Quantum computing with trapped ions

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- Basics of ion trap quantum computing
- Measuring a density matrix
- Quantum gates

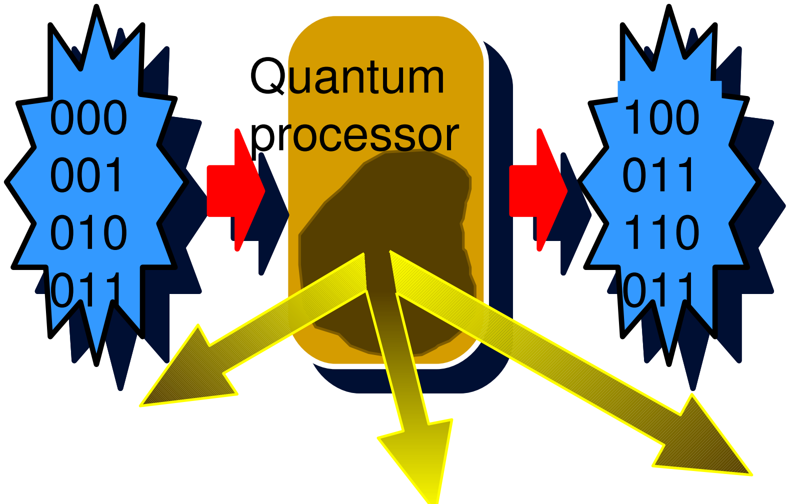
€ FWF SFB SCALA QGATES Industrie Tirol IQI GmbH

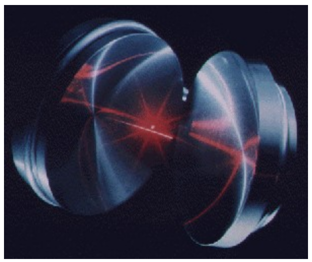
FWF bm:bwk

$ IARPA

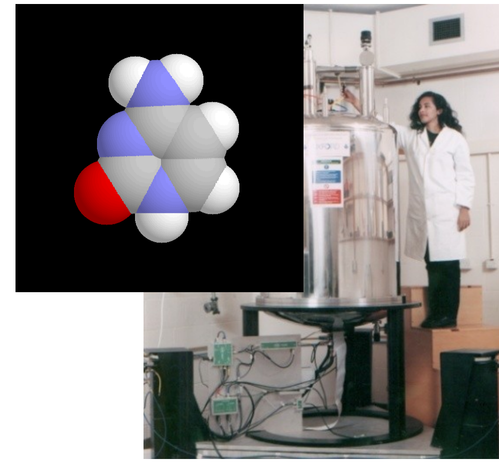
Zürich, Dec 8th 2008

Which technology ?

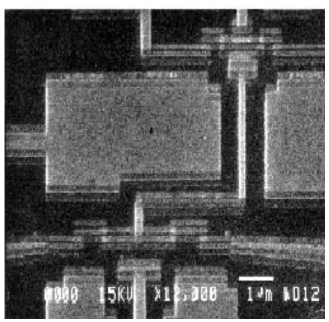




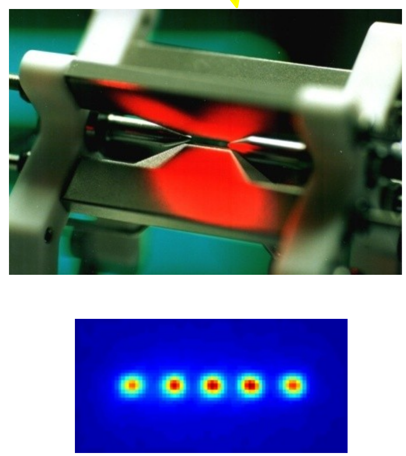
Cavity QED



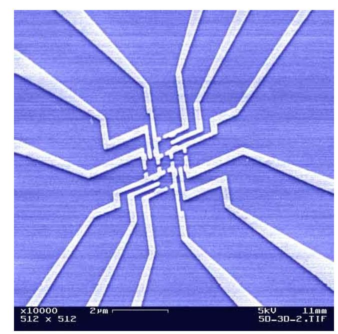
NMR



Superconducting qubits

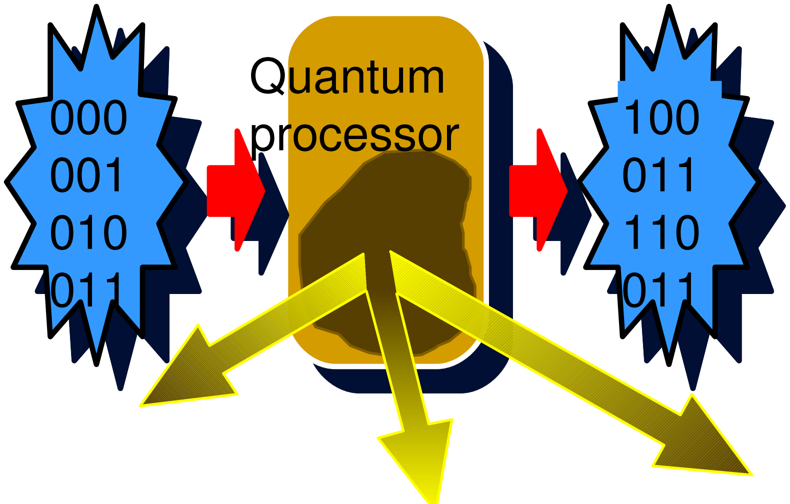


Trapped ions



Quantum dots

Which technology ?



Cavity QED

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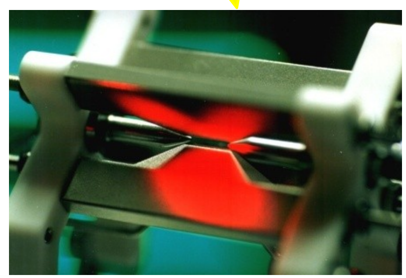
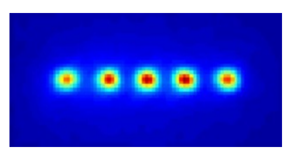
Superconducting qubits





Trapped ions

Quantum dots

Why trapped ions ?

Good things about ion traps:

- Ions are excellent quantum memories; single qubit coherence times > 10 minutes have been demonstrated

- Ions can be controlled very well

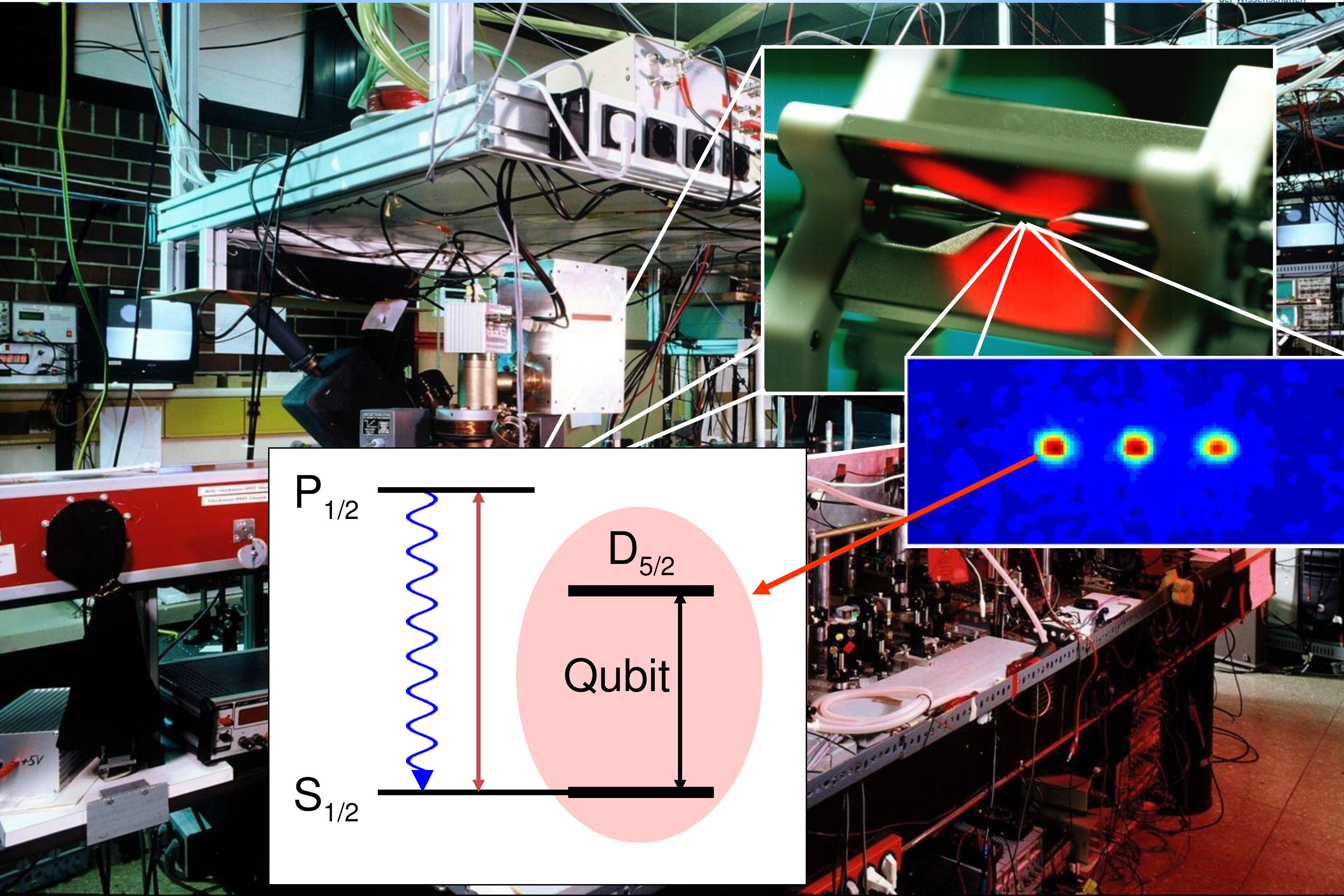
- Many ideas to scale ion traps

Bad things about ion traps:

- Slow (~1 MHz)

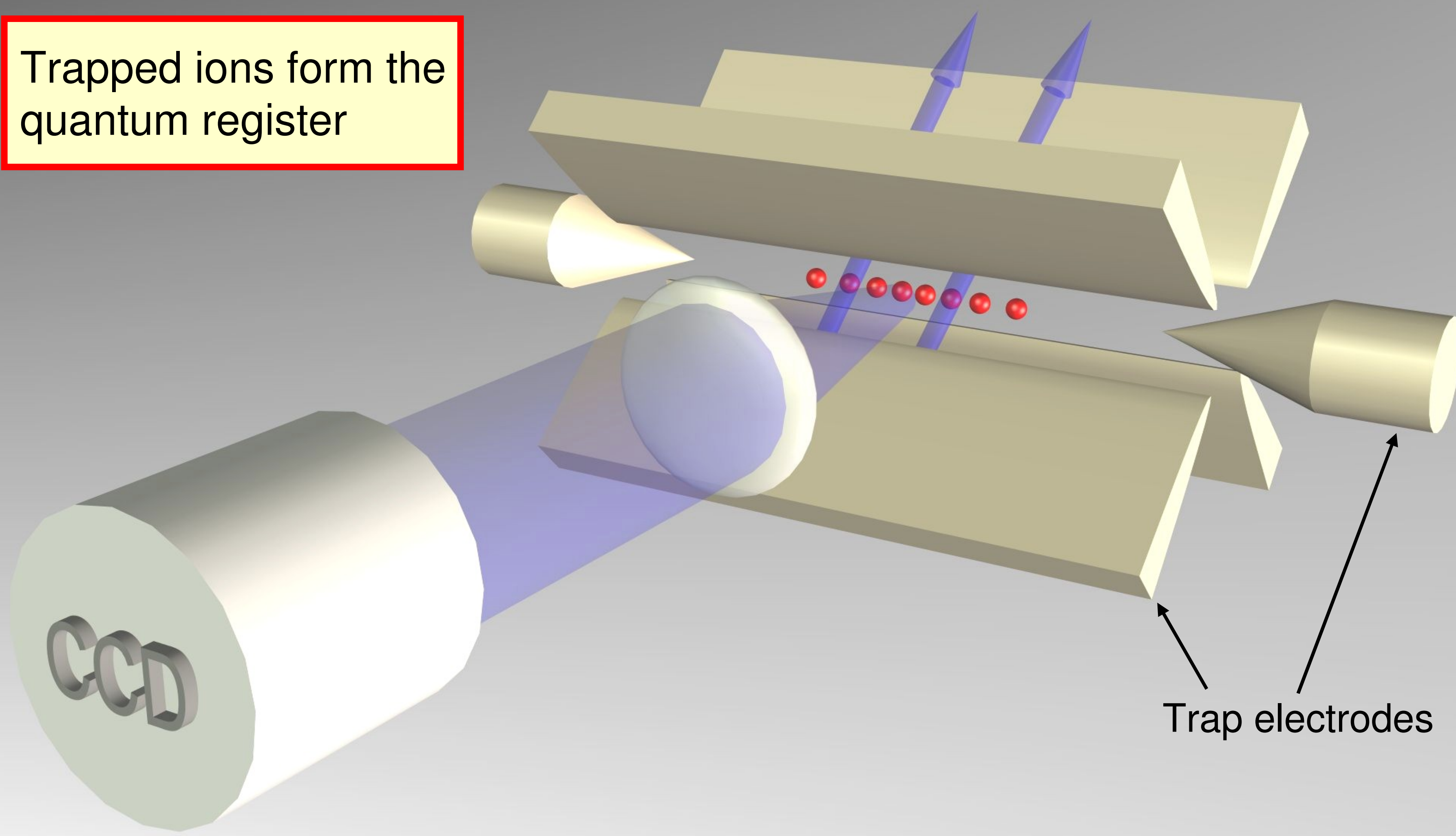
- Technically demanding

The hardware



Innsbruck quantum processor

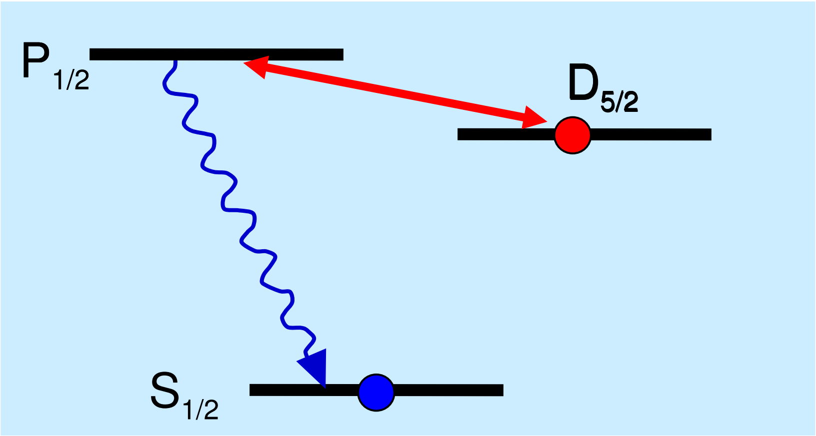
Trapped ions form the quantum register



DiVincenzo criteria

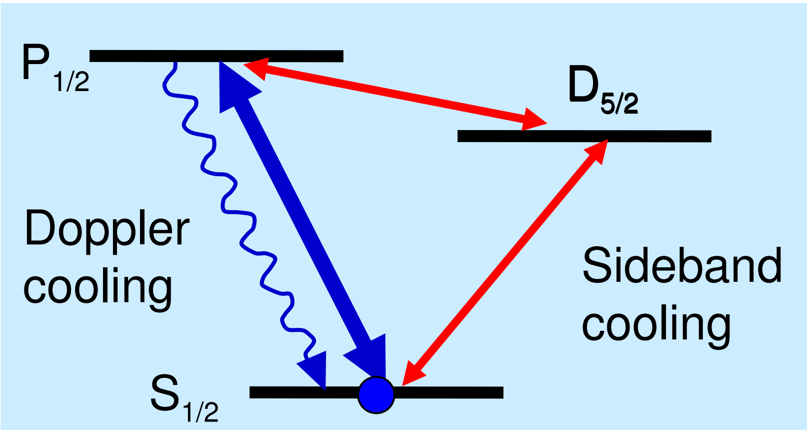
- Scalable physical system, well characterized qubits
- Ability to initialize the state of the qubits
- Long relevant coherence times, much longer than gate operation time
- “Universal” set of quantum gates
- Qubit-specific measurement capability

Experimental procedure



1. Initialization in a pure quantum state

Experimental procedure



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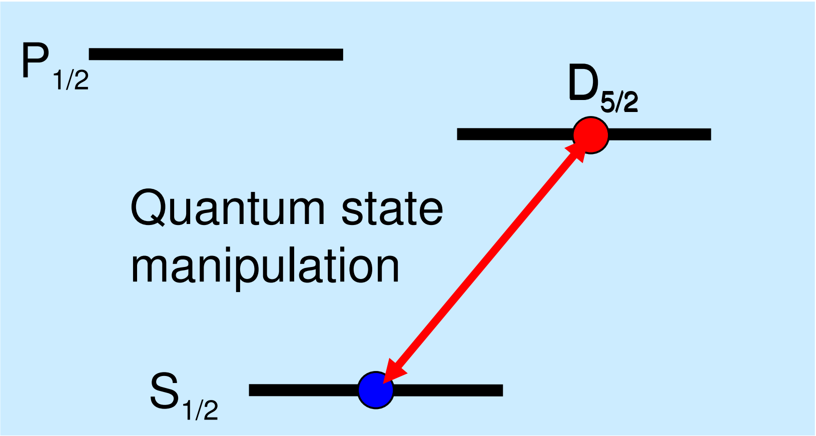
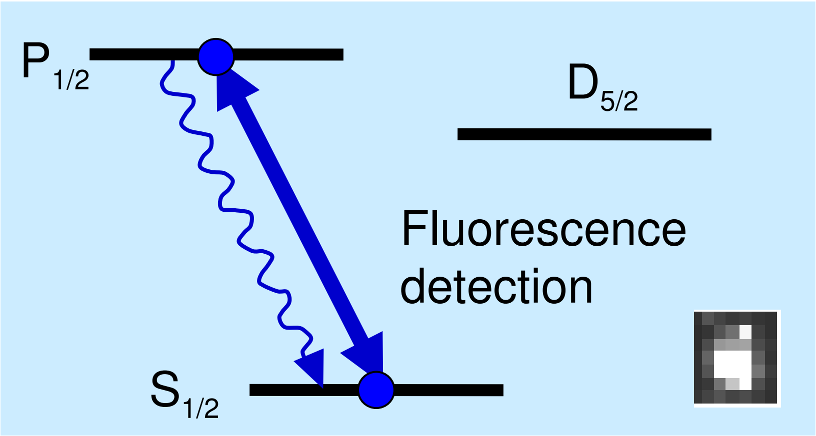


1. Initialization in a pure quantum state

2. Quantum state manipulation on $S_{1/2} - D_{5/2}$ transition

Experimental procedure



1. Initialization in a pure quantum state:

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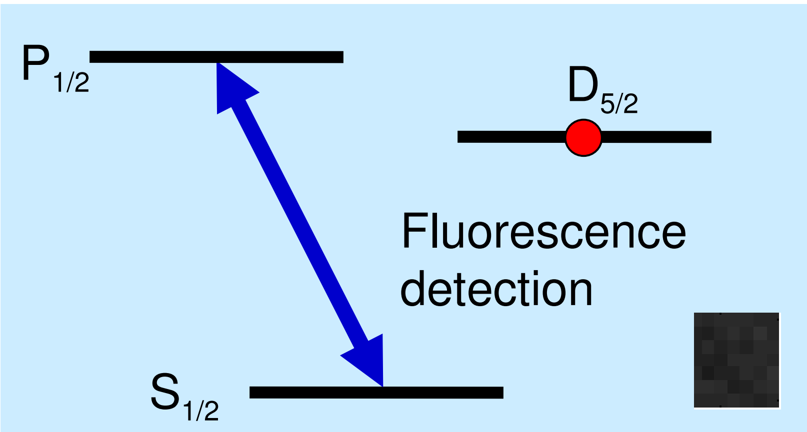
3. Quantum state measurement by fluorescence detection

Experimental procedure



1. Initialization in a pure quantum state:

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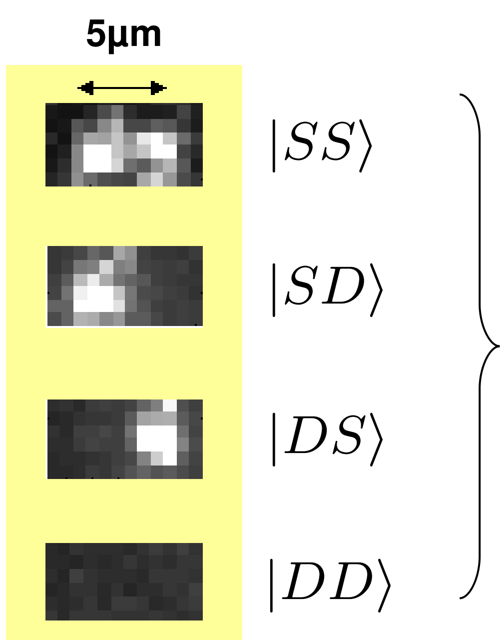
3. Quantum state measurement by fluorescence detection

Two ions:

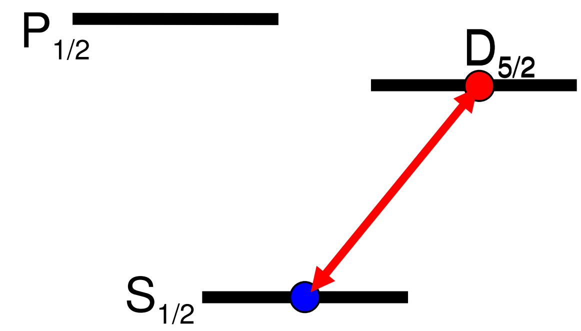
Spatially resolved detection with CCD camera

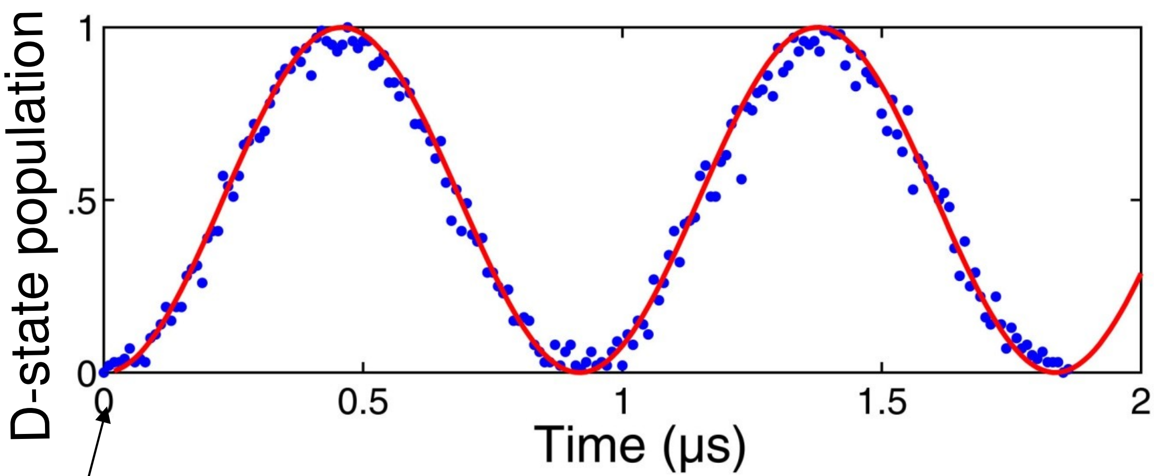


50 experiments / s

Repeat experiments 100-200 times

Rabi oscillations

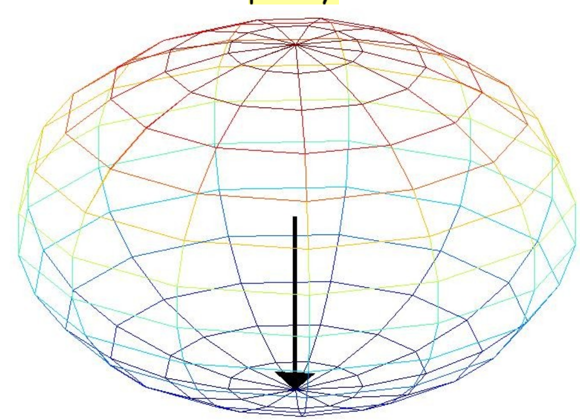




$|S\rangle$

$|D\rangle$

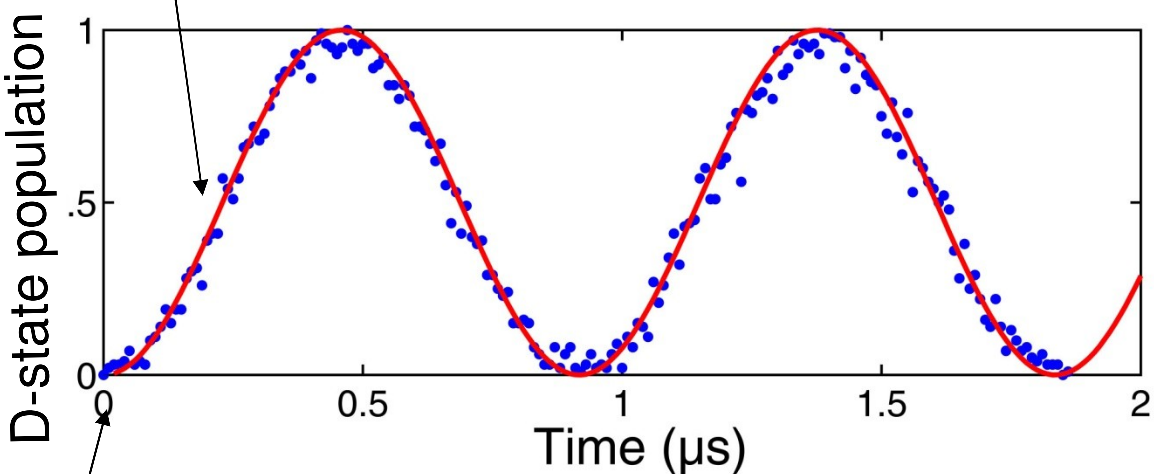
$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$



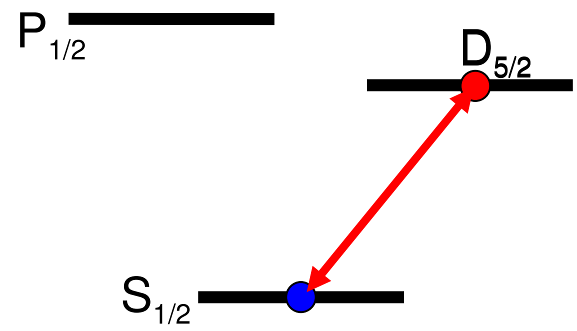
$|S\rangle$

Rabi oscillations

$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$

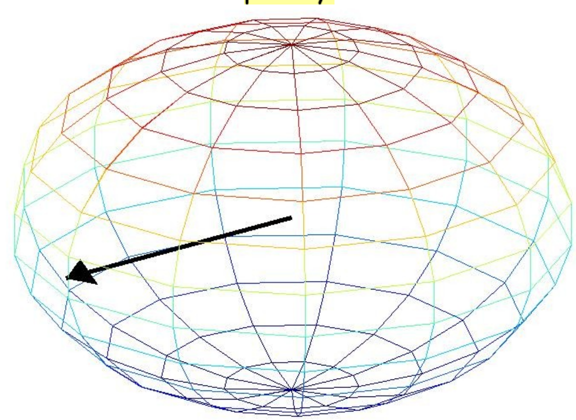


$$|S\rangle$$



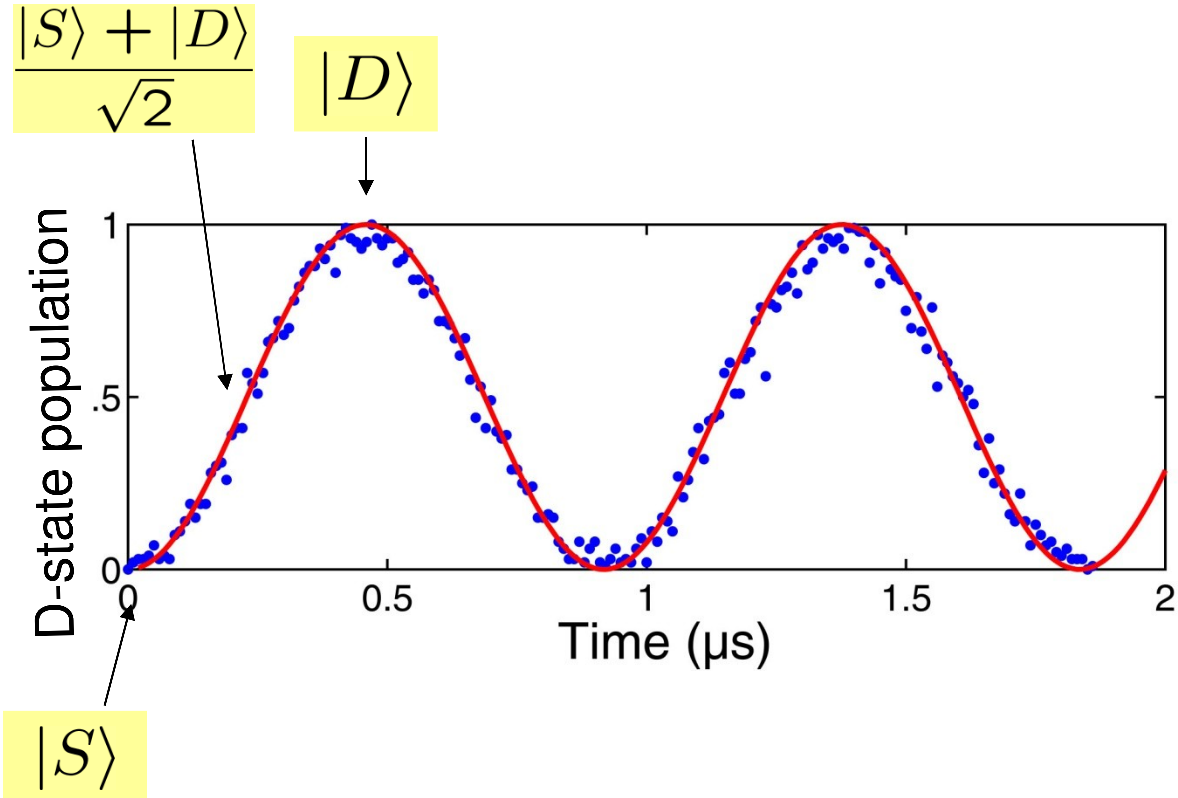
$$|D\rangle$$

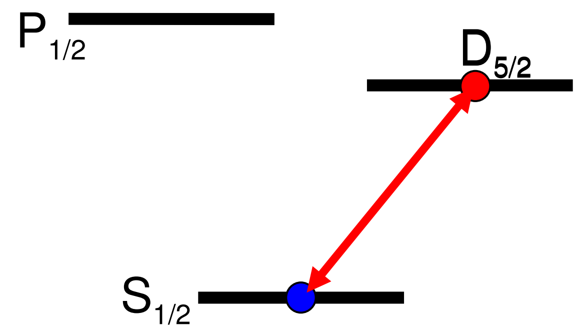
$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$

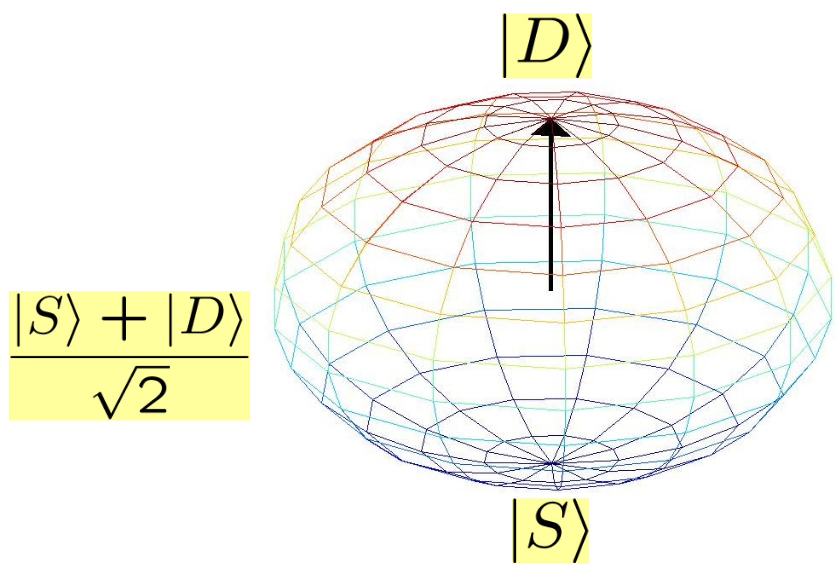


$$|S\rangle$$

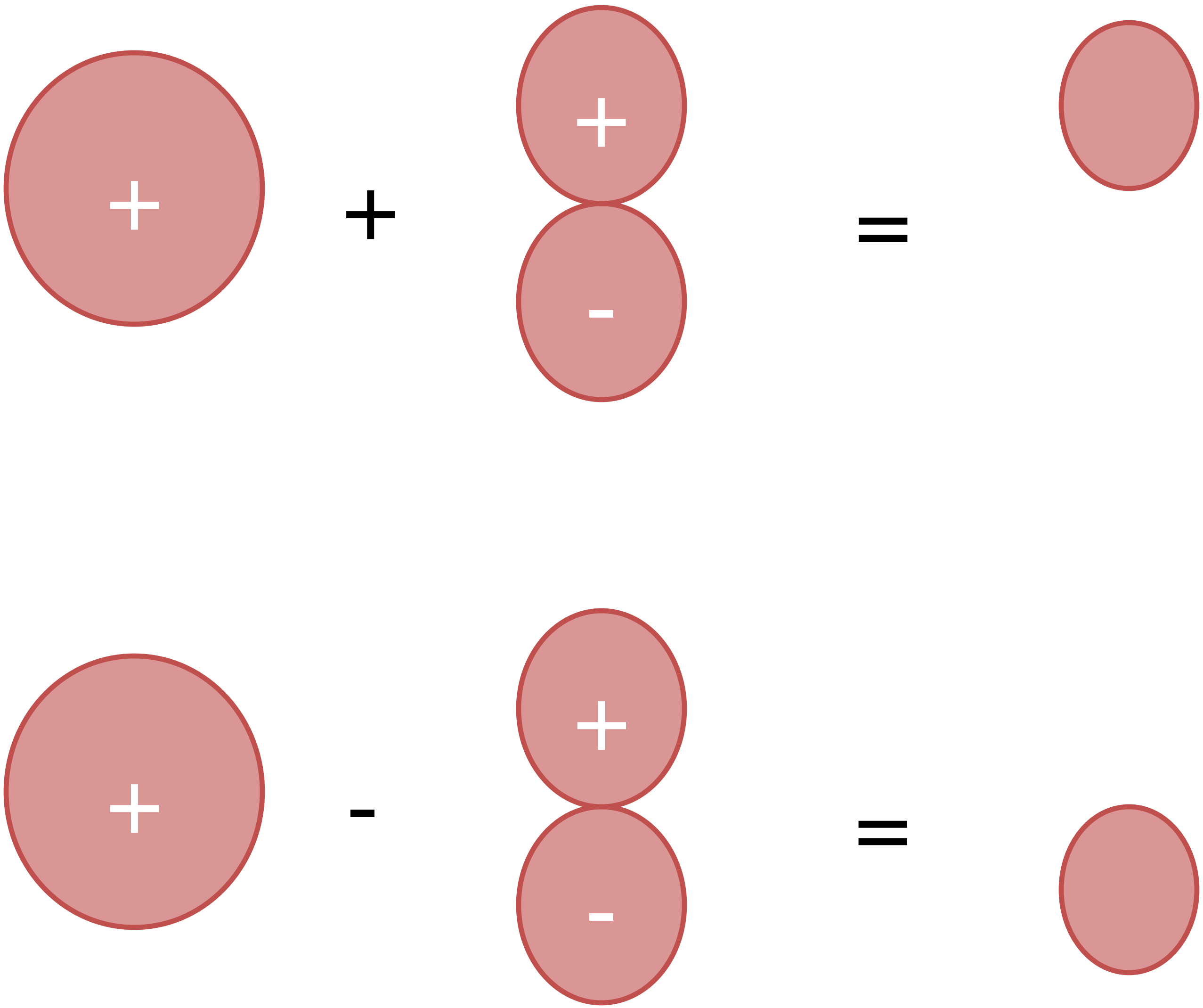
Rabi oscillations







The phase ...



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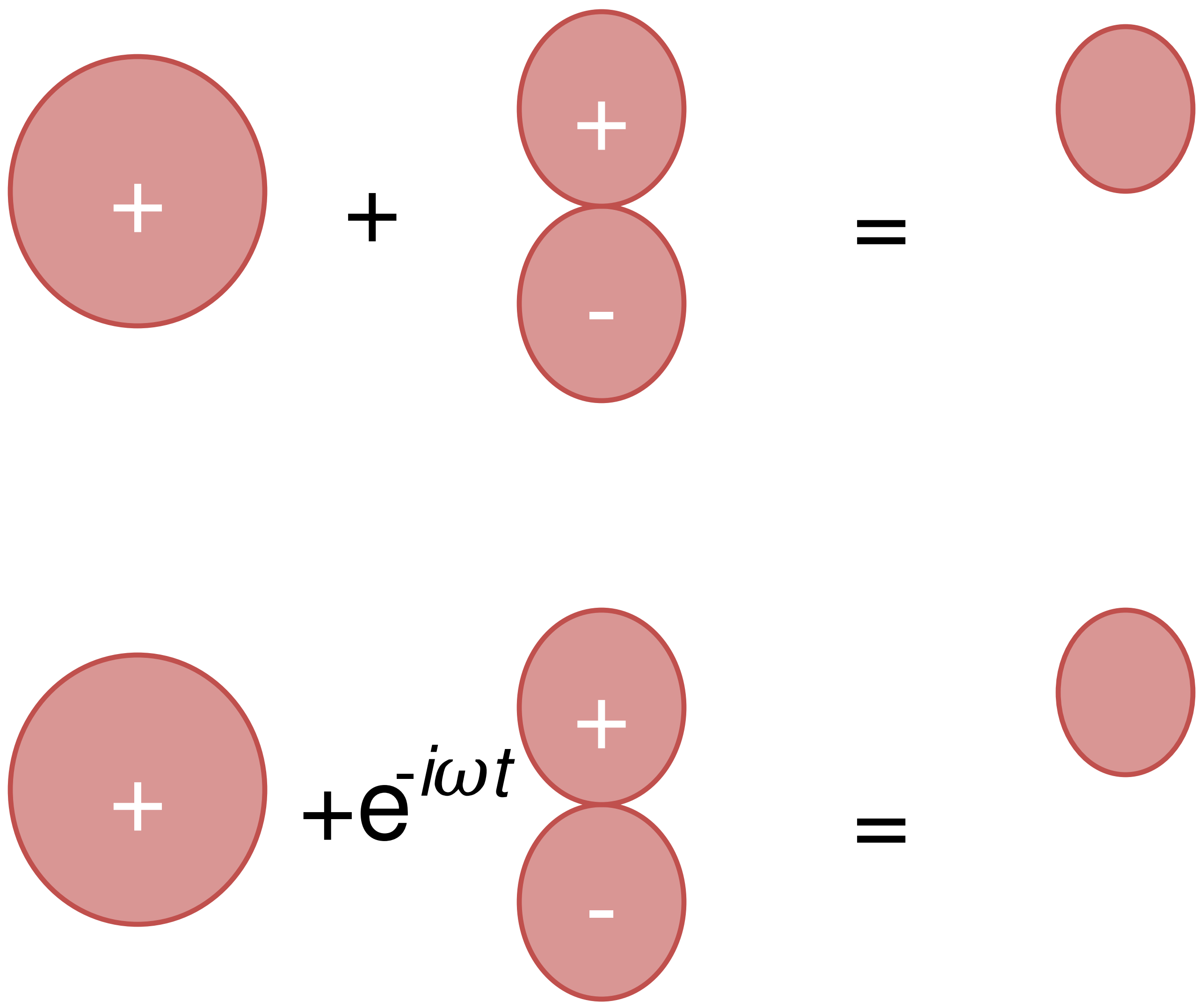
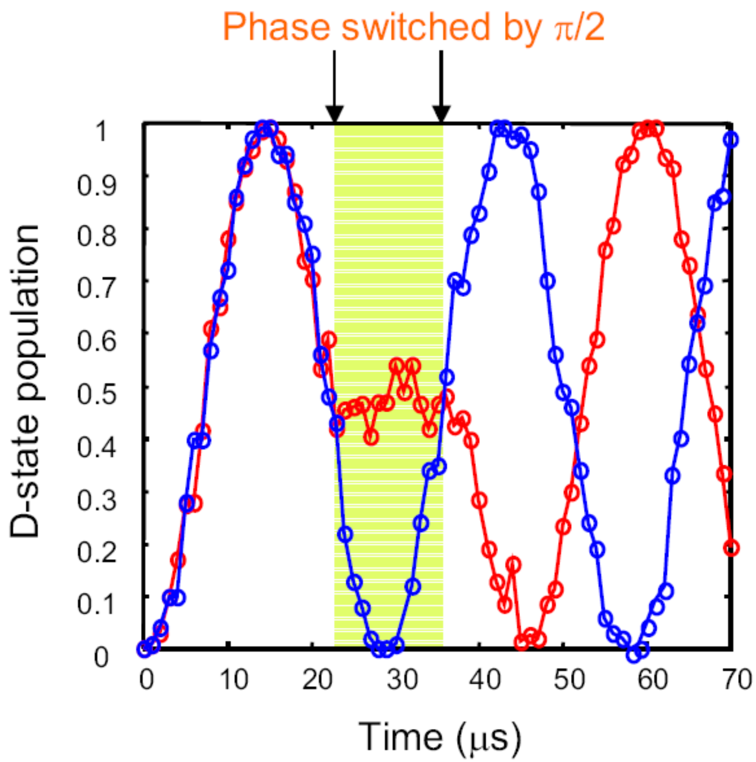
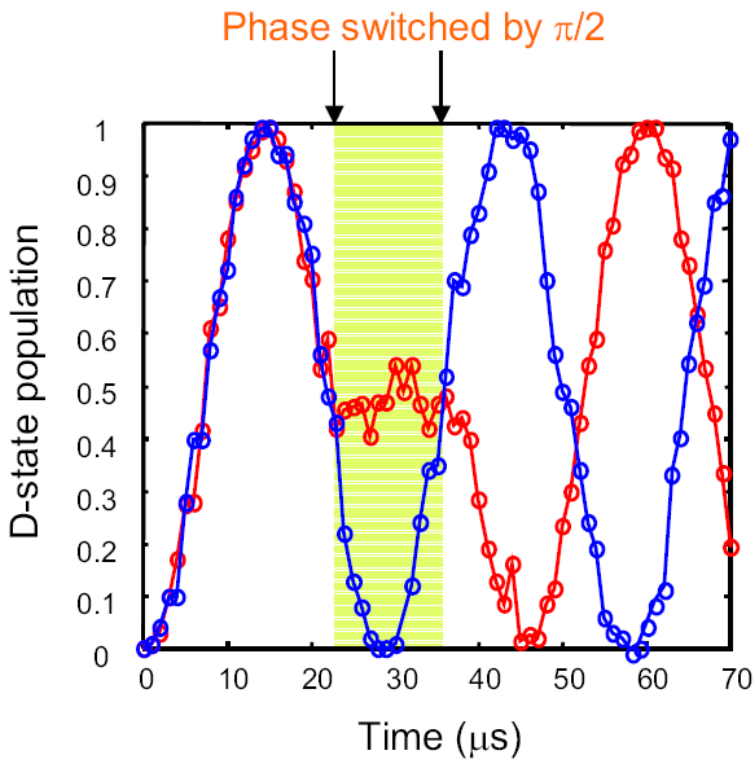


The phase ...



Phase switched by π/2
D-state population
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
60
70
Time (μs)

The phase ...



