Sharif University of Technology
department of computer engineering
ML for bioinformatics, March 2022

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Assignment #1

Question One

a) The integral of PDF across the space must be one, therfore:

$$\int_{-\infty}^{+\infty} f_X(x)\,dx = 1 \implies \int_0^2 cx^2\,dx = 1 \implies \frac{cx^3}{3}\Big]_0^2 = 1 \implies c = \frac{3}{8}$$

b) CDF in a certain point, is the integral of PDF from -∞ to that point, so we have:

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt$$

for x<0 we have :

$$F_X(x) = 0$$

for x>2 we have :

$$F_X(x) = 1$$

for 0≤x≤2 we have :

$$F_X(x) = \int_{-\infty}^{x} \frac{3}{8}t^2\,dt = \int_0^x \frac{3}{8}t^2\,dt = \frac{t^3}{8}\Big]_0^x = \frac{x^3}{8}$$

c)

$$P(1 \le X \le 1.5) = F_X(1.5) - F_X(1) = \frac{1.5^3}{8} - \frac{1^3}{8} = 0.296875$$

d)

$$\mathbb{E}[X] = \int_{-\infty}^{+\infty} x f_X(x)\,dx = \int_0^2 x * \frac{3}{8}x^2\,dx = \frac{3}{8} * \frac{1}{4}x^4\Big]_0^2 = \frac{3}{32}(16-0) = \frac{3}{2}$$

e)

$$\mathbb{V}[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

$$\mathbb{E}[X^2] = \int_{-\infty}^{+\infty} x^2 f_X(x)\,dx = \int_0^2 x^2 * \frac{3}{8}x^2\,dx = \frac{3}{8} * \frac{1}{5}x^5\Big]_0^2 = \frac{3}{40}(32-0) = \frac{12}{5}$$

$$\mathbb{E}[X]^2 = (\frac{3}{2})^2 = \frac{9}{4}$$

$$\mathbb{V}[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2 = \frac{12}{5} - \frac{9}{4} = 0.15$$

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Question Two

$$p(x|\theta) = \theta^2 x e^{-\theta x} u(x)$$

$$L(\theta|x_1,...,x_n) = \prod_{i=1}^{n} p(x_i|\theta) = \prod_{i=1}^{n} \theta^2 x_i e^{-\theta x_i} u(x_i)$$

$$ln(L) = \sum_{i=1}^{n} 2ln(\theta) + ln(x_i) - \theta x_i + ln(u(x_i))$$

to maximize the likelihood we should set $\frac{\partial L}{\partial \theta} = 0$:

$$\frac{\partial L}{\partial \theta} = \sum_{i=1}^{n} \frac{2}{\theta} + 0 - x_i + 0 = 0 \implies \frac{2n}{\theta} = \sum_{i=1}^{n} x_i \implies \theta = \frac{2n}{\sum_{i=1}^{n} x_i}$$

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Question Three ....

$$W^* = (X^T X)^{-1} X^T t$$

$$X = \begin{bmatrix} 0 & -1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, t = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$W^* = \left(\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$0 * 0 + 0 * 0 + 0 * 0 = 0$$
$$0 * (-1) + 0 * 0 + 0 * 1 = 0$$
$$-1 * 0 + 0 * 0 + 1 * 0 = 0$$
$$-1 * (-1) + 0 * 0 + 1 * 1 = 2$$
$$0 * 1 + 0 * (-1) + 0 * 2 = 0$$
$$-1 * 1 + 0 * (-1) + 1 * 2 = 1$$

$$\implies W^* = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

matrix has no inverse so we use its Pseudoinverse :

$$\implies W^* = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}^{+} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 * 0 + 0 * 0 = 0 \\ 0 * 0 + 1/2 * 1 = 1/2 \end{bmatrix}$$
$$\implies \beta_1 = 1/2$$

