



Quantum computing with trapped ions



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- Introduction to quantum information
- Ion trap quantum computing
- Quantum teleportation
- Scaling up ion trap quantum computers
- Wiring up trapped ions



FWF
SFB



SCALA
QGATES



Industrie
Tirol



IQI
GmbH



bm:bwk





Quantum information



Strip down quantum mechanics to bare bones:

- Hilbert space
- unitary operations
- measurement

“New” paradigms:

Computer scientist: *There is nothing but information*

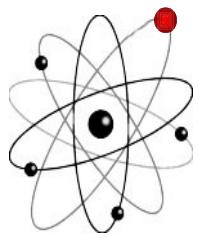
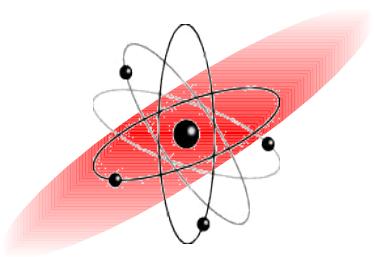
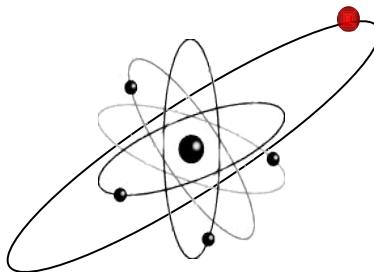
Physicist: *Ultimately information must be quantum*

001101001111010101010010110

physics \longleftrightarrow computer science



Qubits

 $|0\rangle$  $\alpha|0\rangle + \beta|1\rangle$  $|1\rangle$



Information content



$$|\Psi\rangle_{\text{reg}} = \alpha_0 |000\rangle + \alpha_1 |001\rangle + \alpha_2 |010\rangle + \alpha_3 |011\rangle + \\ \alpha_4 |100\rangle + \alpha_5 |101\rangle + \alpha_6 |110\rangle + \alpha_7 |111\rangle$$

# bits	classical	quantum mechanical
1	1	$0.9717 + 0.2364i$
2	01	$0.2044 + 0.4911i, 0.1732 + 0.3855i, 0.2040 + 0.4890i, 0.3193 + 0.3947i$
3	001	$0.2583 + 0.2704i, 0.2310 + 0.1150i, 0.2956 + 0.3118i, 0.3558 + 0.2113i, 0.1943 + 0.1377i, 0.3273 + 0.2613i, 0.0643 + 0.2033i, 0.3643 + 0.1654i$
4	1010	$0.1691 + 0.0891i, 0.1096 + 0.0828i, 0.1420 + 0.2873i, 0.0741 + 0.2419i, 0.1902 + 0.0448i, 0.2495 + 0.0039i, 0.1738 + 0.2933i, 0.2102 + 0.0653i, 0.0686 + 0.0980i$ $0.1246 + 0.2170i, 0.2570 + 0.0933i, 0.2234 + 0.1540i, 0.1513 + 0.0213i, 0.1863 + 0.3243i, 0.2606 + 0.1912i, 0.0194 + 0.1390i$
5	10001	$0.1060 + 0.1416i, 0.0103 + 0.0118i, 0.0064 + 0.0976i, 0.0734 + 0.0716i, 0.0030 + 0.2054i, 0.0902 + 0.0035i, 0.1605 + 0.1804i, 0.0218 + 0.2280i, 0.0083 + 0.2326i, 0.1438 + 0.1853i, 0.1429 + 0.1030i, 0.0037 + 0.1171i, 0.0038 + 0.0503i, 0.0446 + 0.1512i, 0.1379 + 0.0752i, 0.0135 + 0.2255i, 0.0863 + 0.1707i, 0.1483 + 0.0968i, 0.1686 + 0.1749i, 0.1627 + 0.0629i, 0.0197 + 0.1033i, 0.1067 + 0.2192i, 0.1038 + 0.1605i, 0.0830 + 0.0499i, 0.0361 + 0.1971i, 0.1587 + 0.1477i, 0.1642 + 0.0314i, 0.1709 + 0.0487i, 0.1124 + 0.1426i, 0.1303 + 0.1480i, 0.0284 + 0.0870i, 0.1059 + 0.1351i$
6	110101	$0.0595 + 0.1064i, 0.0295 + 0.1327i, 0.0929 + 0.0406i, 0.1090 + 0.0379i, 0.0559 + 0.1286i, 0.0015 + 0.0345i, 0.0624 + 0.1196i, 0.1120 + 0.1350i, 0.1180 + 0.0345i, 0.1367 + 0.0356i, 0.1255 + 0.0074i, 0.0547 + 0.0116i, 0.0923 + 0.0952i, 0.1087 + 0.0284i, 0.0288 + 0.1254i, 0.1345 + 0.0258i, 0.0846 + 0.0254i, 0.0939 + 0.1478i, 0.0348 + 0.0654i, 0.0816 + 0.0505i, 0.1384 + 0.0467i, 0.0498 + 0.0543i, 0.0974 + 0.0584i, 0.0582 + 0.0879i, 0.0932 + 0.0178i, 0.1039 + 0.0057i, 0.0590 + 0.0682i, 0.0615 + 0.1293i, 0.0974 + 0.1388i, 0.1245 + 0.0393i, 0.0537 + 0.0238i, 0.0632 + 0.1297i, 0.0884 + 0.0354i, 0.0841 + 0.0960i, 0.1065 + 0.1437i, 0.0760 + 0.0988i, 0.1154 + 0.1293i, 0.0727 + 0.0015i, 0.0276 + 0.0204i, 0.1041 + 0.1217i, 0.1460 + 0.0639i, 0.1199 + 0.1323i, 0.1046 + 0.1092i, 0.0721 + 0.1021i, 0.0170 + 0.0514i, 0.0988 + 0.0247i, 0.0543 + 0.0231i, 0.0208 + 0.0284i, 0.0842 + 0.0628i, 0.1223 + 0.1272i$ $0.1002 + 0.0729i, 0.1485 + 0.1213i, 0.1429 + 0.0685i, 0.0087 + 0.0680i, 0.0535 + 0.0670i, 0.0815 + 0.0613i, 0.0389 + 0.1340i, 0.0888 + 0.0008i, 0.0073 + 0.0442i, 0.0849 + 0.0073i, 0.1042 + 0.1030i, 0.1430 + 0.0966i, 0.1115 + 0.1461i, 0.1100 + 0.0821i$
7	1001010	$0.0880 + 0.0466i, 0.1054 + 0.0684i, 0.0239 + 0.0866i, 0.0759 + 0.0090i, 0.0563 + 0.1020i, 0.1006 + 0.0988i, 0.0769 + 0.0649i, 0.0246 + 0.0273i, 0.0485 + 0.0942i, 0.0186 + 0.0554i, 0.1045 + 0.0790i, 0.0384 + 0.0455i, 0.0053 + 0.1037i, 0.0815 + 0.0078i, 0.0965 + 0.0597i, 0.0309 + 0.0315i, 0.0221 + 0.0925i, 0.1006 + 0.0362i, 0.0141 + 0.0734i, 0.1015 + 0.0058i, 0.0757 + 0.0385i, 0.0537 + 0.0226i, 0.0468i + 0.0491 + 0.0607i, 0.0087 + 0.0665i, 0.0918 + 0.0122i, 0.0606 + 0.0969i, 0.0344 + 0.0814i, 0.0404 + 0.0853i, 0.0936 + 0.0879i, 0.0401 + 0.0723i, 0.0079 + 0.0217i, 0.0216 + 0.0294i, 0.0053 + 0.0675i, 0.0611 + 0.0579i, 0.0131 + 0.0664i, 0.0563 + 0.0966i, 0.0126 + 0.0293i, 0.0830 + 0.0441i, 0.0404 + 0.0511i, 0.0888 + 0.0980i, 0.0050 + 0.0643i, 0.0645 + 0.0355i, 0.1024 + 0.0516i, 0.0311 + 0.0644i, 0.0959 + 0.0174i, 0.0110 + 0.0894i, 0.0070 + 0.1031i, 0.0253 + 0.0642i, 0.1006 + 0.0031i, 0.0068 + 0.0876i, 0.0285 + 0.0658i, 0.1078 + 0.0756i, 0.0229 + 0.0999i, 0.0337 + 0.0458i, 0.0313 + 0.0405i, 0.0725 + 0.0179i, 0.1033 + 0.0889i, 0.0827 + 0.0904i, 0.0718 + 0.0487i, 0.0141 + 0.1032i, 0.0103 + 0.1599i, 0.0016 + 0.0938i, 0.0311 + 0.0830i, 0.0881 + 0.0479i, 0.1063 + 0.0669i, 0.0019 + 0.1026i, 0.0884 + 0.0690i, 0.0670 + 0.0267i, 0.0604 + 0.0380i, 0.0263 + 0.0203i, 0.0886 + 0.0529i, 0.0284 + 0.0441i, 0.0813 + 0.0500i, 0.0711 + 0.0659i, 0.0231 + 0.0077i, 0.0549 + 0.0339i, 0.0652 + 0.0656i, 0.0711 + 0.0189i, 0.0198 + 0.0670i, 0.0686 + 0.0425i, 0.0184 + 0.0633i, 0.0582 + 0.0546i, 0.0572 + 0.0501i, 0.0740 + 0.0161i, 0.0946 + 0.0369i, 0.0014 + 0.0433i, 0.0335 + 0.0332i, 0.0840 + 0.0444i, 0.0331 + 0.0308i, 0.0999 + 0.0425i, 0.0732 + 0.0542i, 0.0080 + 0.0779i, 0.0076 + 0.0330i, 0.0013 + 0.0121i, 0.0245 + 0.0478i, 0.0557 + 0.0503i, 0.0494 + 0.0166i, 0.0758 + 0.0716i, 0.0628 + 0.0781i, 0.0549 + 0.0304i, 0.0080 + 0.0282i, 0.0208 + 0.0764i, 0.0409 + 0.0845i, 0.0893 + 0.0425i, 0.0562 + 0.0122 + 0.0774i, 0.0876 + 0.0614i, 0.0979 + 0.0497i, 0.0169 + 0.0480i, 0.0132 + 0.0995i, 0.0582 + 0.0478i, 0.0778 + 0.0395i, 0.0703 + 0.0326i, 0.0813 + 0.0919i, 0.0953 + 0.1024i, 0.0293 + 0.0602i, 0.0452 + 0.0151i, 0.0230 + 0.0643i$
8	10101011	$0.0199 + 0.0027i, 0.0033 + 0.0063i, 0.0005 + 0.0656i, 0.0443 + 0.0262i, 0.0573 + 0.0359i, 0.0622 + 0.0704i, 0.0491 + 0.0176i, 0.0194 + 0.0664i, 0.0111 + 0.0506i, 0.0502 + 0.0687i, 0.0729 + 0.0376i, 0.0629 + 0.0765i, 0.0717 + 0.0288i, 0.0239 + 0.0410i, 0.0207 + 0.0140i, 0.0413 + 0.0387i, 0.0126 + 0.0325i, 0.0163 + 0.0509i, 0.0167 + 0.0519i, 0.0502 + 0.0738i, 0.0041 + 0.0177i, 0.0086i + 0.0514i, 0.0436i + 0.0240 + 0.0747i, 0.0236 + 0.0018i, 0.0555 + 0.0672i, 0.0736 + 0.0201i, 0.0101 + 0.0400i, 0.0053 + 0.0148i, 0.0097 + 0.0552i, 0.0128 + 0.0193i, 0.0702 + 0.0720i, 0.0105 + 0.0106i, 0.0476 + 0.0402i, 0.0207 + 0.0690i, 0.0170 + 0.0726i, 0.0549 + 0.0258i, 0.0423 + 0.0373i, 0.0726 + 0.0363i, 0.0254 + 0.0151i, 0.0543 + 0.0105i, 0.0727 + 0.0410i, 0.0448 + 0.0559i, 0.0679 + 0.0307i, 0.0578 + 0.0276i, 0.0293 + 0.0220i, 0.0559 + 0.0670i, 0.0125 + 0.0483i, 0.0737 + 0.0186i, 0.0151 + 0.0754i, 0.0598 + 0.0494i, 0.0473 + 0.0177i, 0.0125 + 0.0525i, 0.0024 + 0.0513i, 0.0222 + 0.0104i, 0.0748 + 0.0017i, 0.0733 + 0.0202i, 0.0176 + 0.0090i, 0.0739 + 0.0053i, 0.0524 + 0.0657i, 0.0042 + 0.0139i, 0.0462 + 0.0025i, 0.0303 + 0.0566i, 0.0166 + 0.0414i, 0.0141 + 0.0213i, 0.0059 + 0.0284i, 0.0006 + 0.0010i, 0.0608 + 0.0685i, 0.0014 + 0.0667i, 0.0677 + 0.0196i, 0.0272 + 0.0439i, 0.0557 + 0.0123i, 0.0746 + 0.0458i, 0.0120 + 0.0255i, 0.0126 + 0.0508i, 0.0242 + 0.0666i, 0.0023 + 0.0437i, 0.0276 + 0.0756i, 0.0021 + 0.0610i, 0.0612 + 0.0118i, 0.0770 + 0.0642i, 0.0085 + 0.0148i, 0.0480 + 0.0493i, 0.0102 + 0.0516i, 0.0240 + 0.0293i, 0.0172 + 0.0074i, 0.0340i + 0.0306 + 0.0572i, 0.0186 + 0.0136i, 0.0715 + 0.0020i, 0.0301 + 0.0369i, 0.0061 + 0.0396i, 0.0072 + 0.0164i, 0.0017 + 0.0080i, 0.0123 + 0.0121i, 0.0061 + 0.0314i, 0.0678 + 0.0314i, 0.0144 + 0.0041i, 0.0764 + 0.0726i, 0.0459 + 0.0116i, 0.0672 + 0.0296i, 0.0370 + 0.0240i, 0.0382 + 0.0130i, 0.0691 + 0.0611i, 0.0047 + 0.0249i, 0.0202 + 0.0566i, 0.0123 + 0.0141i, 0.0371 + 0.0317i, 0.0707 + 0.0308i, 0.0059 + 0.0390i, 0.0010 + 0.0130i, 0.0285 + 0.0404i, 0.0538 + 0.0494i, 0.0685 + 0.0015i, 0.0458 + 0.0645i, 0.0293 + 0.0121i, 0.0619 + 0.0244i, 0.0180 + 0.0358i, 0.0006 + 0.0064i, 0.0633 + 0.0051 + 0.0149i, 0.0066 + 0.0343i, 0.0593 + 0.0010j, 0.0747$



