Quantum information with trapped ions

Hartmut Häffner

Department of Physics, University of California, Berkeley, USA
and
Institute for Quantum Optics and Quantum Information, Innsbruck, Austria

- Physics and information
- Ion trap quantum computing
- Quantum teleportation
- Trends in quantum information

€ FWF SFB · SCALA QGATES · Industrie Tirol · IQI GmbH
FWF bm:bwk
$ IARPA

Freiburg, July 3rd 2009

Physics and information

Information is physical (Rolf Landauer, 1961)

Erasing information generates heat:

0101001101 ➡ 0000000000 + entropy

Physics and information

Physical process

Equivalence ?

Algorithm

Physics and information

Information is physical (Rolf Landauer, 1961)

➡ Is information quantum mechanical ?

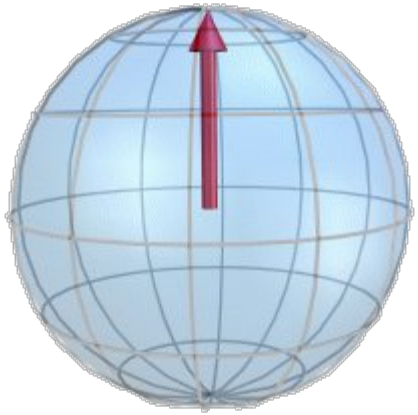
Classical information is a subset of quantum information

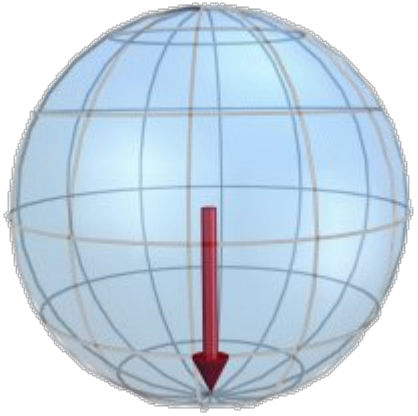
Quantum information: strip down quantum mechanics to bare bones.

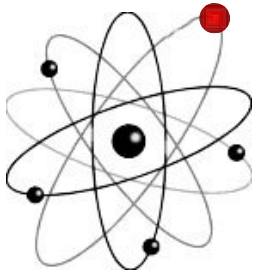
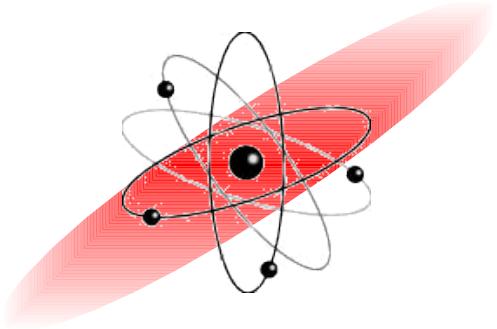
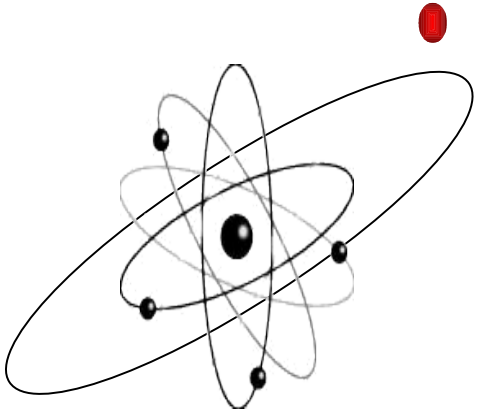
- Hilbert space
- unitary operations
- measurement

Qubits

$|0\rangle$ $\alpha|0\rangle + \beta|1\rangle$ $|1\rangle$





Information content

$$|\Psi\rangle_{\text{reg}} = \alpha_0 |000\rangle + \alpha_1 |001\rangle + \alpha_2 |010\rangle + \alpha_3 |011\rangle + \alpha_4 |100\rangle + \alpha_5 |101\rangle + \alpha_6 |110\rangle + \alpha_7 |111\rangle$$

# bits	classical	quantum mechanical
1	1	0.5208 + 0.7059i, 0.3014 + 0.3736i
2	01	0.2044 + 0.4911i , 0.1732 + 0.3855i, 0.2040 + 0.4890i, 0.3193 + 0.3947i
3	001	0.2583 + 0.2704i , 0.2310 + 0.1150i, 0.2956 + 0.3118i , 0.3558 + 0.2113i , 0.1943 + 0.1377i 0.3273 + 0.2613i , 0.0643 + 0.2033i, 0.3643 + 0.1654i
4	1010	0.1691 + 0.0891i 0.1096 + 0.0828i 0.1420 + 0.2873i 0.0741 + 0.2419i 0.1902 + 0.0448i 0.2495 + 0.0039i 0.1738 + 0.2933i 0.2102 + 0.0653i 0.0686 + 0.0980i 0.1246 + 0.2170i 0.2570 + 0.0933i 0.2234 + 0.1540i 0.1513 + 0.0213i 0.1863 + 0.3243i 0.2606 + 0.1912i 0.0194 + 0.1390i
5	10001	0.1060 + 0.1416i 0.0103 + 0.0118i 0.0064 + 0.0976i 0.0734 + 0.0716i 0.0030 + 0.2054i 0.0902 + 0.0035i 0.1605 + 0.1804i 0.0218 + 0.2280i 0.0083 + 0.2326i 0.1438 + 0.1853i 0.1429 + 0.1030i 0.0037 + 0.1171i 0.0038 + 0.0503i 0.0446 + 0.1512i 0.1379 + 0.0752i 0.0135 + 0.2255i 0.0863 + 0.1707i 0.1483 + 0.0968i 0.1686 + 0.1749i 0.1627 + 0.0629i 0.0197 + 0.1033i 0.1067 + 0.2192i 0.1038 + 0.1605i 0.0830 + 0.0499i 0.0361 + 0.1971i 0.1587 + 0.1477i 0.1642 + 0.0314i 0.1709 + 0.0487i 0.1124 + 0.1426i 0.1303 + 0.1480i 0.0284 + 0.0870i 0.1059 + 0.1351i
6	110101	0.0595 + 0.1064i 0.0295 + 0.1327i 0.0929 + 0.0406i 0.1090 + 0.0379i 0.0559 + 0.1286i 0.0015 + 0.0345i 0.0624 + 0.1196i 0.1120 + 0.1350i 0.1180 + 0.0345i 0.1367 + 0.0356i 0.1255 + 0.0074i 0.0547 + 0.0116i 0.0923 + 0.0952i 0.1087 + 0.0284i 0.0288 + 0.1254i 0.1345 + 0.0258i 0.0846 + 0.0254i 0.0939 + 0.1478i 0.0348 + 0.0654i 0.0816 + 0.0505i 0.1384 + 0.0467i 0.0498 + 0.0543i 0.0974 + 0.0584i 0.0582 + 0.0879i 0.0932 + 0.0178i 0.1039 + 0.0057i 0.0590 + 0.0682i 0.0615 + 0.1293i 0.0974 + 0.1388i 0.1245 + 0.0393i 0.0552 + 0.0238i 0.0632 + 0.1297i 0.0884 + 0.0354i 0.0841 + 0.0960i 0.1065 + 0.1437i 0.0760 + 0.0988i 0.1154 + 0.1293i 0.0727 + 0.0015i 0.0276 + 0.0204i 0.1041 + 0.1217i 0.1460 + 0.0639i 0.1199 + 0.1323i 0.1046 + 0.1092i 0.0721 + 0.1021i 0.0170 + 0.0514i 0.0988 + 0.0247i 0.0543 + 0.0231i 0.0208 + 0.0284i 0.0842 + 0.0628i 0.1223 + 0.1272i 0.1002 + 0.0729i 0.1485 + 0.1213i 0.1429 + 0.0685i 0.0087 + 0.0680i 0.0535 + 0.0670i 0.0815 + 0.0613i 0.0389 + 0.1340i 0.0888 + 0.0008i 0.0073 + 0.0442i 0.0849 + 0.0073i 0.1042 + 0.1030i 0.1430 + 0.0966i 0.1115 + 0.1461i 0.1100 + 0.0821i
7	1001010	0.0880 + 0.0466i 0.1054 + 0.0684i 0.0239 + 0.0866i 0.0759 + 0.0090i 0.0563 + 0.1020i 0.1006 + 0.0988i 0.0769 + 0.0649i 0.0246 + 0.0273i 0.0485 + 0.0942i 0.0186 + 0.0554i 0.1045 + 0.0790i 0.0384 + 0.0455i 0.0053 + 0.1037i 0.0815 + 0.0078i 0.0965 + 0.0597i 0.0309 + 0.0315i 0.0271 + 0.0925i 0.1006 + 0.0362i 0.0141 + 0.0734i 0.1015 + 0.0058i 0.0757 + 0.0385i 0.0914 + 0.0537i 0.0226 + 0.0468i 0.0491 + 0.0607i 0.0087 + 0.0665i 0.0918 + 0.0122i 0.0606 + 0.0969i 0.0344 + 0.0814i 0.0404 + 0.0853i 0.0936 + 0.0879i 0.0401 + 0.0723i 0.0079 + 0.0217i 0.0216 + 0.0294i 0.0053 + 0.0675i 0.0611 + 0.0579i 0.0131 + 0.0064i 0.0563 + 0.0096i 0.0126 + 0.0293i 0.0830 + 0.0441i 0.0404 + 0.0511i 0.0888 + 0.0980i 0.0050 + 0.0643i 0.0645 + 0.0355i 0.1024 + 0.0516i 0.0311 + 0.0644i 0.0959 + 0.0174i 0.0110 + 0.0894i 0.0070 + 0.1031i 0.0253 + 0.0642i 0.1006 + 0.0031i 0.0068 + 0.0876i 0.0285 + 0.0658i 0.1078 + 0.0756i 0.0229 + 0.0099i 0.0537 + 0.0458i 0.0313 + 0.0405i 0.0725 + 0.0179i 0.1033 + 0.0898i 0.0827 + 0.0904i 0.0718 + 0.0487i 0.0141 + 0.1032i 0.0103 + 0.0159i 0.0016 + 0.0938i 0.0311 + 0.0830i 0.0881 + 0.0479i 0.1063 + 0.0669i 0.0019 + 0.1026i 0.0884 + 0.0690i 0.0670 + 0.0267i 0.0604 + 0.0380i 0.0263 + 0.0203i 0.0886 + 0.0529i 0.0284 + 0.0441i 0.0813 + 0.0500i 0.0711 + 0.0659i 0.0231 + 0.0077i 0.0649 + 0.0339i 0.0652 + 0.0656i 0.0711 + 0.0189i 0.0198 + 0.0670i 0.0686 + 0.0265i 0.0184 + 0.0633i 0.0582 + 0.0546i 0.0672 + 0.0501i 0.0740 + 0.0584i 0.0730 + 0.1016i 0.0946 + 0.0369i 0.0014 + 0.0433i 0.0335 + 0.0332i 0.0840 + 0.0444i 0.0331 + 0.0308i 0.0999 + 0.0425i 0.0732 + 0.0542i 0.0080 + 0.0779i 0.0076 + 0.0330i 0.0013 + 0.0121i 0.0245 + 0.0478i 0.0557 + 0.0503i 0.0494 + 0.0016i 0.0758 + 0.0716i 0.0628 + 0.0781i 0.0549 + 0.0304i 0.0080 + 0.0282i 0.0208 + 0.0764i 0.0409 + 0.0845i 0.0893 + 0.0425i 0.0989 + 0.0562i 0.0122 + 0.0774i 0.0876 + 0.0614i 0.0979 + 0.0497i 0.0169 + 0.0480i 0.0132 + 0.0095i 0.0822 + 0.0478i 0.0778 + 0.0395i 0.0703 + 0.0326i 0.0813 + 0.0919i 0.0715 + 0.0819i 0.0953 + 0.1024i 0.0293 + 0.0602i 0.0452 + 0.0015i 0.0230 + 0.0643i
8	10101011	0.0199 + 0.0027i 0.0033 + 0.0063i 0.0005 + 0.0656i 0.0443 + 0.0262i 0.0573 + 0.0359i 0.0622 + 0.0704i 0.0491 + 0.0176i 0.0194 + 0.0664i 0.0111 + 0.0506i 0.0502 + 0.0687i 0.0729 + 0.0376i 0.0629 + 0.0765i 0.0717 + 0.0288i 0.0239 + 0.0410i 0.0207 + 0.0140i 0.0413 + 0.0387i 0.0126 + 0.0325i 0.0163 + 0.0509i 0.0167 + 0.0519i 0.0502 + 0.0738i 0.0041 + 0.0148i 0.0177 + 0.0086i 0.0514 + 0.0436i 0.0240 + 0.0747i 0.0236 + 0.0018i 0.0555 + 0.0671i 0.0736 + 0.0021i 0.0101 + 0.0400i 0.0053 + 0.0148i 0.0097 + 0.0552i 0.0128 + 0.0193i 0.0702 + 0.0720i 0.0105 + 0.0106i 0.0476 + 0.0402i 0.0207 + 0.0690i 0.0170 + 0.0726i 0.0549 + 0.0258i 0.0423 + 0.0337i 0.0726 + 0.0363i 0.0254 + 0.0115i 0.0543 + 0.0105i 0.0727 + 0.0410i 0.0448 + 0.0559i 0.0678 + 0.0307i 0.0578 + 0.0276i 0.0293 + 0.0220i 0.0559 + 0.0670i 0.0125 + 0.0483i 0.0737 + 0.0186i 0.0151 + 0.0754i 0.0598 + 0.0494i 0.0473 + 0.0177i 0.0125 + 0.0525i 0.0024 + 0.0513i 0.0222 + 0.0104i 0.0748 + 0.0017i 0.0733 + 0.0202i 0.0176 + 0.0090i 0.0739 + 0.0053i 0.0524 + 0.0657i 0.0042 + 0.0139i 0.0462 + 0.0025i 0.0303 + 0.0566i 0.0166 + 0.0414i 0.0141 + 0.0213i 0.0059 + 0.0284i 0.0006 + 0.0010i 0.0608 + 0.0685i 0.0014 + 0.0667i 0.0677 + 0.0196i 0.0272 + 0.0439i 0.0557 + 0.0123i 0.0746 + 0.0458i 0.0120 + 0.0255i 0.0126 + 0.0508i 0.0242 + 0.0666i 0.0023 + 0.0437i 0.0276 + 0.0756i 0.0021 + 0.0610i 0.0612 + 0.0118i 0.0770 + 0.0642i 0.0085 + 0.0148i 0.0480 + 0.0493i 0.0102 + 0.0516i 0.0239 + 0.0595i 0.0104 + 0.0293i 0.0172 + 0.0340i 0.0306 + 0.0372i 0.0104 + 0.0469i 0.0186 + 0.0136i 0.0715 + 0.0002i 0.0301 + 0.0609i 0.0394 + 0.0396i 0.0072 + 0.0164i 0.0017 + 0.0080i 0.0123 + 0.0121i 0.0651 + 0.0314i 0.0678 + 0.0314i 0.0144 + 0.0041i 0.0764 + 0.0726i 0.0549 + 0.0116i 0.0672 + 0.0296i 0.0370 + 0.0240i 0.0382 + 0.0130i 0.0222 + 0.0691i 0.0047 + 0.0249i 0.0202 + 0.0566i 0.0144 + 0.0317i 0.0707 + 0.0308i 0.0095 + 0.0390i 0.0010 + 0.0130i 0.0285 + 0.0404i 0.0538 + 0.0494i 0.0685 + 0.0012i 0.0458 + 0.0645i 0.0121 + 0.0619i 0.0244 + 0.0538i 0.0180 + 0.0356i 0.0006 + 0.0064i 0.0306 + 0.0633i 0.0501 + 0.0149i 0.0066 + 0.0343i 0.0593 + 0.0010i 0.0747 + 0.0238i 0.0551 + 0.0675i

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6	110101	0.0595 + 0.1064i 0.0295 + 0.1327i 0.0929 + 0.0406i 0.1090 + 0.0379i 0.0559 + 0.1286i 0.0015 + 0.0345i 0.0624 + 0.1196i 0.1120 + 0.1350i 0.1180 + 0.0345i 0.1367 + 0.0356i 0.1255 + 0.0074i 0.0547 + 0.0116i 0.0923 + 0.0952i 0.1087 + 0.0284i 0.0288 + 0.1254i 0.1345 + 0.0258i 0.0846 + 0.0254i 0.0939 + 0.1478i 0.0348 + 0.0654i 0.0816 + 0.0505i 0.1384 + 0.0467i 0.0498 + 0.0543i 0.0974 + 0.0584i 0.0582 + 0.0879i 0.0932 + 0.0178i 0.1039 + 0.0057i 0.0590 + 0.0682i 0.0615 + 0.1293i 0.0974 + 0.1388i 0.1245 + 0.0393i 0.0552 + 0.0238i 0.0632 + 0.1297i 0.0884 + 0.0354i 0.0841 + 0.0960i 0.1065 + 0.1437i 0.0760 + 0.0988i 0.1154 + 0.1293i 0.0727 + 0.0015i 0.0276 + 0.0204i 0.1041 + 0.1217i 0.1460 + 0.0639i 0.1199 + 0.1323i 0.1046 + 0.1092i 0.0721 + 0.1021i 0.0170 + 0.0514i 0.0988 + 0.0247i 0.0543 + 0.0231i 0.0208 + 0.0284i 0.0842 + 0.0628i 0.1223 + 0.1272i 0.1002 + 0.0729i 0.1485 + 0.1213i 0.1429 + 0.0685i 0.0087 + 0.0680i 0.0535 + 0.0670i 0.0815 + 0.0613i 0.0389 + 0.1340i 0.0888 + 0.0008i 0.0073 + 0.0442i 0.0849 + 0.0073i 0.1042 + 0.1030i 0.1430 + 0.0966i 0.1115 + 0.1461i 0.1100 + 0.0821i
7	1001010	0.0880 + 0.0466i 0.1054 + 0.0684i 0.0239 + 0.0866i 0.0759 + 0.0090i 0.0563 + 0.1020i 0.1006 + 0.0988i 0.0769 + 0.0649i 0.0246 + 0.0273i 0.0485 + 0.0942i 0.0186 + 0.0554i 0.1045 + 0.0790i 0.0384 + 0.0455i 0.0053 + 0.1037i 0.0815 + 0.0078i 0.0965 + 0.0597i 0.0309 + 0.0315i 0.0271 + 0.0925i 0.1006 + 0.0362i 0.0141 + 0.0734i 0.1015 + 0.0058i 0.0757 + 0.0385i 0.0914 + 0.0537i 0.0226 + 0.0468i 0.0491 + 0.0607i 0.0087 + 0.0665i 0.0918 + 0.0122i 0.0606 + 0.0969i 0.0344 + 0.0814i 0.0404 + 0.0853i 0.0936 + 0.0879i 0.0401 + 0.0723i 0.0079 + 0.0217i 0.0216 + 0.0294i 0.0053 + 0.0675i 0.0611 + 0.0579i 0.0131 + 0.0064i 0.0563 + 0.0096i 0.0126 + 0.0293i 0.0830 + 0.0441i 0.0404 + 0.0511i 0.0888 + 0.0980i 0.0050 + 0.0643i 0.0645 + 0.0355i 0.1024 + 0.0516i 0.0311 + 0.0644i 0.0959 + 0.0174i 0.0110 + 0.0894i 0.0070 + 0.1031i 0.0253 + 0.0642i 0.1006 + 0.0031i 0.0068 + 0.0876i 0.0285 + 0.0658i 0.1078 + 0.0756i 0.0229 + 0.0099i 0.0537 + 0.0458i 0.0313 + 0.0405i 0.0725 + 0.0179i 0.1033 + 0.0898i 0.0827 + 0.0904i 0.0718 + 0.0487i 0.0141 + 0.1032i 0.0103 + 0.0159i 0.0016 + 0.0938i 0.0311 + 0.0830i 0.0881 + 0.0479i 0.1063 + 0.0669i 0.0019 + 0.1026i 0.0884 + 0.0690i 0.0670 + 0.0267i 0.0604 + 0.0380i 0.0263 + 0.0203i 0.0886 + 0.0529i 0.0284 + 0.0441i 0.0813 + 0.0500i 0.0711 + 0.0659i 0.0231 + 0.0077i 0.0649 + 0.0339i 0.0652 + 0.0656i 0.0711 + 0.0189i 0.0198 + 0.0670i 0.0686 + 0.0265i 0.0184 + 0.0633i 0.0582 + 0.0546i 0.0672 + 0.0501i 0.0740 + 0.0584i 0.0730 + 0.1016i 0.0946 + 0.0369i 0.0014 + 0.0433i 0.0335 + 0.0332i 0.0840 + 0.0444i 0.0331 + 0.0308i 0.0999 + 0.0425i 0.0732 + 0.0542i 0.0080 + 0.0779i 0.0076 + 0.0330i 0.0013 + 0.0121i 0.0245 + 0.0478i 0.0557 + 0.0503i 0.0494 + 0.0016i 0.0758 + 0.0716i 0.0628 + 0.0781i 0.0549 + 0.0304i 0.0080 + 0.0282i 0.0208 + 0.0764i 0.0409 + 0.0845i 0.0893 + 0.0425i 0.0989 + 0.0562i 0.0122 + 0.0774i 0.0876 + 0.0614i 0.0979 + 0.0497i 0.0169 + 0.0480i 0.0132 + 0.0095i 0.0822 + 0.0478i 0.0778 + 0.0395i 0.0703 + 0.0326i 0.0813 + 0.0919i 0.0715 + 0.0819i 0.0953 + 0.1024i 0.0293 + 0.0602i 0.0452 + 0.0015i 0.0230 + 0.0643i
8	10101011	0.0199 + 0.0027i 0.0033 + 0.0063i 0.0005 + 0.0656i 0.0443 + 0.0262i 0.0573 + 0.0359i 0.0622 + 0.0704i 0.0491 + 0.0176i 0.0194 + 0.0664i 0.0111 + 0.0506i 0.0502 + 0.0687i 0.0729 + 0.0376i 0.0629 + 0.0765i 0.0717 + 0.0288i 0.0239 + 0.0410i 0.0207 + 0.0140i 0.0413 + 0.0387i 0.0126 + 0.0325i 0.0163 + 0.0509i 0.0167 + 0.0519i 0.0502 + 0.0738i 0.0041 + 0.0148i 0.0177 + 0.0086i 0.0514 + 0.0436i 0.0240 + 0.0747i 0.0236 + 0.0018i 0.0555 + 0.0671i 0.0736 + 0.0021i 0.0101 + 0.0400i 0.0053 + 0.0148i 0.0097 + 0.0552i 0.0128 + 0.0193i 0.0702 + 0.0720i 0.0105 + 0.0106i 0.0476 + 0.0402i 0.0207 + 0.0690i 0.0170 + 0.0726i 0.0549 + 0.0258i 0.0423 + 0.0337i 0.0726 + 0.0363i 0.0254 + 0.0115i 0.0543 + 0.0105i 0.0727 + 0.0410i 0.0448 + 0.0559i 0.0678 + 0.0307i 0.0578 + 0.0276i 0.0293 + 0.0220i 0.0559 + 0.0670i 0.0125 + 0.0483i 0.0737 + 0.0186i 0.0151 + 0.0754i 0.0598 + 0.0494i 0.0473 + 0.0177i 0.0125 + 0.0525i 0.0024 + 0.0513i 0.0222 + 0.0104i 0.0748 + 0.0017i 0.0733 + 0.0202i 0.0176 + 0.0090i 0.0739 + 0.0053i 0.0524 + 0.0657i 0.0042 + 0.0139i 0.0462 + 0.0025i 0.0303 + 0.0566i 0.0166 + 0.0414i 0.0141 + 0.0213i 0.0059 + 0.0284i 0.0006 + 0.0010i 0.0608 + 0.0685i 0.0014 + 0.0667i 0.0677 + 0.0196i 0.0272 + 0.0439i 0.0557 + 0.0123i 0.0746 + 0.0458i 0.0120 + 0.0255i 0.0126 + 0.0508i 0.0242 + 0.0666i 0.0023 + 0.0437i 0.0276 + 0.0756i 0.0021 + 0.0610i 0.0612 + 0.0118i 0.0770 + 0.0642i 0.0085 + 0.0148i 0.0480 + 0.0493i 0.0102 + 0.0516i 0.0239 + 0.0595i 0.0104 + 0.0293i 0.0172 + 0.0340i 0.0306 + 0.0372i 0.0104 + 0.0469i 0.0186 + 0.0136i 0.0715 + 0.0002i 0.0301 + 0.0609i 0.0394 + 0.0396i 0.0072 + 0.0164i 0.0017 + 0.0080i 0.0123 + 0.0121i 0.0651 + 0.0314i 0.0678 + 0.0314i 0.0144 + 0.0041i 0.0764 + 0.0726i 0.0549 + 0.0116i 0.0672 + 0.0296i 0.0370 + 0.0240i 0.0382 + 0.0130i 0.0222 + 0.0691i 0.0047 + 0.0249i 0.0202 + 0.0566i 0.0144 + 0.0317i 0.0707 + 0.0308i 0.0095 + 0.0390i 0.0010 + 0.0130i 0.0285 + 0.0404i 0.0538 + 0.0494i 0.0685 + 0.0012i 0.0458 + 0.0645i 0.0121 + 0.0619i 0.0244 + 0.0538i 0.0180 + 0.0356i 0.0006 + 0.0064i 0.0306 + 0.0633i 0.0501 + 0.0149i 0.0066 + 0.0343i 0.0593 + 0.0010i 0.0747 + 0.0238i 0.0551 + 0.0675i 0.0603 + 0.0644i 0.0183 + 0.0257i 0.0151 + 0.0679i 0.0203 + 0.0370i 0.0550 + 0.0432i 0.0753 + 0.0475i 0.0491 + 0.0510i 0.0421 + 0.0475i 0.0654 + 0.0528i 0.0618 + 0.0393i 0.0515 + 0.0550i 0.0517 + 0.0397i 0.0633 + 0.0467i 0.0748 + 0.0745i 0.0375 + 0.0634i 0.0630 + 0.0245i 0.0494 + 0.0453i 0.0236 + 0.0100i 0.0509 + 0.0196i 0.0276 + 0.0619i 0.0723 + 0.0515i 0.0376 + 0.0011i 0.0070 + 0.0433i 0.0519 + 0.0350i 0.0397 + 0.0697i 0.0171 + 0.0217i 0.0559 + 0.0050i 0.0053 + 0.0367i 0.0743 + 0.0758i 0.0160 + 0.0711i 0.0124 + 0.0433i 0.0492 + 0.0503i 0.0000 + 0.0596i 0.0259 + 0.0082i 0.0212 + 0.0001i 0.0034 + 0.0418i 0.0072 + 0.0005i 0.0316 + 0.0348i 0.0630 + 0.0151i 0.0671 + 0.0607i 0.0017 + 0.0477i 0.0560 + 0.0012i 0.0654 + 0.0687i 0.0562 + 0.0587i 0.0736 + 0.0699i 0.0506 + 0.0585i 0.0572 + 0.0293i 0.0266 + 0.0255i 0.0681 + 0.0389i 0.0268 + 0.0435i 0.0670 + 0.0514i 0.0302 + 0.0522i 0.0195 + 0.0726i 0.0273 + 0.0594i 0.0573 + 0.0568i 0.0502 + 0.0668i 0.0724 + 0.0764i 0.0642 + 0.0388i 0.0362 + 0.0485i 0.0485 + 0.0611i 0.0045 + 0.0346i 0.0418 + 0.0404i 0.0351 + 0.0132i 0.0665 + 0.0101i 0.0659 + 0.0169i 0.0364 + 0.0081i 0.0607 + 0.0109i 0.0506 + 0.0352i 0.0000 + 0.0607i 0.0101 + 0.0217i 0.0381 + 0.0173i 0.0030 + 0.0701i 0.0175 + 0.0006i 0.0253 + 0.0454i 0.0693 + 0.0418i 0.0242 + 0.0504i 0.0194 + 0.0242i 0.0334 + 0.0178i 0.0649 + 0.0321i 0.0142 + 0.0230i 0.0392 + 0.0518i 0.0349 + 0.0723i 0.0251 + 0.0264i 0.0293 + 0.0434i 0.0683 + 0.0092i 0.0587 + 0.0130i 0.0681 + 0.0215i 0.0353 + 0.0429i 0.0616 + 0.0374i 0.0103 + 0.0734i 0.0050 + 0.0179i 0.0289 + 0.0369i 0.0288 + 0.0406i 0.0373 + 0.0611i 0.0747 + 0.0149i 0.0264 + 0.0701i 0.0195 + 0.0711i 0.0451 + 0.0010i 0.0404 + 0.0592i 0.0126 + 0.0730i 0.0375 + 0.0627i 0.0382 + 0.0712i 0.0650 + 0.0153i 0.0621 + 0.0520i 0.0661 + 0.0715i 0.0470 + 0.0265i 0.0436 + 0.0458i 0.0472 + 0.0474i 0.0079 + 0.0003i 0.0122 + 0.0757i 0.0319 + 0.0693i 0.0432 + 0.0534i 0.0207 + 0.0339i 0.0604 + 0.0540i 0.0299 + 0.0470i 0.0024 + 0.0231i 0.0451 + 0.0660i 0.0431 + 0.0086i 0.0155 + 0.0225i 0.0067 + 0.0075i 0.0719 + 0.0306i 0.0200 + 0.0257i 0.0157 + 0.0728i 0.0038 + 0.0646i

Information content

40 qubits	10 000 GigaByte
1 additional qubit	Double the memory
300 qubits	Every atom in the Universe would have to hold one classical bit

Why quantum information ?

