Exercise 1 (image Compression):

Given an 16-sample sequence:

S = [200, 202, 208, 210, 195, 190, 180, 175, 120, 115, 100, 98, 85, 80, 75, 75]

A. Apply multi-level 1D Haar encoding transform.
B. Apply the first two stages of 5/3 lifting scheme

Solution:

A)

a= (s1+s2)/2

d=(s1-s2)/2

Stage 1:

Pairs,	Trends (A1),	Details (D1)
(200, 202),	201,	-1
(208, 210),	209,	-1
(195, 190),	192.5,	2.5
(180, 175),	177.5,	2.5
(120, 115),	117.5,	2.5
(100, 98),	99,	1
(85, 80),	82.5,	2.5
(75, 75),	75,	0

A1= [201, 209, 192.5, 177.5, 99, 82.5,75]

D1 =[-1 , -1, 2.5, 2.5, 2.5, 1, 2.5, 0]

Stage 2 :

A1 Pairs,	Trends (A2),	Details (D2)
(201, 209),	205,	-4
(192.5, 177.5),	185,	7.5
(117.5, 99),	108.25,	9.25
(82.5, 75),	78.75,	3.75

A2= [205, 185, 108.25, 87.75]

D2 =[-4 , 7.5, 9.25, 3.75]

Stage 3 :

A2 Pairs,	Trends (A3),	Details (D3)
(205, 185),	195,	10
(108.25, 78.75),	93.5,	14.75

A3= [195, 93.5]

D3 =[10 , 14.75]

Stage 4 :

A3 Pairs,	Trends (A4),	Details (D4)
(195, 93.5),	144.25,	50.75

A4= [144.25]

D4 =[50.75]

Final values = [144.25, 50.75, 10 , 14.75, -4 , 7.5, 9.25, 3.75, -1 , -1, 2.5, 2.5, 2.5, 1, 2.5, 0]

B)

Even (se): [200, 208, 195, 180, 120, 100, 85, 75]

Odd (so): [202, 210, 190, 175, 115, 98, 80, 75]

$$D_i = X_{o,i} - \left\lfloor \frac{X_{e,i} + X_{e,i+1}}{2} \right\rfloor$$

D0 = 202 – (200+208)/2 = -2

D1 = 210 – (208+195)/2 = 0

D2 =190 – (195+180)/2 = 3

D3 = 175 – (180+120)/2 = 25

D4 = 115 – (120+100)/2 = 5

D5 = 98 – (100+85) /2 = 5.5

D6 = 80 – (85+75)/2 = 0

D7 = 75 – (75+75)/2= 0 (using boundary extension/mirroring)

Detail Array D_{stage1} = [-2,9,3,25,5,5.5,0,0]

$$A_i = X_{e,i} + \left\lfloor \frac{D_{i-1} + D_i + 2}{4} \right\rfloor$$

A0 = 200 + [-2 + -2 + 2] / 4 = 200 (mirroring $d_{-1} = d_0$)

A1 = 208 + [-2 + 9 + 2] /4 = 210

A2 = … = 198

A3 = 187

A4 = 128

A5 = 103

A6 = 87

A7 = 75

A_{stage1} = [200, 210, 198, 187, 128, 103, 87, 75]

For stage 2 take A_{stage1} as the new array values and perform the same steps that will produce 2 Arrays D_{stage2} =[11, 24, -4, -12] and A_{stage2} =[206, 207, 133, 84]

Exercise 2 (image processing)

You are given a 5x6 grayscale image. The image is represented in 3-bit grayscale.

$$I = \begin{bmatrix} 7 & 7 & 6 & 6 & 5 & 5 \\ 6 & 5 & 5 & 4 & 4 & 3 \\ 4 & 3 & 3 & 2 & 2 & 2 \\ 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A. Compute the Histogram
B. Compute the PDF
C. Compute the CDF (Cumulative Distribution Function
D. Produce the Equalized Image
E. Given a 4x4 reference image R with the following intensity distribution:

$$R = \begin{bmatrix} 7 & 7 & 7 & 6 \\ 6 & 6 & 5 & 5 \\ 4 & 4 & 3 & 2 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Match the histogram of our original 5x6 image (I) to the histogram of this reference image R.

F. Apply an auto contrast correction to the original image (I)
G. Consider the following two images I and T, correct the contrast of the image I based on the Image T

$$I = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 2 & 2 & 3 \\ 2 & 3 & 3 & 4 \\ 3 & 4 & 4 & 4 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$$

Solution :

A) H = [7, 4,4,3,3,4,3,2]

B) PDF = [7/30, 4/30, 4/30, 3/30, 3/30, 4/30, 3/30, 2/30]

C) CDF = [7/30, 11/30, 15/30, 18/30, 21/30, 25/30, 28/30, 30/30]

D) S_k = round ((L – 1) x CDF_k) where L is image level = 8
S0 = round(7 x 7 /30) = 2 ➔ 0 mapped to 2
S1 = round(7 x 11/30) = 3 ➔ 1 mapped to 3
S2 = round(7 x 15/30) = 4 ➔ 2 mapped to 4
S3 = round(7 x 18 / 30) = 4 ➔ 3 mapped to 4
S4 = round(7 x 21 /30) = 5 ➔ 4 mapped to 5
S5 = round (7 x 25/30)= 6 ➔ 5 mapped to 6
S6 = round (7 x 28/30) = 7 ➔ 6 mapped to 7
S7 = round (7 x 30/30) = 7 ➔ 7 mapped to 7

