

Algorithm

Hadi Asemi

[Book](#)

[Lecture Video for the Book](#)

The Steps for analyzing Algorithm:

- time
- space
- network transformation
- power consumption
- cpu registers

Recursion :

Recursion is the process of defining a problem (or the solution to a problem) in terms of (a simpler version) itself.

Law of Recursive:

- A recursive algorithm must have a base case (when to stop)
- A recursive algorithm must move toward the base case
- A recursive algorithm must call itself recursively

Code:

Example 1:

```
def count_down(n):  
    print(n,end=' ')  
    if n>0:  
        count_down(n-1)
```

Example 2:

```
def sum_list(list):  
    if len(list)==0:  
        return 0  
    return list[0]+sum_list(list[1:])
```

Example 3:

Convert decimal to different base

```
def tostr(n,base):  
    digits='0123456789ABCDEF'  
    if n<base:  
        return digits[n]  
    return tostr(n // base,base) + digits[n % base]
```

Example 4:**Check Palindrome**

- Recursive:

```
def pallidnrome_recursive(num):  
    s=str(num)  
    if len(s) < 1:  
        return True  
    else:  
        if s[0] == s[-1]:  
            return pallidnrome_recursive(s[1:-1])  
        else:  
            return False
```

- Second Way:

```
def reverseDigits(num) :  
  
    rev_num = 0;  
    while (num > 0) :  
        rev_num = rev_num * 10 + num % 10  
        num = num // 10  
  
    return rev_num  
  
# Function to check if n is Palindrome  
def isPalindrome(n) :  
  
    # get the reverse of n  
    rev_n = reverseDigits(n);  
  
    # Check if rev_n and n are same or not.  
    if (rev_n == n) :  
        return 1  
    else :  
        return 0
```

Example 5:**Fibonacci sequence:**

- Recursive:

```
def fib_recursive(num):  
    if num <=1:  
        return num  
    return fib(num-1)+fib(num-2)
```

- Loop:

```
def fib_loop(num):  
    n1,n2=0,1  
    count=0  
    if num==0:  
        return 0  
    elif num==1:  
        return 1  
    else:  
        while count < num:
```

```

nth=n1+n2
n1=n2 # swap
n2=nth # swap
count +=1
return n1

```

Example 6(Check if the item in the node list):

```

def search(item,node):
    if node.item==item:
        return True
    elif node==None:
        return False
    else:
        returnsearch(item,node.rest)

```

Stack(LIFO):

Common Data Structure Operations									
Data Structure	Time Complexity								Space Complexity
	Average				Worst				Worst
	Access	Search	Insertion	Deletion	Access	Search	Insertion	Deletion	
Array	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Stack	$\Theta(n)$	$\Theta(n)$	$\Theta(1)$	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$	$\Theta(1)$	$\Theta(1)$	$\Theta(n)$
Queue	$\Theta(n)$	$\Theta(n)$	$\Theta(1)$	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$	$\Theta(1)$	$\Theta(1)$	$\Theta(n)$
Singly-Linked List	$\Theta(n)$	$\Theta(n)$	$\Theta(1)$	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$	$\Theta(1)$	$\Theta(1)$	$\Theta(n)$
Doubly-Linked List	$\Theta(n)$	$\Theta(n)$	$\Theta(1)$	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$	$\Theta(1)$	$\Theta(1)$	$\Theta(n)$
Skip List	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n \log(n))$
Hash Table	N/A	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$	N/A	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Binary Search Tree	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Cartesian Tree	N/A	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	N/A	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
B-Tree	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(n)$
Red-Black Tree	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(n)$
Splay Tree	N/A	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	N/A	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(n)$
AVL Tree	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(n)$
KD Tree	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$

Figure 1: Common Data Structure Operation

Stack Array:

Stack class implemented with array

class Stack:

"""Implements an efficient last-in first-out Abstract Data Type using a Python List"""

capacity is max number of Nodes, init_items is optional List parameter for initialization
if the length of the init_items List exceeds capacity, raise IndexError

def __init__(self, capacity, init_items=None):

