



Quantum computing with trapped ions - real thought experiments



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- Physics and information
- Ion trap quantum computing
- Quantum teleportation
- Scaling up ion trap quantum computers



FWF
SFB



SCALA
QGATES



Industrie
Tirol



IQI
GmbH



bm:bwk



Reno, Feb 5th 2010



Physics and information



Information is physical (Rolf Landauer, 1961)



Erasing information generates heat:

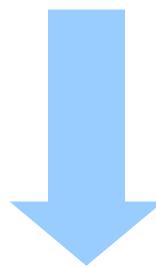
0101001101 → 0000000000 + entropy



Physics and information



Physical process





Physics and information



Information is physical (Rolf Landauer, 1961)

→ Information is quantum mechanical

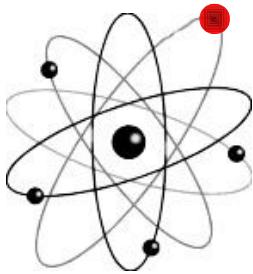
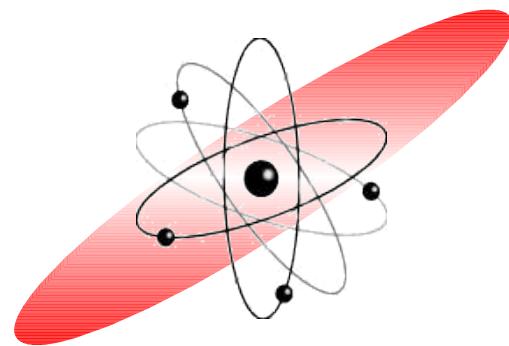
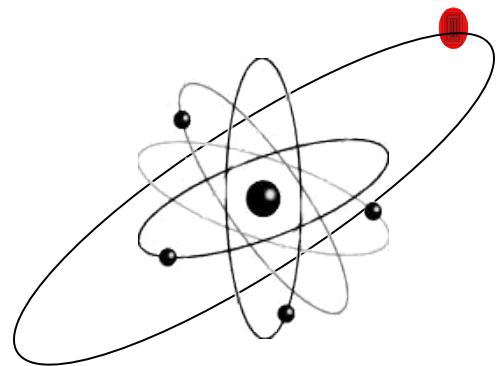
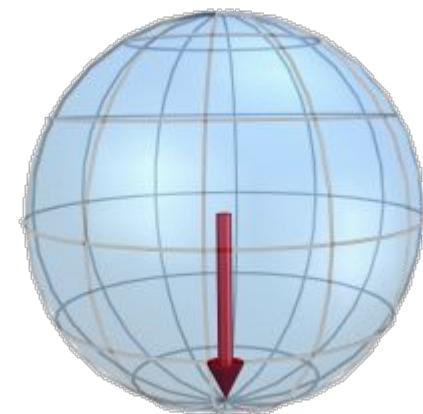
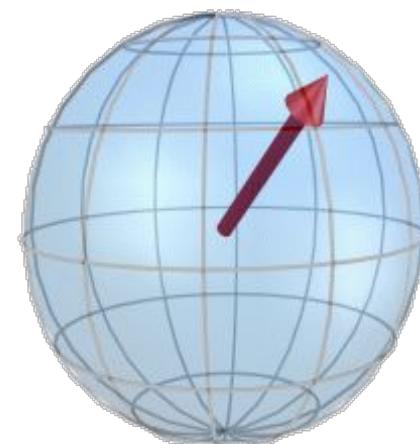
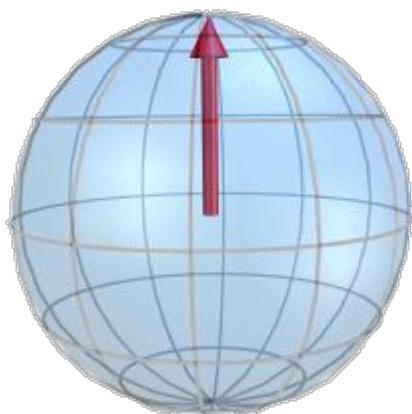
Classical information is a subset in quantum information

Quantum information: strip down quantum mechanics to bare bones.

- Hilbert space
- unitary operations
- measurement



Qubits

 $|0\rangle$  $\alpha|0\rangle + \beta|1\rangle$  $|1\rangle$ 



Information content



$$|\Psi\rangle_{\text{reg}} = \alpha_0 |000\rangle + \alpha_1 |001\rangle + \alpha_2 |010\rangle + \alpha_3 |011\rangle + \alpha_4 |100\rangle + \alpha_5 |101\rangle + \alpha_6 |110\rangle + \alpha_7 |111\rangle$$

| # bits | classical | quantum mechanical |
|--------|-----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1 | 1 | $0.5208 + 0.7059i, 0.3014 + 0.3736i$ |
| 2 | 01 | $0.2044 + 0.4911i, 0.1732 + 0.3855i, 0.2040 + 0.4890i, 0.3193 + 0.3947i$ |
| 3 | 001 | $0.2583 + 0.2704i, 0.2310 + 0.1150i, 0.2956 + 0.3118i, 0.3558 + 0.2113i, 0.1943 + 0.1377i, 0.3273 + 0.2613i, 0.0643 + 0.2033i, 0.3643 + 0.1654i$ |
| 4 | 1010 | $0.1691 + 0.0891i, 0.1096 + 0.0828i, 0.1420 + 0.2873i, 0.0741 + 0.2419i, 0.1902 + 0.0448i, 0.2495 + 0.0039i, 0.1738 + 0.2933i, 0.2102 + 0.0653i, 0.0686 + 0.0980i, 0.1246 + 0.2170i$ $0.2570 + 0.0933i, 0.2234 + 0.1540i, 0.1513 + 0.0213i, 0.1863 + 0.3243i, 0.2606 + 0.1912i, 0.0194 + 0.1390i$ |
| 5 | 10001 | $0.1060 + 0.1416i, 0.0103 + 0.0118i, 0.0064 + 0.0976i, 0.0734 + 0.0716i, 0.0030 + 0.2054i, 0.0902 + 0.0035i, 0.1605 + 0.1804i, 0.0218 + 0.2280i, 0.0083 + 0.2326i, 0.1438 + 0.1853i, 0.1429 + 0.1030i, 0.0037 + 0.1171i, 0.0038 + 0.0503i$ $0.0446 + 0.1512i, 0.1379 + 0.0752i, 0.0135 + 0.2255i, 0.0863 + 0.1707i, 0.1483 + 0.0968i, 0.1686 + 0.1749i, 0.1627 + 0.0629i, 0.0197 + 0.1033i, 0.1067 + 0.2192i, 0.1038 + 0.1605i, 0.0830 + 0.0499i, 0.0361 + 0.1971i, 0.1587 + 0.1477i$ $0.1642 + 0.0314i, 0.1709 + 0.0487i, 0.1124 + 0.1426i, 0.1303 + 0.1480i, 0.0284 + 0.0870i, 0.1059 + 0.1351i$ |
| 6 | 110101 | $0.0595 + 0.1064i, 0.0295 + 0.1327i, 0.0929 + 0.0406i, 0.1090 + 0.0379i, 0.0559 + 0.1286i, 0.0015 + 0.0345i, 0.0624 + 0.1196i, 0.1120 + 0.1350i, 0.1180 + 0.0345i, 0.1367 + 0.0356i, 0.1255 + 0.0074i, 0.0547 + 0.0116i, 0.0923 + 0.0952i$ $0.1087 + 0.0284i, 0.0288 + 0.1254i, 0.1345 + 0.0258i, 0.0846 + 0.0254i, 0.0939 + 0.1478i, 0.0348 + 0.0654i, 0.0816 + 0.0505i, 0.1384 + 0.0467i, 0.0948 + 0.0543i, 0.0974 + 0.0584i, 0.0582 + 0.0879i, 0.0932 + 0.0178i, 0.0109 + 0.0565i$ $0.0590 + 0.0682i, 0.0615 + 0.1293i, 0.0974 + 0.1388i, 0.1245 + 0.0393i, 0.0552 + 0.0238i, 0.0632 + 0.1297i, 0.0884 + 0.0354i, 0.0841 + 0.0960i, 0.1065 + 0.1437i, 0.0760 + 0.0988i, 0.1154 + 0.1293i, 0.0727 + 0.0204i$ $0.1041 + 0.1217i, 0.1460 + 0.0639i, 0.1199 + 0.1323i, 0.1046 + 0.1092i, 0.0721 + 0.1021i, 0.0170 + 0.0514i, 0.0988 + 0.0247i, 0.0543 + 0.0231i, 0.0208 + 0.0284i, 0.0842 + 0.0628i, 0.1223 + 0.1272i$ $0.1002 + 0.0729i, 0.1485 + 0.1213i, 0.1429 + 0.0685i, 0.0087 + 0.0680i, 0.0535 + 0.0670i, 0.0815 + 0.0613i, 0.0389 + 0.1340i, 0.0888 + 0.0008i, 0.0073 + 0.0442i, 0.0849 + 0.0073i, 0.1042 + 0.1030i, 0.1430 + 0.0966i, 0.1115 + 0.1461i$ $0.1100 + 0.0821i$ |
| 7 | 1001010 | $0.0880 + 0.0466i, 0.1054 + 0.0684i, 0.0239 + 0.0866i, 0.0759 + 0.0090i, 0.0563 + 0.1020i, 0.1006 + 0.0988i, 0.0769 + 0.0649i, 0.0246 + 0.0273i, 0.0485 + 0.0942i, 0.0186 + 0.0554i, 0.1045 + 0.0790i, 0.0384 + 0.0455i, 0.0053 + 0.1037i$ $0.0815 + 0.0078i, 0.0965 + 0.0597i, 0.0309 + 0.0315i, 0.0271 + 0.0925i, 0.1006 + 0.0362i, 0.0141 + 0.0734i, 0.1015 + 0.0058i, 0.0757 + 0.0385i, 0.0914 + 0.0537i, 0.0226 + 0.0468i, 0.0491 + 0.0607i, 0.0087 + 0.0665i, 0.0918 + 0.0122i$ $0.0606 + 0.0969i, 0.0344 + 0.0814i, 0.0404 + 0.0853i, 0.0936 + 0.0879i, 0.0401 + 0.0723i, 0.0079 + 0.0217i, 0.0216 + 0.0294i, 0.0553 + 0.0675i, 0.0611 + 0.0579i, 0.0131 + 0.0064i, 0.0491 + 0.0096i, 0.0126 + 0.0293i, 0.0830 + 0.0441i$ $0.0404 + 0.0511i, 0.0888 + 0.0980i, 0.0050 + 0.0643i, 0.0645 + 0.0355i, 0.1024 + 0.0516i, 0.0311 + 0.0644i, 0.0959 + 0.0174i, 0.0110 + 0.0894i, 0.0070 + 0.1031i, 0.0253 + 0.0642i, 0.1006 + 0.0031i, 0.0068 + 0.0876i, 0.0285 + 0.0658i$ $0.1078 + 0.0756i, 0.0229 + 0.0599i, 0.0537 + 0.0458i, 0.0313 + 0.0405i, 0.0725 + 0.0179i, 0.1033 + 0.0898i, 0.0827 + 0.0904i, 0.0718 + 0.0487i, 0.0141 + 0.1032i, 0.0103 + 0.0159i, 0.0016 + 0.0938i, 0.0311 + 0.0830i, 0.0881 + 0.0479i$ $0.1063 + 0.0669i, 0.0019 + 0.1026i, 0.0884 + 0.0690i, 0.0670 + 0.0267i, 0.0604 + 0.0380i, 0.0263 + 0.0203i, 0.0886 + 0.0529i, 0.0284 + 0.0441i, 0.0813 + 0.0500i, 0.0711 + 0.0659i, 0.0231 + 0.0077i, 0.0649 + 0.0339i, 0.0652 + 0.0656i$ $0.0711 + 0.0189i, 0.0198 + 0.0670i, 0.0686 + 0.0265i, 0.0184 + 0.0633i, 0.0582 + 0.0546i, 0.0672 + 0.0501i, 0.0740 + 0.0584i, 0.0730 + 0.1016i, 0.0946 + 0.0369i, 0.0014 + 0.0433i, 0.0335 + 0.0332i, 0.0840 + 0.0444i, 0.0331 + 0.0308i$ $0.0999 + 0.0425i, 0.0732 + 0.0542i, 0.0080 + 0.0779i, 0.0276 + 0.0330i, 0.0013 + 0.0121i, 0.0245 + 0.0478i, 0.0557 + 0.0503i, 0.0494 + 0.0016i, 0.0758 + 0.0716i, 0.0628 + 0.0781i, 0.0549 + 0.0304i, 0.0080 + 0.0282i, 0.0208 + 0.0764i$ $0.0409 + 0.0845i, 0.0893 + 0.0452i, 0.0989 + 0.0562i, 0.0122 + 0.0774i, 0.0876 + 0.0614i, 0.0979 + 0.0497i, 0.0169 + 0.0480i, 0.0132 + 0.0095i, 0.0822 + 0.0478i, 0.0778 + 0.0395i, 0.0703 + 0.0326i, 0.0813 + 0.0919i, 0.0715 + 0.0819i$ $0.0953 + 0.1024i, 0.0293 + 0.0602i, 0.0452 + 0.0015i, 0.0230 + 0.0643i$ |
| 8 | 10101011 | $0.0199 + 0.0027i, 0.0033 + 0.0063i, 0.0005 + 0.0656i, 0.0443 + 0.0262i, 0.0573 + 0.0359i, 0.0622 + 0.0704i, 0.0491 + 0.0176i, 0.0194 + 0.0664i, 0.0111 + 0.0506i, 0.0502 + 0.0687i, 0.0729 + 0.0376i, 0.0629 + 0.0765i, 0.0717 + 0.0288i$ $0.0239 + 0.0410i, 0.0207 + 0.0140i, 0.0413 + 0.0387i, 0.0126 + 0.0325i, 0.0163 + 0.0509i, 0.0167 + 0.0519i, 0.0502 + 0.0738i, 0.0041 + 0.0148i, 0.0177 + 0.0086i, 0.0514 + 0.0436i, 0.0240 + 0.0747i, 0.0236 + 0.0018i, 0.0555 + 0.0671i$ $0.0736 + 0.0021i, 0.0101 + 0.0400i, 0.0053 + 0.0148i, 0.0097 + 0.0552i, 0.0128 + 0.0193i, 0.0702 + 0.0720i, 0.0105 + 0.0106i, 0.0475 + 0.0402i, 0.0207 + 0.0690i, 0.0170 + 0.0726i, 0.0549 + 0.0258i, 0.0423 + 0.0337i, 0.0726 + 0.0363i$ $0.0254 + 0.0115i, 0.0543 + 0.0105i, 0.0727 + 0.0410i, 0.0448 + 0.0559i, 0.0678 + 0.0307i, 0.0578 + 0.0276i, 0.0293 + 0.0220i, 0.0559 + 0.0125, 0.0483 + 0.0737 + 0.0186i, 0.0151 + 0.0754i, 0.0598 + 0.0494i, 0.0473 + 0.0177i$ $0.0125 + 0.0525i, 0.0024 + 0.0513i, 0.0222 + 0.0104i, 0.0748 + 0.0017i, 0.0733 + 0.0202i, 0.0176 + 0.0090i, 0.0739 + 0.0053i, 0.0524 + 0.0657i, 0.0042 + 0.0139i, 0.0462 + 0.0025i, 0.0303 + 0.0303i, 0.0166 + 0.0214i$ |



Information content



| # bits | classical | quantum mechanical |
|--------|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
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| 6 | 110101 | $0.0595 + 0.1064i, 0.0295 + 0.1327i, 0.0929 + 0.0406i, 0.1090 + 0.0379i, 0.0559 + 0.1286i, 0.0015 + 0.0345i, 0.0624 + 0.1196i, 0.1120 + 0.1350i, 0.1180 + 0.0345i, 0.1367 + 0.0356i, 0.1255 + 0.0074i, 0.0547 + 0.0116i, 0.0923 + 0.0952i$ $0.1087 + 0.0284i, 0.0298 + 0.1254i, 0.1345 + 0.0258i, 0.0846 + 0.0258i, 0.0839 + 0.1478i, 0.0348 + 0.0654i, 0.0816 + 0.0505i, 0.1384 + 0.0467i, 0.0498 + 0.0543i, 0.0974 + 0.0584i, 0.0582 + 0.0879i, 0.0932 + 0.0178i, 0.1039 + 0.0057i$ $0.0590 + 0.0682i, 0.0615 + 0.1293i, 0.0974 + 0.1382i, 0.1245 + 0.0393i, 0.0552 + 0.1297i, 0.0632 + 0.0238i, 0.0884 + 0.0354i, 0.0841 + 0.0960i, 0.1065 + 0.1437i, 0.0760 + 0.0988i, 0.1154 + 0.1293i, 0.0727 + 0.0015i, 0.0276 + 0.0204i$ $0.1041 + 0.1217i, 0.1460 + 0.0639i, 0.1199 + 0.1323i, 0.1046 + 0.1092i, 0.0721 + 0.1021i, 0.0170 + 0.0514i, 0.0988 + 0.0247i, 0.0543 + 0.0231i, 0.0208 + 0.0284i, 0.0842 + 0.0628i, 0.1223 + 0.1272i$ $0.1002 + 0.0729i, 0.1485 + 0.1213i, 0.1429 + 0.0685i, 0.0087 + 0.0680i, 0.0535 + 0.0670i, 0.0815 + 0.0613i, 0.0389 + 0.1340i, 0.0888 + 0.0008i, 0.0073 + 0.0442i, 0.0849 + 0.0073i, 0.1042 + 0.1030i, 0.1430 + 0.0966i, 0.1115 + 0.1461i$ $0.1100 + 0.0821i$ |
| 7 | 1001010 | $0.0880 + 0.0466i, 0.1054 + 0.0684i, 0.0239 + 0.0866i, 0.0759 + 0.0090i, 0.0563 + 0.1020i, 0.1006 + 0.0988i, 0.0769 + 0.0649i, 0.0246 + 0.0273i, 0.0485 + 0.0942i, 0.0186 + 0.0554i, 0.1045 + 0.0790i, 0.0384 + 0.0455i, 0.0053 + 0.1037i$ $0.0815 + 0.0078i, 0.0965 + 0.0597i, 0.0309 + 0.0315i, 0.0271 + 0.0925i, 0.1006 + 0.0362i, 0.0141 + 0.0734i, 0.1015 + 0.0058i, 0.0757 + 0.0385i, 0.0914 + 0.0537i, 0.0226 + 0.0468i, 0.0491 + 0.0607i, 0.0087 + 0.0668i, 0.0918 + 0.0122i$ $0.0606 + 0.0969i, 0.0344 + 0.0814i, 0.0404 + 0.0853i, 0.0936 + 0.0879i, 0.0401 + 0.0723i, 0.0079 + 0.0217i, 0.0216 + 0.0294i, 0.0553 + 0.0675i, 0.0611 + 0.0579i, 0.0131 + 0.0064i, 0.0563 + 0.0096i, 0.0126 + 0.0293i, 0.0830 + 0.0441i$ $0.0404 + 0.0511i, 0.0888 + 0.0890i, 0.0050 + 0.0643i, 0.0645 + 0.0355i, 0.1024 + 0.0516i, 0.0311 + 0.0644i, 0.0959 + 0.0174i, 0.0110 + 0.0894i, 0.0070 + 0.1031i, 0.0253 + 0.0462i, 0.1006 + 0.0031i, 0.0068 + 0.0878i, 0.0285 + 0.0658i$ $0.1078 + 0.0756i, 0.0229 + 0.0099i, 0.0537 + 0.0450i, 0.0313 + 0.0405i, 0.0725 + 0.0179i, 0.1033 + 0.0898i, 0.0827 + 0.0904i, 0.0718 + 0.0487i, 0.0141 + 0.1032i, 0.0103 + 0.0159i, 0.0019 + 0.0398i, 0.0311 + 0.0830i, 0.0881 + 0.0479i$ $0.1063 + 0.0669i, 0.0019 + 0.1026i, 0.0884 + 0.0690i, 0.0670 + 0.0267i, 0.0604 + 0.0380i, 0.0263 + 0.0203i, 0.0886 + 0.0529i, 0.0284 + 0.0441i, 0.0813 + 0.0504i, 0.0711 + 0.0659i, 0.0231 + 0.0077i, 0.0649 + 0.0339i, 0.0652 + 0.0656i$ $0.111 + 0.0189i, 0.0001 + 0.0670i, 0.0688 + 0.0268i, 0.0184 + 0.0633i, 0.0582 + 0.0546i, 0.0672 + 0.0501i, 0.0740 + 0.0584i, 0.0730 + 0.1016i, 0.0946 + 0.0369i, 0.0014 + 0.0433i, 0.0338 + 0.0332i, 0.0840 + 0.0444i, 0.0331 + 0.0308i$ $0.0659 + 0.0732i, 0.0001 + 0.0626i, 0.0600 + 0.0529i, 0.0076 + 0.0333i, 0.0013 + 0.0121i, 0.0245 + 0.0478i, 0.0557 + 0.0503i, 0.0494 + 0.0016i, 0.0758 + 0.0716i, 0.0628 + 0.0781i, 0.0549 + 0.0304i, 0.0080 + 0.0282i, 0.0208 + 0.0764i$ $0.0409 + 0.0845i, 0.0693 + 0.0425i, 0.0999 + 0.0562i, 0.0222 + 0.0774i, 0.0870 + 0.0614i, 0.0979 + 0.0497i, 0.0169 + 0.0480i, 0.0322 + 0.0478i, 0.0778 + 0.0395i, 0.0703 + 0.0326i, 0.0613 + 0.0919i, 0.0715 + 0.0619i$ $0.0953 + 0.1024i, 0.0293 + 0.0602i, 0.0452 + 0.0015i, 0.0230 + 0.0643i$ |
| 8 | 10101011 | $0.0199 + 0.0027i, 0.0033 + 0.0063i, 0.0005 + 0.0656i, 0.0443 + 0.0262i, 0.0573 + 0.0359i, 0.0622 + 0.0704i, 0.0491 + 0.0176i, 0.0194 + 0.0664i, 0.0111 + 0.0506i, 0.0502 + 0.0687i, 0.0729 + 0.0376i, 0.0629 + 0.0765i, 0.0717 + 0.0288i$ $0.0239 + 0.0410i, 0.0207 + 0.0140i, 0.0413 + 0.0387i, 0.0126 + 0.0325i, 0.0163 + 0.0509i, 0.0167 + 0.0519i, 0.0502 + 0.0738i, 0.0041 + 0.0148i, 0.0177 + 0.0868i, 0.0514 + 0.0436i, 0.0240 + 0.0747i, 0.0236 + 0.0018i, 0.0555 + 0.0671i$ $0.0736 + 0.0021i, 0.0101 + 0.0400i, 0.0053 + 0.1484i, 0.0097 + 0.0552i, 0.0128 + 0.0193i, 0.0702 + 0.0720i, 0.0105 + 0.0106i, 0.0476 + 0.0402i, 0.0207 + 0.0690i, 0.0170 + 0.0726i, 0.0549 + 0.0258i, 0.0423 + 0.0337i, 0.0726 + 0.0363i$ $0.0254 + 0.0115i, 0.0543 + 0.0105i, 0.0727 + 0.0414i, 0.0448 + 0.0559i, 0.0678 + 0.0307i, 0.0578 + 0.0276i, 0.0293 + 0.0220i, 0.0559 + 0.0670i, 0.0125 + 0.0483i, 0.0737 + 0.0186i, 0.0151 + 0.0754i, 0.0598 + 0.0494i, 0.0473 + 0.0177i$ $0.0125 + 0.0525i, 0.0047 + 0.0513i, 0.0222 + 0.0104i, 0.0748 + 0.0017i, 0.0733 + 0.0202i, 0.0176 + 0.0040i, 0.0739 + 0.0053i, 0.0524 + 0.0657i, 0.0042 + 0.1393i, 0.0462 + 0.0025i, 0.0303 + 0.0566i, 0.0166 + 0.0114i, 0.0441 + 0.0213i$ $0.0559 + 0.0286i, 0.0001 + 0.0604i, 0.0606 + 0.0501i, 0.0014 + 0.0666i, 0.0272 + 0.0439i, 0.0509 + 0.0124i, 0.0526 + 0.0458i, 0.0120 + 0.0559i, 0.0226 + 0.0505i, 0.0038 + 0.0666i, 0.0023 + 0.0049i, 0.0223 + 0.0756i$ $0.0001 + 0.0610i, 0.0162 + 0.0118i, 0.0070 + 0.0642i, 0.0076 + 0.0488i, 0.0498 + 0.0640i, 0.0042 + 0.0516i, 0.0239 + 0.0045i, 0.0164 + 0.0309i, 0.0172 + 0.0550i, 0.0026 + 0.0372i, 0.0014 + 0.0668i, 0.0036 + 0.0136i, 0.0715 + 0.0261i$ $0.0301 + 0.0609i, 0.0394 + 0.0396i, 0.0072 + 0.0164i, 0.0017 + 0.0009i, 0.0123 + 0.0211i, 0.0651 + 0.0314i, 0.0770 + 0.0314i, 0.0144 + 0.0441i, 0.0544 + 0.0726i, 0.0549 + 0.0161i, 0.0672 + 0.0269i, 0.0370 + 0.0324i, 0.0382 + 0.0130i$ $0.0222 + 0.0691i, 0.0047 + 0.0249i, 0.0202 + 0.0566i, 0.0144 + 0.0337i, 0.0707 + 0.0309i, 0.0095 + 0.0390i, 0.0010 + 0.0130i, 0.0285 + 0.0404i, 0.0538 + 0.0494i, 0.0685 + 0.0012i, 0.0459 + 0.0645i, 0.0121 + 0.0519i, 0.0244 + 0.0538i$ $0.0180 + 0.0356i, 0.0006 + 0.0664i, 0.0306 + 0.0633i, 0.0501 + 0.0149i, 0.0593 + 0.0010i, 0.0747 + 0.0238i, 0.0561 + 0.0571i, 0.0603 + 0.0644i, 0.0183 + 0.0257i, 0.0151 + 0.0679i, 0.0203 + 0.0370i, 0.0305 + 0.0423i$ $0.0753 + 0.0475i, 0.0491 + 0.0510i, 0.0421 + 0.0475i, 0.0654 + 0.0528i, 0.0618 + 0.0393i, 0.0515 + 0.0550i, 0.0517 + 0.0397i, 0.0633 + 0.0474i, 0.0748 + 0.0745i, 0.0375 + 0.0634i, 0.0245 + 0.0494i, 0.0494 + 0.0453i, 0.0236 + 0.0100i$ $0.0509 + 0.0196i, 0.0276 + 0.0619i, 0.0723 + 0.0515i, 0.0376 + 0.0011i, 0.0070 + 0.0433i, 0.0519 + 0.0350i, 0.0397 + 0.0697i, 0.0171 + 0.0267i, 0.0559 + 0.0050i, 0.0053 + 0.0367i, 0.0743 + 0.0758i, 0.0160 + 0.0713i, 0.0124 + 0.0433i$ $0.0492 + 0.0503i, 0.0003 + 0.0596i, 0.0259 + 0.0082i, 0.0212 + 0.0001i, 0.0034 + 0.0418i, 0.0072 + 0.0051i, 0.0316 + 0.0348i, 0.0630 + 0.0151i, 0.0671 + 0.0607i, 0.0017 + 0.0477i, 0.0560 + 0.0012i, 0.0654 + 0.0562 + 0.0587i$ $0.0736 + 0.0699i, 0.0503 + 0.0585i, 0.0272 + 0.0239i, 0.0266 + 0.0255i, 0.0681 + 0.0389i, 0.0268 + 0.0435i, 0.0670 + 0.0514i, 0.0302 + 0.0522i, 0.0195 + 0.0726i, 0.0273 + 0.0594i, 0.0573 + 0.0568i, 0.0502 + 0.0668i, 0.0724 + 0.0764i, 0.0642 + 0.0388i, 0.0362 + 0.0485i, 0.0485 + 0.0611i, 0.0045 + 0.0346i, 0.0351 + 0.0132i, 0.0665 + 0.0101i, 0.00671 + 0.0217i, 0.0517 + 0.0381i, 0.0030 + 0.0701i, 0.0171 + 0.0071i, 0.0231 + 0.0271i, 0.0381 + 0.0173i, 0.0030 + 0.0701i, 0.0175 + 0.0066i, 0.0252 + 0.0321i, 0.0392 + 0.0518i, 0.0349 + 0.0723i, 0.0251 + 0.0264i, 0.0293 + 0.0434i, 0.0434 + 0.0683i, 0.0683 + 0.0092i, 0.0587 + 0.0130i, 0.0681 + 0.0215i, 0.0353 + 0.0437i, 0.0418 + 0.0242i, 0.0194 + 0.0242i, 0.0334 + 0.0178i, 0.0649 + 0.0343i, 0.0142 + 0.0230i, 0.0373 + 0.0518i, 0.0251 + 0.0264i, 0.0373 + 0.0191i, 0.0747 + 0.0149i, 0.0264 + 0.0701i, 0.0195 + 0.0711i, 0.0451 + 0.0018i, 0.0204 + 0.0592i, 0.0126 + 0.0703i, 0.0375 + 0.0627i, 0.0382 + 0.0712i, 0.0650 + 0.0153i, 0.0621 + 0.0502i, 0.0061 + 0.0715i, 0.0470 + 0.0265i, 0.0436 + 0.0458i, 0.0472 + 0.0474i, 0.0079 + 0.0003i, 0.0122 + 0.0757i, 0.0757 + 0.0319i, 0.0693 + 0.0432i, 0.0207 + 0.0339i, 0.0604 + 0.0540i, 0.0299 + 0.0470i, 0.0024 + 0.0231i, 0.0451 + 0.0660i, 0.0431 + 0.0086i, 0.0155 + 0.0225i, 0.0067 + 0.0075i, 0.0719 + 0.0306i, 0.0200 + 0.0257i, 0.0157 + 0.0728i$ |



Information content



| | |
|--------------------|------------------------------------------|
| 40 qubits | 10 000 GigaByte |
| 1 additional qubit | Double the memory |
| 300 qubits | Each atom in the universe holds one bit. |



Why quantum information ?



