

Quantum computing with trapped ions



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- Physics and information
- Ion trap quantum computing
- Quantum teleportation
- Scaling up ion trap quantum computers



FWF
SFB



SCALA
QGATES



Industrie
Tirol



IQI
GmbH



bm:bwk



Physics and information

Information is physical (Rolf Landauer, 1961)

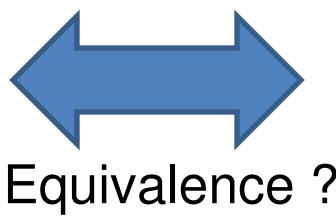
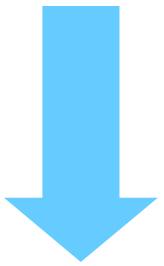


Erasing information generates heat:

0101001101 → 0000000000 + entropy



Physical process



Algorithm



Physics and information

Information is physical (Rolf Landauer, 1961)

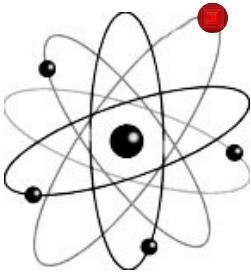
→ Information is quantum mechanical

Classical information is a subset in quantum information

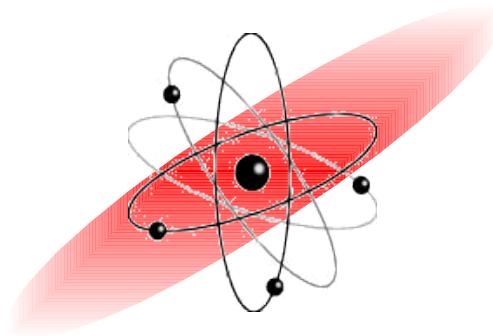
Quantum information: strip down quantum mechanics to bare bones.

- Hilbert space
- unitary operations
- measurement

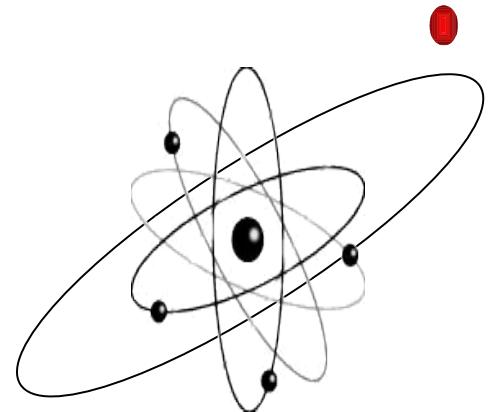
Qubits



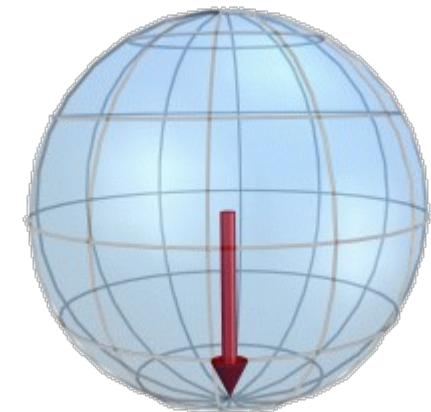
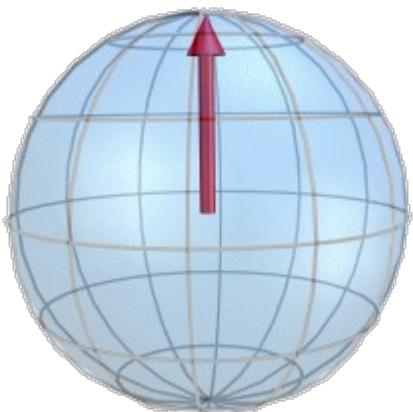
$|0\rangle$



$\alpha|0\rangle + \beta|1\rangle$



$|1\rangle$



Information content

$$|\Psi\rangle_{\text{reg}} = \alpha_0 |000\rangle + \alpha_1 |001\rangle + \alpha_2 |010\rangle + \alpha_3 |011\rangle + \\ \alpha_4 |100\rangle + \alpha_5 |101\rangle + \alpha_6 |110\rangle + \alpha_7 |111\rangle$$

# bits	classical	quantum mechanical
1	1	$0.5208 + 0.7059i, 0.3014 + 0.3736i$
2	01	$0.2044 + 0.4911i, 0.1732 + 0.3855i, 0.2040 + 0.4890i, 0.3193 + 0.3947i$
3	001	$0.2583 + 0.2704i, 0.2310 + 0.1150i, 0.2956 + 0.3118i, 0.3558 + 0.2113i, 0.1943 + 0.1377i, 0.3273 + 0.2613i, 0.0643 + 0.2033i, 0.3643 + 0.1654i$
4	1010	$0.1691 + 0.0891i, 0.1096 + 0.0828i, 0.1420 + 0.2873i, 0.0741 + 0.2419i, 0.1902 + 0.0448i, 0.2495 + 0.0039i, 0.1738 + 0.2933i, 0.2102 + 0.0653i, 0.0686 + 0.0980i$ $0.1246 + 0.2170i, 0.2570 + 0.0933i, 0.2234 + 0.1540i, 0.1513 + 0.0213i, 0.1863 + 0.3243i, 0.2606 + 0.1912i, 0.0194 + 0.1390i$
5	10001	$0.1060 + 0.1416i, 0.0103 + 0.0118i, 0.0064 + 0.0976i, 0.0734 + 0.0716i, 0.0030 + 0.2054i, 0.0902 + 0.0035i, 0.1605 + 0.1804i, 0.0218 + 0.2280i, 0.0083 + 0.2326i, 0.1438 + 0.1853i, 0.1429 + 0.1030i, 0.0037 + 0.1171i, 0.0038 + 0.0503i, 0.0446 + 0.1512i, 0.1379 + 0.0752i, 0.0135 + 0.2255i, 0.0863 + 0.1707i, 0.1483 + 0.0968i, 0.1686 + 0.1749i, 0.1627 + 0.0629i, 0.0197 + 0.1033i, 0.1067 + 0.2192i, 0.1038 + 0.1605i, 0.0830 + 0.0499i, 0.0361 + 0.1971i, 0.1587 + 0.1477i, 0.1642 + 0.0314i, 0.1709 + 0.0487i, 0.1124 + 0.1426i, 0.1303 + 0.1480i, 0.0284 + 0.0870i, 0.1059 + 0.1351i$
6	110101	$0.0595 + 0.1064i, 0.0295 + 0.1327i, 0.0929 + 0.0406i, 0.1090 + 0.0379i, 0.0559 + 0.1286i, 0.0015 + 0.0345i, 0.0624 + 0.1196i, 0.1120 + 0.1350i, 0.1180 + 0.0345i, 0.1367 + 0.0356i, 0.1255 + 0.0074i, 0.0547 + 0.0116i, 0.0923 + 0.0952i, 0.1087 + 0.0284i, 0.0288 + 0.1254i, 0.1345 + 0.0258i, 0.0846 + 0.0254i, 0.0939 + 0.1478i, 0.0348 + 0.0654i, 0.0816 + 0.0505i, 0.1384 + 0.0467i, 0.0498 + 0.0543i, 0.0974 + 0.0584i, 0.0582 + 0.0879i, 0.0932 + 0.0178i, 0.01393 + 0.00571i, 0.0590 + 0.0682i, 0.0615 + 0.1293i, 0.0974 + 0.1388i, 0.1245 + 0.0393i, 0.0552 + 0.0238i, 0.0632 + 0.1297i, 0.0884 + 0.0354i, 0.0841 + 0.0960i, 0.1065 + 0.1437i, 0.0760 + 0.0988i, 0.1154 + 0.1293i, 0.0727 + 0.0015i, 0.0276 + 0.0204i, 0.1041 + 0.1217i, 0.1460 + 0.0639i, 0.1199 + 0.1323i, 0.1046 + 0.1092i, 0.0721 + 0.1021i, 0.0170 + 0.0514i, 0.0988 + 0.0247i, 0.0543 + 0.0231i, 0.0208 + 0.0284i, 0.0842 + 0.0628i, 0.1223 + 0.1272i, 0.1002 + 0.0729i, 0.1485 + 0.1213i, 0.1429 + 0.0685i, 0.0087 + 0.0680i, 0.0535 + 0.0670i, 0.0815 + 0.0613i, 0.0389 + 0.1340i, 0.0888 + 0.0008i, 0.0073 + 0.0442i, 0.0849 + 0.0073i, 0.1042 + 0.1030i, 0.1430 + 0.0966i, 0.1115 + 0.1461i, 0.1100 + 0.0821i$
7	1001010	$0.0880 + 0.0466i, 0.1054 + 0.0684i, 0.0239 + 0.0866i, 0.0759 + 0.0090i, 0.0563 + 0.1020i, 0.1006 + 0.0988i, 0.0769 + 0.0649i, 0.0246 + 0.0273i, 0.0485 + 0.0942i, 0.0186 + 0.0554i, 0.1045 + 0.0790i, 0.0384 + 0.0455i, 0.0053 + 0.1037i, 0.0815 + 0.0078i, 0.0965 + 0.0597i, 0.0309 + 0.0315i, 0.0271 + 0.0925i, 0.1006 + 0.0362i, 0.0141 + 0.0734i, 0.1015 + 0.0058i, 0.0757 + 0.0385i, 0.0914 + 0.0537i, 0.0226 + 0.0468i, 0.0491 + 0.0607i, 0.0087 + 0.0665i, 0.0918 + 0.0122i, 0.0606 + 0.0969i, 0.0344 + 0.0814i, 0.0404 + 0.0853i, 0.0936 + 0.0879i, 0.0401 + 0.0723i, 0.0079 + 0.0217i, 0.0216 + 0.0294i, 0.0053 + 0.0675i, 0.0611 + 0.0579i, 0.0131 + 0.0064i, 0.0563 + 0.0096i, 0.0126 + 0.0293i, 0.0830 + 0.0441i, 0.0404 + 0.0511i, 0.0888 + 0.0050i, 0.0073 + 0.0442i, 0.0849 + 0.0073i, 0.1042 + 0.1030i, 0.1430 + 0.0966i, 0.1115 + 0.0461i, 0.1100 + 0.0821i$
8	10101011	$0.0199 + 0.0027i, 0.0033 + 0.0063i, 0.0005 + 0.0656i, 0.0443 + 0.0262i, 0.0573 + 0.0359i, 0.0622 + 0.0704i, 0.0491 + 0.0176i, 0.0194 + 0.0664i, 0.0111 + 0.0506i, 0.0502 + 0.0687i, 0.0729 + 0.0376i, 0.0629 + 0.0765i, 0.0717 + 0.0288i, 0.0239 + 0.0410i, 0.0207 + 0.0140i, 0.0387i, 0.0126 + 0.0325i, 0.0163 + 0.0509i, 0.0167 + 0.0519i, 0.0502 + 0.0738i, 0.0041 + 0.0148i, 0.0177 + 0.0086i, 0.0514 + 0.0436i, 0.0240 + 0.0747i, 0.0236 + 0.0018i, 0.0555 + 0.0671i, 0.0736 + 0.0021i, 0.0101 + 0.0400i, 0.0053 + 0.0148i, 0.0097 + 0.0552i, 0.0128 + 0.0193i, 0.0702 + 0.0720i, 0.0105 + 0.0106i, 0.0476 + 0.0402i, 0.0207 + 0.0690i, 0.0170 + 0.0726i, 0.0549 + 0.0258i, 0.0423 + 0.0337i, 0.0726 + 0.0363i, 0.0254 + 0.0115i, 0.0543 + 0.0105i, 0.0727 + 0.0410i, 0.0448 + 0.0559i, 0.0678 + 0.0307i, 0.0578 + 0.0276i, 0.0293 + 0.0220i, 0.0559 + 0.0670i, 0.0125 + 0.0483i, 0.0737 + 0.0186i, 0.0151 + 0.0754i, 0.0598 + 0.0494i, 0.0473 + 0.0177i, 0.0125 + 0.0525i, 0.0024 + 0.0513i, 0.0222 + 0.0104i, 0.0748 + 0.0017i, 0.0733 + 0.0202i, 0.0176 + 0.0090i, 0.0739 + 0.0053i, 0.0524 + 0.0657i, 0.0042 + 0.0139i, 0.0462 + 0.0025i, 0.0303 + 0.0566i, 0.0166 + 0.0414i, 0.0141 + 0.0213i, 0.0059 + 0.0284i, 0.0006 + 0.0010i, 0.0608 + 0.0685i, 0.0014 + 0.0667i, 0.0677 + 0.0196i, 0.0272 + 0.0439i, 0.0557 + 0.0123i, 0.0746 + 0.0458i, 0.0120 + 0.0255i, 0.0126 + 0.0508i, 0.0242 + 0.0666i, 0.0023 + 0.0437i, 0.0276 + 0.0756i, 0.0021 + 0.0610i, 0.0612 + 0.0118i, 0.0770 + 0.0642i, 0.0085 + 0.0148i, 0.0480 + 0.0493i, 0.0102 + 0.0516i, 0.0239 + 0.0595i, 0.0104 + 0.0293i, 0.0172 + 0.0340i, 0.0236 + 0.0372i, 0.0104 + 0.0469i, 0.0186 + 0.0136i, 0.0715 + 0.0002i, 0.0301 + 0.0609i, 0.0394 + 0.0396i, 0.0072 + 0.0164i, 0.0017 + 0.0080i, 0.0123 + 0.0121i, 0.0651 + 0.0314i, 0.0678 + 0.0314i, 0.0144 + 0.0041i, 0.0764 + 0.0276i, 0.0549 + 0.0116i, 0.0672 + 0.0296i, 0.0370 + 0.0240i, 0.0382 + 0.0130i, 0.0222 + 0.0091i, 0.0047 + 0.0249i, 0.0202 + 0.0562i, 0.0144 + 0.0314i, 0.0707 + 0.0308i, 0.0095 + 0.0390i, 0.0010 + 0.0130i, 0.0285 + 0.0040i, 0.0543 + 0.0494i, 0.00685 + 0.00121i, 0.0458 + 0.0045i, 0.0121 + 0.0619i, 0.0244 + 0.0358i, 0.0180 + 0.0356i, 0.0006 + 0.0064i, 0.0306 + 0.0633i, 0.0501 + 0.0149i, 0.0066 + 0.0343i, 0.0593 + 0.0010i, 0.0747 + 0.0238i, 0.0551 + 0.0675i$

Information content

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5	10001	$0.1060 + 0.1416i, \ 0.0103 + 0.0118i, \ 0.0064 + 0.0976i, \ 0.0734 + 0.0716i, \ 0.0030 + 0.2054i, \ 0.0902 + 0.0035i, \ 0.1605 + 0.1804i, \ 0.0218 + 0.2280i, \ 0.0083 + 0.2326i, \ 0.1438 + 0.1853i, \ 0.1429 + 0.1030i, \ 0.0037 + 0.1171i, \ 0.0038 + 0.0503i, \ 0.0446 + 0.1512i, \ 0.1379 + 0.0752i, \ 0.0135 + 0.2255i, \ 0.0863 + 0.1707i, \ 0.1483 + 0.0968i, \ 0.1686 + 0.1749i, \ 0.1627 + 0.0629i, \ 0.0197 + 0.1033i, \ 0.1067 + 0.2192i, \ 0.1038 + 0.1605i, \ 0.0830 + 0.0499i, \ 0.0361 + 0.1971i$ $0.1587 + 0.1477i, \ 0.1642 + 0.0314i, \ 0.1709 + 0.0487i, \ 0.1124 + 0.1426i, \ 0.1303 + 0.1480i, \ 0.0284 + 0.0870i, \ 0.1059 + 0.1351i$
6	110101	$0.0595 + 0.1064i, \ 0.0295 + 0.1327i, \ 0.0929 + 0.0406i, \ 0.1090 + 0.0379i, \ 0.0559 + 0.1286i, \ 0.0015 + 0.0345i, \ 0.0624 + 0.1196i, \ 0.1120 + 0.1350i, \ 0.1180 + 0.0345i, \ 0.1367 + 0.0356i, \ 0.1255 + 0.0074i, \ 0.0547 + 0.0116i, \ 0.0923 + 0.0952i, \ 0.1087 + 0.0284i, \ 0.0284 + 0.1254i, \ 0.1345 + 0.0258i, \ 0.0846 + 0.0245i, \ 0.0939 + 0.1478i, \ 0.0348 + 0.0654i, \ 0.0816 + 0.0505i, \ 0.1384 + 0.0467i, \ 0.0498 + 0.0543i, \ 0.0974 + 0.0584i, \ 0.0582 + 0.0879i, \ 0.0197 + 0.1033i, \ 0.1067 + 0.2192i, \ 0.1038 + 0.1605i, \ 0.0830 + 0.0499i, \ 0.0361 + 0.1971i$ $0.0139 + 0.0057i, \ 0.0590 + 0.0682i, \ 0.0615 + 0.1293i, \ 0.0974 + 0.1388i, \ 0.1245 + 0.0393i, \ 0.0552 + 0.0238i, \ 0.0632 + 0.1297i, \ 0.0884 + 0.0354i, \ 0.0841 + 0.0960i, \ 0.1065 + 0.1437i, \ 0.0760 + 0.0988i, \ 0.1154 + 0.1293i, \ 0.0727 + 0.0015i, \ 0.0276 + 0.0204i, \ 0.1041 + 0.1217i, \ 0.1460 + 0.0639i, \ 0.1199 + 0.1323i, \ 0.1046 + 0.1092i, \ 0.0721 + 0.1021i, \ 0.0170 + 0.0514i, \ 0.0988 + 0.0247i, \ 0.0543 + 0.0231i, \ 0.0208 + 0.0284i, \ 0.0842 + 0.0628i, \ 0.1223 + 0.1272i$ $0.1002 + 0.0729i, \ 0.1485 + 0.1213i, \ 0.1429 + 0.0685i, \ 0.0087 + 0.0680i, \ 0.0535 + 0.0670i, \ 0.0815 + 0.0613i, \ 0.0389 + 0.1340i, \ 0.0888 + 0.0008i, \ 0.0073 + 0.0442i, \ 0.0849 + 0.0073i, \ 0.1042 + 0.1030i, \ 0.1430 + 0.0966i, \ 0.1115 + 0.1461i, \ 0.1100 + 0.0821i$
7	1001010	$0.0880 + 0.0466i, \ 0.1054 + 0.0684i, \ 0.0239 + 0.0866i, \ 0.0759 + 0.0090i, \ 0.0563 + 0.1020i, \ 0.1006 + 0.0988i, \ 0.0769 + 0.0649i, \ 0.0246 + 0.0273i, \ 0.0485 + 0.0942i, \ 0.0186 + 0.0554i, \ 0.1045 + 0.0790i, \ 0.0384 + 0.0455i, \ 0.0053 + 0.1037i, \ 0.0815 + 0.0078i, \ 0.0965 + 0.0597i, \ 0.0309 + 0.0315i, \ 0.0271 + 0.0925i, \ 0.1006 + 0.0362i, \ 0.0141 + 0.0734i, \ 0.1015 + 0.0058i, \ 0.0757 + 0.0385i, \ 0.0914 + 0.0537i, \ 0.0226 + 0.0468i, \ 0.0491 + 0.0607i, \ 0.0087 + 0.0665i, \ 0.0918 + 0.0122i, \ 0.0606 + 0.0969i, \ 0.0344 + 0.0814i, \ 0.0404 + 0.0853i, \ 0.0936 + 0.0879i, \ 0.0401 + 0.0723i, \ 0.0079 + 0.0217i, \ 0.0216 + 0.0294i, \ 0.0053 + 0.0675i, \ 0.0611 + 0.0579i, \ 0.0131 + 0.0064i, \ 0.0563 + 0.0096i, \ 0.0126 + 0.0293i, \ 0.0830 + 0.0441i, 0.0404 + 0.0511i, 0.0888 + 0.0980i, 0.0050 + 0.0643i, 0.0645 + 0.0355i, 0.1024 + 0.0516i, 0.0311 + 0.0644i, 0.0959 + 0.0174i, 0.0110 + 0.0894i, 0.0070 + 0.1031i, 0.0253 + 0.0642i, 0.1006 + 0.0031i, 0.0068 + 0.0876i, 0.0285 + 0.0658i, 0.1078 + 0.0756i, 0.0229 + 0.0099i, 0.0537 + 0.0458i, 0.0313 + 0.0405i, 0.0725 + 0.0179i, 0.1033 + 0.0898i, 0.0827 + 0.0904i, 0.0718 + 0.0487i, 0.0141 + 0.1032i, 0.0103 + 0.0159i, 0.0016 + 0.00938i, 0.0311 + 0.0830i, 0.0881 + 0.0479i, 0.1063 + 0.0669i, 0.0019 + 0.1026i, 0.0884 + 0.0690i, 0.0670 + 0.0267i, 0.0604 + 0.0380i, 0.0263 + 0.0203i, 0.0886 + 0.0529i, 0.0284 + 0.0441i, 0.0813 + 0.0500i, 0.0711 + 0.0659i, 0.0231 + 0.0077i, 0.0649 + 0.0339i, 0.0652 + 0.0656i, 0.0711 + 0.0189i, 0.0198 + 0.0670i, 0.0868 + 0.0265i, 0.0184 + 0.0633i, 0.0582 + 0.0546i, 0.0672 + 0.0501i, 0.0740 + 0.0584i, 0.0730 + 0.1016i, 0.0946 + 0.0369i, 0.0014 + 0.0433i, 0.0335 + 0.0332i, 0.0840 + 0.0444i, 0.0331 + 0.0308i, 0.0999 + 0.0425i, 0.0732 + 0.0542i, 0.0080 + 0.0779i, 0.0076 + 0.0330i, 0.0013 + 0.0121i, 0.0245 + 0.0478i, 0.0557 + 0.0503i, 0.0494 + 0.0016i, 0.0758 + 0.0716i, 0.0628 + 0.0781i, 0.0549 + 0.0304i, 0.0808 + 0.0282i, 0.0208 + 0.0764i, 0.0409 + 0.0845i, 0.0893 + 0.0425i, 0.0989 + 0.0562i, 0.0122 + 0.0774i, 0.0876 + 0.0614i, 0.0379 + 0.0497i, 0.0169 + 0.0480i, 0.0132 + 0.0095i, 0.0822 + 0.0478i, 0.0778 + 0.0395i, 0.0703 + 0.0326i, 0.0813 + 0.0919i, 0.0715 + 0.0819i, 0.0953 + 0.1024i, 0.0293 + 0.0602i, 0.0452 + 0.0015i, 0.0230 + 0.0643i$
8	10101011	$0.0199 + 0.0027i, \ 0.0033 + 0.0063i, \ 0.0005 + 0.0656i, \ 0.0443 + 0.0262i, \ 0.0573 + 0.0359i, \ 0.0622 + 0.0704i, \ 0.0491 + 0.0176i, \ 0.0194 + 0.0664i, \ 0.0111 + 0.0506i, \ 0.0502 + 0.0687i, \ 0.0729 + 0.0376i, \ 0.0629 + 0.0765i, \ 0.0717 + 0.0288i, \ 0.0239 + 0.0410i, \ 0.0207 + 0.0140i, \ 0.0413 + 0.0387i, \ 0.0126 + 0.0325i, \ 0.0163 + 0.0509i, \ 0.0167 + 0.0519i, \ 0.0502 + 0.0738i, \ 0.0041 + 0.0148i, \ 0.0177 + 0.0086i, \ 0.0514 + 0.0436i, \ 0.0240 + 0.0747i, \ 0.0236 + 0.0018i, \ 0.0555 + 0.0671i, \ 0.0736 + 0.0021i, \ 0.0101 + 0.0400i, \ 0.0053 + 0.0148i, \ 0.0097 + 0.0552i, \ 0.0218 + 0.0193i, \ 0.0727 + 0.0410i, \ 0.0448 + 0.0559i, \ 0.0678 + 0.0307i, \ 0.0578 + 0.0276i, \ 0.0293 + 0.0220i, \ 0.0559 + 0.0670i, \ 0.0125 + 0.0483i, \ 0.0737 + 0.0186i, \ 0.0151 + 0.0754i, \ 0.0598 + 0.0494i, \ 0.0473 + 0.0177i, \ 0.0125 + 0.0525i, \ 0.0024 + 0.0513i, \ 0.0222 + 0.0104i, \ 0.0748 + 0.0017i, \ 0.0733 + 0.0202i, \ 0.0176 + 0.0090i, \ 0.0739 + 0.0053i, \ 0.0524 + 0.0657i, \ 0.0042 + 0.0139i, \ 0.0462 + 0.0025i, \ 0.0303 + 0.0566i, \ 0.0166 + 0.0414i, \ 0.0141 + 0.0213i, \ 0.0059 + 0.0284i, \ 0.0006 + 0.0010i, \ 0.0608 + 0.0685i, \ 0.0014 + 0.0667i, \ 0.0677 + 0.0196i, \ 0.0272 + 0.0439i, \ 0.0057 + 0.0123i, \ 0.0746 + 0.0458i, \ 0.0120 + 0.0255i, \ 0.0126 + 0.0508i, \ 0.0242 + 0.0666i, \ 0.0023 + 0.0437i, \ 0.0276 + 0.0756i, \ 0.0021 + 0.0610i, \ 0.0612 + 0.0118i, \ 0.0770 + 0.0642i, \ 0.0085 + 0.0148i, \ 0.0480 + 0.0493i, \ 0.0102 + 0.0516i, \ 0.0239 + 0.0595i, \ 0.0104 + 0.0293i, \ 0.0172 + 0.0340i, \ 0.0306 + 0.0372i, \ 0.0104 + 0.0469i, \ 0.0186 + 0.0136i, \ 0.0715 + 0.0002i, \ 0.0301 + 0.0609i, \ 0.0394 + 0.0396i, \ 0.0072 + 0.0164i, \ 0.0017 + 0.0080i, \ 0.0123 + 0.0121i, \ 0.0651 + 0.0314i, \ 0.0678 + 0.0314i, \ 0.0144 + 0.0041i, \ 0.0764 + 0.0726i, \ 0.0549 + 0.0116i, \ 0.0672 + 0.0296i, \ 0.0370 + 0.0240i, \ 0.0382 + 0.0130i, \ 0.0222 + 0.0691i, \ 0.0047 + 0.0249i, \ 0.0202 + 0.0566i, \ 0.0144 + 0.0317i, \ 0.0707 + 0.0308i, \ 0.0095 + 0.0390i, \ 0.0010 + 0.0130i, \ 0.0285 + 0.0404i, \ 0.0538 + 0.0494i, \ 0.0685 + 0.0012i, \ 0.0458 + 0.0645i, \ 0.0121 + 0.0619i, \ 0.0244 + 0.0358i, \ 0.0180 + 0.0356i, \ 0.0006 + 0.0306i, \ 0.0633i + 0.0501i, \ 0.0149 + 0.0343i, \ 0.0593 + 0.0010i, \ 0.0747 + 0.0238i, \ 0.0551 + 0.0675i, \ 0.0603 + 0.0644i, \ 0.0183 + 0.0257i, \ 0.0151 + 0.0679i, \ 0.0203 + 0.0370i, \ 0.0050 + 0.0432i, \ 0.0753 + 0.0475i, \ 0.0491 + 0.0510i, \ 0.0421 + 0.0475i, \ 0.0654 + 0.0528i, \ 0.0618 + 0.0393i, \ 0.0515 + 0.0550i, \ 0.00517 + 0.0397i, \ 0.0633 + 0.0467i, \ 0.0748 + 0.0745i, \ 0.0375 + 0.0634i, \ 0.0630 + 0.0245i, \ 0.0494 + 0.0453i, \ 0.0236 + 0.0100i, \ 0.0509 + 0.0196i, \ 0.0276 + 0.0619i, \ 0.0723 + 0.0515i, \ 0.0376 + 0.0111i, \ 0.0070 + 0.0433i, \ 0.0519 + 0.0350i, \ 0.0397 + 0.0697i, \ 0.0171 + 0.0217i, \ 0.0559 + 0.0050i, \ 0.0053 + 0.0367i, \ 0.0743 + 0.0758i, \ 0.0160 + 0.0711i, \ 0.0124 + 0.0433i, \ 0.0492 + 0.0503i, \ 0.0000 + 0.0596i, \ 0.0259 + 0.0083i, \ 0.0212 + 0.0001i, \ 0.0034 + 0.0418i, \ 0.0072 + 0.0005i, \ 0.0316 + 0.0348i, \ 0.0630 + 0.0151i, \ 0.0671 + 0.0607i, \ 0.0017 + 0.0477i, \ 0.0560 + 0.0012i, \ 0.0654 + 0.0687i, \ 0.0562 + 0.0587i, \ 0.0736 + 0.0699i, \ 0.0506 + 0.0585i, \ 0.0572 + 0.0293i, \ 0.0266 + 0.0255i, \ 0.0681 + 0.0389i, \ 0.0268 + 0.0443i, \ 0.0670 + 0.0514i, \ 0.0242 + 0.0504i, \ 0.0194 + 0.0242i, \ 0.0334 + 0.0178i, \ 0.0649 + 0.0321i, \ 0.0142 + 0.0230i, \ 0.0392 + 0.0518i, \ 0.0349 + 0.0723i, \ 0.0251 + 0.0264i, \ 0.0293 + 0.0092i, \ 0.0587 + 0.0130i, \ 0.0681 + 0.0215i, \ 0.0429i + 0.0616i, \ 0.0161 + 0.0374i, \ 0.0103 + 0.0734i, \ 0.0050 + 0.0179i, \ 0.0289 + 0.0369i, \ 0.0288 + 0.0406i, \ 0.0373 + 0.0611i, \ 0.0747 + 0.0149i, \ 0.0264 + 0.0701i, \ 0.0195 + 0.0711i, \ 0.0451 + 0.0010i, \ 0.0404 + 0.0592i, \ 0.0126 + 0.0730i, \ 0.0375 + 0.0627i, \ 0.0382 + 0.0712i, \ 0.0650 + 0.0153i, \ 0.0621 + 0.0520i, \ 0.0661 + 0.0715i, \ 0.0470 + 0.0265i, \ 0.0436 + 0.0458i, \ 0.0472 + 0.0474i, \ 0.0079 + 0.0033i, \ 0.0122 + 0.0757i, \ 0.0319 + 0.0693i, \ 0.0432 + 0.0534i, \ 0.0207 + 0.0339i, \ 0.0604 + 0.0540i, \ 0.0299 + 0.0470i, \ 0.0024 + 0.0231i, \ 0.0451 + 0.0660i, \ 0.0155 + 0.0225i, \ 0.0067 + 0.0075i, \ 0.0719 + 0.0306i, \ 0.0200 + 0.0257i, \ 0.0157 + 0.0728i, \ 0.038 + 0.0646i$

Information content

40 qubits	10 000 GigaByte
1 additional qubit	Double the memory
300 qubits	Every atom in the Universe would have to hold one classical bit

Why quantum information ?