"""Creates an empty stack with a capacity"""

self.capacity = capacity *# capacity of stack*

self.items = [None]*capacity *# array for stack*

self.num_items = 0 *# number of items in stack*

if init_items **is not** None: *# if init_items is not None, initialize stack*

if len(init_items) > capacity:

raise IndexError

else:

self.num_items = len(init_items)

self.items[:self.num_items] = init_items

```
def __eq__(self, other):
    return ((type(other) == Stack)
            and self.capacity == other.capacity
            and self.items[:self.num_items] == other.items[:other.num_items]
            )

def __repr__(self):
    return "Stack({!r}, {!r})".format(self.capacity, self.items[:self.num_items])

def is_empty(self):
    '''Returns True if the stack is empty, and False otherwise
    MUST have O(1) performance'''
    return self.num_items == 0

def is_full(self):
    '''Returns True if the stack is full, and False otherwise
    MUST have O(1) performance'''
    return self.num_items == self.capacity

def push(self, item):
    '''If stack is not full, pushes item on stack.
    If stack is full when push is attempted, raises IndexError
    MUST have O(1) performance'''
    if self.num_items == self.capacity:
        raise IndexError("The Stack is Full")
    self.items[self.num_items] = item
    self.num_items += 1
    # print(self.items.__repr__())
    # return self.items[self.num_items-1]

def pop(self):
    '''If stack is not empty, pops item from stack and returns item.
    If stack is empty when pop is attempted, raises IndexError
    MUST have O(1) performance'''
    if self.num_items == 0:
        raise IndexError("Index out of range")
    self.num_items -= 1
    # print(self.items[self.num_items].__repr__())
    return self.items[self.num_items]

def peek(self):
    '''If stack is not empty, returns next item to be popped (but does not remove the item)
    If stack is empty, raises IndexError
    MUST have O(1) performance'''
    if self.num_items == 0:
        raise IndexError
    # print(self.items[self.num_items-1].__repr__())
    return self.items[self.num_items-1]

def size(self):
```

```

    '''Returns the number of elements currently in the stack, not the capacity
    MUST have O(1) performance'''
    return self.num_items

```

Stack Model List:

```

# NodeList is one of
# None or
# Node(value, rest), where rest is reference to the rest of the list
class Node:
    def __init__(self, value, rest):
        self.value = value      # object reference stored in Node
        self.rest = rest       # reference to NodeList
    def __eq__(self, other):
        return ((type(other) == Node)
                and self.value == other.value
                and self.rest == other.rest
               )
    def __repr__(self):
        return ("Node({!r}, {!r})".format(self.value, self.rest))

class Stack:
    """Implements an efficient last-in first-out Abstract Data Type using a node list"""

    # top is the top Node of stack
    def __init__(self, top=None):
        self.top = top          # top node of stack
        self.num_items = 0      # number of items in stack
        node = top              # set number of items based on input
        while node is not None:
            self.num_items += 1
            node = node.rest

    def __eq__(self, other):
        return ((type(other) == Stack)
                and self.top == other.top
               )

    def __repr__(self):
        return ("Stack({!r})".format(self.top))

    def is_empty(self):
        '''Returns True if the stack is empty, and False otherwise
        MUST have O(1) performance '''
        return self.num_items==0

    def push(self, item):
        '''Pushes item on stack.
        MUST have O(1) performance'''
        new_stack=Node(item,self.top)
        self.rest=self.top
        self.top=new_stack
        self.num_items += 1

    def pop(self):
        '''If stack is not empty, pops item from stack and returns item.
        If stack is empty when pop is attempted, raises IndexError

```

```

        MUST have  $O(1)$  performance'''
    if self.top is None:
        raise IndexError
    self.num_items -= 1
    temp = self.top.value
    self.top.value = None
    self.top = self.top.rest
    return temp

def peek(self):
    '''If stack is not empty, returns next item to be popped (but does not remove the item)
    If stack is empty, raises IndexError
    MUST have  $O(1)$  performance'''
    if self.num_items == 0:
        raise IndexError
    return self.top.value

def size(self):
    '''Returns the number of elements currently in the stack, not the capacity
    MUST have  $O(1)$  performance'''
    return self.num_items

```

Queue(FIFO):

