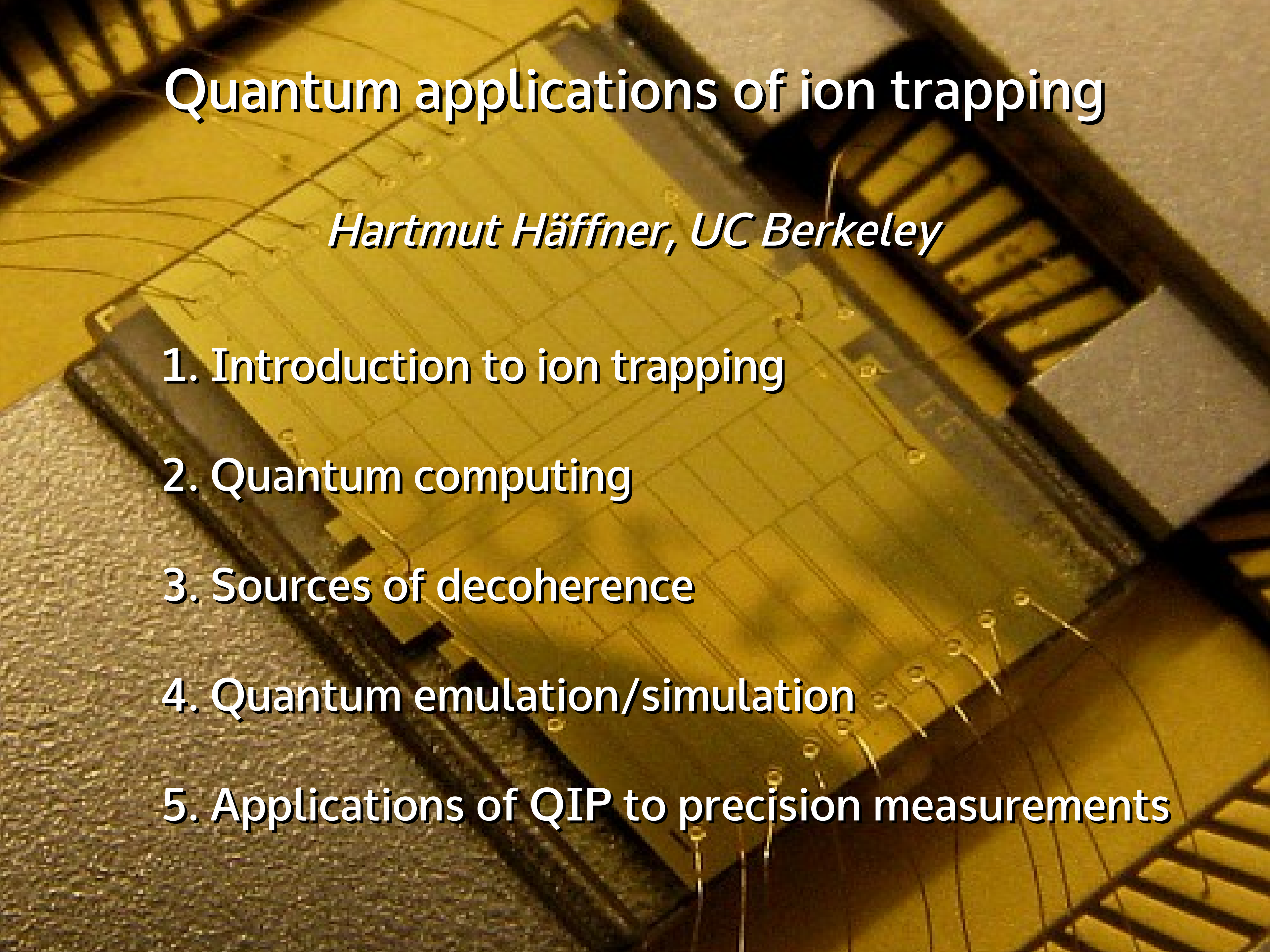

Quantum applications of ion trapping

Hartmut Häffner, UC Berkeley

1. Introduction to ion trapping
2. Quantum computing
3. Sources of decoherence
4. Quantum emulation/simulation
5. Applications of QIP to precision measurements

Plan

Lecture #1: Introduction
- Paul traps
- Laser ion-interaction

Lecture #2: Quantum computing
- Quantum gates
- Quantum state tomography

Lecture #3: Decoherence
- Qubit decoherence
- Anomalous heating

Lecture #4: Quantum simulation/emulation
- Quantum emulation
- Digital quantum simulation

Lecture #5: Applications
- Atomic clocks, quadrupole shifts
- Michelson-Morley experiment

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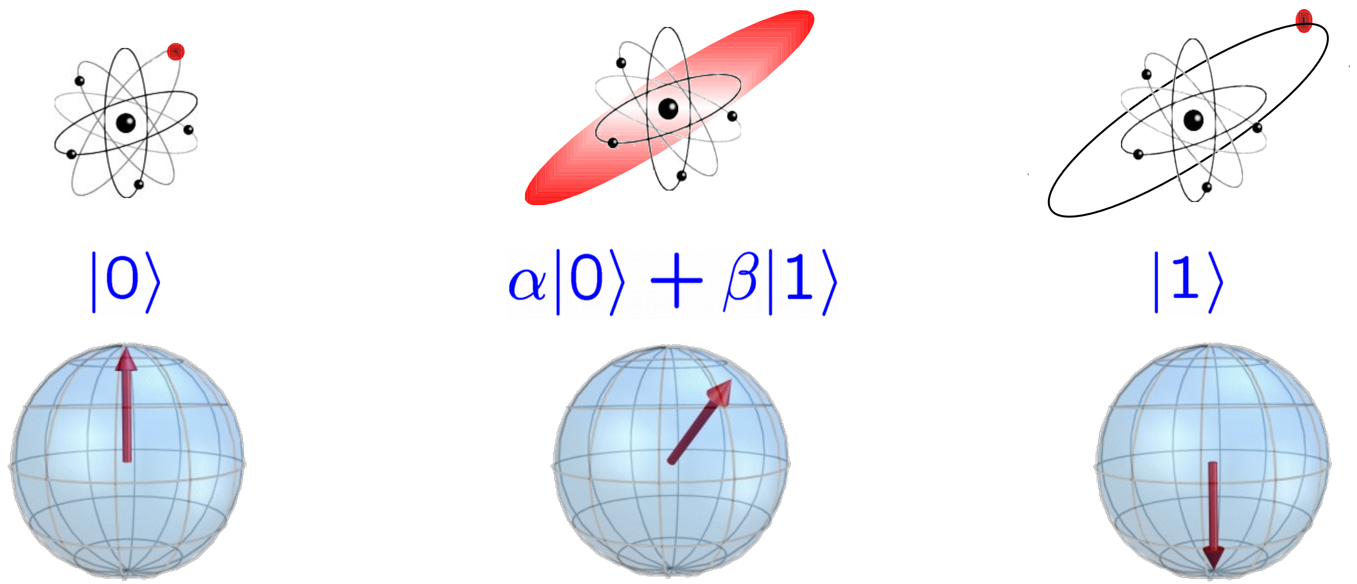
- Michelson-Morley experiment

Physics and quantum information

Qubits



Information content

$$|\Psi\rangle_{\mathrm{reg}} = \alpha_0\,|000\rangle + \alpha_1\,|001\rangle + \alpha_2\,|010\rangle + \alpha_3\,|011\rangle + \alpha_4\,|100\rangle + \alpha_5\,|101\rangle + \alpha_6\,|110\rangle + \alpha_7\,|111\rangle$$

# bits	classical	quantum mechanical
1	1	0.5208 + 0.7059*i*, 0.3014 + 0.3736*i*
2	01	0.2044 + 0.4911*i*, 0.1732 + 0.3855*i*, 0.2040 + 0.4890*i*, 0.3193 + 0.3947*i*
3	001	0.2583 + 0.2704i , 0.2310 + 0.1150i, 0.2956 + 0.3118i , 0.3558 + 0.2113i , 0.1943 + 0.1377i 0.3273 + 0.2613i , 0.0643 + 0.2033i, 0.3643 + 0.1654i
4	1010	0.1691 + 0.0891i 0.1096 + 0.0828i 0.1420 + 0.2873i 0.0741 + 0.2419i 0.1902 + 0.0448i 0.2495 + 0.0039i 0.1738 + 0.2933i 0.2102 + 0.0653i 0.0686 + 0.0980i 0.1246 + 0.2170i 0.2570 + 0.0933i 0.2234 + 0.1540i 0.1513 + 0.0213i 0.1863 + 0.3243i 0.2606 + 0.1912i 0.0194 + 0.1390i
5	10001	0.1060 + 0.1416i 0.0103 + 0.0118i 0.0064 + 0.0976i 0.0734 + 0.0716i 0.0030 + 0.2054i 0.0902 + 0.0035i 0.1605 + 0.1804i 0.0218 + 0.2280i 0.0083 + 0.2326i 0.1438 + 0.1853i 0.1429 + 0.1030i 0.0037 + 0.1171i 0.0038 + 0.0503i 0.0446 + 0.1512i 0.1379 + 0.0752i 0.0135 + 0.2255i 0.0863 + 0.1707i 0.1483 + 0.0968i 0.1686 + 0.1749i 0.1627 + 0.0629i 0.0197 + 0.1033i 0.1067 + 0.2192i 0.1038 + 0.1605i 0.0830 + 0.0499i 0.0361 + 0.1971i 0.1587 + 0.1477i 0.1642 + 0.0314i 0.1709 + 0.0487i 0.1124 + 0.1426i 0.1303 + 0.1480i 0.0284 + 0.0870i 0.1059 + 0.1351i
6	110101	0.0595 + 0.1064i 0.0295 + 0.1327i 0.0929 + 0.0406i 0.1090 + 0.0379i 0.0559 + 0.1286i 0.0015 + 0.0345i 0.0624 + 0.1196i 0.1120 + 0.1350i 0.1180 + 0.0345i 0.1367 + 0.0356i 0.1255 + 0.0074i 0.0547 + 0.0116i 0.0923 + 0.0952i 0.1087 + 0.0284i 0.0288 + 0.1254i 0.1345 + 0.0258i 0.0846 + 0.0254i 0.0939 + 0.1478i 0.0348 + 0.0654i 0.0816 + 0.0505i 0.1384 + 0.0467i 0.0498 + 0.0543i 0.0974 + 0.0584i 0.0582 + 0.0879i 0.0932 + 0.0178i 0.1039 + 0.0057i 0.0590 + 0.0682i 0.0615 + 0.1293i 0.0974 + 0.1388i 0.1245 + 0.0393i 0.0552 + 0.0238i 0.0632 + 0.1297i 0.0884 + 0.0354i 0.0841 + 0.0960i 0.1065 + 0.1437i 0.0760 + 0.0988i 0.1154 + 0.1293i 0.0727 + 0.0015i 0.0276 + 0.0204i 0.1041 + 0.1217i 0.1460 + 0.0639i 0.1199 + 0.1323i 0.1046 + 0.1092i 0.0721 + 0.1021i 0.0170 + 0.0514i 0.0988 + 0.0247i 0.0543 + 0.0231i 0.0208 + 0.0284i 0.0842 + 0.0628i 0.1223 + 0.1272i 0.1002 + 0.0729i 0.1485 + 0.1213i 0.1429 + 0.0685i 0.0087 + 0.0680i 0.0535 + 0.0670i 0.0815 + 0.0613i 0.0389 + 0.1340i 0.0888 + 0.0008i 0.0073 + 0.0442i 0.0849 + 0.0073i 0.1042 + 0.1030i 0.1430 + 0.0966i 0.1115 + 0.1461i 0.1100 + 0.0821i
7	1001010	0.0880 + 0.0466i 0.1054 + 0.0684i 0.0239 + 0.0866i 0.0759 + 0.0090i 0.0563 + 0.1020i 0.1006 + 0.0988i 0.0769 + 0.0649i 0.0246 + 0.0273i 0.0485 + 0.0942i 0.0186 + 0.0554i 0.1045 + 0.0790i 0.0384 + 0.0455i 0.0053 + 0.1037i 0.0815 + 0.0078i 0.0965 + 0.0597i 0.0309 + 0.0315i 0.0271 + 0.0925i 0.1006 + 0.0362i 0.0141 + 0.0734i 0.1015 + 0.0058i 0.0757 + 0.0385i 0.0914 + 0.0537i 0.0226 + 0.0468i 0.0491 + 0.0607i 0.0087 + 0.0665i 0.0918 + 0.0122i 0.0606 + 0.0969i 0.0344 + 0.0814i 0.0404 + 0.0853i 0.0936 + 0.0879i 0.0401 + 0.0723i 0.0079 + 0.0217i 0.0216 + 0.0294i 0.0053 + 0.0675i 0.0611 + 0.0579i 0.0131 + 0.0064i 0.0563 + 0.0096i 0.0126 + 0.0293i 0.0830 + 0.0441i 0.0404 + 0.0511i 0.0888 + 0.0980i 0.0050 + 0.0643i 0.0645 + 0.0355i 0.1024 + 0.0516i 0.0311 + 0.0644i 0.0959 + 0.0174i 0.0110 + 0.0894i 0.0070 + 0.1031i 0.0253 + 0.0642i 0.1006 + 0.0031i 0.0068 + 0.0876i 0.0285 + 0.0658i 0.1078 + 0.0756i 0.0229 + 0.0099i 0.0537 + 0.0458i 0.0313 + 0.0405i 0.0725 + 0.0179i 0.1033 + 0.0898i 0.0827 + 0.0904i 0.0718 + 0.0487i 0.0141 + 0.1032i 0.0103 + 0.0159i 0.0016 + 0.0938i 0.0311 + 0.0830i 0.0881 + 0.0479i 0.1063 + 0.0669i 0.0019 + 0.1026i 0.0884 + 0.0690i 0.0670 + 0.0267i 0.0604 + 0.0380i 0.0263 + 0.0203i 0.0886 + 0.0529i 0.0284 + 0.0441i 0.0813 + 0.0500i 0.0711 + 0.0659i 0.0231 + 0.0077i 0.0649 + 0.0339i 0.0652 + 0.0656i 0.0711 + 0.0189i 0.0198 + 0.0670i 0.0686 + 0.0265i 0.0184 + 0.0633i 0.0582 + 0.0546i 0.0672 + 0.0501i 0.0740 + 0.0584i 0.0730 + 0.1016i 0.0946 + 0.0369i 0.0014 + 0.0433i 0.0335 + 0.0332i 0.0840 + 0.0444i 0.0331 + 0.0308i 0.0999 + 0.0425i 0.0732 + 0.0542i 0.0080 + 0.0779i 0.0076 + 0.0330i 0.0013 + 0.0121i 0.0245 + 0.0478i 0.0557 + 0.0503i 0.0494 + 0.0016i 0.0758 + 0.0716i 0.0628 + 0.0781i 0.0549 + 0.0304i 0.0080 + 0.0282i 0.0208 + 0.0764i 0.0409 + 0.0845i 0.0893 + 0.0425i 0.0989 + 0.0562i 0.0122 + 0.0774i 0.0876 + 0.0614i 0.0979 + 0.0497i 0.0169 + 0.0480i 0.0132 + 0.0095i 0.0822 + 0.0478i 0.0778 + 0.0395i 0.0703 + 0.0326i 0.0813 + 0.0919i 0.0715 + 0.0819i 0.0953 + 0.1024i 0.0293 + 0.0602i 0.0452 + 0.0015i 0.0230 + 0.0643i
8	10101011	0.0199 + 0.0027i 0.0033 + 0.0063i 0.0005 + 0.0656i 0.0443 + 0.0262i 0.0573 + 0.0359i 0.0622 + 0.0704i 0.0491 + 0.0176i 0.0194 + 0.0664i 0.0111 + 0.0506i 0.0502 + 0.0687i 0.0729 + 0.0376i 0.0629 + 0.0765i 0.0717 + 0.0288i 0.0239 + 0.0410i 0.0207 + 0.0140i 0.0413 + 0.0387i 0.0126 + 0.0325i 0.0163 + 0.0509i 0.0167 + 0.0519i 0.0502 + 0.0738i 0.0041 + 0.0148i 0.0177 + 0.0086i 0.0514 + 0.0436i 0.0240 + 0.0747i 0.0236 + 0.0018i 0.0555 + 0.0671i 0.0736 + 0.0021i 0.0101 + 0.0400i 0.0053 + 0.0148i 0.0097 + 0.0552i 0.0128 + 0.0193i 0.0702 + 0.0720i 0.0105 + 0.0106i 0.0476 + 0.0402i 0.0207 + 0.0690i 0.0170 + 0.0726i 0.0549 + 0.0258i 0.0423 + 0.0337i 0.0726 + 0.0363i 0.0254 + 0.0115i 0.0543 + 0.0105i 0.0727 + 0.0410i 0.0448 + 0.0559i 0.0678 + 0.0307i 0.0578 + 0.0276i 0.0293 + 0.0220i 0.0559 + 0.0670i 0.0125 + 0.0483i 0.0737 + 0.0186i 0.0151 + 0.0754i 0.0598 + 0.0494i 0.0473 + 0.0177i 0.0125 + 0.0525i 0.0024 + 0.0513i 0.0222 + 0.0104i 0.0748 + 0.0017i 0.0733 + 0.0202i 0.0176 + 0.0090i 0.0739 + 0.0053i 0.0524 + 0.0657i 0.0042 + 0.0139i 0.0462 + 0.0025i 0.0303 + 0.0566i 0.0166 + 0.0414i 0.0141 + 0.0213i 0.0059 + 0.0284i 0.0006 + 0.0010i 0.0608 + 0.0685i 0.0014 + 0.0667i 0.0677 + 0.0196i 0.0272 + 0.0439i 0.0557 + 0.0123i 0.0746 + 0.0458i 0.0120 + 0.0255i 0.0126 + 0.0508i 0.0242 + 0.0666i 0.0023 + 0.0437i 0.0276 + 0.0756i