The Equalized image

$$I_{eq} = \begin{bmatrix} 7 & 7 & 7 & 7 & 6 & 6 \\ 7 & 6 & 6 & 5 & 5 & 4 \\ 5 & 4 & 4 & 4 & 4 & 4 \\ 4 & 3 & 3 & 3 & 2 & 2 \\ 3 & 2 & 2 & 2 & 2 & 2 \end{bmatrix}$$

E) Compute the CDF of R

H_R=[2, 2, 1, 1, 2, 2, 3, 3]

PDF_R= [2/16, 2/16, 1/16 , 1/16, 2/16, 2/16, 3/16, 3/16]

CDF_R= [2/16, 4/16, 5/16, 6/16, 8/16, 10/16, 13/16, 16/16]

=[0.125, 0.25, 0.3125, 0.375, 0.5, 0.625, 0.8125, 1]

CDF_I = [7/30, 11/30, 15/30, 18/30, 21/30, 25/30, 28/30, 30/30] (from previous part)

CDF_I = [0.2333, 0.366, 0.5, 0.6, 0.7, 0.83, 0.933, 1]

Do the matching to the closest

From CDF_I to R CDF_R

value	CDF I
0	0.2333
1	0.366
2	0.5
3	0.6
4	0.7
5	0.83
6	0.933
7	1

CDF R
0.125
0.25
0.3125
0.375
0.5
0.625
0.8125
1

Replace 0 by 1

Replace 1 by 3

Replace 2 by 4

Replace 3 by 5

Replace 4 by 5

Replace 5 by 6

Replace 6 by 7

Replace 7 by 7

Matched Matrix $I_{matched}$:

$$\begin{bmatrix} 7 & 7 & 7 & 7 & 6 & 6 \\ 7 & 6 & 6 & 5 & 5 & 5 \\ 5 & 5 & 5 & 4 & 4 & 4 \\ 4 & 3 & 3 & 3 & 1 & 1 \\ 3 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

F)

For a 3-bit image, the gray levels are

$$L = 2^3 = 8 \quad \Rightarrow \quad \text{levels } 0 \text{ to } 7$$

1. Find min and max intensities

From the given image I:

- $I_{\min} = 0$
- $I_{\max} = 7$

2. Auto-contrast formula

Auto-contrast (linear contrast stretching) is defined as:

$$I_{\text{ac}}(x, y) = \frac{I(x, y) - I_{\min}}{I_{\max} - I_{\min}} \times (L - 1)$$

Substitute the values:

$$I_{\text{ac}}(x, y) = \frac{I(x, y) - 0}{7 - 0} \times 7 = I(x, y)$$

3. Result

Since the image already spans the **full dynamic range** $[0, 7]$, auto-contrast **does not change any pixel**

G)

I min = 1 I max = 4 T min = 0 T max = 7

The mapping formula is:

$$I_{\text{cc}}(x, y) = \frac{I(x, y) - I_{\min}}{I_{\max} - I_{\min}} \times (T_{\max} - T_{\min}) + T_{\min}$$

Substitute values:

$$I_{\text{cc}} = \frac{I - 1}{4 - 1} \times 7 = \frac{7}{3}(I - 1)$$

Mapping :

Original I	Corrected I_{cc}
1	0
2	2
3	5
4	7

$$I_{\text{cc}} = \begin{bmatrix} 0 & 0 & 2 & 2 \\ 0 & 2 & 2 & 5 \\ 2 & 5 & 5 & 7 \\ 5 & 7 & 7 & 7 \end{bmatrix}$$

Corrected Image

(Rounded to nearest integer)

Exercise 3: (image region processing)

Consider the following **5×5 grayscale image** I:

$$I = \begin{bmatrix} 10 & 10 & 10 & 10 & 10 \\ 10 & 20 & 20 & 20 & 10 \\ 10 & 20 & 40 & 20 & 10 \\ 10 & 20 & 20 & 20 & 10 \\ 10 & 10 & 10 & 10 & 10 \end{bmatrix}$$

The **horizontal Sobel convolution kernel** S_x is given by:

$$S_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

1. Explain briefly the purpose of the Sobel operator.
2. Perform the convolution of image I with the Sobel kernel S_x
   Assume zero padding. Compute only the valid inner pixels (only pixels that doesn't involve padded pixels).
3. Write the resulting gradient image G_x
4. Identify the location of strong edges in the image.

Solution:

1. The Sobel operator estimates the **image gradient**.

   Sx approximates the derivative in the **x-direction**, so it highlights **vertical edges** (intensity changes from left to right).

2. Convolution with S_x (valid inner pixels only)

With zero padding assumed but **only valid inner pixels computed**, the output size is 3×3.

Example (center pixel at position (3,3)):

$$\begin{bmatrix} 20 & 20 & 20 \\ 20 & 40 & 20 \\ 20 & 20 & 20 \end{bmatrix} \odot \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = 0$$

Performing this for all valid positions gives:

$$G_x = \begin{bmatrix} 50 & 0 & -50 \\ 80 & 0 & -80 \\ 50 & 0 & -50 \end{bmatrix}$$

3. Resulting gradient image G_x

$$\boxed{G_x = \begin{bmatrix} 50 & 0 & -50 \\ 80 & 0 & -80 \\ 50 & 0 & -50 \end{bmatrix}}$$

Positive values correspond to transitions from **dark → bright** (left to right),

negative values to **bright → dark.**

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4. Location of strong edges

- **Strong vertical edges** appear in the **middle row**, left and right of the center (values $+80$ and -80).
- This corresponds to the sharp intensity change around the central bright pixel (40) relative to its neighbors.
- The **center column is zero**, meaning no left–right intensity change there (symmetry).

Conclusion: the strongest edges are vertical edges surrounding the bright central region.