Quantum computing with trapped ions

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- Introduction to quantum information
- Ion trap quantum computing
- Quantum teleportation
- Scaling up ion trap quantum computers

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$ IARPA

Merced, Apr 3rd 2009

Quantum information

Strip down quantum mechanics to bare bones:

- Hilbert space
- unitary operations
- measurement

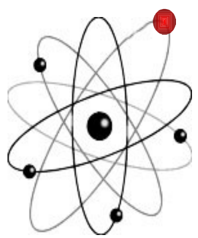
"New" paradigms:

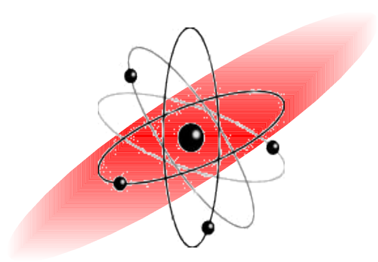
Computer scientist: *There is nothing but information*

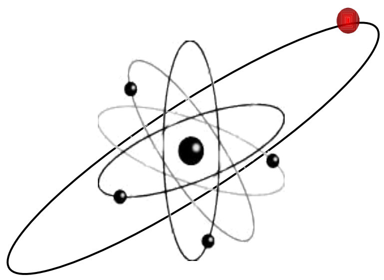
Physicist: *Ultimately information must be quantum*

physics $\longleftrightarrow$ computer science

Qubits







$|0\rangle$ $\qquad$ $\alpha|0\rangle + \beta|1\rangle$ $\qquad$ $|1\rangle$

Information content

$$|\Psi\rangle_{\text{reg}} = \alpha_0 |000\rangle + \alpha_1 |001\rangle + \alpha_2 |010\rangle + \alpha_3 |011\rangle + \alpha_4 |100\rangle + \alpha_5 |101\rangle + \alpha_6 |110\rangle + \alpha_7 |111\rangle$$

# bits	classical	quantum mechanical
1	1	0.9717 + 0.2364i
2	01	0.2044 + 0.4911i , 0.1732 + 0.3855i, 0.2040 + 0.4890i, 0.3193 + 0.3947i
3	001	0.2583 + 0.2704i , 0.2310 + 0.1150i, 0.2956 + 0.3118i , 0.3558 + 0.2113i , 0.1943 + 0.1377i 0.3273 + 0.2613i , 0.0643 + 0.2033i, 0.3643 + 0.1654i
4	1010	0.1691 + 0.0891i 0.1096 + 0.0828i 0.1420 + 0.2873i 0.0741 + 0.2419i 0.1902 + 0.0448i 0.2495 + 0.0039i 0.1738 + 0.2933i 0.2102 + 0.0653i 0.0686 + 0.0980i 0.1246 + 0.2170i 0.2570 + 0.0933i 0.2234 + 0.1540i 0.1513 + 0.0213i 0.1863 + 0.3243i 0.2606 + 0.1912i 0.0194 + 0.1390i
5	10001	0.1060 + 0.1416i 0.0103 + 0.0118i 0.0064 + 0.0976i 0.0734 + 0.0716i 0.0030 + 0.2054i 0.0902 + 0.0035i 0.1605 + 0.1804i 0.0218 + 0.2280i 0.0083 + 0.2326i 0.1438 + 0.1853i 0.1429 + 0.1030i 0.0037 + 0.1171i 0.0038 + 0.0503i 0.0446 + 0.1512i 0.1379 + 0.0752i 0.0135 + 0.2255i 0.0863 + 0.1707i 0.1483 + 0.0968i 0.1686 + 0.1749i 0.1627 + 0.0629i 0.0197 + 0.1033i 0.1067 + 0.2192i 0.1038 + 0.1605i 0.0830 + 0.0499i 0.0361 + 0.1971i 0.1587 + 0.1477i 0.1642 + 0.0314i 0.1709 + 0.0487i 0.1124 + 0.1426i 0.1303 + 0.1480i 0.0284 + 0.0870i 0.1059 + 0.1351i
6	110101	0.0595 + 0.1064i 0.0295 + 0.1327i 0.0929 + 0.0406i 0.1090 + 0.0379i 0.0559 + 0.1286i 0.0015 + 0.0345i 0.0624 + 0.1196i 0.1120 + 0.1350i 0.1180 + 0.0345i 0.1367 + 0.0356i 0.1255 + 0.0074i 0.0547 + 0.0116i 0.0923 + 0.0952i 0.1087 + 0.0284i 0.0288 + 0.1254i 0.1345 + 0.0258i 0.0846 + 0.0254i 0.0939 + 0.1478i 0.0348 + 0.0654i 0.0816 + 0.0505i 0.1384 + 0.0467i 0.0498 + 0.0543i 0.0974 + 0.0584i 0.0582 + 0.0879i 0.0932 + 0.0178i 0.1039 + 0.0057i 0.0590 + 0.0682i 0.0615 + 0.1293i 0.0974 + 0.1388i 0.1245 + 0.0393i 0.0552 + 0.0238i 0.0632 + 0.1297i 0.0884 + 0.0354i 0.0841 + 0.0960i 0.1065 + 0.1437i 0.0760 + 0.0988i 0.1154 + 0.1293i 0.0727 + 0.0015i 0.0276 + 0.0204i 0.1041 + 0.1217i 0.1460 + 0.0639i 0.1199 + 0.1323i 0.1046 + 0.1092i 0.0721 + 0.1021i 0.0170 + 0.0514i 0.0988 + 0.0247i 0.0543 + 0.0231i 0.0208 + 0.0284i 0.0842 + 0.0628i 0.1223 + 0.1272i 0.1002 + 0.0729i 0.1485 + 0.1213i 0.1429 + 0.0685i 0.0087 + 0.0680i 0.0535 + 0.0670i 0.0815 + 0.0613i 0.0389 + 0.1340i 0.0888 + 0.0008i 0.0073 + 0.0442i 0.0849 + 0.0073i 0.1042 + 0.1030i 0.1430 + 0.0966i 0.1115 + 0.1461i 0.1100 + 0.0821i
7	1001010	0.0880 + 0.0466i 0.1054 + 0.0684i 0.0239 + 0.0866i 0.0759 + 0.0090i 0.0563 + 0.1020i 0.1006 + 0.0988i 0.0769 + 0.0649i 0.0246 + 0.0273i 0.0485 + 0.0942i 0.0186 + 0.0554i 0.1045 + 0.0790i 0.0384 + 0.0455i 0.0053 + 0.1037i 0.0815 + 0.0078i 0.0965 + 0.0597i 0.0309 + 0.0315i 0.0271 + 0.0925i 0.1006 + 0.0362i 0.0141 + 0.0734i 0.1015 + 0.0058i 0.0757 + 0.0385i 0.0914 + 0.0537i 0.0226 + 0.0468i 0.0491 + 0.0607i 0.0087 + 0.0665i 0.0918 + 0.0122i 0.0606 + 0.0969i 0.0344 + 0.0814i 0.0404 + 0.0853i 0.0936 + 0.0879i 0.0401 + 0.0723i 0.0079 + 0.0217i 0.0216 + 0.0294i 0.0053 + 0.0675i 0.0611 + 0.0579i 0.0131 + 0.0064i 0.0563 + 0.0096i 0.0126 + 0.0293i 0.0830 + 0.0441i 0.0404 + 0.0511i 0.0888 + 0.0980i 0.0050 + 0.0643i 0.0645 + 0.0355i 0.1024 + 0.0516i 0.0311 + 0.0644i 0.0959 + 0.0174i 0.0110 + 0.0894i 0.0070 + 0.1031i 0.0253 + 0.0642i 0.1006 + 0.0031i 0.0068 + 0.0876i 0.0285 + 0.0658i 0.1078 + 0.0756i 0.0229 + 0.0099i 0.0537 + 0.0458i 0.0313 + 0.0405i 0.0725 + 0.0179i 0.1033 + 0.0898i 0.0827 + 0.0904i 0.0718 + 0.0487i 0.0141 + 0.1032i 0.0103 + 0.0159i 0.0016 + 0.0938i 0.0311 + 0.0830i 0.0881 + 0.0479i 0.1063 + 0.0669i 0.0019 + 0.1026i 0.0884 + 0.0690i 0.0670 + 0.0267i 0.0604 + 0.0380i 0.0263 + 0.0203i 0.0886 + 0.0529i 0.0284 + 0.0441i 0.0813 + 0.0500i 0.0711 + 0.0659i 0.0231 + 0.0077i 0.0649 + 0.0339i 0.0652 + 0.0656i 0.0711 + 0.0189i 0.0198 + 0.0670i 0.0686 + 0.0265i 0.0184 + 0.0633i 0.0582 + 0.0546i 0.0672 + 0.0501i 0.0740 + 0.0584i 0.0730 + 0.1016i 0.0946 + 0.0369i 0.0014 + 0.0433i 0.0335 + 0.0332i 0.0840 + 0.0444i 0.0331 + 0.0308i 0.0999 + 0.0425i 0.0732 + 0.0542i 0.0080 + 0.0779i 0.0076 + 0.0330i 0.0013 + 0.0121i 0.0245 + 0.0478i 0.0557 + 0.0503i 0.0494 + 0.0016i 0.0758 + 0.0716i 0.0628 + 0.0781i 0.0549 + 0.0304i 0.0080 + 0.0282i 0.0208 + 0.0764i 0.0409 + 0.0845i 0.0893 + 0.0425i 0.0989 + 0.0562i 0.0122 + 0.0774i 0.0876 + 0.0614i 0.0979 + 0.0497i 0.0169 + 0.0480i 0.0132 + 0.0095i 0.0822 + 0.0478i 0.0778 + 0.0395i 0.0703 + 0.0326i 0.0813 + 0.0919i 0.0715 + 0.0819i 0.0953 + 0.1024i 0.0293 + 0.0602i 0.0452 + 0.0015i 0.0230 + 0.0643i
8	10101011	0.0199 + 0.0027i 0.0033 + 0.0063i 0.0005 + 0.0656i 0.0443 + 0.0262i 0.0573 + 0.0359i 0.0622 + 0.0704i 0.0491 + 0.0176i 0.0194 + 0.0664i 0.0111 + 0.0506i 0.0502 + 0.0687i 0.0729 + 0.0376i 0.0629 + 0.0765i 0.0717 + 0.0288i 0.0239 + 0.0410i 0.0207 + 0.0140i 0.0413 + 0.0387i 0.0126 + 0.0325i 0.0163 + 0.0509i 0.0167 + 0.0519i 0.0502 + 0.0738i 0.0041 + 0.0148i 0.0177 + 0.0086i 0.0514 + 0.0436i 0.0240 + 0.0747i 0.0236 + 0.0018i 0.0555 + 0.0671i 0.0736 + 0.0021i 0.0101 + 0.0400i 0.0053 + 0.0148i 0.0097 + 0.0552i 0.0128 + 0.0193i 0.0702 + 0.0720i 0.0105 + 0.0106i 0.0476 + 0.0402i 0.0207 + 0.0690i 0.0170 + 0.0726i 0.0549 + 0.0258i 0.0423 + 0.0337i 0.0726 + 0.0363i 0.0254 + 0.0115i 0.0543 + 0.0105i 0.0727 + 0.0410i 0.0448 + 0.0559i 0.0678 + 0.0307i 0.0578 + 0.0276i 0.0293 + 0.0220i 0.0559 + 0.0670i 0.0125 + 0.0483i 0.0737 + 0.0186i 0.0151 + 0.0754i 0.0598 + 0.0494i 0.0473 + 0.0177i 0.0125 + 0.0525i 0.0024 + 0.0513i 0.0222 + 0.0104i 0.0748 + 0.0017i 0.0733 + 0.0202i 0.0176 + 0.0090i 0.0739 + 0.0053i 0.0524 + 0.0657i 0.0042 + 0.0139i 0.0462 + 0.0025i 0.0303 + 0.0566i 0.0166 + 0.0414i 0.0141 + 0.0213i 0.0059 + 0.0284i 0.0006 + 0.0010i 0.0608 + 0.0685i 0.0014 + 0.0667i 0.0677 + 0.0196i 0.0272 + 0.0439i 0.0557 + 0.0123i 0.0746 + 0.0458i 0.0120 + 0.0255i 0.0126 + 0.0508i 0.0242 + 0.0666i 0.0023 + 0.0437i 0.0276 + 0.0756i 0.0021 + 0.0610i 0.0612 + 0.0118i 0.0770 + 0.0642i 0.0085 + 0.0148i 0.0480 + 0.0493i 0.0102 + 0.0516i 0.0239 + 0.0595i 0.0104 + 0.0293i 0.0172 + 0.0340i 0.0306 + 0.0372i 0.0104 + 0.0469i 0.0186 + 0.0136i 0.0715 + 0.0002i 0.0301 + 0.0609i 0.0394 + 0.0396i 0.0072 + 0.0164i 0.0017 + 0.0080i 0.0123 + 0.0121i 0.0651 + 0.0314i 0.0678 + 0.0314i 0.0144 + 0.0041i 0.0764 + 0.0726i 0.0549 + 0.0116i 0.0672 + 0.0296i 0.0370 + 0.0240i 0.0382 + 0.0130i 0.0222 + 0.0691i 0.0047 + 0.0249i 0.0202 + 0.0566i 0.0144 + 0.0317i 0.0707 + 0.0308i 0.0095 + 0.0390i 0.0010 + 0.0130i 0.0285 + 0.0404i 0.0538 + 0.0494i 0.0685 + 0.0012i 0.0458 + 0.0645i 0.0121 + 0.0619i 0.0244 + 0.0538i 0.0180 + 0.0356i 0.0006 + 0.0064i 0.0306 + 0.0633i 0.0501 + 0.0149i 0.0066 + 0.0343i 0.0593 + 0.0010i 0.0747

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1	1	0.9717 + 0.2364i
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6	110101	0.0595 + 0.1064i 0.0295 + 0.1327i 0.0929 + 0.0406i 0.1090 + 0.0379i 0.0559 + 0.1286i 0.0015 + 0.0345i 0.0624 + 0.1196i 0.1120 + 0.1350i 0.1180 + 0.0345i 0.1367 + 0.0356i 0.1255 + 0.0074i 0.0547 + 0.0116i 0.0923 + 0.0952i 0.1087 + 0.0284i 0.0288 + 0.1254i 0.1345 + 0.0258i 0.0846 + 0.0254i 0.0939 + 0.1478i 0.0348 + 0.0654i 0.0816 + 0.0505i 0.1384 + 0.0467i 0.0498 + 0.0543i 0.0974 + 0.0584i 0.0582 + 0.0879i 0.0932 + 0.0178i 0.1039 + 0.0057i 0.0590 + 0.0682i 0.0615 + 0.1293i 0.0974 + 0.1388i 0.1245 + 0.0393i 0.0552 + 0.0238i 0.0632 + 0.1297i 0.0884 + 0.0354i 0.0841 + 0.0960i 0.1065 + 0.1437i 0.0760 + 0.0988i 0.1154 + 0.1293i 0.0727 + 0.0015i 0.0276 + 0.0204i 0.1041 + 0.1217i 0.1460 + 0.0639i 0.1199 + 0.1323i 0.1046 + 0.1092i 0.0721 + 0.1021i 0.0170 + 0.0514i 0.0988 + 0.0247i 0.0543 + 0.0231i 0.0208 + 0.0284i 0.0842 + 0.0628i 0.1223 + 0.1272i 0.1002 + 0.0729i 0.1485 + 0.1213i 0.1429 + 0.0685i 0.0087 + 0.0680i 0.0535 + 0.0670i 0.0815 + 0.0613i 0.0389 + 0.1340i 0.0888 + 0.0008i 0.0073 + 0.0442i 0.0849 + 0.0073i 0.1042 + 0.1030i 0.1430 + 0.0966i 0.1115 + 0.1461i 0.1100 + 0.0821i
7	1001010	0.0880 + 0.0466i 0.1054 + 0.0684i 0.0239 + 0.0866i 0.0759 + 0.0090i 0.0563 + 0.1020i 0.1006 + 0.0988i 0.0769 + 0.0649i 0.0246 + 0.0273i 0.0485 + 0.0942i 0.0186 + 0.0554i 0.1045 + 0.0790i 0.0384 + 0.0455i 0.0053 + 0.1037i 0.0815 + 0.0078i 0.0965 + 0.0597i 0.0309 + 0.0315i 0.0271 + 0.0925i 0.1006 + 0.0362i 0.0141 + 0.0734i 0.1015 + 0.0058i 0.0757 + 0.0385i 0.0914 + 0.0537i 0.0226 + 0.0468i 0.0491 + 0.0607i 0.0087 + 0.0665i 0.0918 + 0.0122i 0.0606 + 0.0969i 0.0344 + 0.0814i 0.0404 + 0.0853i 0.0936 + 0.0879i 0.0401 + 0.0723i 0.0079 + 0.0217i 0.0216 + 0.0294i 0.0053 + 0.0675i 0.0611 + 0.0579i 0.0131 + 0.0064i 0.0563 + 0.0096i 0.0126 + 0.0293i 0.0830 + 0.0441i 0.0404 + 0.0511i 0.0888 + 0.0980i 0.0050 + 0.0643i 0.0645 + 0.0355i 0.1024 + 0.0516i 0.0311 + 0.0644i 0.0959 + 0.0174i 0.0110 + 0.0894i 0.0070 + 0.1031i 0.0253 + 0.0642i 0.1006 + 0.0031i 0.0068 + 0.0876i 0.0285 + 0.0658i 0.1078 + 0.0756i 0.0229 + 0.0099i 0.0537 + 0.0458i 0.0313 + 0.0405i 0.0725 + 0.0179i 0.1033 + 0.0898i 0.0827 + 0.0904i 0.0718 + 0.0487i 0.0141 + 0.1032i 0.0103 + 0.0159i 0.0016 + 0.0938i 0.0311 + 0.0830i 0.0881 + 0.0479i 0.1063 + 0.0669i 0.0019 + 0.1026i 0.0884 + 0.0690i 0.0670 + 0.0267i 0.0604 + 0.0380i 0.0263 + 0.0203i 0.0886 + 0.0529i 0.0284 + 0.0441i 0.0813 + 0.0500i 0.0711 + 0.0659i 0.0231 + 0.0077i 0.0649 + 0.0339i 0.0652 + 0.0656i 0.0711 + 0.0189i 0.0198 + 0.0670i 0.0686 + 0.0265i 0.0184 + 0.0633i 0.0582 + 0.0546i 0.0672 + 0.0501i 0.0740 + 0.0584i 0.0730 + 0.1016i 0.0946 + 0.0369i 0.0014 + 0.0433i 0.0335 + 0.0332i 0.0840 + 0.0444i 0.0331 + 0.0308i 0.0999 + 0.0425i 0.0732 + 0.0542i 0.0080 + 0.0779i 0.0076 + 0.0330i 0.0013 + 0.0121i 0.0245 + 0.0478i 0.0557 + 0.0503i 0.0494 + 0.0016i 0.0758 + 0.0716i 0.0628 + 0.0781i 0.0549 + 0.0304i 0.0080 + 0.0282i 0.0208 + 0.0764i 0.0409 + 0.0845i 0.0893 + 0.0425i 0.0989 + 0.0562i 0.0122 + 0.0774i 0.0876 + 0.0614i 0.0979 + 0.0497i 0.0169 + 0.0480i 0.0132 + 0.0095i 0.0822 + 0.0478i 0.0778 + 0.0395i 0.0703 + 0.0326i 0.0813 + 0.0919i 0.0715 + 0.0819i 0.0953 + 0.1024i 0.0293 + 0.0602i 0.0452 + 0.0015i 0.0230 + 0.0643i
8	10101011	0.0199 + 0.0027i 0.0033 + 0.0063i 0.0005 + 0.0656i 0.0443 + 0.0262i 0.0573 + 0.0359i 0.0622 + 0.0704i 0.0491 + 0.0176i 0.0194 + 0.0664i 0.0111 + 0.0506i 0.0502 + 0.0687i 0.0729 + 0.0376i 0.0629 + 0.0765i 0.0717 + 0.0288i 0.0239 + 0.0410i 0.0207 + 0.0140i 0.0413 + 0.0387i 0.0126 + 0.0325i 0.0163 + 0.0509i 0.0167 + 0.0519i 0.0502 + 0.0738i 0.0041 + 0.0148i 0.0177 + 0.0086i 0.0514 + 0.0436i 0.0240 + 0.0747i 0.0236 + 0.0018i 0.0555 + 0.0671i 0.0736 + 0.0021i 0.0101 + 0.0400i 0.0053 + 0.0148i 0.0097 + 0.0552i 0.0128 + 0.0193i 0.0702 + 0.0720i 0.0105 + 0.0106i 0.0476 + 0.0402i 0.0207 + 0.0690i 0.0170 + 0.0726i 0.0549 + 0.0258i 0.0423 + 0.0337i 0.0726 + 0.0363i 0.0254 + 0.0115i 0.0543 + 0.0105i 0.0727 + 0.0410i 0.0448 + 0.0559i 0.0678 + 0.0307i 0.0578 + 0.0276i 0.0293 + 0.0220i 0.0559 + 0.0670i 0.0125 + 0.0483i 0.0737 + 0.0186i 0.0151 + 0.0754i 0.0598 + 0.0494i 0.0473 + 0.0177i 0.0125 + 0.0525i 0.0024 + 0.0513i 0.0222 + 0.0104i 0.0748 + 0.0017i 0.0733 + 0.0202i 0.0176 + 0.0090i 0.0739 + 0.0053i 0.0524 + 0.0657i 0.0042 + 0.0139i 0.0462 + 0.0025i 0.0303 + 0.0566i 0.0166 + 0.0414i 0.0141 + 0.0213i 0.0059 + 0.0284i 0.0006 + 0.0010i 0.0608 + 0.0685i 0.0014 + 0.0667i 0.0677 + 0.0196i 0.0272 + 0.0439i 0.0557 + 0.0123i 0.0746 + 0.0458i 0.0120 + 0.0255i 0.0126 + 0.0508i 0.0242 + 0.0666i 0.0023 + 0.0437i 0.0276 + 0.0756i 0.0021 + 0.0610i 0.0612 + 0.0118i 0.0770 + 0.0642i 0.0085 + 0.0148i 0.0480 + 0.0493i 0.0102 + 0.0516i 0.0239 + 0.0595i 0.0104 + 0.0293i 0.0172 + 0.0340i 0.0306 + 0.0372i 0.0104 + 0.0469i 0.0186 + 0.0136i 0.0715 + 0.0002i 0.0301 + 0.0609i 0.0394 + 0.0396i 0.0072 + 0.0164i 0.0017 + 0.0080i 0.0123 + 0.0121i 0.0651 + 0.0314i 0.0678 + 0.0314i 0.0144 + 0.0041i 0.0764 + 0.0726i 0.0549 + 0.0116i 0.0672 + 0.0296i 0.0370 + 0.0240i 0.0382 + 0.0130i 0.0222 + 0.0691i 0.0047 + 0.0249i 0.0202 + 0.0566i 0.0144 + 0.0317i 0.0707 + 0.0308i 0.0095 + 0.0390i 0.0010 + 0.0130i 0.0285 + 0.0404i 0.0538 + 0.0494i 0.0685 + 0.0012i 0.0458 + 0.0645i 0.0121 + 0.0619i 0.0244 + 0.0538i 0.0180 + 0.0356i 0.0006 + 0.0064i 0.0306 + 0.0633i 0.0501 + 0.0149i 0.0066 + 0.0343i 0.0593 + 0.0010i 0.0747 + 0.0238i 0.0551 + 0.0675i 0.0603 + 0.0644i 0.0183 + 0.0257i 0.0151 + 0.0679i 0.0203 + 0.0370i 0.0550 + 0.0432i 0.0753 + 0.0475i 0.0491 + 0.0510i 0.0421 + 0.0475i 0.0654 + 0.0528i 0.0618 + 0.0393i 0.0515 + 0.0550i 0.0517 + 0.0397i 0.0633 + 0.0467i 0.0748 + 0.0745i 0.0375 + 0.0634i 0.0630 + 0.0245i 0.0494 + 0.0453i 0.0236 + 0.0100i 0.0509 + 0.0196i 0.0276 + 0.0619i 0.0723 + 0.0515i 0.0376 + 0.0011i 0.0070 + 0.0433i 0.0519 + 0.0350i 0.0397 + 0.0697i 0.0171 + 0.0217i 0.0559 + 0.0050i 0.0053 + 0.0367i 0.0743 + 0.0758i 0.0160 + 0.0711i 0.0124 + 0.0433i 0.0492 + 0.0503i 0.0000 + 0.0596i 0.0259 + 0.0082i 0.0212 + 0.0001i 0.0034 + 0.0418i 0.0072 + 0.0005i 0.0316 + 0.0348i 0.0630 + 0.0151i 0.0671 + 0.0607i 0.0017 + 0.0477i 0.0560 + 0.0012i 0.0654 + 0.0687i 0.0562 + 0.0587i 0.0736 + 0.0699i 0.0506 + 0.0585i 0.0572 + 0.0293i 0.0266 + 0.0255i 0.0681 + 0.0389i 0.0268 + 0.0435i 0.0670 + 0.0514i 0.0302 + 0.0522i 0.0195 + 0.0726i 0.0273 + 0.0594i 0.0573 + 0.0568i 0.0502 + 0.0668i 0.0724 + 0.0764i 0.0642 + 0.0388i 0.0362 + 0.0485i 0.0485 + 0.0611i 0.0045 + 0.0346i 0.0418 + 0.0404i 0.0351 + 0.0132i 0.0665 + 0.0101i 0.0659 + 0.0169i 0.0364 + 0.0081i 0.0607 + 0.0109i 0.0506 + 0.0352i 0.0000 + 0.0607i 0.0101 + 0.0217i 0.0381 + 0.0173i 0.0030 + 0.0701i 0.0175 + 0.0006i 0.0253 + 0.0454i 0.0693 + 0.0418i 0.0242 + 0.0504i 0.0194 + 0.0242i 0.0334 + 0.0178i 0.0649 + 0.0321i 0.0142 + 0.0230i 0.0392 + 0.0518i 0.0349 + 0.0723i 0.0251 + 0.0264i 0.0293 + 0.0434i 0.0683 + 0.0092i 0.0587 + 0.0130i 0.0681 + 0.0215i 0.0353 + 0.0429i 0.0616 + 0.0374i 0.0103 + 0.0734i 0.0050 + 0.0179i 0.0289 + 0.0369i 0.0288 + 0.0406i 0.0373 + 0.0611i 0.0747 + 0.0149i 0.0264 + 0.0701i 0.0195 + 0.0711i 0.0451 + 0.0010i 0.0404 + 0.0592i 0.0126 + 0.0730i 0.0375 + 0.0627i 0.0382 + 0.0712i 0.0650 + 0.0153i 0.0621 + 0.0520i 0.0661 + 0.0715i 0.0470 + 0.0265i 0.0436 + 0.0458i 0.0472 + 0.0474i 0.0079 + 0.0003i 0.0122 + 0.0757i 0.0319 + 0.0693i 0.0432 + 0.0534i 0.0207 + 0.0339i 0.0604 + 0.0540i 0.0299 + 0.0470i 0.0024 + 0.0231i 0.0451 + 0.0660i 0.0431 + 0.0086i 0.0155 + 0.0225i 0.0067 + 0.0075i 0.0719 + 0.0306i 0.0200 + 0.0257i 0.0157 + 0.0728i 0.0038 + 0.0646i

Information content

40 qubits	10 000 GigaByte
1 additional qubit	Double the memory
300 qubits	Every atom in the Universe would have to hold one classical bit

Why quantum information ?