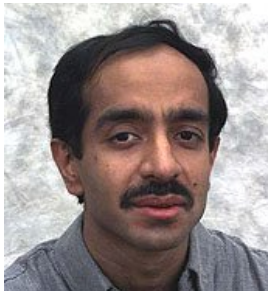
Schrödinger equation for 300 interacting spins.

Classical computation needs more bits than there are atoms in the universe.

➡ Quantum computers can solve certain tasks much more efficiently than classical computers.

Other prominent examples:

- Factoring of large integers (P. Shor 1994)
- Search in an unsorted data base (L. Grover, 1997)



- …

Quantum computing

Classical computer

- Initialization
- 1-bit operations (NOT)
- 2-bit gates (e.g. NAND)

Computational space:

00
01
10
11

- Read out
  ➡ result

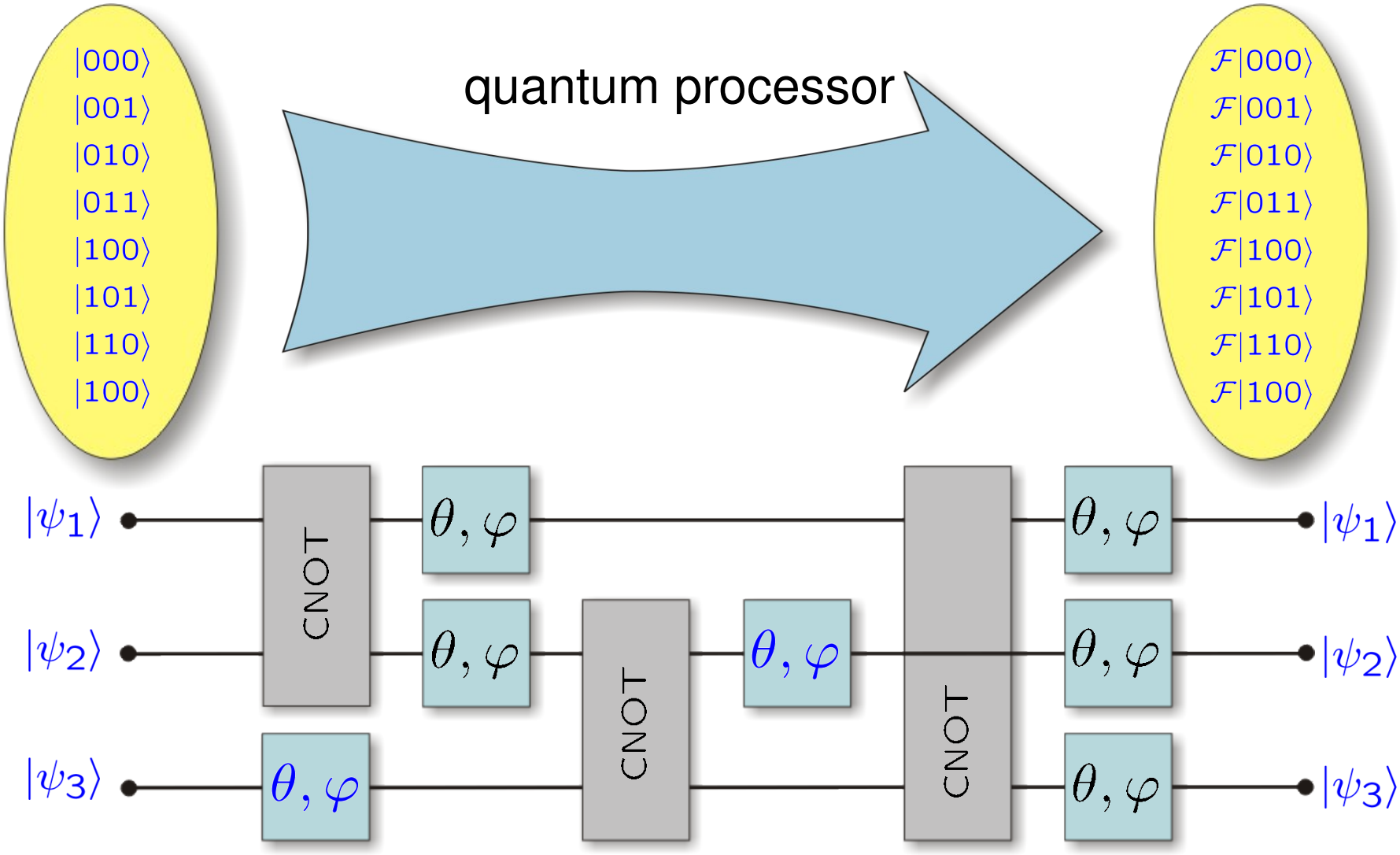
Quantum computer

- Initialization
- 1-qubit rotations
  ➡ superpositions
- 2-qubit gates (CNOT gate)
  ➡ entanglement

Computational space: Hilbert space 2^n dimensional

- Read out of qubits
  ➡ gain of classical information

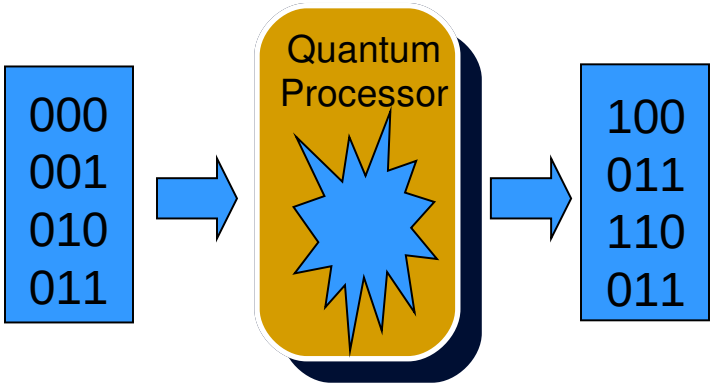
Quantum computing



Input ➡ computation: sequence of quantum gates ➡ output

Quantum computing

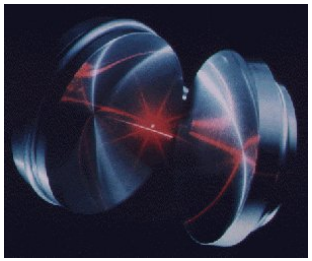
Long term goal: A universal quantum computer



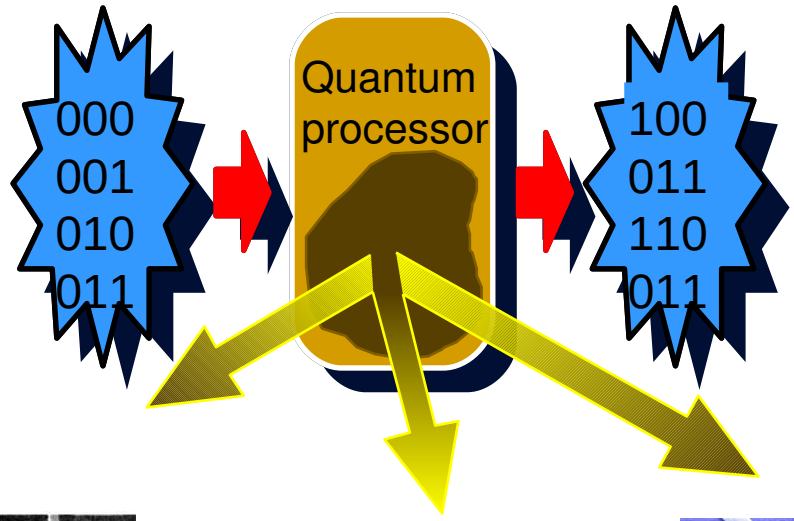
In the mean time:

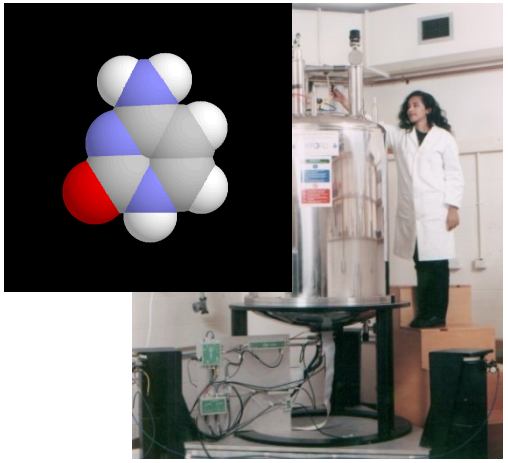
- “understand” quantum mechanics
- apply quantum mechanics
- where does quantum mechanics fail?

Which technology ?

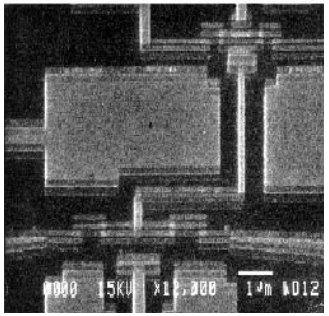


Cavity QED

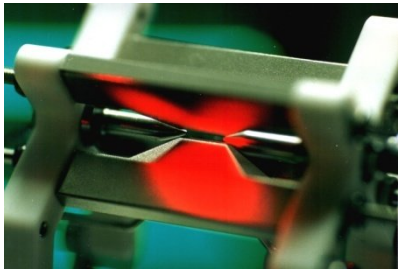


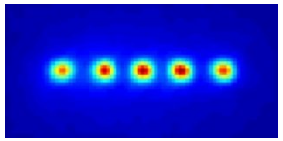


NMR

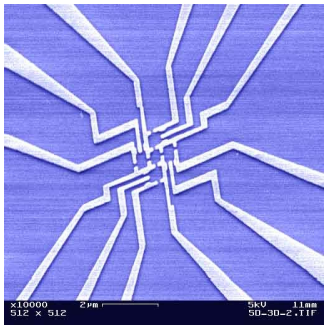


Superconducting qubits





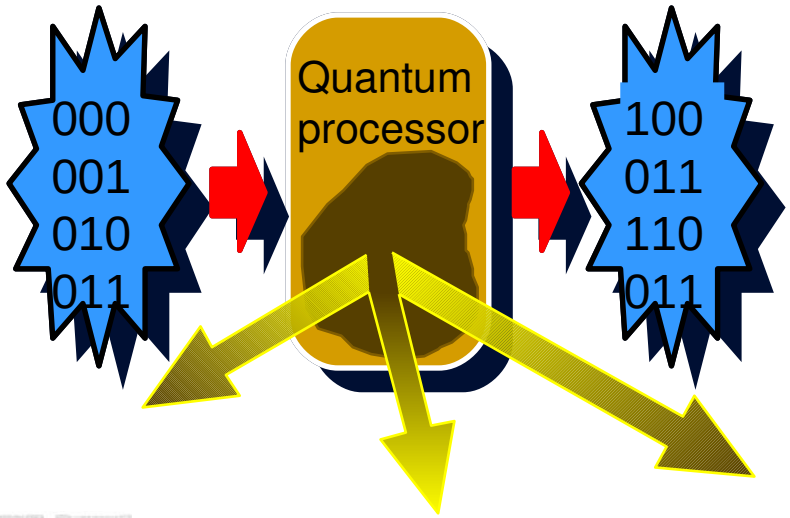
Trapped ions



Quantum dots

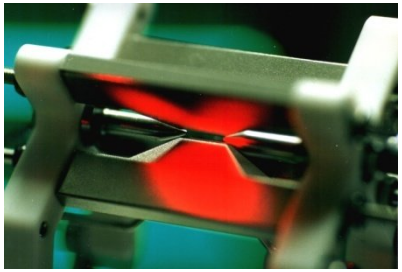
Which technology ?

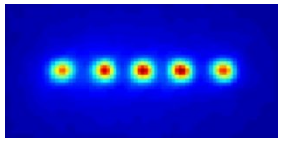
Cavity QED



NMR

Superconducting qubits



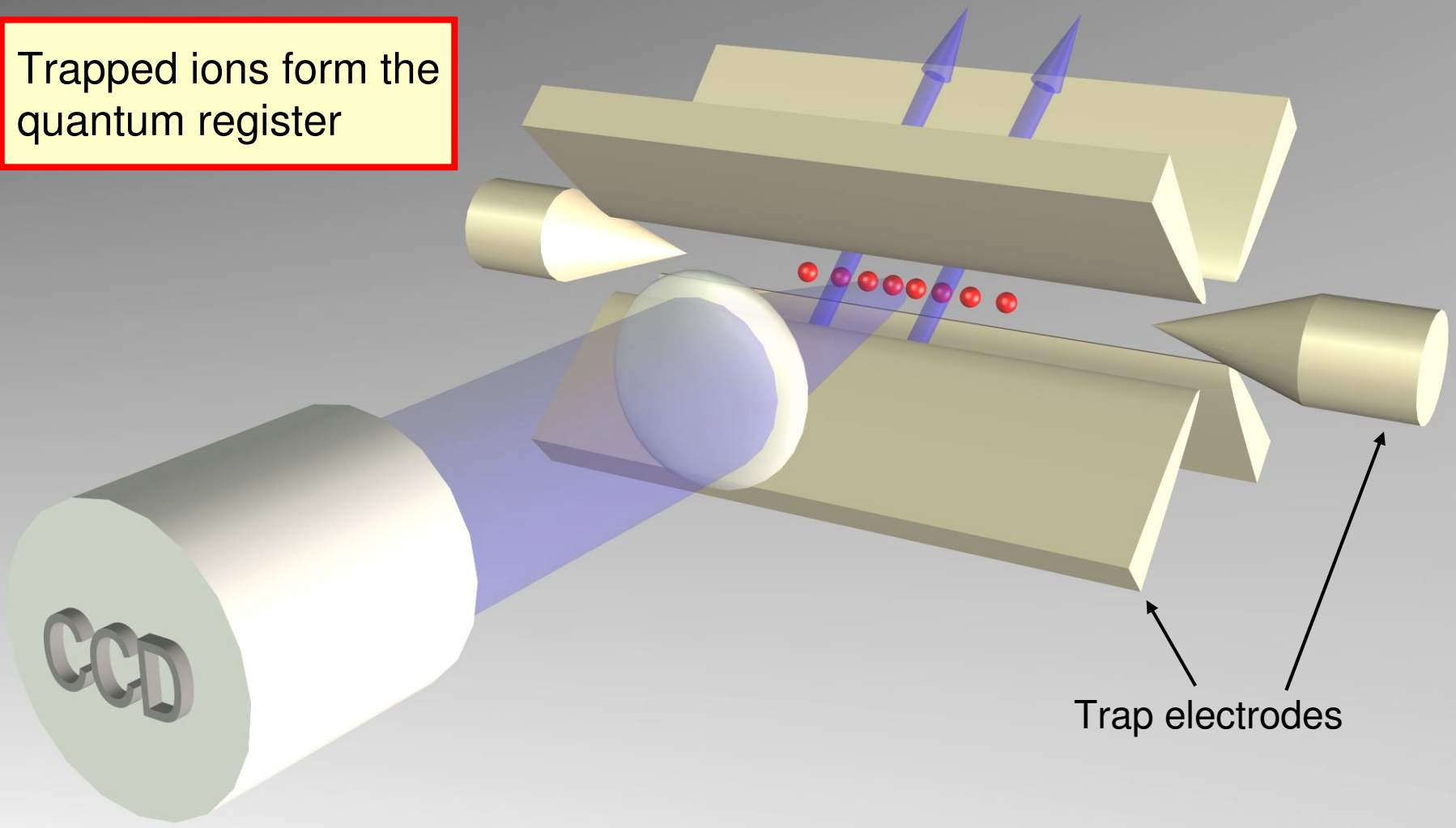


Trapped ions

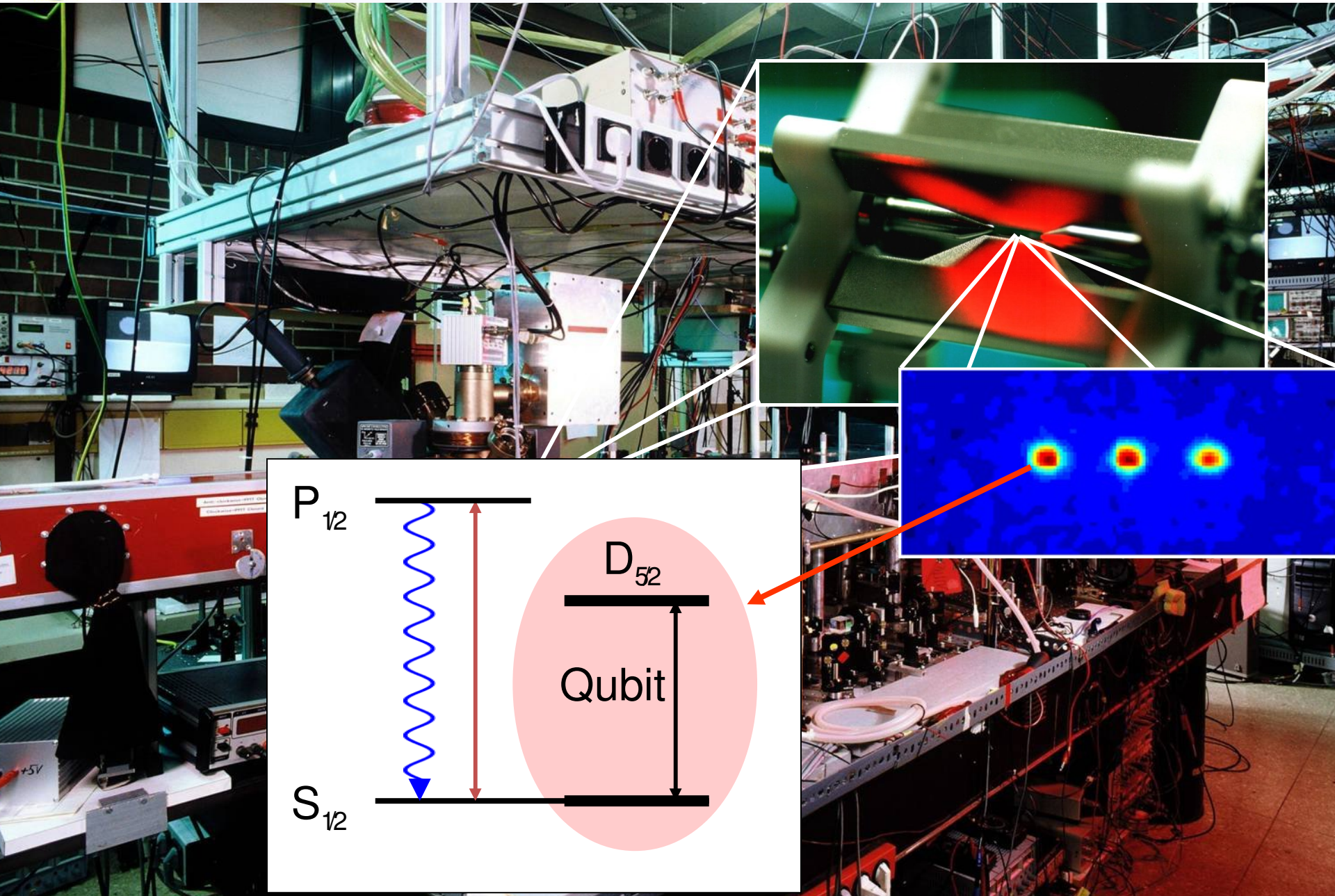
Quantum dots

Ion trap quantum computing

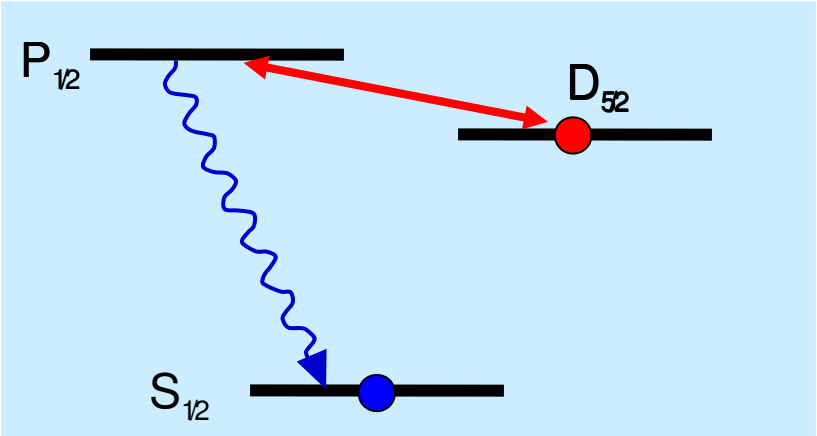
Trapped ions form the quantum register



The hardware

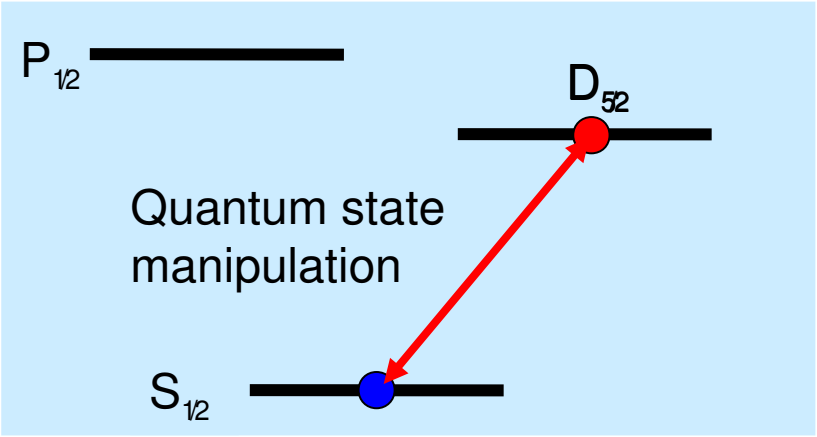


Experimental procedure



1. Initialization in a pure quantum state

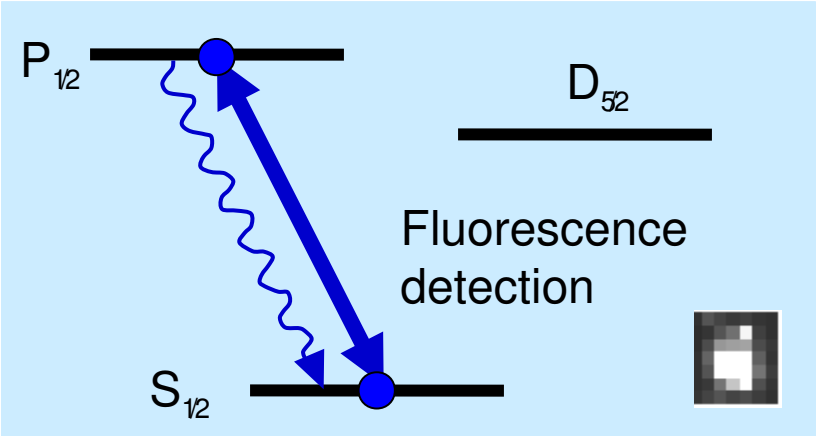
Experimental procedure



1. Initialization in a pure quantum state

2. Quantum state manipulation on $S_{1/2}$ – $D_{5/2}$ transition

Experimental procedure

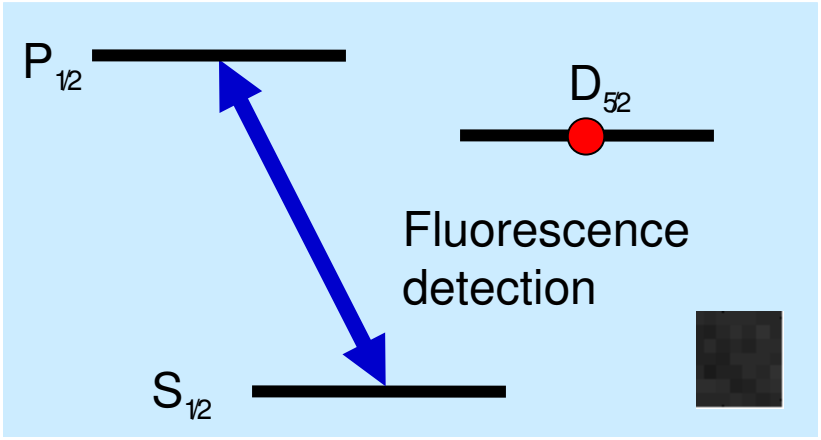


1. Initialization in a pure quantum state:

2. Quantum state manipulation on $S_{1/2} - D_{5/2}$ transition

3. Quantum state measurement by fluorescence detection

Experimental procedure



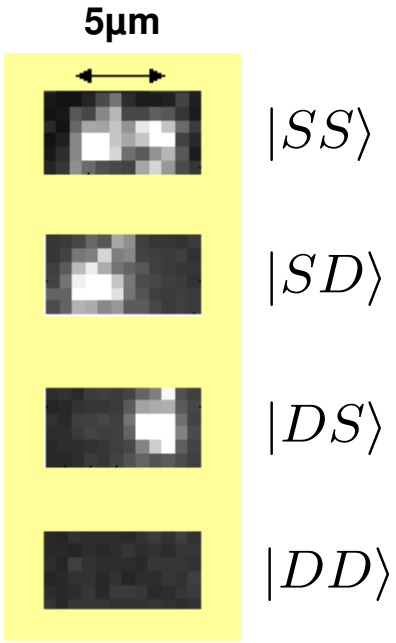
1. Initialization in a pure quantum state:

2. Quantum state manipulation on $S_{1/2}$ – $D_{5/2}$ transition

3. Quantum state measurement by fluorescence detection

Two ions:

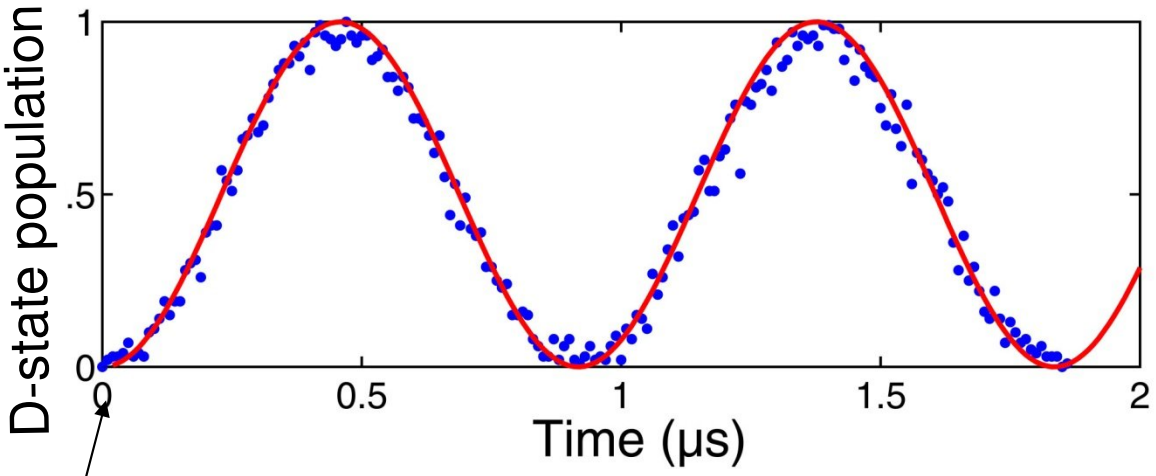
Spatially resolved detection with CCD camera