Schrödinger equation for 300 interacting spins.

Classical computation needs more bits than there are atoms in the universe.

- Quantum computers can solve certain tasks much more efficiently than classical computers.

Other prominent examples:

- Factoring of large integers (P. Shor 1994)
- Search in an unsorted data base (L. Grover, 1997)
- ...



Quantum computing

Classical computer

- Initialization
- 1-bit operations (NOT)
- 2-bit gates (e.g. NAND)

Computational space:

00
01
10
11

- Read out
→ result

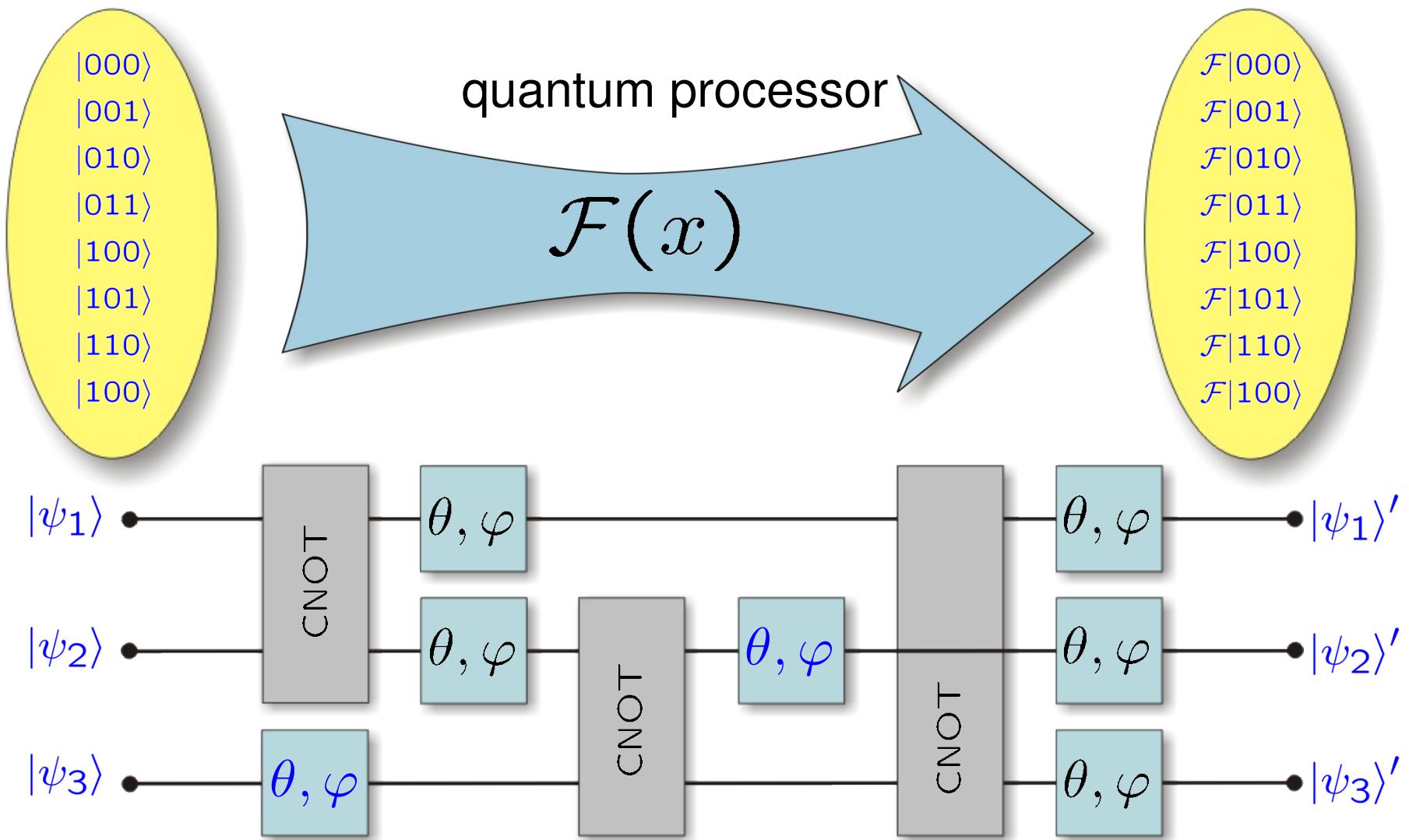
Quantum computer

- Initialization
- 1-qubit rotations
→ superpositions
- 2-qubit gates (CNOT gate)
→ entanglement

Computational space: Hilbert space
 2^n dimensional

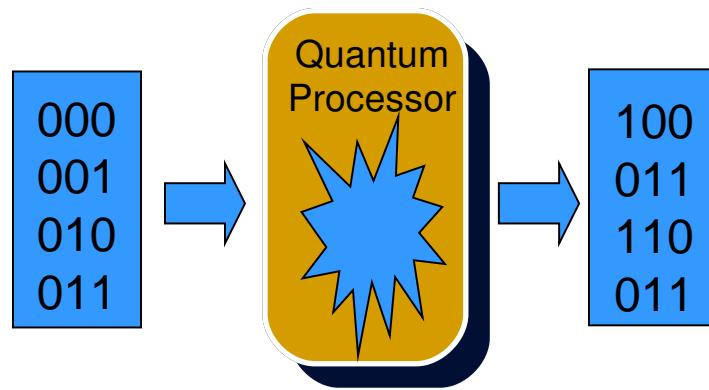
- Read out of qubits
→ gain of classical information

Quantum computing



Quantum computing

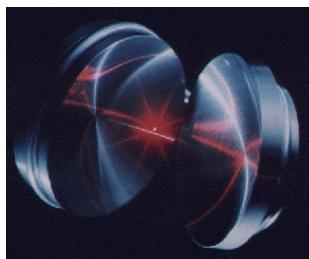
Long term goal: A universal quantum computer



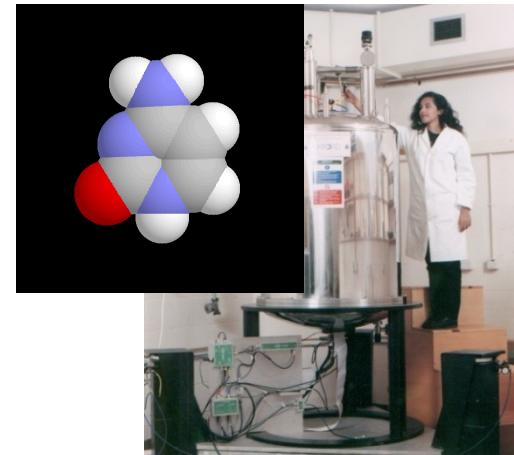
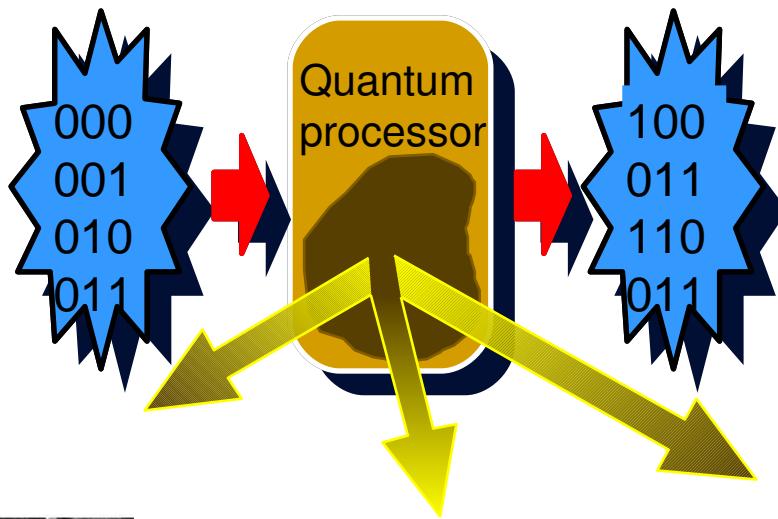
In the mean time:

- “understand” quantum mechanics
- apply quantum mechanics
- where does quantum mechanics fail?

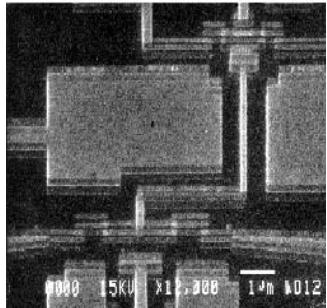
Which technology ?



Cavity QED



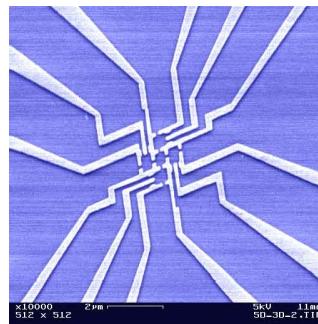
NMR



Superconducting qubits

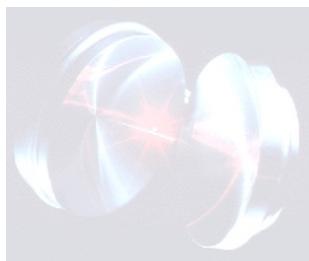


Trapped ions

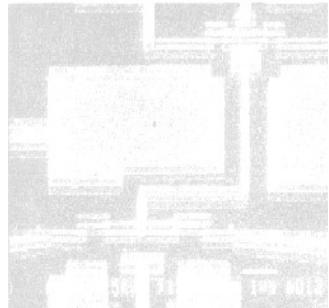
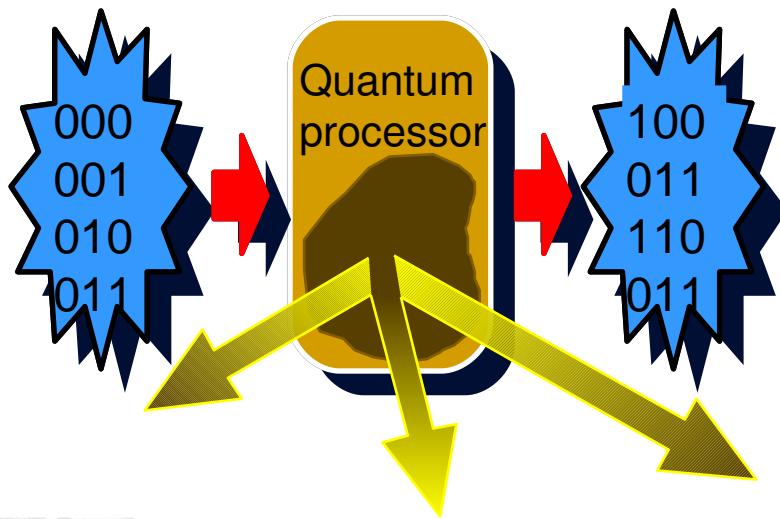


Quantum dots

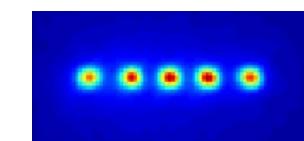
Which technology ?



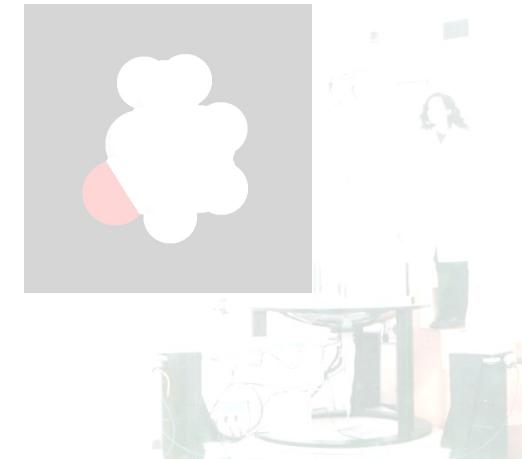
Cavity QED



Superconducting qubits



Trapped ions



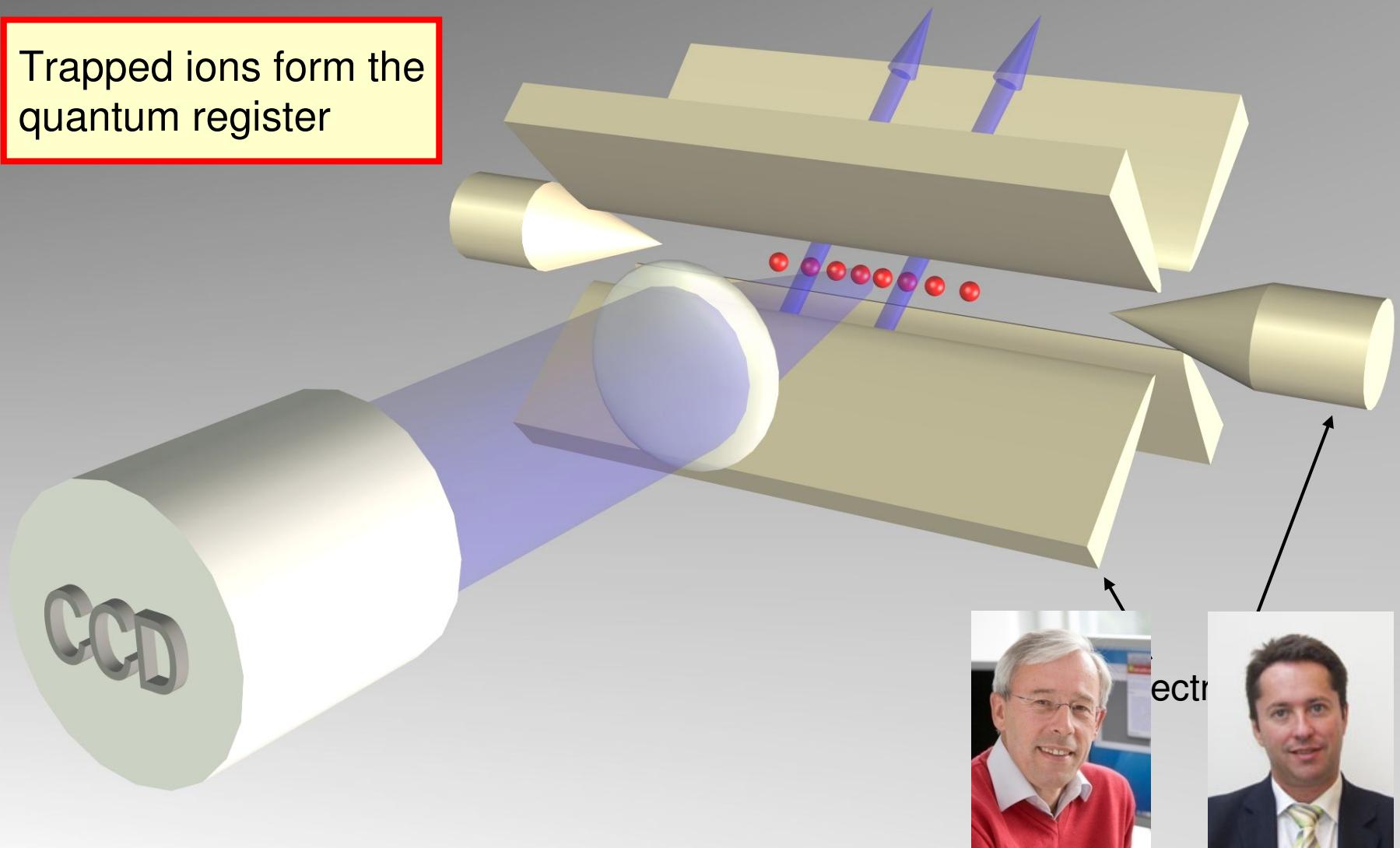
NMR



Quantum dots

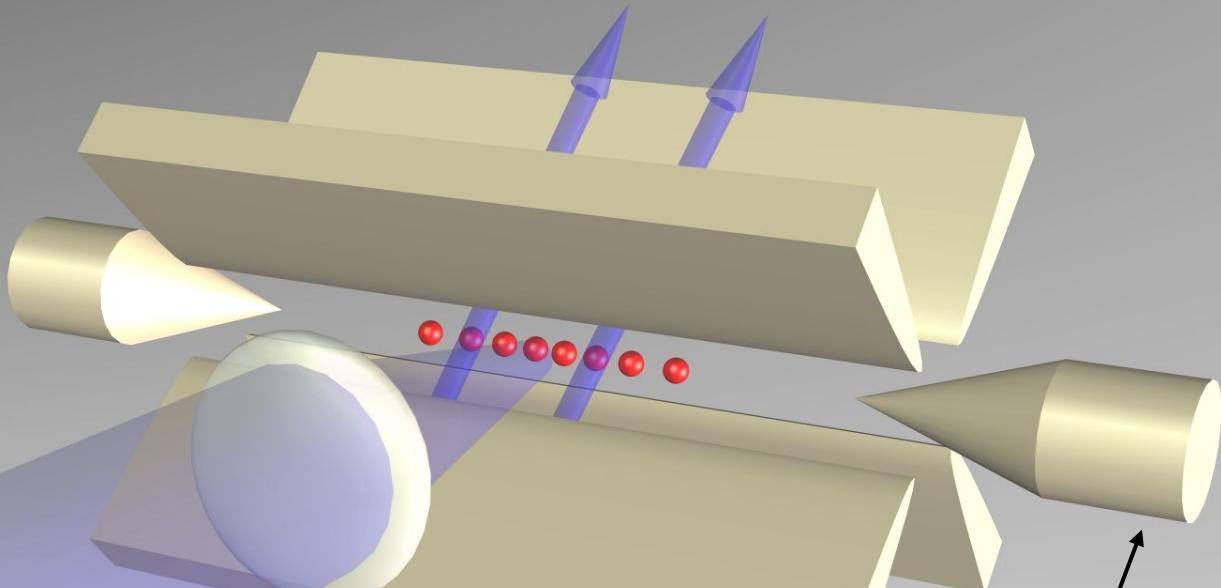
Ion trap quantum computing

Trapped ions form the quantum register



Ion trap quantum computing

Trapped ions form the quantum register



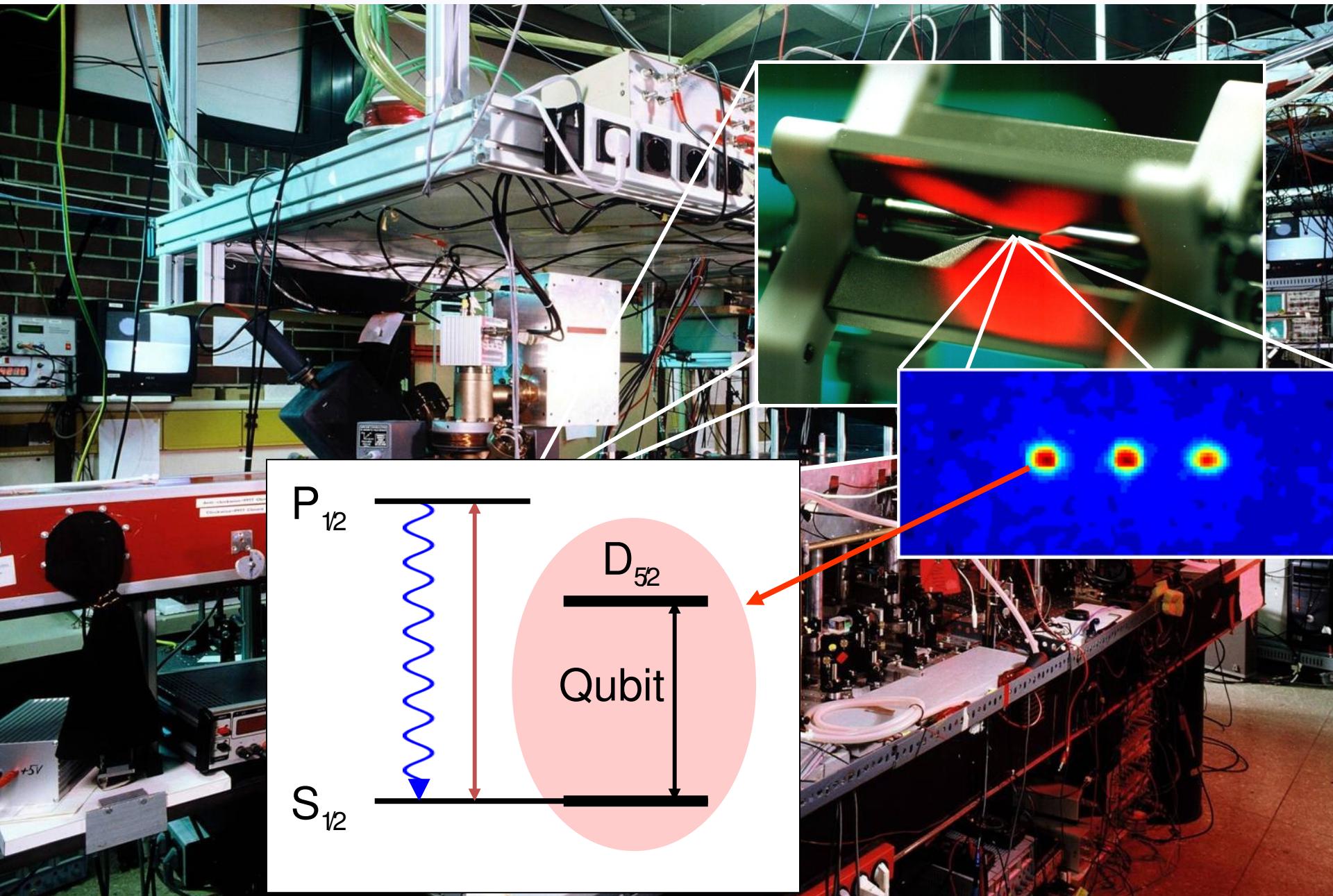
DiVincenzo criteria (2001)

- I. Scalable physical system, well characterized qubits
- II. Ability to initialize the state of the qubits
- III. Long relevant coherence times, much longer than gate operation time
- IV. “Universal” set of quantum gates
- V. Qubit-specific measurement capability

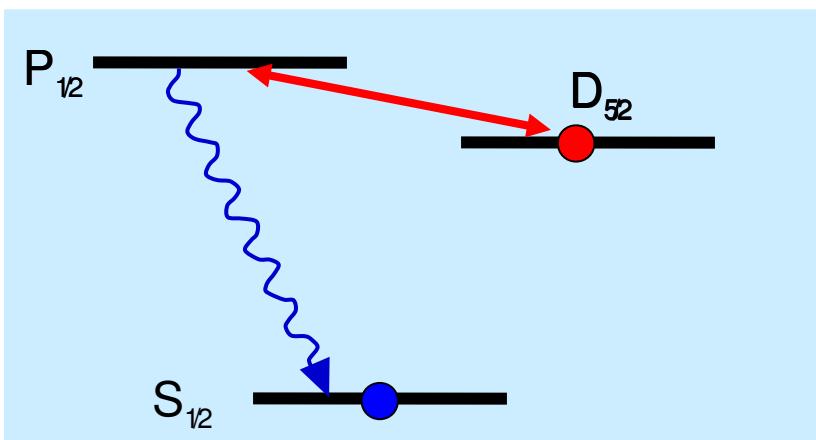


- Introduction to quantum information
- Ion trap quantum computing
- Teleportation and more
- Scaling of ion trap quantum computers

