Eng 260

Circuits

"I guess I always wanted to be an electrician. As a little boy, I ran away to join the circuits."

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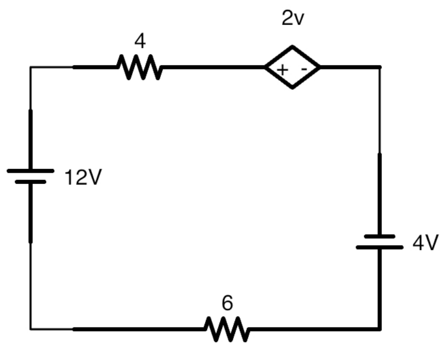
Circuits
Jan 27,2019

Contents

1 Circuit Analysis:

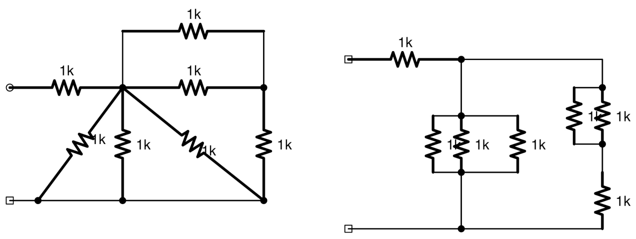
1.1 Basics Law

Example 1:



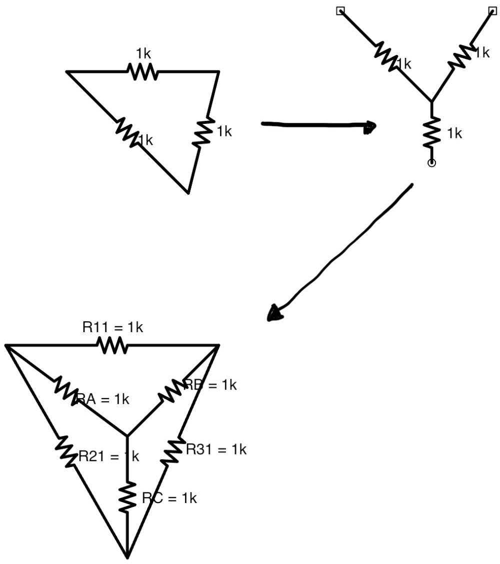
$$12 - 4I - 2v_0 + 4 - 6I = 0 \tag{1}$$
$$I = \frac{v}{R} = \frac{-v_0}{6} \rightarrow v_0 = -6I \tag{2}$$
$$12 - 4I - 2(-6I) + 4 - 6I = 0 \tag{3}$$
$$2I = -16 \tag{4}$$
$$I = 8A \qquad v0 = 42v \tag{5}$$

Example 2:

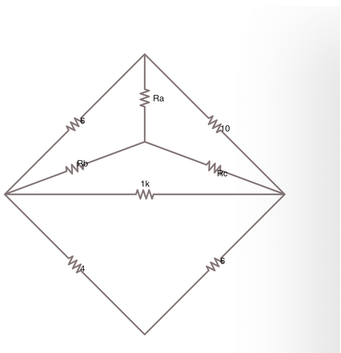


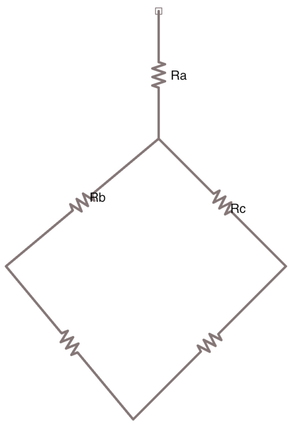
1.1.1 Delta to Y conversion:

$$R_A = \frac{R11 \times R21}{R11 + R21 + R31} \tag{1}$$
$$R_B = \frac{R31 \times R21}{R11 + R21 + R31} \tag{2}$$
$$R_C = \frac{R31 \times R21}{R11 + R21 + R31} \tag{3}$$



Example:



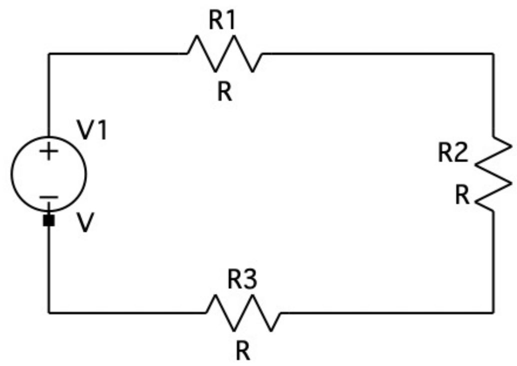


$$Ra = \frac{6 \times 10}{6 + 1 + 10} \tag{1}$$

$$Rb = \frac{6 \times 1}{6 + 1 + 10} \tag{2}$$

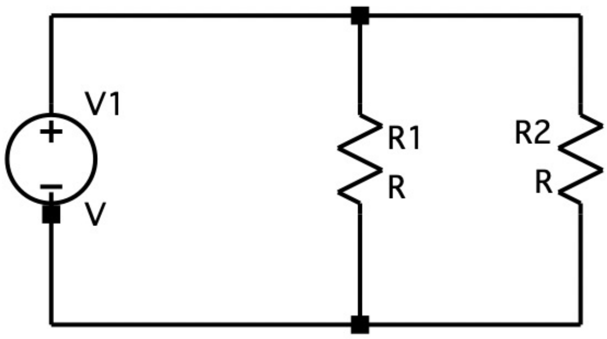
$$Rc = \frac{1 \times 10}{6 + 1 + 10} \tag{3}$$

1.2 Voltage Divider (for series circuit):



$$V_{R_1} = \frac{V_s \times R_1}{R_1 + R_2 + R_3}$$

1.3 Current Divider:

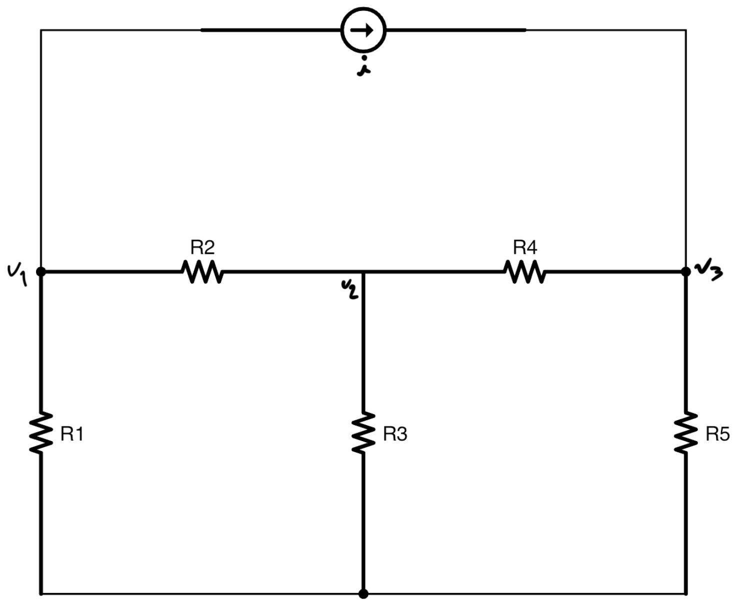


$$I_1 = \frac{I_T \times R_2}{R_1 + R_2 + R_3}$$

$$I_2 = \frac{I_T \times R_1}{R_1 + R_2 + R_3}$$

1.4 Nodal Analysis:

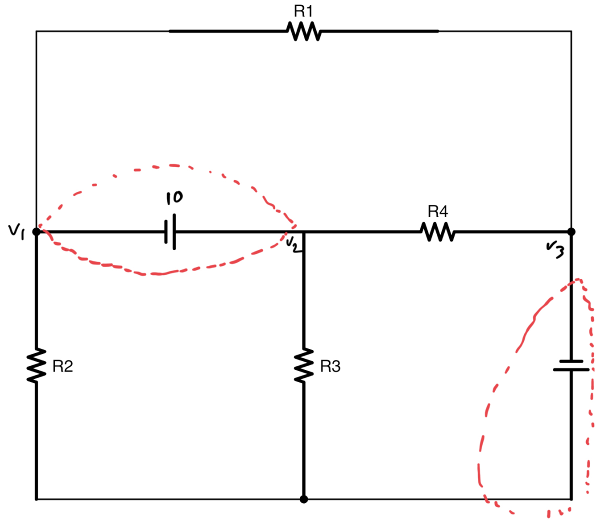
1. Do the KCL at nodes to get the equation.
2. Consider all current leaving nodes except if specified.
3. Select a reference point to compare to usually negative of the power supply or ground.



