

Algorithm

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[Book](#)

[Lecture Video for the Book](#)

Recursion :

Recursion is the process of defining a problem (or the solution to a problem) in terms of (a simpler version) itself.

Law of Recursive:

- A recursive algorithm must have a base case (when to stop)
- A recursive algorithm must move toward the base case
- A recursive algorithm must call itself recursively

Code:

Example 1:

```
def count_down(n):  
    print(n,end=' ')  
    if n>0:  
        count_down(n-1)
```

Example 2:

```
def sum_list(list):  
    if len(list)==0:  
        return 0  
    return list[0]+sum_list(list[1:])
```

Example 3:

Convert decimal to different base

```
def tostr(n,base):  
    digits='0123456789ABCDEF'  
    if n<base:  
        return digits[n]  
    return tostr(n // base,base) + digits[n % base]
```

Example 4:

Check Palindrome

- Recursive:

```
def pallidnrome_recursive(num):  
    s=str(num)  
    if len(s) < 1:
```

```

        return True
    else:
        if s[0] == s[-1]:
            return pallidnrome_recursive(s[1:-1])
        else:
            return False

```

- Second Way:

```

def reverseDigits(num) :

    rev_num = 0;
    while (num > 0) :
        rev_num = rev_num * 10 + num % 10
        num = num // 10

    return rev_num

# Function to check if n is Palindrome
def isPalindrome(n) :

    # get the reverse of n
    rev_n = reverseDigits(n);

    # Check if rev_n and n are same or not.
    if (rev_n == n) :
        return 1
    else :
        return 0

```

Example 5:

Fibonacci sequence:

- Recursive:

```

def fib_recursive(num):
    if num <=1:
        return num
    return fib(num-1)+fib(num-2)

```

- Loop:

```

def fib_loop(num):
    n1,n2=0,1
    count=0
    if num==0:
        return 0
    elif num==1:
        return 1
    else:
        while count < num:
            nth=n1+n2
            n1=n2 # swap
            n2=nth # swap
            count +=1
        return n1

```

Stack(LIFO):

Stack Array:

```
# Stack class implemented with array
class Stack:
    """Implements an efficient last-in first-out Abstract Data Type using a Python List"""

    # capacity is max number of Nodes, init_items is optional List parameter for initialization
    # if the length of the init_items List exceeds capacity, raise IndexError
    def __init__(self, capacity, init_items=None):
        """Creates an empty stack with a capacity"""
        self.capacity = capacity          # capacity of stack
        self.items = [None]*capacity      # array for stack
        self.num_items = 0                # number of items in stack
        if init_items is not None:        # if init_items is not None, initialize stack
            if len(init_items) > capacity:
                raise IndexError
            else:
                self.num_items = len(init_items)
                self.items[:self.num_items] = init_items

    def __eq__(self, other):
        return ((type(other) == Stack)
                and self.capacity == other.capacity
                and self.items[:self.num_items] == other.items[:other.num_items]
                )

    def __repr__(self):
        return ("Stack({!r}, {!r})".format(self.capacity, self.items[:self.num_items]))

    def is_empty(self):
        '''Returns True if the stack is empty, and False otherwise
        MUST have O(1) performance'''
        return self.num_items == 0

    def is_full(self):
        '''Returns True if the stack is full, and False otherwise
        MUST have O(1) performance'''
        return self.num_items==self.capacity

    def push(self, item):
        '''If stack is not full, pushes item on stack.
        If stack is full when push is attempted, raises IndexError
        MUST have O(1) performance'''
        if self.num_items==self.capacity:
            raise IndexError("The Stack is Full")
        self.items[self.num_items]=item
        self.num_items +=1
        # print(self.items.__repr__())
        # return self.items[self.num_items-1]

    def pop(self):
        '''If stack is not empty, pops item from stack and returns item.
```

```

        If stack is empty when pop is attempted, raises IndexError
        MUST have O(1) performance'''
    if self.num_items==0:
        raise IndexError("Index out of range")
    self.num_items -=1
    # print(self.items[self.num_items].__repr__())
    return self.items[self.num_items]

def peek(self):
    '''If stack is not empty, returns next item to be popped (but does not remove the item)
    If stack is empty, raises IndexError
    MUST have O(1) performance'''
    if self.num_items==0:
        raise IndexError
    # print(self.items[self.num_items-1].__repr__())
    return self.items[self.num_items-1]

def size(self):
    '''Returns the number of elements currently in the stack, not the capacity
    MUST have O(1) performance'''
    return self.num_items