The hardware

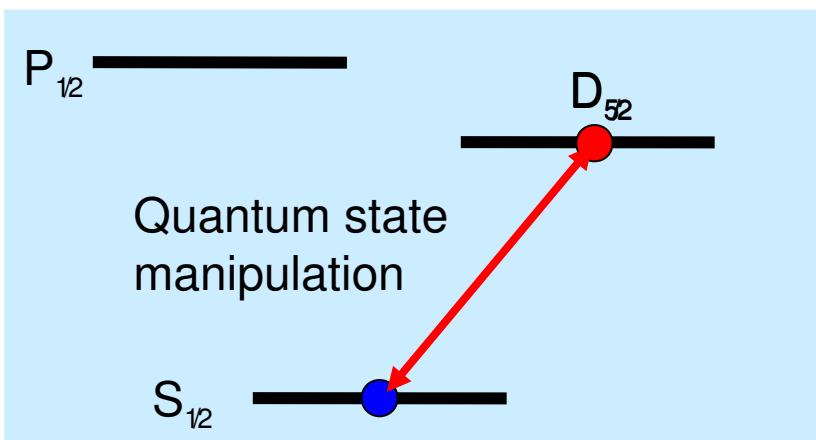


Experimental procedure



1. Initialization in a pure quantum state

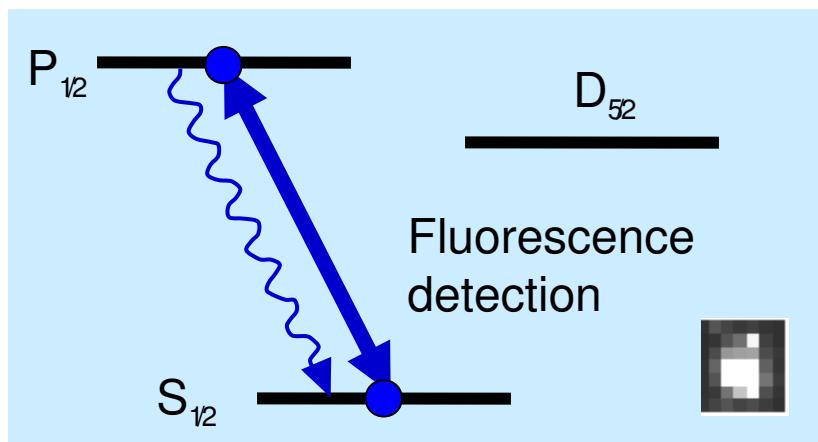
Experimental procedure



1. Initialization in a pure quantum state

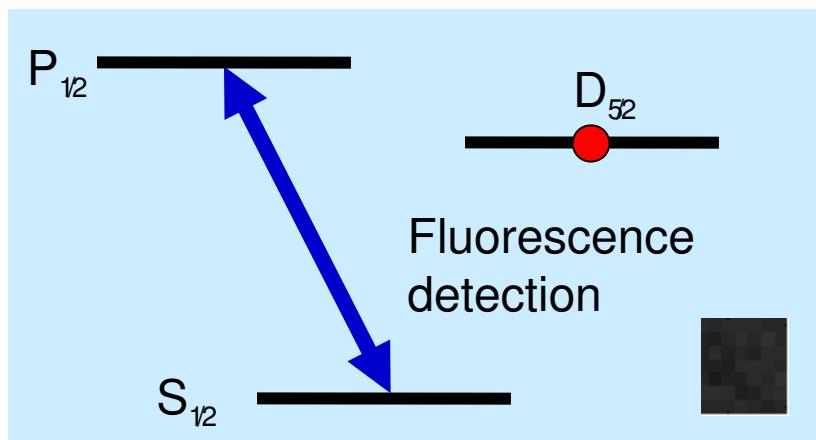
2. Quantum state manipulation on
 $S_{1/2} - D_{5/2}$ transition

Experimental procedure



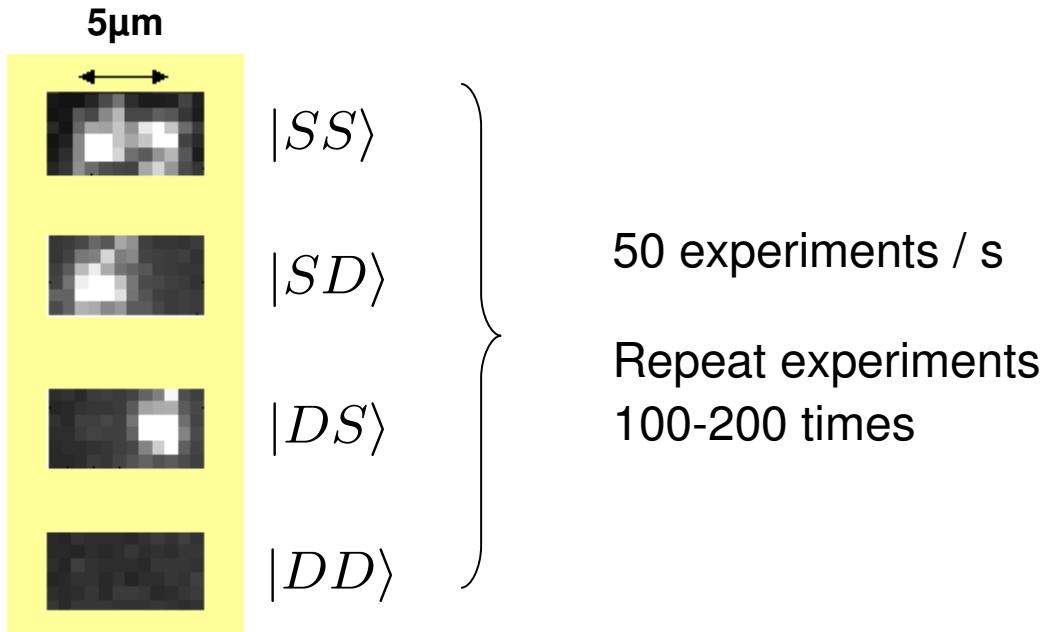
1. Initialization in a pure quantum state:
2. Quantum state manipulation on $S_{1/2} - D_{5/2}$ transition
3. Quantum state measurement by fluorescence detection

Experimental procedure



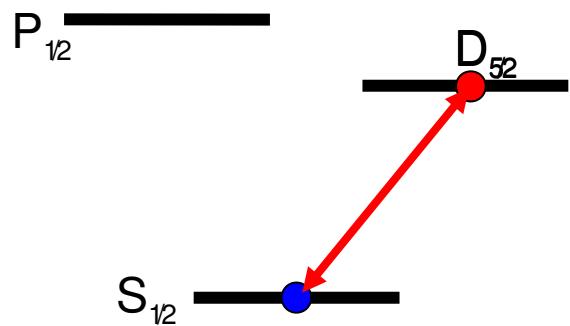
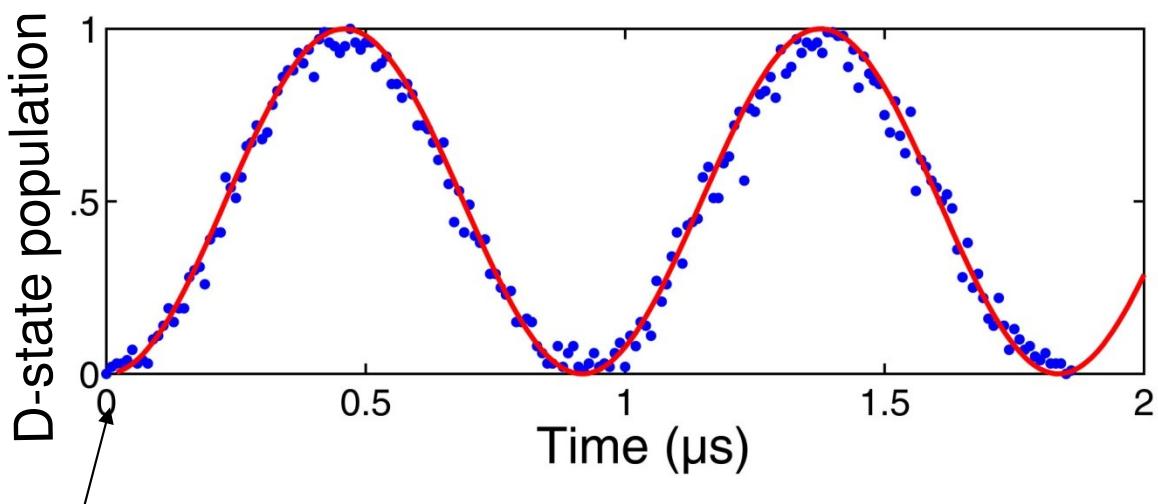
Two ions:

Spatially resolved
detection with
CCD camera

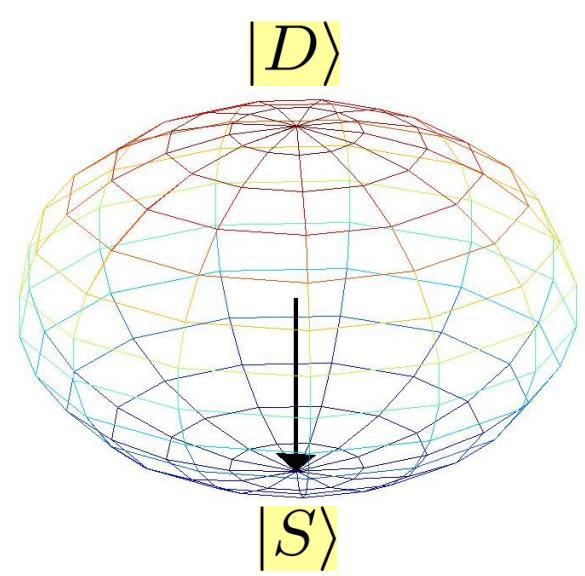


1. Initialization in a pure quantum state:
2. Quantum state manipulation on $S_{1/2} - D_{52}$ transition
3. Quantum state measurement by fluorescence detection

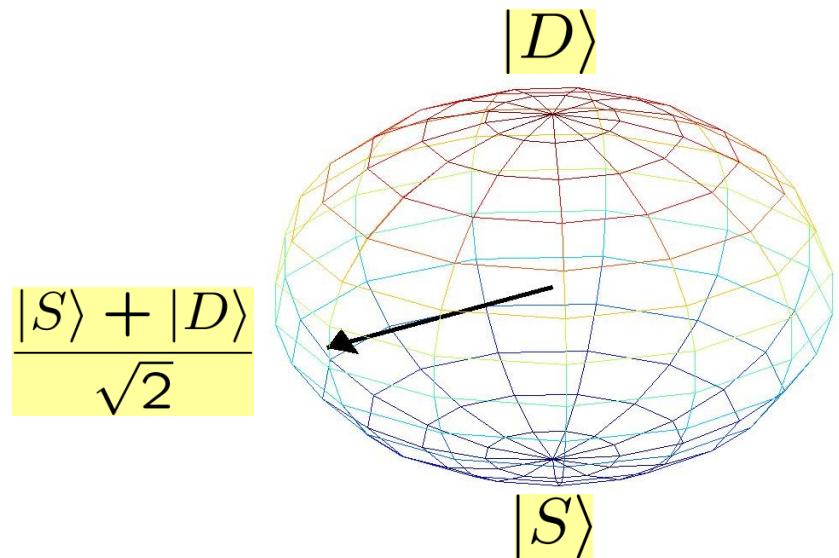
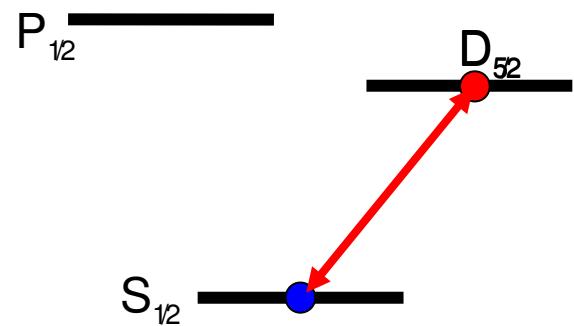
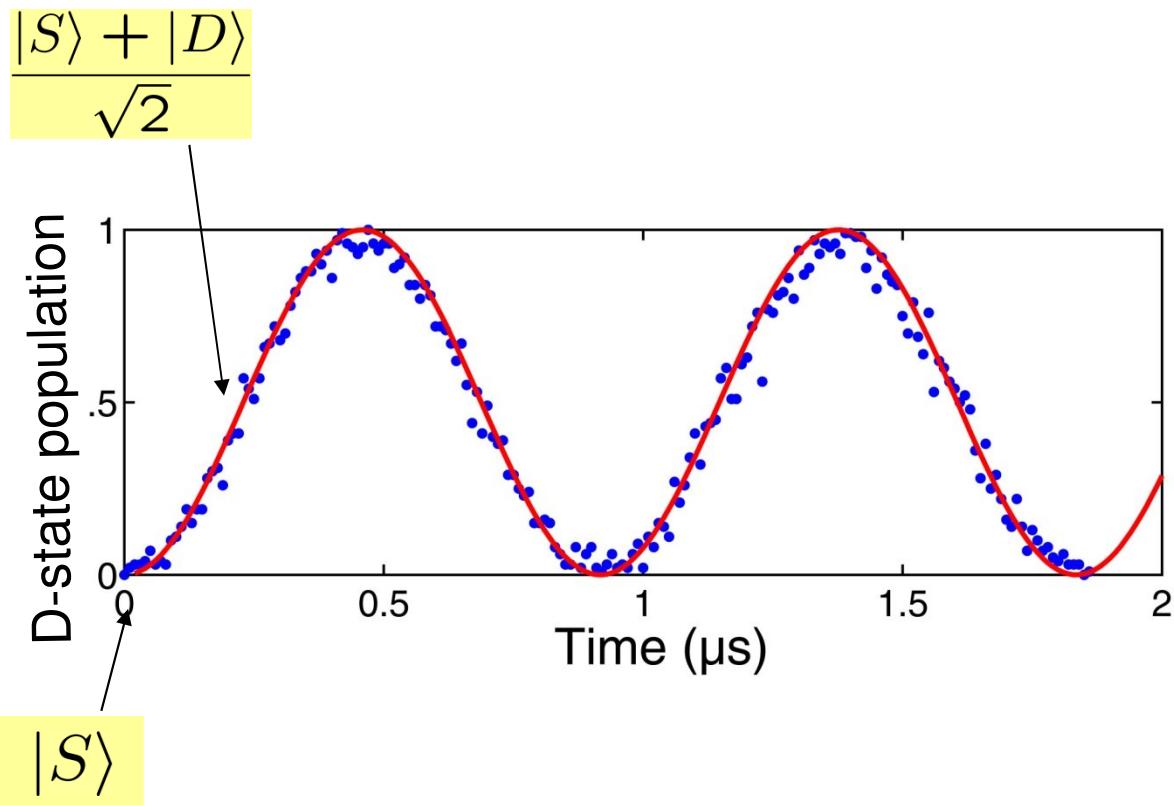
Rabi oscillations



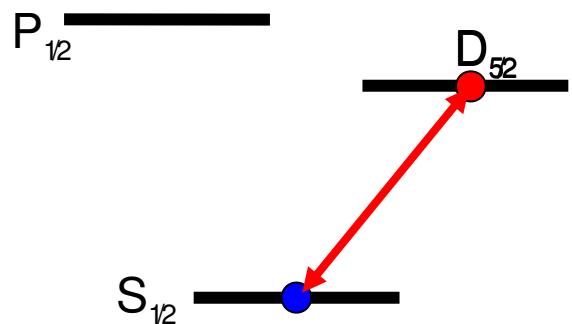
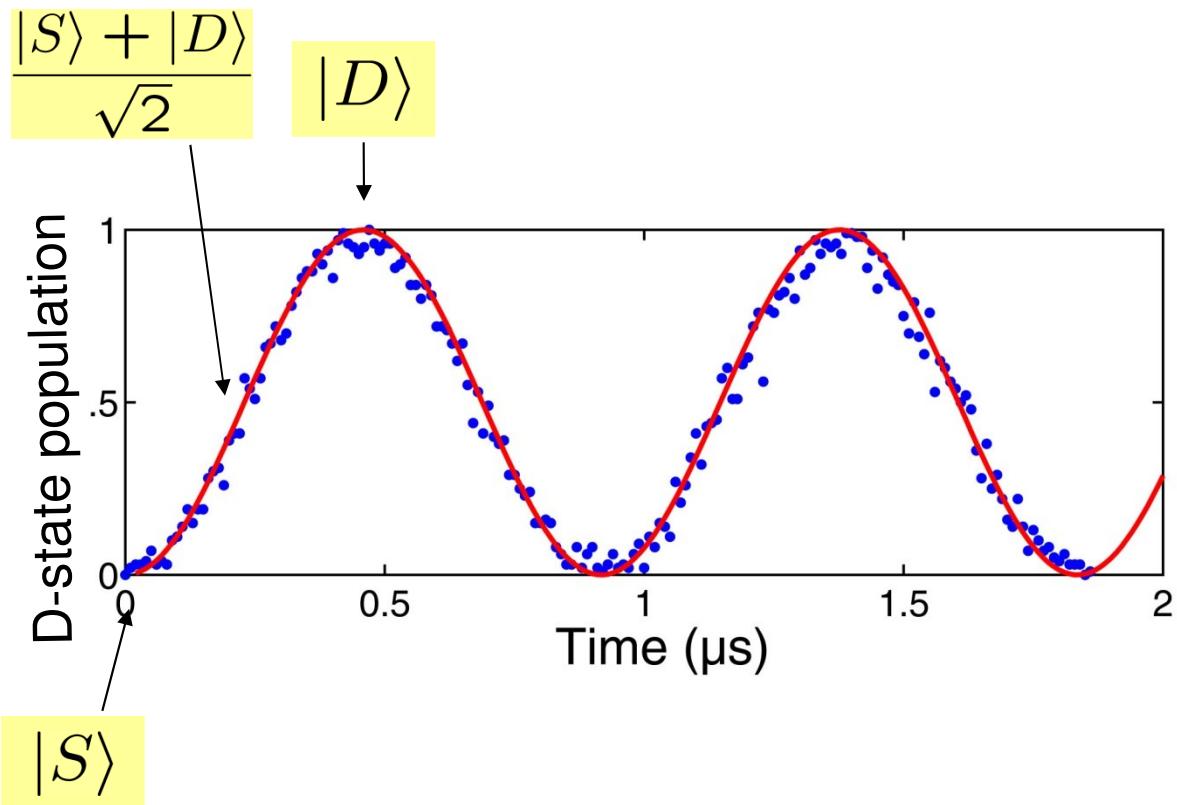
$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$



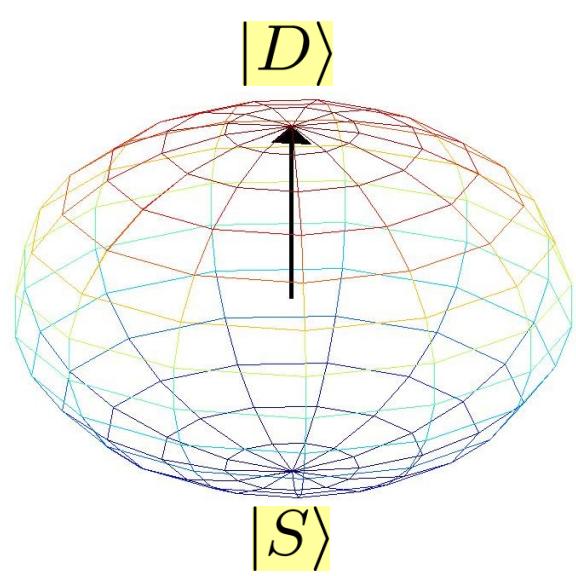
Rabi oscillations



Rabi oscillations



$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$



Requirements for quantum computing

Classical computer

- Initialization
- 1-bit operations (NOT)
- 2-bit gates (e.g. NAND)

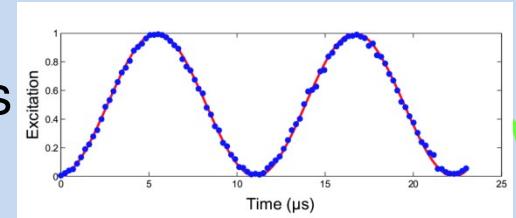
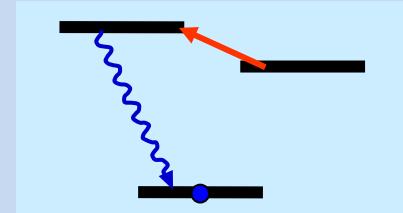
Computational space:

00
01
10
11

- Read out
→ result

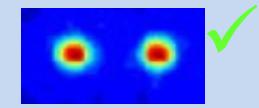
Quantum computer

- Initialization
- 1-qubit rotations
→ superpositions
- 2-qubit gates (CNOT gate)
→ entanglement



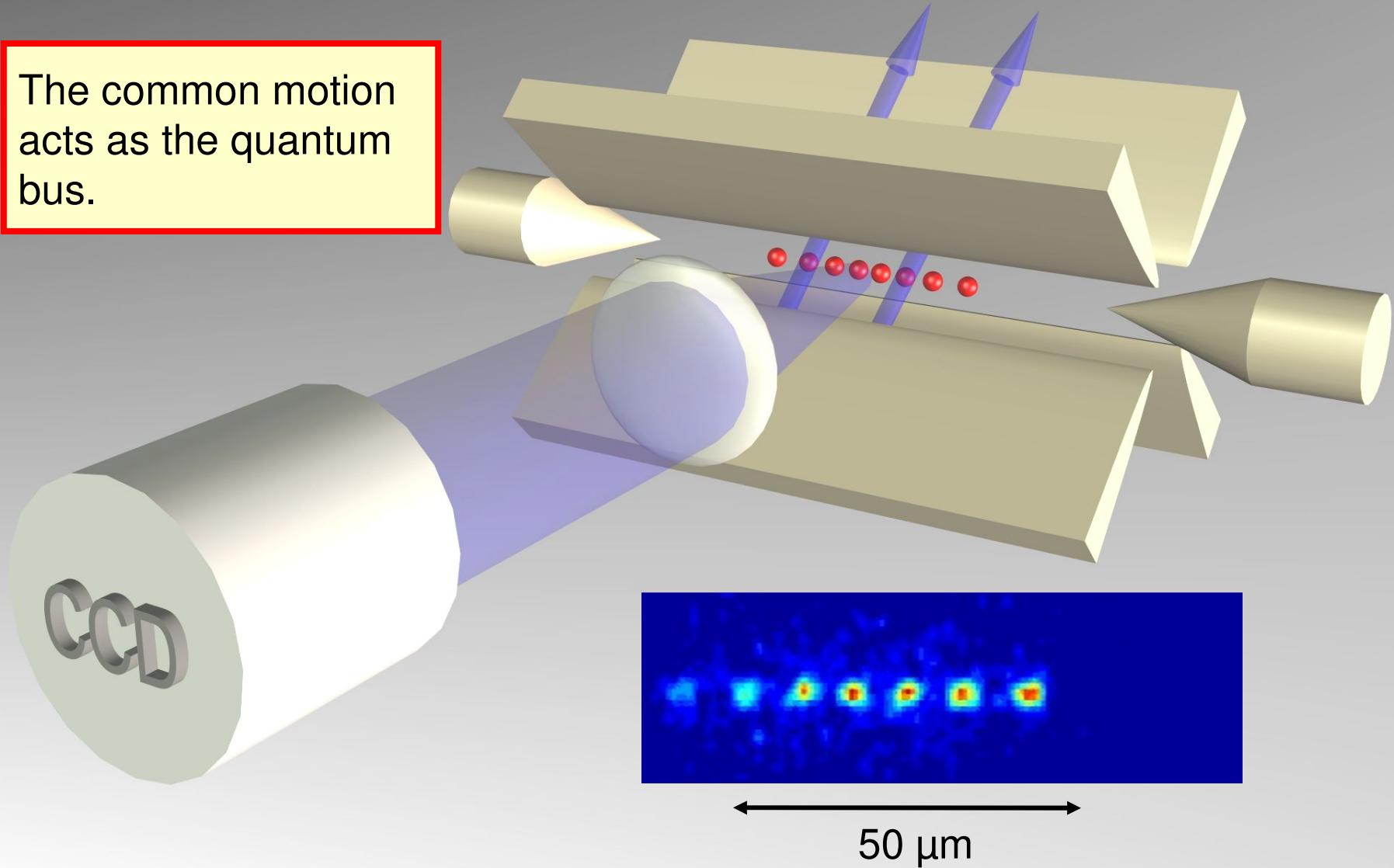
Computational space:

- Read out of qubits
→ gain of classical information

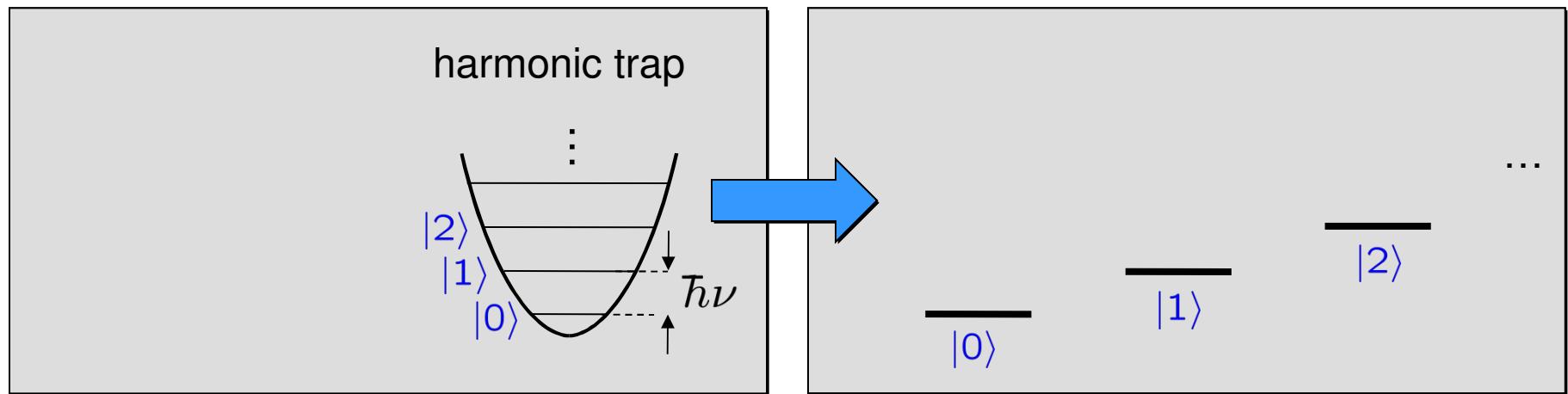


Having the qubits interact

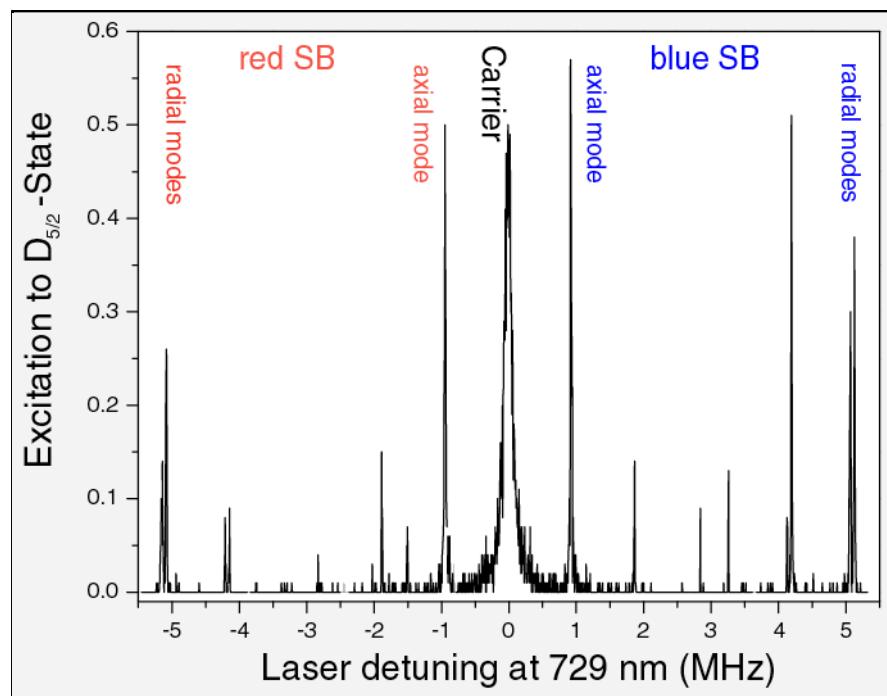
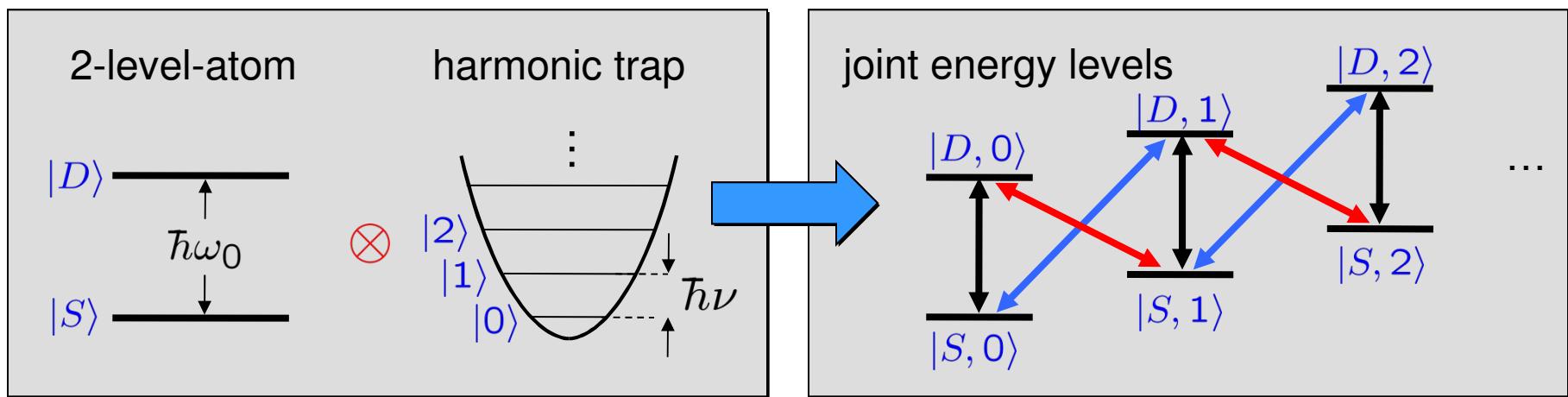
The common motion acts as the quantum bus.