Information content

# bits	classical	quantum mechanical
1	1	0.5208 + 0.7059*i*, 0.3014 + 0.3736*i*
2	01	0.2044 + 0.4911*i*, 0.1732 + 0.3855*i*, 0.2040 + 0.4890*i*, 0.3193 + 0.3947*i*
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4	1010	0.1691 + 0.0891i 0.1096 + 0.0828i 0.1420 + 0.2873i 0.0741 + 0.2419i 0.1902 + 0.0448i 0.2495 + 0.0039i 0.1738 + 0.2933i 0.2102 + 0.0653i 0.0686 + 0.0980i 0.1246 + 0.2170i 0.2570 + 0.0933i 0.2234 + 0.1540i 0.1513 + 0.0213i 0.1863 + 0.3243i 0.2606 + 0.1912i 0.0194 + 0.1390i
5	10001	0.1060 + 0.1416i 0.0103 + 0.0118i 0.0064 + 0.0976i 0.0734 + 0.0716i 0.0030 + 0.2054i 0.0902 + 0.0035i 0.1605 + 0.1804i 0.0218 + 0.2280i 0.0083 + 0.2326i 0.1438 + 0.1853i 0.1429 + 0.1030i 0.0037 + 0.1171i 0.0038 + 0.0503i 0.0446 + 0.1512i 0.1379 + 0.0752i 0.0135 + 0.2255i 0.0863 + 0.1707i 0.1483 + 0.0968i 0.1686 + 0.1749i 0.1627 + 0.0629i 0.0197 + 0.1033i 0.1067 + 0.2192i 0.1038 + 0.1605i 0.0830 + 0.0499i 0.0361 + 0.1971i 0.1587 + 0.1477i 0.1642 + 0.0314i 0.1709 + 0.0487i 0.1124 + 0.1426i 0.1303 + 0.1480i 0.0284 + 0.0870i 0.1059 + 0.1351i
6	110101	0.0595 + 0.1064i 0.0295 + 0.1327i 0.0929 + 0.0406i 0.1090 + 0.0379i 0.0559 + 0.1286i 0.0015 + 0.0345i 0.0624 + 0.1196i 0.1120 + 0.1350i 0.1180 + 0.0345i 0.1367 + 0.0356i 0.1255 + 0.0074i 0.0547 + 0.0116i 0.0923 + 0.0952i 0.1087 + 0.0284i 0.0288 + 0.1254i 0.1345 + 0.0258i 0.0846 + 0.0254i 0.0939 + 0.1478i 0.0348 + 0.0654i 0.0816 + 0.0505i 0.1384 + 0.0467i 0.0498 + 0.0543i 0.0974 + 0.0584i 0.0582 + 0.0879i 0.0932 + 0.0178i 0.1039 + 0.0057i 0.0590 + 0.0682i 0.0615 + 0.1293i 0.0974 + 0.1388i 0.1245 + 0.0393i 0.0552 + 0.0238i 0.0632 + 0.1297i 0.0884 + 0.0354i 0.0841 + 0.0960i 0.1065 + 0.1437i 0.0760 + 0.0988i 0.1154 + 0.1293i 0.0727 + 0.0015i 0.0276 + 0.0204i 0.1041 + 0.1217i 0.1460 + 0.0639i 0.1199 + 0.1323i 0.1046 + 0.1092i 0.0721 + 0.1021i 0.0170 + 0.0514i 0.0988 + 0.0247i 0.0543 + 0.0231i 0.0208 + 0.0284i 0.0842 + 0.0628i 0.1223 + 0.1272i 0.1002 + 0.0729i 0.1485 + 0.1213i 0.1429 + 0.0685i 0.0087 + 0.0680i 0.0535 + 0.0670i 0.0815 + 0.0613i 0.0389 + 0.1340i 0.0888 + 0.0008i 0.0073 + 0.0442i 0.0849 + 0.0073i 0.1042 + 0.1030i 0.1430 + 0.0966i 0.1115 + 0.1461i 0.1100 + 0.0821i
7	1001010	0.0880 + 0.0466i 0.1054 + 0.0684i 0.0239 + 0.0866i 0.0759 + 0.0090i 0.0563 + 0.1020i 0.1006 + 0.0988i 0.0769 + 0.0649i 0.0246 + 0.0273i 0.0485 + 0.0942i 0.0186 + 0.0554i 0.1045 + 0.0790i 0.0384 + 0.0455i 0.0053 + 0.1037i 0.0815 + 0.0078i 0.0965 + 0.0597i 0.0309 + 0.0315i 0.0271 + 0.0925i 0.1006 + 0.0362i 0.0141 + 0.0734i 0.1015 + 0.0058i 0.0757 + 0.0385i 0.0914 + 0.0537i 0.0226 + 0.0468i 0.0491 + 0.0607i 0.0087 + 0.0665i 0.0918 + 0.0122i 0.0606 + 0.0969i 0.0344 + 0.0814i 0.0404 + 0.0853i 0.0936 + 0.0879i 0.0401 + 0.0723i 0.0079 + 0.0217i 0.0216 + 0.0294i 0.0053 + 0.0675i 0.0611 + 0.0579i 0.0131 + 0.0064i 0.0563 + 0.0096i 0.0126 + 0.0293i 0.0830 + 0.0441i 0.0404 + 0.0511i 0.0888 + 0.0980i 0.0050 + 0.0643i 0.0645 + 0.0355i 0.1024 + 0.0516i 0.0311 + 0.0644i 0.0959 + 0.0174i 0.0110 + 0.0894i 0.0070 + 0.1031i 0.0253 + 0.0642i 0.1006 + 0.0031i 0.0068 + 0.0876i 0.0285 + 0.0658i 0.1078 + 0.0756i 0.0229 + 0.0099i 0.0537 + 0.0458i 0.0313 + 0.0405i 0.0725 + 0.0179i 0.1033 + 0.0898i 0.0827 + 0.0904i 0.0718 + 0.0487i 0.0141 + 0.1032i 0.0103 + 0.0159i 0.0016 + 0.0938i 0.0311 + 0.0830i 0.0881 + 0.0479i 0.1063 + 0.0669i 0.0019 + 0.1026i 0.0884 + 0.0690i 0.0670 + 0.0267i 0.0604 + 0.0380i 0.0263 + 0.0203i 0.0886 + 0.0529i 0.0284 + 0.0441i 0.0813 + 0.0500i 0.0711 + 0.0659i 0.0231 + 0.0077i 0.0649 + 0.0339i 0.0652 + 0.0656i 0.0711 + 0.0189i 0.0198 + 0.0670i 0.0686 + 0.0265i 0.0184 + 0.0633i 0.0582 + 0.0546i 0.0672 + 0.0501i 0.0740 + 0.0584i 0.0730 + 0.1016i 0.0946 + 0.0369i 0.0014 + 0.0433i 0.0335 + 0.0332i 0.0840 + 0.0444i 0.0331 + 0.0308i 0.0999 + 0.0425i 0.0732 + 0.0542i 0.0080 + 0.0779i 0.0076 + 0.0330i 0.0013 + 0.0121i 0.0245 + 0.0478i 0.0557 + 0.0503i 0.0494 + 0.0016i 0.0758 + 0.0716i 0.0628 + 0.0781i 0.0549 + 0.0304i 0.0080 + 0.0282i 0.0208 + 0.0764i 0.0409 + 0.0845i 0.0893 + 0.0425i 0.0989 + 0.0562i 0.0122 + 0.0774i 0.0876 + 0.0614i 0.0979 + 0.0497i 0.0169 + 0.0480i 0.0132 + 0.0095i 0.0822 + 0.0478i 0.0778 + 0.0395i 0.0703 + 0.0326i 0.0813 + 0.0919i 0.0715 + 0.0819i 0.0953 + 0.1024i 0.0293 + 0.0602i 0.0452 + 0.0015i 0.0230 + 0.0643i
8	10101011	0.0199 + 0.0027i 0.0033 + 0.0063i 0.0005 + 0.0656i 0.0443 + 0.0262i 0.0573 + 0.0359i 0.0622 + 0.0704i 0.0491 + 0.0176i 0.0194 + 0.0664i 0.0111 + 0.0506i 0.0502 + 0.0687i 0.0729 + 0.0376i 0.0629 + 0.0765i 0.0717 + 0.0288i 0.0239 + 0.0410i 0.0207 + 0.0140i 0.0413 + 0.0387i 0.0126 + 0.0325i 0.0163 + 0.0509i 0.0167 + 0.0519i 0.0502 + 0.0738i 0.0041 + 0.0148i 0.0177 + 0.0086i 0.0514 + 0.0436i 0.0240 + 0.0747i 0.0236 + 0.0018i 0.0555 + 0.0671i 0.0736 + 0.0021i 0.0101 + 0.0400i 0.0053 + 0.0148i 0.0097 + 0.0552i 0.0128 + 0.0193i 0.0702 + 0.0720i 0.0105 + 0.0106i 0.0476 + 0.0402i 0.0207 + 0.0690i 0.0170 + 0.0726i 0.0549 + 0.0258i 0.0423 + 0.0337i 0.0726 + 0.0363i 0.0254 + 0.0115i 0.0543 + 0.0105i 0.0727 + 0.0410i 0.0448 + 0.0559i 0.0678 + 0.0307i 0.0578 + 0.0276i 0.0293 + 0.0220i 0.0559 + 0.0670i 0.0125 + 0.0483i 0.0737 + 0.0186i 0.0151 + 0.0754i 0.0598 + 0.0494i 0.0473 + 0.0177i 0.0125 + 0.0525i 0.0024 + 0.0513i 0.0222 + 0.0104i 0.0748 + 0.0017i 0.0733 + 0.0202i 0.0176 + 0.0090i 0.0739 + 0.0053i 0.0524 + 0.0657i 0.0042 + 0.0139i 0.0462 + 0.0025i 0.0303 + 0.0566i 0.0166 + 0.0414i 0.0141 + 0.0213i 0.0059 + 0.0284i 0.0006 + 0.0010i 0.0608 + 0.0685i 0.0014 + 0.0667i 0.0677 + 0.0196i 0.0272 + 0.0439i 0.0557 + 0.0123i 0.0746 + 0.0458i 0.0120 + 0.0255i 0.0126 + 0.0508i 0.0242 + 0.0666i 0.0023 + 0.0437i 0.0276 + 0.0756i 0.0021 + 0.0610i 0.0612 + 0.0118i 0.0770 + 0.0642i 0.0085 + 0.0148i 0.0480 + 0.0493i 0.0102 + 0.0516i 0.0239 + 0.0595i 0.0104 + 0.0293i 0.0172 + 0.0340i 0.0306 + 0.0372i 0.0104 + 0.0469i 0.0186 + 0.0136i 0.0715 + 0.0002i 0.0301 + 0.0609i 0.0394 + 0.0396i 0.0072 + 0.0164i 0.0017 + 0.0080i 0.0123 + 0.0121i 0.0651 + 0.0314i 0.0678 + 0.0314i 0.0144 + 0.0041i 0.0764 + 0.0726i 0.0549 + 0.0116i 0.0672 + 0.0296i 0.0370 + 0.0240i 0.0382 + 0.0130i 0.0222 + 0.0691i 0.0047 + 0.0249i 0.0202 + 0.0566i 0.0144 + 0.0317i 0.0707 + 0.0308i 0.0095 + 0.0390i 0.0010 + 0.0130i 0.0285 + 0.0404i 0.0538 + 0.0494i 0.0685 + 0.0012i 0.0458 + 0.0645i 0.0121 + 0.0619i 0.0244 + 0.0538i 0.0180 + 0.0356i 0.0006 + 0.0064i 0.0306 + 0.0633i 0.0501 + 0.0149i 0.0066 + 0.0343i 0.0593 + 0.0010i 0.0747 + 0.0238i 0.0551 + 0.0675i 0.0603 + 0.0644i 0.0183 + 0.0257i 0.0151 + 0.0679i 0.0203 + 0.0370i 0.0550 + 0.0432i 0.0753 + 0.0475i 0.0491 + 0.0510i 0.0421 + 0.0475i 0.0654 + 0.0528i 0.0618 + 0.0393i 0.0515 + 0.0550i 0.0517 + 0.0397i 0.0633 + 0.0467i 0.0748 + 0.0745i 0.0375 + 0.0634i 0.0630 + 0.0245i 0.0494 + 0.0453i 0.0236 + 0.0100i 0.0509 + 0.0196i 0.0276 + 0.0619i 0.0723 + 0.0515i 0.0376 + 0.0011i 0.0070 + 0.0433i 0.0519 + 0.0350i 0.0397 + 0.0697i 0.0171 + 0.0217i 0.0559 + 0.0050i 0.0053 + 0.0367i 0.0743 + 0.0758i 0.0160 + 0.0711i 0.0124 + 0.0433i 0.0492 + 0.0503i 0.0000 + 0.0596i 0.0259 + 0.0082i 0.0212 + 0.0001i 0.0034 + 0.0418i 0.0072 + 0.0005i 0.0316 + 0.0348i 0.0630 + 0.0151i 0.0671 + 0.0607i 0.0017 + 0.0477i 0.0560 + 0.0012i 0.0654 + 0.0687i 0.0562 + 0.0587i 0.0736 + 0.0699i 0.0506 + 0.0585i 0.0572 + 0.0293i 0.0266 + 0.0255i 0.0681 + 0.0389i 0.0268 + 0.0435i 0.0670 + 0.0514i 0.0302 + 0.0522i 0.0195 + 0.0726i 0.0273 + 0.0594i 0.0573 + 0.0568i 0.0502 + 0.0668i 0.0724 + 0.0764i 0.0642 + 0.0388i 0.0362 + 0.0485i 0.0485 + 0.0611i 0.0045 + 0.0346i 0.0418 + 0.0404i 0.0351 + 0.0132i 0.0665 + 0.0101i 0.0659 + 0.0169i 0.0364 + 0.0081i 0.0607 + 0.0109i 0.0506 + 0.0352i 0.0000 + 0.0607i 0.0101 + 0.0217i 0.0381 + 0.0173i 0.0030 + 0.0701i 0.0175 + 0.0006i 0.0253 + 0.0454i 0.0693 + 0.0418i 0.0242 + 0.0504i 0.0194 + 0.0242i 0.0334 + 0.0178i 0.0649 + 0.0321i 0.0142 + 0.0230i 0.0392 + 0.0518i 0.0349 + 0.0723i 0.0251 + 0.0264i 0.0293 + 0.0434i 0.0683 + 0.0092i 0.0587 + 0.0130i 0.0681 + 0.0215i 0.0353 + 0.0429i 0.0616 + 0.0374i 0.0103 + 0.0734i 0.0050 + 0.0179i 0.0289 + 0.0369i 0.0288 + 0.0406i 0.0373 + 0.0611i 0.0747 + 0.0149i 0.0264 + 0.0701i 0.0195 + 0.0711i 0.0451 + 0.0010i 0.0404 + 0.0592i 0.0126 + 0.0730i 0.0375 + 0.0627i 0.0382 + 0.0712i 0.0650 + 0.0153i 0.0621 + 0.0520i 0.0661 + 0.0715i 0.0470 + 0.0265i 0.0436 + 0.0458i 0.0472 + 0.0474i 0.0079 + 0.0003i 0.0122 + 0.0757i 0.0319 + 0.0693i 0.0432 + 0.0534i 0.0207 + 0.0339i 0.0604 + 0.0540i 0.0299 + 0.0470i 0.0024 + 0.0231i 0.0451 + 0.0660i 0.0431 + 0.0086i 0.0155 + 0.0225i 0.0067 + 0.0075i 0.0719 + 0.0306i 0.0200 + 0.0257i 0.0157 + 0.0728i 0.0038 + 0.0646i

Taking advantage of quantum information ?

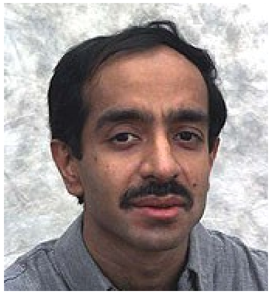
Schrödinger equation for 300 interacting spins.

Naive classical computation needs more bits than there are atoms in the universe.

➡ Quantum computers can solve *certain* tasks much more efficiently than classical computers.

Other prominent examples:

- Factoring of large integers (P. Shor 1994)
- Search in an unsorted data base (L. Grover, 1997)
- ...



Taking advantage of quantum information ?

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➡ Quantum computers can solve *certain* tasks much more efficiently than classical computers.

Physics motivations:

- quantum mechanics.
- impact on condensed matter physics, Chemistry, Biology.
- impact on cosmology.
- unforeseen applications ...

Quantum computing

Classical computer

- Initialization
- 1-bit operations (NOT)
- 2-bit gates (e.g. NAND)

Computational space: 00 01 10 11

- Read out
  → result

Quantum computer

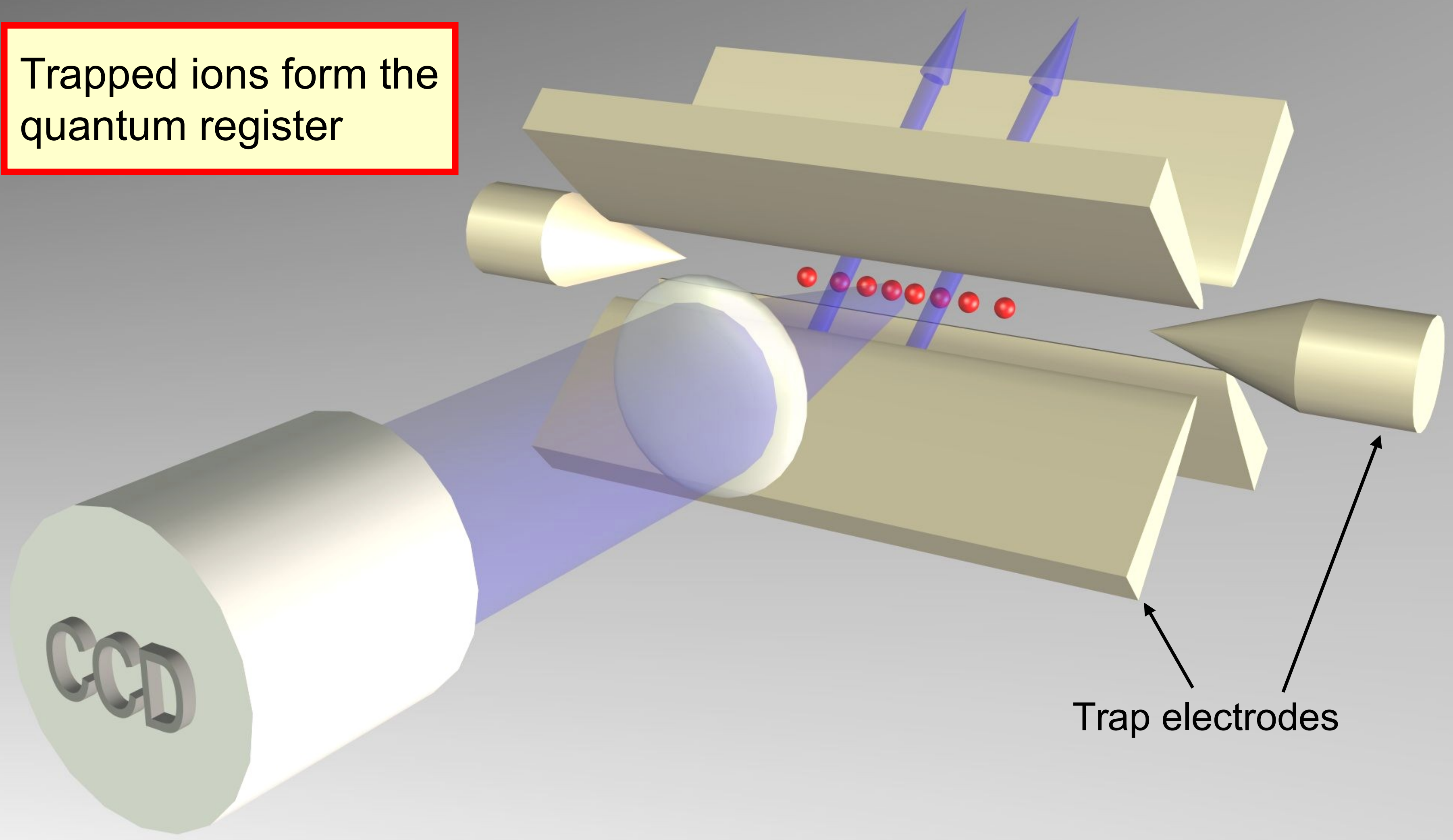
- Initialization
- 1-qubit rotations
  → superpositions
- 2-qubit gates (CNOT gate)
  → entanglement

Computational space: Hilbert space
2^n dimensional

- Read out of qubits
  → gain of classical information

Ion trap quantum computing

Trapped ions form the quantum register



The DiVincenzo criteria for quantum computing

I. Scalable physical system, well characterized qubits

II. Ability to initialize the state of the qubits

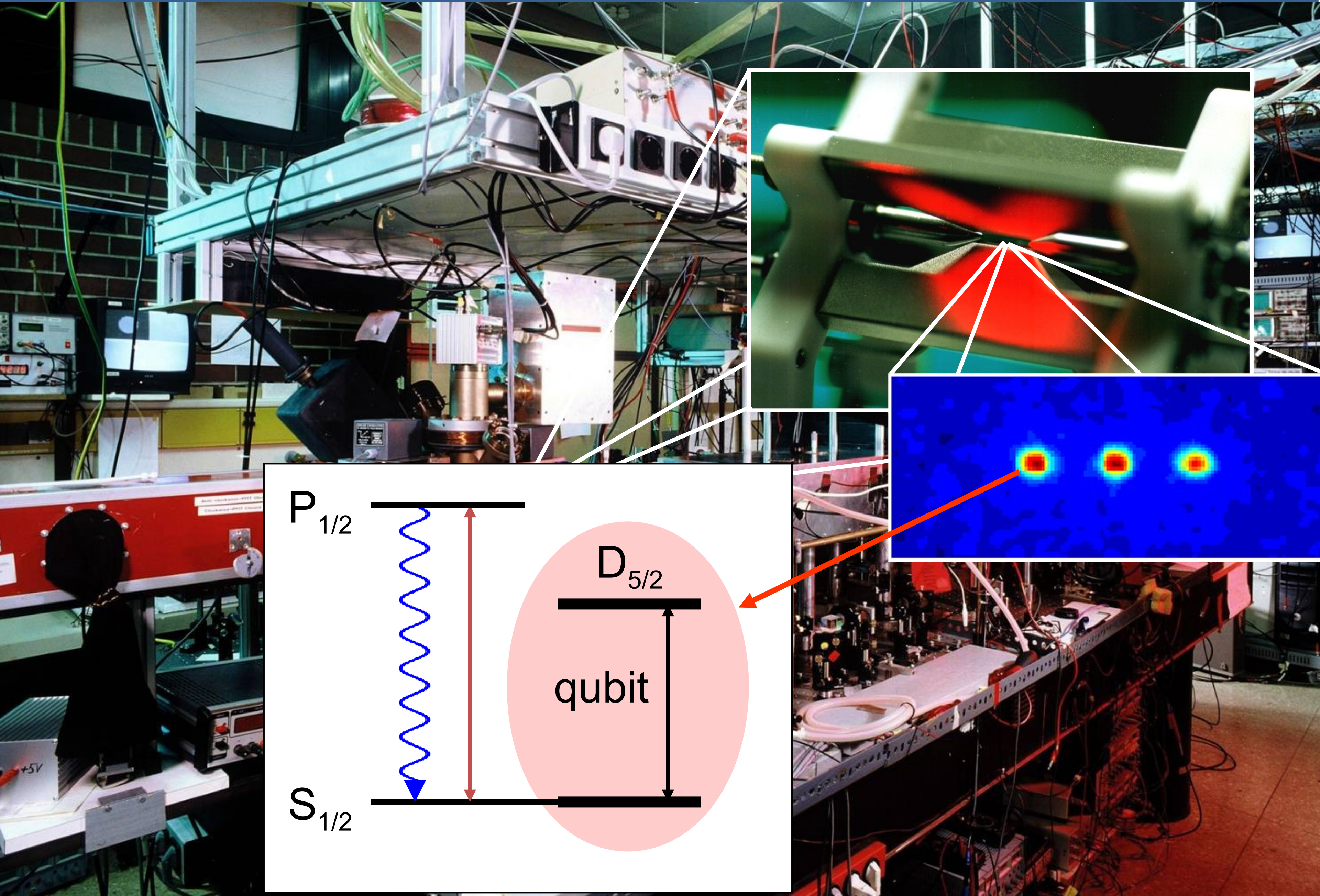
III. Long relevant coherence times, much longer than gate operation time

IV. "Universal" set of quantum gates

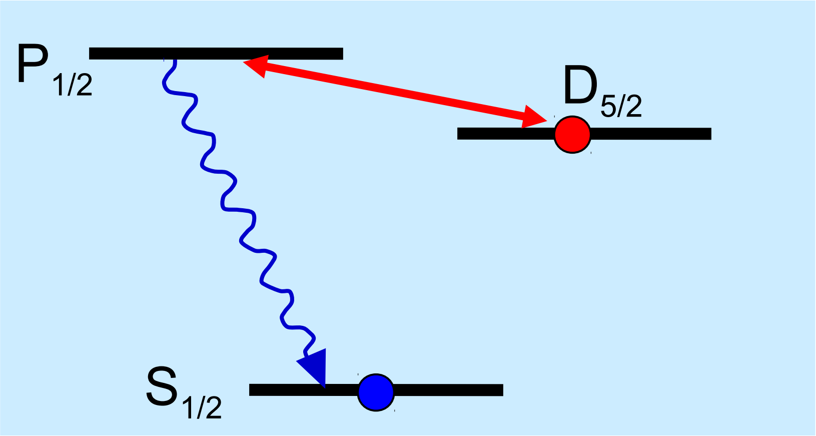
V. Qubit-specific measurement capability

DiVincenzo, D. P. & Loss, D. Quantum information is physical, Superlattices Microstruct. 23,419–432 (1998).