Schrödinger equation for 300 interacting spins.

Classical computation needs more bits than there are atoms in the universe.

➡ Quantum computers can solve certain tasks much more efficiently than classical computers.

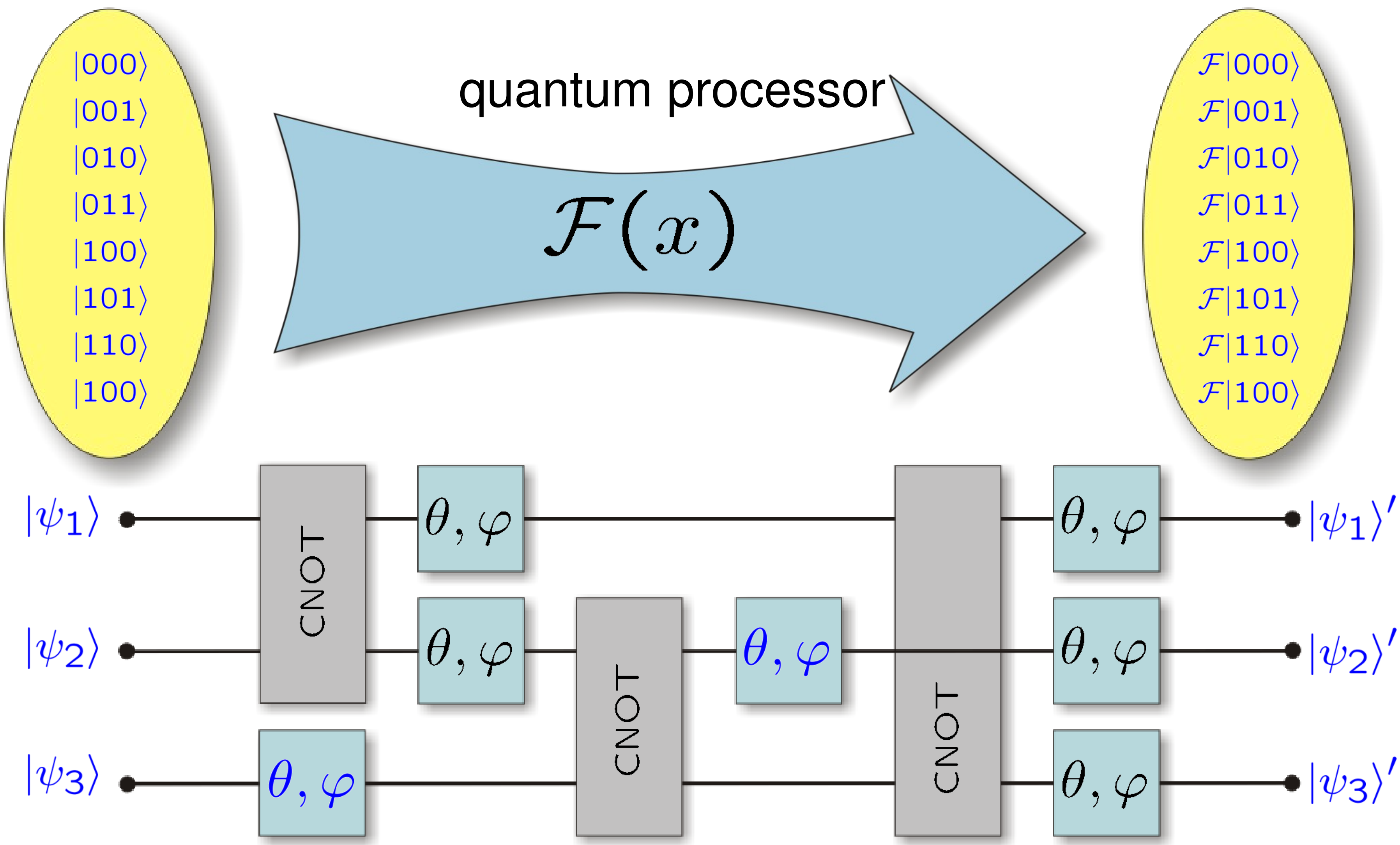
Other prominent examples:

- Factoring of large integers (P. Shor 1994)
- Search in an unsorted data base (L. Grover, 1997)
- …

Quantum computing

Classical computer	Quantum computer
Initialization	Initialization
1-bit operations (NOT)	1-qubit rotations → superpositions
2-bit gates (e.g. NAND)	2-qubit gates (CNOT gate) → entanglement
Computational space: 00 01 10 11	Computational space: Hilbert space 2^n dimensional
Read out → result	Read out of qubits → gain of classical information

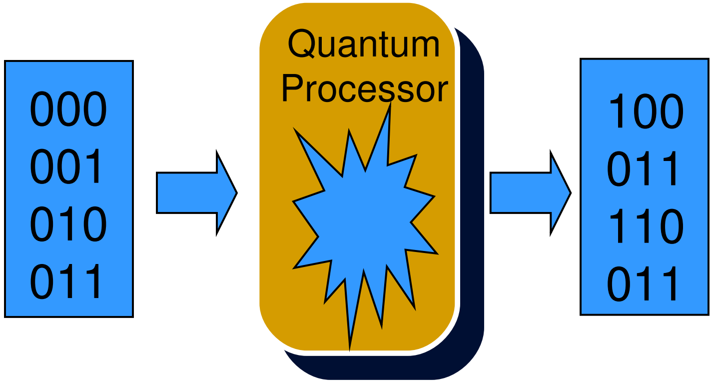
Quantum computing



Input → **computation: sequence of quantum gates** → **output**

Quantum computing

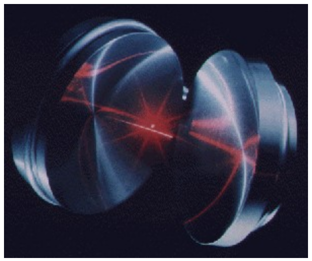
Long term goal: A universal quantum computer



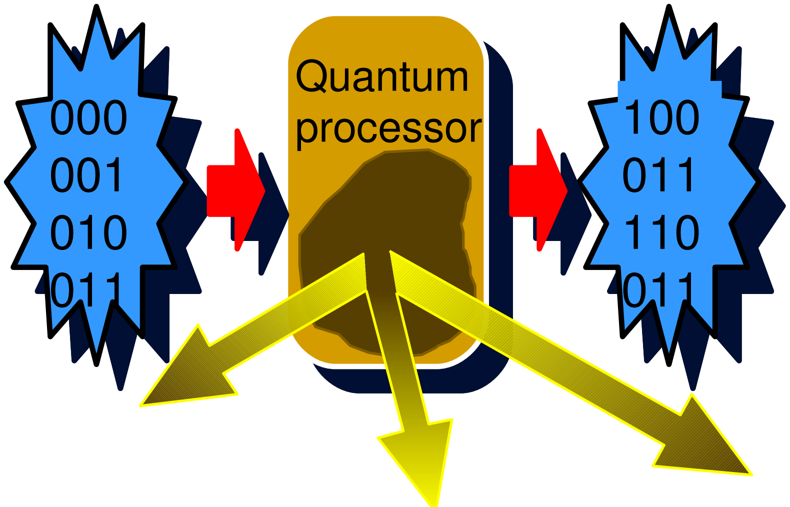
In the mean time:

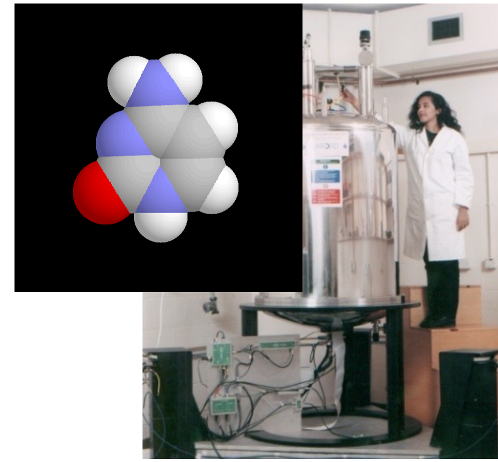
- "understand" quantum mechanics
- apply quantum mechanics
- where does quantum mechanics fail?

Which technology ?

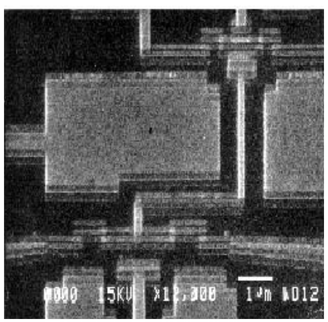


Cavity QED

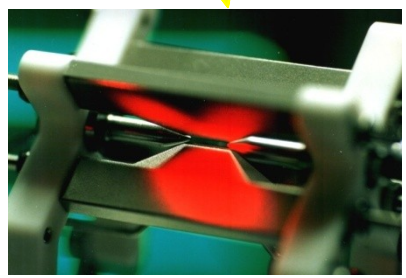


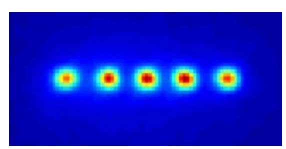


NMR

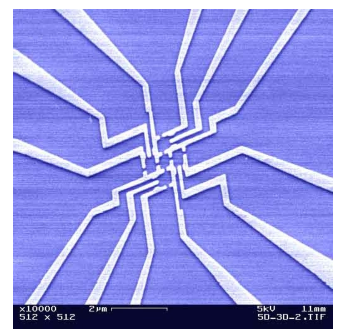


Superconducting qubits



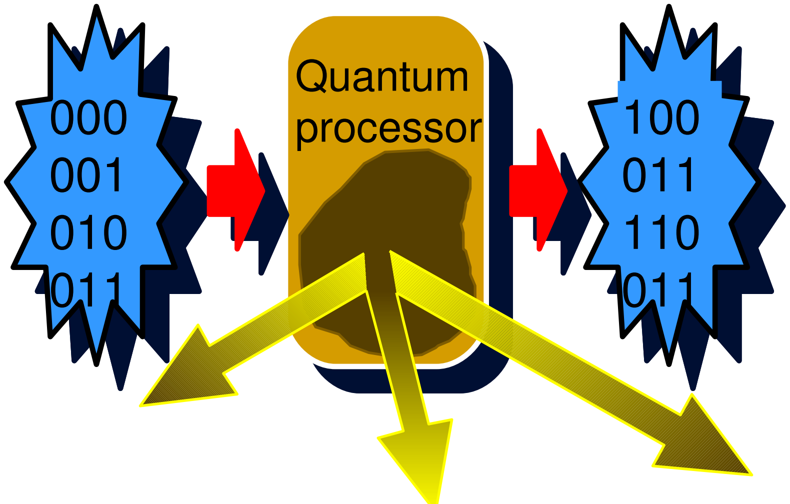


Trapped ions



Quantum dots

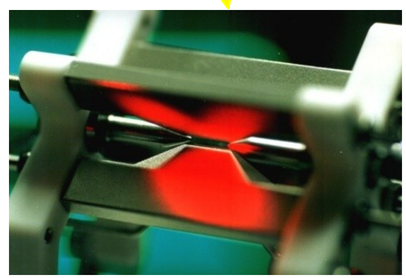
Which technology ?

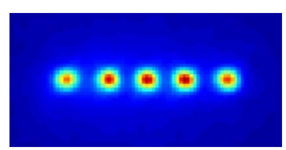


Cavity QED

NMR

Superconducting qubits



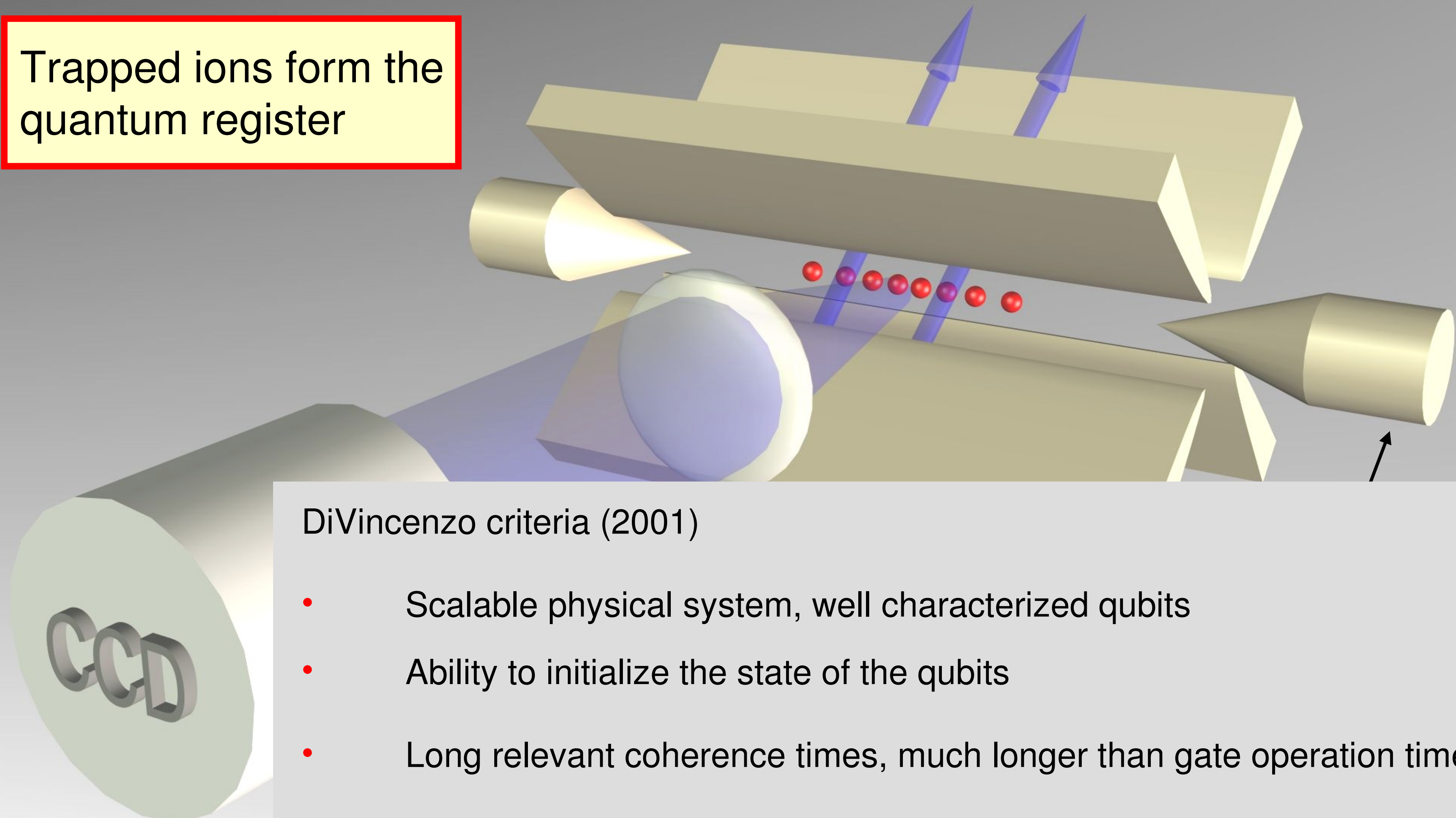


Trapped ions

Quantum dots

Ion trap quantum computing

Trapped ions form the quantum register



DiVincenzo criteria (2001)

- Scalable physical system, well characterized qubits
- Ability to initialize the state of the qubits
- Long relevant coherence times, much longer than gate operation time
- “Universal” set of quantum gates
- Qubit-specific measurement capability

- Introduction to quantum information
- Ion trap quantum computing
- Teleportation and more
- Scaling of ion trap quantum computers

Why trapped ions ?

Good things about ion traps:

- Ions are excellent quantum memories; single qubit coherence times > 10 minutes have been demonstrated

- Ions can be controlled very well

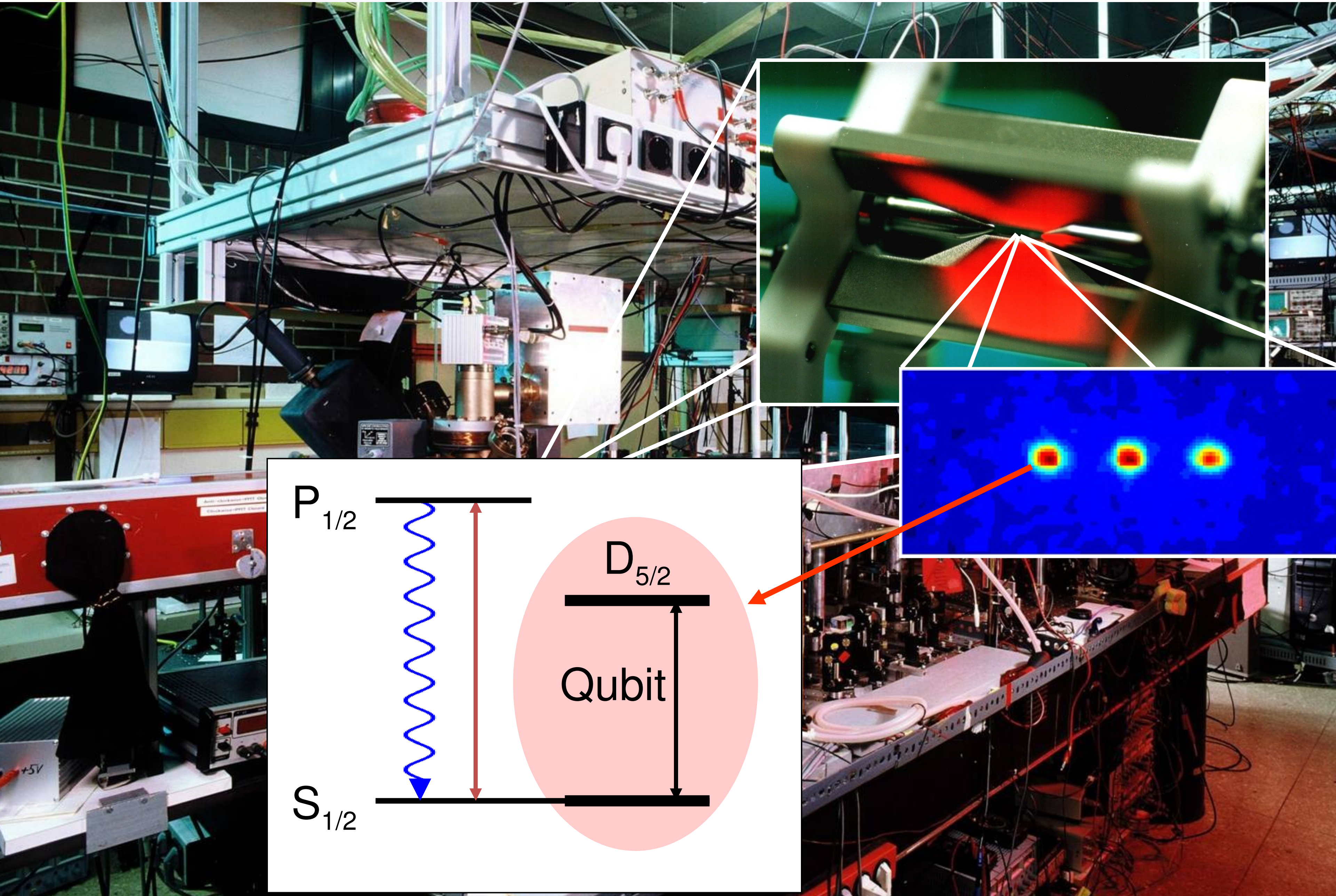
- Many ideas to scale ion traps

Bad things about ion traps:

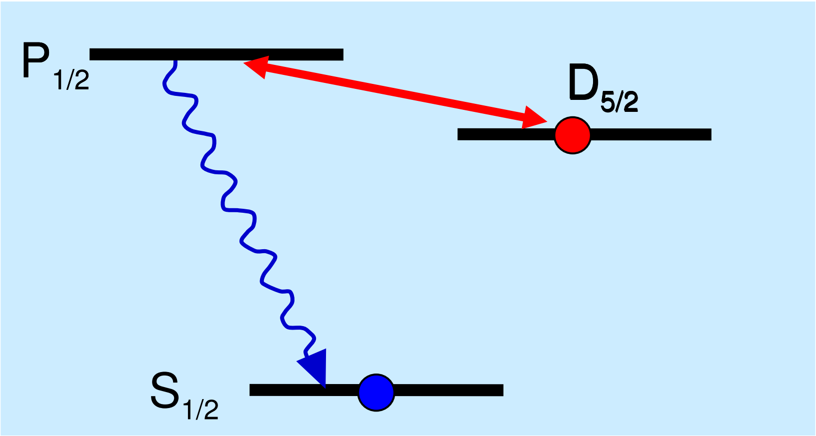
- Slow (~1 MHz)

- Technically demanding

The hardware

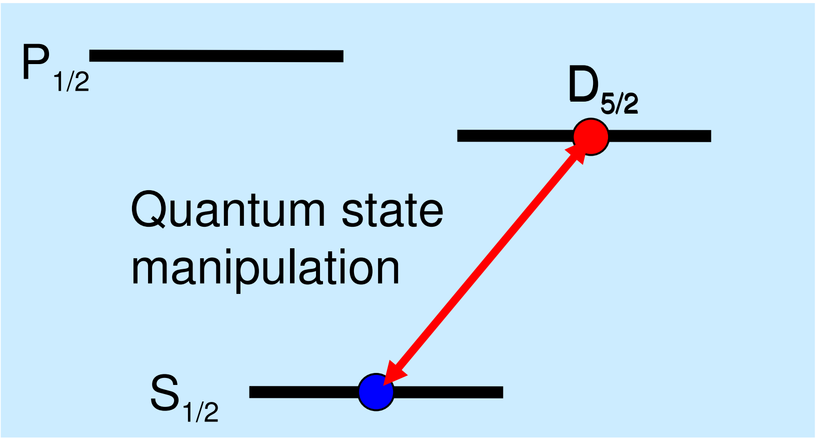


Experimental procedure



1. Initialization in a pure quantum state

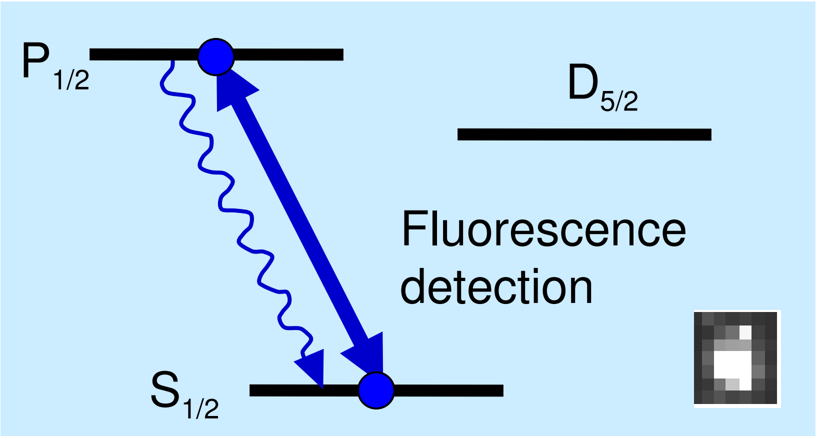
Experimental procedure



1. Initialization in a pure quantum state

2. Quantum state manipulation on $S_{1/2} - D_{5/2}$ transition

Experimental procedure

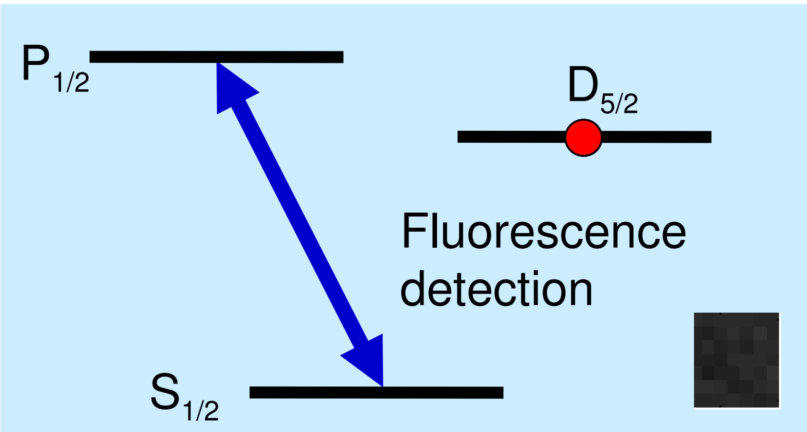


1. Initialization in a pure quantum state:

2. Quantum state manipulation on $S_{1/2} – D_{5/2}$ transition

3. Quantum state measurement by fluorescence detection

Experimental procedure



1. Initialization in a pure quantum state:

2. Quantum state manipulation on $S_{1/2} - D_{5/2}$ transition

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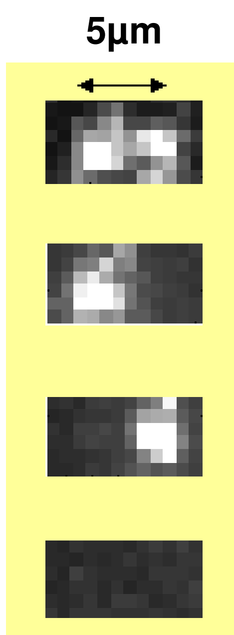
Two ions:

Spatially resolved detection with CCD camera



$|SS\rangle$

$|SD\rangle$

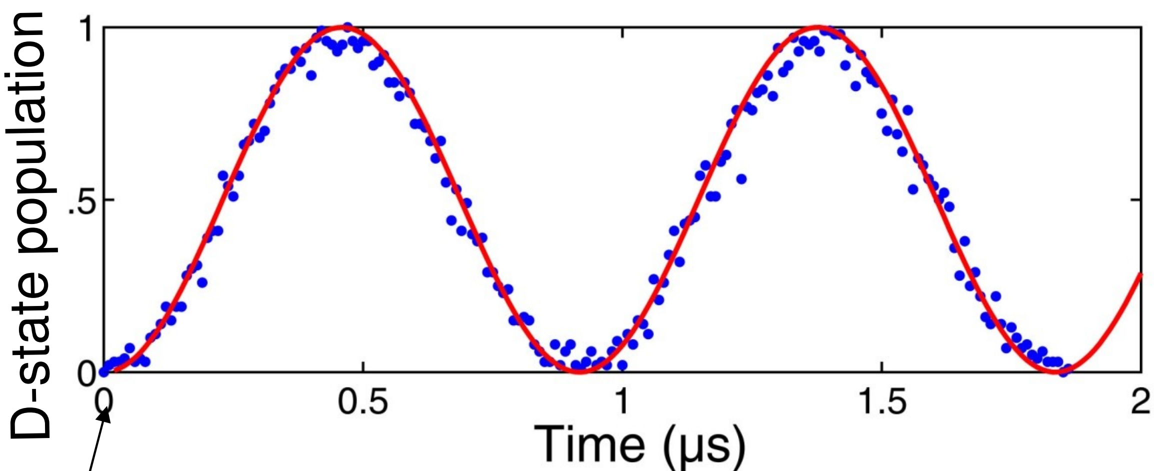
$|DS\rangle$

$|DD\rangle$

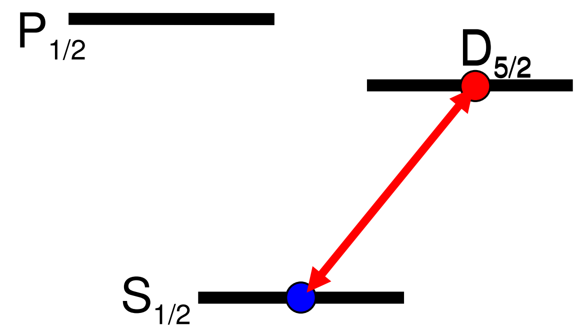
50 experiments / s

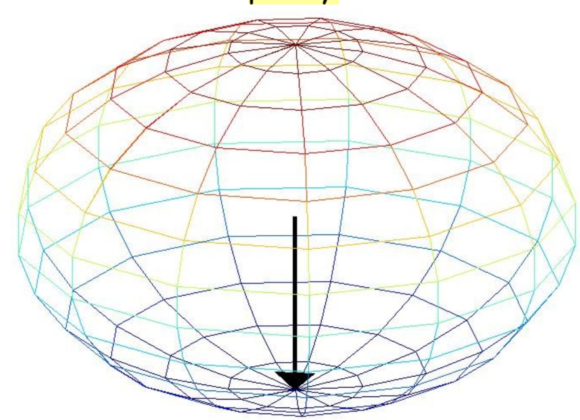
Repeat experiments 100-200 times

Rabi oscillations

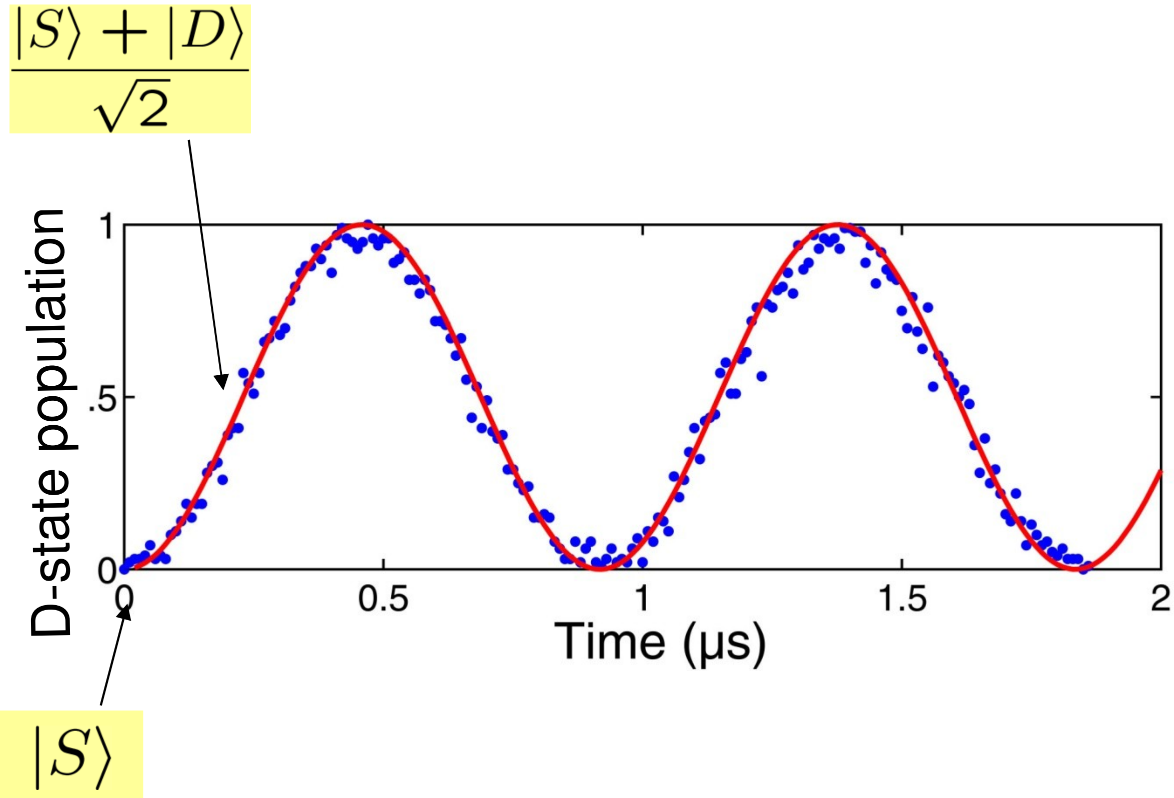


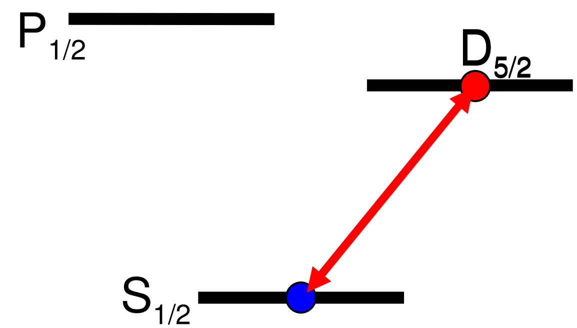
$|S\rangle$

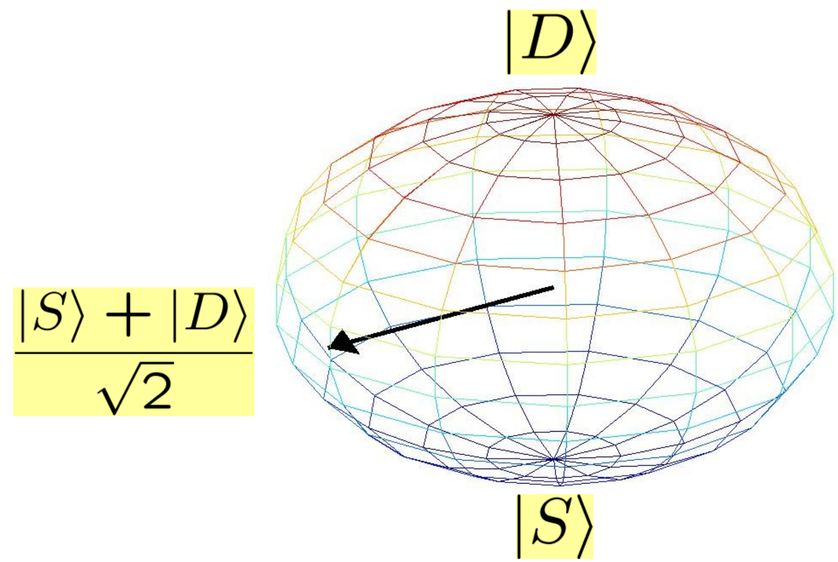




Rabi oscillations







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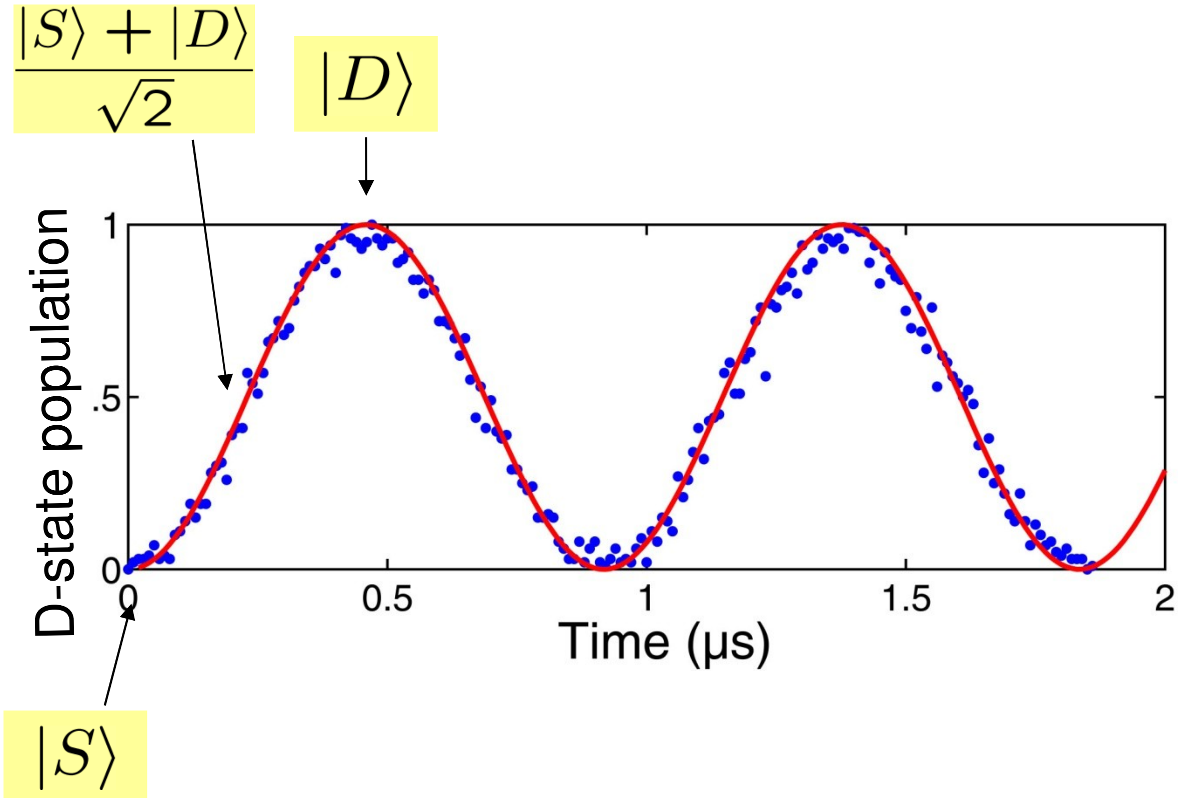
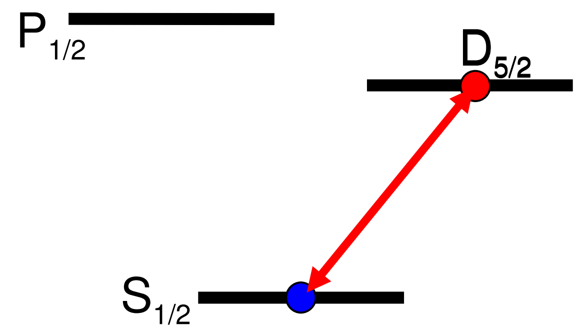
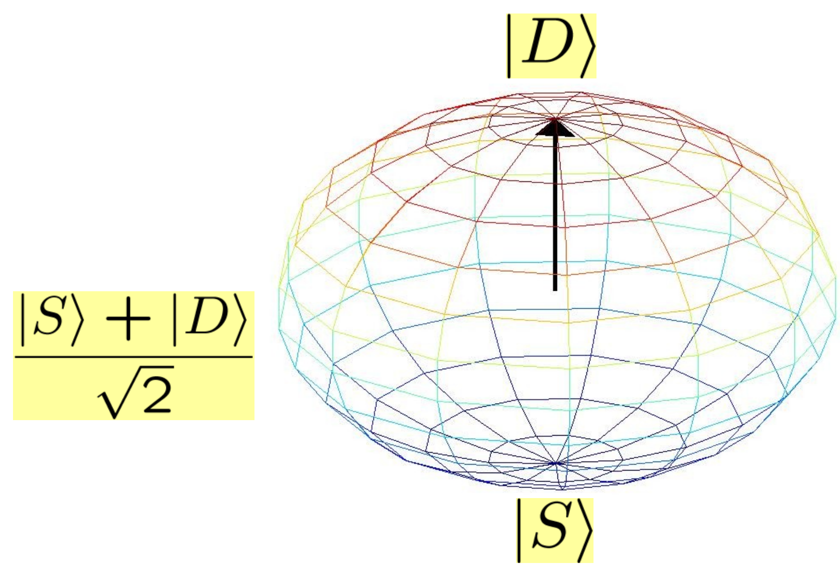
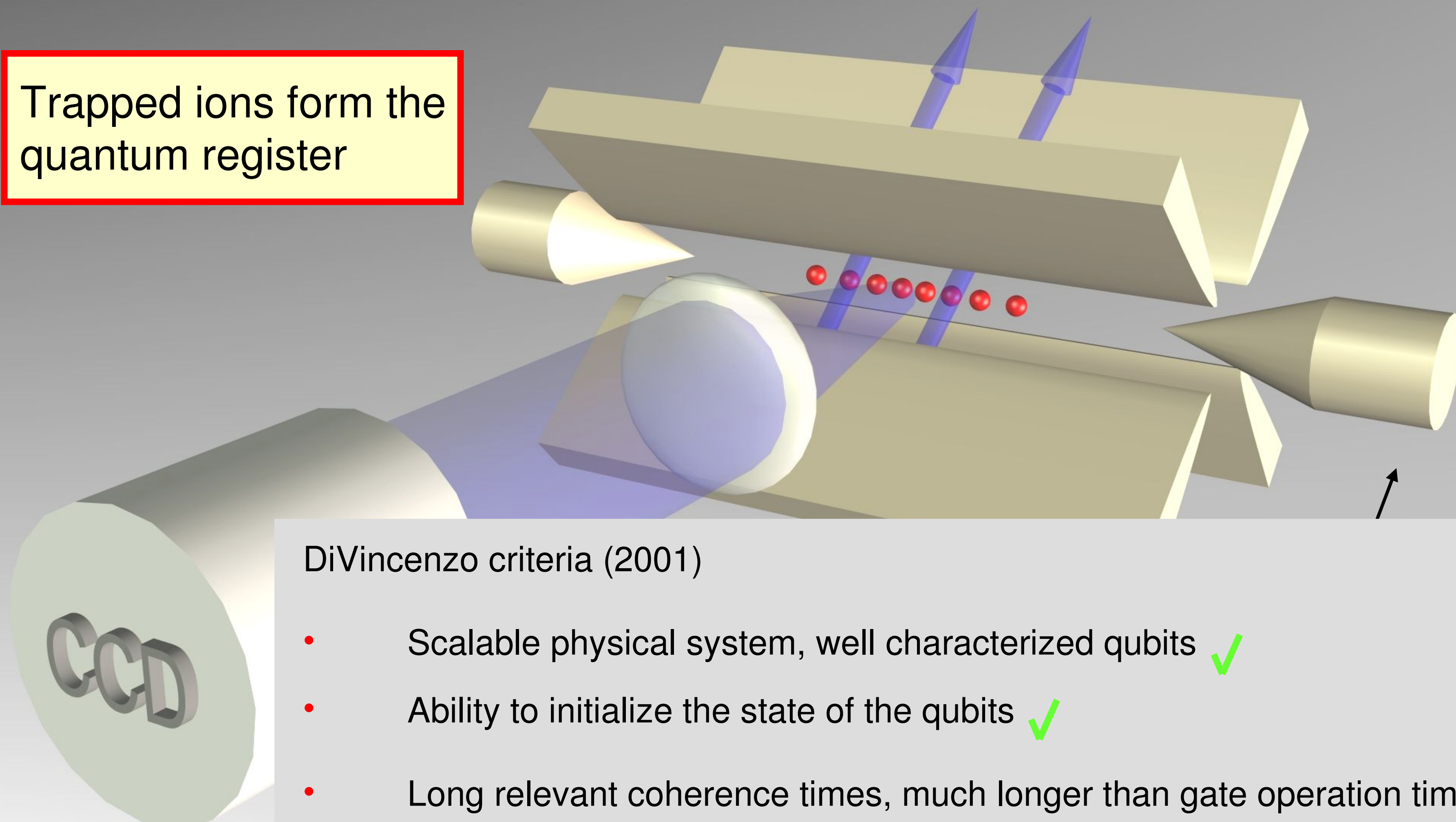






Ion trap quantum computing



Trapped ions form the quantum register

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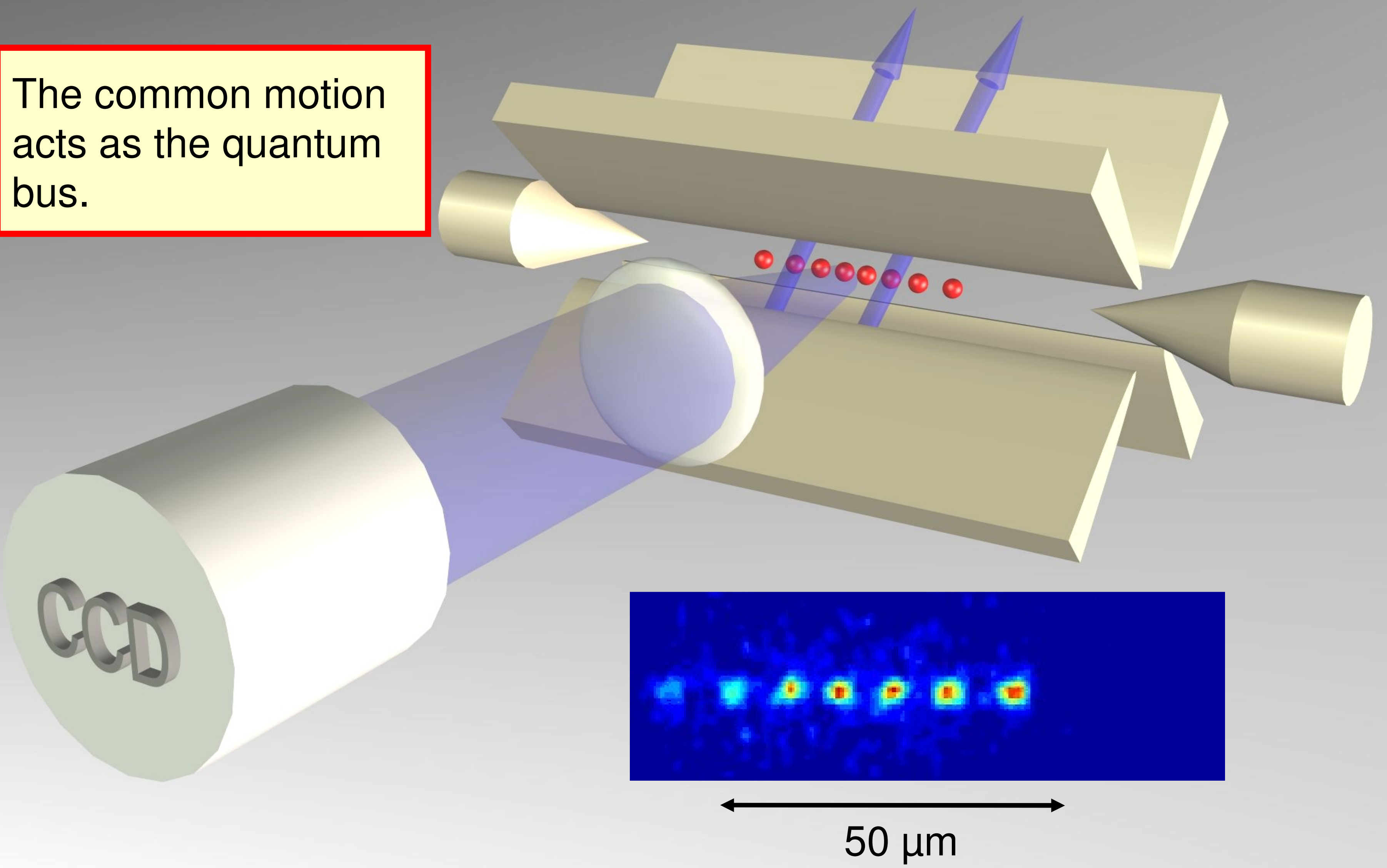
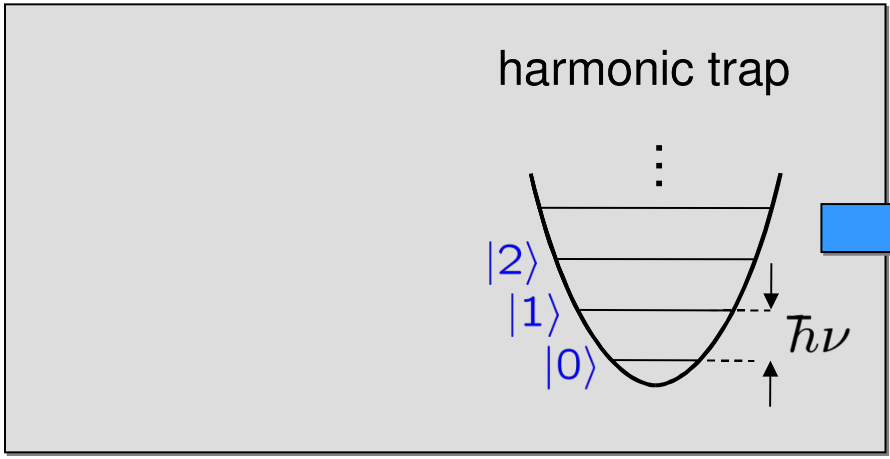
- Ability to initialize the state of the qubits ✓
- Long relevant coherence times, much longer than gate operation time ✓
- "Universal" set of quantum gates ✓
- Qubit-specific measurement capability

Having the qubits interact

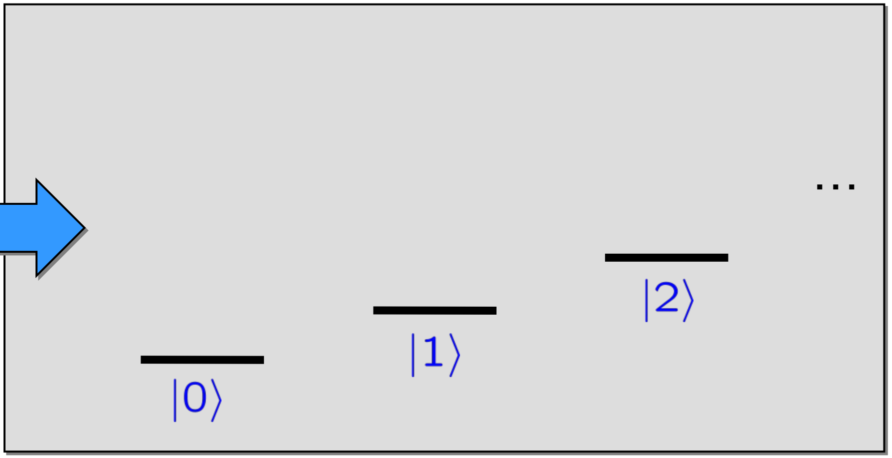
The common motion acts as the quantum bus.



