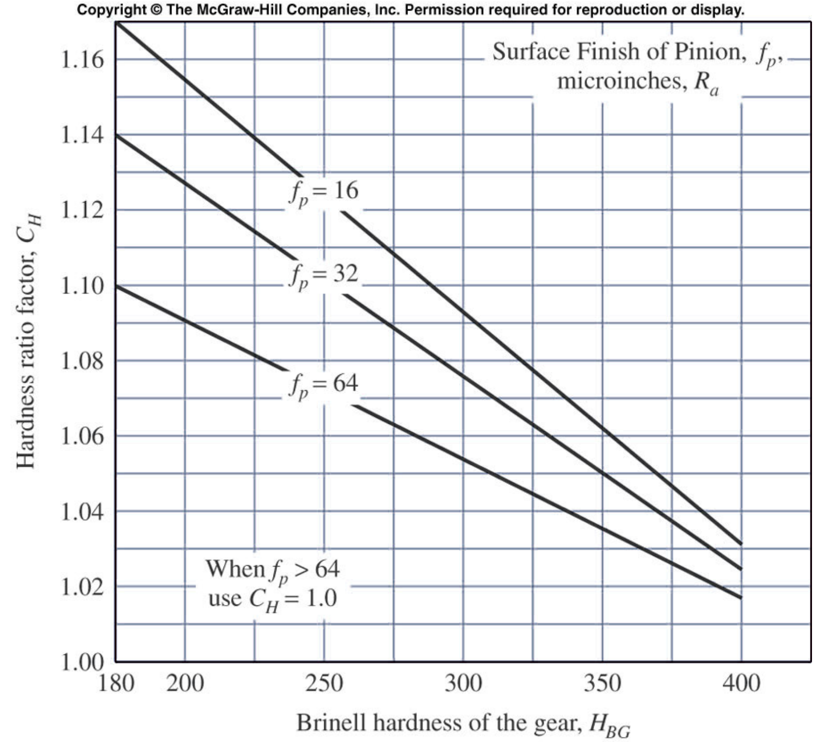
The value of C_H is obtained from equations of the form :

$$C_H = 1 + A^{\backslash} (m_G - 1)$$

Where m_G is the speed ratio and $A^{\backslash}$ is a function of (H_{BP} / H_{BG}), H_{BP} And H_{BG} are the Brinell hardness of the pinion and gear.

But mainly we select the value of C_H from figures for different conditions.





Safety Factor for Surface Fatigue

If we use the well known definition of safety factor which is a ratio of the strength to the stress then the safety factor will be

$$n_c^{\backslash} = \frac{\sigma_{HP}}{\sigma_c} = \frac{\text{the fully corrected contact strength}}{\text{contact stress}}$$

But since σc is not linearly related to W_t, this safety factor can not be Compared with the bending fatigue safety factor. For this reason the safety Factor for contact fatigue is usually taken as:

$$n_c = \left(\frac{\sigma_{HP}}{\sigma_c}\right)^2 = \left(\frac{\text{the fully corrected contact strength}}{\text{contact stress}}\right)^2$$

Example: A 17-tooth 20° pressure angle spur pinion rotates at 1800 rpm and transmits 3 kW to a 52-tooth disk gear. The module is 2.5 mm, the face width is 38 mm and the quality standard is No. 6. the gears are straddle-mounted with bearings immediately adjacent. The pinion is grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is grade 1 steel, through-hardened, with a Brinell hardness of 200, tooth surface and core. Poisson's ratio is 0.3 and Young's modulus is 207 GPa. $J_p = 0.3$ and $J_G = 0.4$. The loading is smooth because of motor and load. It is a commercial enclosed gear unit and the tooth profile is uncrowned.

For a reliability of 90% and a pinion life of 10^8 cycles (use $Y_N = 1.3558\ N^{-0.0178}$ and $Z_N = 1.4488\ N^{-0.023}$) Find:

a) The factor of safety of gears against bending fatigue

b) The factor of safety of gears against surface fatigue.

c) Discuss the results.

Solution :

$$d_p = mN_p = 2.5 \times 17 = 42.5 \ \text{mm}$$

$$d_G = mN_G = 2.5 \times 52 = 130 \ \text{mm}$$

$$V_t = \omega \times r = \frac{2\pi n_p}{60}\frac{d_p}{2} = \frac{2\pi \times 1800}{60}\frac{42.5}{2} = 4005.5 \ \text{mm/s} = 4.005 \ \text{m/s}$$

$$W_t = \frac{H}{V_t} = \frac{3 \times 1000}{4.005} = 749.06 \ \text{N}$$

the dynamic factor $\mathrm{K_v}$:

$$K_v = \left(\frac{A + \sqrt{200V}}{A}\right)^B$$

$$B = 0.25(12 - Q_v)^{\frac{2}{3}} = 0.25(12 - 6)^{\frac{2}{3}} = 0.8255$$

$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.8255) = 59.772$$

$$K_v = \left(\frac{59.772 + \sqrt{200 \times 4.005}}{59.772}\right)^{0.8255} = 1.377$$

since the load is uniform $K_o = 1$

Assuming constant t hickness gears $K_B = 1$

The size factor $K_s = \frac{1}{k_b} = \frac{1}{0.974} = 1.03$

The load distribution factor:

$$K_H = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$K_H = 1 + 1(0.0695 \times 1 + 0.15 \times 1) = 1.22$$

