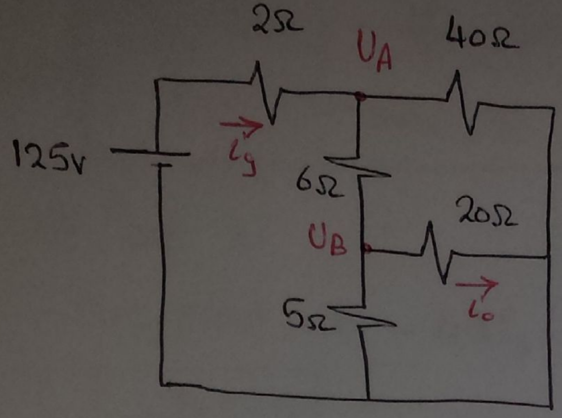
Sheet. 1.

Problem 1:

Get i_o, i_g ?



Using nodal analysis:

@ node U_A:

$$\frac{U_A - 125}{2} + \frac{U_A}{40} + \frac{U_A - U_B}{6} = 0$$

@ node U_B:

$$\frac{U_B}{5} + \frac{U_B}{20} + \frac{U_B - U_A}{6} = 0$$

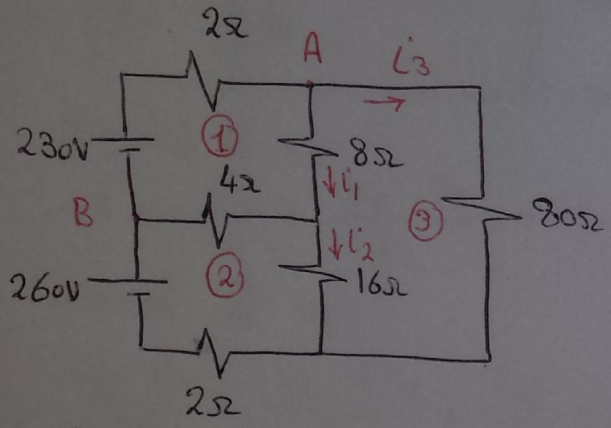
Solving to get U_A and U_B $\Rightarrow$ $U_A = 100V$, $U_B = 50V$

$$i_g = \frac{125 - U_A}{2} \Rightarrow \boxed{i_g = 12.5A}$$

$$i_o = U_B / 20 \Rightarrow \boxed{i_o = 2.5A}$$

Problem 2:

Get i_1, i_2, U_{AB} ?



using KVL analysis:

@ loop 1:

$$-230 + (i_1 + i_3)(2) + i_1(8) + (i_1 - i_2)(4) = 0$$

@ loop 2:

$$-260 + (i_1 - i_2)(4) + i_2(16) + (i_2 + i_3)(2) = 0$$

@ loop 3:

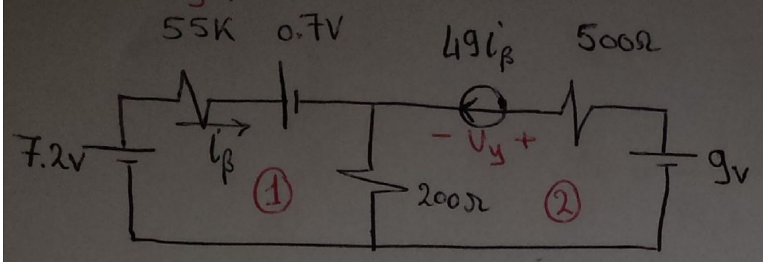
$$-i_1(8) + i_3(80) - i_2(16) = 0$$

Solving to get i_1, i_2, i_3 $\Rightarrow$ $\boxed{i_1 = 20A}$, $\boxed{i_2 = 15A}$, $i_3 = 5A$

$$U_{AB} = 8i_1 + 4(i_1 - i_2) \Rightarrow \boxed{U_{AB} = 180V}$$

Problem 3:

Get V_y ?



Using KVL:

@ loop 1:

$$-7.2 + 55K i_\beta + 0.7 + 50 i_\beta (200) = 0$$

$$\therefore i_\beta = 0.1 mA$$

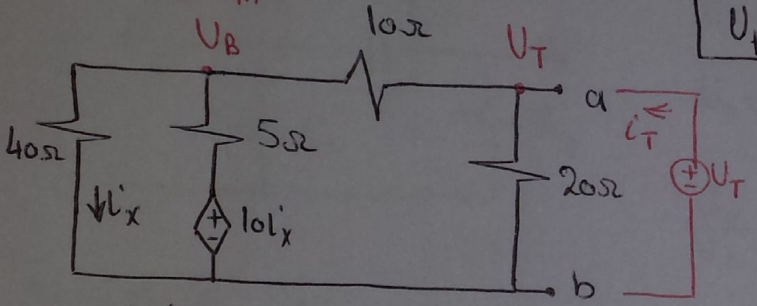
@ loop 2:

$$-50 i_\beta (200) - V_y - 49 i_\beta (500) + 9 = 0$$

$$\boxed{\therefore V_y = 5.55V}$$

Problem 4:

Get V_{th}, R_{th} ?