Information content



# bits	classical	quantum mechanical
1	1	$0.9717 + 0.2364i$
2	10	$0.2044 + 0.4911i, 0.1732 + 0.3855i, 0.2040 + 0.4890i, 0.3193 + 0.3947i$
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6	110101	$0.0595 + 0.1064i, 0.0295 + 0.1327i, 0.0929 + 0.0406i, 0.1090 + 0.0379i, 0.0559 + 0.1286i, 0.0015 + 0.0345i, 0.0624 + 0.1196i, 0.1120 + 0.1350i, 0.1180 + 0.0345i, 0.1367 + 0.0356i, 0.1255 + 0.0074i, 0.0547 + 0.0116i, 0.0923 + 0.0592i, 0.1087 + 0.0284i, 0.0288 + 0.1254i, 0.1345 + 0.0258i, 0.0846 + 0.0254i, 0.0939 + 0.1478i, 0.0348 + 0.0654i, 0.0816 + 0.0505i, 0.1384 + 0.0467i, 0.0498 + 0.0543i, 0.0974 + 0.0584i, 0.0582 + 0.0879i, 0.0932 + 0.0178i, 0.1039 + 0.0057i, 0.0590 + 0.0682i, 0.0615 + 0.1293i, 0.0974 + 0.1388i, 0.1245 + 0.0393i, 0.0552 + 0.0238i, 0.0632 + 0.1297i, 0.0884 + 0.0354i, 0.0841 + 0.0960i, 0.1065 + 0.1437i, 0.0760 + 0.0988i, 0.1154 + 0.1293i, 0.0727 + 0.0015i, 0.0276 + 0.0204i, 0.1041 + 0.1217i, 0.1460 + 0.0639i, 0.1199 + 0.1323i, 0.1046 + 0.1092i, 0.0721 + 0.1021i, 0.0170 + 0.0514i, 0.0988 + 0.0247i, 0.0543 + 0.0231i, 0.0208 + 0.0284i, 0.0842 + 0.0628i, 0.1223 + 0.1272i, 0.1002 + 0.0729i, 0.1485 + 0.1213i, 0.1429 + 0.0685i, 0.0087 + 0.0680i, 0.0535 + 0.0670i, 0.0815 + 0.0613i, 0.0389 + 0.1340i, 0.0888 + 0.0008i, 0.0073 + 0.0442i, 0.0849 + 0.0073i, 0.1042 + 0.1030i, 0.1430 + 0.0966i, 0.1115 + 0.1461i, 0.1109 + 0.0921i$
7	1001010	$0.0880 + 0.0466i, 0.1054 + 0.0684i, 0.0239 + 0.0866i, 0.0759 + 0.0090i, 0.0563 + 0.1020i, 0.1006 + 0.0988i, 0.0769 + 0.0649i, 0.0246 + 0.0273i, 0.0485 + 0.0942i, 0.0186 + 0.0554i, 0.1045 + 0.0790i, 0.0384 + 0.0455i, 0.0053 + 0.1037i, 0.0815 + 0.0078i, 0.0965 + 0.0597i, 0.0309 + 0.0315i, 0.0271 + 0.0925i, 0.1006 + 0.0362i, 0.0141 + 0.0734i, 0.1015 + 0.0058i, 0.0757 + 0.0385i, 0.0914 + 0.0537i, 0.0226 + 0.0468i, 0.0491 + 0.0607i, 0.0087 + 0.0665i, 0.0918 + 0.0122i, 0.0606 + 0.0969i, 0.0344 + 0.0814i, 0.0404 + 0.0853i, 0.0936 + 0.0879i, 0.0401 + 0.0723i, 0.0079 + 0.0217i, 0.0216 + 0.0294i, 0.0053 + 0.0675i, 0.0611 + 0.0579i, 0.0131 + 0.0064i, 0.0563 + 0.0096i, 0.0126 + 0.0293i, 0.0830 + 0.0253i, 0.0253 + 0.0642i, 0.1006 + 0.0016, 0.0583 + 0.0441i, 0.0404 + 0.0511i, 0.0888 + 0.0980i, 0.0050 + 0.0643i, 0.0645 + 0.0355i, 0.1024 + 0.0516i, 0.0311 + 0.0646i, 0.0959 + 0.0174i, 0.0110 + 0.0894i, 0.0070 + 0.1031i, 0.0253 + 0.0642i, 0.1006 + 0.0031i, 0.0068 + 0.0876i, 0.0285 + 0.0658i, 0.1078 + 0.0756i, 0.0229 + 0.0099i, 0.0537 + 0.0458i, 0.0313 + 0.0403i, 0.0405 + 0.0725i, 0.0179 + 0.1033i, 0.0898 + 0.0089i, 0.0827 + 0.0904i, 0.0718 + 0.0487i, 0.0141 + 0.1032i, 0.0103 + 0.0159i, 0.0016 + 0.0938i, 0.0311 + 0.0830i, 0.0881 + 0.0479i, 0.1063 + 0.0669i, 0.0019 + 0.1026i, 0.0884 + 0.0690i, 0.0670 + 0.0267i, 0.0604 + 0.0380i, 0.0263 + 0.0203i, 0.0886 + 0.0529i, 0.0284 + 0.0441i, 0.0813 + 0.0500i, 0.0711 + 0.0659i, 0.0231 + 0.0377i, 0.0649 + 0.0539i, 0.0652 + 0.0656i, 0.0711 + 0.0118i, 0.0198 + 0.0670i, 0.0686 + 0.0265i, 0.0184 + 0.0563i, 0.0582 + 0.0524i, 0.0762 + 0.0501i, 0.0740 + 0.0584i, 0.0730 + 0.1016i, 0.0946 + 0.0369i, 0.0014 + 0.0433i, 0.0335 + 0.0332i, 0.0840 + 0.0444i, 0.0331 + 0.0308i, 0.0999 + 0.0425i, 0.0732 + 0.0542i, 0.0080 + 0.0779i, 0.0076 + 0.0330i, 0.0013 + 0.0121i, 0.0245 + 0.0478i, 0.0557 + 0.0503i, 0.0494 + 0.0016i, 0.0758 + 0.0716i, 0.0628 + 0.0781i, 0.0549 + 0.0304i, 0.0080 + 0.0282i, 0.0208 + 0.0764i, 0.0409 + 0.0845i, 0.0893 + 0.0425i, 0.0989 + 0.0562i, 0.0122 + 0.0774i, 0.0876 + 0.0614i, 0.0979 + 0.0497i, 0.0169 + 0.0480i, 0.0132 + 0.0095i, 0.0822 + 0.0447i, 0.0778 + 0.0395i, 0.0703 + 0.0326i, 0.0813 + 0.0919i, 0.0953 + 0.1024i, 0.0293 + 0.0602i, 0.0452 + 0.0015i, 0.0230 + 0.0643i$
8	10101011	$0.0199 + 0.0027i, 0.0033 + 0.0063i, 0.0005 + 0.0656i, 0.0443 + 0.0262i, 0.0573 + 0.0359i, 0.0622 + 0.0704i, 0.0491 + 0.0176i, 0.0194 + 0.0664i, 0.0111 + 0.0506i, 0.0502 + 0.0687i, 0.0729 + 0.0376i, 0.0629 + 0.0765i, 0.0717 + 0.0288i, 0.0239 + 0.0410i, 0.0207 + 0.0140i, 0.0413 + 0.0387i, 0.0126 + 0.0325i, 0.0163 + 0.0509i, 0.0167 + 0.0519i, 0.0502 + 0.0738i, 0.0041 + 0.0418i, 0.0177 + 0.0086i, 0.0514 + 0.0436i, 0.0240 + 0.0747i, 0.0236 + 0.0018i, 0.0555 + 0.0671i, 0.0736 + 0.0021i, 0.0101 + 0.0400i, 0.0053 + 0.0148i, 0.0097 + 0.0552i, 0.0128 + 0.0193i, 0.0702 + 0.0720i, 0.0105 + 0.0106i, 0.0476 + 0.0402i, 0.0207 + 0.0690i, 0.0170 + 0.0726i, 0.0549 + 0.0258i, 0.0423 + 0.0371i, 0.0276 + 0.0363i, 0.0543 + 0.0105i, 0.0448 + 0.0559i, 0.0678 + 0.0307i, 0.0578 + 0.0276i, 0.0293 + 0.0670i, 0.0125 + 0.0483i, 0.0737 + 0.0166i, 0.0754i, 0.0598 + 0.0494i, 0.0473 + 0.0177i, 0.0125 + 0.0525i, 0.0222 + 0.0104i, 0.0748 + 0.0017i, 0.0733 + 0.0202i, 0.0176 + 0.0090i, 0.0739 + 0.0053i, 0.0524 + 0.0657i, 0.0704 + 0.0422i, 0.0462 + 0.0025i, 0.0561 + 0.0441i, 0.0141 + 0.0213i, 0.0059 + 0.0284i, 0.0006 + 0.0010i, 0.0608 + 0.0685i, 0.0014 + 0.0667i, 0.0677 + 0.0196i, 0.0272 + 0.0439i, 0.0557 + 0.0123i, 0.0746 + 0.0458i, 0.0120 + 0.2255i, 0.0126 + 0.0508i, 0.0242 + 0.0666i, 0.0023 + 0.0437i, 0.0276 + 0.0756i, 0.0021 + 0.0610i, 0.0612 + 0.0118i, 0.0770 + 0.0642i, 0.0085 + 0.0148i, 0.0480 + 0.0493i, 0.0102 + 0.0516i, 0.0239 + 0.0595i, 0.0104 + 0.0293i, 0.0172 + 0.0340i, 0.0306 + 0.0306i, 0.0072 + 0.0372i, 0.0104 + 0.0602i, 0.0301 + 0.0609i, 0.0394 + 0.0396i, 0.0072 + 0.0164i, 0.0017 + 0.0080i, 0.0123 + 0.0121i, 0.0651 + 0.0314i, 0.0578 + 0.0314i, 0.0144 + 0.0041i, 0.0764 + 0.0726i, 0.0549 + 0.0116i, 0.0672 + 0.0296i, 0.0370 + 0.0240i, 0.0382 + 0.0130i, 0.0222 + 0.0691i, 0.0047 + 0.0249i, 0.0220 + 0.0566i, 0.0144 + 0.0317i, 0.0707 + 0.0308i, 0.0095 + 0.0390i, 0.0010 + 0.0130i, 0.0285 + 0.0404i, 0.0538 + 0.0494i, 0.0685 + 0.0012i, 0.0121 + 0.0619i, 0.0244 + 0.0538i, 0.0180 + 0.0356i, 0.0006 + 0.0064i, 0.0306 + 0.0633i, 0.0501 + 0.0149i, 0.0066 + 0.0343i, 0.0593 + 0.0010i, 0.0747 + 0.0238i, 0.0551 + 0.0675i, 0.0603 + 0.0644i, 0.0183 + 0.0257i, 0.0151 + 0.0679i, 0.0203 + 0.0370i, 0.0050 + 0.0432i, 0.0753 + 0.0475i, 0.0491 + 0.0510i, 0.0421 + 0.0475i, 0.0654 + 0.0528i, 0.0618 + 0.0393i, 0.0515 + 0.0550i, 0.0517 + 0.0397i, 0.0633 + 0.0467i, 0.0748 + 0.0745i, 0.0375 + 0.0634i, 0.0630 + 0.0245i, 0.0494 + 0.0453i, 0.0239 + 0.0100i, 0.0509 + 0.0196i, 0.0276 + 0.0619i, 0.0723 + 0.0515i, 0.0376 + 0.0011i, 0.0070 + 0.0433i, 0.0519 + 0.0350i, 0.0397 + 0.0171i, 0.0217 + 0.0559i, 0.0030 + 0.0503i, 0.0053 + 0.0367i, 0.0743 + 0.0758i, 0.0160 + 0.0711i, 0.0124 + 0.0433i, 0.0492 + 0.0503i, 0.0000 + 0.0596i, 0.0259 + 0.0028i, 0.0212 + 0.0001i, 0.0034 + 0.0418i, 0.0072 + 0.0005i, 0.0316 + 0.0348i, 0.0630 + 0.0151i, 0.0671 + 0.0071i, 0.0477 + 0.0051i, 0.0654 + 0.0587i, 0.0736 + 0.0059i, 0.0562 + 0.0293i, 0.0572 + 0.0293i, 0.0266 + 0.0255i, 0.0681 + 0.0389i, 0.0248 + 0.0435i, 0.0670 + 0.0514i, 0.0302 + 0.0522i, 0.0073 + 0.0594i, 0.0572 + 0.0485i, 0.0266 + 0.0411i, 0.0045 + 0.0346i, 0.0418 + 0.0404i, 0.0351 + 0.0132i, 0.0665 + 0.0101i, 0.0659 + 0.0169i, 0.0364 + 0.0081i, 0.0607 + 0.0109i, 0.0506 + 0.0352i, 0.0000 + 0.0607i, 0.0101 + 0.0217i, 0.0381 + 0.0173i, 0.0030 + 0.0701i, 0.0175 + 0.0006i, 0.0253 + 0.0454i, 0.0693 + 0.0418i, 0.0242 + 0.0504i, 0.0194 + 0.0242i, 0.0334 + 0.0178i, 0.0649 + 0.0321i, 0.0142 + 0.0230i, 0.0392 + 0.0518i, 0.0349 + 0.0723i, 0.0251 + 0.0264i, 0.0293 + 0.0434i, 0.0683 + 0.0092i, 0.0587 + 0.0130i, 0.0681 + 0.0215i, 0.0616 + 0.0374i, 0.0103 + 0.0734i, 0.0289 + 0.0406i, 0.0373 + 0.0611i, 0.0747 + 0.0149i, 0.0264 + 0.0701i, 0.0195 + 0.0711i, 0.0451 + 0.0010i, 0.0404 + 0.0592i, 0.0126 + 0.0730i, 0.0375 + 0.0607i, 0.0382 + 0.0712i, 0.0650 + 0.0153i, 0.0621 + 0.0502i, 0.0061 + 0.0715i, 0.0470 + 0.0265i, 0.0436 + 0.0458i, 0.0472 + 0.0474i, 0.0079 + 0.0003i, 0.0122 + 0.0075i, 0.0457 + 0.0709i, 0.0092 + 0.0646i$



Information content



40 qubits	10 000 GigaByte
1 additional qubit	Double the memory
300 qubits	Every atom in the Universe would have to hold one classical bit



Why quantum information ?



Schrödinger equation for 300 interacting spins.

Classical computation needs more bits than there are atoms in the universe.

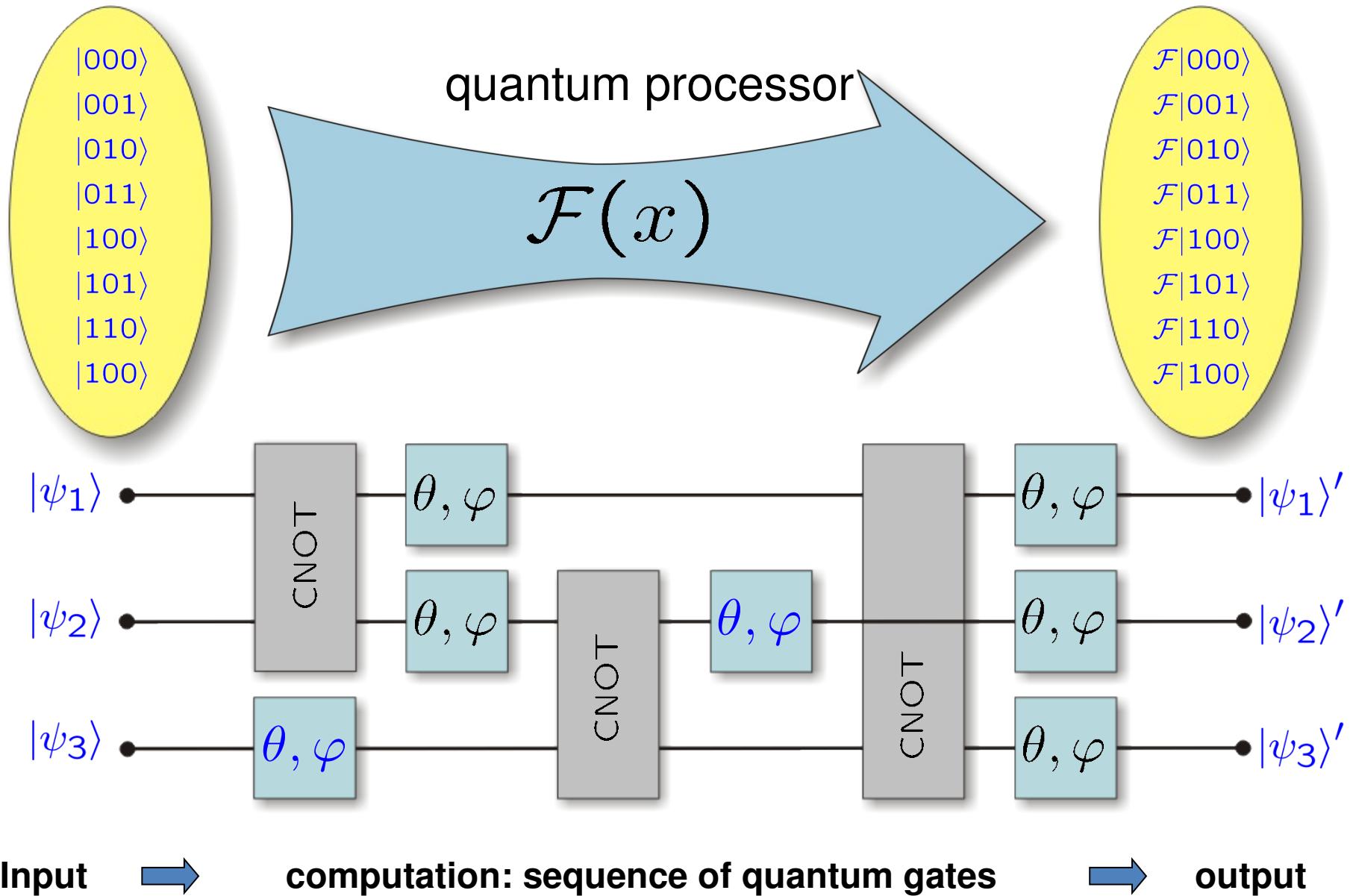
- Quantum computers can solve certain tasks much more efficiently than classical computers.

Other prominent examples:

- Factoring of large integers (P. Shor 1994)
- Search in an unsorted data base (L. Grover, 1997)
- ...