Ion motion

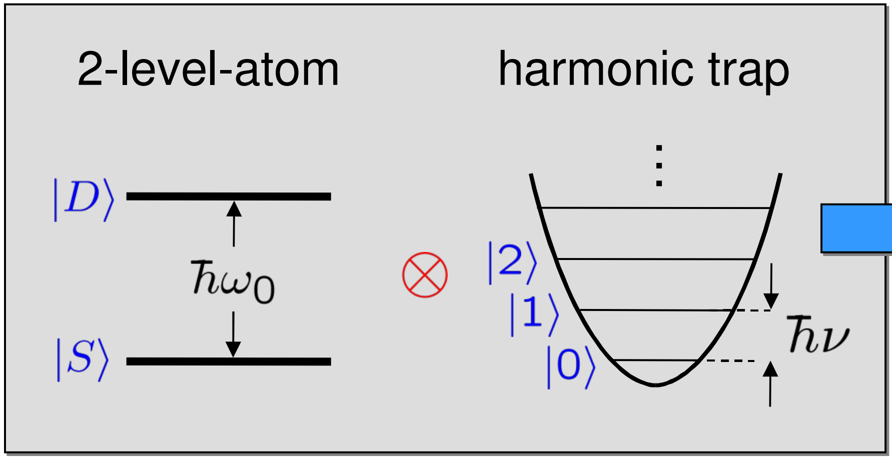


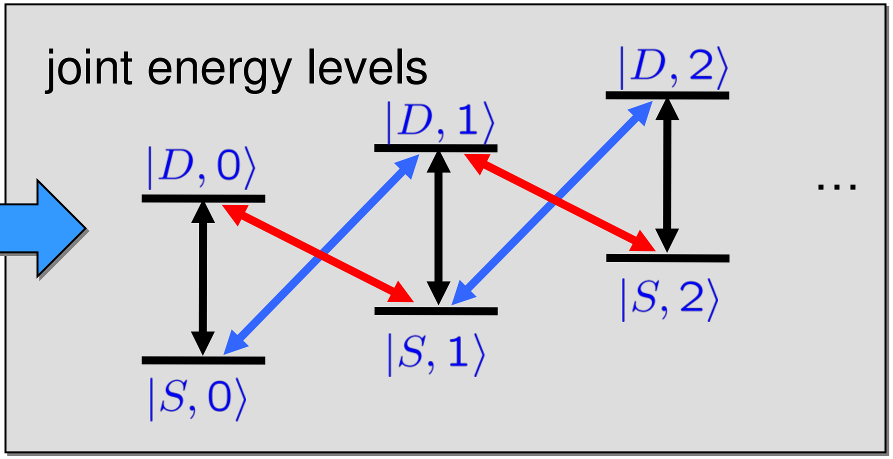
harmonic trap
$|2\rangle$
$|1\rangle$
$|0\rangle$
$\hbar\nu$

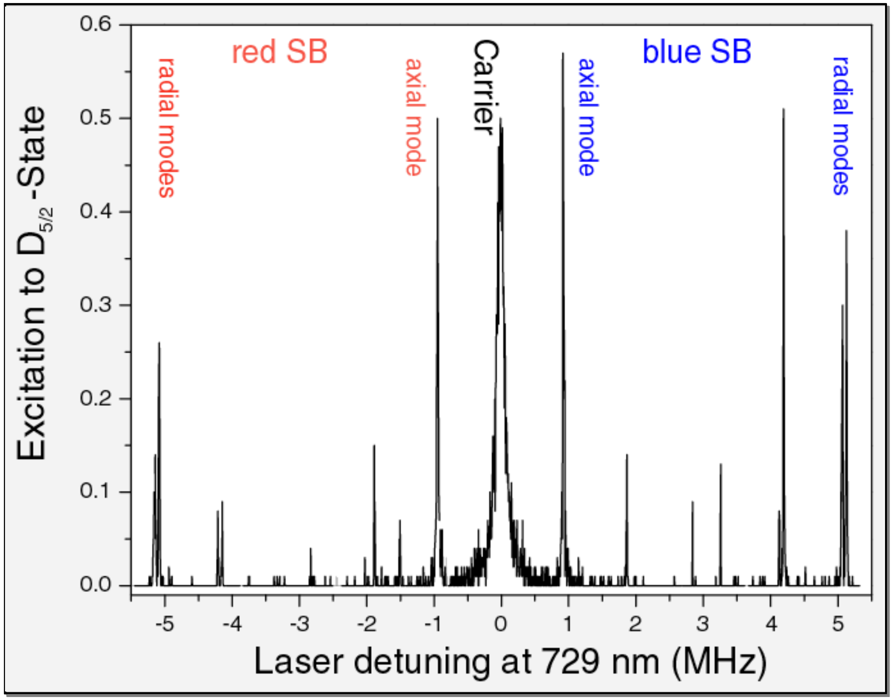


$|0\rangle$
$|1\rangle$
$|2\rangle$
...

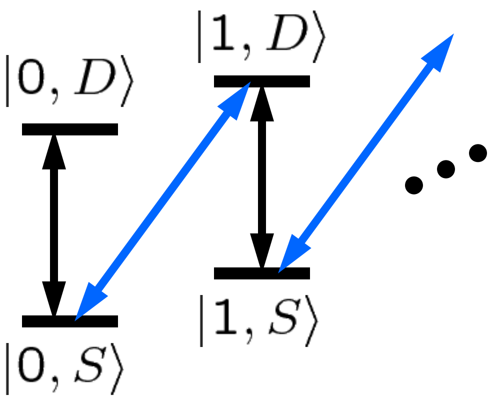
Ion motion


2-level-atom
harmonic trap
$|D\rangle$
$|S\rangle$
$\hbar\omega_0$
$\otimes$
$|2\rangle$
$|1\rangle$
$|0\rangle$
$\hbar\nu$

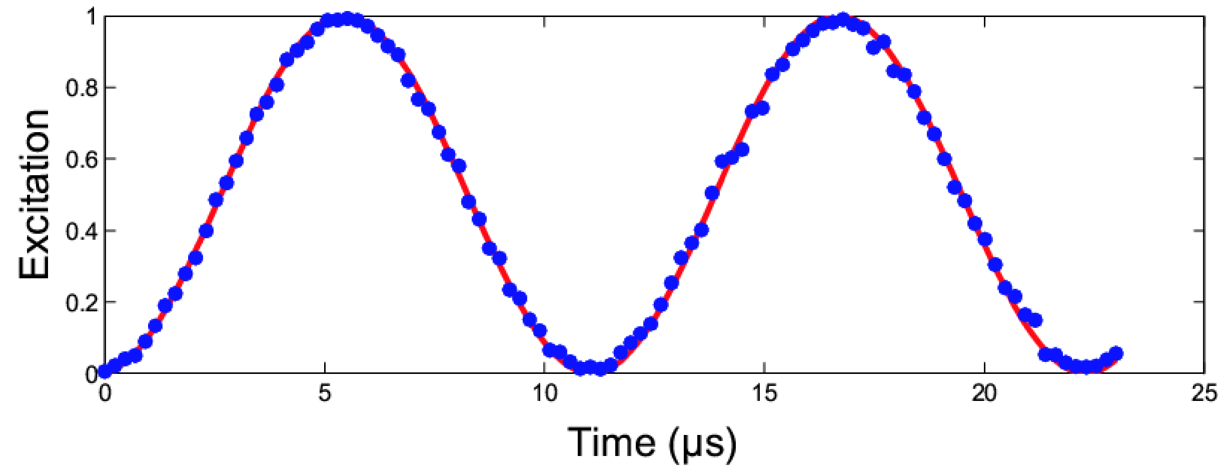

joint energy levels
$|D,0\rangle$
$|D,1\rangle$
$|D,2\rangle$
$|S,0\rangle$
$|S,1\rangle$
$|S,2\rangle$
...


red SB
blue SB
radial modes
axial mode
Carrier
axial mode
radial modes
Excitation to $D_{5/2}$ -State
Laser detuning at 729 nm (MHz)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
-5
-4
-3
-2
-1
0
1
2
3
4
5

Coherent manipulation

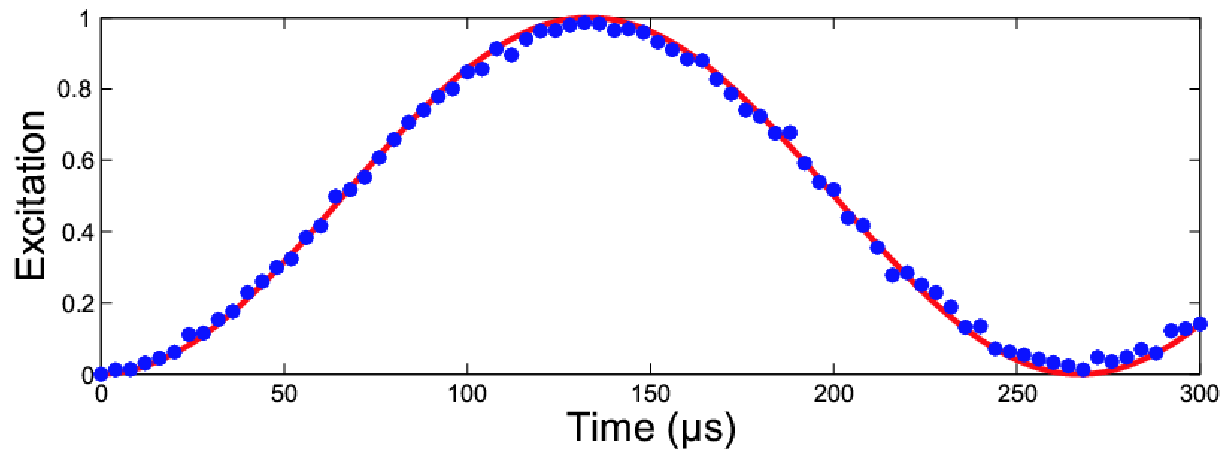


carrier



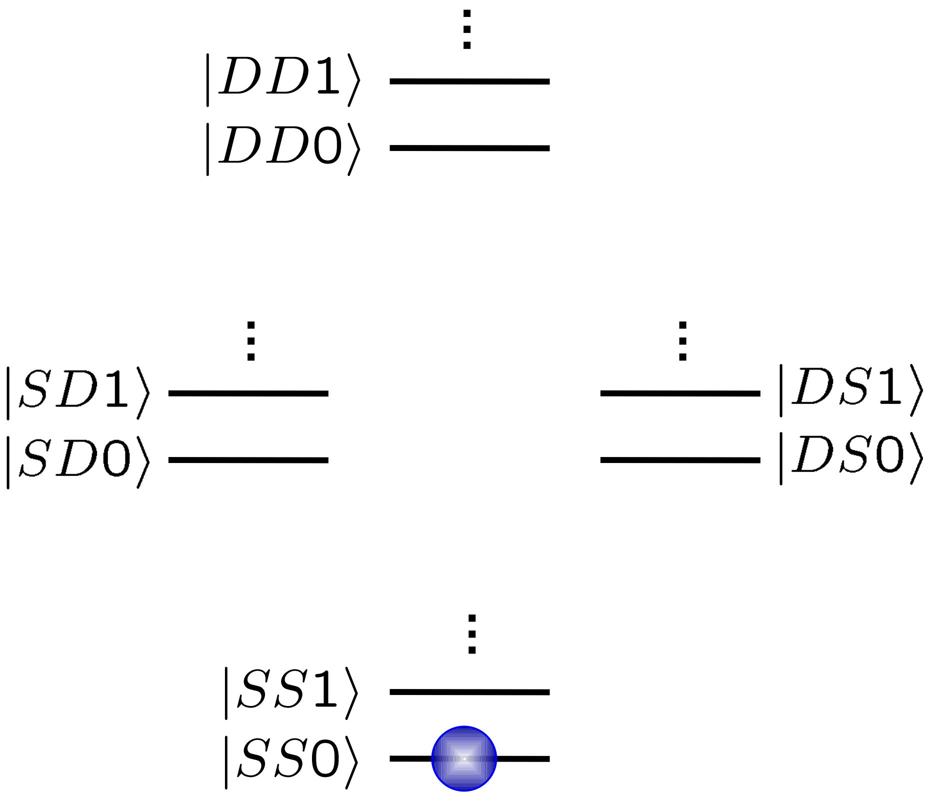
carrier and sideband
Rabi oscillations
with Rabi frequencies

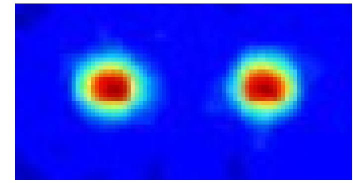
$$\Omega, \quad \eta\Omega$$



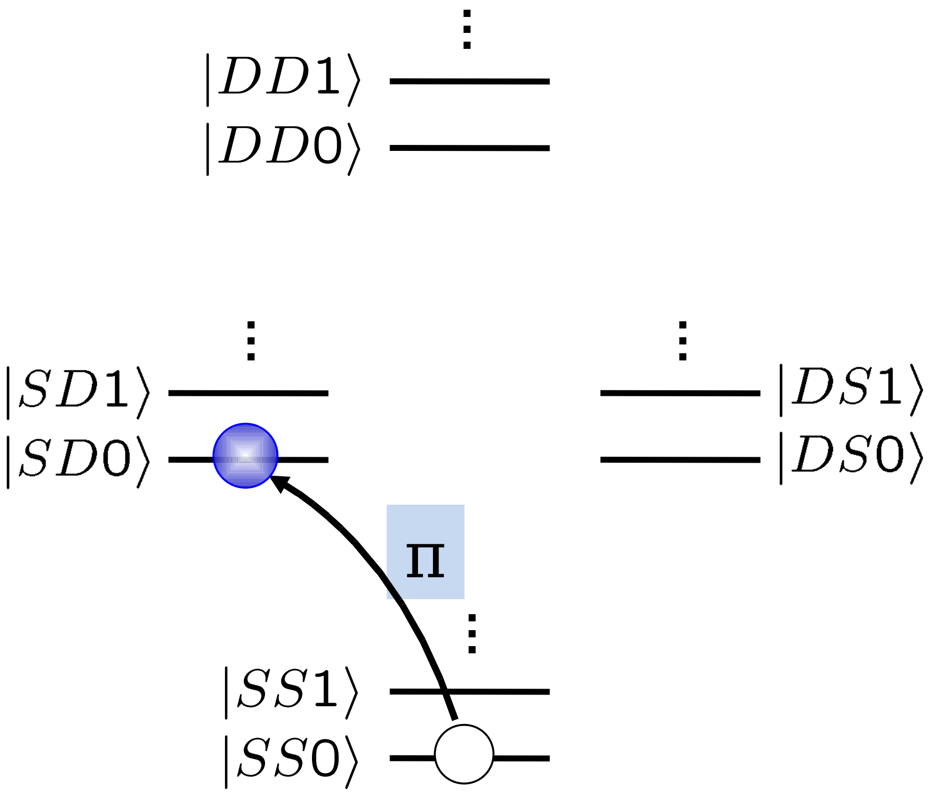
$\eta = kx_0$ Lamb-Dicke parameter

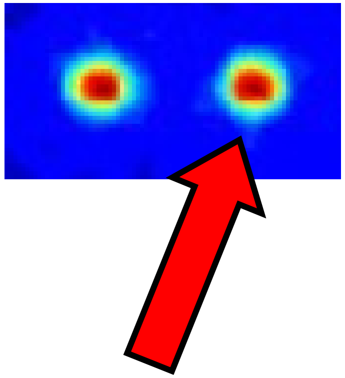
Generation of Bell states



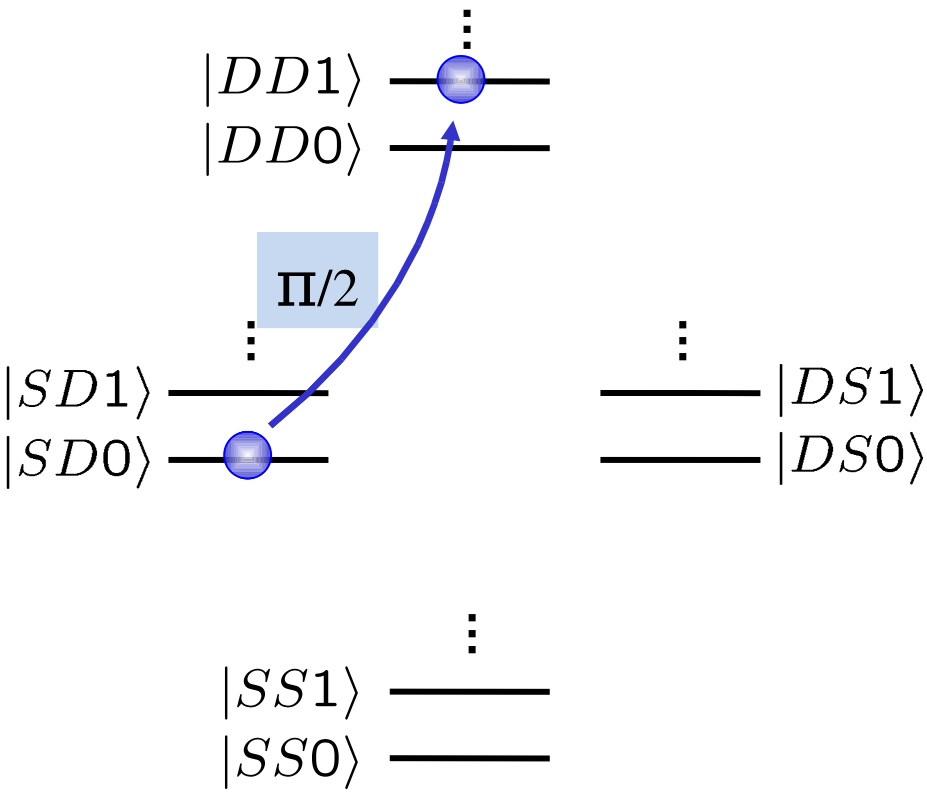


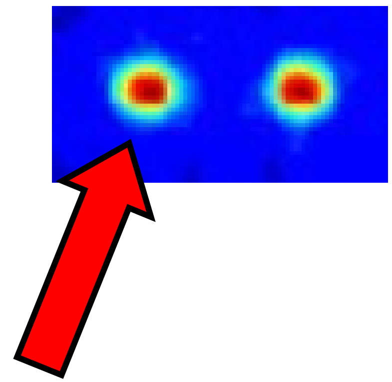
Generation of Bell states



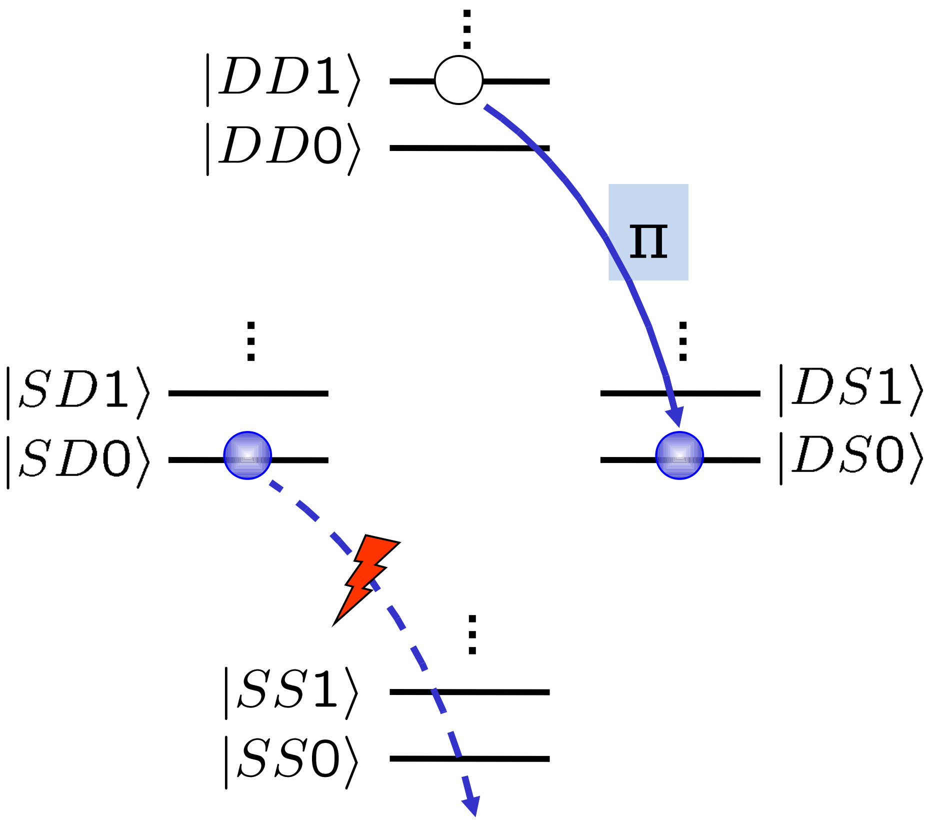


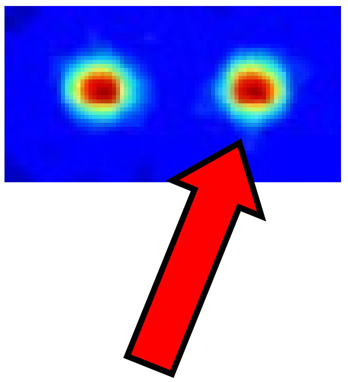
Generation of Bell states



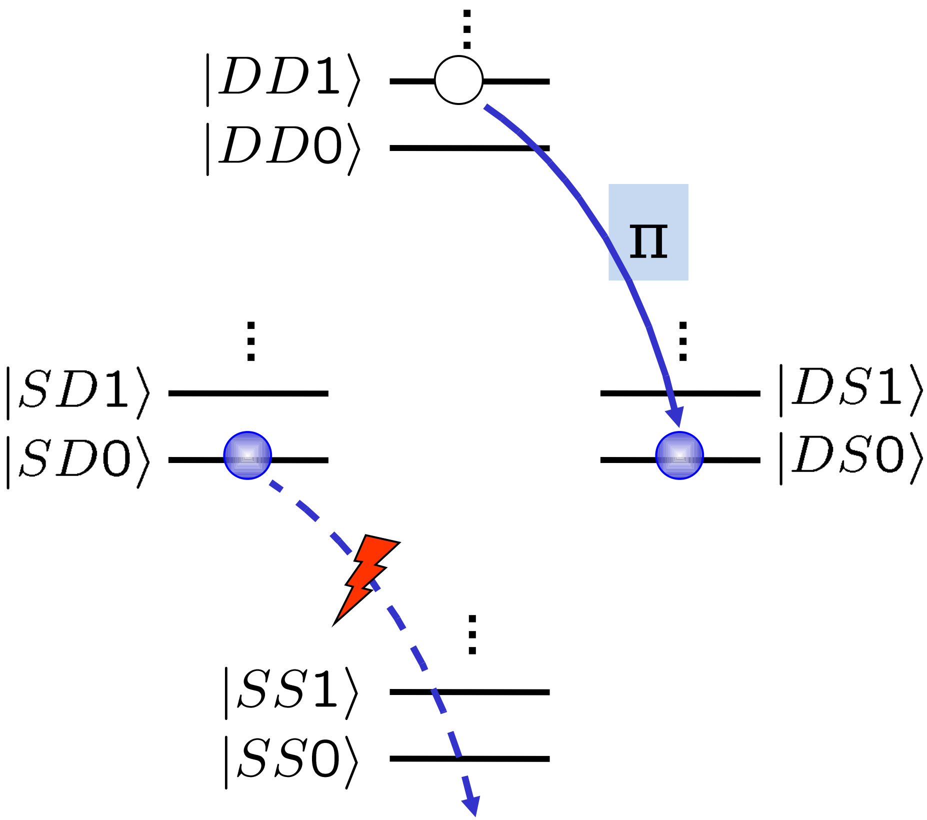


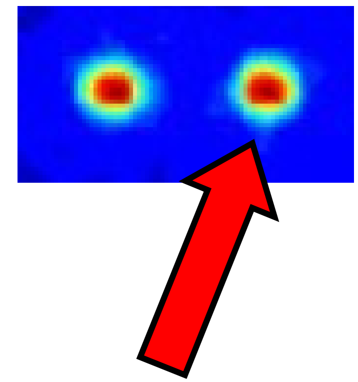
Generation of Bell states





Generation of Bell states





<u>Bell states with atoms</u>

- $^{9}Be^{+}$: NIST (fidelity: 97 %)
- $^{40}Ca^{+}$: Oxford (83%)
- $^{111}Cd^{+}$: Ann Arbor (79%)
- $^{25}Mg^{+}$: Munich
- $^{40}Ca^{+}$: Innsbruck (99%)