```

Stack Node List:

```

# NodeList is one of
# None or
# Node(value, rest), where rest is reference to the rest of the list
class Node:
    def __init__(self, value, rest):
        self.value = value        # object reference stored in Node
        self.rest = rest         # reference to NodeList
    def __eq__(self, other):
        return ((type(other) == Node)
                and self.value == other.value
                and self.rest == other.rest
                )
    def __repr__(self):
        return ("Node{!r}, {!r}".format(self.value, self.rest))

class Stack:
    """Implements an efficient last-in first-out Abstract Data Type using a node list"""

    # top is the top Node of stack
    def __init__(self, top=None):
        self.top = top            # top node of stack
        self.num_items = 0        # number of items in stack
        node = top                # set number of items based on input
        while node is not None:
            self.num_items += 1
            node = node.rest

    def __eq__(self, other):
        return ((type(other) == Stack)
                and self.top == other.top)

```

```

    )

def __repr__(self):
    return ("Stack({!r})".format(self.top))

def is_empty(self):
    '''Returns True if the stack is empty, and False otherwise
    MUST have O(1) performance'''
    return self.num_items==0

def push(self, item):
    '''Pushes item on stack.
    MUST have O(1) performance'''
    new_stack=Node(item,self.top)
    self.rest=self.top
    self.top=new_stack
    self.num_items += 1

def pop(self):
    '''If stack is not empty, pops item from stack and returns item.
    If stack is empty when pop is attempted, raises IndexError
    MUST have O(1) performance'''
    if self.top is None:
        raise IndexError
    self.num_items -=1
    temp=self.top.value
    self.top.value=None
    self.top=self.top.rest
    return temp

def peek(self):
    '''If stack is not empty, returns next item to be popped (but does not remove the item)
    If stack is empty, raises IndexError
    MUST have O(1) performance'''
    if self.num_items==0:
        raise IndexError
    return self.top.value

def size(self):
    '''Returns the number of elements currently in the stack, not the capacity
    MUST have O(1) performance'''
    return self.num_items

```

Queue(FIFO):

Array:

Queue ADT - circular array implementation

```

class Queue:
    """Implements an efficient first-in first-out Abstract Data Type using a Python List"""

    def __init__(self, capacity, init_items=None):
        """Creates a queue with a capacity and initializes with init_items"""
        self.capacity= capacity          # capacity of queue
        self.items = [None]*capacity     # array for queue