Quantum computing





Quantum computing



Classical computer

- Initialization
- 1-bit operations (NOT)
- 2-bit gates (e.g. NAND)

Computational space:

00
01
10
11

- Read out
→ result

Quantum computer

- Initialization
- 1-qubit rotations
→ superpositions
- 2-qubit gates (CNOT gate)
→ entanglement

Computational space: Hilbert space
 2^n dimensional

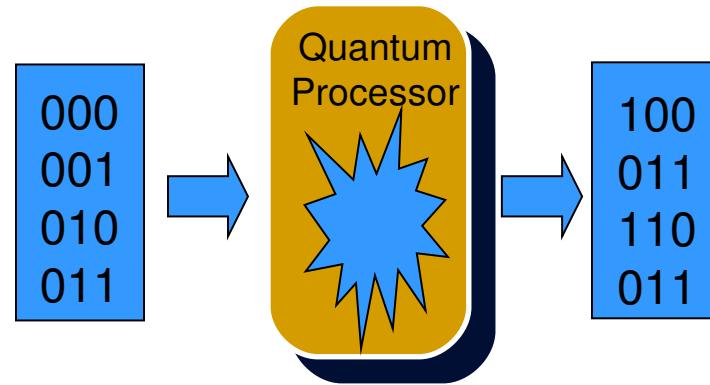
- Read out of qubits
→ gain of classical information



Quantum computing



Long term goal: A universal quantum computer

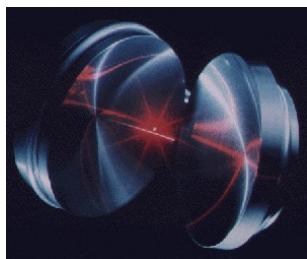


In the mean time:

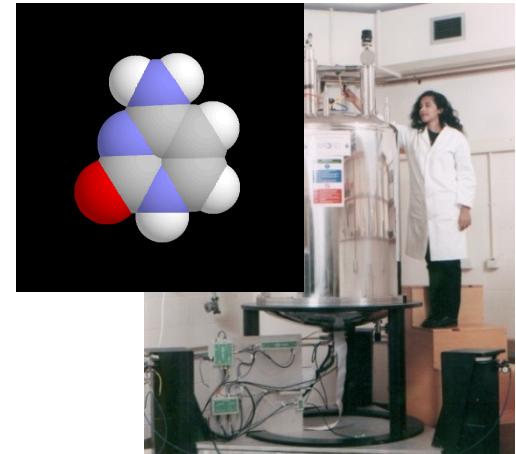
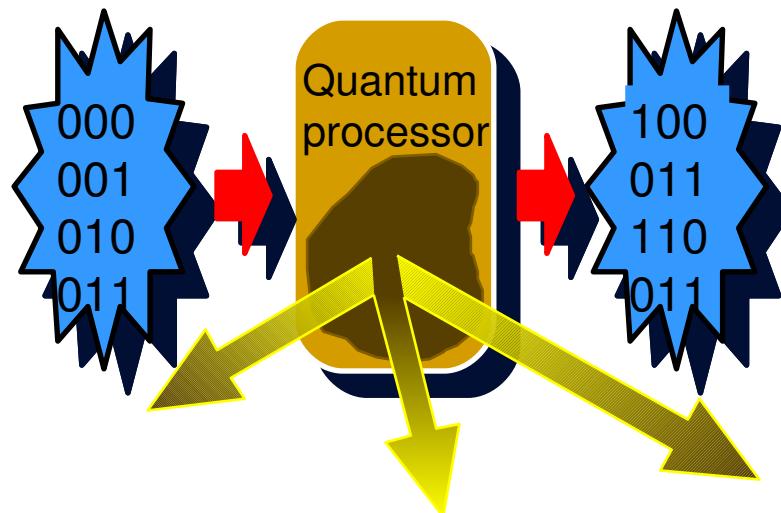
- “understand” quantum mechanics
- apply quantum mechanics
- where does quantum mechanics fail?



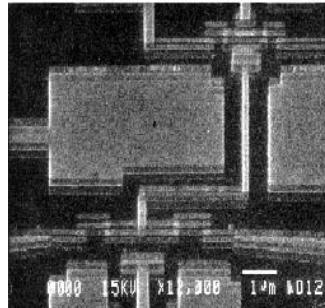
Which technology ?



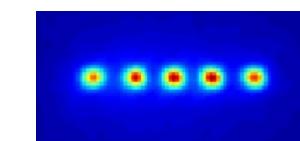
Cavity QED



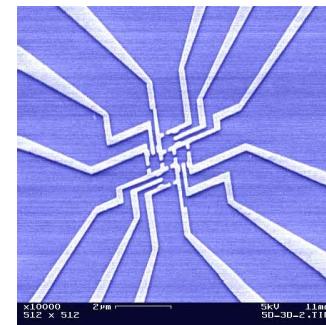
NMR



Superconducting qubits



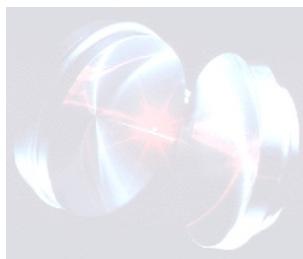
Trapped ions



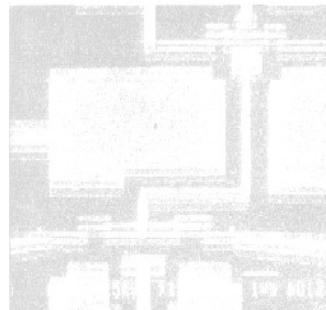
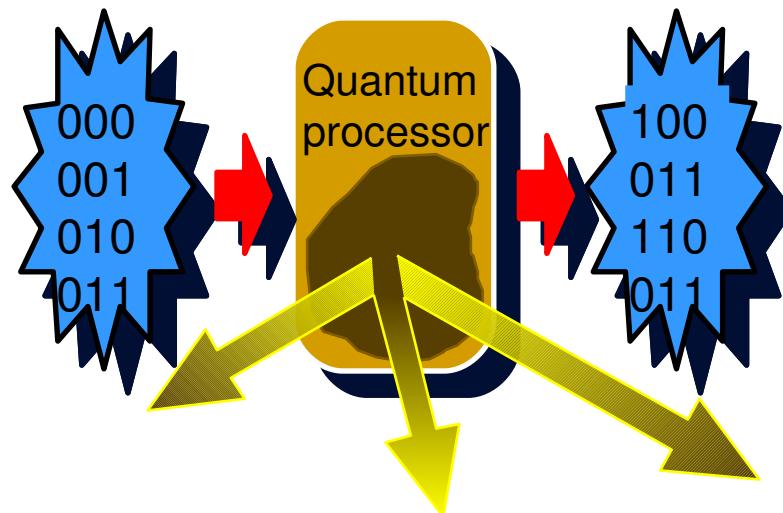
Quantum dots



Which technology ?



Cavity QED



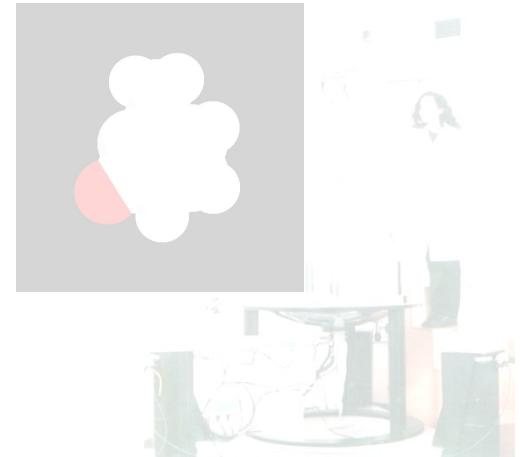
Superconducting qubits



Trapped ions



Quantum dots



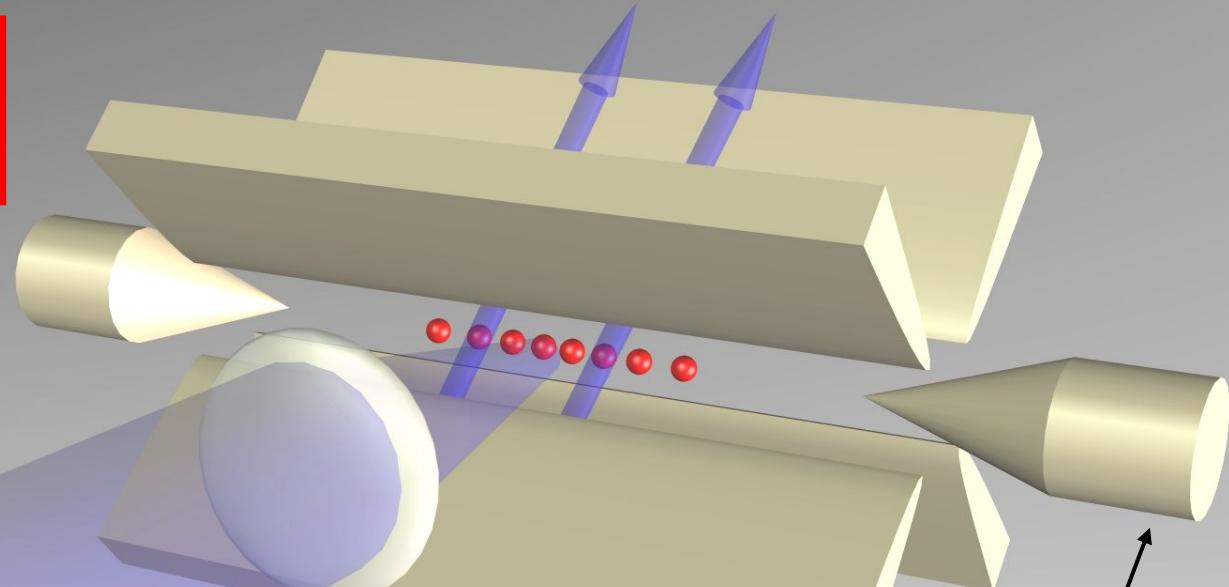
NMR



Ion trap quantum computing



Trapped ions form the quantum register



CCD

DiVincenzo criteria (2001)

- Scalable physical system, well characterized qubits
- Ability to initialize the state of the qubits
- Long relevant coherence times, much longer than gate operation time
- “Universal” set of quantum gates
- Qubit-specific measurement capability

- Introduction to quantum information
- Ion trap quantum computing
- Teleportation and more
- Scaling of ion trap quantum computers
- Wiring up trapped ions



Why trapped ions ?

Good things about ion traps:

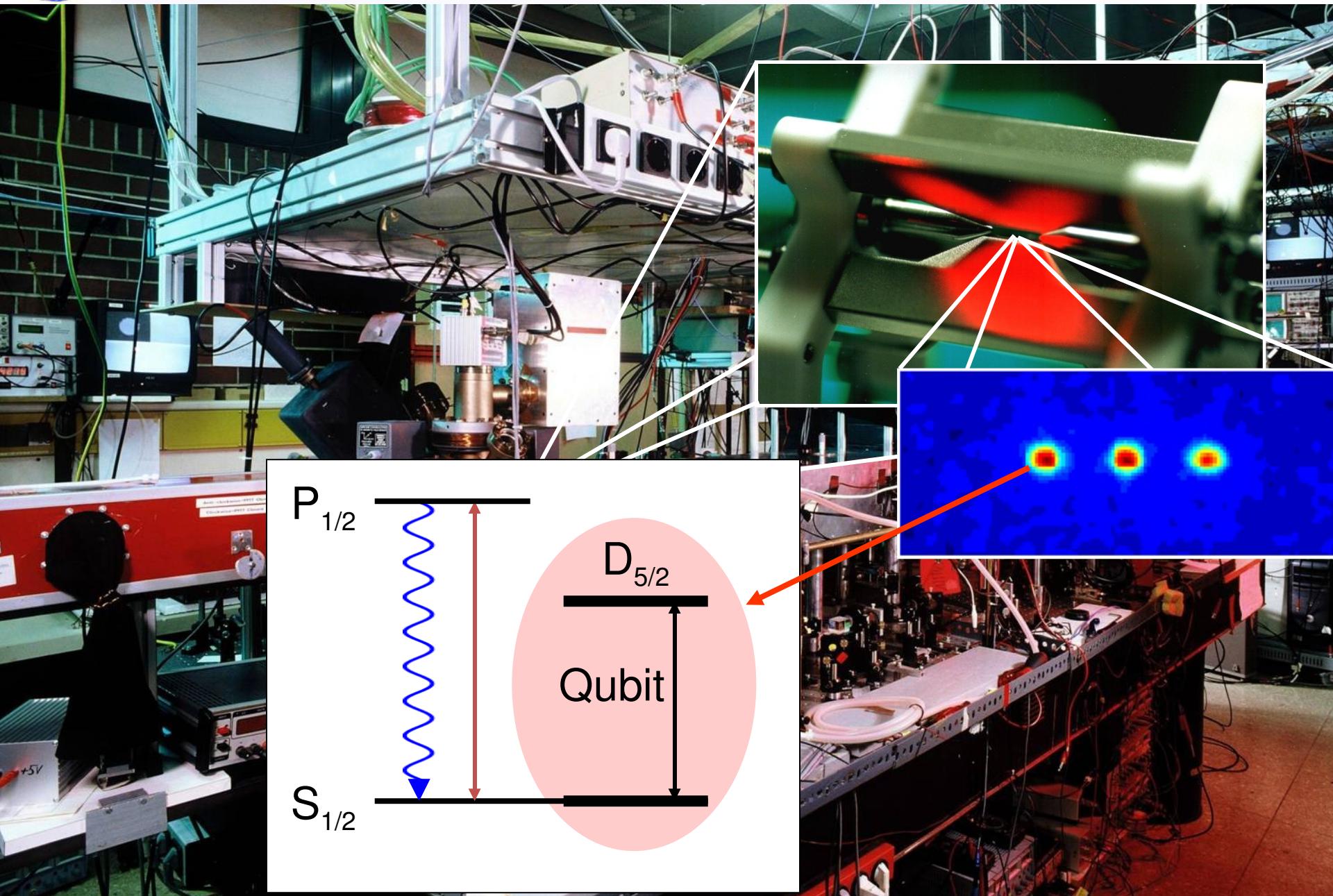
- Ions are excellent quantum memories; single qubit coherence times > 10 minutes have been demonstrated
- Ions can be controlled very well
- Many ideas to scale ion traps

Bad things about ion traps:

- Slow (~1 MHz)
- Technically demanding

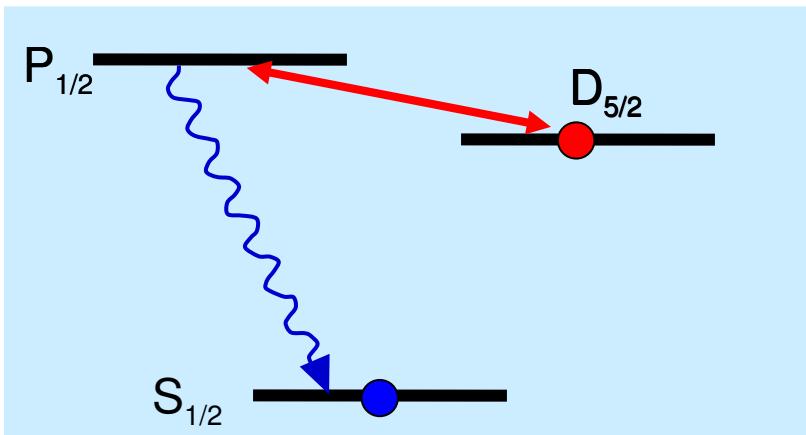


The hardware





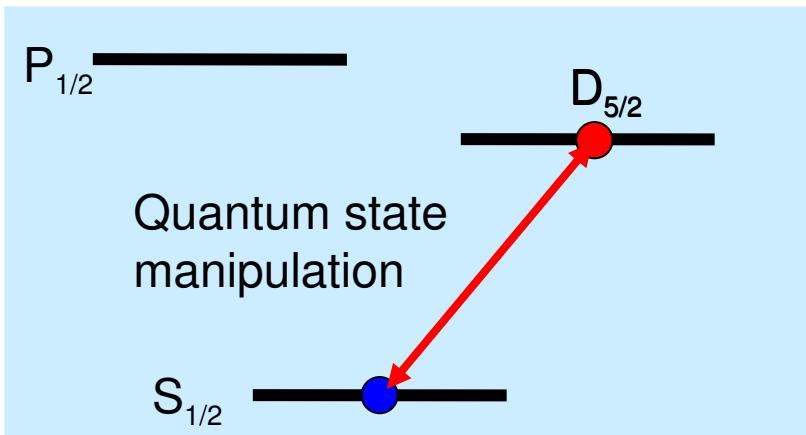
Experimental procedure



1. Initialization in a pure quantum state



Experimental procedure

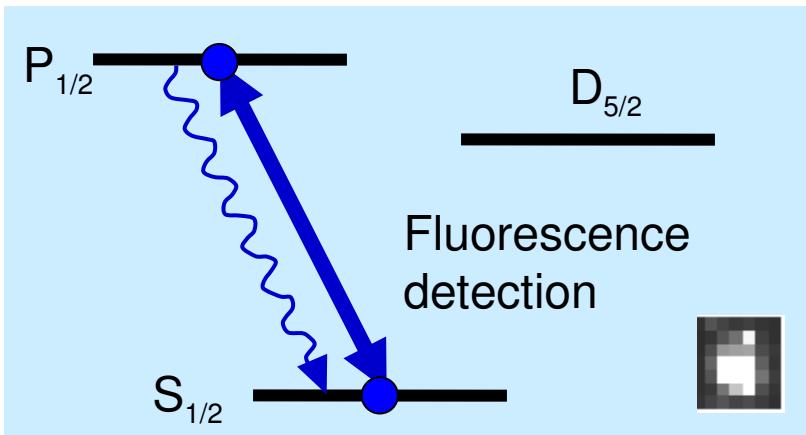


1. Initialization in a pure quantum state

2. Quantum state manipulation on
 $S_{1/2} - D_{5/2}$ transition



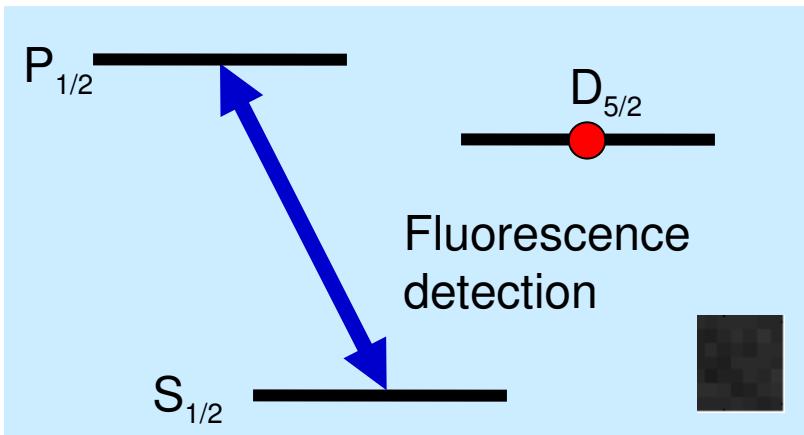
Experimental procedure



1. Initialization in a pure quantum state:
2. Quantum state manipulation on $S_{1/2} - D_{5/2}$ transition
3. Quantum state measurement by fluorescence detection