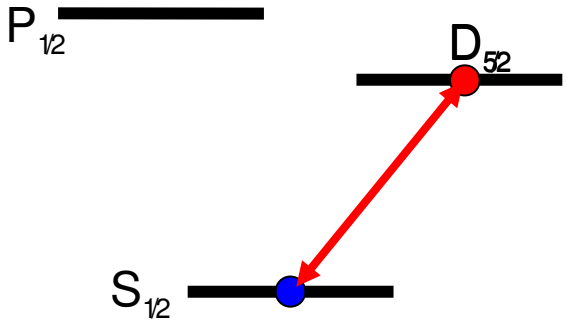
50 experiments / s

Repeat experiments 100-200 times

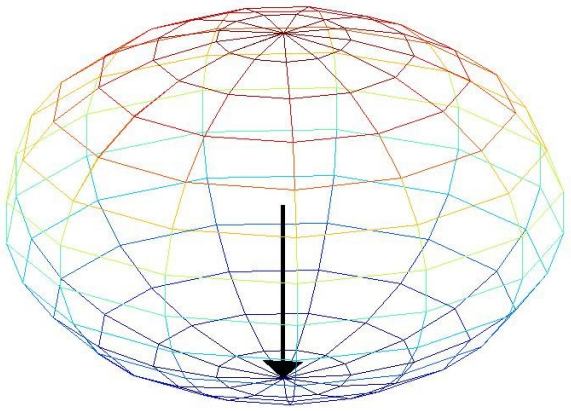
Rabi oscillations



$|S\rangle$



$|D\rangle$

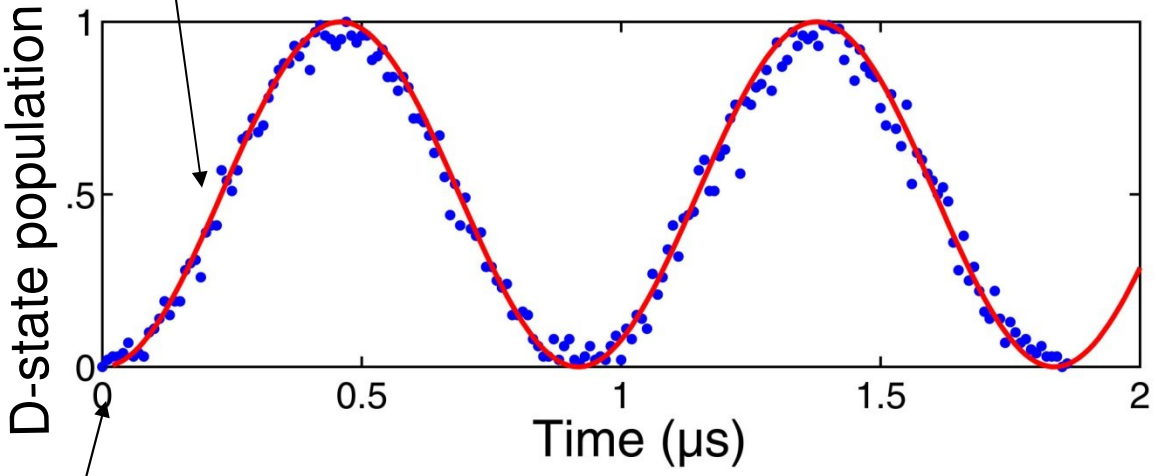


$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$

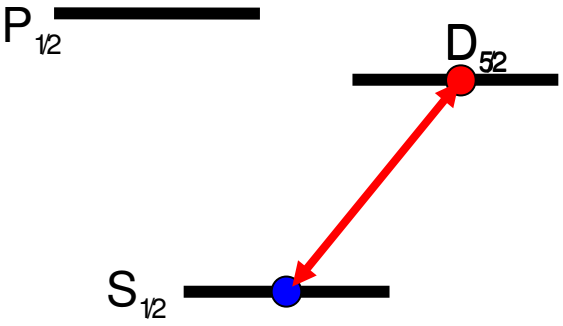
$|S\rangle$

Rabi oscillations

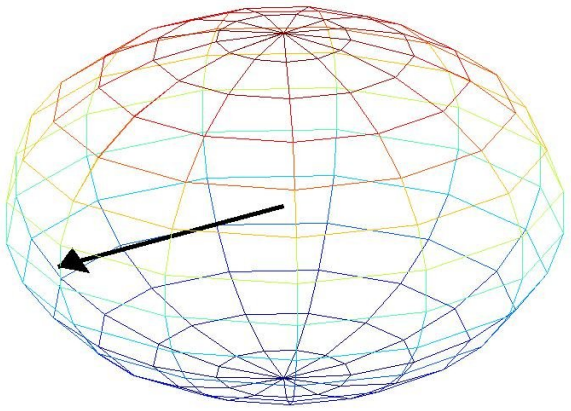
$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$



$|S\rangle$



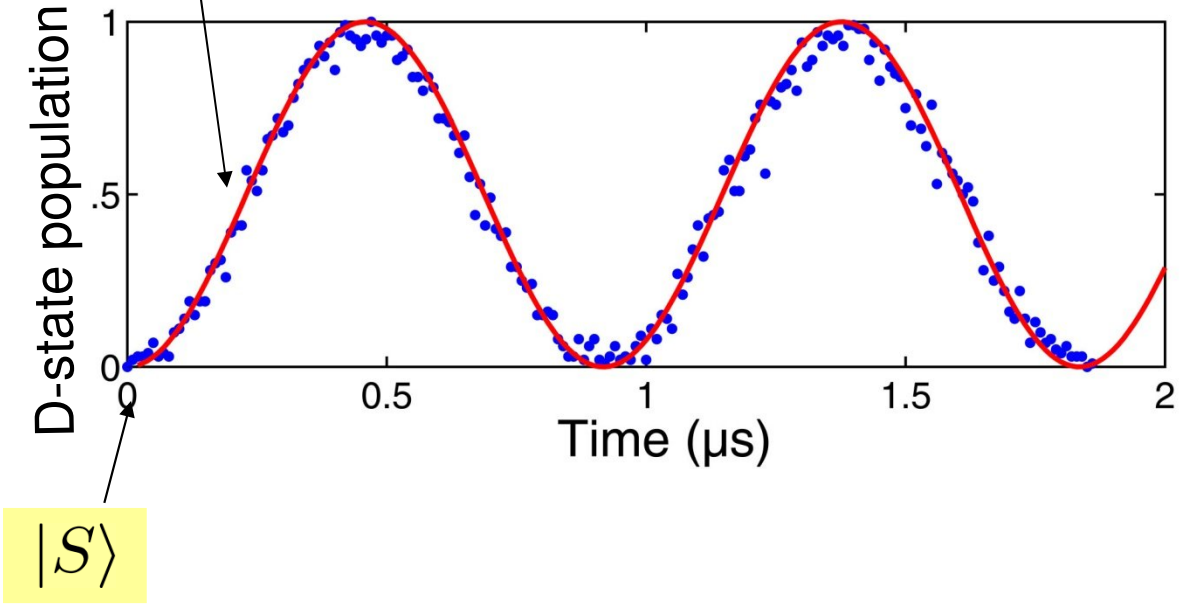
$|D\rangle$

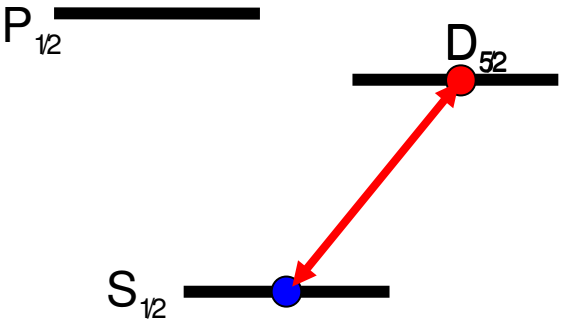


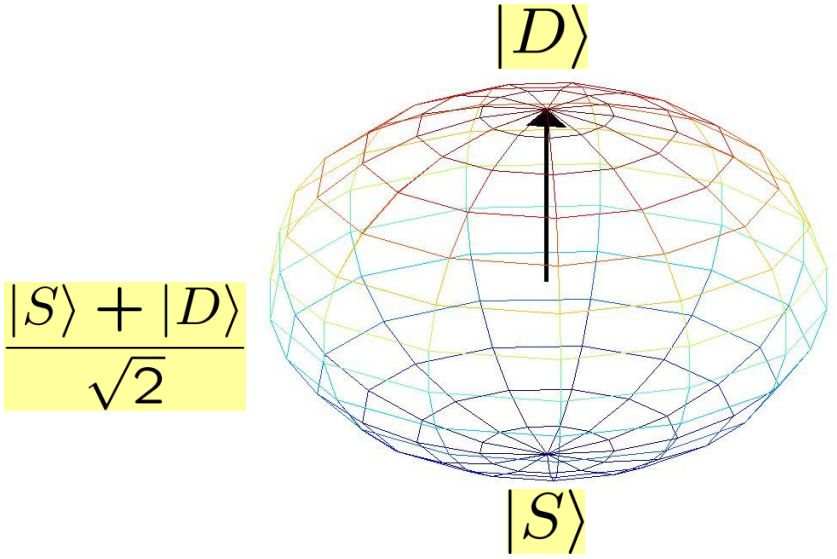
$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$

$|S\rangle$

Rabi oscillations







Requirements for quantum computing

Classical computer

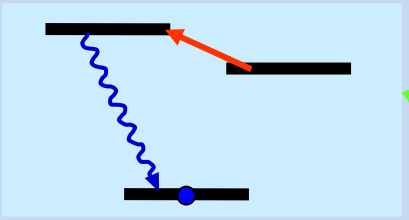
- Initialization
- 1-bit operations (NOT)
- 2-bit gates (e.g. NAND)

Computational space:

00
01
10
11

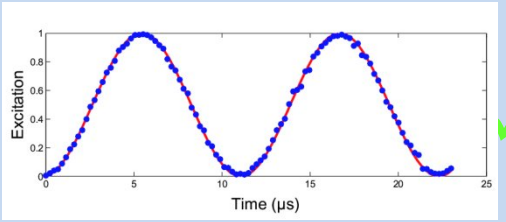
- Read out
  ➡ result

Quantum computer



- Initialization

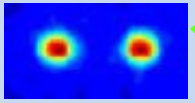
- 1-qubit rotations
  ➡ superpositions



- 2-qubit gates (CNOT gate)
  ➡ entanglement

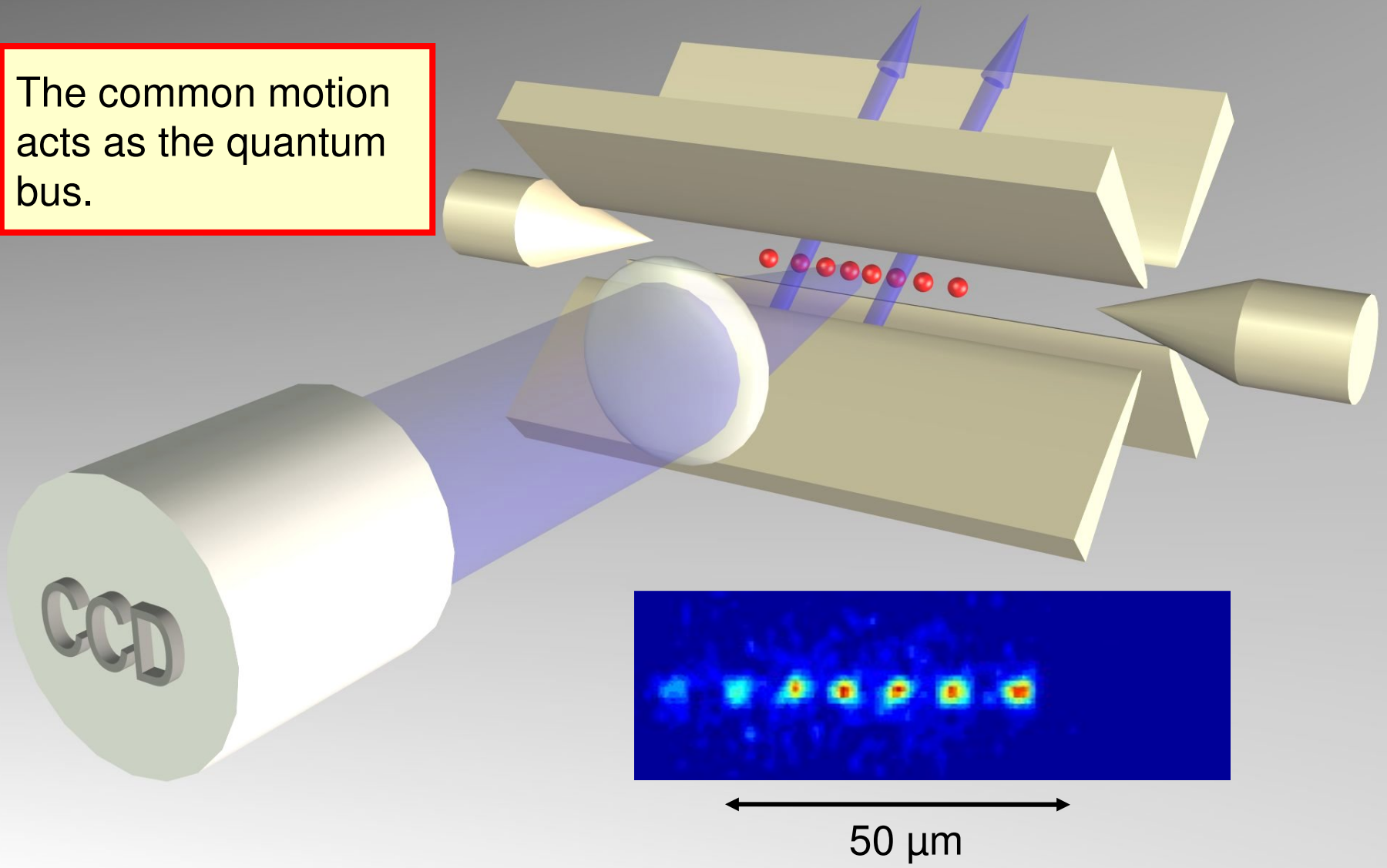
Computational space:

- Read out of qubits
  ➡ gain of classical information

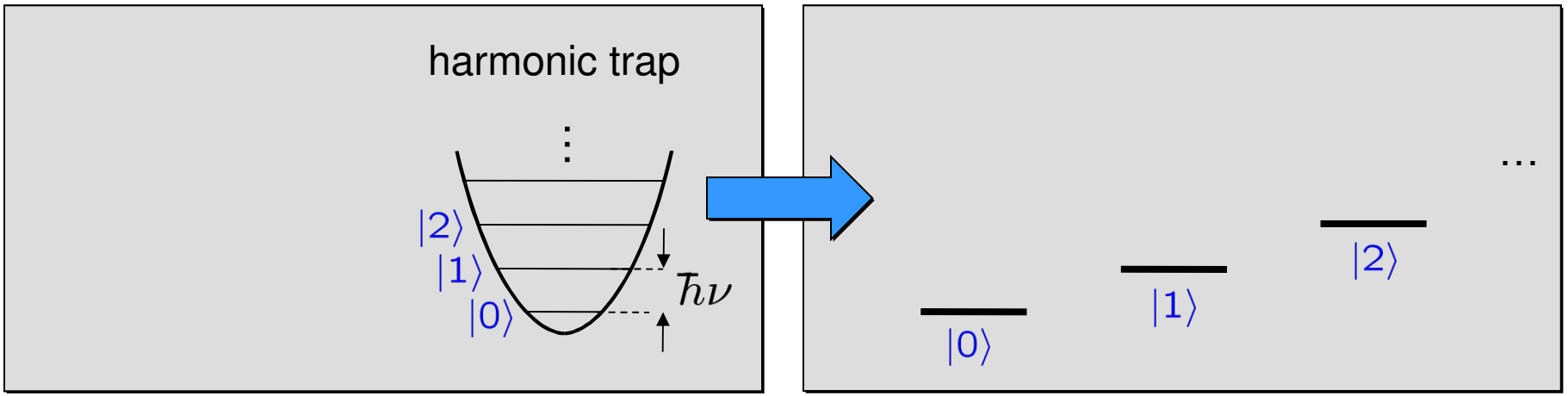


Having the qubits interact

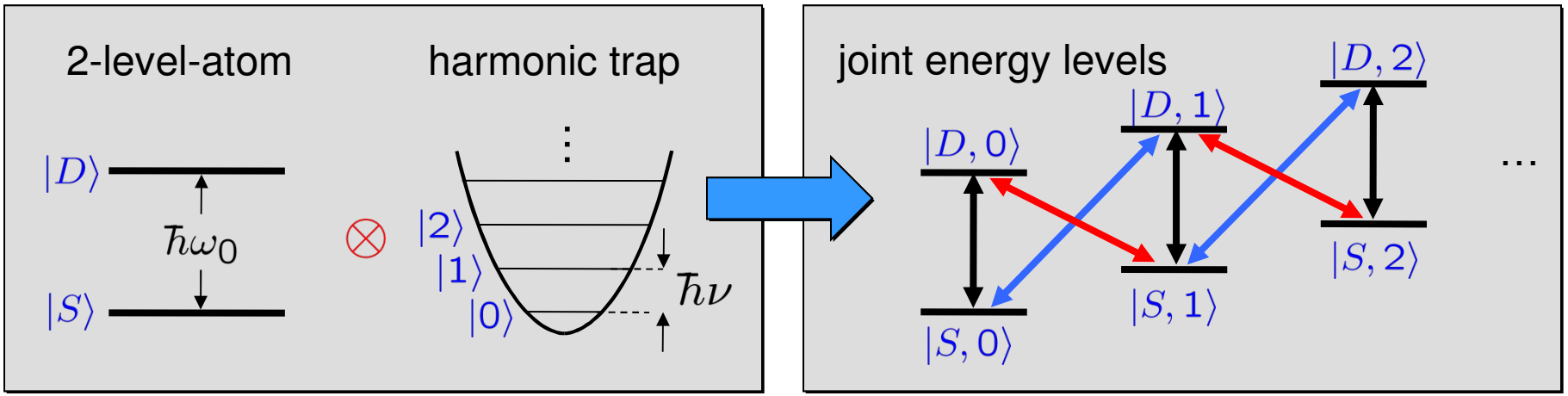
The common motion acts as the quantum bus.

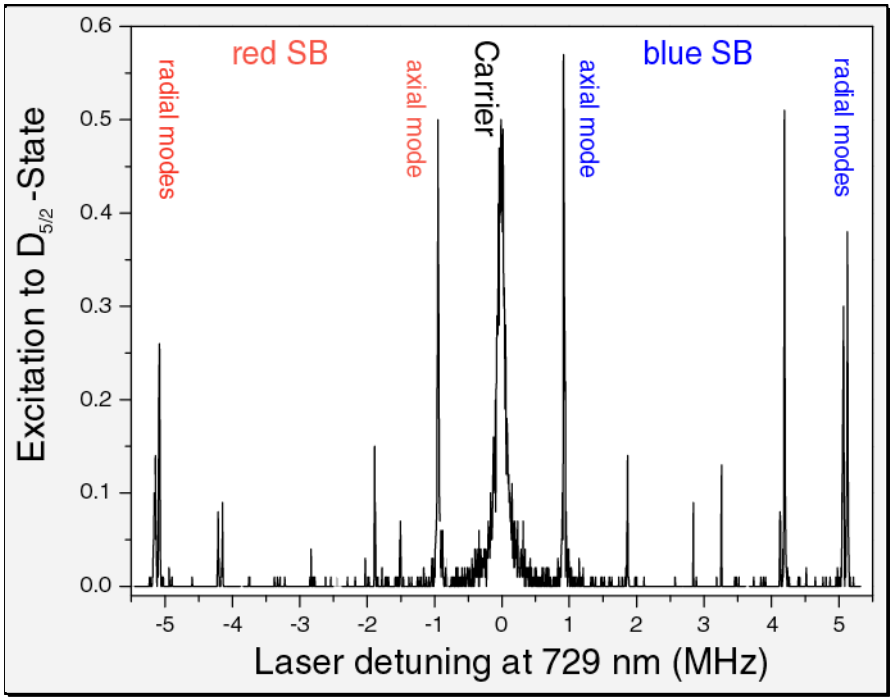


Ion motion

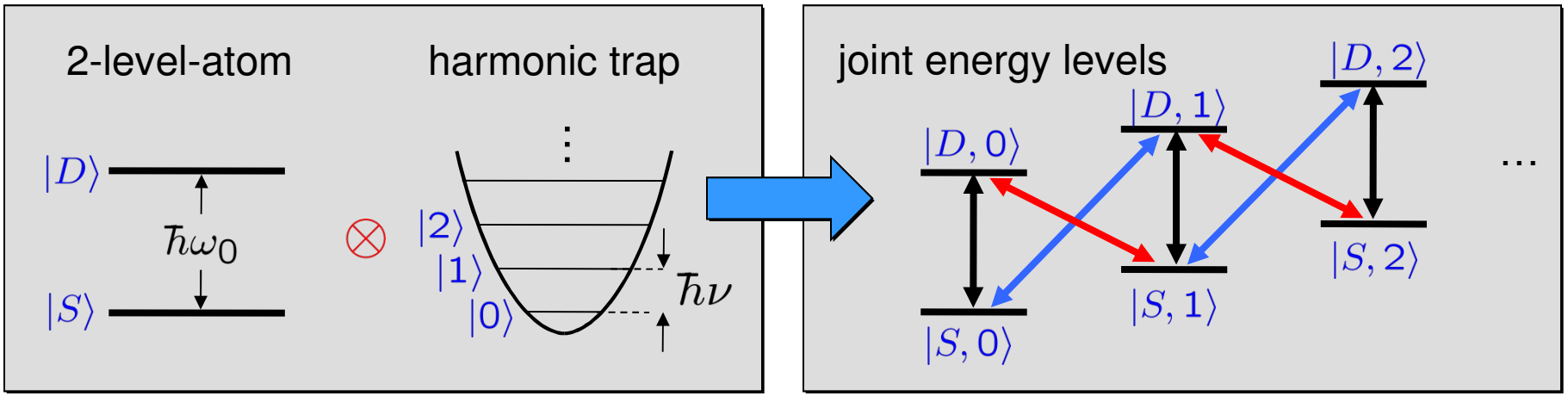


Ion motion

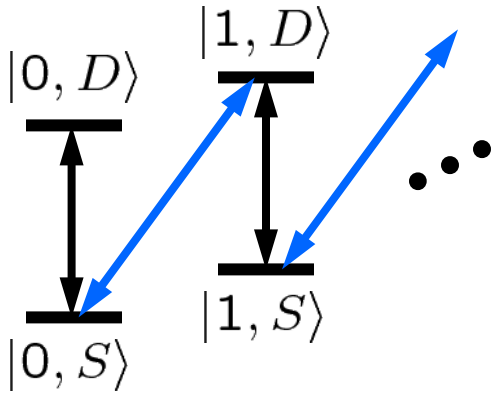

2-level-atom
harmonic trap
joint energy levels
$|D\rangle$
$|S\rangle$
$\hbar\omega_0$
$|2\rangle$
$|1\rangle$
$|0\rangle$
$\hbar\nu$
$|D,0\rangle$
$|D,1\rangle$
$|D,2\rangle$
$|S,0\rangle$
$|S,1\rangle$
$|S,2\rangle$
...


red SB
blue SB
radial modes
axial mode
Carrier
axial mode
radial modes
Excitation to $D_{5/2}$-State
0.0
0.1
0.2
0.3
0.4
0.5
0.6
-5
-4
-3
-2
-1
0
1
2
3
4
5
Laser detuning at 729 nm (MHz)

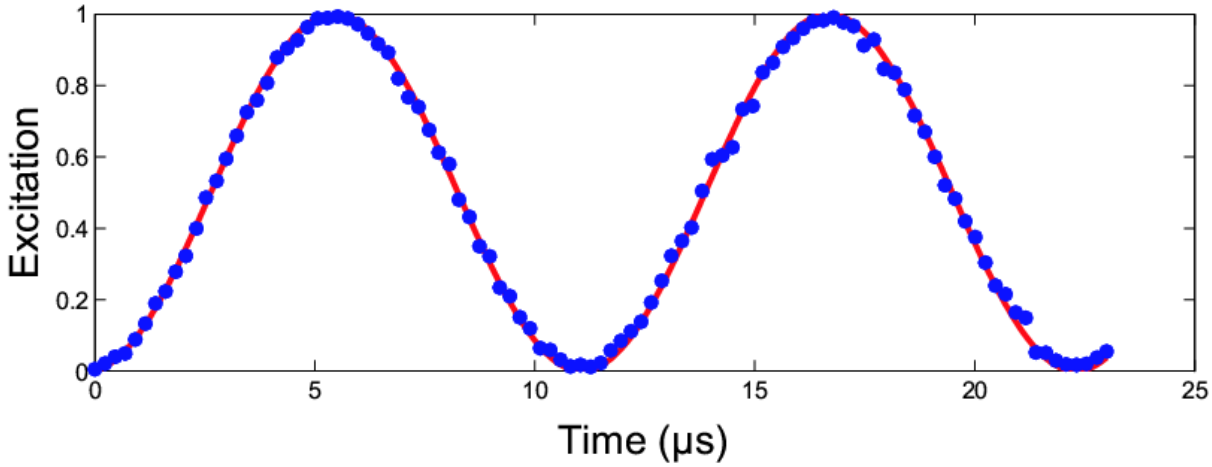
Ion motion


2-level-atom
harmonic trap
$|D\rangle$
$\hbar\omega_0$
$|S\rangle$
$|2\rangle$
$|1\rangle$
$|0\rangle$
$\hbar\nu$
joint energy levels
$|D,0\rangle$
$|D,1\rangle$
$|D,2\rangle$
$|S,0\rangle$
$|S,1\rangle$
$|S,2\rangle$
...

Coherent manipulation

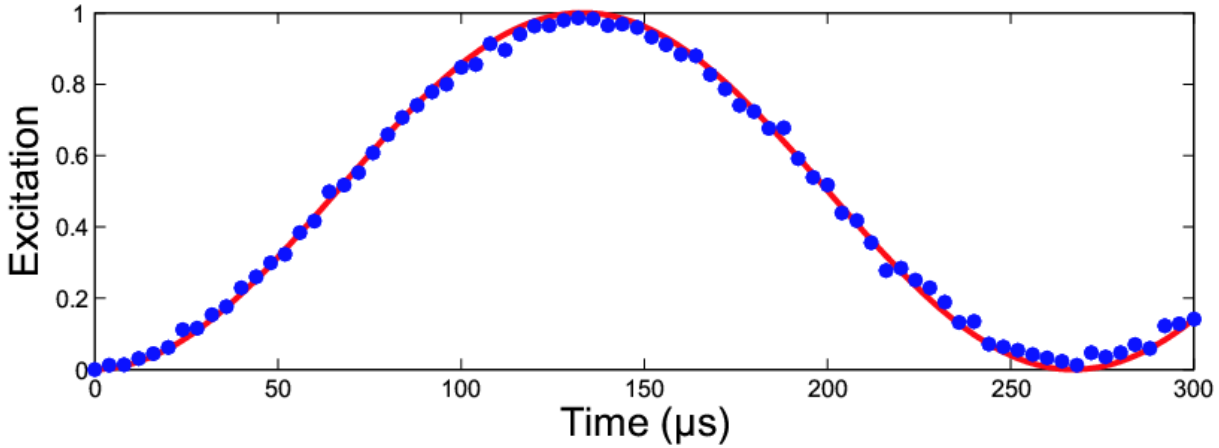


carrier



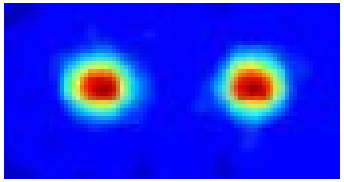
carrier and sideband
Rabi oscillations
with Rabi frequencies