Phase switched by π/2
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0
0.1
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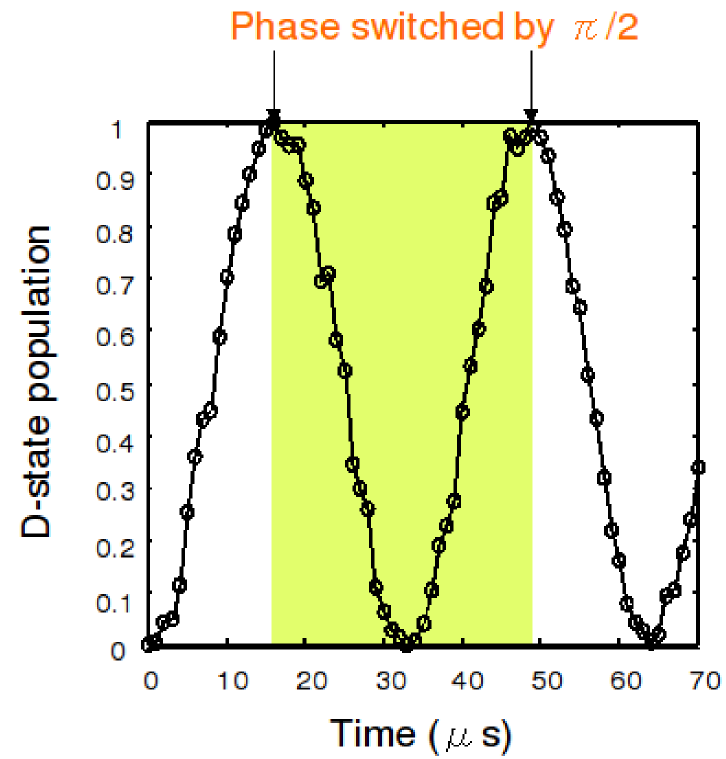
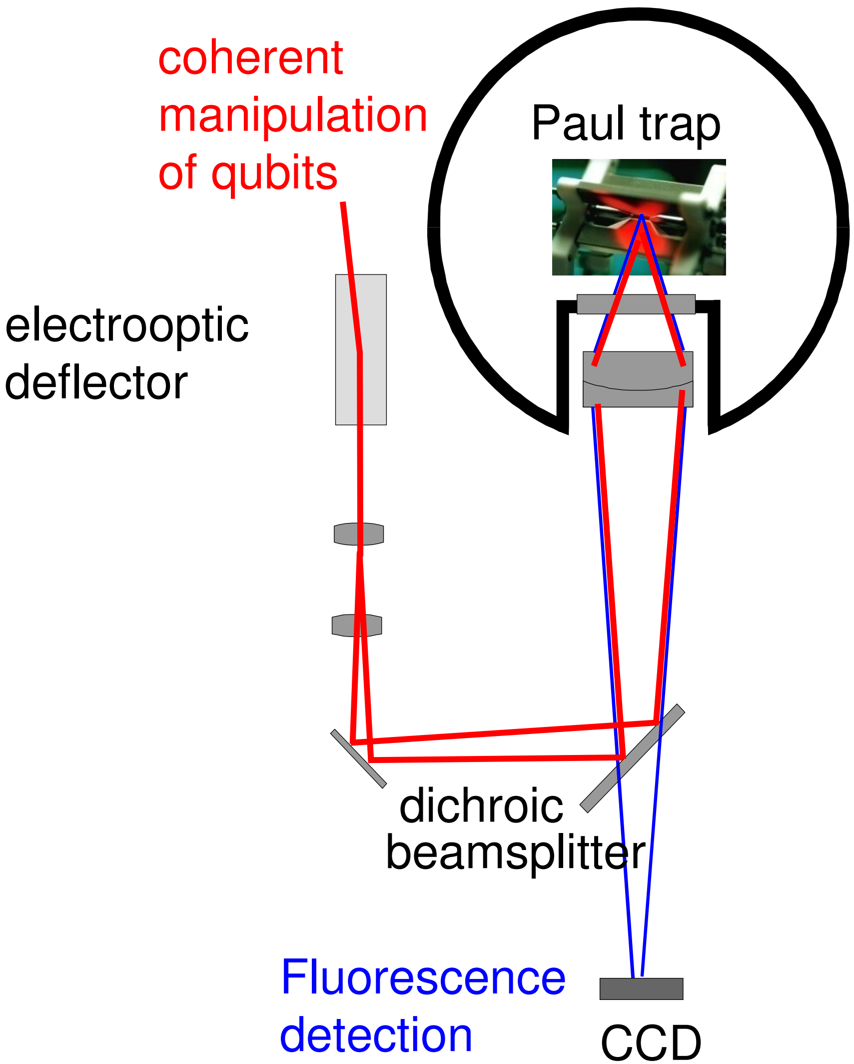
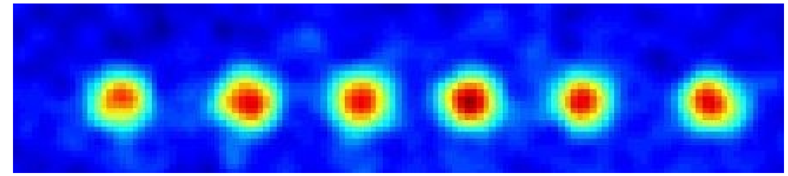
0.9
1
0
10
20
30
40
50
60
70
Time (μs)

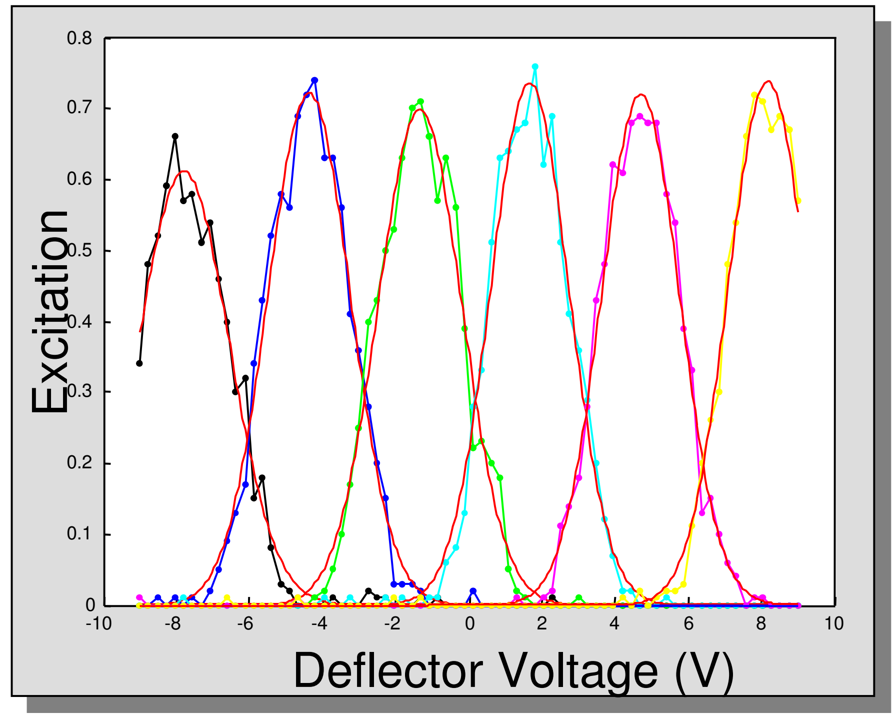


Phase switched by π /2
D-state population
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
10
20
30
40
50
60
70
Time (μ s)

Addressing single qubits







- inter ion distance: ~ 4 µm
- addressing waist: ~ 2 µm

< 0.1% intensity on neighbouring ions

Decoherence mechanisms

Memory errors:

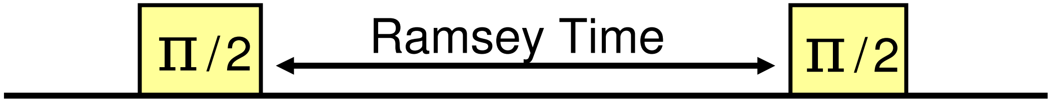
- Bit-flips
- Dephasing

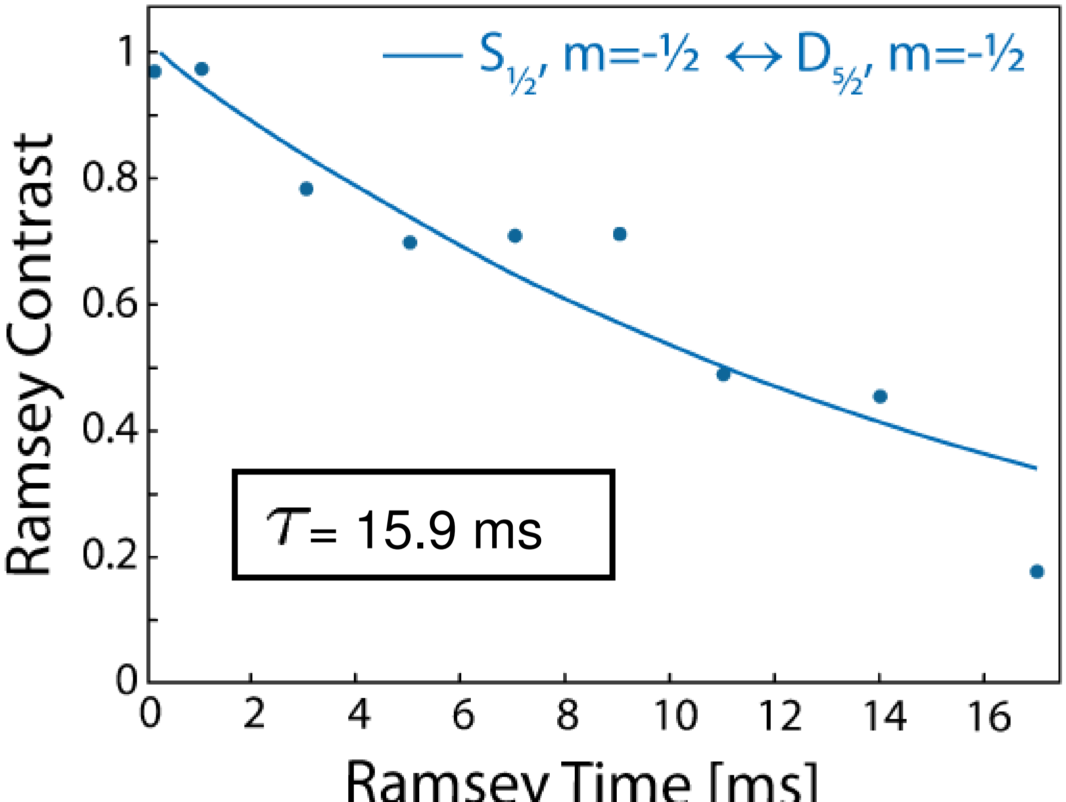
Operationial errors

- technical imperfections …

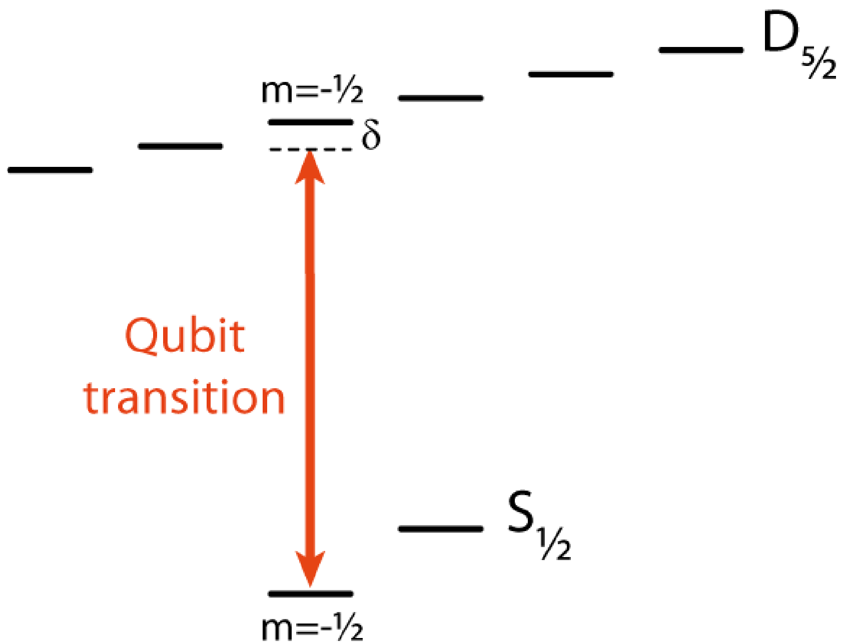
Dephasing of qubits

Ramsey Experiment



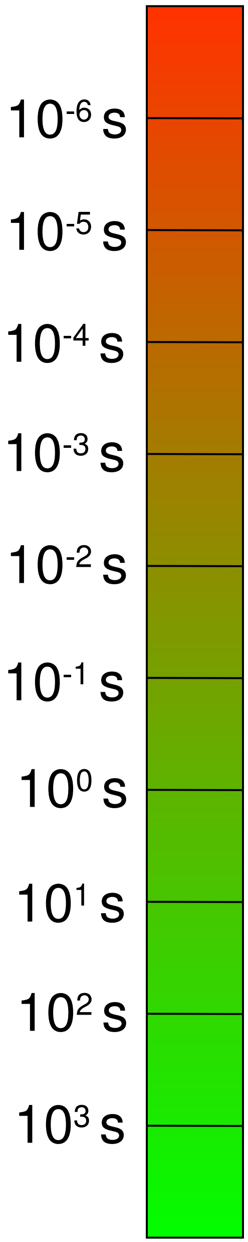


Zeeman shift



$\Rightarrow$ Detuning causes a linear drift of the phase ϕ with $e^{i\phi} = e^{i\delta t}$

Realized time scales



Single qubit gates

Two qubit gates (Geometric phase gates)

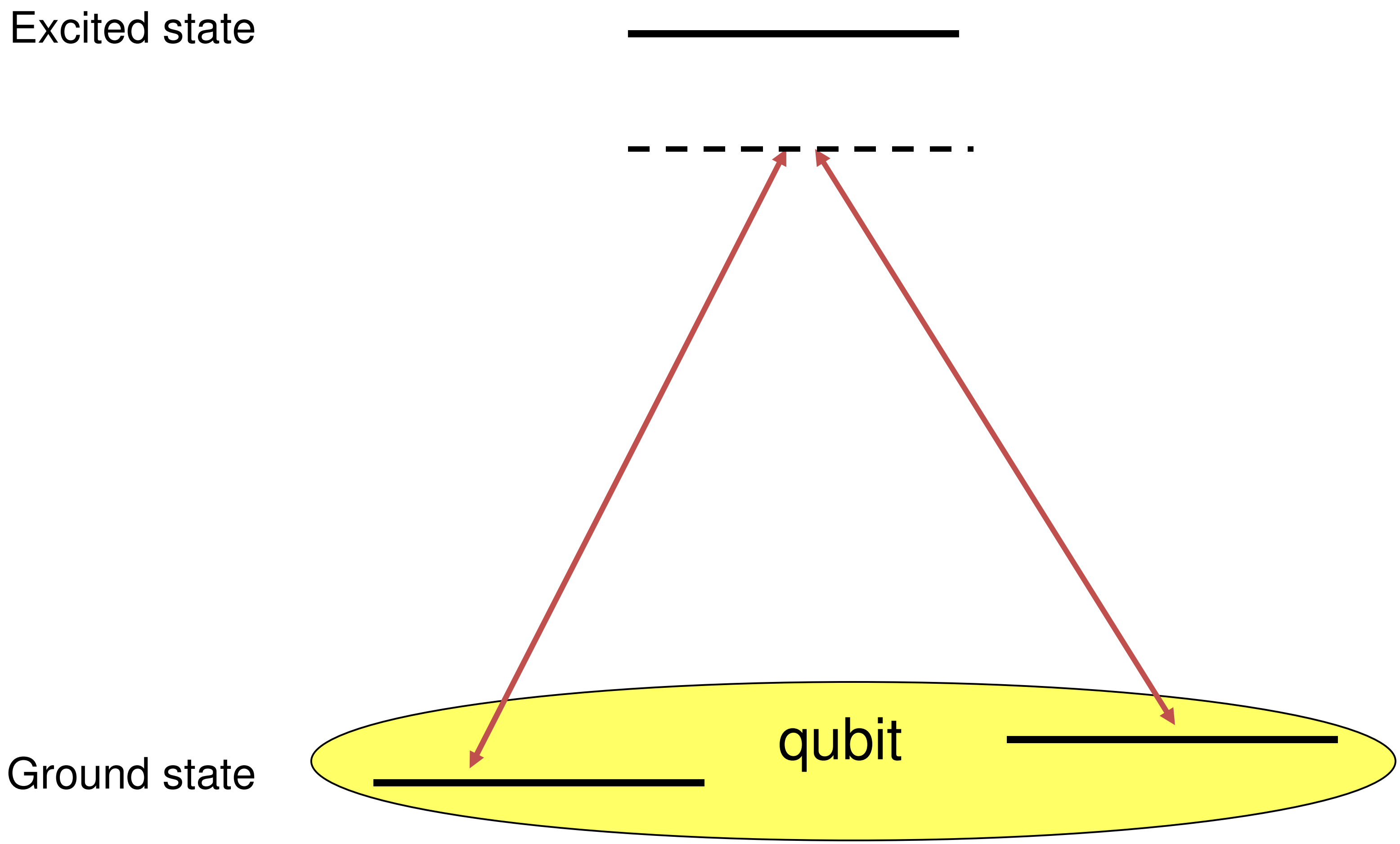
Two qubit gates (Cirac-Zoller approach)

Single qubit coherence (magnetic field sensitive)

Coherence of the motion

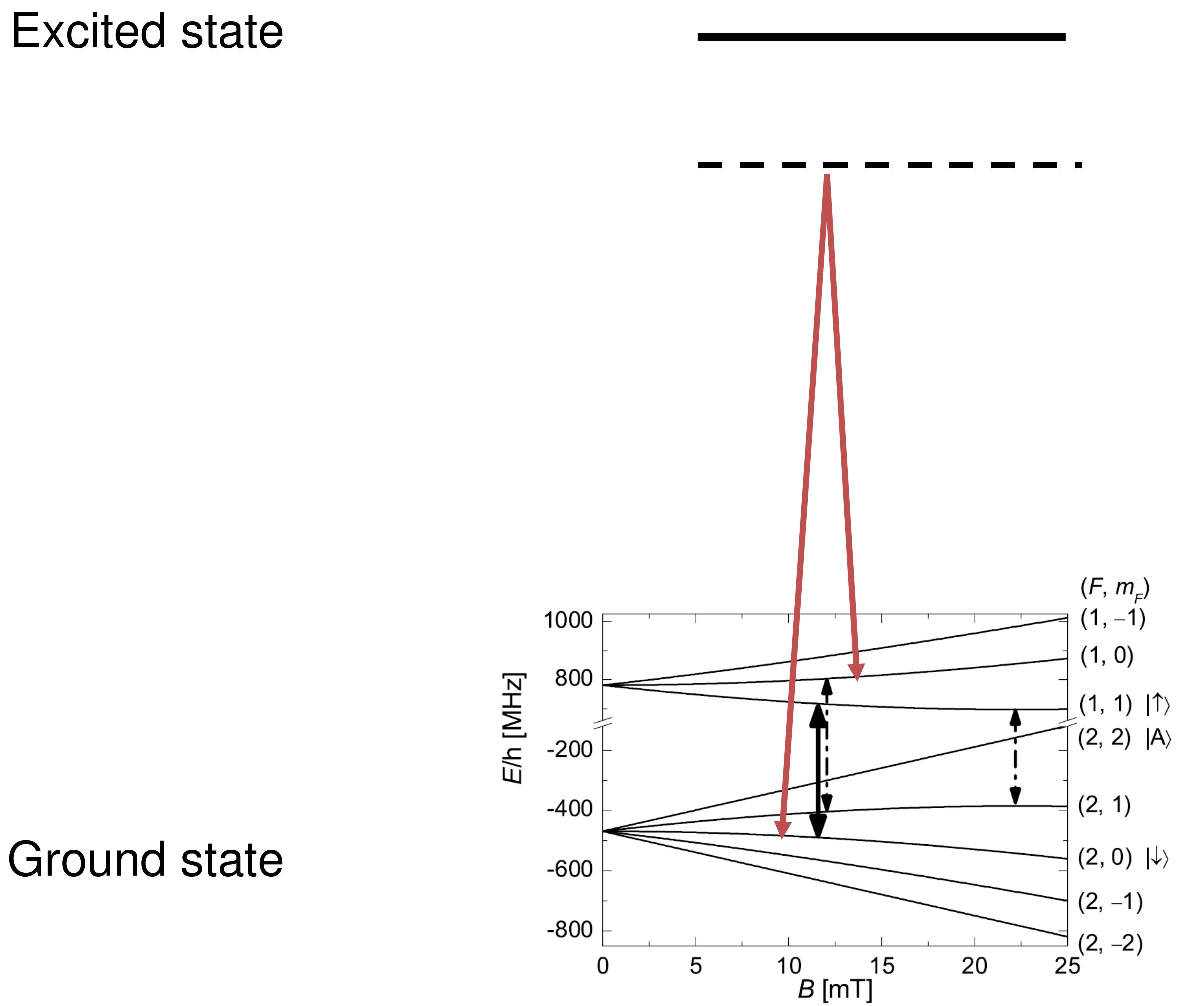
Long lived qubits

Raman transitions:



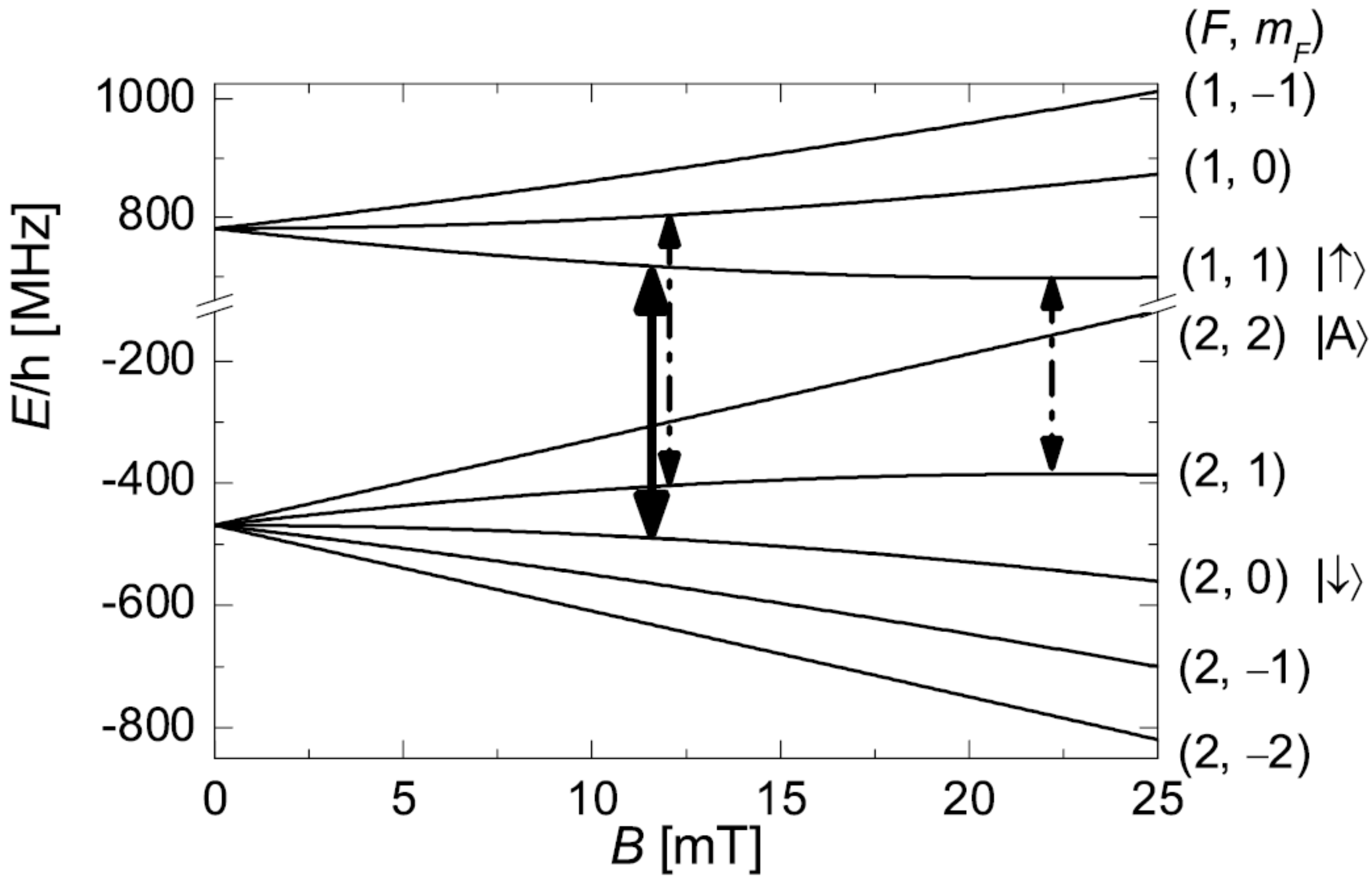
Long lived qubits

Raman transitions:



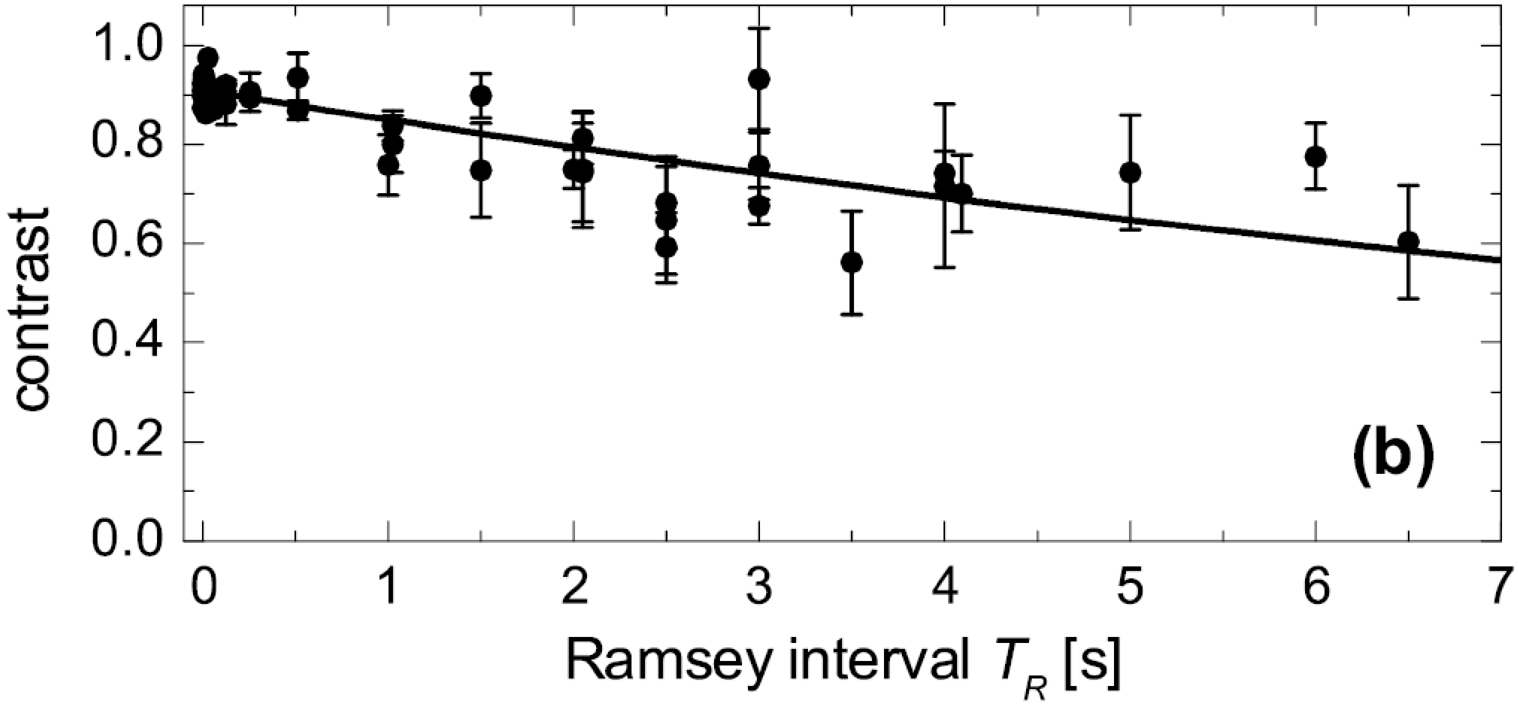
Long lived qubits

Level scheme of $^9Be^+$:



From: C. Langer *et al.*, PRL **95**, 060502 (2005), NIST

Long lived qubits



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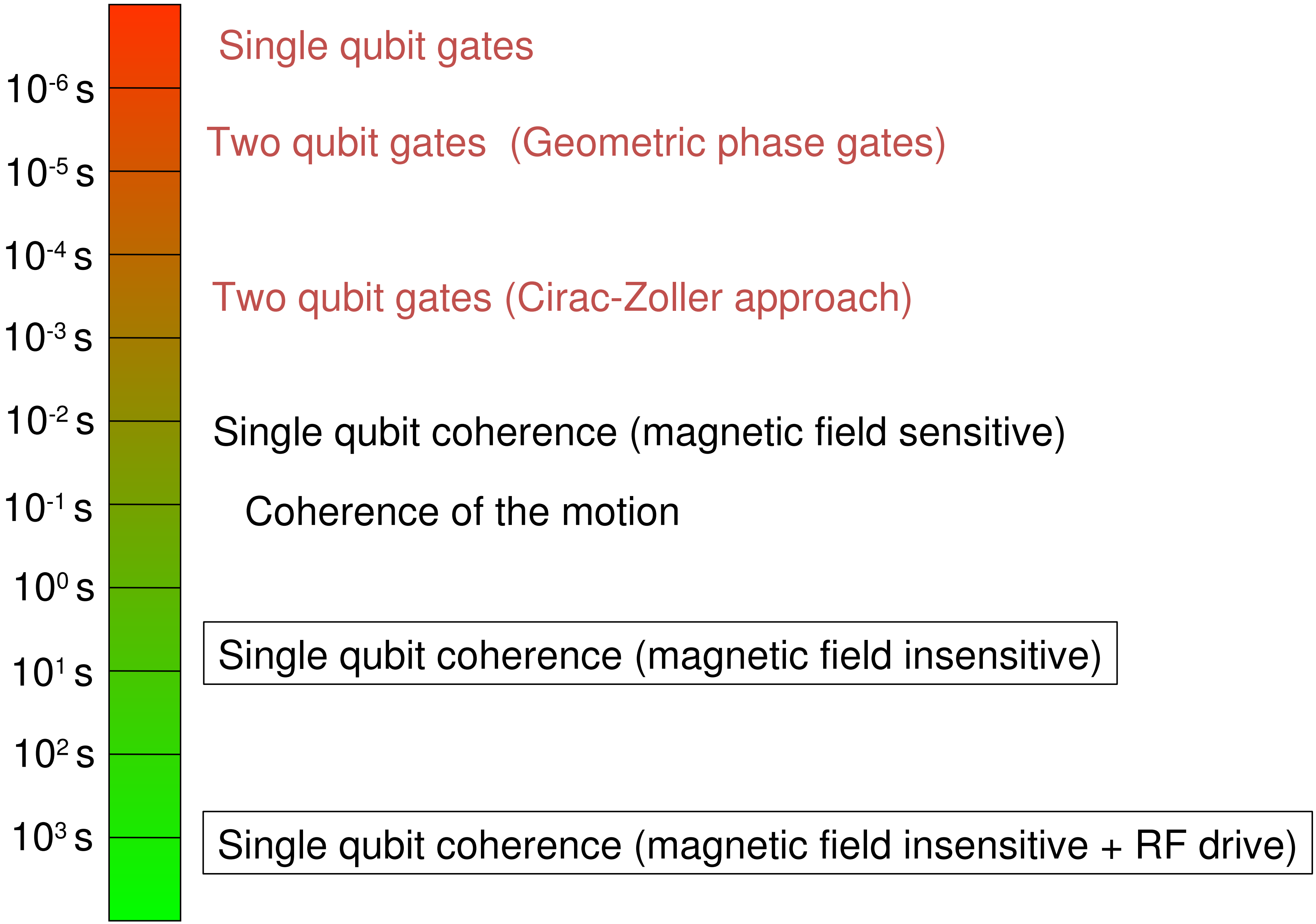
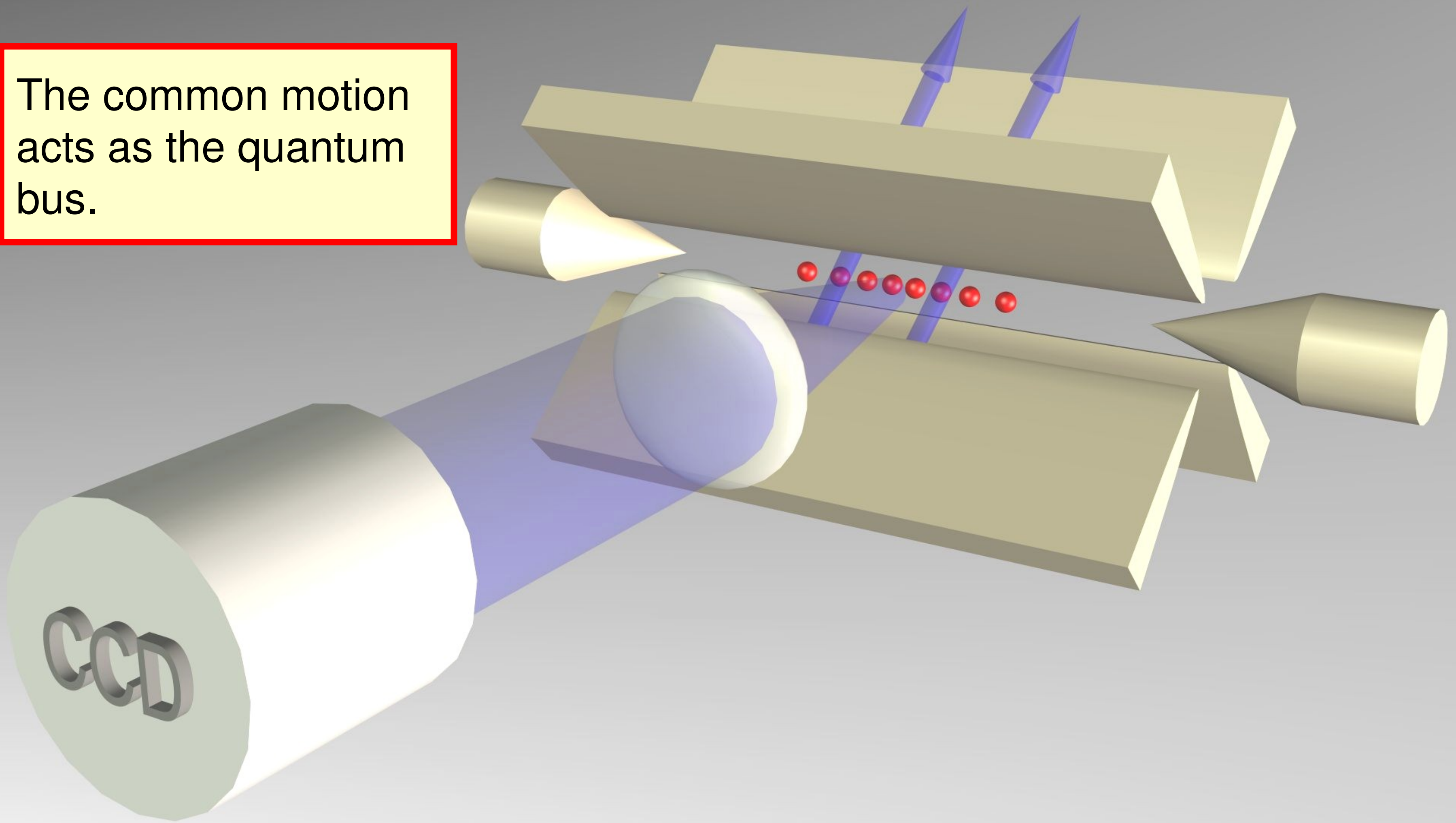
Realized time scales



Having the qubits interact

The common motion acts as the quantum bus.



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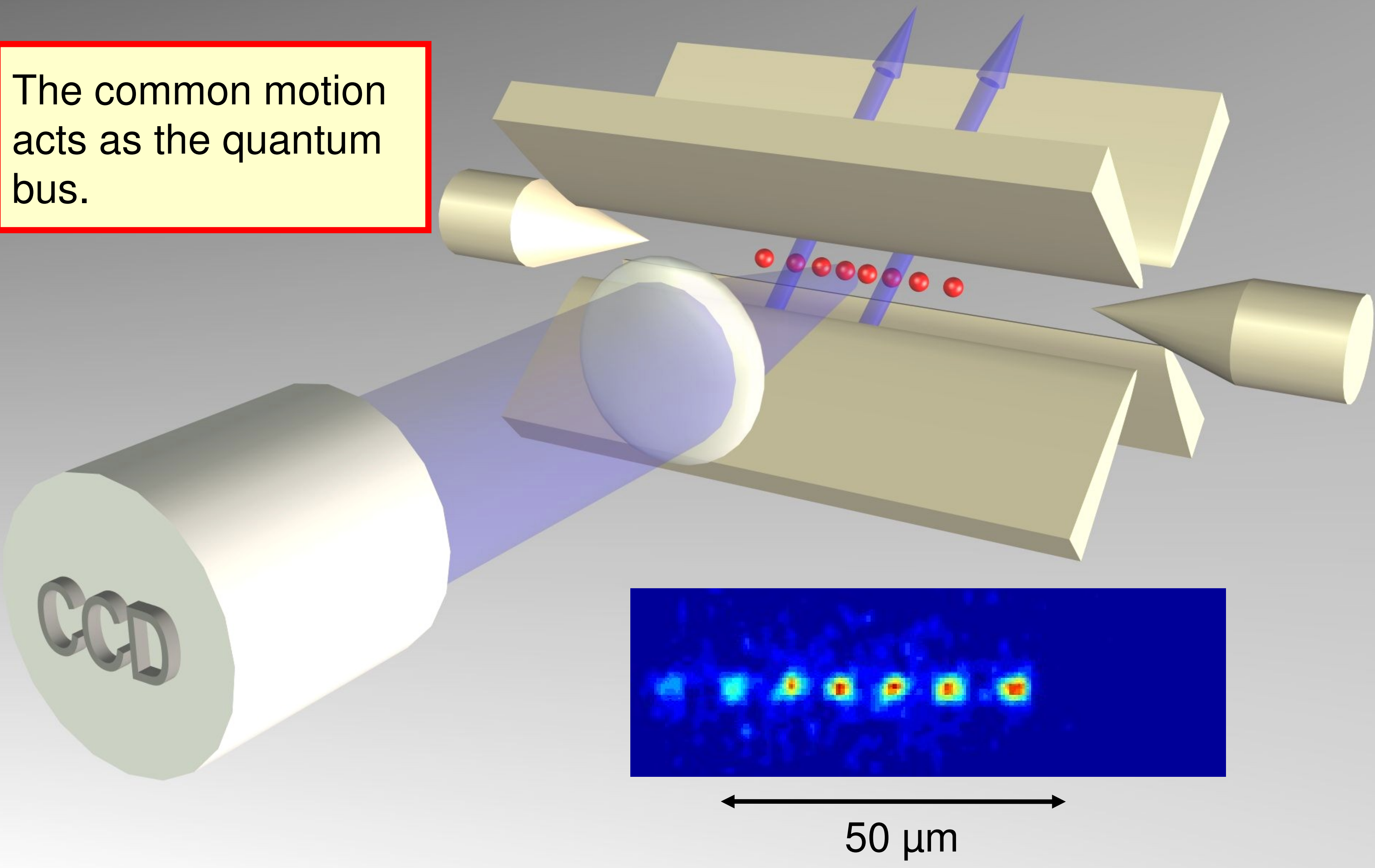
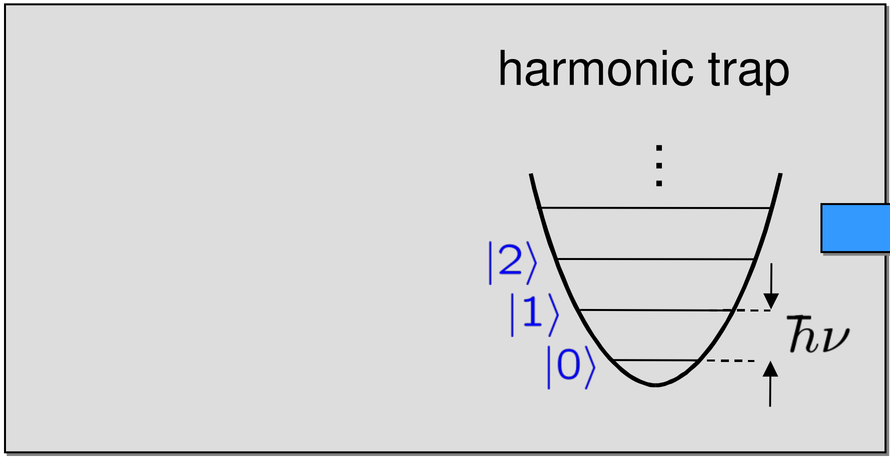
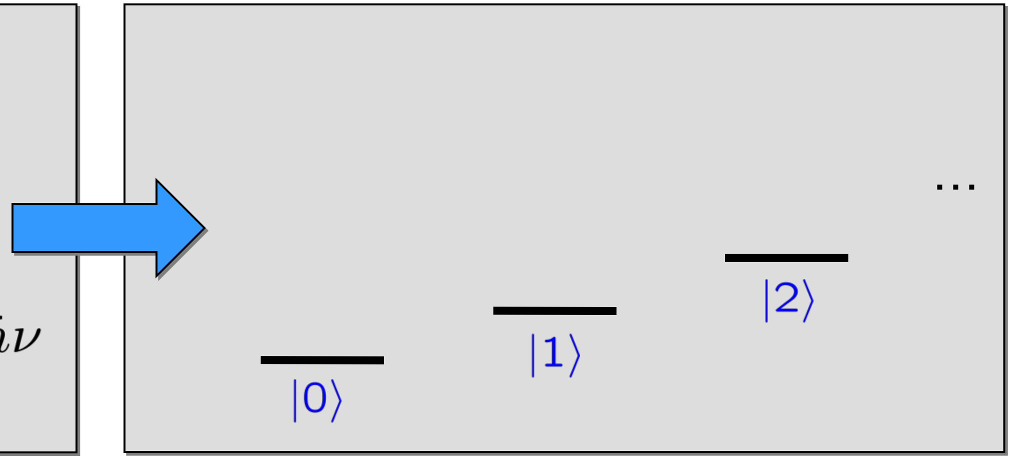


Ion motion

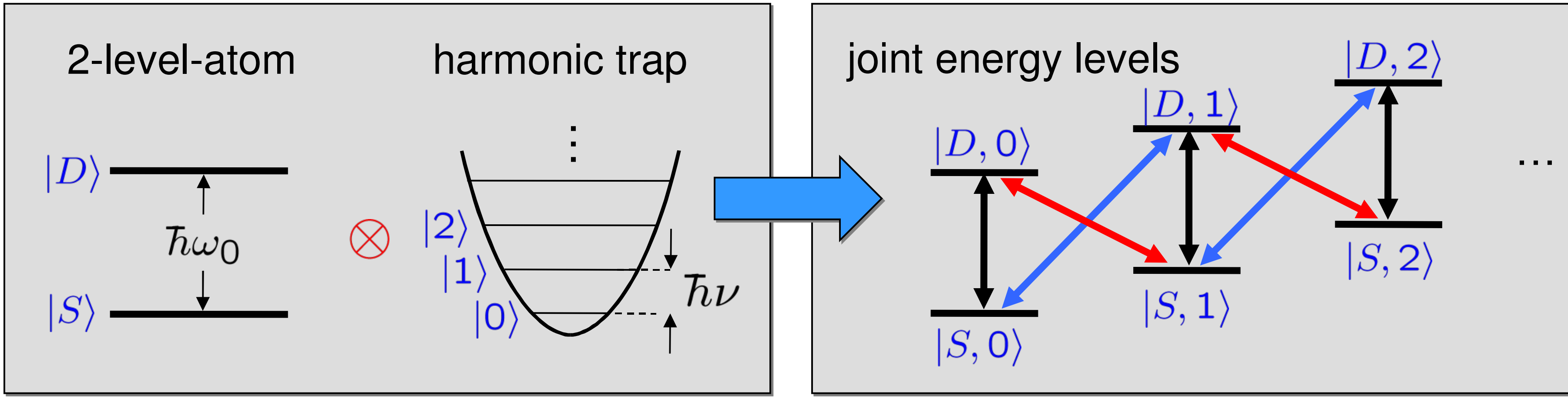


harmonic trap
$|2\rangle$
$|1\rangle$
$|0\rangle$
$\hbar\nu$

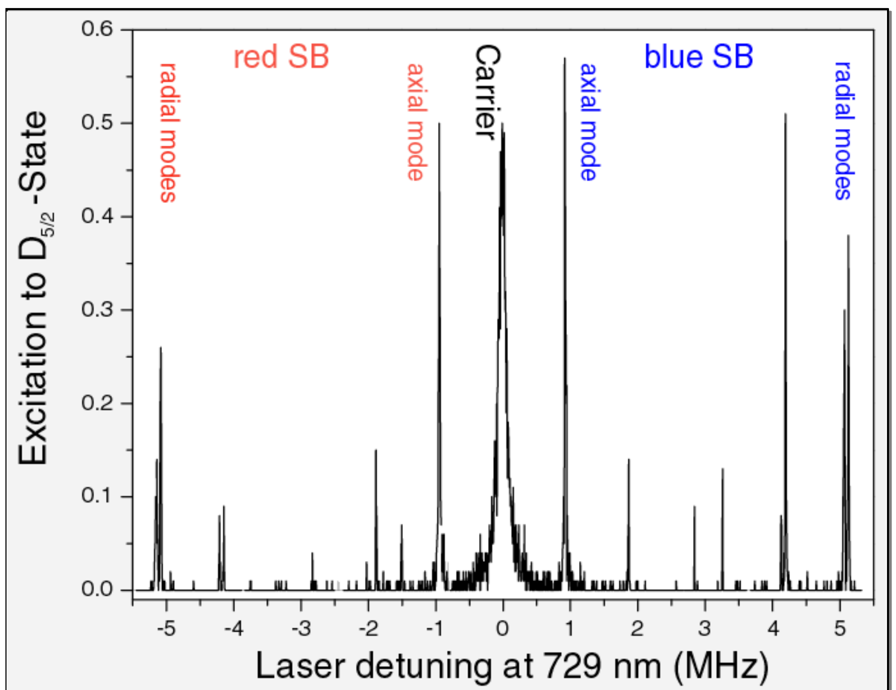


$|0\rangle$
$|1\rangle$
$|2\rangle$
...