The hardware

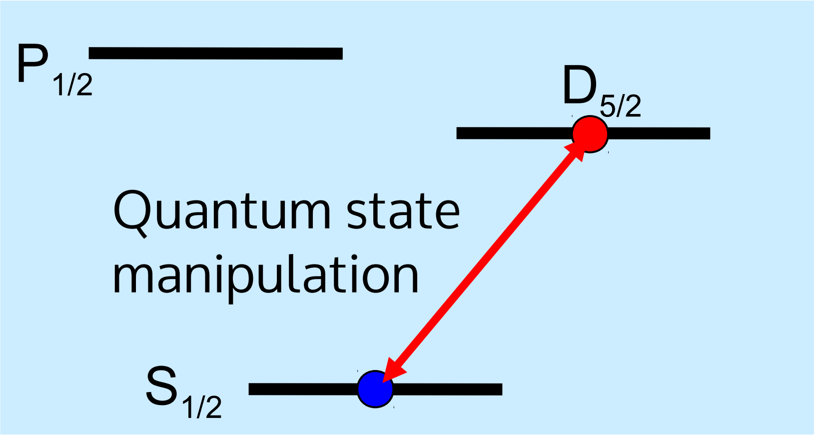


Experimental procedure



1. Initialization in a pure quantum state

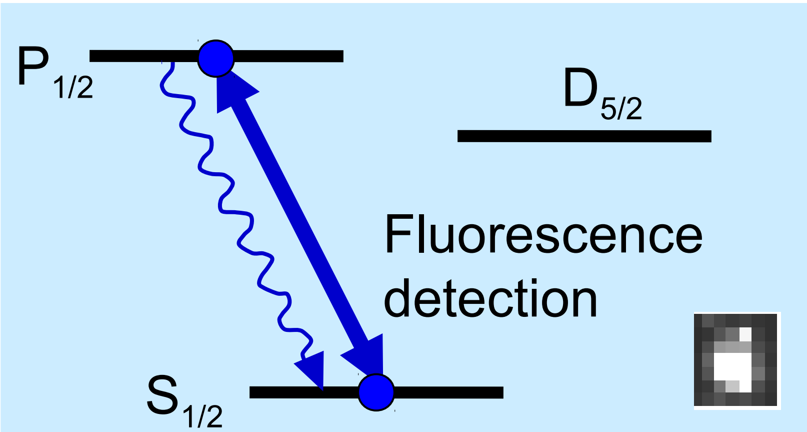
Experimental procedure



1. Initialization in a pure quantum state

2. Quantum state manipulation on $S_{1/2} – D_{5/2}$ transition

Experimental procedure



1. Initialization in a pure quantum state

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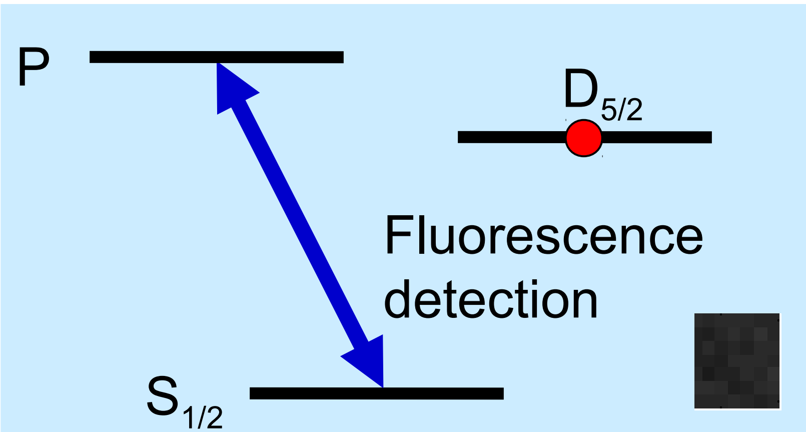
3. Quantum state measurement by fluorescence detection

Experimental procedure



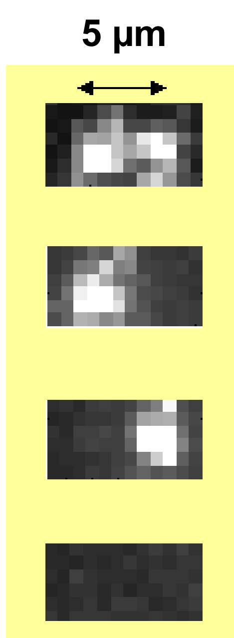
1. Initialization in a pure quantum state

2. Quantum state manipulation on $S_{1/2} - D_{5/2}$ transition

3. Quantum state measurement by fluorescence detection

<u>Two ions:</u>

Spatially resolved detection with CCD camera



$|SS\rangle$

$|SD\rangle$

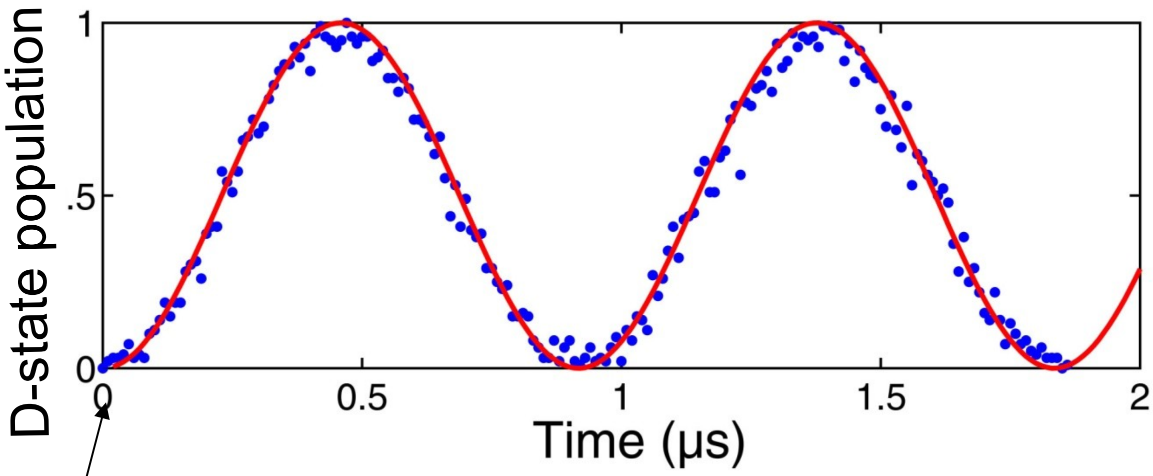
$|DS\rangle$

$|DD\rangle$

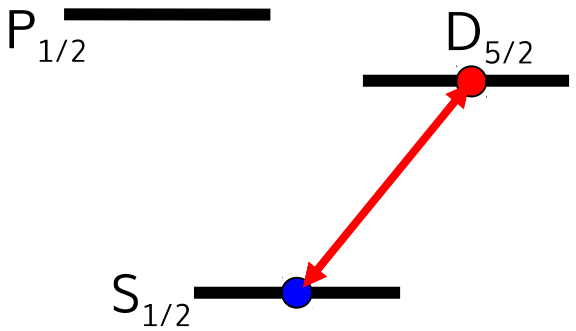
50 experiments / s

Repeat experiments 100-200 times

Single qubit gates

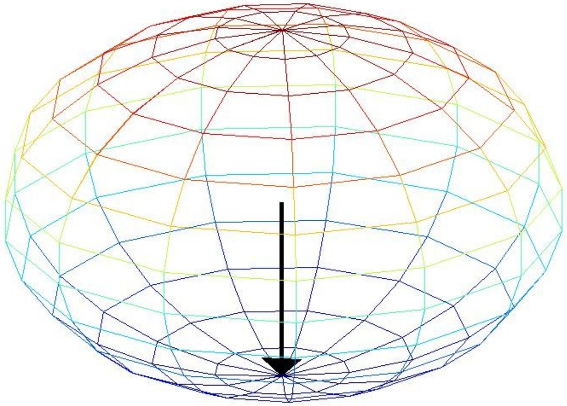


$|S\rangle$



$|D\rangle$

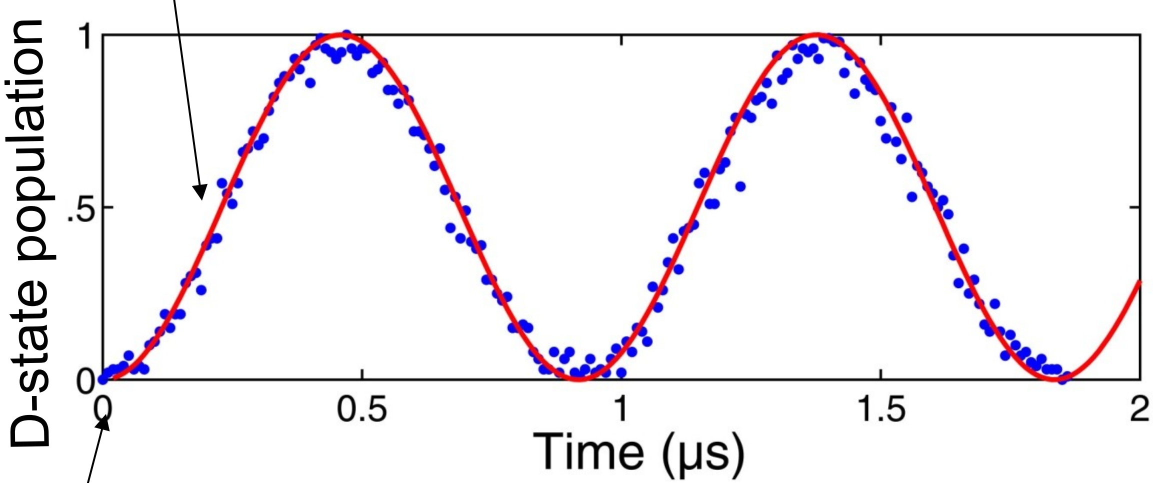
$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$



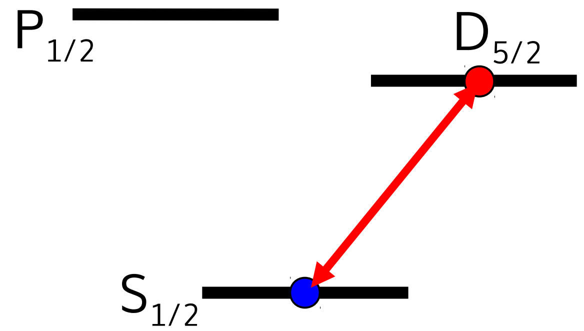
$|S\rangle$

Single qubit gates

$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$

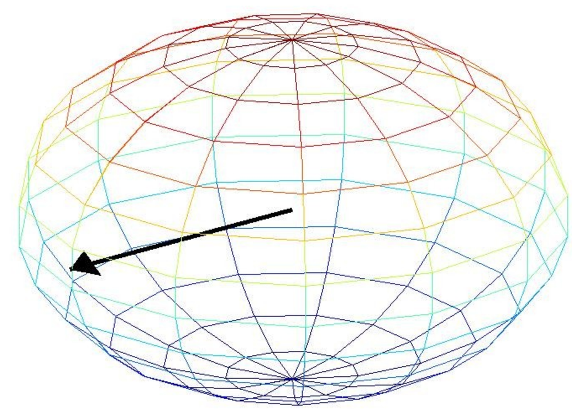


$|S\rangle$



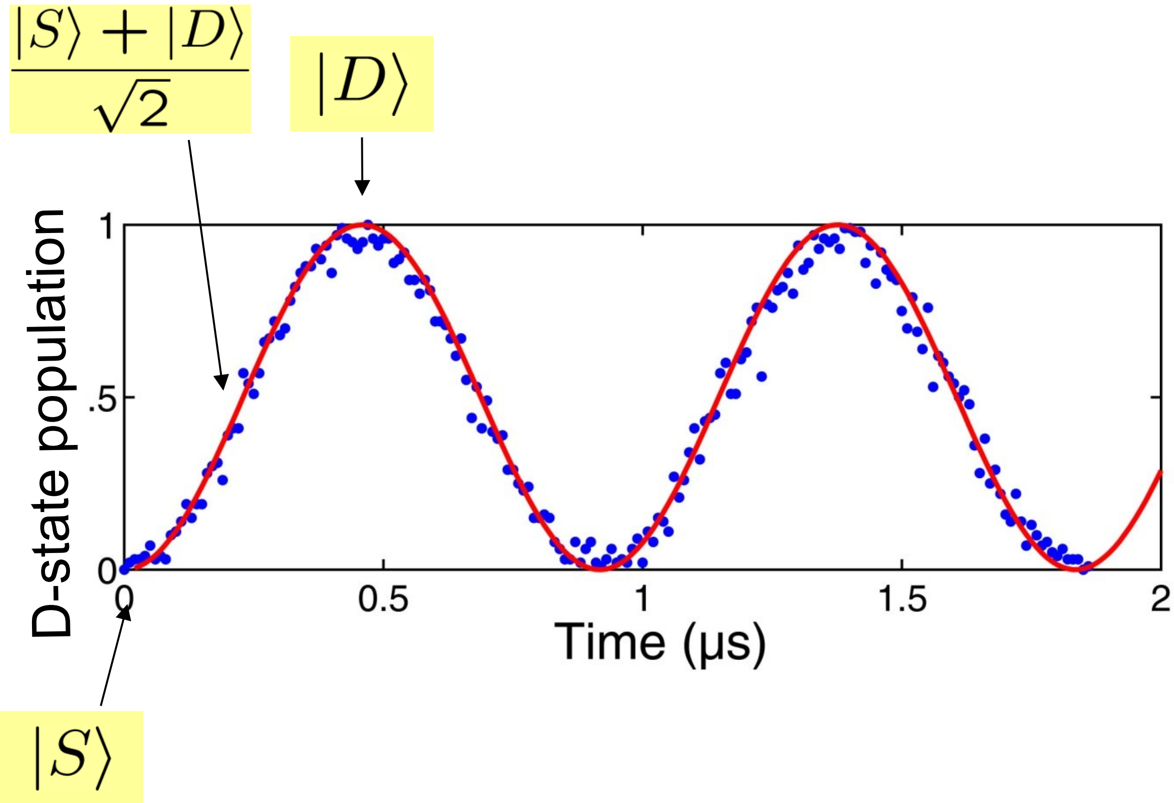
$|D\rangle$

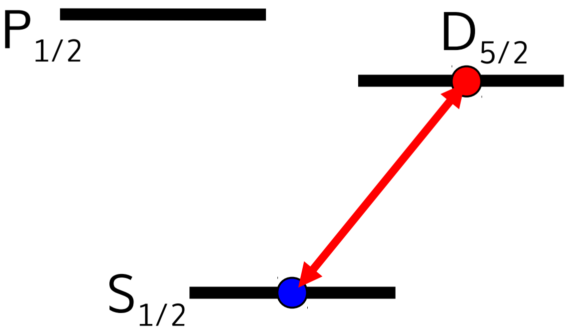
$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$

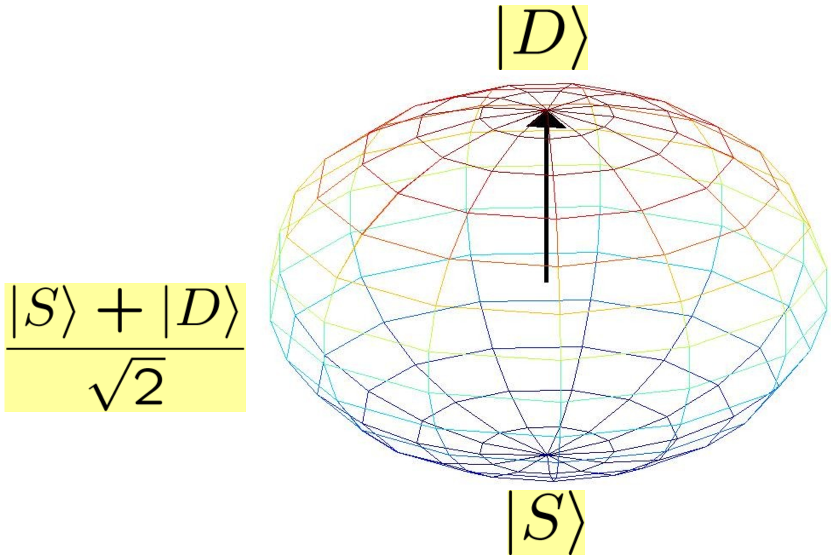


$|S\rangle$

Single qubit gates







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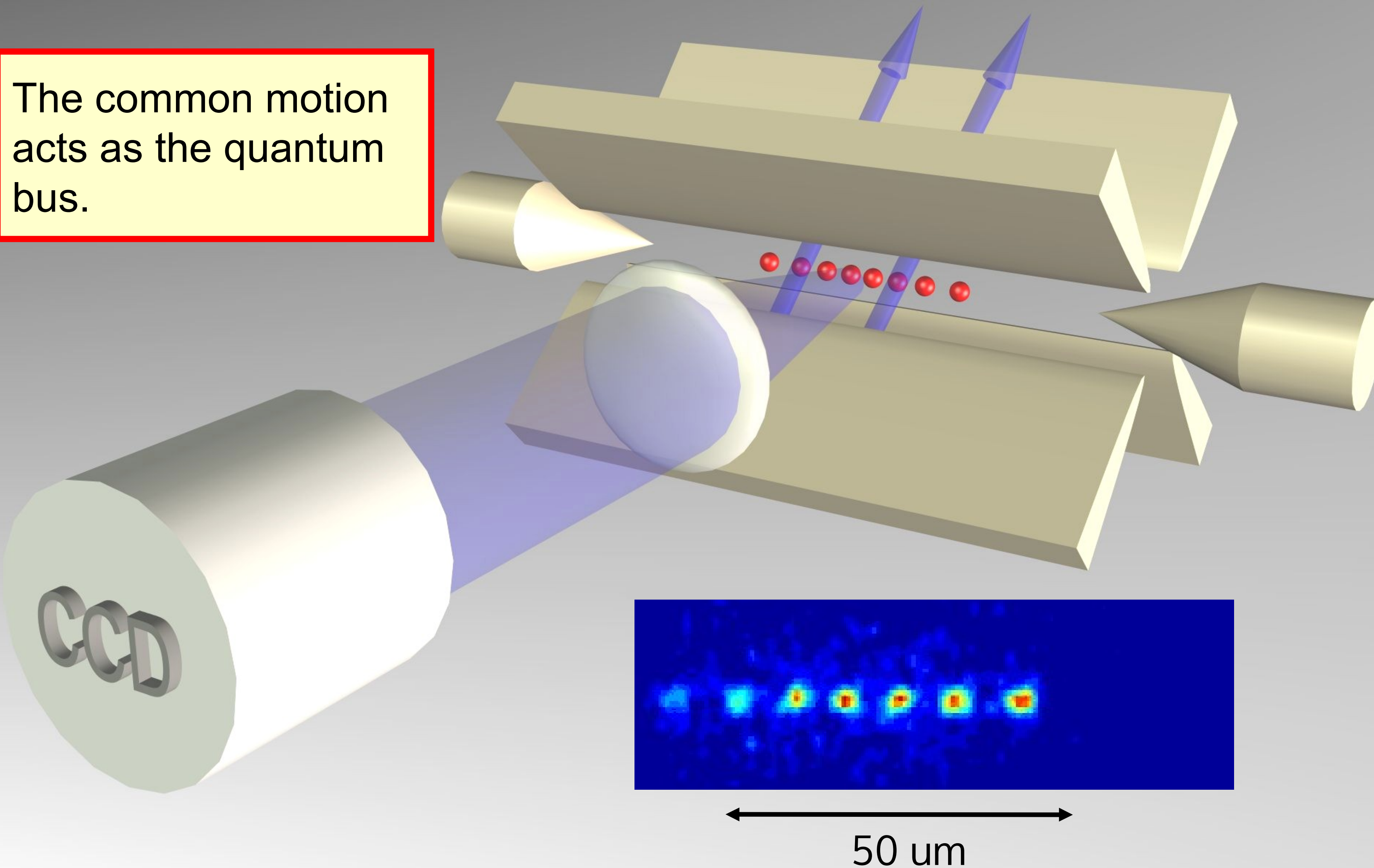
IV. "Universal" set of quantum gates
V. Qubit-specific measurement capability √

Having the qubits interact

The common motion acts as the quantum bus.



