

Binomial Probabilities and Hypothesis testing

1. Control and Treatment Groups

The control group is the one that received the placebo.

The treatment group is the one that received the vaccine.

2. Hypothesis

Null Hypothesis (H0): The Pfizer-BioNTech COVID-19 vaccination has no effect on the risk of COVID-19 infection among adolescents.

$$H_0: P(\text{Infection}|\text{Vaccine}) = P(\text{Infection}|\text{Placebo})$$

$$H_a: P(\text{Infection}|\text{Vaccine}) < P(\text{Infection}|\text{Placebo})$$

Alternative Hypothesis (H_A): The Pfizer-BioNTech COVID-19 vaccination lowers the risk of COVID-19 infection among adolescents.

3. Hypothesis Testing

Alpha Level: At 0.05 is chosen for the alpha level, indicating a 5% risk of concluding that a difference exists when there is no actual difference.

Test Selection: Used a left-tailed exact binomial test to compare the proportion of positive COVID-19 cases in the vaccinated group versus the placebo group.

- The binomial test showed a statistically significant difference

4. Results

Receiving the Pfizer-BioNTech vaccine was correlated with a statistically significant reduced risk of getting COVID-19.

Mathematical interpretation: The extremely low p-value indicates a significant decrease in COVID-19 instances among the vaccinated group compared to the placebo group.

- $p\text{-value} \leq 0.05$ so the null hypothesis is rejected

Written Interpretation: This finding significantly supports the effectiveness of the Pfizer-BioNTech COVID-19 vaccination in preventing COVID-19 infection in adolescents.

5. Causation vs. Correlation

The randomized controlled trial (RCT) design of this study allows for the inference of causation, rather than just correlation, about the vaccine's efficacy. Random assignment to treatment or placebo groups reduces confounding variables, allowing variations in infection rates to be attributed to the vaccination itself.

6. Risk Analysis

The risk analysis compares the frequency and severity of adverse events following immunization to the hazards associated with COVID-19 infection, such as hospitalization and ICU admission rates. Given the low frequency of major adverse events associated with the vaccination, as well as the benefits of herd immunity and disease prevention, the Pfizer-BioNTech vaccine for teenagers outweighs the dangers of COVID-19.