```

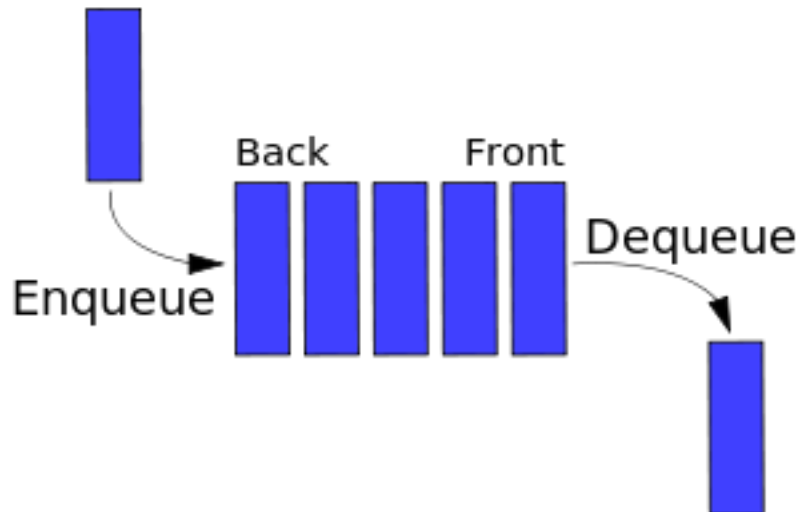


Figure 1: Queue

```

self.num_items = 0          # number of items in queue
self.front = 0              # front index of queue (items removed from front)
self.rear = 0               # rear index of queue (items enter at rear)
if init_items is not None:  # if init_items is not None, initialize queue
    if len(init_items) > capacity:
        raise IndexError
    else:
        self.num_items = len(init_items)
        self.items[:self.num_items] = init_items
        self.rear = self.num_items % self.capacity # % capacity addresses length=capacity

def __eq__(self, other):
    return ((type(other) == Queue)
            and self.capacity == other.capacity
            and self.get_items() == other.get_items()
            )

def __repr__(self):
    return "Queue({!r}, {!r})".format(self.capacity, self.get_items())

# get_items returns array (Python list) of items in Queue
# first item in the list will be front of queue, last item is rear of queue
def get_items(self):
    if self.num_items == 0:
        return []
    if self.front < self.rear:
        return self.items[self.front:self.rear]
    else:
        return self.items[self.front:] + self.items[:self.rear]

def is_empty(self):
    """Returns true if the queue is empty and false otherwise
    Must be O(1)"""
    return self.num_items==0

def is_full(self):

```

```

    """Returns true if the queue is full and false otherwise
    Must be  $O(1)$ """
    return self.num_items==self.capacity

def enqueue(self, item):
    """enqueues item, raises IndexError if Queue is full
    Must be  $O(1)$ """
    if self.is_full():
        raise IndexError
    self.items[self.rear]=item
    self.rear=(self.rear+1)%self.capacity # give the location which next time we need to be
    self.num_items += 1

def dequeue(self):
    """dequeues and returns item, raises IndexError if Queue is empty
    Must be  $O(1)$ """
    if self.is_empty():
        raise IndexError
    value=self.items[self.front]
    self.front=(self.front+1)%self.capacity
    self.num_items -=1
    return value

def size(self):
    """Returns the number of items in the queue
    Must be  $O(1)$ """
    return self.num_items

```

NodeList:

```

# NodeList version of ADT Queue

# Node class for use with Queue implemented with linked list
# NodeList is one of
# None or
# Node(value, rest), where rest is the rest of the list
class Node:
    def __init__(self, value, rest):
        self.value = value # value
        self.rest = rest # NodeList
    def __eq__(self, other):
        return ((type(other) == Node)
            and self.value == other.value
            and self.rest == other.rest
        )
    def __repr__(self):
        return ("Node{!r}, {!r}".format(self.value, self.rest))

class Queue:
    def __init__(self):
        self.rear = None # rear NodeList
        self.front = None # front NodeList
        self.num_items = 0 # number of items in Queue

    def __eq__(self, other):

```

```

    return ((type(other) == Queue)
            and self.get_items() == other.get_items()
            )

def __repr__(self):
    return ("Queue({!r}, {!r})".format(self.rear, self.front))

# get_items returns array (Python list) of items in Queue
# first item in the list will be front of queue, last item is rear of queue
def get_items(self):
    items = []
    front = self.front
    while front is not None:
        items.append(front.value)
        front = front.rest
    if self.rear is not None:
        rear_items = []
        rear = self.rear
        while rear is not None:
            rear_items.append(rear.value)
            rear = rear.rest
        rear_items.reverse()
        items.extend(rear_items)
    return items

def is_empty(self):
    """Returns true if the queue is empty and false otherwise
Must be O(1)"""
    return self.num_items==0

def enqueue(self, item):
    """enqueues item, adding it to the rear NodeList
Must be O(1)"""

    que=Node(item,self.rear)

    self.rear=que

    self.num_items+=1

def dequeue(self):
    """dequeues item, removing first item from front NodeList
If front NodeList is empty, remove items from rear NodeList
and add to front NodeList until rear NodeList is empty
If front NodeList and rear NodeList are both empty, raise IndexError
Must be O(1) - general case"""
    if self.is_empty():
        raise IndexError
    self.num_items -= 1
    if self.front is not None:
        temp=self.front.value
        self.front=self.front.rest
        return temp
    if self.front is None:
        i=self.rear