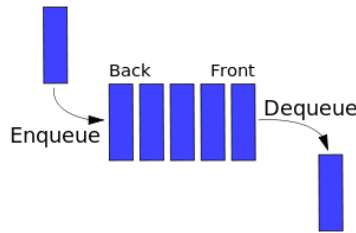


Figure 2: Queue

Array:

Queue ADT - circular array implementation

```

class Queue:
    """Implements an efficient first-in first-out Abstract Data Type using a Python List"""

    def __init__(self, capacity, init_items=None):
        """Creates a queue with a capacity and initializes with init_items"""
        self.capacity = capacity          # capacity of queue
        self.items = [None] * capacity    # array for queue
        self.num_items = 0                # number of items in queue
        self.front = 0                    # front index of queue (items removed from front)
        self.rear = 0                     # rear index of queue (items enter at rear)
        if init_items is not None:        # if init_items is not None, initialize queue
            if len(init_items) > capacity:
                raise IndexError
            else:
                self.num_items = len(init_items)
                self.items[:self.num_items] = init_items
                self.rear = self.num_items % self.capacity # % capacity addresses length=capacity

```

```

def __eq__(self, other):
    return ((type(other) == Queue)
            and self.capacity == other.capacity
            and self.get_items() == other.get_items()
            )

def __repr__(self):
    return ("Queue({!r}, {!r})".format(self.capacity, self.get_items()))

# get_items returns array (Python list) of items in Queue
# first item in the list will be front of queue, last item is rear of queue
def get_items(self):
    if self.num_items == 0:
        return []
    if self.front < self.rear:
        return self.items[self.front:self.rear]
    else:
        return self.items[self.front:] + self.items[:self.rear]

def is_empty(self):
    """Returns true if the queue is empty and false otherwise
Must be O(1)"""
    return self.num_items==0

def is_full(self):
    """Returns true if the queue is full and false otherwise
Must be O(1)"""
    return self.num_items==self.capacity

def enqueue(self, item):
    """enqueues item, raises IndexError if Queue is full
Must be O(1)"""
    if self.is_full():
        raise IndexError
    self.items[self.rear]=item
    self.rear=(self.rear+1)%self.capacity # give the location which next time we need to be
    self.num_items += 1

def dequeue(self):
    """dequeues and returns item, raises IndexError if Queue is empty
Must be O(1)"""
    if self.is_empty():
        raise IndexError
    value=self.items[self.front]
    self.front=(self.front+1)%self.capacity
    self.num_items -=1
    return value

def size(self):
    """Returns the number of items in the queue
Must be O(1)"""
    return self.num_items

```

NodeList:

```

# NodeList version of ADT Queue

# Node class for use with Queue implemented with linked list
# NodeList is one of
# None or
# Node(value, rest), where rest is the rest of the list
class Node:
    def __init__(self, value, rest):
        self.value = value      # value
        self.rest = rest       # NodeList
    def __eq__(self, other):
        return ((type(other) == Node)
                and self.value == other.value
                and self.rest == other.rest
                )
    def __repr__(self):
        return ("Node({!r}, {!r})".format(self.value, self.rest))

class Queue:
    def __init__(self):
        self.rear = None      # rear NodeList
        self.front = None     # front NodeList
        self.num_items = 0    # number of items in Queue

    def __eq__(self, other):
        return ((type(other) == Queue)
                and self.get_items() == other.get_items()
                )

    def __repr__(self):
        return ("Queue({!r}, {!r})".format(self.rear, self.front))

# get_items returns array (Python list) of items in Queue
# first item in the list will be front of queue, last item is rear of queue
def get_items(self):
    items = []
    front = self.front
    while front is not None:
        items.append(front.value)
        front = front.rest
    if self.rear is not None:
        rear_items = []
        rear = self.rear
        while rear is not None:
            rear_items.append(rear.value)
            rear = rear.rest
        rear_items.reverse()
        items.extend(rear_items)
    return items

def is_empty(self):
    """Returns true if the queue is empty and false otherwise
    Must be O(1)"""
    return self.num_items==0