Ion motion

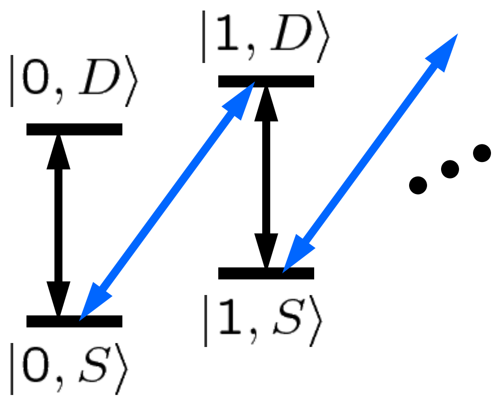


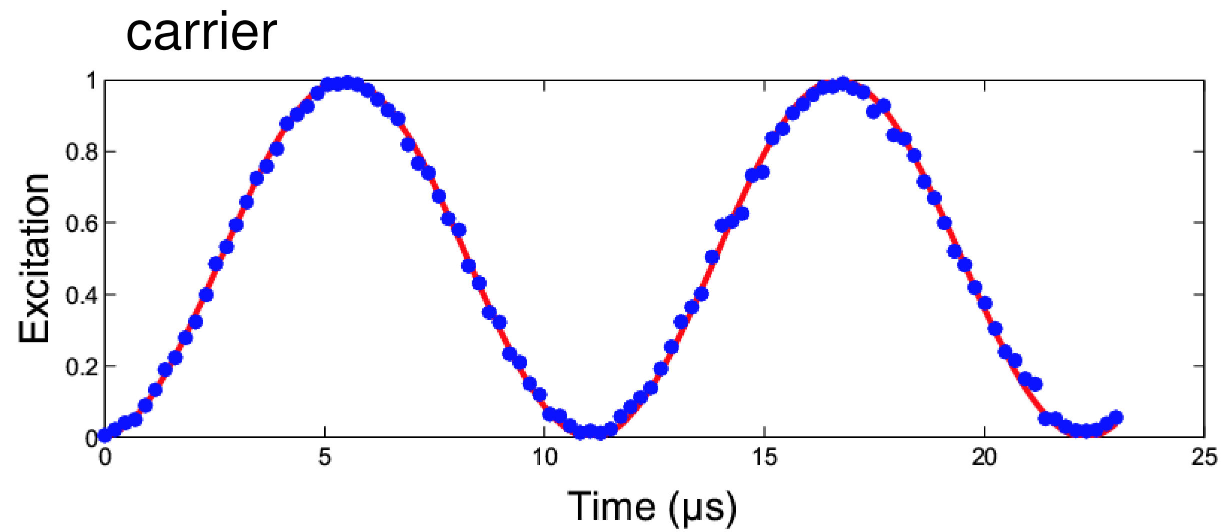
2-level-atom
harmonic trap
$|D\rangle$
$|S\rangle$
$\hbar\omega_0$
$\otimes$
$|2\rangle$
$|1\rangle$
$|0\rangle$
$\hbar\nu$
joint energy levels
$|D,0\rangle$
$|D,1\rangle$
$|D,2\rangle$
$|S,0\rangle$
$|S,1\rangle$
$|S,2\rangle$
...



red SB
blue SB
Carrier
radial modes
axial mode
axial mode
radial modes
Excitation to $D_{5/2}$ -State
Laser detuning at 729 nm (MHz)
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-5
-4
-3
-2
-1
0
1
2
3
4
5

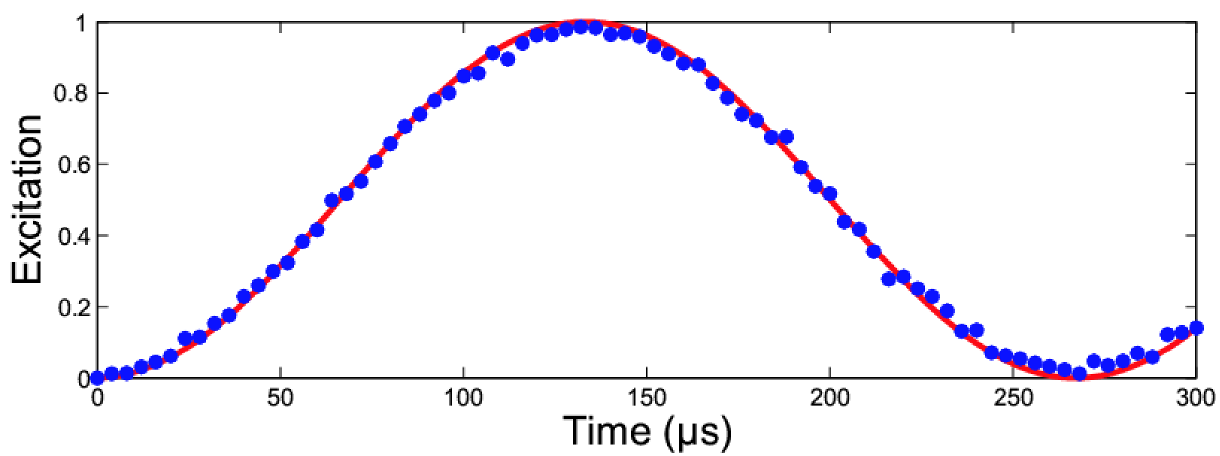
Coherent manipulation





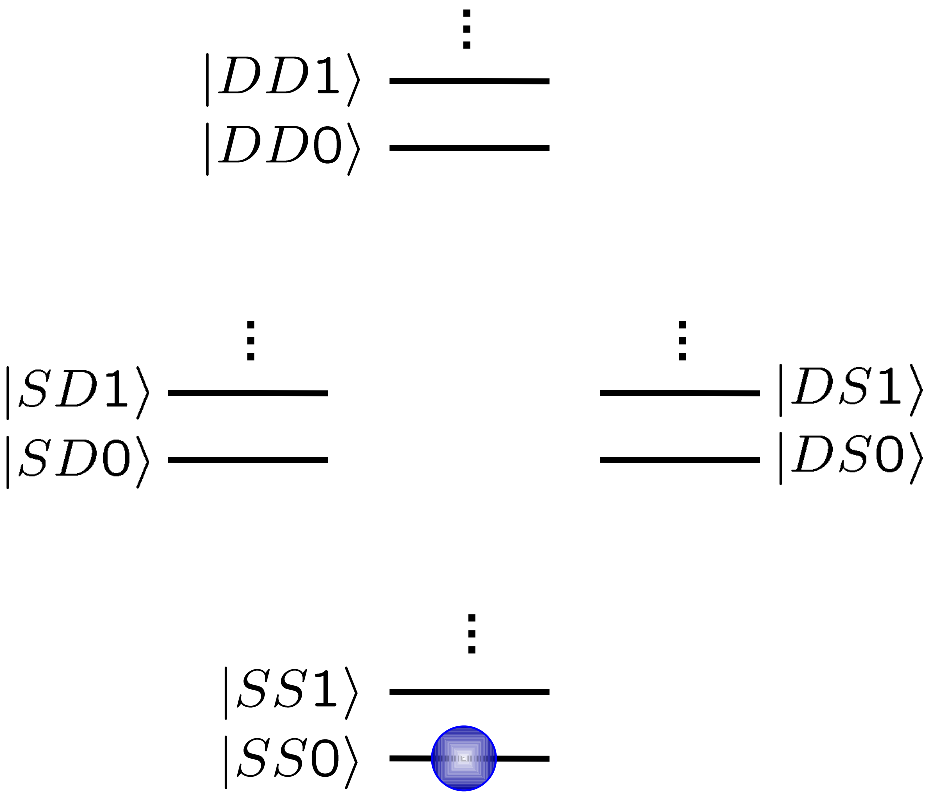
carrier and sideband
Rabi oscillations
with Rabi frequencies

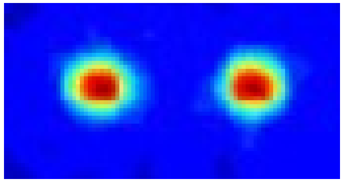
$$\Omega, \quad \eta\Omega$$



$\eta = kx_0$ Lamb-Dicke parameter

Generation of Bell states





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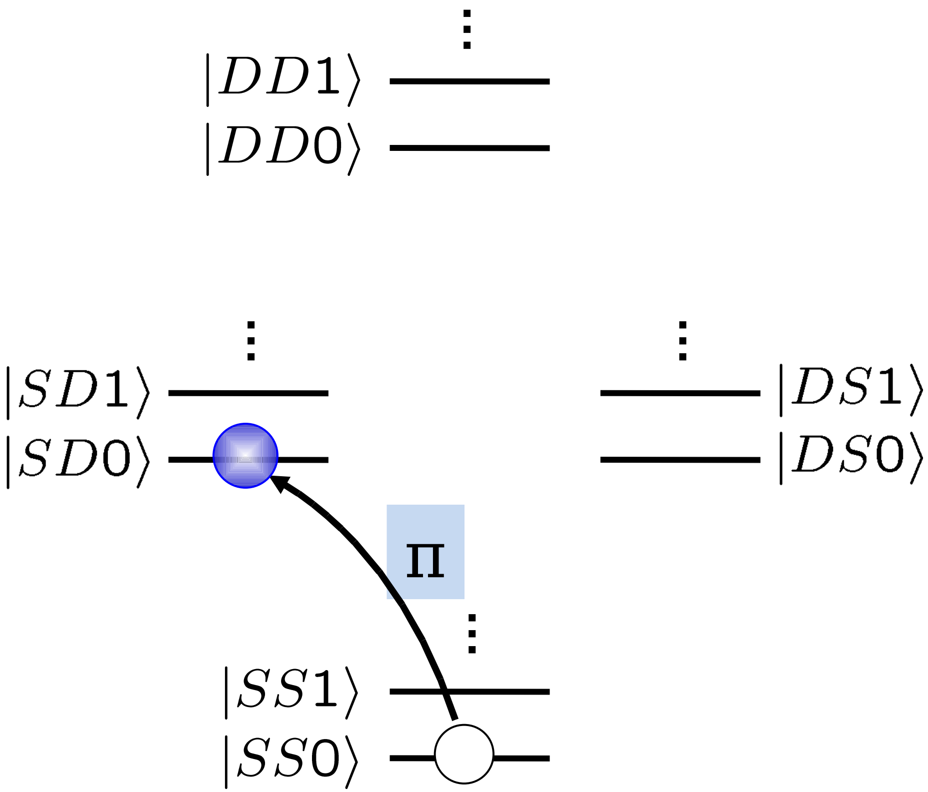

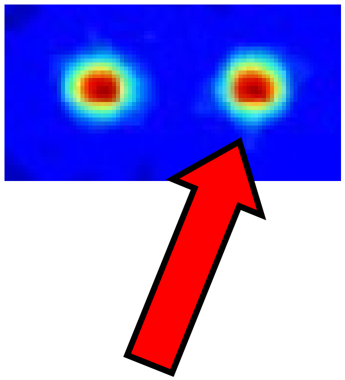

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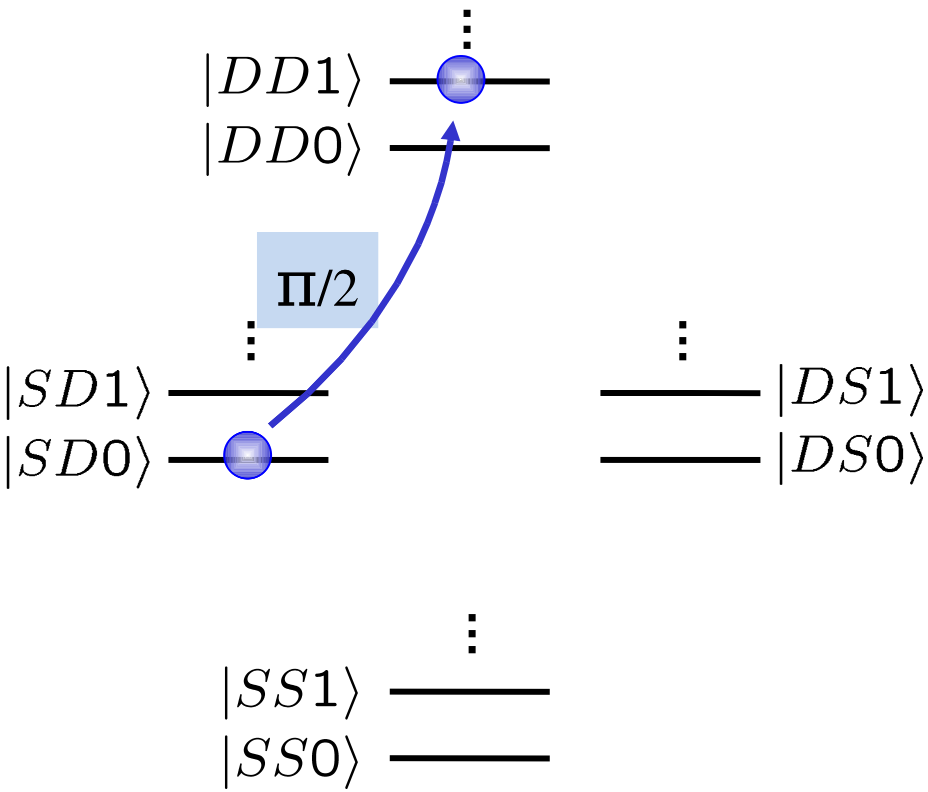

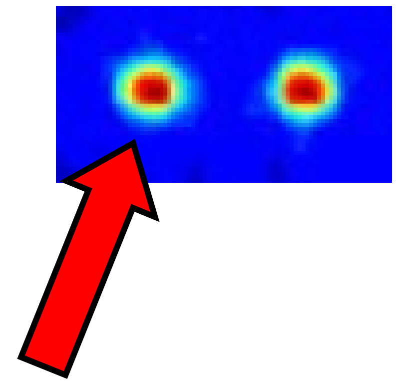

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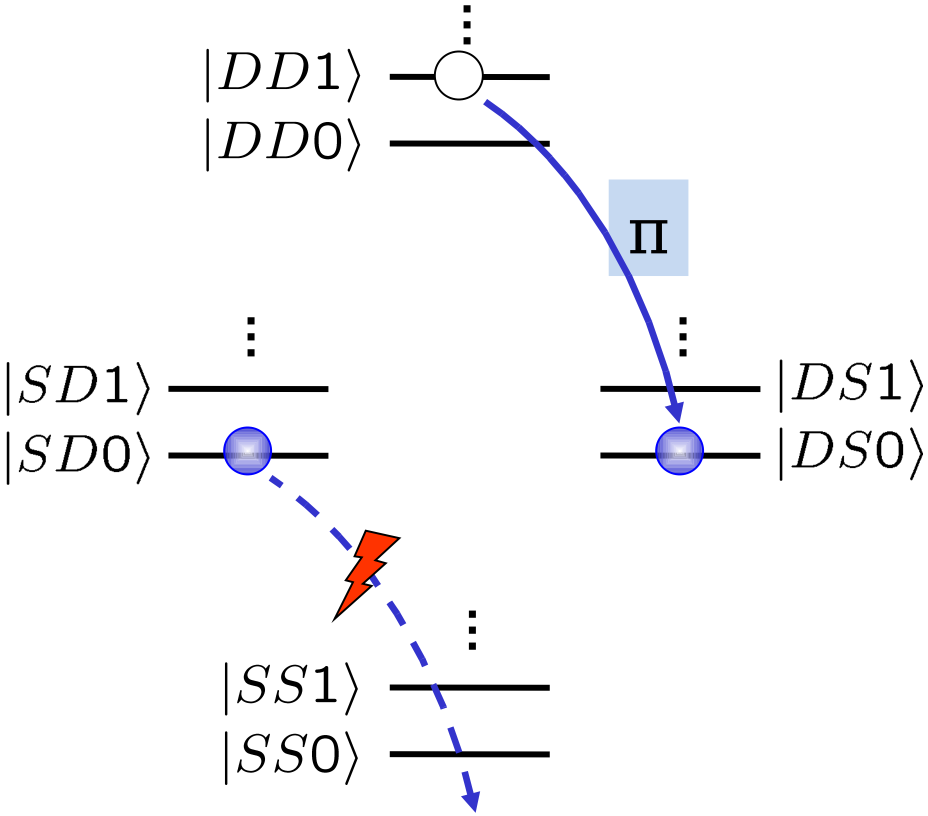
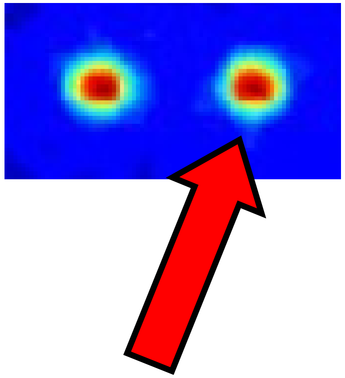
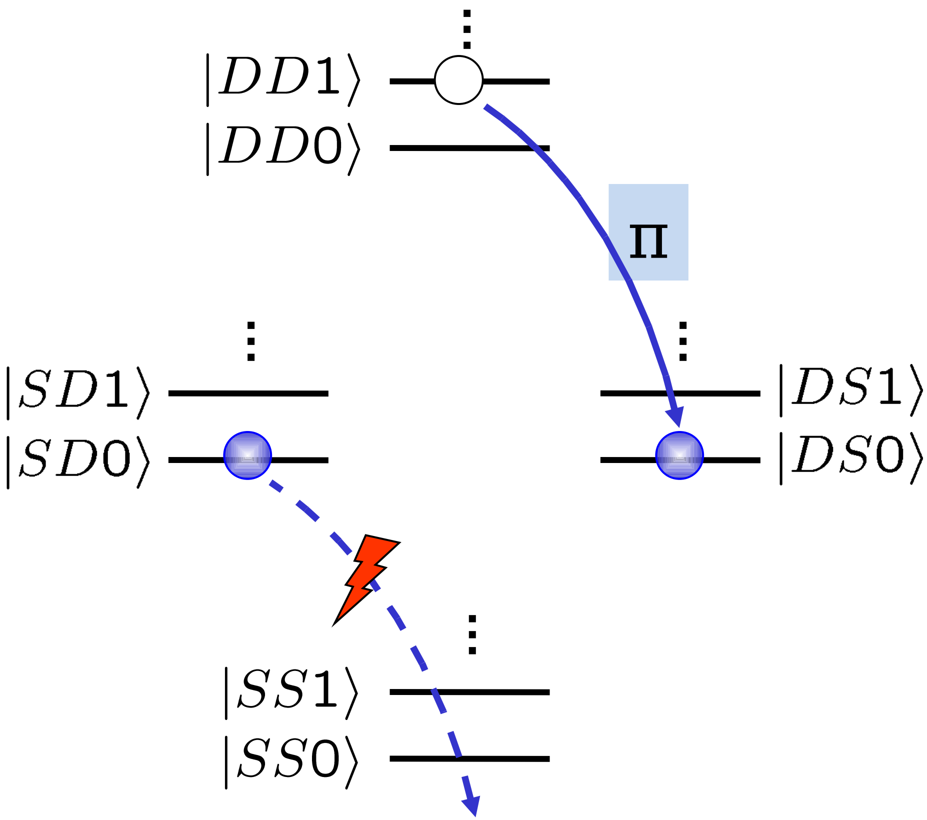
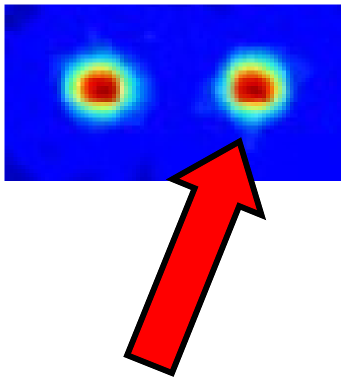




Generation of Bell states





<u>Bell states with atoms</u>

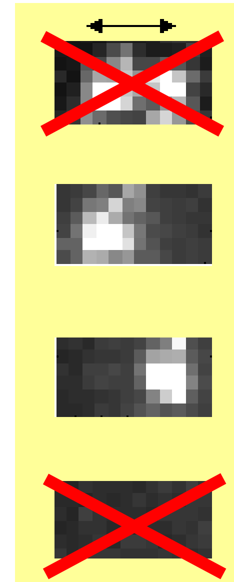
- $^{9}Be^{+}$: NIST (fidelity: 97 %)
- $^{40}Ca^{+}$: Oxford (83%)
- $^{111}Cd^{+}$: Ann Arbor (79%)
- $^{25}Mg^{+}$: Munich
- $^{40}Ca^{+}$: Innsbruck (99%)

Analysis of Bell states

$$|SD\rangle + |DS\rangle$$

Fluorescence detection with CCD camera:

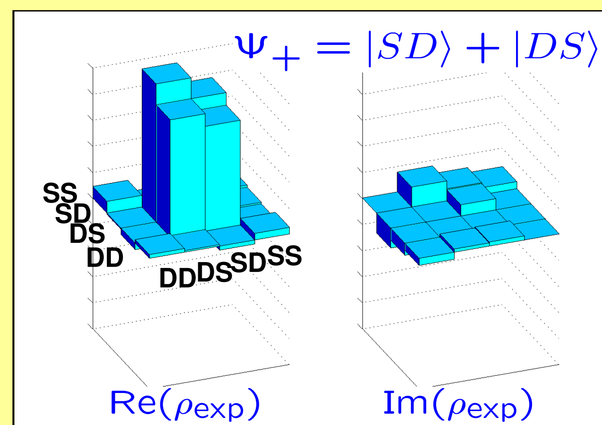
$$\left\{ \begin{array}{l} |SS\rangle \\ |SD\rangle \\ |DS\rangle \\ |DD\rangle \end{array} \right.$$



Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

➡ Measurement of the density matrix:



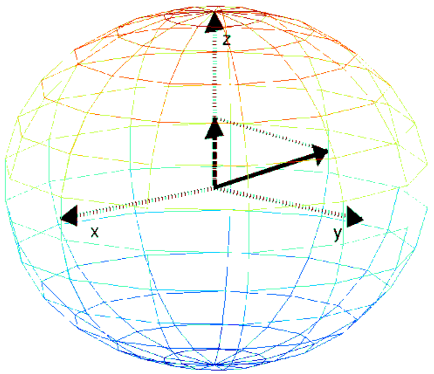
Measuring a density matrix

A measurement yields the *z*-component of the Bloch vector

=> Diagonal of the density matrix

$$\rho = \begin{pmatrix} P_S & C - iD \\ C + iD & P_D \end{pmatrix}$$

$|D\rangle$



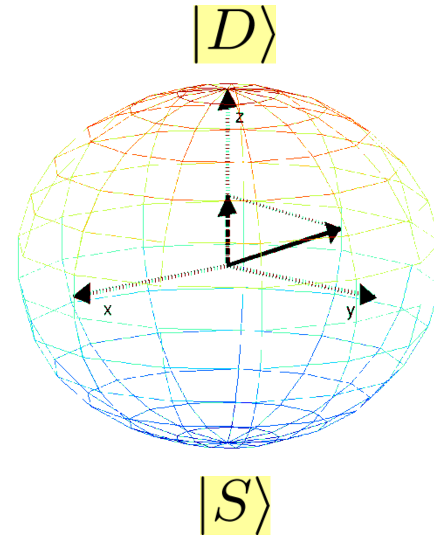
$|S\rangle$

Measuring a density matrix

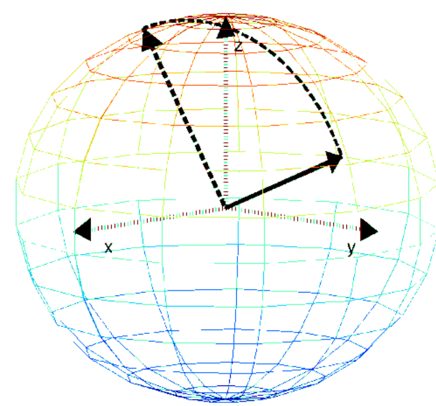
A measurement yields the *z*-component of the Bloch vector

=> Diagonal of the density matrix

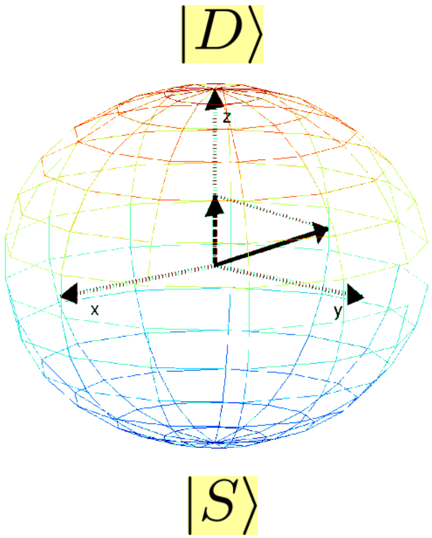
$$\rho = \begin{pmatrix} P_S & C - i{\color{blue}D} \\ C + i{\color{blue}D} & P_D \end{pmatrix}$$



Rotation around the *x*- or the *y*-axis prior to the measurement yields the phase information of the qubit.



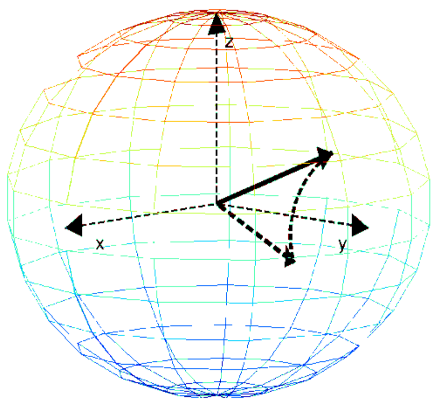
Measuring a density matrix



A measurement yields the *z*-component of the Bloch vector

=> Diagonal of the density matrix

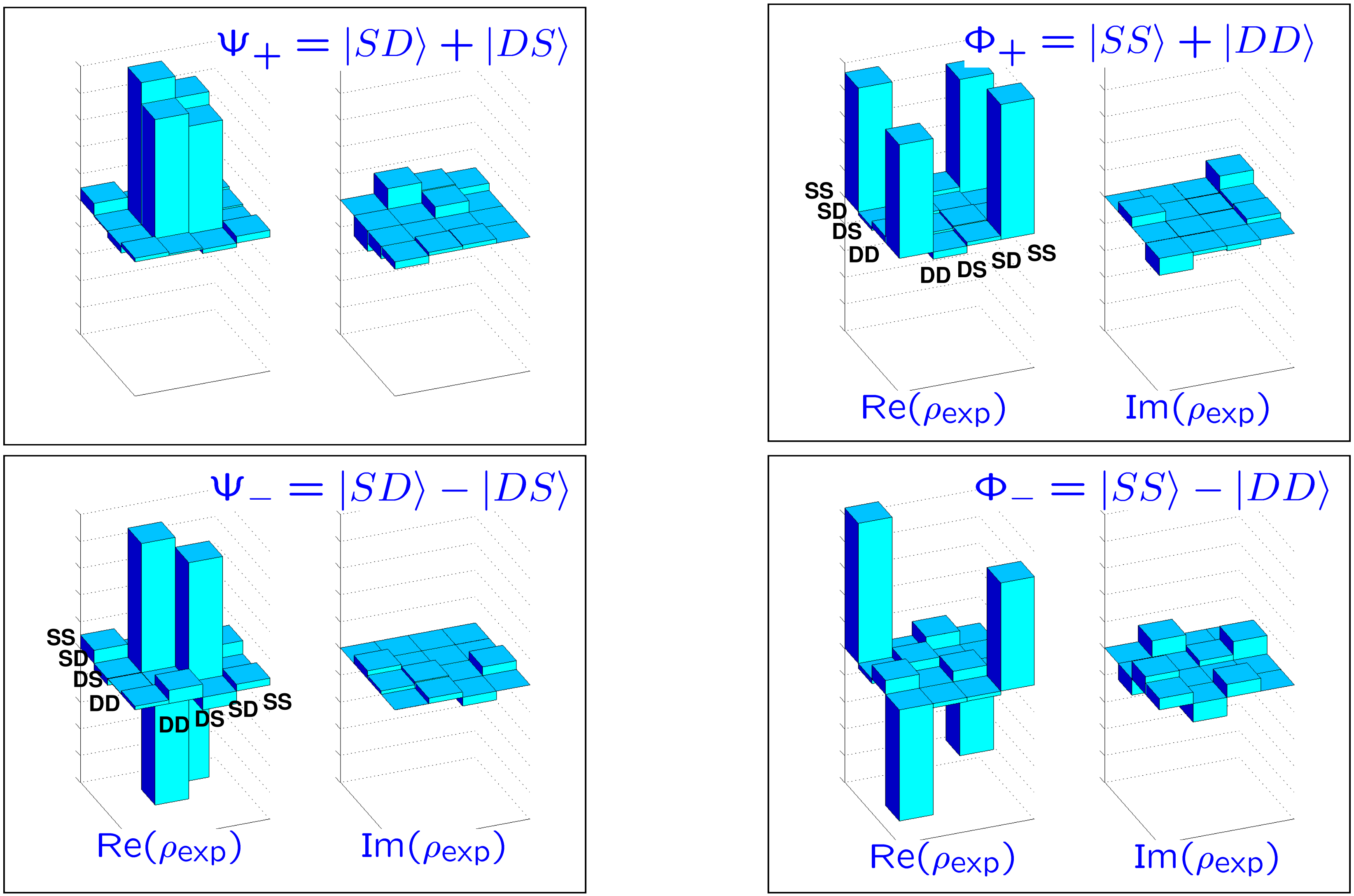
$$\rho = \begin{pmatrix} P_S & C - iD \\ C + iD & P_D \end{pmatrix}$$



Rotation around the *x*- or the *y*-axis prior to the measurement yields the phase information of the qubit.

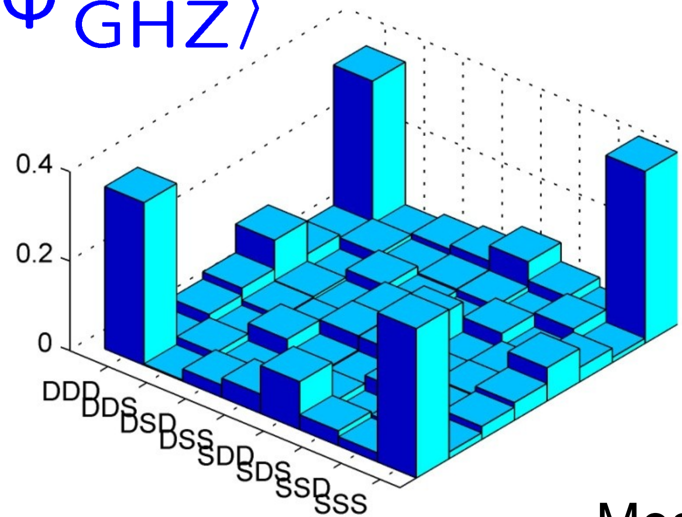
=> coherences of the density ma

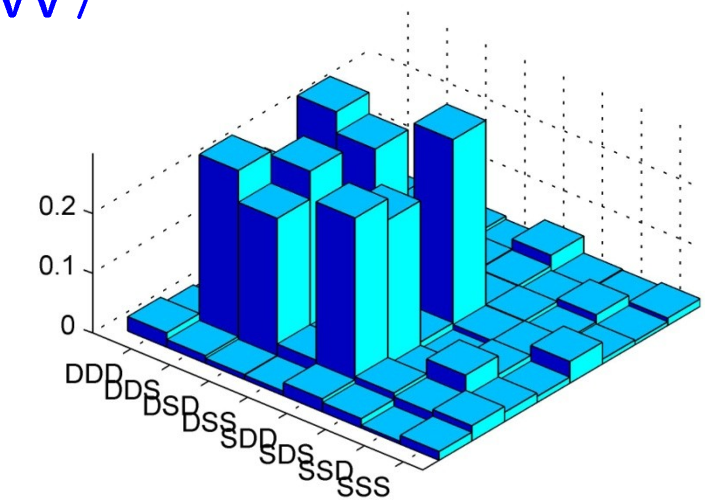
Decoherence properties of qubits



A “real” thought experiment

$|\Psi_{\mathrm{GHZ}}\rangle$ $|\Psi_{\mathrm{W}}\rangle$

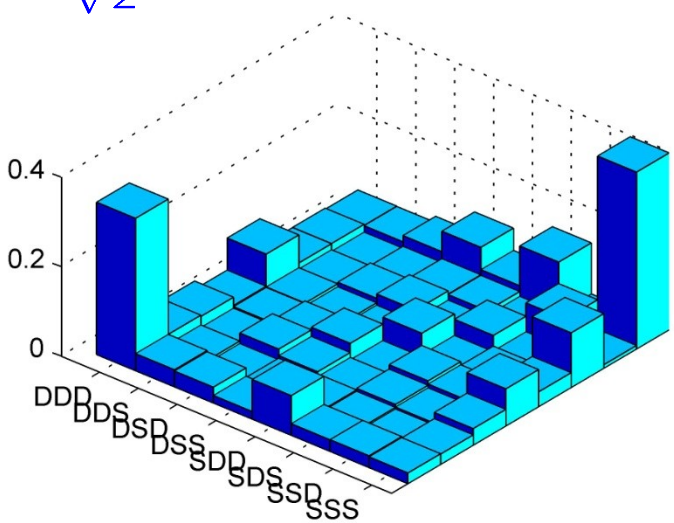




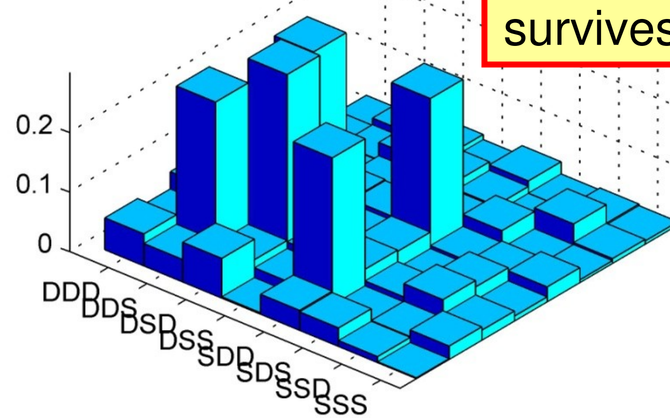
$\frac{1}{\sqrt{2}}(|SSS\rangle + |DDD\rangle)$

Measurement
of the center ion

$\frac{1}{\sqrt{3}}(|SDD\rangle + |DSD\rangle + |DDS\rangle)$



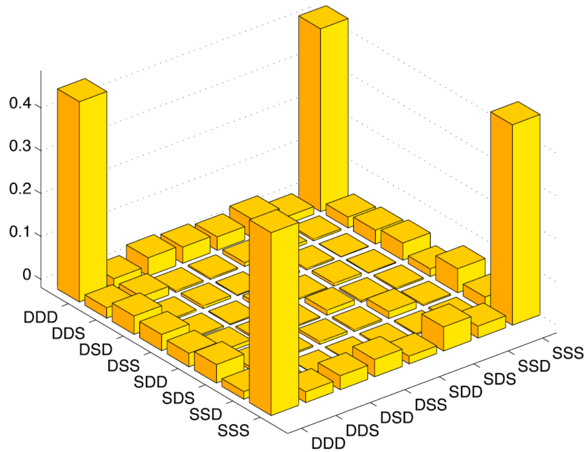
Bell-state
survives



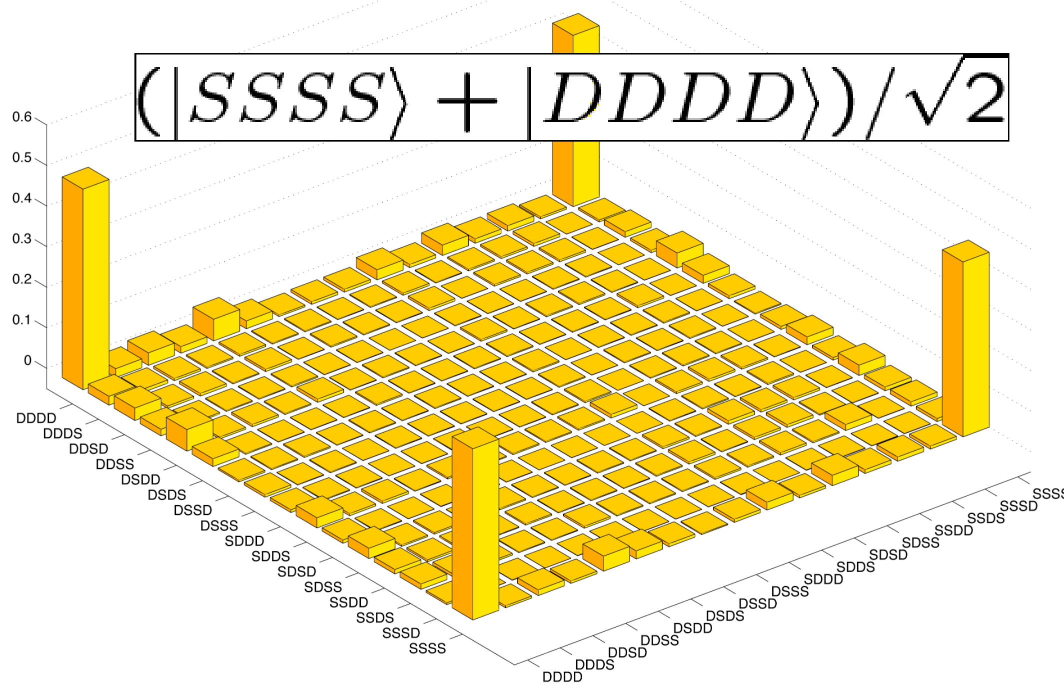
Roos et al., *Science* **304**, 1478 (200

Generalized Bell states

$$(|SSS\rangle + |DDD\rangle)/\sqrt{2}$$

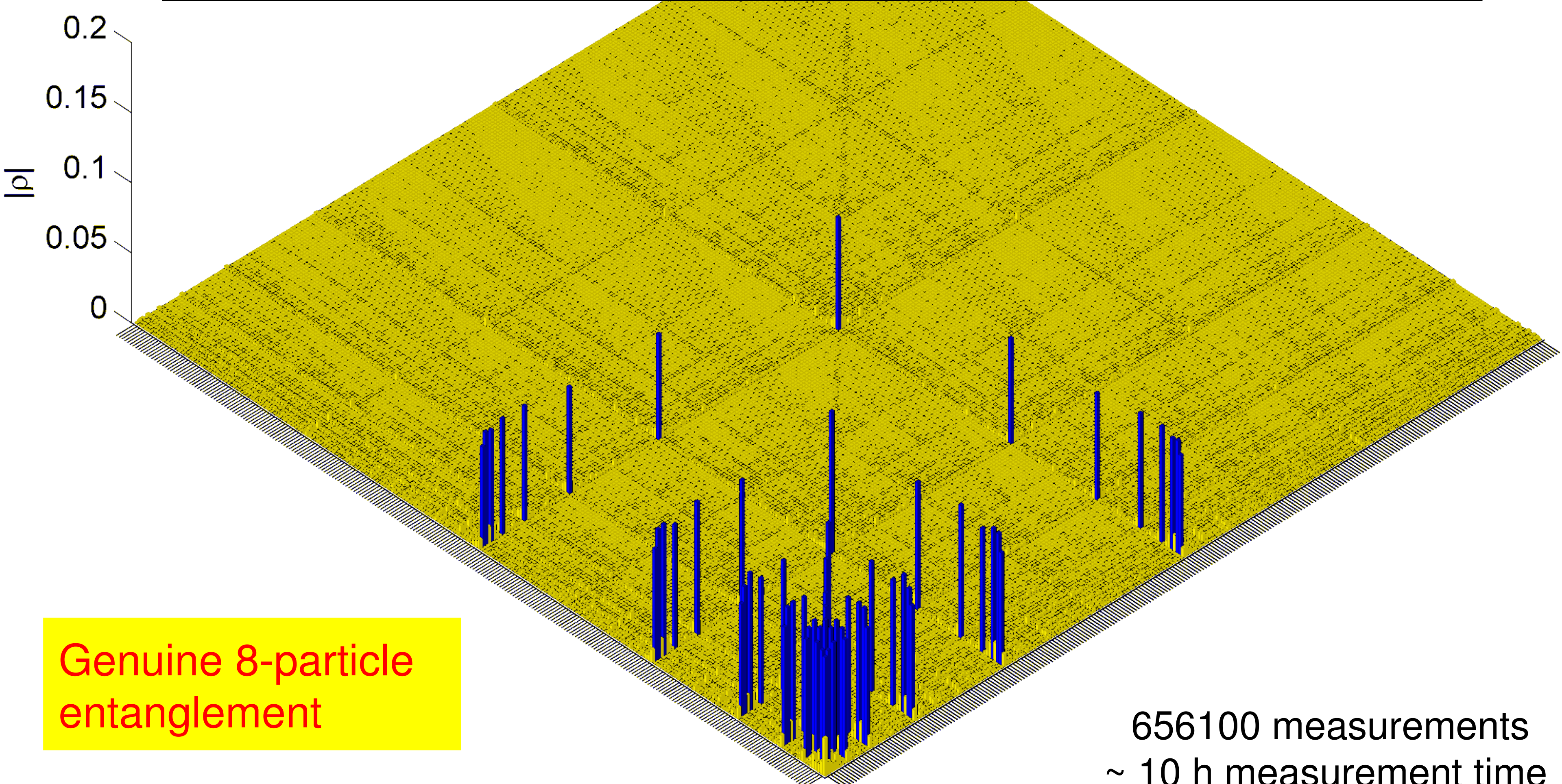


$$(|SSSS\rangle + |DDDD\rangle)/\sqrt{2}$$

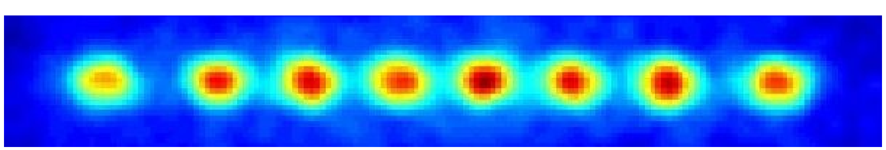


Generalized Bell states

$$\frac{1}{\sqrt{8}}(|DDDDDDDS\rangle + |DDDDDDSD\rangle + \ldots + |SDDDDDDD\rangle)$$



Genuine 8-particle entanglement



656100 measurements
~ 10 h measurement time

Häffner et al., Nature **438**, 643 (2005)

DiVincenzo criteria

- Scalable physical system, well characterized qubits
- Ability to initialize the state of the qubits
- Long relevant coherence times, much longer than gate operation time
- “Universal” set of quantum gates
- Qubit-specific measurement capability

Quantum gates …

Having the qubits interact

VOLUME 74, NUMBER 20 PHYSICAL REVIEW LETTERS 15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universiät Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

PACS numbers: 89.80.+h, 03.65.Bz, 12.20.Fv, 32.80.Pj

...allows the realization of a ***universal*** quantum computer !

$$|D\rangle|D\rangle \rightarrow |D\rangle|D\rangle$$

$$|D\rangle|S\rangle \rightarrow |D\rangle|S\rangle$$

$$|S\rangle|D\rangle \rightarrow |D\rangle|S\rangle$$

$$|S\rangle|S\rangle \rightarrow |S\rangle|D\rangle$$

control target

Having the qubits interact

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...allows the realization of a
universal quantum computer !

$$|D\rangle|D\rangle \longrightarrow |D\rangle|D\rangle$$

$$|D\rangle|S\rangle \longrightarrow |D\rangle|S\rangle$$

$$|S\rangle|D\rangle \longrightarrow |D\rangle|S\rangle$$

$$|S\rangle|S\rangle \longrightarrow |S\rangle|D\rangle$$

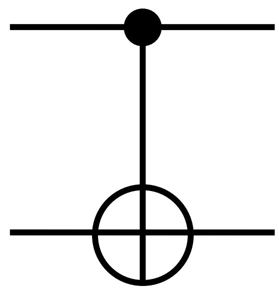
control target

Most popular gates:
- Cirac-Zoller gate (Schmidt-Kaler et al., Nature **422,** 408 (2003)).
- Geometric phase gate (Leibfried et al., Nature **422,** 412 (2003)).
- Mølmer-Sørensen gate (Sackett et al., Nature **404**, 256 (2000)).