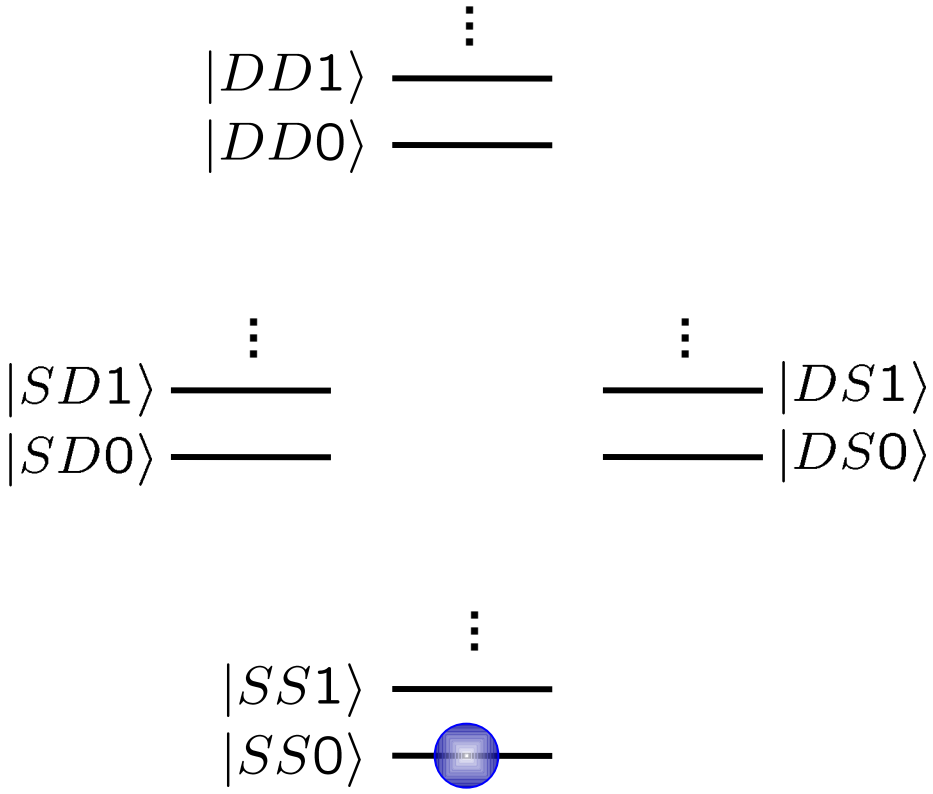
$\Omega, \quad \eta\Omega$



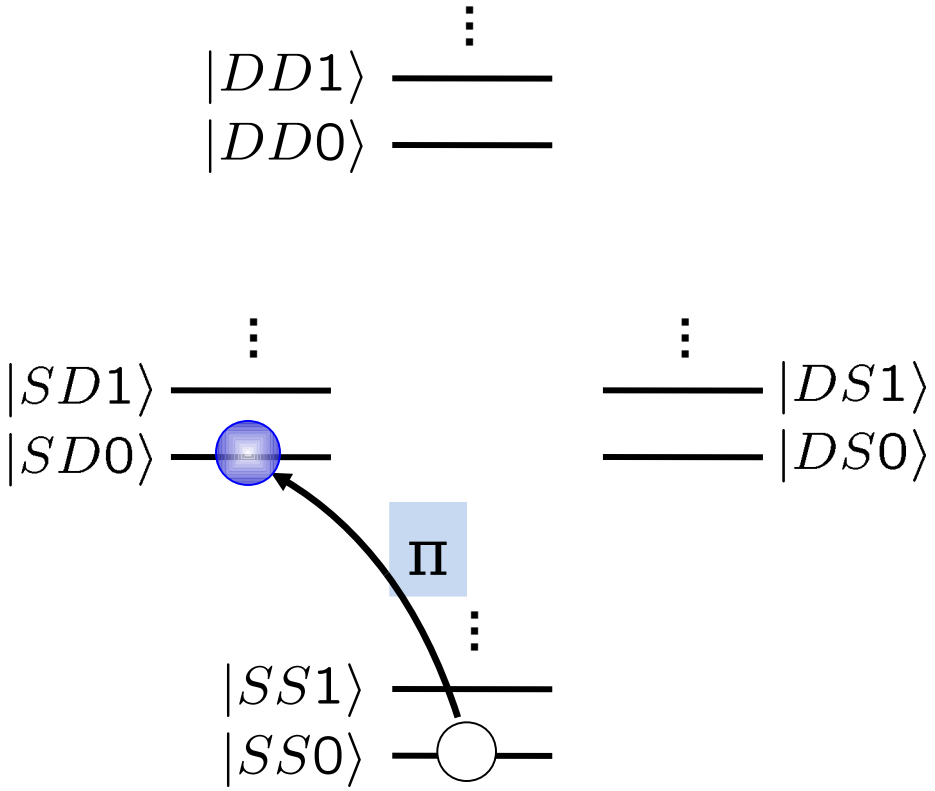
$\eta = kx_0$ Lamb-Dicke parameter

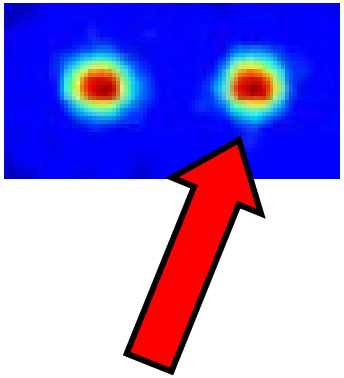
Generation of Bell states



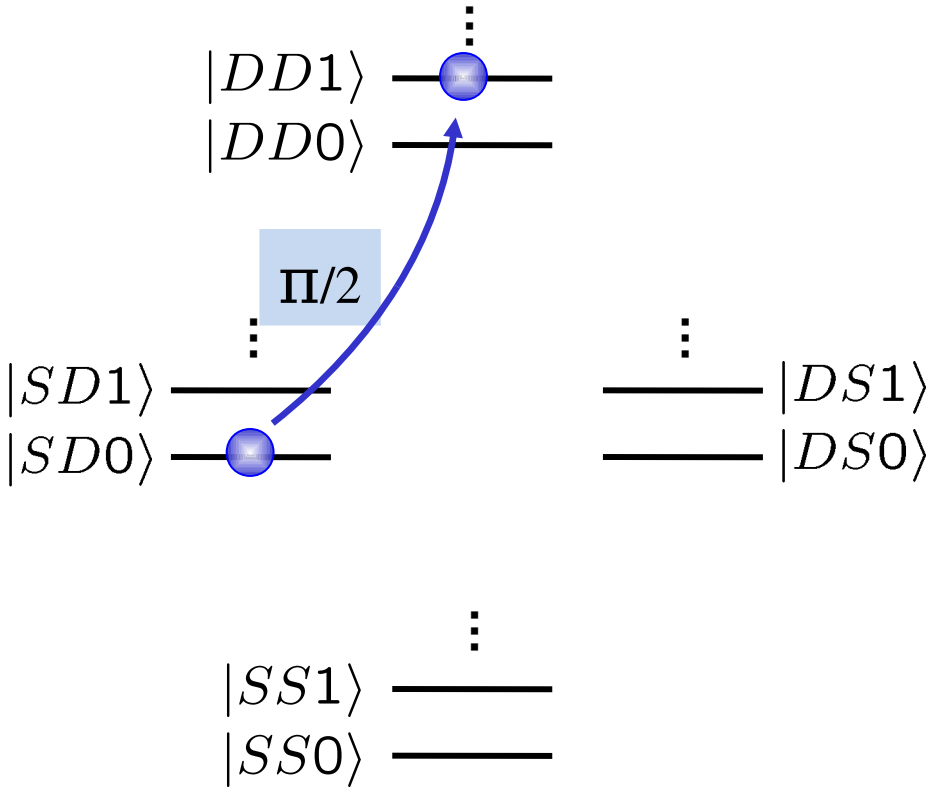


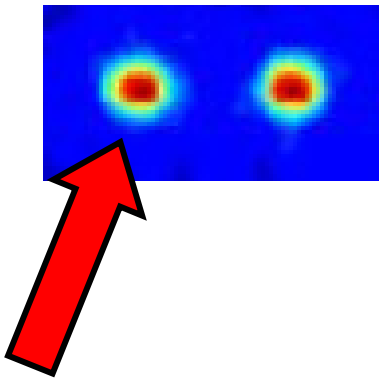
Generation of Bell states



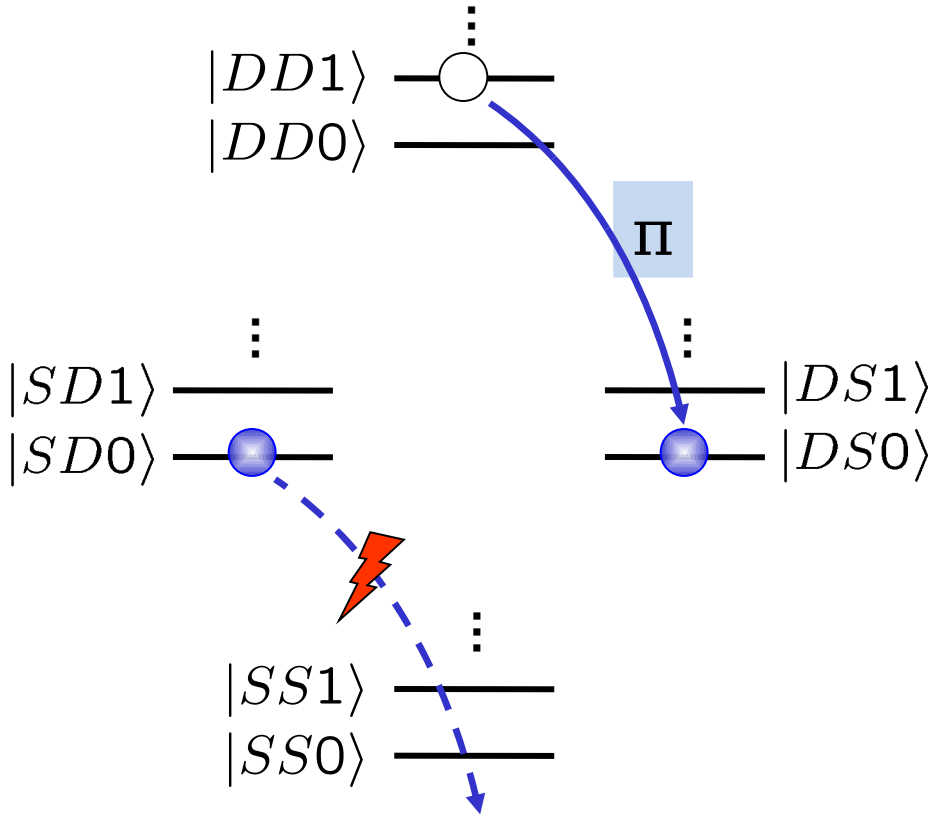


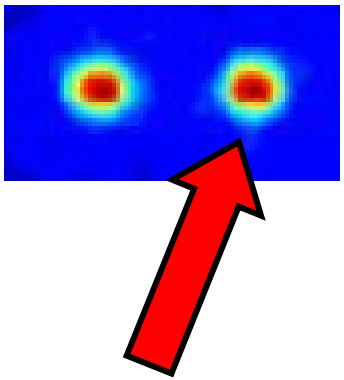
Generation of Bell states



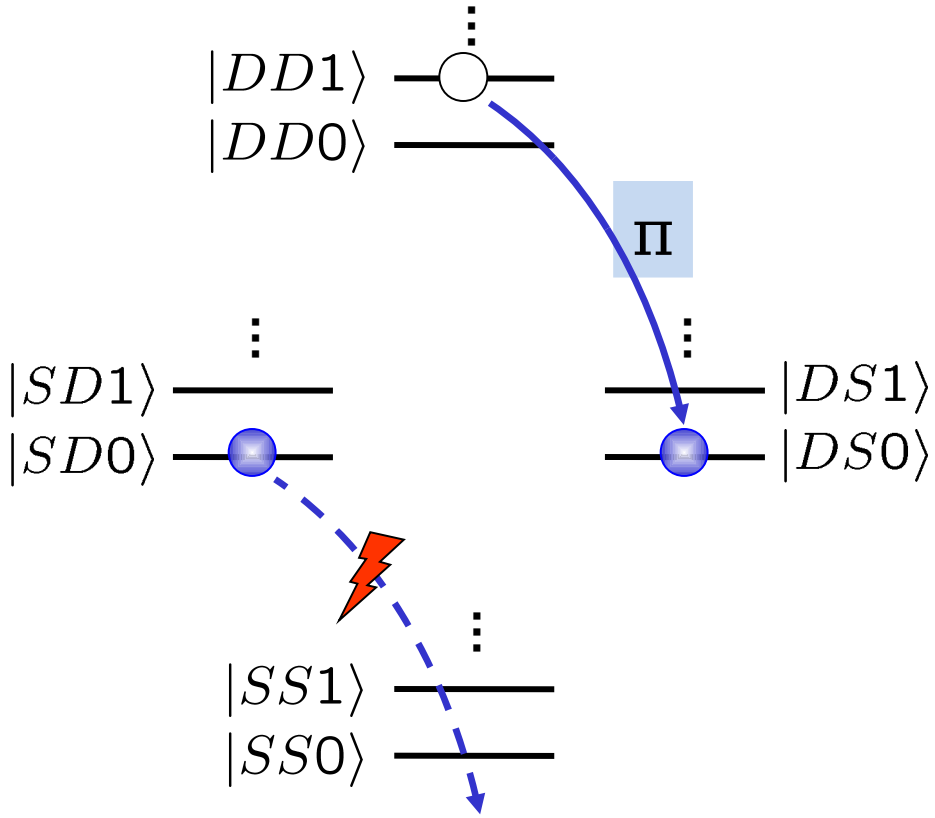


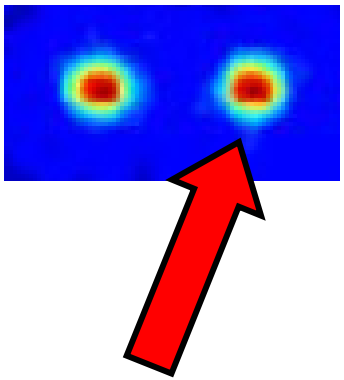
Generation of Bell states





Generation of Bell states





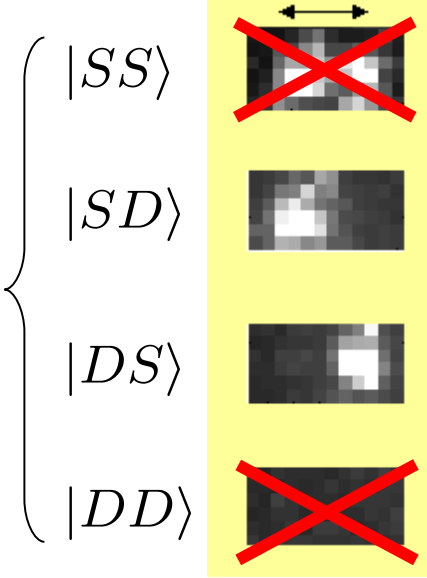
Bell states with atoms

- $^{9}Be^{+}$: NIST (fidelity: 97 %)
- $^{40}Ca^{+}$: Oxford (83%)
- $^{111}Cd^{+}$: Ann Arbor (79%)
- $^{25}Mg^{+}$: Munich (97%)
- $^{40}Ca^{+}$: Innsbruck (99%)

Analysis of Bell states

$$|SD\rangle + |DS\rangle$$

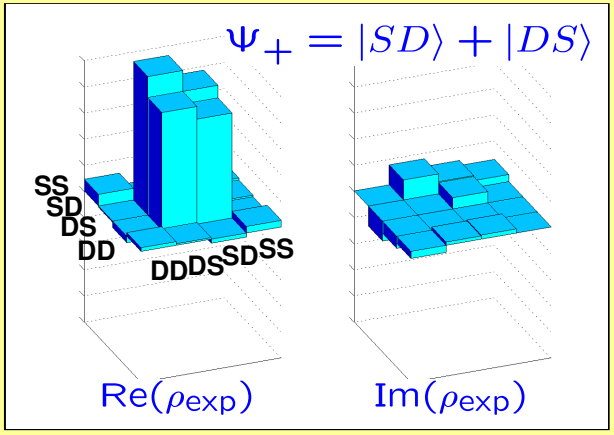
Fluorescence detection with CCD camera:



Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

➡ Measurement of the density matrix:

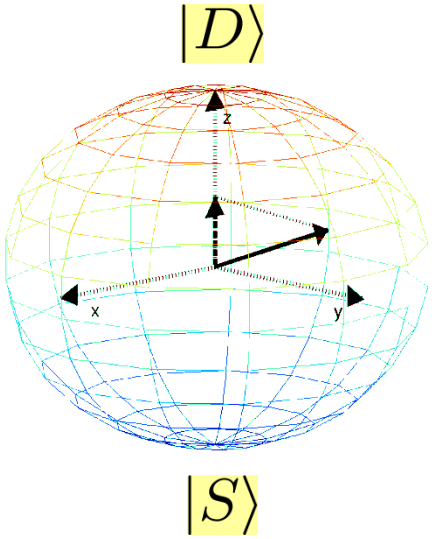


Measuring a density matrix

A measurement yields the *z*-component of the Bloch vector

=> Diagonal of the density matrix

$$\rho = \begin{pmatrix} P_S & C - iD \\ C + iD & P_D \end{pmatrix}$$



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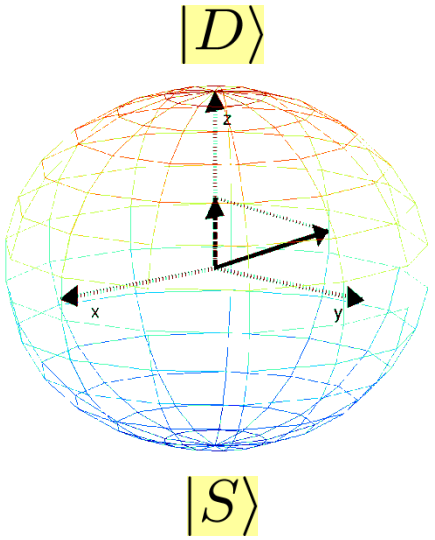
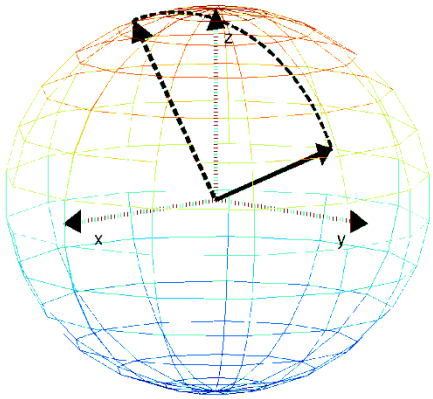


Rotation around the *x*- or the *y*-axis prior to the measurement yields the phase information of the qubit.



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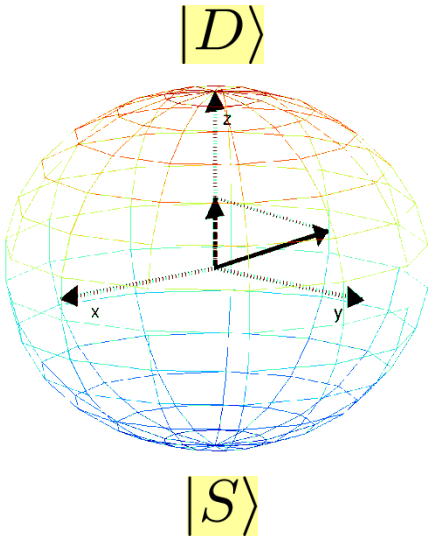
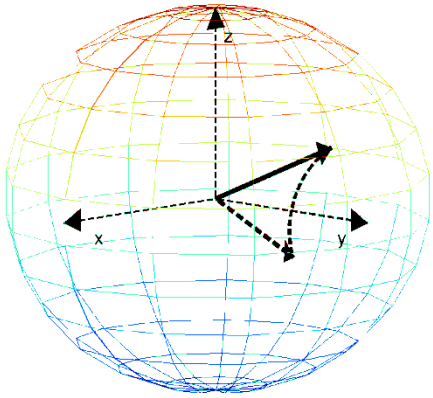
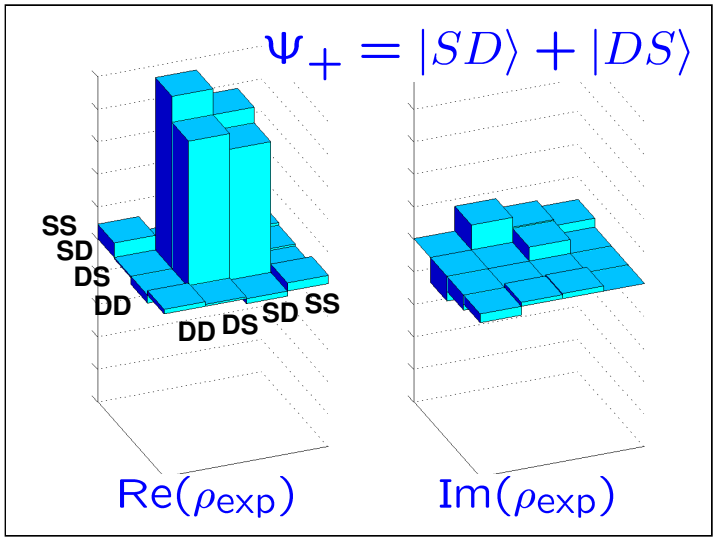


Rotation around the *x*- or the *y*-axis prior to the measurement yields the phase information of the qubit.

=> coherences of the density matrix



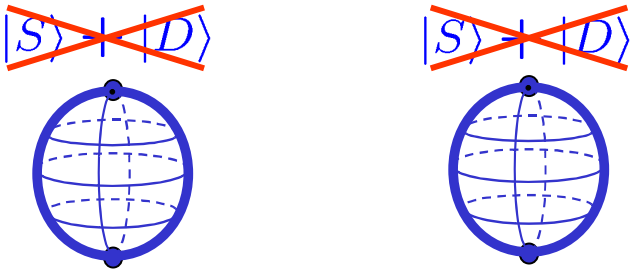
Entanglement



States are fully characterized

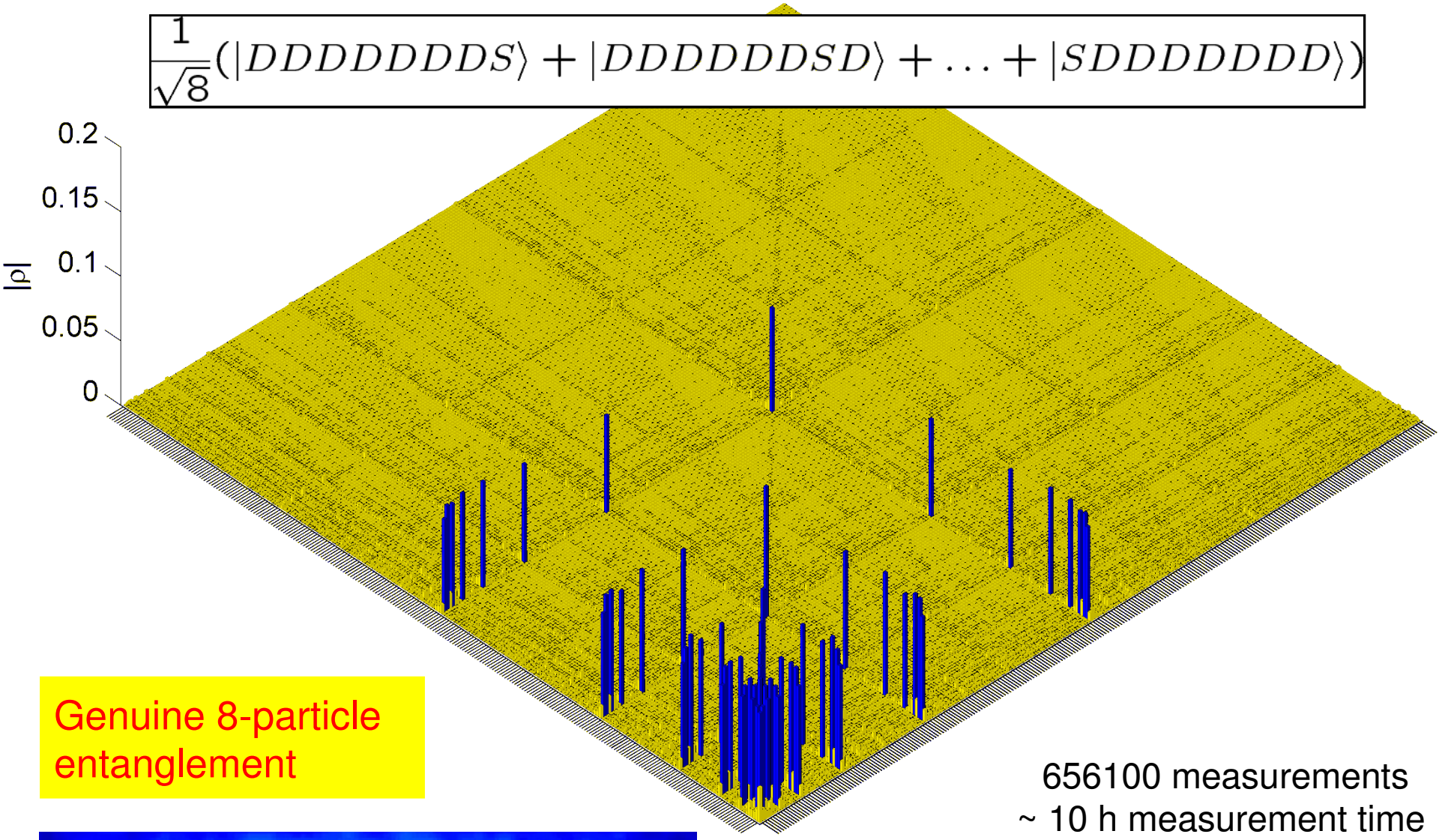
Fidelities: up to 0.99

Entanglement: the state of each qubit is not defined!

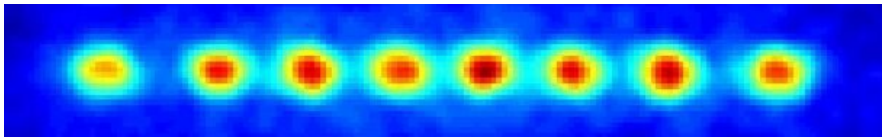


“Large” entangled states

$$\frac{1}{\sqrt{8}}(|DDDDDDDS\rangle + |DDDDDDSD\rangle + \ldots + |SDDDDDDD\rangle)$$



Genuine 8-particle entanglement

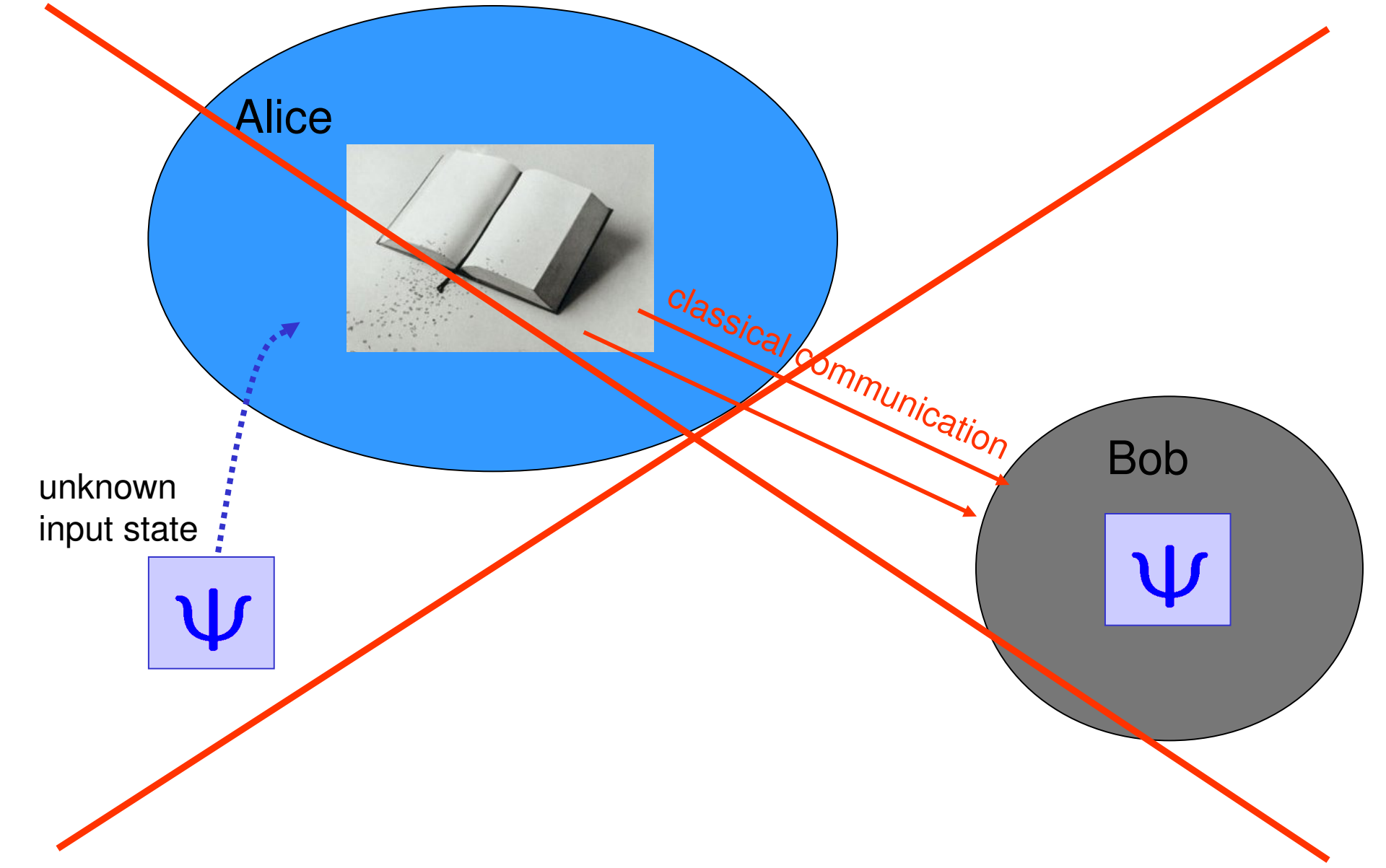


656100 measurements
~ 10 h measurement time

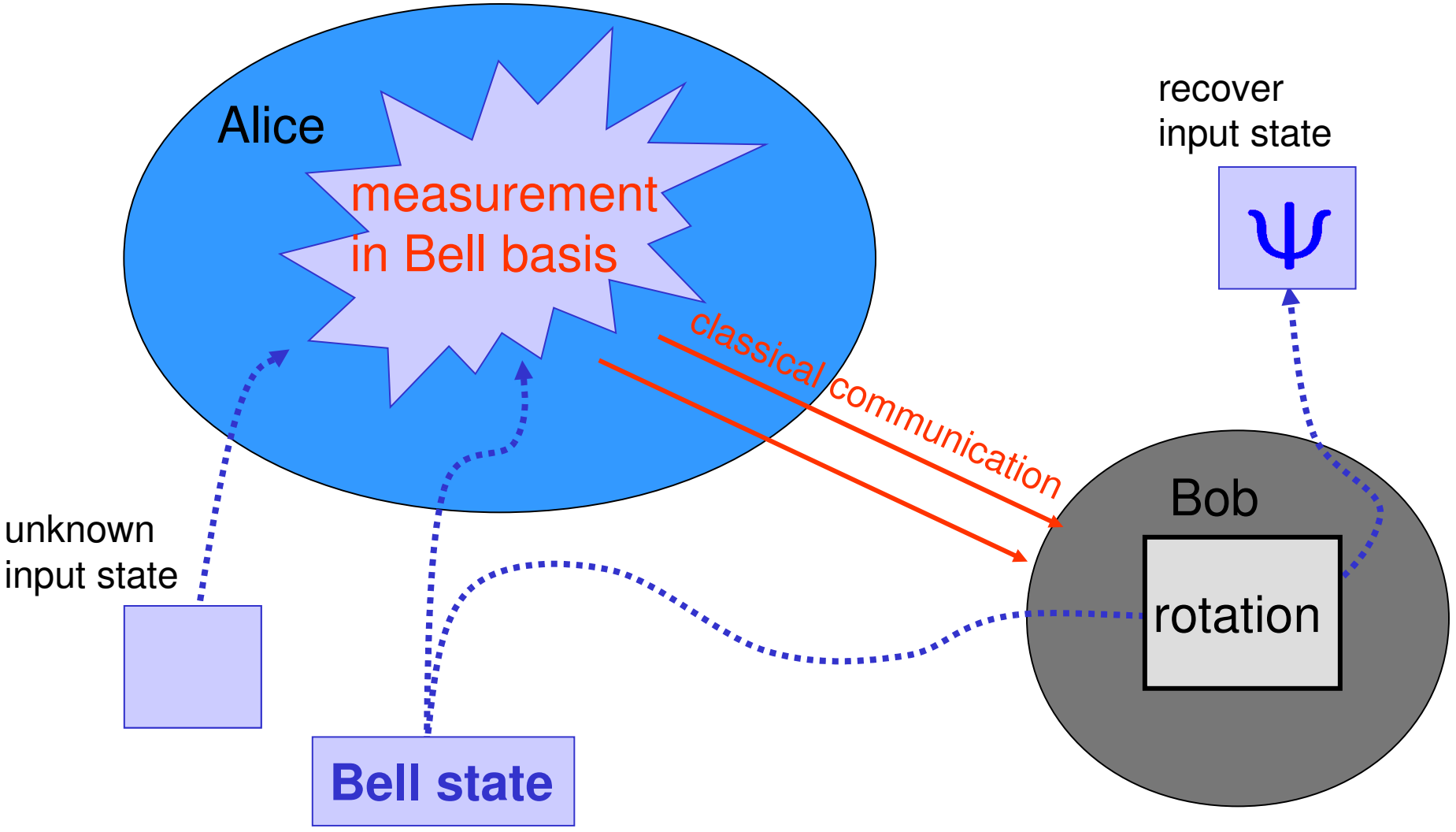
Häffner et al., Nature **438**, 643 (2005)

- Physics and information
- Ion trap quantum computing
- Teleportation
- Trends in quantum information

Teleportation

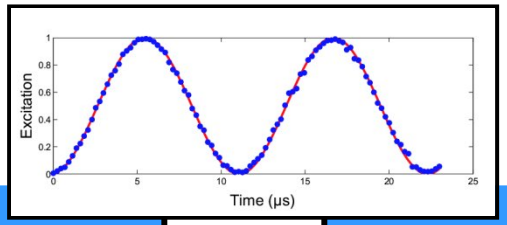


Teleportation



Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).
Bouwmeester *et al.*, Nature **390**, 575 (1997).

