Homework Problems due May 28, 2021 08:50 +03

Unit 4: Continuous random variables

Adapted from Blitzstein-Hwang Chapter 5.

Problem 1

1/1 point (graded)

A circle with a random radius $R \sim \text{Unif}(0, 1)$ is generated. Let A be its area. Find the PDF of A.

- ( ) $\frac{1}{\pi}$, for $0 < a < \pi$ (and 0 otherwise)
- ( ) $\sqrt{\pi/a}$, for $0 < a < \pi$ (and 0 otherwise)
- ( ) $\sqrt{a/\pi}$, for $0 < a < \pi$ (and 0 otherwise)
- (•) $\frac{1}{2\sqrt{\pi a}}$, for $0 < a < \pi$ (and 0 otherwise)

✔

Solution:

The CDF of A is

$$P(A \leq a) = P(\pi R^2 \leq a) = P(R \leq \sqrt{a/\pi}) = \sqrt{a/\pi},$$

for $0 < a < \pi$ (and the CDF is 0 for $a \leq 0$ and 1 for $a \geq \pi$). So the PDF of A is

$$f(a) = \frac{1}{2\sqrt{\pi a}},$$

for $0 < a < \pi$ (and 0 otherwise).

Submit

You have used 1 of 2 attempts

Answers are displayed within the problem

Problem 2

1/1 point (graded)

Let $U \sim \text{Unif}(0, 1)$. Suppose that we want to construct X as a function of U, such that $X \sim \text{Expo}(\lambda)$. Which of the following ways to define X gives the desired distribution?

- (selected) $$-\frac{1}{\lambda}\log(1-U)$$
- $$1 - e^{-\lambda U}$$
- $$\frac{\lambda}{1-U}$$
- $$\frac{\lambda}{1-e^{-U}}$$

✓

Solution:

The desired CDF is $F(x) = 1 - e^{-\lambda x}$ for $x > 0$. The inverse function is $F^{-1}(u) = -\frac{1}{\lambda}\log(1-u)$, for $0 < u < 1$. So by universality of the Uniform,

$$X = -\frac{1}{\lambda}\log(1-U) \sim \text{Expo}(\lambda).$$

Submit

You have used 1 of 2

attempts

Answers are displayed within the problem

Problem 3

1/1 point (graded)

The distance between two points needs to be measured, in meters. The true distance between the points is 10 meters, but due to measurement error we can't measure the distance exactly. Instead, we will observe a value of $10 + \epsilon$, where the error ϵ is distributed $\mathcal{N}(0, 0.04)$. Find the probability that the observed distance is within 0.4 meters of the true distance (10 meters). Give an approximate numerical answer.

0.95

Answer: 0.95

0.95

Solution:

Standardizing ϵ (which has mean 0 and standard deviation 0.2), the desired probability is

$$\begin{aligned} P(|\epsilon| \leq 0.4) &= P(-0.4 \leq \epsilon \leq 0.4) \\ &= P(-\frac{0.4}{0.2} \leq \frac{\epsilon}{0.2} \leq \frac{0.4}{0.2}) \\ &= P(-2 \leq \frac{\epsilon}{0.2} \leq 2) \\ &= \Phi(2) - \Phi(-2) \\ &= 2\Phi(2) - 1. \end{aligned}$$

By the 68-95-99.7% rule, this is approximately 0.95.

Submit

You have used 1 of 5 attempts

Answers are displayed within the problem

Problem 4

1/1 point (graded)

Emails arrive in an inbox according to a Poisson process with rate 20 emails per hour. Let T be the time at which the third email arrives, measured in hours after a certain fixed starting time. Find $P(T > 0.1)$ without using calculus.

0.6767

✔ **Answer:** 0.6767

0.6767

Solution:

By the count-time duality, $P(T > 0.1) = P(N \leq 2)$, where N is the number of emails that arrive in the first 0.1 hours. We have $N \sim \text{Pois}(2)$, so

$$P(T > 0.1) = P(N \leq 2) = e^{-2} + e^{-2} \cdot 2 + e^{-2} \cdot 2^2/2! \approx 0.6767.$$

Submit

You have used 1 of 5 attempts

Answers are displayed within the problem