$$\frac{v_1 - v_2}{R_2} + \frac{v_1}{R_1} + i = 0 \tag{1}$$
$$\frac{v_3}{R_5} - i + \frac{v_3 - v_2}{R_4} = 0 \tag{2}$$
$$\frac{v_2 - v_3}{R_4} + \frac{v_2 - v_1}{R_2} + \frac{v_2}{R_4} = 0 \tag{3}$$

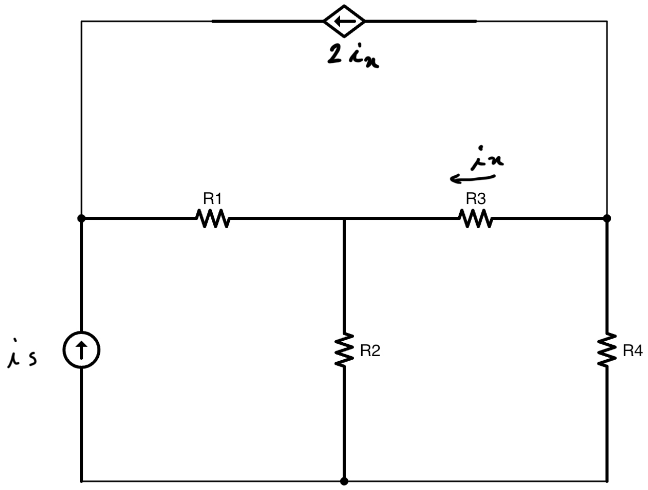
1.5 Node voltage with Supper Node:

1. When you have voltage source between two nodes.
2. enclose the two nodes with dashed line positive consider that as super node.



$$\frac{v_1}{R_2} + \frac{v_1 - v_3}{R_1} + \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} = 0 \tag{1}$$
$$kvl \rightarrow v1 - 10 + v_2 = 0 \tag{2}$$

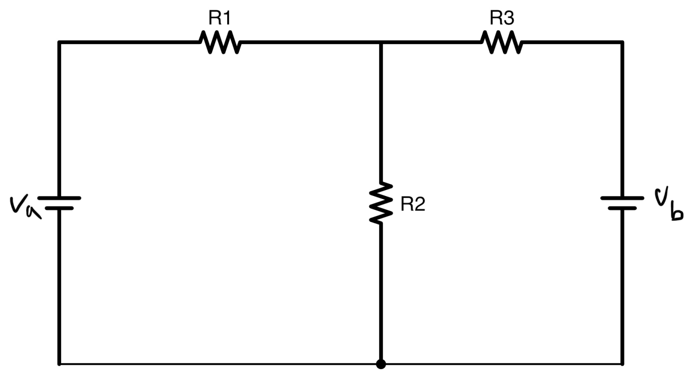
1.6 Node Voltage Analysis with Controlled Source:



$$-i_s - 2i_x + \frac{v_1 - v_2}{R_1} = 0 \tag{1}$$
$$\frac{v_2}{R_2} + \frac{v_2 - v_1}{R_1} - i_x = 0 \tag{2}$$
$$v_x + \frac{v_3}{R_4} + 2i_x = 0 \tag{3}$$
$$i_x = \frac{v_3 - v_2}{R_3} \tag{4}$$

1.7 Mesh Current Analysis:

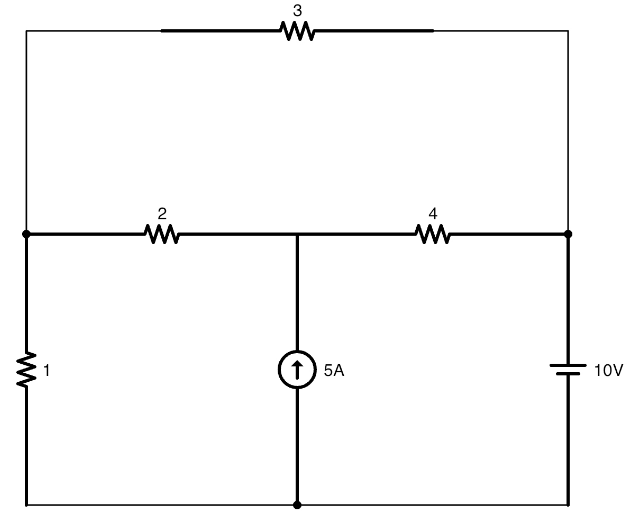
1. Assume loop current direction(clockwise)
2. Current going to source from **negative** to **positive** is negative volt
3. Current going in load or Resistor results, in positive; neighboring current goes negative



$$-v_a + i_1R_1 + (i_1 - i_2)R_2 = 0 \tag{1}$$
$$v_b + (i_2 - i_1)R_2 + i_2R_3 = 0 \tag{2}$$

1.8 Mesh Current Analysis with Shared Current Source Between Loops:(Supper Mesh)

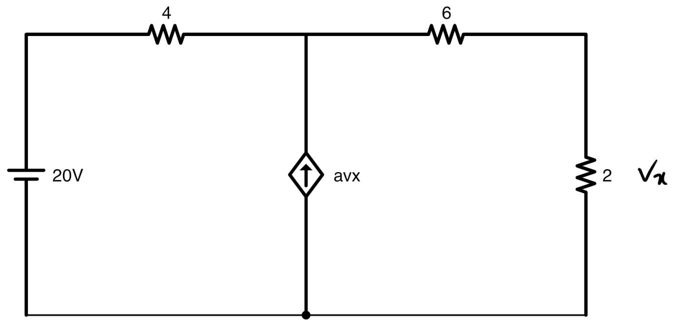
Treat the two loops with current source as one supper mesh.



KVL for supper mesh 1,2

$$i_1 1\Omega + (i_1 - i_3)2\Omega + (i_2 - i_3)4\Omega + 10v = 0 \quad (1)$$
$$(i_3 - i_1)2\Omega + i_3 3\Omega + (i_3 - i_2)4\Omega = 0 \quad (2)$$
$$i_2 - i_1 = 5A \quad (3)$$

1.9 Mesh Current Analysis with Controlled Source:



$$V_x = i_2 2\Omega \quad (1)$$
$$i_2 - i_1 = aV_x \quad (2)$$
$$-20 + i_1 4\Omega + i_2 6\Omega + i_2 2\Omega = 0 \quad (3)$$
$$i_2 - i_1 = a(i_2 \times 2\Omega) \quad (4)$$
$$i_1 = 1A \quad (5)$$
$$i_2 = 2A \quad (6)$$

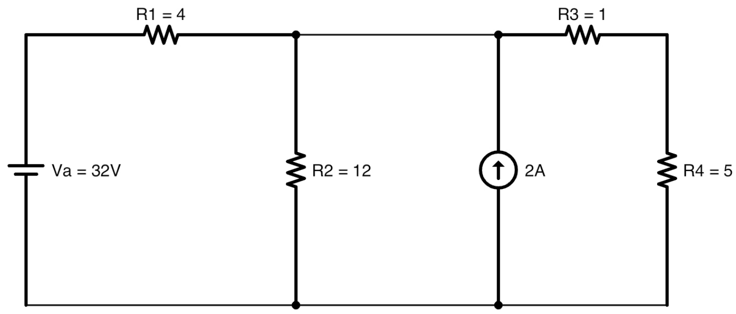
1.10 Thevenin Equivalent Circuit:

Any circuit can be reduced to voltage source V_{oc} or V_{TH} and a series resistor to R_{TH}. V_{oc}=open circuit voltage.
Steps:

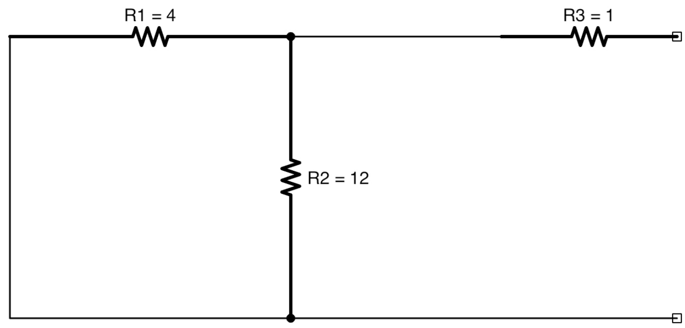
1. To find R_{TH} Remove current sources and short voltage sources
2. To find V_{TH} Remove R_L and find v across open

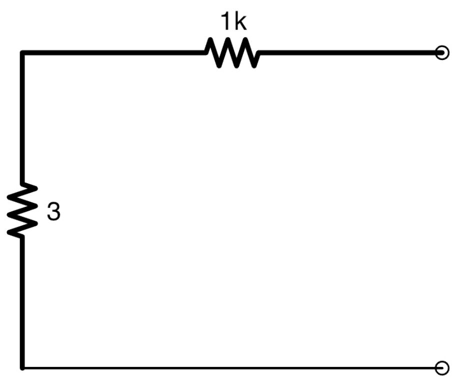
$$R_{TH} = \frac{v_{oc}}{I_{sc}}$$

Example:



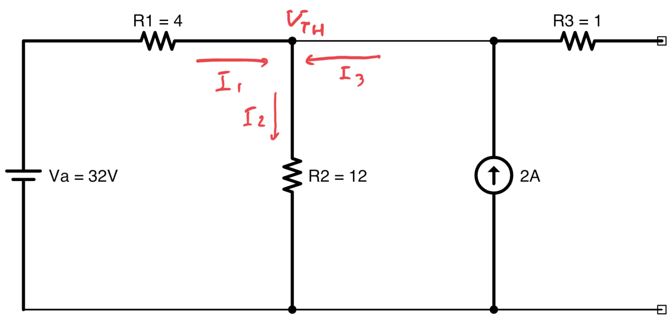
First step:



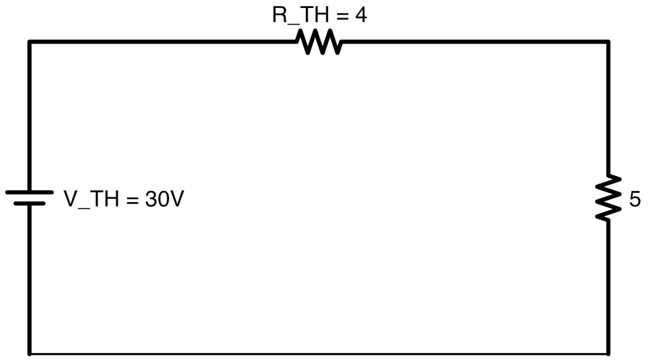


$$R_{TH} = 4\Omega$$

Step two:



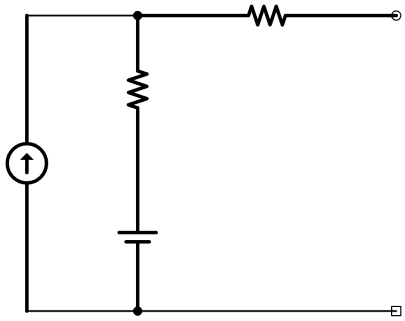
$$I_1 + I_3 = I_2 \tag{1}$$
$$\frac{32 - V_{TH}}{4} + 2 = \frac{V_{TH}}{12} \tag{2}$$
$$94 - 3V_{TH} + 24 = V_{TH} \tag{3}$$
$$120 = 4V_{TH} \tag{4}$$
$$V_{TH} = \frac{120}{4} = 30V \tag{5}$$

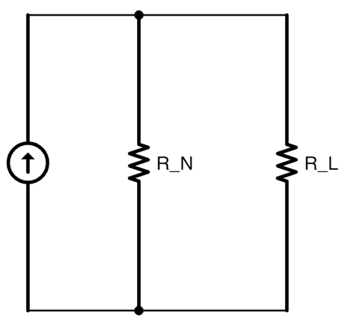


$$I = \frac{V}{R_{TH}} = \frac{30V}{(4+5)\Omega} = 33.3$$

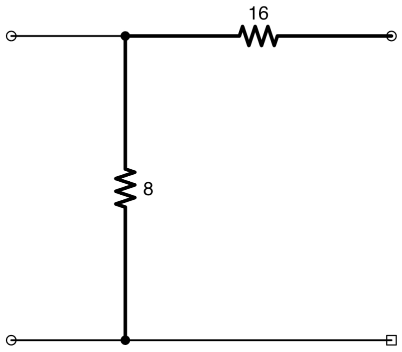
1.11 Norton's Theorem:

1. determine $V_{OC} = V_{TH}$. Determine short circuit $I_{SC} = I_{IN}$. Zero the independent source and find R_{TH}(Do not zero of the dependent sources)
2. Use the equation $V_{TH} = R_{TH} I_N$
3. Thevenin equivalent consist of V_{TH} in R_{TH}. Norton equivalent I_{SC}parallel with R_{TH}

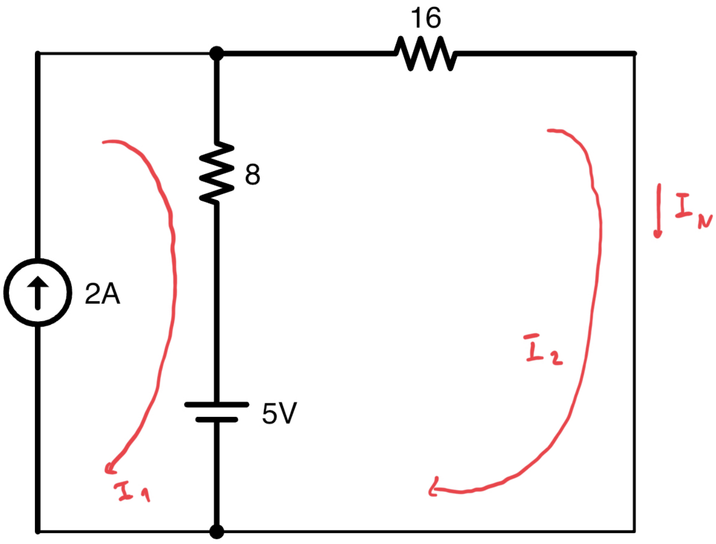




$$R_N = R_{TH}$$
$$I_L = I_N(\frac{R_N}{R_N + R_L})$$
$$I_N = \frac{V_{TH}}{R_{TH}} = \frac{V_{TH}}{ER_N}$$



$$R_N = 16\Omega + 8\Omega = 24$$



$$\sum V_{Loop2} = 8 - 8I_2 + 8I_1 - 16I_2 = 0 \tag{1}$$
$$8 - 8I_2 + (8)(2) - 16I_2 = 0 \tag{2}$$
$$24I_2 = 24 \tag{3}$$
$$I_2 = 1A \tag{4}$$
$$I_L = (1A)(\frac{24\Omega}{24\Omega + R_L}) \tag{5}$$
$$\textbf{Second Method:} \tag{6}$$
$$\sum I \rightarrow I_1 + I_2 + I_3 = 0 \tag{7}$$
$$2 + \frac{8 - V_1}{8} + 0 = 0 \tag{8}$$
$$16 + 8 - V_1 = 0 \tag{9}$$
$$V_1 = 24V \tag{10}$$
$$I_N = \frac{24V}{24\Omega} = 1A \tag{11}$$
$$I_L = 1A(\frac{24\Omega}{24\Omega + R_L}) \tag{12}$$

1.12 Summery of Circuit Analysis:

Method	Summery	When to Apply
Mesh Analysis	KVL to obtain simultaneous equation for the current	-Multiple currents are needed -Current source are present
Node Analysis	KCL to obtain simultaneous equation for the voltages	-Multiple voltage are needed -Voltage source are present
Thevenin and Norton Equivalent circuit	Simple equivalent circuits, source transformation	Intermediate values not important, only output voltage or current

1.13 Source Transformation:

1.14 Maximum Power Transfer:

We need to match the load resistor to system resistor for maximum power transfer.

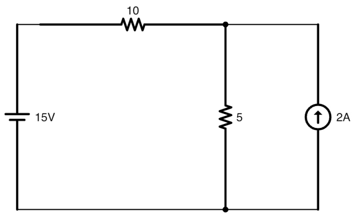
$$P = RI^2 \tag{1}$$

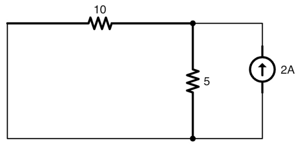
$$P = \frac{v_{TH}^2}{(R_{TH} + R_L)} R_L \tag{2}$$

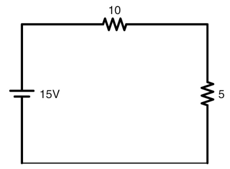
$$P = \frac{v_{TH}^2}{4R_{TH}} \rightarrow R_{TH} = R_L \tag{3}$$

1.15 Superposition:

Total response a current on an element is a circuit is the sum of each independent source acting individually. **Open the current source and short the voltage source.**







$$I_{R_1} = \frac{2A \times R_2}{R_2 + R_1} = \frac{2A \times 5\Omega}{15\Omega} = \overleftarrow{0.667A} \tag{1}$$

$$I_{R_2} = 2A - 0.667A = 1.33A \downarrow \tag{2}$$

$$i = \frac{15v}{15\Omega} = 1A \tag{3}$$

$$I_{R_1} = \overrightarrow{1A} \tag{4}$$

$$I_{R_2} = 1A \downarrow \tag{5}$$

$$I_{R_1} = 1A - 0.667A = \overrightarrow{0.33A} \tag{6}$$

$$I_{R_2} = 1.33A + 1 = 2.33A \downarrow \tag{7}$$

2 Capacitor and Inductors:

2.1 First Order Differential Equation:

$$\frac{dy}{dt} + ay = K, y(0) \tag{1}$$
$$y(t) = \frac{K}{a}(1 - e^{-at}) + y(0)e^{-at}, t > 0 \tag{2}$$

Note: We can only use this formula if K and a our constant.