Teleportation



ψ

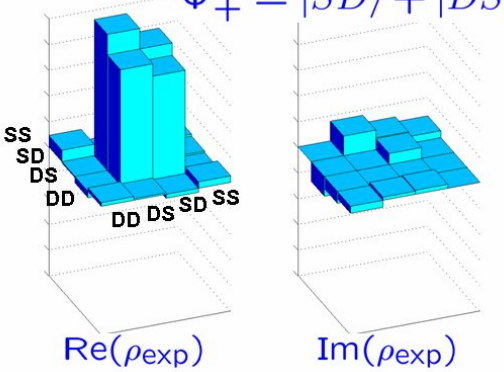
Bell state

$\pi/2$

Z

X

$\Psi_+ = |SD\rangle + |DS\rangle$



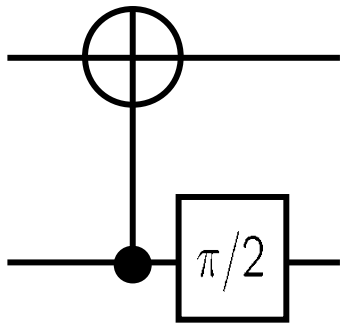
Bell measurement

$|SS + DD\rangle$

$|SD + DS\rangle$

$|SS - DD\rangle$

$|SD - DS\rangle$



$|SS\rangle$

$|SD\rangle$

$|DS\rangle$

$|DD\rangle$

Ion trap quantum computing

VOLUME 74, NUMBER 20 PHYSICAL REVIEW LETTERS 15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universiät Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

PACS numbers: 89.80.+h, 03.65.Bz, 12.20.Fv, 32.80.Pj

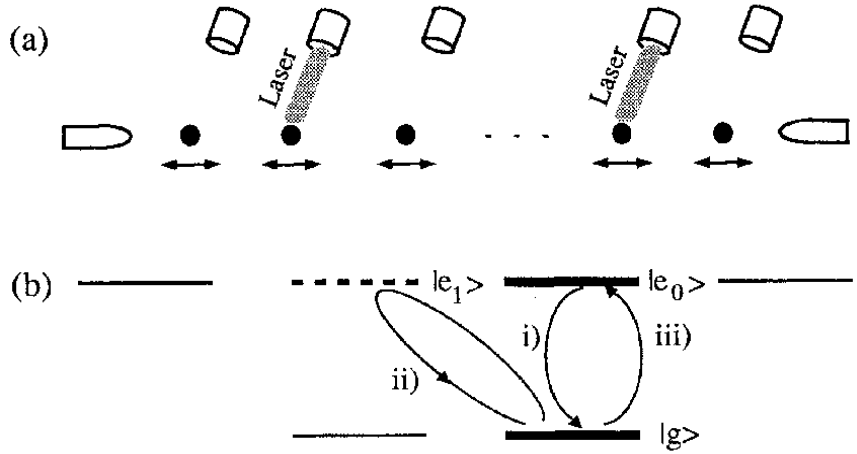


FIG. 1. (a) N ions in a linear trap interacting with N different laser beams; (b) atomic level scheme.

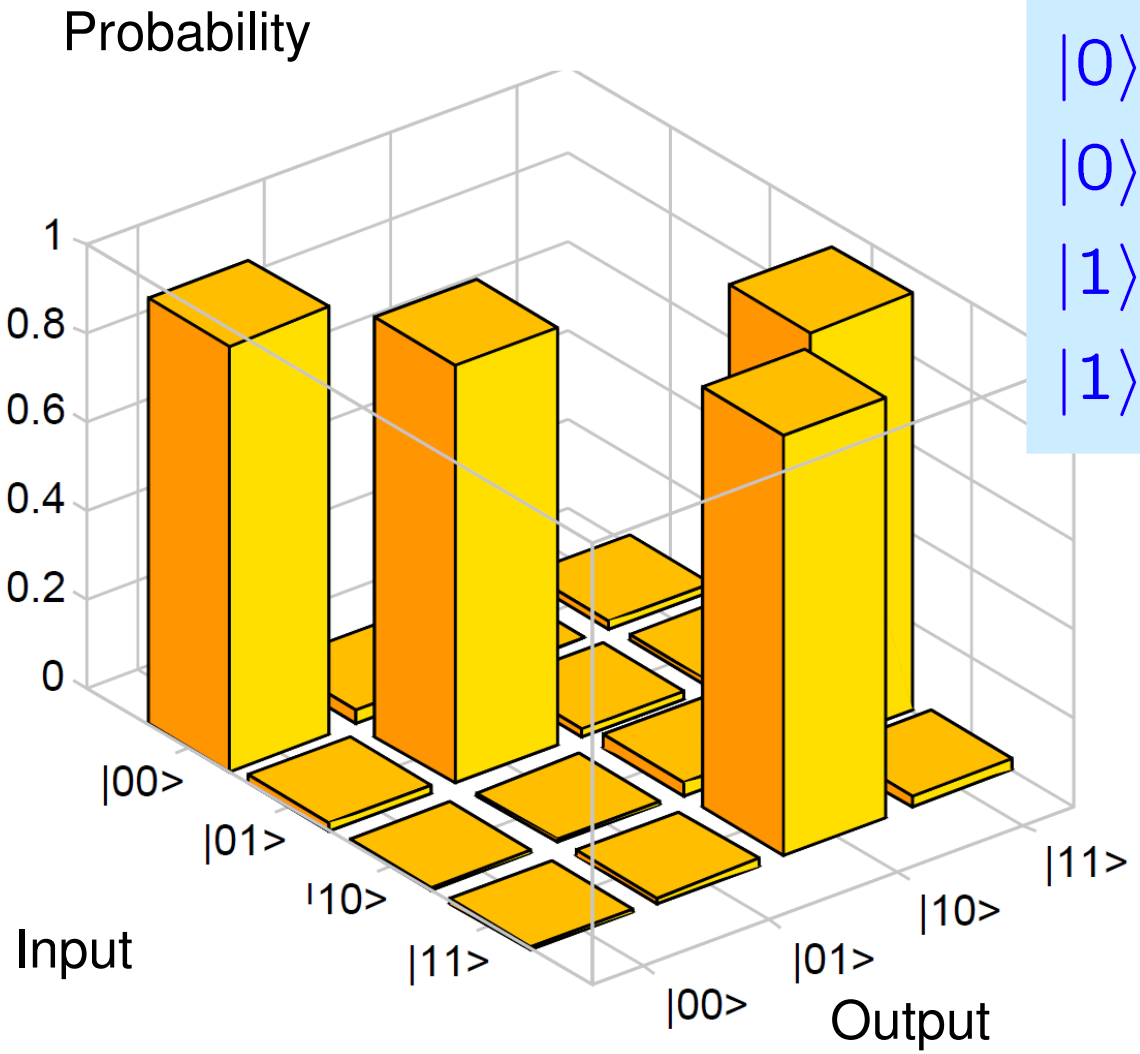
controlled – NOT :

$$|\varepsilon_1\rangle|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle|\varepsilon_1 \oplus \varepsilon_2\rangle$$

$$\begin{aligned} |0\rangle|0\rangle &\rightarrow |0\rangle|0\rangle \\ |0\rangle|1\rangle &\rightarrow |0\rangle|1\rangle \\ |1\rangle|0\rangle &\rightarrow |1\rangle|1\rangle \\ |1\rangle|1\rangle &\rightarrow |1\rangle|0\rangle \end{aligned}$$

control bit target bit

Truth table of a controlled NOT gate



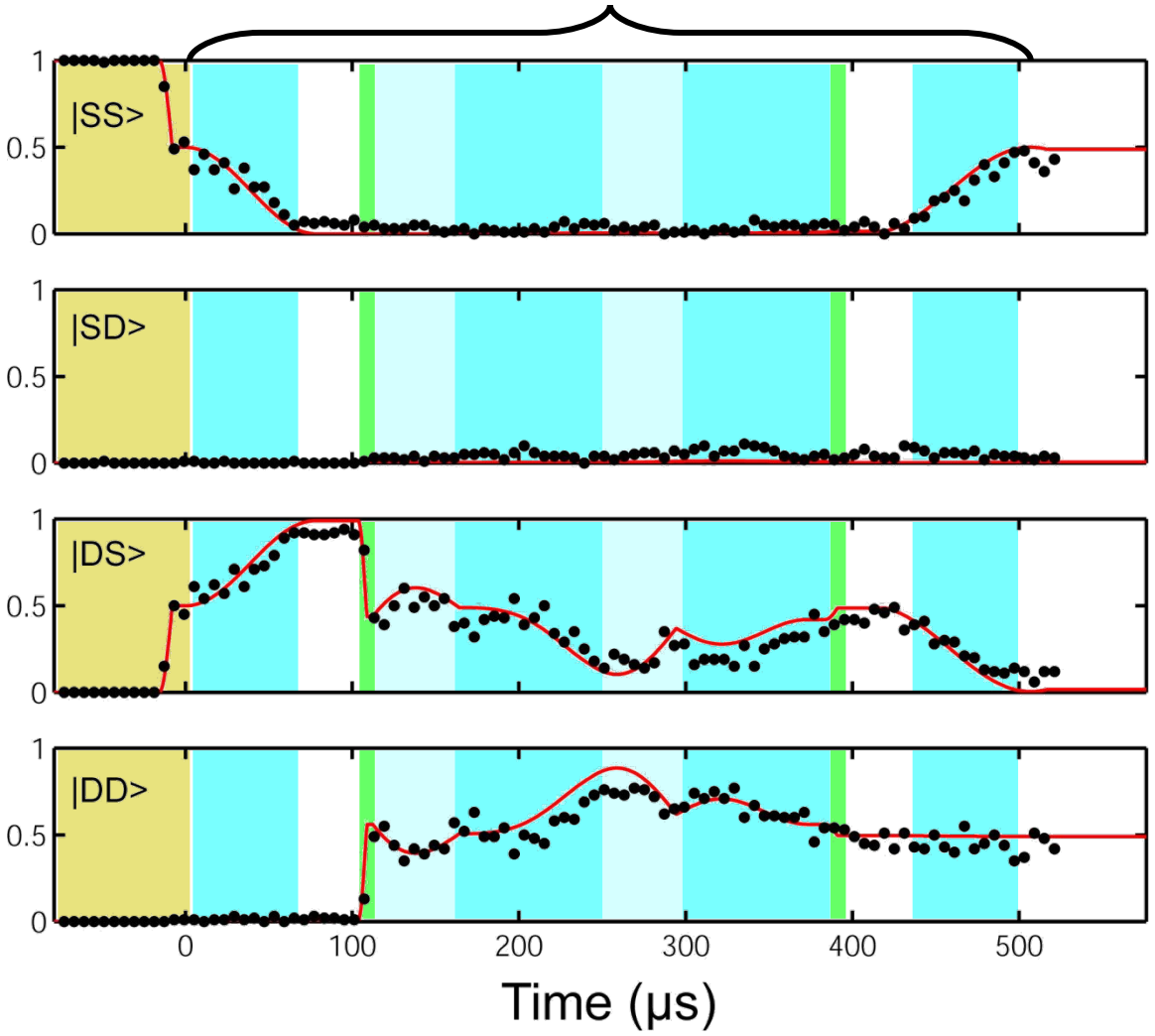
$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

➡ universal set of quantum gates

Another way to generate Bell states

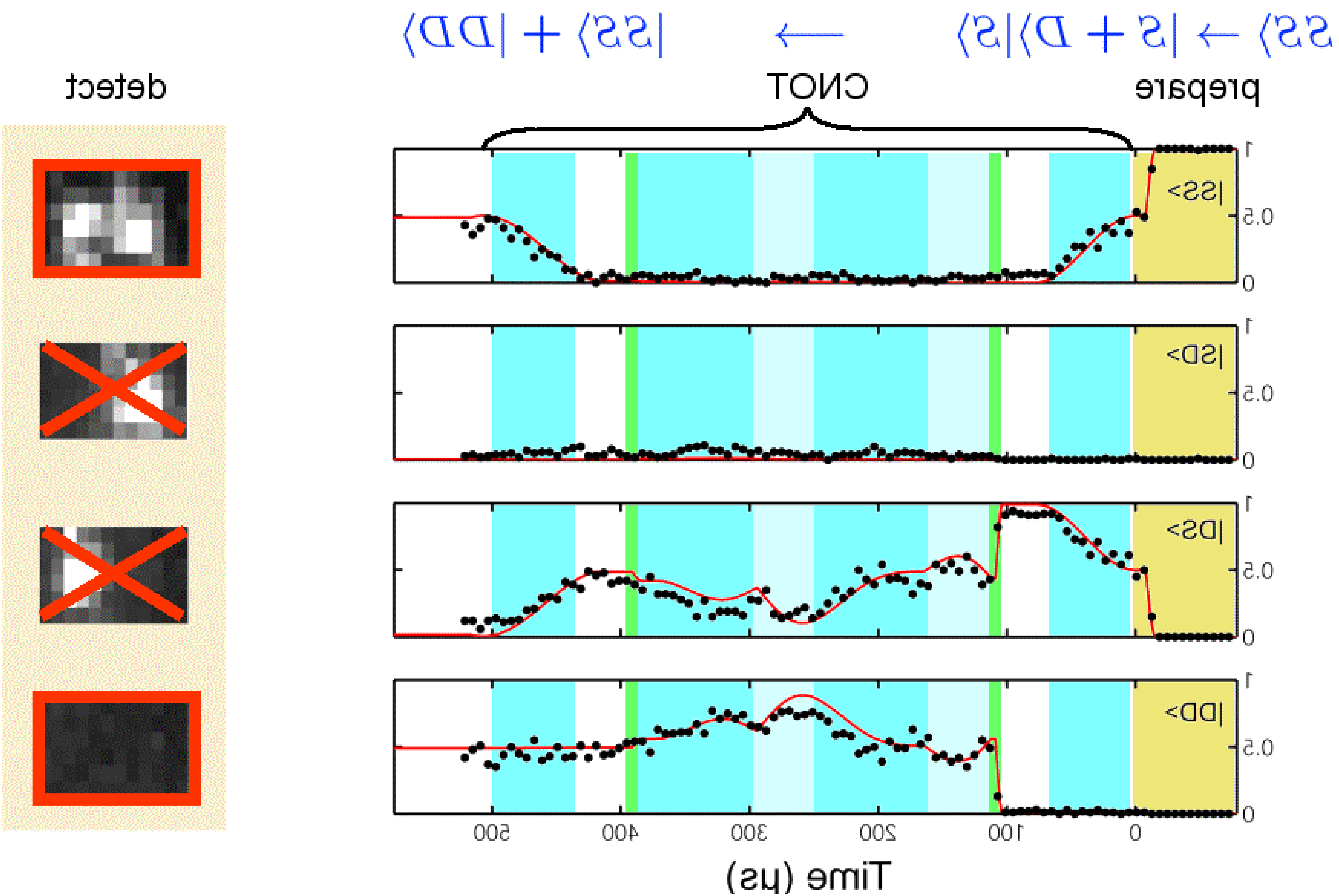
$$|SS\rangle \rightarrow |S+D\rangle|S\rangle \longrightarrow |SS\rangle + |DD\rangle$$

prepare CNOT



output

And now backwards



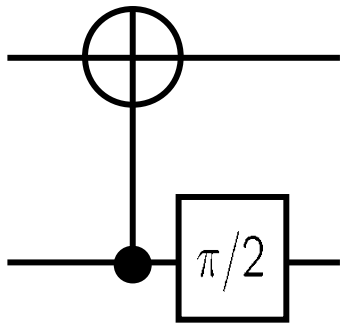
Bell measurement

$|SS + DD\rangle$

$|SD + DS\rangle$

$|SS - DD\rangle$

$|SD - DS\rangle$



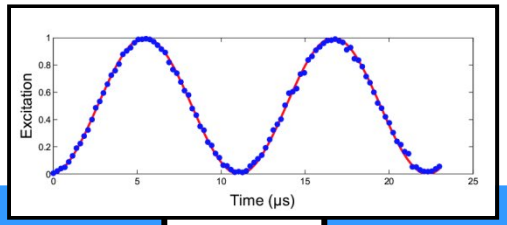
$|SS\rangle$

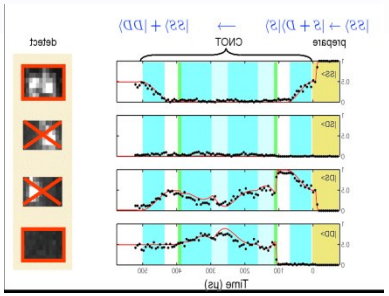
$|SD\rangle$

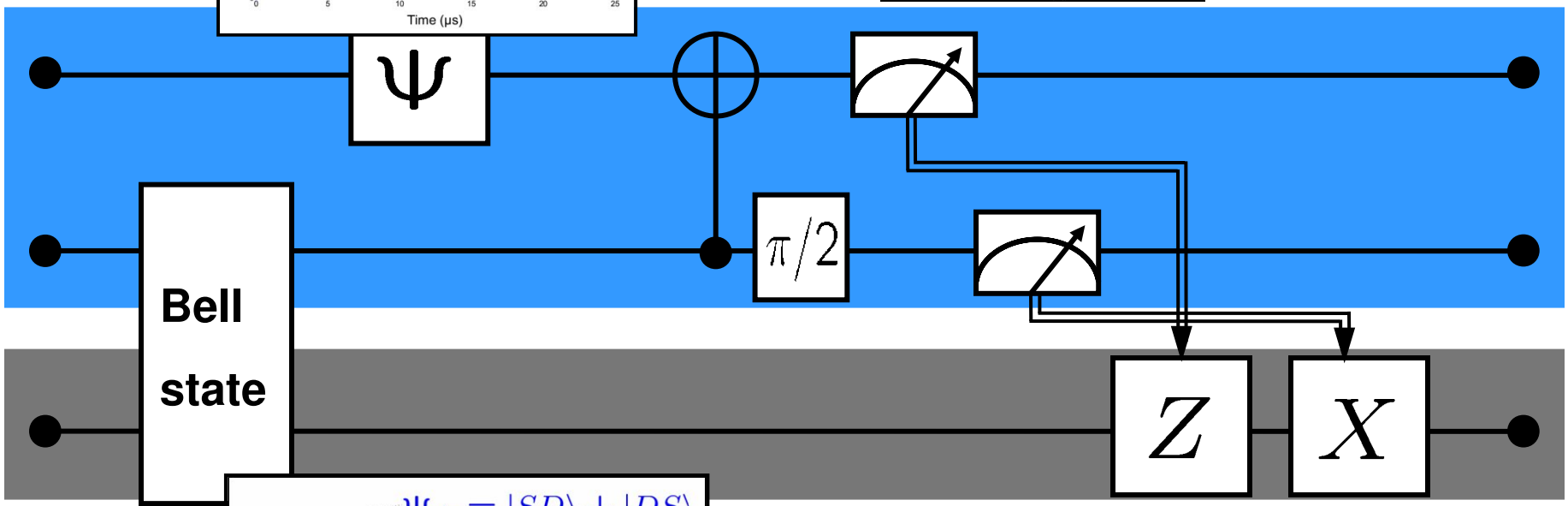
$|DS\rangle$

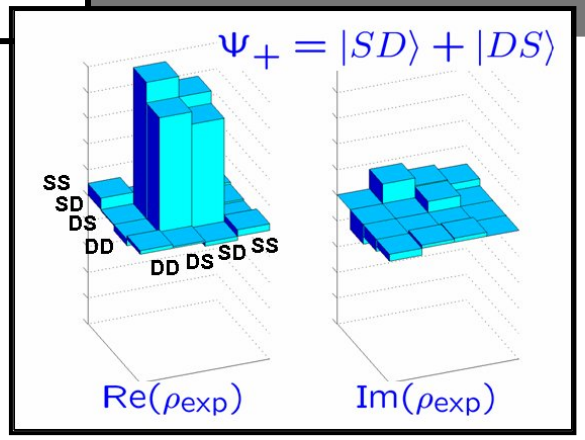
$|DD\rangle$

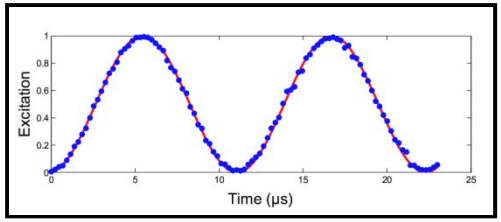
Teleportation



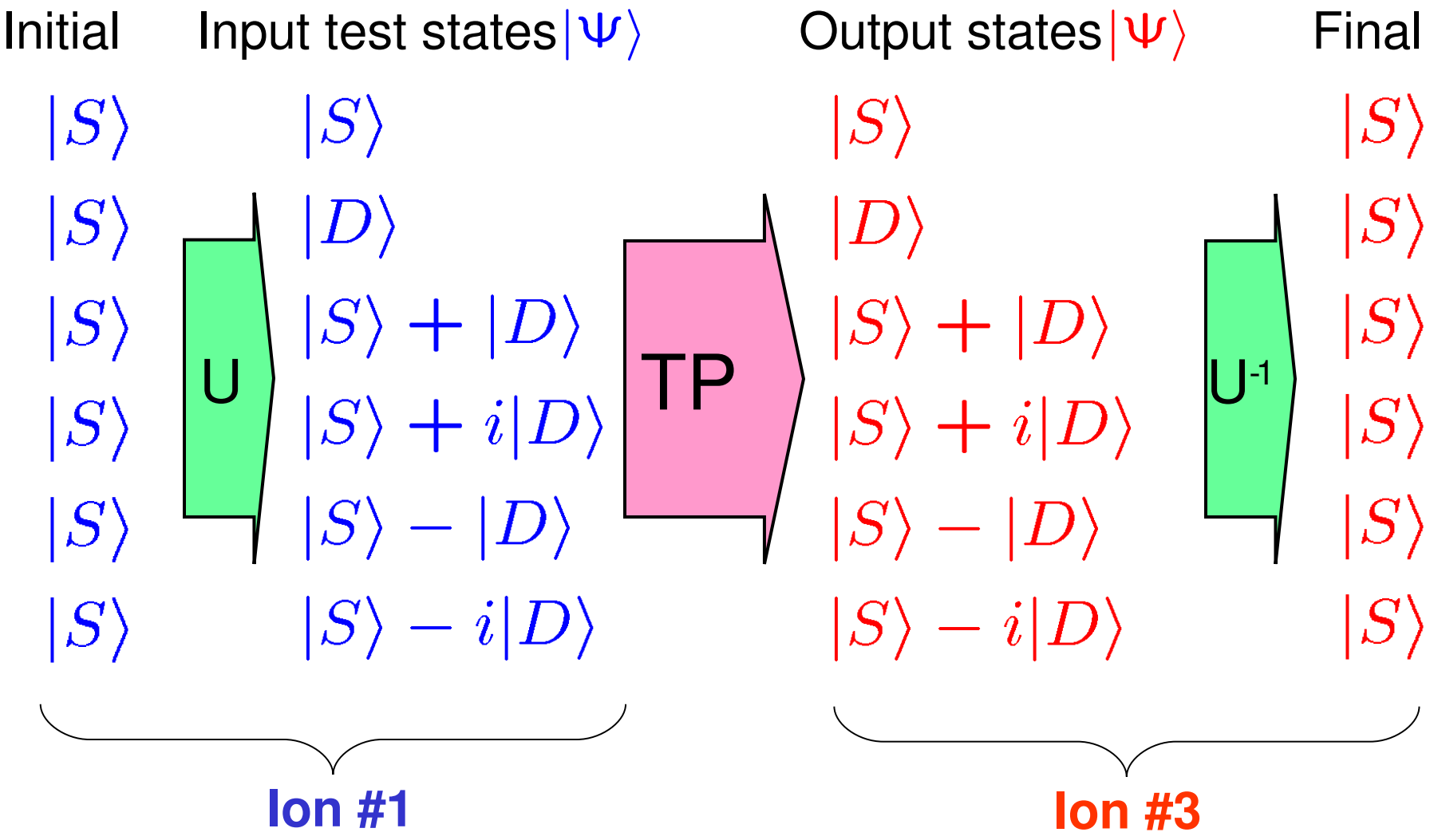




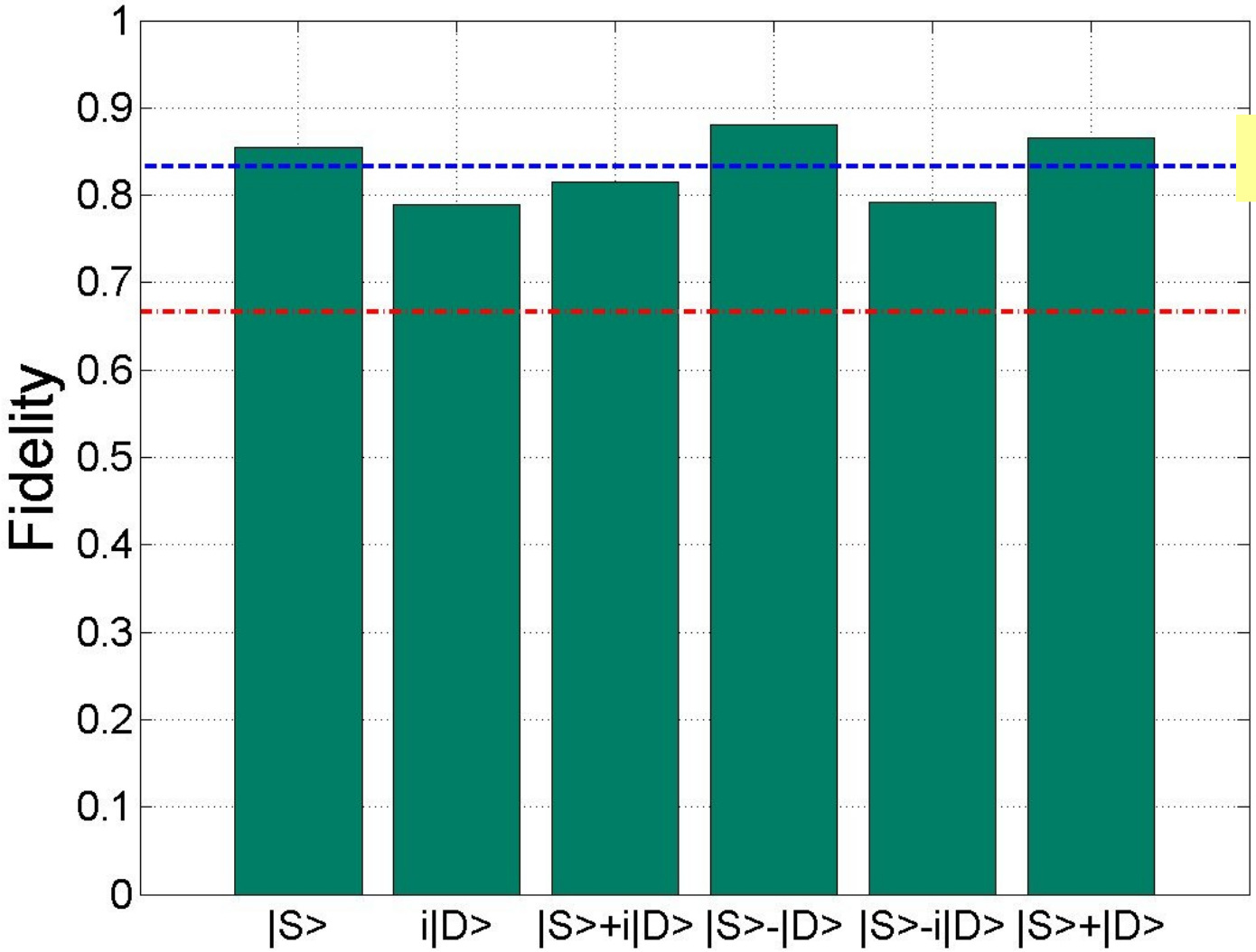




Teleportation analysis



Deterministic teleportation



Fidelity: 0.83

Classical threshold

"Deterministic teleportation with atoms"
Barrett et al., Nature **429**, 737 (2004) and Riebe et al., Nature **429**, 734 (2004)