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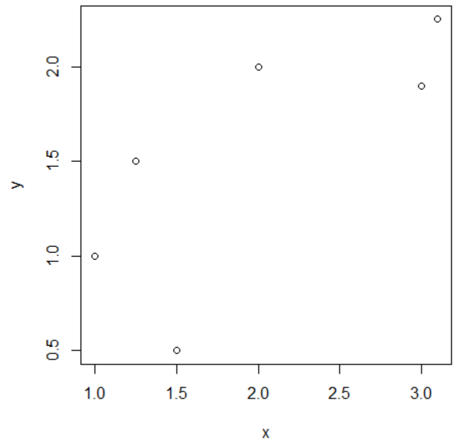
Question Four

using command `plot(x,y)` in R, we plot X and Y to see their realation.



Finding covariance matrix:

$$\mathbb{V}[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$
$$\mathbb{E}[X]^2 = ((1 + 1.25 + 1.5 + 2 + 3 + 3.1)/6)^2 = 3.900625$$
$$EX[X^2] = ((1 + 1.5625 + 2.25 + 4 + 9 + 9.61)/6) = 4.570417$$
$$\mathbb{V}[X] = 4.570417 - 3.900625 = 0.6697917$$

$$\mathbb{V}[Y] = \mathbb{E}[Y^2] - \mathbb{E}[Y]^2$$
$$\mathbb{E}[Y]^2 = ((1 + 1.5 + 0.5 + 2 + 1.9 + 2.25)/6)^2 = 2.325625$$
$$EX[Y^2] = ((1 + 2.25 + 0.25 + 4 + 3.61 + 5.0625)/6) = 2.695417$$
$$\mathbb{V}[X] = 2.695417 - 2.325625 = 0.369792$$

$$\mathrm{Cov}[X,Y] = \mathrm{Cov}[Y,X] = \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y] =$$
$$((1 + 1.875 + 0.75 + 4 + 5.7 + 6.975)/6) - 1.975 * 1.525 = 0.3714583$$

$$CovMatrix = \begin{bmatrix} 0.6697917 & 0.3714583 \\ 0.3714583 & 0.369792 \end{bmatrix}$$

Finding eigenvalues:

$$A*v = \lambda*v \implies A*v - \lambda*v = (A - \lambda*I)*v = 0 \implies det(A - \lambda*I) = 0(non-zero-solution)$$

For simplification in further calculations, I round the numbers to 2 decimal places:

$$\implies \begin{vmatrix} 0.66\text{-}\lambda & 0.37 \\ 0.37 & 0.36\text{-}\lambda \end{vmatrix} = 0 \implies (0.66 - \lambda) * (0.36 - \lambda) - (0.37) * (0.37) = 0 \implies$$
$$\lambda^2 - 1.02\lambda + 0.1007 = 0 \implies \lambda_1 = 1.41, \lambda_2 = 0.62$$

finding eigenvector for λ_1 :

$$(A - \lambda_1 * I) * X = \begin{bmatrix} 0.66\text{-}1.41 & 0.37 \\ 0.37 & 0.36\text{-}1.41 \end{bmatrix} *X = \begin{bmatrix} \text{-}0.75 & 0.37 \\ 0.37 & \text{-}1.05 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\implies \begin{cases} -0.75x_1 + 0.37x_2 = 0 \\ 0.37x_1 - 1.05x_2 = 0 \end{cases} \implies -0.75x_1 + 0.37x_2 + 0.37x_1 - 1.05x_2 = 0$$

$$\implies -0.38x_1 = 0.68x_2 \implies X = \begin{bmatrix} x_1 \\ \frac{-0.38}{0.68}x_1 \end{bmatrix} = \begin{bmatrix} x_1 \\ -0.55x_1 \end{bmatrix}$$

finding eigenvector for λ_2 :