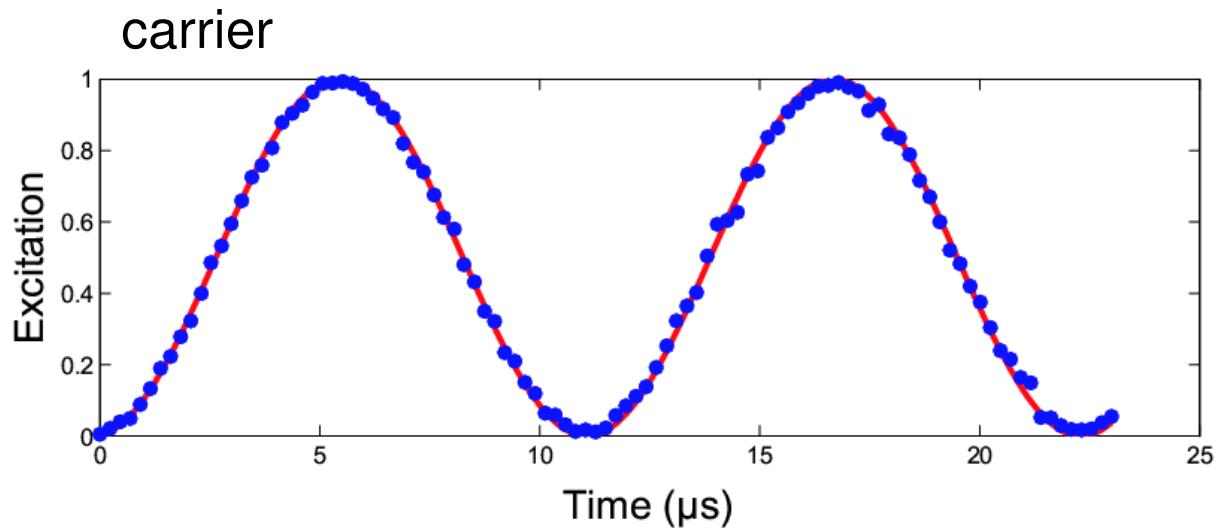
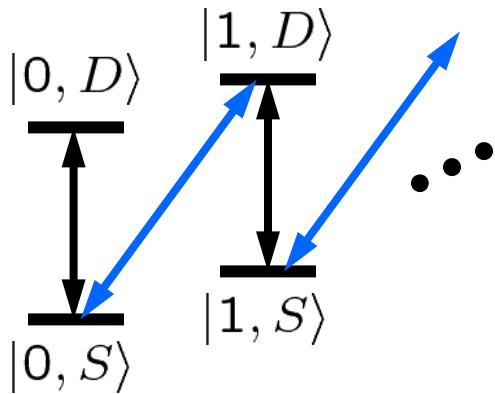
Ion motion



Ion motion

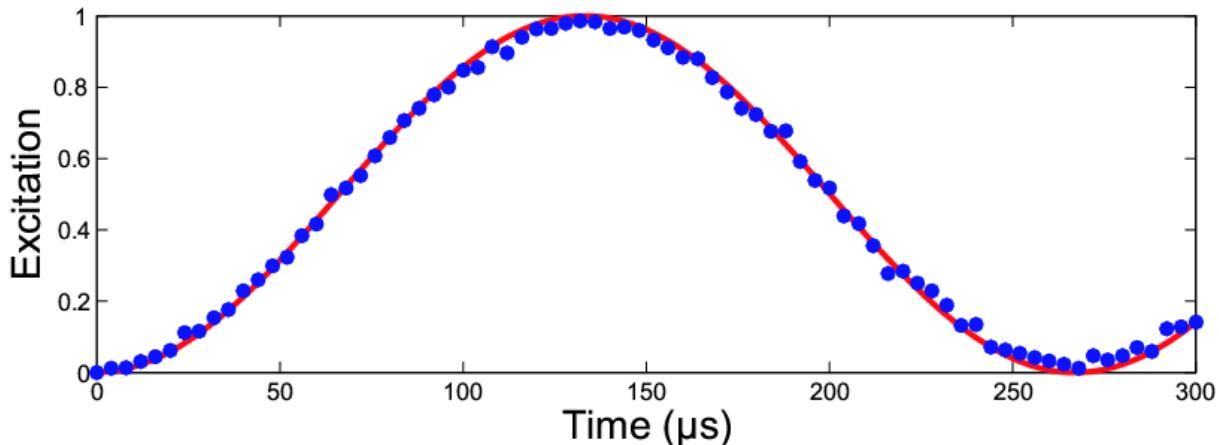


Coherent manipulation



carrier and sideband
Rabi oscillations
with Rabi frequencies

$\Omega, \eta\Omega$

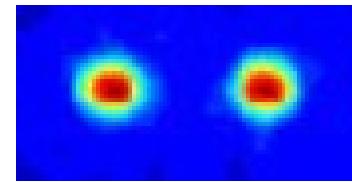


$\eta = kx_0$ Lamb-Dicke parameter

Generation of Bell states

$|DD1\rangle$ 

$|DD0\rangle$ 



$|SD1\rangle$ 

$|SD0\rangle$

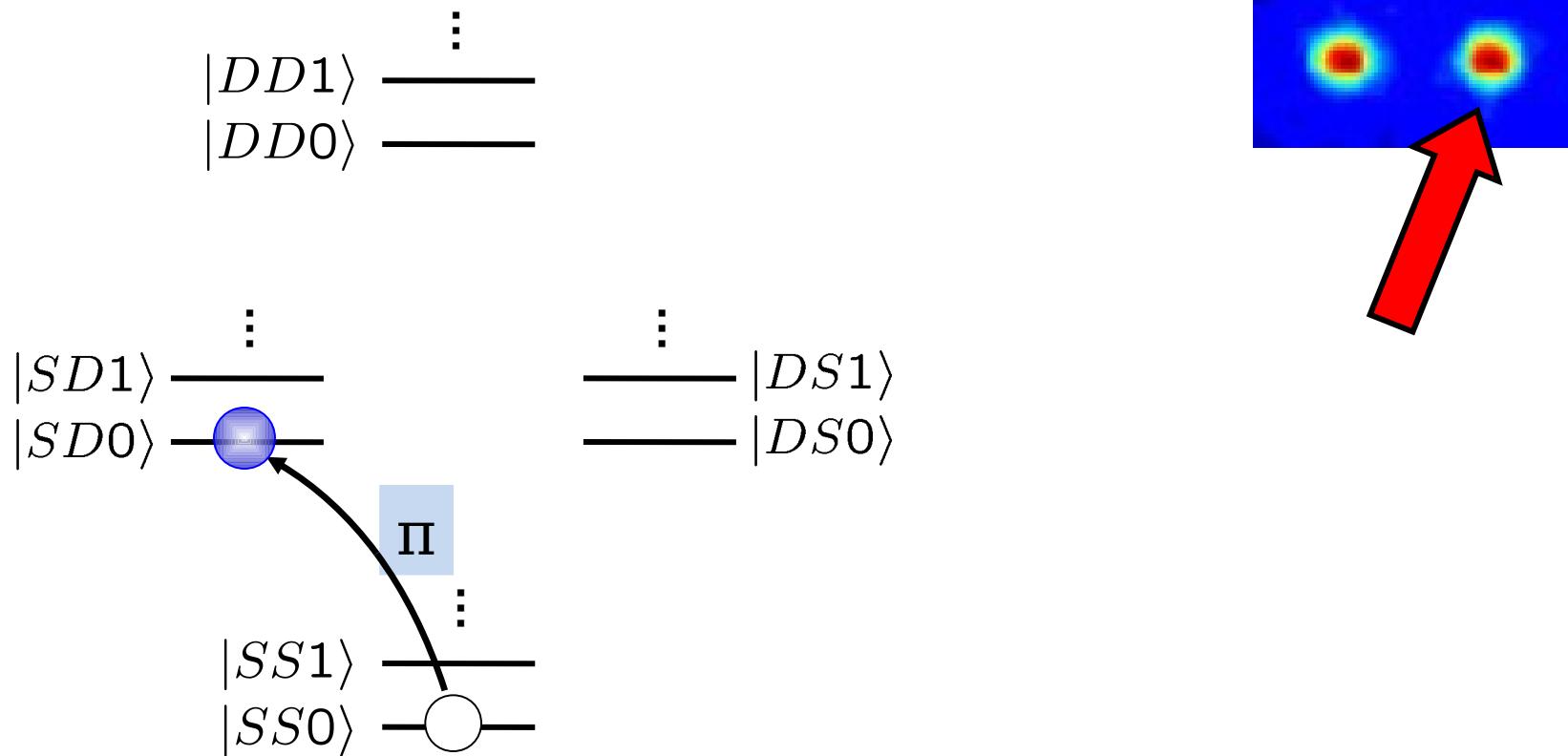
 $|DS1\rangle$

 $|DS0\rangle$

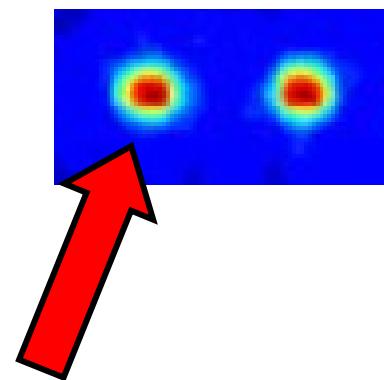
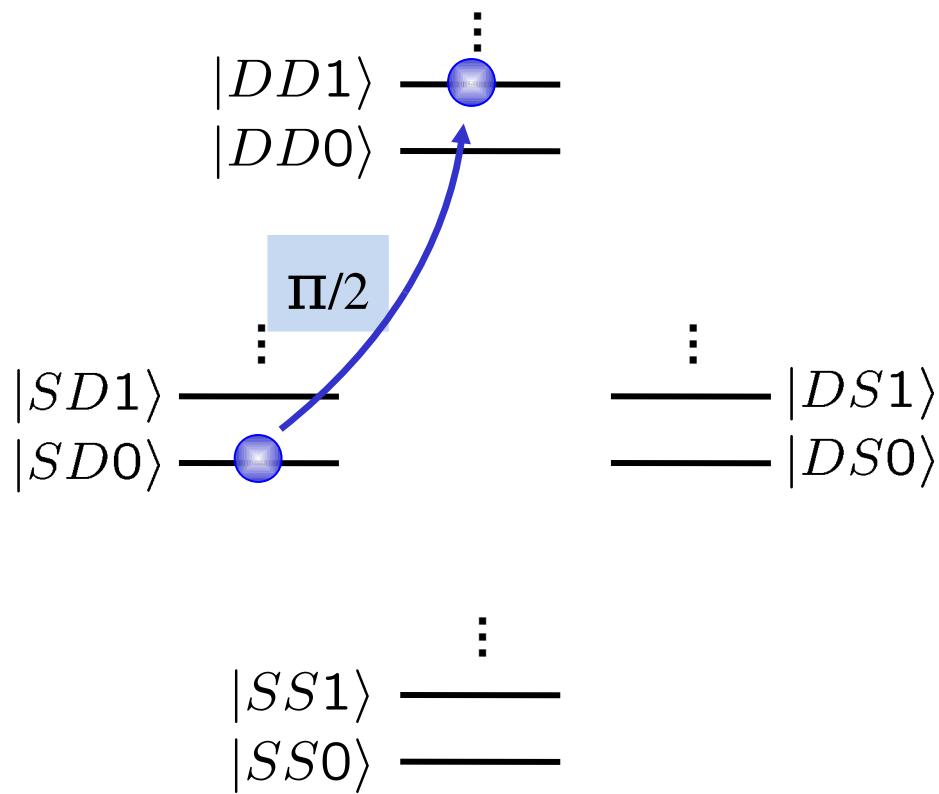
$|SS1\rangle$ 

$|SS0\rangle$ 

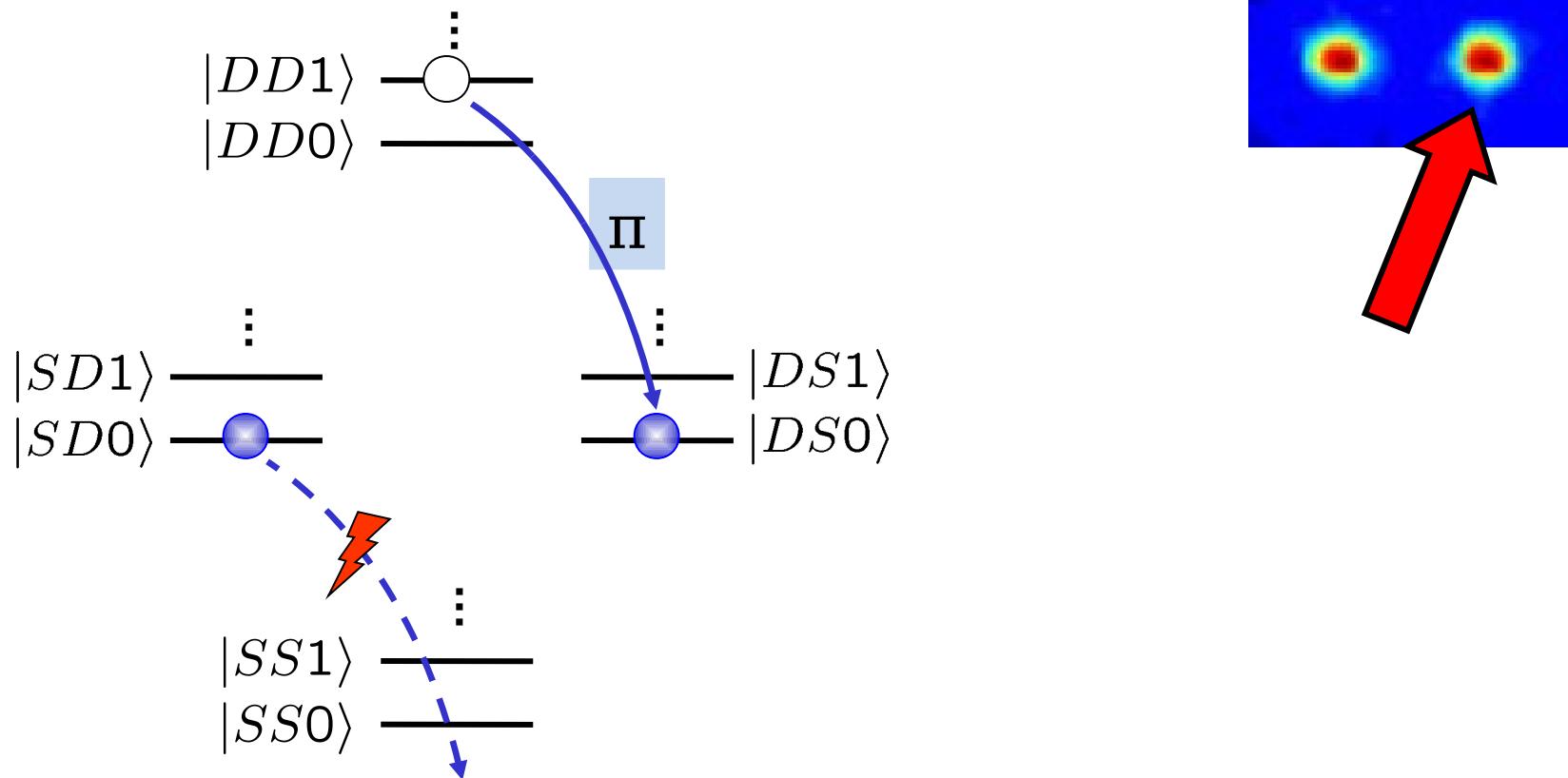
Generation of Bell states



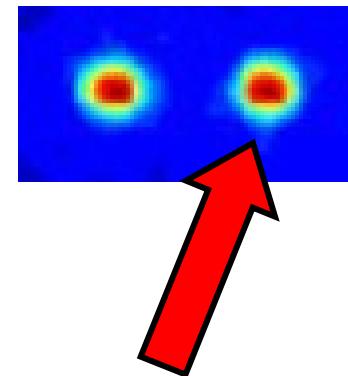
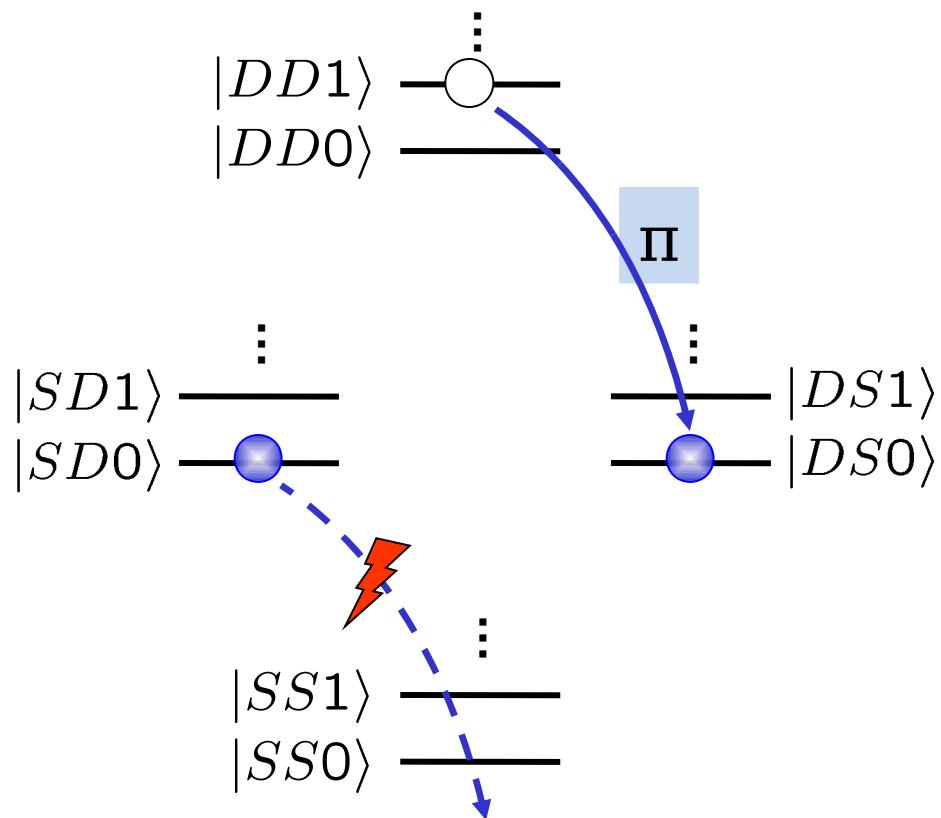
Generation of Bell states



Generation of Bell states



Generation of Bell states



Bell states with atoms

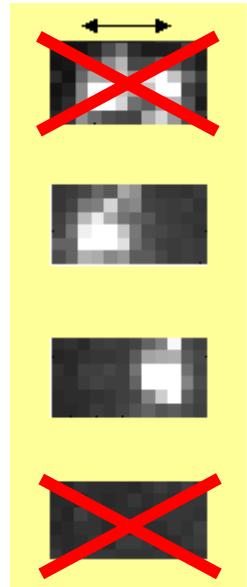
- ${}^9\text{Be}^+$: NIST (fidelity: 97 %)
- ${}^{40}\text{Ca}^+$: Oxford (83%)
- ${}^{111}\text{Cd}^+$: Ann Arbor (79%)
- ${}^{25}\text{Mg}^+$: Munich (97%)
- ${}^{40}\text{Ca}^+$: Innsbruck (99%)

Analysis of Bell states

$$|SD\rangle + |DS\rangle$$

Fluorescence
detection with
CCD camera:

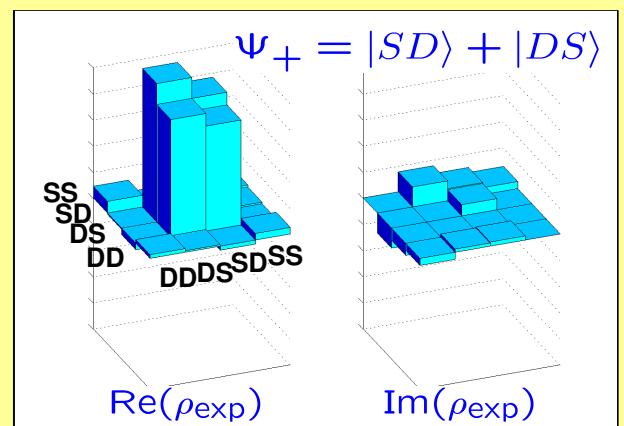
$$\left\{ \begin{array}{l} |SS\rangle \\ |SD\rangle \\ |DS\rangle \\ |DD\rangle \end{array} \right.$$



Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

→ Measurement of the density matrix:

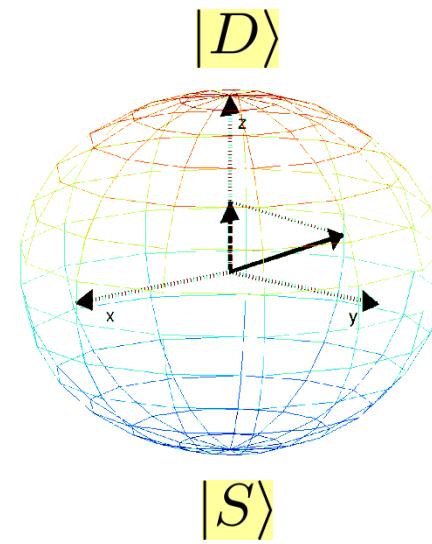


Measuring a density matrix

A measurement yields the z -component of the Bloch vector

=> Diagonal of the density matrix

$$\rho = \begin{pmatrix} P_S & C - iD \\ C + iD & P_D \end{pmatrix}$$

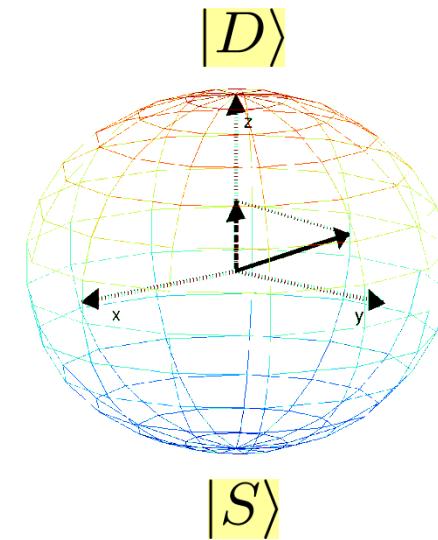


Measuring a density matrix

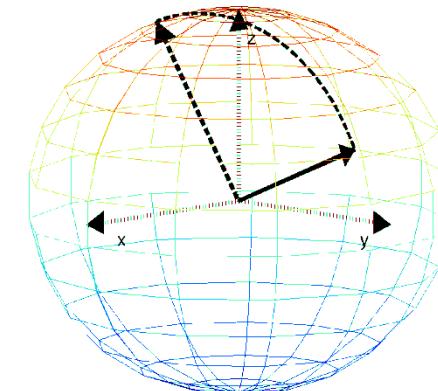
A measurement yields the z -component of the Bloch vector

=> Diagonal of the density matrix

$$\rho = \begin{pmatrix} P_S & C - iD \\ C + iD & P_D \end{pmatrix}$$



Rotation around the x - or the y -axis prior to the measurement yields the phase information of the qubit.