Requirements for quantum computing

Classical computer

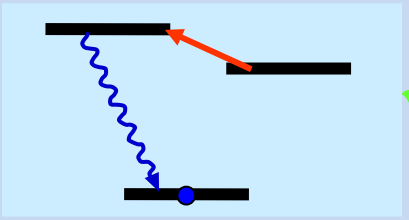
- Initialization
- 1-bit operations (NOT)
- 2-bit gates (e.g. NAND)

Computational space:

00
01
10
11

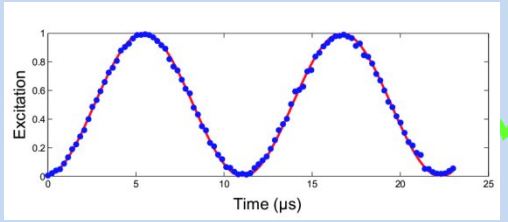
- Read out
  ➡ result

Quantum computer

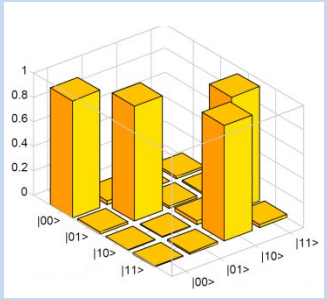


- Initialization

- 1-qubit rotations
  ➡ superpositions

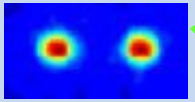


- 2-qubit gates (CNOT gate)
  ➡ entanglement

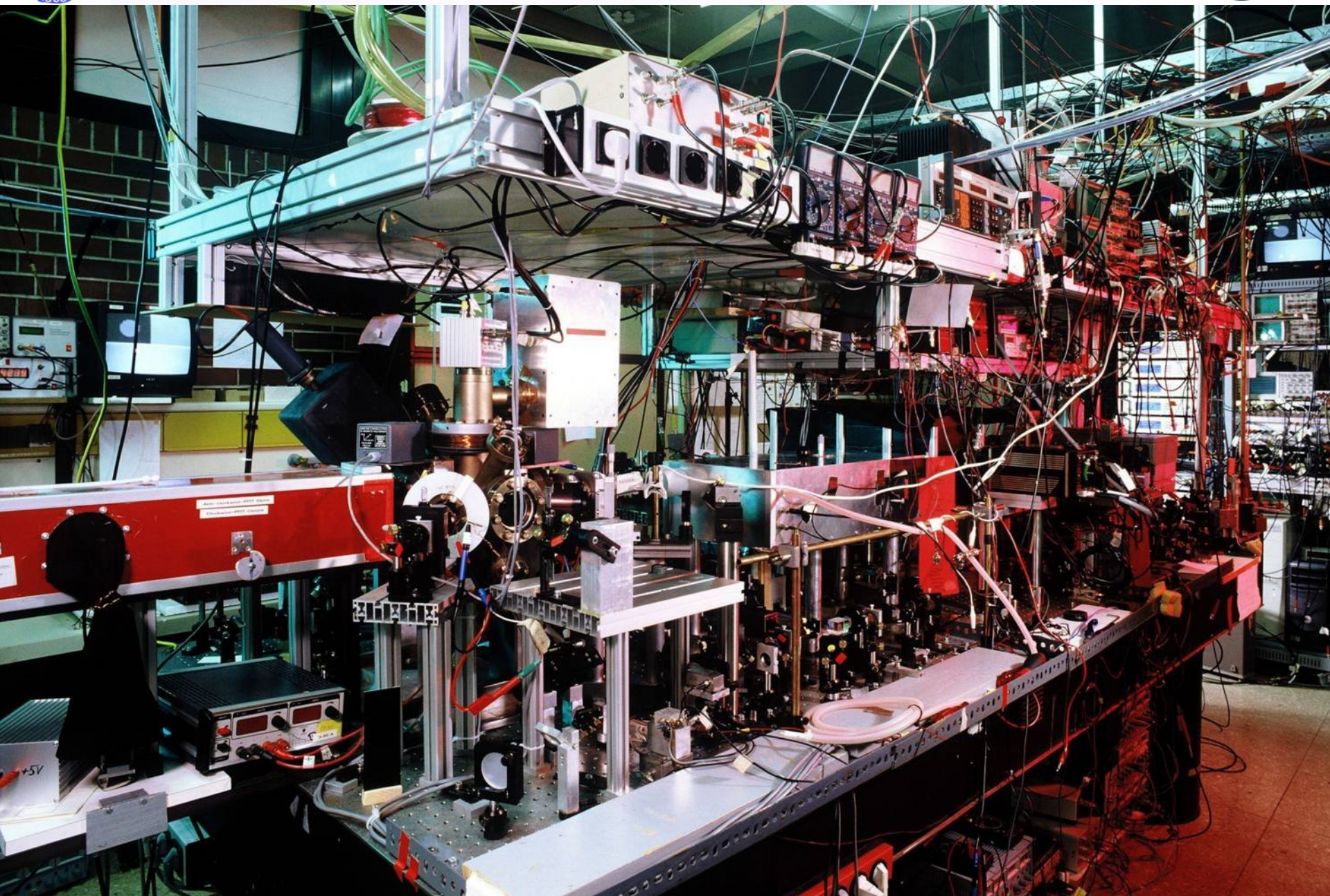


Computational space:

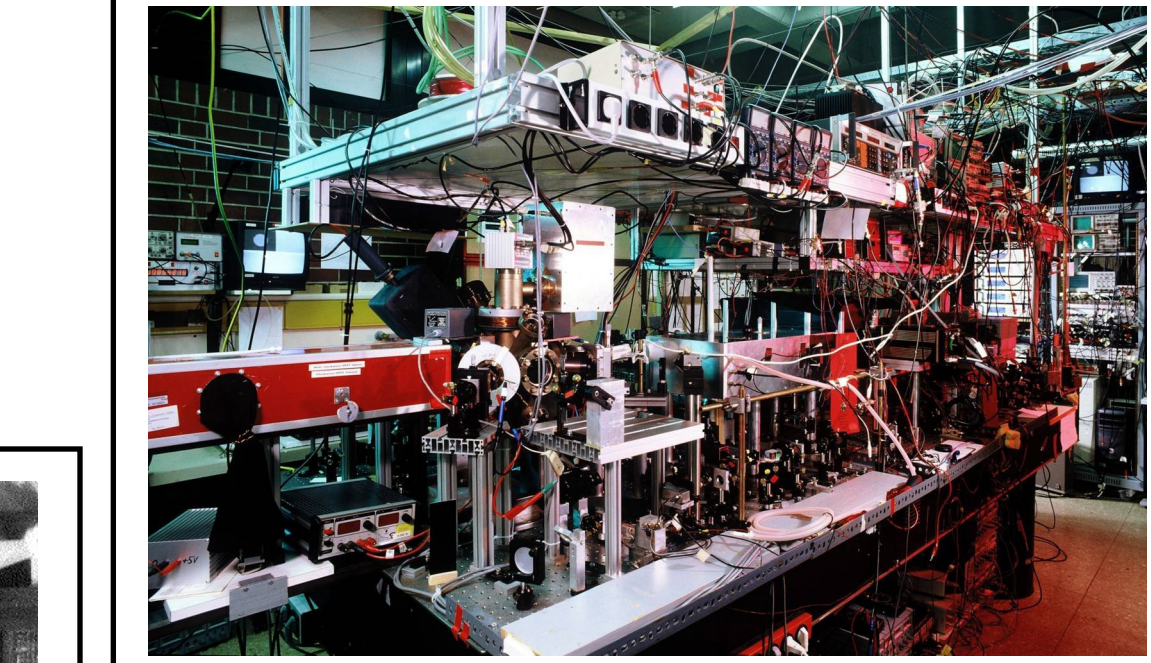
- Read out of qubits
  ➡ gain of classical information



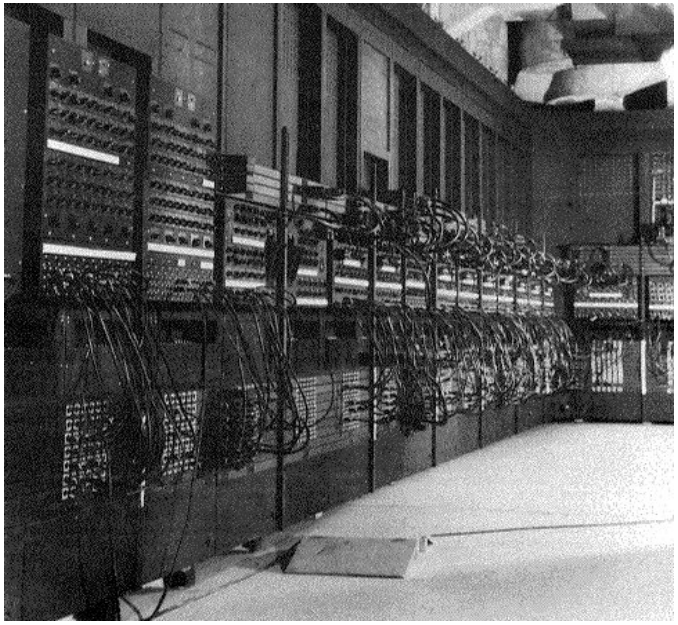
The hardware



The hardware

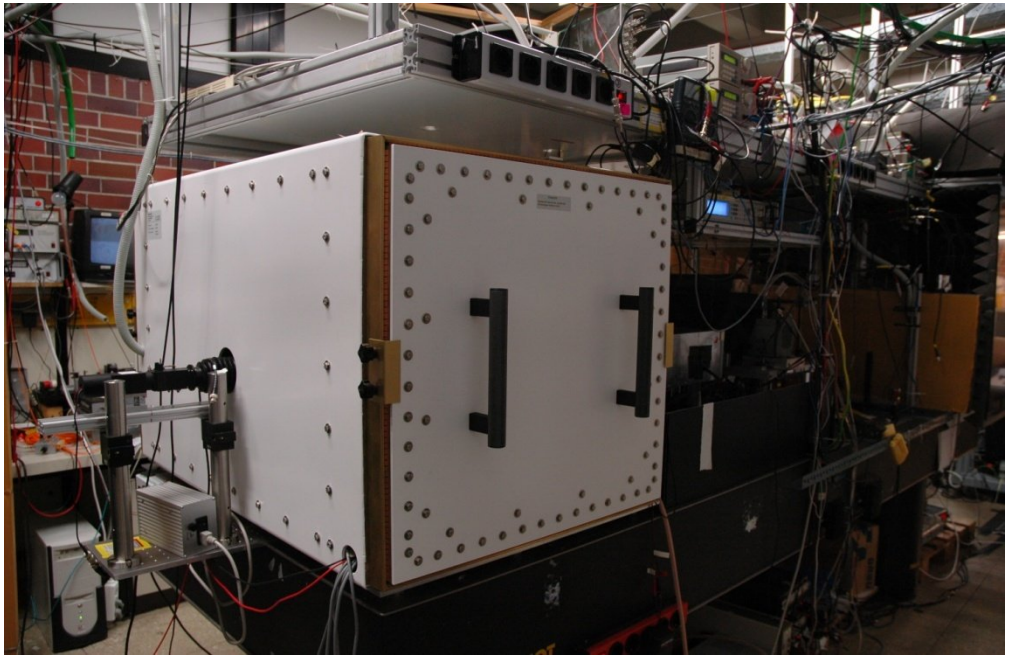


Innsbruck quantum computer, 2005

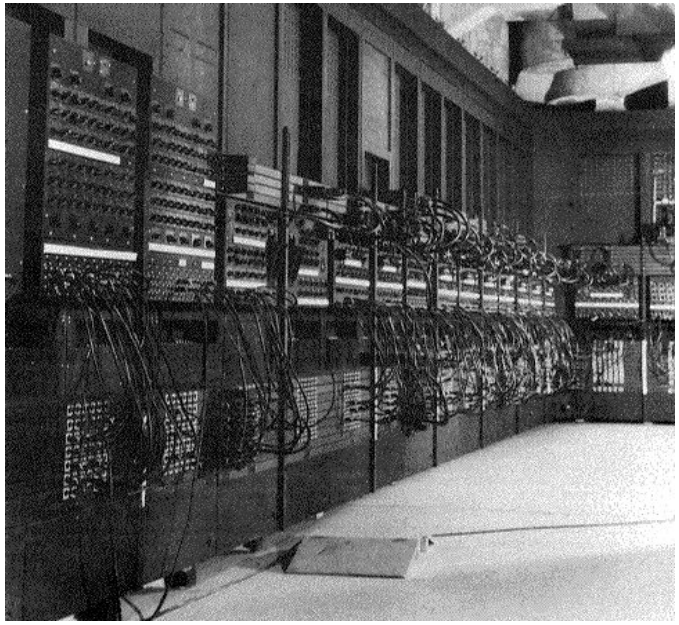


ENIAC, 1950

The hardware



Innsbruck quantum computer, 2009



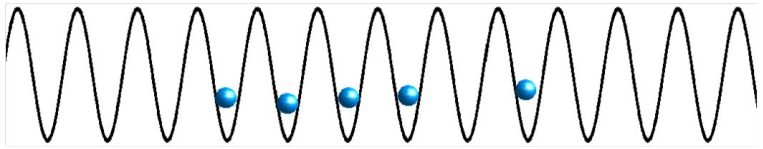
ENIAC, 1950

- Physics and information
- Ion trap quantum computing
- Teleportation
- Trends in quantum information

Trends in quantum information

Application to useful tasks

- Quantum simulations

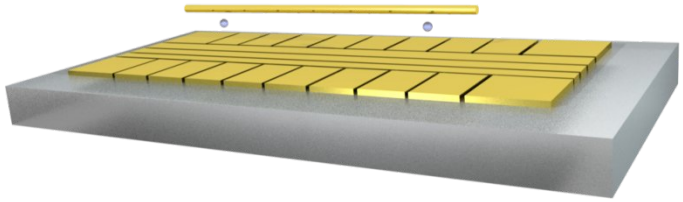


Coherent control for large systems

- Scalable quantum computing

Interconnecting quantum systems

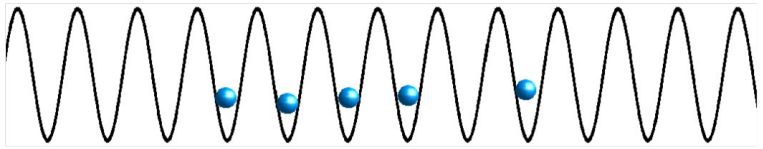
- Wiring up ion traps and other systems
- Single ions as quantum sensors



Trends in quantum information

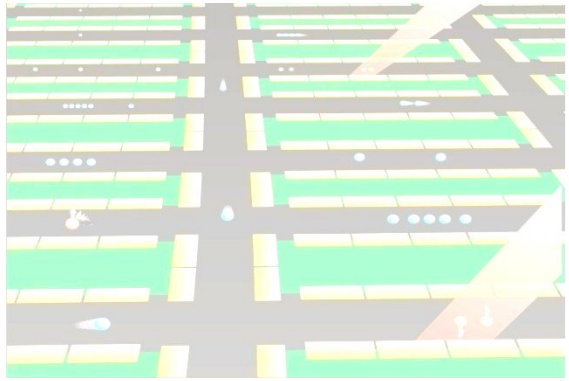
Application to useful tasks

- Quantum simulations



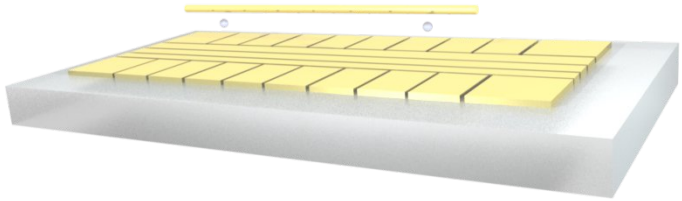
Coherent control for large systems

- Scalable quantum computing



Interconnecting quantum systems

- Wiring up ion traps and other systems
- Single ions as quantum sensors



Quantum simulation

Frenkel-Kontorova model:

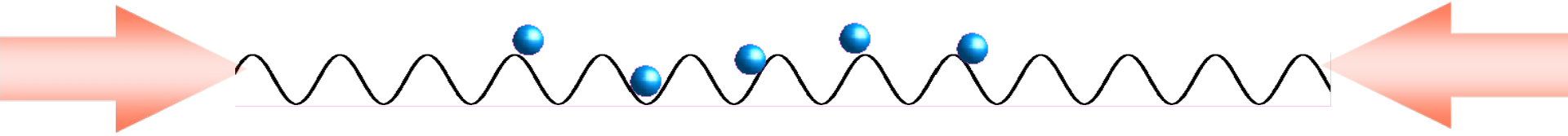
How does chain of atoms fit into a periodic potential ?

Ions move collectively

Quantum simulation

Frenkel-Kontorova model:

How does chain of atoms fit into a periodic potential ?

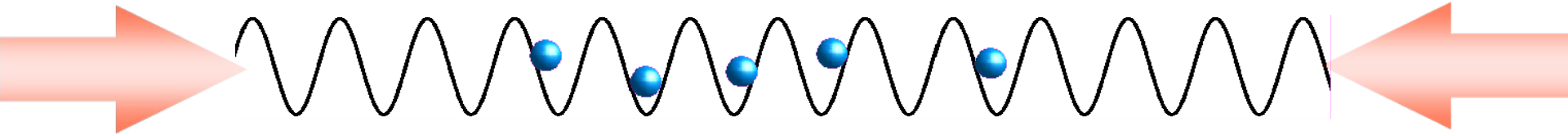


Ions move collectively

Quantum simulation

Frenkel-Kontorova model:

How does chain of atoms fit into a periodic potential ?

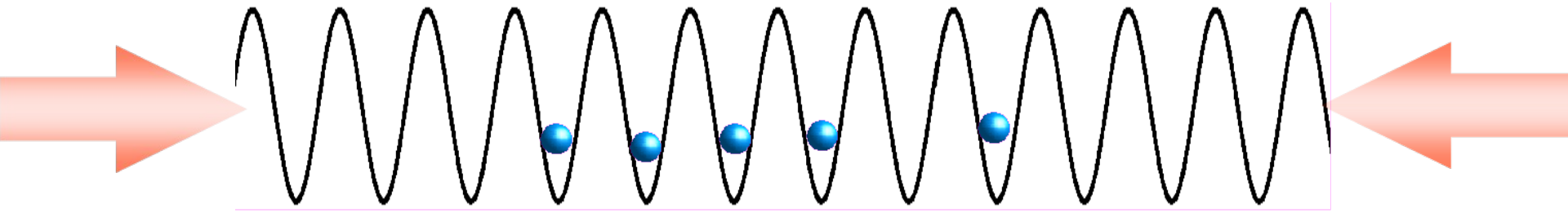


Ions move collectively

Quantum simulation

Frenkel-Kontorova model:

How does chain of atoms fit into a periodic potential ?



Ions are pinned

→ Quantum phase transition

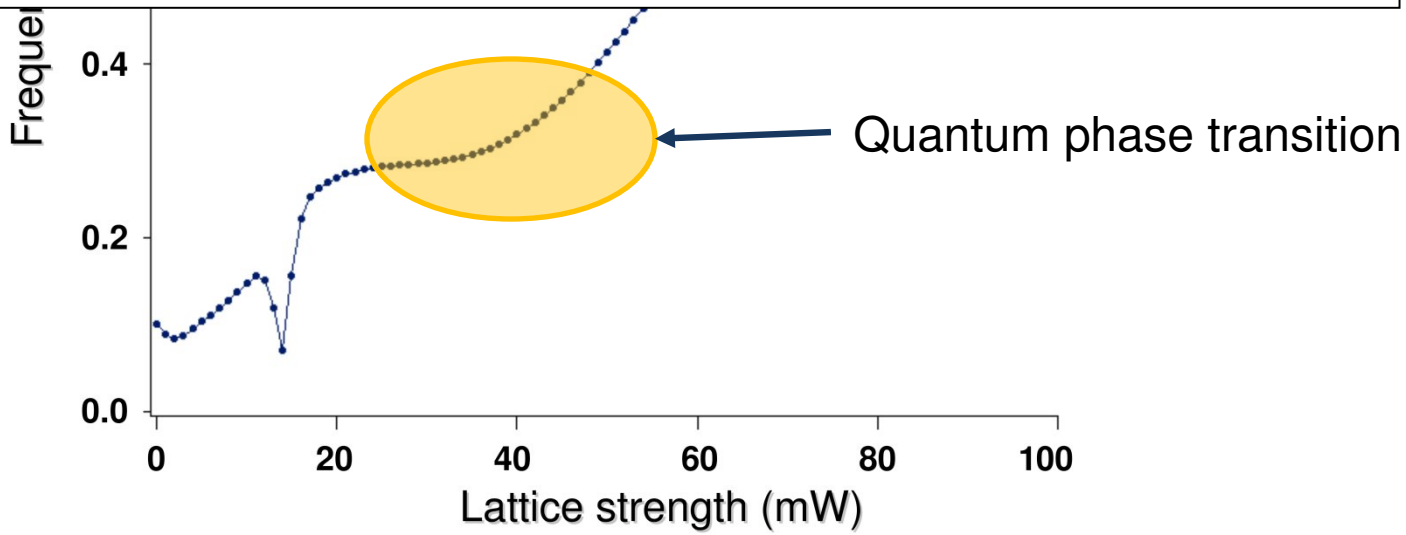
Classical analysis

Frenkel-Kontorova model describes:

- dislocations in crystals
- dry friction
- epitaxial growth
- transport properties in Josephson Junction arrays
- elasticity of DNA

Braun and Kivshar, 1st Edition, Springer, 2004:
"The Frenkel-Kontorova Model: Concepts, Methods, and Applications"

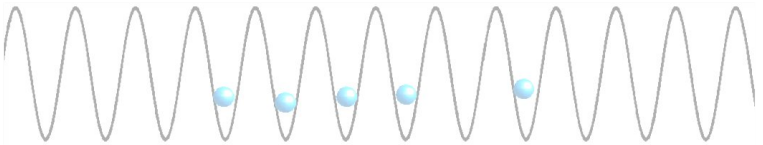
I. Garcia-Mata et al., Eur. Phys. J. D 41, **325** (2007).



Trends in quantum information

Application to useful tasks

- Quantum simulations

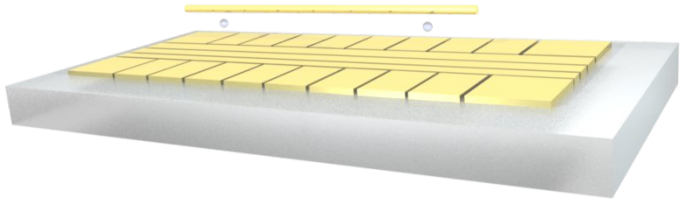


Coherent control for large systems

- Scalable quantum computing

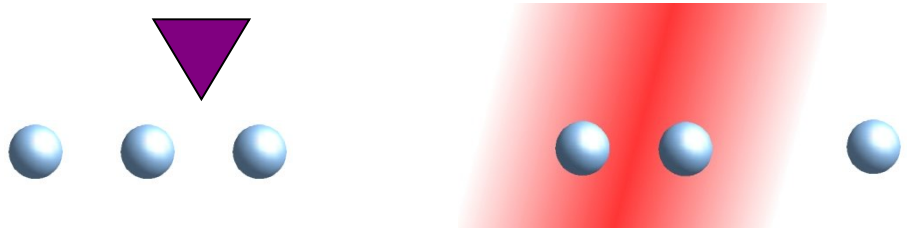
Interconnecting quantum systems

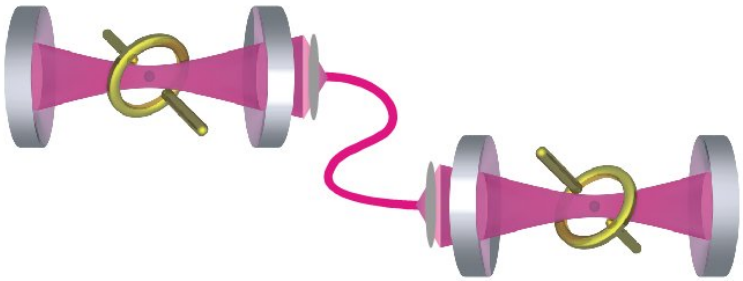
- Wiring up ion traps and other systems
- Single ions as quantum sensors



