TP_0_Analyse_données

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Exercice 1

```
# order c'est pour les indices
# Creations de trois vecteurs
x \leftarrow c(1, 3, 5, 7, 9)
y \leftarrow c(2, 3, 5, 7, 11, 13)
z \leftarrow c(9, 3, 2, 5, 9, 2, 3, 9, 1)
x + 2
## [1] 3 5 7 9 11
y * 3
## [1] 6 9 15 21 33 39
length(x)
## [1] 5
x + y # erreur
## Warning in x + y: la taille d'un objet plus long n'est pas multiple de la taille
## d'un objet plus court
## [1] 3 6 10 14 20 14
sum(x > 5)
## [1] 2
sum(x[x > 5])
## [1] 16
sum(x > 5 \mid x < 3)
## [1] 3
y[3]
## [1] 5
y[<del>-3</del>]
## [1] 2 3 7 11 13
y[x]; (y > 7); y[y > 7]; sort(z); sort(z, dec = TRUE); rev(z);
## [1] 2 5 11 NA NA
## [1] FALSE FALSE FALSE TRUE TRUE
```

```
## [1] 11 13
## [1] 1 2 2 3 3 5 9 9 9
## [1] 9 9 9 5 3 3 2 2 1
## [1] 1 9 3 2 9 5 2 3 9
order(z); unique(z); duplicated(z); table(z); rep(z, 3)
## [1] 9 3 6 2 7 4 1 5 8
## [1] 9 3 2 5 1
## [1] FALSE FALSE FALSE TRUE TRUE TRUE TRUE FALSE
## 1 2 3 5 9
## 1 2 2 1 3
## [1] 9 3 2 5 9 2 3 9 1 9 3 2 5 9 2 3 9 1 9 3 2 5 9 2 3 9 1
Exercice 2
diag1 <- diag(1,nrow=9,ncol=9) # creation d'une matrice diagonale</pre>
x1 \leftarrow rep(x=1,times=9)
matrix1 <- matrix(x1,nrow=9,ncol=9)</pre>
matdiag0 <- matrix1 - diag1</pre>
Exercice 3
# Creation de deux vecteurs
vect1 <- c(1:10)
vect2 <- c(11:20)
vectconcat <- c(1,vect1[2],vect2,vect1[c(3:10)])</pre>
Exercice 4
x \leftarrow c (4.12, 1.84, 4.28, 4.23, 1.74, 2.06, 3.37, 3.83, 5.15, 3.76, 3.23, 4.87,
5.96, 2.29, 4.58)
x_extract1 <- x[(4:15)] # extraction</pre>
x_{extract2} \leftarrow x[(2:14)]
x_{extract3} < x[x > 2.57 & x < 3.48]
x_{extract4} \leftarrow x[x > 4.07 | | x < 1.48]
indice_min <- which.min(x)</pre>
row1A \leftarrow c(-2,1,-3,-2)
row2B \leftarrow c(1,2,1,-1)
row3C \leftarrow c(-2,1,1,-1)
row4D \leftarrow c(-1, -3, 1, 1)
A <- matrix(c(row1A,row2B,row3C,row4D),nrow=4,ncol=4,byrow = TRUE)
rowA <- c(2,-1,3,-4)
rowB <-c(2,-2,4,-5)
rowC <- c(-2,1,3,-1)
rowD <- c(-1,-3,1,-1)
B <- matrix(c(rowA,rowB,rowC,rowD),ncol = 4,nrow = 4,byrow = T)</pre>
# Montrons que A et B sont inversible
det(A)
## [1] 25
```

```
det(B)
## [1] 4
\# \det(A) et \det(B) ne sont pas nuls donc A et B sont inversible
# l'inverse des matrices
solve(A)
        [,1] [,2] [,3] [,4]
## [1,] 0.08 0.48 -0.44 0.2
## [2,] -0.24 -0.44 0.32 -0.6
## [3,] -0.12 0.28 0.16 0.2
## [4,] -0.52 -1.12 0.36 -0.8
solve(B)
        [,1] [,2] [,3] [,4]
## [1,] -7.50 6.50 -0.50 -2.0
## [2,] 3.25 -2.75 0.25 0.5
## [3,] -10.25 8.75 -0.25 -2.5
## [4,] -12.50 10.50 -0.50 -3.0
#2.
