The SAT is a standardized college admissions test used in the United States. The following two multi-part questions will ask you some questions about SAT testing.

This is a 6-part question asking you to determine some probabilities of what happens when a student guessed for all of their answers on the SAT. Use the information below to inform your answers for the following questions.

An old version of the SAT college entrance exam had a -0.25 point penalty for every incorrect answer and awarded 1 point for a correct answer. The quantitative test consisted of 44 multiple-choice questions each with 5 answer choices. Suppose a student chooses answers by guessing for all questions on the test.

Question 1a

1/1 point (graded)
What is the probability of guessing correctly for one question?

| 0.2 | ✔ **Answer:** 0.2 |

0.2

Explanation
The following code can be used to calculate the probability:

```
p <- 1/5 # one correct choice of 5 options
p
```

| Submit | You have used 1 of 10 attempts |

ⓘ   Answers are displayed within the problem

Question 1b

1/1 point (graded)

What is the expected value of points for guessing on one question?

0

✔ **Answer:** 0

0

Explanation

The following code can be used to calculate the expected value:

```
        a <- 1
b <- -0.25
mu <- a*p + b*(1-p)
mu
```

Submit   You have used 1 of 10 attempts

ⓘ   Answers are displayed within the problem

Question 1c

1/1 point (graded)

What is the expected score of guessing on all 44 questions?

0

✔ **Answer:** 0

0

Explanation

The following code can be used to calculate the expected score:

```
        n <- 44
n*mu
```

ⓘ Answers are displayed within the problem

Question 1d

1/1 point (graded)
What is the standard error of guessing on all 44 questions?

3.32

✔ **Answer:** 3.32

| 3.32 |

Explanation

The following code can be used to calculate the standard error:

```
        sigma <- sqrt(n) * abs(b-a) * sqrt(p*(1-p))
 sigma
```

ⓘ Answers are displayed within the problem

Question 1e

1/1 point (graded)
Use the Central Limit Theorem to determine the probability that a guessing student scores 8 points or higher on the test.

0.00793

✔ **Answer:** 0.00793

| 0.00793 |

Explanation

The following code can be used to calculate the probability:

```
        1-pnorm(8, mu, sigma)
```

Submit   You have used 1 of 10
          attempts

ℹ Answers are displayed within the problem

Question 1f

1/1 point (graded)
Set the seed to 21, then run a Monte Carlo simulation of 10,000 students
guessing on the test.

(IMPORTANT! If you use R 3.6 or later, you will need to use the command
`set.seed(x, sample.kind = "Rounding")` instead of `set.seed(x)`. Your R
version will be printed at the top of the Console window when you start
RStudio.)

What is the probability that a guessing student scores 8 points or higher?

```
0.008
```
✔ **Answer:** 0.008

```
0.008
```

Explanation
The following code can be used to calculate the probability:

```
        set.seed(21, sample.kind = "Rounding")
B <- 10000
n <- 44
p <- 0.2
tests <- replicate(B, {
  X <- sample(c(1, -0.25), n, replace = TRUE, prob = c(p, 1-p))
  sum(X)
})
mean(tests >= 8)
```

ⓘ   Answers are displayed within the problem

The SAT was recently changed to reduce the number of multiple choice options from 5 to 4 and also to eliminate the penalty for guessing.

In this two-part question, you'll explore how that affected the expected values for the test.

Question 2a

1/1 point (graded)
Suppose that the number of multiple choice options is 4 and that there is no penalty for guessing - that is, an incorrect question gives a score of 0.

What is the expected value of the score when guessing on this new test?

| 11 | ✔ **Answer:** 11

11

Explanation
The following code can be used to calculate the expected value:

```
        p <- 1/4
a <- 1
b <- 0
n <- 44
mu <- n * a*p + b*(1-p)
mu
```

Question 2b

1/1 point (graded)

Consider a range of correct answer probabilities
`p <- seq(0.25, 0.95, 0.05)` representing a range of student skills.

What is the lowest `p` such that the probability of scoring over 35 exceeds 80%?

| 0.85 |
✔ **Answer:** 0.85

0.85

Explanation

The following code can be used to calculate the value for `p` :

```
        p <- seq(0.25, 0.95, 0.05)
exp_val <- sapply(p, function(x){
  mu <- n * a*x + b*(1-x)
  sigma <- sqrt(n) * abs(b-a) * sqrt(x*(1-x))
  1-pnorm(35, mu, sigma)
})

min(p[which(exp_val > 0.8)])
```

| Submit | You have used 1 of 10 attempts |