

Exercise 1: Core Concepts (6 pts)

$$\frac{TP}{TP+FN}$$

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1. Definitions (4 pts)

- What is *overfitting* and how can it be detected?
- Explain the difference between *bias* and *variance*.
- What is the purpose of *regularization* in machine learning?
- Which metric (Precision or Recall) is more important for Spam detection, and why?

2. True / False (2 pts)

Statement	T/F
Increasing polynomial degree increases variance.	
Logistic regression has a linear decision boundary in the feature space.	

Exercise 2 (5 pts): Given the dataset

X ₁	X ₂	Y
0	0	0
0	1	1
1	0	1
1	1	1

- Compute the **information gain** for splitting on X₁ and on X₂.
- Which feature will the tree choose first? Why?

Exercise 3 (8 pts): A binary classification model outputs probability

Sample	True y	Predicted P(y=1)
1	1	0.9
2	0	0.2
3	1	0.7
4	0	0.6

- (2 pt) Compute the **binary cross-entropy loss** (average over all samples).
- (1 pt) If threshold = 0.5, compute the predicted classes.
- (4 pts) Build the **confusion matrix**, and compute: Accuracy, Precision, Recall and F1-score

4. (1 pts) Interpret what happens if we lower the threshold to 0.3 in terms of precision vs recall.

Exercise 4 (6 pts): Given the following true function

$$f(x) = \sin(x) + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, 0.1^2)$$

We focus at $x=\pi/2$, where: $f(\pi/2)=1.0$. Irreducible error (variance of noise): $\sigma^2=0.01$
Predictions at $x=\pi/2$ across 5 datasets (D1–D5):

Model	Predictions	Expected Prediction	Bias ²	Variance	Total Error
Linear	[0.9, 1.0, 0.8, 1.1, 0.95]	?	0.0025	0.012	0.0245
Polynomial	[1.05, 0.95, 1.15, 0.85, 1.1]	1	?	0.02	0.03
Neural Net	[1.02, 1.03, 0.98, 1.01, 0.99]	1.006	0.000036	?	?

- For each of the three models, calculate the missing indicators
- What is your conclusion about the results? Discuss and justify

Exercise 5 (5 pts): Given the following table

Threshold	TPR	FPR
0.9	0.2	0.01
0.7	0.5	0.05
0.5	0.8	0.20
0.3	0.95	0.50

- Plot the ROC Curve
- Which threshold minimizes false positives while keeping $\text{TPR} \geq 0.8$?
- If the cost of missing fraud (FN) is high, which threshold is best?