```



```

def enqueue(self, item):
    """enqueues item, adding it to the rear NodeList
    Must be  $O(1)$ """

    que=Node(item,self.rear)

    self.rear=que

    self.num_items+=1

def dequeue(self):
    """dequeues item, removing first item from front NodeList
    If front NodeList is empty, remove items from rear NodeList
    and add to front NodeList until rear NodeList is empty
    If front NodeList and rear NodeList are both empty, raise IndexError
    Must be  $O(1)$  - general case"""
    if self.is_empty():
        raise IndexError
    self.num_items -= 1
    if self.front is not None:
        temp=self.front.value
        self.front=self.front.rest
        return temp
    if self.front is None:
        i=self.rear
        while i is not None:
            temp=i.value
            i=i.rest
            self.front=Node(temp,self.front)
            self.rear=self.rear.rest
        temp = self.front.value
        self.front = self.front.rest
        return temp

def size(self):
    """Returns the number of items in the queue
    Must be  $O(1)$ """
    return self.num_items

```

Doubly Link List:

```

class Node:
    """Node for use with doubly-linked list"""
    def __init__(self, item, next=None, prev=None):
        self.item = item # item held by Node
        self.next = next # reference to next Node
        self.prev = prev # reference to previous Node

class OrderedList:
    """A doubly-linked ordered list of integers,
    from lowest (head of list, sentinel.next) to highest (tail of list, sentinel.prev)"""
    def __init__(self, sentinel=None):
        """Use only a sentinel Node. No other instance variables"""

```

```
self.sentinel = Node(None)
self.sentinel.next = self.sentinel
self.sentinel.prev = self.sentinel

def is_empty(self):
    """Returns back True if OrderedList is empty"""
    return self.sentinel.next==self.sentinel

def add(self, item):
    """Adds an item to OrderedList, in the proper location based on ordering of items
    from lowest (at head of list) to highest (at tail of list)
    If item is already in list, do not add again (no duplicate items)"""
    cur=self.sentinel.next
    while cur is not self.sentinel and item >cur.item:
        cur=cur.next
    if cur.item != item:
        temp=Node(item)
        temp.prev=cur.prev
        temp.next=cur
        cur.prev.next=temp
        cur.prev=temp

def remove(self, item):
    """Removes an item from OrderedList. If item is removed (was in the list) returns True
    If item was not removed (was not in the list) returns False"""
    cur=self.sentinel
    if self.is_empty():
        return False
    else:
        while cur.next != self.sentinel:
            if cur.next.item == item:
                cur.next=cur.next.next
                cur.next.prev=cur
                return True
            else:
                cur=cur.next
        return False

def index(self, item):
    """Returns index of an item in OrderedList (assuming head of list is index 0).
    If item is not in list, return None"""
    if self.is_empty():
        raise IndexError
    cur=self.sentinel.next
    num_item =0
    while cur.item != item:
        cur=cur.next
        num_item +=1
    return num_item

def pop(self, index):
```

```

"""Removes and returns item at index (assuming head of list is index 0).
If index is negative or >= size of list, raises IndexError"""
cur = self.sentinel.next
num_items = 0
if self.is_empty():
    raise IndexError
if index < 0:
    raise IndexError

while cur != self.sentinel and num_items < index:
    cur = cur.next
    num_items += 1
if cur == self.sentinel:
    raise IndexError
else:
    ret_val = cur.item
    cur.next.prev = cur.prev
    cur.prev.next = cur.next
    return ret_val

def search(self, item):
    """Searches OrderedList for item, returns True if item is in list, False otherwise recursion"""
    def helper(cur, values):
        if cur == self.sentinel:
            return False
        if cur.item > values:
            return False
        elif cur.item == values:
            return True
        else:
            return helper(cur.next, values)
    cur = self.sentinel.next
    return helper(cur, item)

def python_list(self):
    """Return a Python list representation of OrderedList, from head to tail
For example, list with integers 1, 2, and 3 would return [1, 2, 3]"""
    list = []
    cur = self.sentinel.next
    while cur is not self.sentinel:
        list.append(cur.item)
        cur = cur.next
    return list

def python_list_reversed(self):
    """Return a Python list representation of OrderedList, from tail to head, using recursion
For example, list with integers 1, 2, and 3 would return [3, 2, 1] recursion"""
    def helper(cur):
        if cur.next == self.sentinel:
            return [cur.item]
        else:
            return helper(cur.next) + [cur.item]
    cur = self.sentinel.next
    return helper(cur)