Analysis of Bell states

$$|SD\rangle + |DS\rangle$$

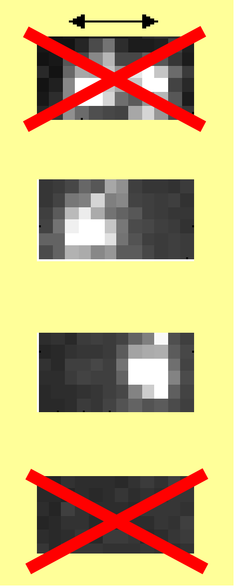
Fluorescence detection with CCD camera:

$|SS\rangle$

$|SD\rangle$

$|DS\rangle$

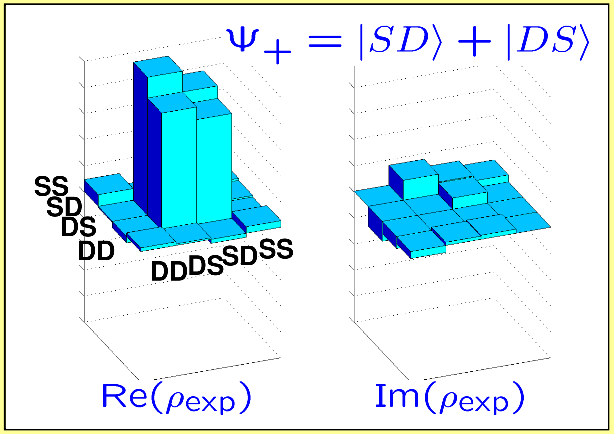
$|DD\rangle$



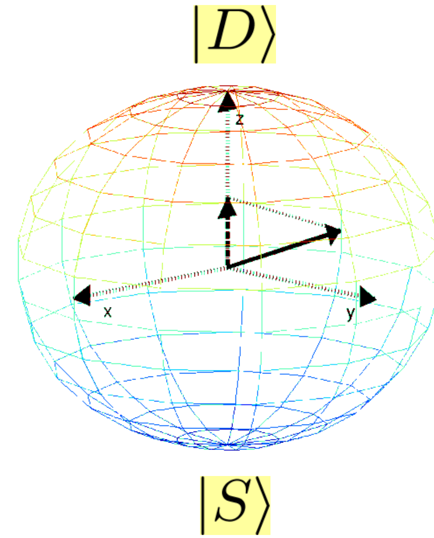
Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

⇨ Measurement of the density matrix:



Measuring a density matrix

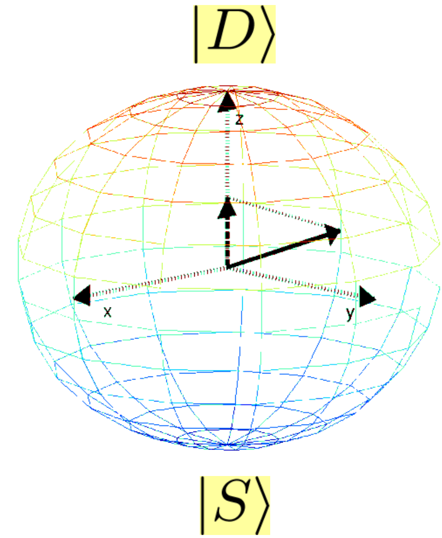


A measurement yields the *z*-component of the Bloch vector

=> Diagonal of the density matrix

$$\rho = \begin{pmatrix} P_S & C - iD \\ C + iD & P_D \end{pmatrix}$$

Measuring a density matrix

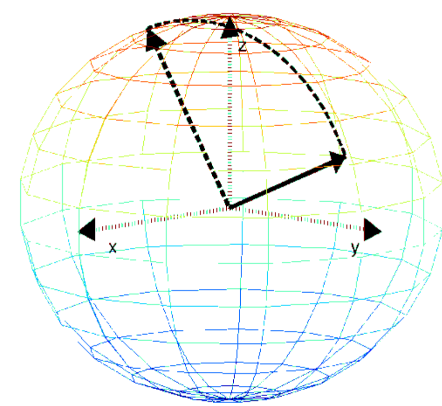


A measurement yields the *z*-component of the Bloch vector

=> Diagonal of the density matrix

$$\rho = \begin{pmatrix} P_S & C - iD \\ C + iD & P_D \end{pmatrix}$$

Rotation around the *x*- or the *y*-axis prior to the measurement yields the phase information of the qubit.

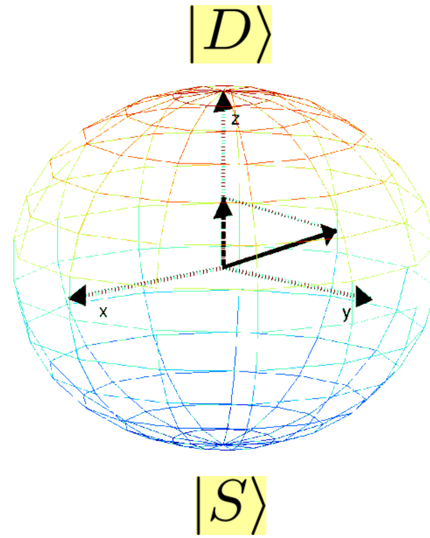


Measuring a density matrix

A measurement yields the *z*-component of the Bloch vector

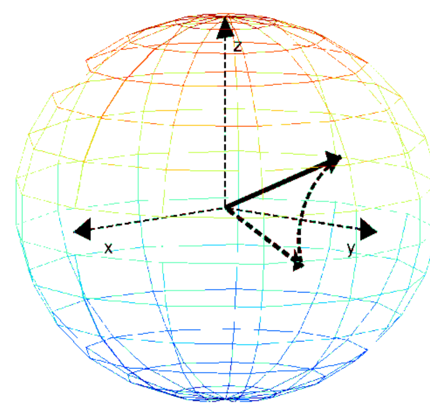
=> Diagonal of the density matrix

$$\rho = \begin{pmatrix} P_S & C - iD \\ C + iD & P_D \end{pmatrix}$$

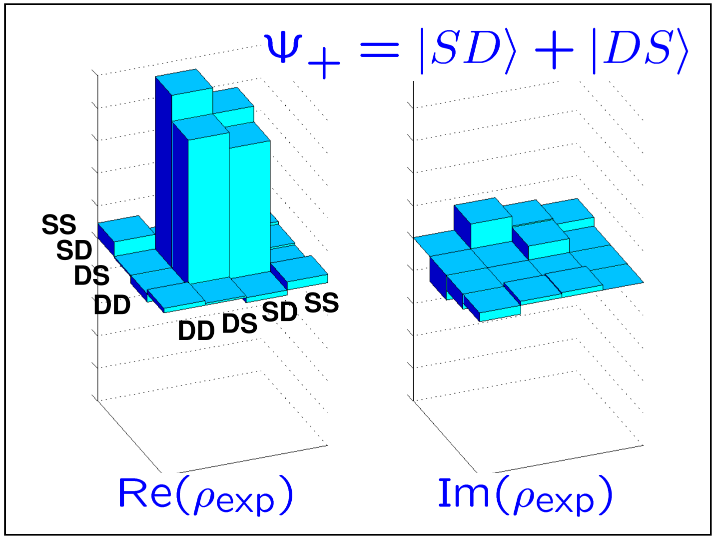


Rotation around the *x*- or the *y*-axis prior to the measurement yields the phase information of the qubit.

=> coherences of the density ma



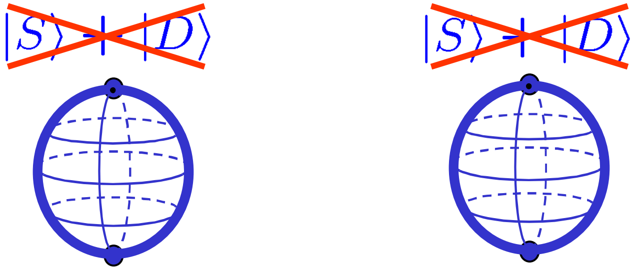
Entanglement



States are fully characterized

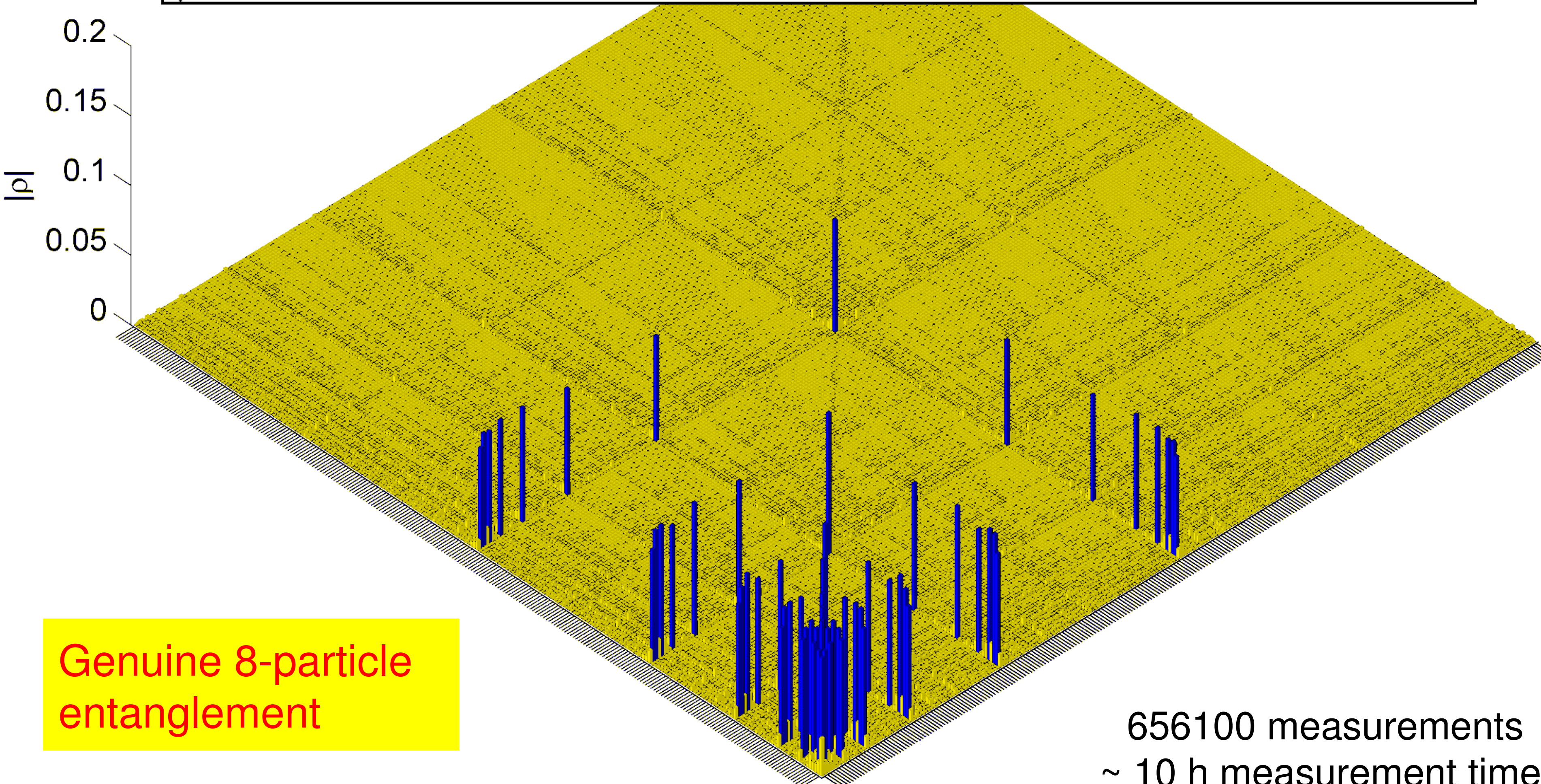
Fidelities: up to 0.99

Entanglement: the state of each qubit is not defined!

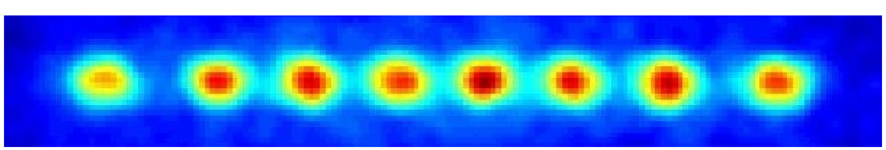


“Large” entangled states

$$\frac{1}{\sqrt{8}}(|DDDDDDDS\rangle + |DDDDDDSD\rangle + \ldots + |SDDDDDDD\rangle)$$



Genuine 8-particle entanglement

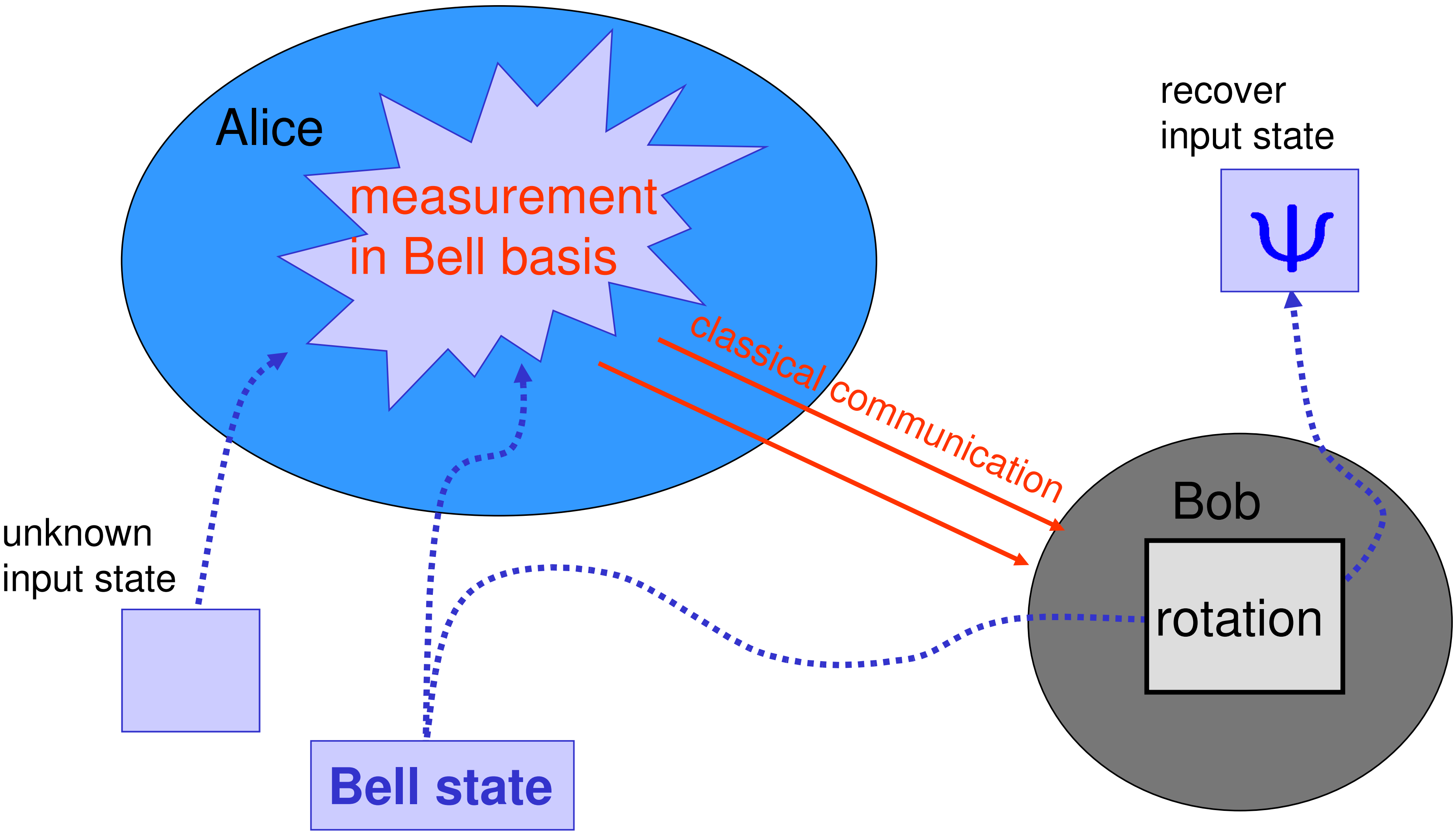


656100 measurements
~ 10 h measurement time

Häffner et al., Nature **438**, 643 (2005)

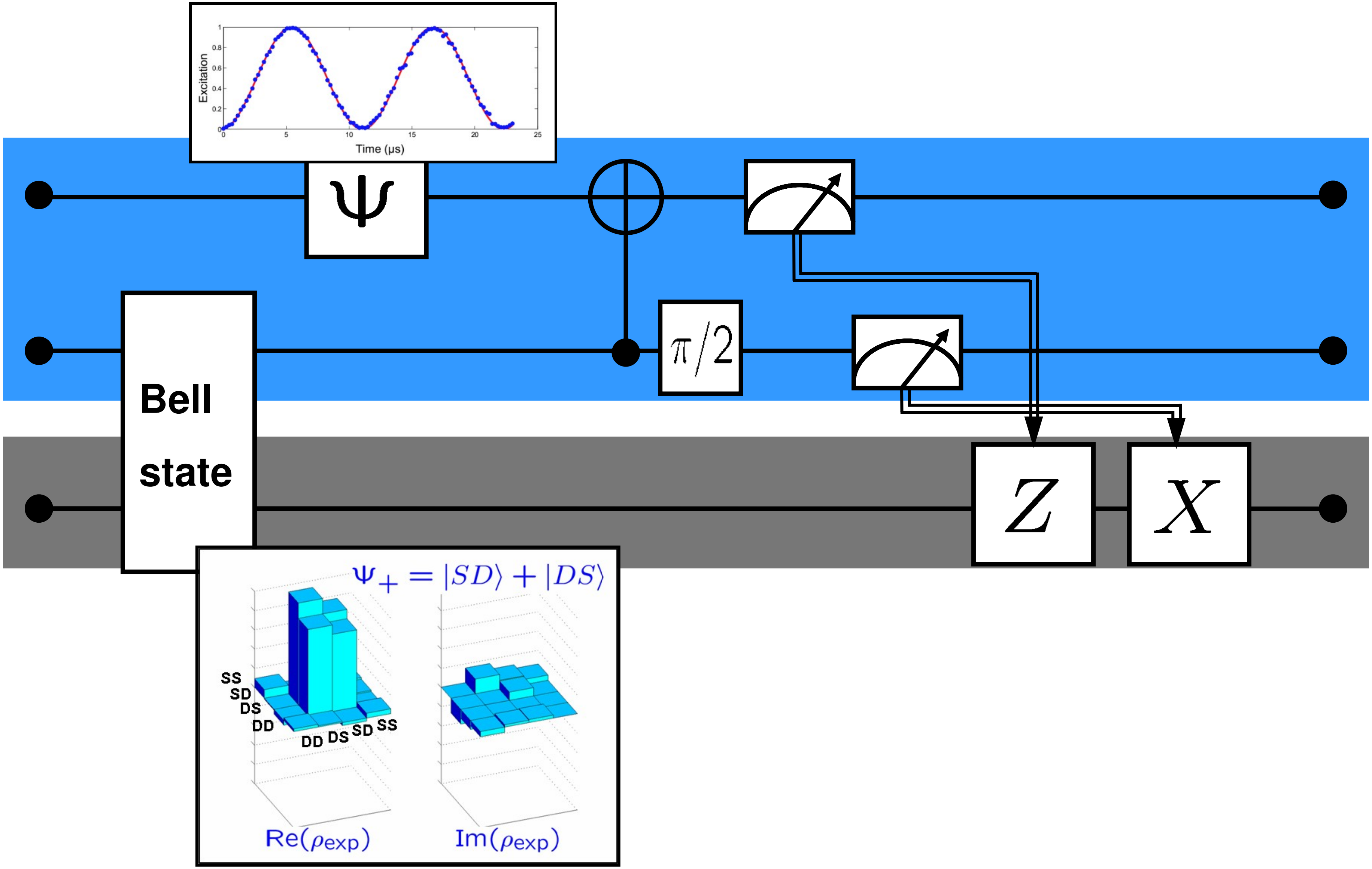
- Introduction to quantum information
- Ion trap quantum computing
- Teleportation and more
- Scaling of ion trap quantum computers

Teleportation

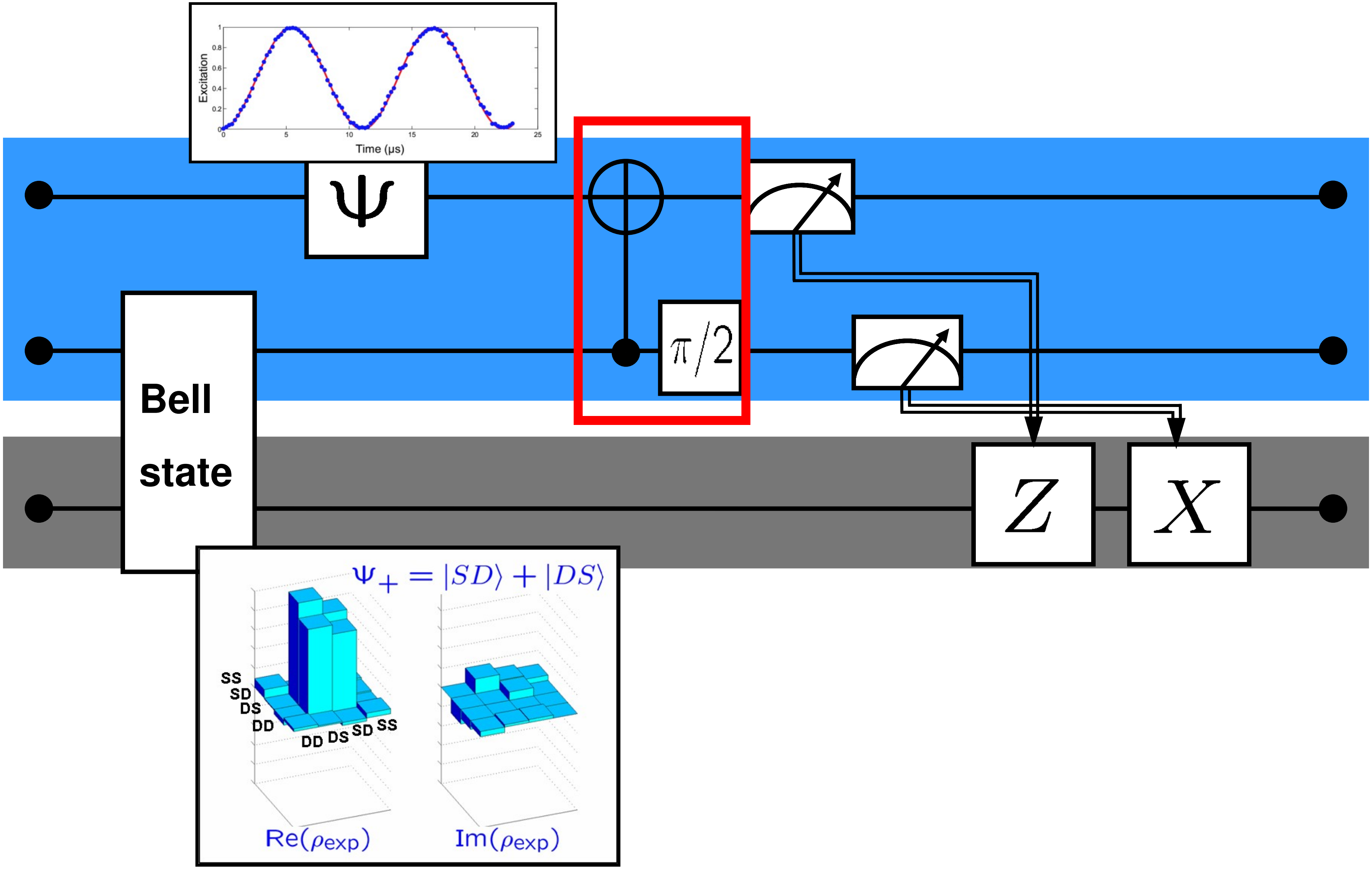


Bennett et al., Phys. Rev. Lett. **70**, 1895 (1993)

Teleportation

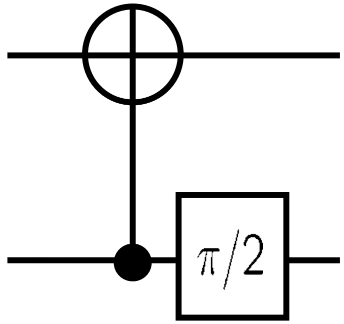


Teleportation



Bell measurement

$$\left.\begin{matrix} |SS+DD\rangle \\ |SD+DS\rangle \\ |SS-DD\rangle \\ |SD+DS\rangle \end{matrix}\right\}$$



$$\left\{\begin{matrix} |SS\rangle \\ |SD\rangle \\ |DS\rangle \\ |DD\rangle \end{matrix}\right.$$