det(t(A))
## [1] 25
det(solve(A))
## [1] 0.04
1 / det(A)
## [1] 0.04
det(A %*% B)
## [1] 100
det(A) * det(B)
## [1] 100
#3.
t(solve(A))
       [,1] [,2] [,3] [,4]
## [1,] 0.08 -0.24 -0.12 -0.52
## [2,] 0.48 -0.44 0.28 -1.12
## [3,] -0.44 0.32 0.16 0.36
## [4,] 0.20 -0.60 0.20 -0.80
solve(t(A))
       [,1] [,2] [,3] [,4]
## [1,] 0.08 -0.24 -0.12 -0.52
## [2,] 0.48 -0.44 0.28 -1.12
## [3,] -0.44 0.32 0.16 0.36
## [4,] 0.20 -0.60 0.20 -0.80
t(A %*% B)
```

```
## [,1] [,2] [,3] [,4]
## [1,] 6 5 -3 -11
## [2,]
       3 -1
                 4 5
## [3,] -13 13
                 0 -11
## [4,]
       8 -14
t(B) %*% t(A)
      [,1] [,2] [,3] [,4]
## [1,] 6 5 -3 -11
       3 -1
## [2,]
                  4 5
## [3,] -13 13
                 0 -11
                 3 17
## [4,]
       8 -14
solve(A %*% B)
       [,1] [,2] [,3] [,4]
## [1,] -1.06 -4.36 4.58 -3.90
## [2,] 0.63 2.28 -2.09 1.95
## [3,] -1.59 -6.04 6.37 -5.35
## [4,] -1.90 -7.40 7.70 -6.50
solve(B) %*% solve(A)
       [,1] [,2] [,3] [,4]
## [1,] -1.06 -4.36 4.58 -3.90
## [2,] 0.63 2.28 -2.09 1.95
## [3,] -1.59 -6.04 6.37 -5.35
## [4,] -1.90 -7.40 7.70 -6.50
Exercice 6
r1 \leftarrow c(1,1,3)
r2 < c(5,2,6)
r3 < c(-2,-1,-3)
A \leftarrow matrix(c(r1,r2,r3),nrow = 3,ncol = 3,byrow = TRUE)
# 1.
A %*% A %*% A
## [,1] [,2] [,3]
## [1,] 0 0 0
## [2,]
        0
              0
## [3,]
       0
            0
                 0
# Donc A est nilpotent avev n = 3
#2.