Lamb-Dicke regime

$$H_{int} = \frac{\hbar\Omega}{2}\sigma_+\{\mathbb{1}+i\eta(e^{-i\nu t}a+e^{i\nu t}a^\dagger)\}e^{-i\delta t+i\phi} + c.c.$$

1. Carrier resonance: $\delta = 0$ $\quad H_{int} = \frac{\hbar\Omega}{2}\{\sigma_+ e^{+i\phi} + \sigma_- e^{-i\phi}\}$

The laser couples ground and excited state without affecting the motional state.

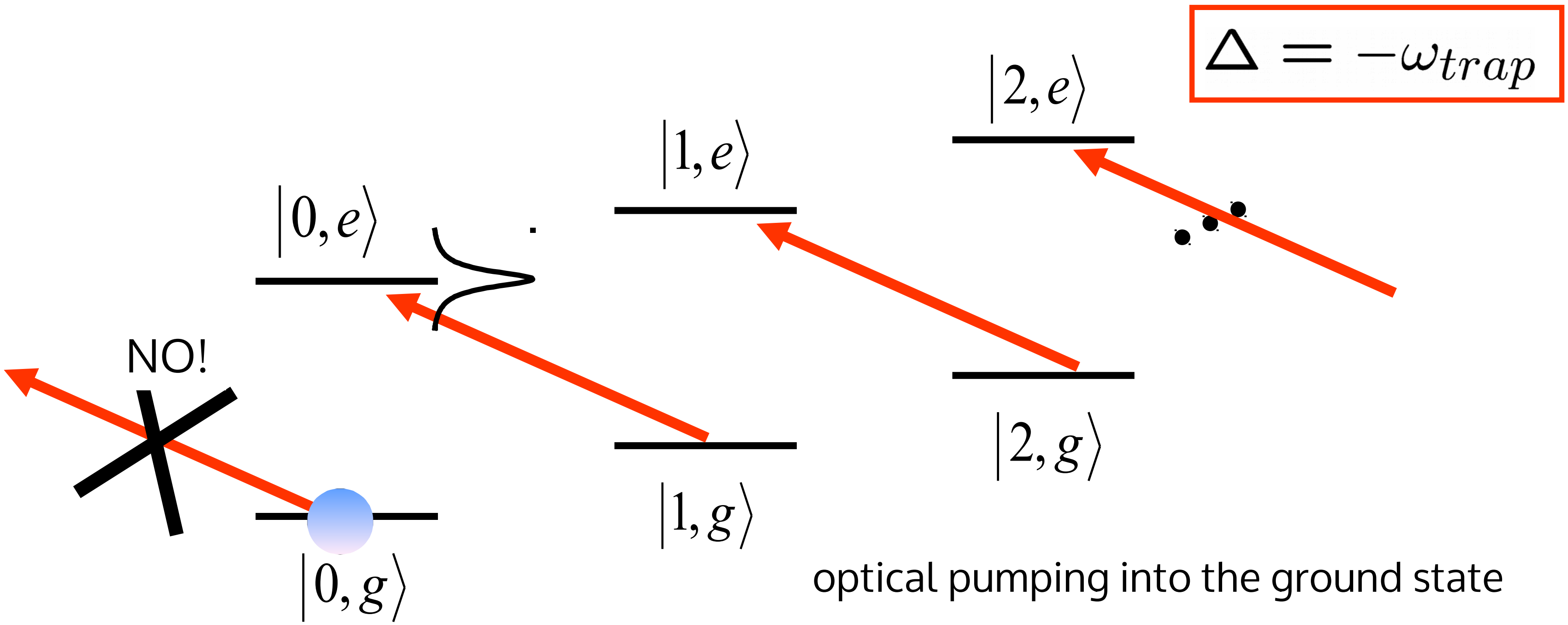
The coupling does not depend on |*n*>. $\quad |g,n\rangle \longleftrightarrow |e,n\rangle$

2. Red sideband: $\delta = -\nu$ $\quad H_{int} = \frac{\hbar\Omega}{2} i\eta\{\sigma_+ a\, e^{+i\phi} - \sigma_- a^\dagger e^{-i\phi}\}$

$$|g,n\rangle \longleftrightarrow |e,n-1\rangle$$

As compared to the carrier resonance, the coupling strength on the sidebands is reduced.
The coupling strength depends on the motional quantum number.

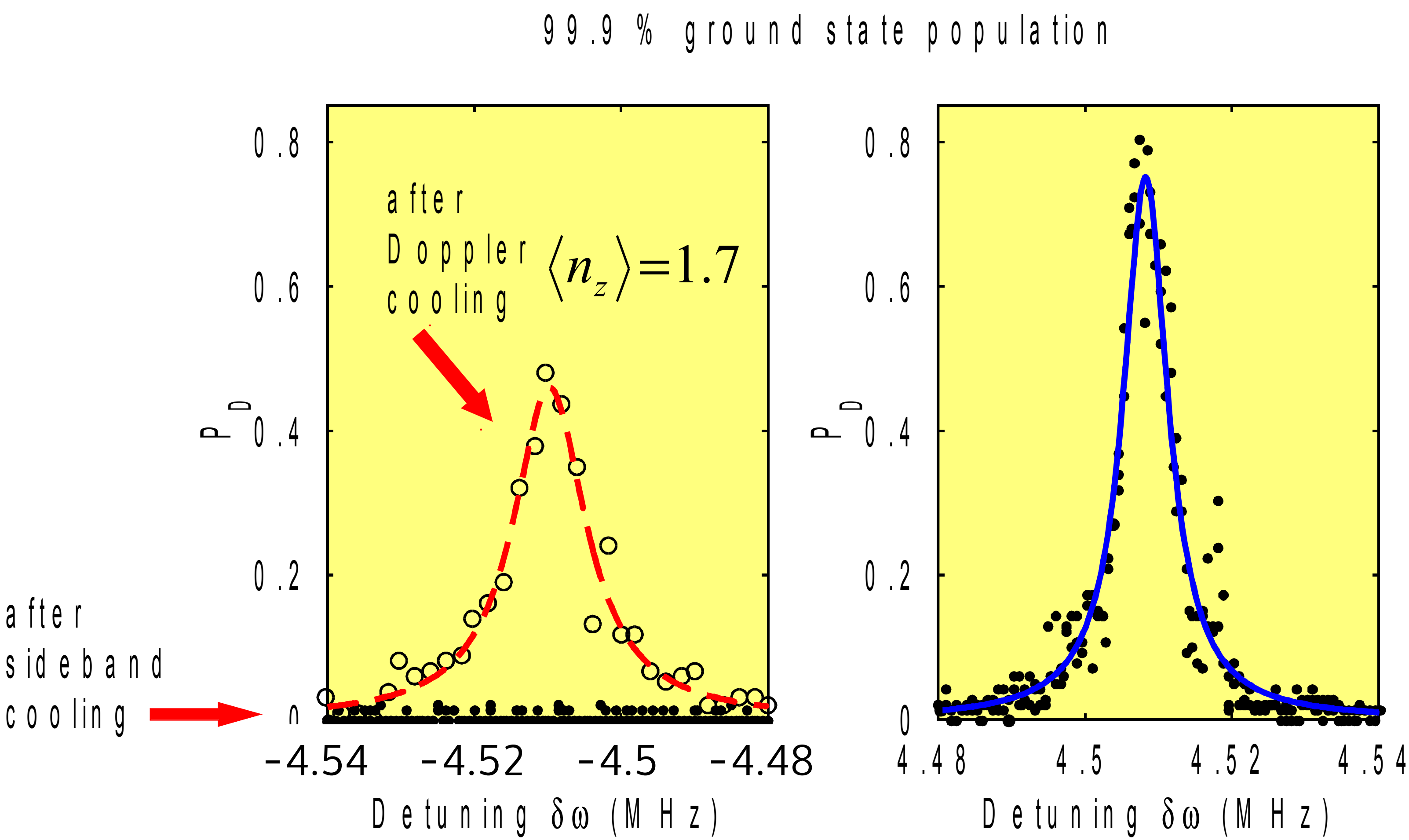
Sideband cooling



optical pumping into the ground state

Signature: no further excitation possible
„dark state“ |0>

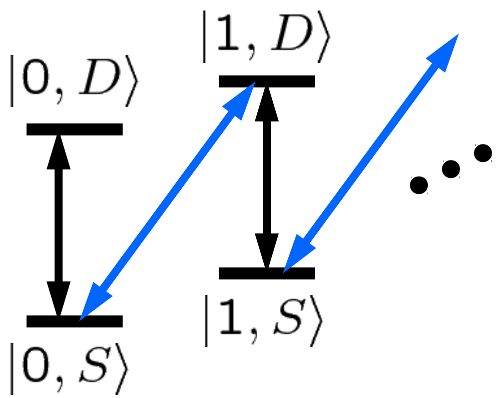
Sideband absorption spectra



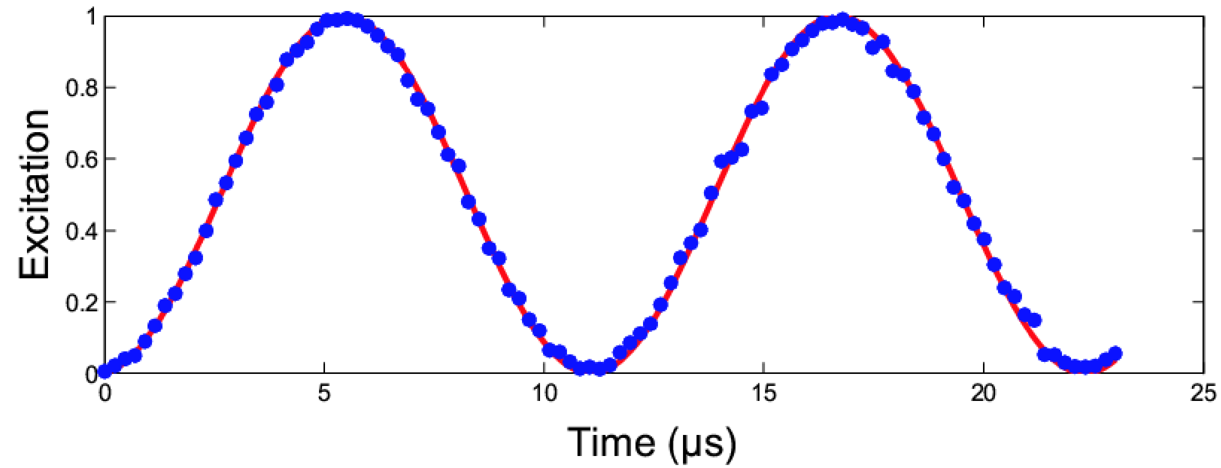
red sideband

blue sideband

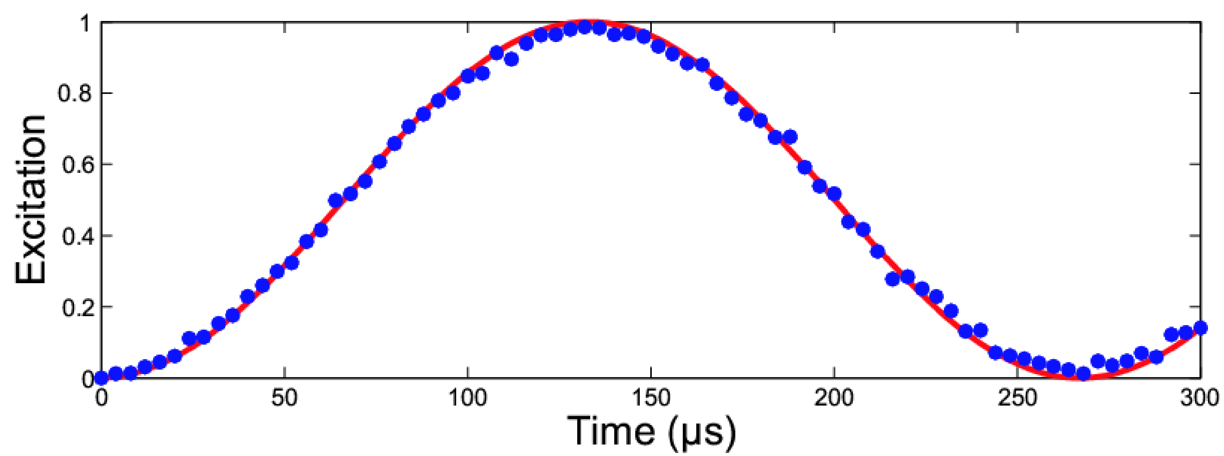
Ion motion



carrier



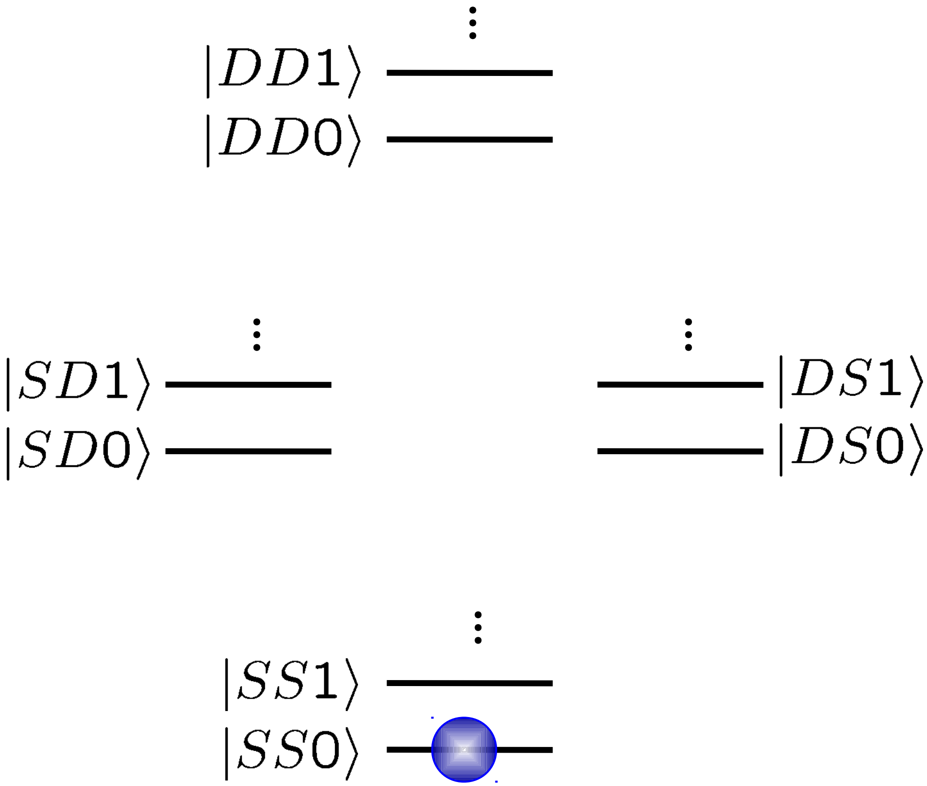
carrier and sideband
Rabi oscillations
with Rabi frequencies