Schrödinger equation for 300 interacting spins.

Classical computation needs more bits than there are atoms in the universe.

- Quantum computers can solve certain tasks much more efficiently than classical computers.

Other prominent examples:

- Factoring of large integers (P. Shor 1994)
- Search in an unsorted data base (L. Grover, 1997)
- ...





Quantum computing



Classical computer

- Initialization
- 1-bit operations (NOT)
- 2-bit gates (e.g. NAND)

Computational space:

00
01
10
11

- Read out
→ result

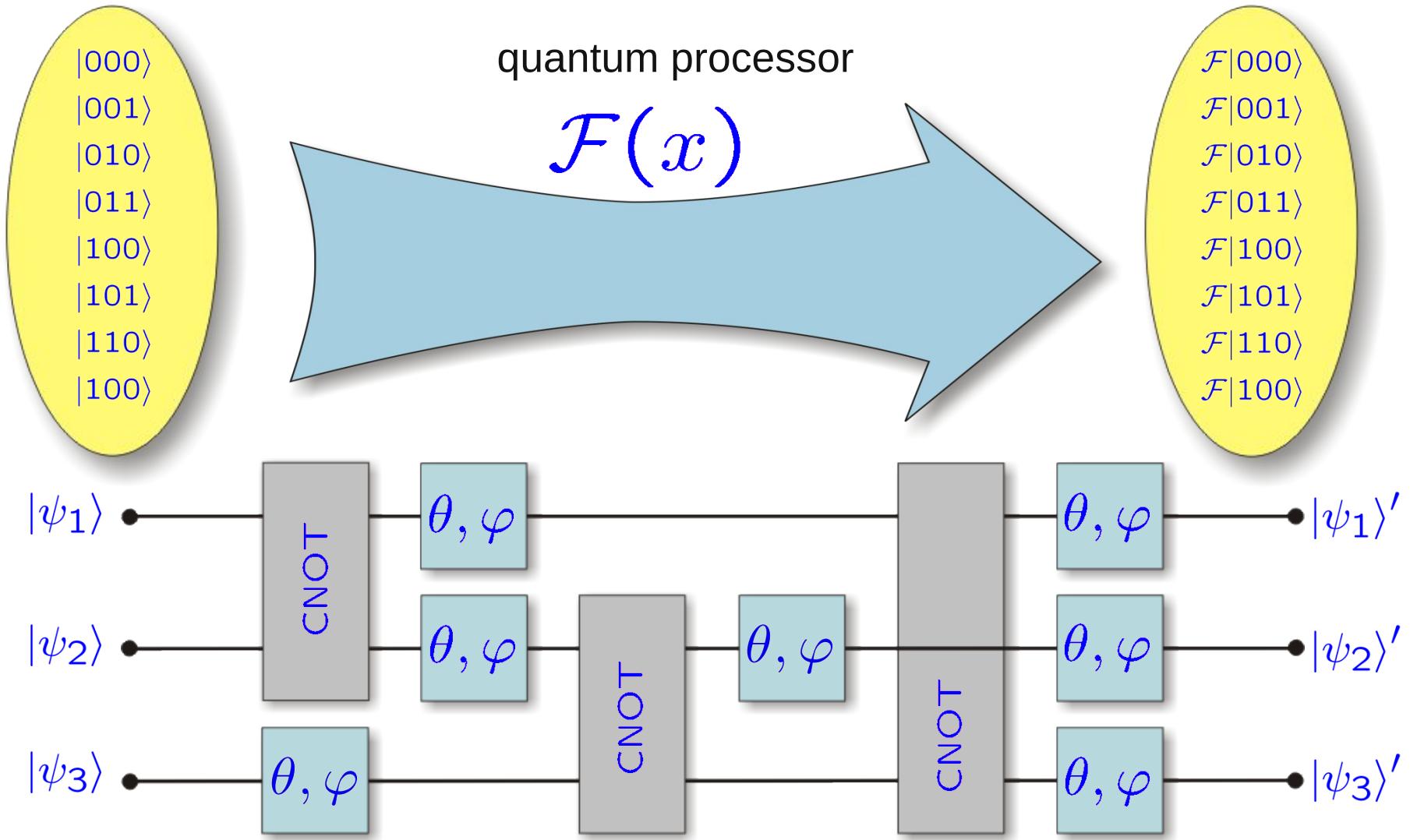
Quantum computer

- Initialization
- 1-qubit rotations
→ superpositions
- 2-qubit gates (CNOT gate)
→ entanglement

Computational space: Hilbert space
 2^n dimensional

- Read out of qubits
→ gain of classical information

Quantum computing



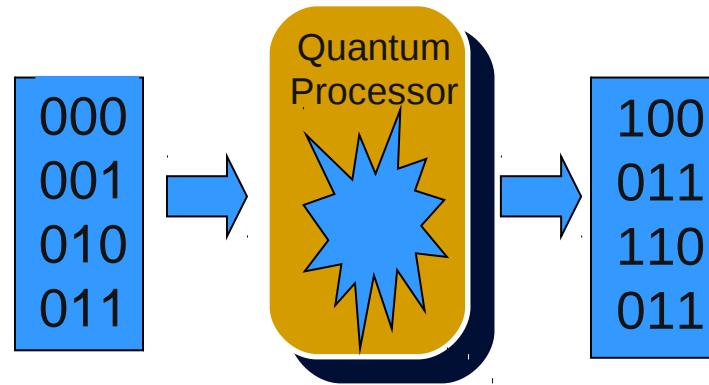
Input 

computation: sequence of quantum gates



output

Long term goal: A universal quantum computer

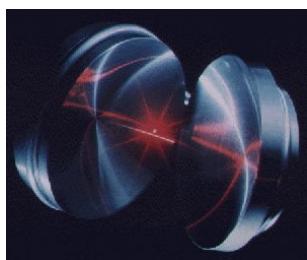


In the mean time:

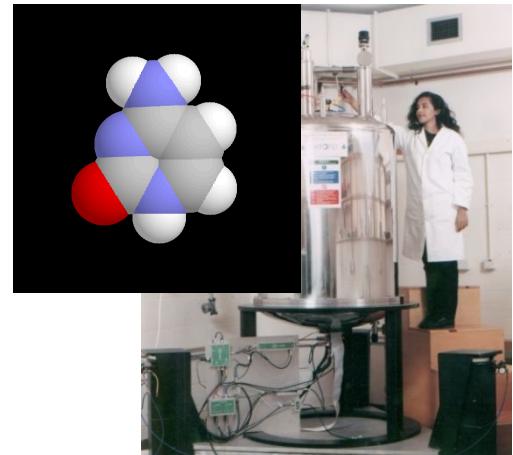
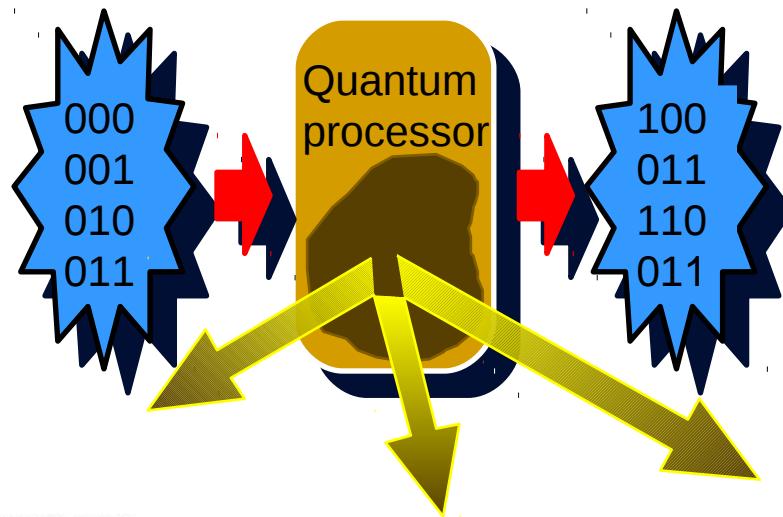
- “understand” quantum mechanics
- apply quantum mechanics
- where does quantum mechanics fail?



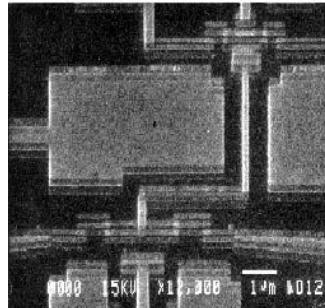
Which technology ?



Cavity QED



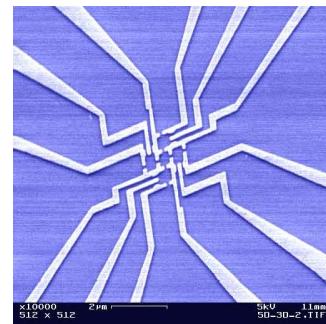
NMR



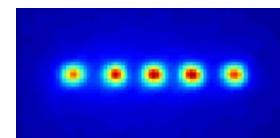
Superconducting qubits



Trapped atoms/ions

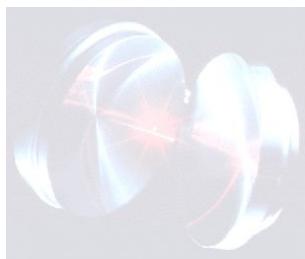


Quantum dots

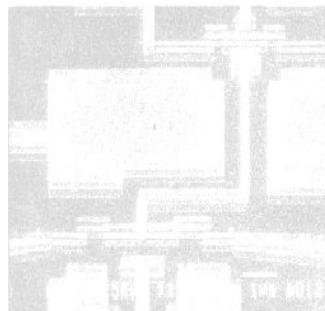
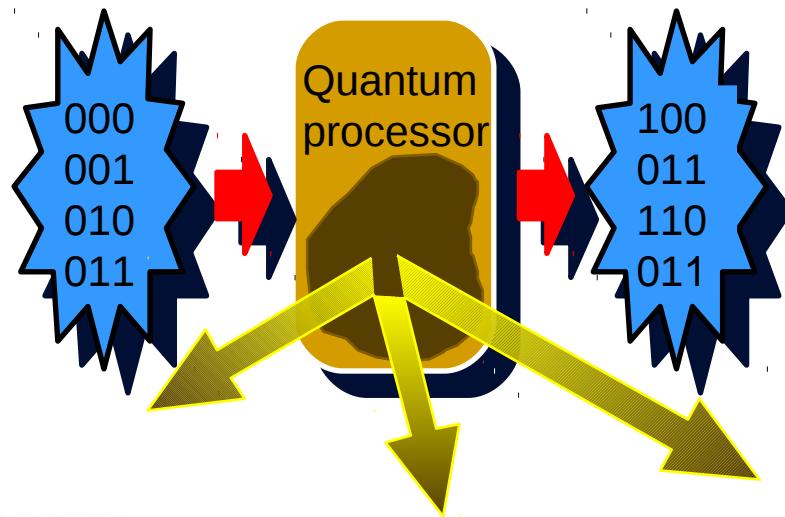




Which technology ?



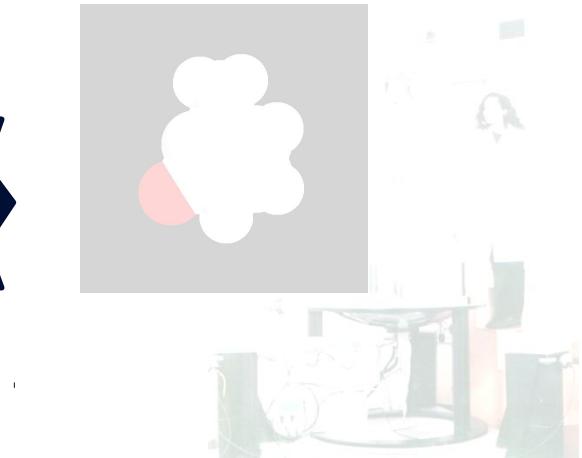
Cavity QED



Superconducting qubits



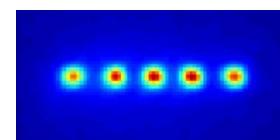
Trapped atoms/ions



NMR



Quantum dots

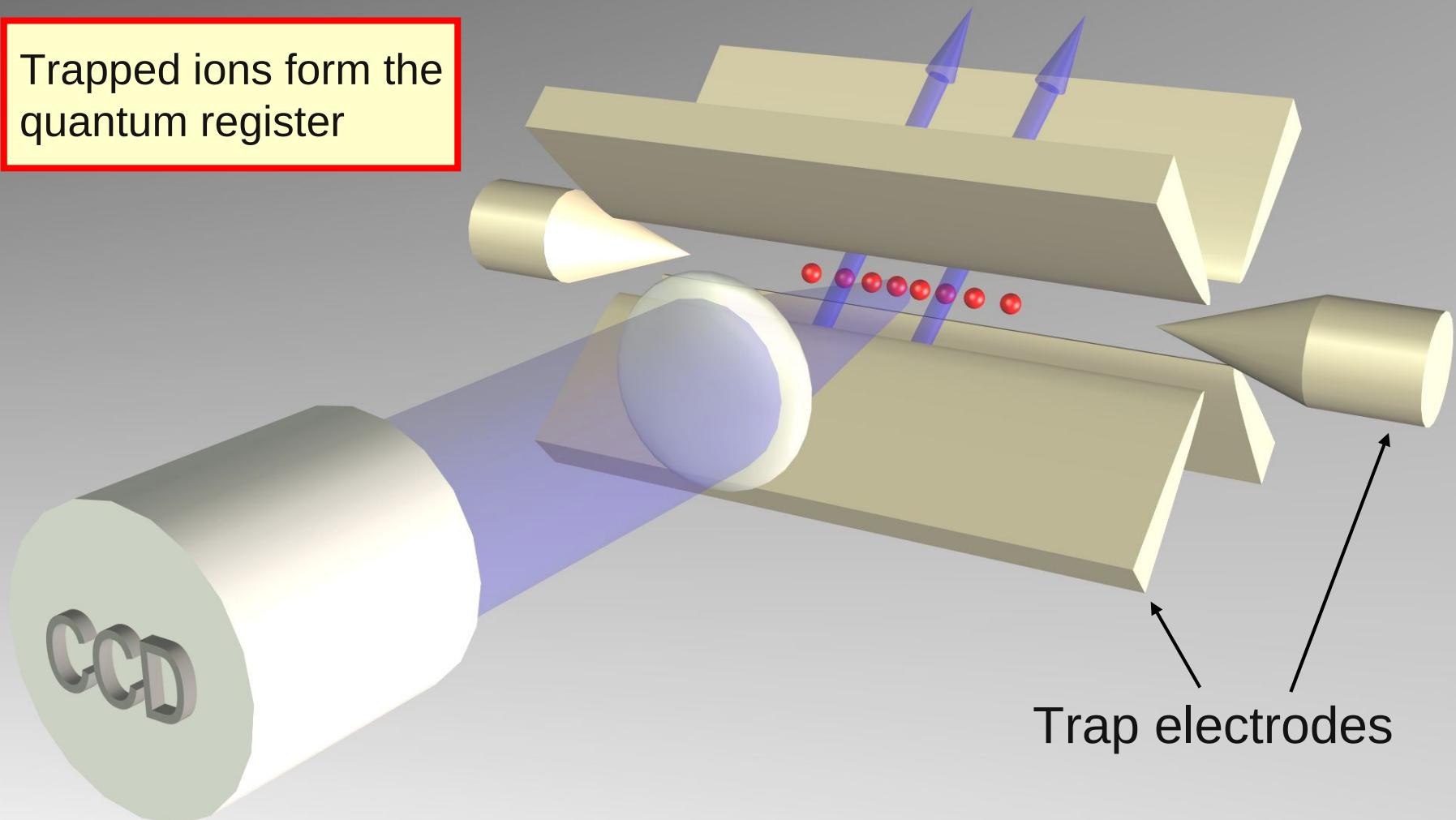




Ion trap quantum computing

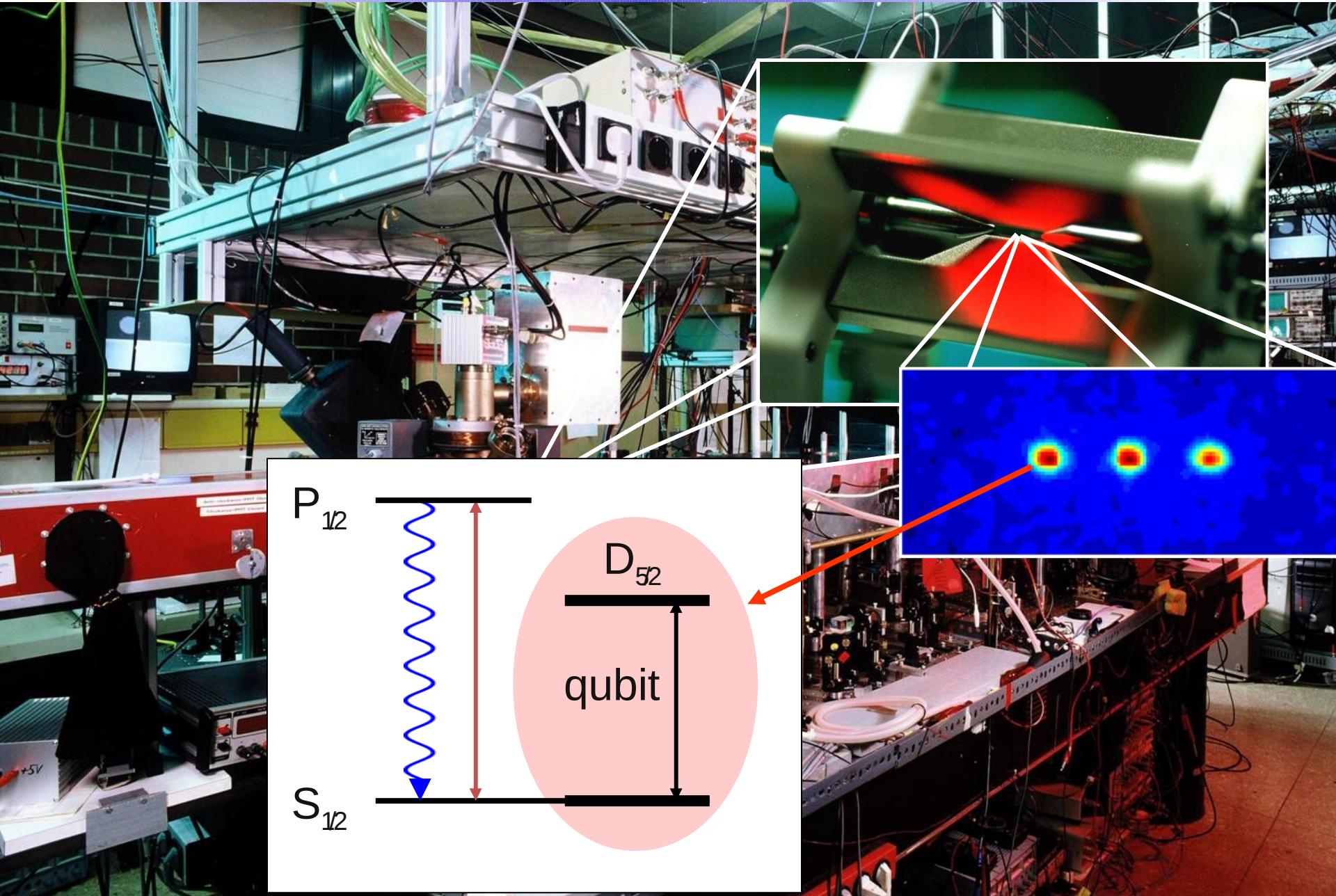


Trapped ions form the quantum register

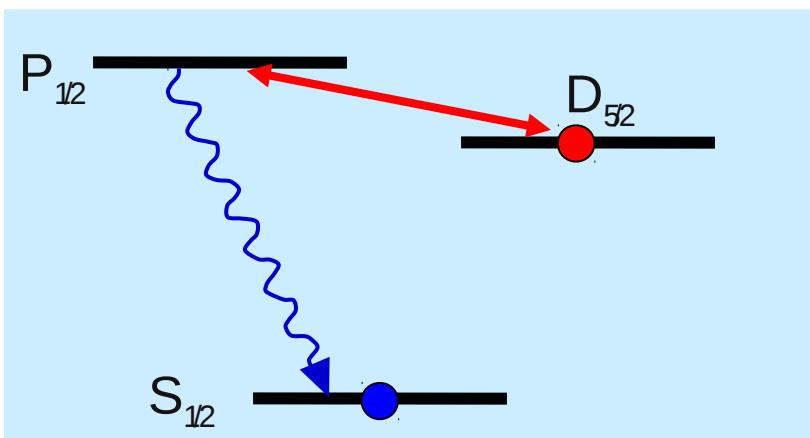




The hardware

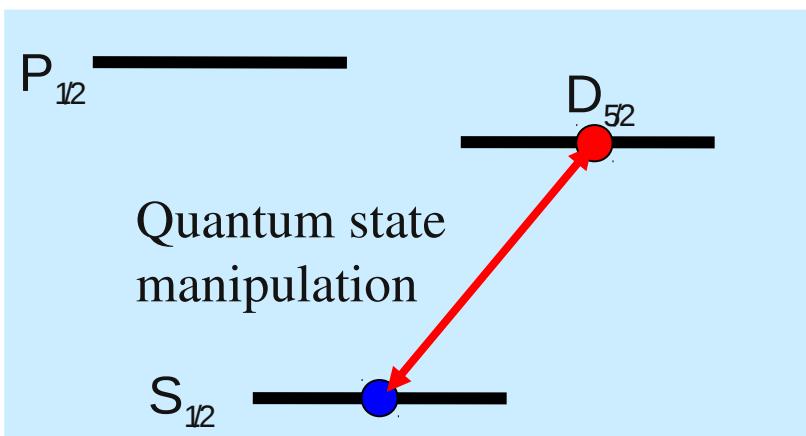


Experimental procedure



1. Initialization in a pure quantum state

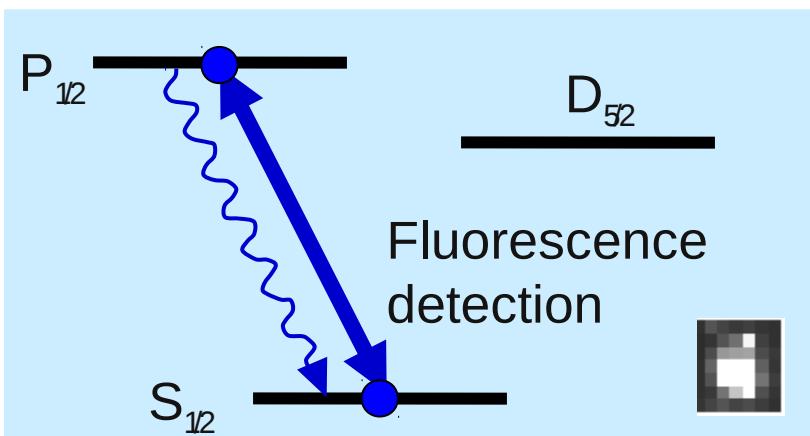
Experimental procedure



1. Initialization in a pure quantum state

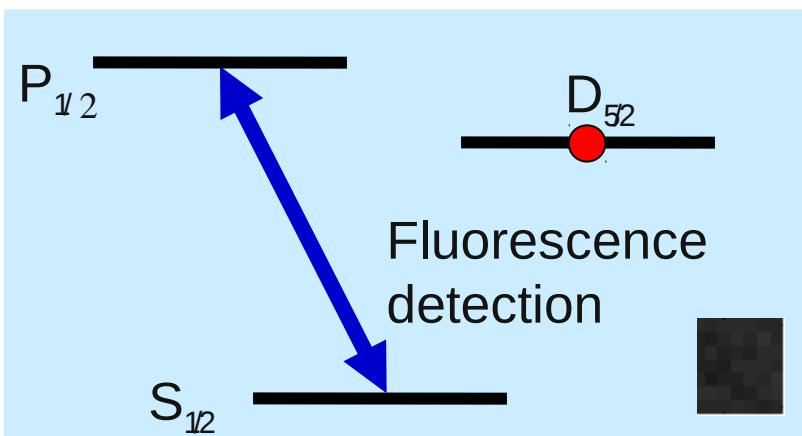
2. Quantum state manipulation on
 $S_{1/2} - D_{5/2}$ transition

Experimental procedure



1. Initialization in a pure quantum state
2. Quantum state manipulation on $S_{1/2} - D_{5/2}$ transition
3. Quantum state measurement by fluorescence detection

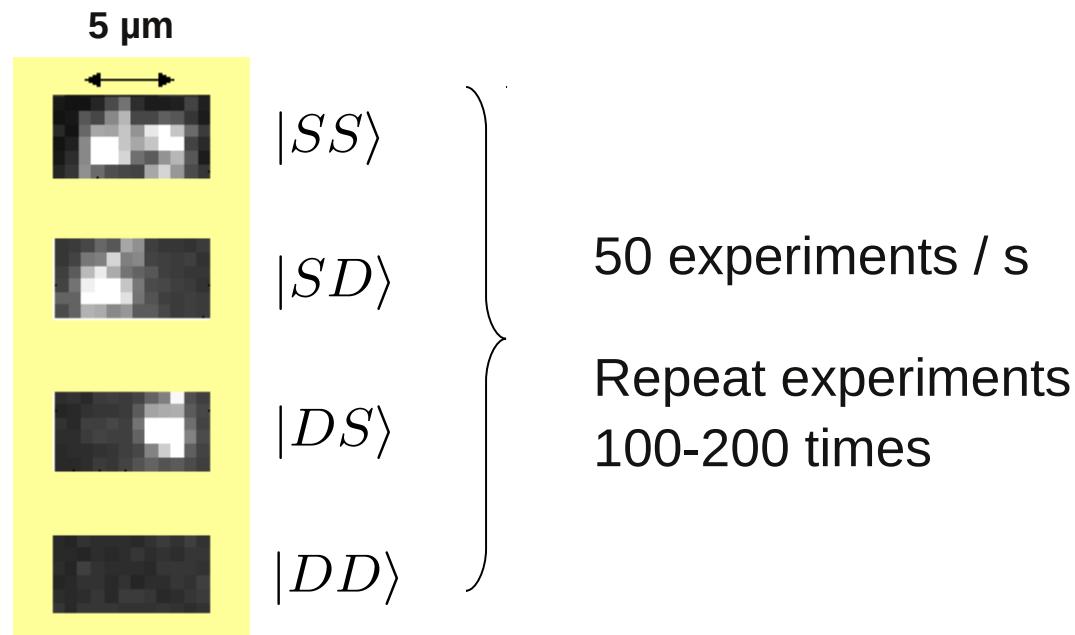
Experimental procedure



1. Initialization in a pure quantum state
2. Quantum state manipulation on $S_{1/2} - D_{5/2}$ transition
3. Quantum state measurement by fluorescence detection