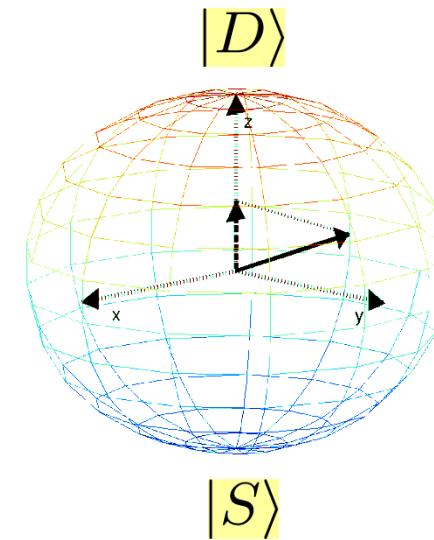


Measuring a density matrix

A measurement yields the z -component of the Bloch vector

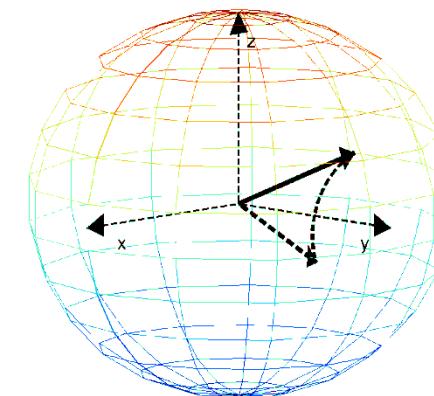
=> Diagonal of the density matrix

$$\rho = \begin{pmatrix} P_S & \mathcal{C} - iD \\ \mathcal{C} + iD & P_D \end{pmatrix}$$

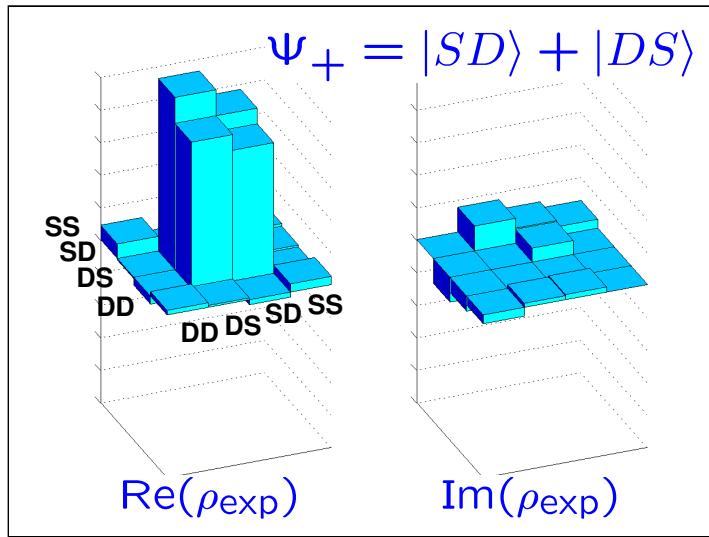


Rotation around the x - or the y -axis prior to the measurement yields the phase information of the qubit.

=> coherences of the density matrix

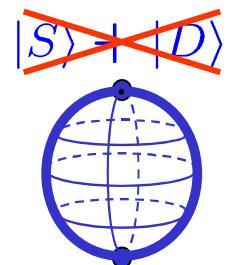
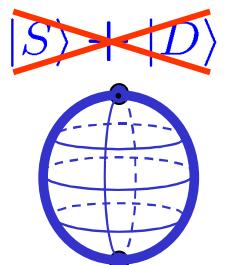


Entanglement



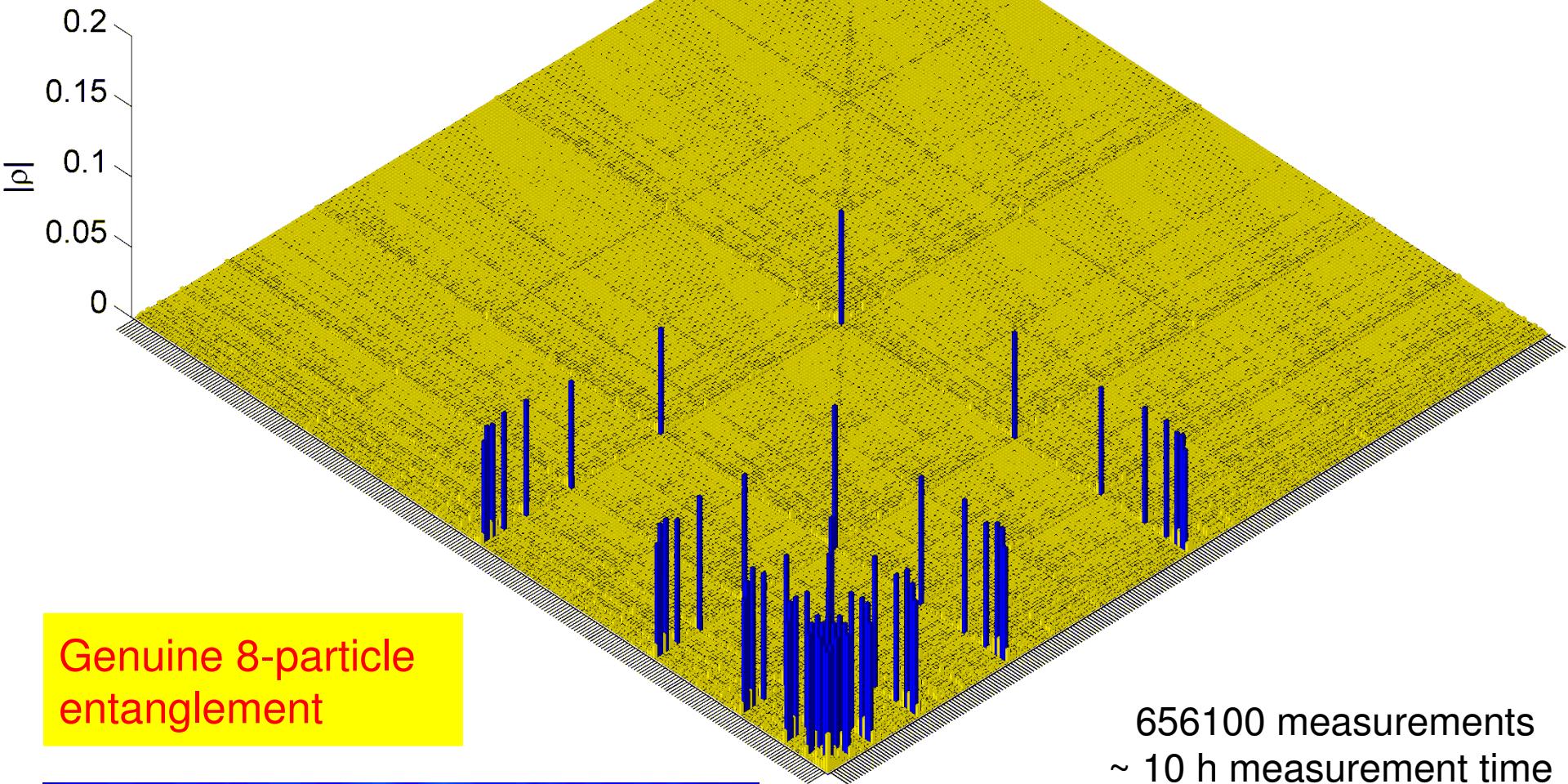
States are fully characterized
Fidelities: up to 0.99

Entanglement: the state of each qubit is not defined!

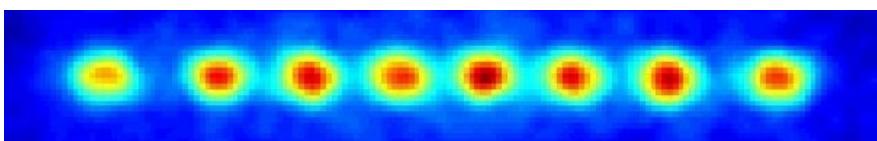


“Large” entangled states

$$\frac{1}{\sqrt{8}}(|DDDDDDDDS\rangle + |DDDDDDDS\rangle + \dots + |SDDDDDDD\rangle)$$



Genuine 8-particle entanglement

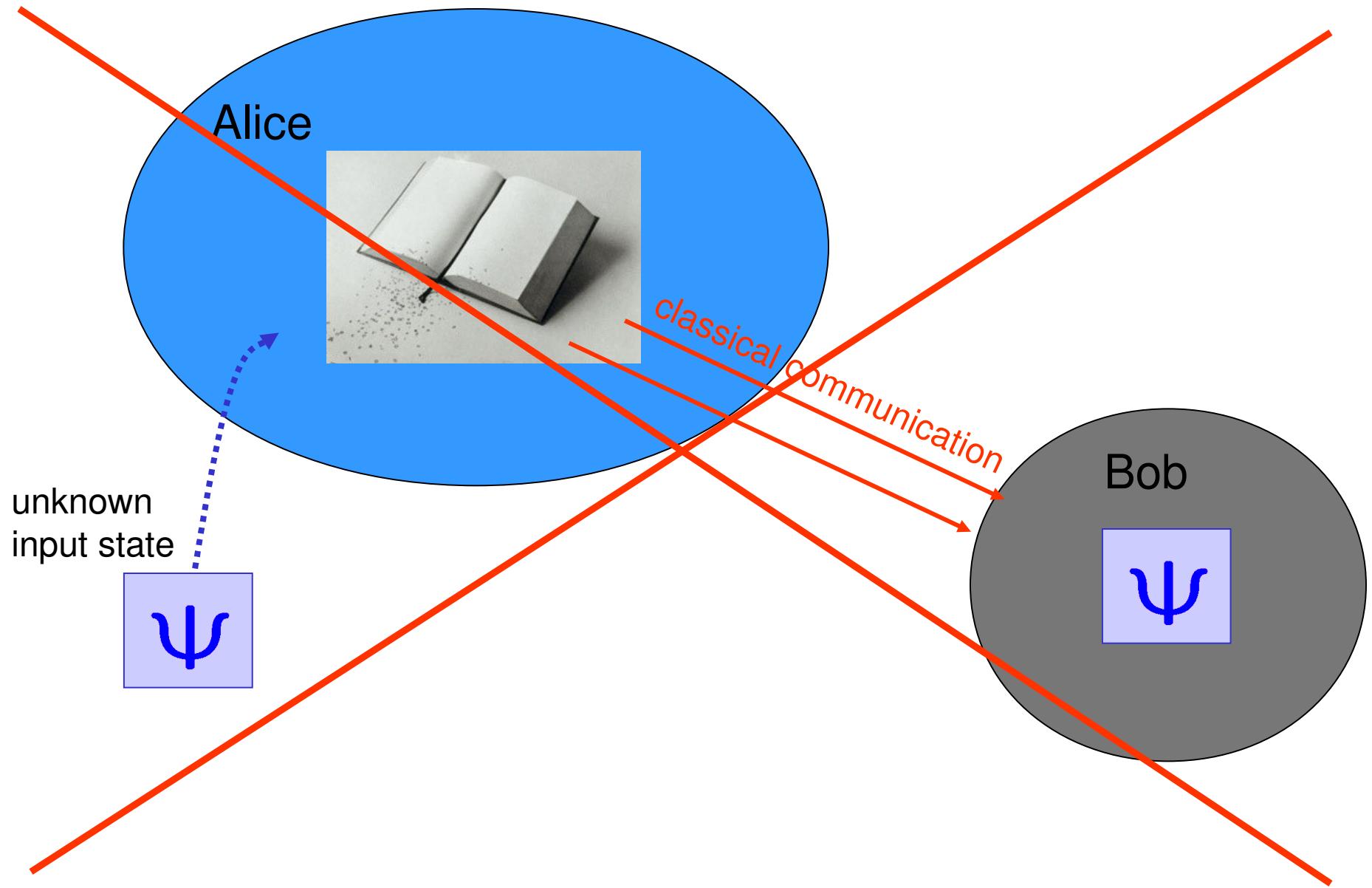


656100 measurements
~ 10 h measurement time

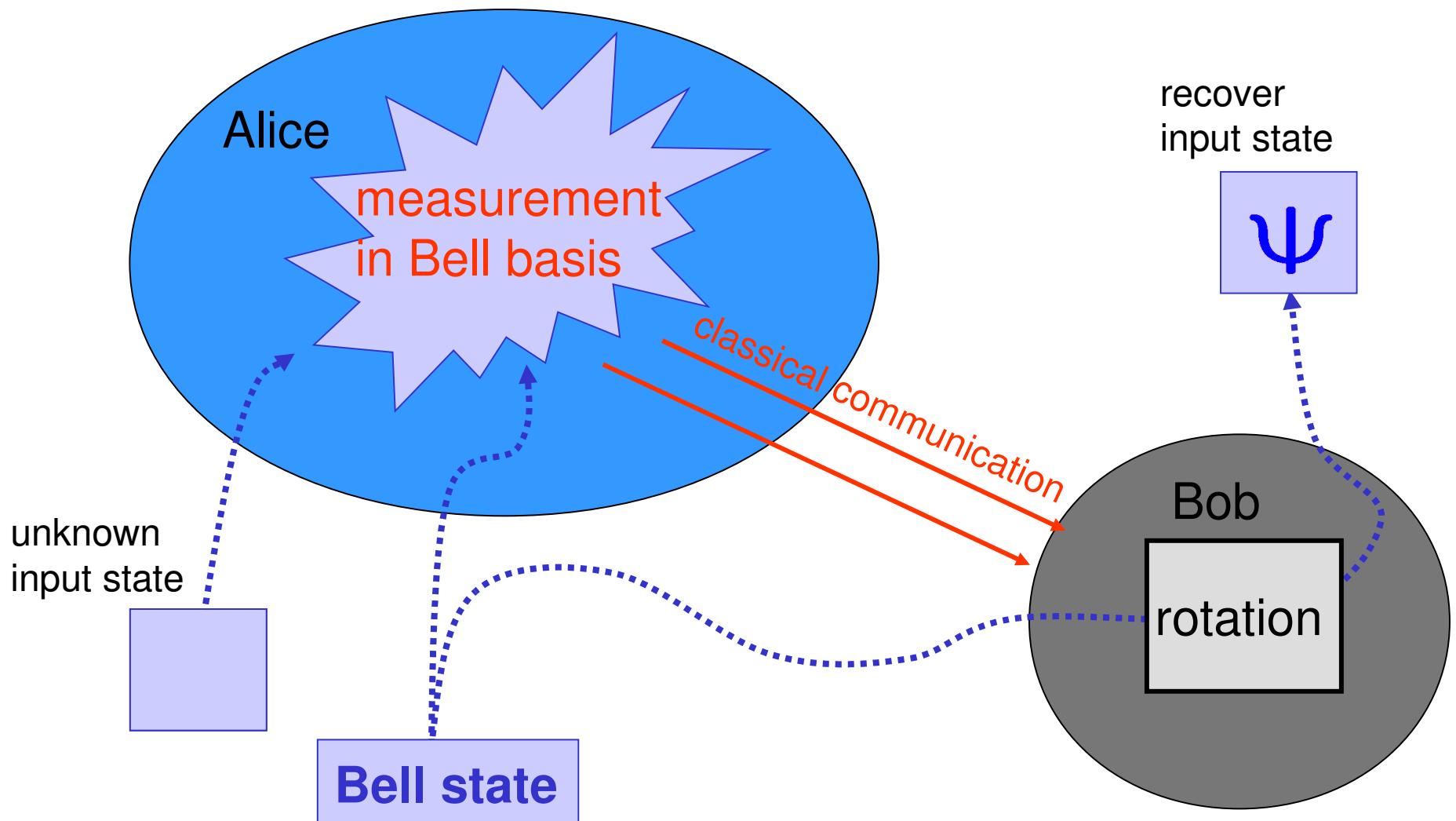
Häffner et al., Nature 438, 643 (2005)

- Physics and information
- Ion trap quantum computing
- Teleportation
- Scaling of ion trap quantum computers

Teleportation

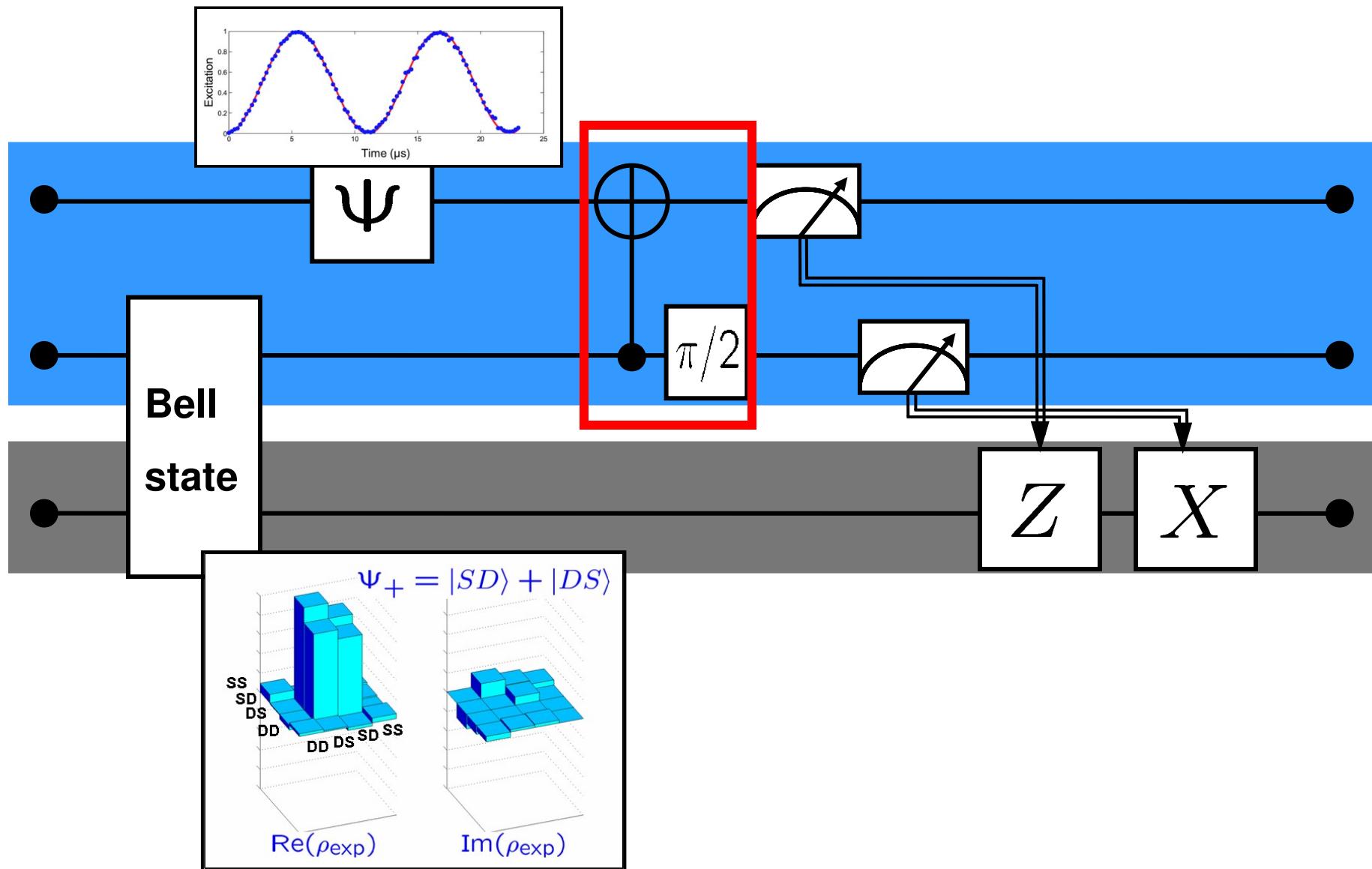


Teleportation

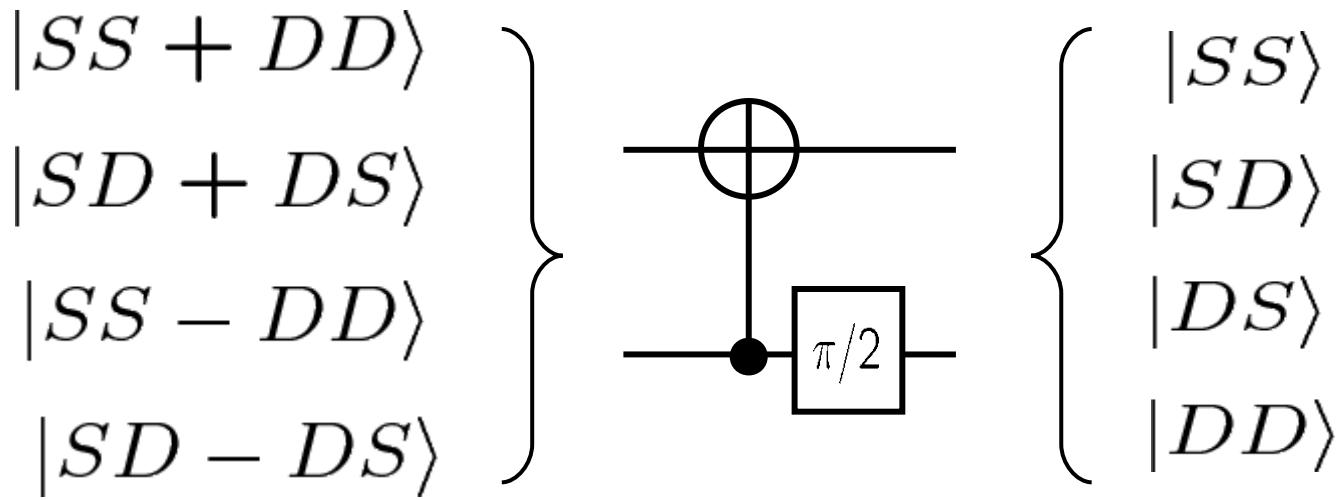


Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).
Bouwmeester *et al.*, Nature **390**, 575 (1997).

Teleportation



Bell measurement



Ion trap quantum computing

VOLUME 74, NUMBER 20

PHYSICAL REVIEW LETTERS

15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

PACS numbers: 89.80.+h, 03.65.Bz, 12.20.Fv, 32.80.Pj

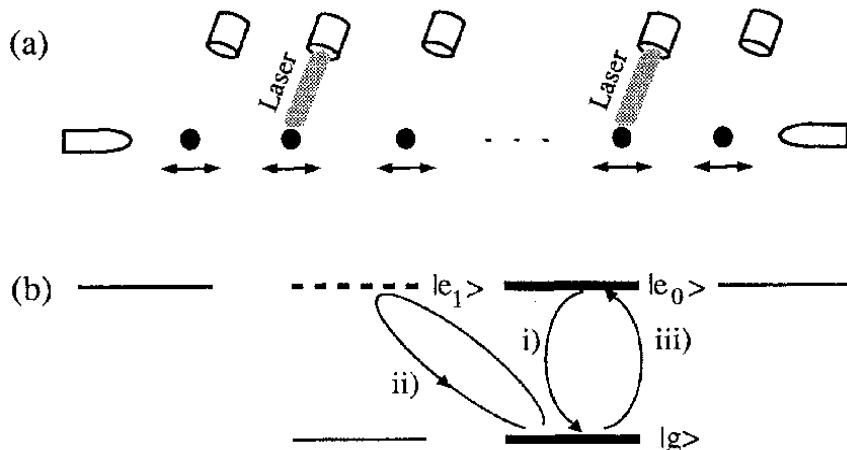


FIG. 1. (a) N ions in a linear trap interacting with N different laser beams; (b) atomic level scheme.

controlled – NOT :

$$|\varepsilon_1\rangle|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle|\varepsilon_1 \oplus \varepsilon_2\rangle$$

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$

$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$

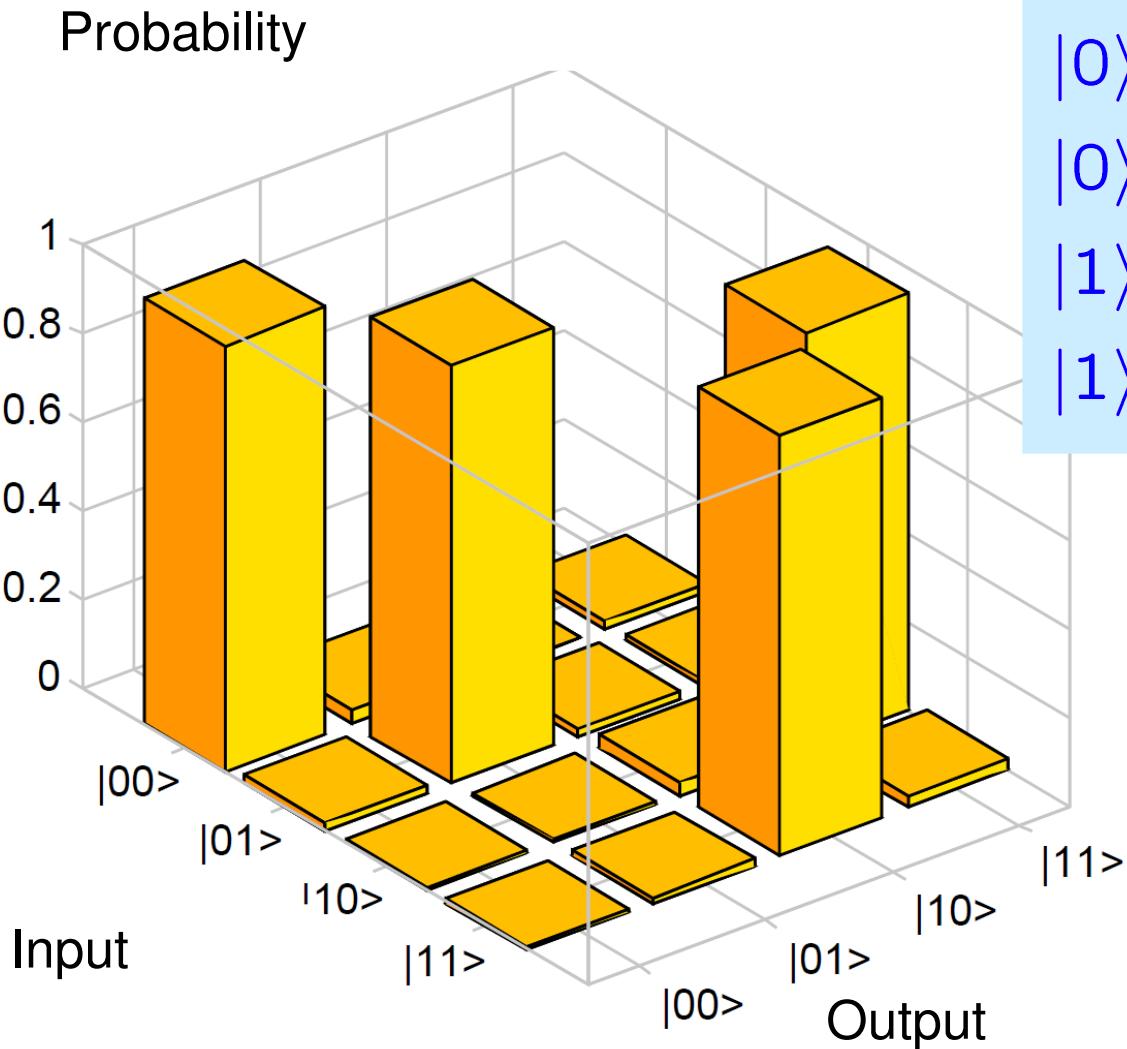
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$