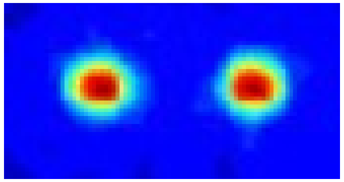


$$\Omega, \quad \eta\Omega$$

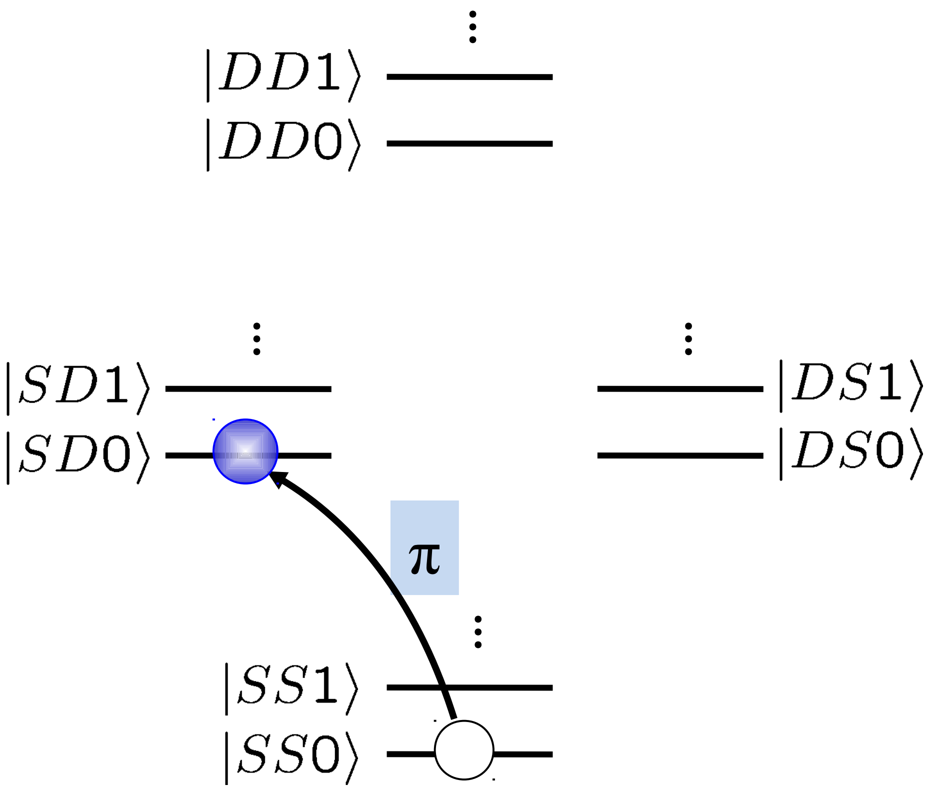
$\eta = kx_0$ Lamb-Dicke parameter

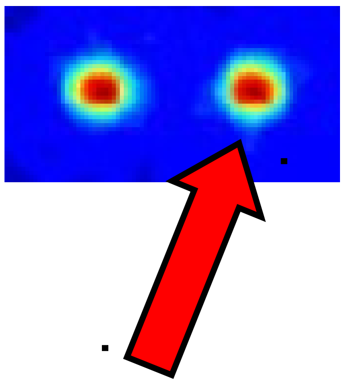
Bell state generation



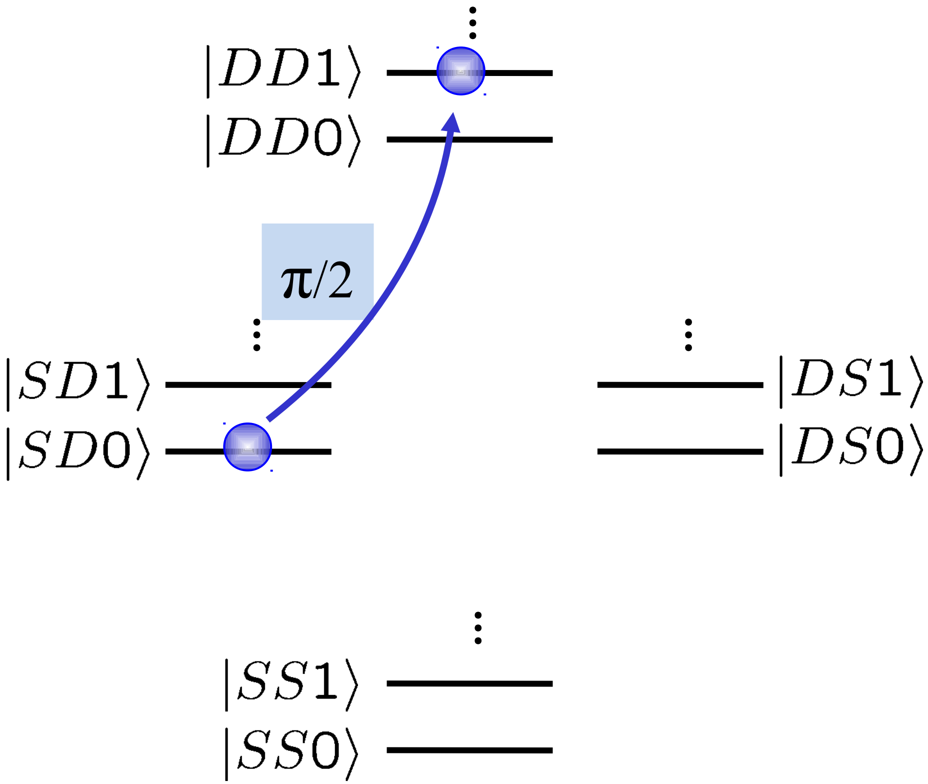


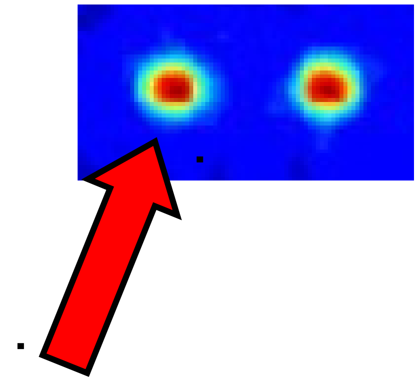
Bell state generation



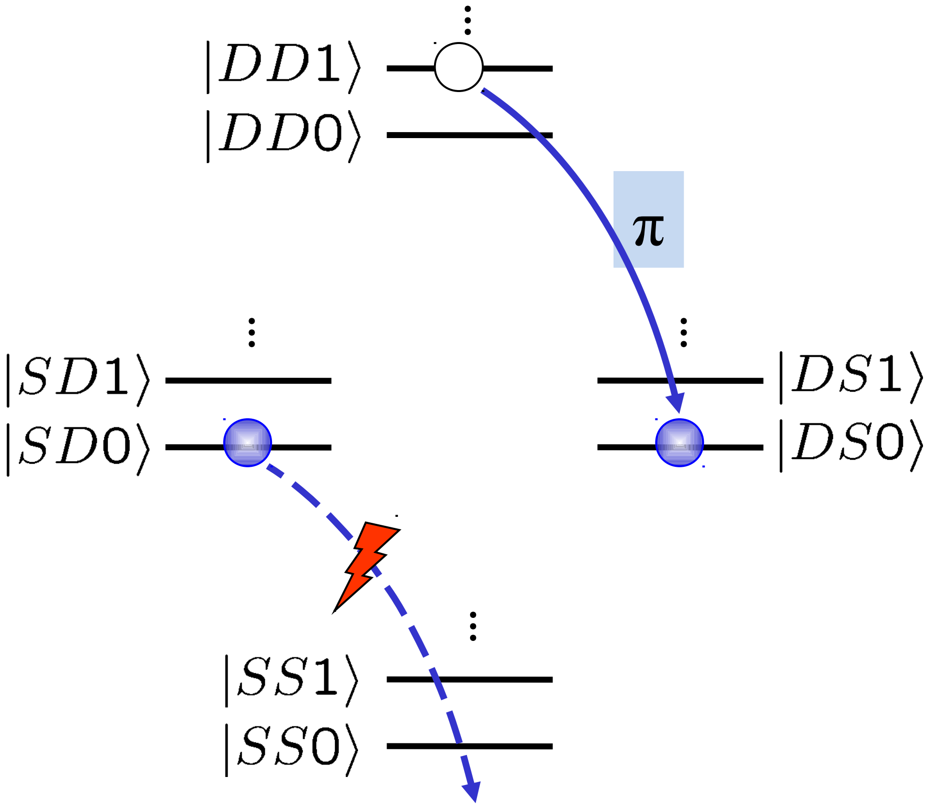


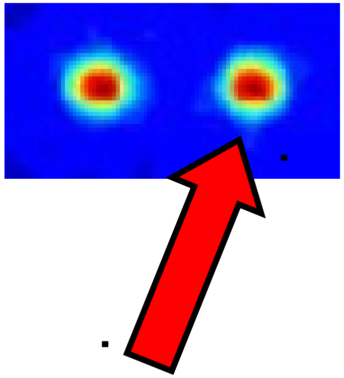
Bell state generation



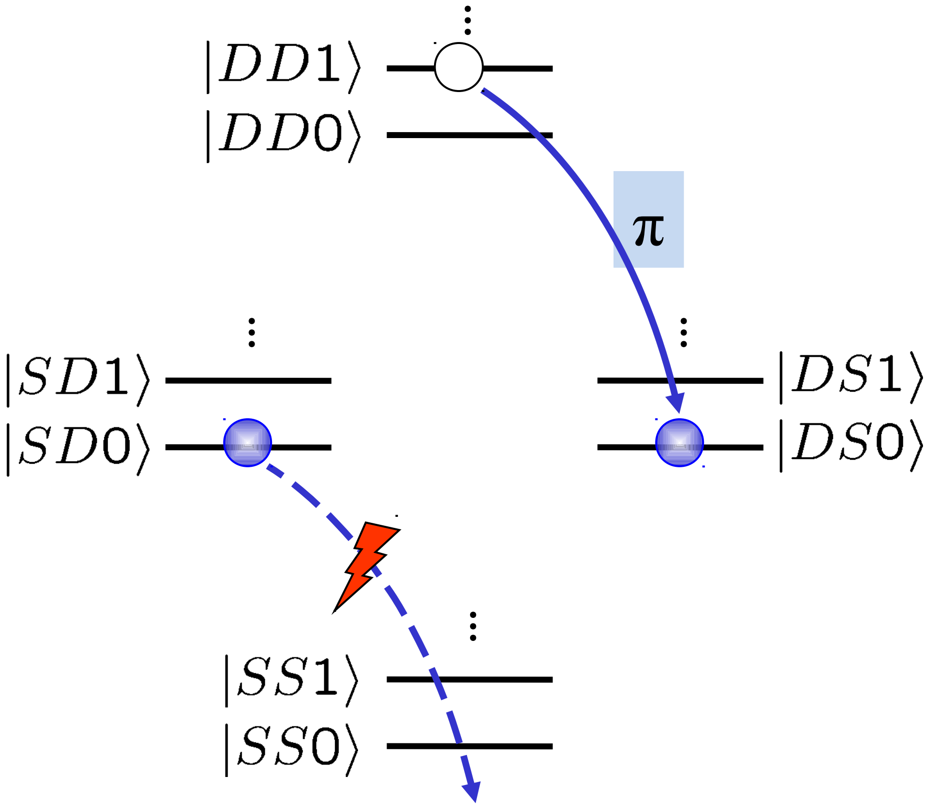


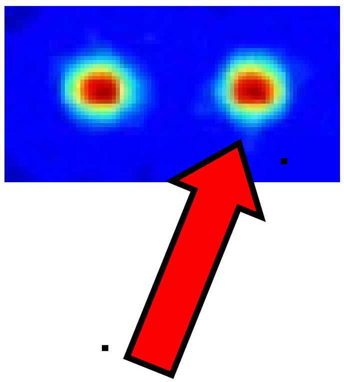
Bell state generation





Bell state generation





Bell states with atoms

- $^{9}Be^{+}$: NIST (fidelity: 97 %)
- $^{40/43}Ca^{+}$: Oxford (99.7%)
- $^{111}Cd^{+}$: Ann Arbor (79%)
- $^{25}Mg^{+}$: Munich
- $^{40}Ca^{+}$: Innsbruck (99.3%)

Analysing the Bell state

$$|SD\rangle + |DS\rangle$$

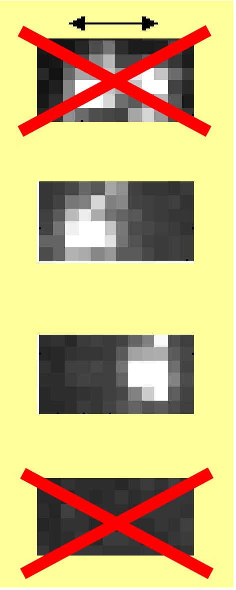
Fluorescence detection with CCD camera:

$|SS\rangle$

$|SD\rangle$

$|DS\rangle$

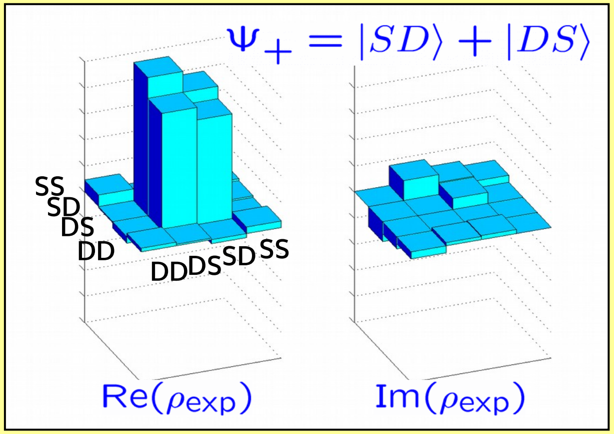
$|DD\rangle$



Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

Measurement of the density matrix:



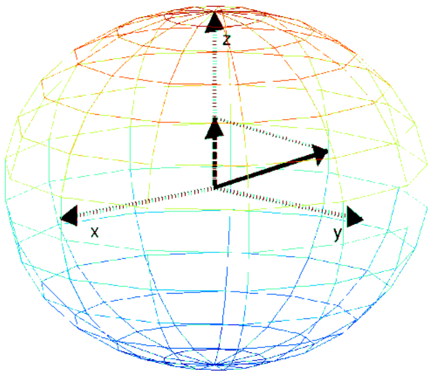
State tomography

A measurement yields the *z*-component of the Bloch vector

=> Diagonal of the density matrix

$$\rho = \begin{pmatrix} P_S & C - iD \\ C + iD & P_D \end{pmatrix}$$

$|D\rangle$



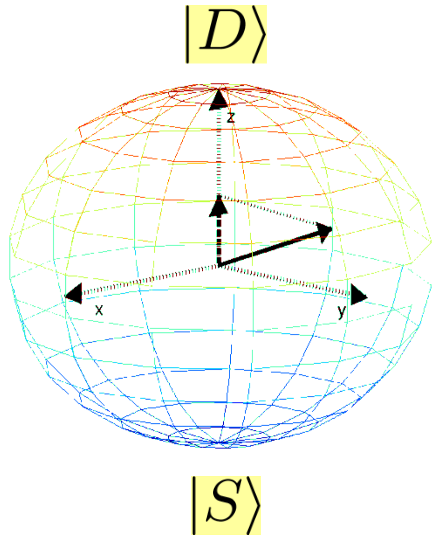
$|S\rangle$

State tomography

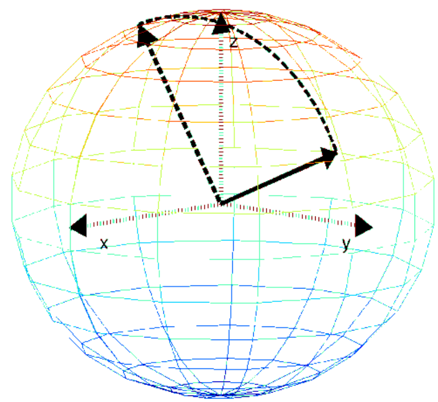
A measurement yields the *z*-component of the Bloch vector

=> Diagonal of the density matrix

$$\rho = \begin{pmatrix} P_S & C - i\color{blue}{D} \\ C + i\color{blue}{D} & P_D \end{pmatrix}$$



Rotation around the *x*- or the *y*-axis prior to the measurement yields the phase information of the qubit.



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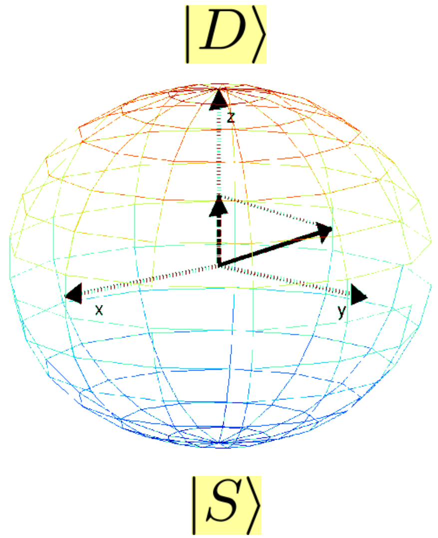
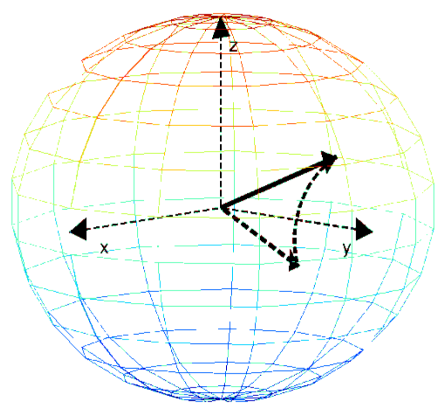
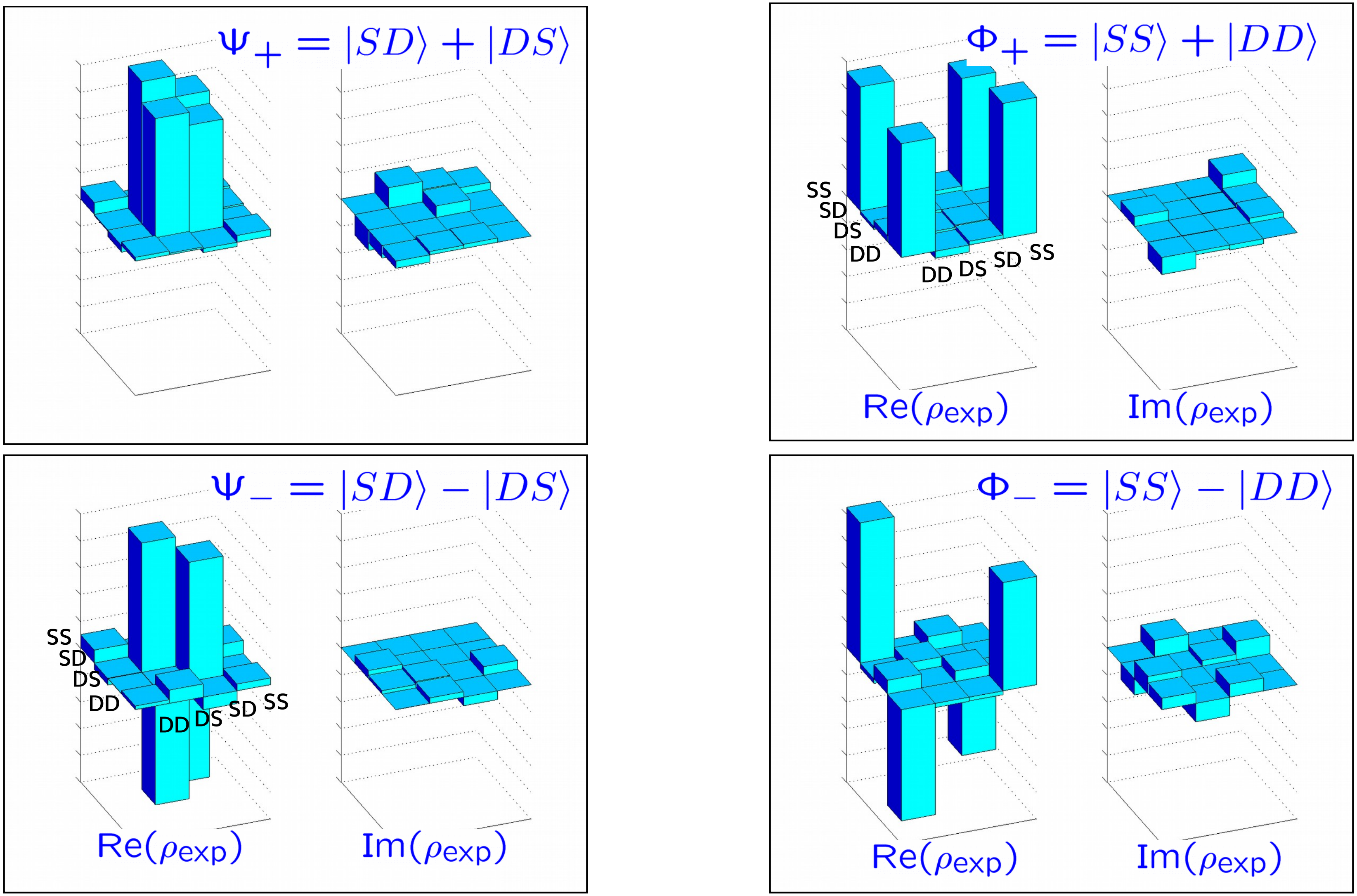


Rotation around the *x*- or the *y*-axis prior to the measurement yields the phase information of the qubit.