$$(A - \lambda_2 * I) * X = \begin{bmatrix} 0.66\text{-}0.62 & 0.37 \\ 0.37 & 0.36\text{-}0.62 \end{bmatrix} * X = \begin{bmatrix} 0.04 & 0.37 \\ 0.37 & \text{-}0.26 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\implies \begin{cases} 0.04x_1 + 0.37x_2 = 0 \\ 0.37x_1 - 0.26x_2 = 0 \end{cases} \implies 0.04x_1 + 0.37x_2 + 0.37x_1 - 0.26x_2 = 0$$

$$\implies 0.41x_1 = -0.11x_2 \implies X = \begin{bmatrix} x_1 \\ \frac{0.41}{-0.11}x_1 \end{bmatrix} = \begin{bmatrix} x_1 \\ -3.72x_1 \end{bmatrix}$$

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Question Five

CK7 :

$H(S) = -(3/4)log(3/4) - (1/4)log(1/4) = 0.8112781244591328$ bits
$H(S, CK7 = +) = 0$ bits
$H(S, CK7 = -) = 0$ bits
$Gain(s, CK7) = 0.8112781244591328 - ((3/4) * 0) - ((1/4) * 0) = 0.8112781244591328$ bits

CK20 :

$H(S) = -(3/4)log(3/4) - (1/4)log(1/4) = 0.8112781244591328$ bits
$H(S, CK20 = +) = 0$ bits
$H(S, CK20 = -) = 0$ bits
$Gain(s, CK20) = 0.8112781244591328 - ((1/4) * 0) - ((3/4) * 0) = 0.8112781244591328$ bits

TTF1 :

$H(S) = -(3/4)log(3/4) - (1/4)log(1/4) = 0.8112781244591328$ bits
$H(S, TTF1 = +) = 0$ bits
$H(S, TTF1 = -) = -(2/3)log(2/3) - (1/3)log(1/3) = 0.9182958340544896$ bits
$Gain(s, TTF1) = 0.8112781244591328 - ((1/4) * 0) - ((3/4) * 0.9182958340544896) = 0.12255624891826566$ bits

CDX-II :

$H(S) = -(3/4)log(3/4) - (1/4)log(1/4) = 0.8112781244591328$ bits
$H(S, CDX - II = +) = -(3/4)log(3/4) - (1/4)log(1/4) = 0.8112781244591328$ bits
$H(S, CDX - II = -) = 0$ bits
$Gain(s, CDX - II) = 0.8112781244591328 - ((4/4) * 0.8112781244591328) - ((0/4) * 0) = 0.0$ bits

CEA :

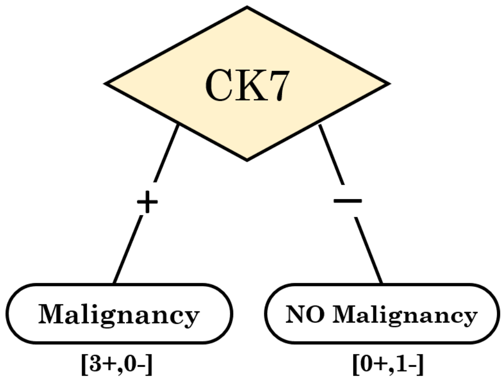
$H(S) = -(3/4)log(3/4) - (1/4)log(1/4) = 0.8112781244591328$ bits
$H(S, CEA = +) = -(2/3)log(2/3) - (1/3)log(1/3) = 0.9182958340544896$ bits
$H(S, CEA = -) = 0$ bits

$Gain(s, CEA) = 0.8112781244591328 - ((3/4)*0.9182958340544896) - ((1/4)*0) = 0.12255624891826566$ bits

....

