Practice Problems due May 10, 2021 20:50 +03

Unit 2: Conditioning

Adapted from Blitzstein-Hwang Chapter 2.

Problem 1

1/1 point (graded)

A spam filter is designed by looking at commonly occurring phrases in spam. Suppose that 80% of email is spam. In 10% of the spam emails, the phrase "free money" is used, whereas this phrase is only used in 1% of non-spam emails. A new email has just arrived, which does mention "free money". What is the probability that it is spam?

0.9756

✔ **Answer:** 0.9756

0.9756

Solution

Let S be the event that an email is spam and F be the event that an email has the "free money" phrase. By Bayes' rule,

$$P(S|F) = \frac{P(F|S)\,P(S)}{P(F)} = \frac{0.1 \cdot 0.8}{0.1 \cdot 0.8 + 0.01 \cdot 0.2} = \frac{80/1000}{82/1000} = \frac{80}{82} \approx 0.9756.$$

Submit

You have used 1 of 99 attempts

Answers are displayed within the problem

Problem 2

1/1 point (graded)

The screens used for a certain type of cell phone are manufactured by 3 companies, A, B, and C. The proportions of screens supplied by A, B, and C are 0.5, 0.3, and 0.2, respectively, and their screens are defective with probabilities 0.01, 0.02, and 0.03, respectively.

Given that the screen on such a phone is defective, what is the probability that Company A manufactured it?

0.294

0.294

Submit You have used 1 of 99 attempts

Correct (1/1 point)

For Problem 3

A family has 3 children, creatively named A, B, and C.

Problem 3a

1/1 point (graded)

(a) Discuss intuitively whether the event "A is older than B" is independent of the event "A is older than C".

```
They are not independent: understanding that A is older than
B makes it more likely that A is older than C as if A is
older than B, then the only way that A can be younger than C
is if the birth order of CAB, whereas the birthday orders ABC
and ACB are both compatible with A being older than B.
```

Thank you for your response.

Solution

They are not independent: knowing that A is older than B makes it more likely that A is older than C, as if A is older than B, then the only way that A can be younger than C is if the birth order is CAB, whereas the birth orders ABC and ACB are both compatible with A being older than B. To make this more intuitive, think of an extreme case where there are 100 children instead of 3, call them $A_1, \ldots, A_{100}$. Given that A_1 is older than all of $A_2, A_3, \ldots, A_{99}$, it's clear that A_1 is very old (relatively), whereas there isn't evidence about where A_{100} fits into the birth order.

Submit You have used 1 of 99 attempts

Answers are displayed within the problem

Problem 3b

1/1 point (graded)

(b) Find the probability that A is older than B, given that A is older than C.

2/3

Answer: 2/3

$\frac{2}{3}$

Solution

Writing $x > y$ to mean that x is older than y,

$$P(A > B | A > C) = \frac{P(A > B, A > C)}{P(A > C)} = \frac{1/3}{1/2} = \frac{2}{3}$$

since $P(A > B, A > C) = P(A \text{ is the eldest child}) = 1/3$ (unconditionally, any of the 3 children is equally likely to be the eldest).

Submit

You have used 1 of 99 attempts

Answers are displayed within the problem

For Problem 4

Consider the Monty Hall problem, except that Monty enjoys opening door 2 more than he enjoys opening door 3, and if he has a choice between opening these two doors, he opens door 2 with probability $p = \frac{3}{4}$.

To recap: there are three doors, behind one of which there is a car (which you want), and behind the other two of which there are goats (which you don't want). Initially, all possibilities are equally likely for where the car is. You choose a door, which for concreteness we assume is door 1. Monty Hall then opens a door to reveal a goat, and offers you the option of switching. Assume that Monty Hall knows which door has the car, will always open a goat door and offer the option of switching, and as above assume that if Monty Hall has a choice between opening door 2 and door 3, he chooses door 2 with probability $p = \frac{3}{4}$.

Problem 4a

1/1 point (graded)

(a) Find the unconditional probability that the strategy of always switching succeeds (unconditional in the sense that we do not condition on which of doors 2 or 3 Monty opens).

2/3

✔ **Answer:** 2/3

$\frac{2}{3}$

Solution

Let C_j be the event that the car is hidden behind door j and let W be the event that we win using the switching strategy. Using the law of total probability, we can find the unconditional probability of winning:

$$\begin{aligned} P(W) &= P(W|C_1)\,P(C_1) + P(W|C_2)\,P(C_2) + P(W|C_3)\,P(C_3) \\ &= 0 \cdot 1/3 + 1 \cdot 1/3 + 1 \cdot 1/3 \\ &= 2/3 \end{aligned}$$

Submit

You have used 1 of 99 attempts

Answers are displayed within the problem

Problem 4b

1/1 point (graded)

(b) Find the probability that the strategy of always switching succeeds, given that Monty opens door 2.

4/7

✔

$\frac{4}{7}$

Submit

You have used 1 of 99 attempts

✔ Correct (1/1 point)

Problem 4c

1/1 point (graded)

(c) Find the probability that the strategy of always switching succeeds, given that Monty opens door 3.

4/5

$\frac{4}{5}$

Submit You have used 1 of 99 attempts

Correct (1/1 point)