Example:

$$\frac{dy}{dt} + 10y = 20, y(0) = 1 \tag{1}$$
$$y(t) = 2(1 - e^{-10t}) + e^{-10t} \tag{2}$$
$$\text{steady state} = 2 \quad \& \quad \tau = \frac{1}{10} \tag{3}$$

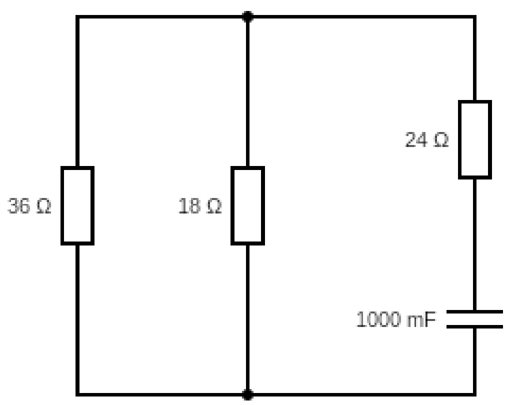
2.2 RC & RL

$$i_C(t) = C\frac{dV}{dt} \tag{1}$$
$$V_L(t) = L\frac{di}{dt} \tag{2}$$
$$C\frac{dv}{dt} + \frac{V}{R} = 0 \tag{3}$$
$$L\frac{di}{dt} + Ri = 0 \tag{4}$$
$$\tag{5}$$

Simple RC Circuit:

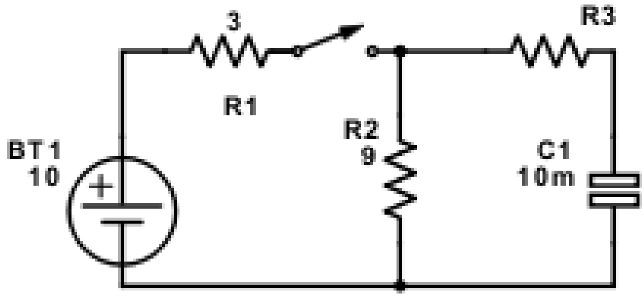
$$i_C + i_R = 0 \tag{1}$$
$$C\frac{dv}{dt} + \frac{V}{R} = 0 \tag{2}$$
$$V(t) = V_0 e^{\frac{-t}{RC}} \tag{3}$$
$$\tau = RC \tag{4}$$

Example:



$$Req = \frac{36*18}{36+18} + 24 = 36\Omega \quad (1)$$
$$\tau = RC = 36\Omega \times 1F = 36s \quad (2)$$
$$V_C = 10ve^{\frac{-t}{36}} \quad (3)$$
$$V_B = \frac{12}{12+24} * 10ve^{\frac{-t}{36}} = 3.33e^{\frac{-t}{36}} \quad (4)$$

Example:

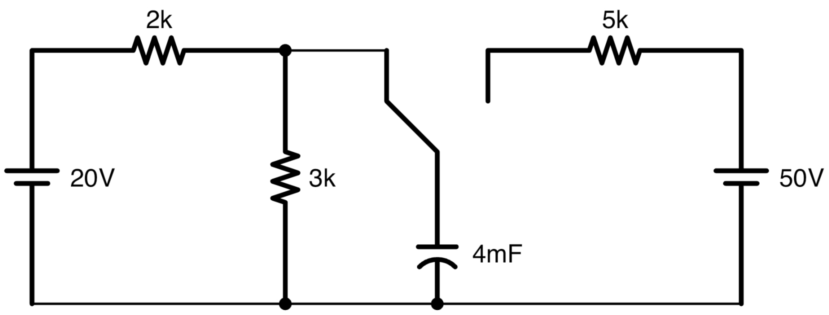


$$V_9 = 10 * \frac{9\Omega}{3\Omega + 9\Omega} = 7.5V \quad (1)$$
$$open \rightarrow V_c = 7.5e^{\frac{-t}{0.1}} \quad (2)$$

Steps For Solving RC Circuit:

$$Steps \begin{cases} V_C(t=0) \quad V_C(t=\infty) \quad \tau = RC \\ V_C(t) = V_C(\infty) + [V_C(0) - V_C(\infty)]e^{\frac{-t}{RC}} \end{cases}$$

Example:



$$V_C(t=0) = 12v \quad (1)$$
$$V_C(t=\infty) = 50 \quad (2)$$
$$\tau = RC = 5k\Omega * 04mF = 2sec \quad (3)$$
$$V_C(t) = 50v - [12-50]e^{\frac{-t}{2}} 50 - 38e^{\frac{-t}{2}} \quad (4)$$

2.3 Inductor:

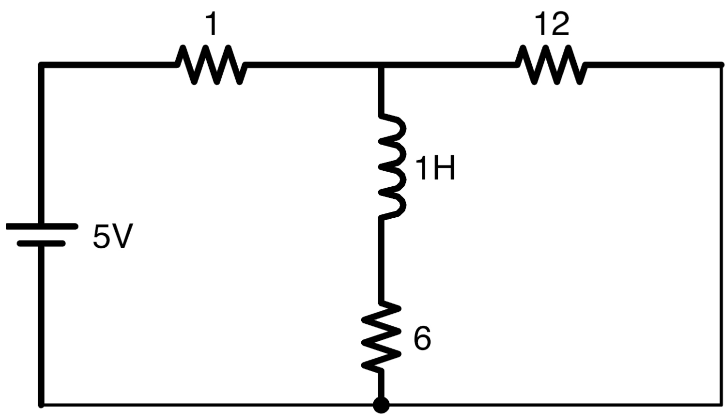
$$v = L\frac{di}{dt}$$

$$i(t) = \frac{V}{R} + (I_0 - \frac{V}{R})e^{\frac{-t}{\tau}}$$

Note: The inductor in **series** will $L_1 + L_2$ to each other and in **parallel** $\frac{1}{L_1} + \frac{1}{L_2} = \frac{1}{L_T}$.
General Strategy:

$$Steps = \begin{cases} i(t=0) \quad i(t=\infty) \quad \tau = \frac{L}{R} \\ i(t) = i(t=\infty) + [i(t=0) - i(t=\infty)]e^{\frac{-t}{\tau}} \end{cases}$$

Note: if i(t=0)=0 we need use this formula: $i(t) = \frac{V}{R}(1 - e^{\frac{-t}{\tau}})$
Example:



After while is act as a wire and current move to the inductor.

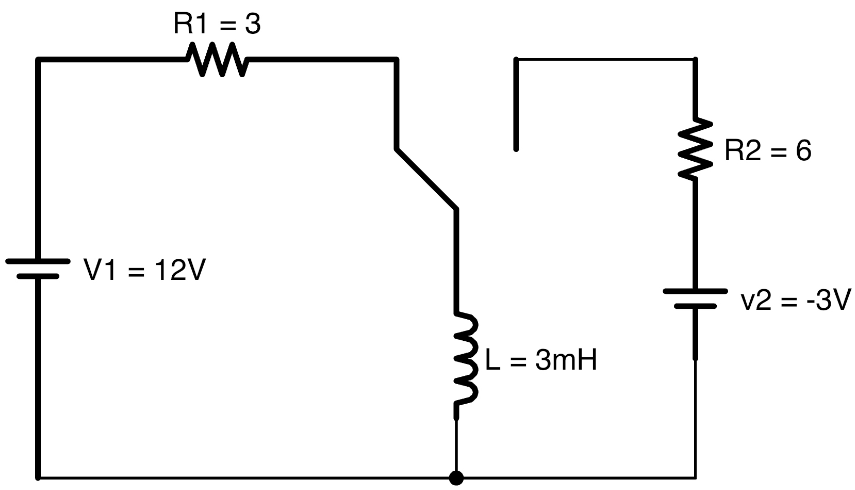
$$P(t) = v(t)v(t) \tag{1}$$

$$w(t) = \int_{t_0}^{t} p(\tau)d\tau + w(t_0) \tag{2}$$

$$v = L\frac{di}{dt} \tag{3}$$

$$w = \frac{1}{2}Li^2 \tag{4}$$

Example:



$$i_L = \frac{v_2 - v_L}{R_2} \tag{1}$$

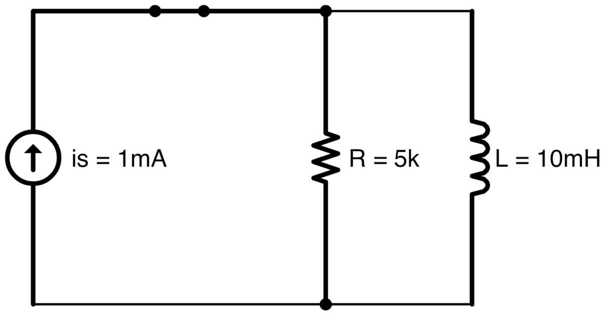
$$i_L = \frac{v_2 - L\frac{di_L}{dt}}{R_2} \tag{2}$$

$$\frac{di_L}{dt} + \frac{t_2}{L} i_L = \frac{v_2}{L} \tag{3}$$

$$\tau = \frac{L}{R_2} \tag{4}$$

$$i_L = \frac{v_2}{R_2}[1 - e^{\frac{-t}{\tau}}] + \frac{v_1}{R_1} e^{\frac{-t}{\tau}} \tag{5}$$