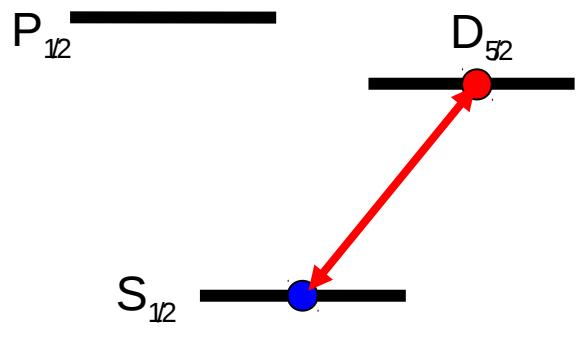
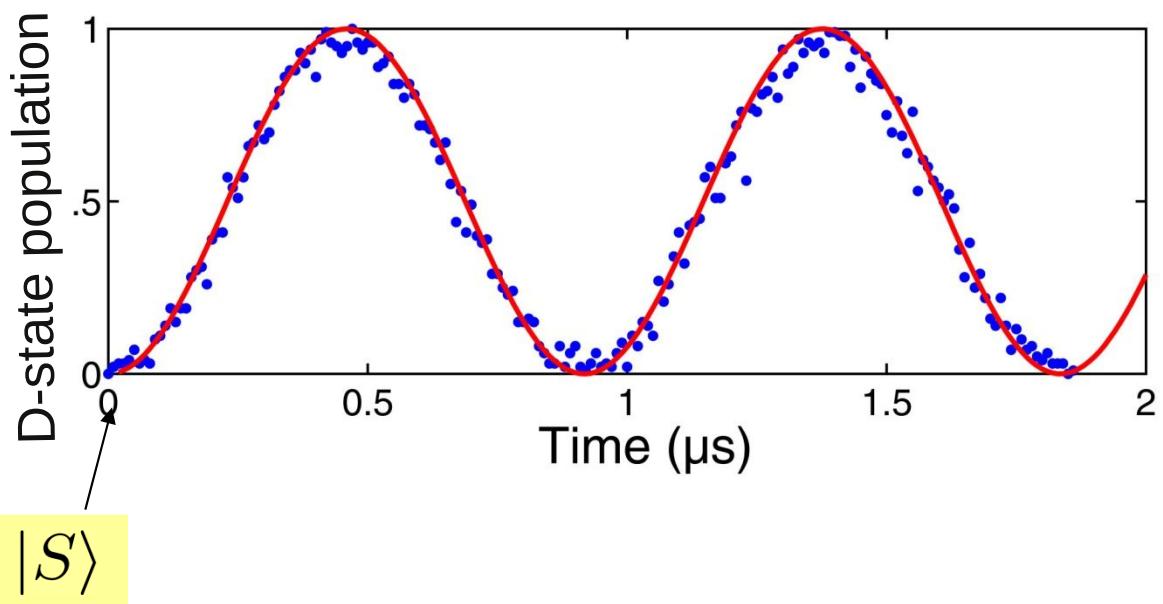
Two ions:

Spatially resolved
detection with
CCD camera

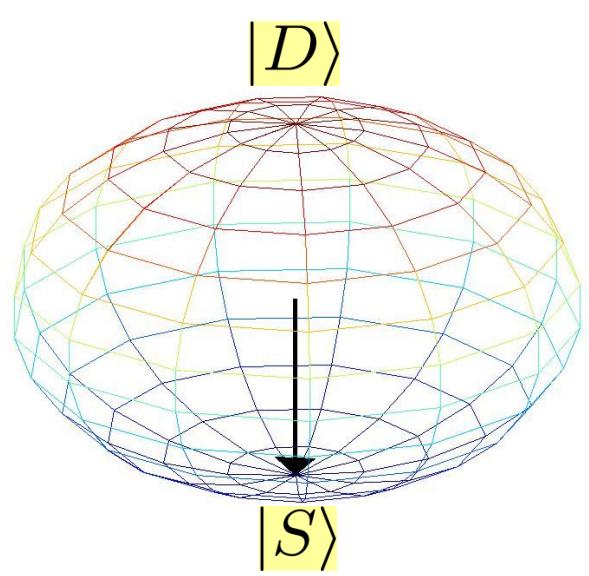




Rabi oscillations



$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$

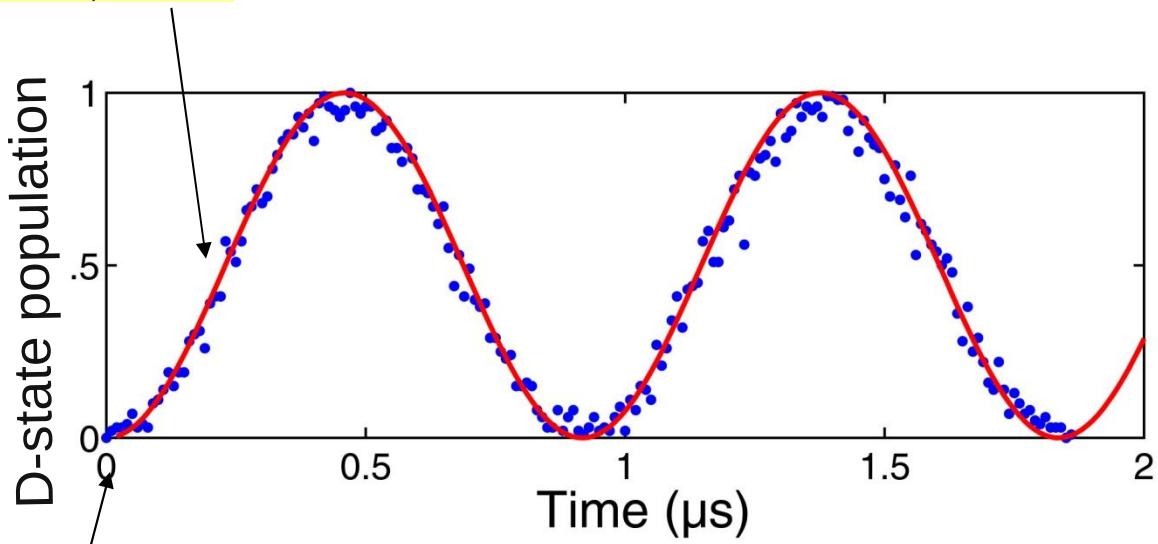




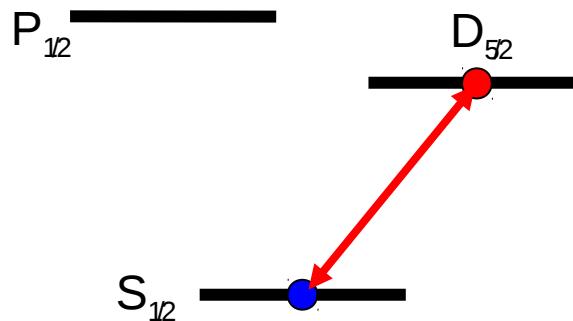
Rabi oscillations



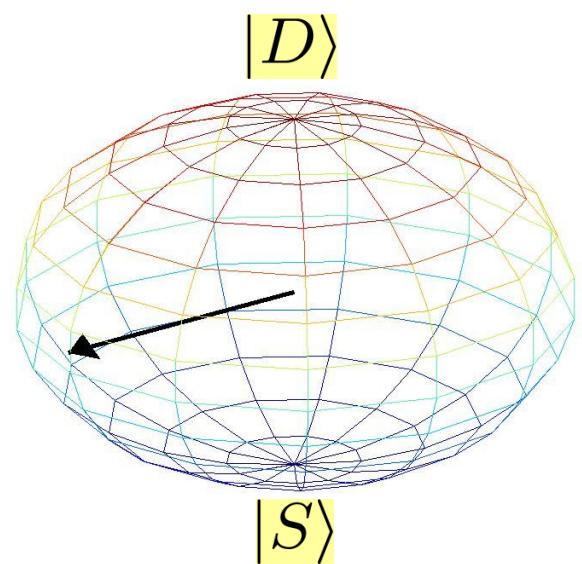
$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$



$$|S\rangle$$

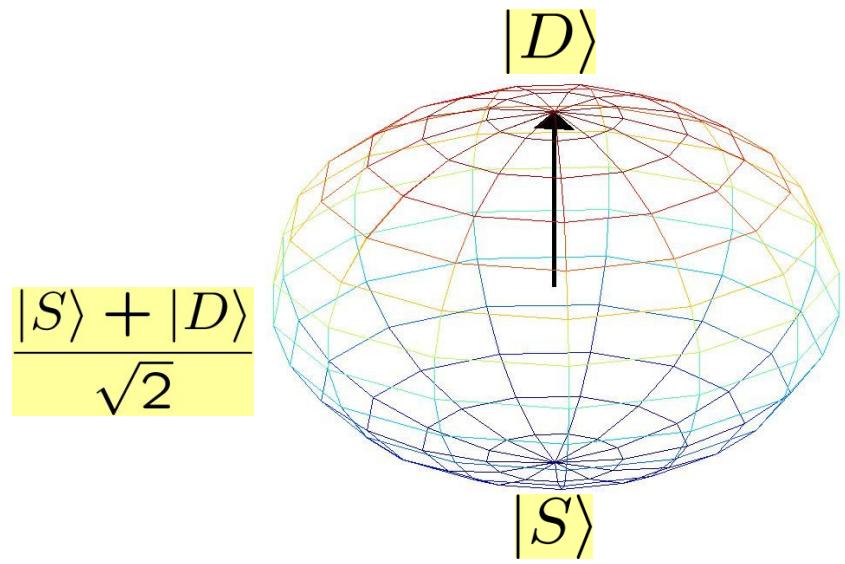
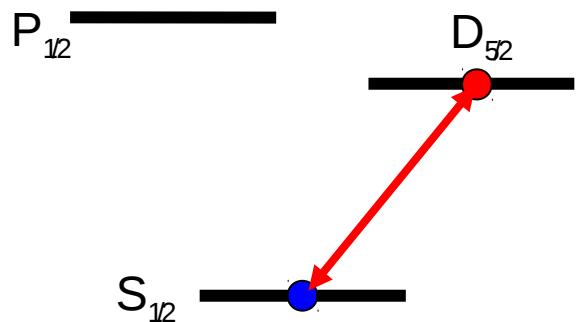
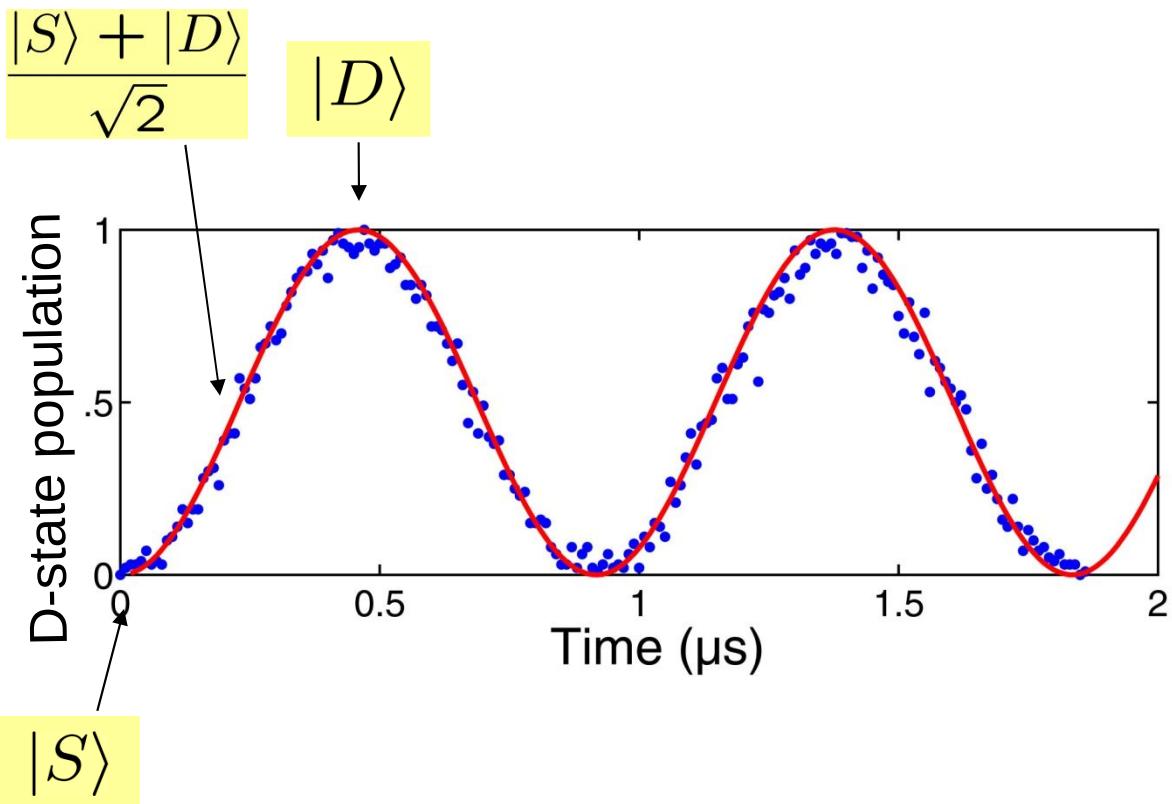


$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$





Rabi oscillations



$$|S\rangle + |D\rangle / \sqrt{2}$$

Classical computer

- Initialization
- 1-bit operations (NOT)
- 2-bit gates (e.g. NAND)

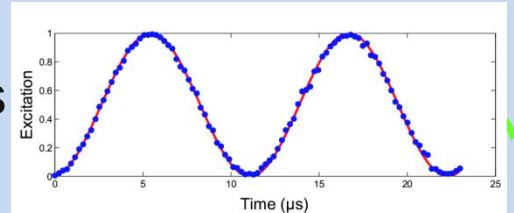
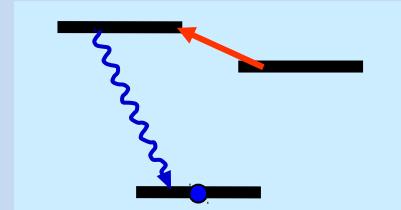
Computational space:

00
01
10
11

- Read out
→ result

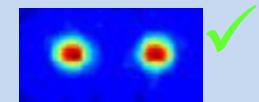
Quantum computer

- Initialization
- 1-qubit rotations
→ superpositions
- 2-qubit gates (CNOT gate)
→ entanglement



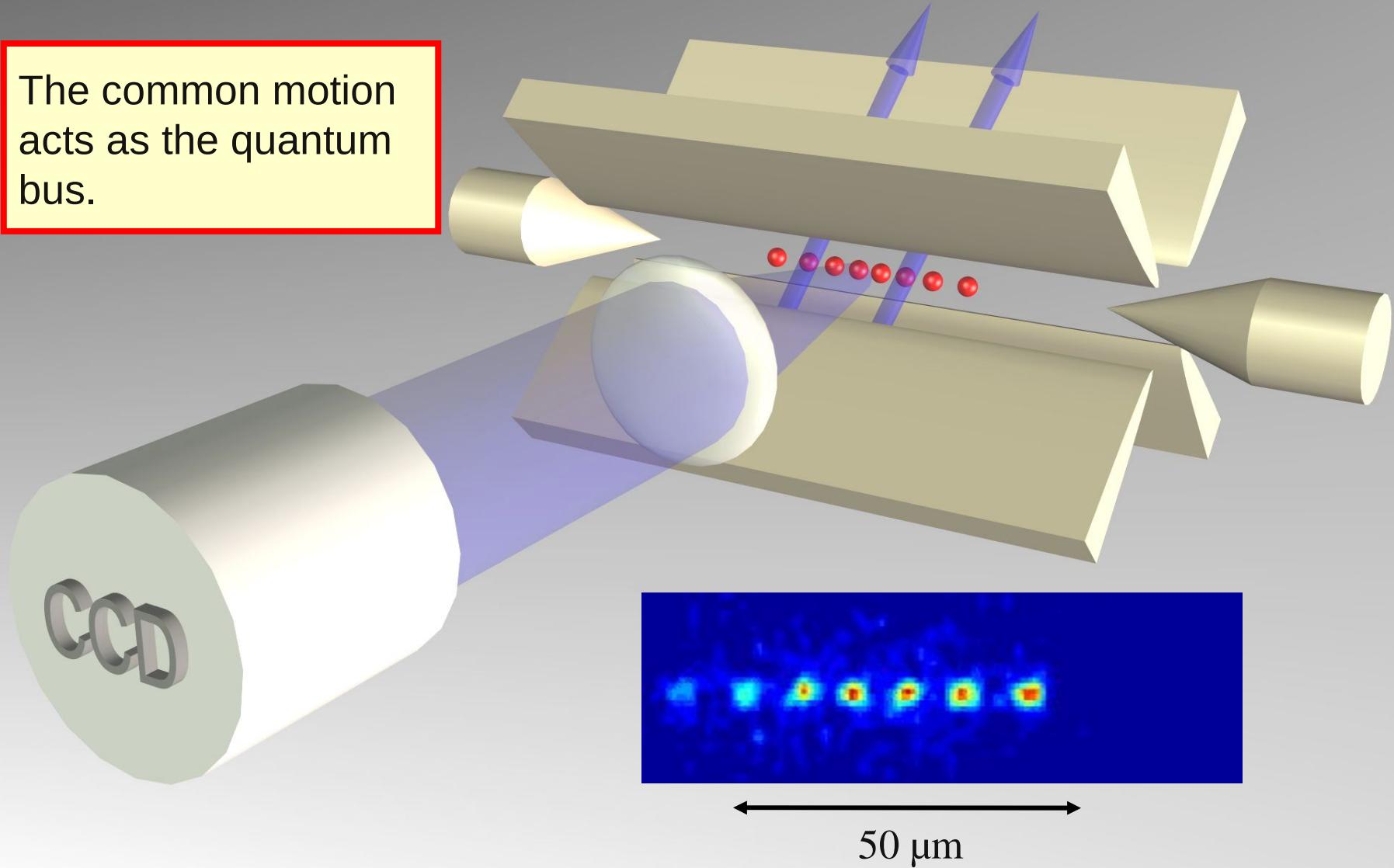
Computational space:

- Read out of qubits
→ gain of classical information



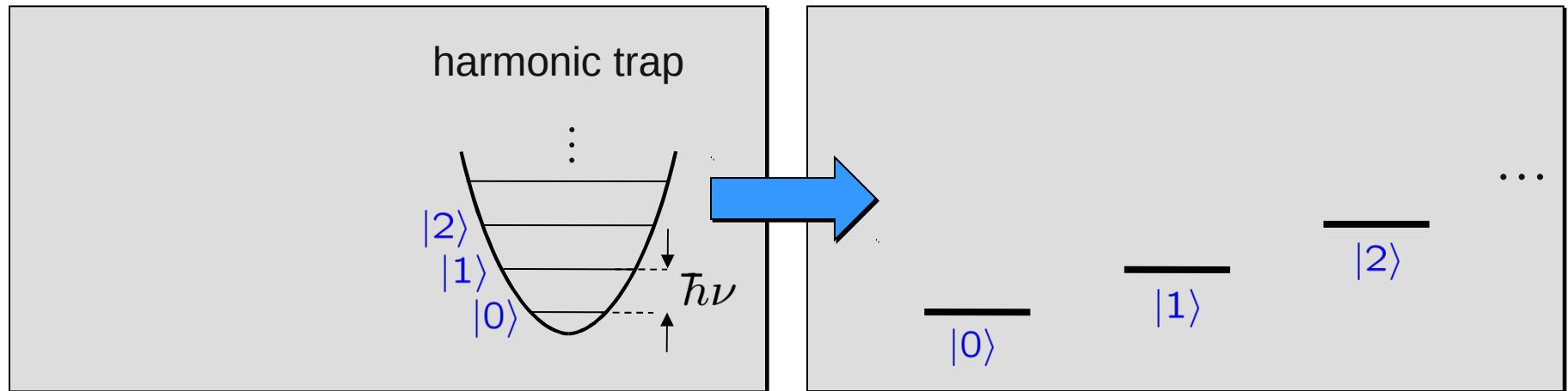
Having the qubits interact

The common motion acts as the quantum bus.



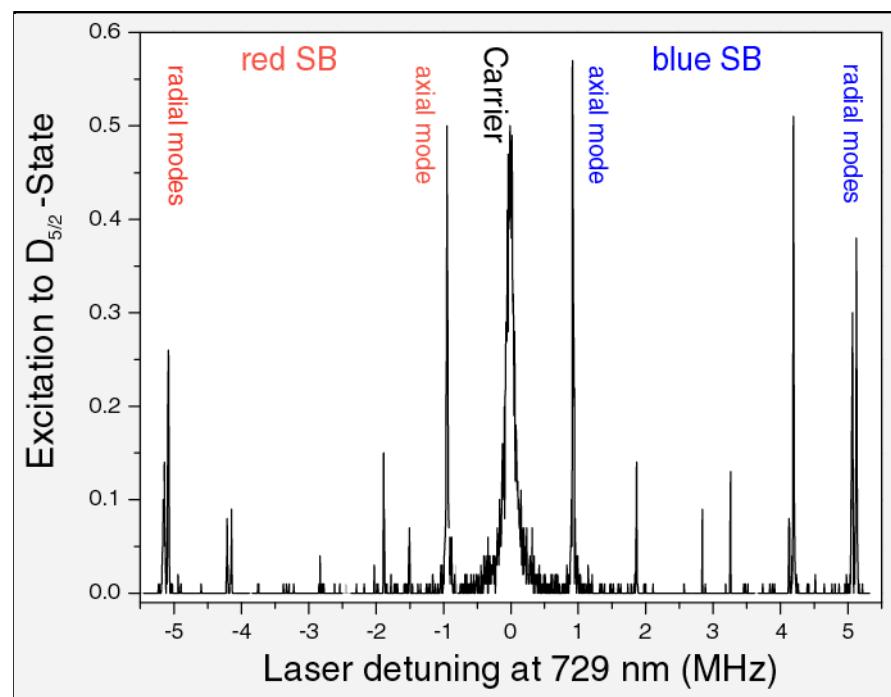
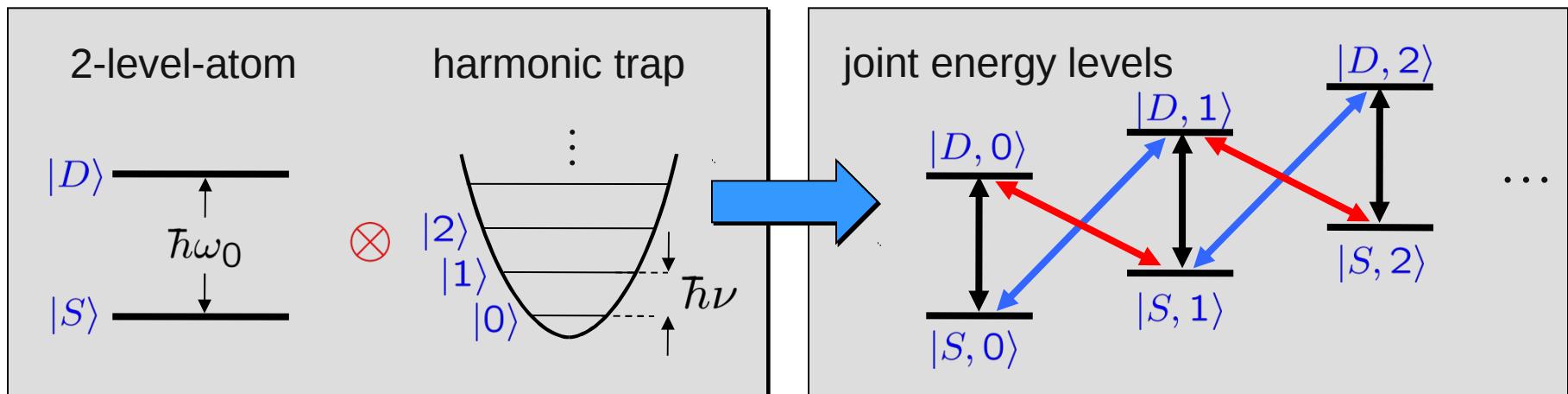