P63 :

$H(S) = -(3/4)log(3/4) - (1/4)log(1/4) = 0.8112781244591328$ bits
$H(S, P63 = +) = -(1/2)log(1/2) - (1/2)log(1/2) = 1.0$ bits
$H(S, P63 = -) = 0$ bits
$Gain(s, P63) = 0.8112781244591328 - ((2/4) * 1.0) - ((2/4) * 0) = 0.31127812445913283$ bits



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CK7 :

$H(S) = -(3/5)log(3/5) - (2/5)log(2/5) = 0.9709505944546686$ bits
$H(S, CK7 = +) = -(3/4)log(3/4) - (1/4)log(1/4) = 0.8112781244591328$ bits
$H(S, CK7 = -) = 0$ bits
$Gain(s, CK7) = 0.9709505944546686 - ((4/5)*0.8112781244591328) - ((1/5)*0) = 0.3219280948873623$ bits

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CK20 :

$H(S) = -(3/5)log(3/5) - (2/5)log(2/5) = 0.9709505944546686$ bits
$H(S, CK20 = +) = 0$ bits
$H(S, CK20 = -) = -(3/4)log(3/4) - (1/4)log(1/4) = 0.8112781244591328$ bits
$Gain(s, CK20) = 0.9709505944546686 - ((1/5)*0) - ((4/5)*0.8112781244591328) = 0.3219280948873623$ bits

....

TTF1 :

$H(S) = -(3/5)log(3/5) - (2/5)log(2/5) = 0.9709505944546686$ bits
$H(S, TTF1 = +) = -(1/2)log(1/2) - (1/2)log(1/2) = 1.0$ bits
$H(S, TTF1 = -) = -(2/3)log(2/3) - (1/3)log(1/3) = 0.9182958340544896$ bits
$Gain(s, TTF1) = 0.9709505944546686 - ((2/5)*1.0) - ((3/5)*0.9182958340544896) = 0.01997309402197489$ bits

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CDX-II :

$H(S) = -(3/5)log(3/5) - (2/5)log(2/5) = 0.9709505944546686$ bits
$H(S, CDX-II = +) = -(3/4)log(3/4) - (1/4)log(1/4) = 0.8112781244591328$ bits

$H(S, CDX - II = -) = 0$ bits
$Gain(s, CDX - II) = 0.9709505944546686 - ((4/5) * 0.8112781244591328) - ((1/5) * 0) = 0.3219280948873623$ bits

....

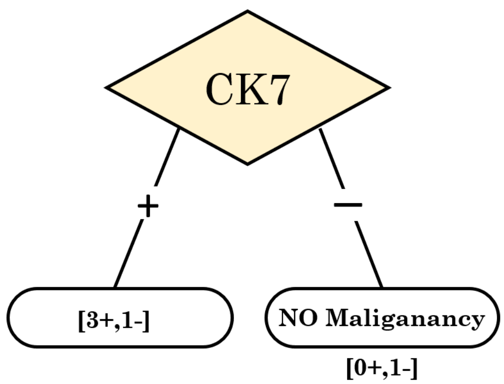
CEA :

$H(S) = -(3/5)log(3/5) - (2/5)log(2/5) = 0.9709505944546686$ bits
$H(S, CEA = +) = -(2/4)log(2/4) - (2/4)log(2/4) = 1.0$ bits
$H(S, CEA = -) = 0$ bits
$Gain(S, CEA) = 0.9709505944546686 - ((4/5) * 1.0) - ((1/5) * 0) = 0.17095059445466854$ bits

....

P63 :

$H(S) = -(3/5)log(3/5) - (2/5)log(2/5) = 0.9709505944546686$ bits
$H(S, P63 = +) = -(1/2)log(1/2) - (1/2)log(1/2) = 1.0$ bits
$H(S, P63 = -) = -(2/3)log(2/3) - (1/3)log(1/3) = 0.9182958340544896$ bits
$Gain(S, P63) = 0.9709505944546686 - ((2/5) * 1.0) - ((3/5) * 0.9182958340544896) = 0.01997309402197489$ bits



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CK20 :

$H(S_{CK7}) = -(3/4)log(3/4) - (1/4)log(1/4) = 0.8112781244591328$ bits
$H(S_{CK7}, CK20 = +) = 0$ bits
$H(S_{CK7}, CK20 = -) = -(3/4)log(3/4) - (1/4)log(1/4) = 0.8112781244591328$ bits
$Gain(S_{CK7}, CK20) = 0.8112781244591328 - ((0/4) * 0) - ((4/4) * 0.8112781244591328) = 0.0$ bits

....