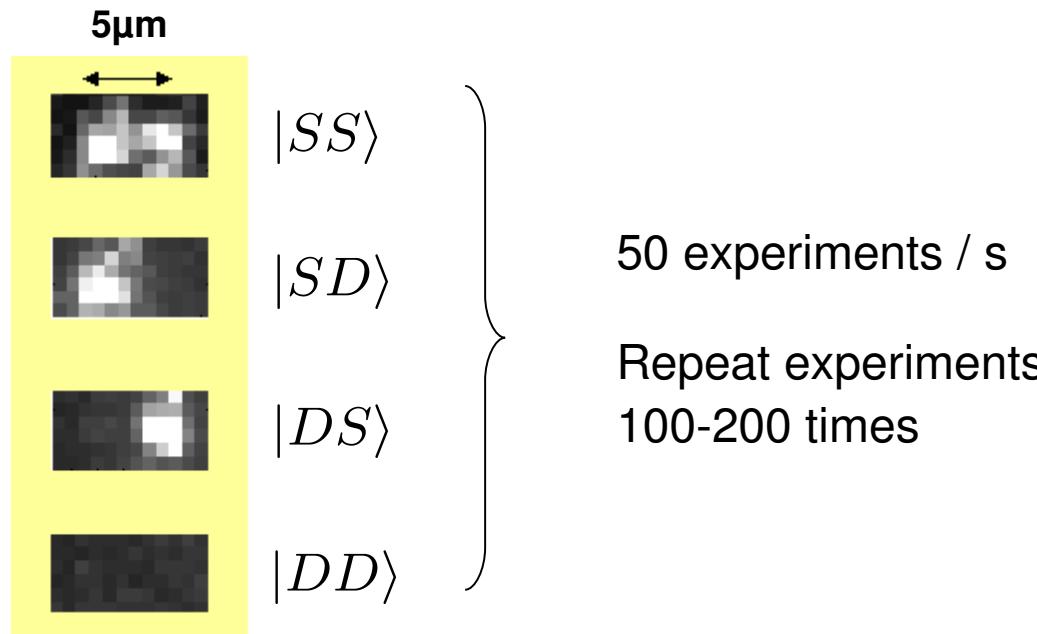


Experimental procedure



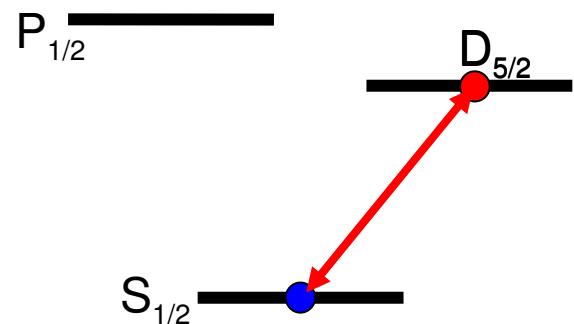
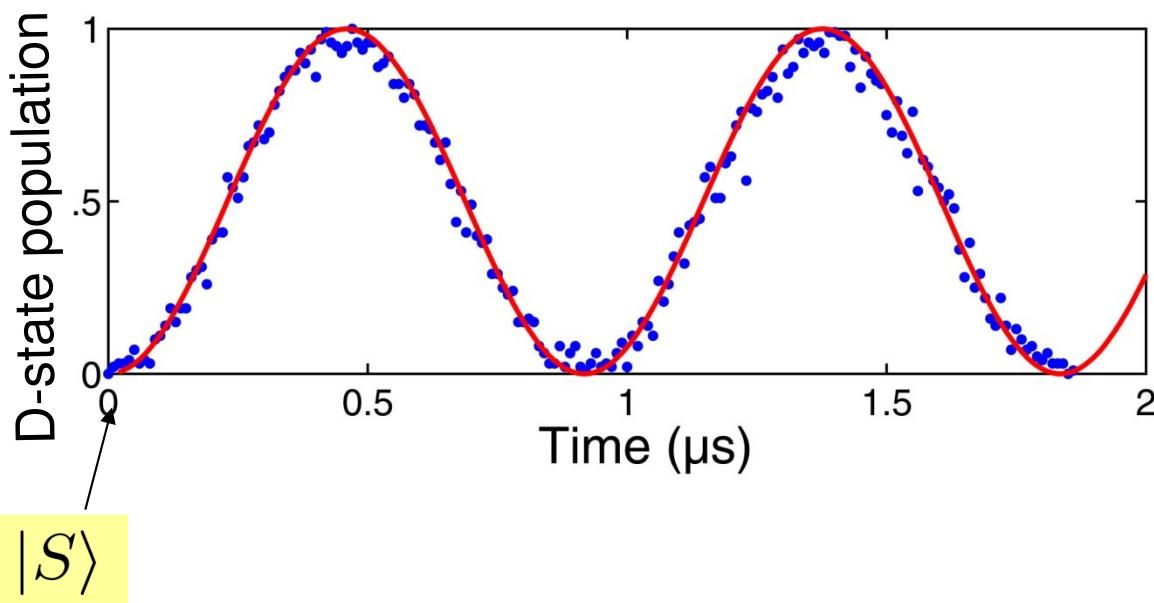
Two ions:

Spatially resolved
detection with
CCD camera

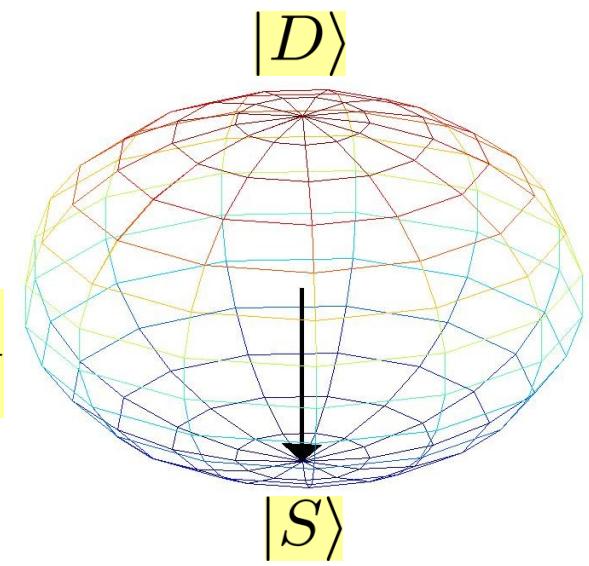




Rabi oscillations



$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$

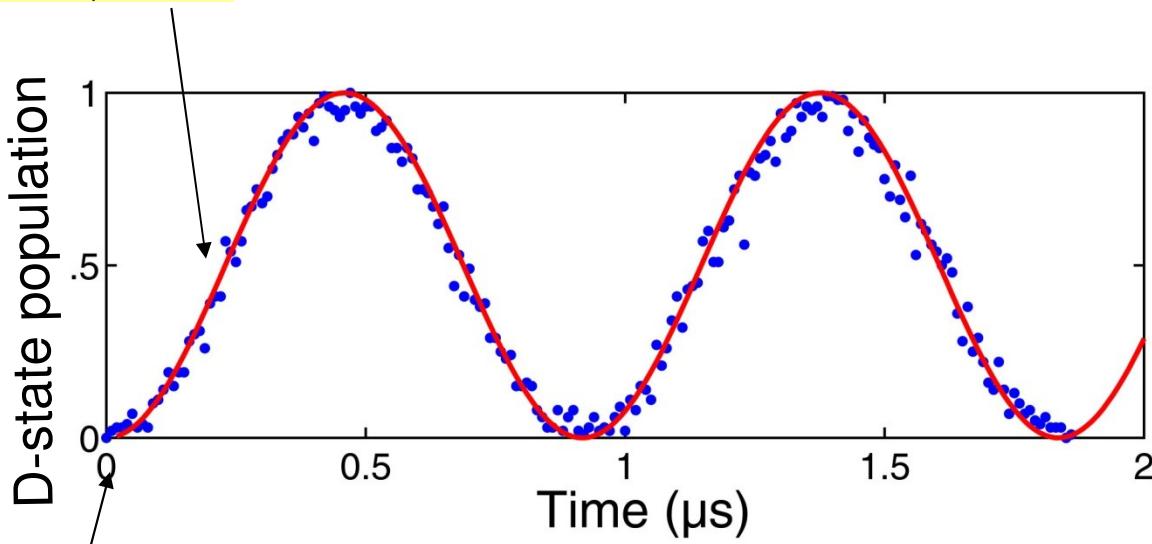




Rabi oscillations

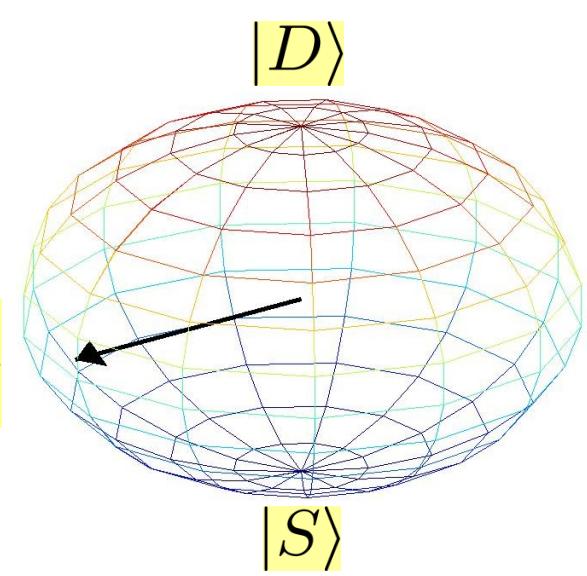
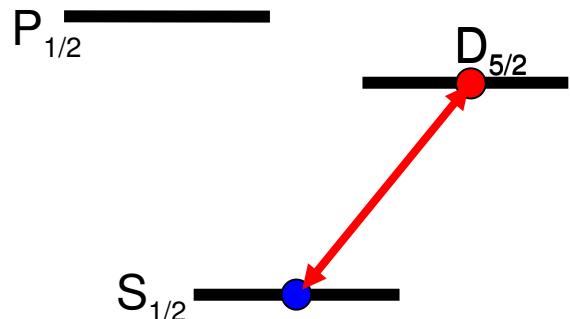


$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$



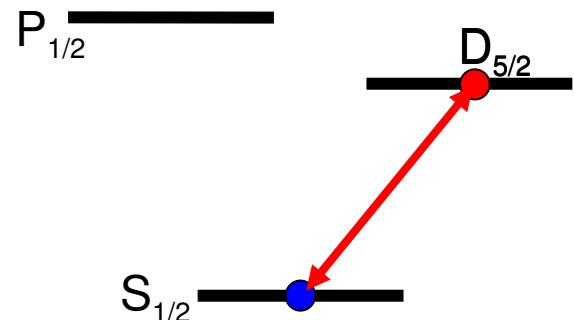
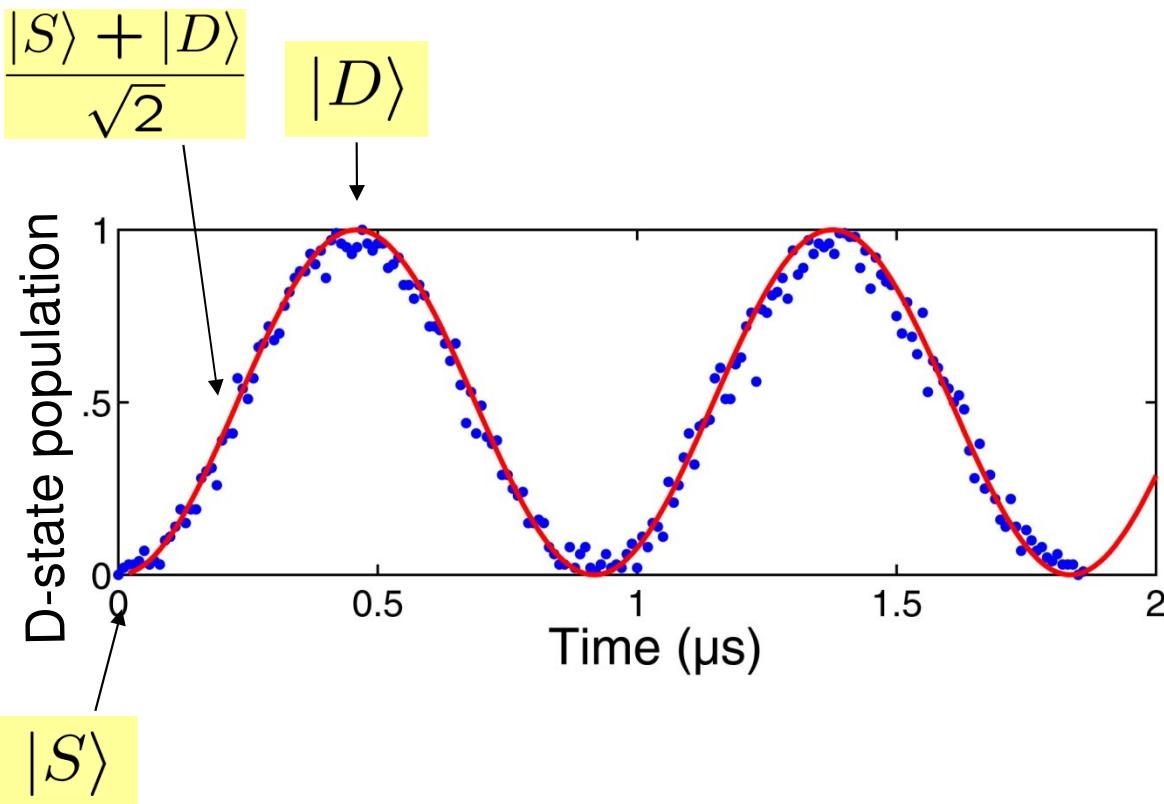
$$|S\rangle$$