Ion trap quantum computing

VOLUME 74, NUMBER 20 PHYSICAL REVIEW LETTERS 15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*
Institut für Theoretische Physik, Universiät Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria
(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

PACS numbers: 89.80.+h, 03.65.Bz, 12.20.Fv, 32.80.Pj

(a)

(b)

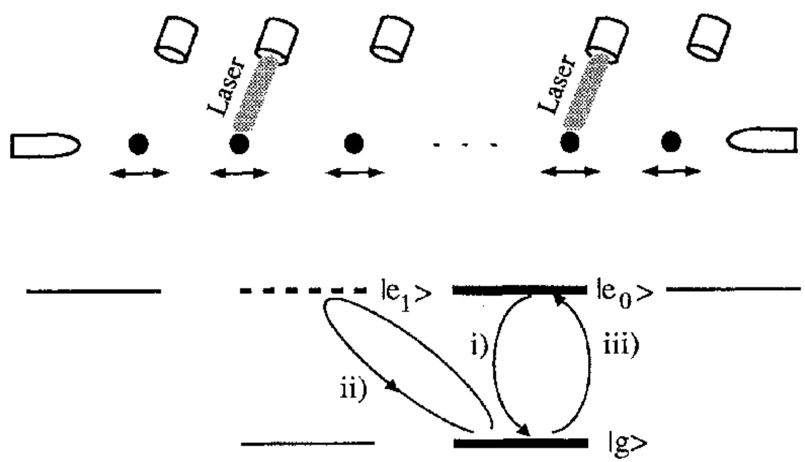


FIG. 1. (a) *N* ions in a linear trap interacting with *N* different laser beams; (b) atomic level scheme.

controlled – NOT :

$$|\varepsilon_1\rangle|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle|\varepsilon_1 \oplus \varepsilon_2\rangle$$

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

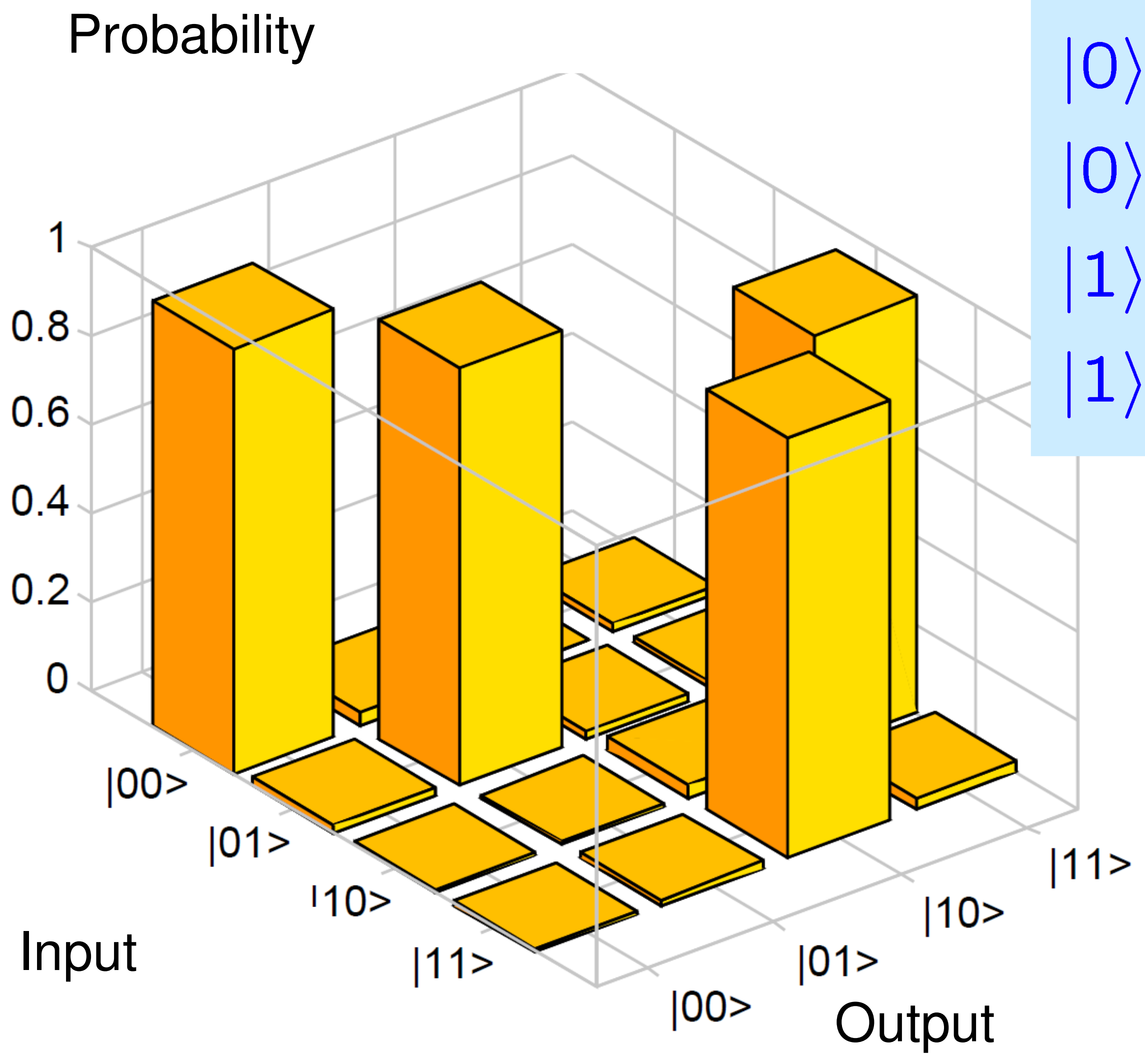
control bit

target bit

other gate proposals (and more):

- Cirac & Zoller
- Mølmer & Sørensen, Milburn
- Jonathan & Plenio & Knight
- Geometric phases
- Leibfried & Wineland

Truth table of a controlled NOT gate



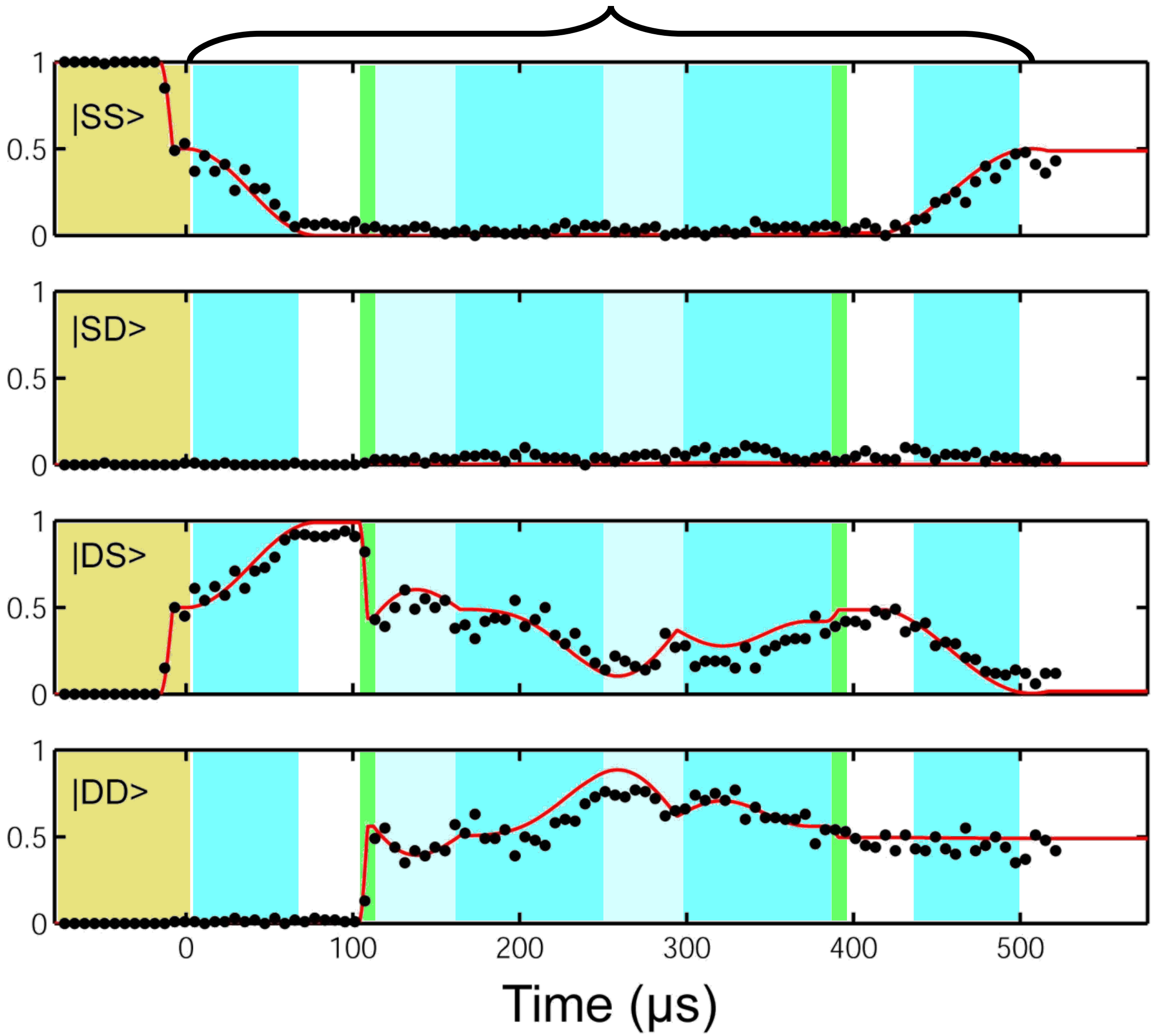
$$
\begin{array}{lcl}
|0\rangle|0\rangle & \rightarrow & |0\rangle|0\rangle \\
|0\rangle|1\rangle & \rightarrow & |0\rangle|1\rangle \\
|1\rangle|0\rangle & \rightarrow & |1\rangle|1\rangle \\
|1\rangle|1\rangle & \rightarrow & |1\rangle|0\rangle
\end{array}
$$

→ universal set of quantum gates

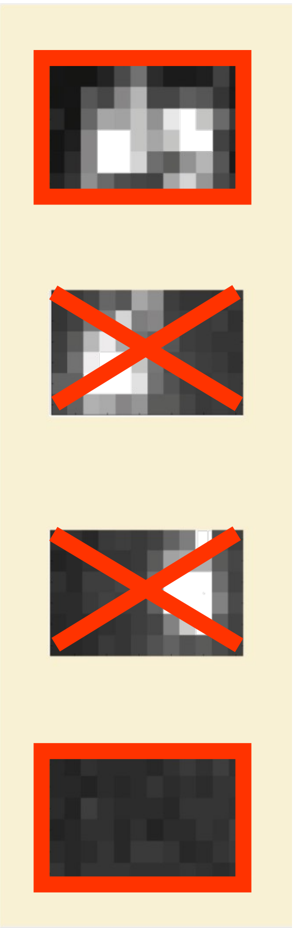
Another way to generate Bell states

$$|SS\rangle \rightarrow |S + D\rangle|S\rangle \quad \longrightarrow \quad |SS\rangle + |DD\rangle$$

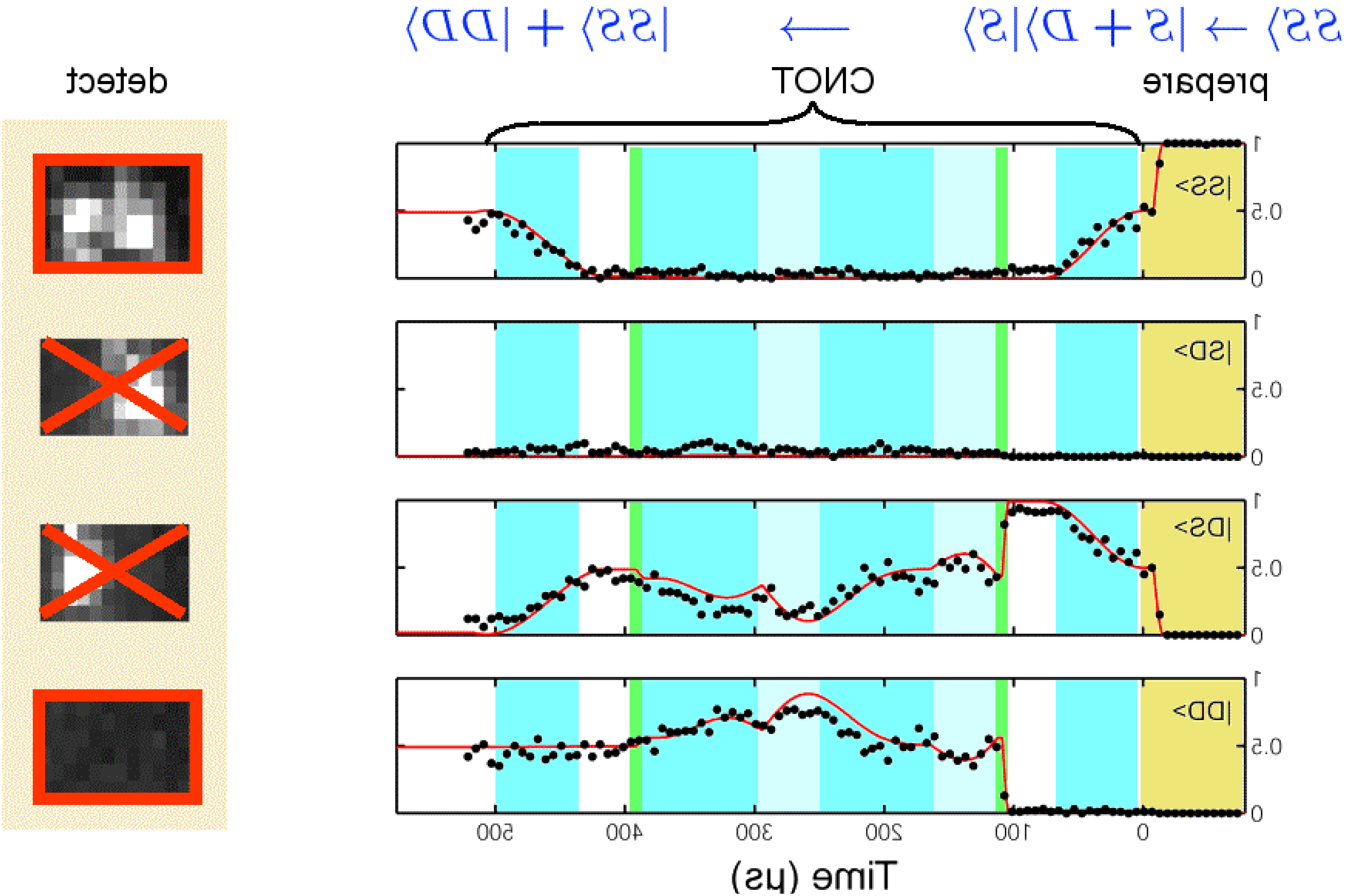
prepare CNOT



output

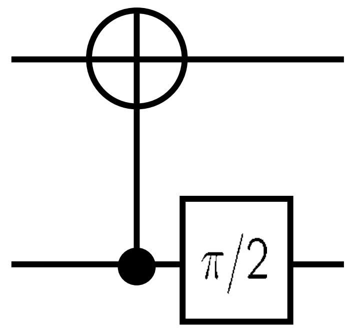


And now backwards



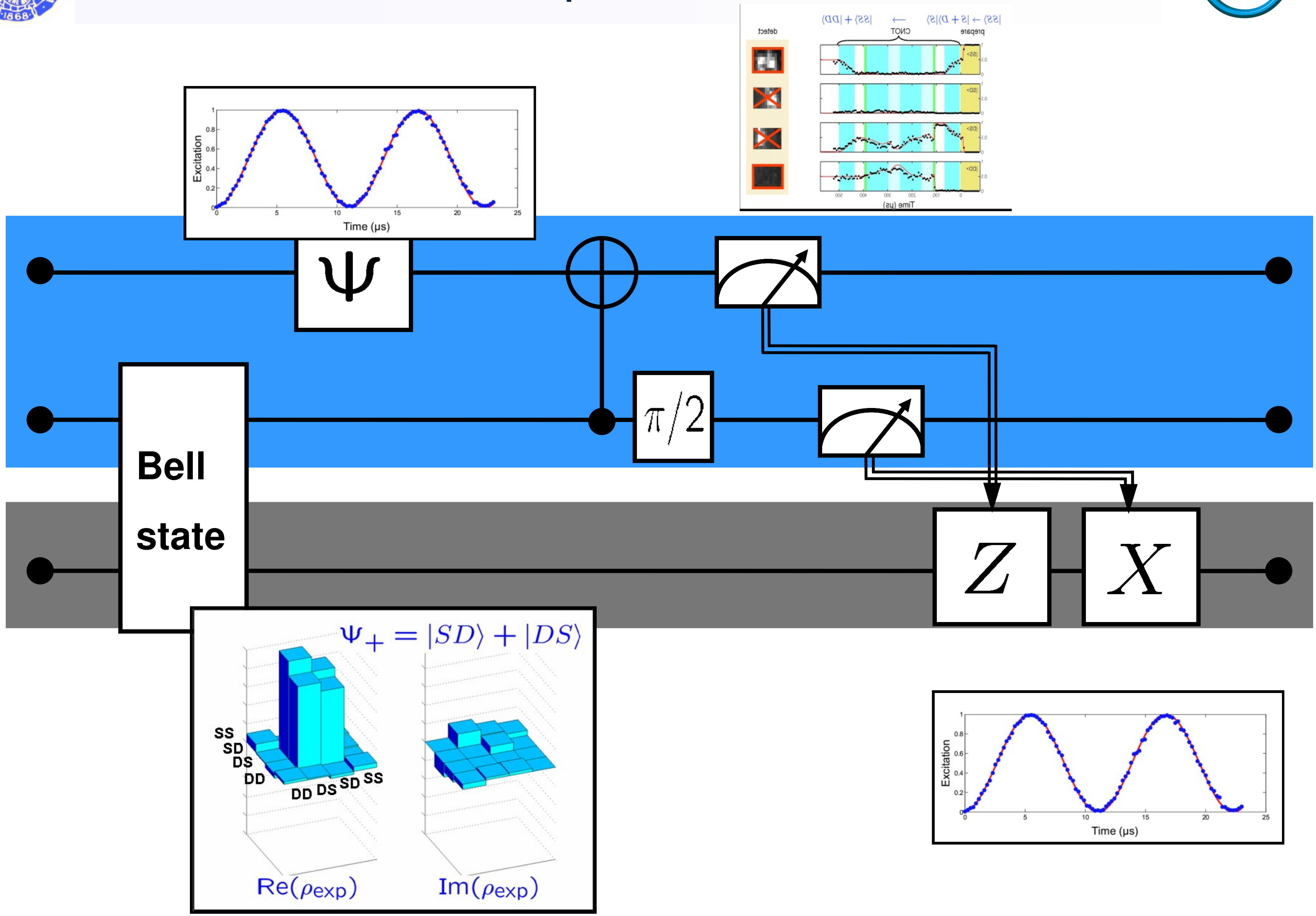
Bell measurement

$$\left.\begin{array}{c} |SS+DD\rangle \\ |SD+DS\rangle \\ |SS-DD\rangle \\ |SD+DS\rangle \end{array}\right\}$$

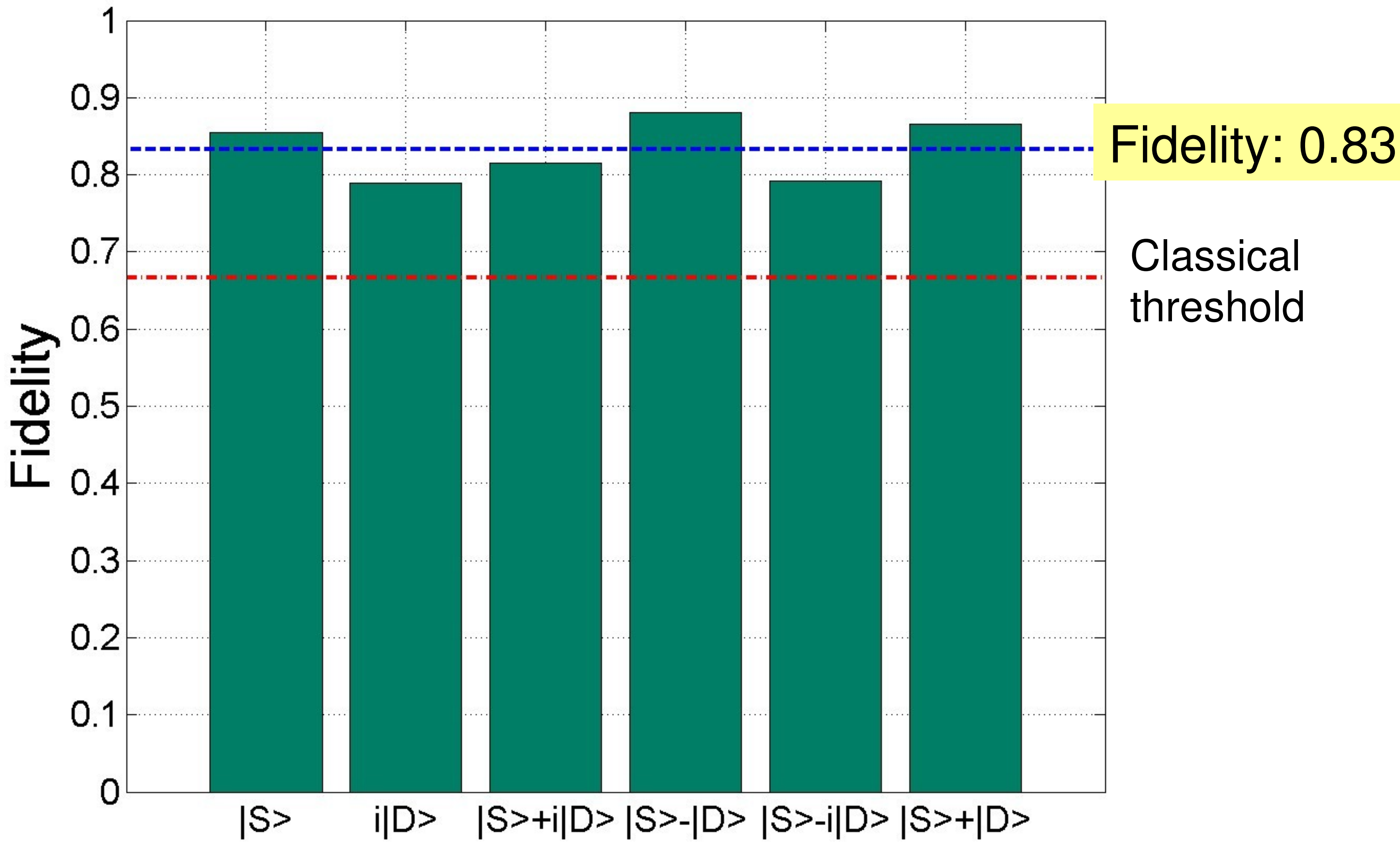


$$\left\{\begin{array}{c} |SS\rangle \\ |SD\rangle \\ |DS\rangle \\ |DD\rangle \end{array}\right.$$