TTF1 :

$H(S_{CK7}) = -(3/4)log(3/4) - (1/4)log(1/4) = 0.8112781244591328$ bits
$H(S_{CK7}, TTF1 = +) = -(1/2)log(1/2) - (1/2)log(1/2) = 1.0$ bits
$H(S_{CK7}, TTF1 = -) = 0$ bits
$Gain(S_{CK7}, TTF1) = 0.8112781244591328 - ((2/4) * 1.0) - ((2/4) * 0) = 0.31127812445913283$ bits

....

CDX-II :

$H(S_{CK7}) = -(3/4)log(3/4) - (1/4)log(1/4) = 0.8112781244591328$ bits
$H(S_{CK7}, CDX - II = +) = 0$ bits
$H(S_{CK7}, CDX - II = -) = 0$ bits

$Gain(s_{CK7}, CDX-II) = 0.8112781244591328 - ((3/4)*0) - ((1/4)*0) = 0.8112781244591328$ bits

....

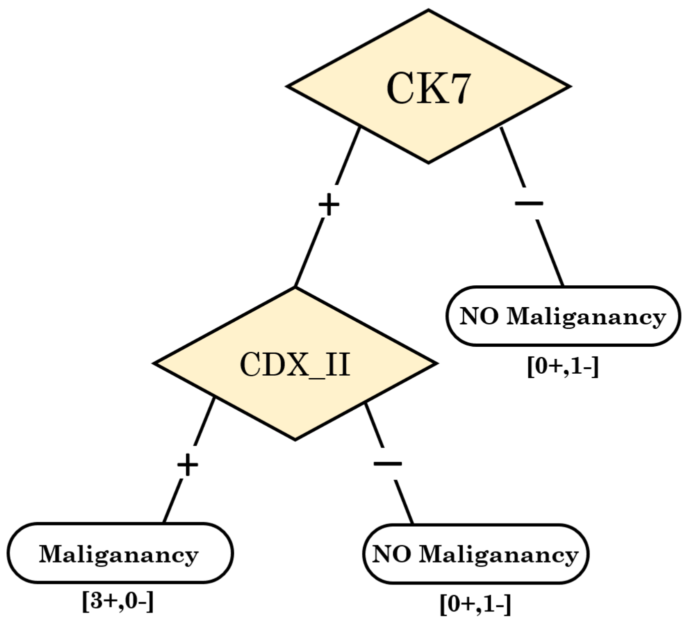
CEA :

$H(S_{CK7}) = -(3/4)log(3/4) - (1/4)log(1/4) = 0.8112781244591328$ bits
$H(S_{CK7}, CEA = +) = -(2/3)log(2/3) - (1/3)log(1/3) = 0.9182958340544896$ bits
$H(S_{CK7}, CEA = -) = 0$ bits
$Gain(S_{CK7}, CEA) = 0.8112781244591328 - ((3/4)*0.9182958340544896) - ((1/4)*0) = 0.12255624891826566$ bits

....

P63 :

$H(S_{CK7}) = -(3/4)log(3/4) - (1/4)log(1/4) = 0.8112781244591328$ bits
$H(S_{CK7}, P63 = +) = 0$ bits
$H(S_{CK7}, P63 = -) = -(2/3)log(2/3) - (1/3)log(1/3) = 0.9182958340544896$ bits
$Gain(S_{CK7}, P63) = 0.8112781244591328 - ((1/4)*0) - ((3/4)*0.9182958340544896) = 0.12255624891826566$ bits



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QUESTIONS 6 & 7 ....

These Questions are implemented in jupyter notebook environment and their needed explanations and reports are also with codes.

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