=> coherences of the density matrix

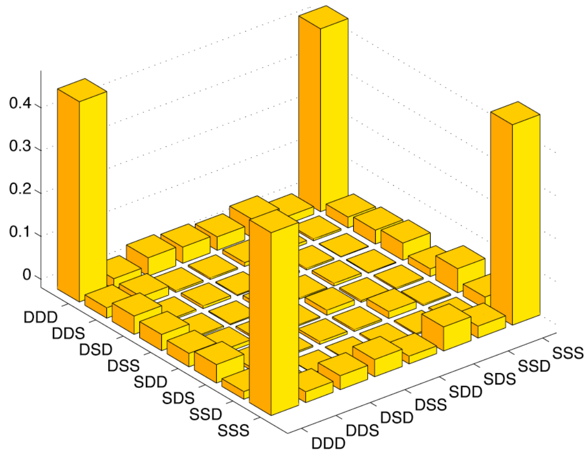


State tomography of Bell states

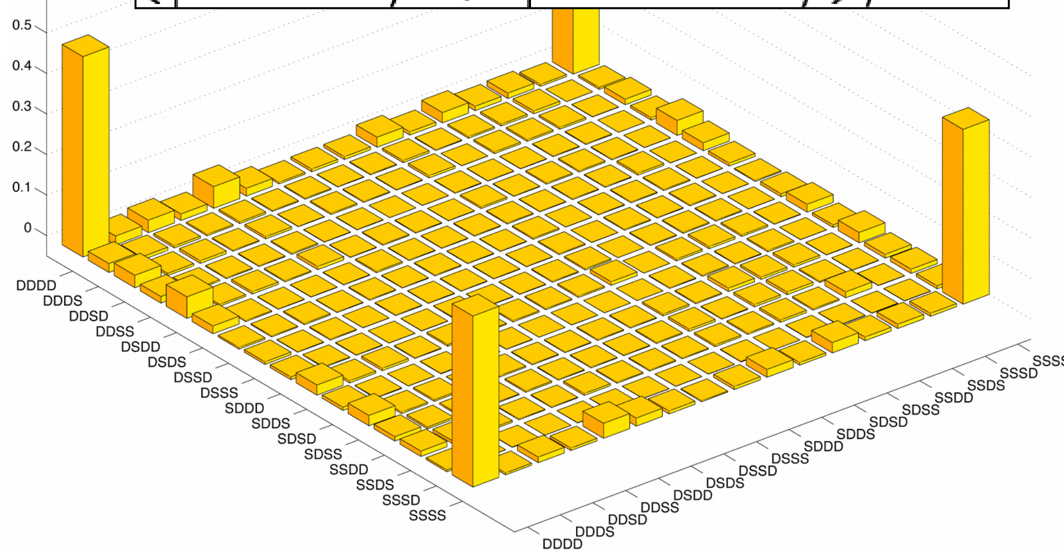


State tomography

$$(|SSS\rangle + |DDD\rangle)/\sqrt{2}$$

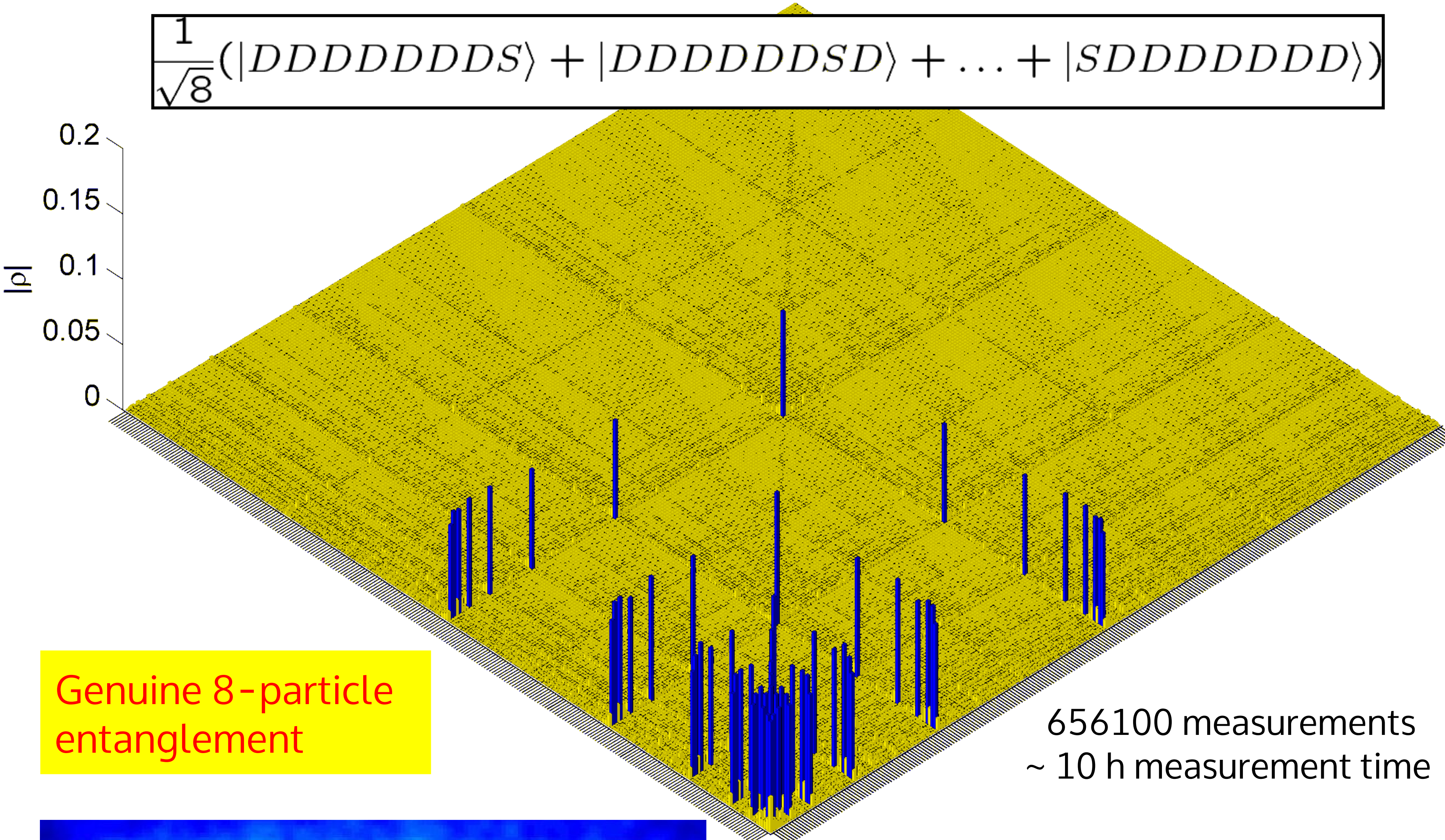


$$(|SSSS\rangle + |DDDD\rangle)/\sqrt{2}$$



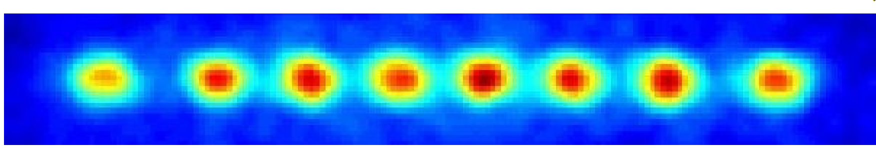
State tomography

$$\frac{1}{\sqrt{8}}(|DDDDDDDS\rangle + |DDDDDDSD\rangle + \ldots + |SDDDDDDD\rangle)$$



Genuine 8-particle entanglement

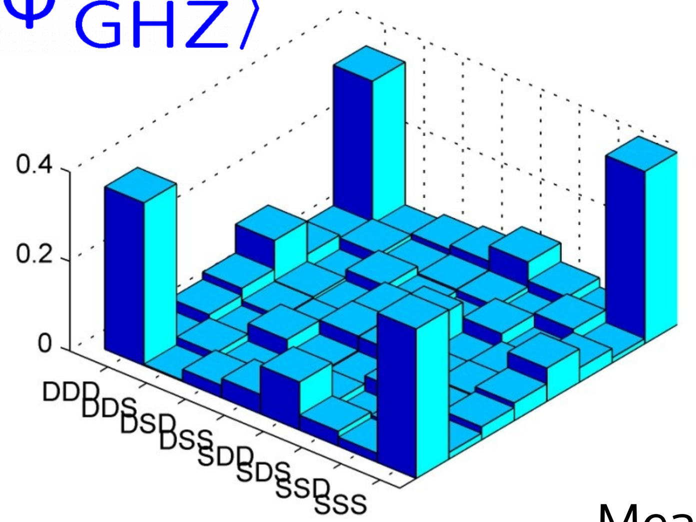
656100 measurements
~ 10 h measurement time



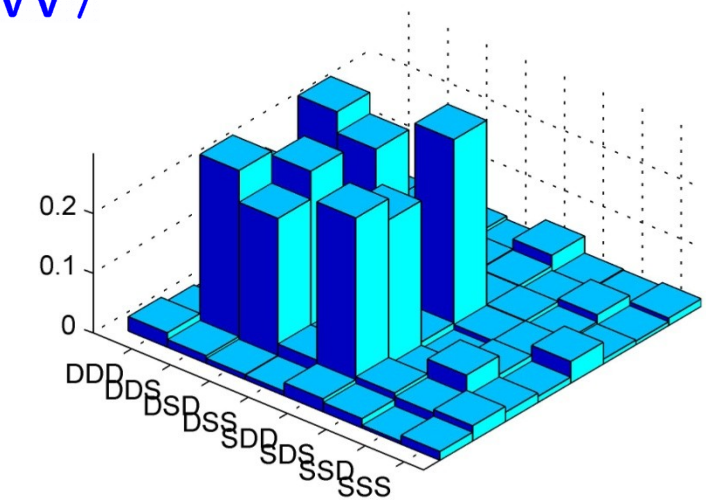
Häffner et al., Nature **438**, 643 (2005)

A "real" thought experiment

$|\Psi_{GHZ}\rangle$



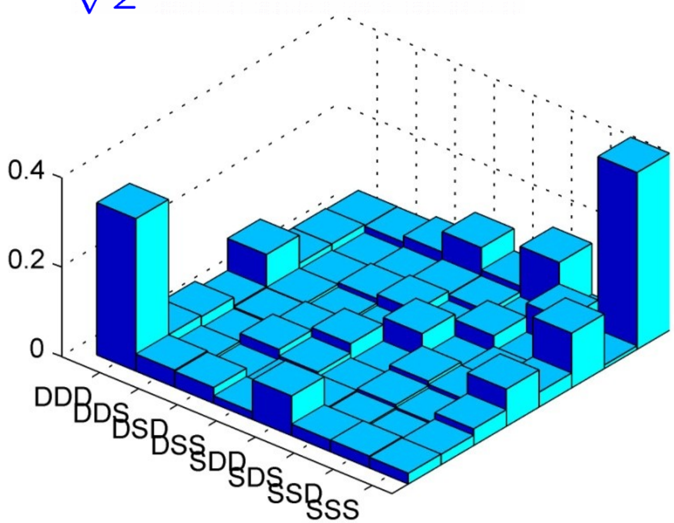
$|\Psi_{W}\rangle$



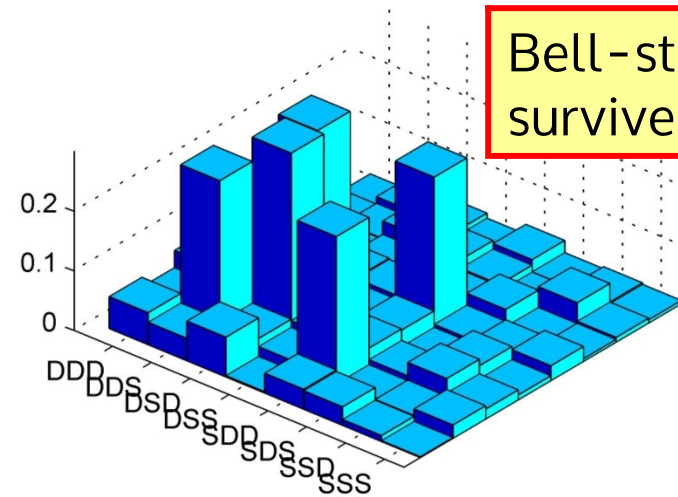
$\frac{1}{\sqrt{2}}(|SSS\rangle + |DDD\rangle)$

Measurement
of the center ion

$\frac{1}{\sqrt{3}}(|SDD\rangle + |DSD\rangle + |DDS\rangle)$



Bell-state
survives



Roos et al., *Science* **304**, 1478 (2004)

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V. Qubit-specific measurement capability ✓

Mølmer - Sørensen gate

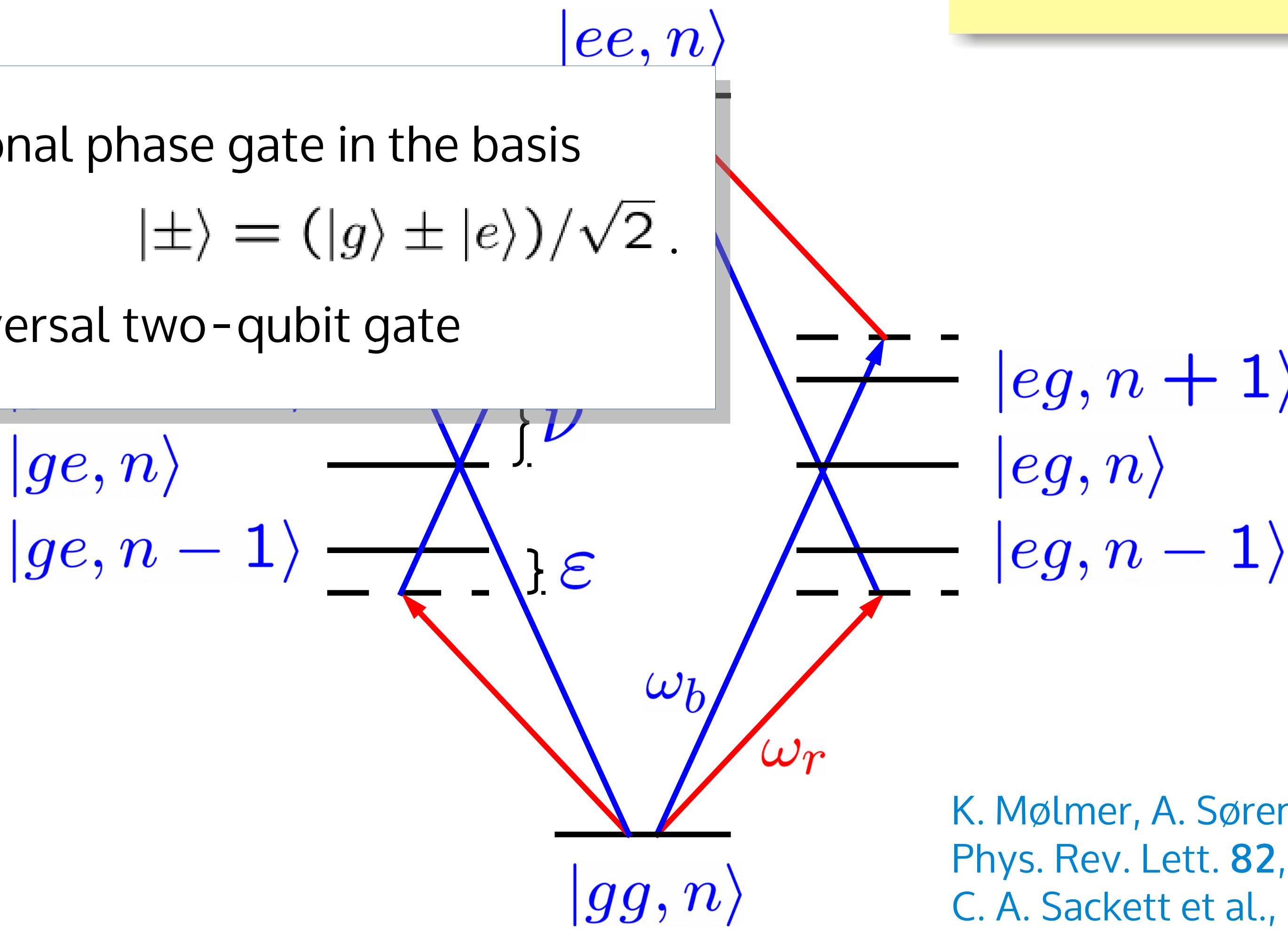
$$|gg\rangle \rightarrow |ee\rangle, \quad |ge\rangle \rightarrow |eg\rangle$$

$$\omega_b = \omega_0 + (\nu - \varepsilon)$$
$$\omega_r = \omega_0 - (\nu - \varepsilon)$$