Example:



$$i_R = \frac{v_L}{R} \tag{1}$$

$$i_s = i_r + i_L \tag{2}$$

$$i_s = \frac{L}{R}\frac{di_L}{dt} + i_L \tag{3}$$

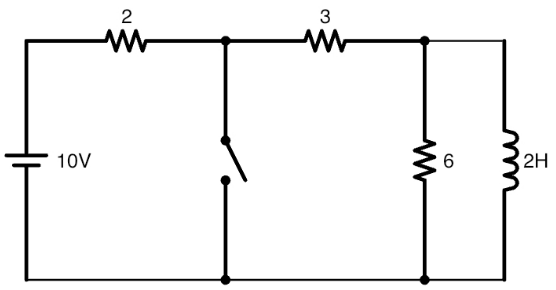
$$\frac{di_L}{dt} + \frac{R}{L} i_L = \frac{R}{L} i_s \tag{4}$$

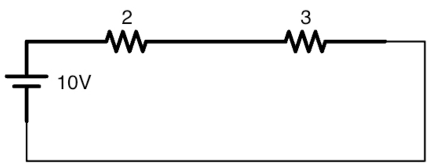
$$\tau = \frac{L}{R} \tag{5}$$

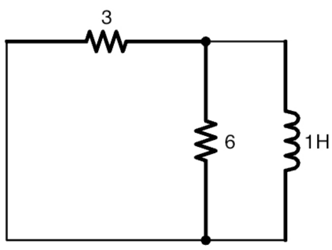
$$i_L = i_s[1 - e^{\frac{-t}{\tau}}] \tag{6}$$

$$v_L = i_s R e^{\frac{-t}{\tau}} \tag{7}$$

Example:

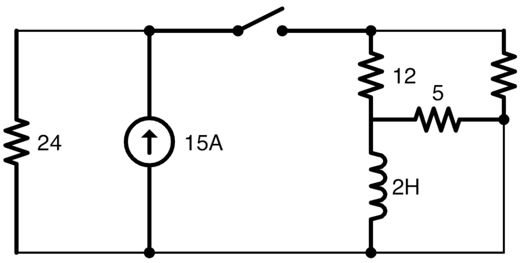


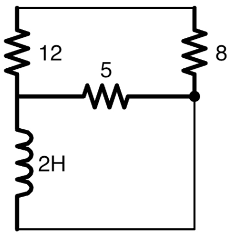


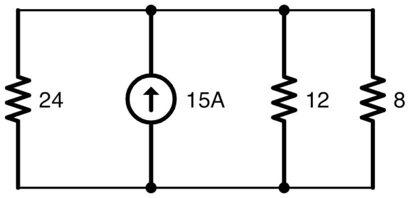


$$I_L = \frac{V}{R} = \frac{10V}{5\Omega} = 2A \tag{1}$$
$$\tau = \frac{L}{R} = \frac{2H}{2\Omega} = 1sec \tag{2}$$
$$I_L(t) = 2Ae^{-t} \tag{3}$$
$$V_L = L\frac{di}{dt} \tag{4}$$
$$V_L = 2H\frac{d}{dt}[2Ae^{-t}] = -4Ve^{-t} \tag{5}$$
$$I_6 = \frac{V_6}{R} = \frac{-4Ve^{-t}}{6\Omega} = \frac{-2}{3} * e^{-t} \tag{6}$$

Example:

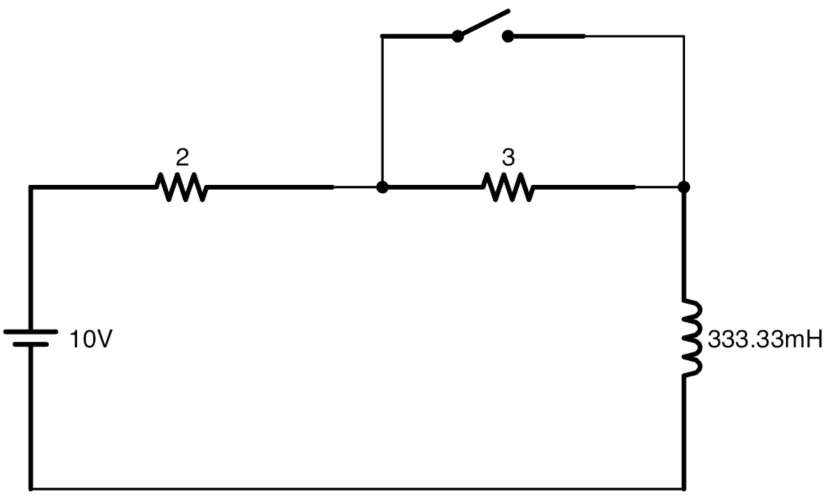






$$I_1 = 15 * \frac{12 * 8}{12 * 8 + 12 * 24 + 8 * 24} = 2.5A \tag{1}$$
$$I_2 = 15 * \frac{24 * 8}{12 * 8 + 12 * 24 + 8 * 24} = 5A \tag{2}$$
$$I_3 = 15 * \frac{12 * 24}{12 * 8 + 12 * 24 + 8 * 24} = 7.5A \tag{3}$$
$$\tau = \frac{2H}{4\Omega} = 0.5sec \tag{4}$$
$$I_L(t) = 5Ae^{-2t} \tag{5}$$
$$V_L(t) = 2 * (-10ve^{-2t}) = -20ve^{-2t} \tag{6}$$

Example:



$$i(t=0) = \frac{10}{2} = 5 \tag{1}$$

$$i(t=\infty) = \frac{10}{5} = 2 \tag{2}$$

$$\tau = \frac{1}{15} sec \tag{3}$$

$$i(t) = i(t=\infty) + [i(t=0) - i(t=\infty)]e^{\frac{-t}{\tau}} \tag{4}$$

$$i(t) = 2 + (5-2)e^{-15t} \tag{5}$$

$$i(t) = 2A + 3e^{-15t} \tag{6}$$

$$10V - (2\Omega + 3\Omega)i - L\frac{di}{dt} \tag{7}$$

$$10v - 5\Omega(2A + 3e^{-15t}) - \frac{1}{3}(-45Ae^{-15t}) \tag{8}$$

$$0 = 0 \tag{9}$$

2.4 Formula Sheet for Capacitance and Inductor:

Capacitance:

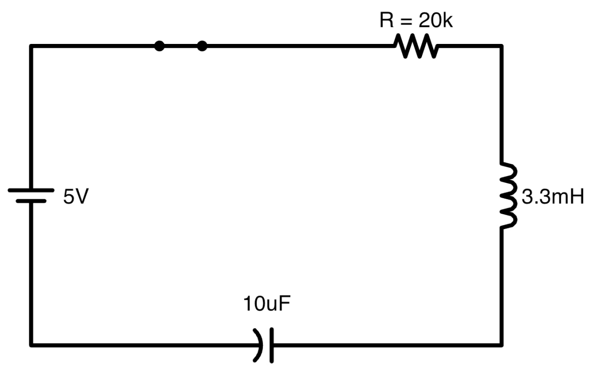
$$q = cv \quad (1)$$
$$i(t) = c\frac{dv}{dt} \quad (2)$$
$$v(t) = \frac{1}{c}\int_0^t i(t)dt + v_0 \quad (3)$$
$$P = cv\frac{dv}{dt} \quad (4)$$
$$w(t) = \frac{1}{2}cv^2(t) \quad (5)$$
$$discharging \rightarrow v(t) = v_i e^{-\frac{t}{\tau}} \quad (6)$$
$$charging \rightarrow v(t) = v_s(1 - e^{\frac{-t}{\tau}}) \quad (7)$$
$$\tau = RC \quad (8)$$