NOTE: K_H may be taken from the to be 1.3

The load cycle factor:

the speed ratio $\mathrm{m_G} = \frac{N_G}{N_p} = \frac{52}{17} = 3.059$

$$(Y_N)_p = 1.3558(10^8)^{-0.0178} = 0.977$$

$$(Y_N)_G = 1.3558\left(\frac{10^8}{3.059}\right)^{-0.0178} = 0.996$$

The bending stress :

$$\sigma = \frac{W_t}{bmJ} K_v K_0 K_s K_H K_B$$

$$\sigma_p = \frac{749.06}{38 \times 2.5 \times 0.3} 1.377 \times 1 \times 1.03 \times 1.3 \times 1$$

$$= 48.46 \quad \text{MPa}$$

$$\sigma_G = \frac{749.06}{38 \times 2.5 \times 0.4} 1.377 \times 1 \times 1.03 \times 1.3 \times 1$$

$$= 36.345 \quad \text{MPa}$$

The bending strength :

$$\sigma_{\text{FP}}^{\backslash} = 0.533 H_B + 88.3 \quad \text{MPa}$$

$$(\sigma_{\text{FP}}^{\backslash})_p = 0.533 \times 240 + 88.3 = 216.22 \quad \text{MPa}$$

$$(\sigma_{\text{FP}}^{\backslash})_G = 0.533 \times 200 + 88.3 = 194.9 \quad \text{MPa}$$

The corrected bending strength :

$$\sigma_{FP} = \sigma^{\backslash}_{\mathrm{FP}} \frac{Y_N}{Y_\theta Y_Z}$$

$$(\sigma_{FP})_p = 216.22 \times \frac{0.977}{1 \times 0.85} = 248.52 \quad \mathrm{MPa}$$

$$(\sigma_{FP})_G = 194.9 \times \frac{0.996}{1 \times 0.85} = 228.38 \quad \mathrm{MPa}$$

The safety factor against bending fatigue :

$$n = \frac{\sigma_{FP}}{\sigma}$$

$$n_p = \frac{248.52}{48.46} = 5.12$$

$$n_G = \frac{228.38}{36.345} = 6.28$$

The contact stress:

$$\sigma_c = C_p \sqrt{\frac{W_t}{b d_p I} K_v K_o K_s K_H C_H}$$

where K_v, K_o, K_s, K_H as obtained for bending stress.

$C_p = 191\ \sqrt{MPa}$ from tables

$$I = \frac{\cos 20^\circ \times \sin 20^\circ}{2} \frac{3.059}{3.059+1} = 0.121$$

$let \quad C_f = 1$

$$\therefore (\sigma_c)_p = 191\sqrt{\frac{749.06}{38 \times 42.5 \times 0.121} \times 1.377 \times 1 \times 1.03 \times 1.22 \times 1}$$

$$= 491.9 \quad \text{MPa}$$

and $(\sigma_c)_G = (\sigma_c)_p$ in this problem.

The contact Strength:

$$\sigma^{\backslash}_{\mathrm{HP}} = 2.22\,\mathrm{H_B} + 200 \qquad \mathrm{MPa}$$

$$(\sigma^{\backslash}_{\mathrm{HP}})_p = 2.22 \times 240 + 200 = 732.8 \quad \mathrm{MPa}$$

$$(\sigma^{\backslash}_{\mathrm{HP}})_G = 2.22 \times 200 + 200 = 644 \qquad \mathrm{MPa}$$

The corrected contact strength:

$$\sigma_{\mathrm{HP}} = \sigma^{\backslash}_{HP} \frac{Z_N C_H}{Y_\theta Y_Z}$$

$Y_\theta = 1, \quad Y_Z = 0.85$ as before

$\mathrm{C}_H = 1.005$ from the figure (for gear only)

$$(Z_N)_p = 1.4488(10^8)^{-0.023} = 0.948$$

$$(Z_N)_G = 1.4488\left(\frac{10^8}{3.059}\right)^{-0.023} = 0.973$$

$$\therefore (\sigma_{HP})_p = 732.8\frac{0.948}{1\times 0.85} = 817.28 \quad \text{MPa}$$

$$\therefore (\sigma_{HP})_G = 644\frac{0.972\times 1.005}{1\times 0.85} = 740.87 \quad \text{MPa}$$

The safety factors:

$$(n_c)_p = \left(\frac{(\sigma_{HP})_p}{(\sigma_c)_p}\right)^2 = \left(\frac{817.28}{491.9}\right)^2 = (1.66)^2 = 2.76$$

$$(n_c)_G = \left(\frac{(\sigma_{HP})_G}{(\sigma_c)_G}\right)^2 = \left(\frac{740.87}{491.9}\right)^2 = (1.506)^2 = 2.268$$