$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$

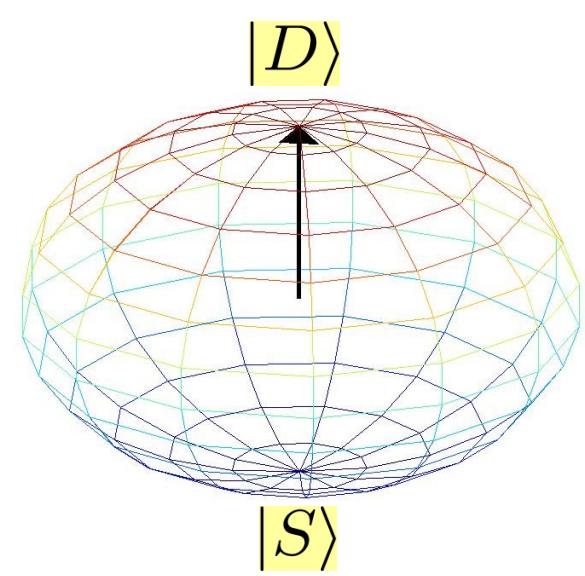




Rabi oscillations



$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$

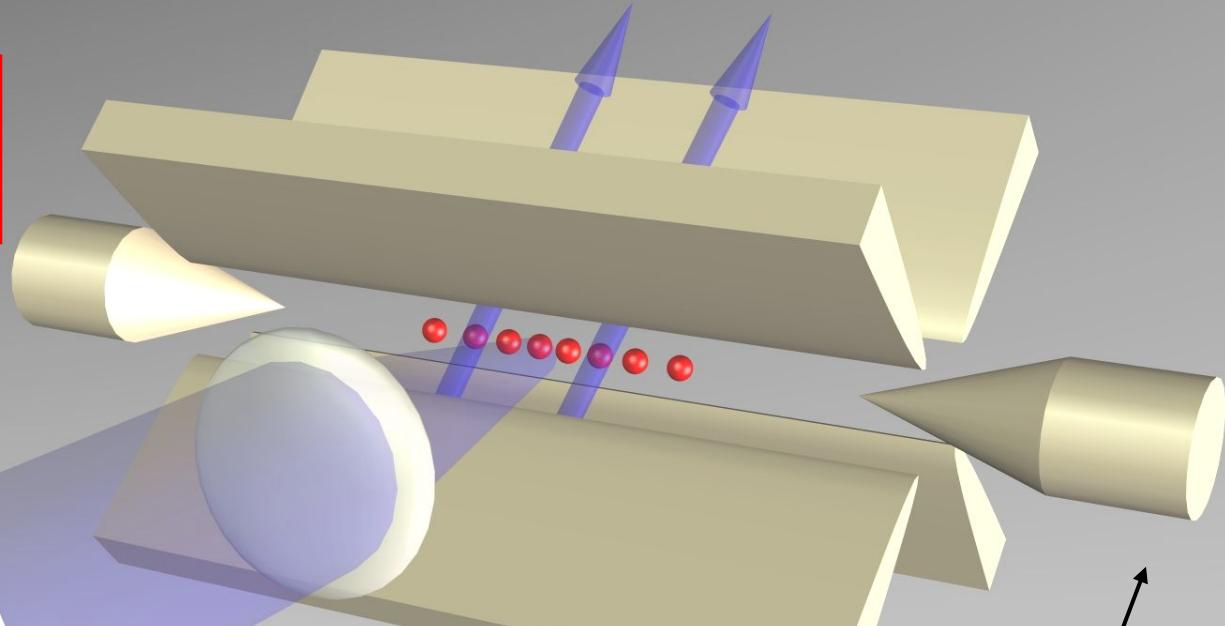




Ion trap quantum computing



Trapped ions form the quantum register



DiVincenzo criteria (2001)

- Scalable physical system, well characterized qubits ✓
- Ability to initialize the state of the qubits ✓
- Long relevant coherence times, much longer than gate operation time ✓
- “Universal” set of quantum gates ✓
- Qubit-specific measurement capability ✓

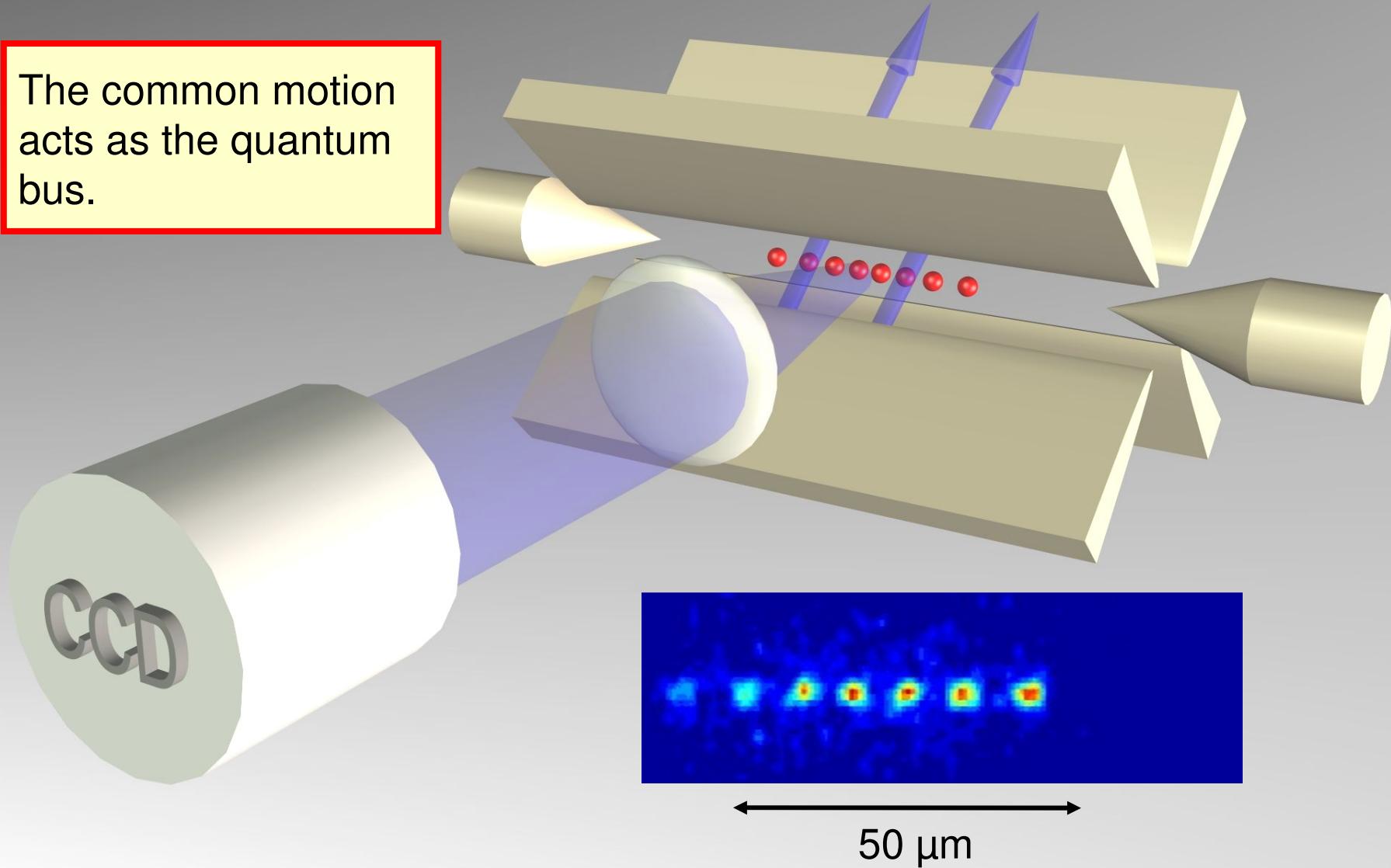




Having the qubits interact

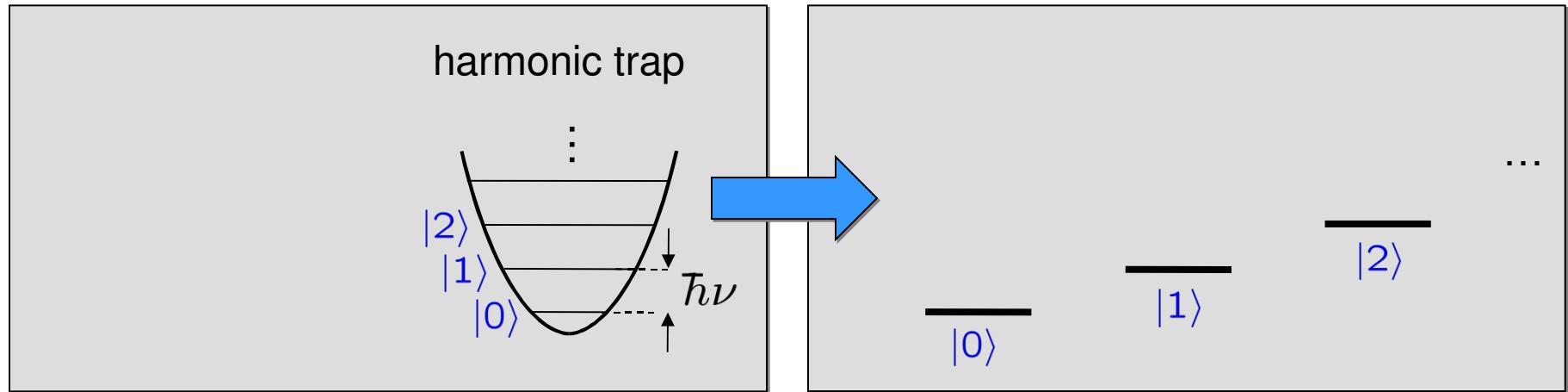


The common motion
acts as the quantum
bus.



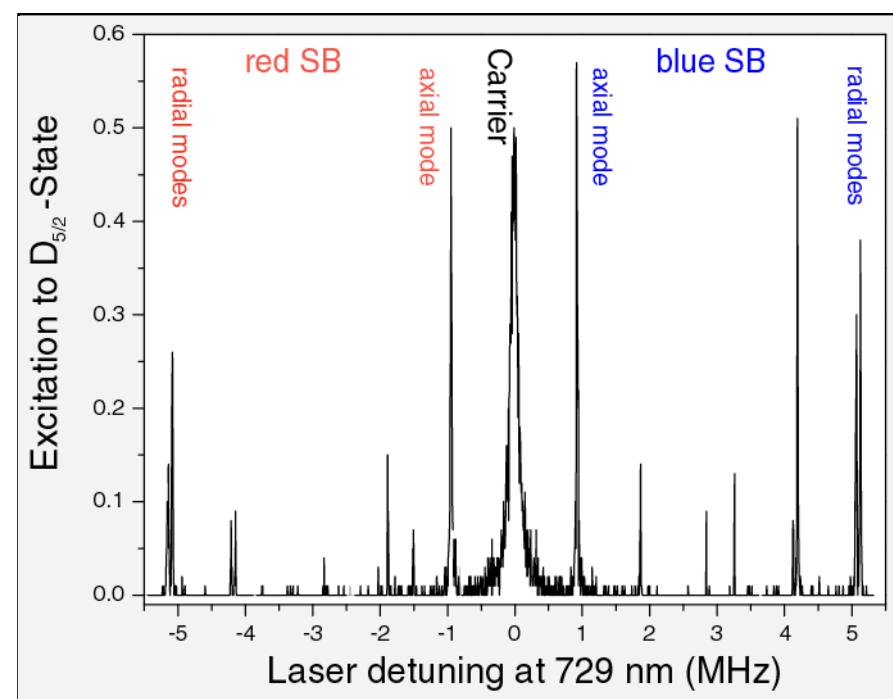
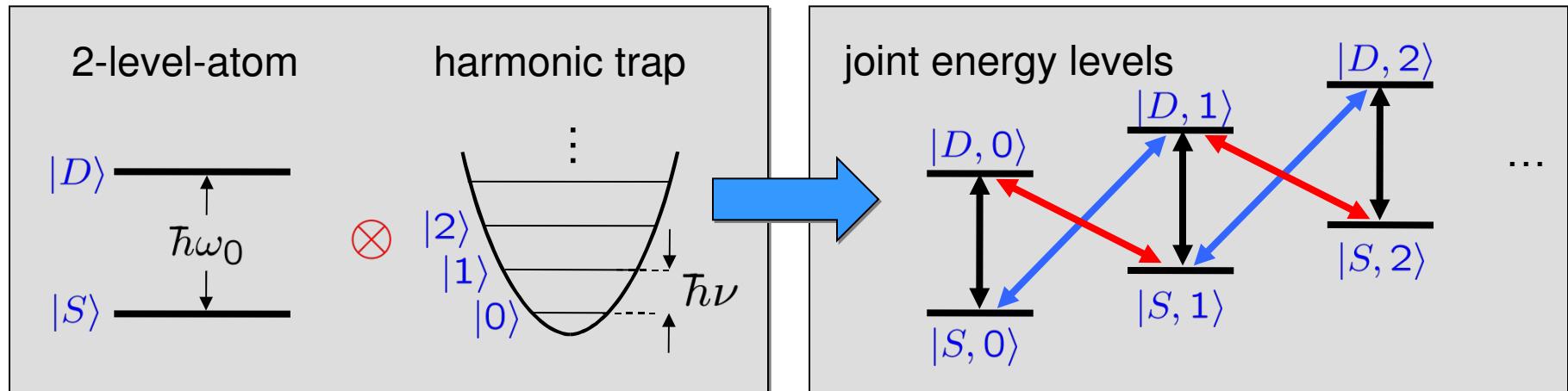


Ion motion



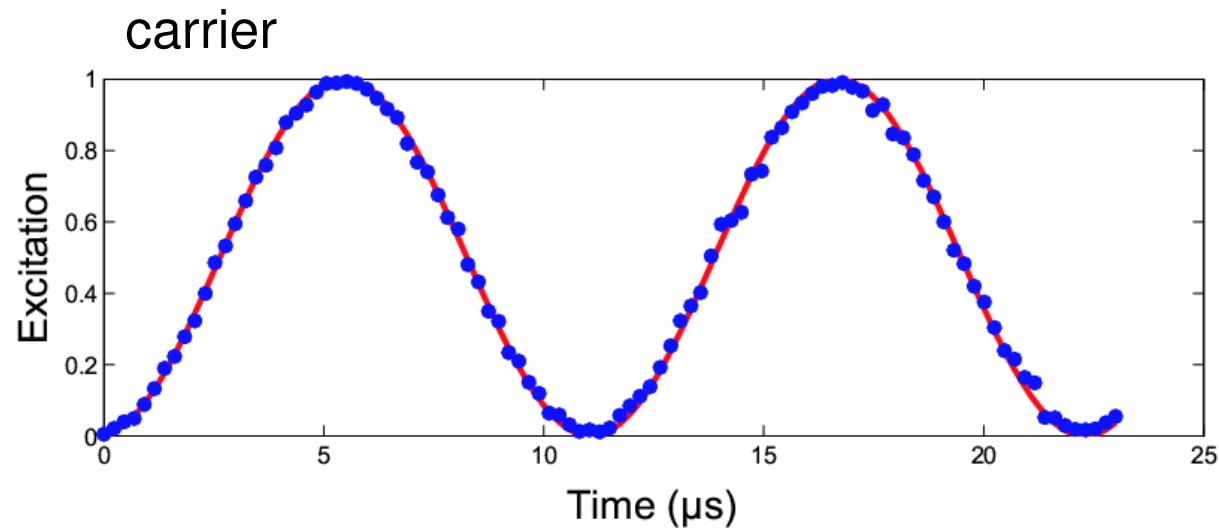
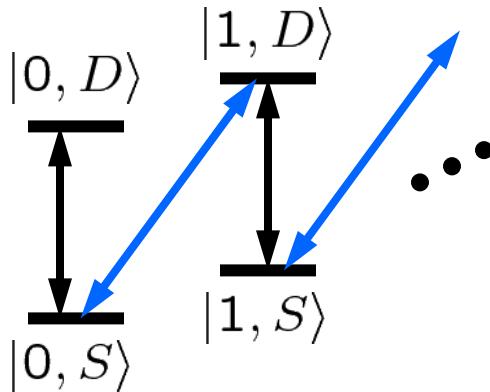