```



```

        while i is not None:
            temp=i.value
            i=i.rest
            self.front=Node(temp,self.front)
            self.rear=self.rear.rest
        temp = self.front.value
        self.front = self.front.rest
        return temp

    def size(self):
        """Returns the number of items in the queue
        Must be O(1)"""
        return self.num_items

```

Doubly Link List:

```

class Node:
    """Node for use with doubly-linked list"""
    def __init__(self, item, next=None, prev=None):
        self.item = item # item held by Node
        self.next = next # reference to next Node
        self.prev = prev # reference to previous Node

class OrderedList:
    """A doubly-linked ordered list of integers,
    from lowest (head of list, sentinel.next) to highest (tail of list, sentinel.prev)"""
    def __init__(self, sentinel=None):
        """Use only a sentinel Node. No other instance variables"""
        self.sentinel = Node(None)
        self.sentinel.next = self.sentinel
        self.sentinel.prev = self.sentinel

    def is_empty(self):
        """Returns back True if OrderedList is empty"""
        return self.sentinel.next==self.sentinel

    def add(self, item):
        """Adds an item to OrderedList, in the proper location based on ordering of items
        from lowest (at head of list) to highest (at tail of list)
        If item is already in list, do not add again (no duplicate items)"""
        cur=self.sentinel.next
        while cur is not self.sentinel and item >cur.item:
            cur=cur.next
        if cur.item != item:
            temp=Node(item)
            temp.prev=cur.prev
            temp.next=cur
            cur.prev.next=temp
            cur.prev=temp

    def remove(self, item):

```

```

"""Removes an item from OrderedList. If item is removed (was in the list) returns True
If item was not removed (was not in the list) returns False"""
cur=self.sentinel
if self.is_empty():
    return False
else:
    while cur.next != self.sentinel:
        if cur.next.item == item:
            cur.next=cur.next.next
            cur.next.prev=cur
            return True
        else:
            cur=cur.next
    return False

def index(self, item):
    """Returns index of an item in OrderedList (assuming head of list is index 0).
    If item is not in list, return None"""
    if self.is_empty():
        raise IndexError
    cur=self.sentinel.next
    num_item =0
    while cur.item != item:
        cur=cur.next
        num_item +=1
    return num_item

def pop(self, index):
    """Removes and returns item at index (assuming head of list is index 0).
    If index is negative or >= size of list, raises IndexError"""
    cur = self.sentinel.next
    num_items = 0
    if self.is_empty():
        raise IndexError
    if index < 0:
        raise IndexError

    while cur != self.sentinel and num_items < index:
        cur = cur.next
        num_items += 1
    if cur == self.sentinel:
        raise IndexError
    else:
        ret_val = cur.item
        cur.next.prev = cur.prev
        cur.prev.next = cur.next
        return ret_val

def search(self, item):
    """Searches OrderedList for item, returns True if item is in list, False otherwise recursion"""
    def helper(cur,values):
        if cur == self.sentinel:
            return False