A[3,] \leftarrow A[1,] + A[2,]
     [,1] [,2] [,3]
## [1,] 1 1 3
## [2,]
       5 2
                   6
## [3,]
         6
            3
                   9
Exercice 7
# 1. Creation de la matrice X
X \leftarrow \text{matrix}(c(0.5^2, 0.5, 1, 1^2, 1, 1, 1.5^2, 1.5, 1), \text{ncol} = 3, \text{byrow} = \text{TRUE})
det(X)
```

```
## [1] -0.25
# donc X est inversible
# inverse de X
invX <- solve(X)</pre>
invX
## [,1] [,2] [,3]
## [1,] 2 -4 2
             8
## [2,]
        -5
                  -3
## [3,]
       3 -3 1
# determinons a,b et c
r < c(1,4,5)
sol <- invX %*% r
rownames(sol) <- c("a", "b", "c")
sol
##
    [,1]
## a -4
## b 12
## c
# 4 determination des valeurs propres
eigen(X)$values # valeurs propres
## [1] 3.10873548 -0.94393086 0.08519538
eigen(X)$vectors # vecteurs propres
             [,1]
                        [,2]
                                   [,3]
## [1,] -0.3608737 -0.61917748 0.4165621
## [2,] -0.5337897 -0.08316023 -0.8387339
## [3,] -0.7647475  0.78083521  0.3507156
Exercice 8
A \leftarrow matrix(0, nrow = 5, ncol = 5)
B \leftarrow abs(col(A) - row(A)) + 1
## [,1] [,2] [,3] [,4] [,5]
## [1,]
       1 2 3 4 5
         2
## [2,]
            1
                 2
                        3
       3 2 1 2 3
## [3,]
## [4,]
       4 3 2 1 2
          5 4 3 2
## [5,]
det(B)
## [1] 48
# Donc B est inversible
# inverse de B
solve(B)
                             [,2]
                [,1]
                                          [,3]
                                                       [, 4]
## [1,] -4.166667e-01 5.000000e-01 -8.881784e-17 3.256654e-16 8.333333e-02
## [2,] 5.000000e-01 -1.000000e+00 5.000000e-01 -7.401487e-16 1.850372e-16
## [3,] -1.110223e-16 5.000000e-01 -1.000000e+00 5.000000e-01 0.000000e+00
## [4,] 2.775558e-17 -1.110223e-16 5.000000e-01 -1.000000e+00 5.000000e-01
```

```
## [5,] 8.333333e-02 2.465190e-32 -1.110223e-16 5.000000e-01 -4.166667e-01
# Resolution du systeme
r \leftarrow c(1,rep(2,2),3,2)
sol <- solve(B) %*% r</pre>
rownames(sol) <- c("a", "b", "c", "d", "e")
sol
##
      [,1]
## a 0.75
## b -0.50
## c 0.50
## d -1.00
## e 0.75
Exercice 9
x < -1:6
y < -5:10
xplusy \leftarrow x + y
xplusy[xplusy > 11]
## [1] 12 14 16
# produit scalaire de x et y
prodxy <- x * y
M \leftarrow matrix(1:36, nrow = 6)
M %*% x # Calcul de M*x
##
        [,1]
## [1,] 441
## [2,] 462
## [3,] 483
## [4,] 504
## [5,] 525
## [6,] 546
x %*% M # Calcul de x*M
       [,1] [,2] [,3] [,4] [,5] [,6]
## [1,] 91 217 343 469 595 721
M %*% t(M) # Calcul de M*t(M)
        [,1] [,2] [,3] [,4] [,5] [,6]
## [1,] 2166 2262 2358 2454 2550 2646
## [2,] 2262 2364 2466 2568 2670 2772
## [3,] 2358 2466 2574 2682 2790 2898
## [4,] 2454 2568 2682 2796 2910 3024
## [5,] 2550 2670 2790 2910 3030 3150
## [6,] 2646 2772 2898 3024 3150 3276
Exercice 10
# Creation d'une de 16 lignes et trois colonnes
r \leftarrow rep(seq(3,1,by = -1),16)
A <- matrix(r,nrow = 16,ncol = 3,byrow = TRUE)
dim(A)
## [1] 16 3
```

```
head(A,5)
        [,1] [,2] [,3]
## [1,]
         3
                2
## [2,]
           3
                 2
                      1
## [3,]
                 2
           3
                      1
## [4,]
         3
                 2
                      1
## [5,]
            3
                 2
                      1
Exercice 11