Ion motion



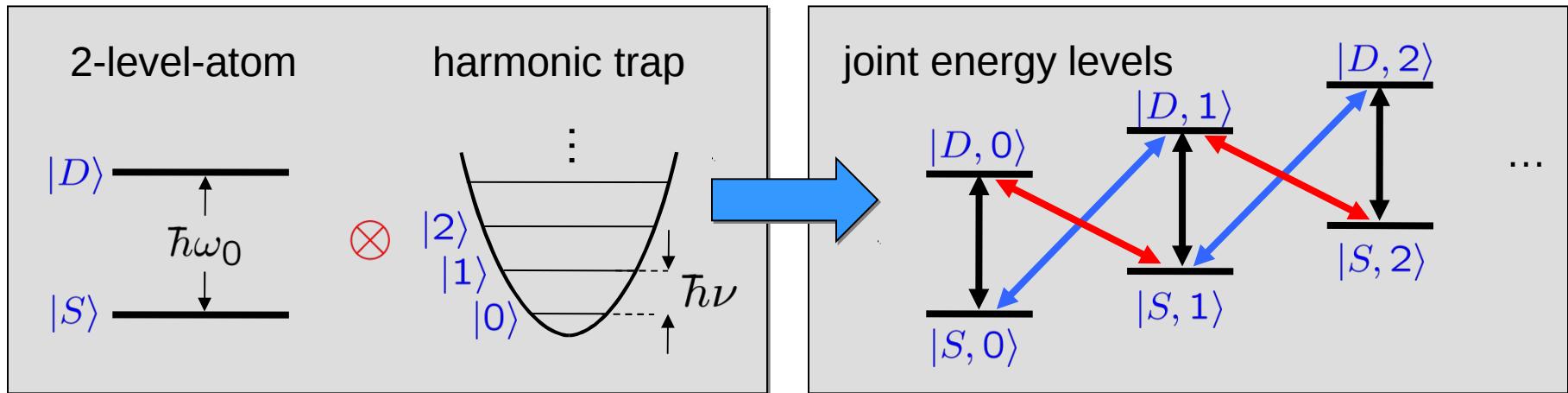


Ion motion

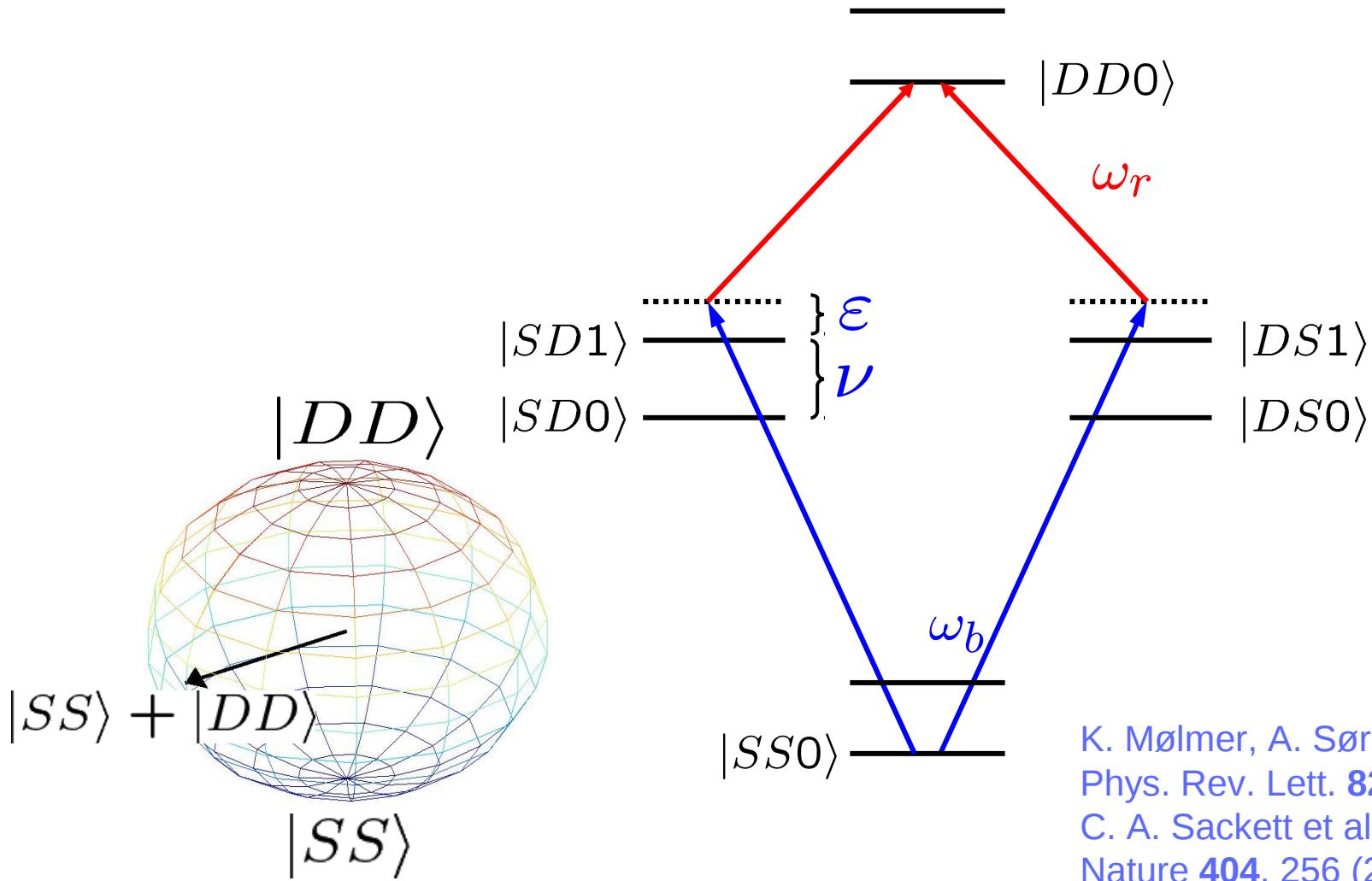




Ion motion

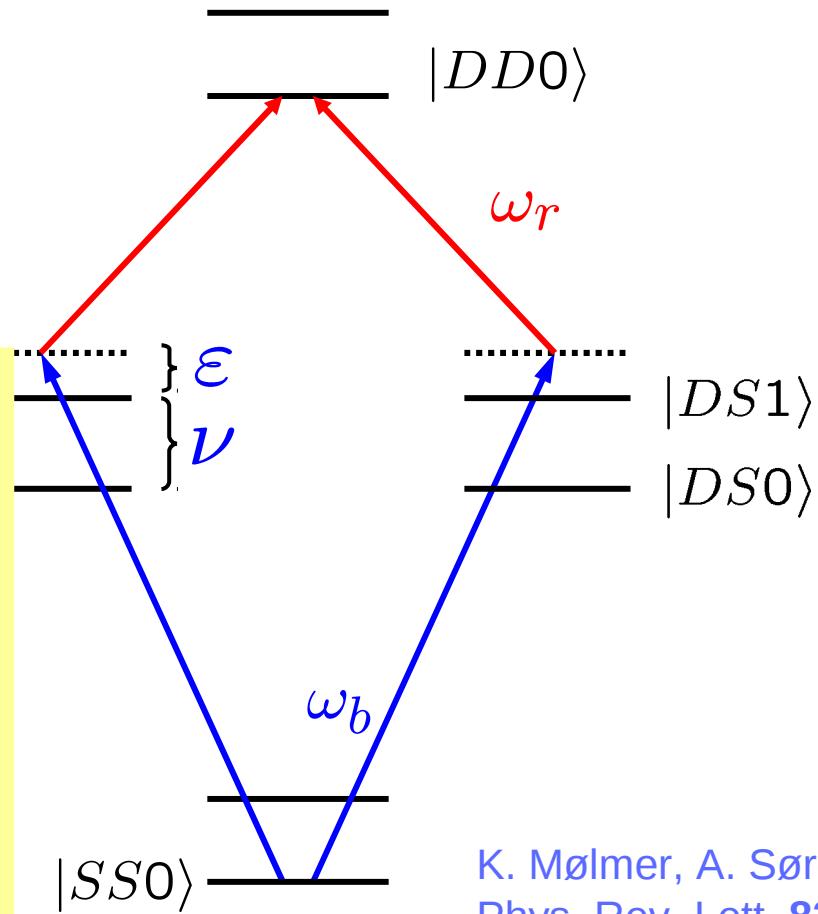


All atoms flip their state together



K. Mølmer, A. Sørensen,
Phys. Rev. Lett. **82**, 1971 (1999)
C. A. Sackett et al.,
Nature **404**, 256 (2000)

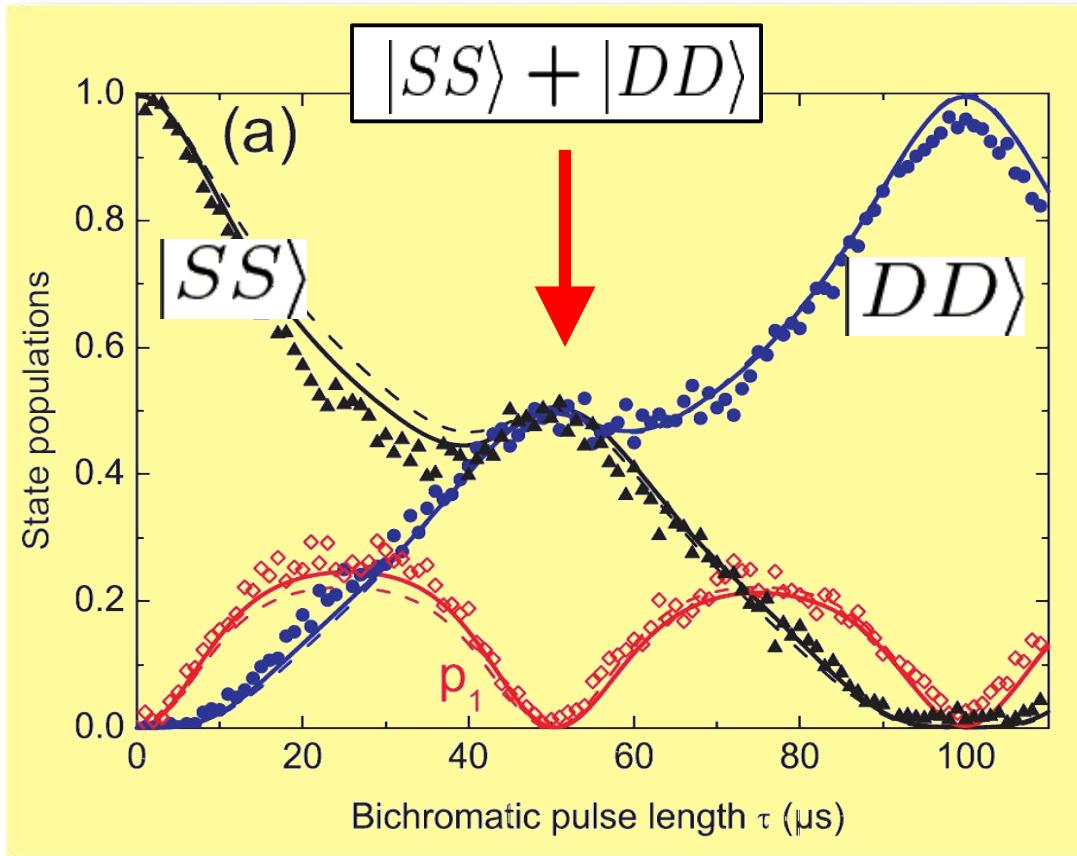
All atoms flip their state together



Bell states with atoms

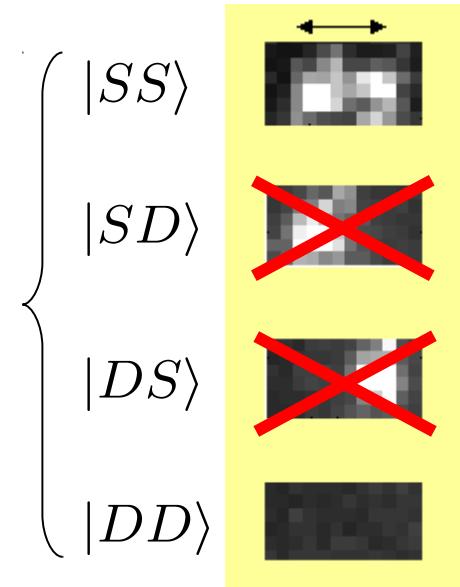
- ${}^9\text{Be}^+$: NIST (fidelity: 97 %)
- ${}^{40}\text{Ca}^+$: Oxford (83%)
- ${}^{111}\text{Cd}^+$: Ann Arbor (79%)
- ${}^{171}\text{Yb}$: Maryland (96%)
- ${}^{25}\text{Mg}^+$: Munich (97%)
- ${}^{40}\text{Ca}^+$: Innsbruck (99%)

K. Mølmer, A. Sørensen,
Phys. Rev. Lett. **82**, 1971 (1999)
C. A. Sackett et al.,
Nature **404**, 256 (2000)



$$|SS\rangle + |DD\rangle$$

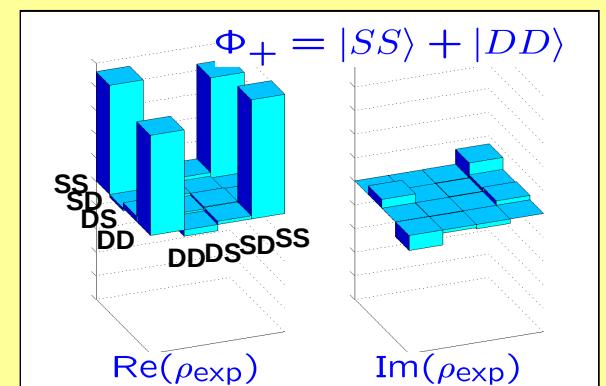
Fluorescence
detection with
CCD camera:

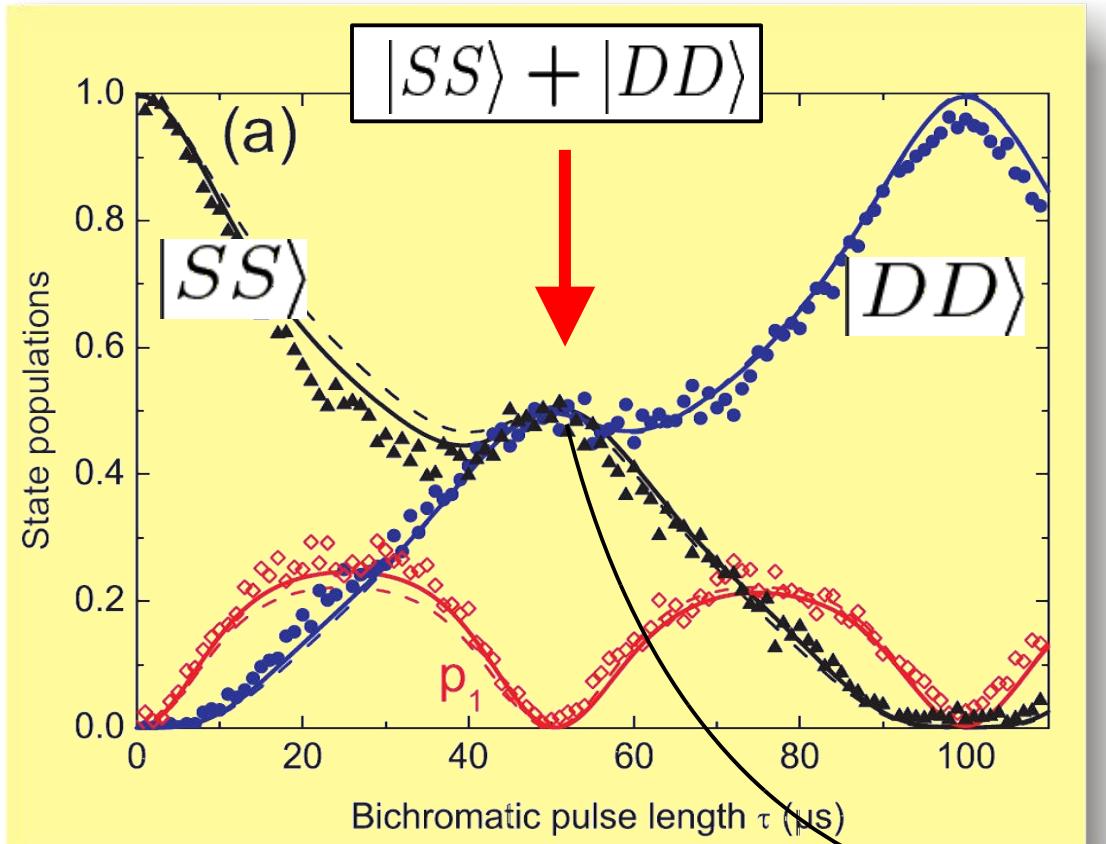


Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

→ Measurement of the density matrix:



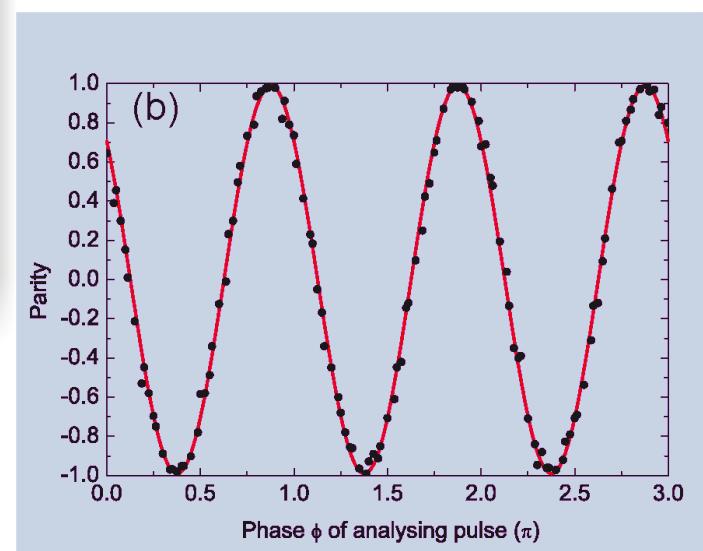


gate duration $51 \mu\text{s}$
 average fidelity

$$F_{\text{MS}} = 99.3(0.2)\%$$

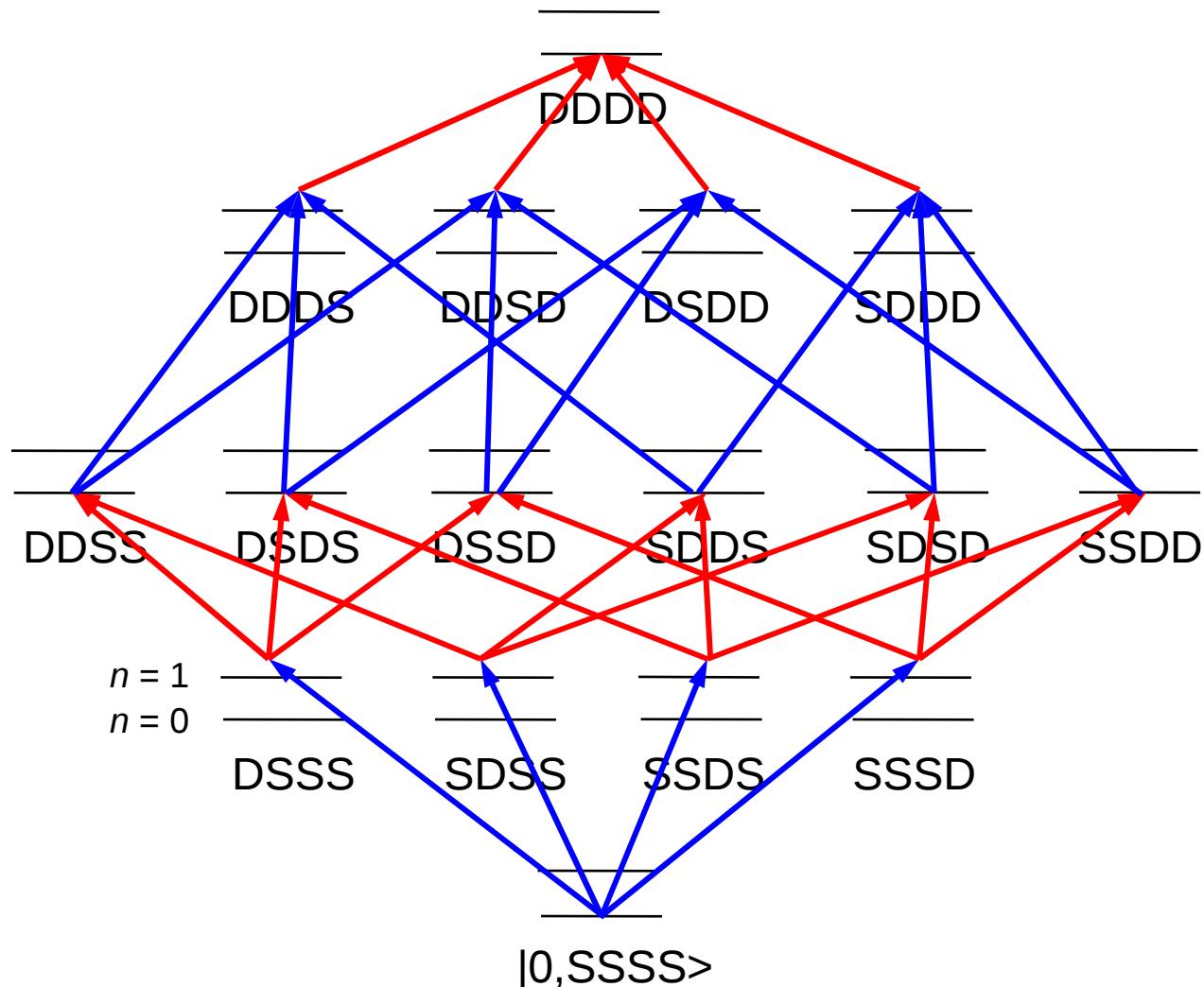
J. Benhelm, G. Kirchmair,
 C. Roos
 Theory: C. Roos,
 New J. Phys. **10**,
 013002 (2008)

measure entanglement
 via parity oscillations





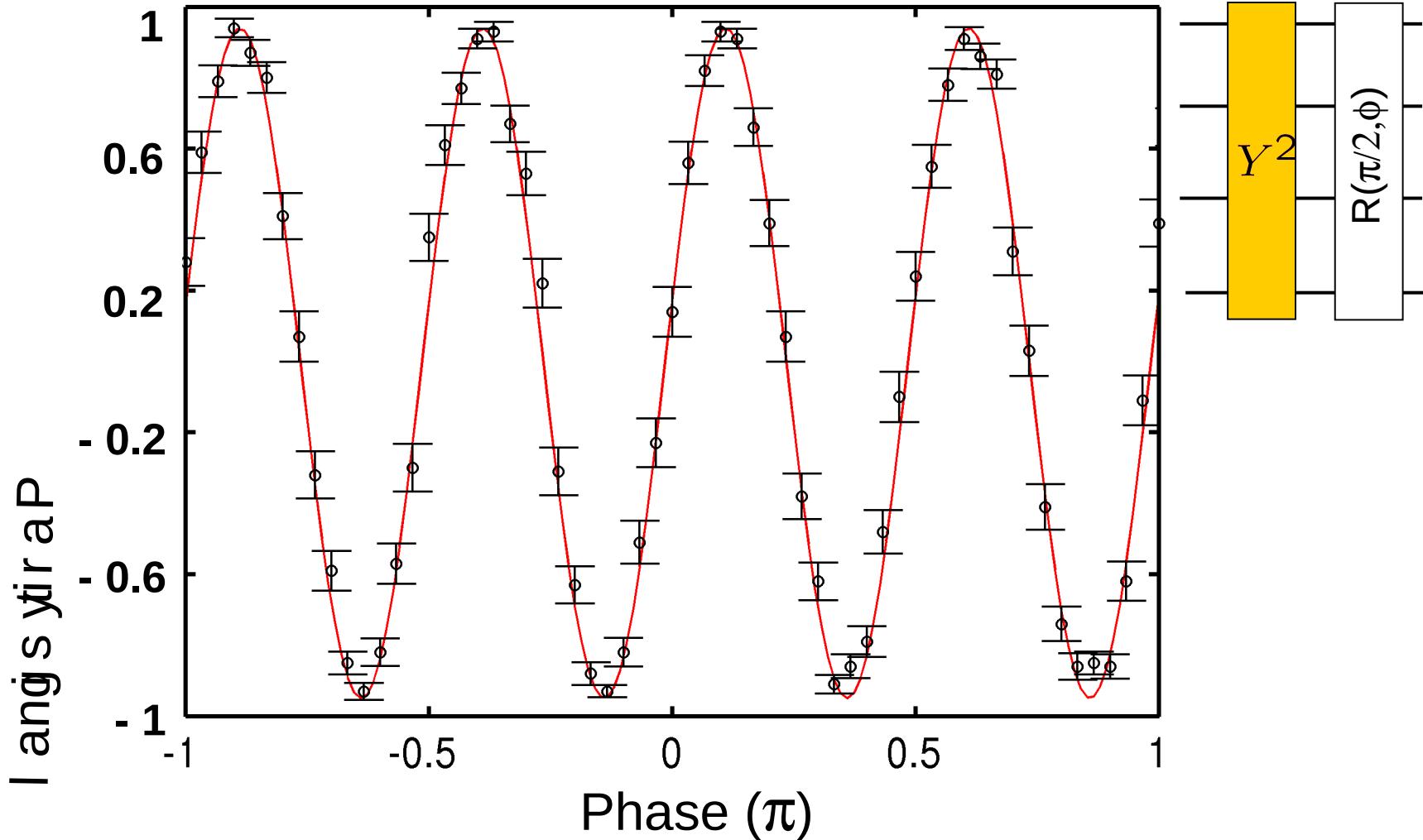
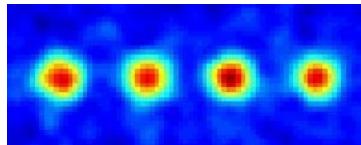
Entangling four ions



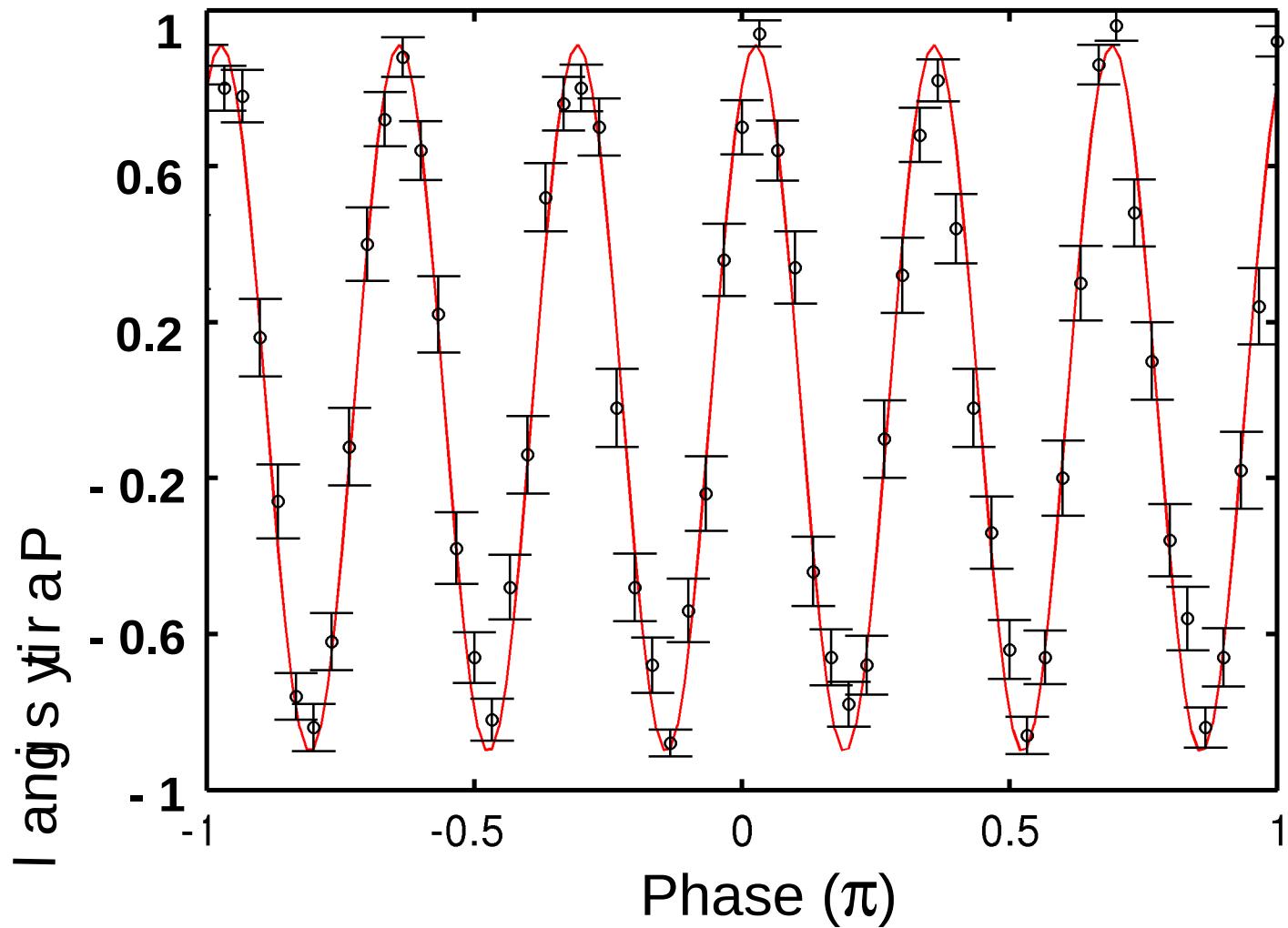
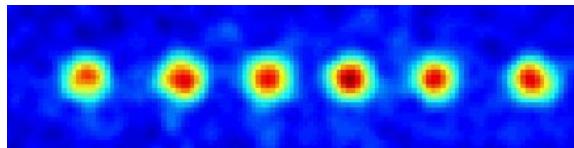
$$\rightarrow (|SSSS\rangle + |DDDD\rangle)/\sqrt{2}$$



Four-ion GHZ state

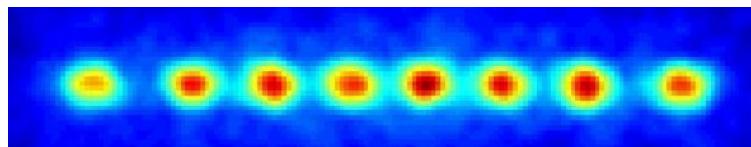


See also: Leibfried et al., Nature 438, 639 (2005)

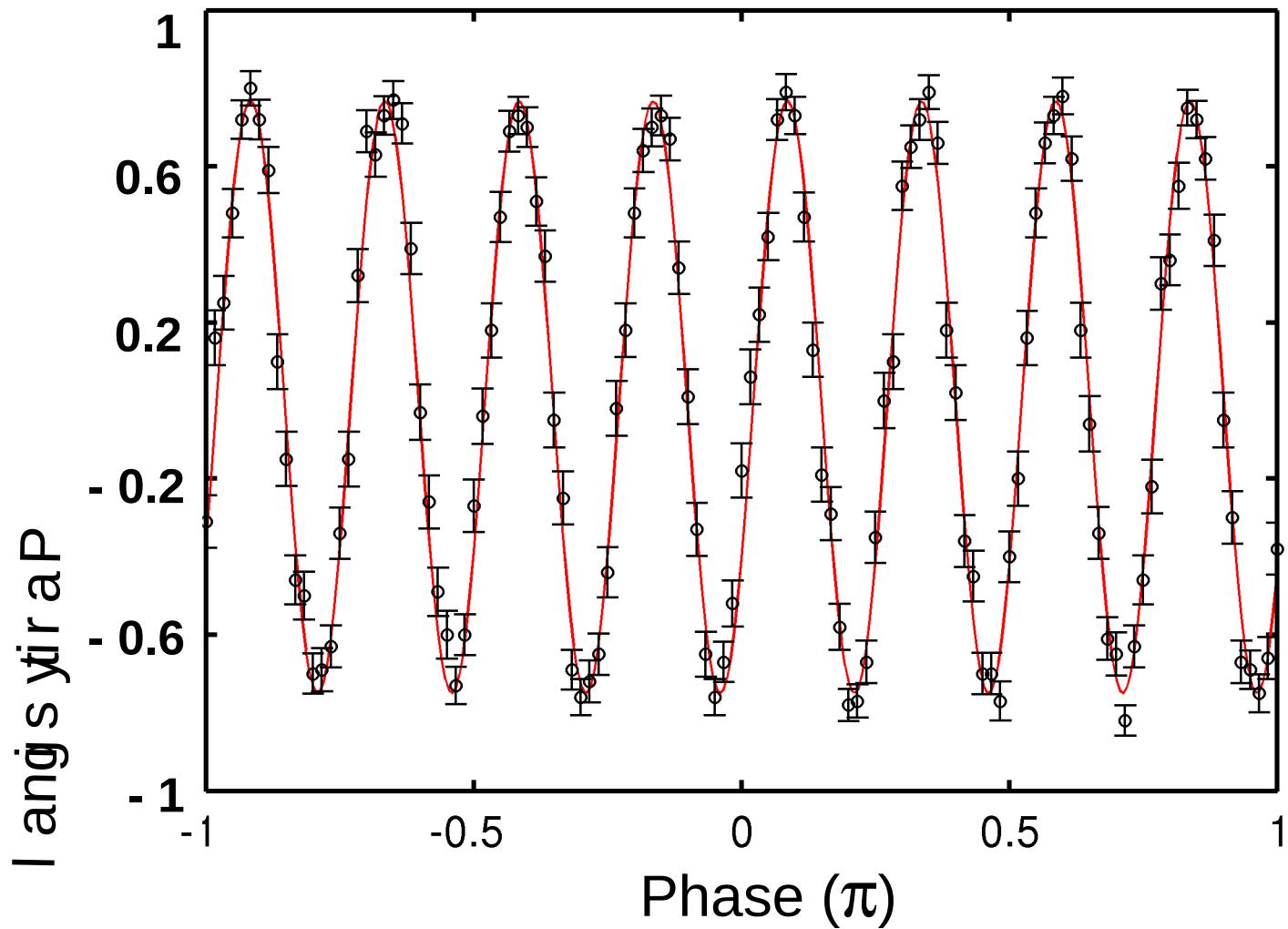


See also: Leibfried *et al.*, Nature 438, 639 (2005)