```

```

    if cur.item > values:
        return False
    elif cur.item == values:
        return True
    else:
        return helper(cur.next, values)
cur = self.sentinel.next
return helper(cur, item)

def python_list(self):
    """Return a Python list representation of OrderedList, from head to tail
    For example, list with integers 1, 2, and 3 would return [1, 2, 3]"""
    list = []
    cur = self.sentinel.next
    while cur is not self.sentinel:
        list.append(cur.item)
        cur = cur.next
    return list

def python_list_reversed(self):
    """Return a Python list representation of OrderedList, from tail to head, using recursion
    For example, list with integers 1, 2, and 3 would return [3, 2, 1] recursion"""
    def helper(cur):
        if cur.next == self.sentinel:
            return [cur.item]
        else:
            return helper(cur.next) + [cur.item]
    cur = self.sentinel.next
    return helper(cur)

def size(self):
    """Returns number of items in the OrderedList. O(n) is OK recursion"""
    def helper(cur):
        if cur == self.sentinel:
            return 0
        return helper(cur.next) + 1
    cur = self.sentinel
    return helper(cur.next)

```

Insertion Sort

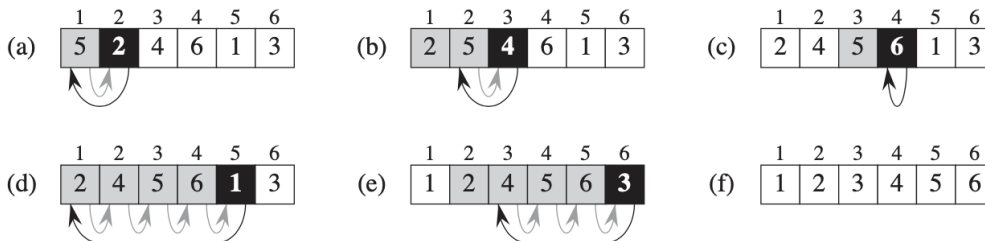


Figure 2: Insertion Sort

Code:

```
def insertion_sort(A):
    for j in range(2, len(A)):
        key = A[j]
        i = j - 1
        while i > 0 and A[i] > key:
            A[i + 1] = A[i]
            i = i - 1
        A[i + 1] = key
    return A
```

The $\theta(n)$ steps. Each steps have $\theta(n)$ swaps.

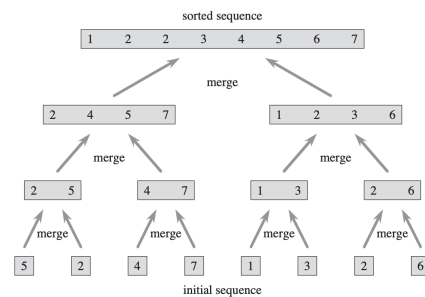
Merge Sort:

Figure 3: Merge Sort

Code:

```
def mergeSort(myList):
    if len(myList) > 1:
        mid = len(myList) // 2
        left = myList[:mid]
        right = myList[mid:]

        # Recursive call on each half
        mergeSort(left)
        mergeSort(right)

        # Two iterators for traversing the two halves
        i = 0
        j = 0

        # Iterator for the main list
        k = 0

        while i < len(left) and j < len(right):
            if left[i] < right[j]:
                # The value from the left half has been used
                myList[k] = left[i]
                # Move the iterator forward
                i += 1
            else:
                myList[k] = right[j]
                j += 1
            k += 1
```

```

        # Move to the next slot
        k += 1

    # For all the remaining values
    while i < len(left):
        myList[k] = left[i]
        i += 1
        k += 1

    while j < len(right):
        myList[k]=right[j]
        j += 1
        k += 1

myList = [54,26,93,17,77,31,44,55,20]
mergeSort(myList)
print(myList)

```

The complexity $\theta(n)$.

$$T(n) = c_1 + 2T\left(\frac{n}{2}\right) + c.n$$

Selection Sort:

Code:

```

def selection_sort(A):
    # Traverse through all array elements
    for i in range(len(A)):

        # Find the minimum element in remaining
        # unsorted array
        min_idx = i
        for j in range(i+1, len(A)):
            if A[min_idx] > A[j]:
                min_idx = j

        # Swap the found minimum element with
        # the first element
        A[i], A[min_idx] = A[min_idx], A[i]
    return A

```