Inductor:

$$v(t) = L\frac{di}{dt} \quad (1)$$
$$i(t) = \frac{1}{L}\int_0^t v(t)dt + i_0 \quad (2)$$
$$P = Li\frac{di}{dt} \quad (3)$$
$$W = \frac{1}{2}Li^2(t) \quad (4)$$
$$Discharging \rightarrow i(t) = \frac{v_s}{R_1}e^{\frac{-t}{\tau}} \quad (5)$$
$$Discharging \rightarrow v(t) = L\frac{v_s}{R_1\tau}e^{\frac{-t}{\tau}} \quad (6)$$
$$Discharging \rightarrow \tau = \frac{L}{R_2} \quad (7)$$
$$charging \rightarrow i(t) = \frac{v_s}{R_1}(1 - e^{\frac{-t}{\tau}}) \quad (8)$$
$$charging \rightarrow v(t) = v_s e^{\frac{-t}{\tau}} \quad (9)$$

2.5 RLC Circuit:

Example:



Transient Response:

$$i(0^-) = 0 \quad (1)$$
$$v_c(0^+) = 0 \quad (2)$$
$$v_s = v_R + v_L + v_c \quad (3)$$
$$v_R = iR \quad (4)$$
$$v_L = L\frac{di}{dt} \quad (5)$$
$$i = C\frac{dv_c}{dt} \quad (6)$$
$$v_S = LC\frac{d^2v_c}{dt^2} + RC\frac{dv_c}{dt} + v_c \quad (7)$$
$$\frac{d^2v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{1}{L}v_c = \frac{v_s}{LC} \quad (8)$$
$$\frac{d^2v_c}{dt^2} + 6.06e - 6L\frac{dv_c}{dt} + 30.3e9v_c = 151.5e9 \quad (9)$$
$$v_{c,t} = K_1e^{-5.00e3t} + K_2e^{-6.06e6t} \quad (10)$$

Steady State:

$$v_c \to \frac{k}{a_2} = LC\frac{v_s}{LC} = v_s \tag{1}$$
$$v_{c,s} = 5 \tag{2}$$

Sum of the V_c:

$$v_c = v_{c,t} + v_{c,s} \tag{1}$$
$$v_c = K_1e^{-5.00e3t} + K_2e^{-6.06e6t} + 5 \tag{2}$$
$$v_c(0^-) = v_c(0) = 0 \tag{3}$$
$$K_1 + K_2 + 5 = 0 \tag{4}$$
$$i(0^-) = 0 \tag{5}$$
$$i = C\frac{dv_c}{dt} \tag{6}$$
$$-50e - 6K_1 - 60.6e - 3K_2 = \tag{7}$$
$$K_1 = -5.00413 \quad \& \quad K_2 = 0.00413 \tag{8}$$

2.6 Second Order circuit:

$$\frac{d^2x}{dt^2} + 2\alpha\frac{dx}{dt} + \omega^2 = f(t) \tag{1}$$
$$s^2 + 2\alpha s + \omega_0 = 0 \tag{2}$$
$$\zeta = \frac{\alpha}{\omega} \tag{3}$$
$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \tag{4}$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \tag{5}$$
$$x_c = K_1e^{s_1t} + K_2e^{s_2t} \to \zeta > 1 \text{ overdamped} \tag{6}$$
$$x_c = K_1e^{s_1t} + K_2te^{s_1t} \to \zeta = 1 \text{ critically damped} \tag{7}$$
$$x_c = K_1e^{-\alpha t}cos(\omega_n t) + K_2e^{-\alpha t}sin(\omega_n t) \to \zeta < 1 \text{ underdapmped} \tag{8}$$
$$\omega_n = \sqrt{\omega_0^2 - \alpha^2} \tag{9}$$
$$\alpha = \frac{1}{2RC} \tag{10}$$
$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{11}$$

2.7 Steps for Solving RLC:

$$\begin{cases} i(t=0^-) & V(t=0^-) \\ i(t=0^+) & V(t=0^+) \\ \frac{di}{dt}(t=0^+) & \frac{dv}{dt}(t=0^+) \\ i_L(t\to\infty) & V_C(t\to\infty) \end{cases} \tag{12}$$

3 Steady State Sinusoidal Analysis:

3.1 sinusoidal currents and voltage:

$$v(t) = V_m cos(\omega t_\theta)$$

V_m is the **peak value** and ω is the **angular frequency** in radians per second and θ is the **phase angle**.

$$\omega = \frac{2\pi}{T} \tag{1}$$

$$f = \frac{1}{T} \to \omega = 2\pi f \tag{2}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2(\omega t + \theta) dt} \tag{3}$$

$$V_{rms} = 0.707 V_m \to sinusoidal \tag{4}$$

3.2 Phasor Representation of Sinusoidal:

$$V(c) = v_m cos(\omega t + \theta) \tag{1}$$

$$\text{Phasor} \to v_m \angle\theta \tag{2}$$

$$Regtangular \to V_m \cos\theta + jV_m \sin\theta \tag{3}$$

Note: sin wave represented by phasor allows easy multiplication and division.
Note: sin wave represented by rectangular allows easy addition and subtraction.
Example: our supply PGE 60HZ and 120 V_{rms}. Write instantaneous, polar, and rectangular?

1. convert frequency to radian.
2. calc V_p or $V_m ax$.
3. write instantenous
4. write polar
5. write rectangular

$$\omega = 2\pi * 60HZ = 377 \tag{1}$$

$$V_p = \frac{V_{rms}}{0.707} = \frac{120}{0.707} = 170V \tag{2}$$

$$polar : 170\angle 0^\circ \tag{3}$$

$$rect : 170 \cos 0 + j170 \sin 0 \tag{4}$$

$$inst : v = 170 cos(377t + 0) \tag{5}$$

Example: give assume rect,polar,instantaneous.

$$V_1 = 200cos(\omega t + 60) \tag{1}$$
$$V_{1polar} = 200\angle 60 \tag{2}$$
$$V_2 = 100sin(\omega t - 20) \tag{3}$$
$$V_2 = 100cos(\omega t - 20 - 90) = 100cos(\omega t - 110) \tag{4}$$
$$V_{2Polar} = 100\angle - 110 \tag{5}$$
$$V_1 = 200cos(60) + j200sin(60) \tag{6}$$
$$V_2 = 100cos(-110) + j100sin(-110) \tag{7}$$
$$V_1 + V_2 = (100 - 34) + (173 - 94)j \tag{8}$$
$$\boxed{V_1 + V_2 = 66 + 79j} \tag{9}$$
$$V_T = \sqrt{66^2 + 79^2} = 102.94 \tag{10}$$
$$\boxed{V_{add} = 102.94cos(\omega t + 50.12)} \tag{11}$$
$$\boxed{polar : 102.94\angle 50.12} \tag{12}$$
$$V_1 - V_2 = (100 + 34) + (173 + 94)j \tag{13}$$
$$\boxed{V_{sub} = 134 + 267j} \tag{14}$$
$$V_{maxSub} = \sqrt{134^2 + 267^2} = 298.73 \tag{15}$$
$$\boxed{V = 298.73cos(\omega t + 63.35)} \tag{16}$$
$$\boxed{polar : 298.73\angle 63.35} \tag{17}$$
$$\boxed{Division : \frac{200\angle 60}{100\angle - 110} = 2\angle 60 - (-110) = 2\angle 170} \tag{18}$$
$$\boxed{V = 2cos(\omega t + 170)} \tag{19}$$
$$\boxed{2cos(170) + j2sin(170) = -1.96 + 0.348j} \tag{20}$$
$$\boxed{Multiplication : 200\angle 60 * 100\angle - 110 = 20000\angle - 50} \tag{21}$$
$$\boxed{V_{multiplication} = 20000cos(\omega t - 50)} \tag{22}$$
$$\boxed{20000cos(-50) + 20000sin(-50)j = 12855 - 15321j} \tag{23}$$

3.3 Complex Impedances:

$$i_L(t) = I_m sin(\omega t + \theta) \rightarrow I_L = I_m\angle\theta - 90^\circ \tag{1}$$
$$V_L(t) = \omega L I_m cos(\omega t + \theta) \rightarrow V_L = \omega L I_m \angle\theta = V_m\angle\theta \tag{2}$$
$$Z_L = j\omega L = \omega L\angle 90 \tag{3}$$
$$V_L = Z_L I_L \tag{4}$$