Eight-ion GHZ state

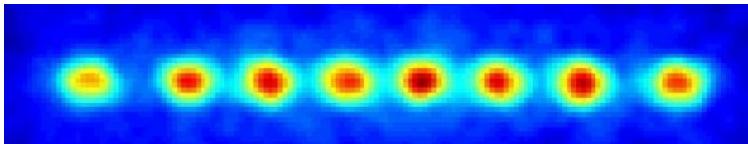


T. Monz, P. Schindler, J. Barreiro,
M. Hennrich, R. Blatt





Eight-ion GHZ state



T. Monz, P. Schindler, J. Barreiro,
M. Hennrich, R. Blatt

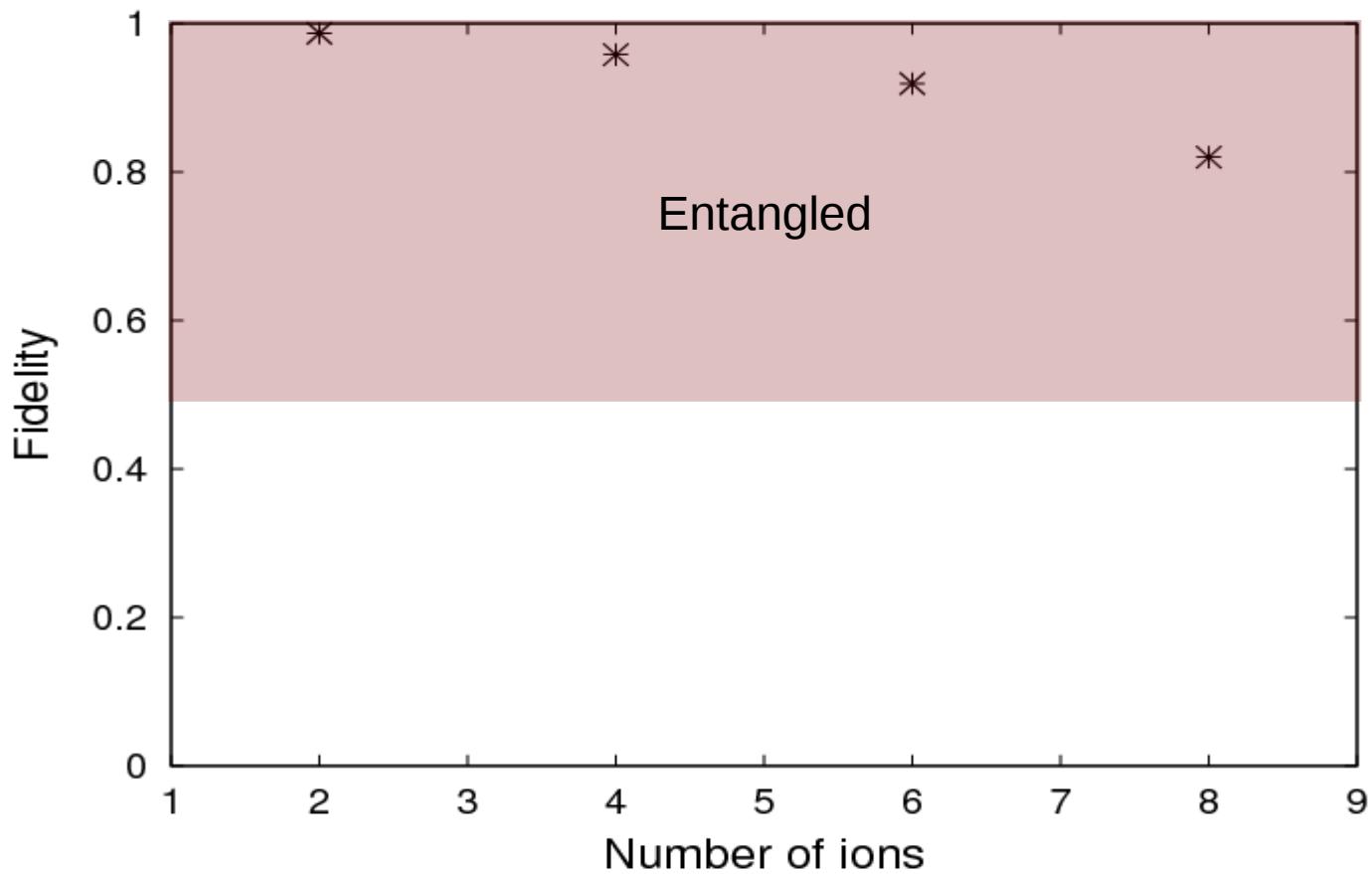
$$\begin{aligned} |\Psi\rangle &= |SSSSSSSS\rangle + |DDDDDDDD\rangle \\ &= |S\rangle |\text{alive}\rangle + |D\rangle |\text{dead}\rangle \end{aligned}$$



Image-source: wikipedia



GHZ-state fidelities

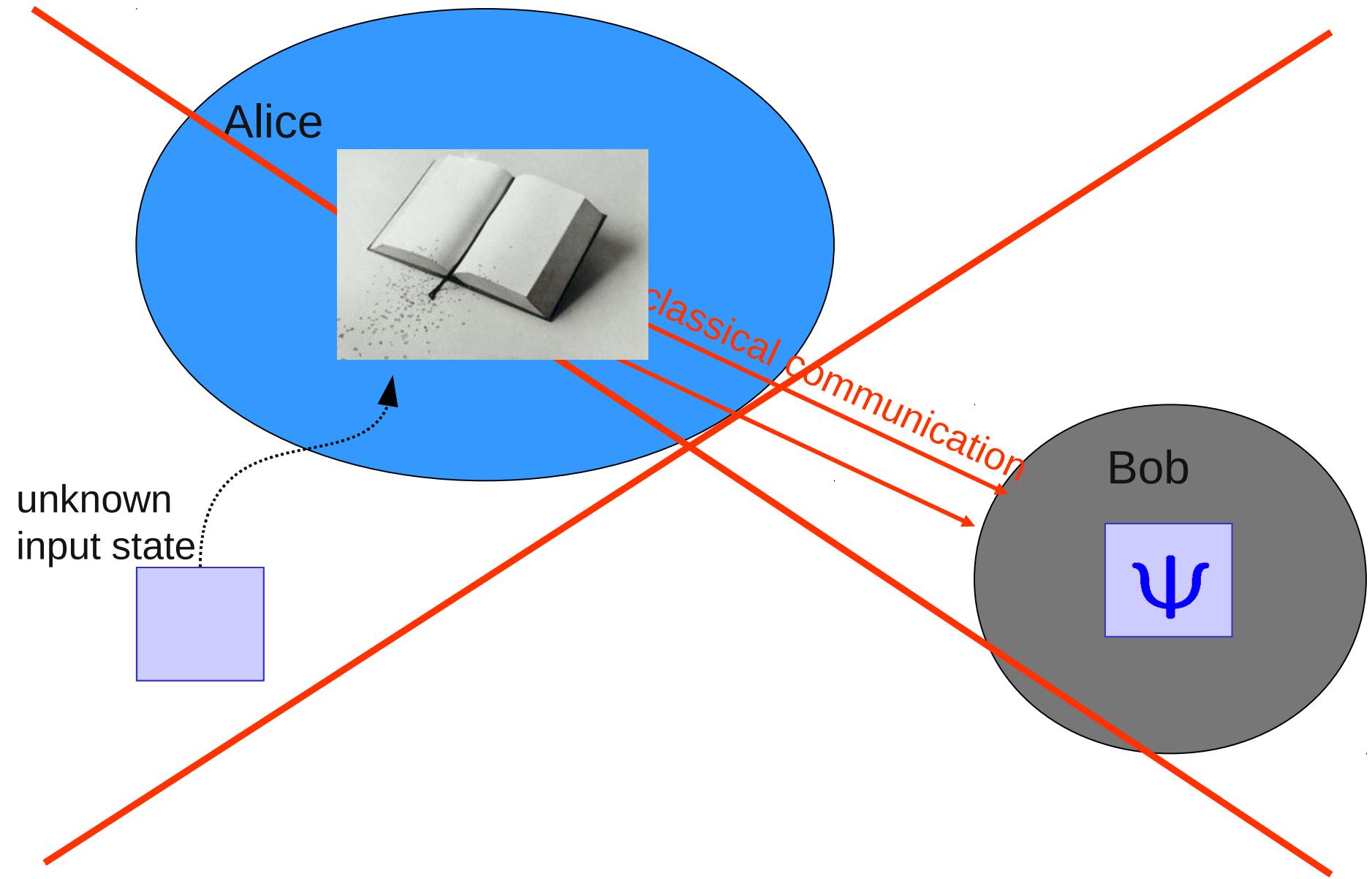




- Physics and information
- Ion trap quantum computing
- Teleportation
- Scaling of ion trap quantum computers

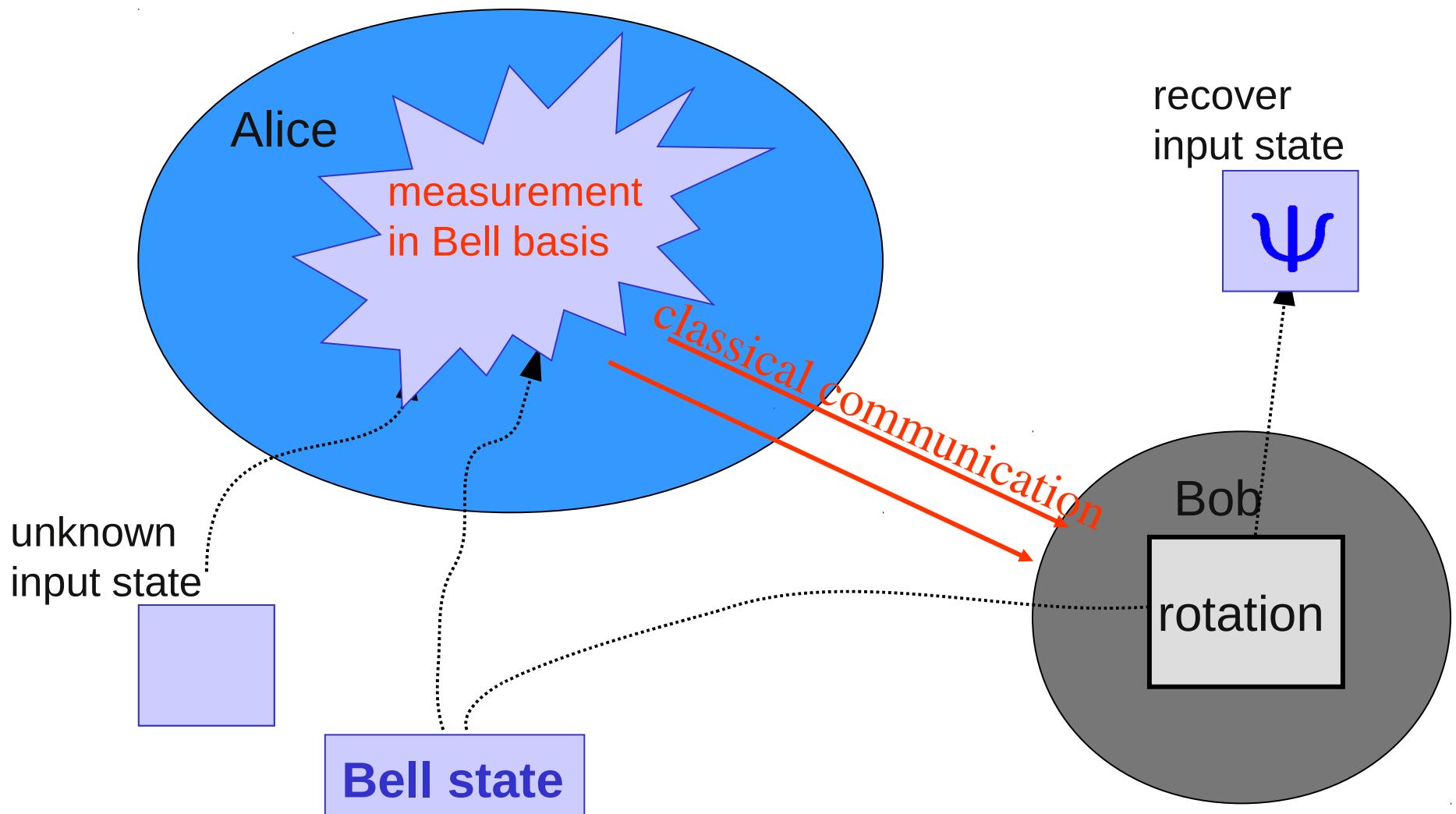


Teleportation





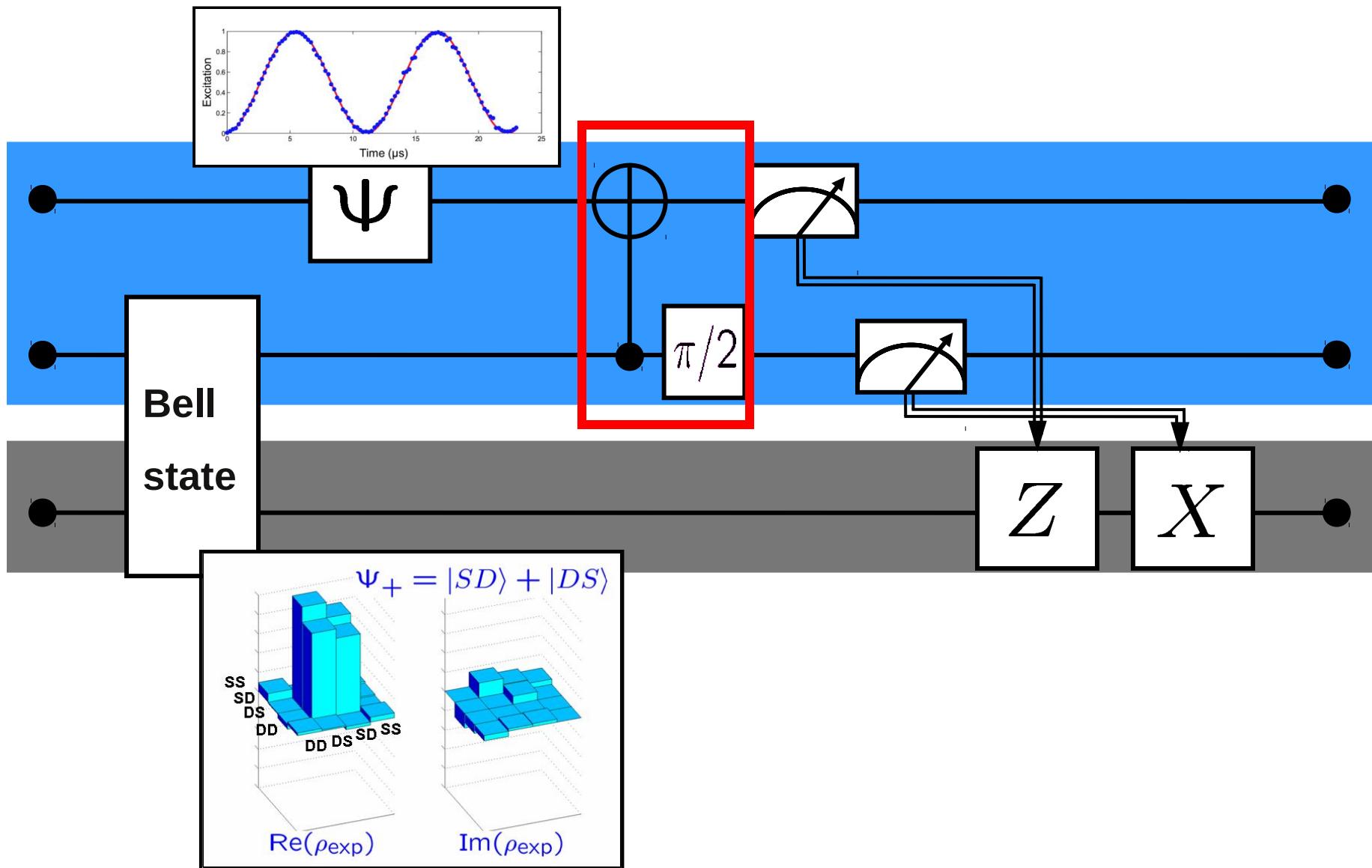
Teleportation



Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).
Bouwmeester *et al.*, Nature **390**, 575 (1997).

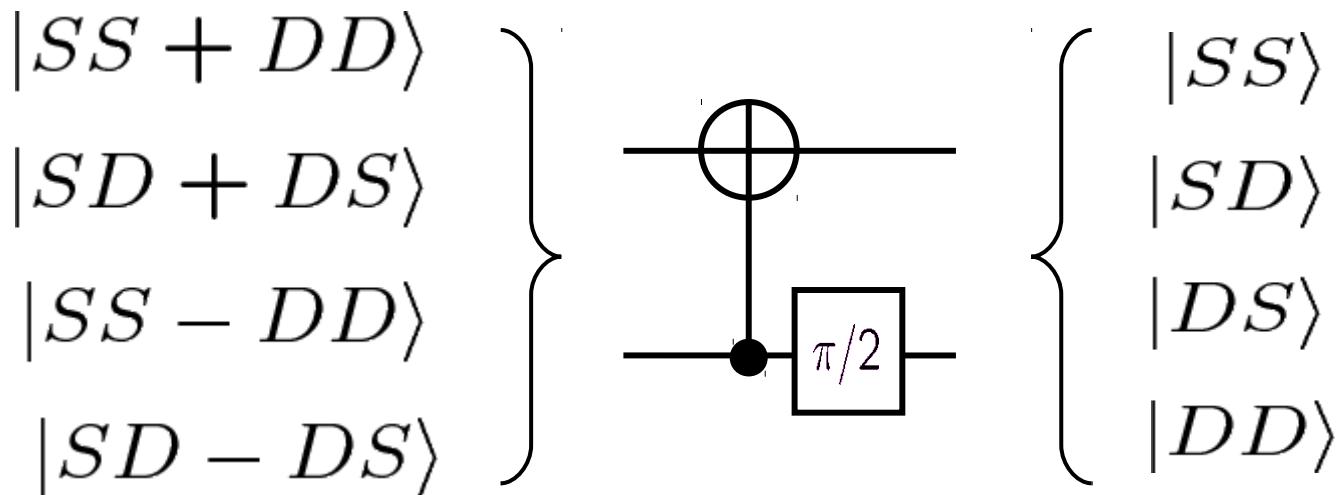


Teleportation





Bell measurement





Ion trap quantum computing



VOLUME 74, NUMBER 20

PHYSICAL REVIEW LETTERS

15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

PACS numbers: 89.80.+h, 03.65.Bz, 12.20.Fv, 32.80.Pj

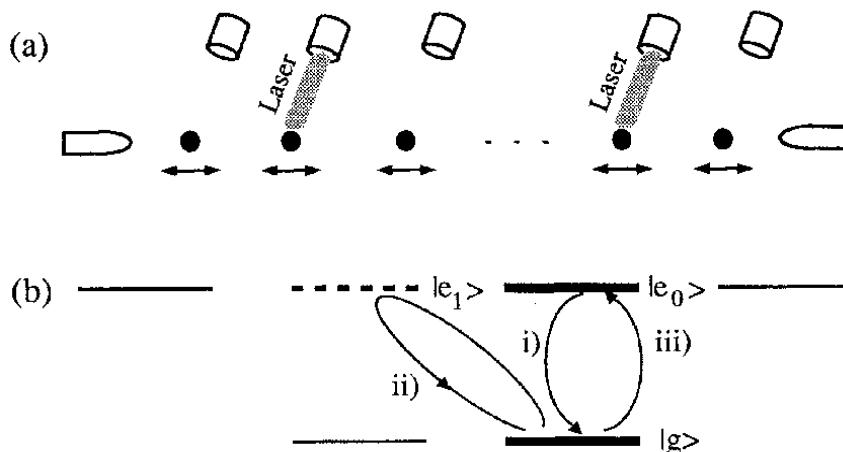


FIG. 1. (a) N ions in a linear trap interacting with N different laser beams; (b) atomic level scheme.

controlled – NOT :

$$|\varepsilon_1\rangle|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle|\varepsilon_1 \oplus \varepsilon_2\rangle$$

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$

$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$

$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$

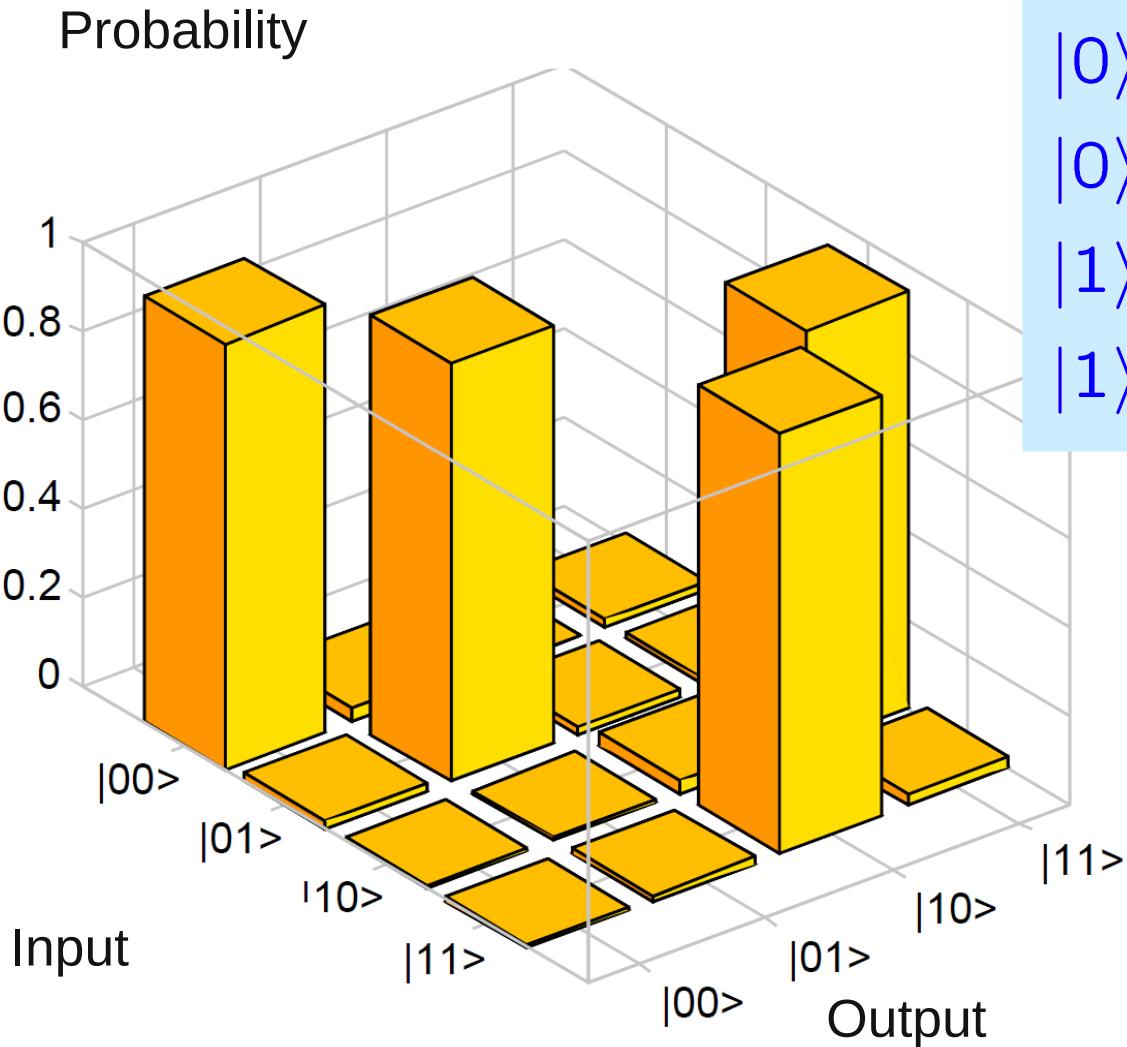
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

control bit target bit





Truth table of a controlled NOT gate



| | | |
|----------------------|---------------|----------------------|
| $ 0\rangle 0\rangle$ | \rightarrow | $ 0\rangle 0\rangle$ |
| $ 0\rangle 1\rangle$ | \rightarrow | $ 0\rangle 1\rangle$ |
| $ 1\rangle 0\rangle$ | \rightarrow | $ 1\rangle 1\rangle$ |
| $ 1\rangle 1\rangle$ | \rightarrow | $ 1\rangle 0\rangle$ |

→ universal set of quantum gates



Another way to generate Bell states

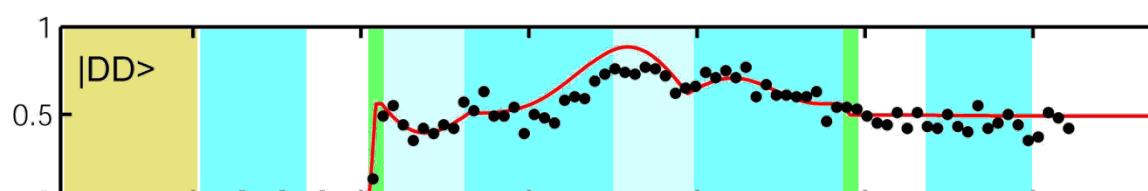
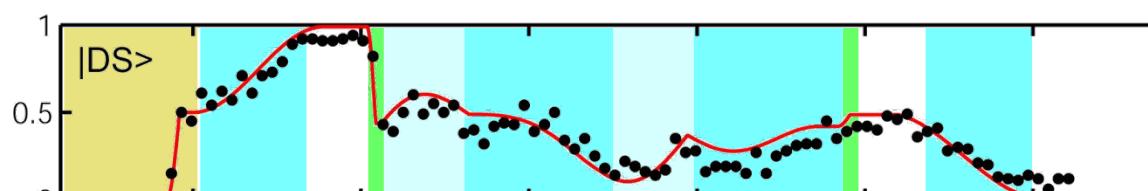
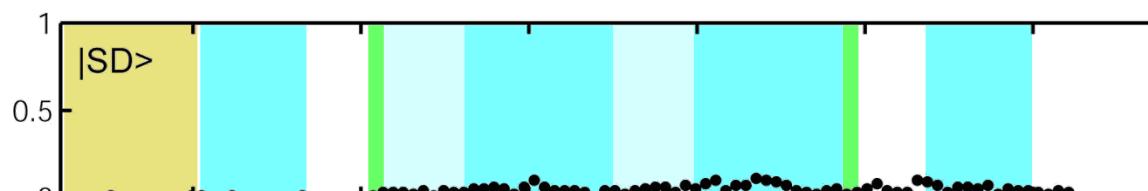
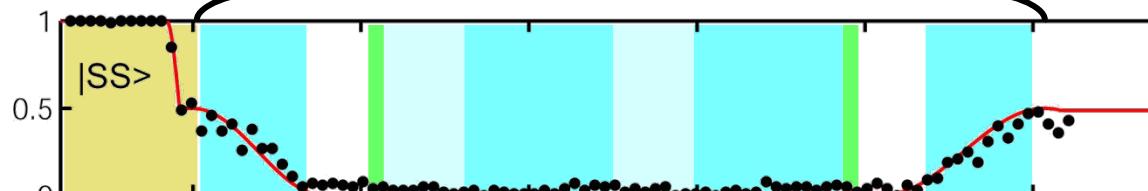


$$|SS\rangle \rightarrow |S+D\rangle|S\rangle \quad \xrightarrow{\text{CNOT}} \quad |SS\rangle + |DD\rangle$$

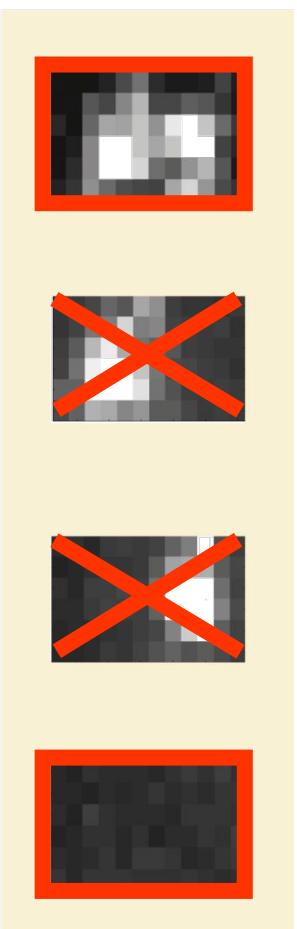
prepare

CNOT

output

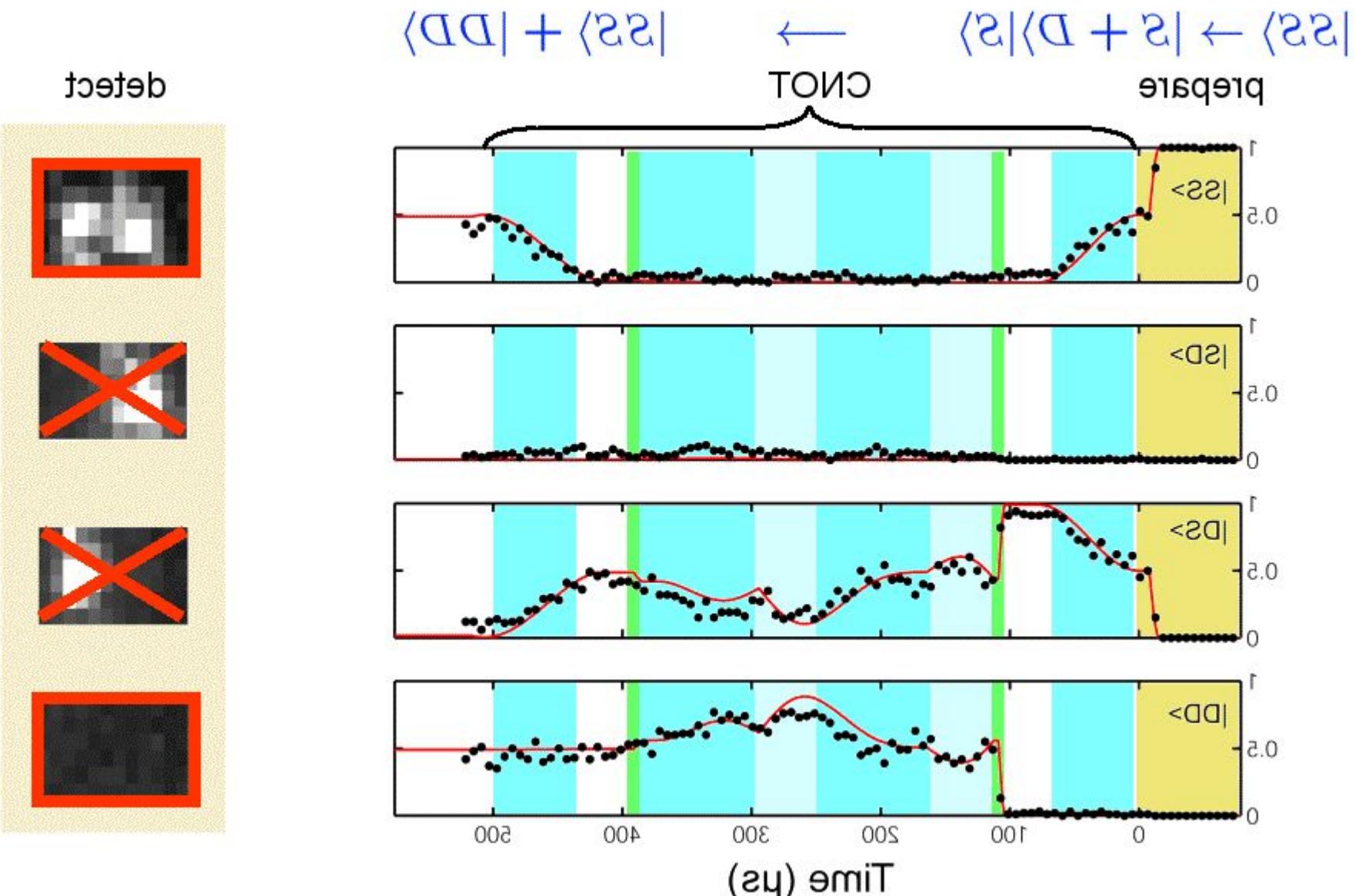


Time (μs)