Ion motion



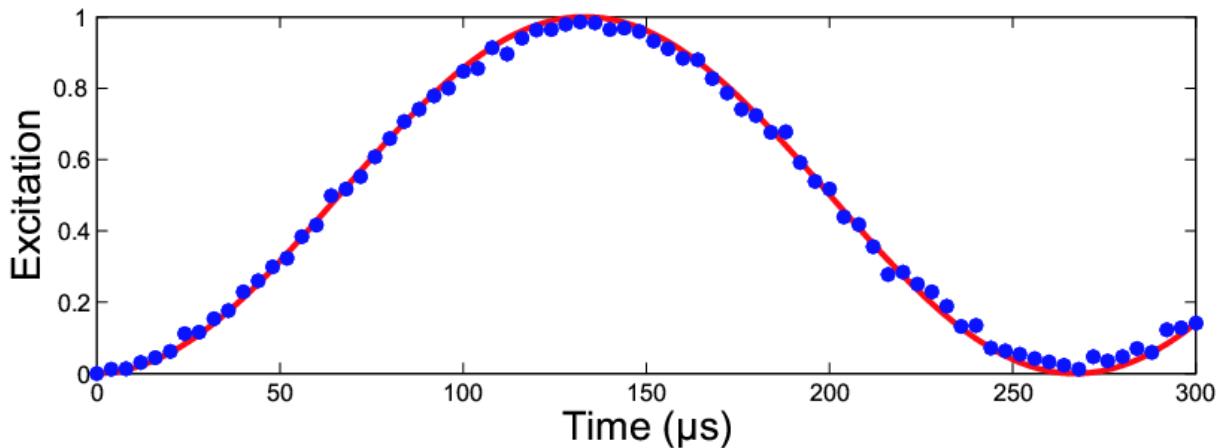


Coherent manipulation



carrier and sideband
Rabi oscillations
with Rabi frequencies

$\Omega, \eta\Omega$



$\eta = kx_0$

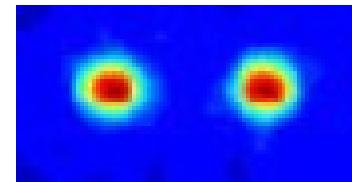
Lamb-Dicke parameter



Generation of Bell states



$|DD1\rangle$
 $|DD0\rangle$



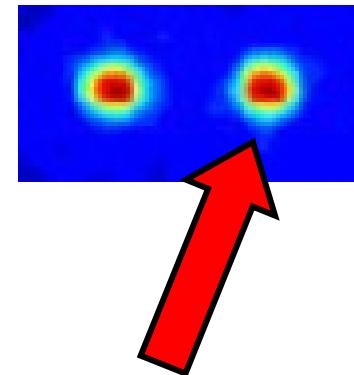
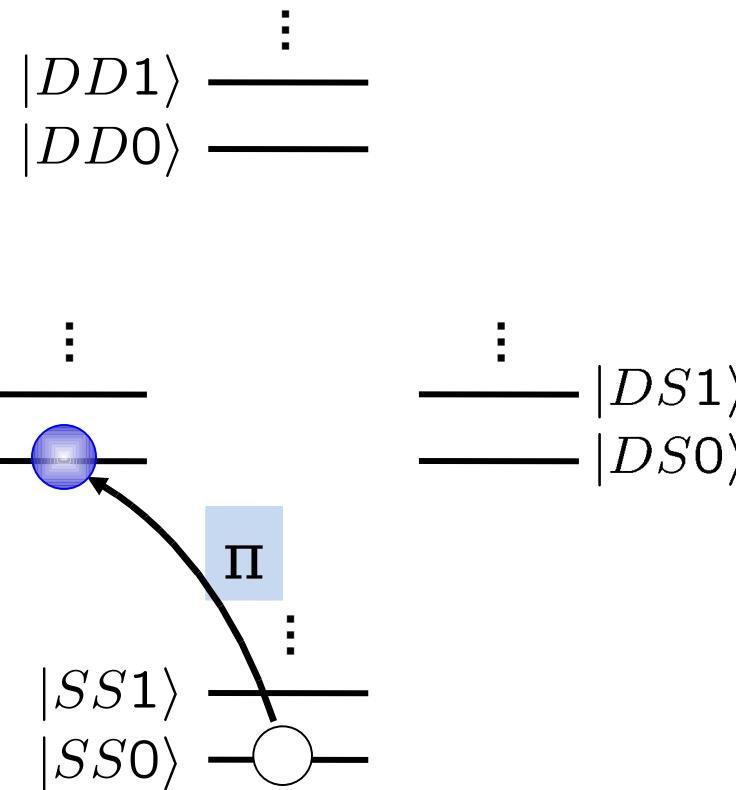
$|SD1\rangle$
 $|SD0\rangle$

$|DS1\rangle$
 $|DS0\rangle$

$|SS1\rangle$
 $|SS0\rangle$

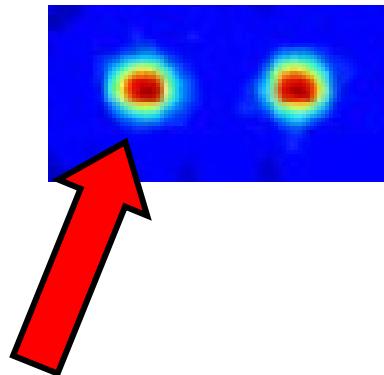
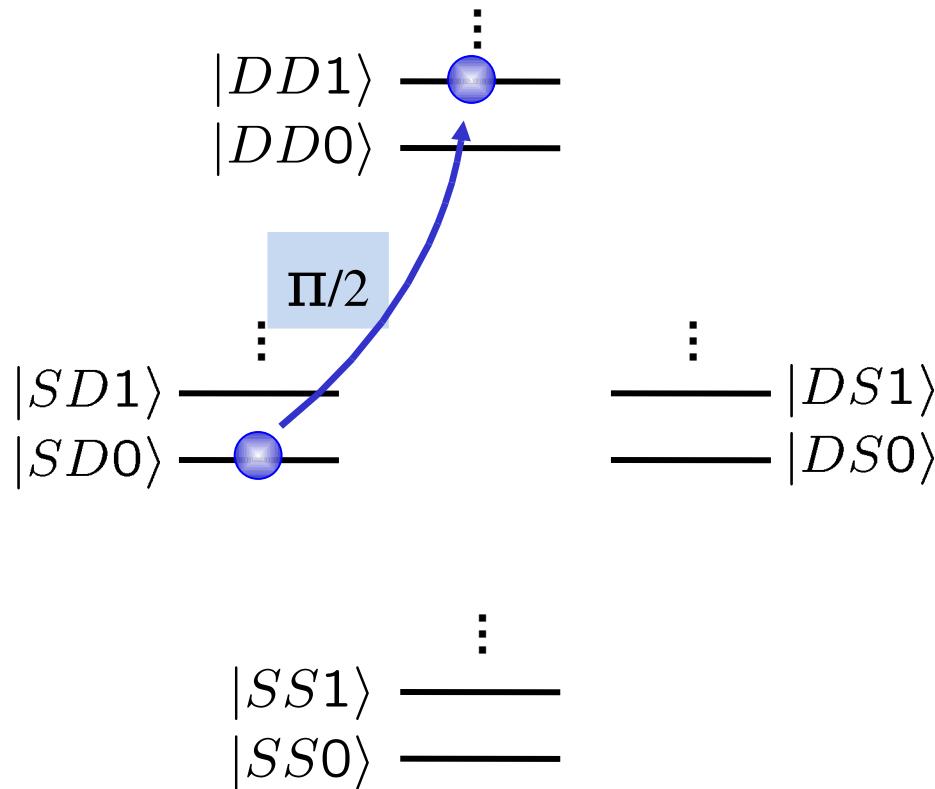


Generation of Bell states



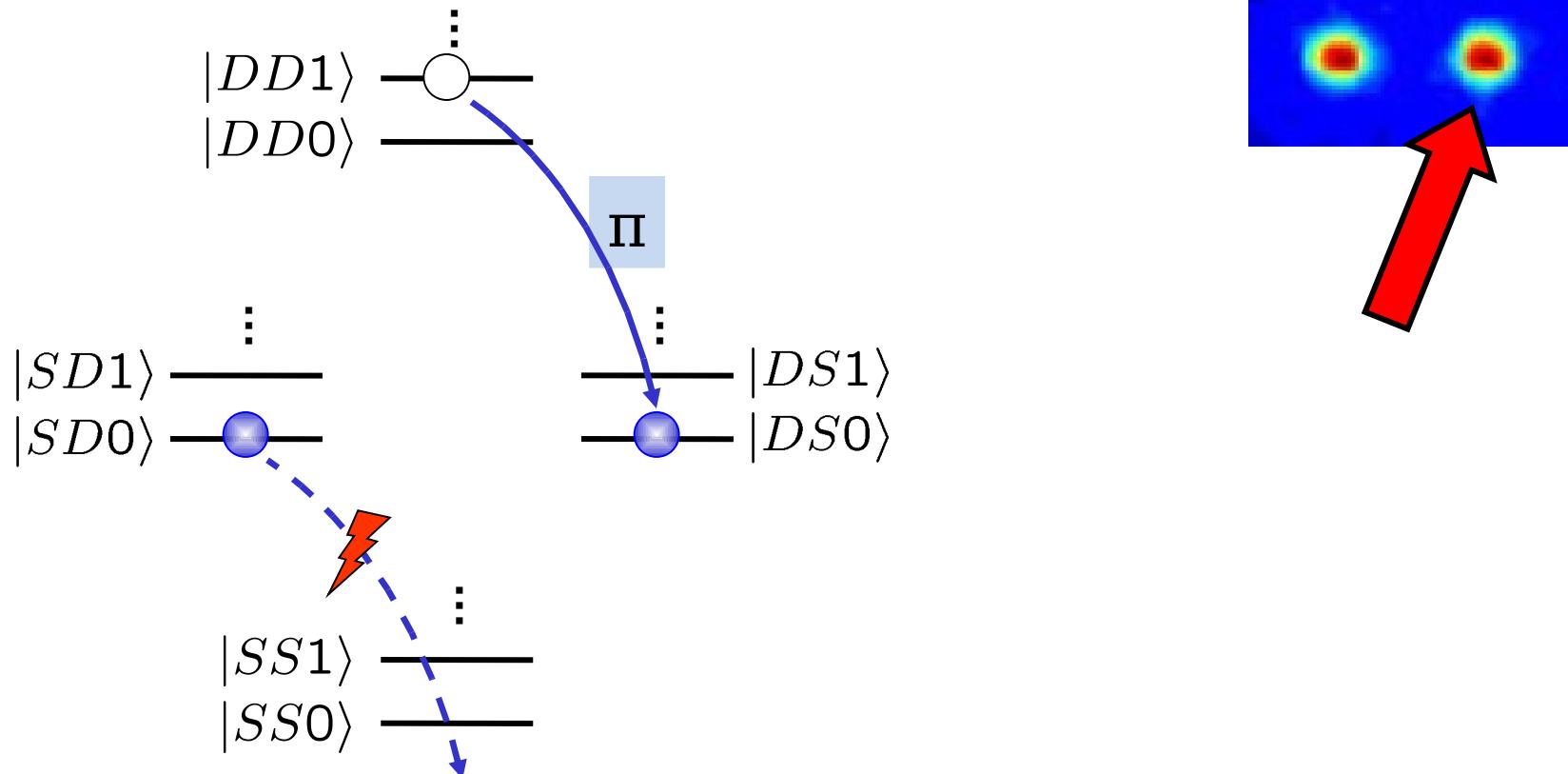


Generation of Bell states

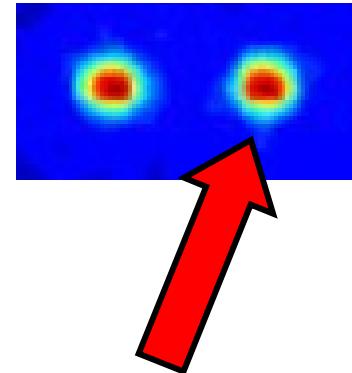
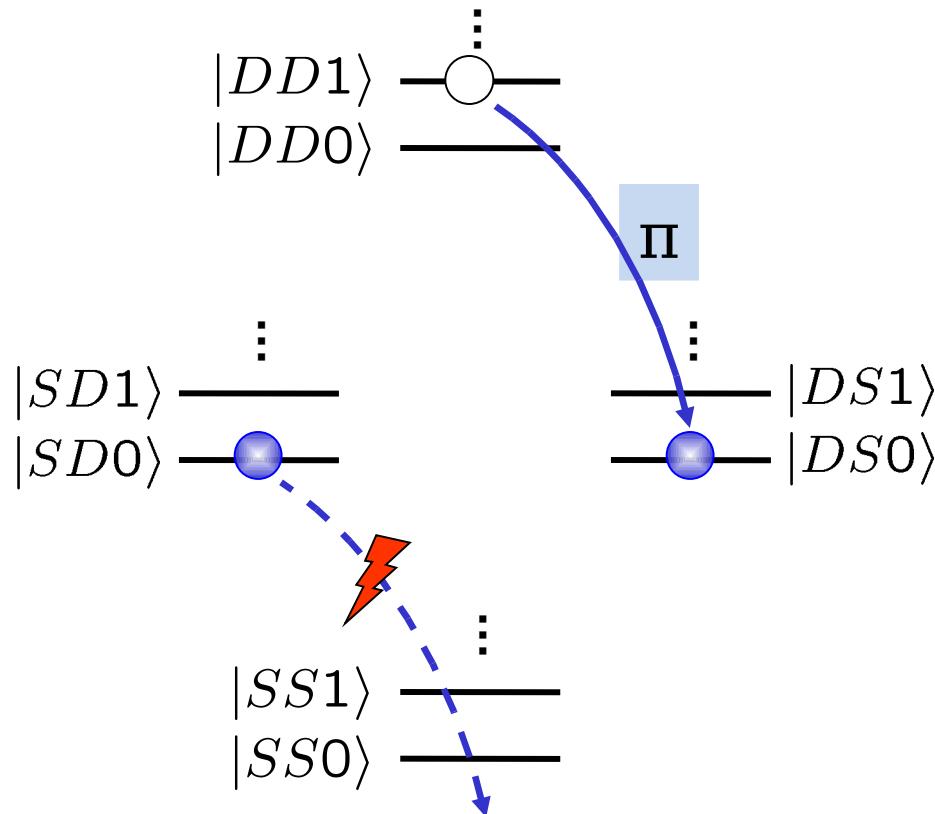




Generation of Bell states



Generation of Bell states



Bell states with atoms

- ${}^9\text{Be}^+$: NIST (fidelity: 97 %)
- ${}^{40}\text{Ca}^+$: Oxford (83%)
- ${}^{111}\text{Cd}^+$: Ann Arbor (79%)
- ${}^{25}\text{Mg}^+$: Munich (97%)
- ${}^{40}\text{Ca}^+$: Innsbruck (99%)



Analysis of Bell states

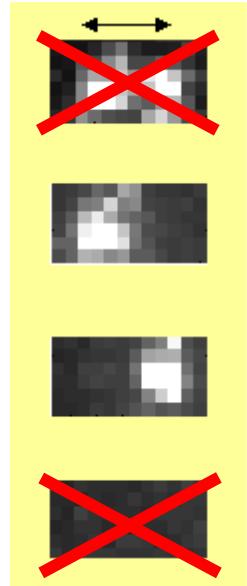
$$|SD\rangle + |DS\rangle$$

Fluorescence
detection with
CCD camera:

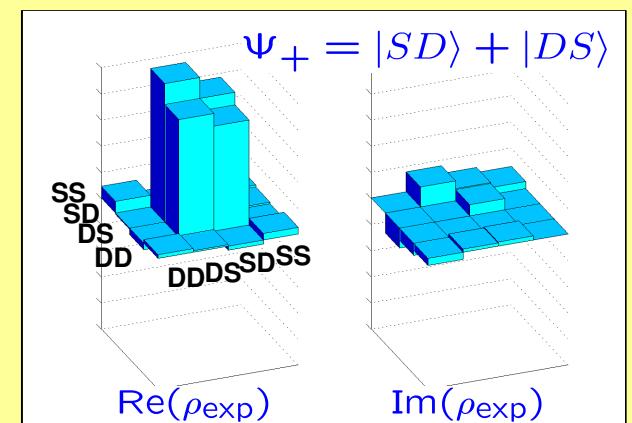
Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

$$\left\{ \begin{array}{l} |SS\rangle \\ |SD\rangle \\ |DS\rangle \\ |DD\rangle \end{array} \right.$$



→ Measurement of the density matrix:





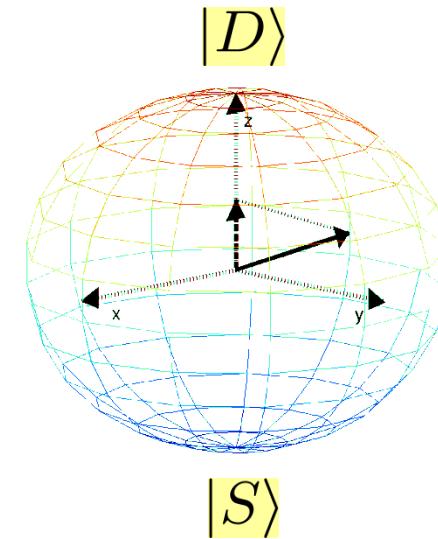
Measuring a density matrix



A measurement yields the z -component of the Bloch vector

=> Diagonal of the density matrix

$$\rho = \begin{pmatrix} P_S & C - iD \\ C + iD & P_D \end{pmatrix}$$





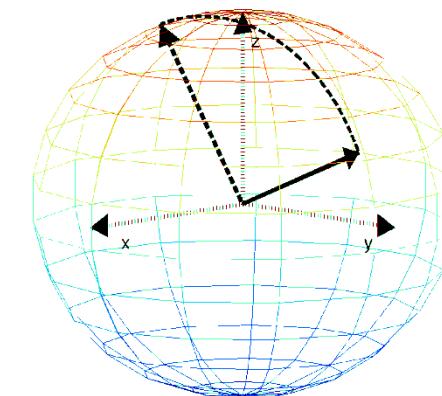
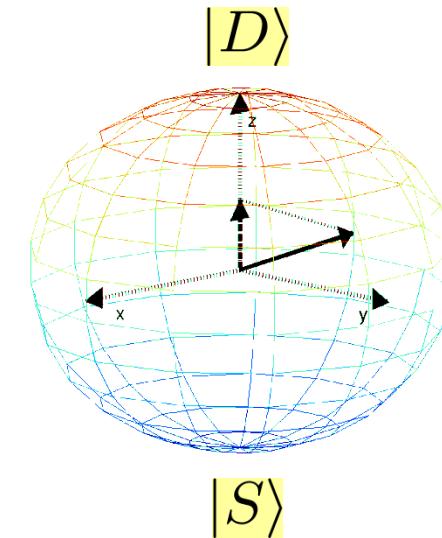
Measuring a density matrix



A measurement yields the z -component of the Bloch vector
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Rotation around the x - or the y -axis prior to the measurement yields the phase information of the qubit.