$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

control bit target bit



Truth table of a controlled NOT gate



$ 0\rangle 0\rangle$	\rightarrow	$ 0\rangle 0\rangle$
$ 0\rangle 1\rangle$	\rightarrow	$ 0\rangle 1\rangle$
$ 1\rangle 0\rangle$	\rightarrow	$ 1\rangle 1\rangle$
$ 1\rangle 1\rangle$	\rightarrow	$ 1\rangle 0\rangle$

→ universal set of quantum gates

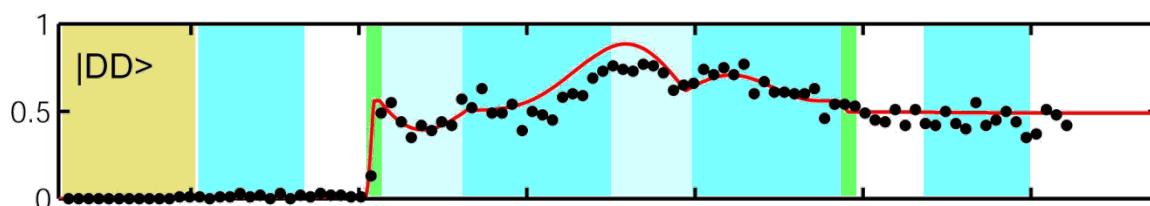
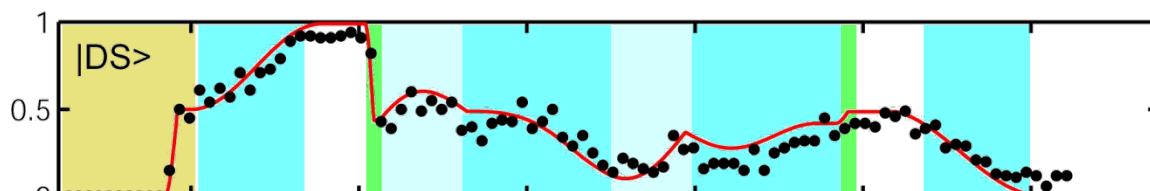
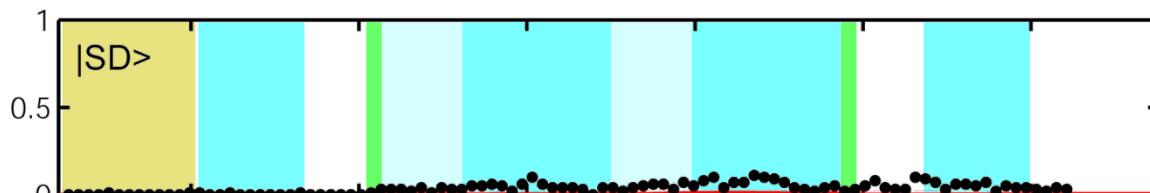
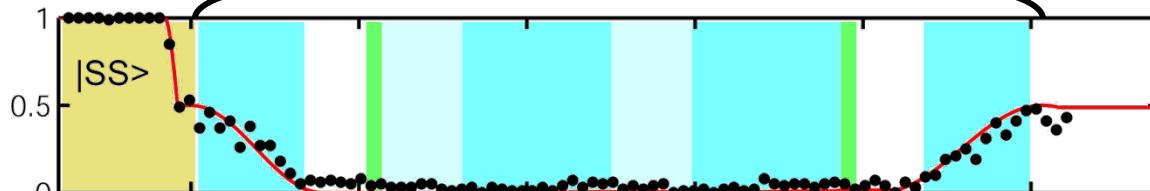
Another way to generate Bell states

$$|SS\rangle \rightarrow |S+D\rangle|S\rangle \quad \xrightarrow{\text{CNOT}} \quad |SS\rangle + |DD\rangle$$

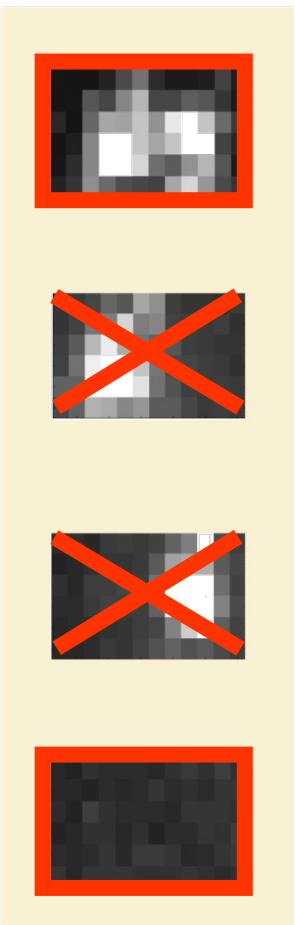
prepare

CNOT

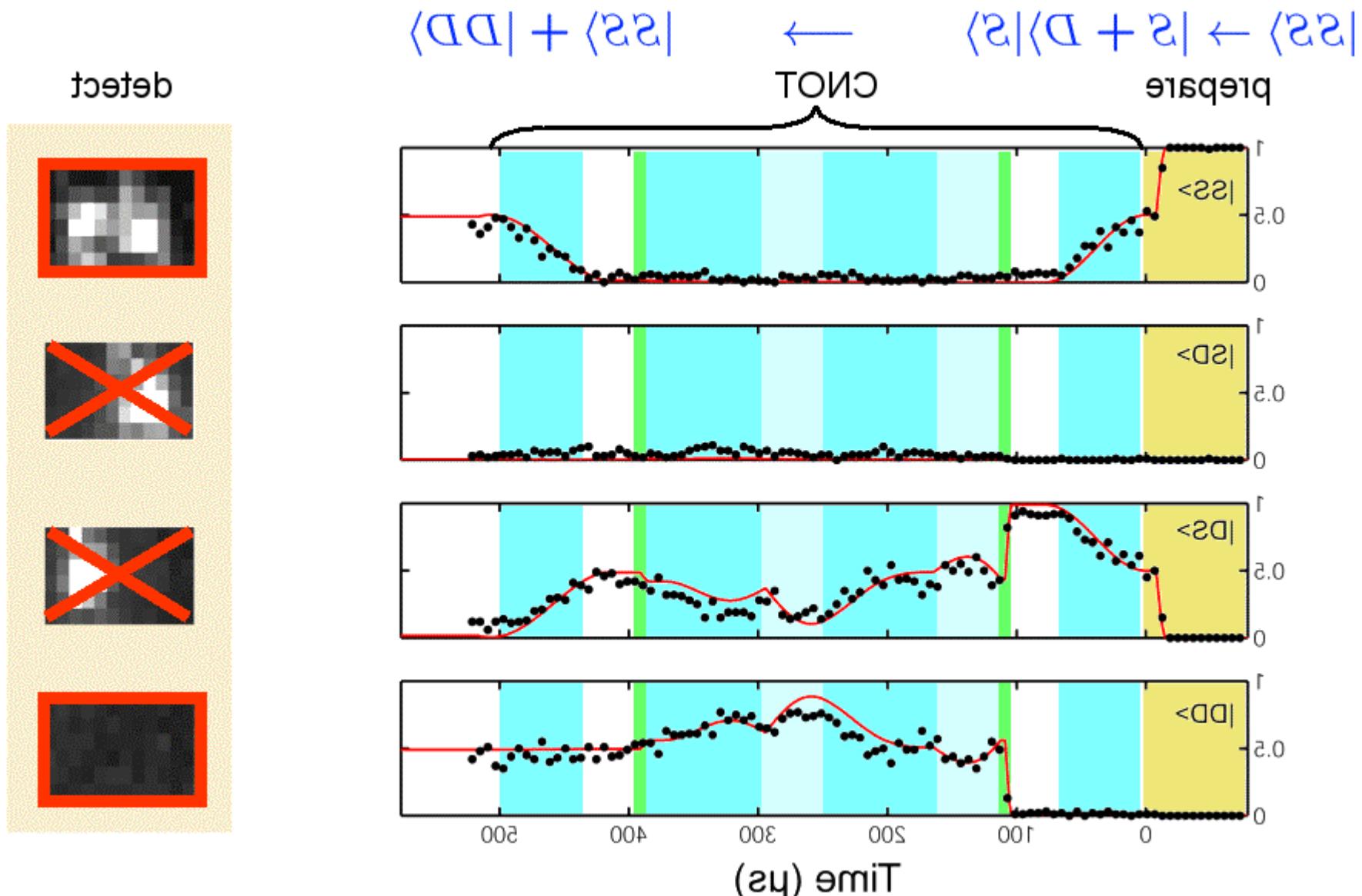
output



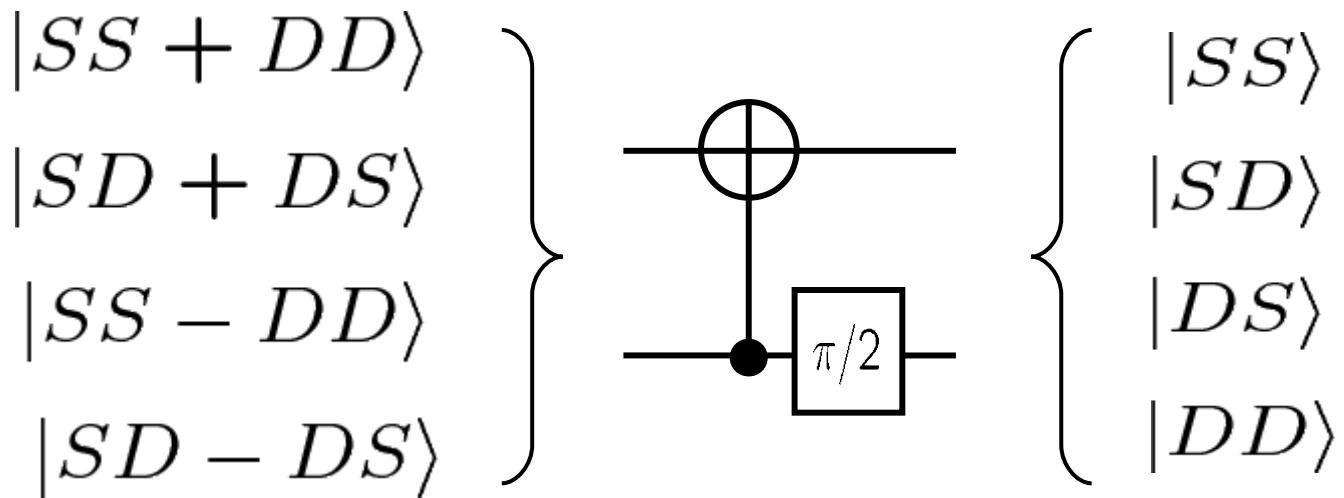
Time (μs)



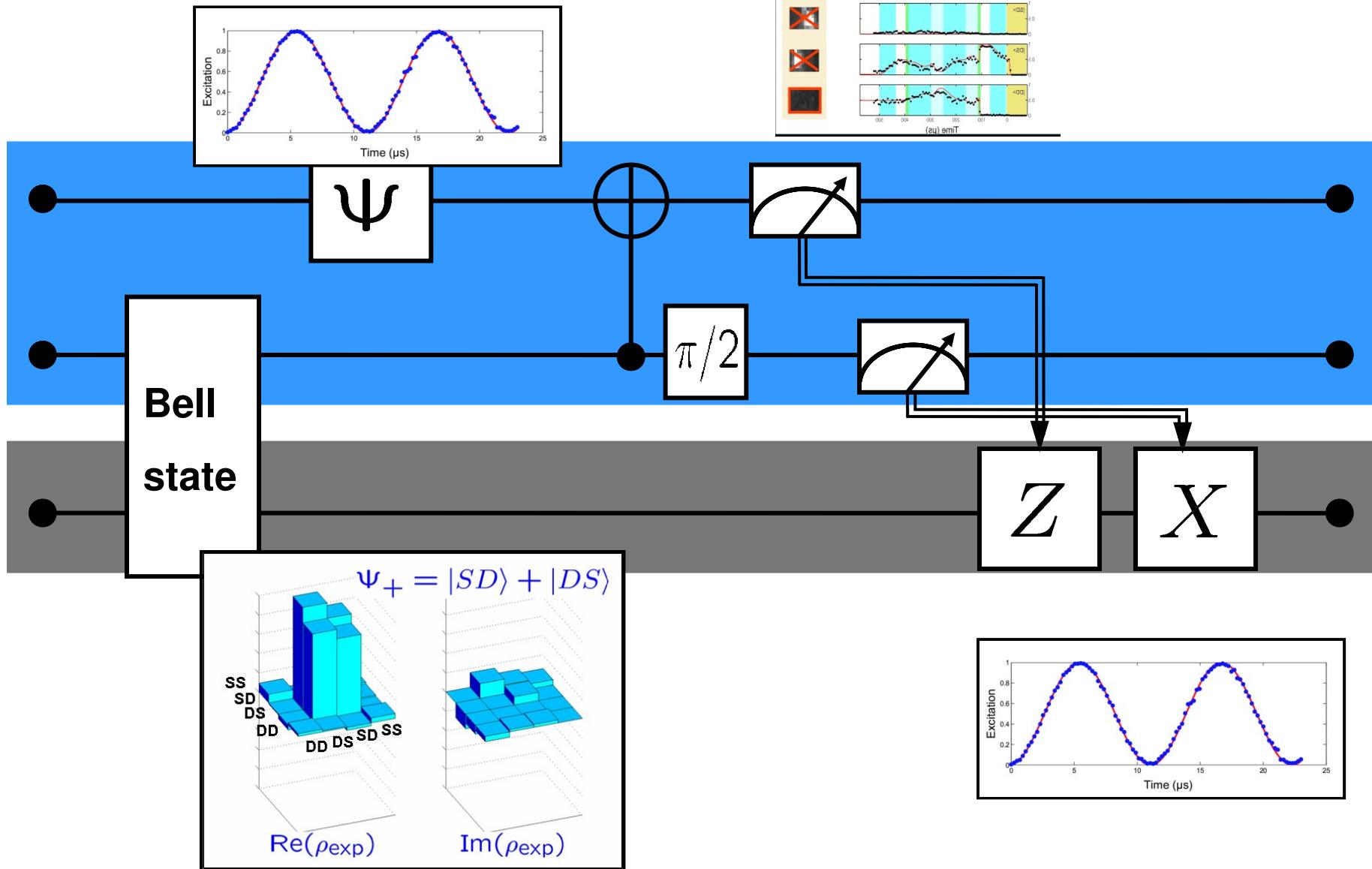
And now backwards



Bell measurement



Teleportation

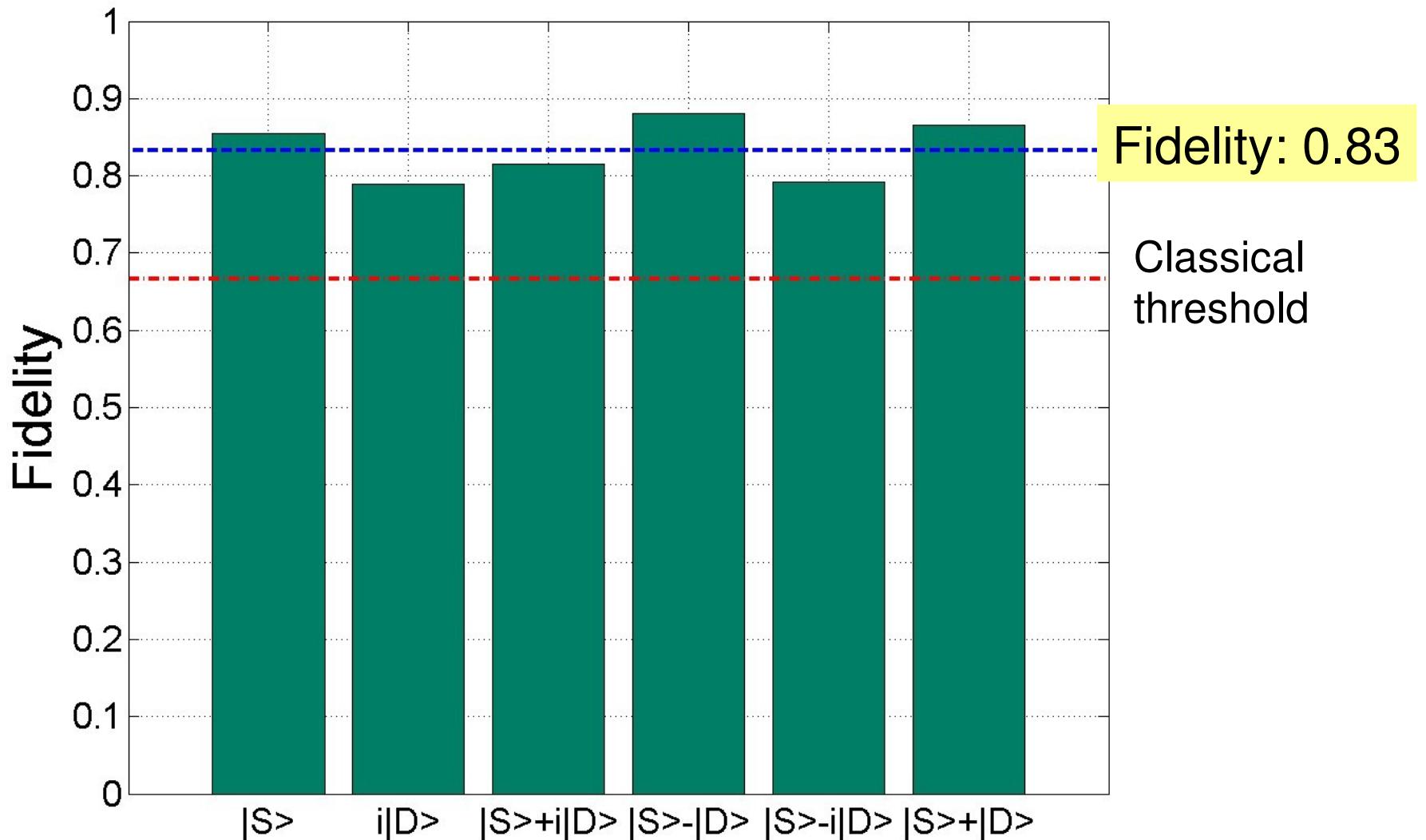


Teleportation analysis

Initial	Input test states $ \Psi\rangle$	Output states $ \Psi\rangle$	Final
$ S\rangle$	$ S\rangle$	$ S\rangle$	$ S\rangle$
$ S\rangle$	$ D\rangle$	$ D\rangle$	$ S\rangle$
$ S\rangle$	$ S\rangle + D\rangle$	$ S\rangle + D\rangle$	$ S\rangle$
$ S\rangle$	$ S\rangle + i D\rangle$	$ S\rangle + i D\rangle$	$ S\rangle$
$ S\rangle$	$ S\rangle - D\rangle$	$ S\rangle - D\rangle$	$ S\rangle$
$ S\rangle$	$ S\rangle - i D\rangle$	$ S\rangle - i D\rangle$	$ S\rangle$

{ **Ion #1** { **Ion #3**

Deterministic teleportation



“Deterministic teleportation with atoms”

Barrett et al., Nature 429, 737 (2004) and Riebe et al., Nature 429, 734 (2004)

Requirements for quantum computing

Classical computer

- Initialization
- 1-bit operations (NOT)
- 2-bit gates (e.g. NAND)

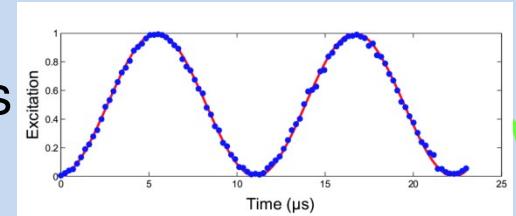
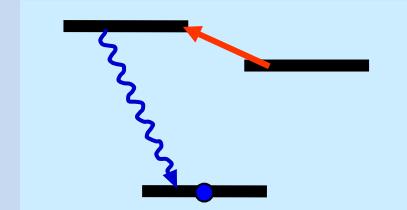
Computational space:

00
01
10
11

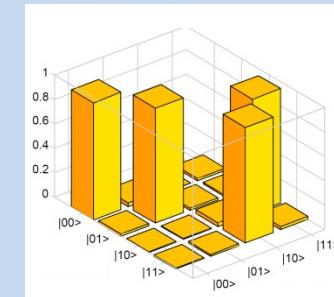
- Read out
→ result

Quantum computer

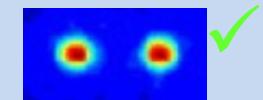
- Initialization
- 1-qubit rotations
→ superpositions
- 2-qubit gates
(CNOT gate)
→ entanglement



Computational space:

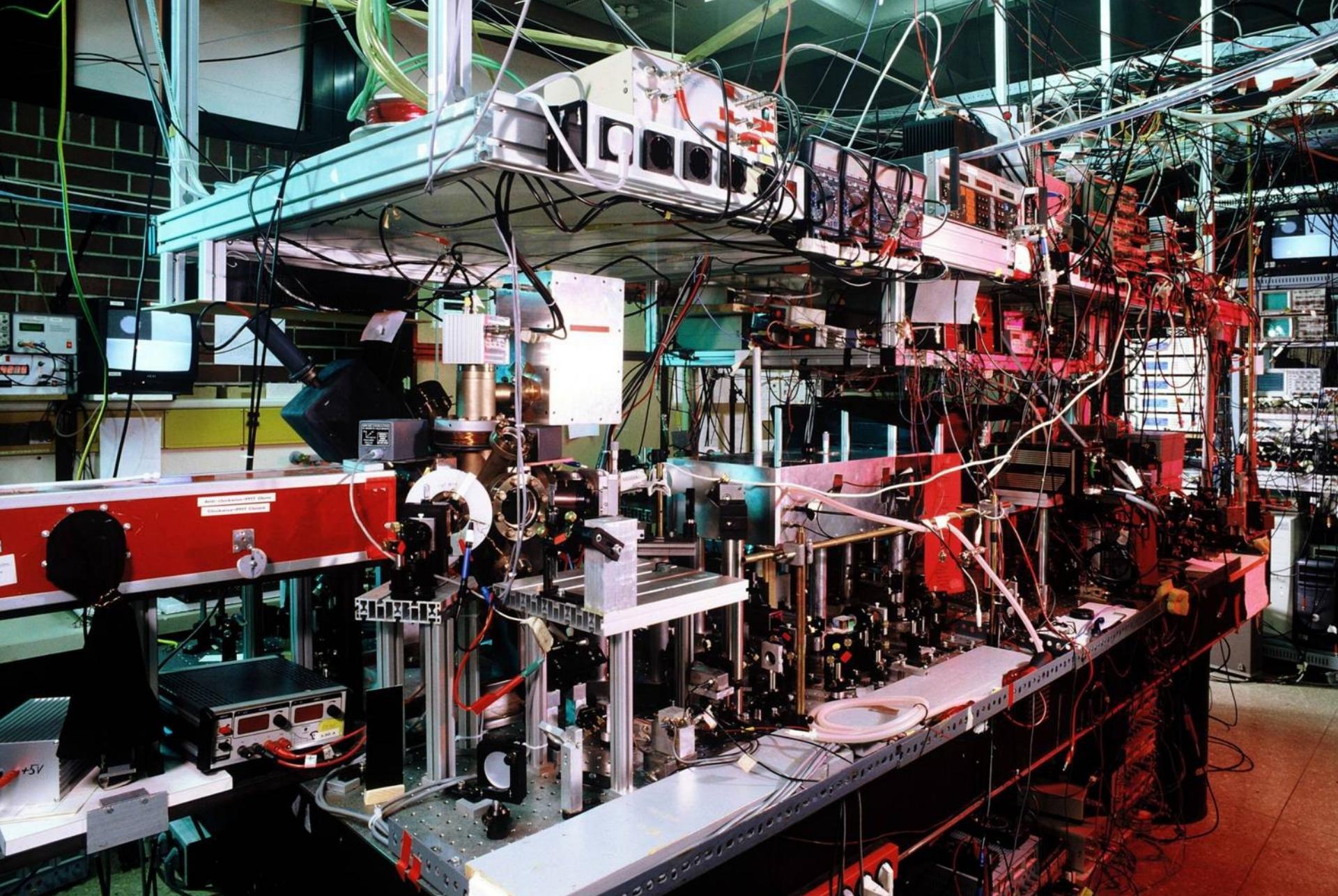


- Read out of qubits
→ gain of classical information

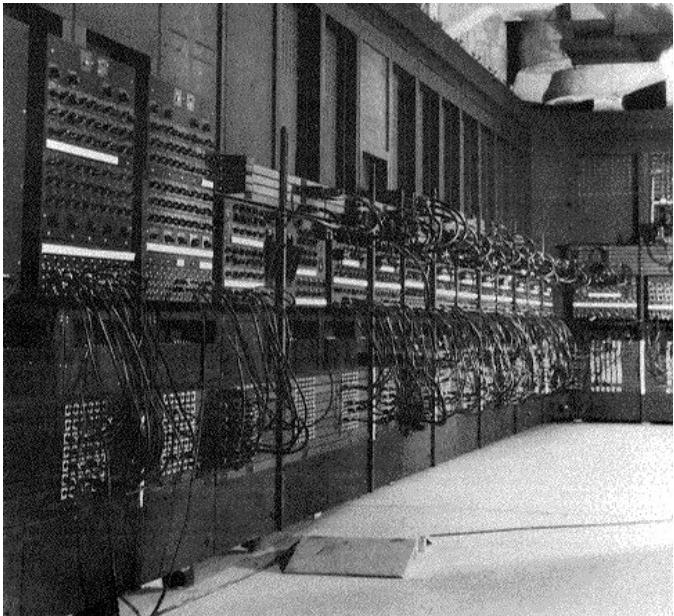


- Physics and information
- Ion trap quantum computing
- Teleportation
- Scaling of ion trap quantum computers