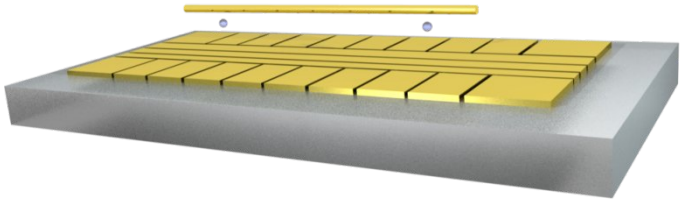
Scaling of ion trap quantum computers

Kielpinski, Monroe, Wineland





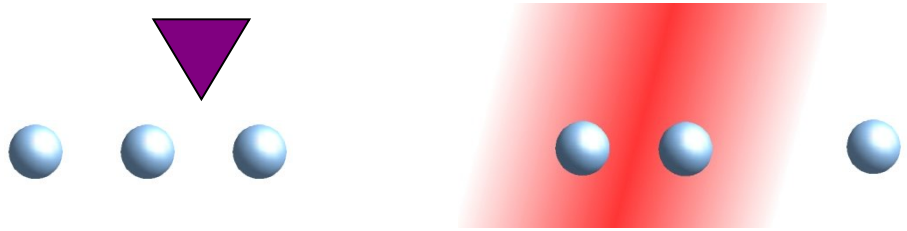
Cirac, Zoller, Kimble, Mabuchi



Zoller, Tian, Blatt

Scaling of ion trap quantum computers

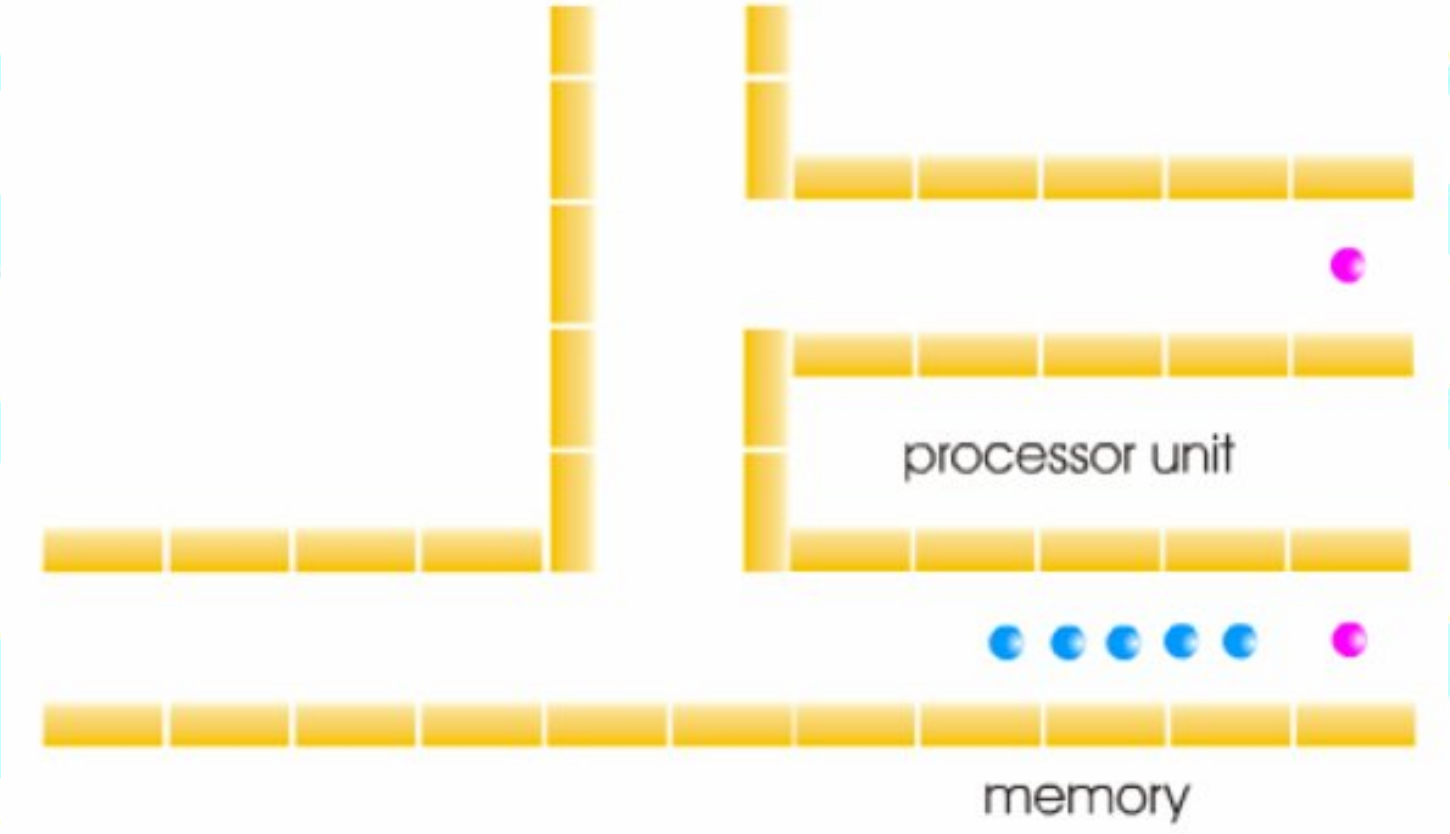
Kielpinski, Monroe, Wineland



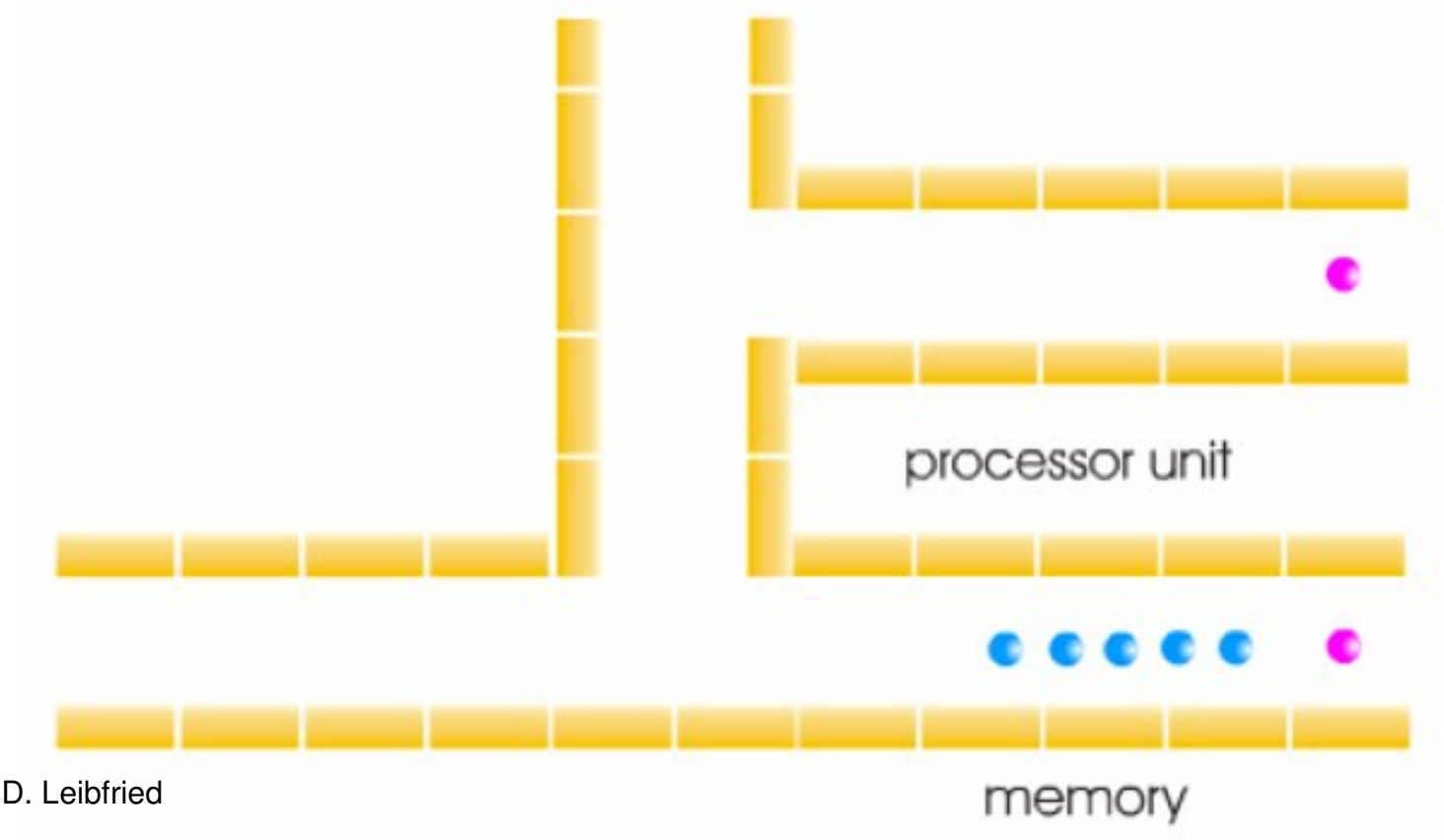
Scaling ion trap quantum computing

D. Leibfried, D. Wineland et al., NIST

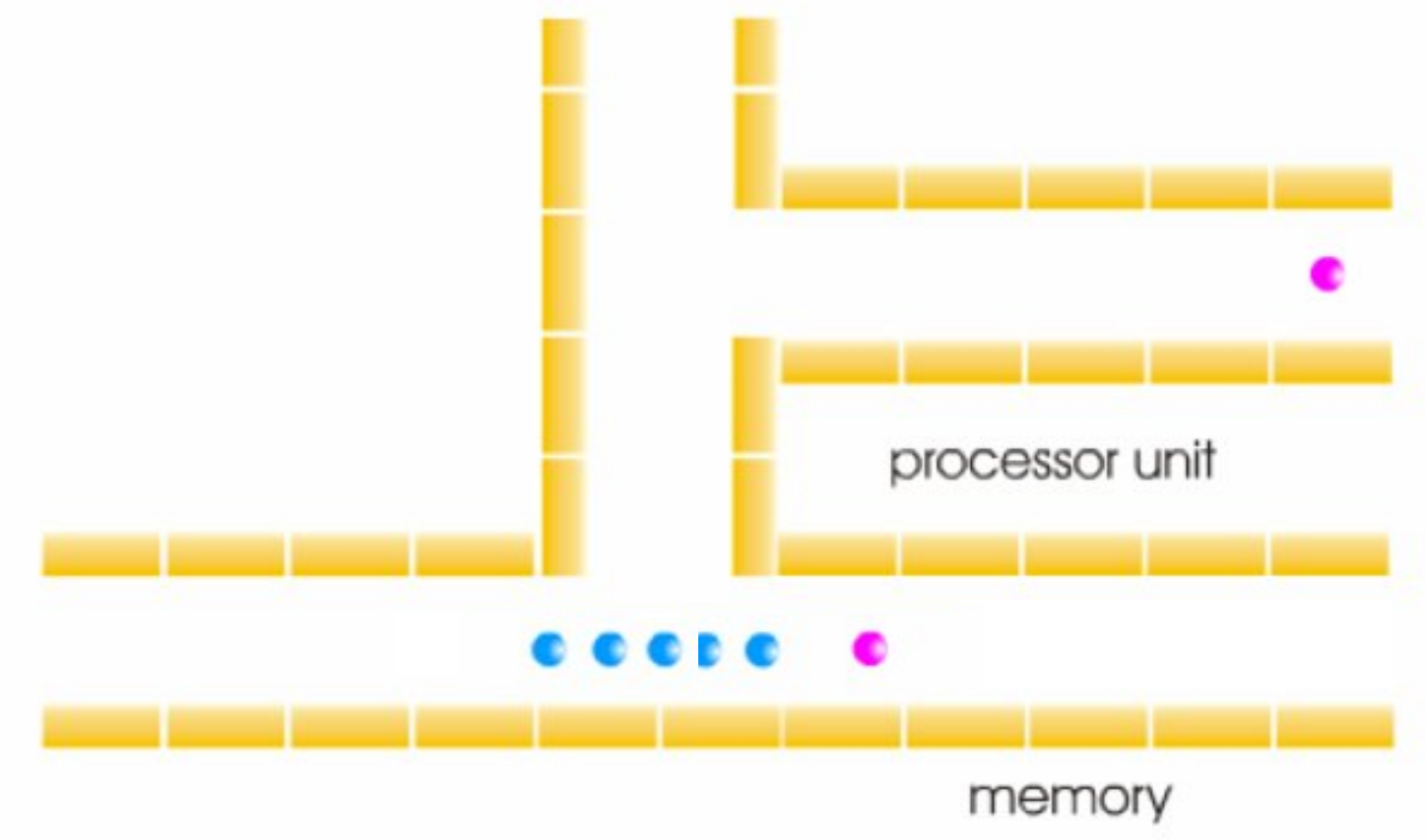
Scaling ion trap quantum computing



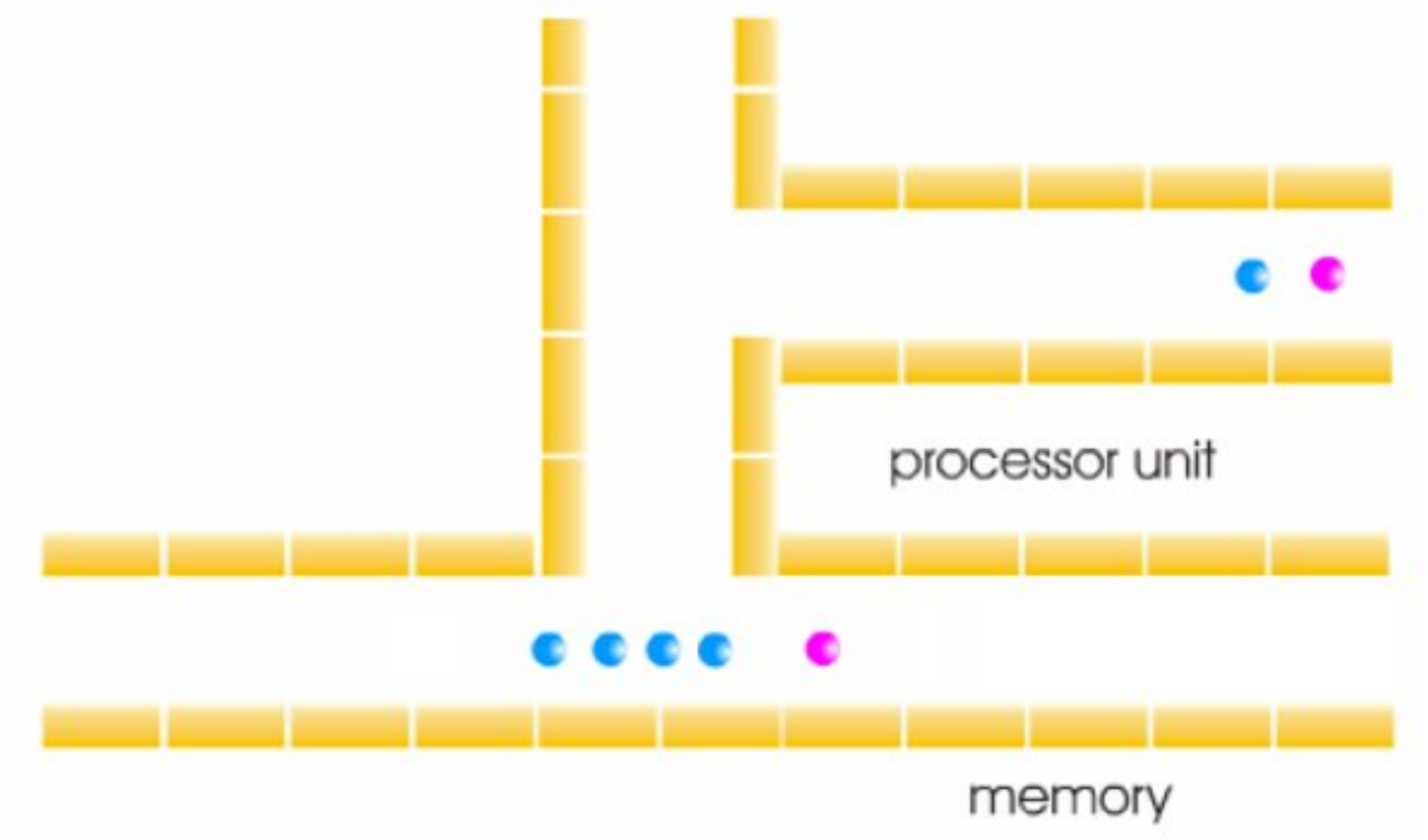
Scaling ion trap quantum computing



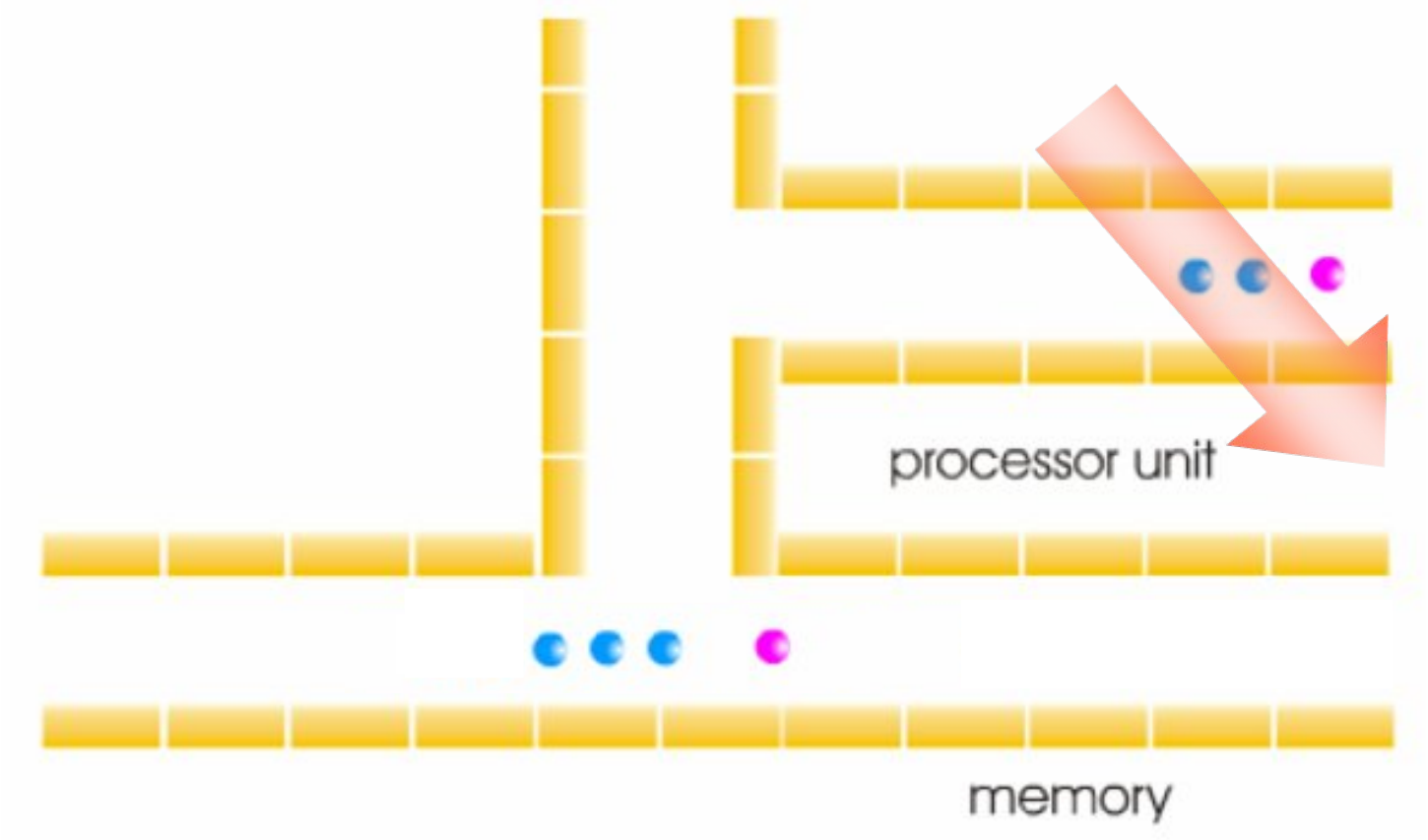
Scaling ion trap quantum computing



Scaling ion trap quantum computing

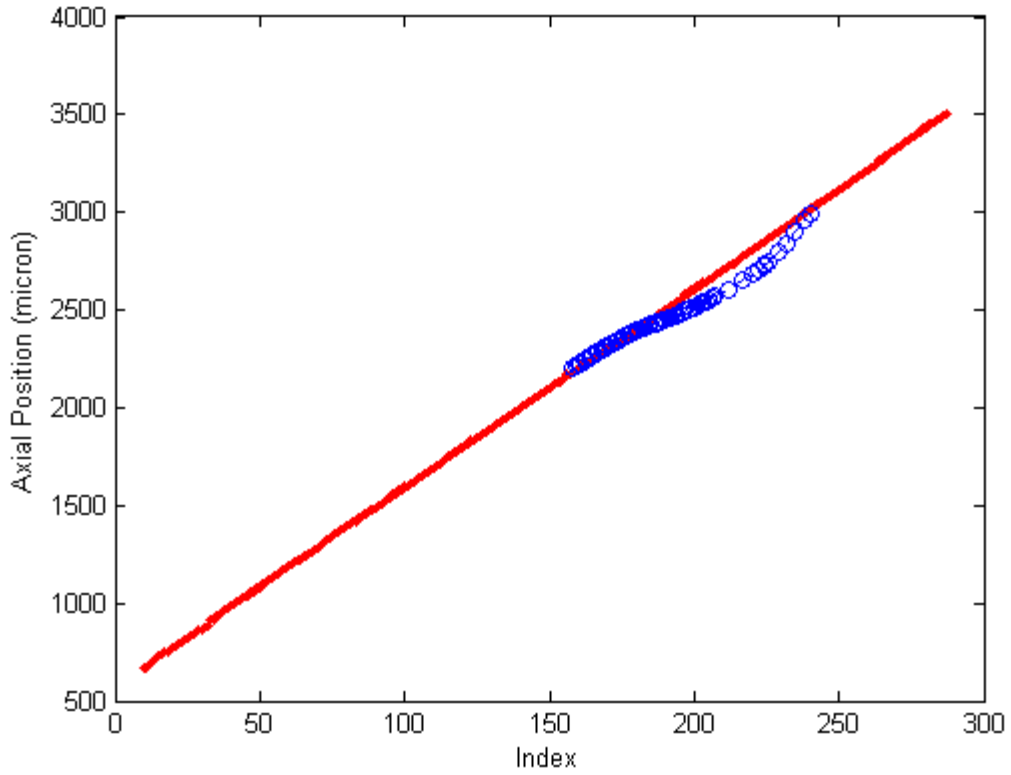


Scaling ion trap quantum computing

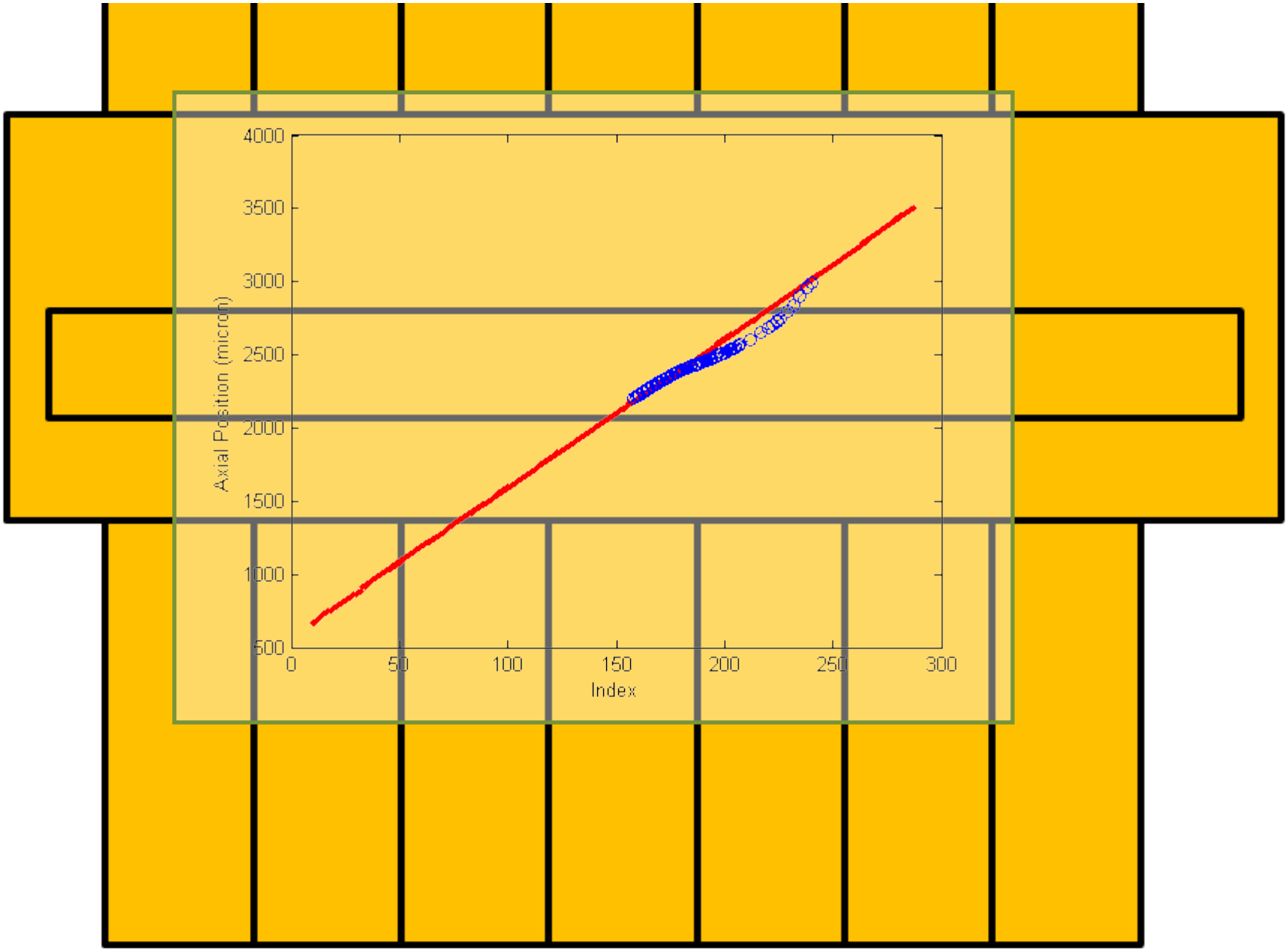


„Architecture for a large-scale ion-trap quantum computer“,
D. Kielpinski et al., Nature **417**, 709 (2002).