Measuring a density matrix



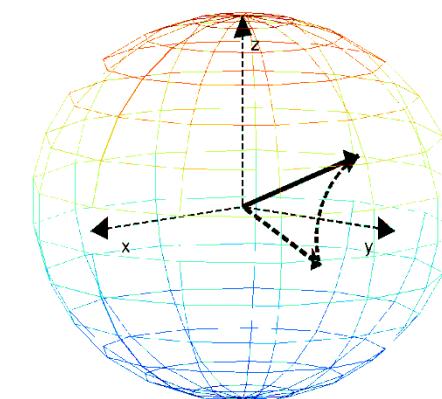
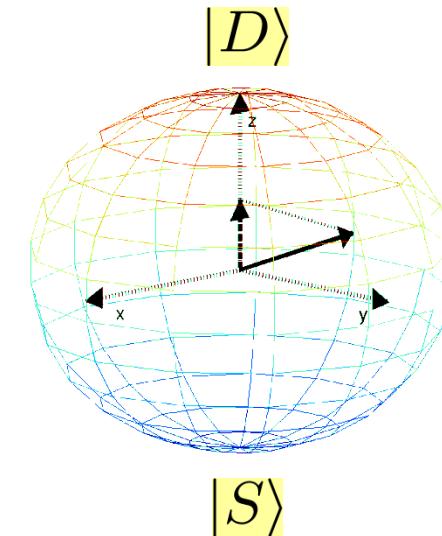
A measurement yields the z -component of the Bloch vector

=> Diagonal of the density matrix

$$\rho = \begin{pmatrix} P_S & \mathcal{C} - iD \\ \mathcal{C} + iD & P_D \end{pmatrix}$$

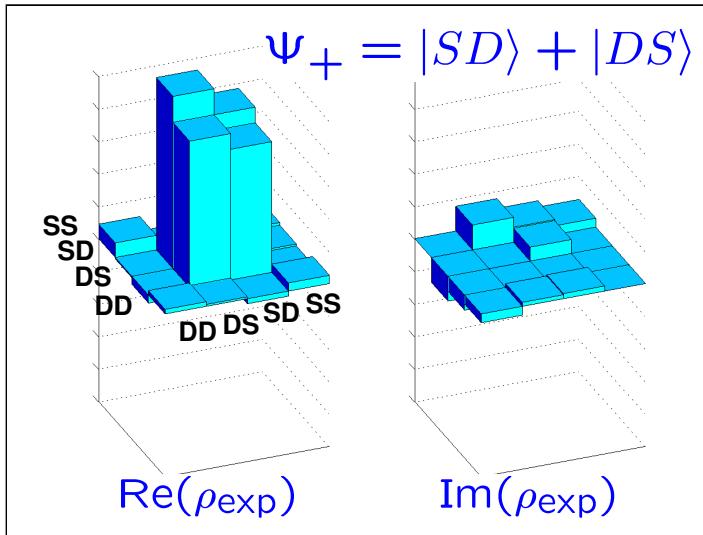
Rotation around the x - or the y -axis prior to the measurement yields the phase information of the qubit.

=> coherences of the density ma



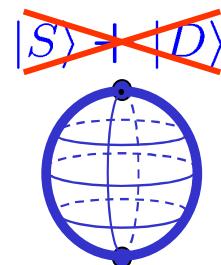
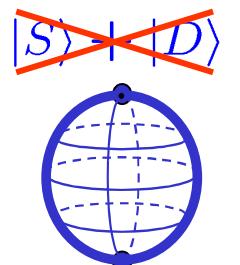


Entanglement



States are fully characterized
Fidelities: up to 0.99

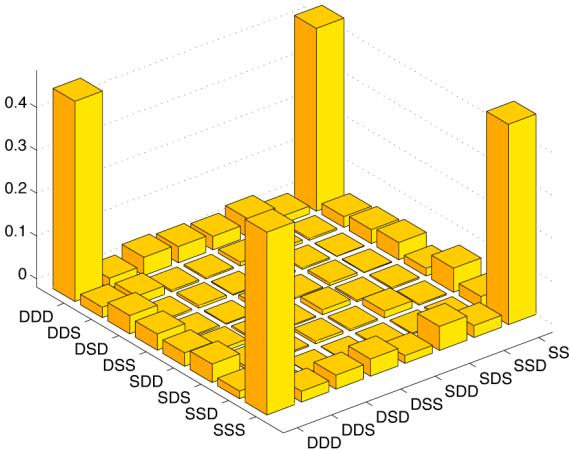
Entanglement: the state of each qubit is not defined!



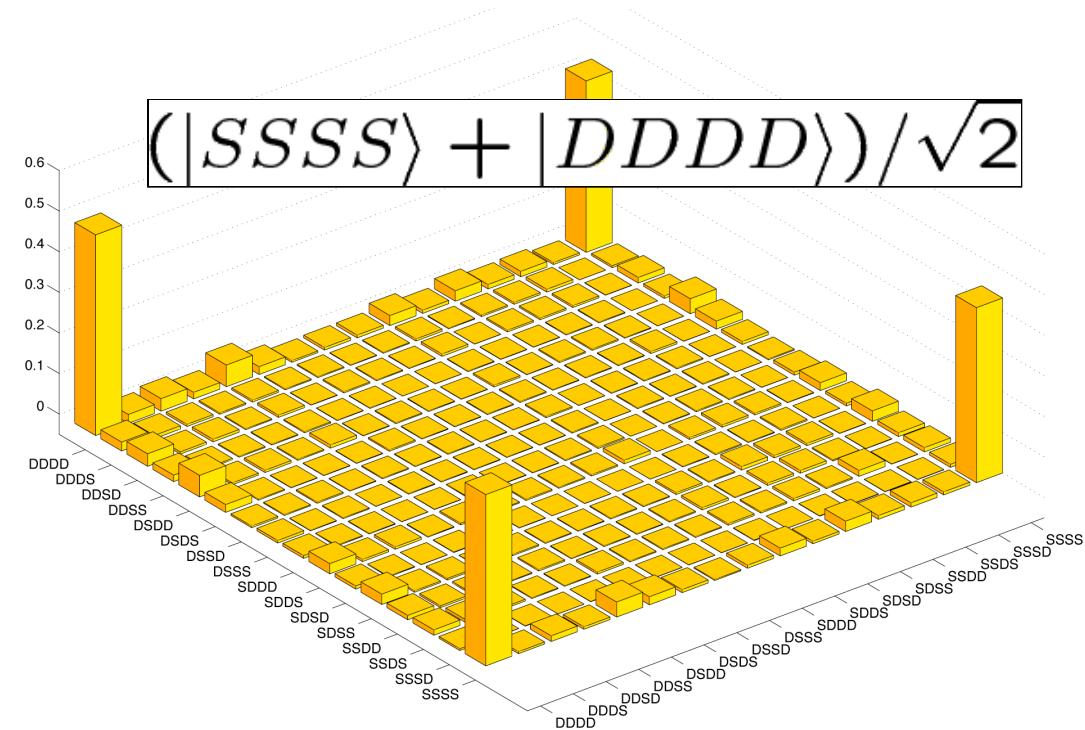


More entangled states

$$(|SSS\rangle + |DDD\rangle)/\sqrt{2}$$



$$(|SSSS\rangle + |DDDD\rangle)/\sqrt{2}$$

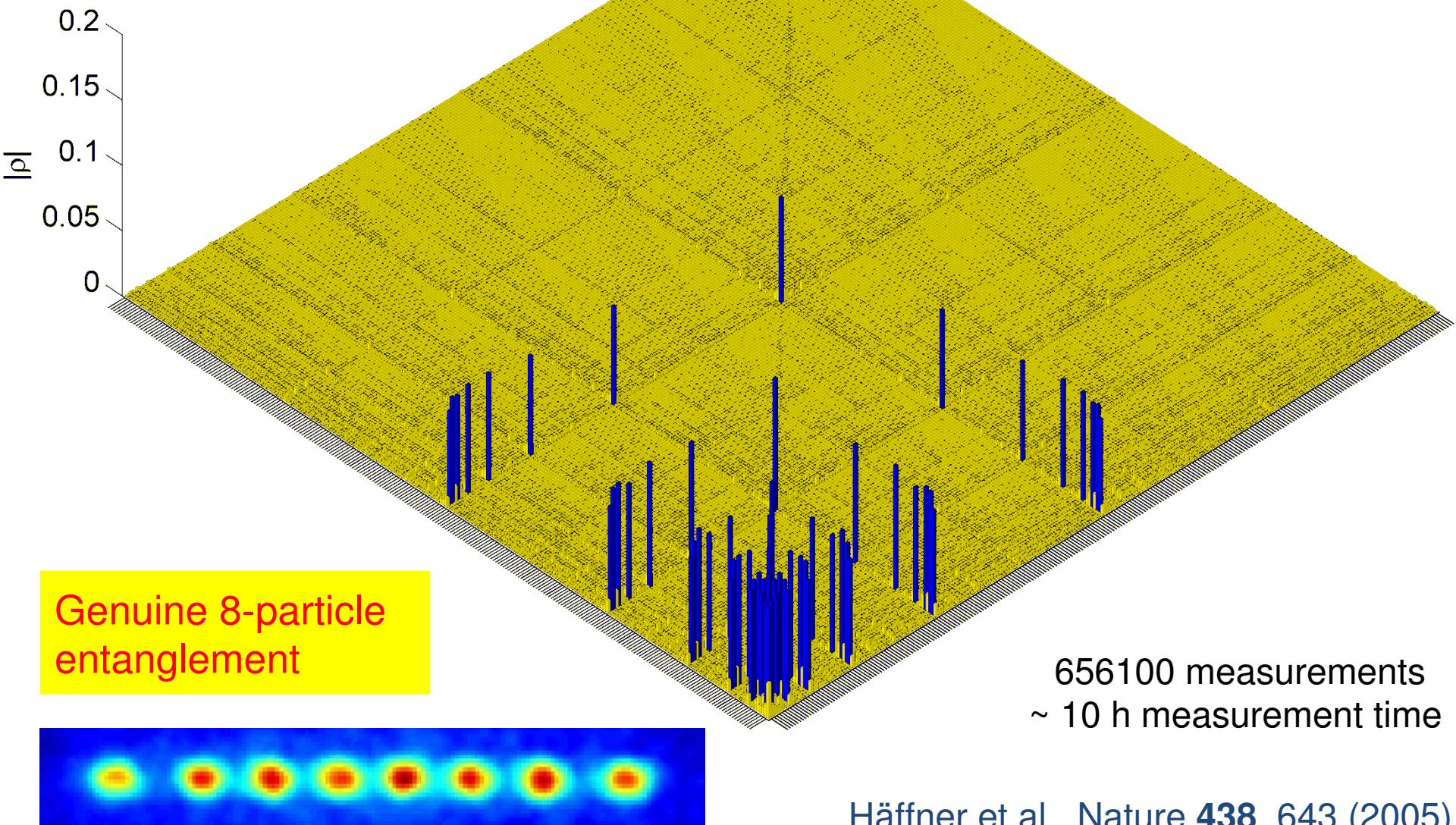




“Large” entangled states



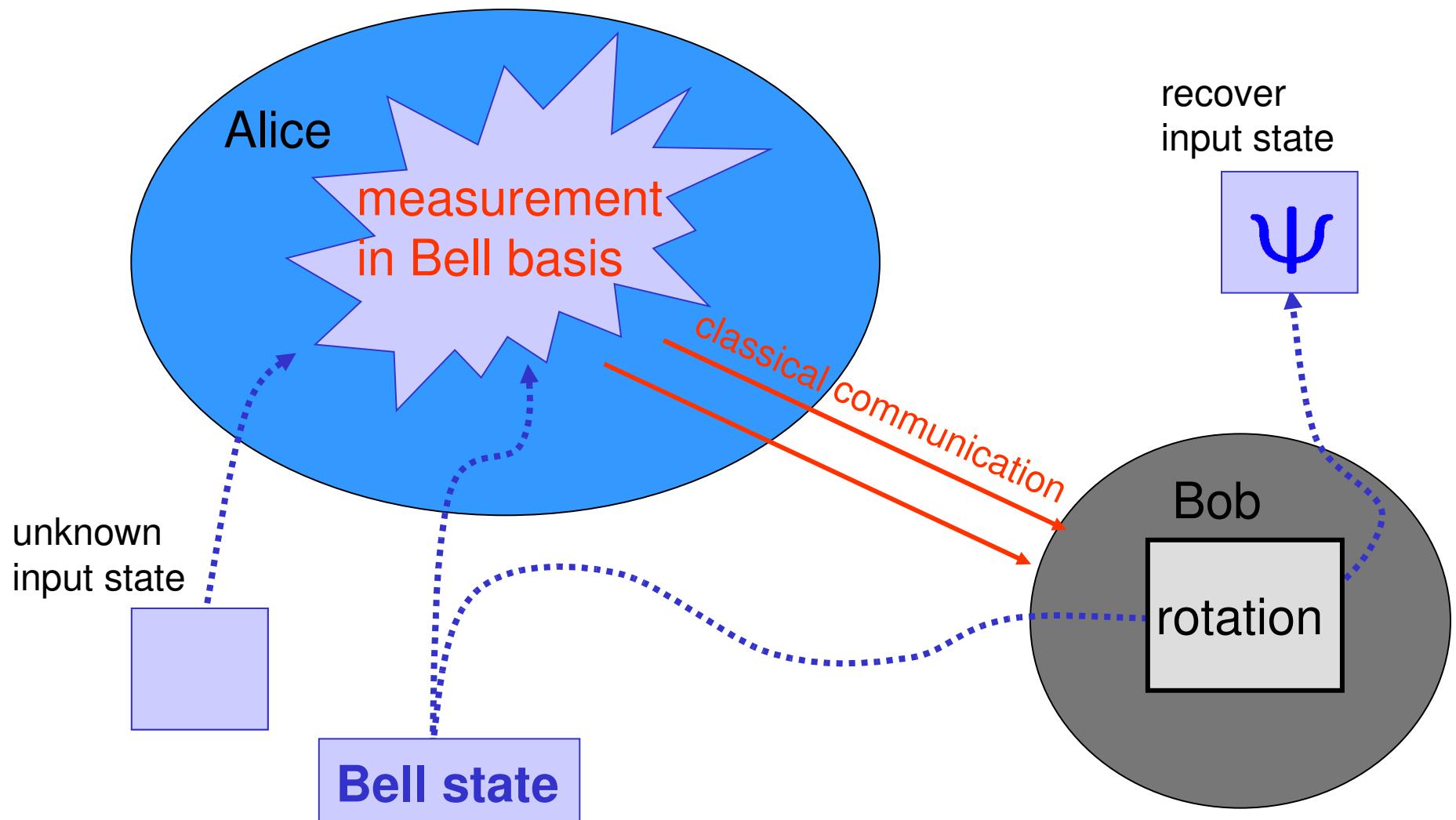
$$\frac{1}{\sqrt{8}}(|DDDDDDDDS\rangle + |DDDDDDDS\rangle + \dots + |SDDDDDDD\rangle)$$



- Introduction to quantum information
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- Wiring up trapped ions