3.4 Capacitance:

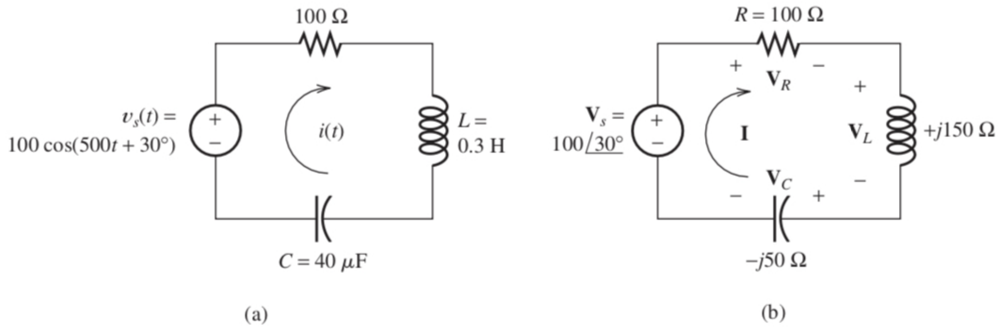
$$V_C = Z_C I_C \tag{1}$$
$$Z_C = -j\frac{1}{\omega C} = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle - 90^\circ \tag{2}$$
$$\tag{3}$$

3.5 Circuit Analysis Using Phasors and Impedances:

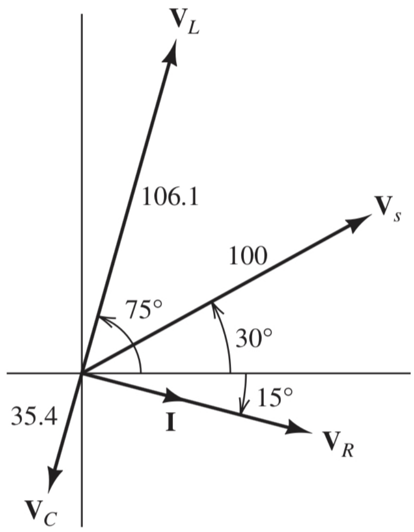
1. Replace the time descriptions of the voltage and current sources with the corresponding phasors.(All of the sources must have the same frequency.)
2. Replace inductances by their complex their complex impedances Z_L. Replace campcitances by their complex impedances Z_C.Resistance have impedances equal to their resistance.

3. Analyze the circuit by using any of the techniques which we studied.

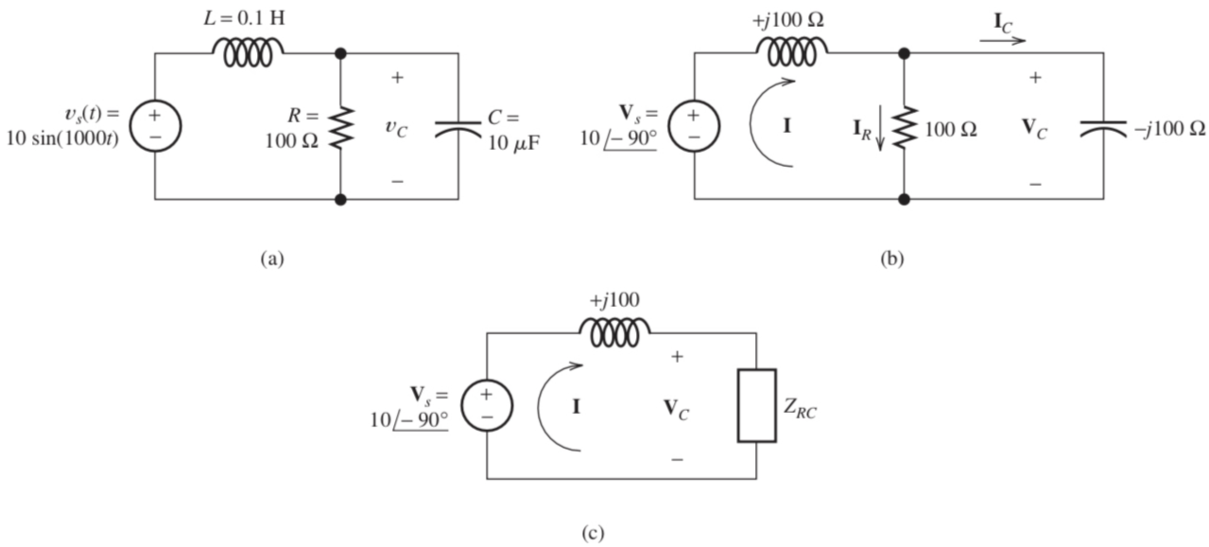
Example: Find the steady-state current for the circuit shown. Also, find the phasor voltage across each element and construct a phasor diagram.



$$V_s = 100\angle 30° \tag{1}$$
$$Z_L = j\omega L = j500 \times 0.3 = j150\Omega \tag{2}$$
$$Z_c = -j\frac{1}{500 \times 40 \times 10^{-6}} = -50j\Omega \tag{3}$$
$$Z_{eq} = R + Z_L + Z_C = 100 + j150 - j50 = 100 + j100 \tag{4}$$
$$Z_{eq} = 141.4\angle 45° \tag{5}$$
$$I = \frac{V_s}{Z} = \frac{100\angle 30°}{141.4\angle 45°} = 0.707\angle - 15° \tag{6}$$
$$i(t) = 0.707cos(500t - 15°) \tag{7}$$
$$\star V_R = R \times I = 100 \times 0.707\angle - 15° = 70.7\angle - 15° \tag{8}$$
$$\star V_L = \omega L\angle 90° I = 150\angle 90° \times 0.707\angle - 15° = 106.1\angle 75° \tag{9}$$
$$\star V_C = \frac{1}{\omega C}\angle - 90° I = 50\angle - 90° \times 0.707\angle - 15° = 35.4\angle - 105° \tag{10}$$



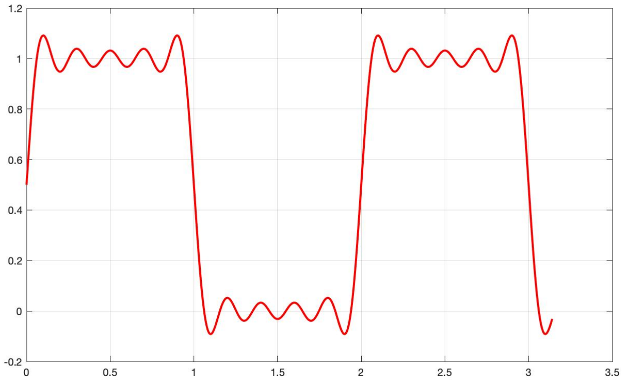
Example: Find the voltage $V_C(t)$ in the steady state. Also, find the phasor voltage across each element and construct a phasor diagram.



$$V_s = 10\angle -90^\circ \tag{1}$$
$$Z_L = j\omega L = j1000 \times 0.1 = j100\Omega \tag{2}$$
$$Z_C = -j\frac{1}{1000 \times 10 \times 10^{-6}} = -j100\Omega \tag{3}$$
$$Z_{RC} = \frac{1}{\frac{1}{R} + \frac{1}{Z_C}} = \frac{1}{\frac{1}{100} + \frac{1}{-j100}} = \frac{1}{0.1 + j0.01} = \frac{1\angle 0^\circ}{0.01414\angle 45^\circ} = 70.71\angle -45^\circ \tag{4}$$
$$V_C = V_S\frac{Z_{RC}}{Z_L + Z_{RC}} = 10\angle -90\frac{70.71\angle -45}{j100 + 50 - 50j} = 10\angle -180^\circ \tag{5}$$
$$V_C(t) = 10cos(1000t - 180^\circ) = -10cos(1000t) \tag{6}$$
$$I = \frac{V_S}{Z_L + Z_{RC}} = \frac{10\angle -90}{j100 + 50 - j50} = 0.1414\angle -135^\circ \tag{7}$$

4 Fourier Series:

$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n cos(n\omega t) + b_n sin(n\omega t)) \tag{1}$$
$$a_0 = \frac{1}{T}\int_1^T f(t)dt \tag{2}$$
$$a_n = \frac{2}{T}\int_0^T f(t)cos(n\omega t)dt \tag{3}$$
$$b_n = \frac{2}{T}\int_0^T f(t)sin(n\omega t)dt \tag{4}$$
$$f(t) = a_0 + \sum_{n=1}^{\infty} A_n cos(n\omega t + \phi_n) \tag{5}$$
$$A_n = \sqrt{a_n^2 + b_n^2} \tag{6}$$
$$\phi_n = -tan^{-1}(\frac{b_n}{a_n}) \tag{7}$$
$$A_n\angle\phi_n = a_n - jb_n \tag{8}$$



5 Filter:

6 Logic Circuits

6.1 Combination of Logic Circuits:

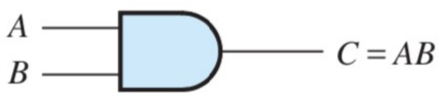
6.1.1 AND Gate:

The **AND** operation on two logic variables, A and B, is represented as AB, read as "A and B." The **AND** operation is also called logical multiplication.