Since there are no independent sources:

$$\boxed{V_{th} = 0}$$

For R_{th} we apply a test source V_T with a current drawn i_T

Nodal @ V_T:

$$-i_T + \frac{V_T}{20} + \frac{V_T - V_B}{10} = 0$$

Nodal @ V_B:

$$\frac{V_B - V_T}{10} + \frac{V_B - 10 i_x}{5} + \frac{V_B}{40} = 0 \quad , \quad i_x = \frac{V_B}{40}$$

simplifying

$$V_B \left(\frac{1}{10} + \frac{3}{20} + \frac{1}{40} \right) = \frac{V_T}{10} \Rightarrow V_B = \frac{11}{400} V_T$$

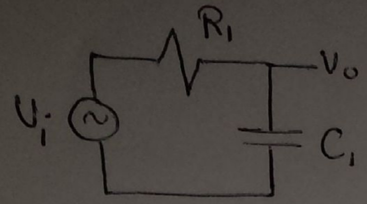
Simplifying

$$-i_T = -V_T \left(\frac{1}{20} + \frac{1}{10} - \frac{11}{400} \right)$$

$$\boxed{\therefore \frac{V_T}{i_T} = 8.16\ \Omega} \rightarrow R_{th}$$

Problem 5:

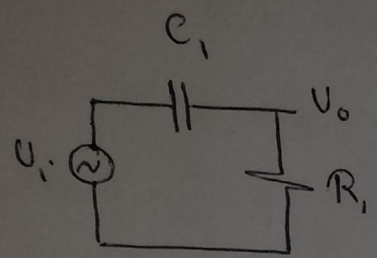
Get V_o / V_i ?



Using Voltage division rule:

$$V_o = V_i \frac{Z_{C_1}}{R_1 + Z_{C_1}}$$

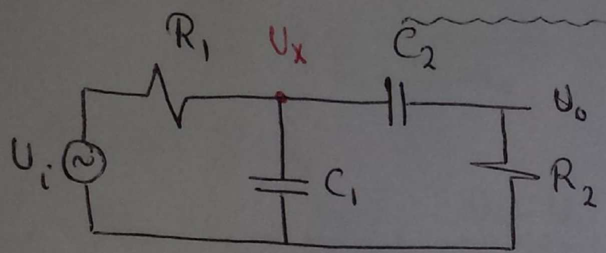
$$\frac{V_o}{V_i} = \frac{1/i\omega C_1}{R_1 + 1/i\omega C_1} = \frac{1}{1 + i\omega C_1 R_1}$$



Using Voltage division rule:

$$V_o = V_i \frac{R_1}{R_1 + Z_{C_1}}$$

$$\frac{V_o}{V_i} = \frac{i\omega C_1 R_1}{1 + i\omega C_1 R_1} \rightarrow \text{After simplification}$$



Using Voltage division rule:

$$V_o = V_x \frac{R_2}{R_2 + 1/i\omega C_2}$$

$$\frac{V_o}{V_x} = \frac{i\omega C_2 R_2}{1 + i\omega C_2 R_2} \rightarrow ①$$

Nodal @ V_x:

$$\frac{V_x - V_i}{R_1} + \frac{V_x}{1/i\omega C_1} + \frac{V_x}{1/i\omega C_2 + R_2} = 0$$

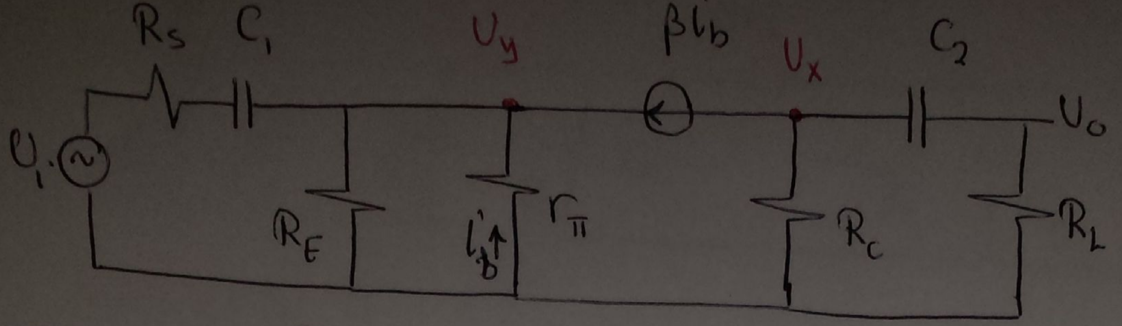
simplification

$$V_x \left(\frac{1}{R_1} + i\omega C_1 + \frac{1}{1/i\omega C_2 + R_2} \right) = \frac{V_i}{R_2}$$

$$\frac{V_x}{V_i} = \frac{1/R_2}{\left(1/R_1 + i\omega C_1 + 1/\left(\frac{1}{i\omega C_2} + R_2\right)\right)} \rightarrow ②$$