A controlled-NOT operation

Control bit

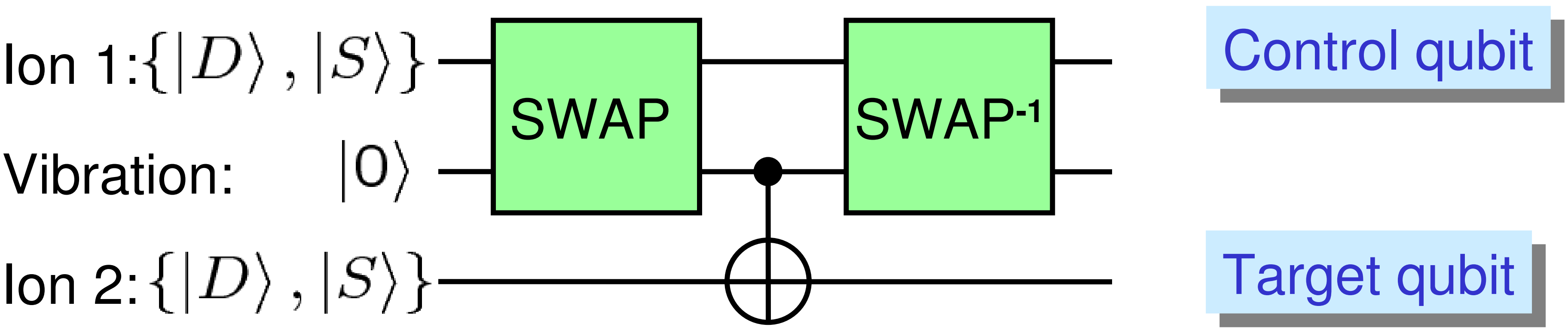
Target bit



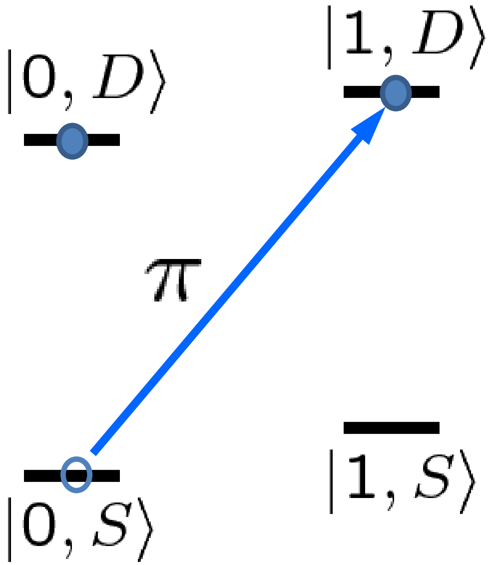
$$
\begin{array}{lcl}
|0\rangle|0\rangle & \rightarrow & |0\rangle|0\rangle \\
|0\rangle|1\rangle & \rightarrow & |0\rangle|1\rangle \\
|1\rangle|0\rangle & \rightarrow & |1\rangle|1\rangle \\
|1\rangle|1\rangle & \rightarrow & |1\rangle|0\rangle
\end{array}
$$

Target

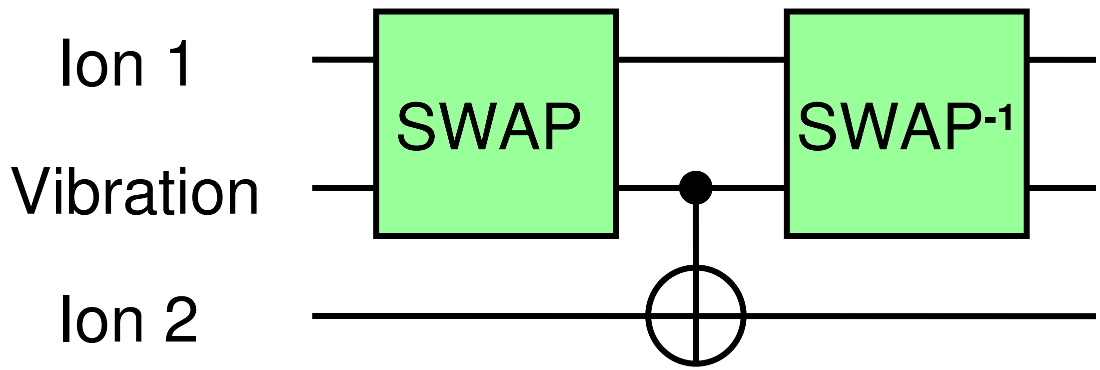
A controlled-NOT operation



Mapping the qubit to the bus



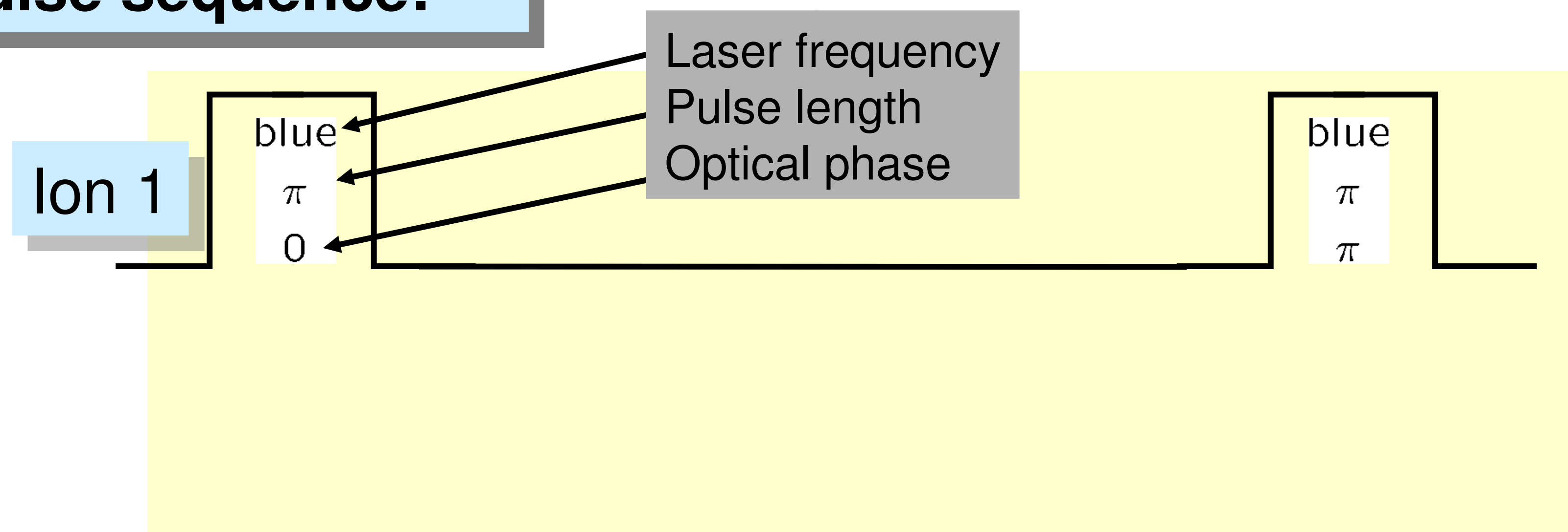
A controlled-NOT operation



Control qubit

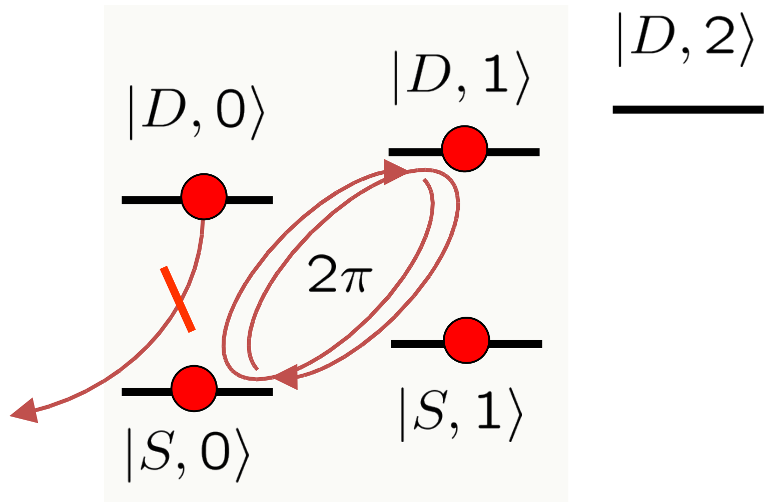
Target qubit

Pulse sequence:

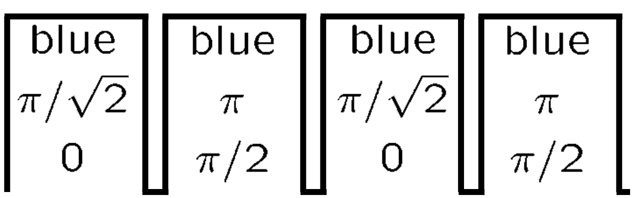


A phase gate

$$\begin{array}{cc} & \begin{array}{cccc} |D,0\rangle & |S,0\rangle & |D,1\rangle & |S,1\rangle \end{array} \\ U_\Phi = & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \end{array}$$

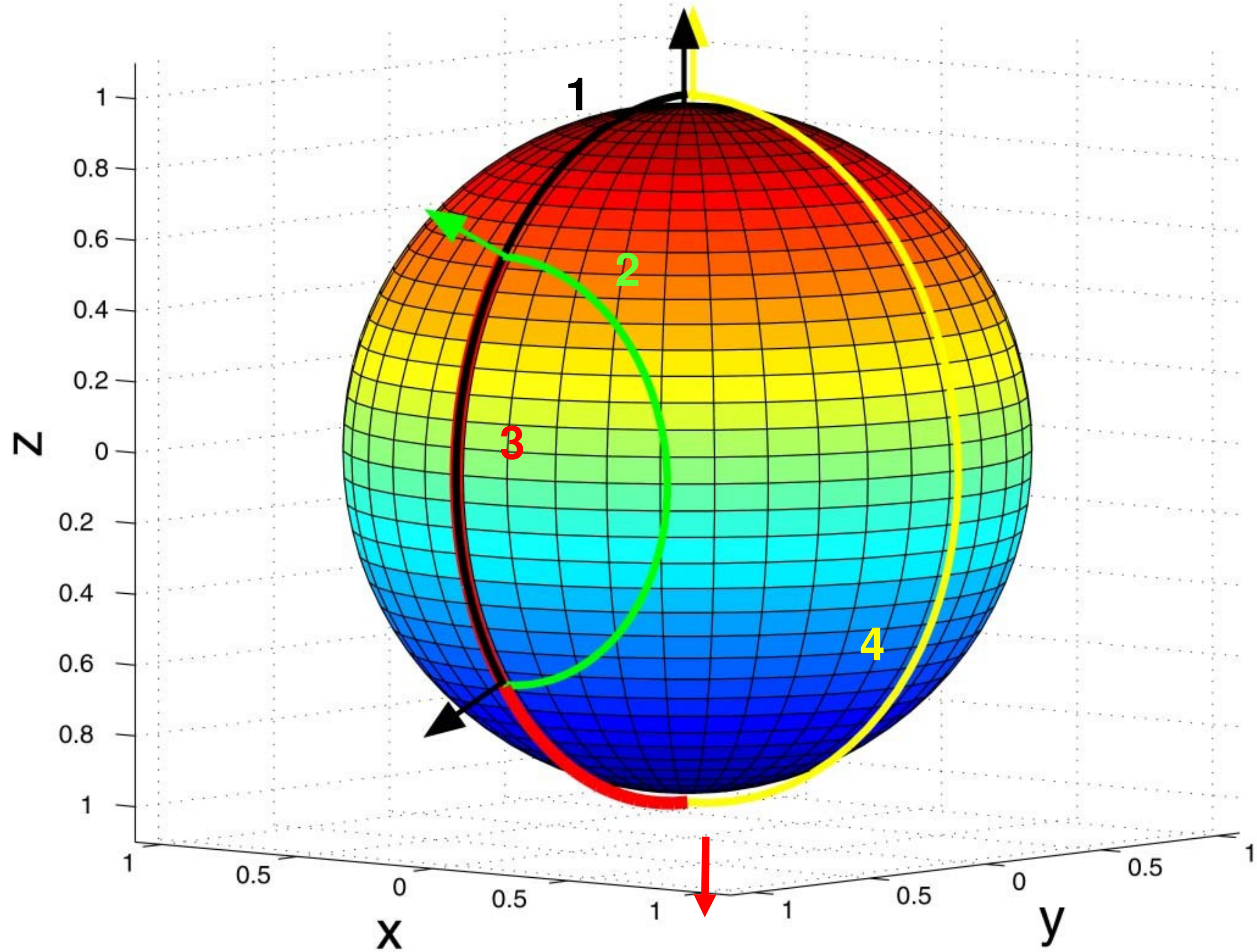


Composite 2π-rotation:

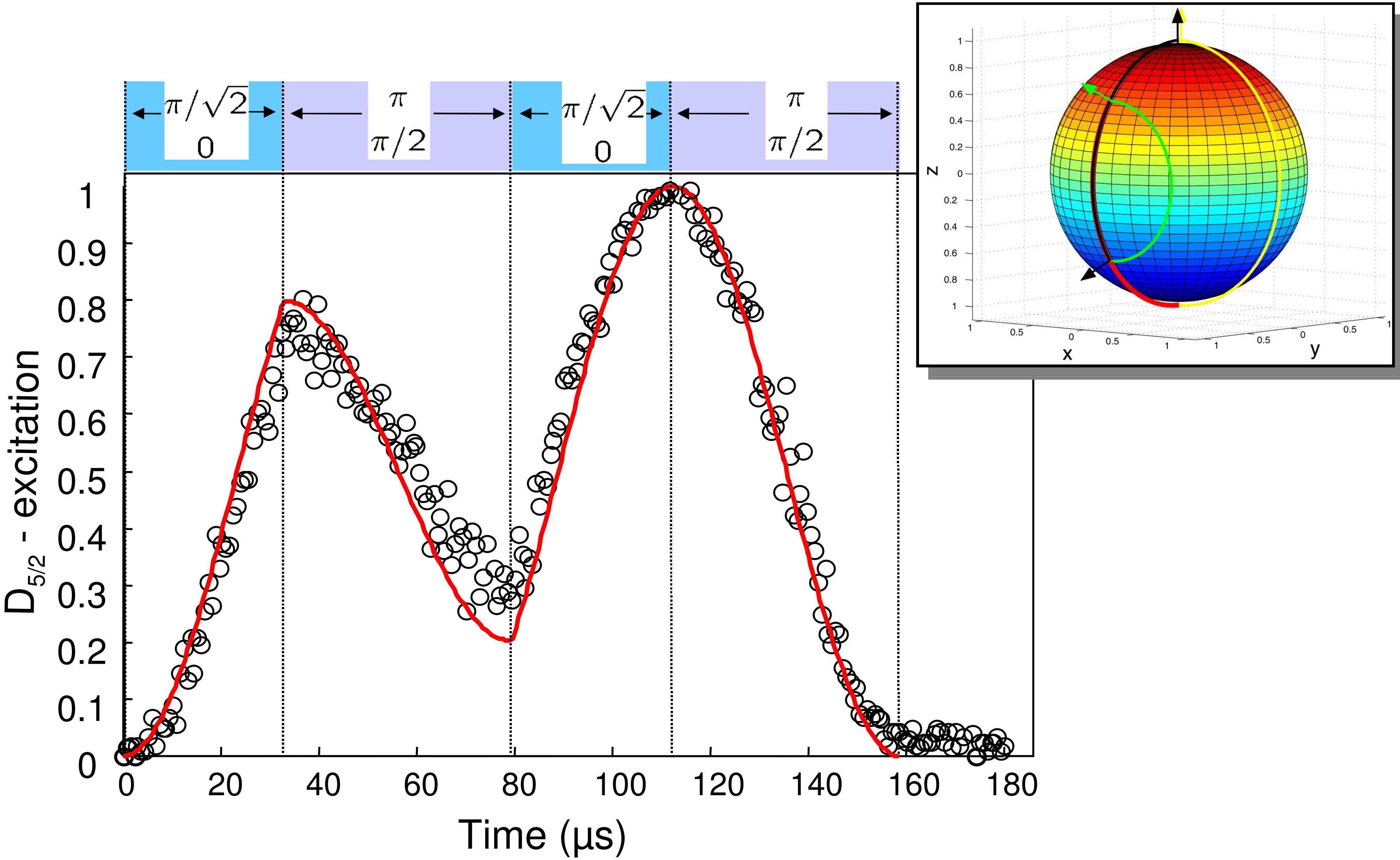


Composite phase gate

$$R^{+}(\pi,\pi/2)\,R^{+}\left(\pi/\sqrt{2},0\right)\,R^{+}(\pi,\pi/2)\,R^{+}\left(\pi/\sqrt{2},0\right)$$

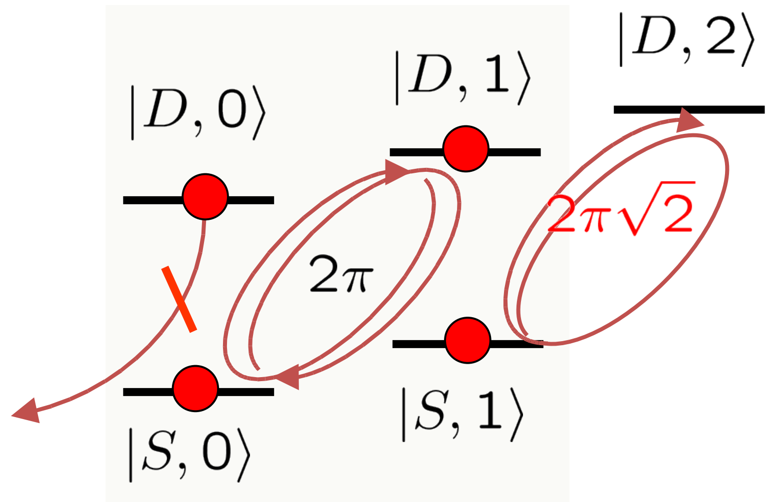


Composite phase gate



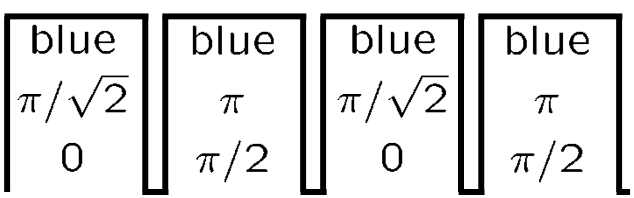
A phase gate

$$
\begin{array}{c} \\ U_\Phi = \end{array}
\begin{array}{c}
\begin{array}{cccc} |D,0\rangle & |S,0\rangle & |D,1\rangle & |S,1\rangle \end{array} \\
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & ? \end{pmatrix}
\end{array}
$$

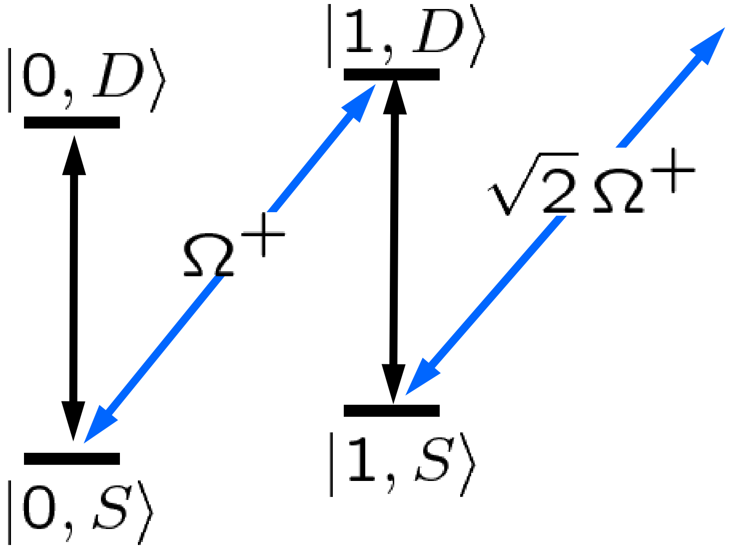


Composite 2π-rotation:

$$a^\dagger |n\rangle = \sqrt{n+1}\,|n+1\rangle$$



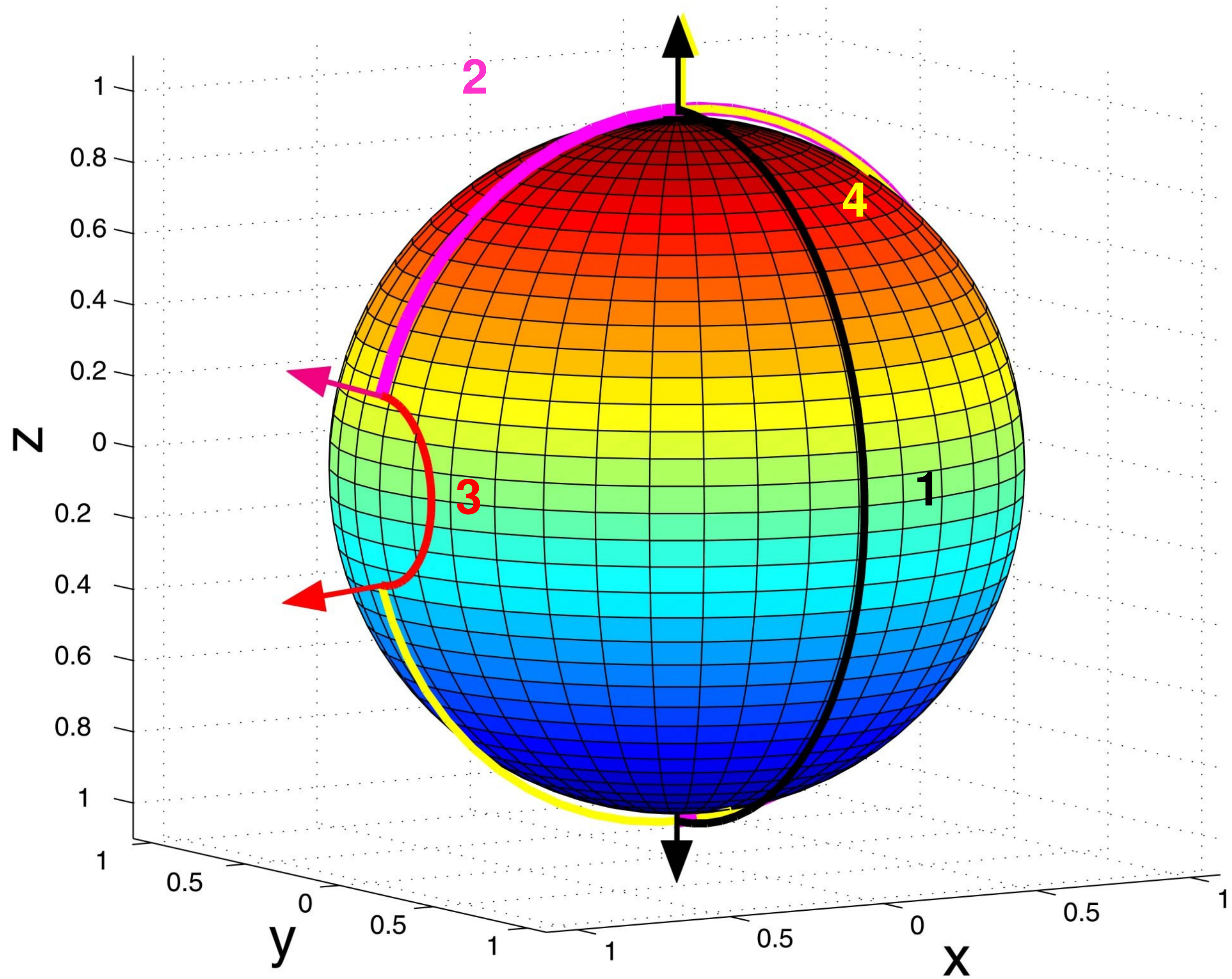
Coherent manipulation of the oscillator



$$a^\dagger |n\rangle = \sqrt{n+1}\,|n+1\rangle$$

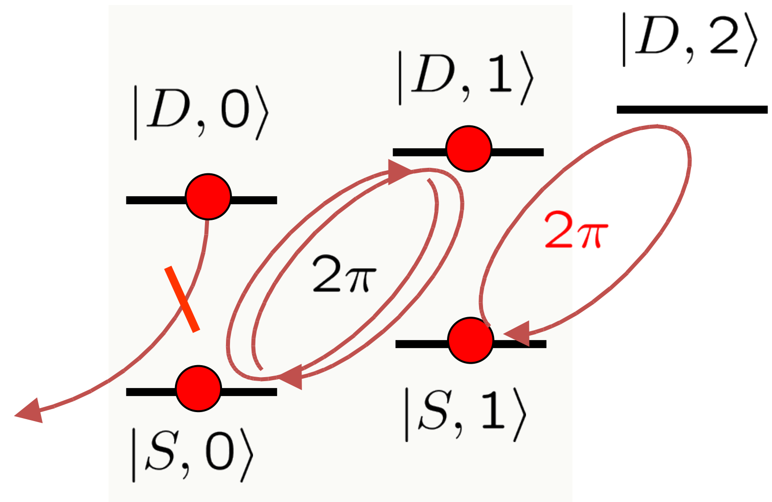
Action on the |1,S>↔ |2,D> manifold

$$R^+(\pi,\pi/2)\,R^+\left(\pi/\sqrt{2},0\right)\,R^+(\pi,\pi/2)\,R^+\left(\pi/\sqrt{2},0\right)$$

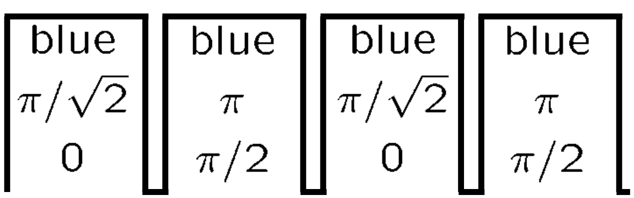


A phase gate

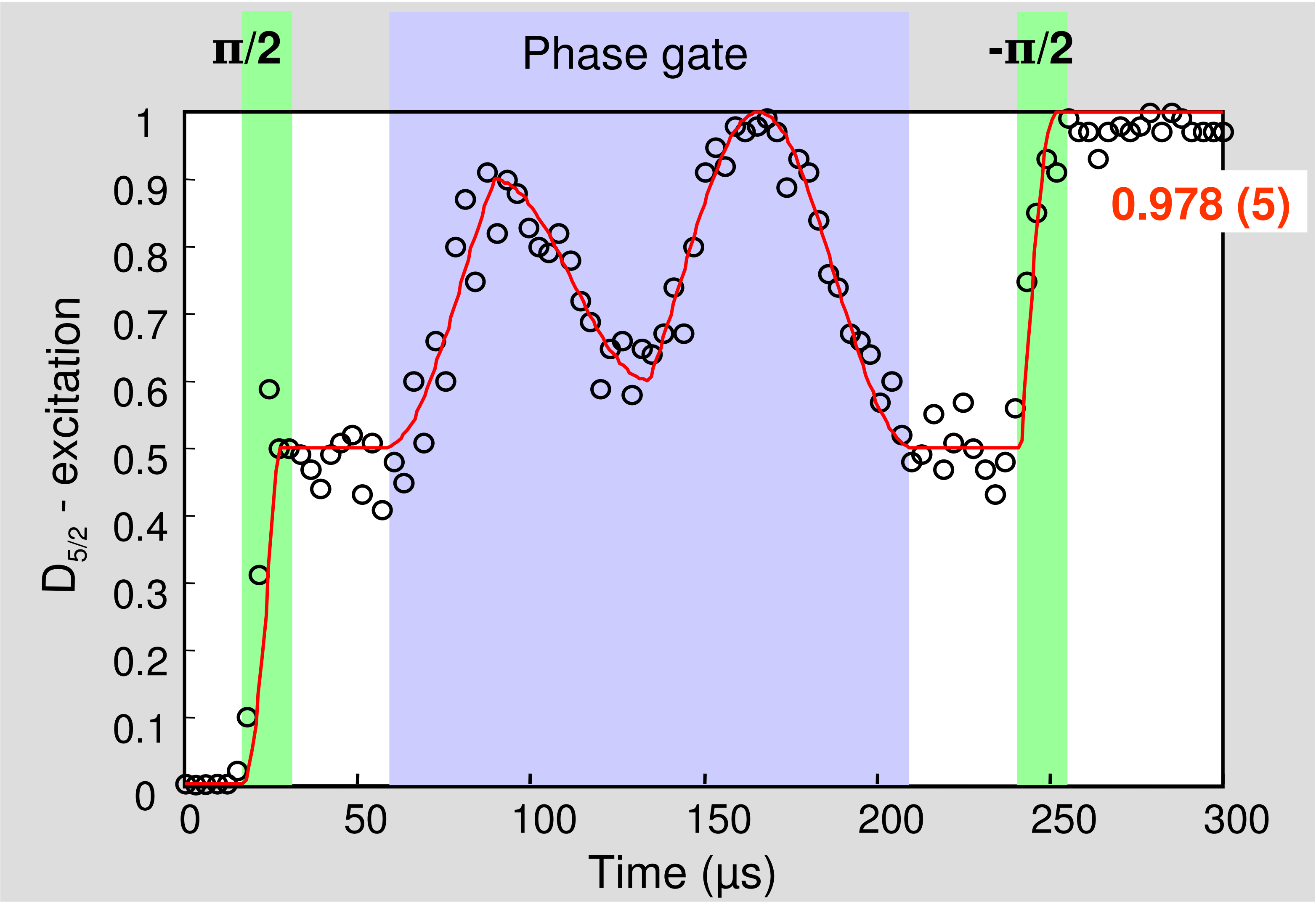
$$
\begin{array}{cc}
 & \begin{array}{cccc} |D,0\rangle & |S,0\rangle & |D,1\rangle & |S,1\rangle \end{array} \\
U_\Phi = & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}
\end{array}
$$



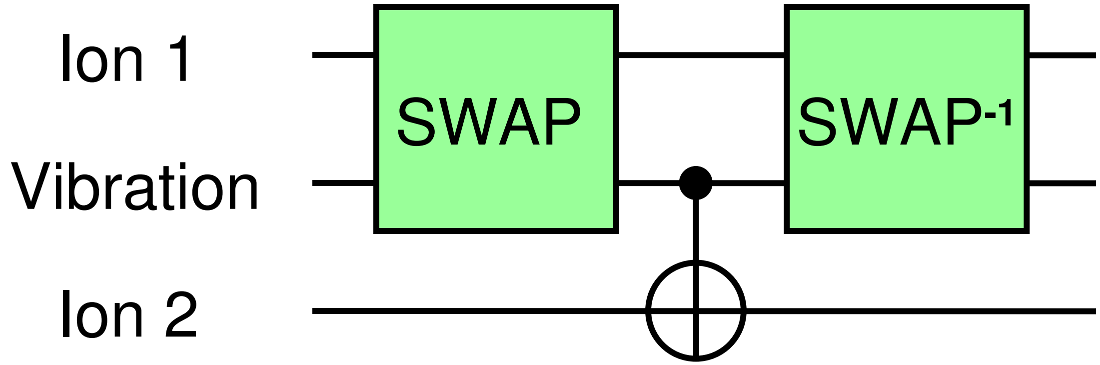
Composite 2π-rotation:



Single-ion CNOT



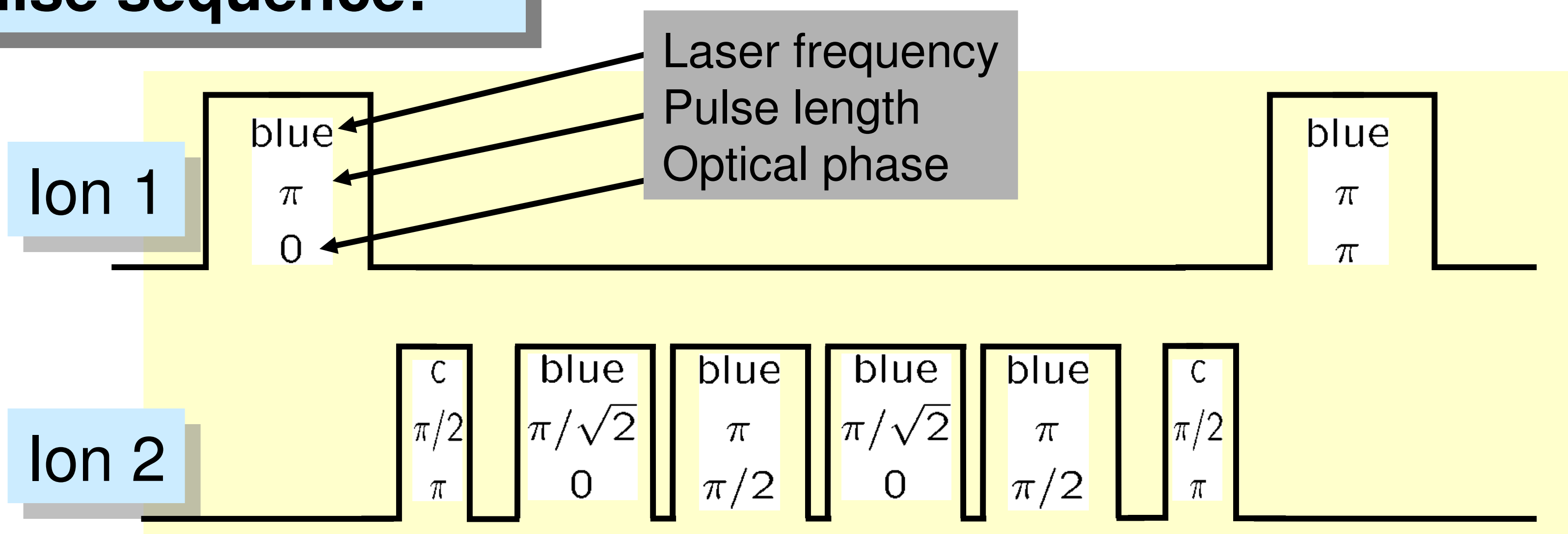
A controlled-NOT operation



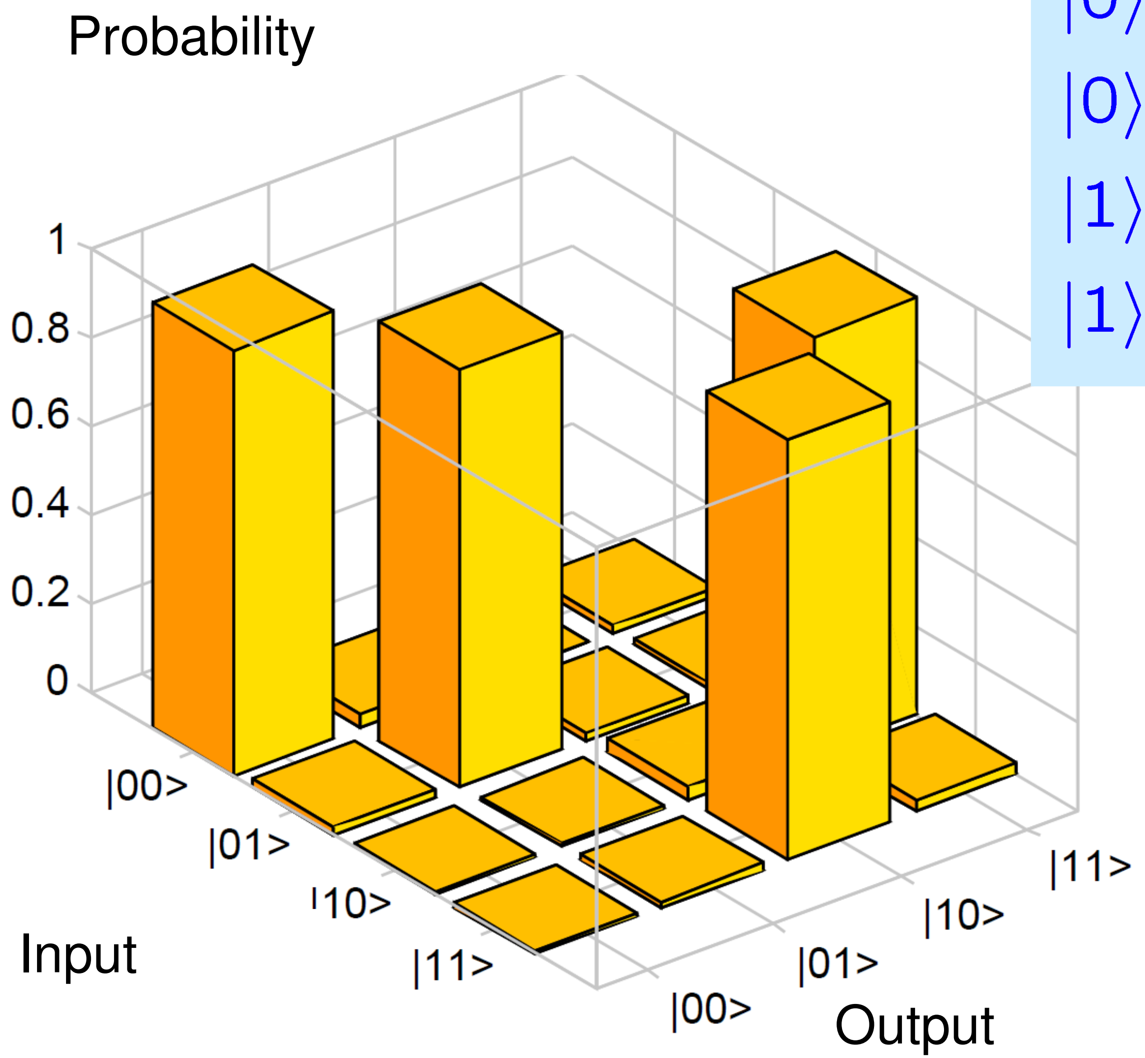
Control qubit

Target qubit

Pulse sequence:



Truth table of the CNOT

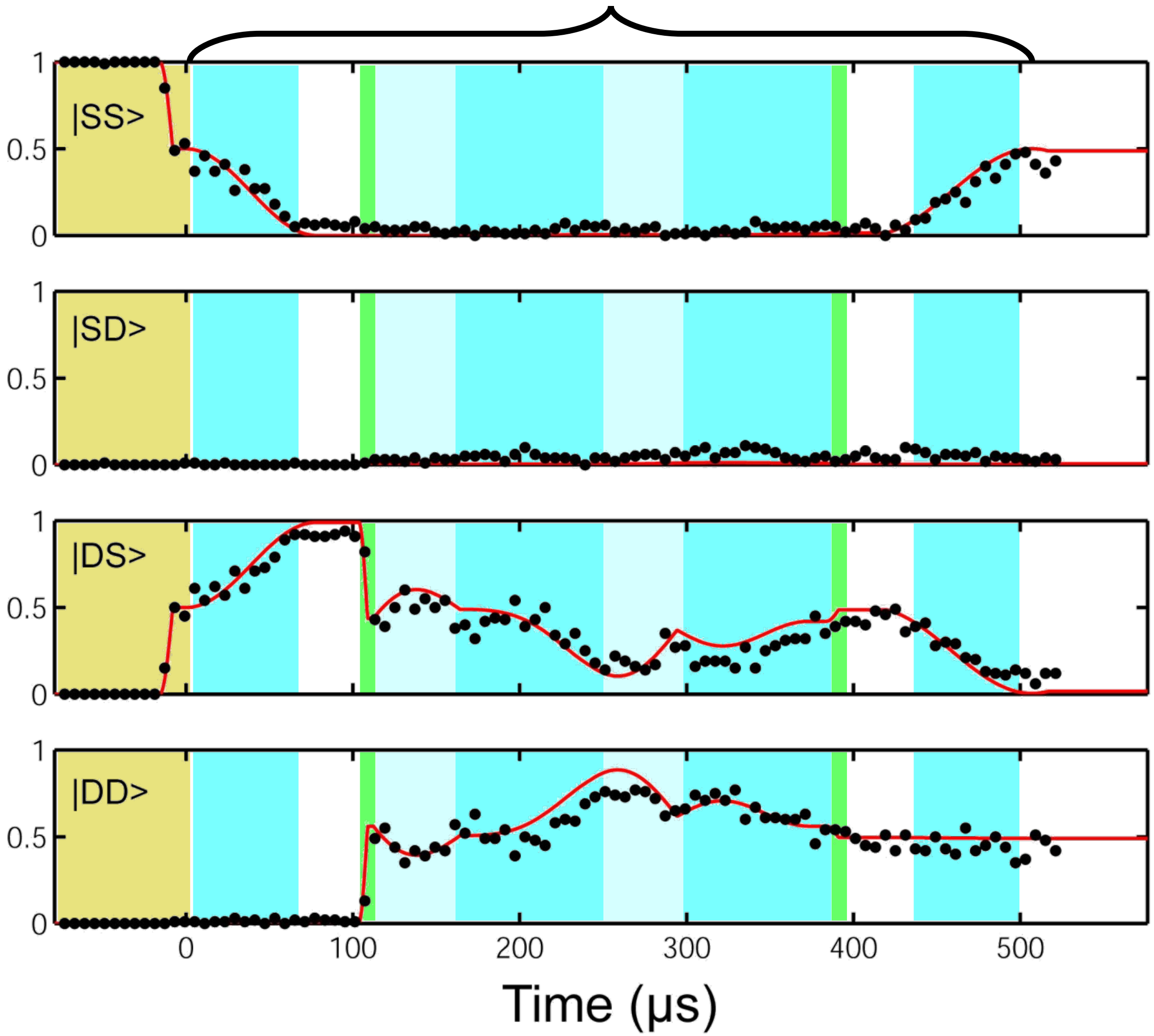


$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

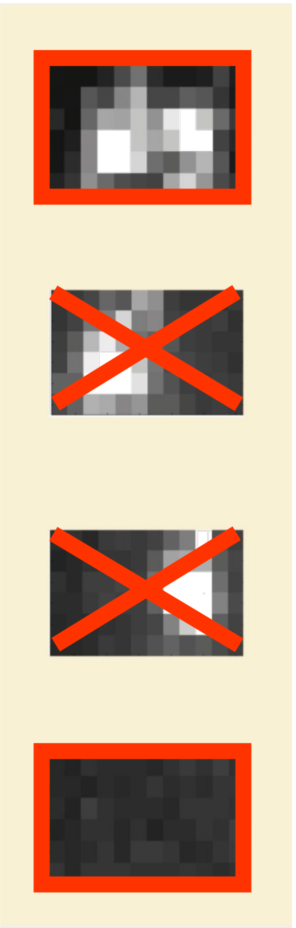
Using a CNOT to create a Bell state

$$|SS\rangle \rightarrow |S+D\rangle|S\rangle \quad \longrightarrow \quad |SS\rangle + |DD\rangle$$

prepare CNOT



output

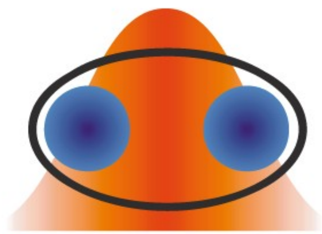


Cirac-Zoller gate

Draw backs of the Cirac-Zoller gate:

- slow (200 trap periods)
- single ion addressing required

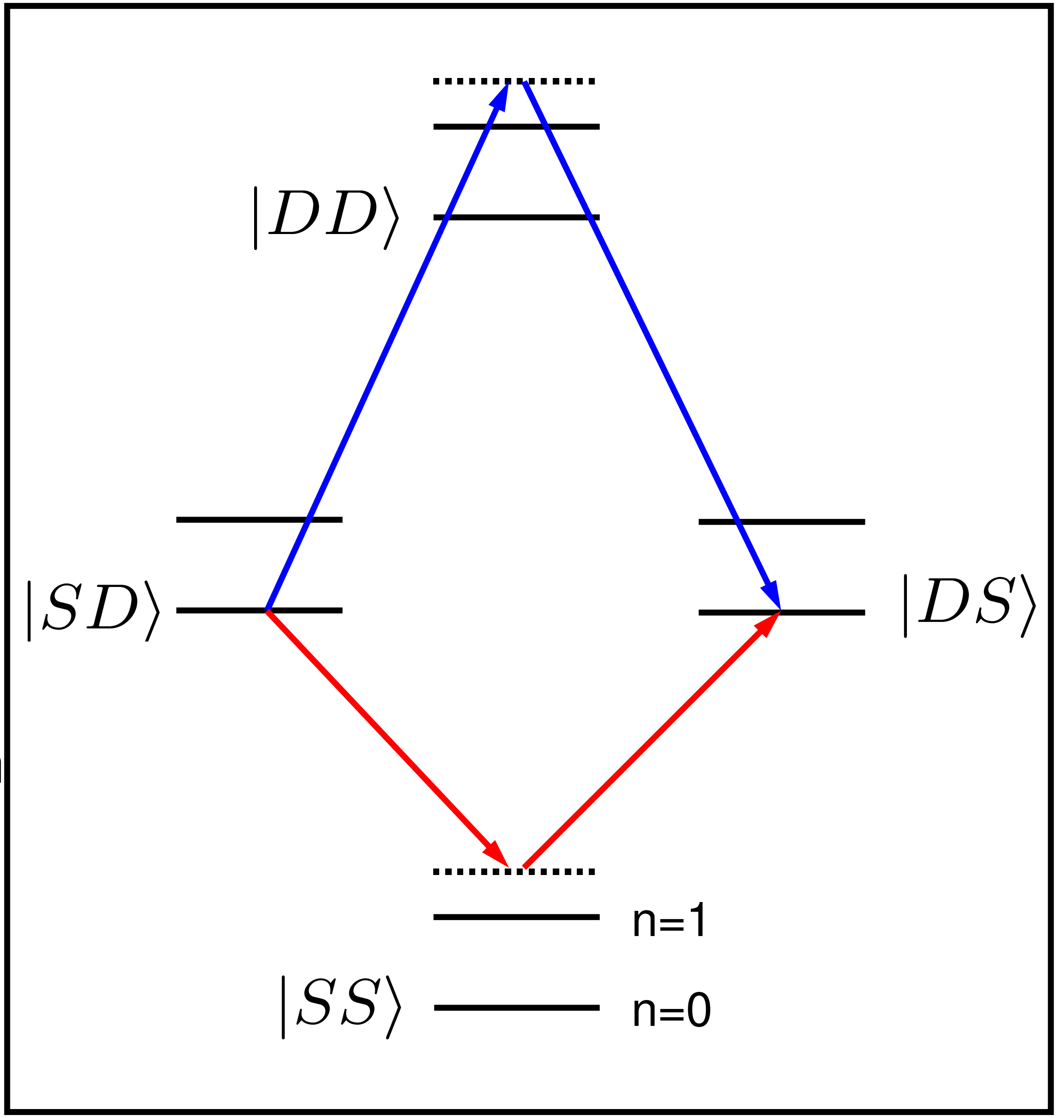
Mølmer-Sørensen gate



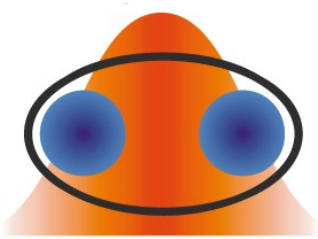
Raman transitions between

$$|SD\rangle \quad \Leftrightarrow \quad |DS\rangle$$

Interaction of two ions via common motion.