$$AA = A \tag{1}$$
$$A1 = A \tag{2}$$
$$A0 = A \tag{3}$$
$$AB = BA \tag{4}$$
$$A(BC) = (AB)C = ABC \tag{5}$$

A	B	$C = AB$
0	0	0
0	1	0
1	0	0
1	1	1

(a) Truth table



Symbol for two-input AND gate

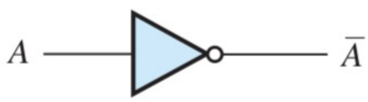
6.1.2 Logic Inverteer:

The NOT operation on a logic variable is represented by placing a bar over the symbol for the logic variable.

$$A\bar{A} = 0 \tag{1}$$
$$\bar{\bar{A}} = A \tag{2}$$

A	$\bar{A}$
0	1
1	0

(a) Truth table



(b) Symbol for an inverter

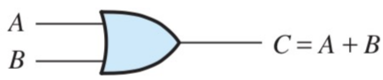
6.2 OR Gate:

The OR operation of logic variables is written as A + B, which is read as "A or B."

$$(A + B) + C = A + (B + C) = A + B + C \tag{1}$$
$$A(B + C) = AB + AC \tag{2}$$
$$A + 0 = A \tag{3}$$
$$A + 1 = 1 \tag{4}$$
$$A + \bar{A} = 1 \tag{5}$$
$$A + A = A \tag{6}$$

A	B	$C = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

(a) Truth table

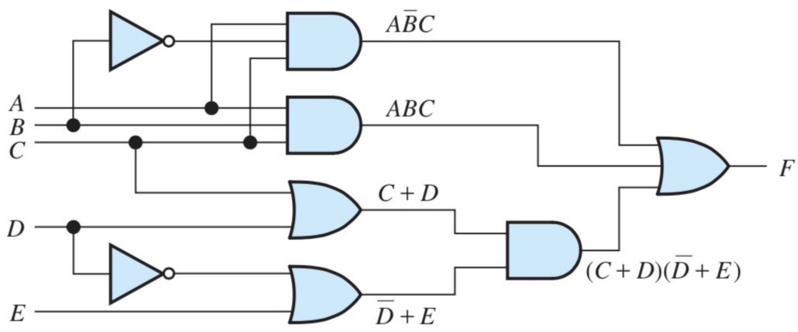


OR	AND
$A + 0 = A$	$A0 = 0$
$A + 1 = 1$	$A1 = A$
$A + A = A$	$AA = A$
$A + \bar{A} = 1$	$A\bar{A} = 0$
$A + (B + C) = (A + B) + C$	$A(BC) = (AB)C$
$AB + AC = A(B + C)$	

Example:

Table 7.5. Truth Table for $D = AB + C$

A	B	C	AB	$D = AB + C$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1



$$F = A\bar{B}C + ABC + (C + D)(\bar{D} + E)$$

6.2.1 De Morgan's Laws:

Another way to state these laws is as follows: If the variables in a logic expression are replaced by their inverses, the AND operation is replaced by OR, the OR operation is replaced by AND, and the entire expression is inverted, the resulting logic expression yields the same values as before the changes.

$$A + B = \overline{\bar{A} + \bar{B}} \quad (1)$$

$$AB = \overline{\bar{A} + \bar{B}} \quad (2)$$

6.3 SOP(sum of products):

1. We need concentrate on the rows of the truth table that is 1.
2. In writing product for each row, we invert the logic variables that are 0 in that row.
3. Product terms that includes all of the input variables are called minterms.

Table 7.6. Truth Table Used to Illustrate SOP and POS Logical Expressions

Row	A	B	C	D
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

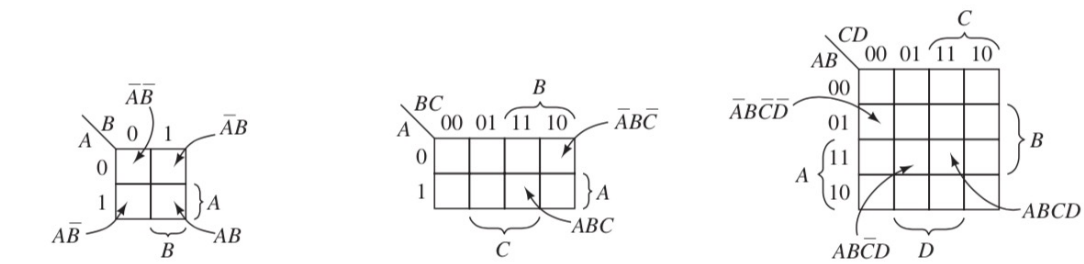
$$D = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + AB\bar{C} + ABC$$
$$\sum m(0, 2, 6, 7)$$

6.4 POS (product of sum)

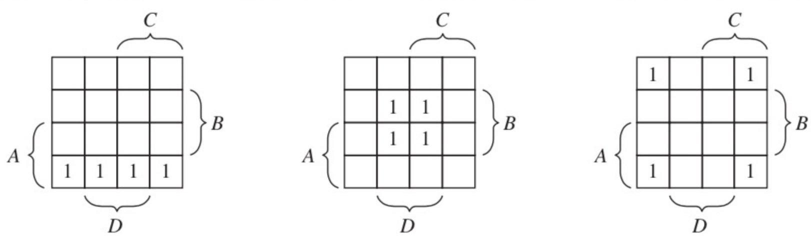
1. identify row with 0 output(Max term)
2. write logical sum for each one
3. invert logic variables that are 1 in that row

$$D = (A + B + \bar{C})(A + \bar{B} + \bar{C})(\bar{A} + B + C)(\bar{A} + B + \bar{C})$$
$$D = \Pi M(1, 2, 3, 4, 5)$$

6.5 Karnaugh Maps:

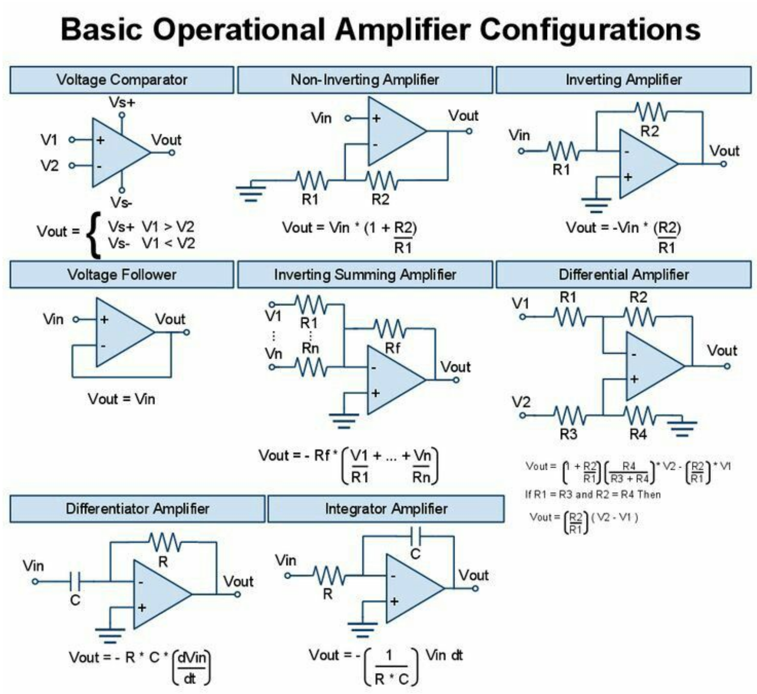


(a) Two-variable Karnaugh map (b) Three-variable Karnaugh map (c) Four-variable Karnaugh map



(a) Map of $A\bar{B}$ (b) Map of BD (c) Map of $\bar{B}\bar{D}$

7 Amplifier:



8 Chapter 9: Review Calpoly

$$V = V_m \cos(\omega t + \phi) \tag{1}$$

$$V_{rms} = V_m * \frac{\sqrt{2}}{2} \tag{2}$$

$$\sin(\omega t + \theta) = cos(\omega t + \theta - 90^\circ) \tag{3}$$

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{4}$$

$$cos(\omega t) + i\sin(\omega t) = e^{i\omega t} \tag{5}$$

9 Chapter 10

$$v = V_m \cos(\omega t + \theta_v - \theta_i)$$
$$i = I_m cos(\omega t)$$
$$p = \begin{cases} power & = V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos \omega t \\ p & = \frac{V_m I_m}{2} cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} cos(\theta_v - \theta_i) cos2\omega t - \frac{V_m I_m}{2} sin(\theta_v - \theta_i) sin2\omega t) \\ p & = P + Pcos2\omega t - Qsin2\omega t \\ P_{avg} & = \frac{V_m I_m}{2} cos(\theta_v - \theta_i) \\ Q_{reactivepower} & = \frac{V_m I_m}{2} sin(\theta_v - \theta_i) \end{cases}$$