From 1, 2:

$$\frac{V_o}{V_i} = \frac{V_o}{V_x} \times \frac{V_x}{V_i}$$



$$U_o = U_x \frac{R_L}{R_L + 1/i\omega C_2} \rightarrow (1) \quad \text{from voltage division rule}$$

Nodal @ U_x:

$$\beta i_b + \frac{U_x}{R_C} + \frac{U_x}{R_L + 1/i\omega C_2} = 0$$

Simplifying

$$\frac{U_x}{i_b} = \frac{-\beta}{\left(\frac{1}{R_C} + \frac{1}{R_L + 1/i\omega C_2}\right)} \rightarrow (2)$$

Nodal @ U_y:

$$-\beta i_b - i_b + \frac{U_y}{R_E} + \frac{U_y - U_i}{R_s + 1/i\omega C_1} = 0 \quad , \; U_y = -i_b r_\pi$$

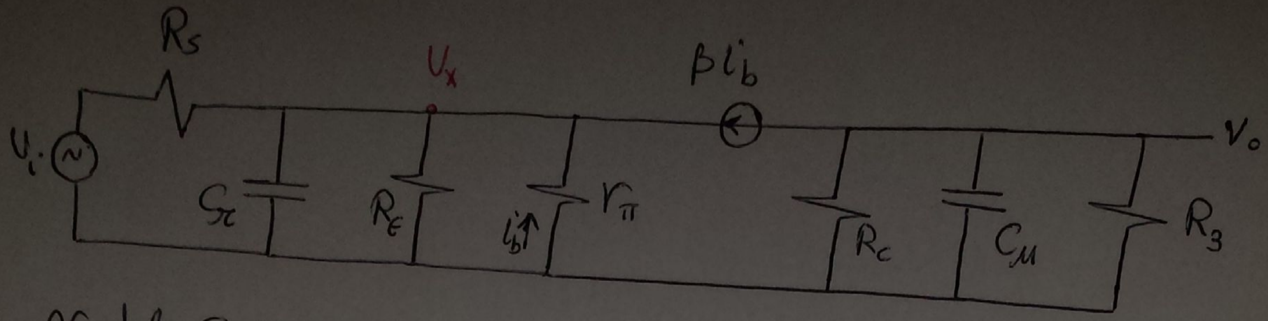
simplifying

$$-i_b\left(\beta + 1 + \frac{r_\pi}{R_E} + \frac{r_\pi}{R_s + 1/i\omega C_1}\right) = \frac{U_i}{R_s + 1/i\omega C_1}$$

$$\frac{i_b}{U_i} = \frac{-1/(R_s + 1/i\omega C_1)}{\left(\beta + 1 + r_\pi\left(\frac{1}{R_E} + \frac{1}{R_s + 1/i\omega C_1}\right)\right)} \rightarrow (3)$$

From 1, 2, 3:

$$\frac{U_o}{U_i} = \frac{U_o}{U_x} * \frac{U_x}{i_b} * \frac{i_b}{U_i}$$



Nodal @ V_o:

$$\beta i_b + \frac{V_o}{R_c} + \frac{V_o}{1/i\omega C_\mu} + \frac{V_o}{R_3} = 0$$

simplifying

$$\frac{V_o}{i_b} = \frac{-\beta}{\left(\frac{1}{R_c} + \frac{1}{i\omega C_\mu} + \frac{1}{R_3}\right)} \rightarrow (1)$$

Nodal @ V_x:

$$-\beta i_b - i_b + \frac{V_x}{R_E} + \frac{V_x}{1/i\omega C_\pi} + \frac{V_x - V_i}{R_s} = 0 \quad , \; V_x = -i_b r_\pi$$

simplifying

$$-i_b\left(\beta + 1 + \frac{r_\pi}{R_E} + r_\pi i\omega C_\pi + \frac{r_\pi}{R_s}\right) = \frac{V_i}{R_s}$$

$$\frac{i_b}{V_i} = \frac{1/R_s}{\left(\beta+1+r_\pi\left(\frac{1}{R_E} + i\omega C_\pi + \frac{1}{R_s}\right)\right)} \rightarrow (2)$$

From 1, 2:

$$\frac{V_o}{V_i} = \frac{V_o}{i_b} * \frac{i_b}{V_i}$$