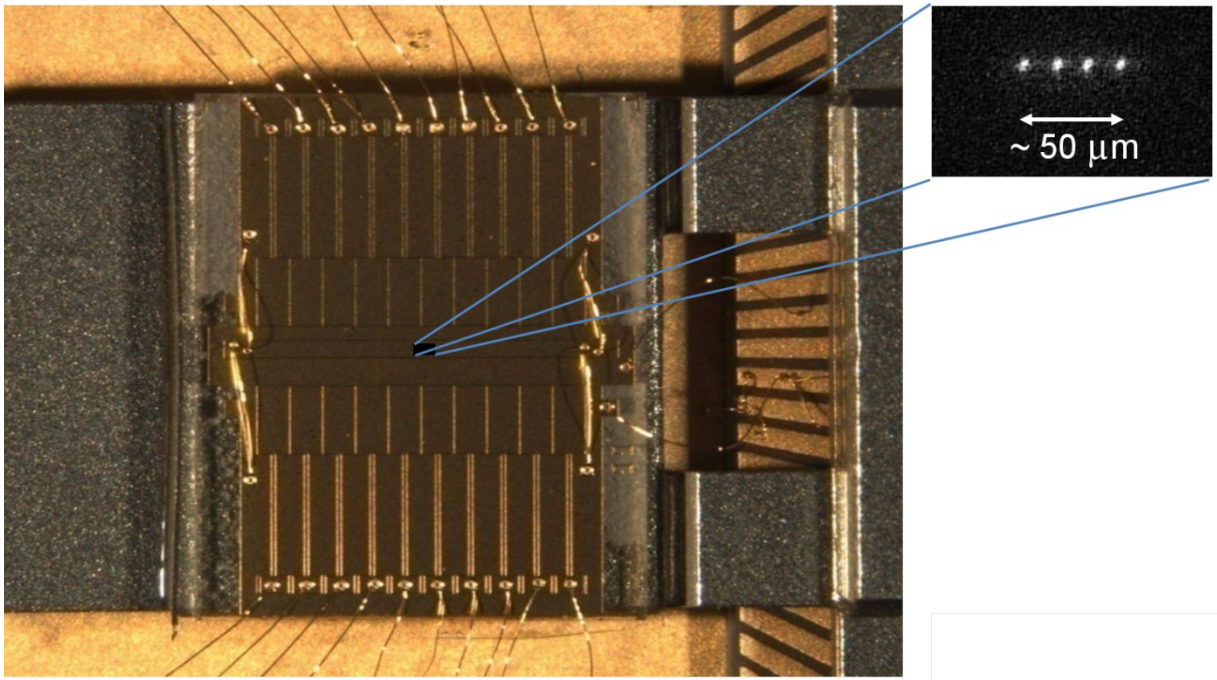
Moving an ion

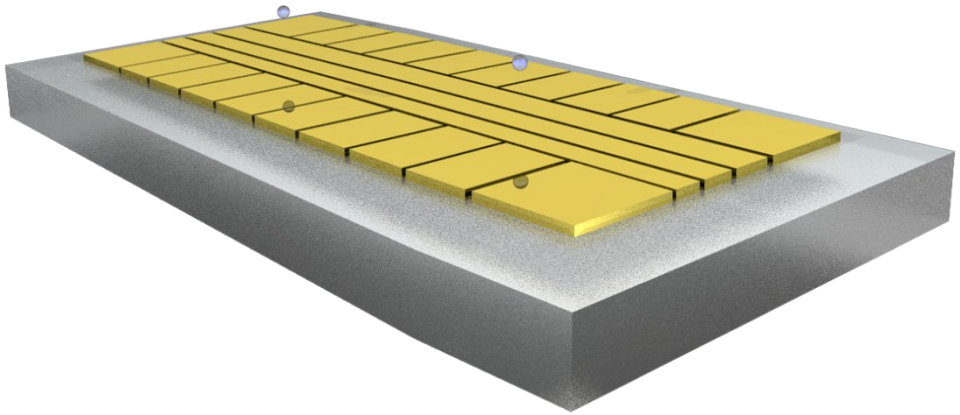


Moving an ion



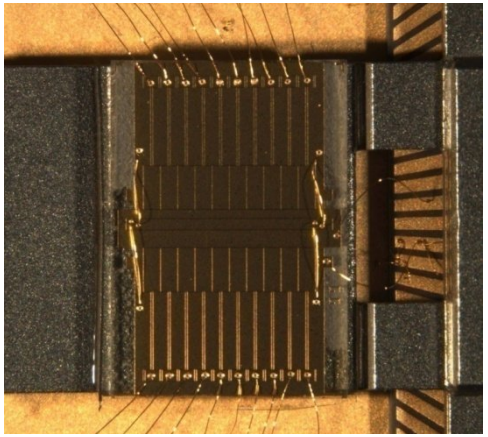
A planar microfabricated trap



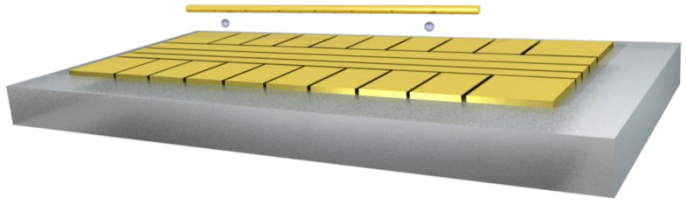


Scaling of ion trap quantum computers

Kielpinski, Monroe, Wineland



Cirac, Zoller, Kimble, Mabuchi

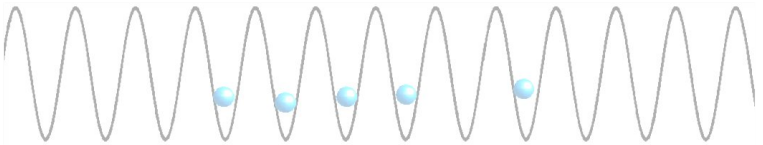


Zoller, Tian, Blatt

Trends in quantum information

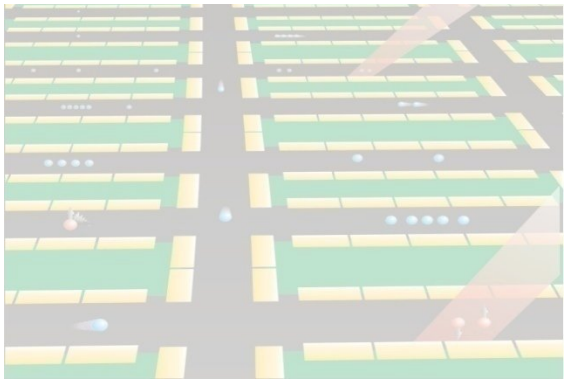
Application to useful tasks

- Quantum simulations



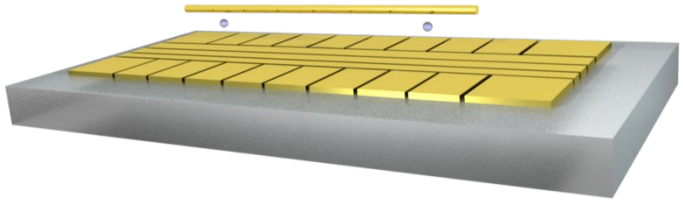
Coherent control for large systems

- Scalable quantum computing



Interconnecting quantum systems

- Wiring up ion traps and other systems
- Single ions as quantum sensors

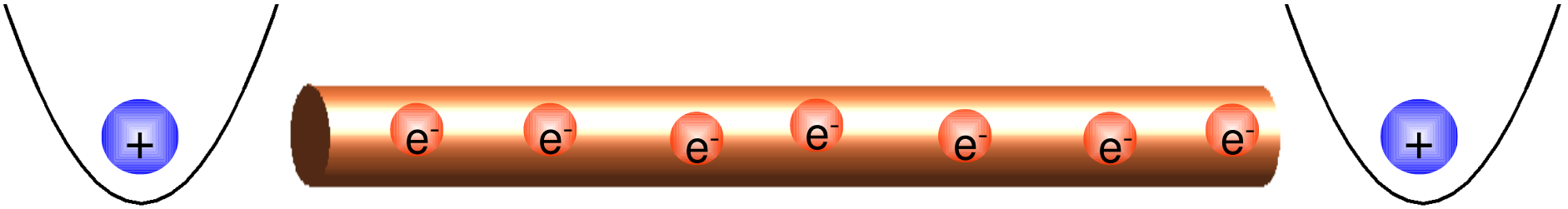


Wiring up trapped ions

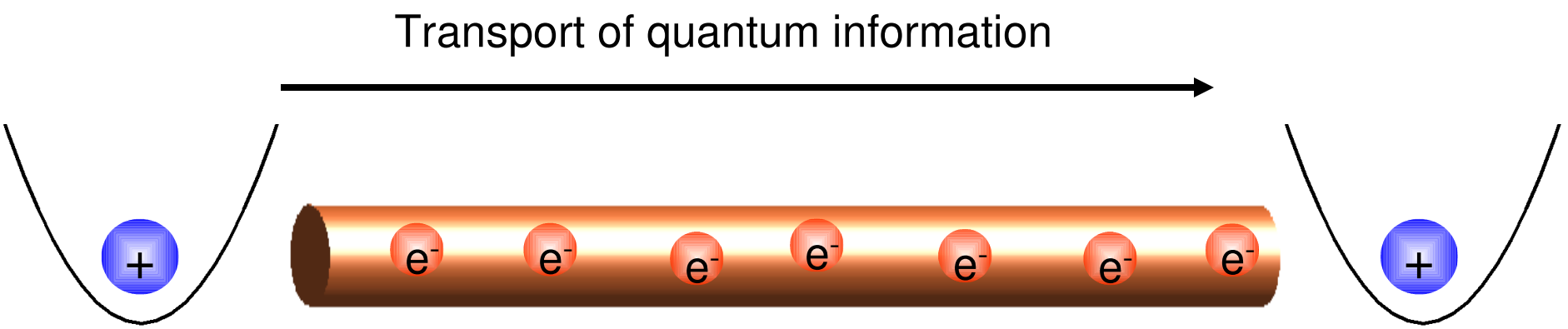
Two trapped ions …

Wiring up trapped ions

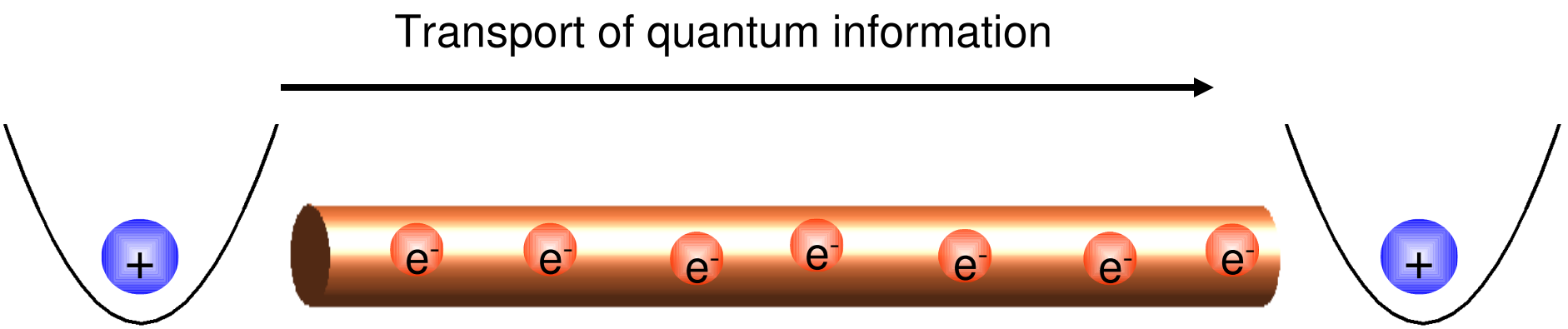
Two trapped ions + a wire



Wiring up trapped ions



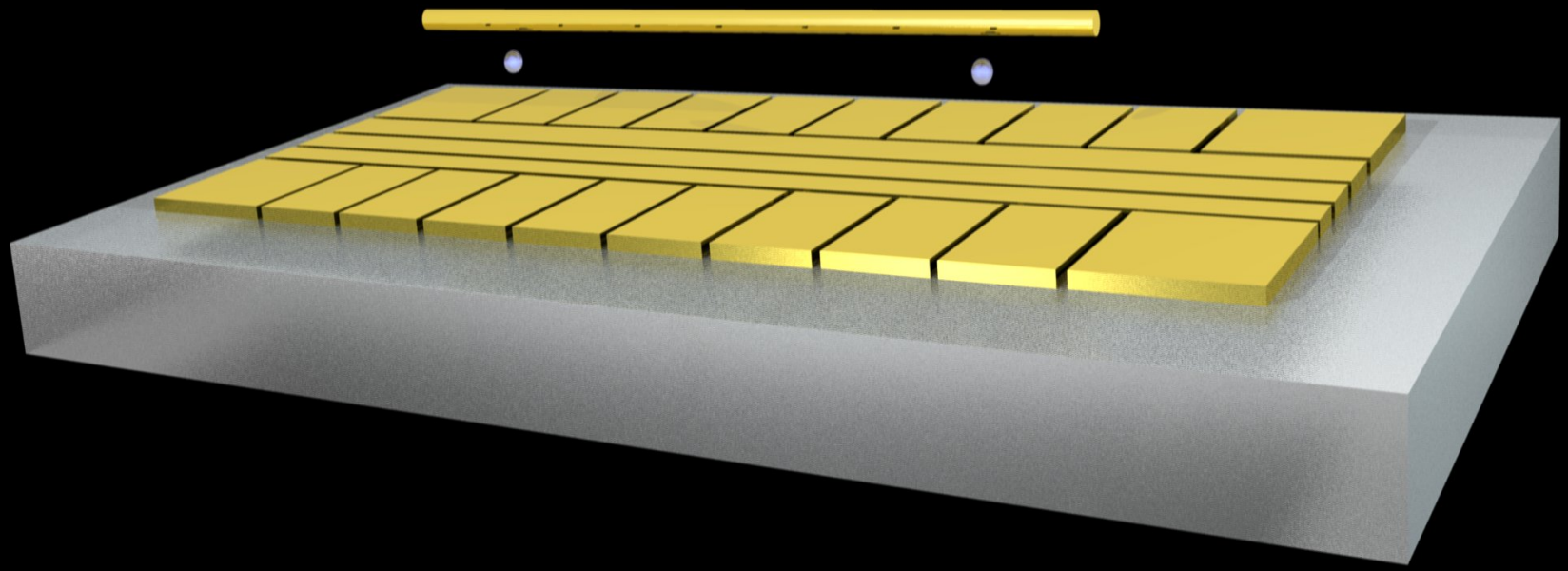
Wiring up trapped ions



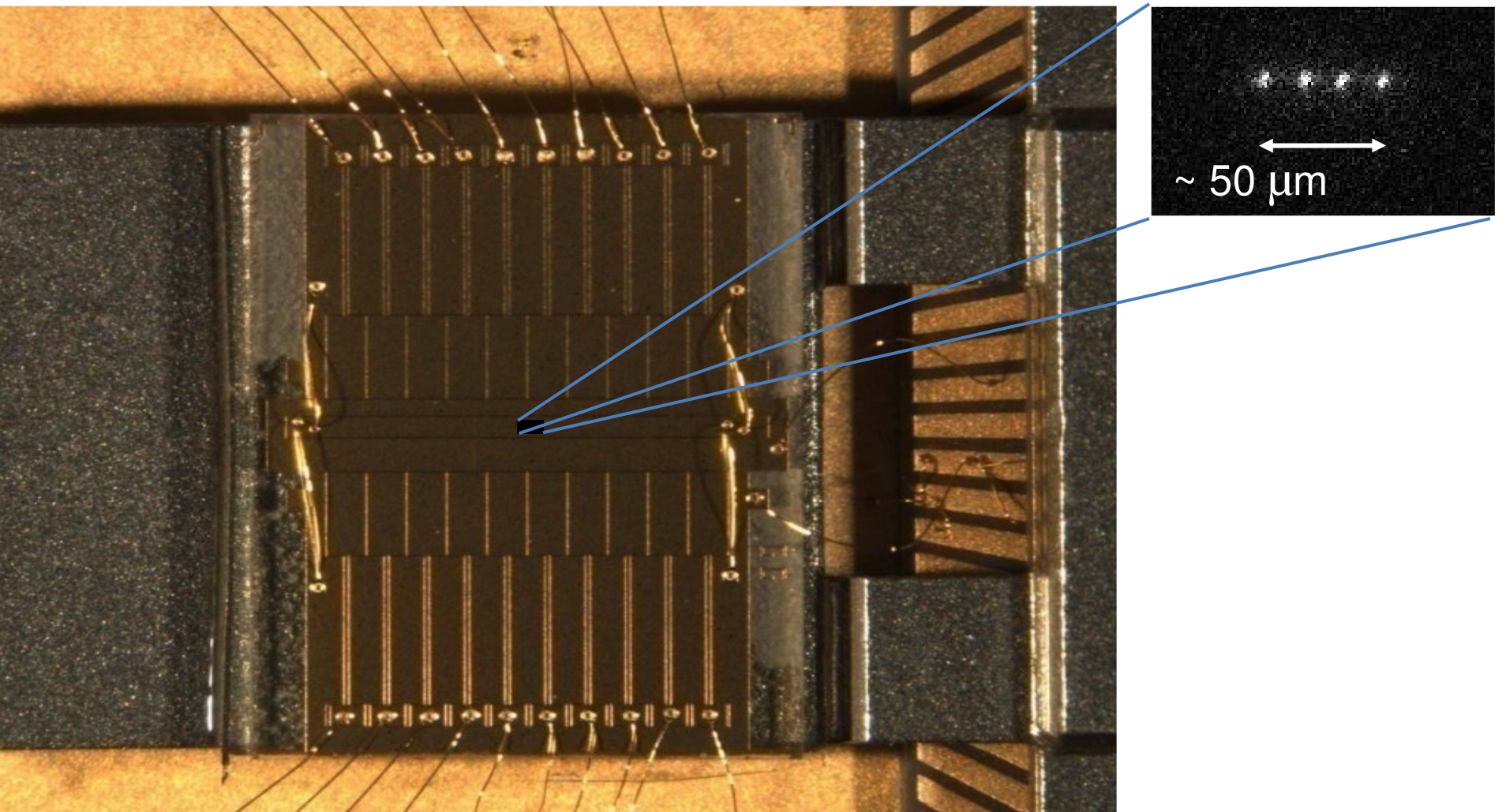
No trace of the quantum information should remain in the wire

~~super conducting~~ wire

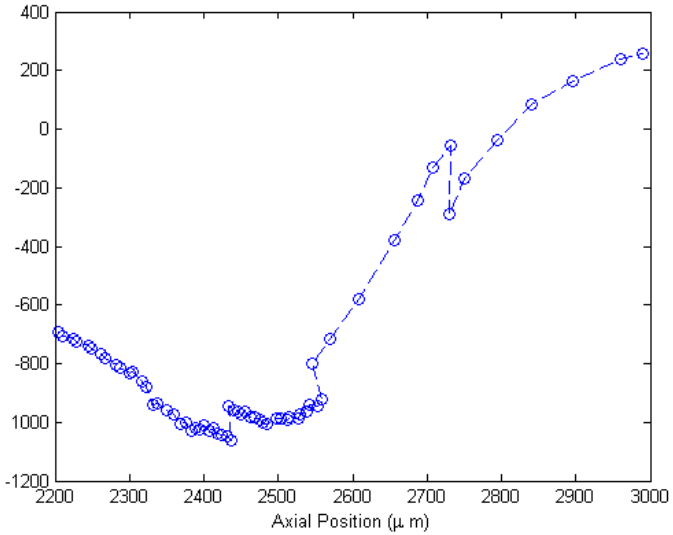
Experimental set-up

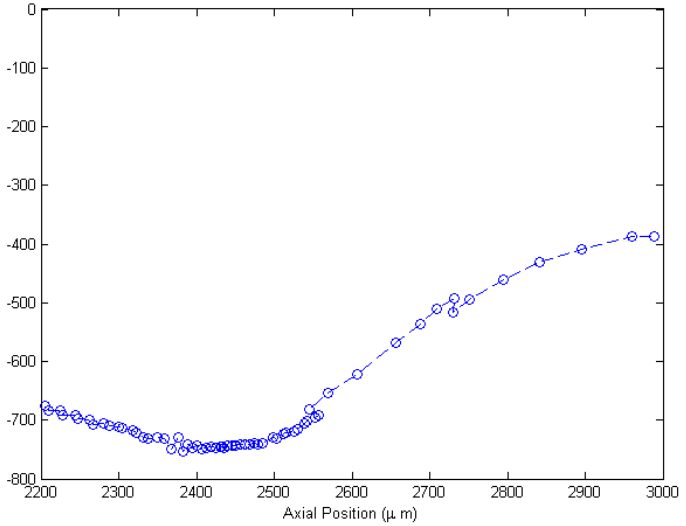


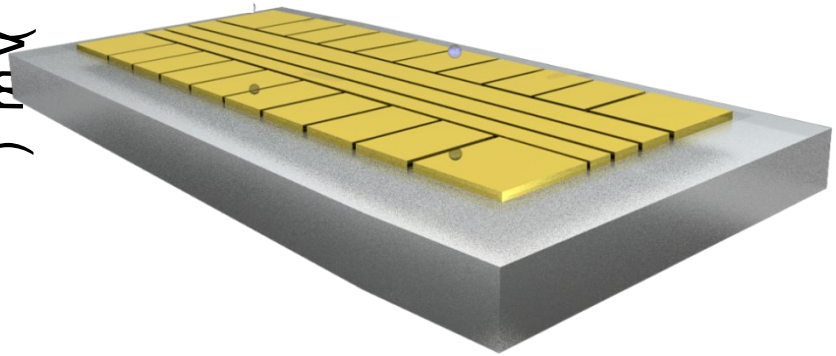
Experimental set-up

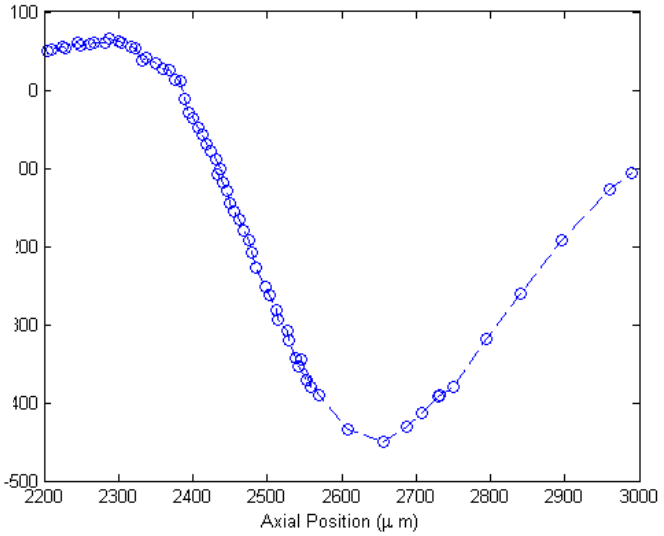


A single-ion sensor

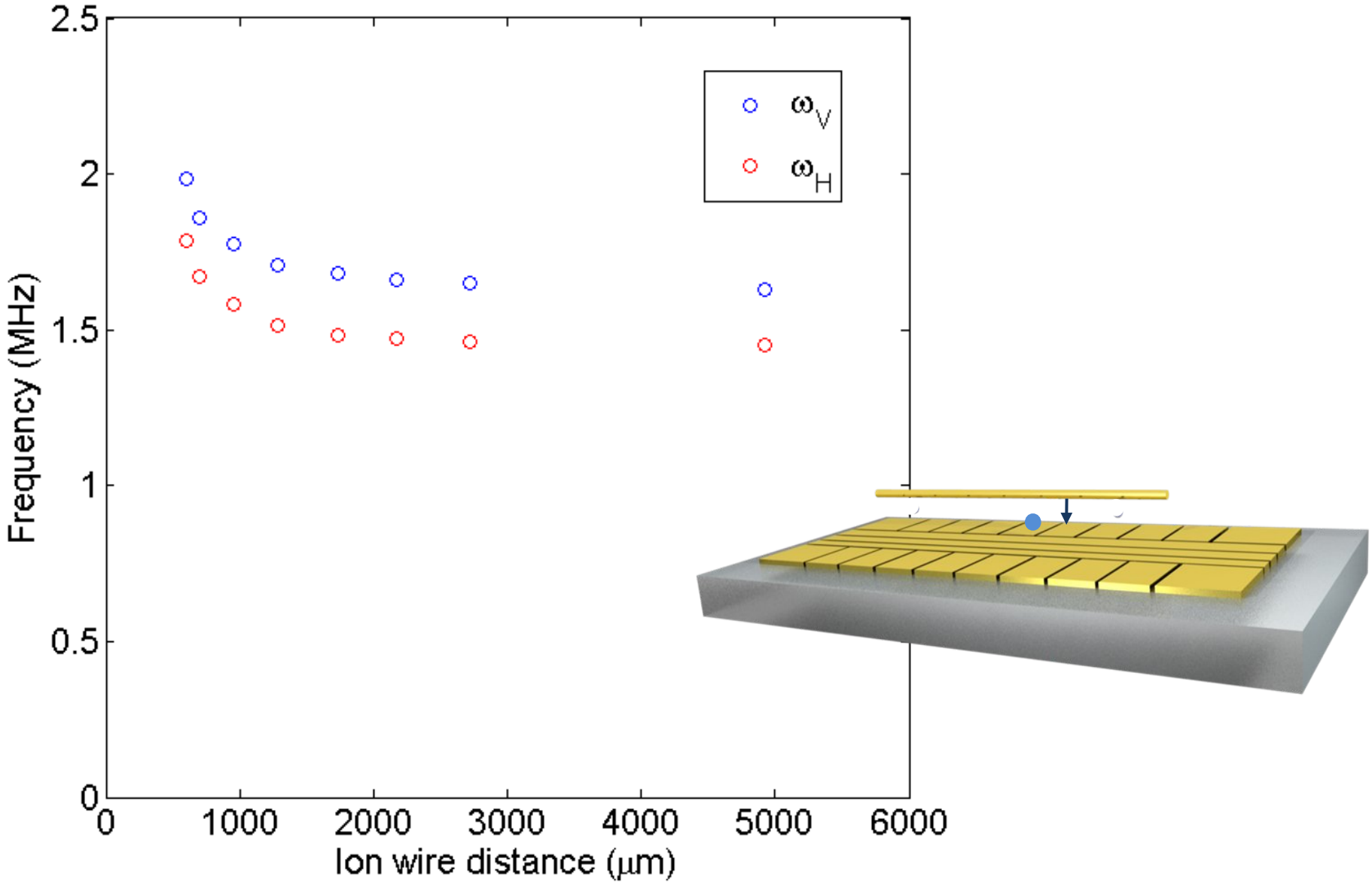




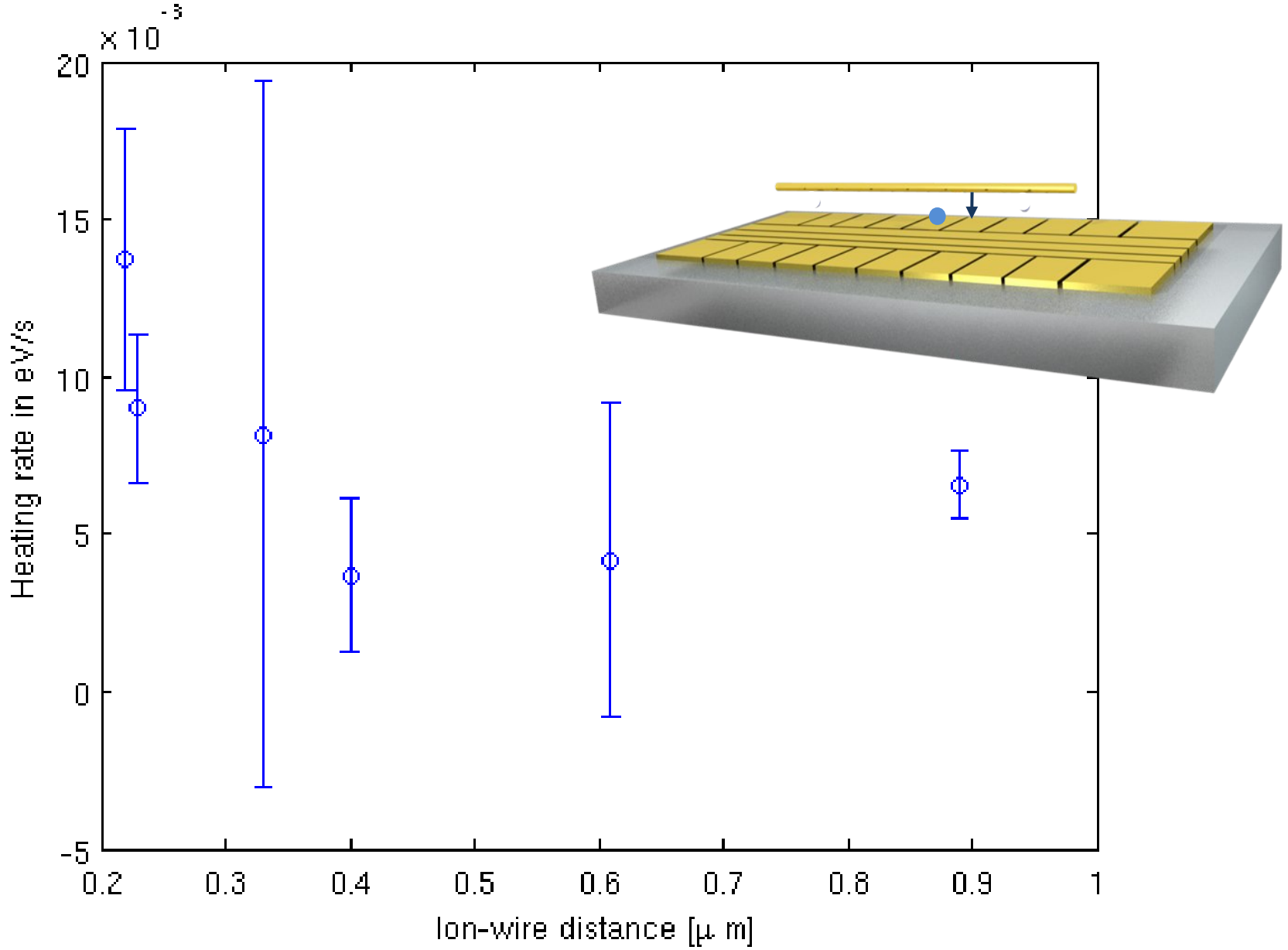




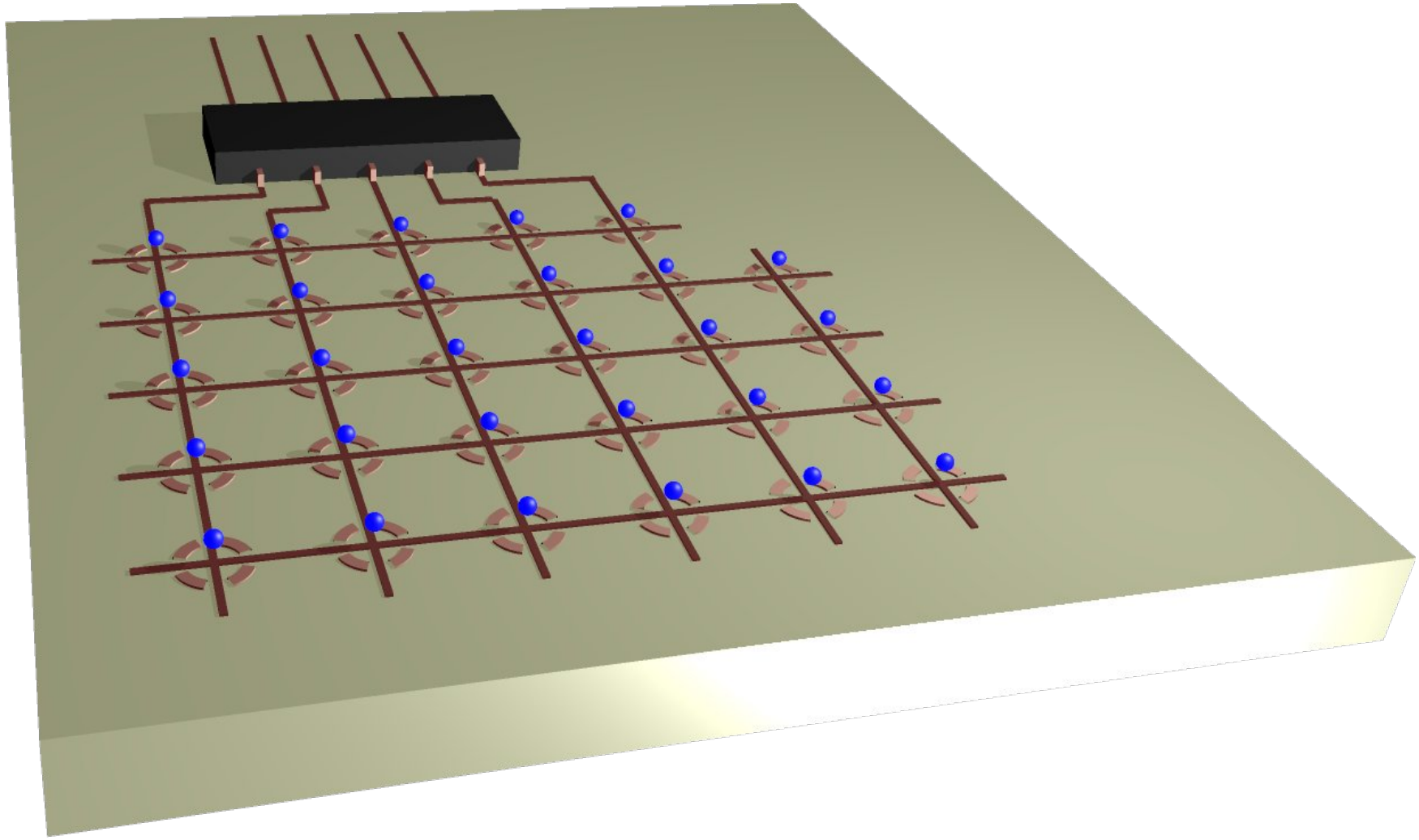
Moving the wire in



Moving the wire in

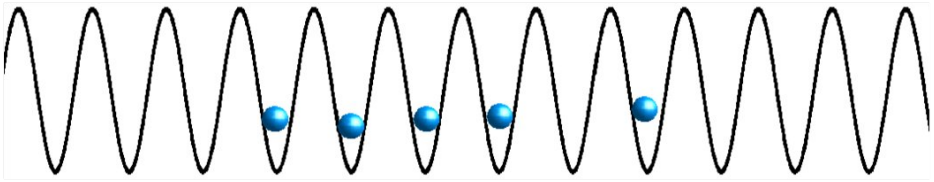


A vision

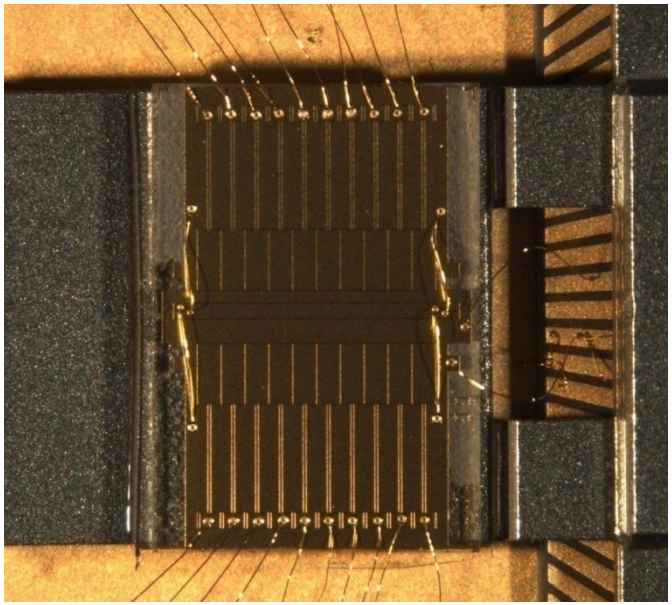


Summary

Quantum simulations



Scaling ion traps



Electrical coupling of ion-qubits