Teleportation

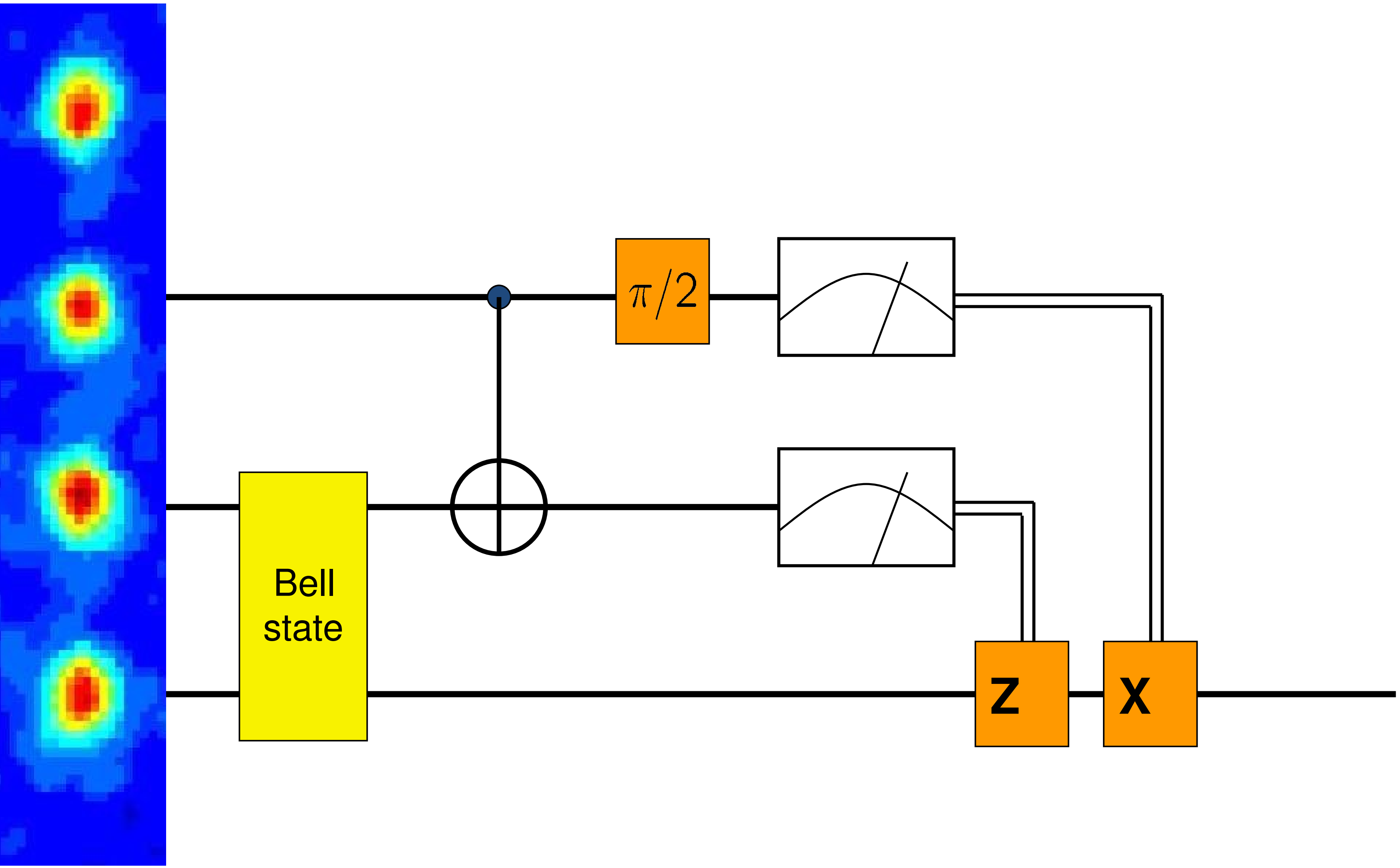


Deterministic teleportation

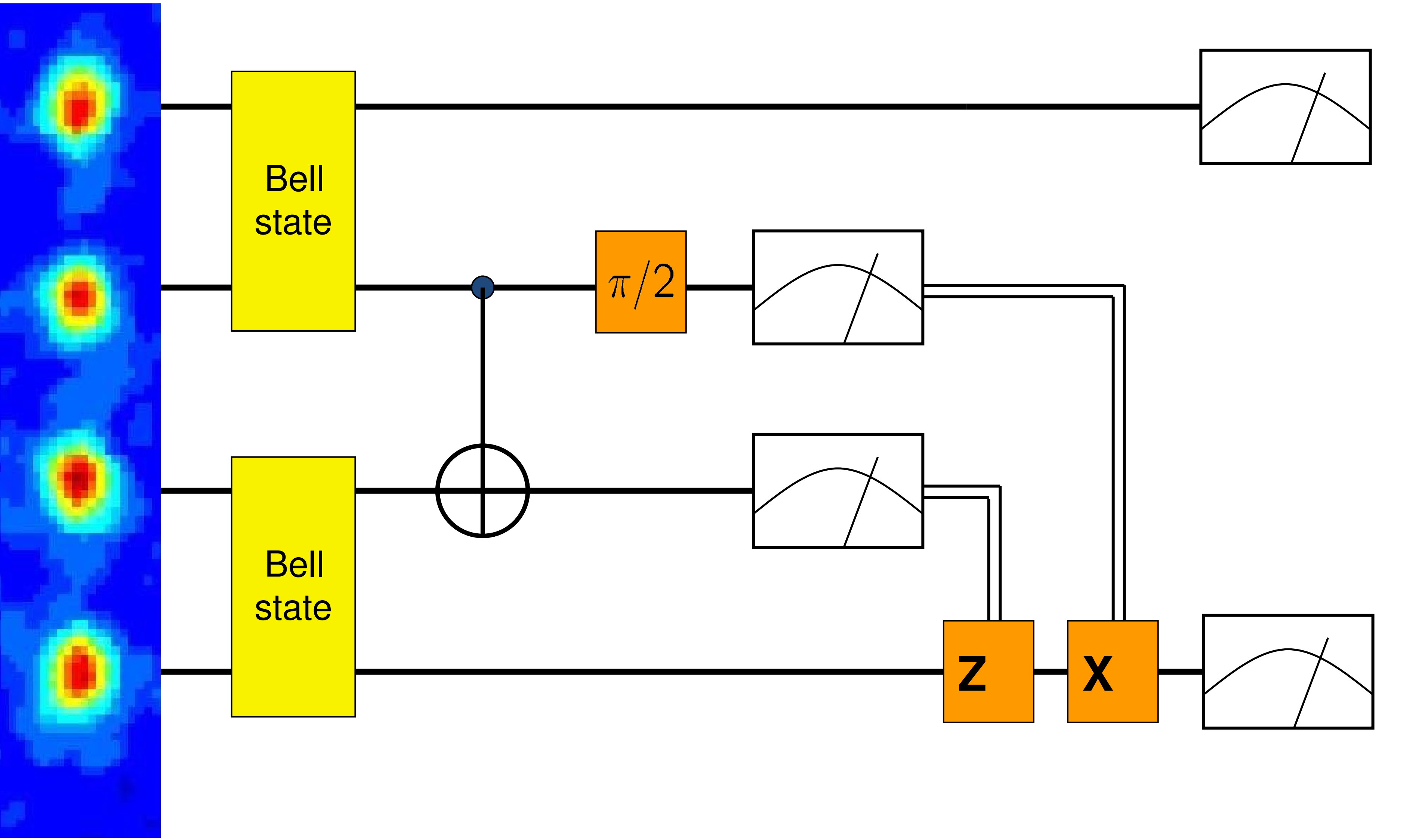


"Deterministic teleportation with atoms"
Barrett et al., Nature **429**, 737 (2004) and Riebe et al., Nature **429**, 734 (2004)

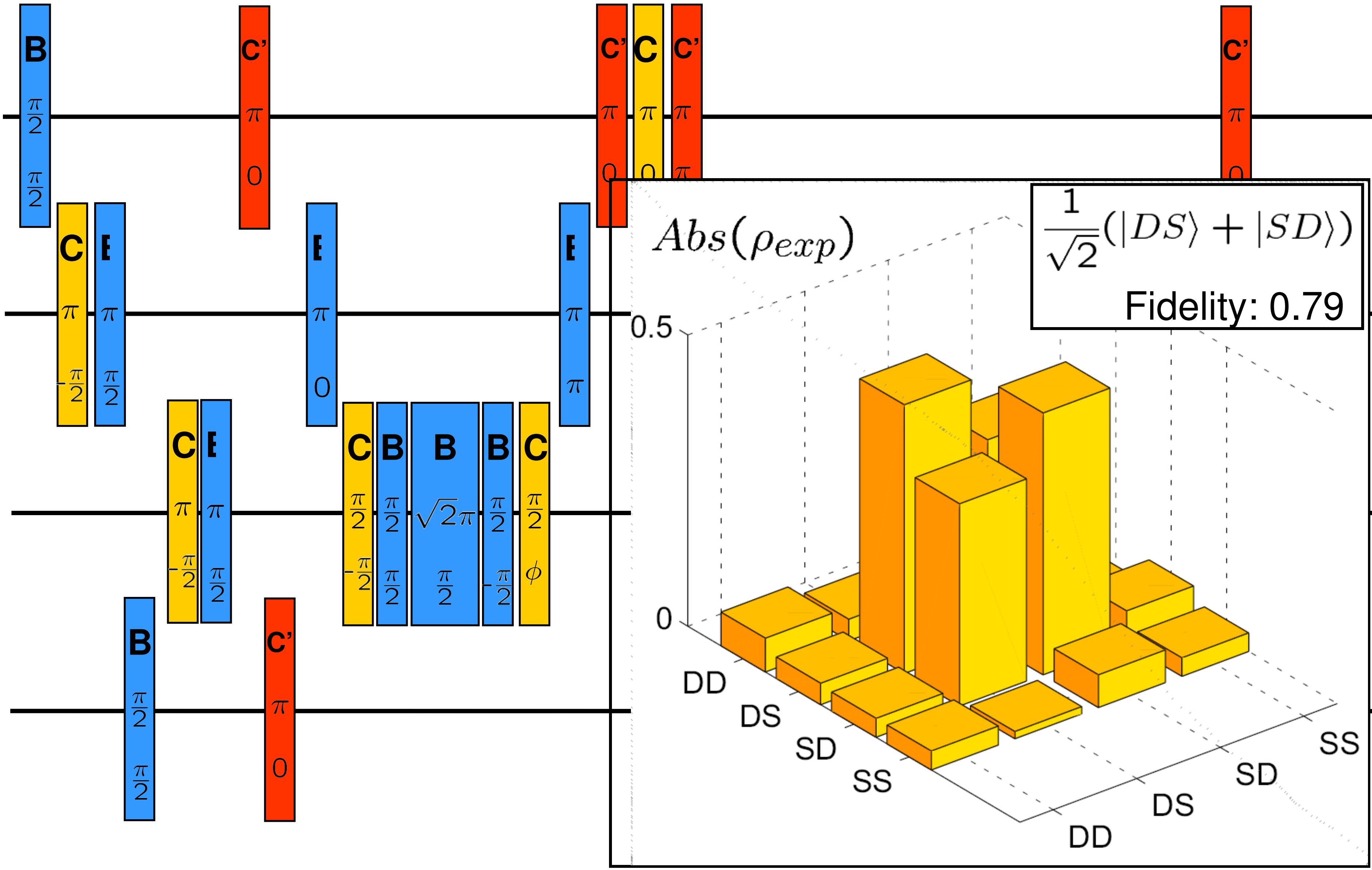
Teleporting entanglement



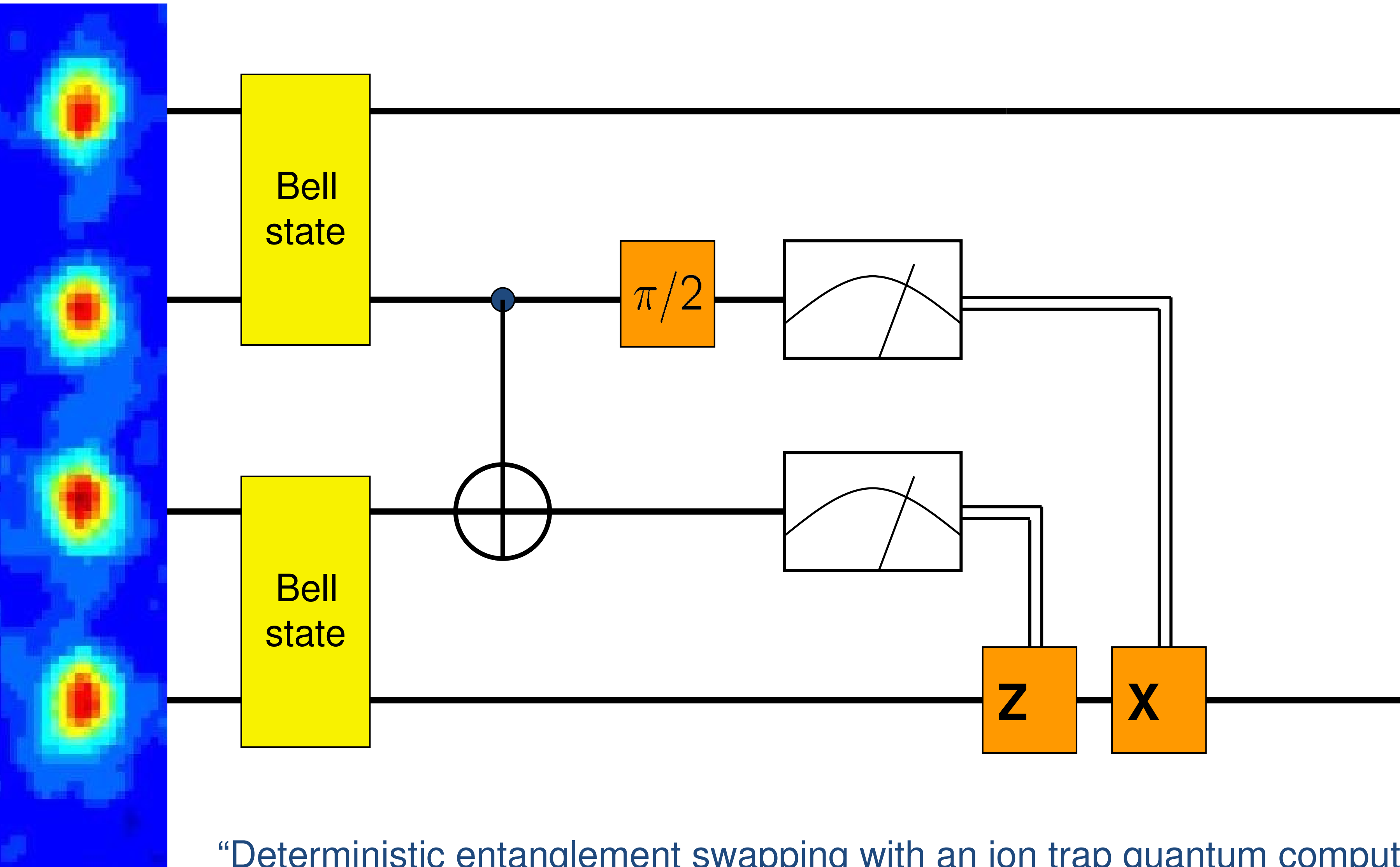
Teleporting entanglement



Teleporting entanglement



Entanglement swapping



"Deterministic entanglement swapping with an ion trap quantum computer"
Riebe et al., Nature Physics **4**, 839 (2008)

Requirements for quantum computing

Classical computer

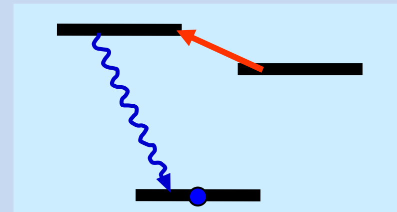
- Initialization
- 1-bit operations (NOT)
- 2-bit gates (e.g. NAND)

Computational space:

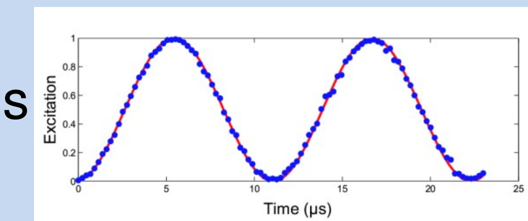
00
01
10
11

- Read out
  ➡ result

Quantum computer

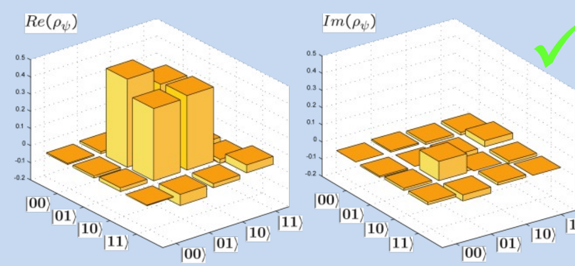


- Initialization
- 1-qubit rotations
  ➡ superpositions

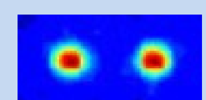


- 2-qubit gates (CNOT gate)
  ➡ entanglement

Computational space:

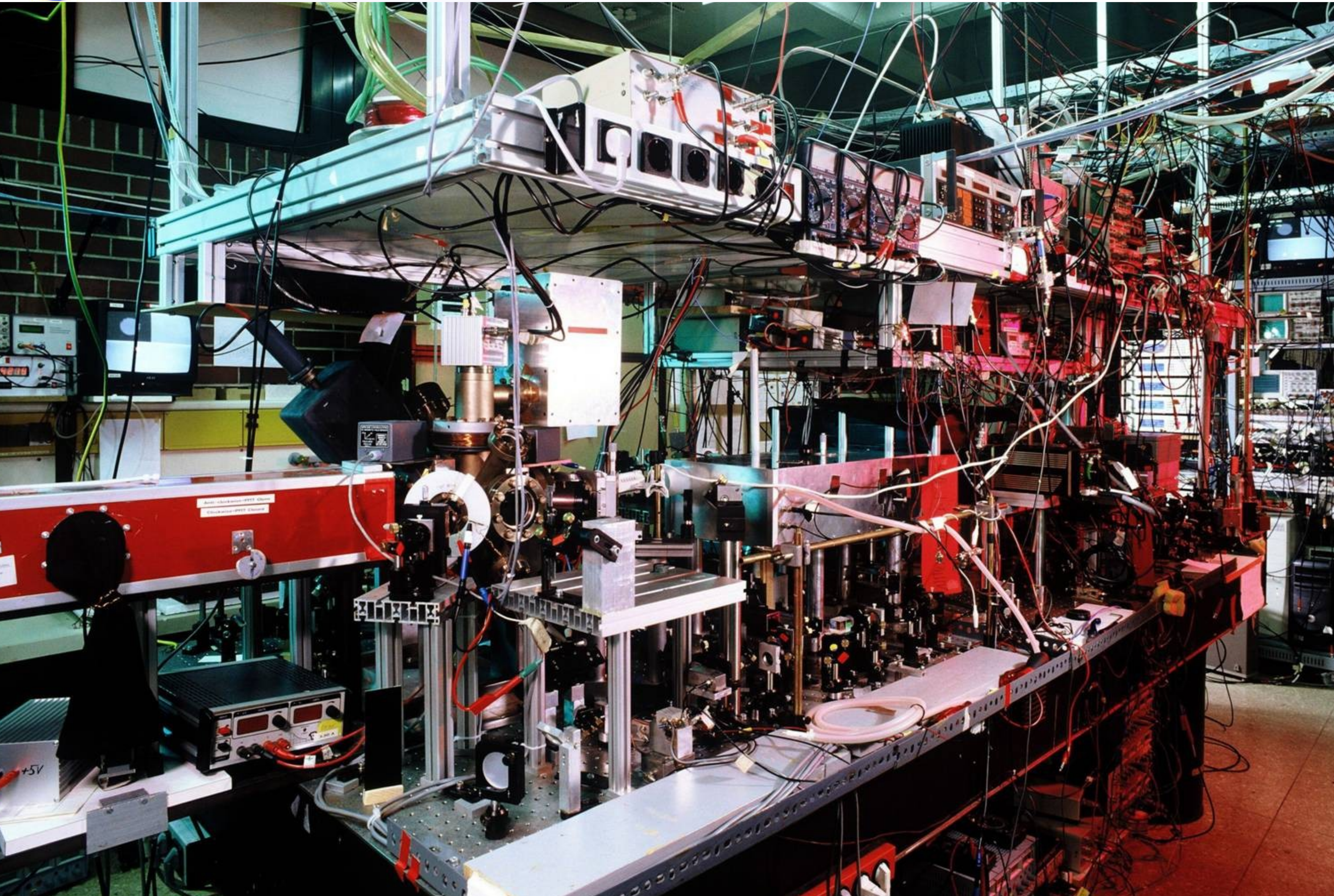


- Read out of qubits
  ➡ gain of classical information

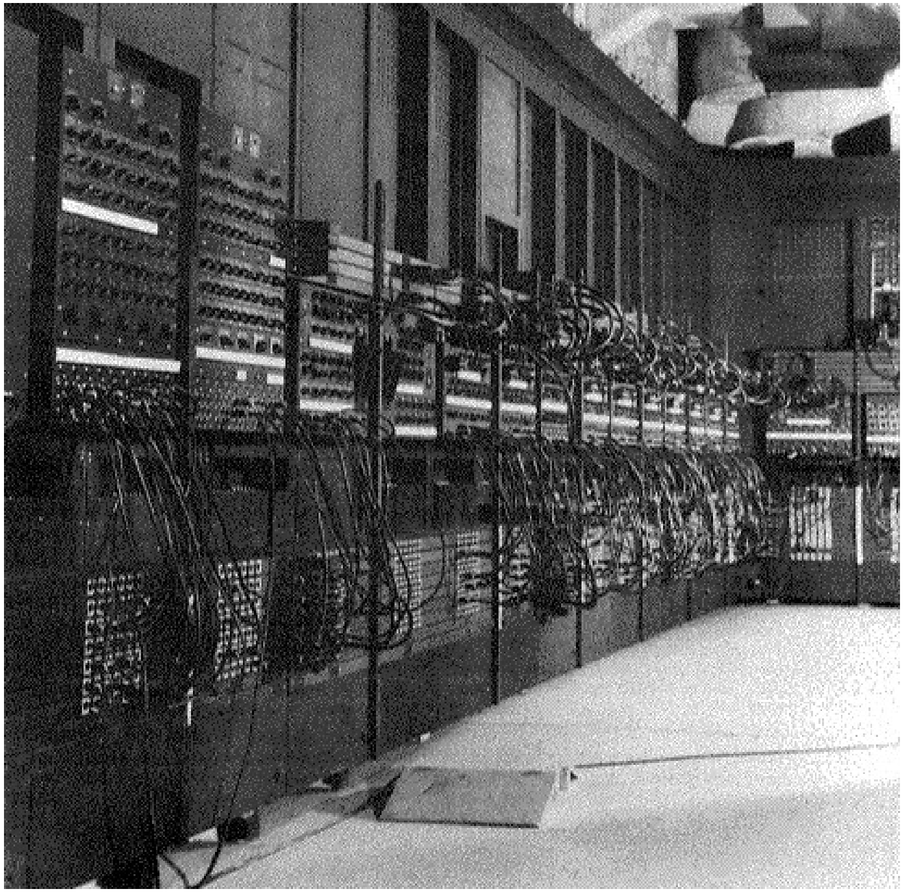


- Introduction to quantum information
- Ion trap quantum computing
- Teleportation and more
- Scaling of ion trap quantum computers

The hardware



The fate of visionaries



20^{th} century

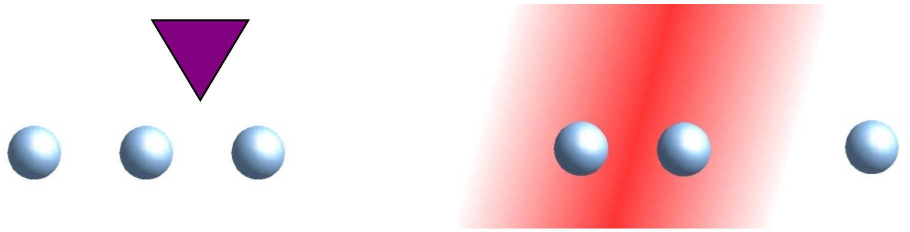
about the ENIAC:

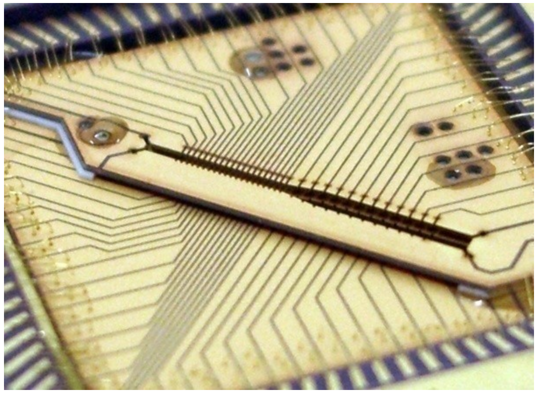
„Where a calculator on the ENIAC is equipped with 18000 vacuum tubes and weighs 30 tons, computers in the future may have only 1000 tubes and weigh only 1 ½ tons.“