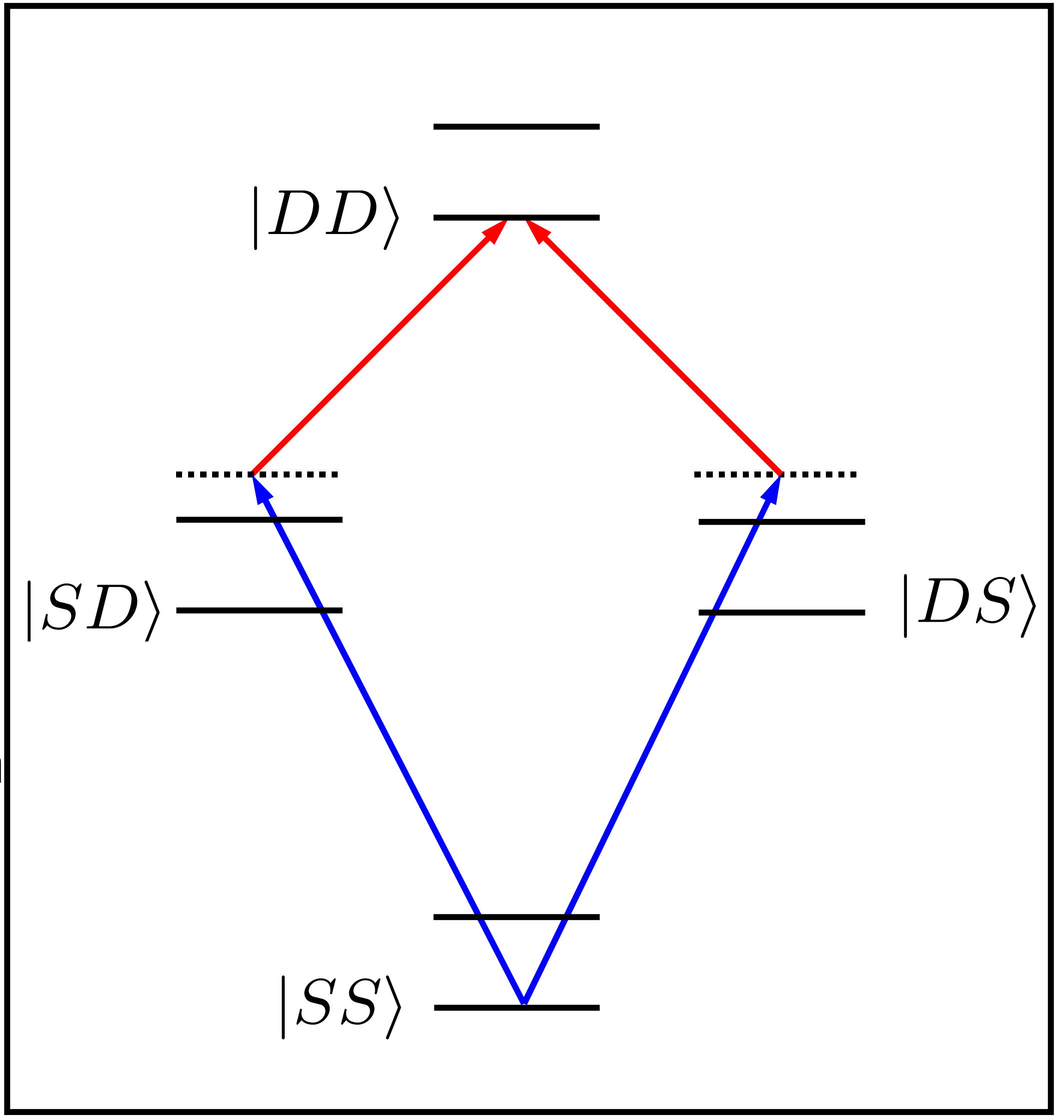
Mølmer-Sørensen gate



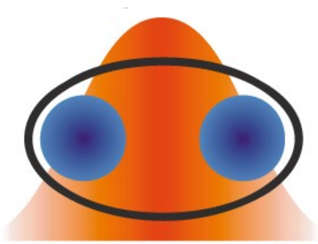
Raman transitions between

$$|SS\rangle \quad \Leftrightarrow \quad |DD\rangle$$

Interaction of two ions via common motion.



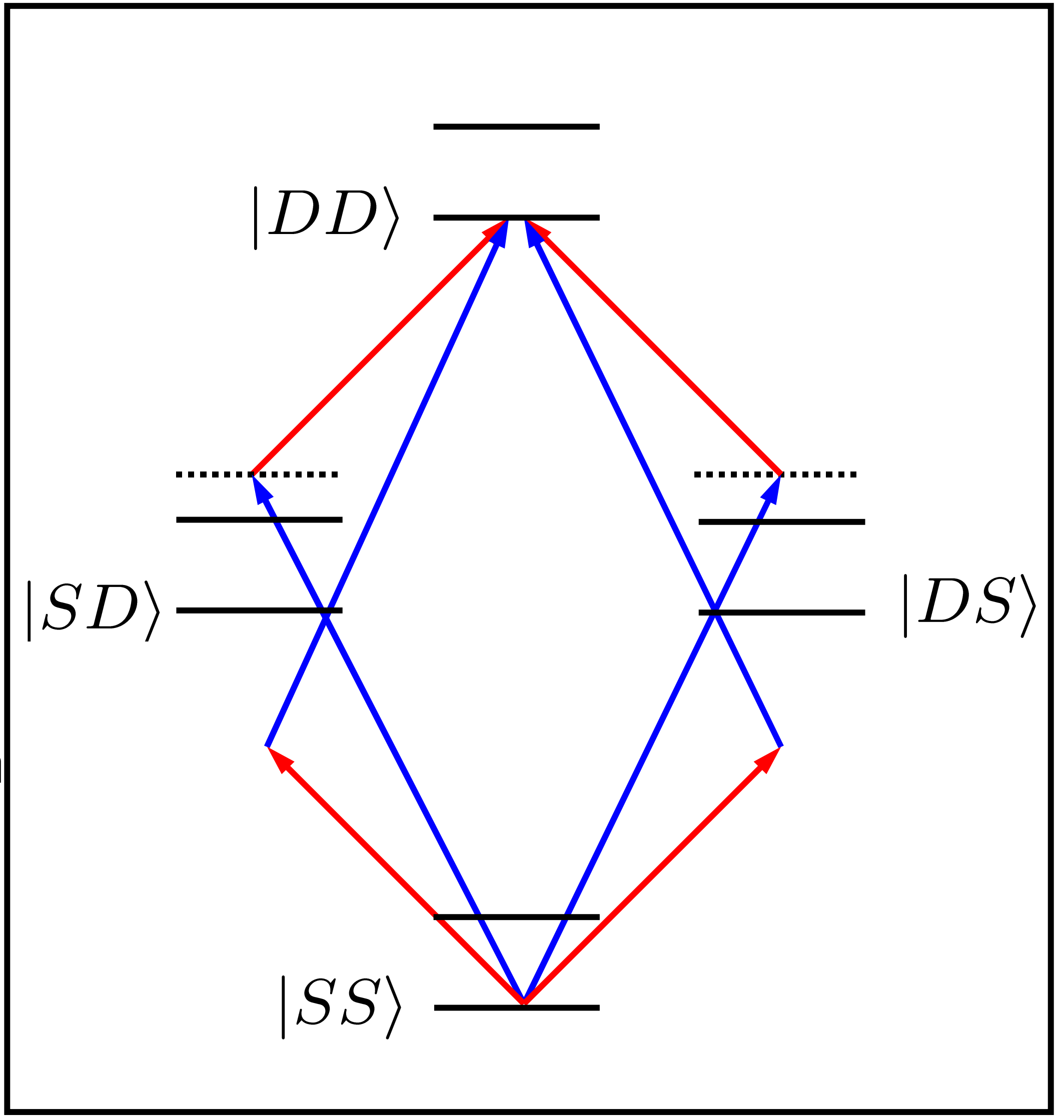
Mølmer-Sørensen gate



Raman transitions between

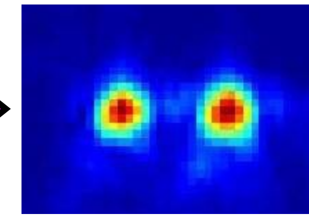
$$|SS\rangle \quad \Leftrightarrow \quad |DD\rangle$$

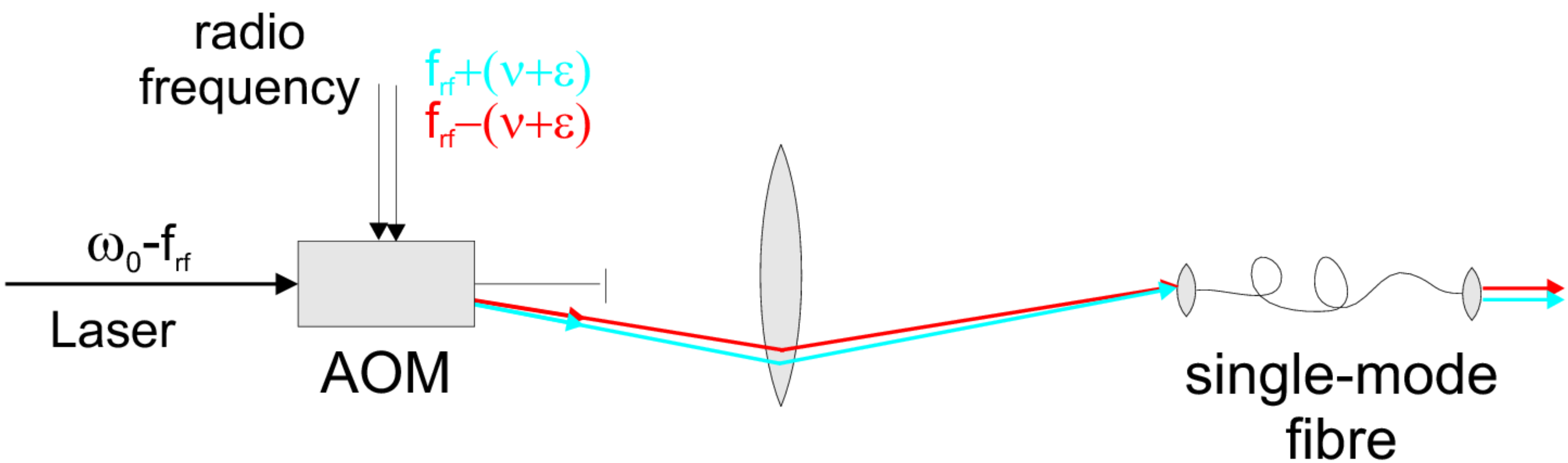
Interaction of two ions via common motion.

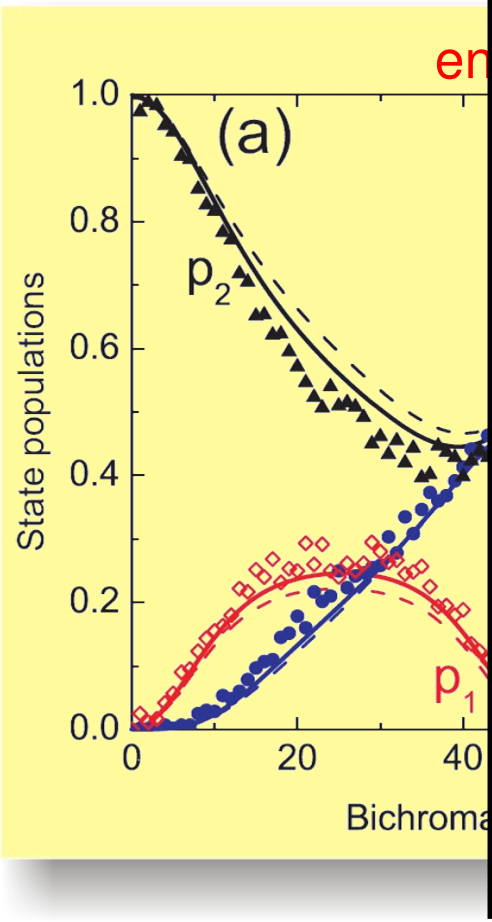


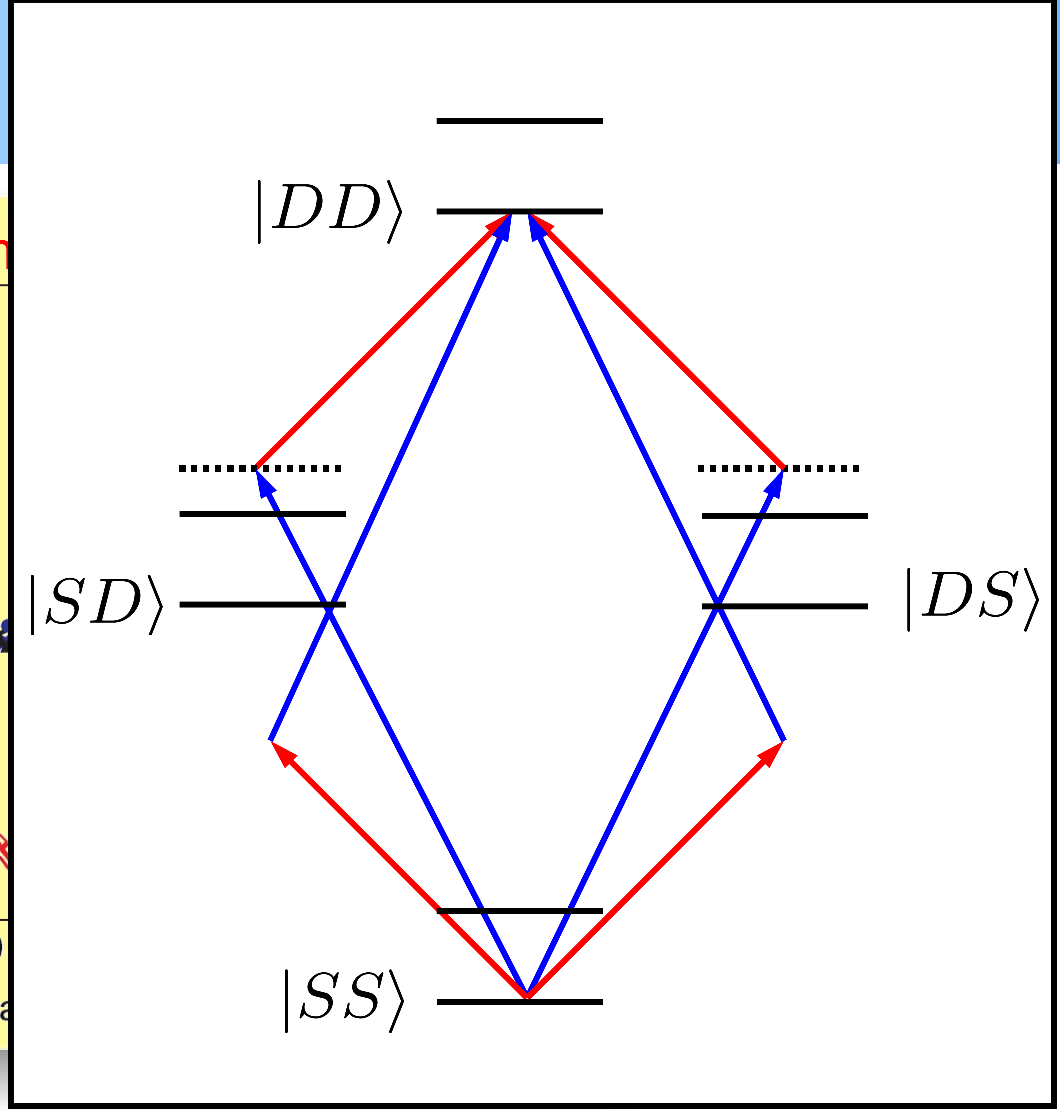
Technical realization

bicromatic beam
applied to both ions





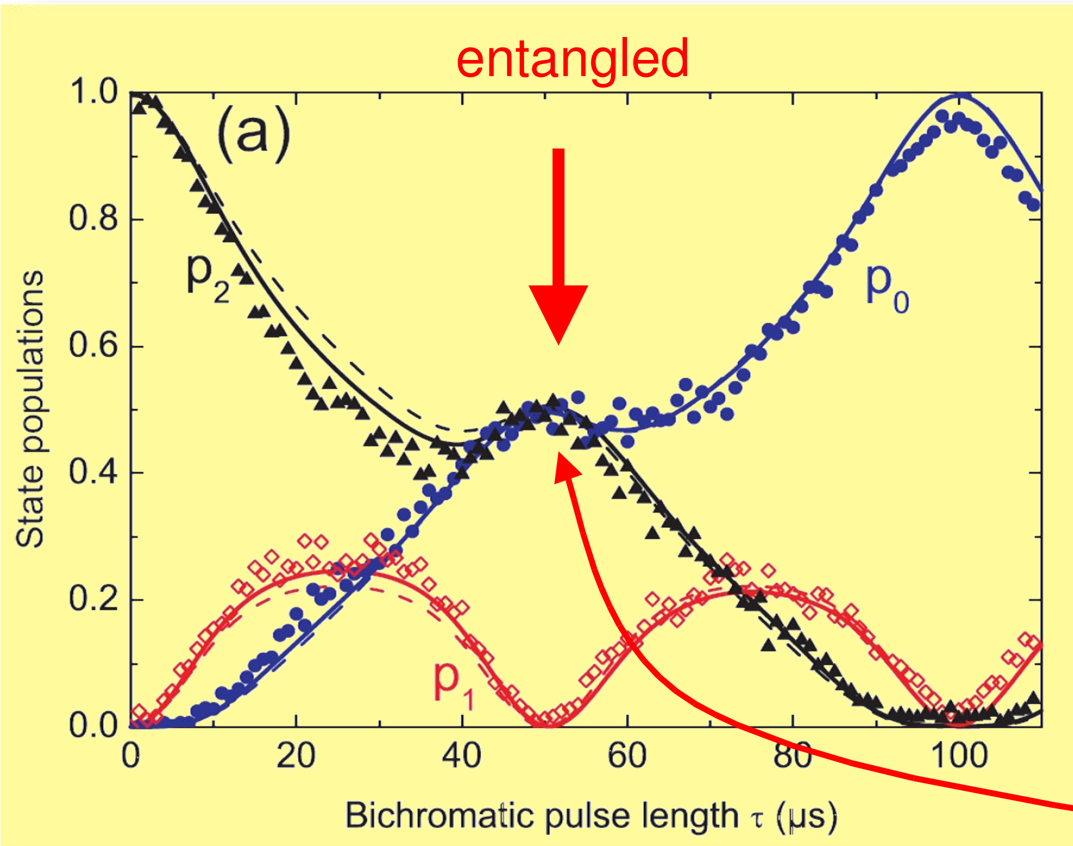




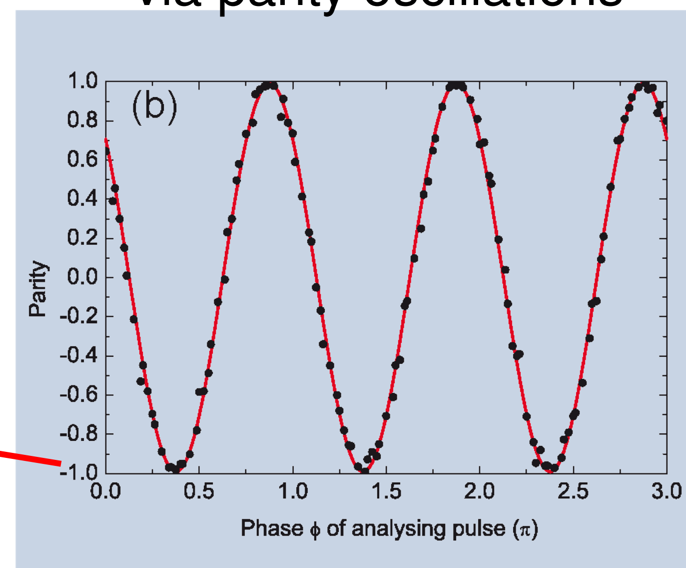
J. Benhelm et al., Nature Physics **4**, 463 (2008)

Theory: C. Roos, NJP **10,** 013002 (2008)

Entangling ions



measure entanglement
via parity oscillations



gate duration $51\mu s$

average fidelity: 99.3 (2) %

J. Benhelm et al., Nature Physics **4**, 463 (2008)

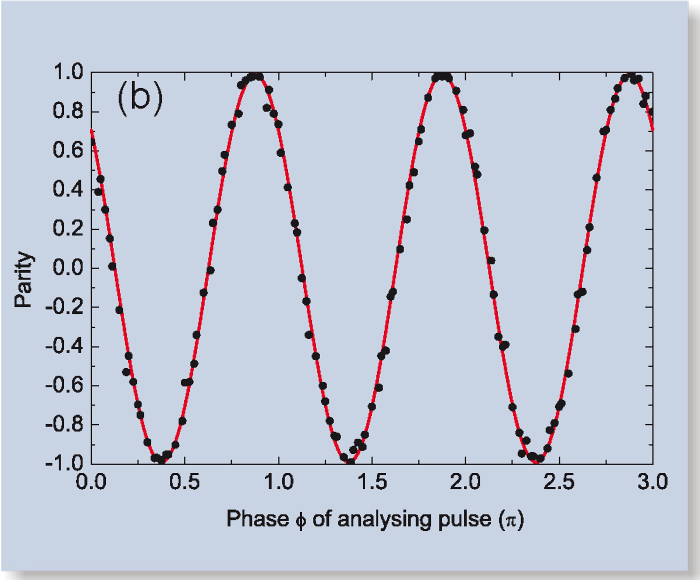
Theory: C. Roos, NJP **10,** 013002 (2008)

$$|00\rangle + |11\rangle \quad \xrightarrow{R_2^C(\pi/2,\varphi),R_1^C(\pi/2,\varphi)}$$

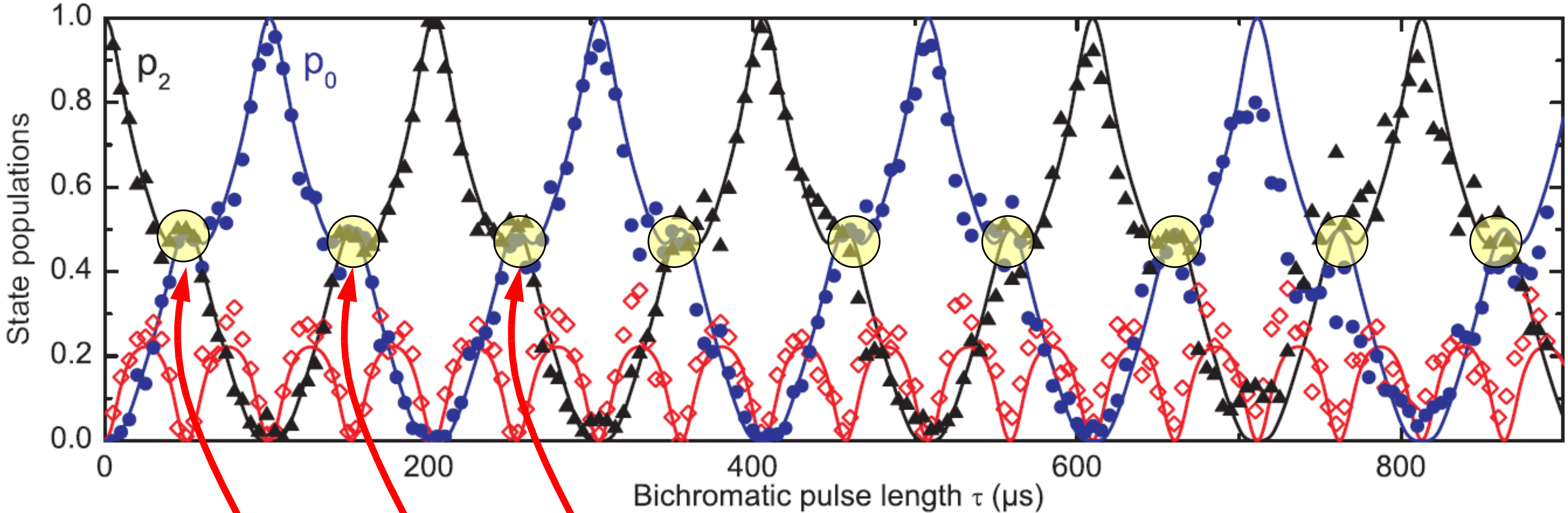
$$\left(|0\rangle + ie^{i\varphi}|1\rangle\right)\left(|0\rangle + ie^{i\varphi}|1\rangle\right) + \left(|1\rangle + ie^{-i\varphi}|0\rangle\right)\left(|1\rangle + ie^{-i\varphi}|0\rangle\right)$$

$$= (1 - e^{-2i\varphi})|00\rangle + ie^{i\varphi}(1 + e^{-2i\varphi})|01\rangle$$

$$+ ie^{i\varphi}(1 + e^{-2i\varphi})|10\rangle + (1 - e^{-2i\varphi})|11\rangle \, ,$$

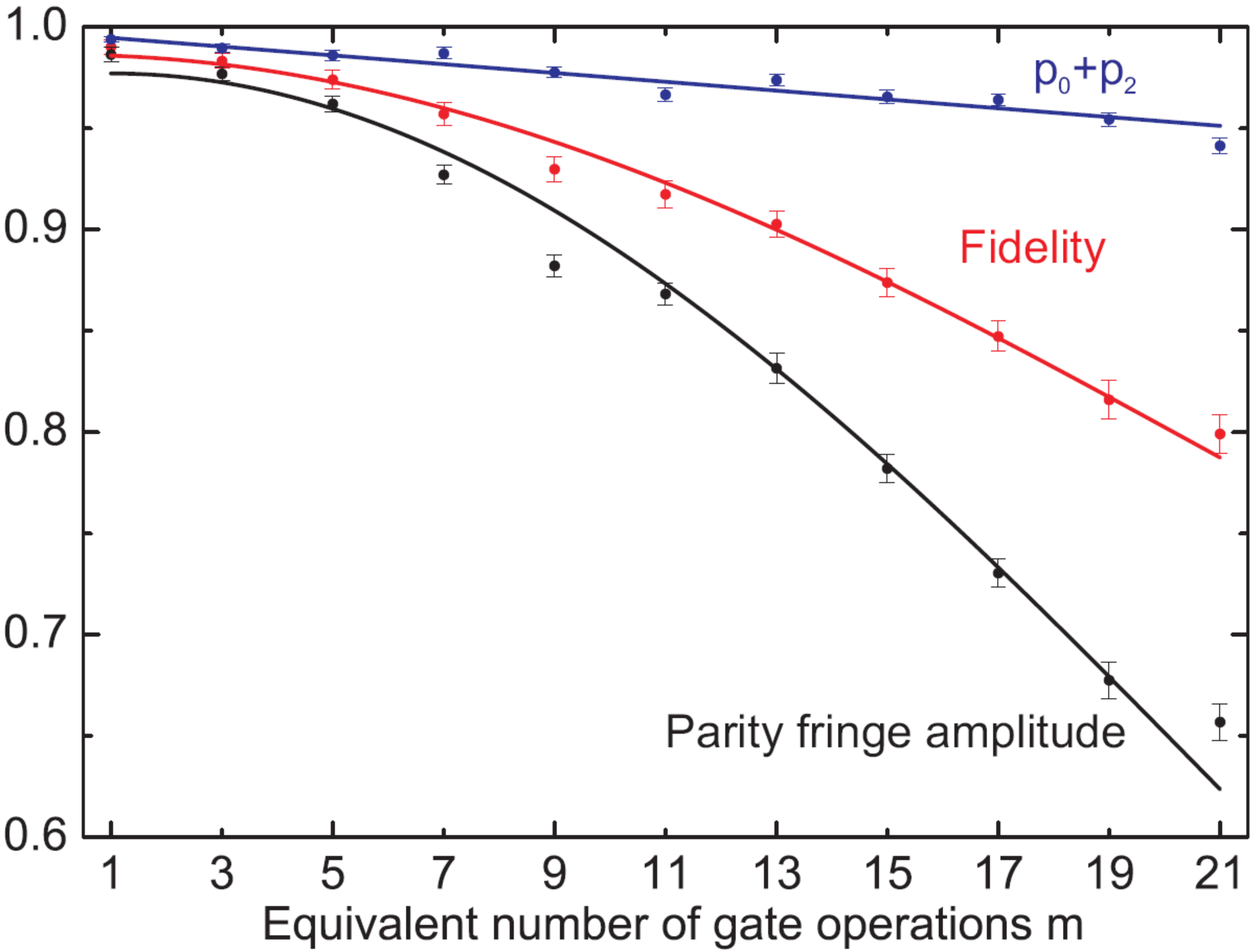


Gate concatenation



maximally entangled states

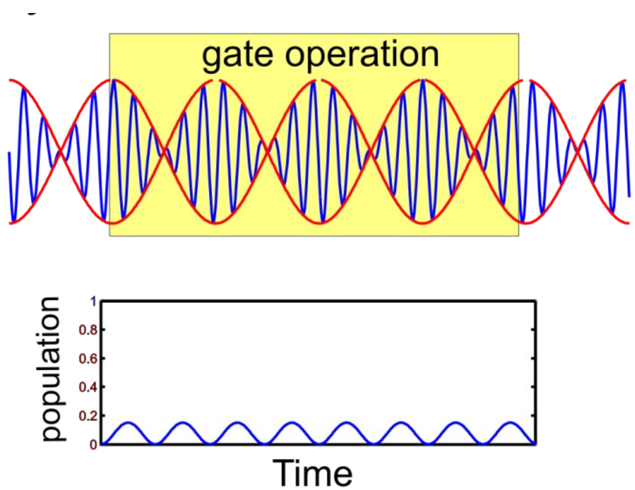
Gate performance

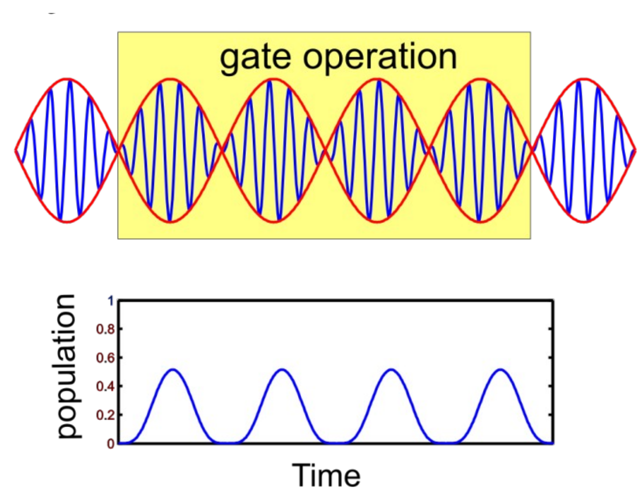


Optimization of the gate

The MS gate operation is optimized by

- pulse shaping, minimizes off-resonant excitation
- phase relation of amplitude modulated dual-frequency beam





C. Roos, NJP **10,** 013002 (2008).

Scaling of this approach?

Problems :

- Coupling strength between internal and motional states of a N-ion string decreases as

$$\eta \propto \frac{1}{\sqrt{N}}$$

(momentum transfer from photon to ion string becomes more difficult)

-> Gate operation speed slows down

- More vibrational modes increase risk of spurious excitation of unwanted modes

- Distance between neighbouring ions decreases -> addressing more difficult

-> Use flexible trap potentials to split long ion string into smaller segments and perform operations on these smaller strings

DiVincenzo criteria

- Scalable physical system, well characterized qubits ✓ / ?
- Ability to initialize the state of the qubits ✓
- Long relevant coherence times, much longer than gate operation time ✓
- "Universal" set of quantum gates ✓
- Qubit-specific measurement capability ✓

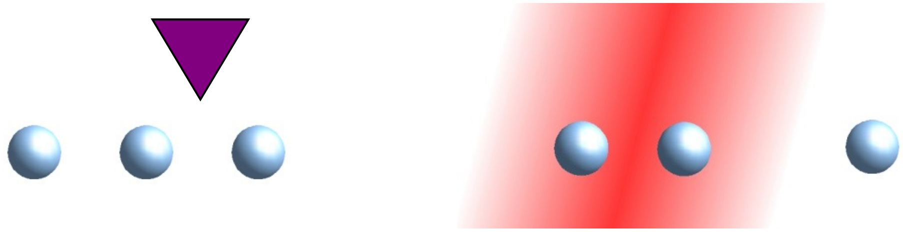
Often neglected:

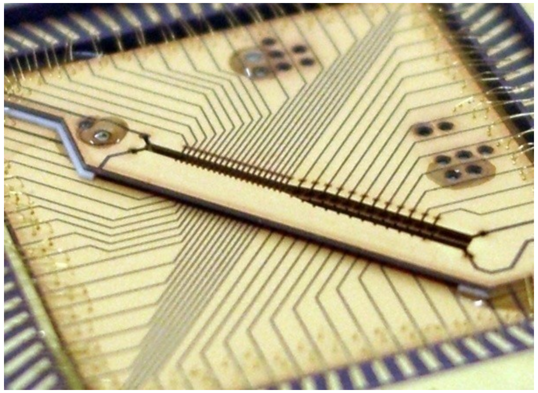
- exceptional fidelity of operations
- low error rate also for large quantum systems
- all requirements have to met at the same time

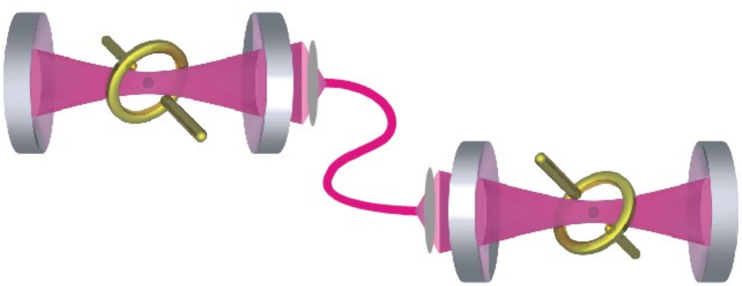
Scaling of ion trap quantum computers

Its easy to have thousands of coherent qubits …
but hard to control their interaction

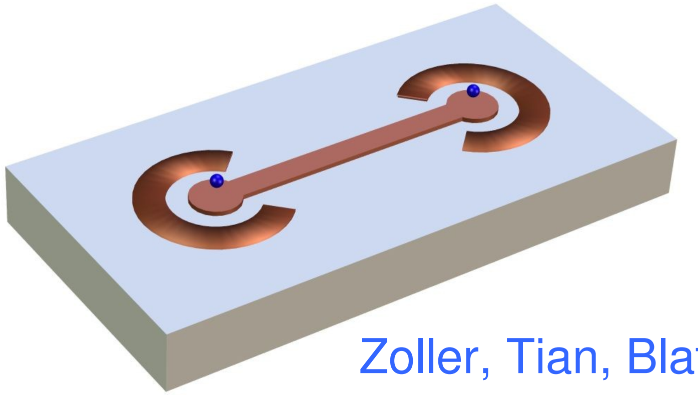
Kielpinski, Monroe, Wineland





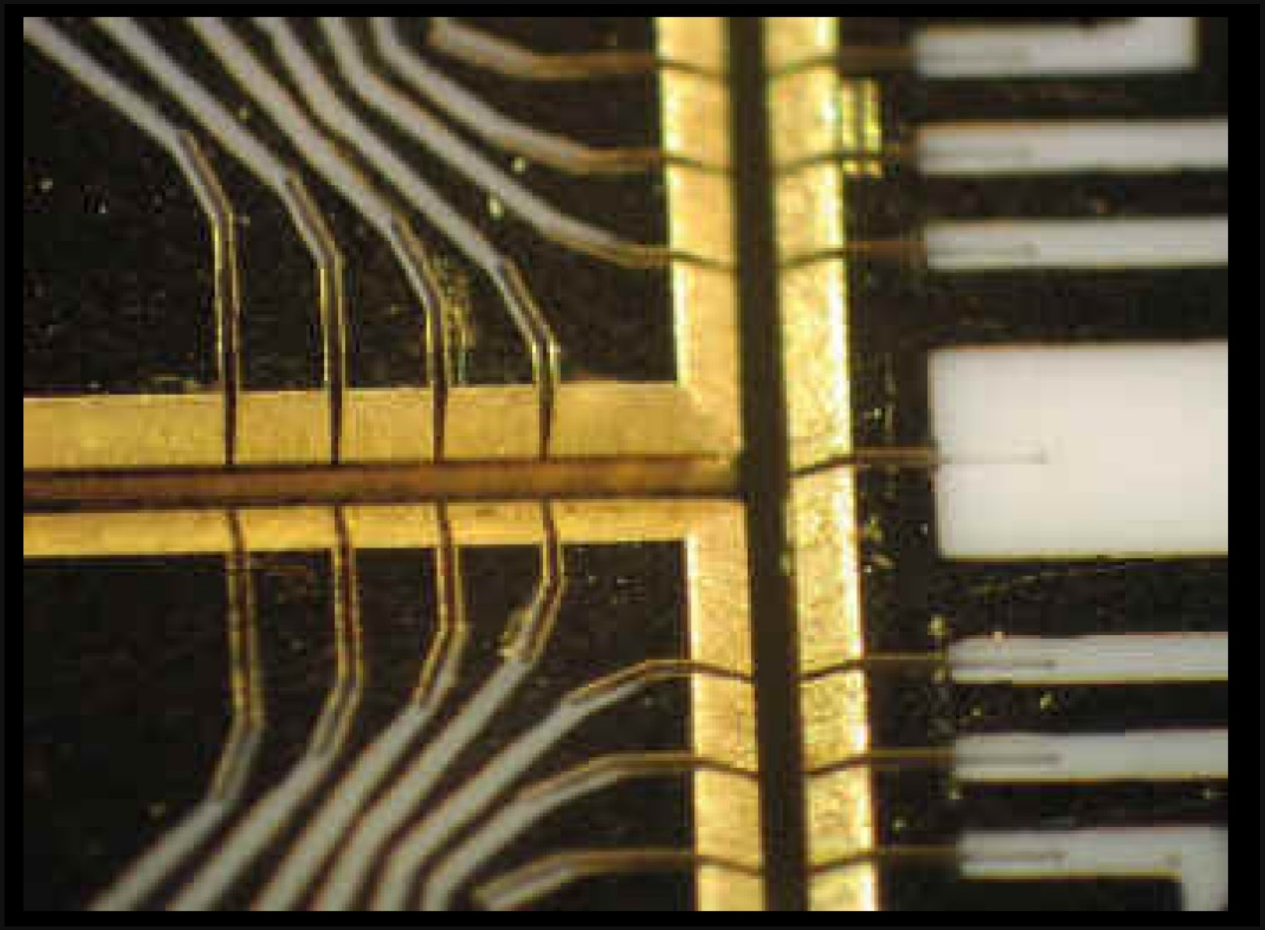


Cirac, Zoller, Kimble, Mabuchi



Zoller, Tian, Blatt

The Michigan T trap



An implementation of the Deutsch-algoritm …

Deutsch's problem: Introduction

Decide which class the coin is:

False (equal sides) or ***Fair***

Front

Back

A single measurement does NOT give the right answer

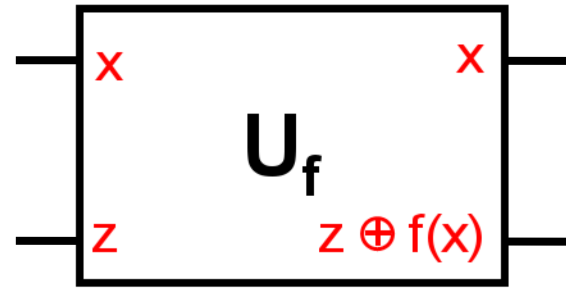
Deutsch‘s problem: Mathematical formulation

4 possible coins are representend by 4 functions

	Constant		**Balanced**	
	Case 1	**Case 2**	**Case 3**	**Case 4**
***f*(0)**	**0**	**1**	**0**	**1**
***f*(1)**	**0**	**1**	**1**	**0**

Deutsch‘s problem: Mathematical formulation

4 possible coins are representend by 4 functions



	Constant		Bal	
	Case 1	Case 2	Case 3	
$f(0)$	0	1	0	1
$f(1)$	0	1	1	0
$z \oplus f(x)$	ID	NOT	CNOT	Z-CNOT

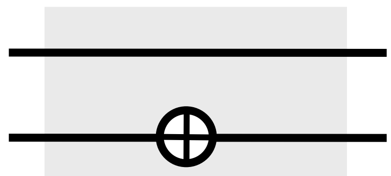
$$U_{f_n}\left|x, z\right\rangle = \left|x, f_n(x) \oplus z\right\rangle$$

Physically reversible process
realized by a unitary transformation

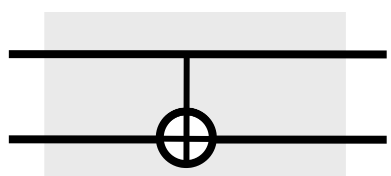
Deutsch Jozsa quantum circuit

<table>
<tr><th>Case</th><th>Logic</th><th>Quantum circuit</th><th>Matrix U_{fn}</th></tr>
<tr><td>f_1</td><td>ID</td><td>

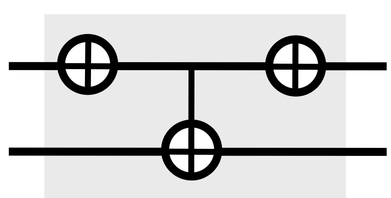
</td><td>1000
0100
0010
0001</td></tr>
<tr><td>f_2</td><td>NOT</td><td>



</td><td>0100
1000
0001
0010</td></tr>
<tr><td>f_3</td><td>CNOT</td><td>



</td><td>1000
0100
0001
0010</td></tr>
<tr><td>f_4</td><td>Z-CNOT</td><td>

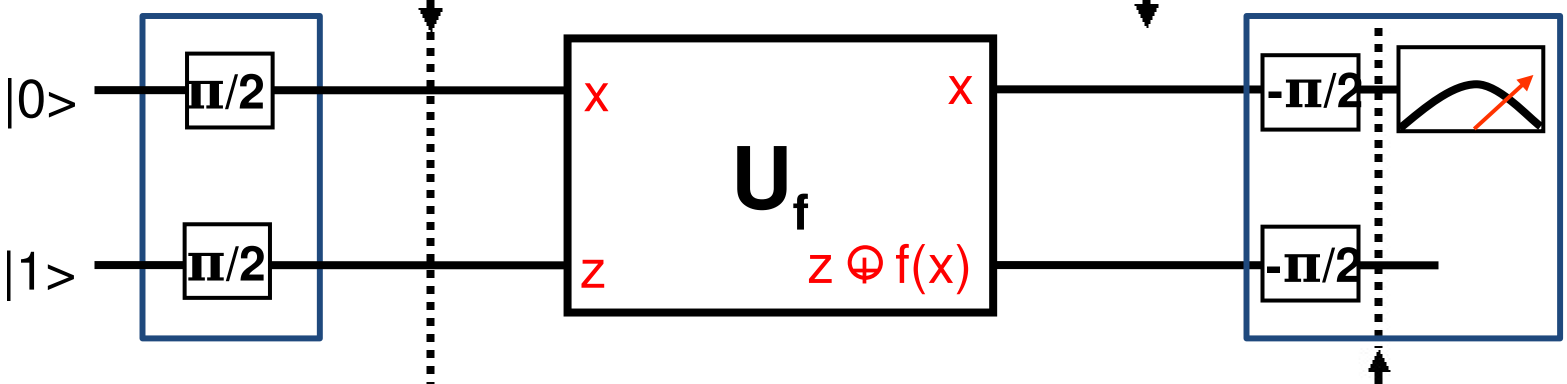


</td><td>0100
1000
0010
0001</td></tr>
</table>

Deutsch Jozsa quantum circuit

$$|\psi_1\rangle = |0+1\rangle|0-1\rangle$$

$$|\psi_2\rangle = \pm|0+1\rangle|0-1\rangle \quad \text{if } f(0) = f(1)$$
$$= \pm|0-1\rangle|0-1\rangle \quad \text{if } f(0) \neq f(1)$$

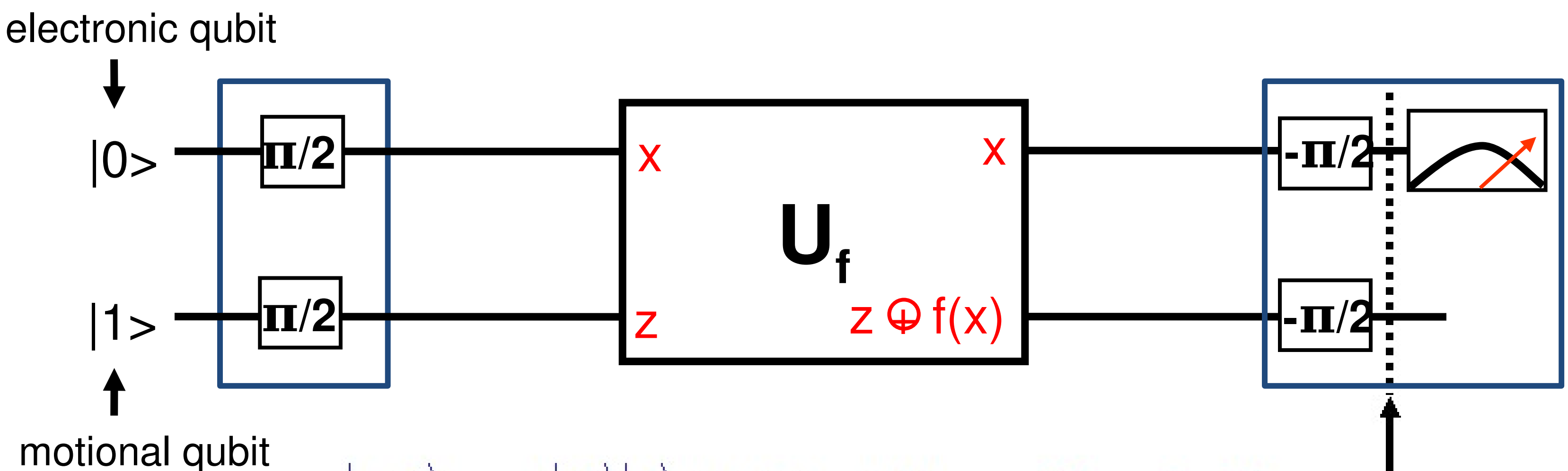


$$|\psi_3\rangle = \pm|0\rangle|1\rangle \quad \text{if } f(0) = f(1) \quad \textit{(Constant)}$$
$$= \pm|1\rangle|1\rangle \quad \text{if } f(0) \neq f(1) \quad \textit{(Balanced)}$$

Quantum analysis gives the right answer after a single measurement!