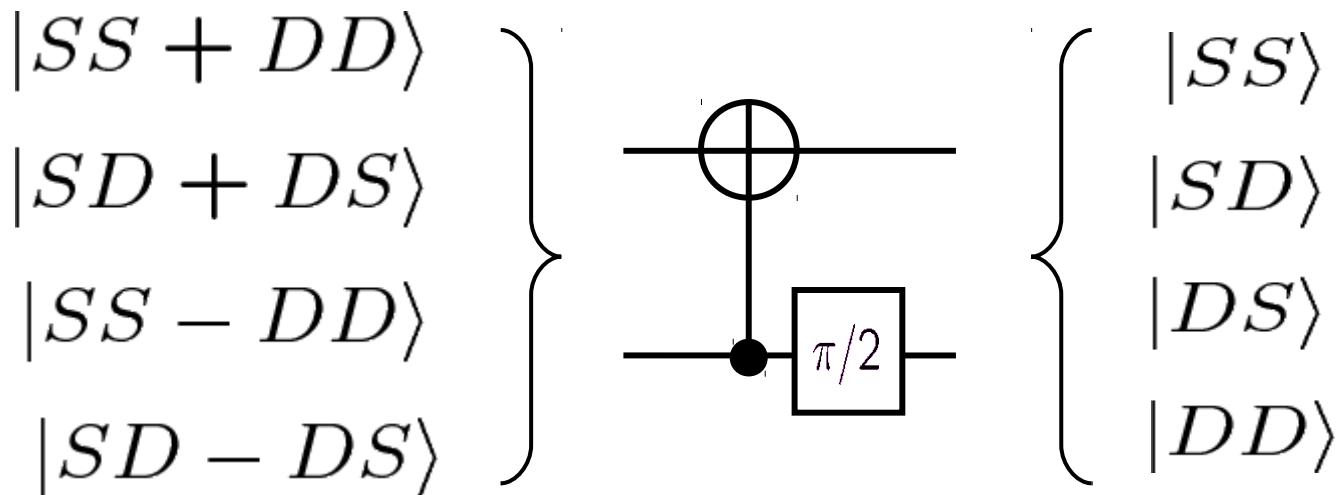


And now backwards



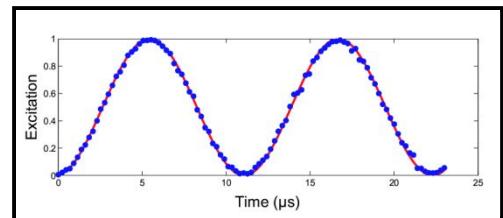
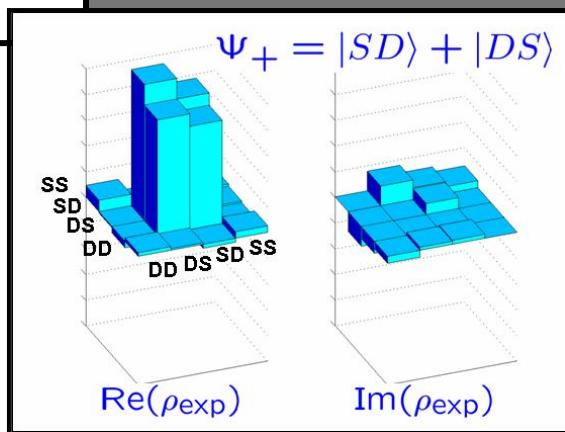
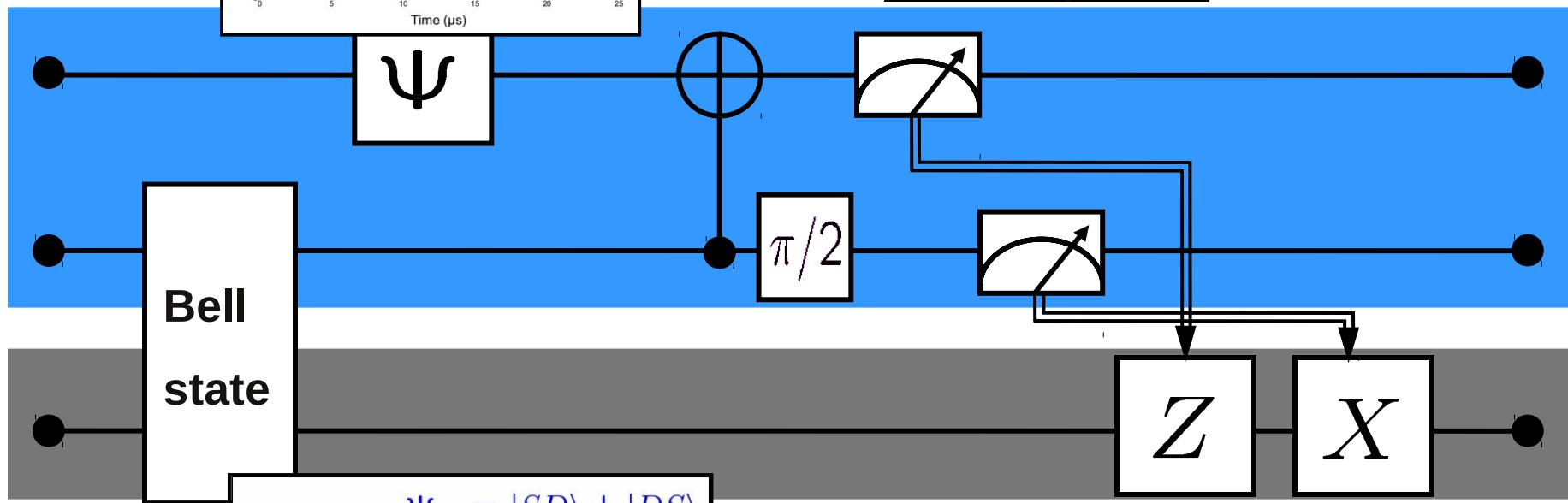
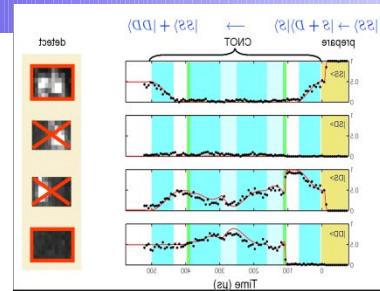
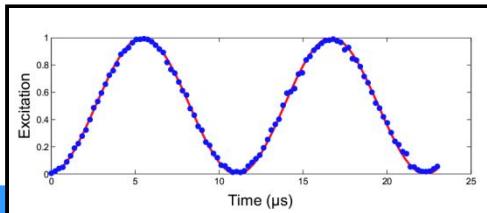


Bell measurement



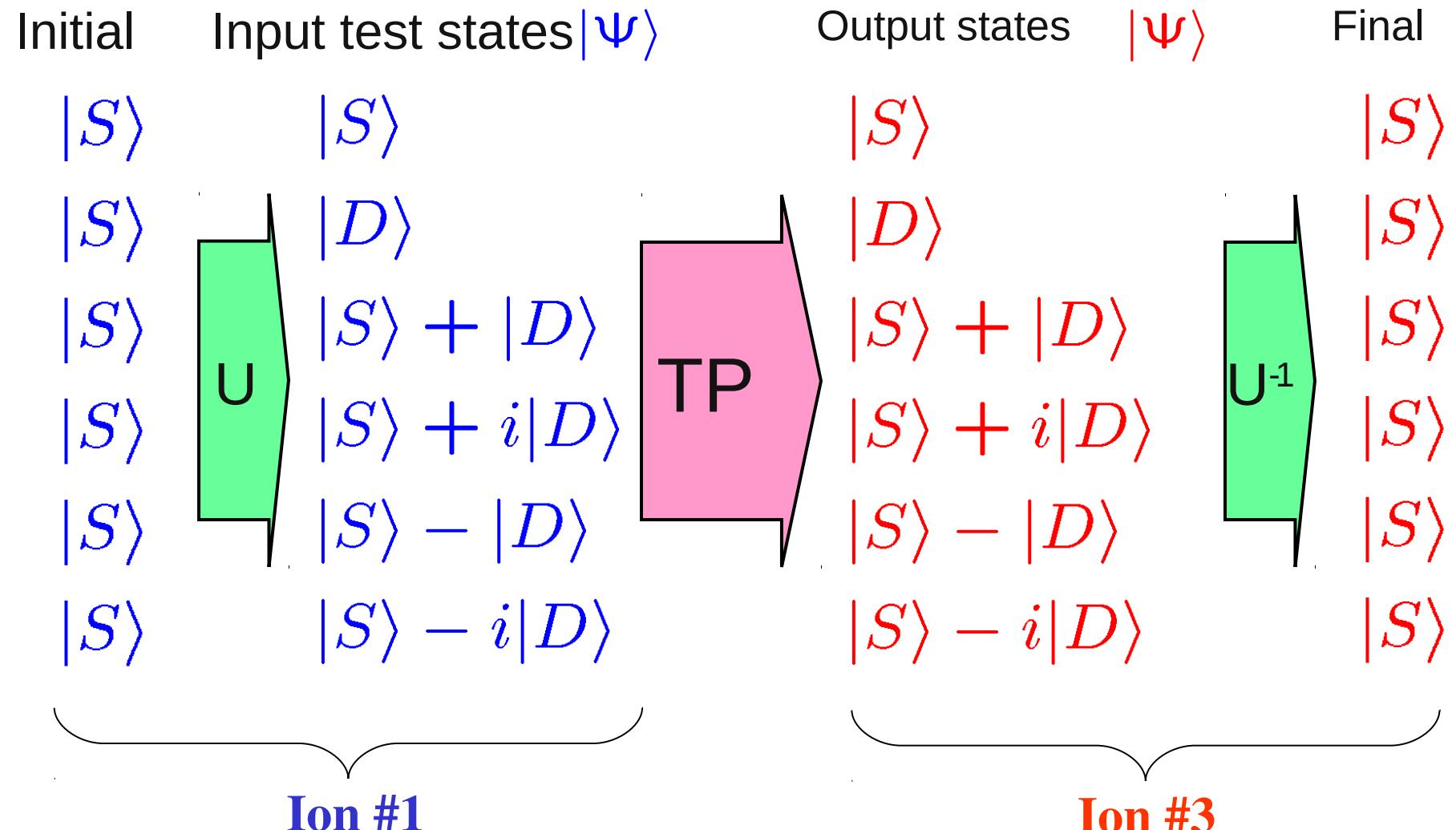


Teleportation

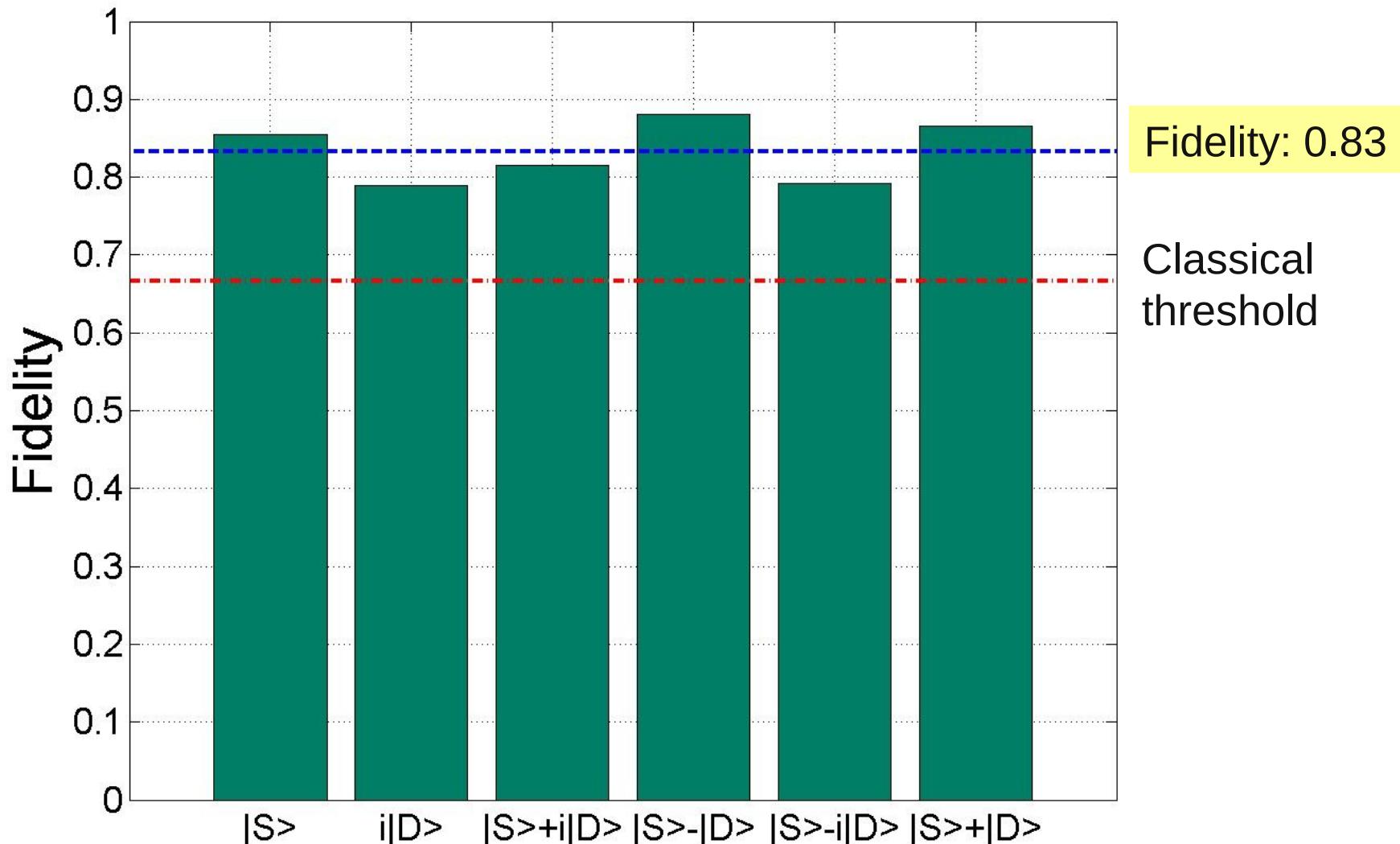




Teleportation analysis



Deterministic teleportation



“Deterministic teleportation with atoms”

Barrett et al., Nature **429**, 737 (2004) and Riebe et al., Nature **429**, 734 (2004)

Classical computer

- Initialization
- 1-bit operations (NOT)
- 2-bit gates (e.g. NAND)

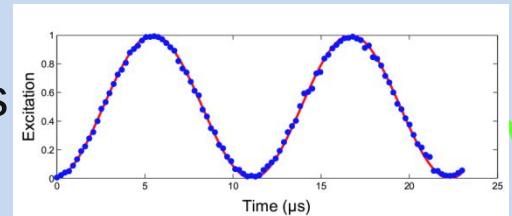
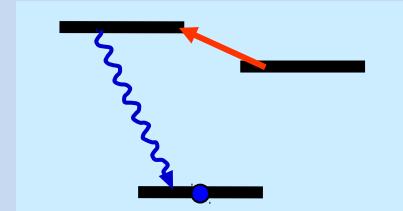
Computational space:

00
01
10
11

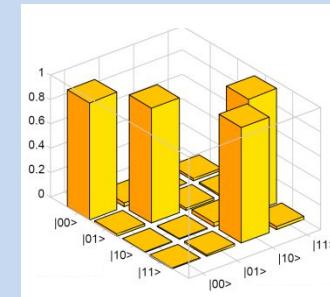
- Read out
→ result

Quantum computer

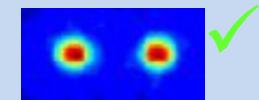
- Initialization
- 1-qubit rotations
→ superpositions
- 2-qubit gates (CNOT gate)
→ entanglement



Computational space:

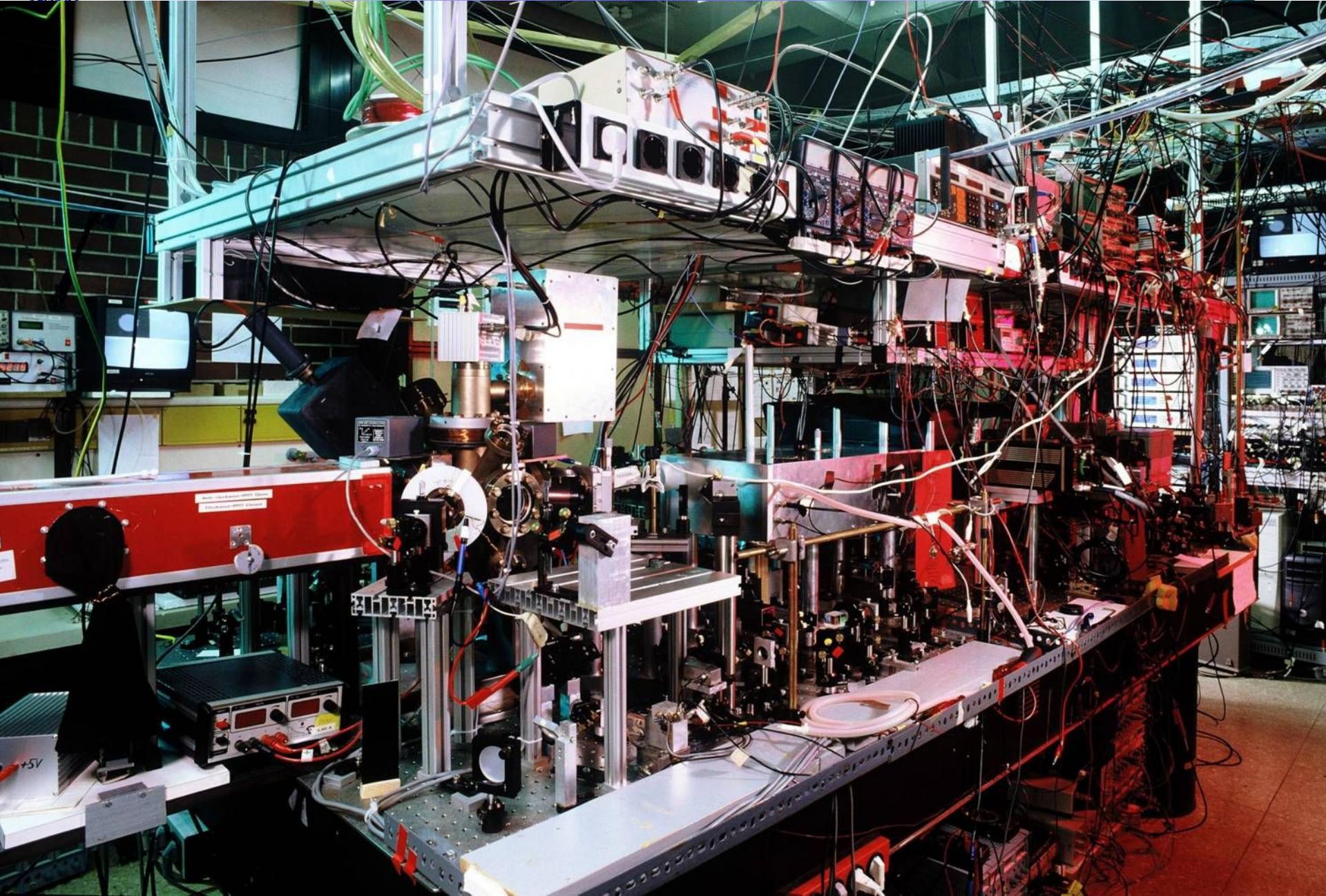


- Read out of qubits
→ gain of classical information





The hardware

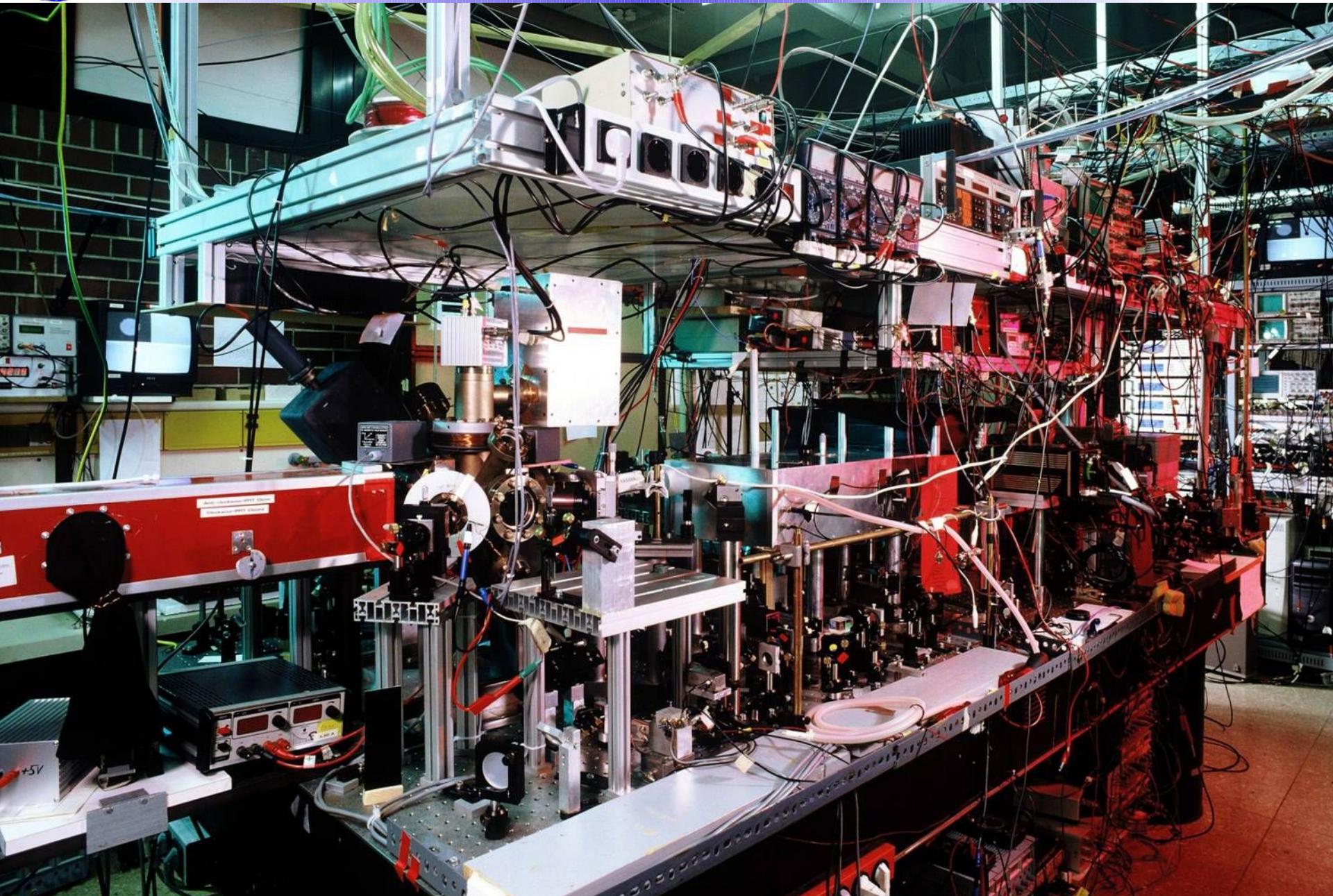




- Physics and information
- Ion trap quantum computing
- Teleportation
- Scaling of ion trap quantum computers