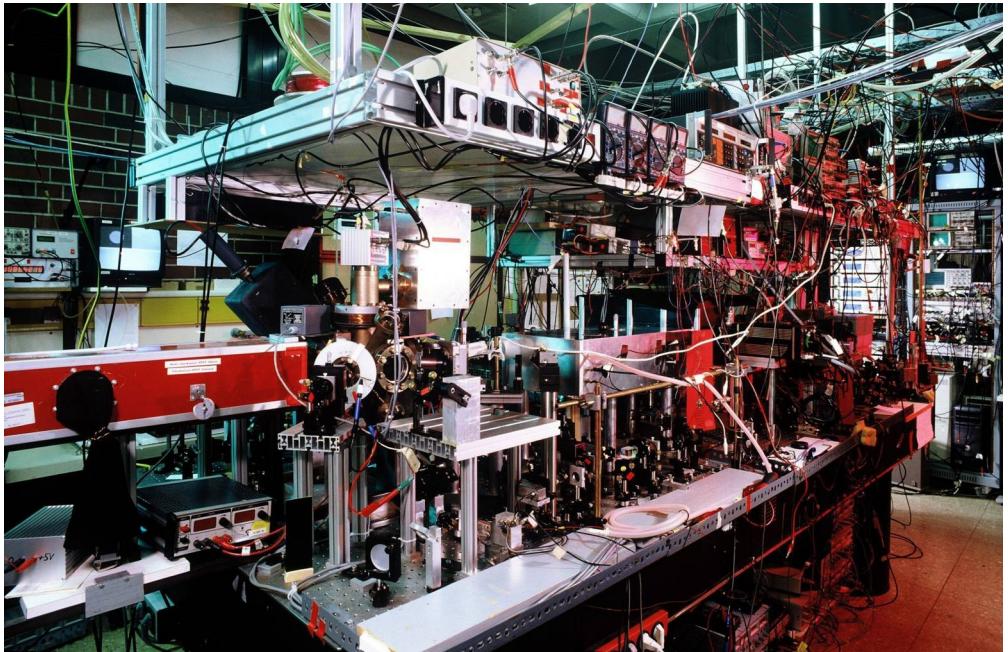
The hardware



The hardware

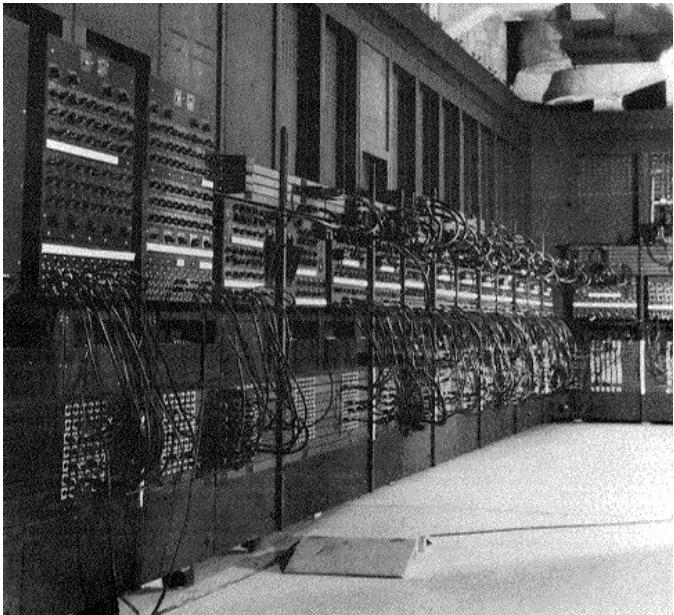


ENIAC, 1950

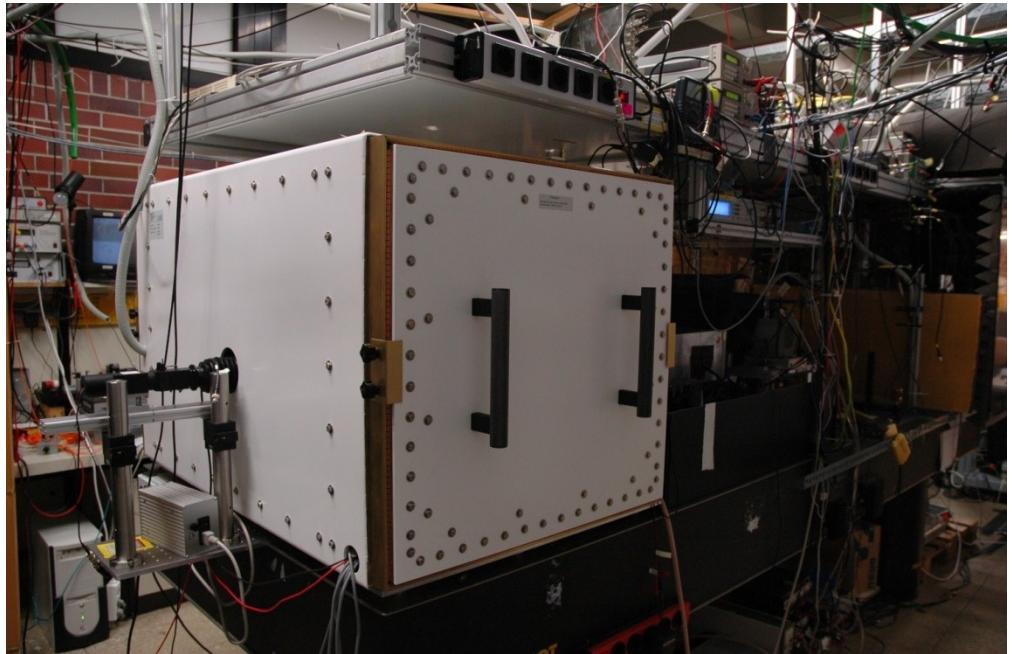


Innsbruck quantum computer, 2005

The hardware



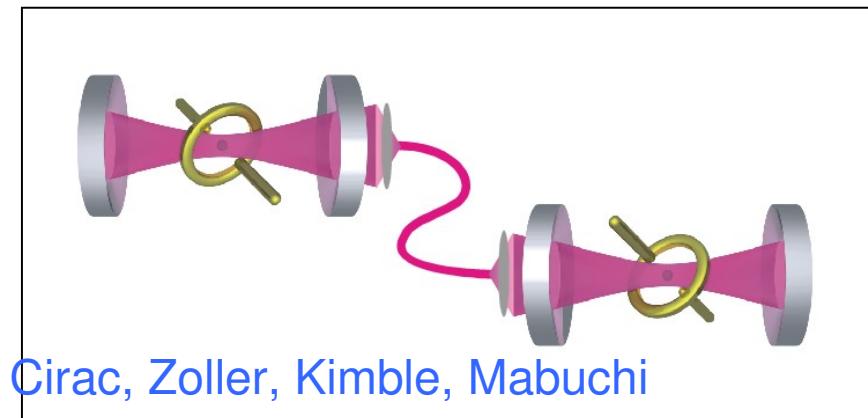
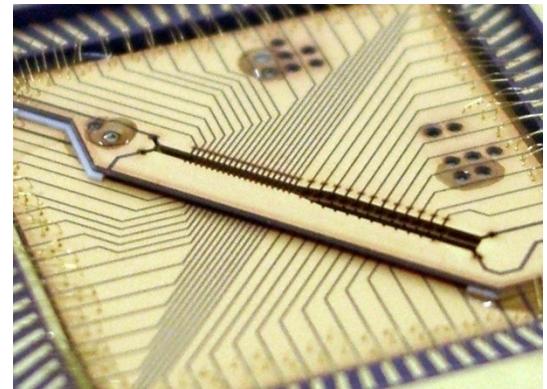
ENIAC, 1950



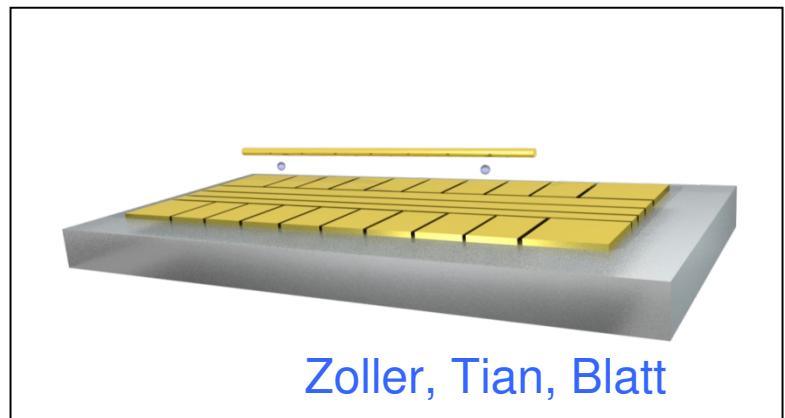
Innsbruck quantum computer, 2009

Scaling of ion trap quantum computers

Kielpinski, Monroe, Wineland



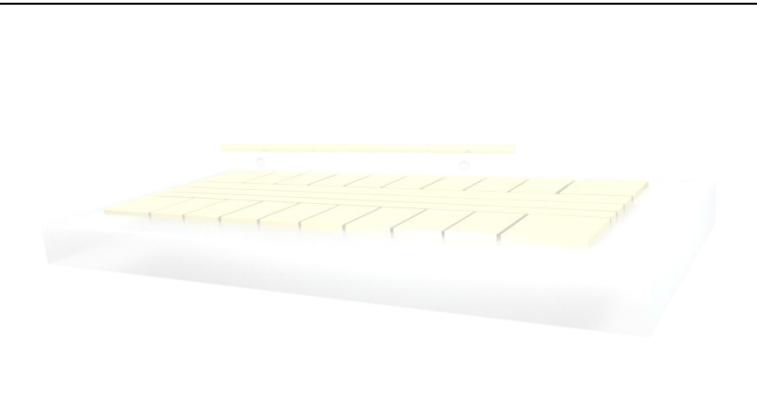
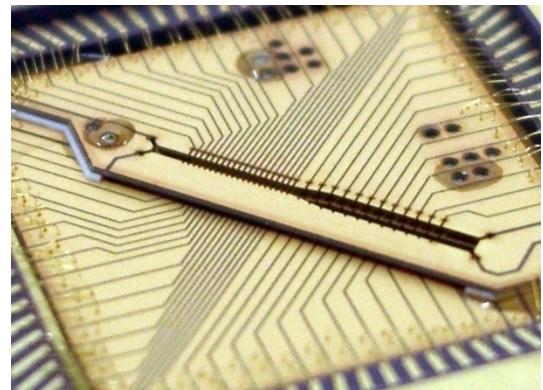
Cirac, Zoller, Kimble, Mabuchi