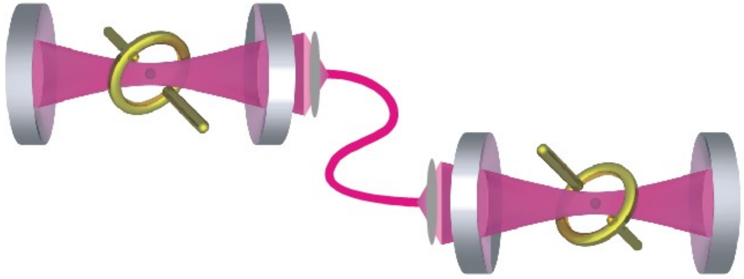
Popular Mechanics, March 1949

Scaling of ion trap quantum computers

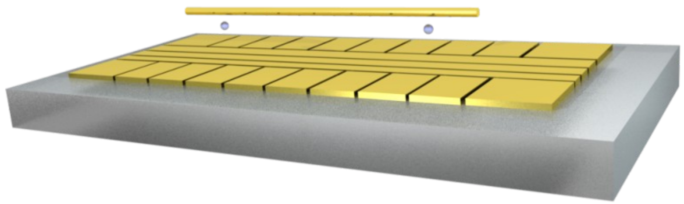
Kielpinski, Monroe, Wineland







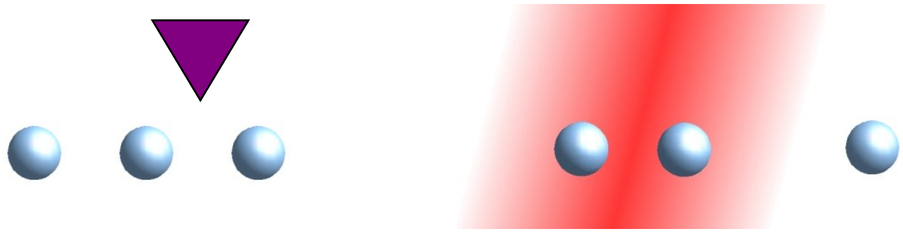
Cirac, Zoller, Kimble, Mabuchi

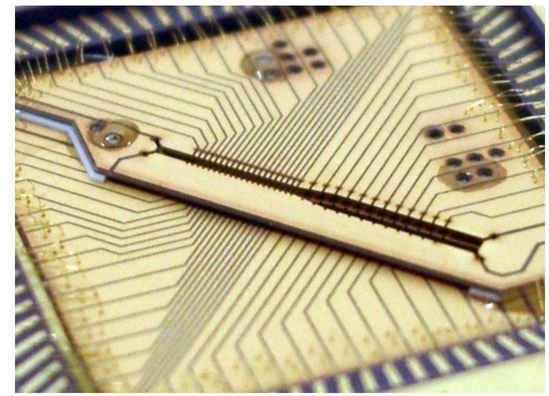


Zoller, Tian, Blatt

Scaling of ion trap quantum computers

Kielpinski, Monroe, Wineland





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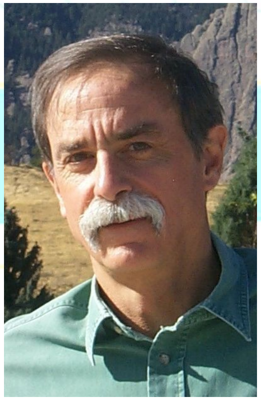
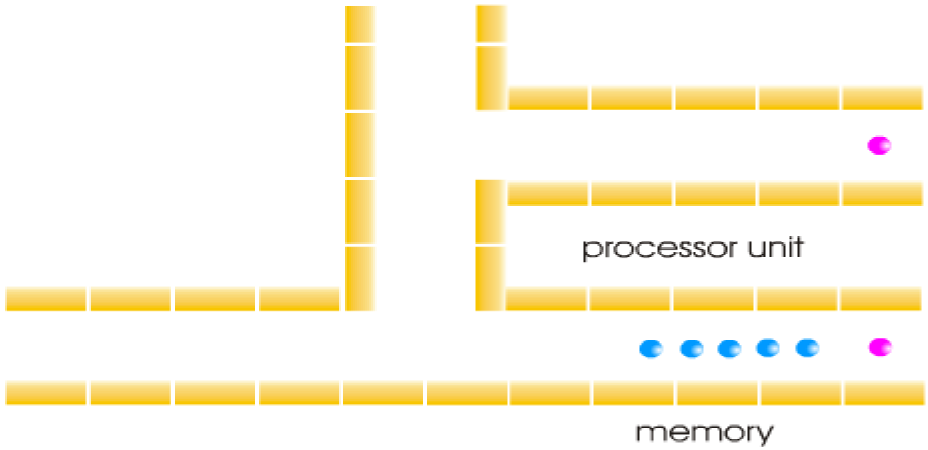


D. Leibfried, D. Wineland et al., NIST

Scaling ion trap quantum computing



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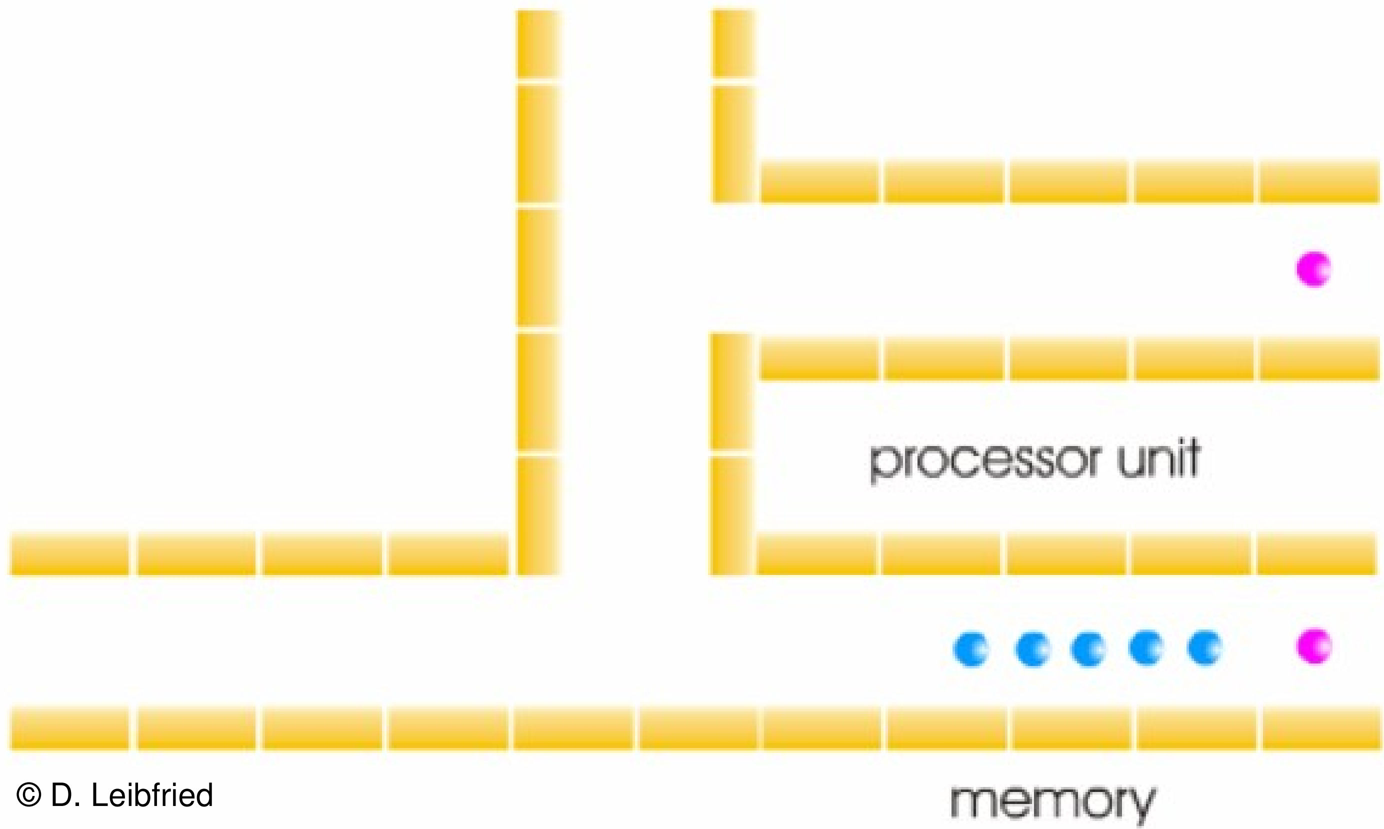

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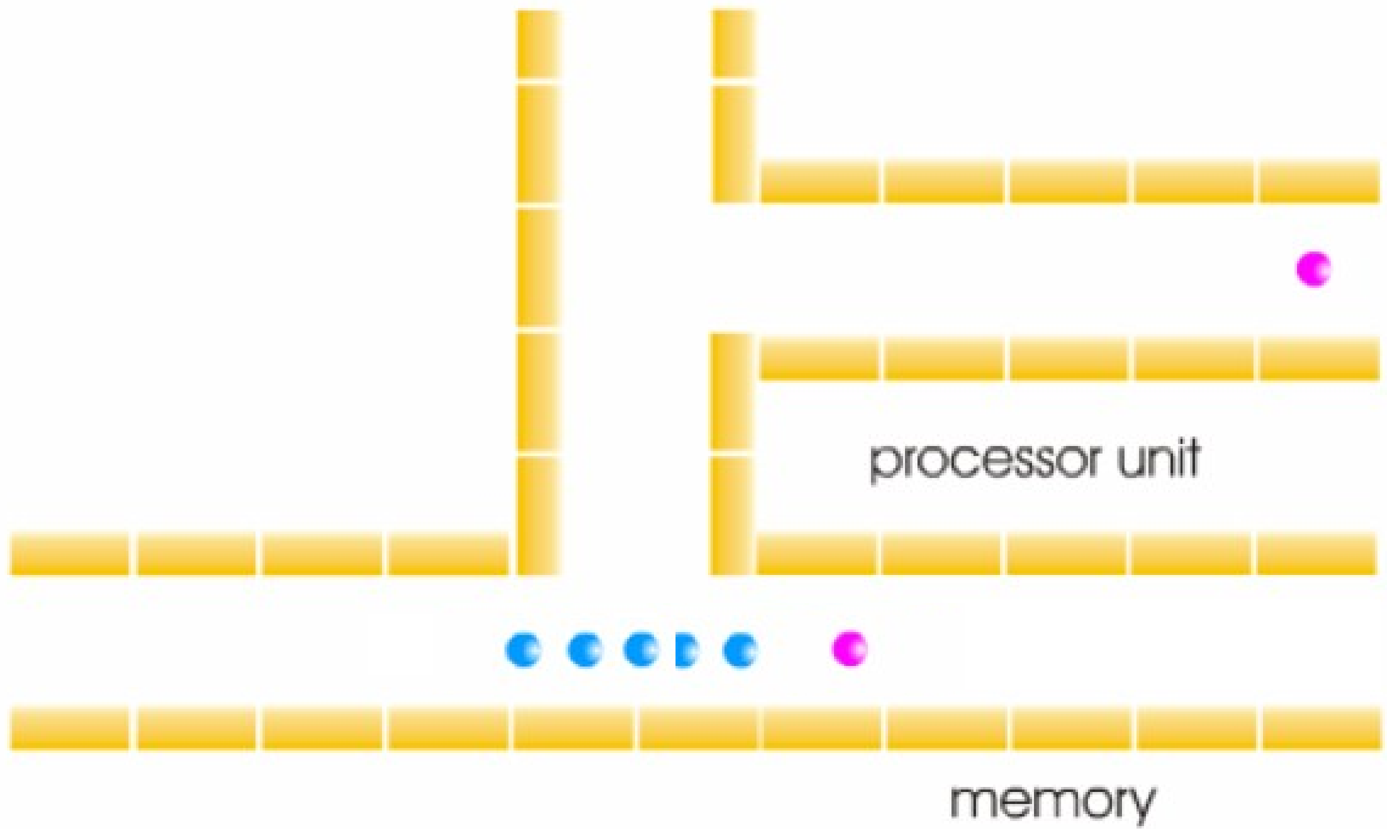

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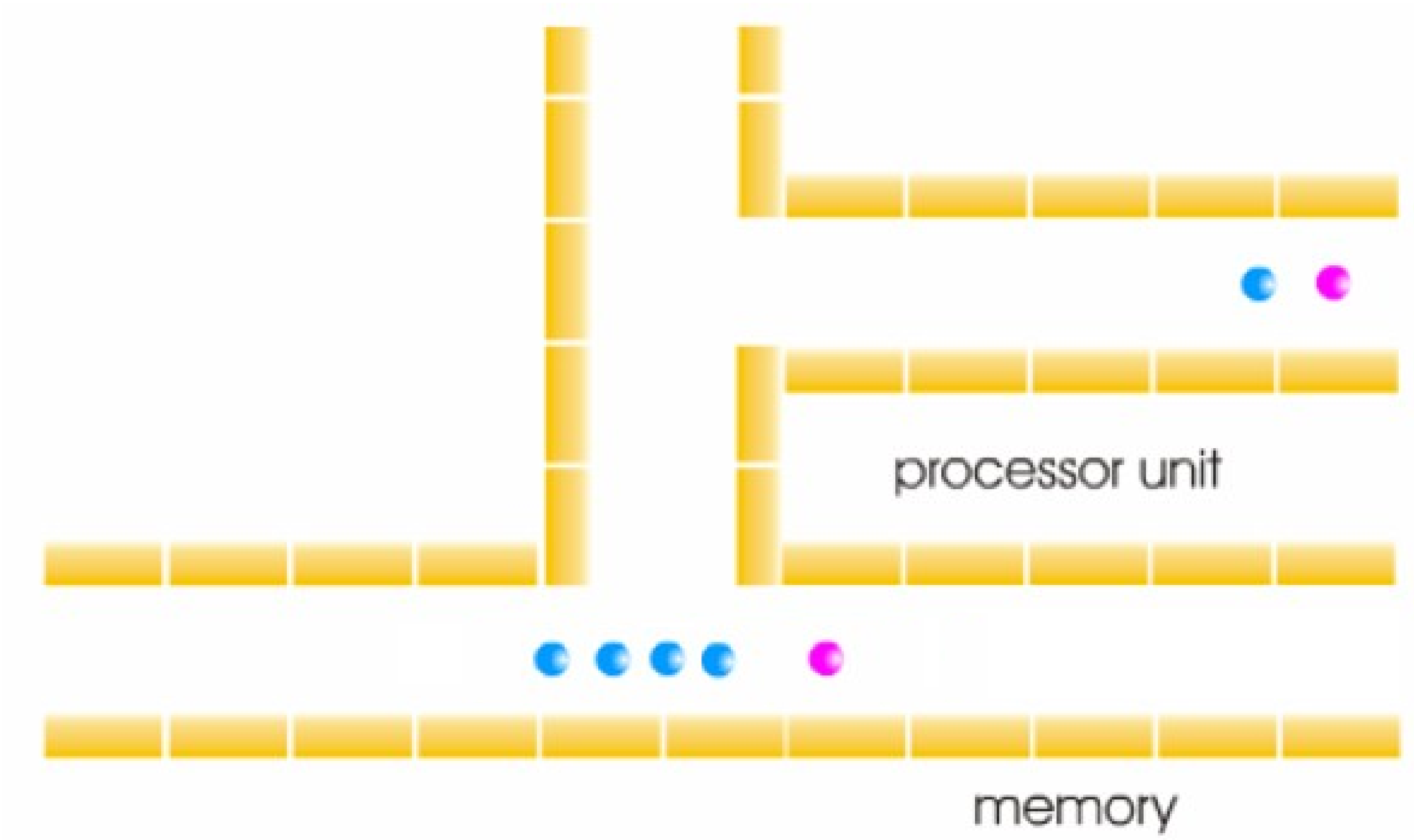

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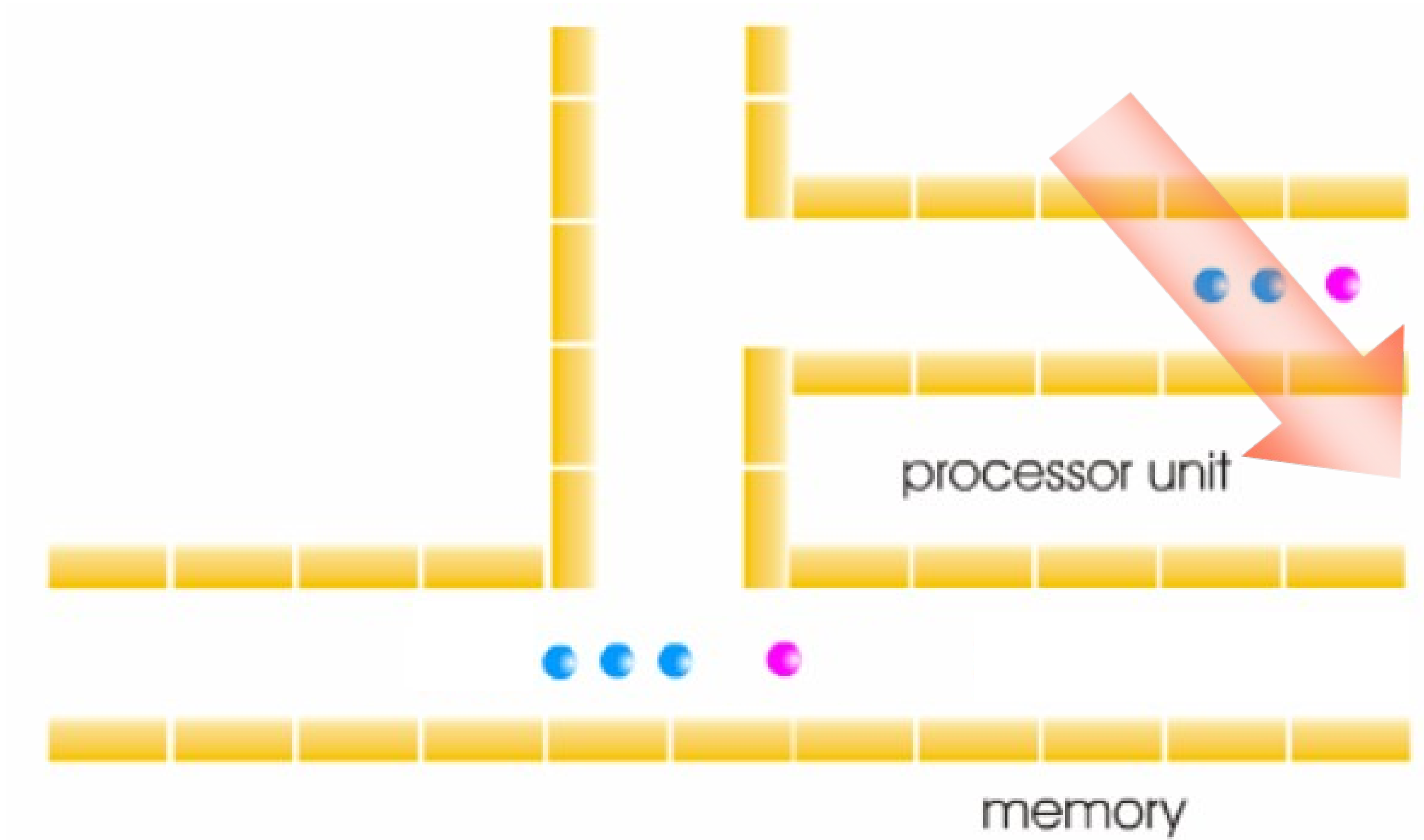
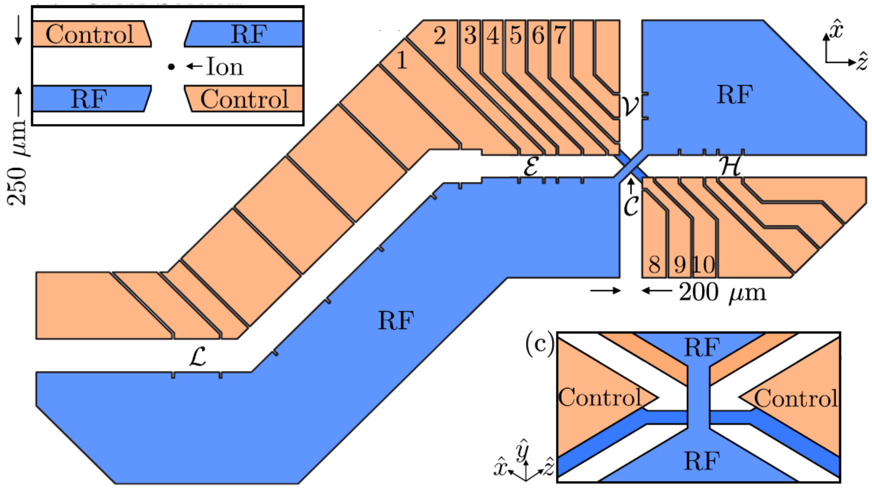
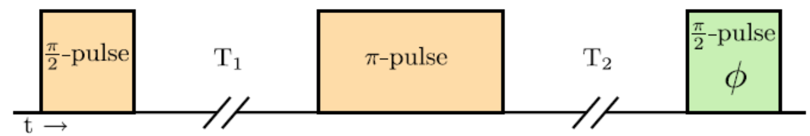


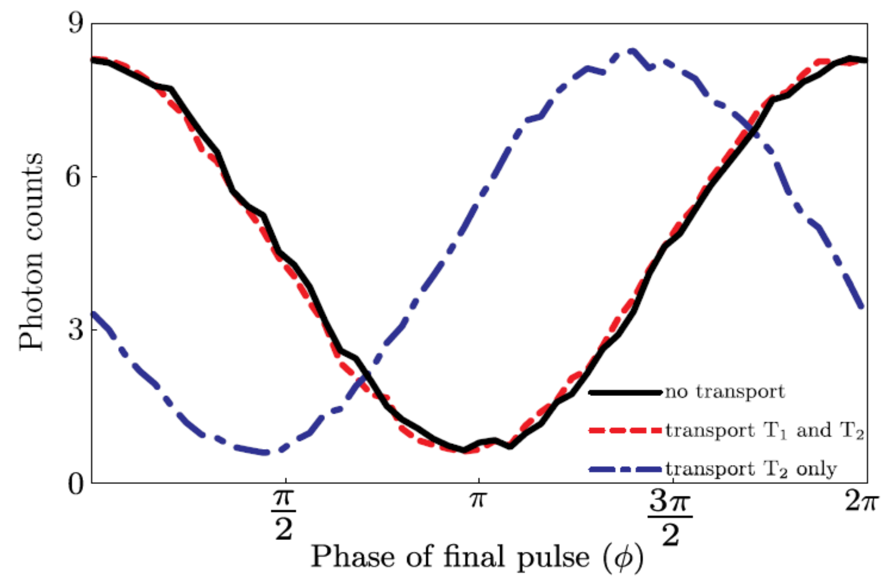
„Architecture for a large-scale ion-trap quantum computer“,
D. Kielpinski et al., Nature **417**, 709 (2002).

Coherent transport through a junction



Transport		Energy Gain (recooling method) quanta/trip
$\mathcal{E}$-$\mathcal{C}$-$\mathcal{E}$	1 ion	3.2 ± 1.8
$\mathcal{E}$-$\mathcal{C}$-$\mathcal{H}$-$\mathcal{C}$-$\mathcal{E}$	1 ion	7.9 ± 1.5
$\mathcal{E}$-$\mathcal{C}$-$\mathcal{V}$-$\mathcal{C}$-$\mathcal{E}$	1 ion	14.5 ± 2.0
$\mathcal{E}$-$\mathcal{C}$-$\mathcal{E}$	2 ions	5.4 ± 1.2
$\mathcal{E}$-$\mathcal{C}$-$\mathcal{H}$-$\mathcal{C}$-$\mathcal{E}$	2 ions	16.6 ± 1.8
$\mathcal{E}$-$\mathcal{C}$-$\mathcal{V}$-$\mathcal{C}$-$\mathcal{E}$	2 ions	53.0 ± 1.2

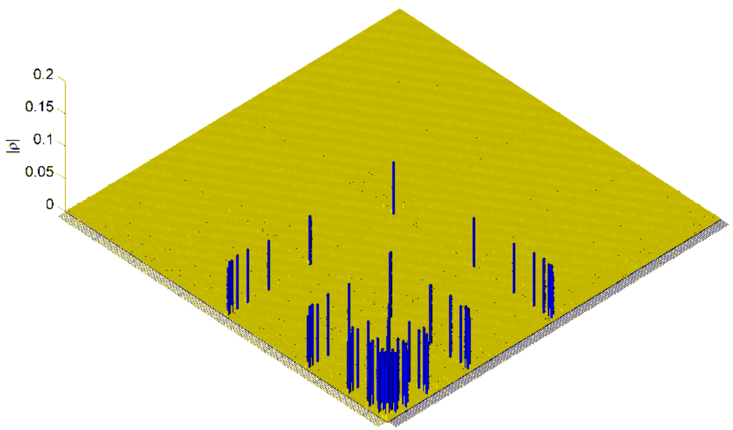




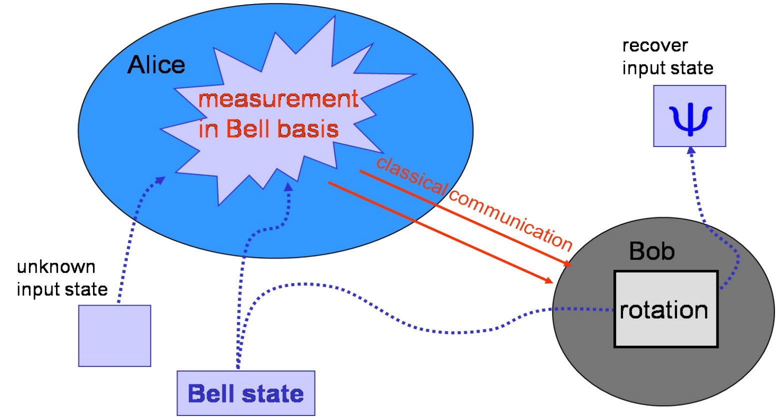
NIST:
Blakestad, *et al.,"High fidelity transport of trapped-ion qubits through an X-junction trap array"*, arXiv:0901.0533v1

Conclusions

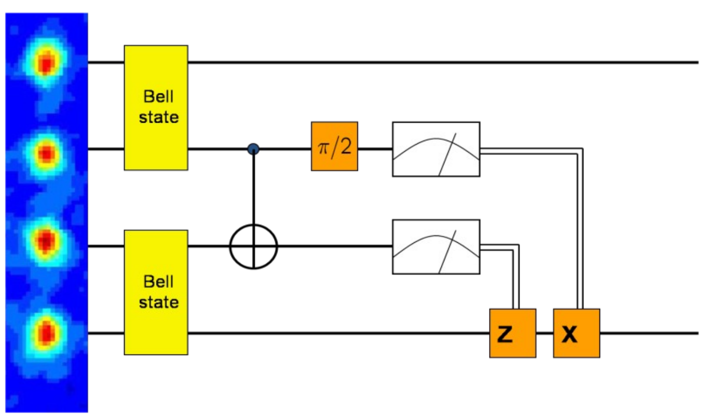
Entanglement



Teleportation



Entanglement swapping



Scaling of ion traps

AG Quantenoptik und Spektroskopie

The Innsbruck ion trap group

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€ FWF SFB

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