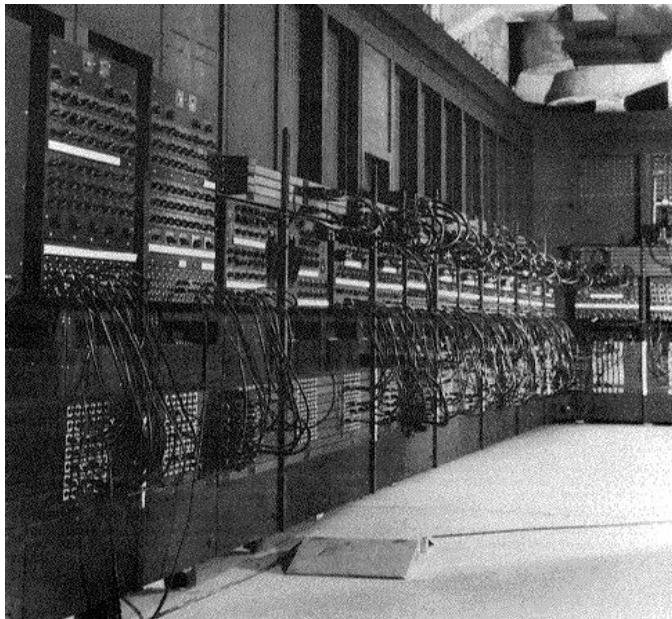


The hardware

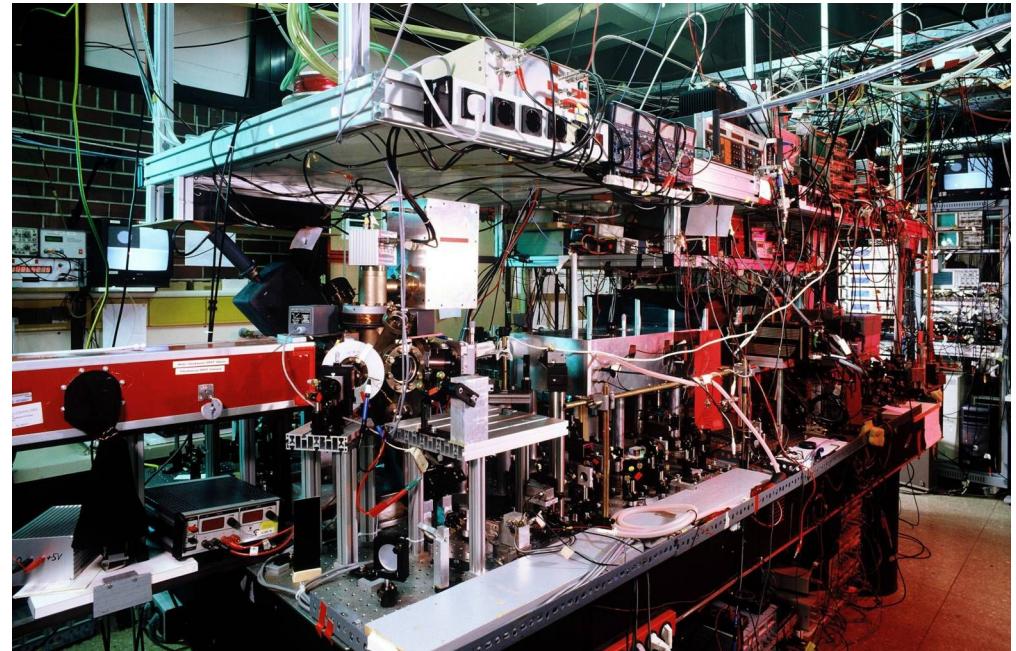




The hardware



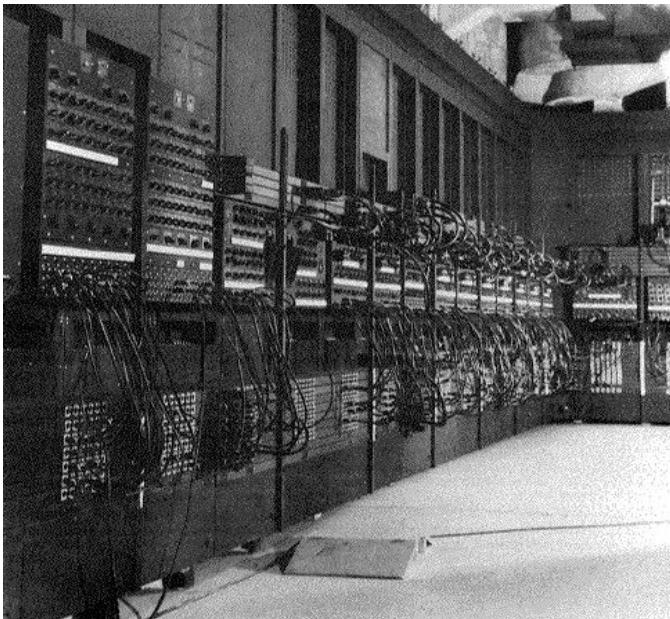
ENIAC, 1950



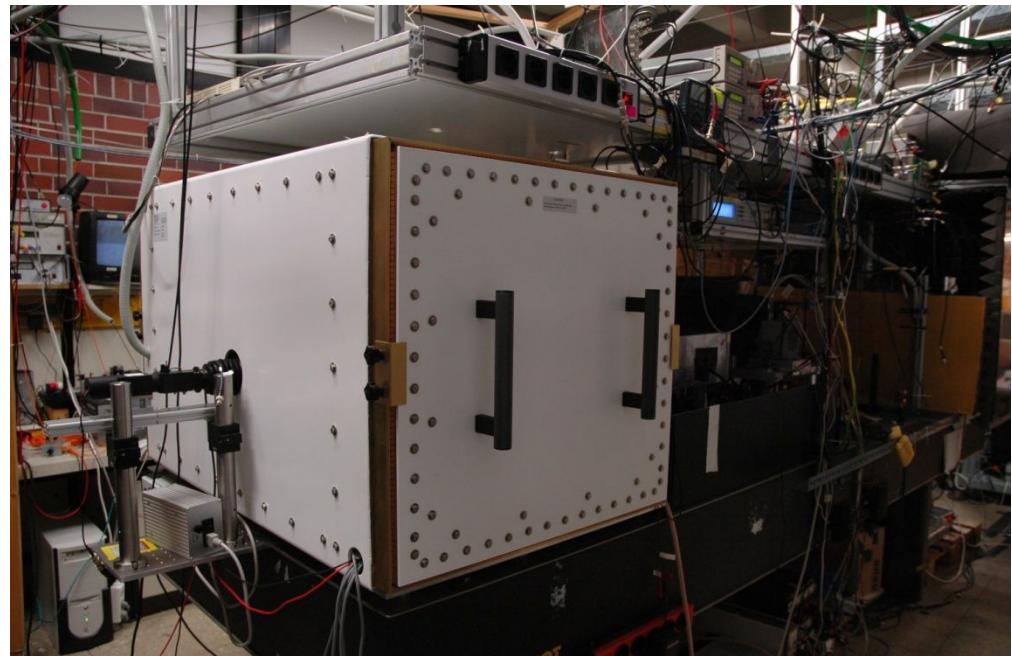
Innsbruck quantum computer, 2005



The hardware



ENIAC, 1950



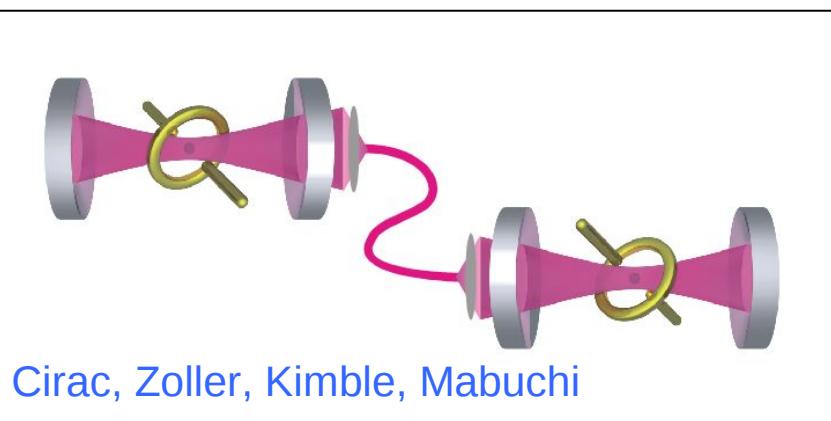
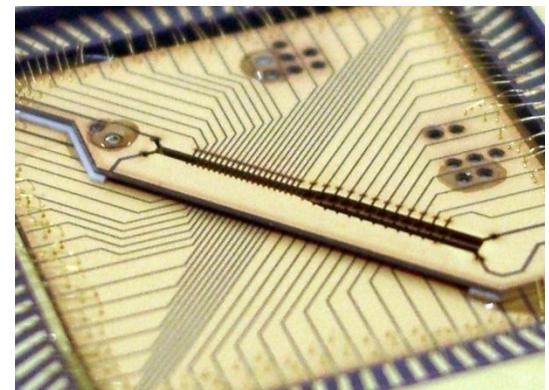
Innsbruck quantum computer, 2009



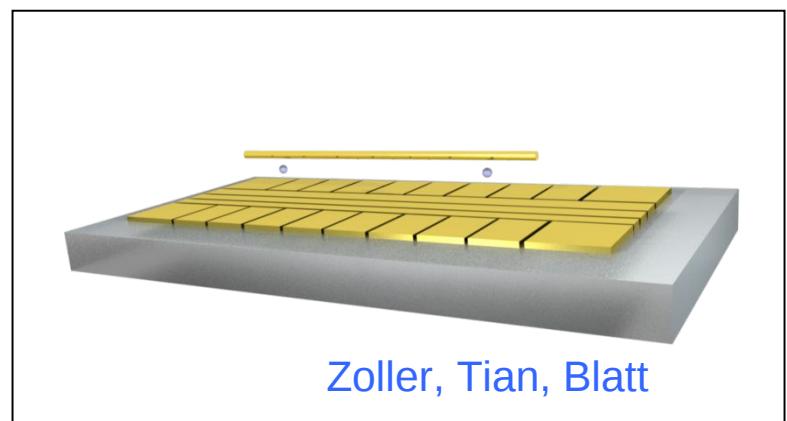
Scaling of ion trap quantum computers



Kielpinski, Monroe, Wineland



Cirac, Zoller, Kimble, Mabuchi



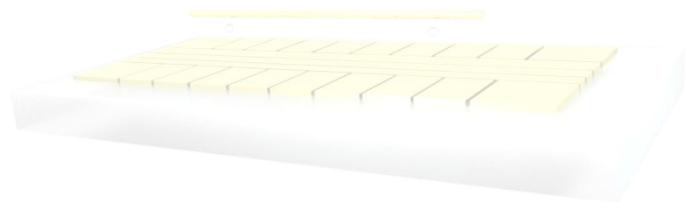
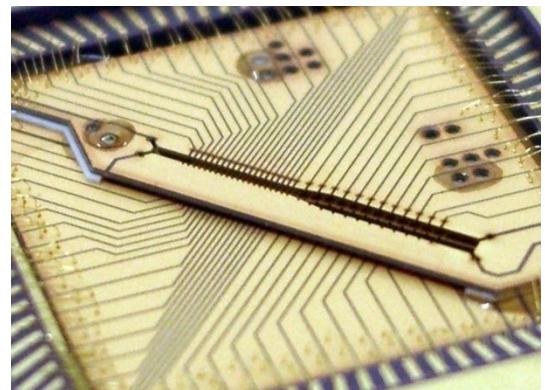
Zoller, Tian, Blatt