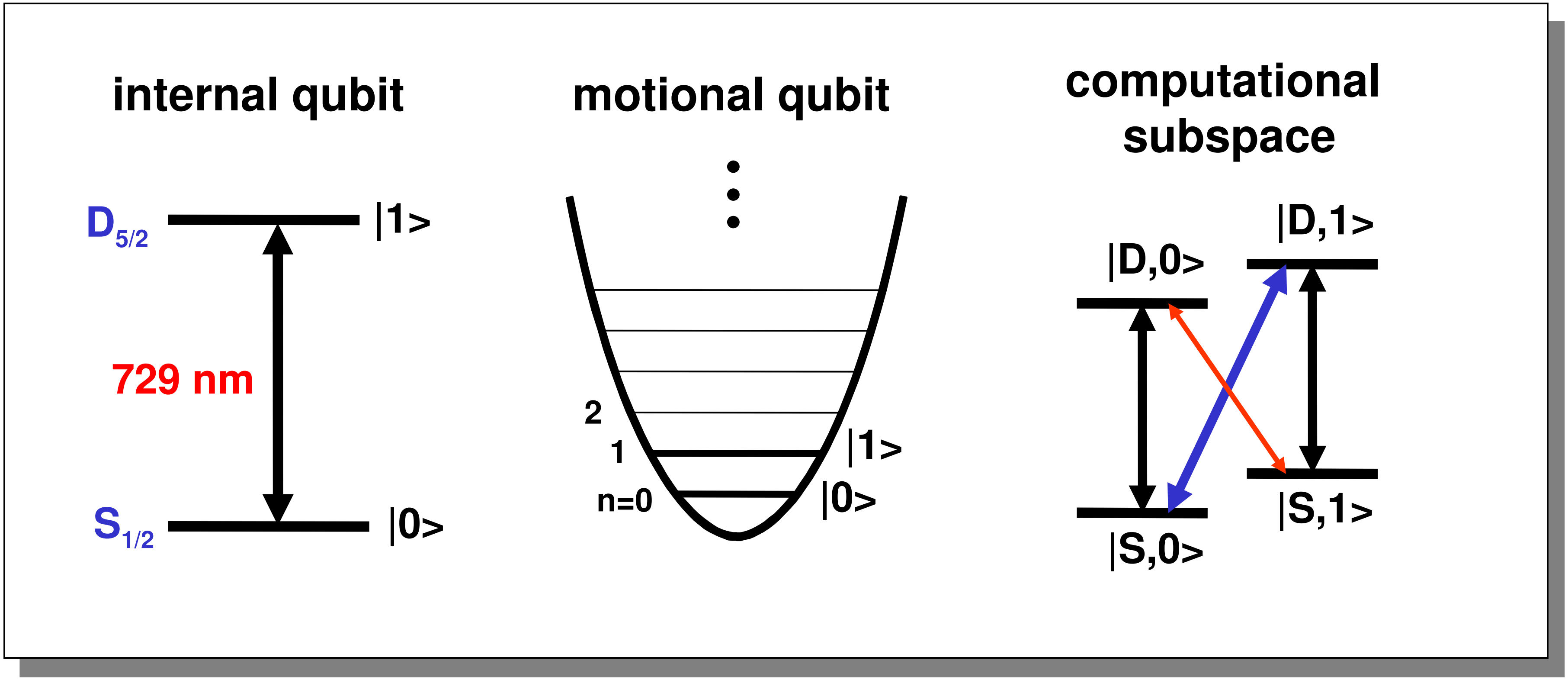
- *D. Deutsch, R. Josza, Proc. R. Soc. London A439, 553 (1992)*
- *M. Nielsen, I. Chuang, QC and QI, Cambridge (2000)*

No information in the second qubit

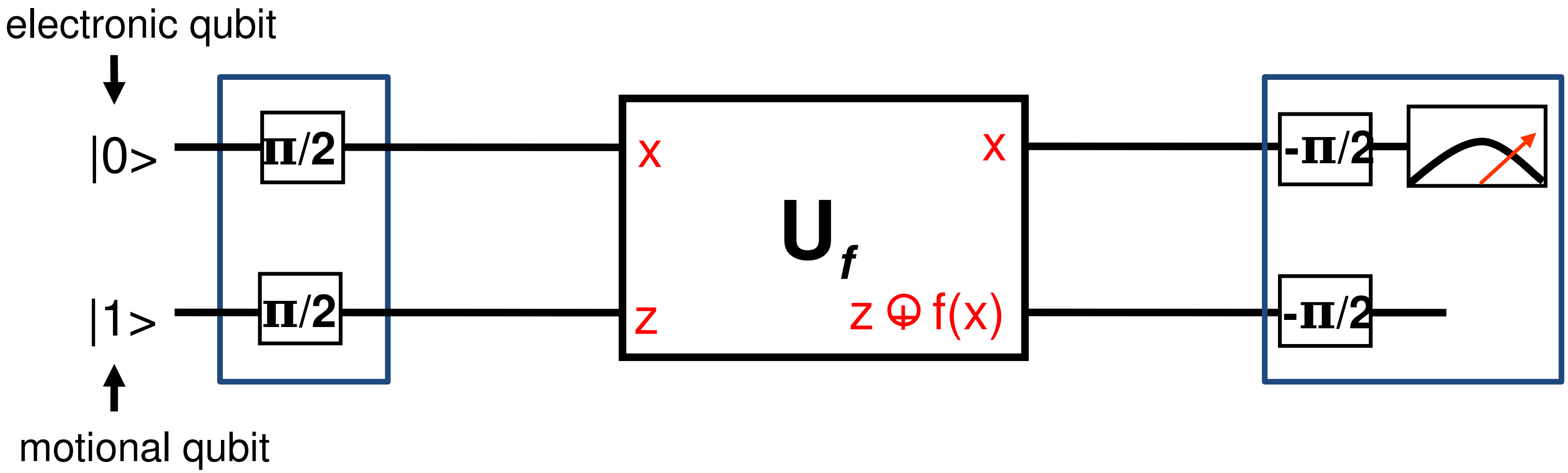


$$|\psi_3\rangle = \pm|0\rangle|1\rangle \quad \text{if } f(0) = f(1) \quad \textit{(Constant)}$$
$$= \pm|1\rangle|1\rangle \quad \text{if } f(0) \neq f(1) \quad \textit{(Balanced)}$$

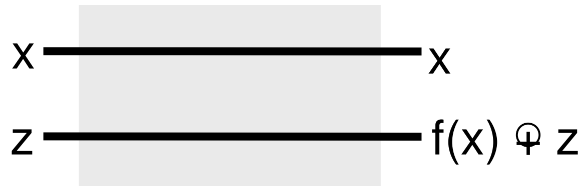
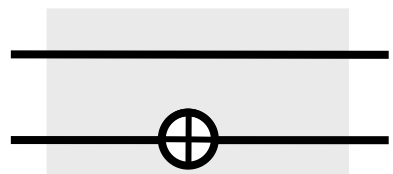
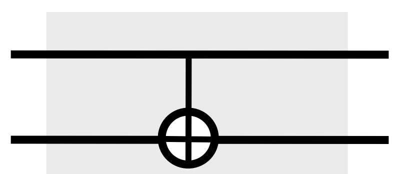
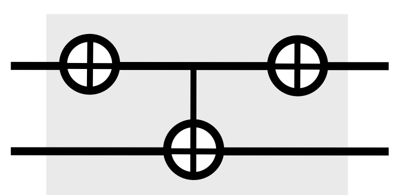
Qubits in $^{40}Ca^{+}$



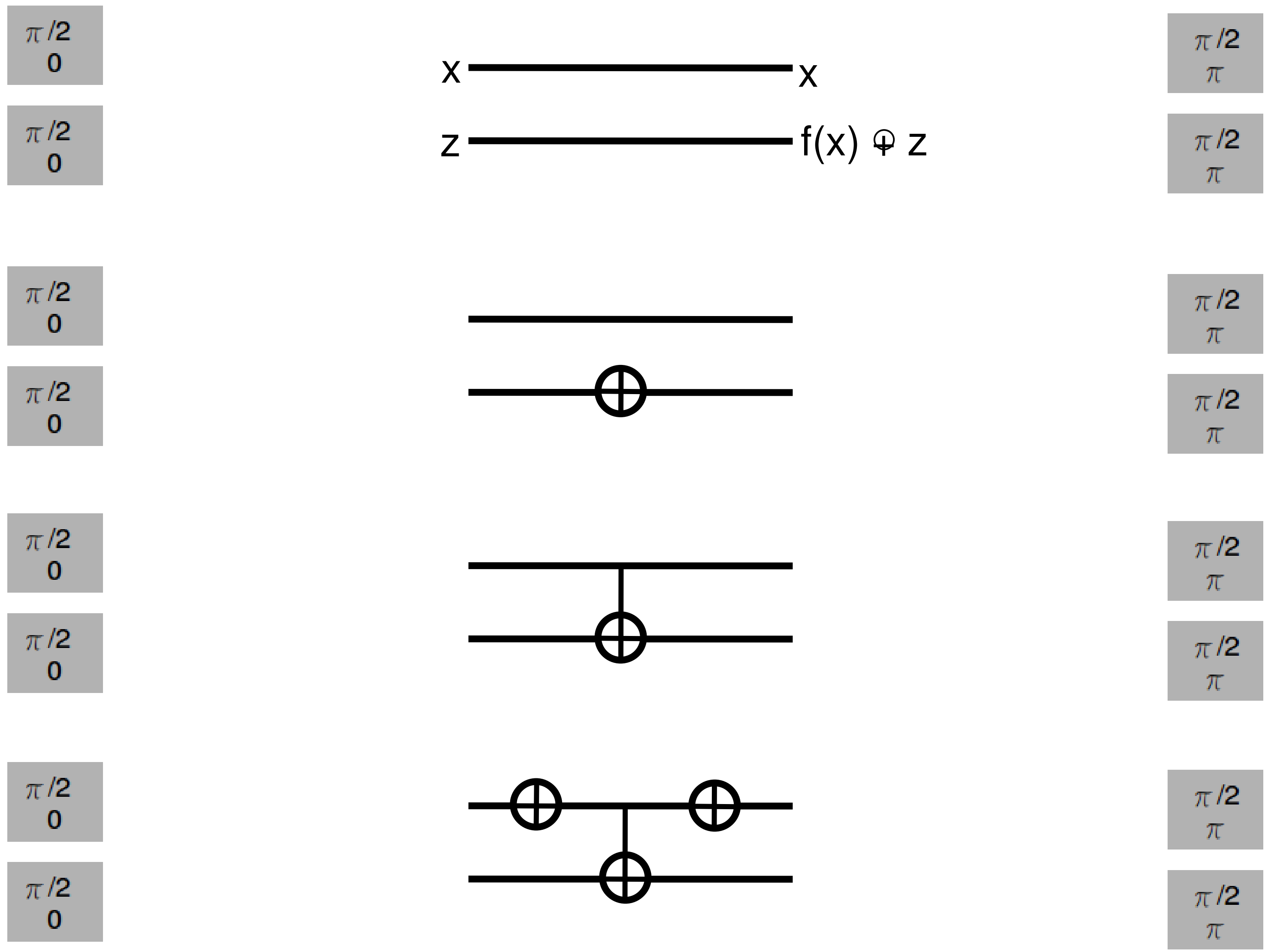
No information in the second qubit



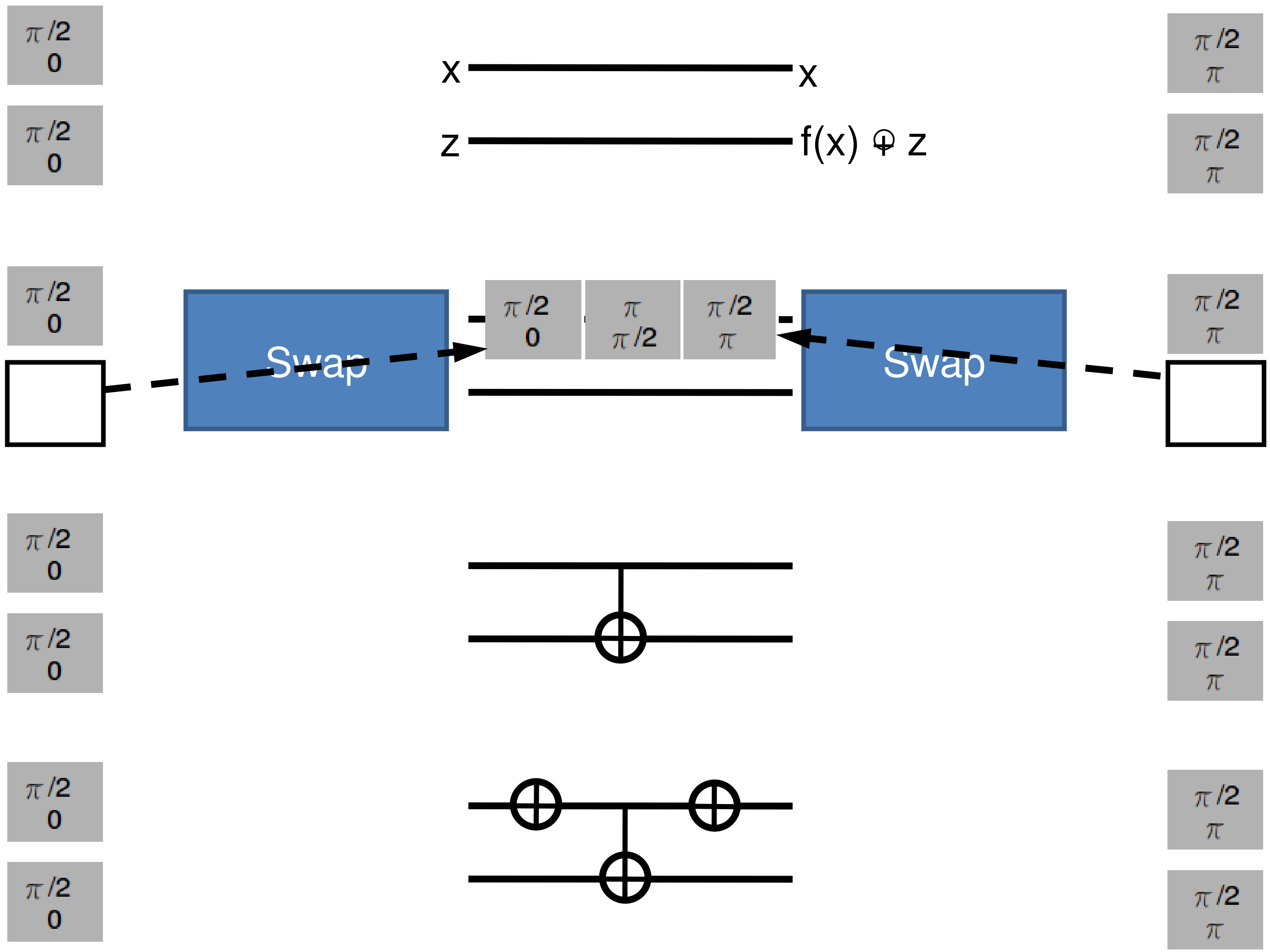
Deutsch Jozsa quantum circuit

Case	Logic	Quantum circuit	Matrix U_{fn}
f_1	***ID***		1000 0100 0010 0001
f_2	***NOT***		0100 1000 0001 0010
f_3	***CNOT***		1000 0100 0001 0010
f_4	***Z-CNOT***		0100 1000 0010 0001

Deutsch Jozsa: Realization

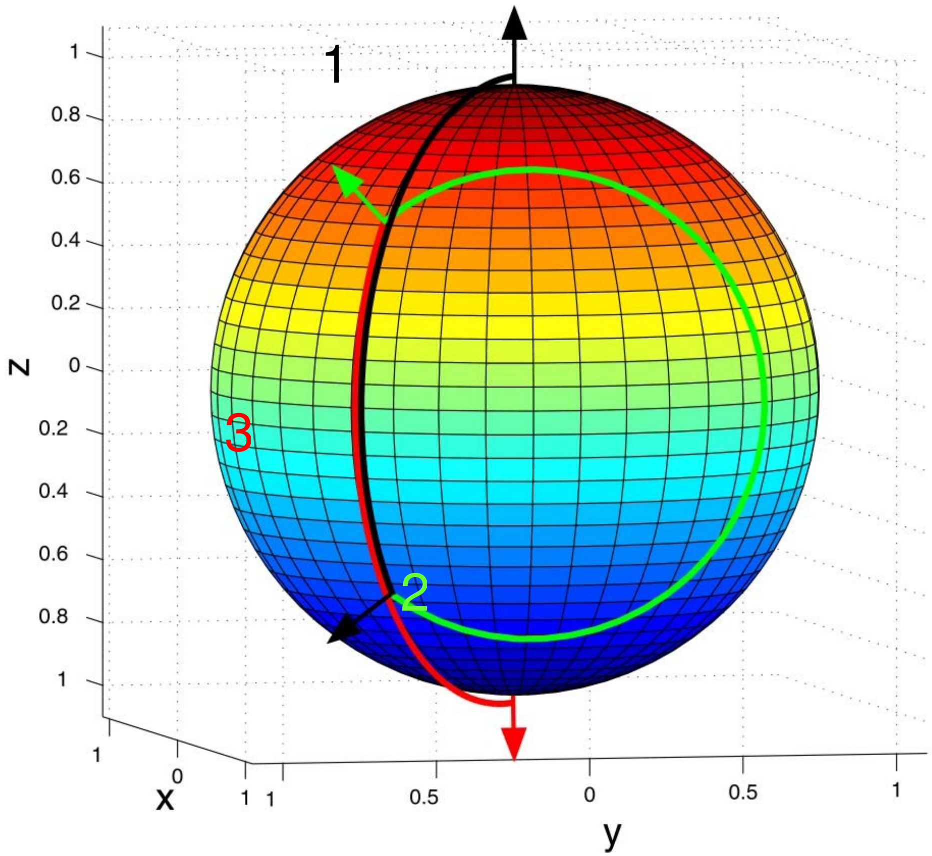


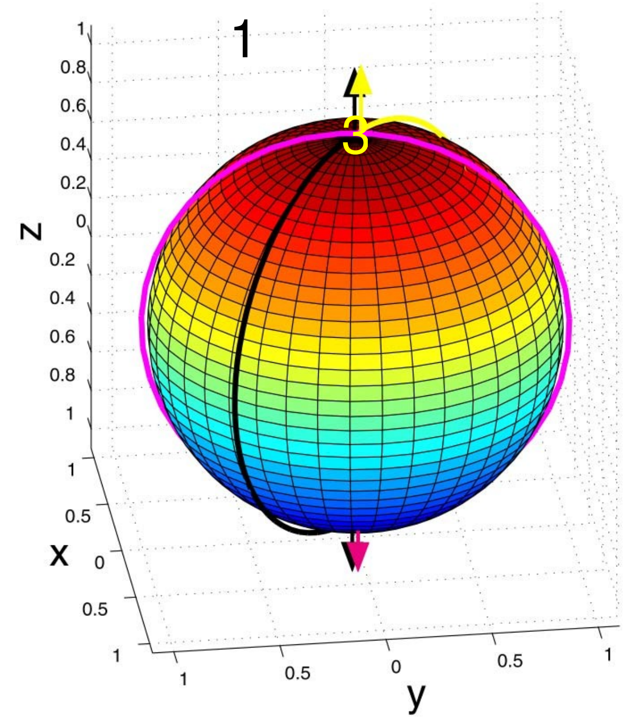
Deutsch Jozsa: Realization



3-step composite SWAP operation

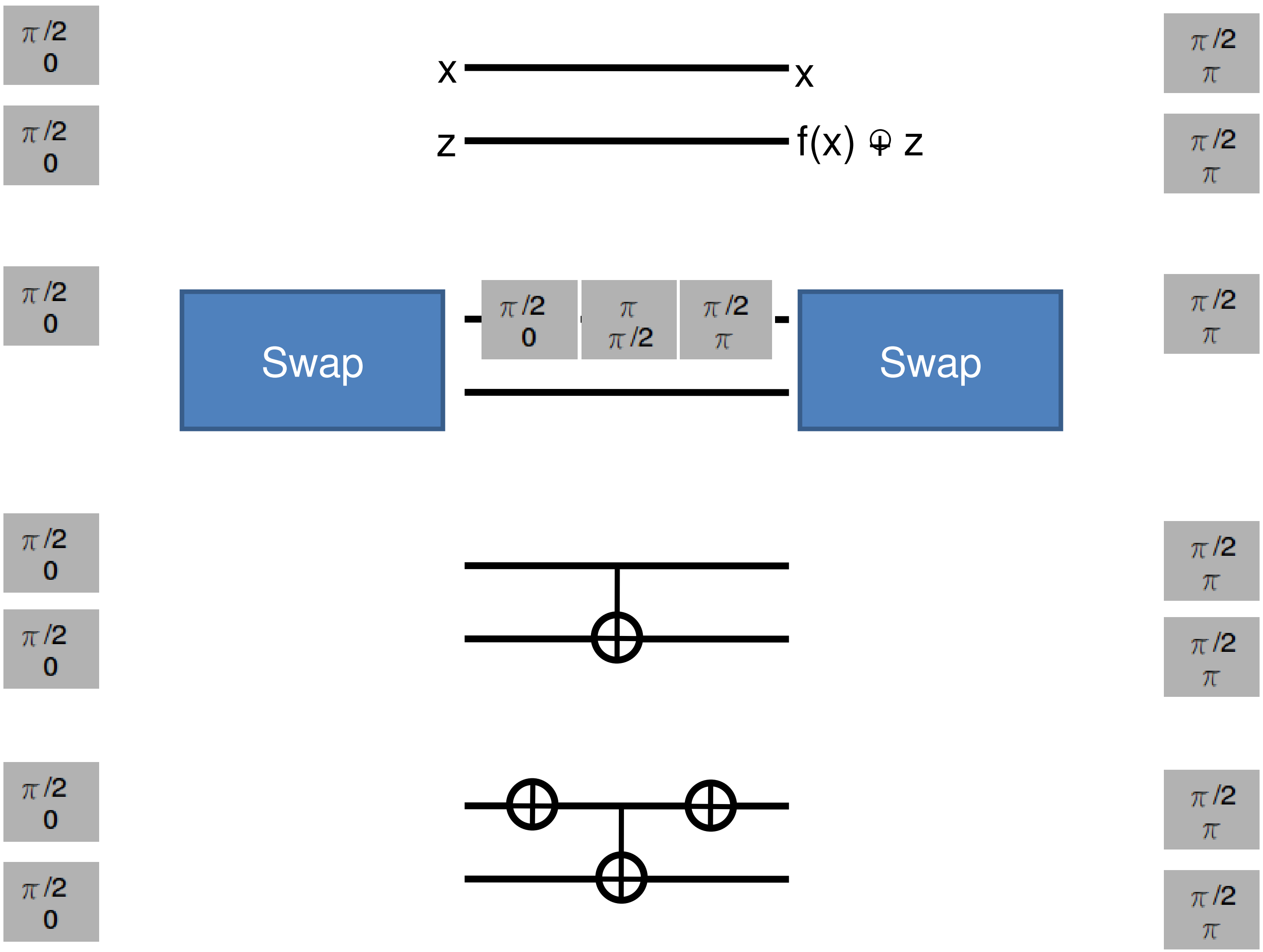
$$R^+\left(\frac{\pi}{\sqrt{2}},\pi\right) R^+\left(\frac{2\pi}{\sqrt{2}},\pi+\varphi_{\mathsf{swap}}\right) R^+\left(\frac{\pi}{\sqrt{2}},\pi\right)$$



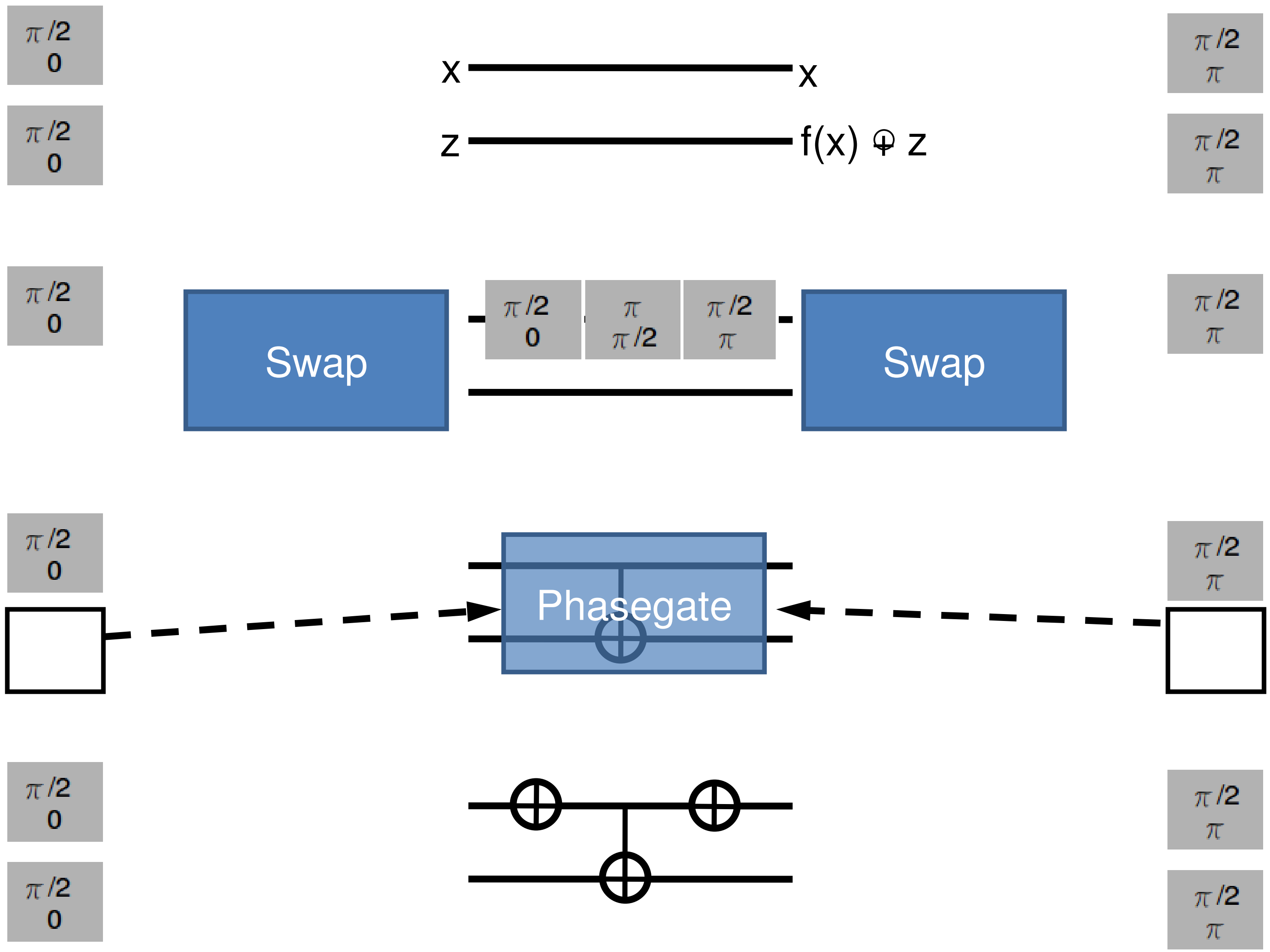


I. Chuang et al., Innsbruck (2002)

Deutsch Jozsa: Realization

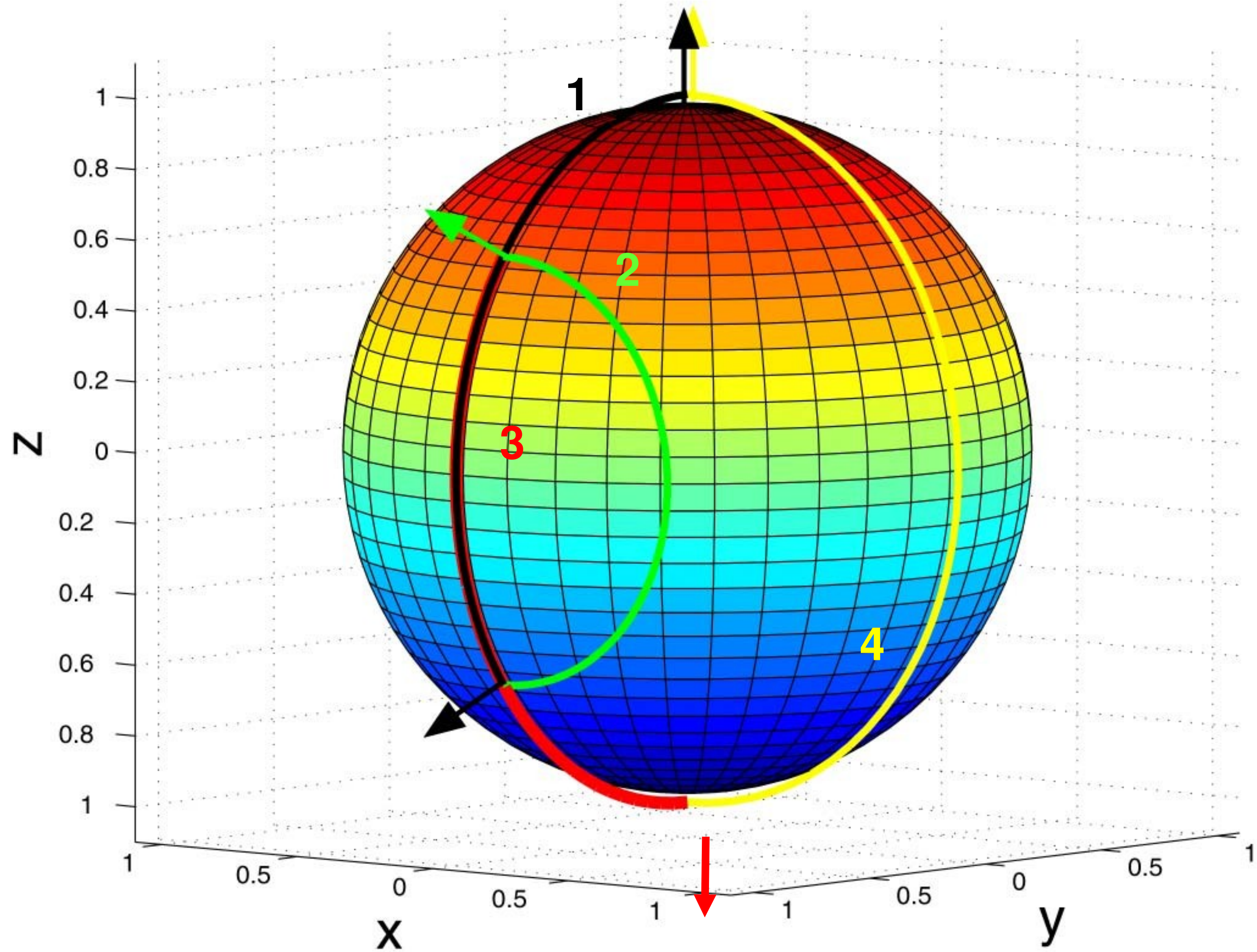


Deutsch Jozsa: Realization



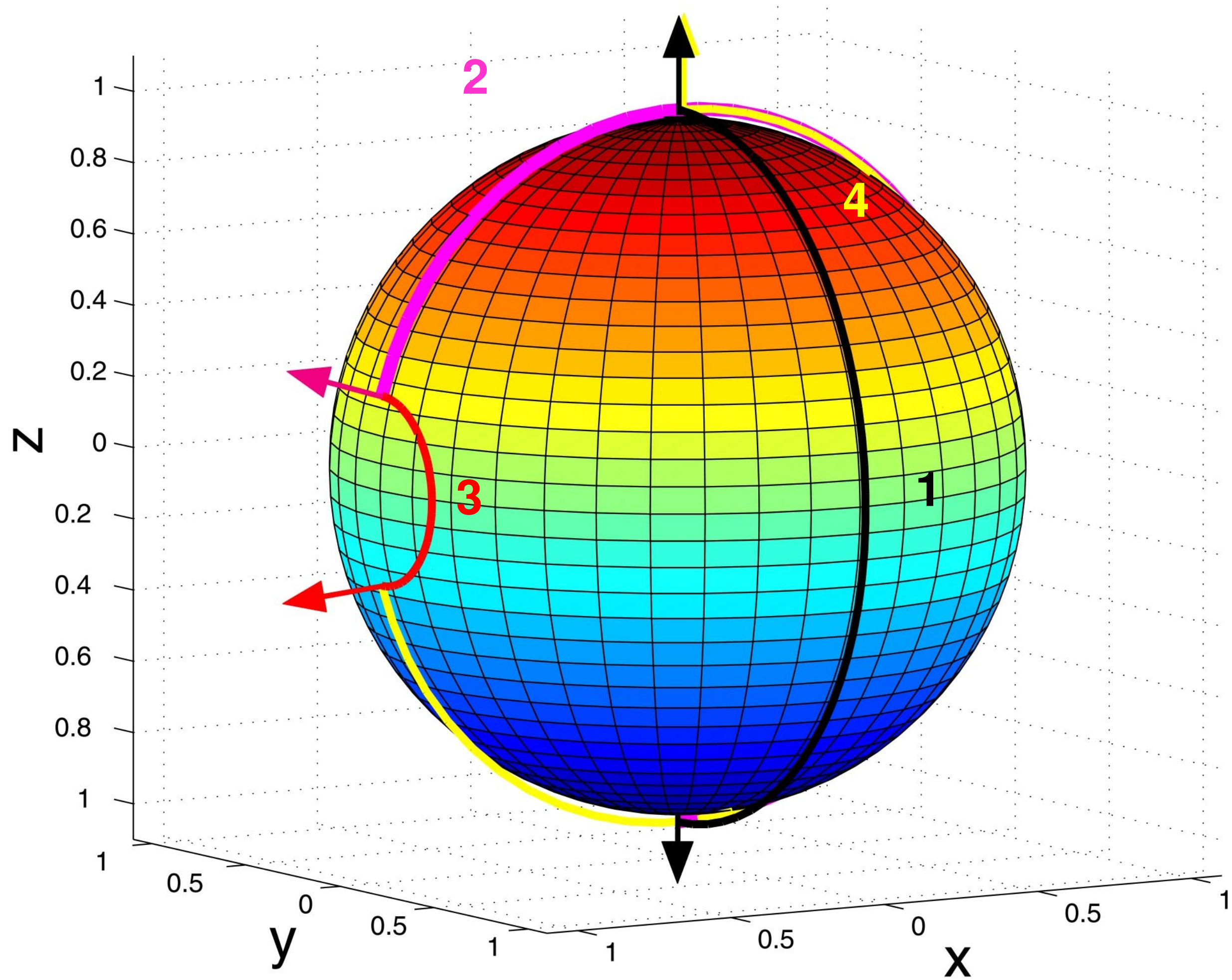
Composite phase gate (2π rotation)

$$R^{+}(\pi,\pi/2)\, R^{+}\left(\pi/\sqrt{2},0\right) R^{+}(\pi,\pi/2)\, R^{+}\left(\pi/\sqrt{2},0\right)$$

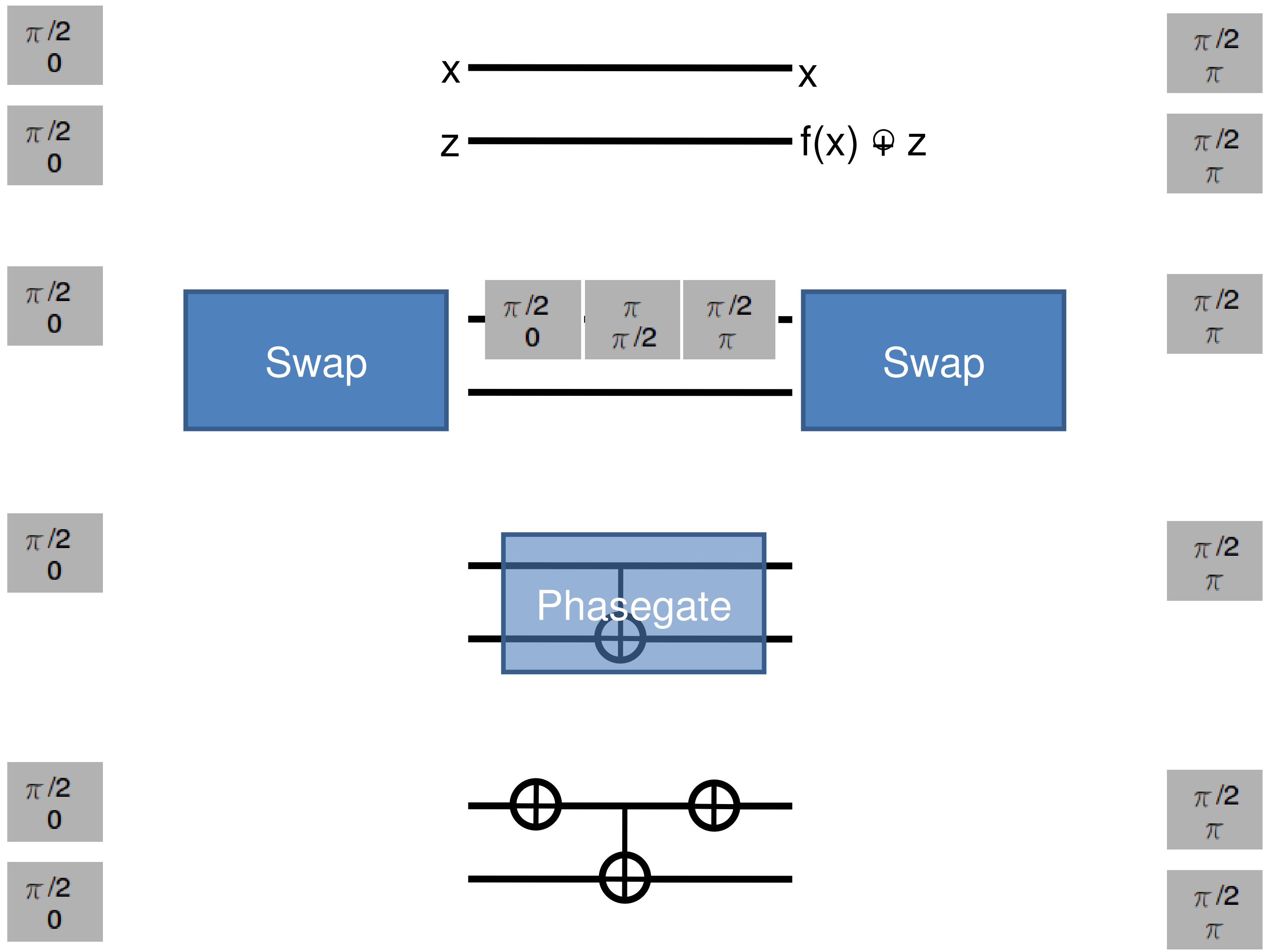


Action on |S,1> - |D,2>

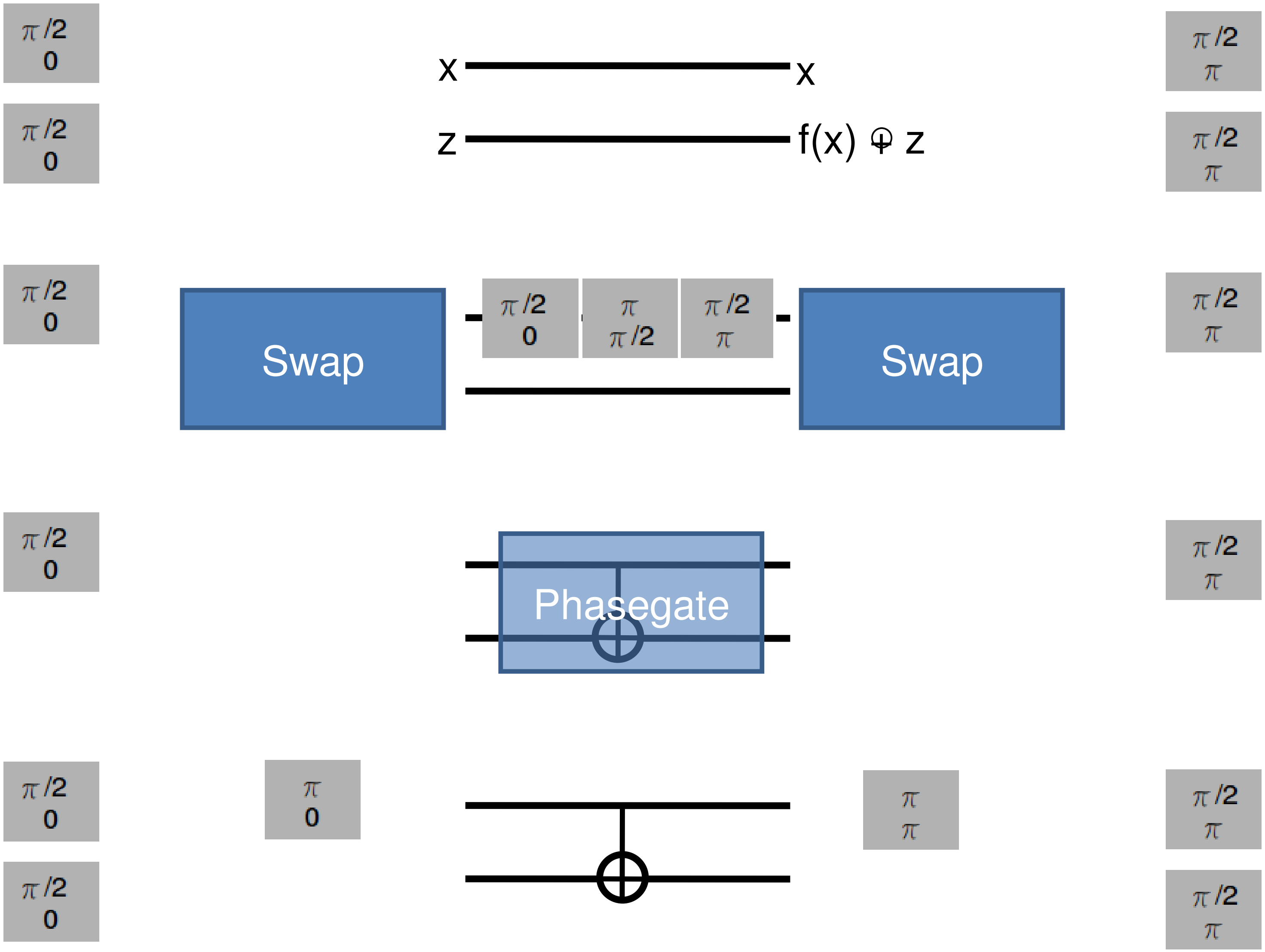
$$R^{+}(\pi,\pi/2)\,R^{+}\left(\pi/\sqrt{2},0\right)\,R^{+}(\pi,\pi/2)\,R^{+}\left(\pi/\sqrt{2},0\right)$$