```

```
def size(self):
    """Returns number of items in the OrderedList. O(n) is OK recursion"""
    def helper(cur):
        if cur == self.sentinel:
            return 0
        return helper(cur.next)+1
    cur=self.sentinel
    return helper(cur.next)
```

Binary Tree:

Three type of trees:

- Full: leaf with no children or with to leaves
- Complete: fill up top to bottom and left to right
- Perfect: all leaves and nodes are at the same level

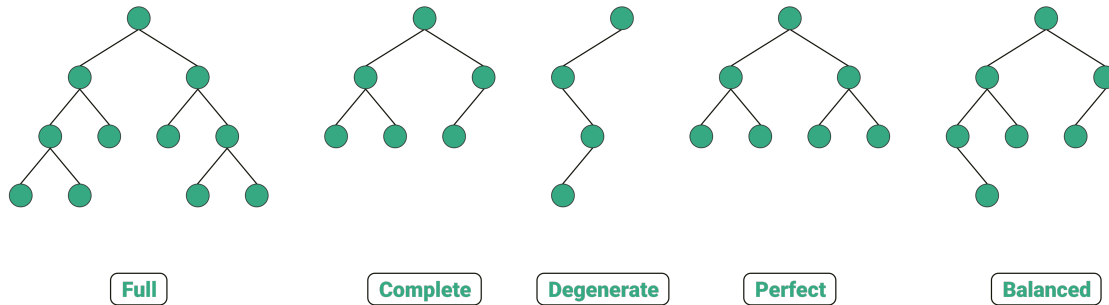


Figure 3: Trees

Traversal:

- Pre Order : n l r
- In Order : l n r
- Post Order: l r n

Calculate the Hight:

```
def height(self,node):
    if node==None:
        return 0
    left=self.height(node.left)
    right=self.height(node.right)
    return 1 + max(left,right)
```

Sorting:

Name	Time Complexity (Best)	Time Complexity (Average)	Time Complexity (Worst)	Space Complexity	Stability
Bubble Sort	$\Omega(n)$	$\Theta(n^2)$	$O(n^2)$	$O(1)$	Stable
Selection Sort	$\Omega(n^2)$	$\Theta(n^2)$	$O(n^2)$	$O(1)$	Unstable
Insertion Sort	$\Omega(n)$	$\Theta(n^2)$	$O(n^2)$	$O(1)$	Stable
Merge Sort	$\Omega(n \log(n))$	$\Theta(n \log(n))$	$O(n \log(n))$	$O(n)$	Stable
Quick Sort	$\Omega(n \log(n))$	$\Theta(n \log(n))$	$O(n^2)$	$O(\log(n))$	Unstable
Heap Sort	$\Omega(n \log(n))$	$\Theta(n \log(n))$	$O(n \log(n))$	$O(1)$	Unstable
Counting Sort	$\Omega(n+k)$	$\Theta(n+k)$	$O(n+k)$	$O(k)$	Stable
Radix Sort	$\Omega(nk)$	$\Theta(nk)$	$O(nk)$	$O(n+k)$	Stable

Figure 4: Sorting Table:

[Summary with animation](#)

Bubble Sort:

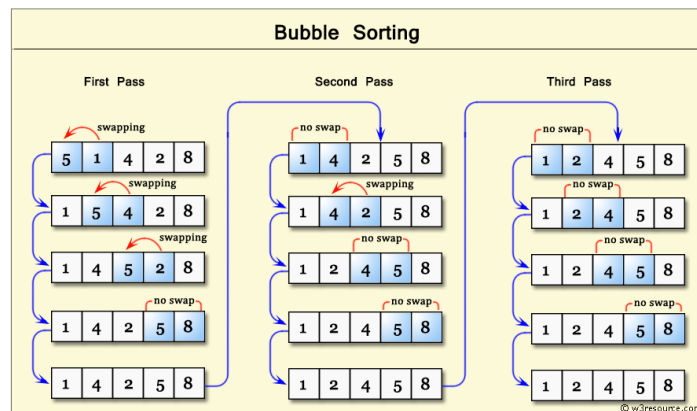


Figure 5: Bubble Sort

```
def bubble_sort(A):
    for k in range(len(A)):
        flag=0 # check if one time run but we did not swap anything for code efficiency
        for i in range(len(A)-k-1):
            if A[i]>A[i+1]:
                A[i],A[i+1]=A[i+1],A[i]
                flag=1
        if flag==0:
            break
```