Conditional phase gate in the basis

$$|\pm\rangle = (|g\rangle \pm |e\rangle)/\sqrt{2}\,.$$

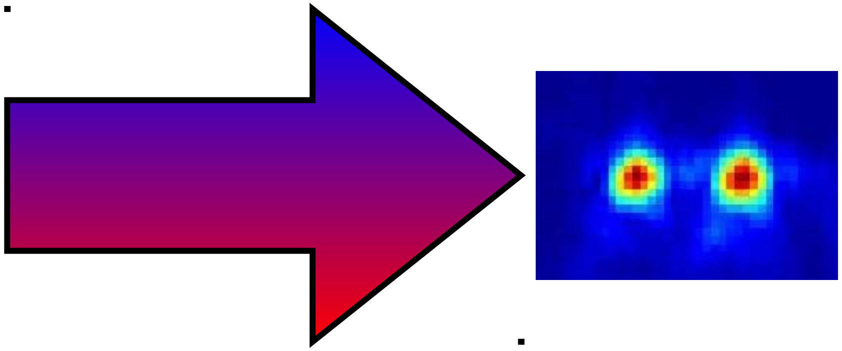
➡ universal two-qubit gate

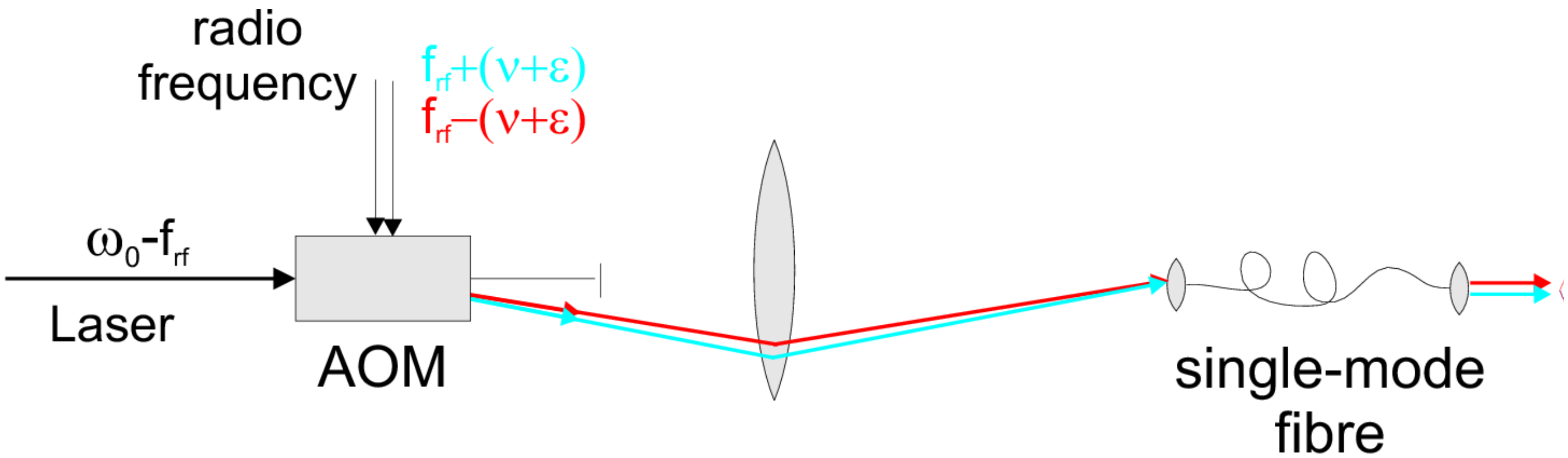


K. Mølmer, A. Sørensen,
Phys. Rev. Lett. **82**, 1971 (1999)
C. A. Sackett et al.,
Nature **404**, 256 (2000)

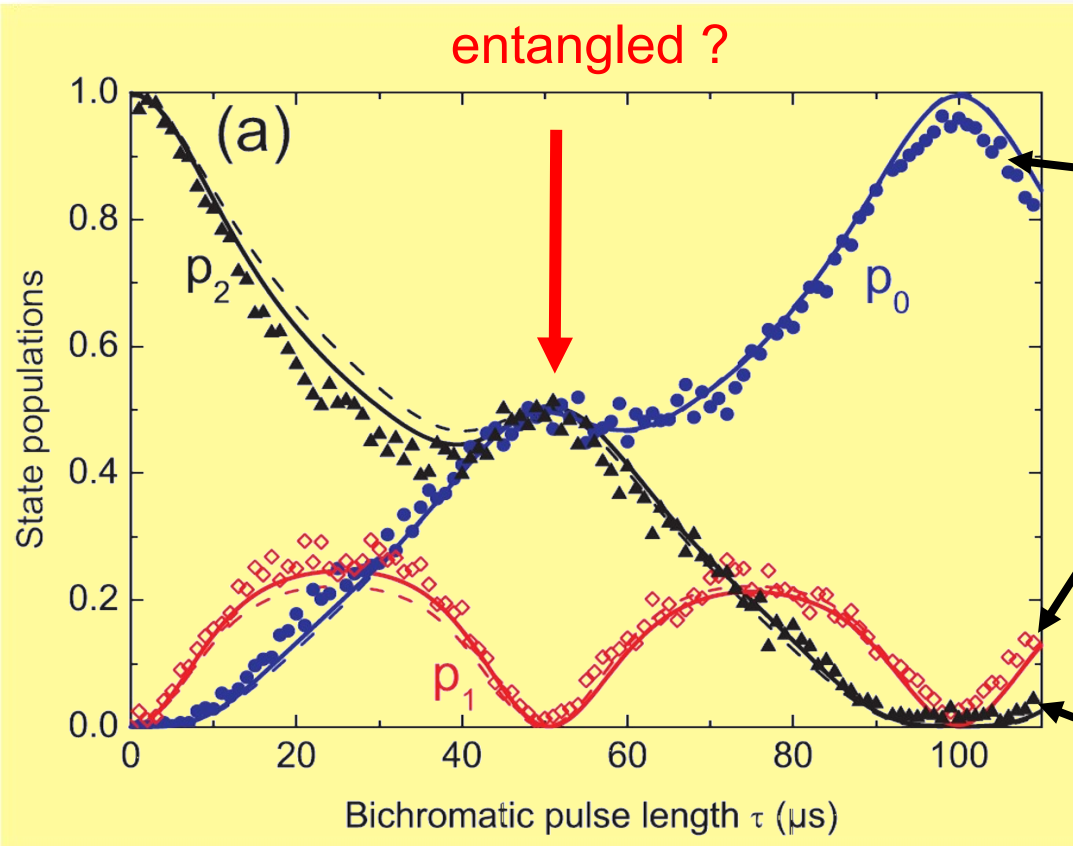
Mølmer - Sørensen gate

bicromatic beam
applied to both ions





Mølmer - Sørensen gate



$|gg,n\rangle$

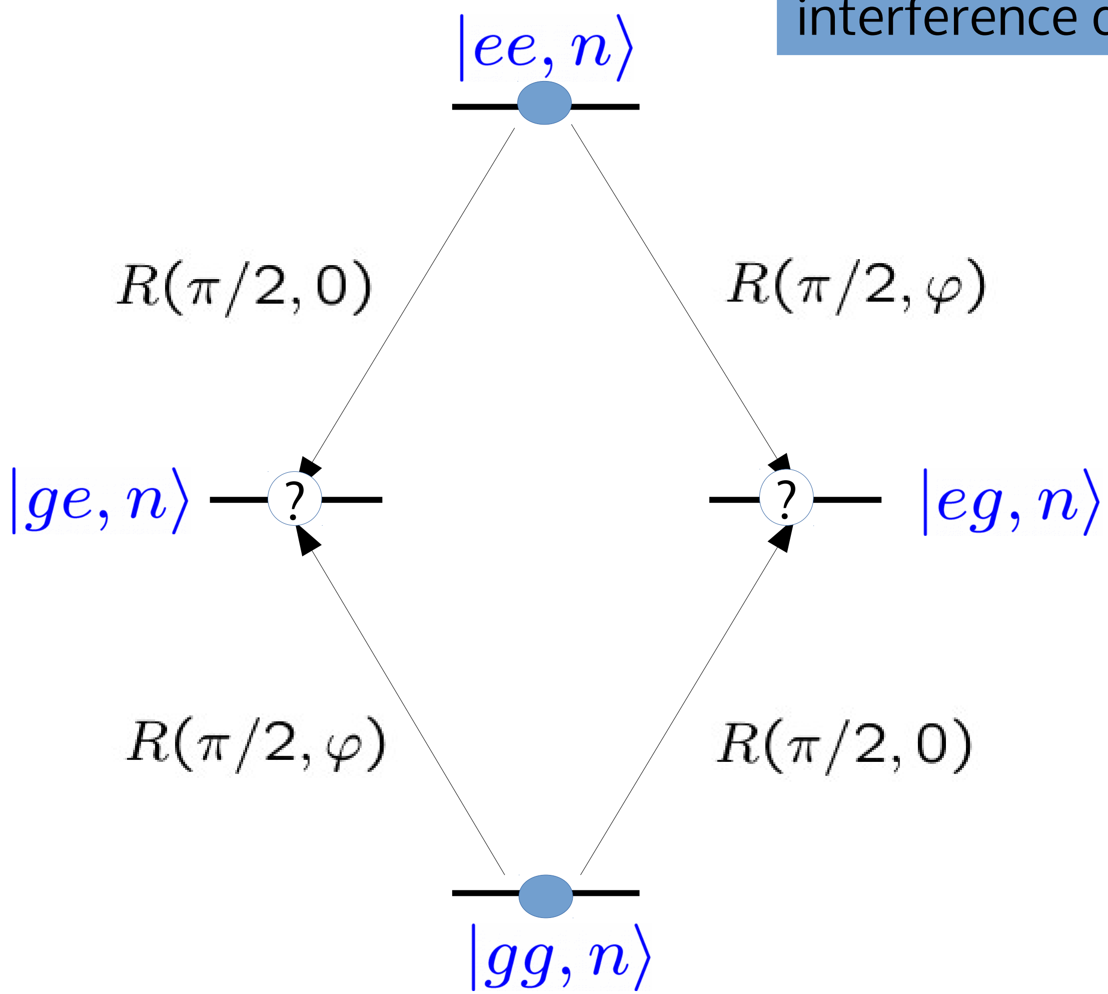
$\left\{ \begin{array}{l} |eg,n+1\rangle, |eg,n-1\rangle \\ |ge,n+1\rangle, |ge,n-1\rangle \end{array} \right.$

$|ee,n\rangle$

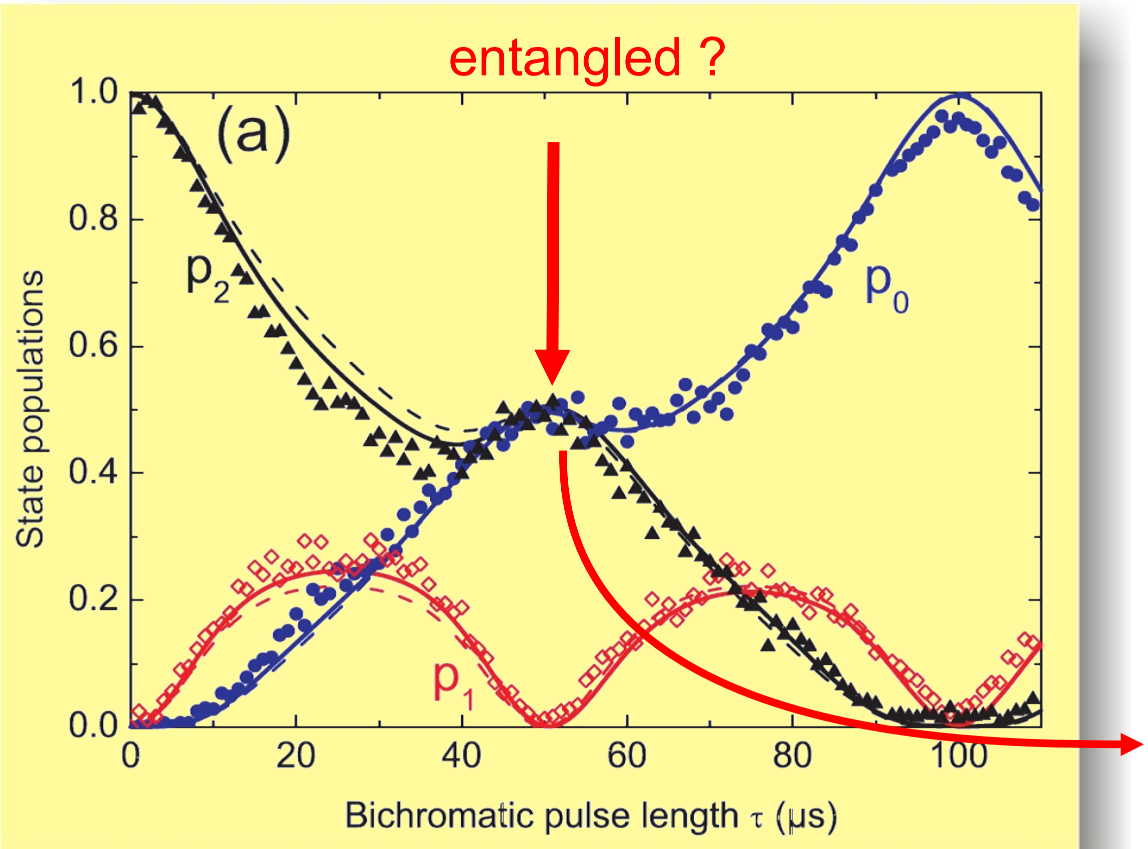
Towards fault-tolerant quantum computing with trapped ions"J. Benhelm *et al.*, Nature Phys. **4**, 463 (2008).

Test interference capability

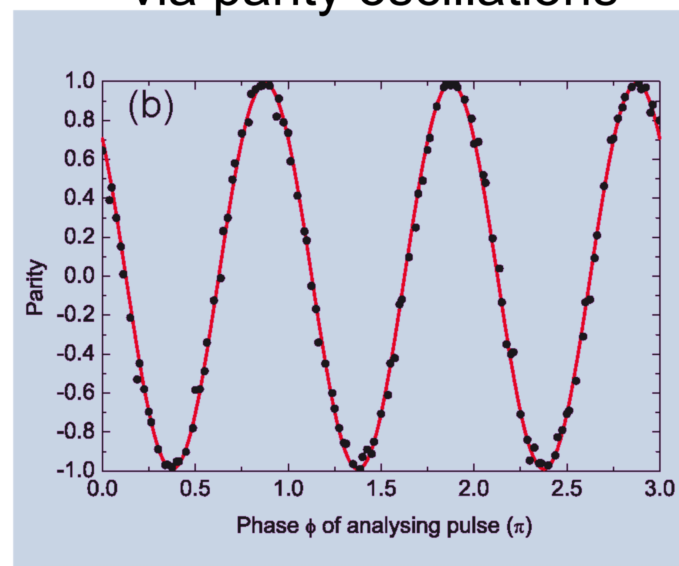
Constructive or destructive interference depending on the phase



Mølmer - Sørensen gate



measure entanglement
via parity oscillations

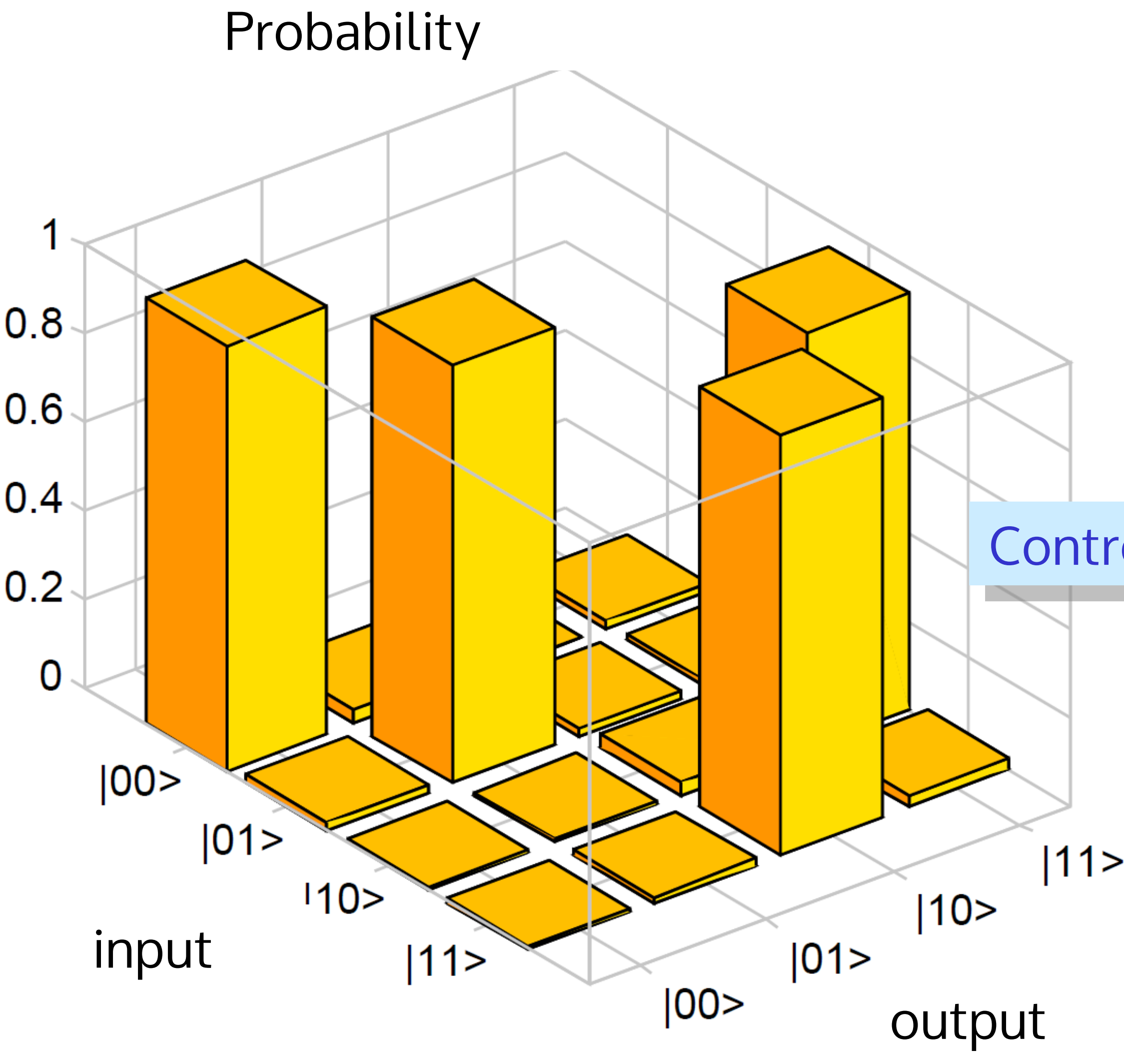


gate duration 50 μs

average fidelity: 99.3 (2) %

Infidelity: detection and initialization, qubit dephasing, trap frequency stability

Can be turned into a CNOT



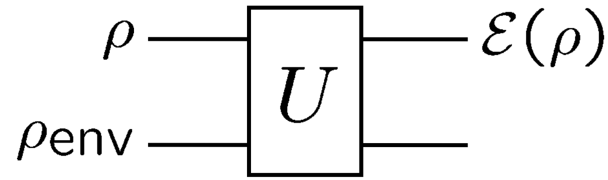
$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

Control bit

Target bit

Quantum process tomography

How to describe action of an operation acting on system of qubits which are coupled to their noisy environment ?



Use completely positive map / quantum operations.
Poyatos, Cirac, Zoller PRL 78, 390 (1997),
Chuang & Nielsen J. Mod. Opt. **44**, 2455 (1997)

Operator sum $\mathcal{E}(\rho)$:

$$\mathcal{E}(\rho) = \sum_{mn} \underbrace{\chi_{mn}}_{\text{process matrix}} \cdot A_m \rho A_n^{\dagger}$$

Operators A are set of
4^N basis operators
for the space of matrices !

Quantum process tomography

Recipe for quantum process tomography:

A) Choose input states for process such that corresponding density matrices form a basis set for the space of matrices.

B) For all these input states do a quantum state tomography of the output state

C) Estimate transfer matrix using your prefered method

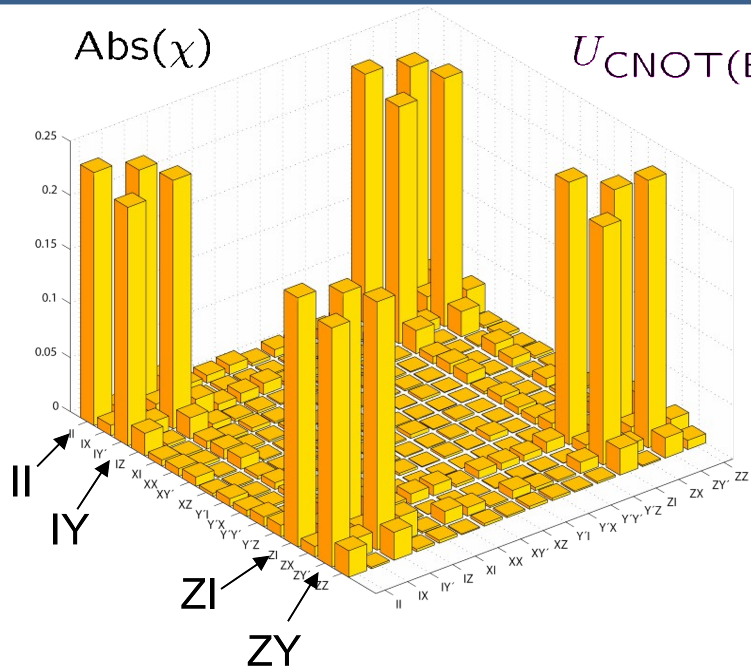
2 qubit QPT in our experimental setup:

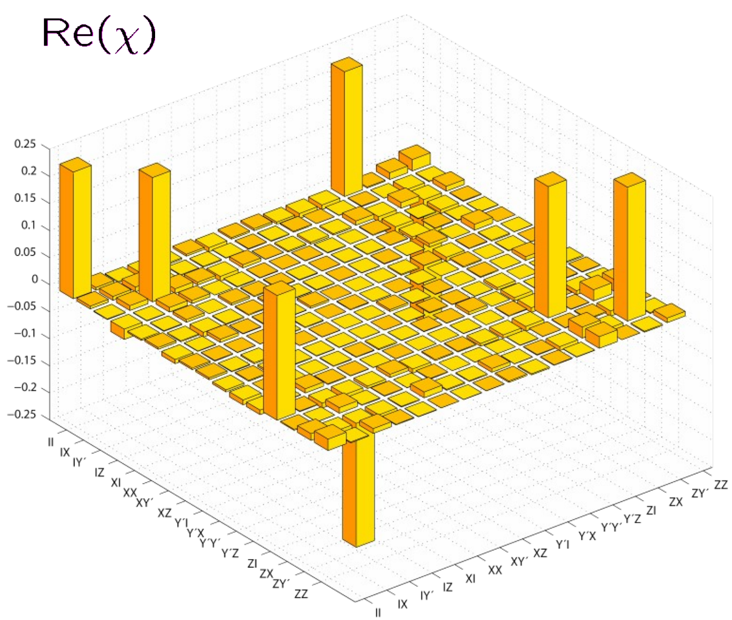
For a two qubit process we use:

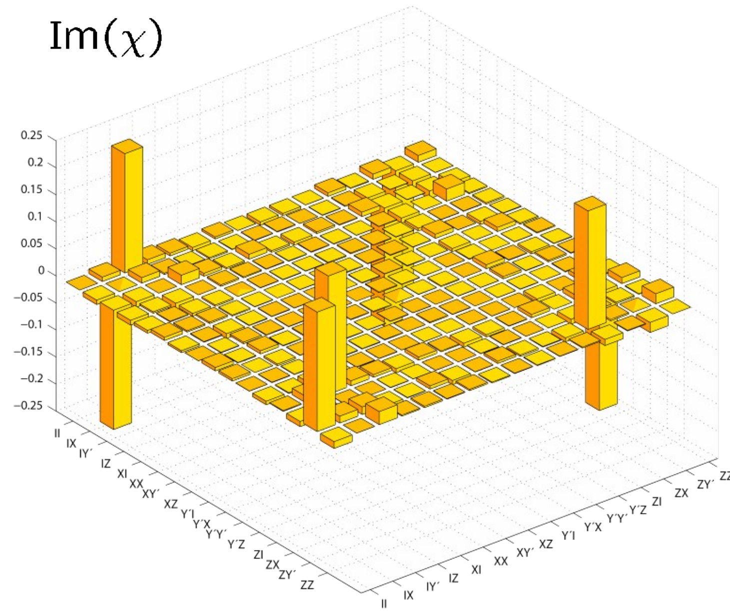
16 input states
9 tomographic measurement settings for each state
total number of measurements is 144

Quantum process tomography of a CNOT

$$U_{\mathrm{CNOT(B)}} = = -\frac{1}{2}\hat{U}_Z \cdot \left(I \otimes I + Z \otimes I - iI \otimes Y' + iZ \otimes Y'\right).$$







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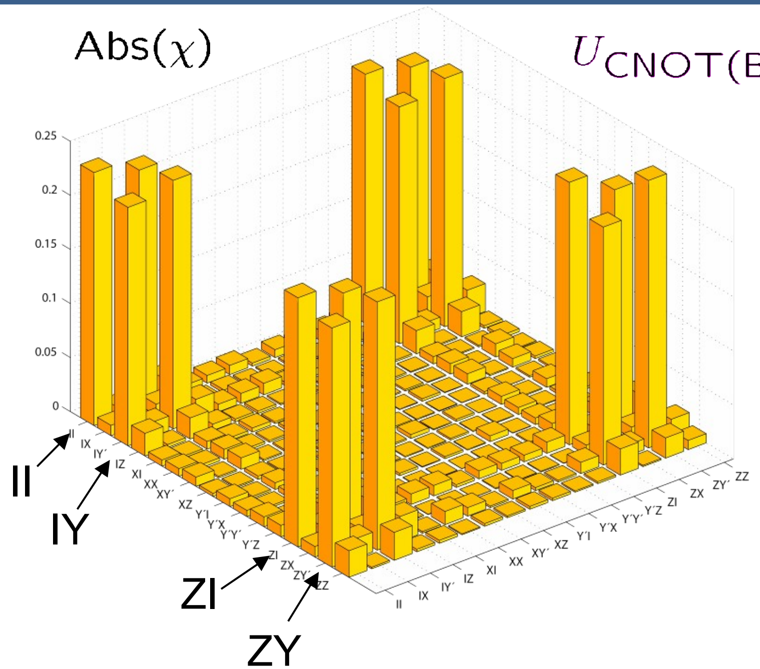
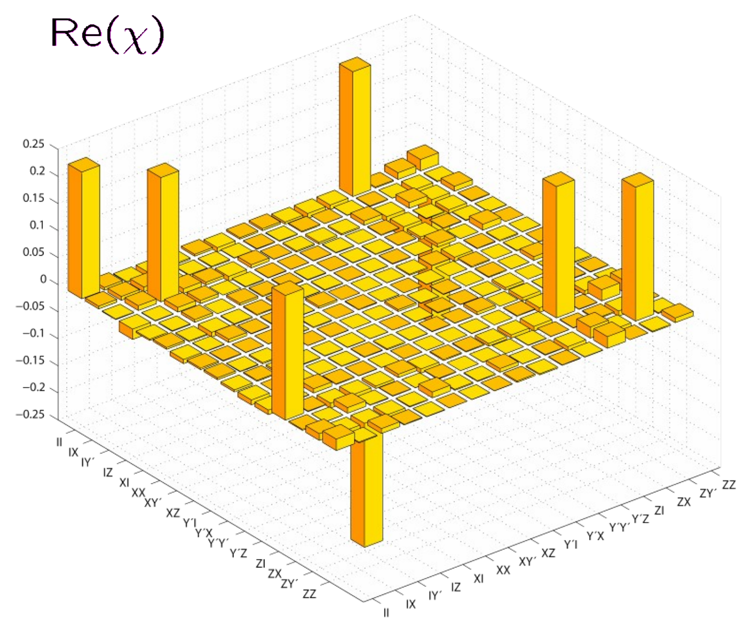
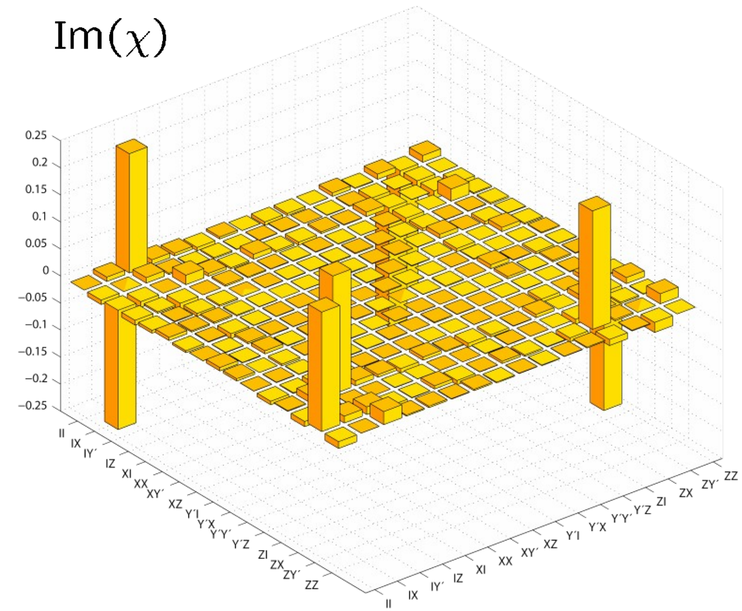