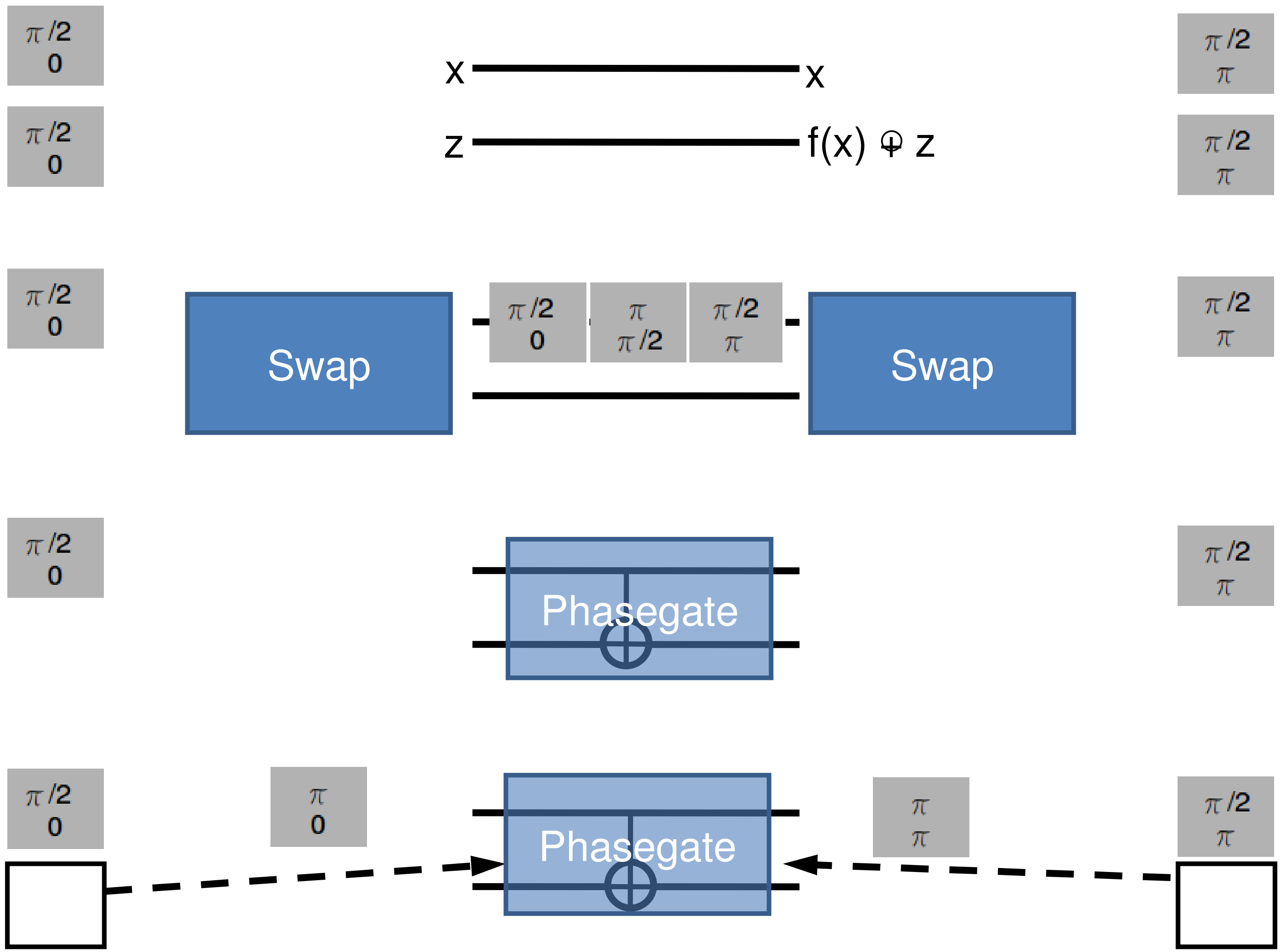
Deutsch Jozsa: Realization



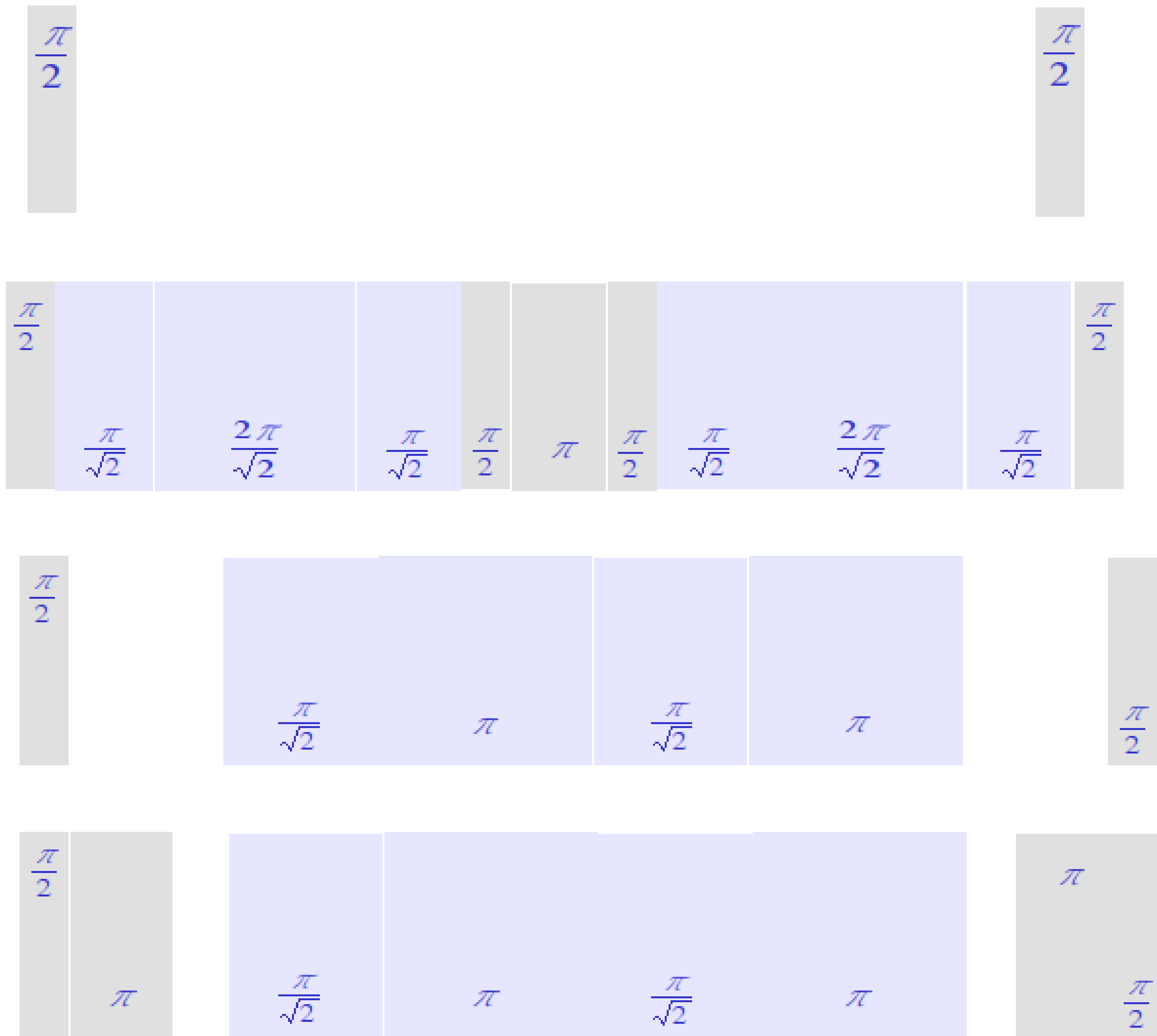
Deutsch Jozsa: Realization



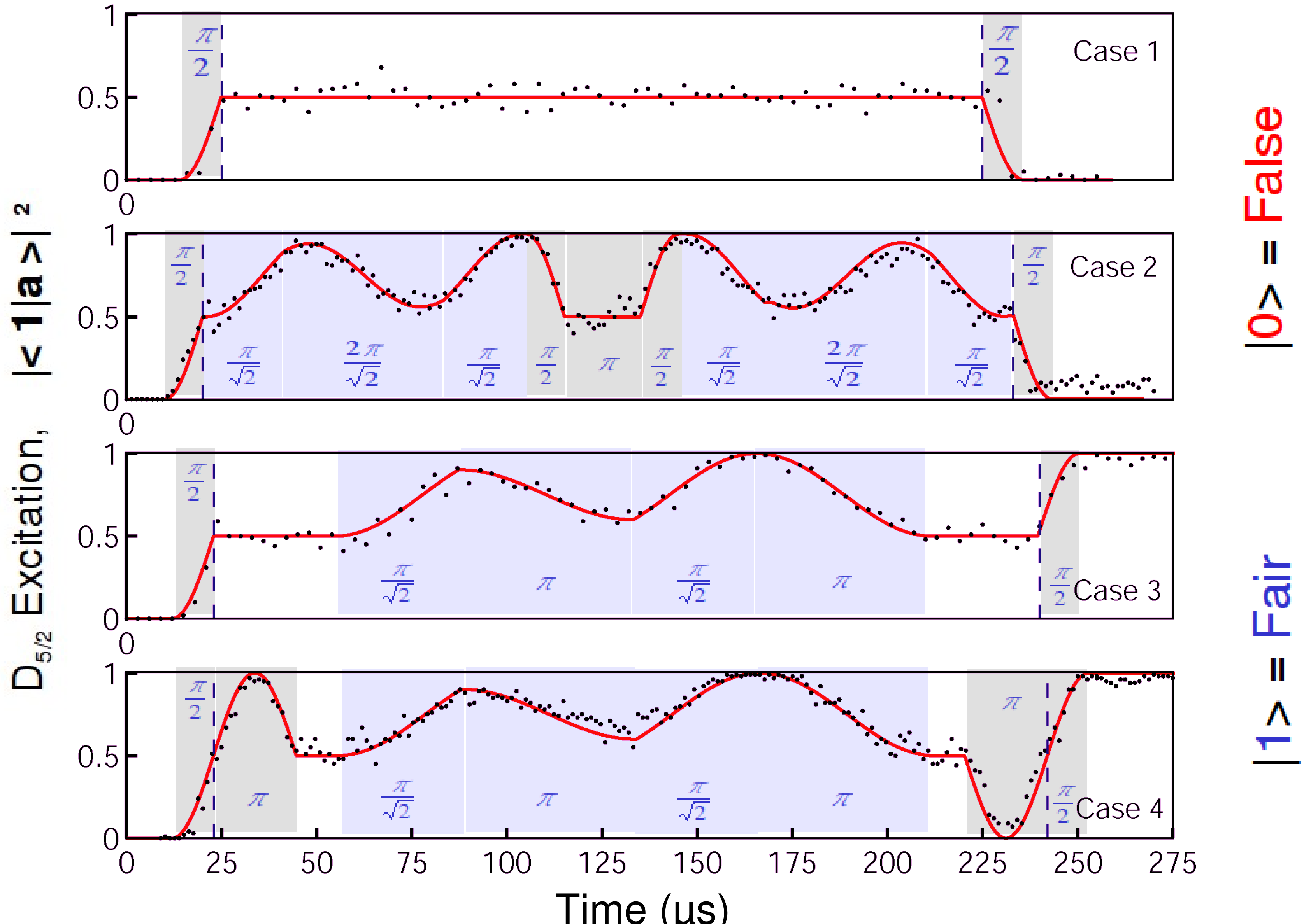
Deutsch Jozsa: Realization



Deutsch Jozsa: Realization



Deutsch Jozsa: Realization



Deutsch Jozsa: Result

	Constant		Balanced	
	Case 1	Case 2	Case 3	Case 4
expected / $\lvert\langle 1 \vert a \rangle\rvert^2$	0	0	1	1
measured / $\lvert\langle 1 \vert a \rangle\rvert^2$	0.019(6)	0.087(6)	0.975(4)	0.975(2)
expected / $\lvert\langle 1 \vert w \rangle\rvert^2$	1	1	1	1
measured / $\lvert\langle 1 \vert w \rangle\rvert^2$	--	0.90(1)	0.931(9)	0.986(4)

S. Gulde et al., Nature 412, 48 (2003)

Conclusions

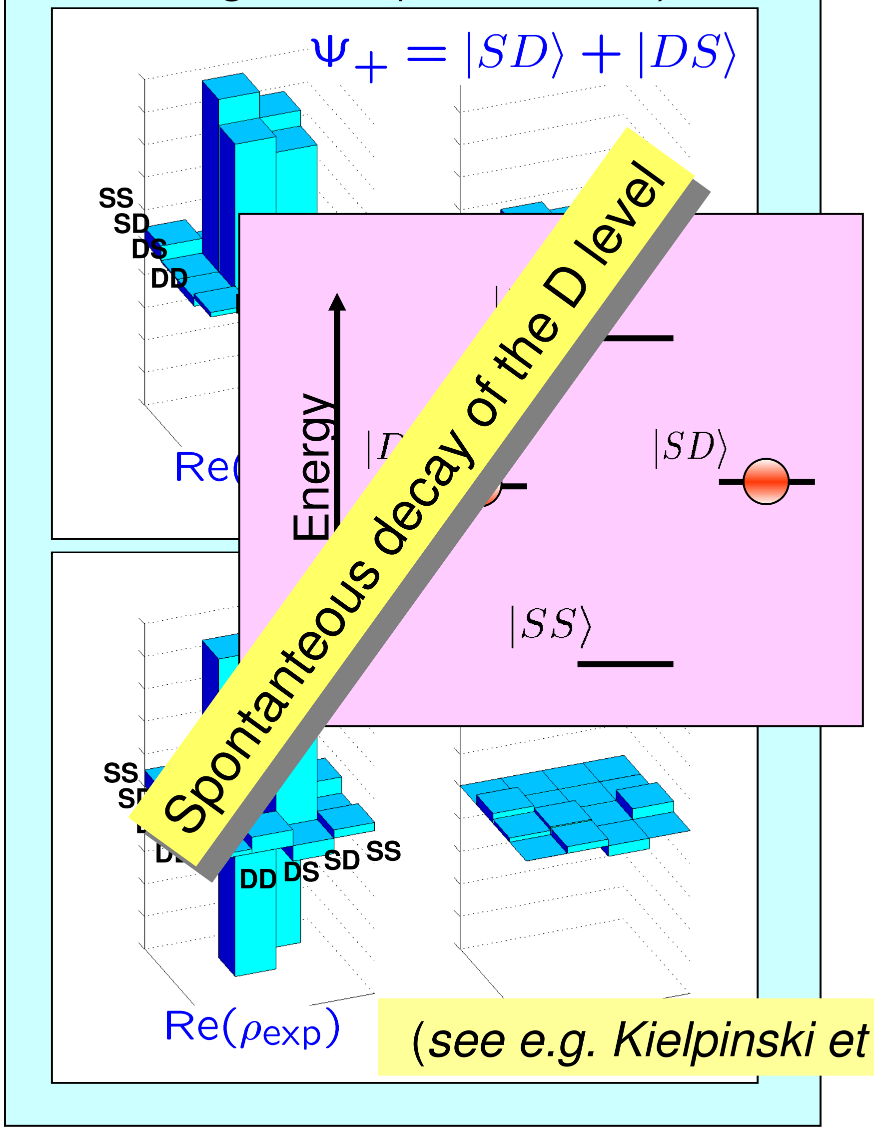
- Basics of ion trap quantum computing
- Measuring a density matrix
- Quantum gates
- Deutsch Algorithm

€ FWF SFB SCALA QGATES Industrie Tirol IQI GmbH

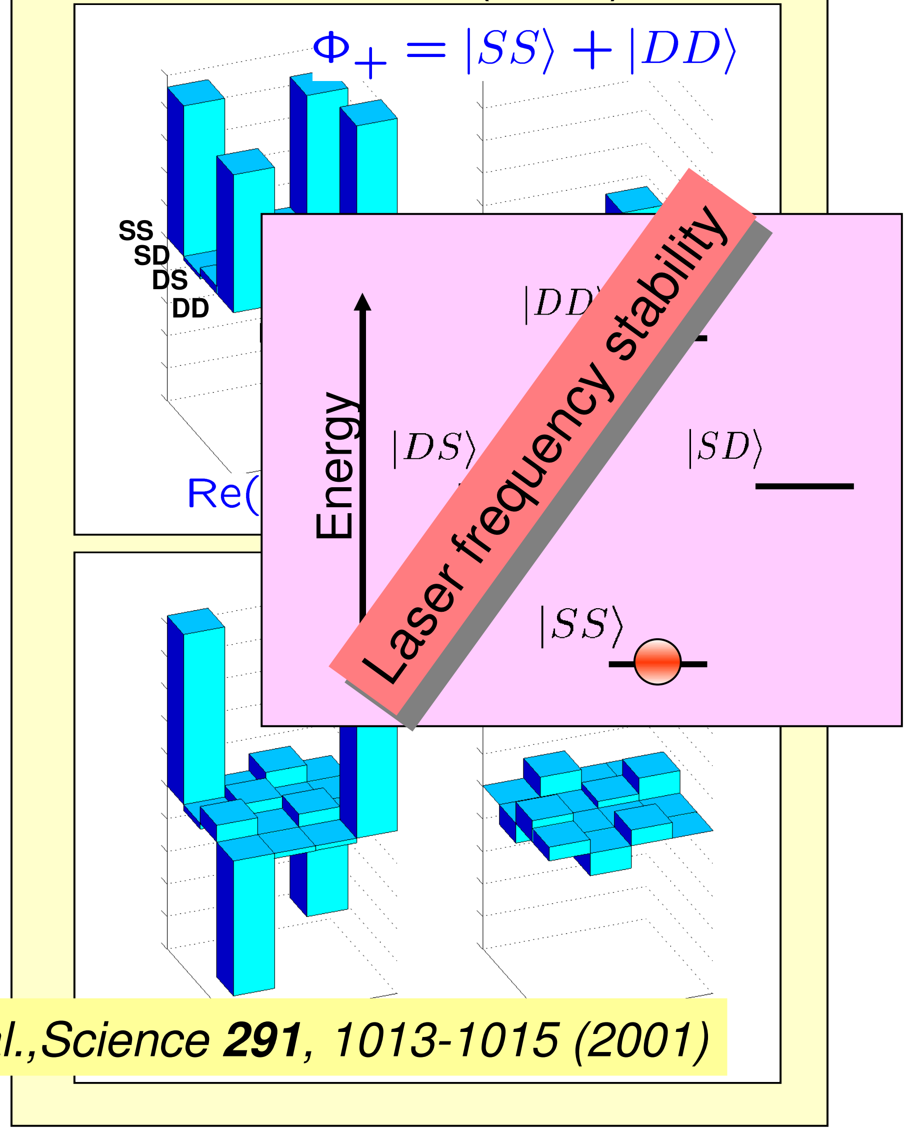
FWF bm:bwk $ IARPA

Decoherence properties of qubits

long lived (~ 1000 ms)



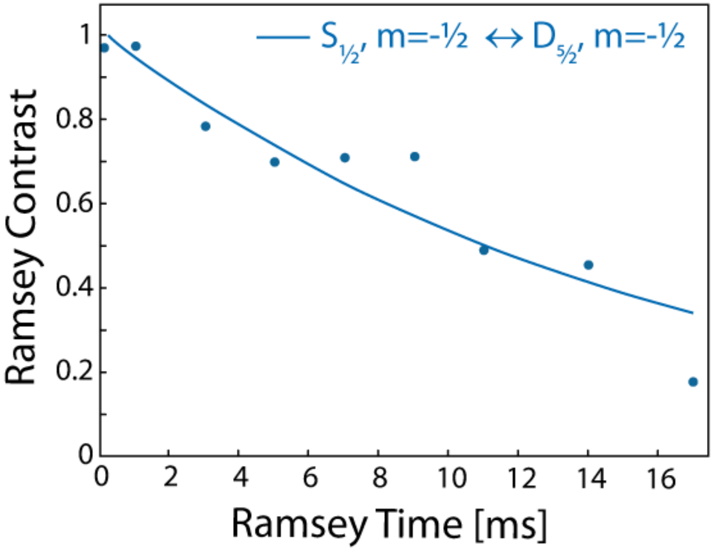
short lived (~ ms)



(*see e.g. Kielpinski et al.,Science **291**, 1013-1015 (2001)*)

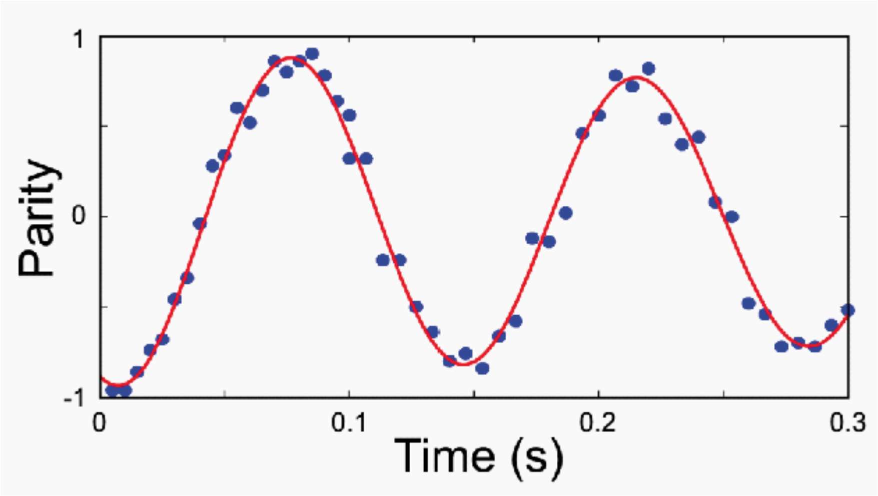
Memory performance of qubits

Physical Qubit
Ramsey Experiment



τ = 16 ms

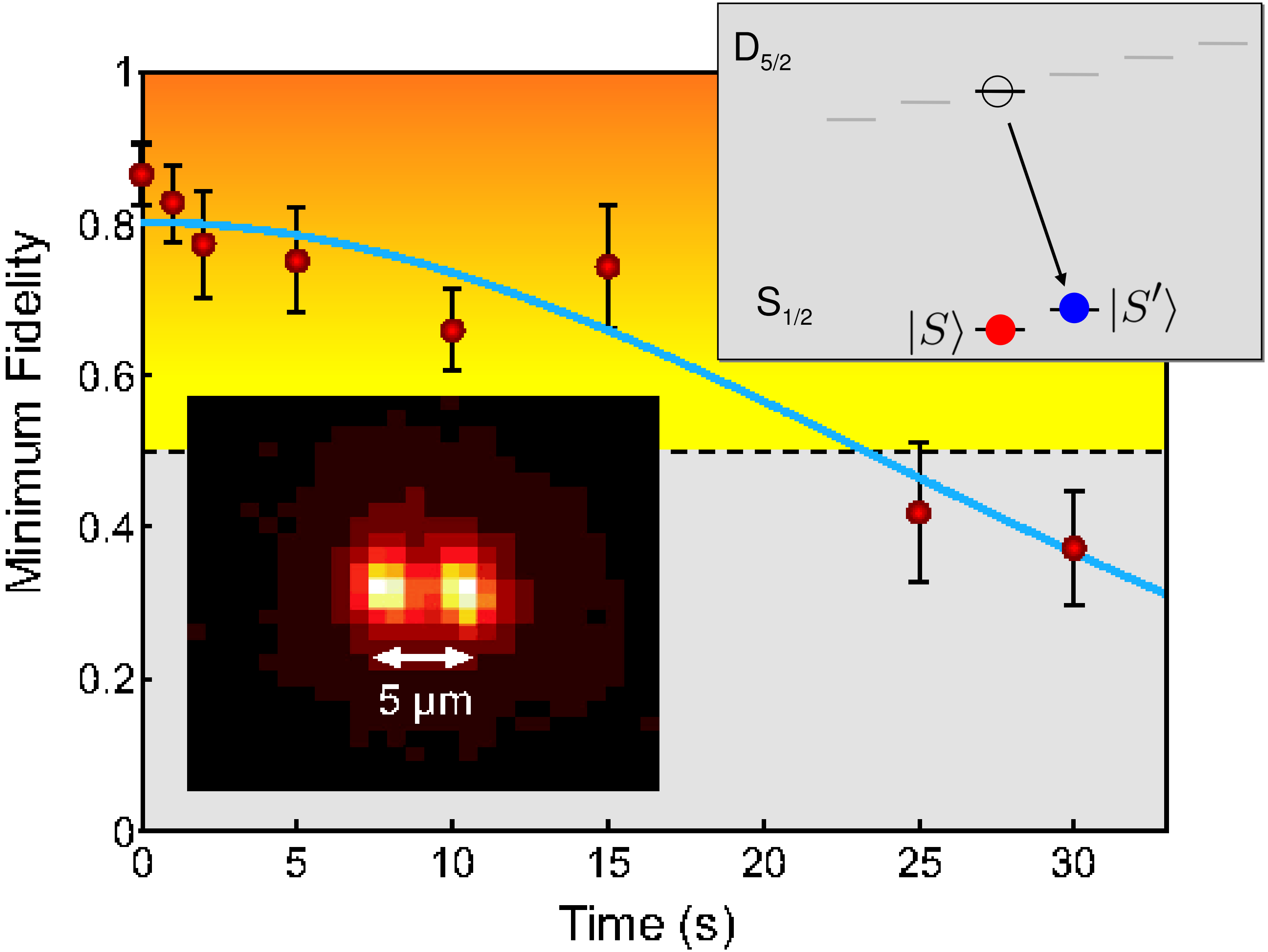
Logical Qubit
Parity Oscillations



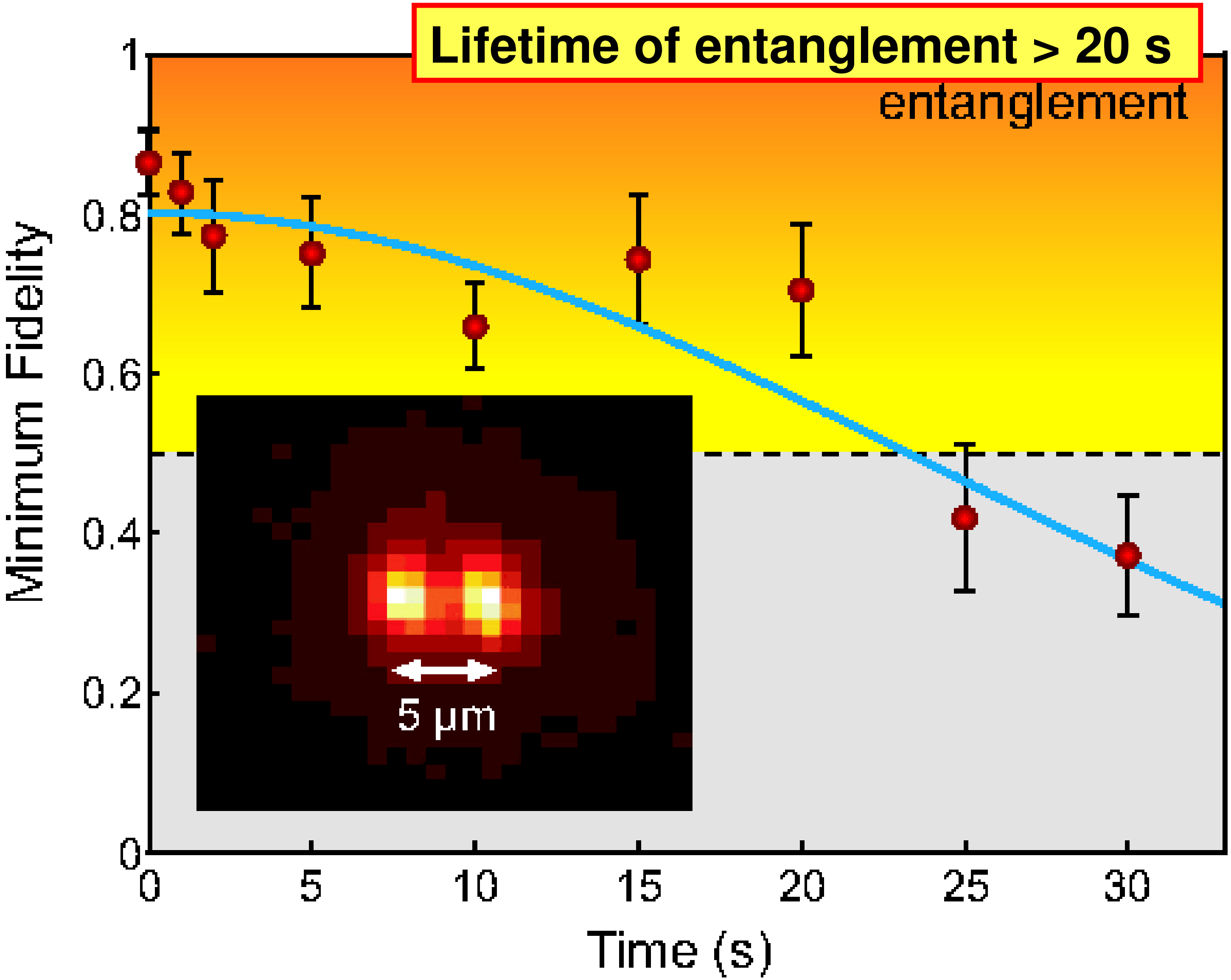
τ = 1050 ms

C.F. Roos et al., PRL 92, 220402 (2004).

Ultra long lived entanglement



Ultra long lived entanglement



H. Häffner et al., App. Phys. B **81** 151 (2005).

Realized time scales

Single qubit gates

10^{-6} s

Two qubit gates (Geometric phase gates: Boulder, Innsbruck)

10^{-5} s

10^{-4} s

Two qubit gates (Cirac-Zoller approach)

10^{-3} s

Single qubit coherence (magnetic field sensitive)

10^{-2} s

Coherence of the motion (Innsbruck)

10^{-1} s

10^{0} s

10^{1} s

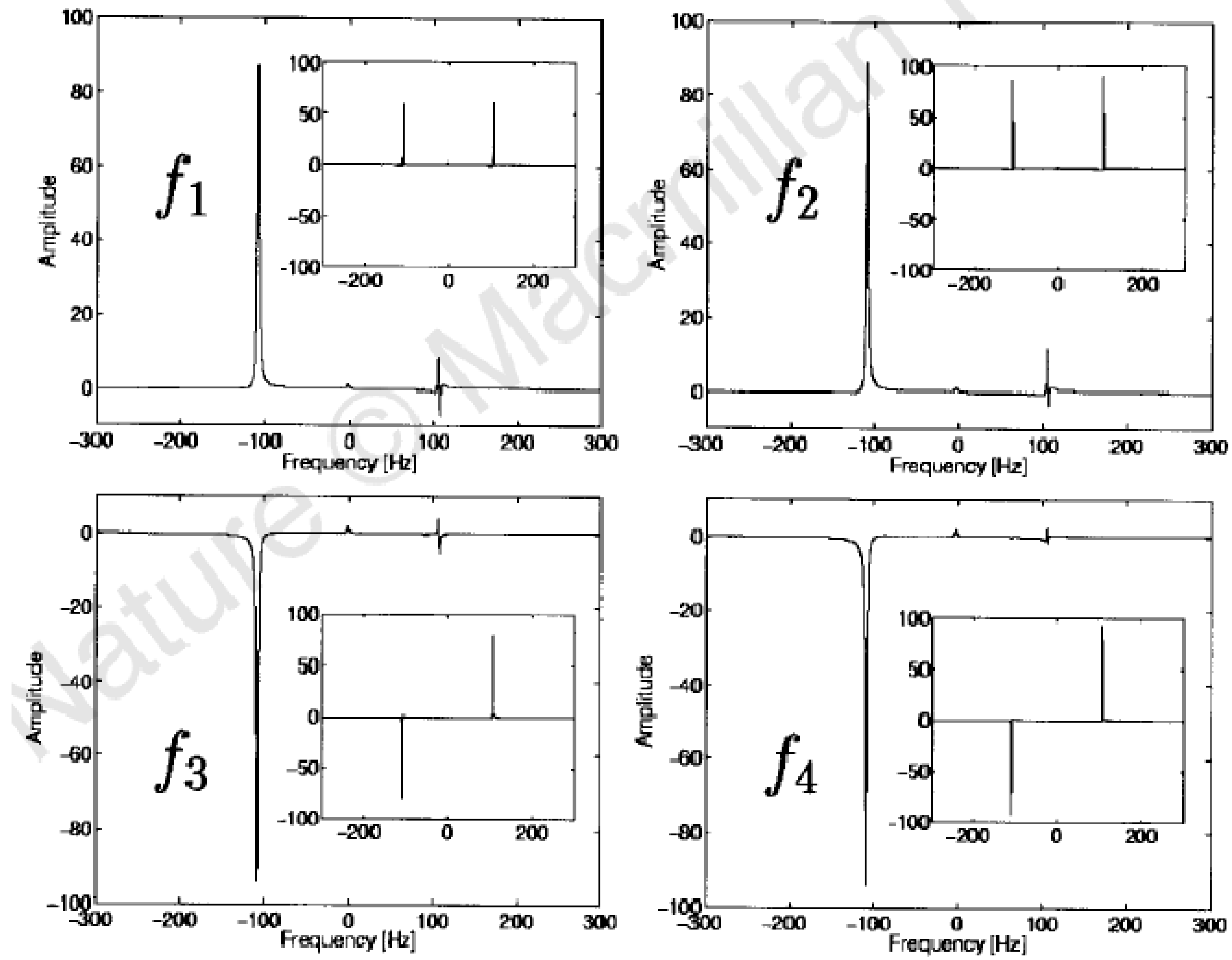
Single qubit coherence (magnetic field insensitive, Boulder)
Decoherence free subspaces

10^{2} s

10^{3} s

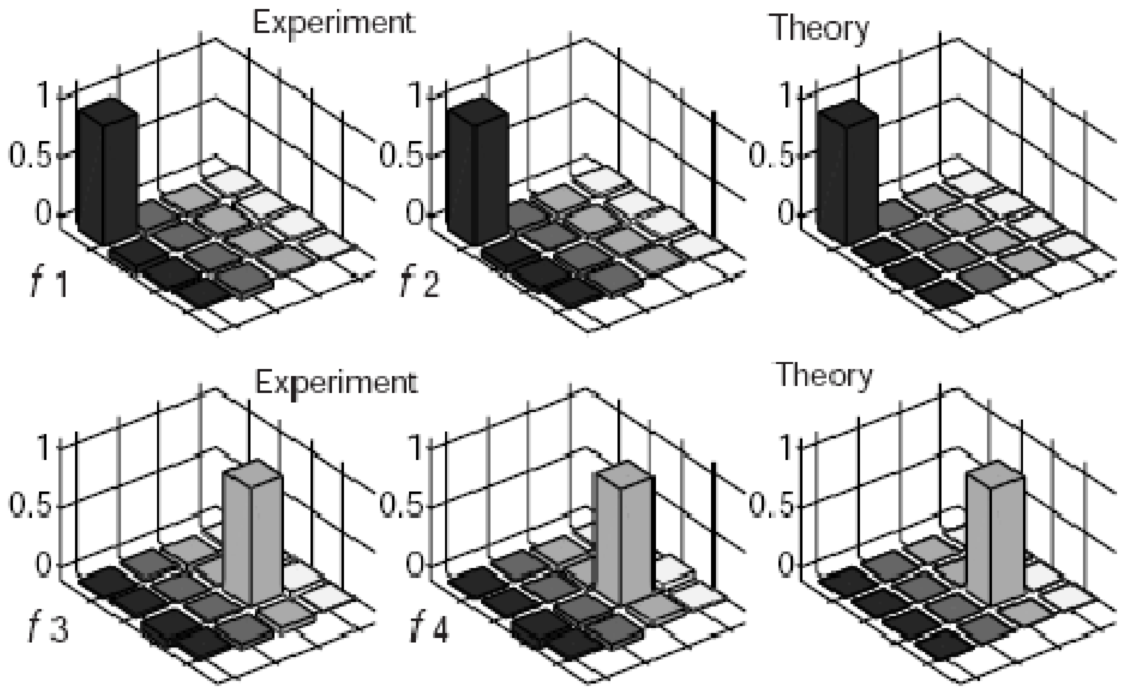
Well chosen single qubit coherence (Boulder)

NMR



Molecule used: Chloroform: $CHCl_3$,
qubits: the ^{13}C (125 MHz) and the ^{13}H (500 MHz)-nuclei,
spin-spin coupling: 200 Hz.

NMR



Fiber optics implementation

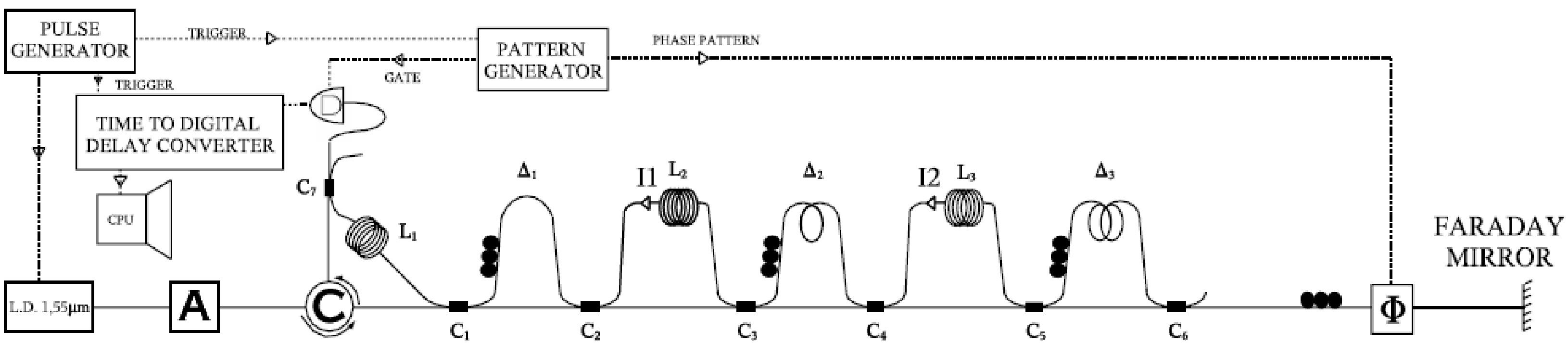
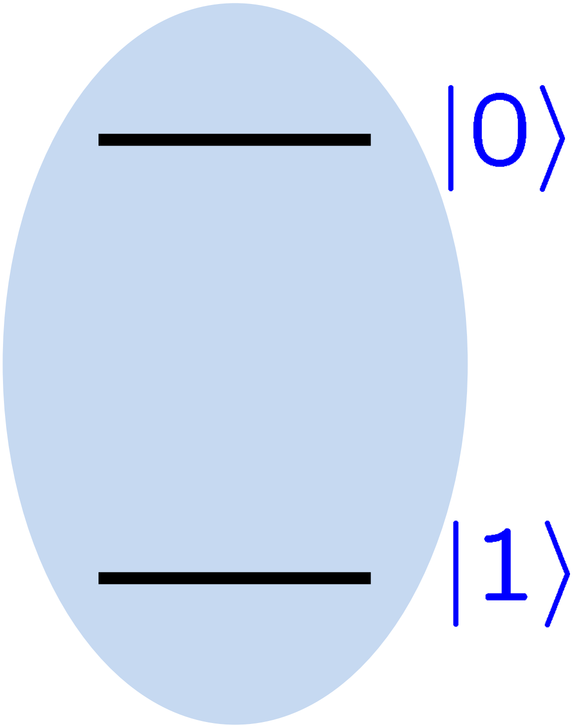


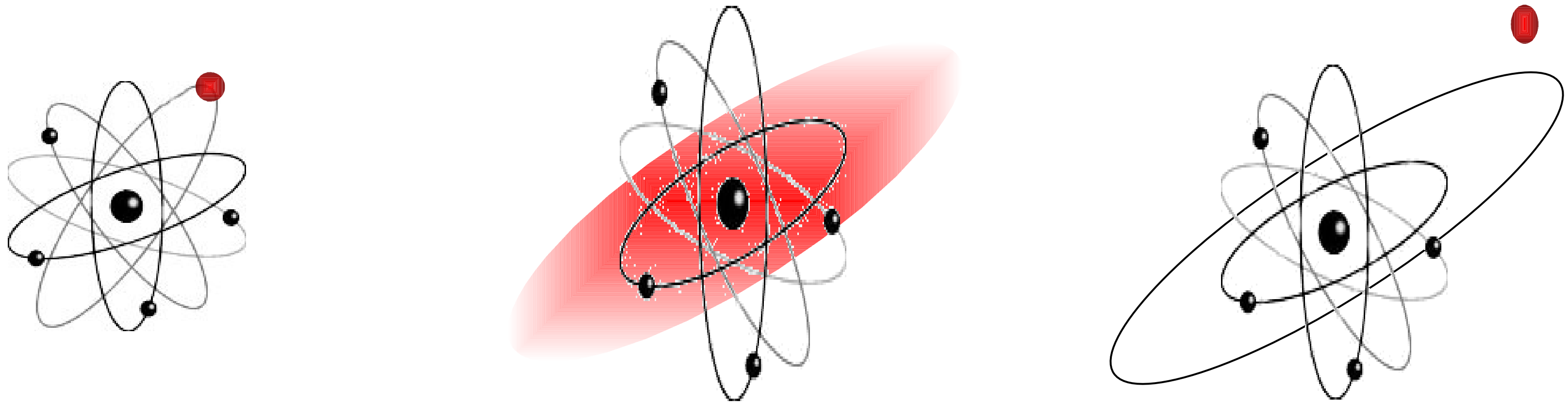
FIG. 1. Fiber optics setup implementing the eight-dimensional Deutsch-Jozsa and Bernstein-Vazirani quantum algorithms.

A quantum bit

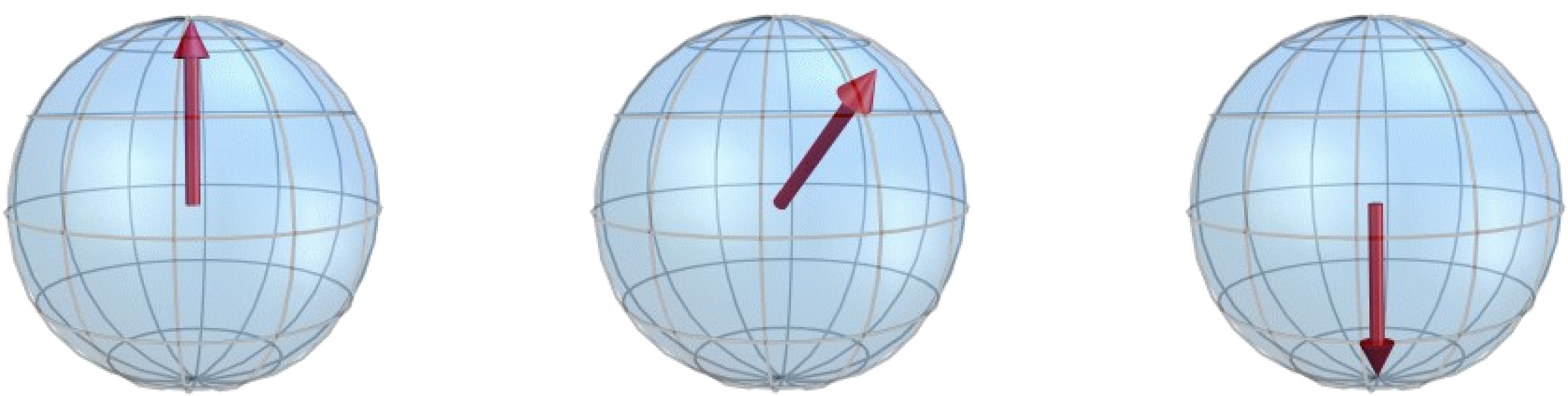
Two level system:



Qubits



$$|0\rangle \qquad \alpha|0\rangle + \beta|1\rangle \qquad |1\rangle$$



Possible qubit encodings

Physical Qubit $\longleftrightarrow$ **Logical Qubit**

$$|0\rangle_P = |D\rangle \qquad |0\rangle_L = |SD\rangle$$

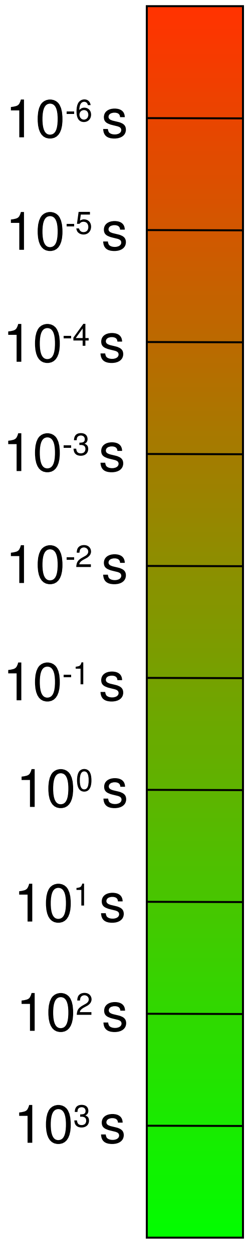
$$|1\rangle_P = |S\rangle \qquad |1\rangle_L = |DS\rangle$$

Effect of magnetic field or laser frequency fluctuations on qubits

$$\begin{array}{ccc} |D\rangle + |S\rangle & & |SD\rangle + |DS\rangle \\ \Downarrow & \longleftrightarrow & \Downarrow \\ e^{i\phi}|D\rangle + |S\rangle & & e^{i\phi}(|SD\rangle + |DS\rangle) \end{array}$$

Logical qubit experiences global phase only

Realized time scales



Single qubit gates

Two qubit gates (Geometric phase gates: Boulder, Innsbruck)

Two qubit gates (Cirac-Zoller approach)

Single qubit coherence (magnetic field sensitive)

Coherence of the motion (Innsbruck)

Single qubit coherence (magnetic field insensitive)

Decoherence free subspaces

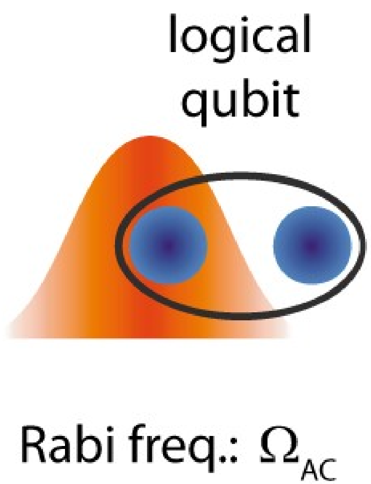
Well chosen single qubit (Boulder)

Universal set of gates in a DFS

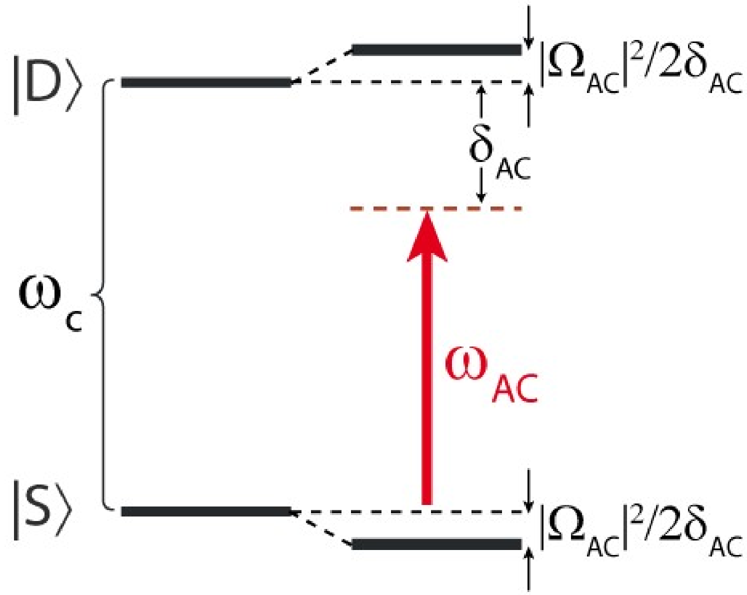
- single qubit operations
  - Z gates
  - X gates

- two –qubit operations
  - phase gate

Z gate

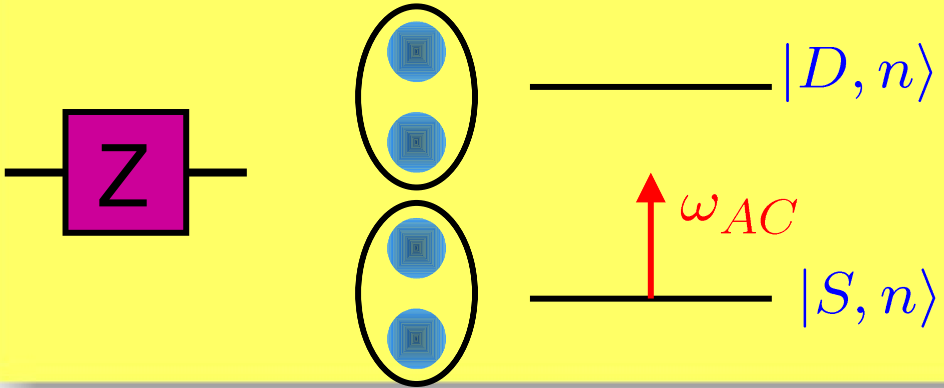


logical
qubit
Rabi freq.: Ω_{AC}

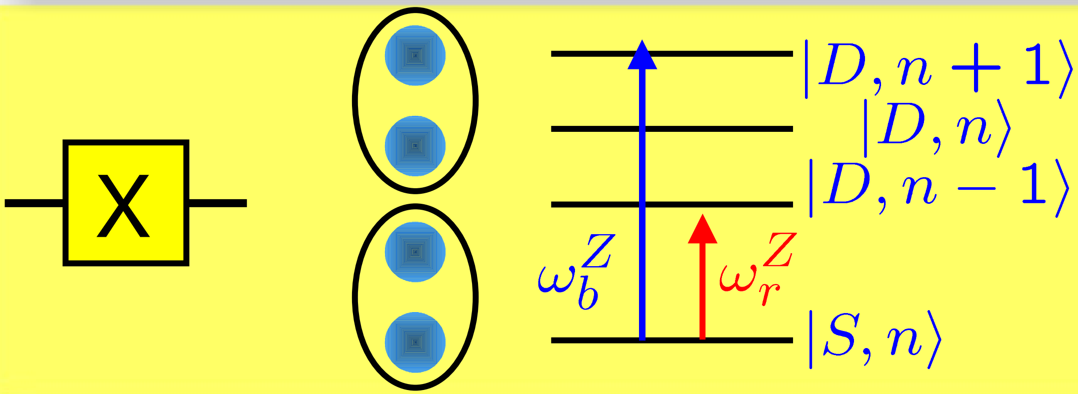


$|D\rangle$
$|\Omega_{AC}|^2/2\delta_{AC}$
δ_{AC}
ω_c
ω_{AC}
$|S\rangle$
$|\Omega_{AC}|^2/2\delta_{AC}$

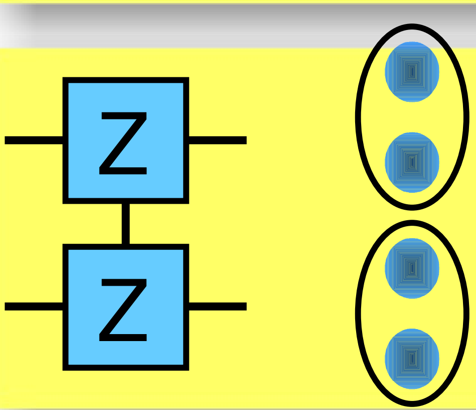
Universal set of gates in a DFS



single qubit phaseshift by AC Stark shifts



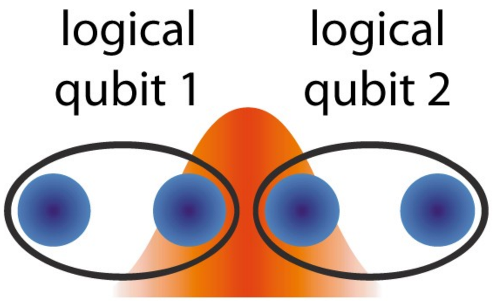
$$\omega_b^{MS} = \omega_c + \nu + \delta_{MS}$$
$$\omega_r^{MS} = \omega_c - \nu - \delta_{MS}$$



?