Insertion Sort:

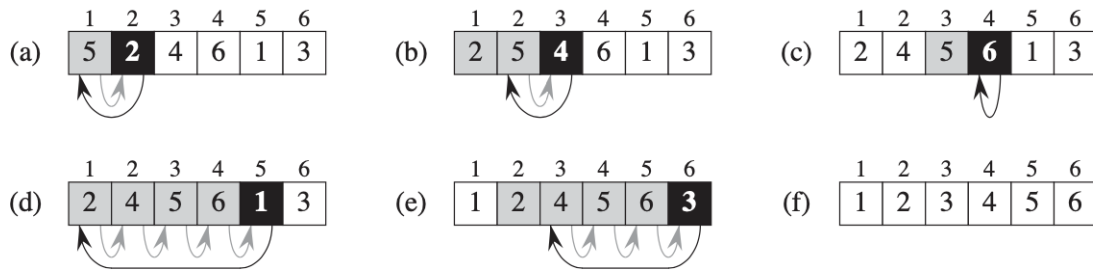


Figure 6: Insertion Sort

Code:

```
def insertion_sort(alist):
    com = 0 # How many comparison the code do
    for i in range(1, len(alist)):
        value = alist[i]
        j = i - 1
        while j >= 0:
            com += 1
            if value < alist[j]:
                alist[j + 1] = alist[j]
                alist[j] = value
                j = j - 1
            else:
                break
    return com
```

The $\theta(n)$ steps. Each steps have $\theta(n)$ swaps.

Selection Sort:

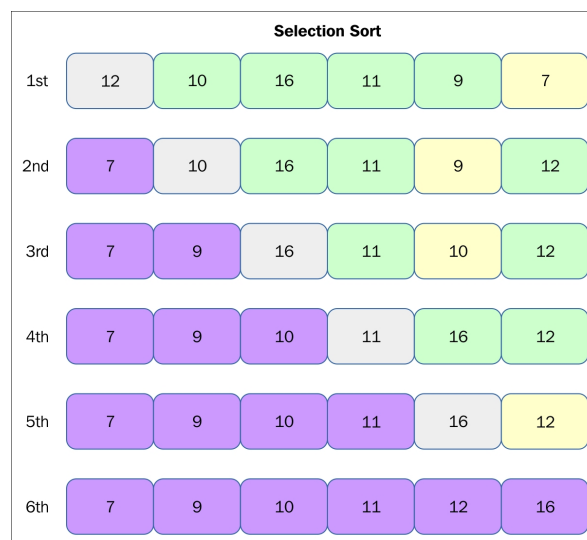


Figure 7: Selection Sort:

- Find the minimum value in the list

- Swap it with the value in the first position
- Repeat the steps above for the remainder of the list (starting at the second position and advancing each time)

Code:

```
def selection_sort(A):
    # Traverse through all array elements
    for i in range(len(A)):

        # Find the minimum element in remaining
        # unsorted array
        min_idx = i
        for j in range(i+1, len(A)):
            if A[min_idx] > A[j]:
                min_idx = j

        # Swap the found minimum element with
        # the first element
        A[i], A[min_idx] = A[min_idx], A[i]
    return A
```

Merge Sort:

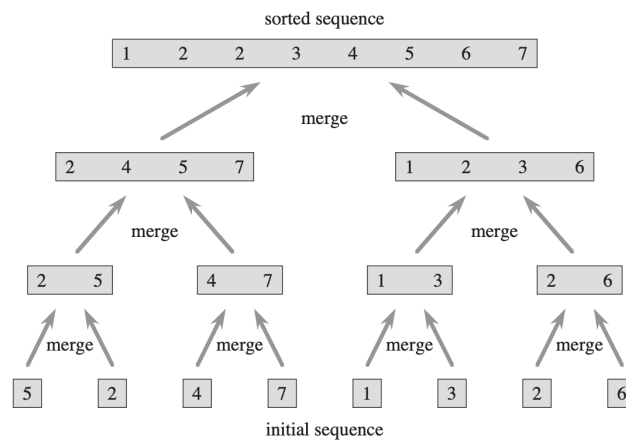


Figure 8: Merge Sort

Code:

```
def mergeSort(myList):
    if len(myList) > 1:
        mid = len(myList) // 2
        left = myList[:mid]
        right = myList[mid:]

        # Recursive call on each half
        mergeSort(left)
        mergeSort(right)

        # Two iterators for traversing the two halves
        i = 0
        j = 0
```

```

# Iterator for the main list
k = 0

while i < len(left) and j < len(right):
    if left[i] < right[j]:
        # The value from the left half has been used
        myList[k] = left[i]
        # Move the iterator forward
        i += 1
    else:
        myList[k] = right[j]
        j += 1
    # Move to the next slot
    k += 1

# For all the remaining values
while i < len(left):
    myList[k] = left[i]
    i += 1
    k += 1

while j < len(right):
    myList[k]=right[j]
    j += 1
    k += 1

myList = [54,26,93,17,77,31,44,55,20]
mergeSort(myList)
print(myList)

```

The complexity $\theta(n)$.

$$T(n) = c_1 + 2T\left(\frac{n}{2}\right) + c.n$$

Quick Sort:

The time complexity in best way is $O(n\log(n))$ and the worst case scenario is when is whole list already sorted so the time complexity is $O(n^2)$.

Code:

```

def partition(arr,low,high):
    i = ( low-1 )           # index of smaller element
    pivot = arr[high]      # pivot

    for j in range(low , high):

        # If current element is smaller than or
        # equal to pivot
        if arr[j] <= pivot:

            # increment index of smaller element
            i = i+1
            arr[i],arr[j] = arr[j],arr[i]

    arr[i+1],arr[high] = arr[high],arr[i+1]
    return ( i+1 )

```

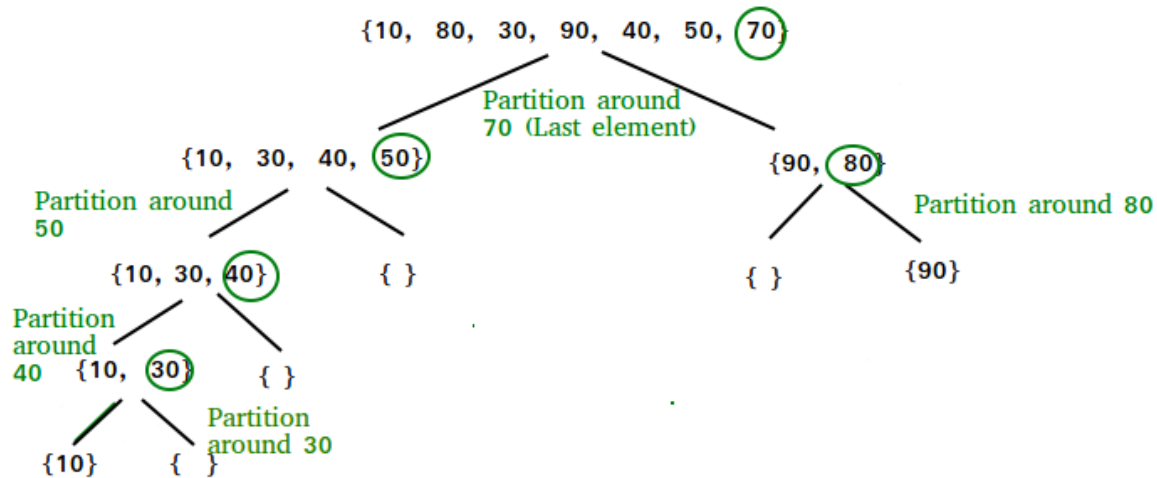



Figure 9: Quick Sort

```

# The main function that implements QuickSort
# arr[] --> Array to be sorted,
# low --> Starting index,
# high --> Ending index

# Function to do Quick sort
def quickSort(arr, low, high):
    if low < high:

        # pi is partitioning index, arr[p] is now
        # at right place
        pi = partition(arr, low, high)

        # Separately sort elements before
        # partition and after partition
        quickSort(arr, low, pi-1)
        quickSort(arr, pi+1, high)

```