Zoller, Tian, Blatt

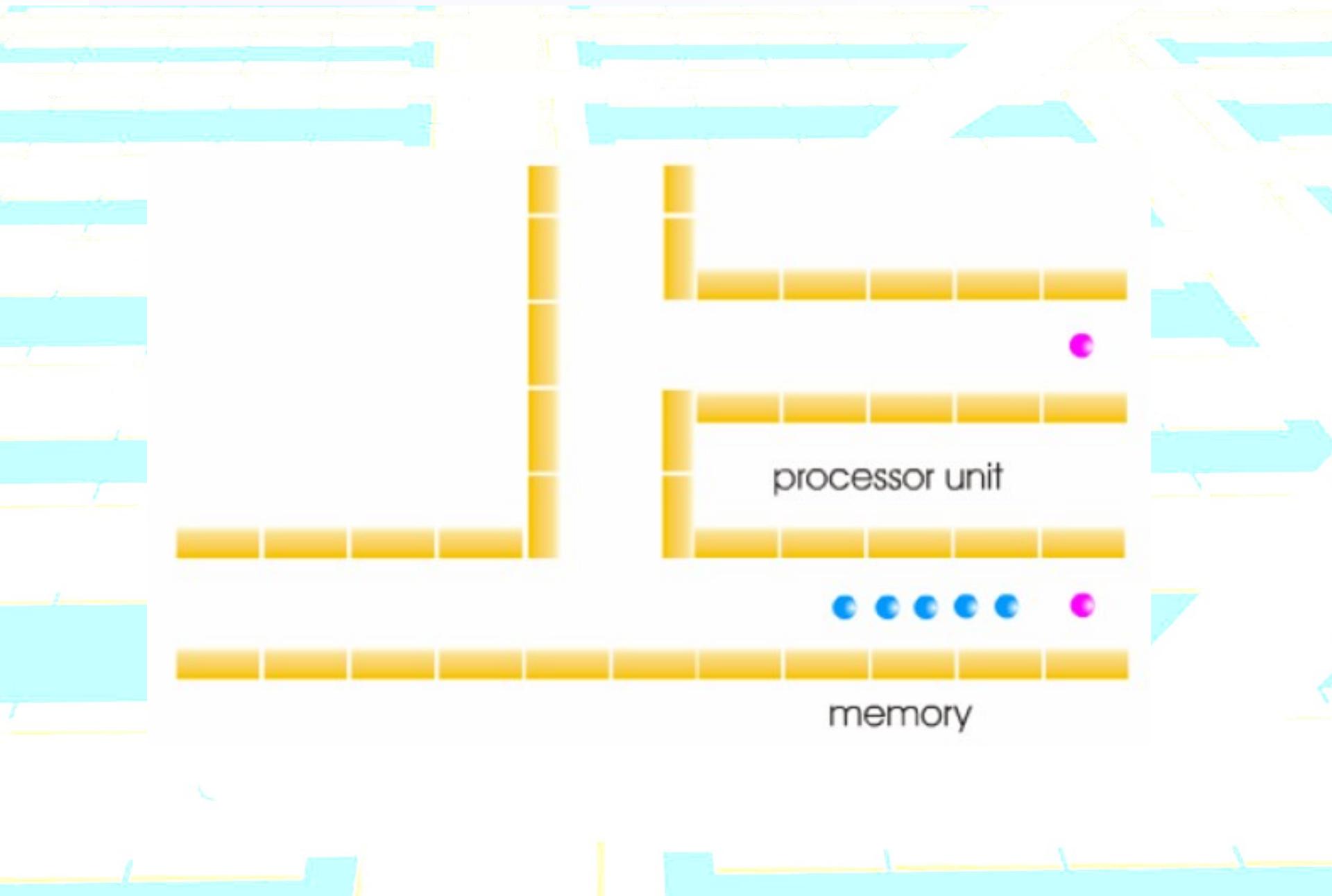
Scaling of ion trap quantum computers

Kielpinski, Monroe, Wineland

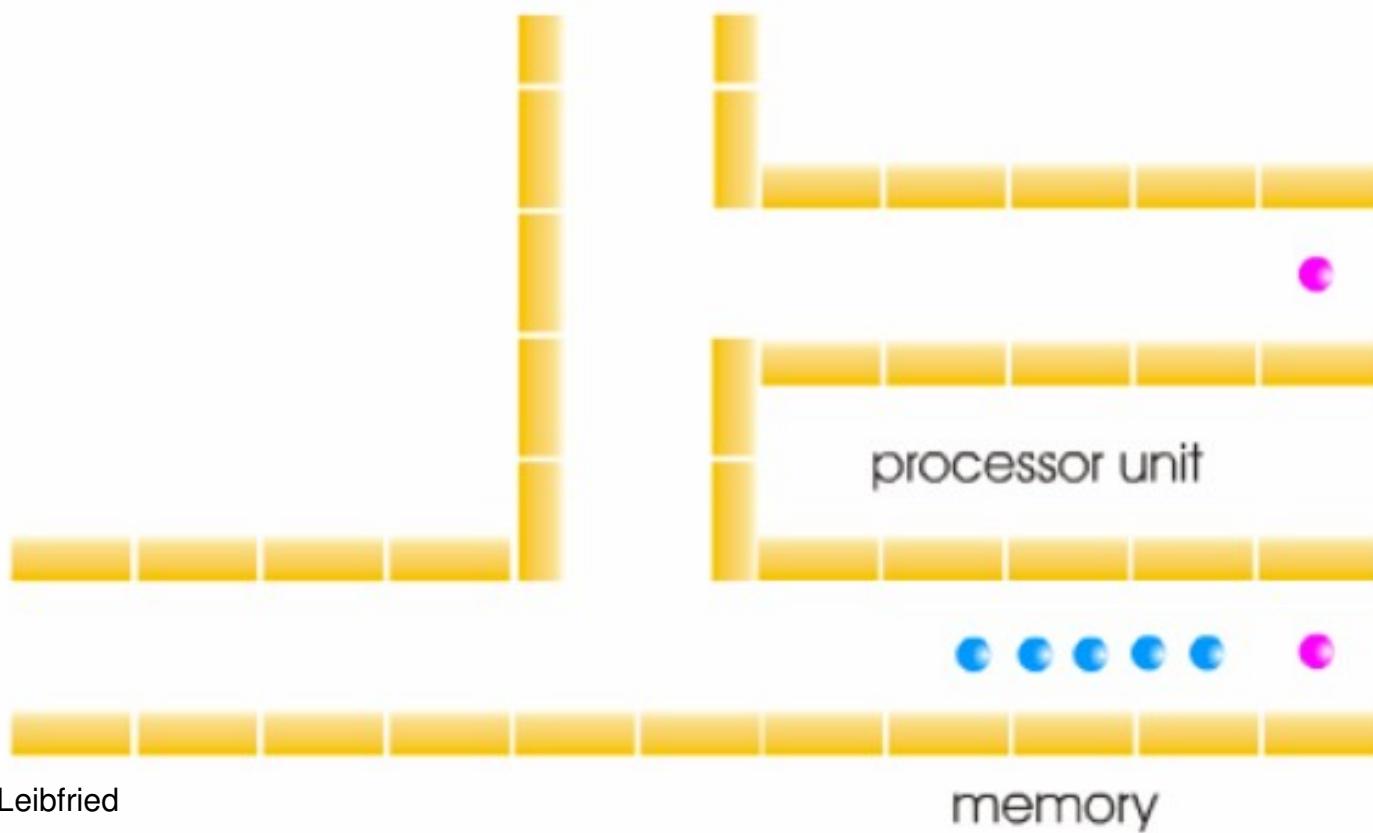


Scaling ion trap quantum computing

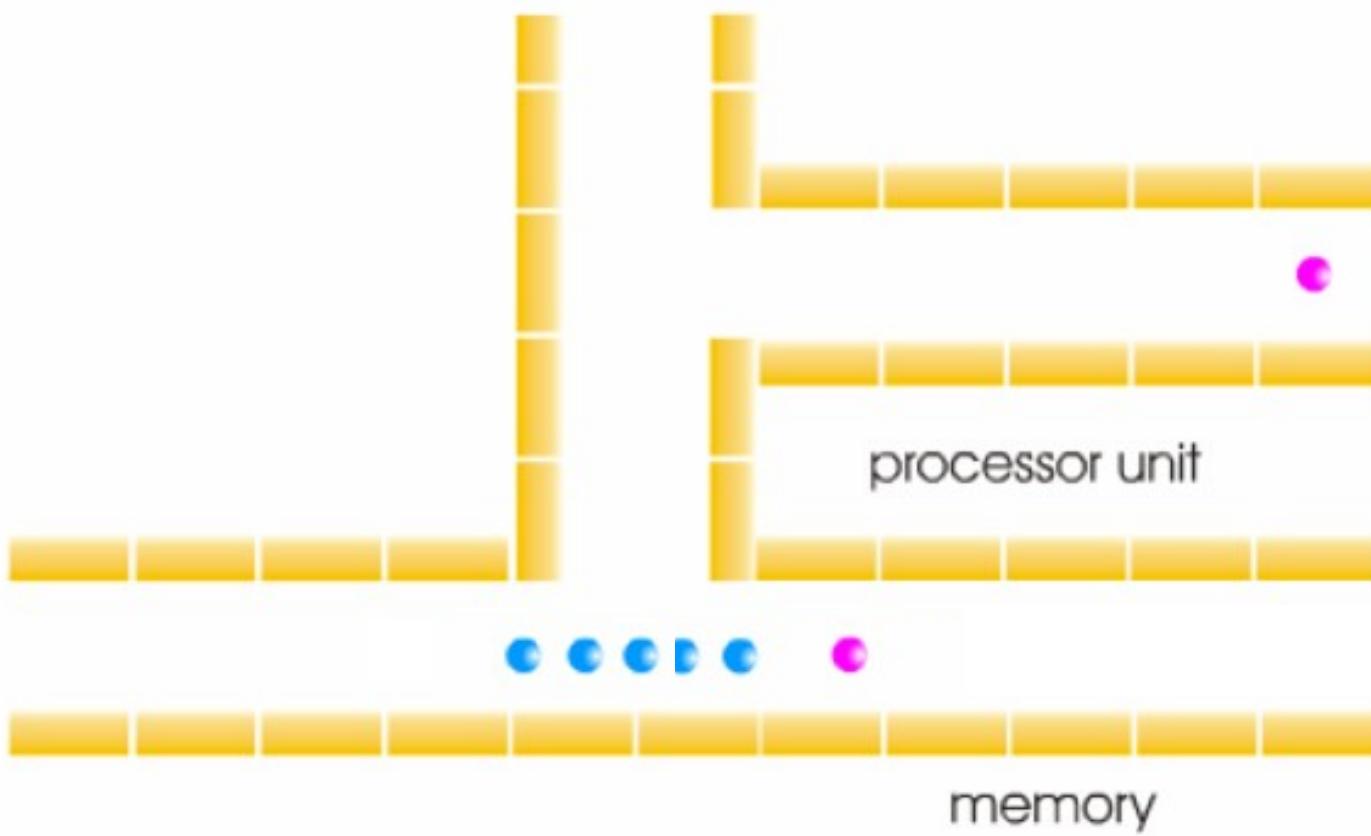
Scaling ion trap quantum computing



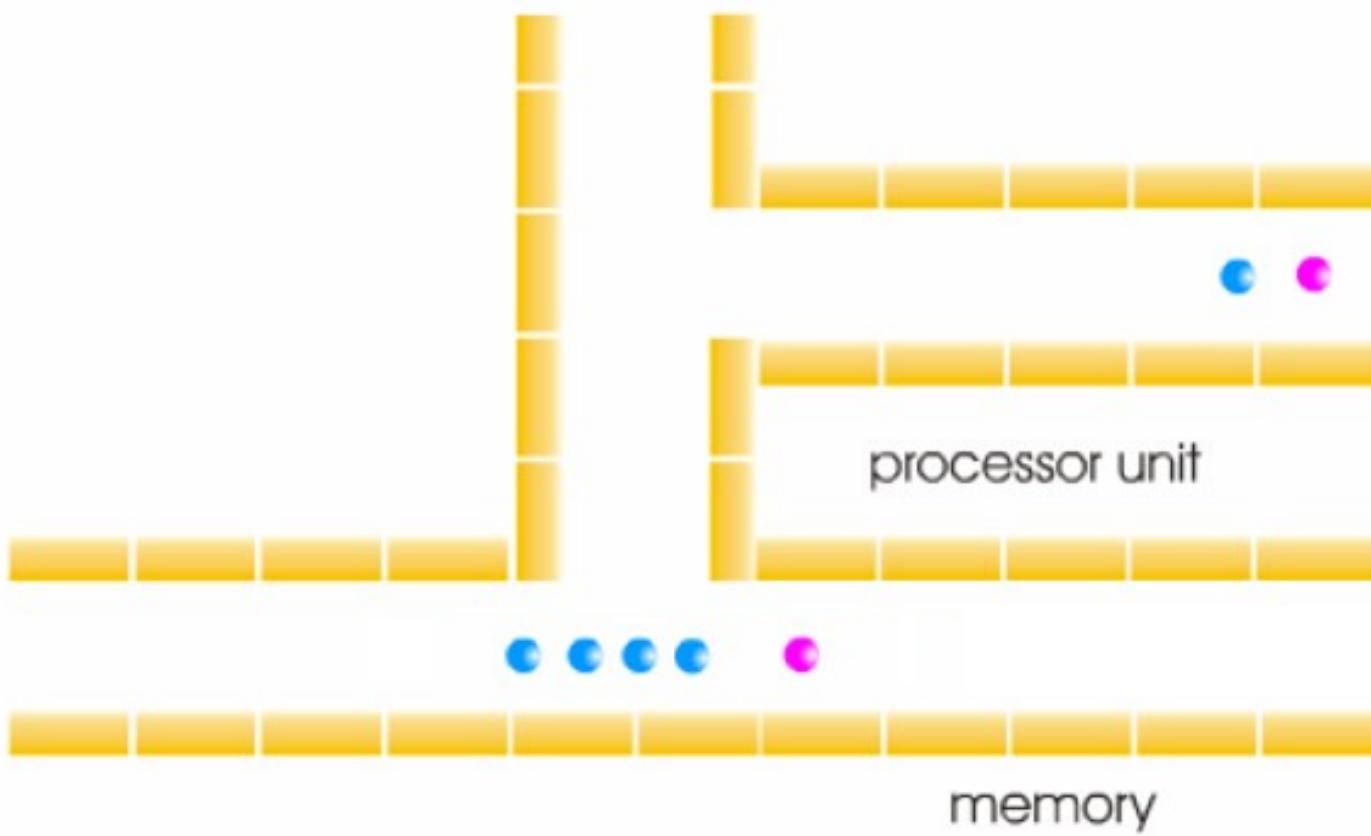
Scaling ion trap quantum computing



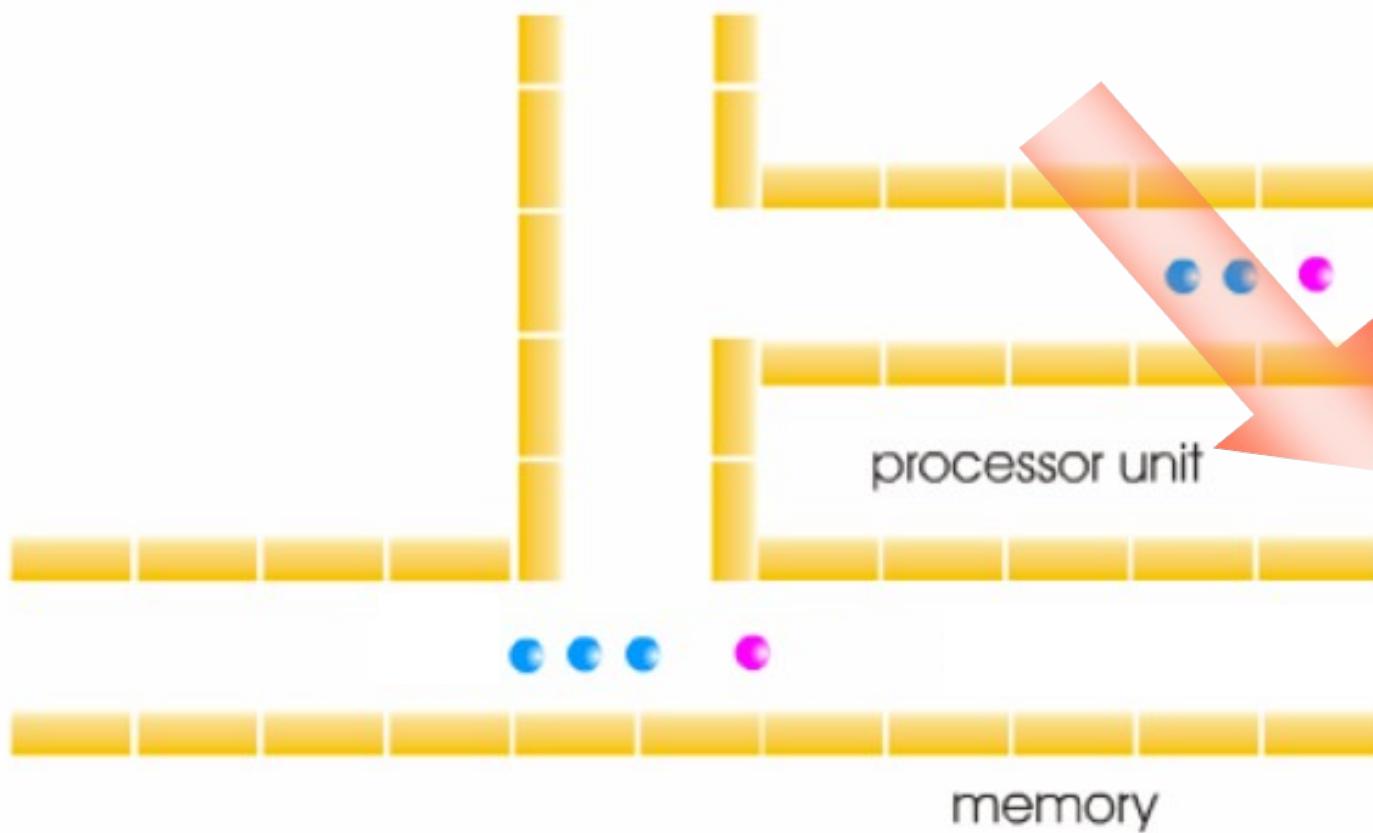
Scaling ion trap quantum computing



Scaling ion trap quantum computing

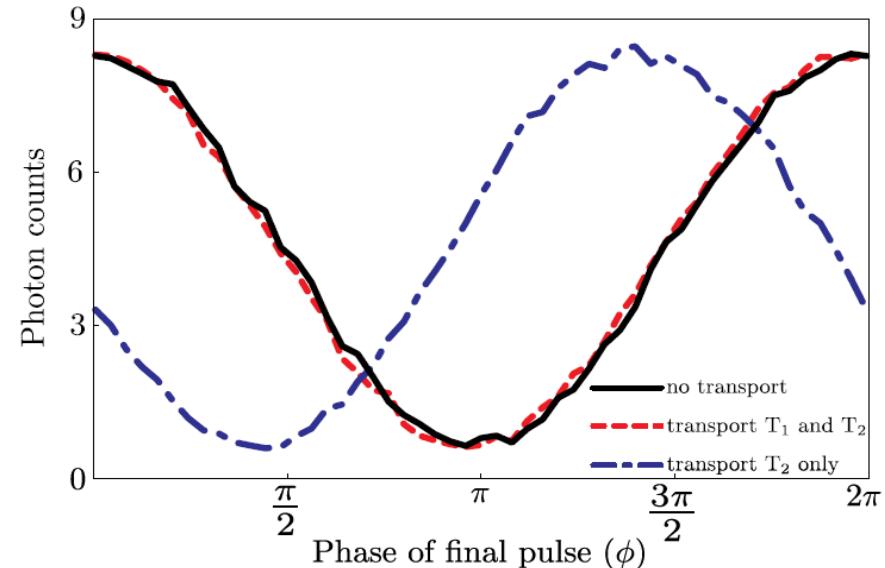
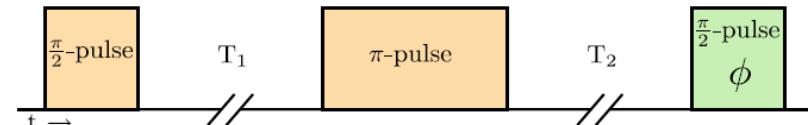
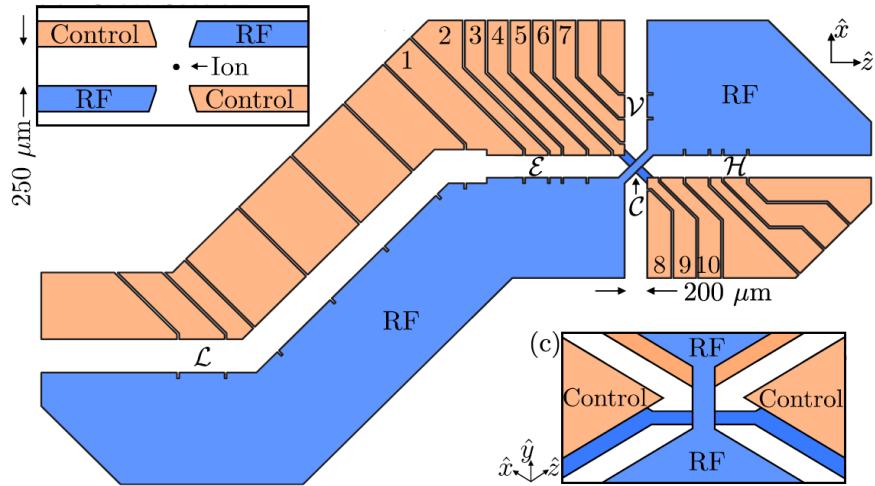


Scaling ion trap quantum computing



„Architecture for a large-scale ion-trap quantum computer“,
D. Kielpinski et al., Nature **417**, 709 (2002).

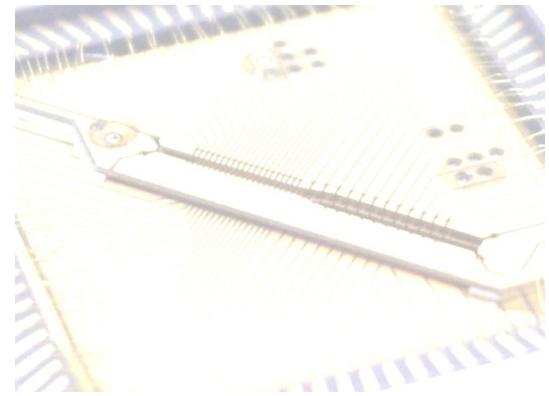
Coherent transport through a junction



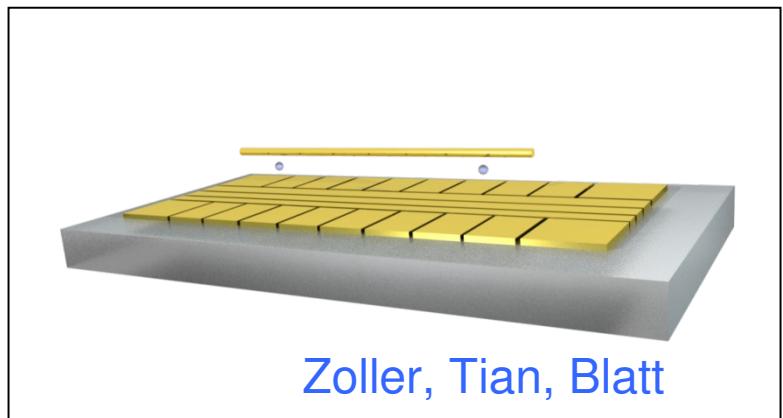
Transport	Energy Gain (recooling method)	
	quanta/ion	quanta/trip
$\mathcal{E}-\mathcal{C}-\mathcal{E}$	1 ion	3.2 ± 1.8
$\mathcal{E}-\mathcal{C}-\mathcal{H}-\mathcal{C}-\mathcal{E}$	1 ion	7.9 ± 1.5
$\mathcal{E}-\mathcal{C}-\mathcal{V}-\mathcal{C}-\mathcal{E}$	1 ion	14.5 ± 2.0
$\mathcal{E}-\mathcal{C}-\mathcal{E}$	2 ions	5.4 ± 1.2
$\mathcal{E}-\mathcal{C}-\mathcal{H}-\mathcal{C}-\mathcal{E}$	2 ions	16.6 ± 1.8
$\mathcal{E}-\mathcal{C}-\mathcal{V}-\mathcal{C}-\mathcal{E}$	2 ions	53.0 ± 1.2

Scaling of ion trap quantum computers

Kielpinski, Monroe, Wineland



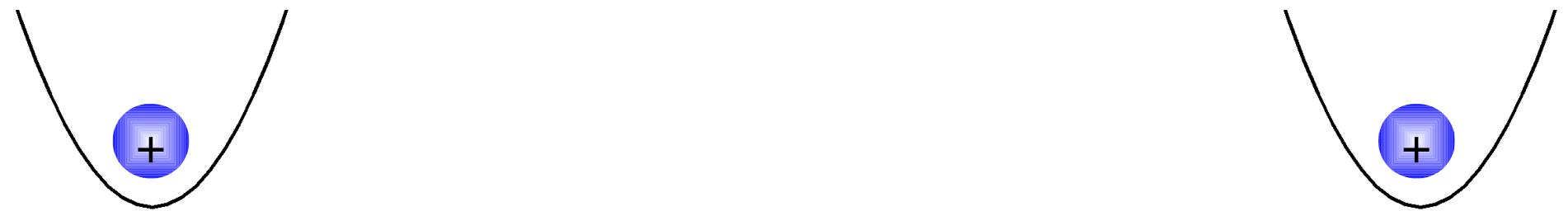
Cirac, Zoller, Kimble, Mabuchi



Zoller, Tian, Blatt

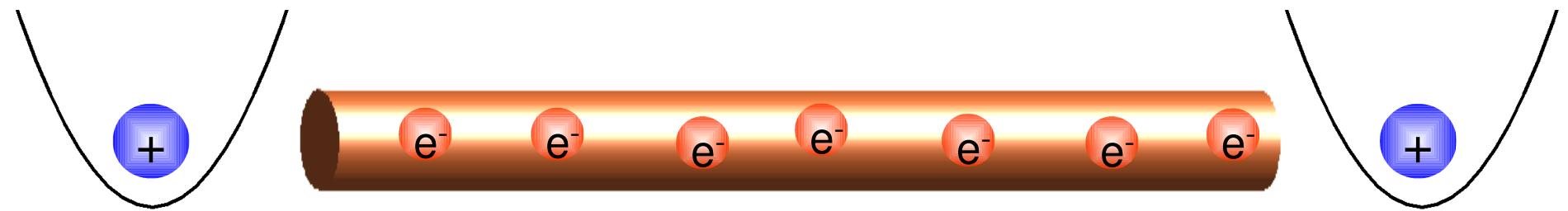
Wiring up trapped ions

Two trapped ions ...



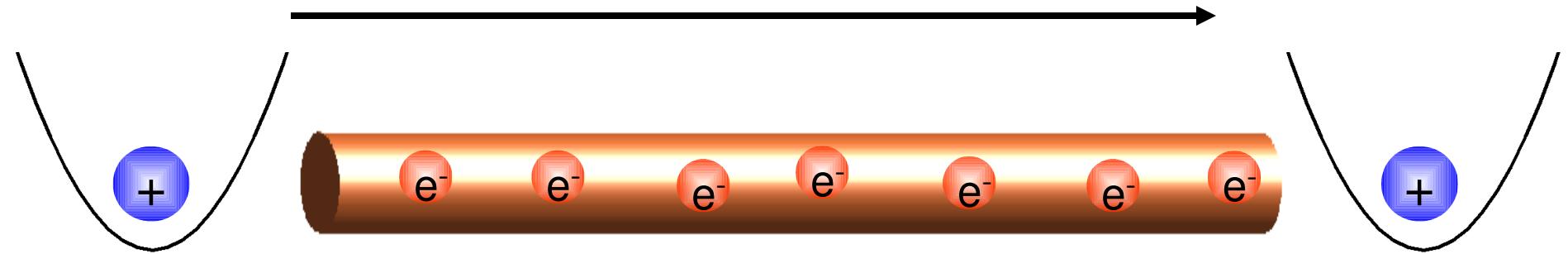
Wiring up trapped ions

Two trapped ions + a wire



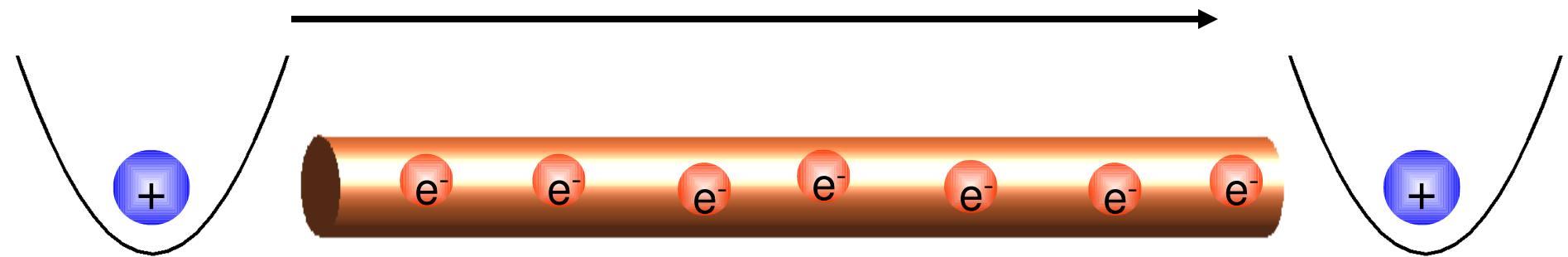
Wiring up trapped ions

Transport of quantum information



Wiring up trapped ions

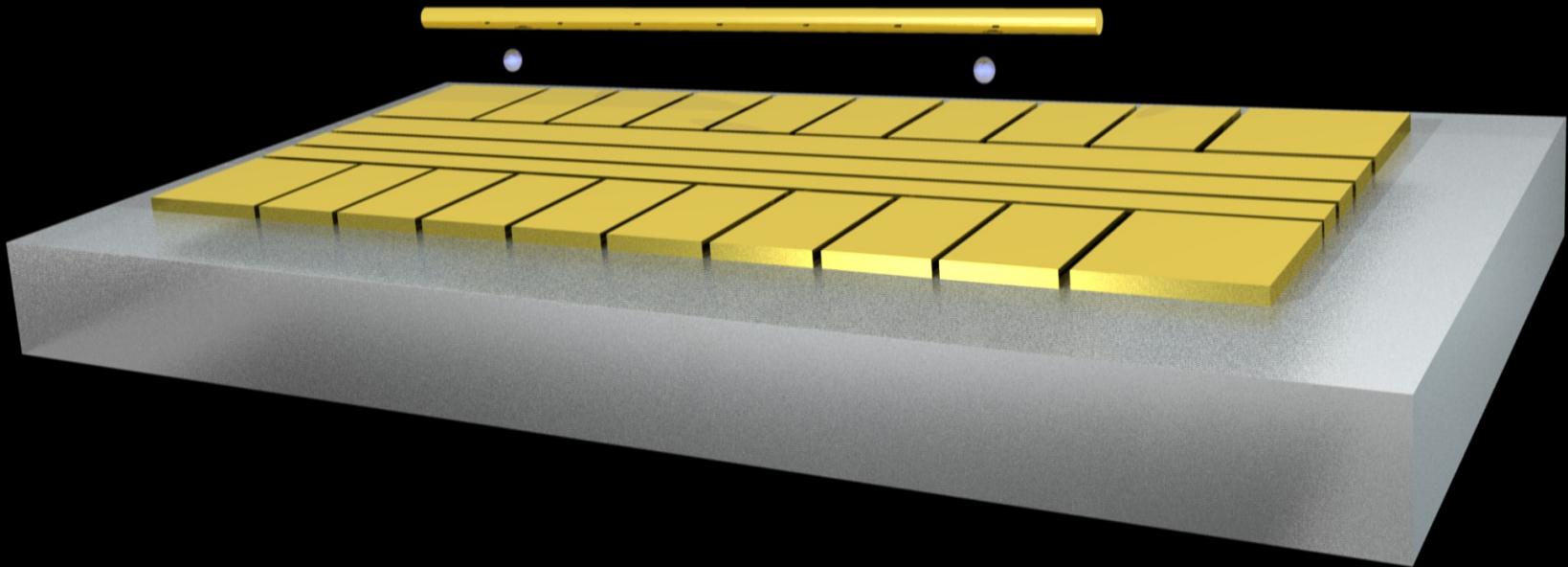
Transport of quantum information



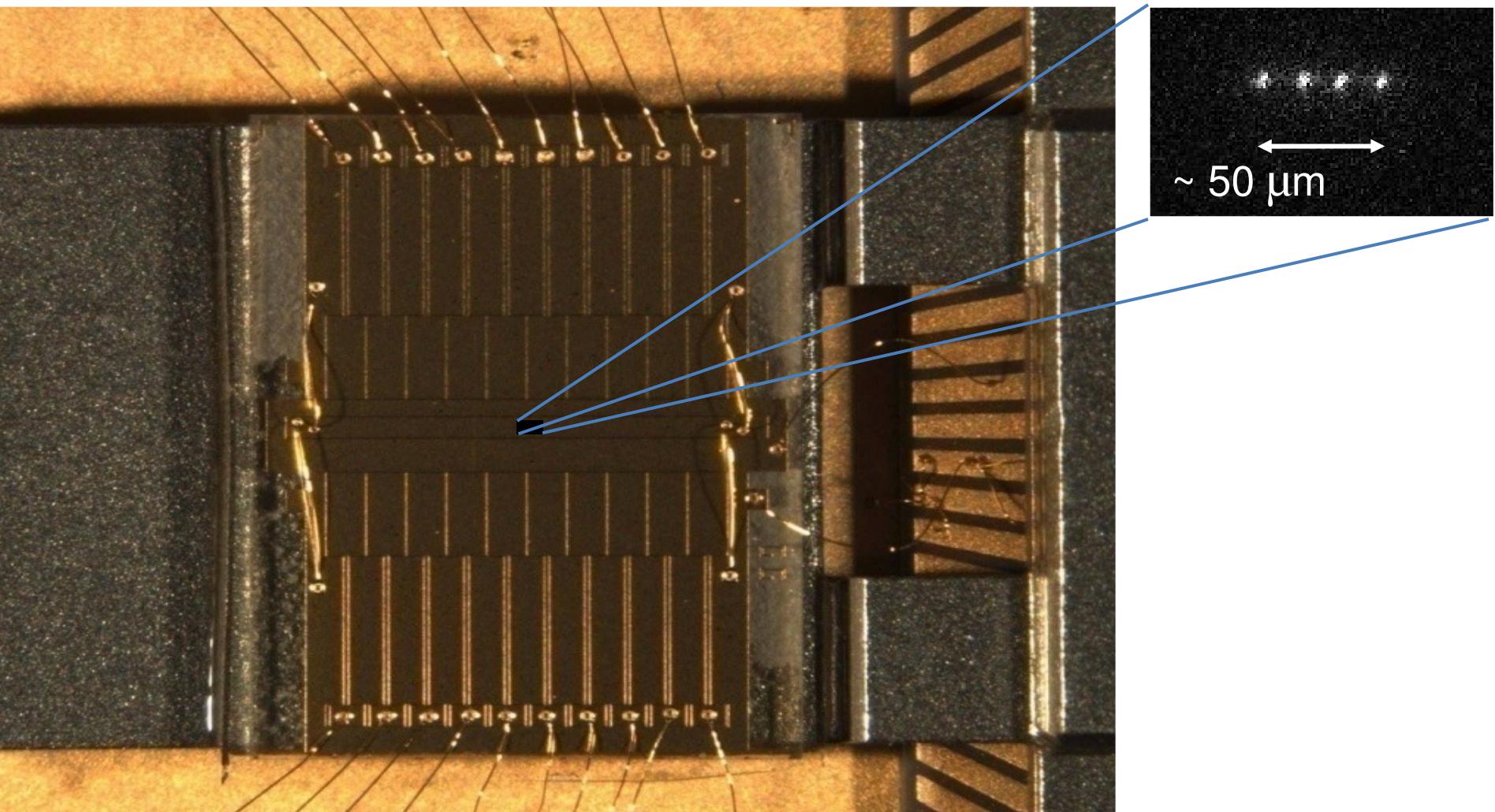
No trace of the quantum information should remain in the wire

→ ~~superconducting wire~~

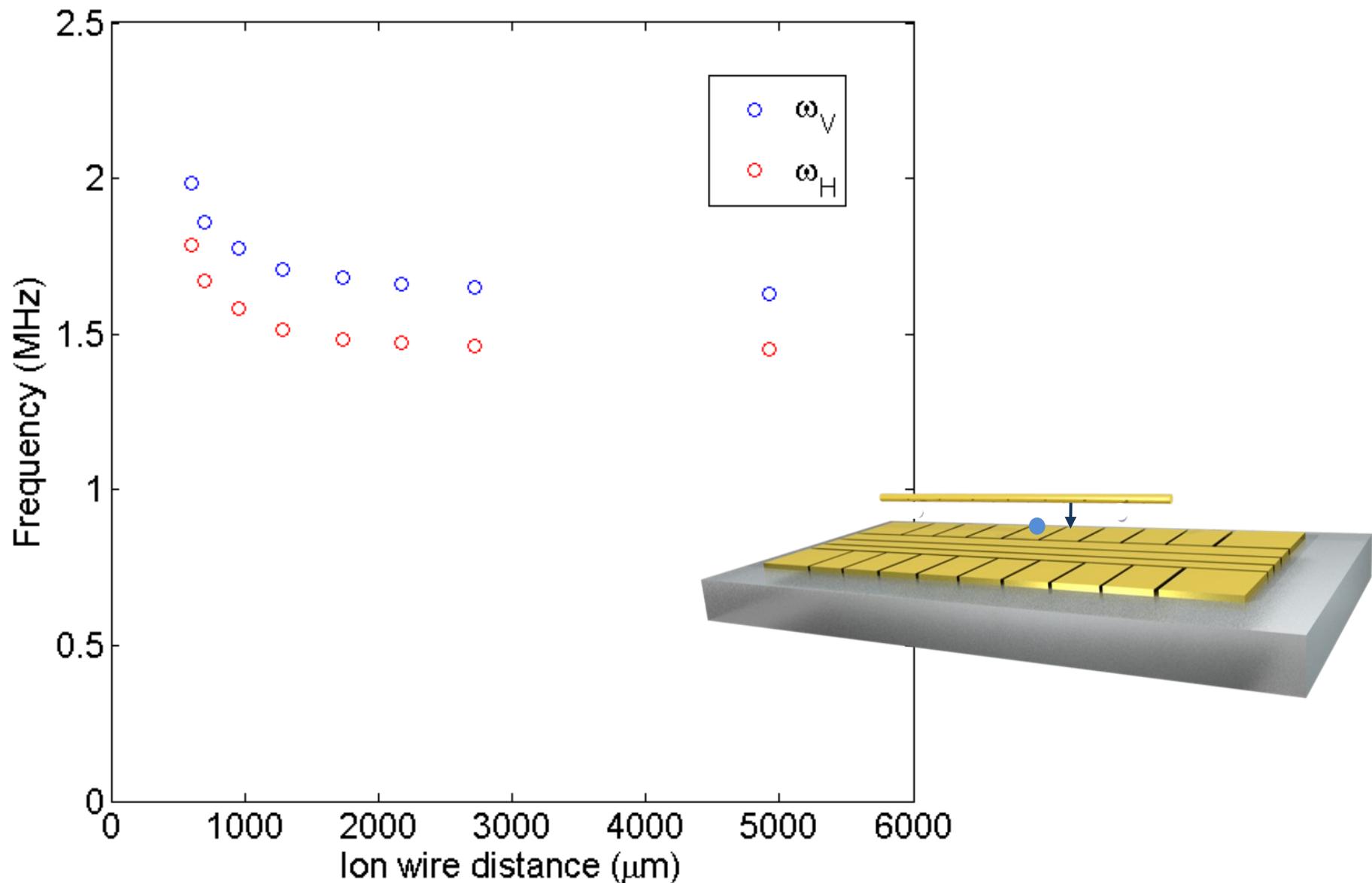
Experimental set-up



Experimental set-up

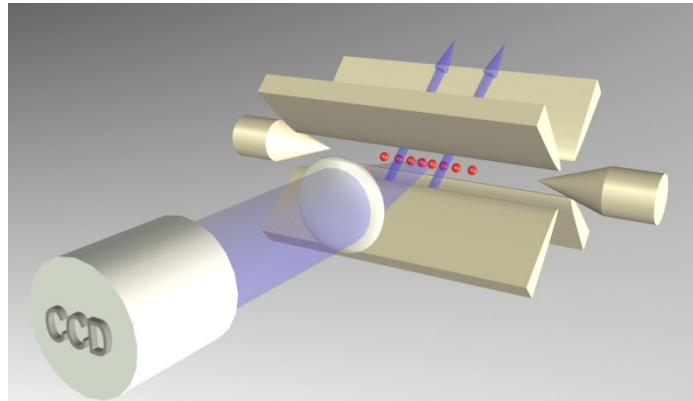


Moving the wire in

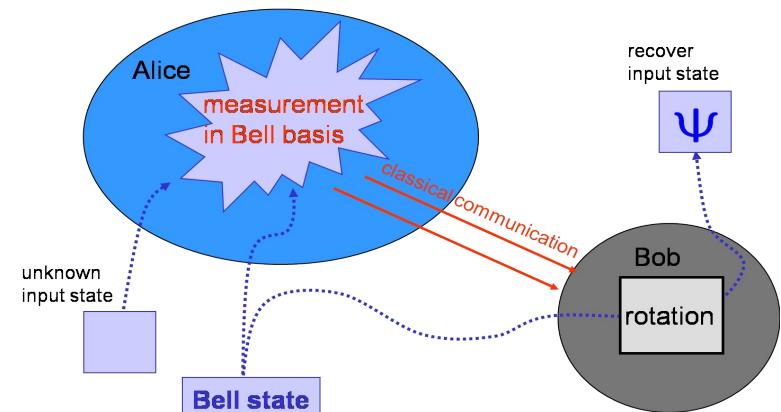


Conclusions

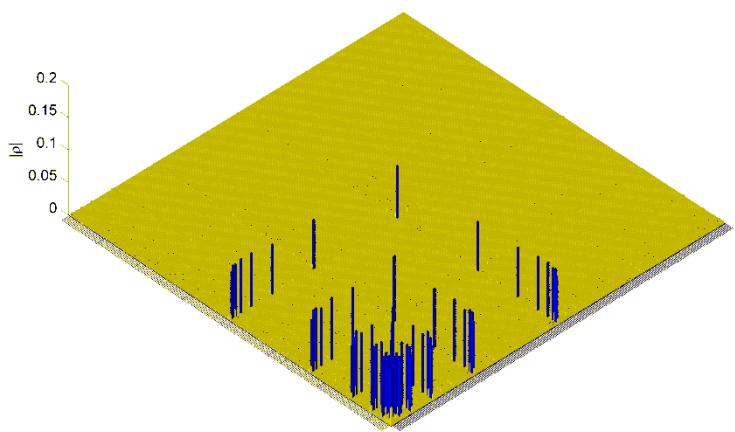
Ion trap quantum computing



Teleportation

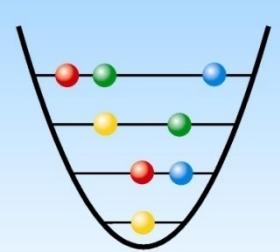


Entanglement

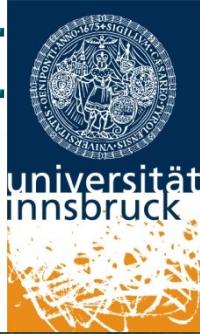


Scaling of ion traps





AG Quantenoptik
und Spektroskopie



The Innsbruck ion trap group



FWF
SFB



CONQUEST
SCALA



Industrie
Tirol



IQI
GmbH

FWF | bm:bwk