Scaling of ion trap quantum computers

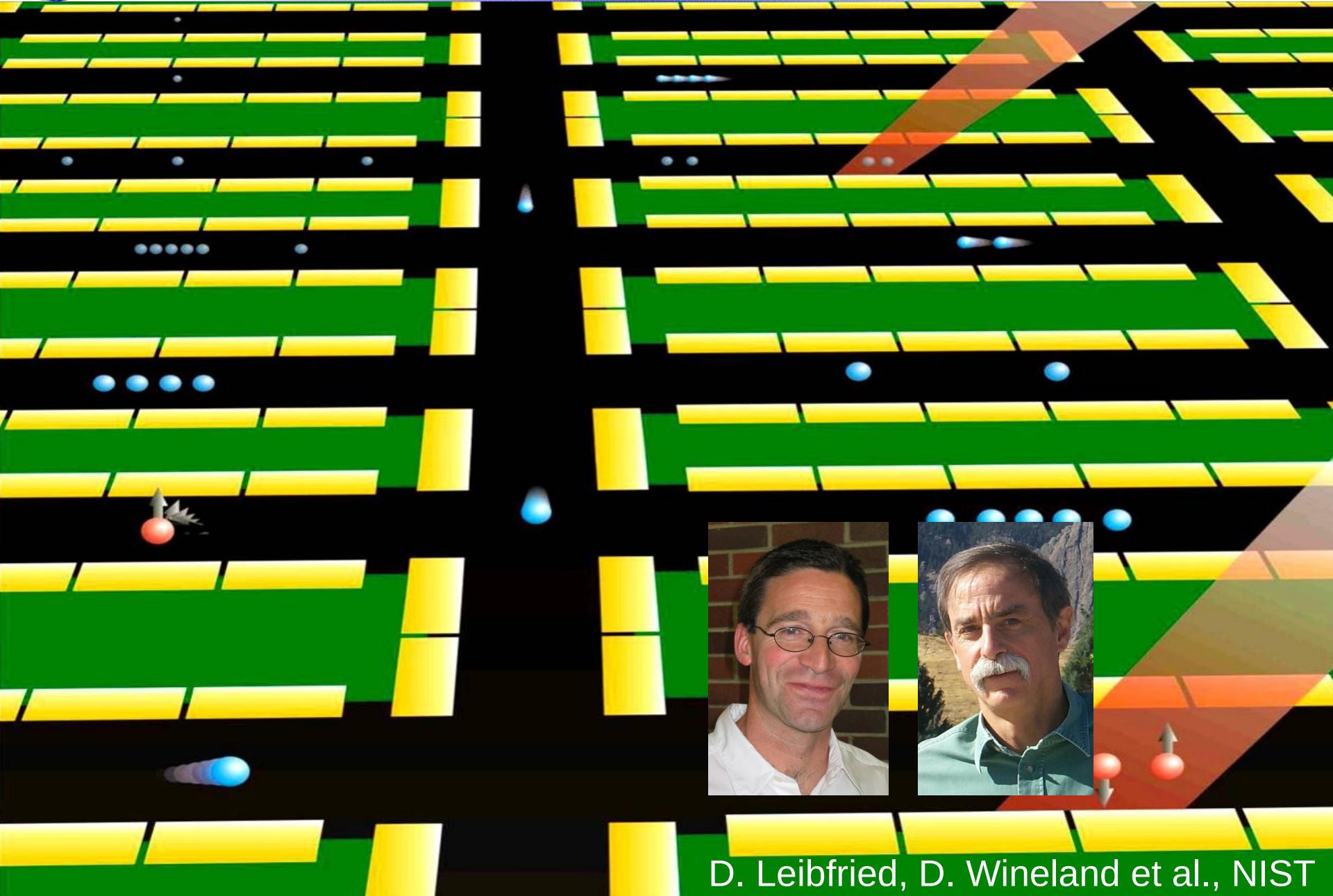


Kielpinski, Monroe, Wineland





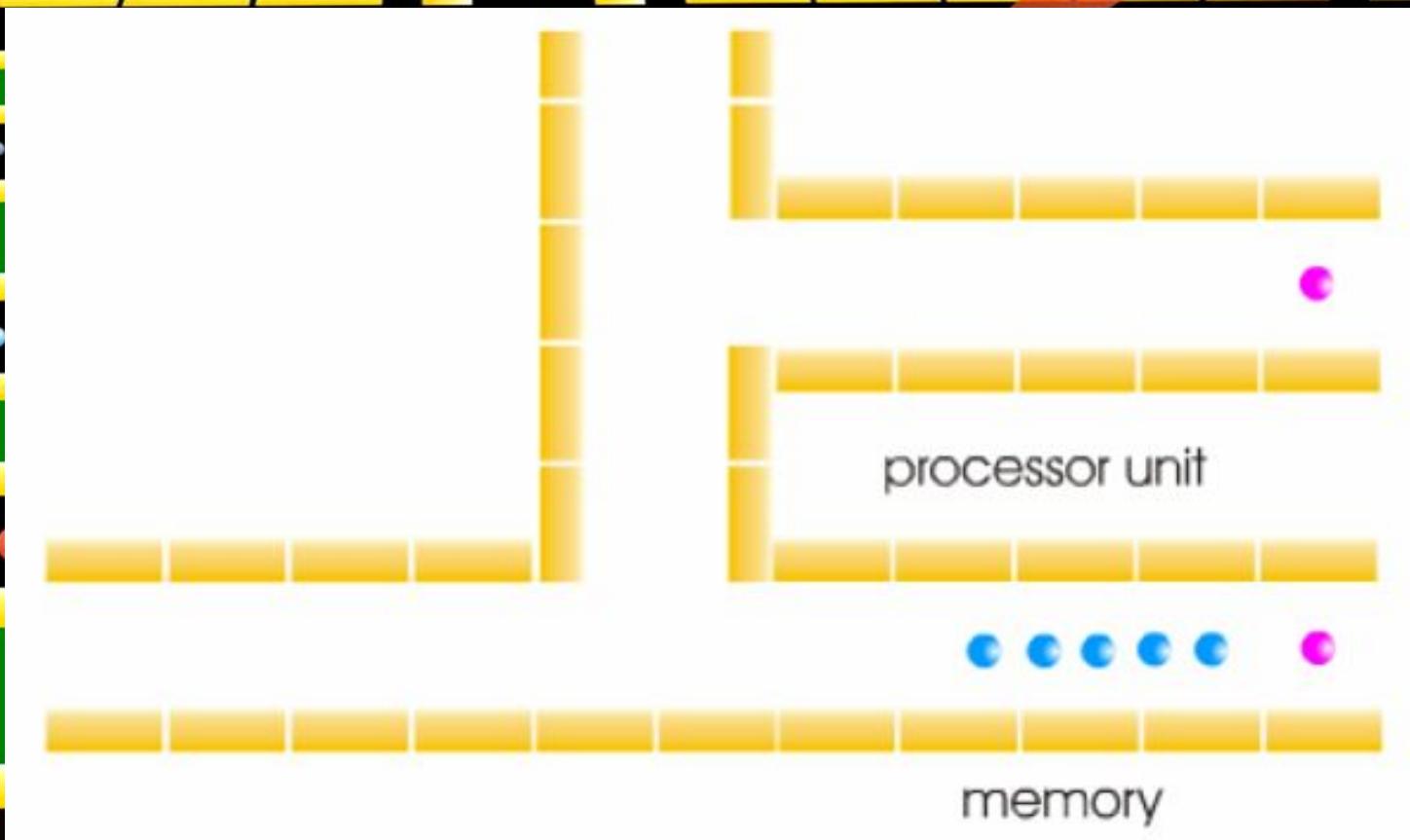
Scaling of ion trap quantum computers



D. Leibfried, D. Wineland et al., NIST

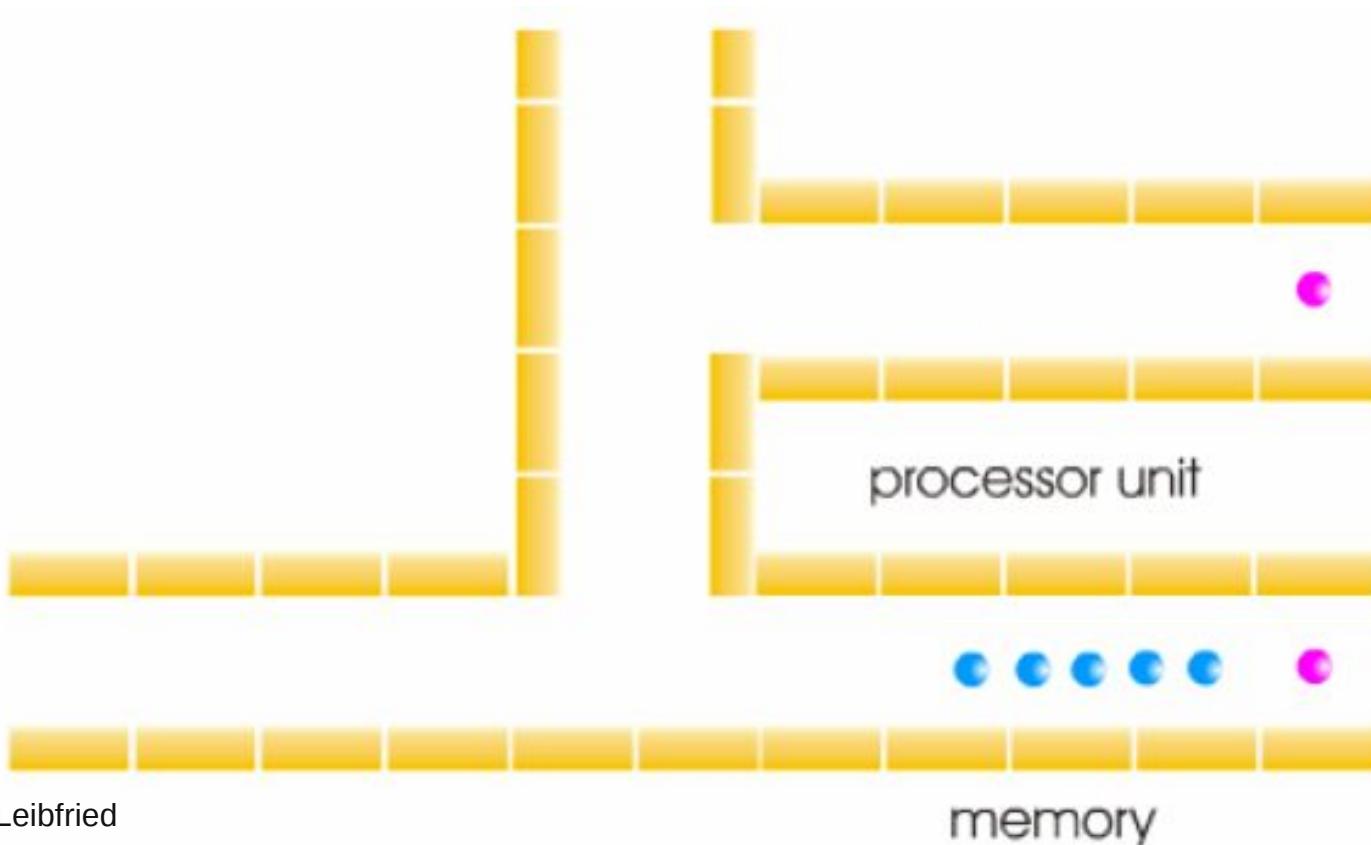


Scaling of ion trap quantum computers





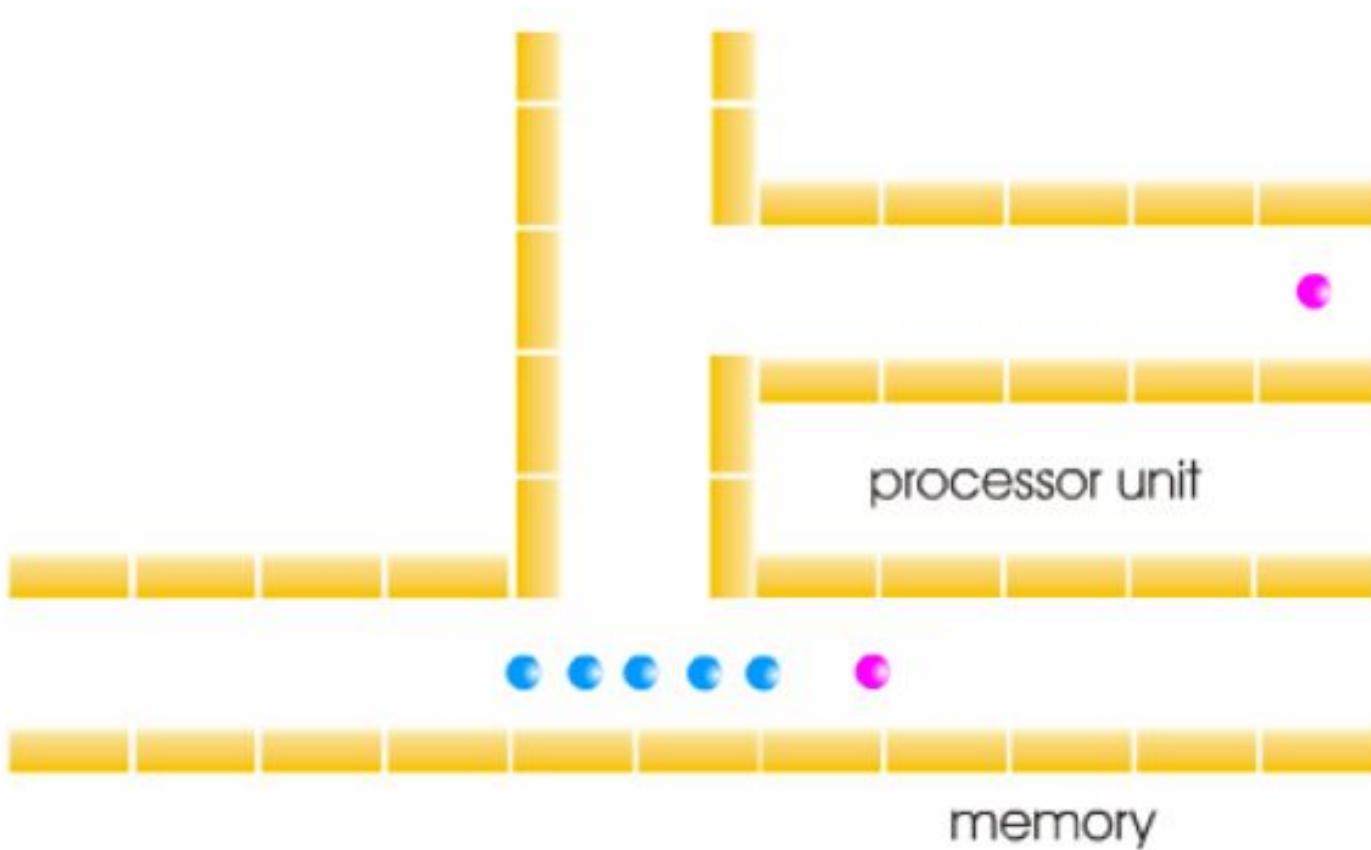
Scaling of ion trap quantum computers



© D. Leibfried

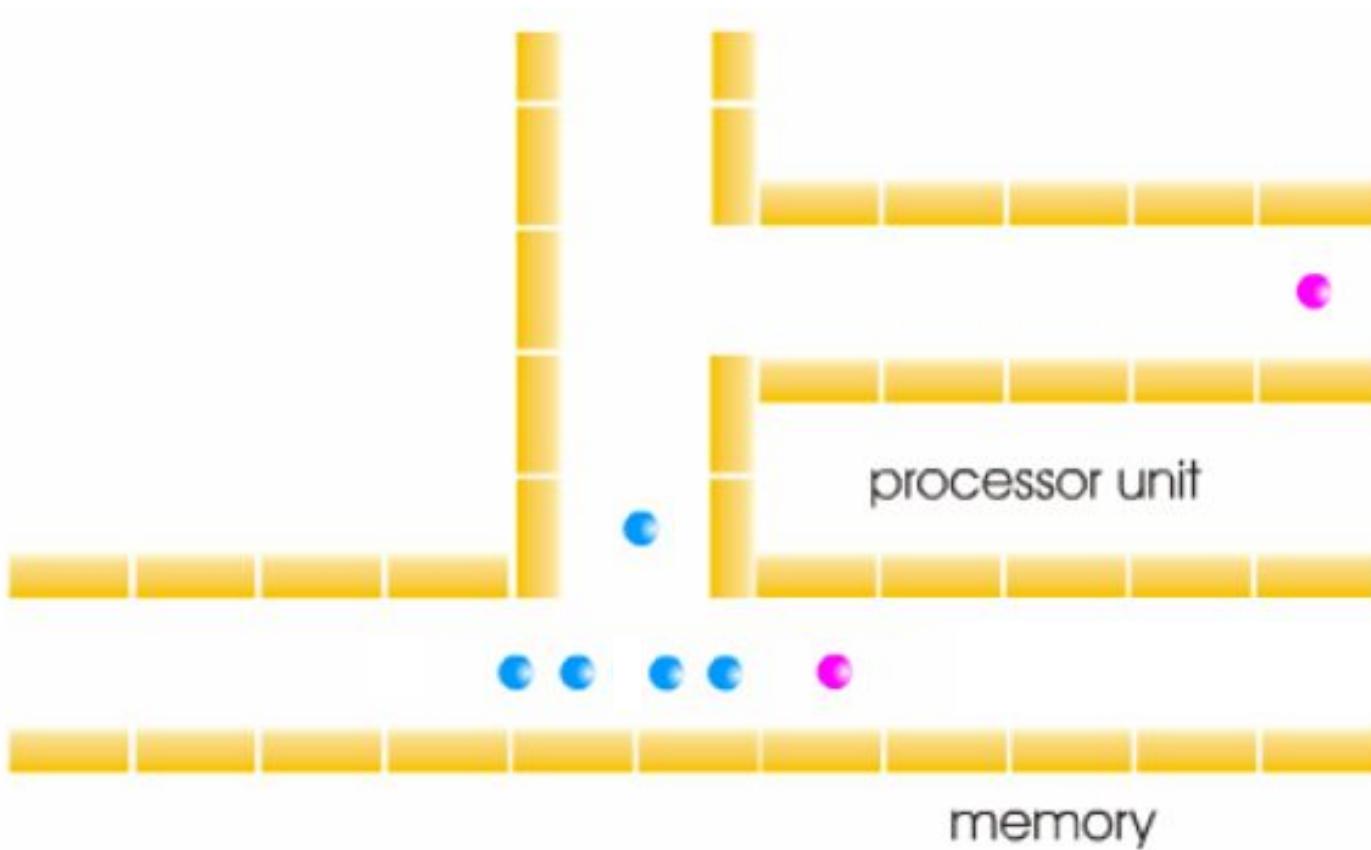


Scaling of ion trap quantum computers



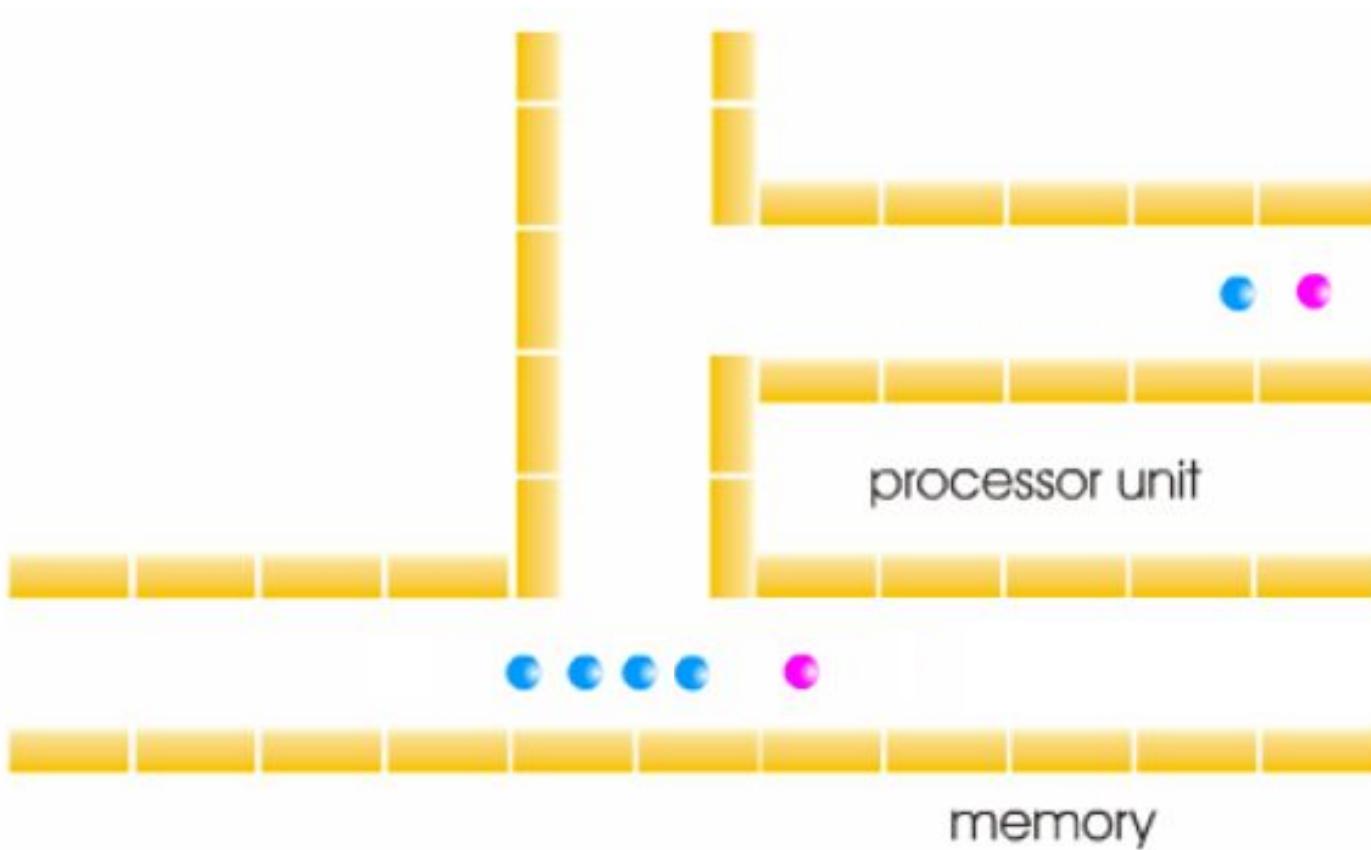


Scaling of ion trap quantum computers



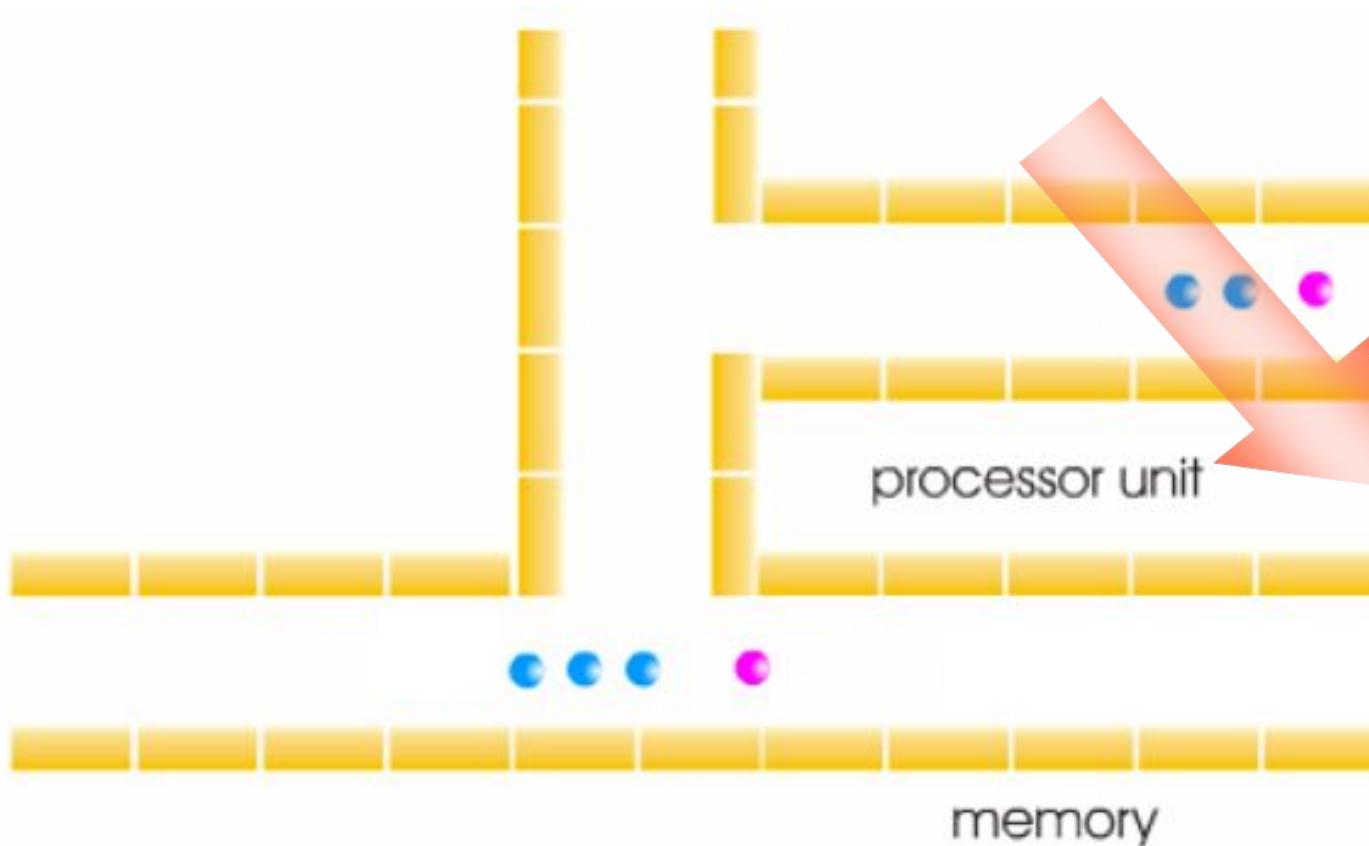


Scaling of ion trap quantum computers



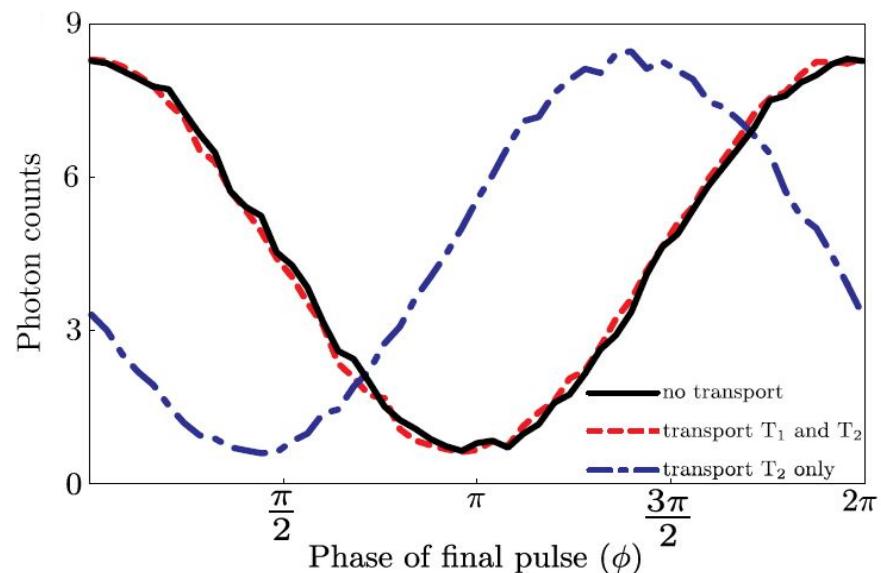
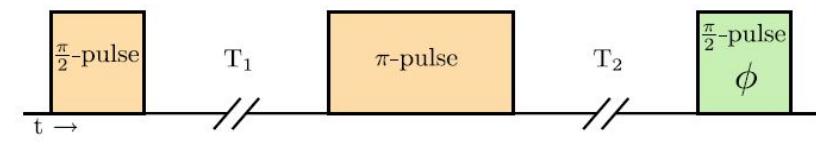
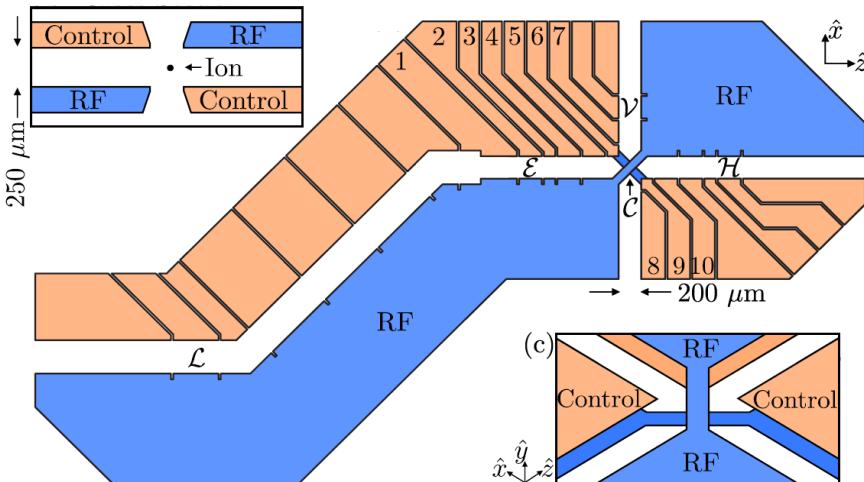


Scaling of ion trap quantum computers



„Architecture for a large-scale ion-trap quantum computer“,
D. Kielpinski et al., Nature **417**, 709 (2002).

Coherent transport through a junction



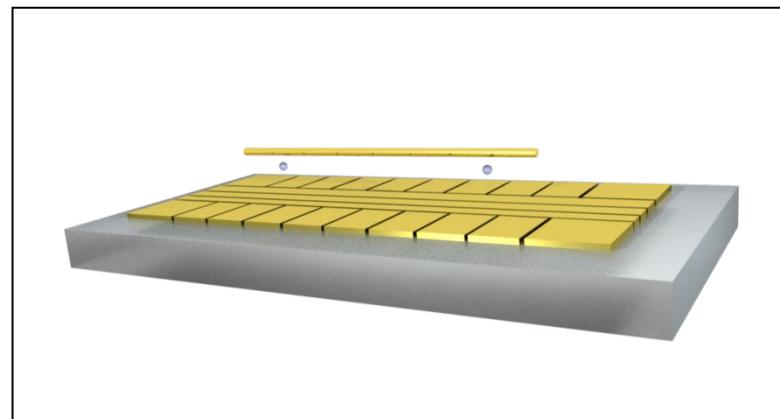
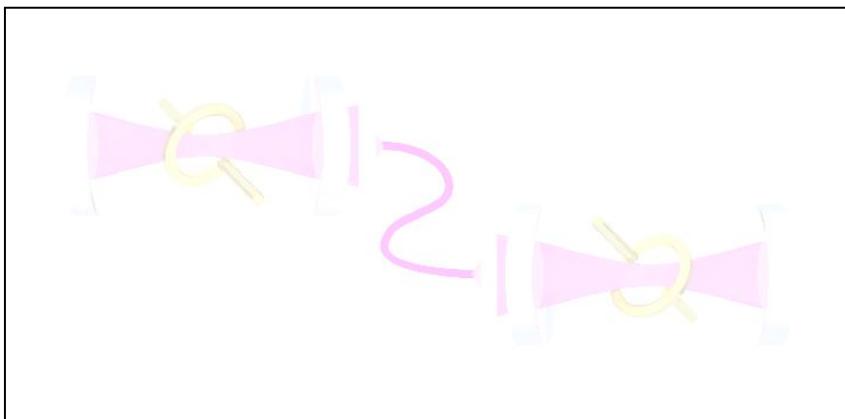
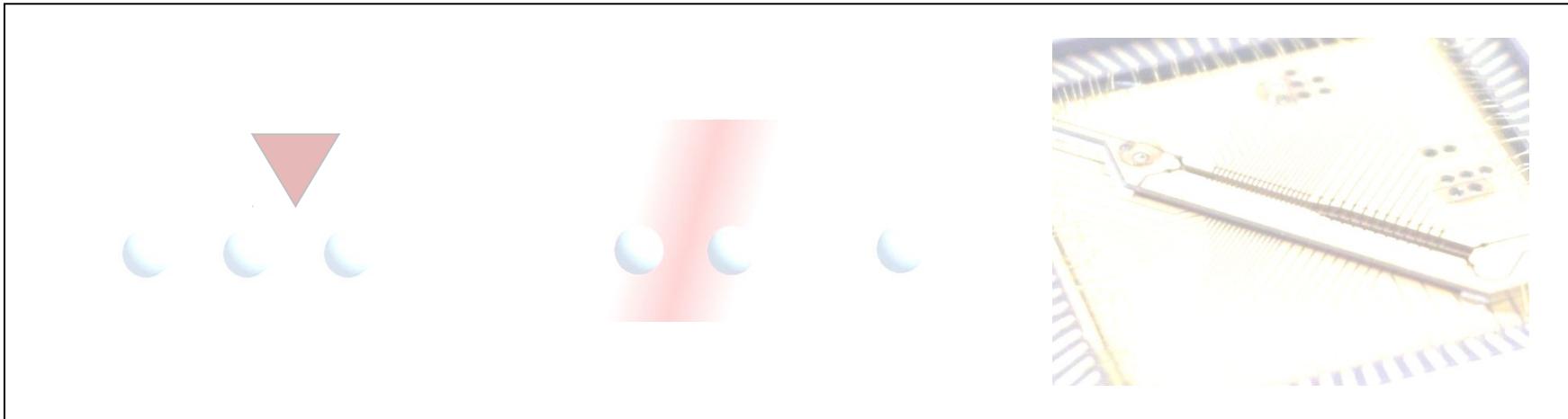
| Transport | Energy Gain (recooling method) quanta/trip | |
|---------------------------------------------------------------|--------------------------------------------------|----------------|
| $\mathcal{E}-\mathcal{C}-\mathcal{E}$ | 1 ion | 3.2 ± 1.8 |
| $\mathcal{E}-\mathcal{C}-\mathcal{H}-\mathcal{C}-\mathcal{E}$ | 1 ion | 7.9 ± 1.5 |
| $\mathcal{E}-\mathcal{C}-\mathcal{V}-\mathcal{C}-\mathcal{E}$ | 1 ion | 14.5 ± 2.0 |
| $\mathcal{E}-\mathcal{C}-\mathcal{E}$ | 2 ions | 5.4 ± 1.2 |
| $\mathcal{E}-\mathcal{C}-\mathcal{H}-\mathcal{C}-\mathcal{E}$ | 2 ions | 16.6 ± 1.8 |
| $\mathcal{E}-\mathcal{C}-\mathcal{V}-\mathcal{C}-\mathcal{E}$ | 2 ions | 53.0 ± 1.2 |

NIST:

Blakestad, et al., "High fidelity transport of trapped-ion qubits through an X-junction trap array", arXiv:0901.0533v1



Scaling of ion trap quantum computers





Wiring up trapped ions



Two trapped ions ...

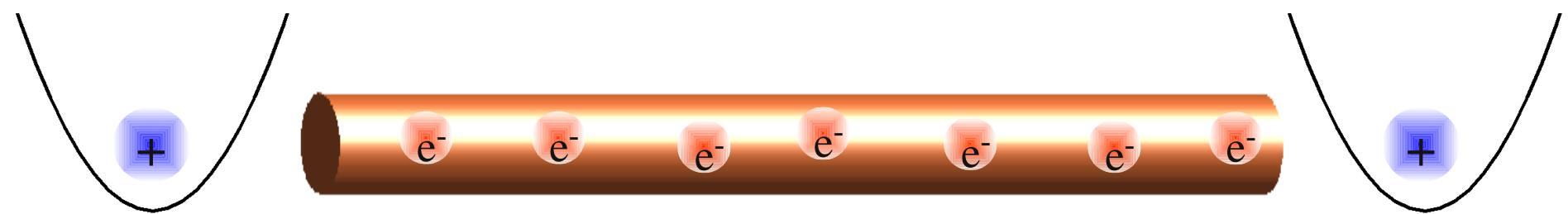




Wiring up trapped ions



Two trapped ions + a wire

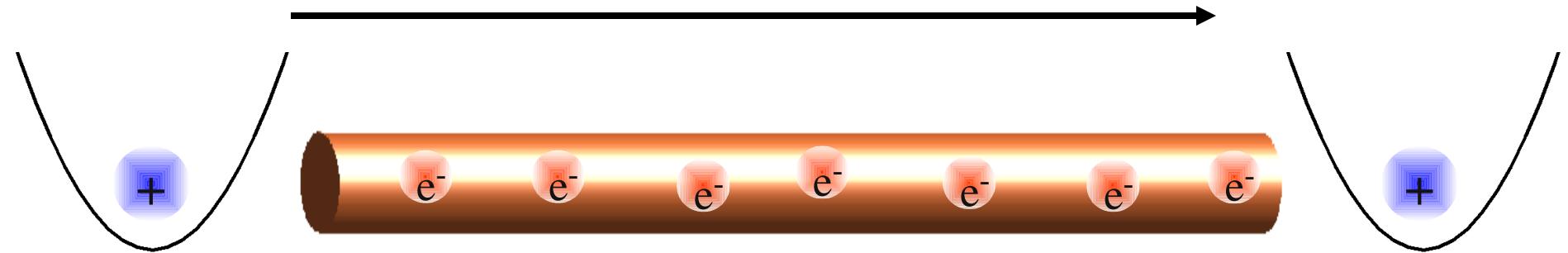




Wiring up trapped ions



Transport of quantum information

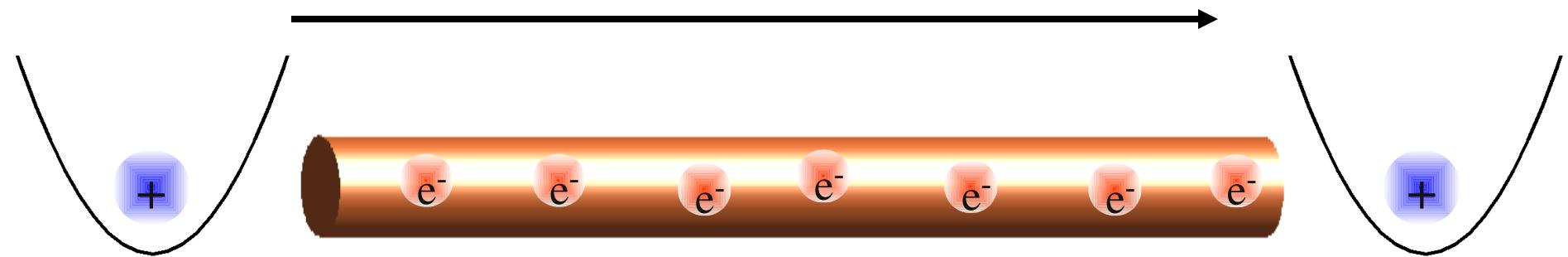




Wiring up trapped ions



Transport of quantum information

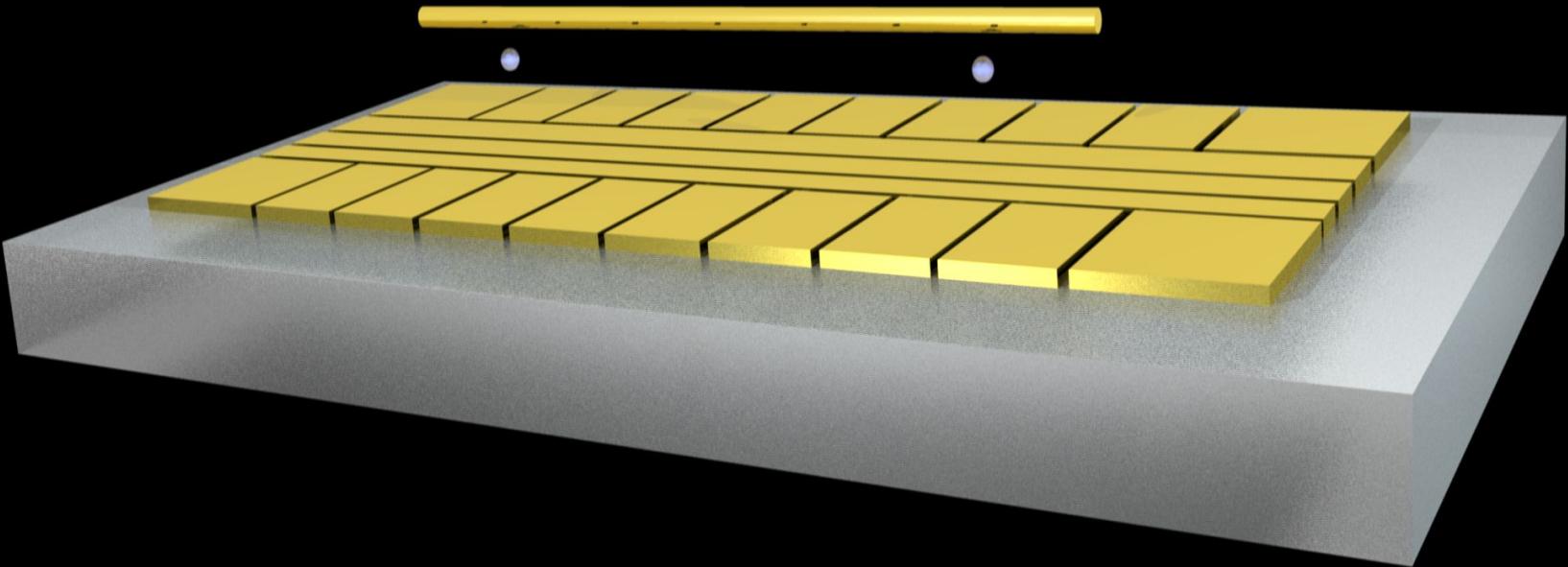


No trace of the quantum information should remain in the wire

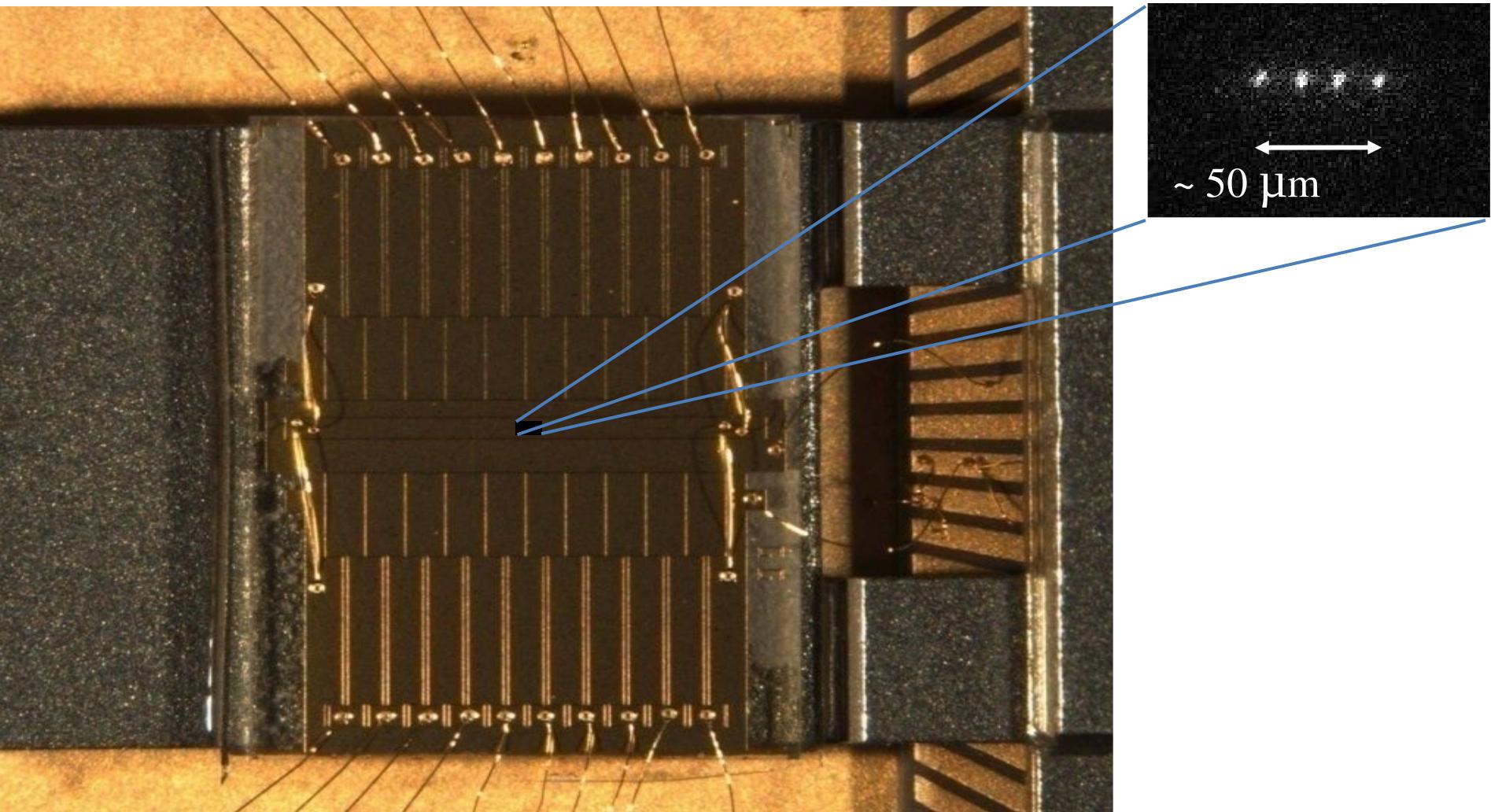
→ ~~super conducting~~ wire



Experimental set-up

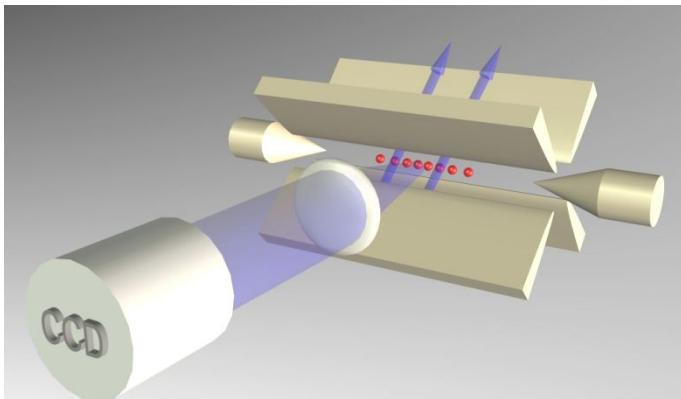


Experimental set-up

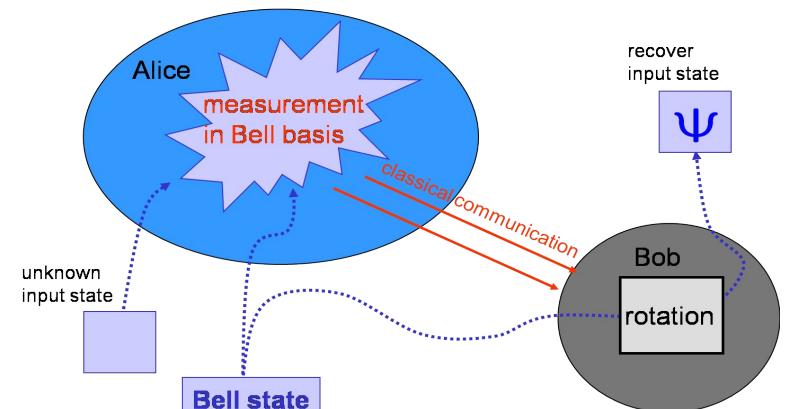


Conclusions

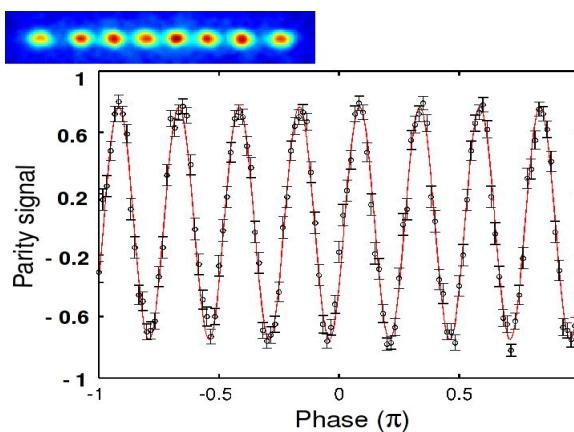
Ion trap quantum computing



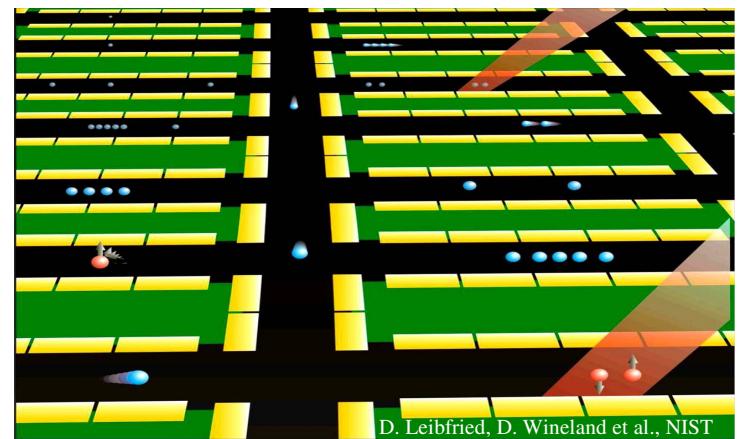
Teleportation

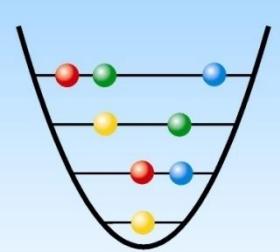


Schrödinger kitten



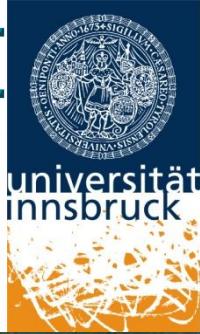
Scaling of ion traps





AG Quantenoptik
und Spektroskopie

IQI



The Innsbruck ion trap group



€



FWF
SFB



CONQUEST
SCALA



Industrie
Tirol



IQI
GmbH

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\$