Ligne1 \leftarrow c(95,68,85,72,55,86,115)
Ligne2 \leftarrow c(189,169,179,167,171,178,179)
mat <- matrix(c(Ligne1,Ligne2),nrow = 2,ncol=7,byrow = TRUE)</pre>
rownames(mat) <- c("Poids", "Taille")</pre>
colnames(mat) <- c("John","Lilly","Stef","Bob","Anna", "Marik","Boris")</pre>
mat
##
           John Lilly Stef Bob Anna Marik Boris
## Poids
             95
                   68
                       85 72
                                  55
                                         86
                                              115
## Taille 189
                  169 179 167 171
                                        178
                                              179
Exercice 12
r1A \leftarrow c(3,1,sqrt(6))
r2A \leftarrow c(1,3,-sqrt(6))
r3A \leftarrow c(-sqrt(6), sqrt(6), 2)
A <- matrix(c(r1A,r2A,r3A),nrow = 3,ncol = 3,byrow = TRUE)
A < -1 / 4 * A
r1B \leftarrow c(-2,-1,2)
r2B \leftarrow c(2,-2,1)
r3B \leftarrow c(1,2,2)
B \leftarrow matrix(c(r1B,r2B,r3B),nrow = 3,ncol = 3,byrow = TRUE)
B <- 1 / 3 * B
A %*% t(A)
                 [,1]
                               [,2] [,3]
## [1,] 1.000000e+00 5.551115e-17
## [2,] 5.551115e-17 1.000000e+00
## [3,] 0.000000e+00 0.000000e+00
# donc A est orthogonal
#2.verifions que inv(A)=t(A)
solve(A)
                          [,2]
                                      [,3]
##
              [,1]
## [1,] 0.7500000 0.2500000 -0.6123724
## [2,] 0.2500000 0.7500000 0.6123724
## [3,] 0.6123724 -0.6123724 0.5000000
t(A)
##
              [,1]
                          [,2]
                                      [,3]
## [1,] 0.7500000 0.2500000 -0.6123724
## [2,] 0.2500000 0.7500000 0.6123724
## [3,] 0.6123724 -0.6123724 0.5000000
# 3. Montrons que B est orthogonal
B %*% t(B)
```

```
## [,1] [,2] [,3]
        1 0
## [1,]
## [2,]
           0
## [3,]
           0
                0
                     1
# 4. Est ce que que A et B commutent
ab <- A %*% B
ba <- B %*% A
ab - ba
                        [,2]
##
             [,1]
                                    [,3]
## [1,] 0.8623724 0.0000000 0.8623724
## [2,] 0.0000000 -0.8623724 -0.9747449
## [3,] 0.9747449 -0.8623724 0.0000000
# Donc A et B ne commute pas
# 5. Le determinant de A
det(A)
## [1] 1
# 6. Valeurs et vecteurs propres de A
vpA <- eigen(A)$values</pre>
VpA
## [1] 0.5+0.8660254i 0.5-0.8660254i 1.0+0.0000000i
vectpA <- eigen(A)$vectors</pre>
vectpA
##
                   [,1]
                                   [,2]
## [1,] 0.0000000+0.5i 0.0000000-0.5i 7.071068e-01+0i
## [2.] 0.0000000-0.5i 0.0000000+0.5i 7.071068e-01+0i
## [3,] -0.7071068+0.0i -0.7071068+0.0i -3.716104e-18+0i
vpB <- eigen(B)$values</pre>
vpB
## [1] 1.0000000+0.0000000i -0.8333333+0.5527708i -0.8333333-0.5527708i
vectpB <- eigen(B)$vectors</pre>
vectpB
                [,1]
                                       [,2]
                                                             [,3]
## [1,] 0.3015113+0i 0.6741999+0.0000000i 0.6741999+0.0000000i
## [2,] 0.3015113+0i -0.0674200-0.6708204i -0.0674200+0.6708204i
## [3,] 0.9045340+0i -0.2022600+0.2236068i -0.2022600-0.2236068i
# 7. Creation d'une nouvelle matrice
C <- A
C[3,] \leftarrow C[1,] + C[2,]
        [,1] [,2]
## [1,] 0.75 0.25 0.6123724
## [2,] 0.25 0.75 -0.6123724
## [3,] 1.00 1.00 0.0000000
det(C)
## [1] 0
```

Donc C n'est pas inversible