Measure	Fidelity
$F_p = \mathrm{Tr}\left(\chi_{\mathrm{id}}\chi_{\mathrm{CNOT}}\right),$	90.8(6)%
Mean fidelity of output state $\bar{F}$	92.6(6)%



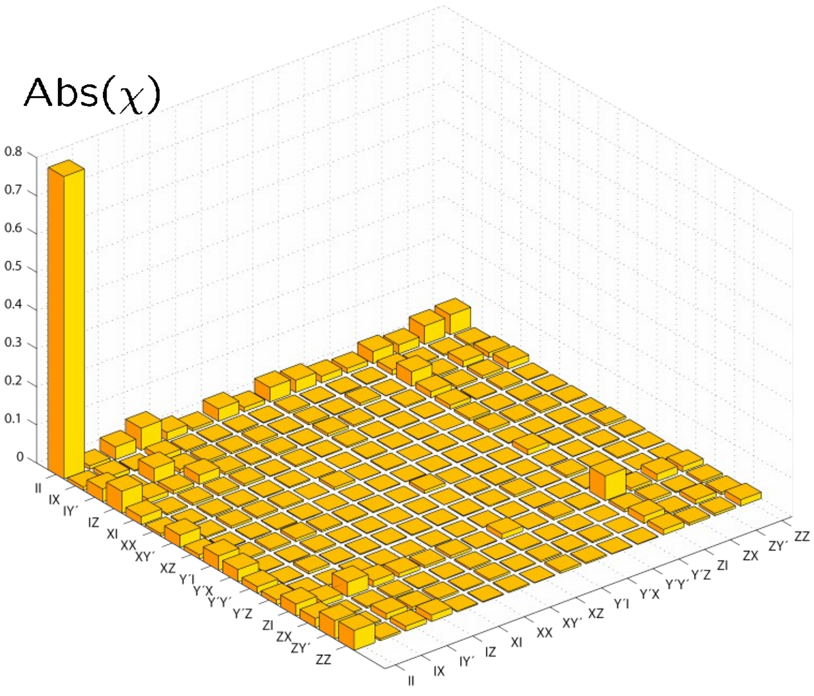


Markovian environment ?

Most quantum algorithms contain more than a single gate operation.

Can we infer the performance of combined gate operation from a QPT of a single gate operation ?

Experiment: QPT of two subsequently applied CNOT gates



Experimental result	2x single gate result
$F_p = 79(1)\%$	$F_p = 82.8\%$
$\bar{F} = 83.4(8)\%$	$\bar{F} = 86.2\%$

Is quantum process tomography practical?

How will we asses our quantum computers?

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II. Ability to initialize the state of the qubits ✓

III. Long relevant coherence times, much longer than gate operation time ✓

IV. "Universal" set of quantum gates ✓

V. Qubit-specific measurement capability ✓

The DiVincenczo criteria for quantum computing

I. Scalable physical system, well characterized qubits

II. Ability to initialize the state of the qubits with sufficient fidelity

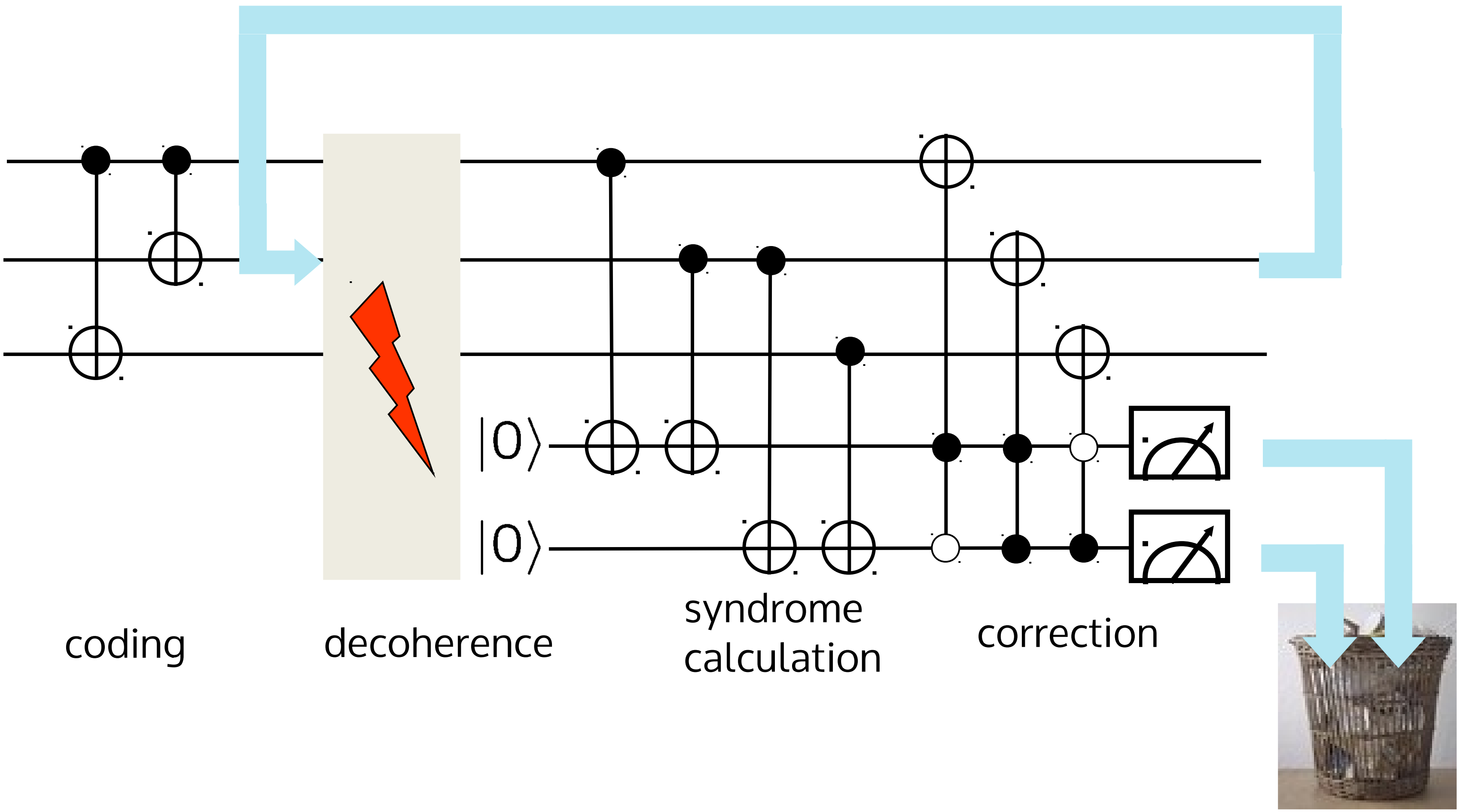
III. Long relevant coherence times, much longer than gate operation time

IV. "Universal" set of quantum gates with sufficient fidelity

V. Qubit-specific measurement capability with sufficient fidelity

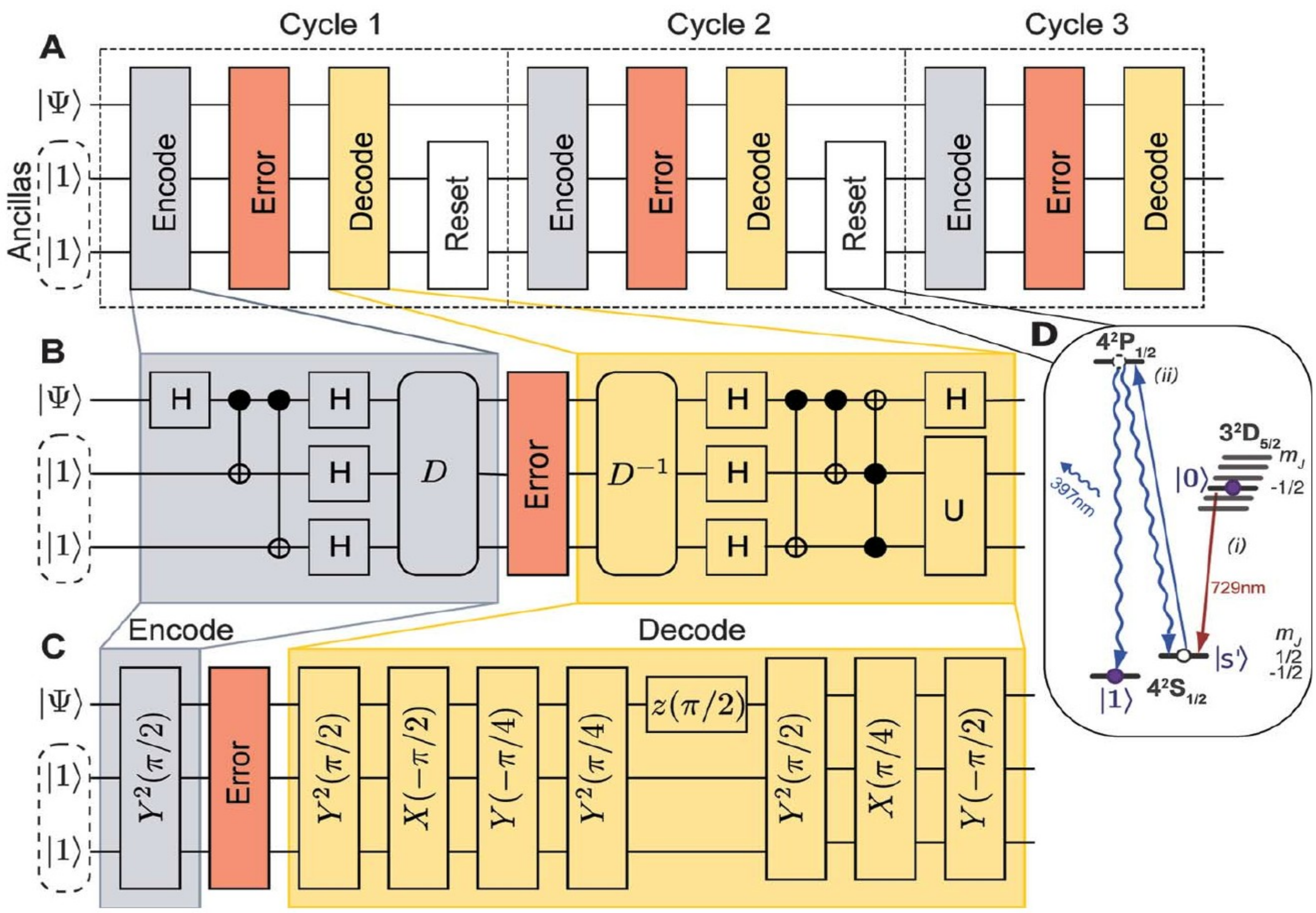
⇒ need to beat the fault-tolerant "threshold"

Quantum error correction



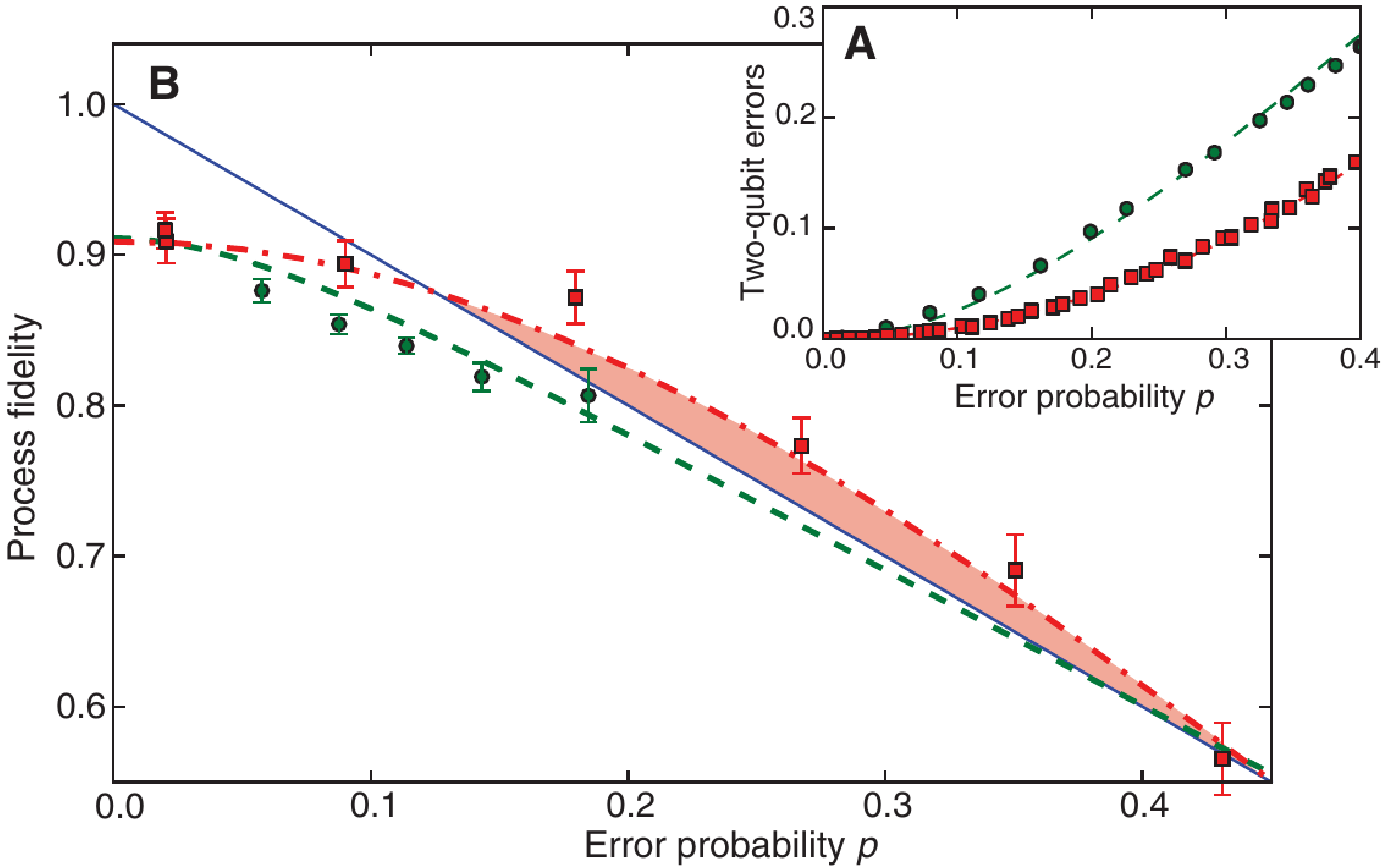
First experiment on three ions by Chiaverini et al., Nature **432**, 602 (2004).

Error correction implementation



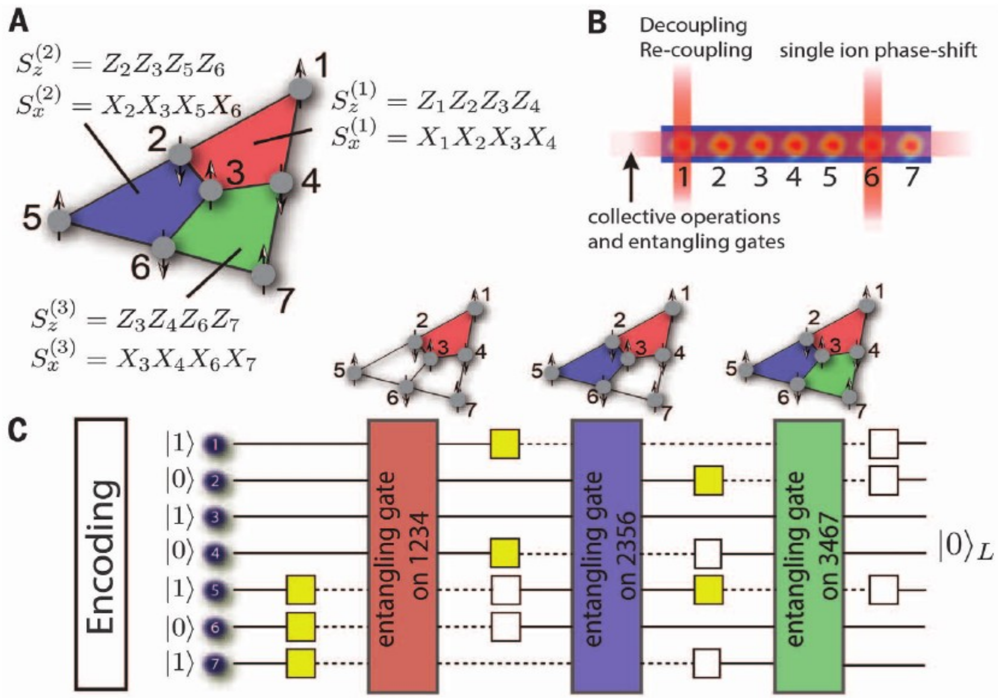
Repeatable quantum error correction:
P. Schindler et al, Science 332, 1059 (2011)

Error correction implementation



P. Schindler et al, Science 332, 1059 (2011)

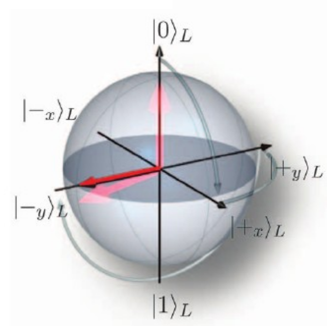
Operating on a topologically corrected qubit

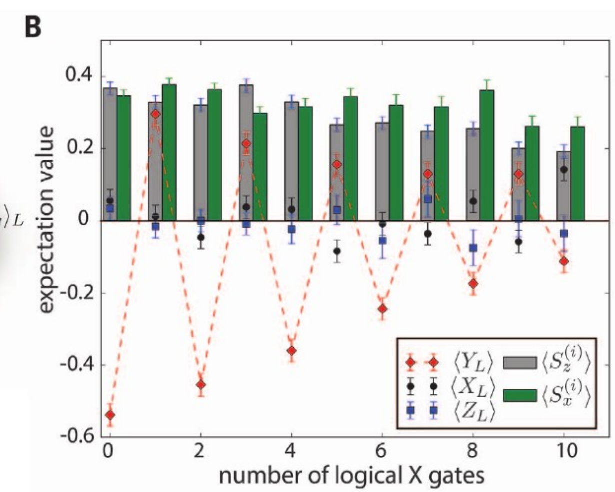


Single qubit operations

Fully protected qubit

D. Nigg *et al.*, Science 345, 302 (2014)





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- Paul traps
- Laser ion-interaction

Lecture #2: Quantum computing
- Quantum gates
- Quantum state tomography

Lecture #3: Decoherence
- Qubit decoherence
- Anomalous heating

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- Digital quantum simulation

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- Atomic clocks, quadrupole shifts
- Michelson-Morley experiment

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