Two logical qubit gate

Two body interactions preferred:



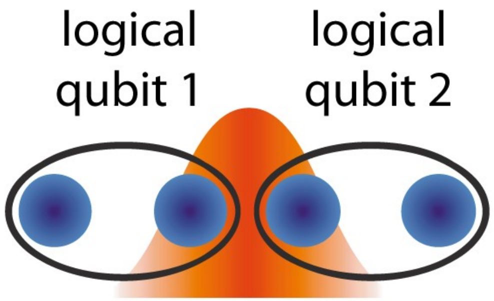
Most interactions cause the state to leave the decoherence free subspace.

Some solutions: L. Aolita et al., PRA **75** 052337 (2007)

Two logical qubit phase gate

Action of the phase gate on two physical qubits:

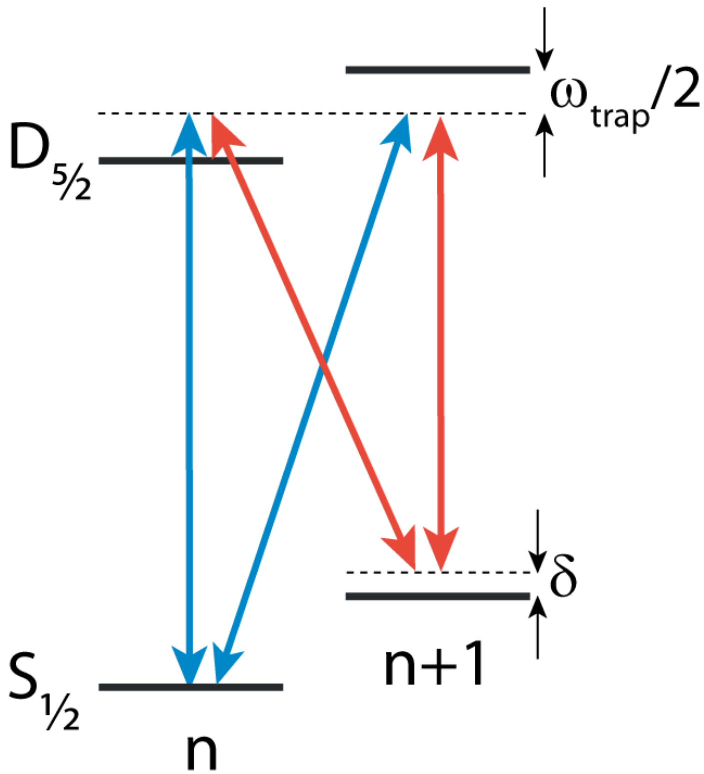
$$
\begin{array}{c} |DD\rangle \\ |DS\rangle \\ |SD\rangle \\ |SS\rangle \end{array}
\Rightarrow
\begin{array}{c} e^{i\phi}|DD\rangle \\ |DS\rangle \\ |SD\rangle \\ e^{i\phi}|SS\rangle \end{array}
$$

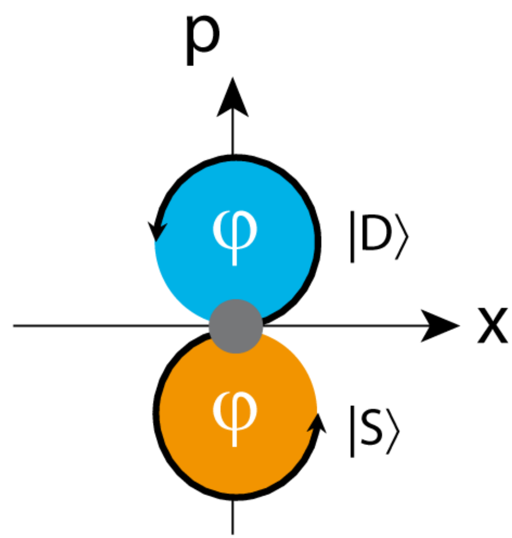


...and on the logical qubits:

$$
\begin{array}{c} |00\rangle_l \\ |01\rangle_l \\ |10\rangle_l \\ |11\rangle_l \end{array}
=
\begin{array}{c} |S\rangle|DS\rangle|D\rangle \\ |S\rangle|DD\rangle|S\rangle \\ |D\rangle|SS\rangle|D\rangle \\ |D\rangle|SD\rangle|S\rangle \end{array}
\Rightarrow
\begin{array}{c} |S\rangle|DS\rangle|D\rangle \\ |S\rangle e^{i\phi}|DD\rangle|S\rangle \\ |D\rangle e^{i\phi}|SS\rangle|D\rangle \\ |D\rangle|SD\rangle|S\rangle \end{array}
=
\begin{array}{c} |00\rangle_l \\ e^{i\phi}|01\rangle_l \\ e^{i\phi}|10\rangle_l \\ |11\rangle_l \end{array}
$$

Two qubit phase gate



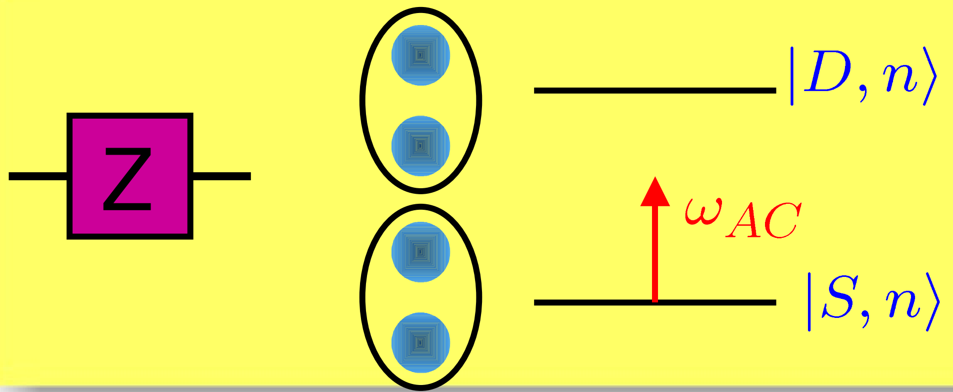


Obtained phase ~ area

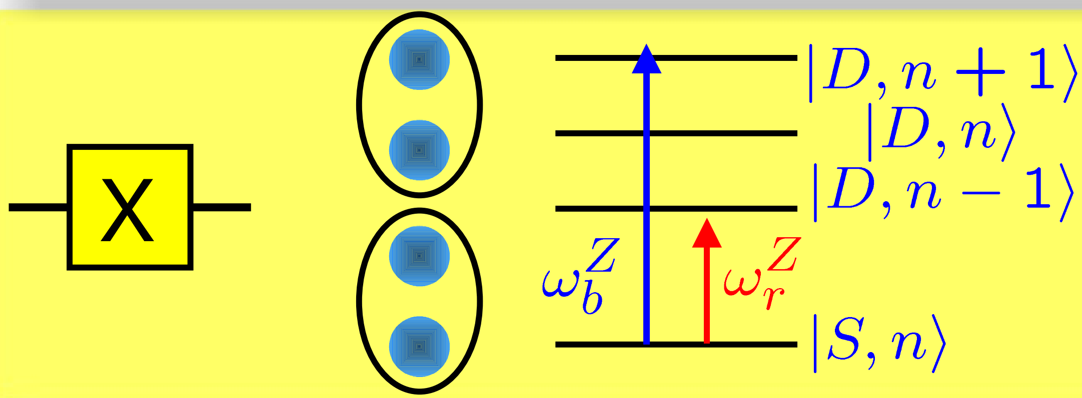
D. Leibfried, et al., Nature **422** 412 (2003)

K. Kim et. al., Phys. Rev. A**77**, 050303 (2008)

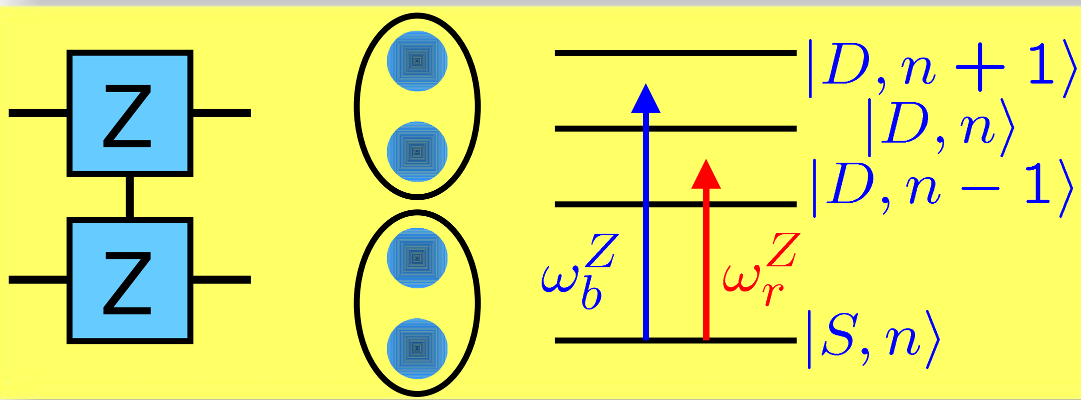
Universal set of gates in a DFS



single qubit phaseshift by AC Stark shifts

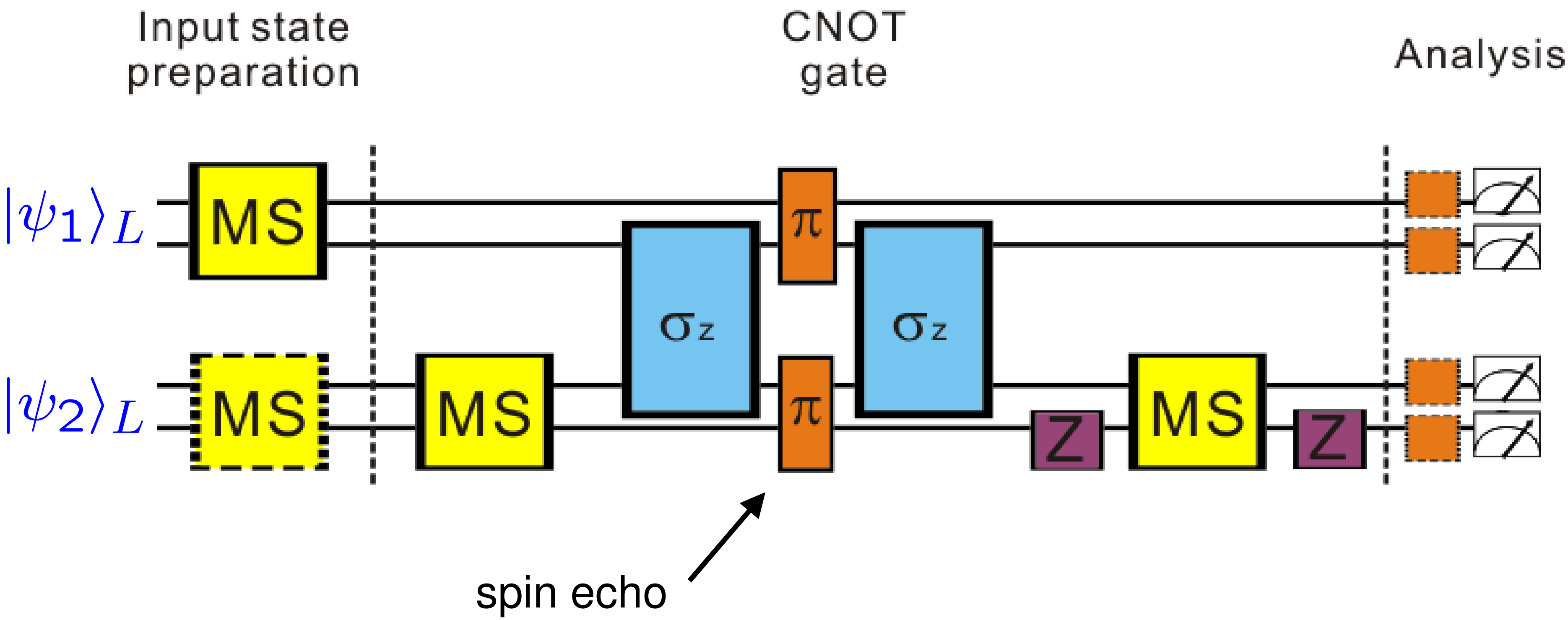


$$\omega_b^{MS} = \omega_c + \nu + \delta_{MS}$$
$$\omega_r^{MS} = \omega_c - \nu - \delta_{MS}$$

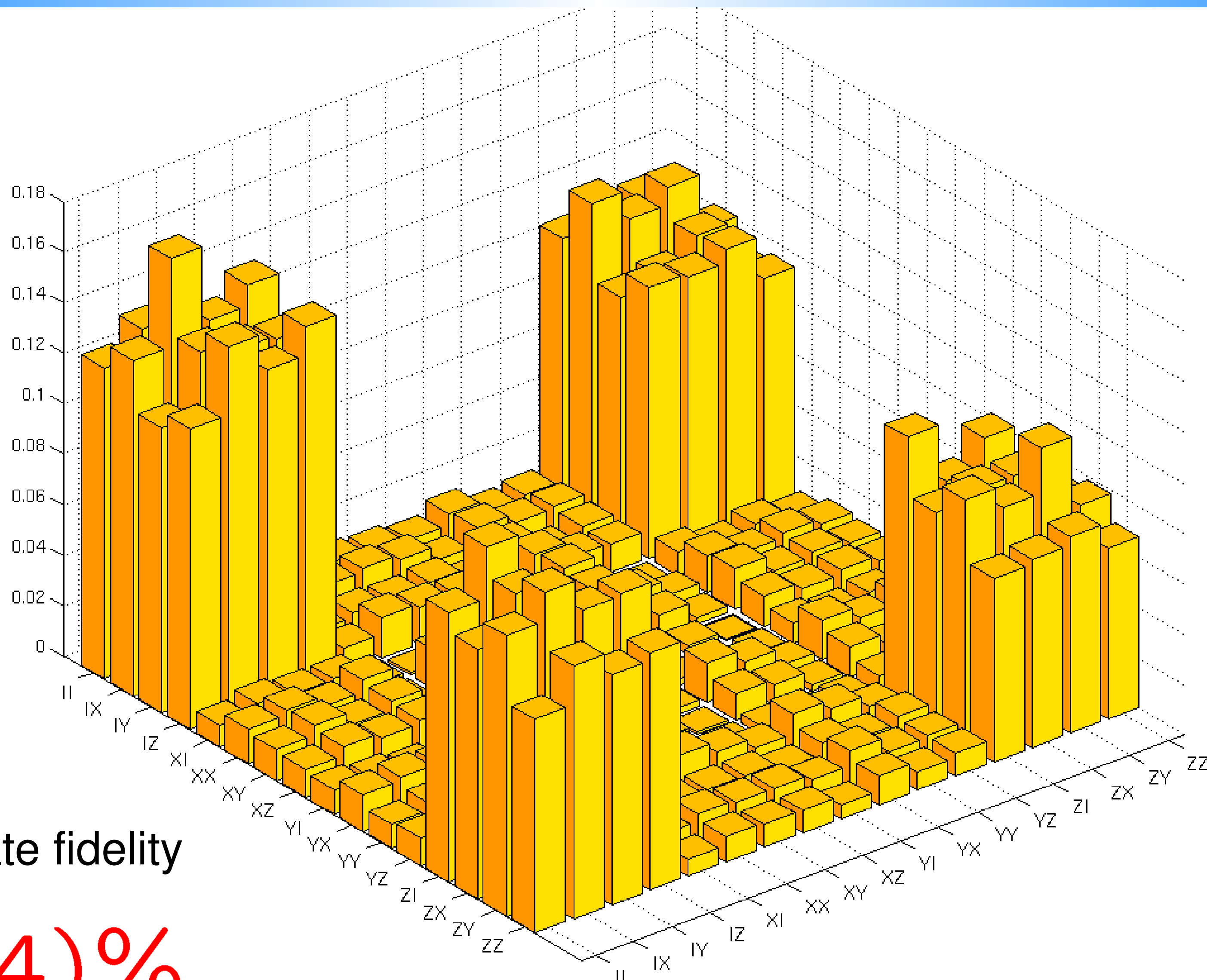


$$\omega_b^Z = \omega_c + \nu/2 + \delta_z$$
$$\omega_r^Z = \omega_c - \nu/2 - \delta_z$$

A CNOT in a DFS



Process tomography of the CNOT



mean gate fidelity

89(4)%

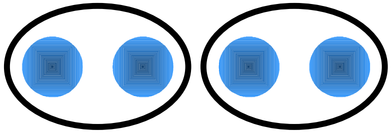
Discussion

mean gate fidelity: 89(4)%

(after DFS postselection)

Main limitations:

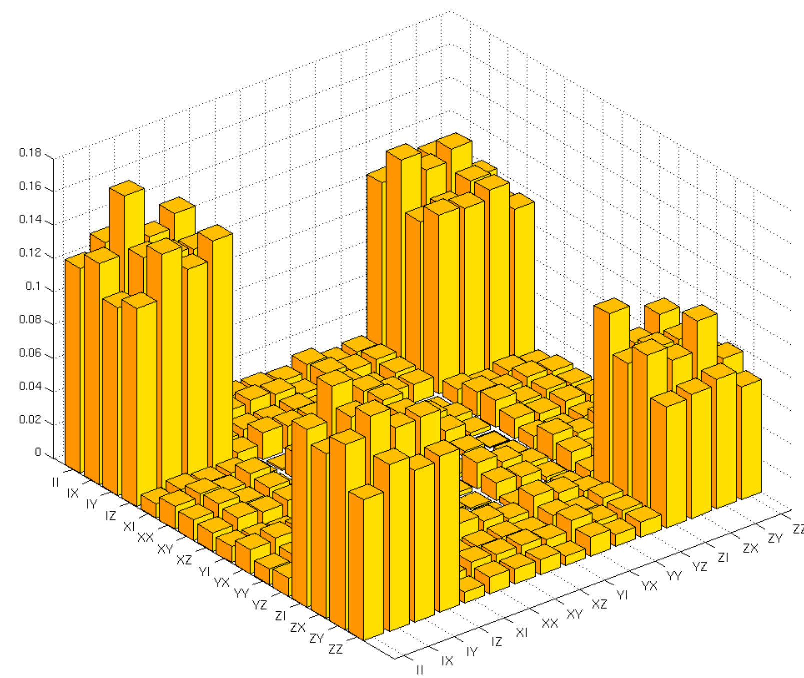
- spurious laser frequency components
- off-resonant coupling to other levels
- intensity stability on ions
- addressing errors



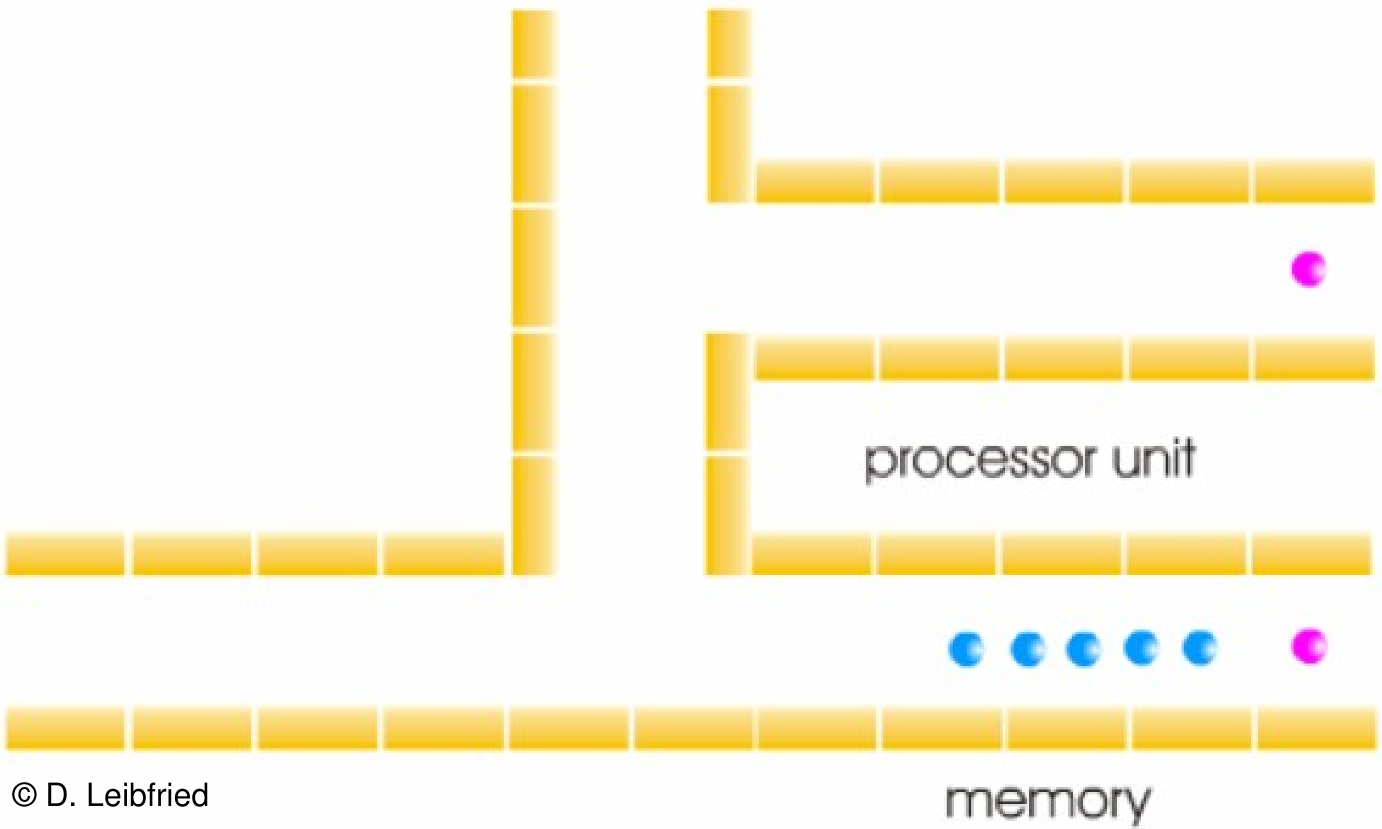
Discussion

Advantages:

- lifetime limited coherence time
- insensitive to laser linewidth
- insensitive to AC-Stark shifts



Scaling of ion trap quantum computers



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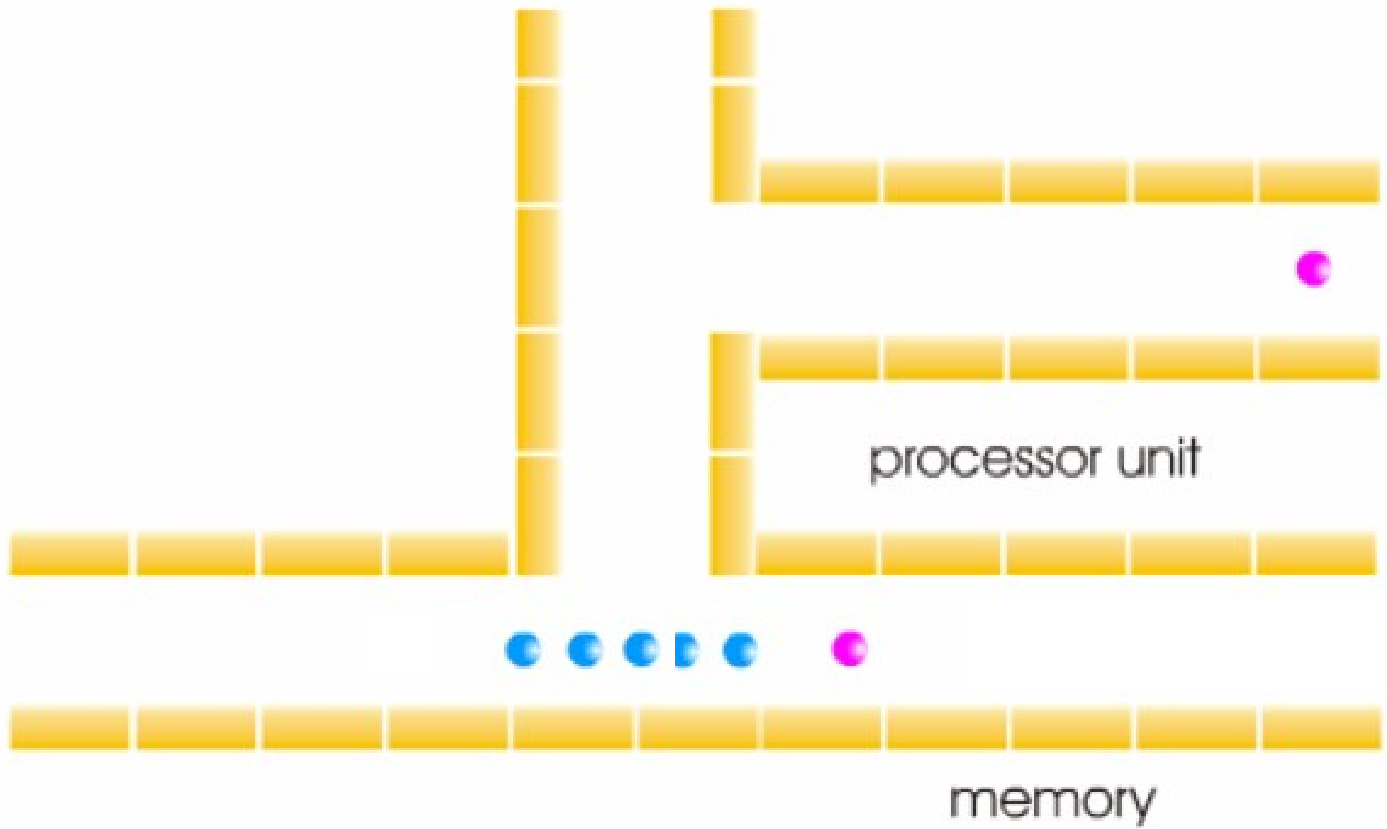
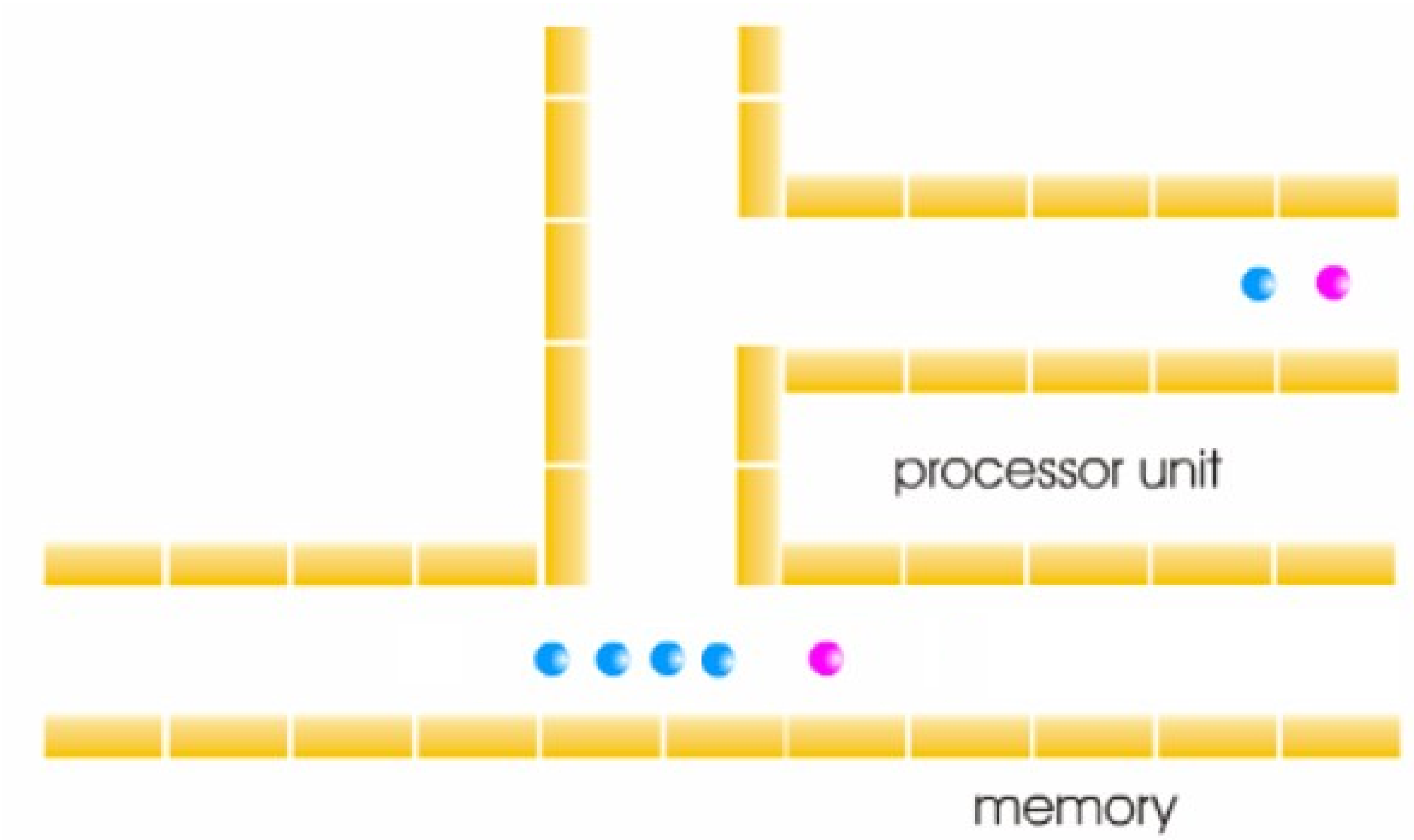
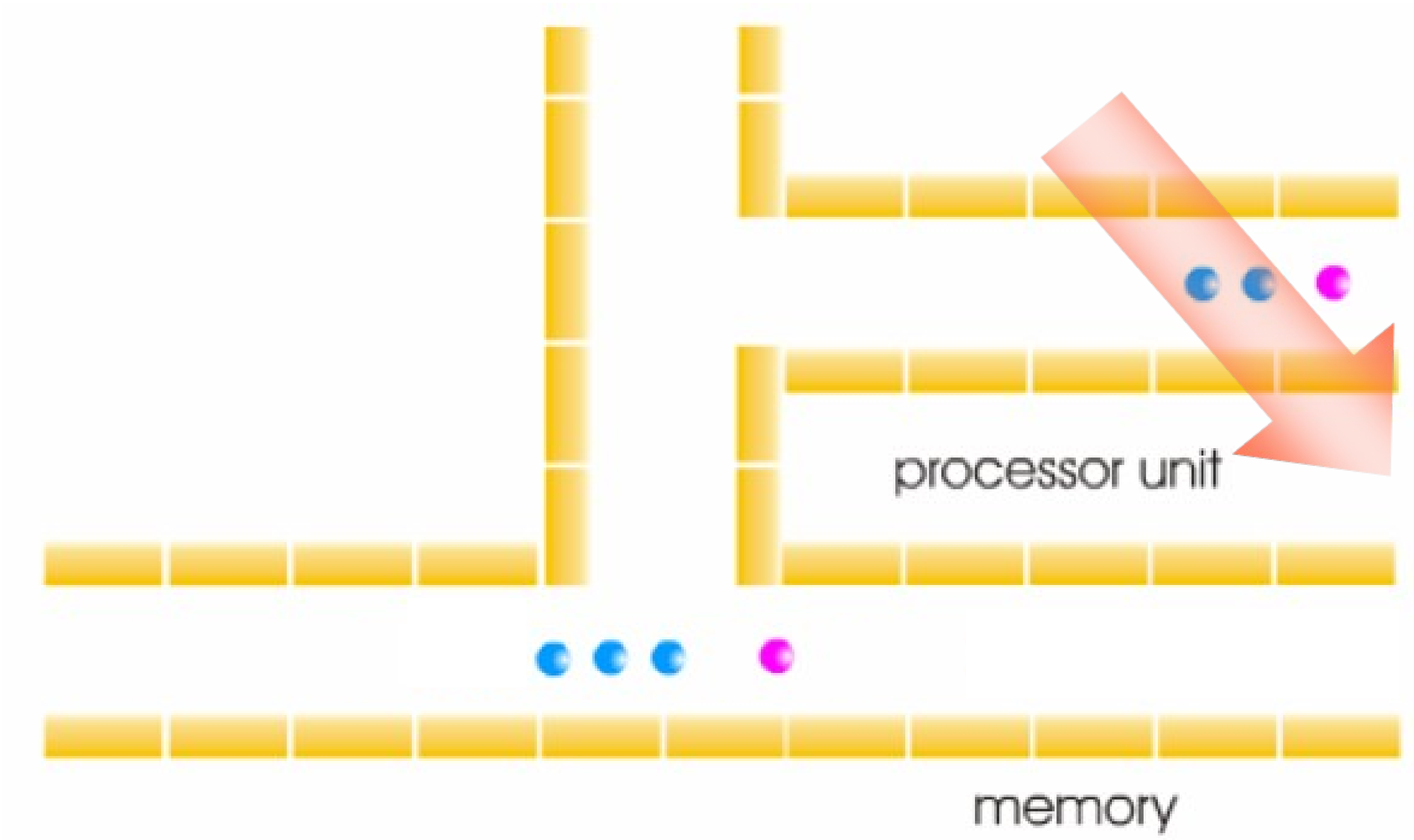
Scaling of ion trap quantum computers



Scaling of ion trap quantum computers

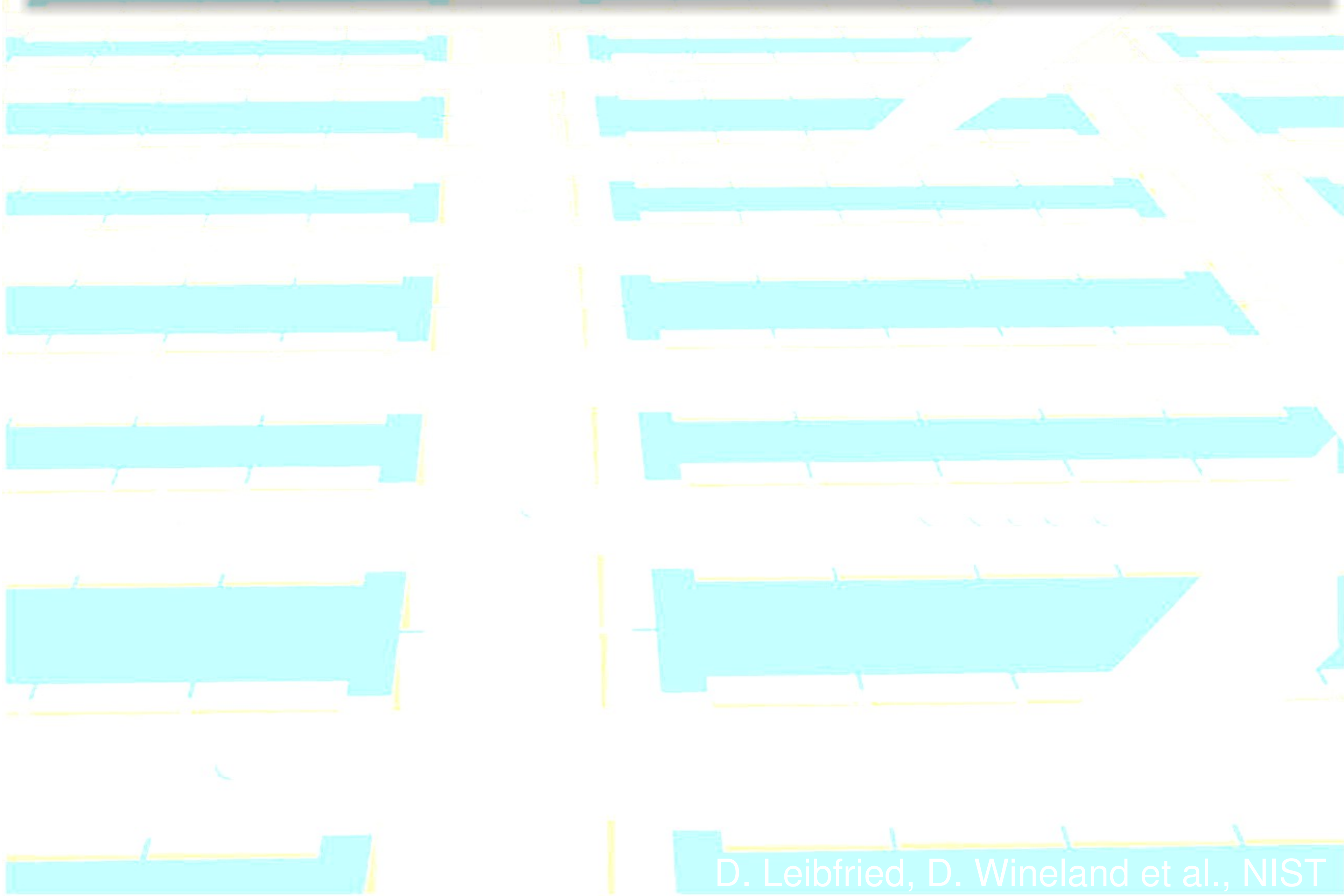


Scaling of ion trap quantum computers



„Architecture for a large-scale ion-trap quantum computer“, D. Kielpinski et al, Nature **417**, 709 (2002)

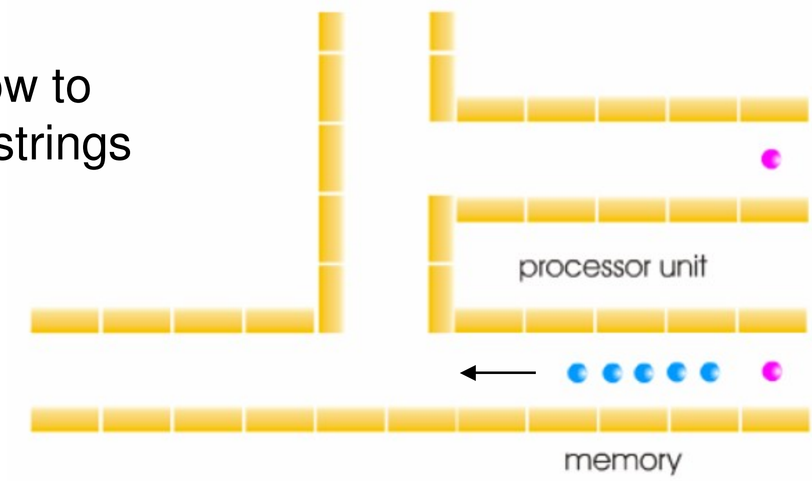
Multiplexed trap structure: NIST Boulder



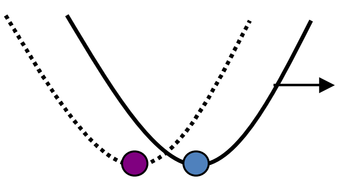
Segmented ion traps as scalable trap architecture

(ideas pioneered by D. Wineland, NIST)

Segmented trap electrode allow to transport ions and to split ion strings



State of the art:



<u>Transport of ions</u>

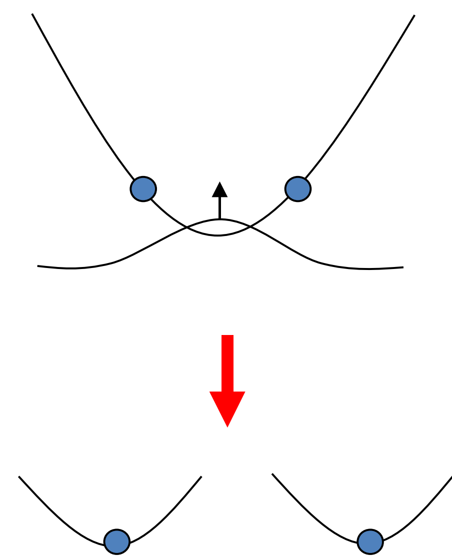
1 mm within 50 µs

no motional heating

<u>Splitting of two-ion crystal</u>

$t_{separation} \approx 200\ \mu s$

small heating $n \approx 1$



„Architecture for a large-scale ion-trap quantum computer“, D. Kielpinski et al, Nature **417**, 709 (2002)

„Transport of quantum states“, M. Rowe et al, quant-ph/0205084

Scaling of this approach?

Problems :

- Coupling strength between internal and motional states of a N-ion string decreases as

$$\eta \propto \frac{1}{\sqrt{N}}$$

(momentum transfer from photon to ion string becomes more difficult)

-> Gate operation speed slows down

- More vibrational modes increase risk of spurious excitation of unwanted modes

- Distance between neighbouring ions decreases -> addressing more difficult

-> Use flexible trap potentials to split long ion string into smaller segments and perform operations on these smaller strings