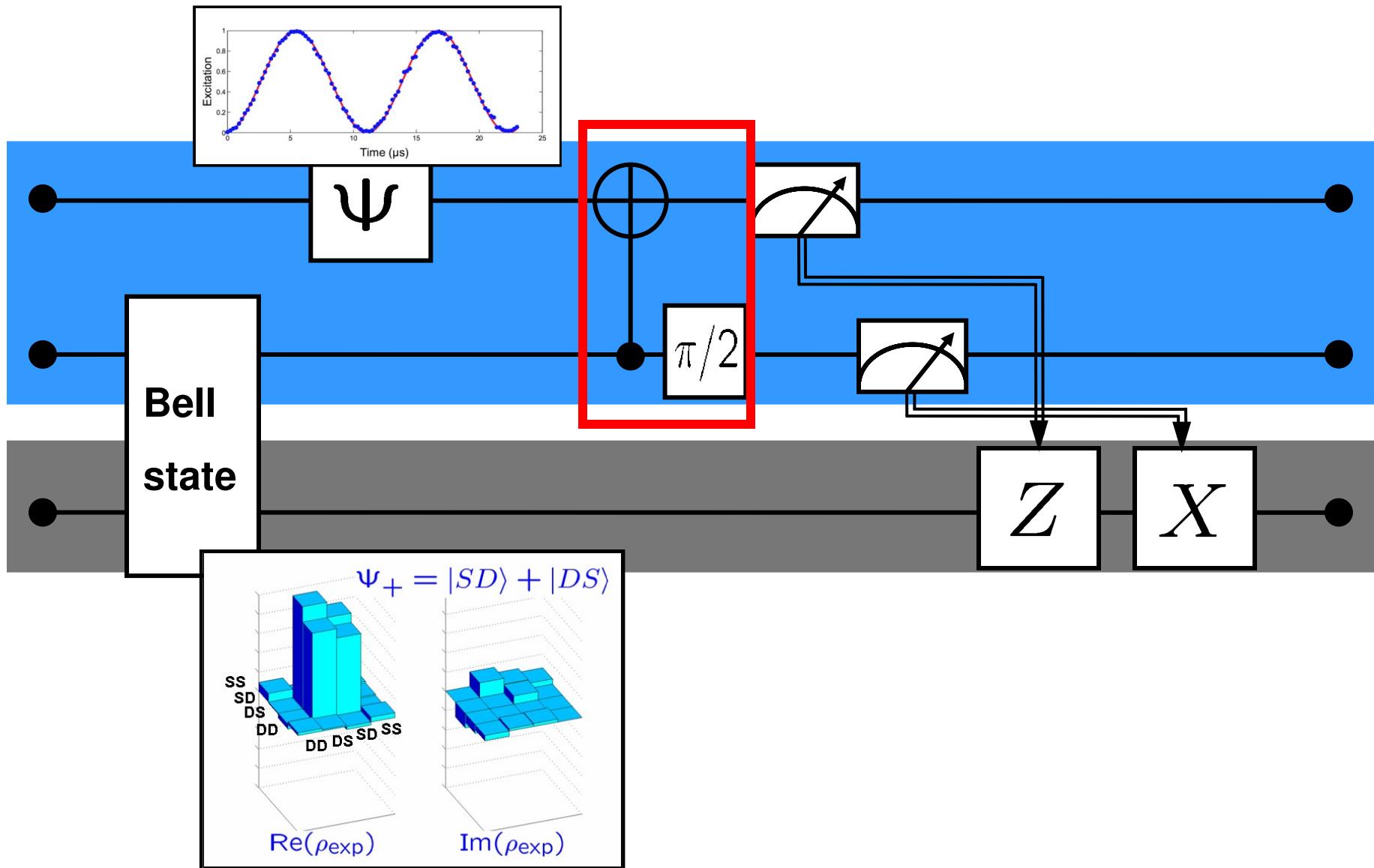


Teleportation



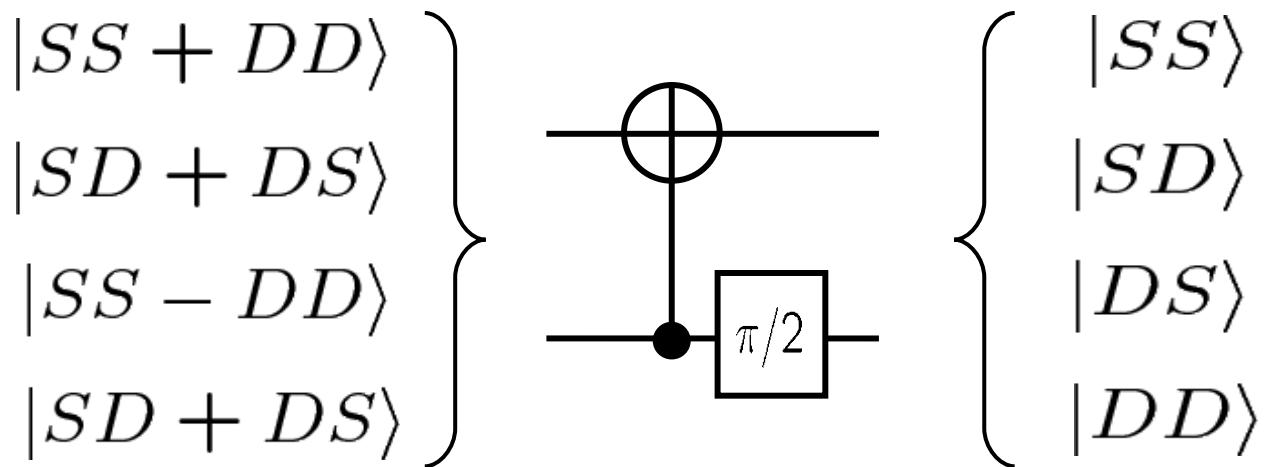


Teleportation





Bell measurement





Ion trap quantum computing



VOLUME 74, NUMBER 20

PHYSICAL REVIEW LETTERS

15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

PACS numbers: 89.80.+h, 03.65.Bz, 12.20.Fv, 32.80.Pj

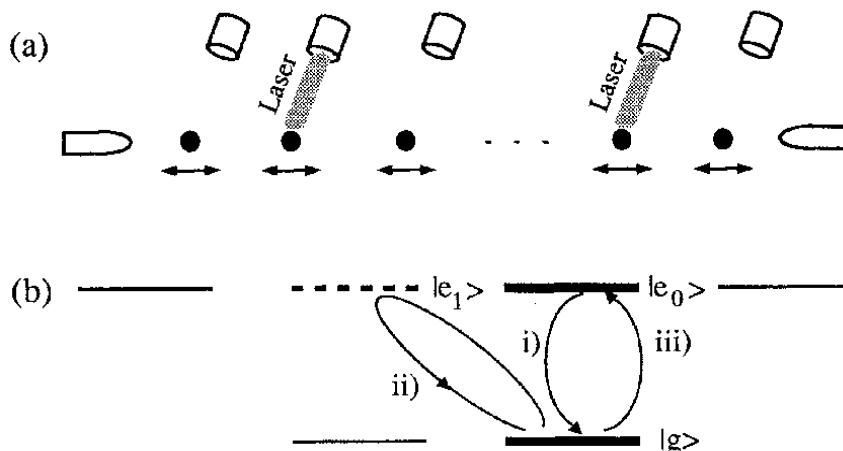


FIG. 1. (a) N ions in a linear trap interacting with N different laser beams; (b) atomic level scheme.

controlled – NOT :

$$|\varepsilon_1\rangle|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle|\varepsilon_1 \oplus \varepsilon_2\rangle$$

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$

$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$

$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$

$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

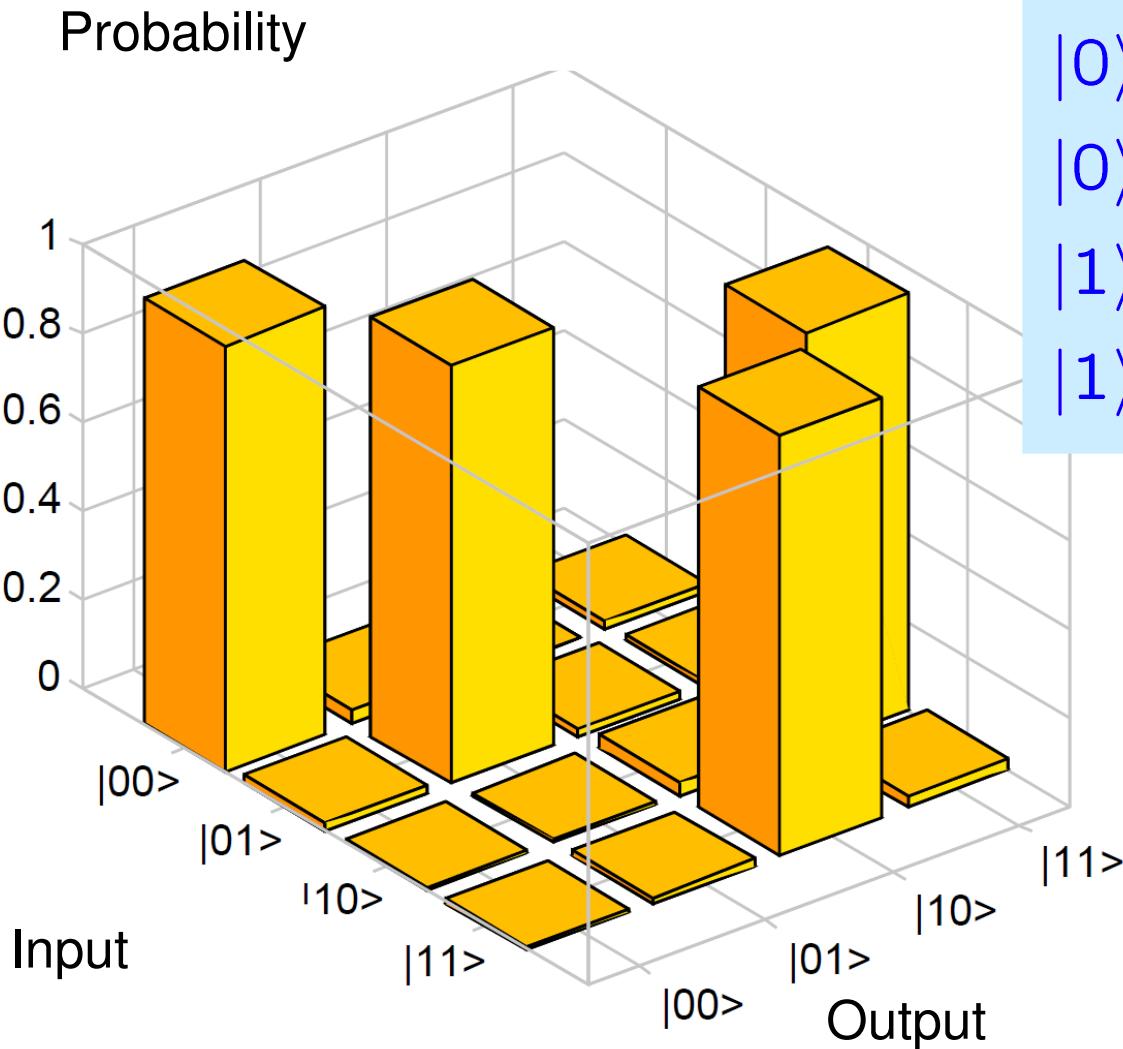
control bit target bit

other gate proposals (and more):

- Cirac & Zoller
- Mølmer & Sørensen, Milburn
- Jonathan & Plenio & Knight
- Geometric phases
- Leibfried & Wineland



Truth table of a controlled NOT gate



$ 0\rangle 0\rangle$	\rightarrow	$ 0\rangle 0\rangle$
$ 0\rangle 1\rangle$	\rightarrow	$ 0\rangle 1\rangle$
$ 1\rangle 0\rangle$	\rightarrow	$ 1\rangle 1\rangle$
$ 1\rangle 1\rangle$	\rightarrow	$ 1\rangle 0\rangle$

→ universal set of quantum gates



Another way to generate Bell states

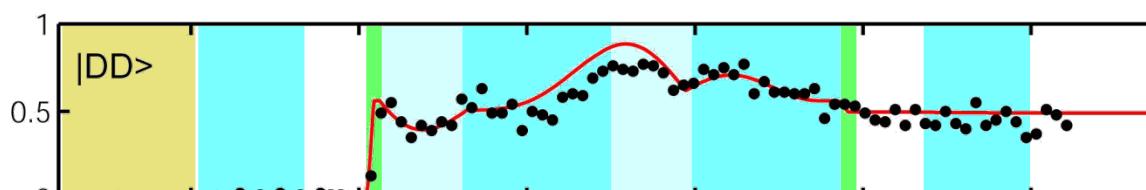
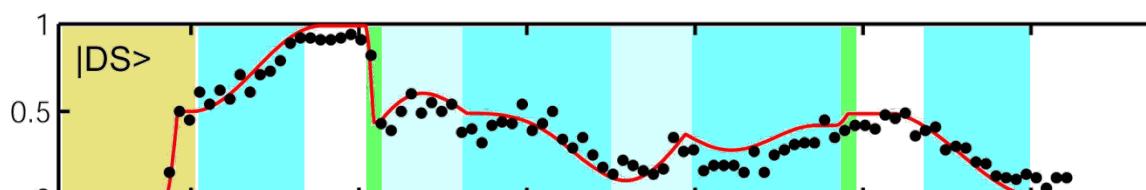
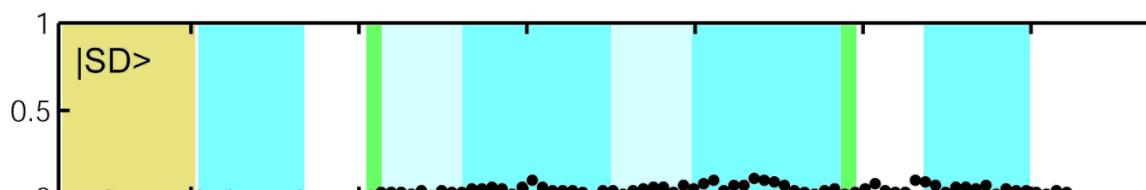
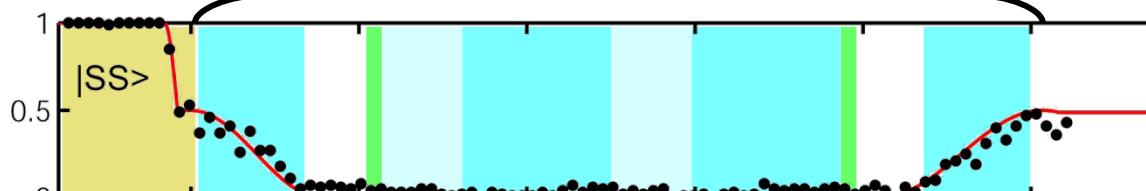


$$|SS\rangle \rightarrow |S+D\rangle|S\rangle \quad \xrightarrow{\text{CNOT}} \quad |SS\rangle + |DD\rangle$$

prepare

CNOT

output

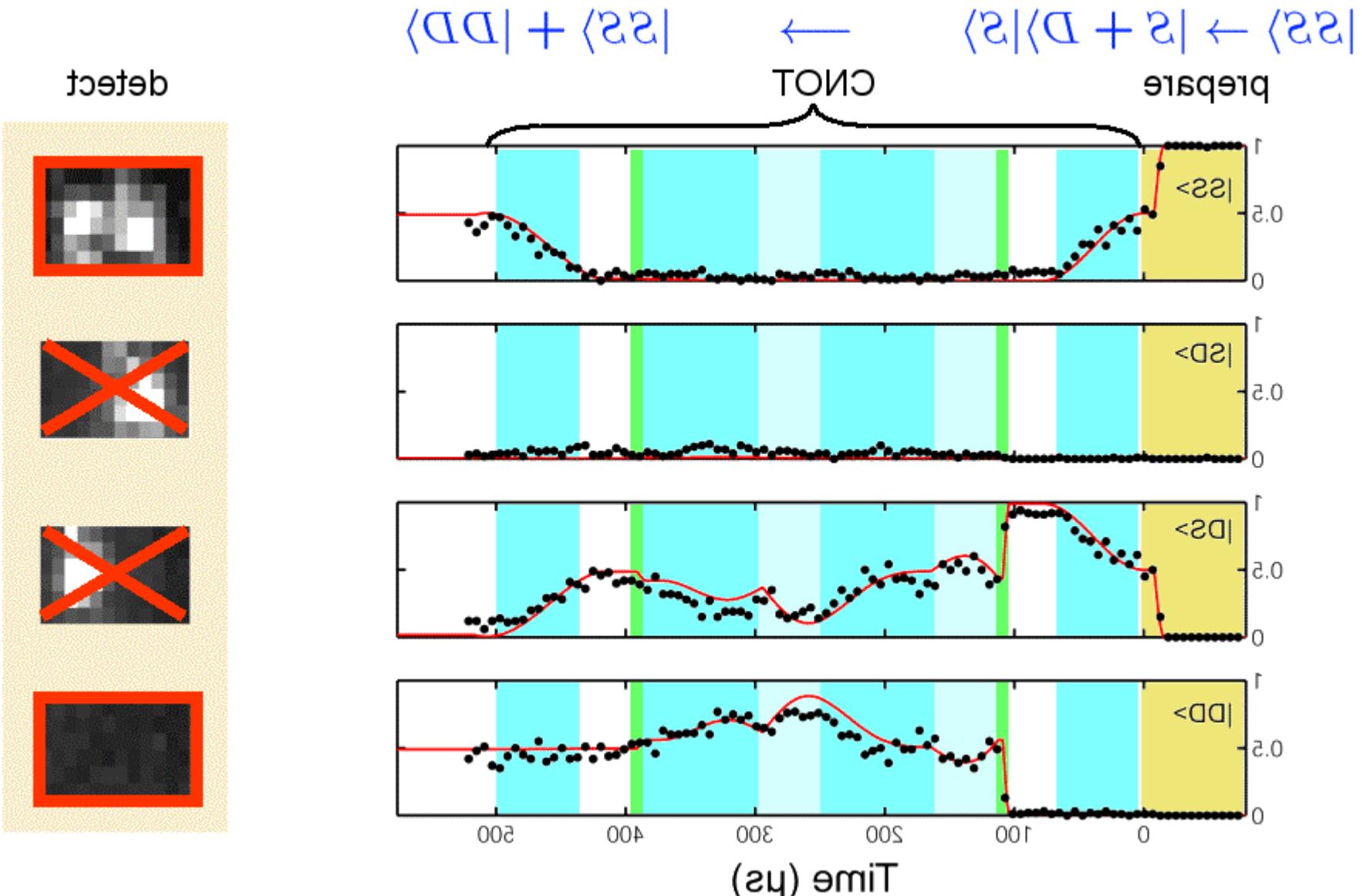


Time (μs)



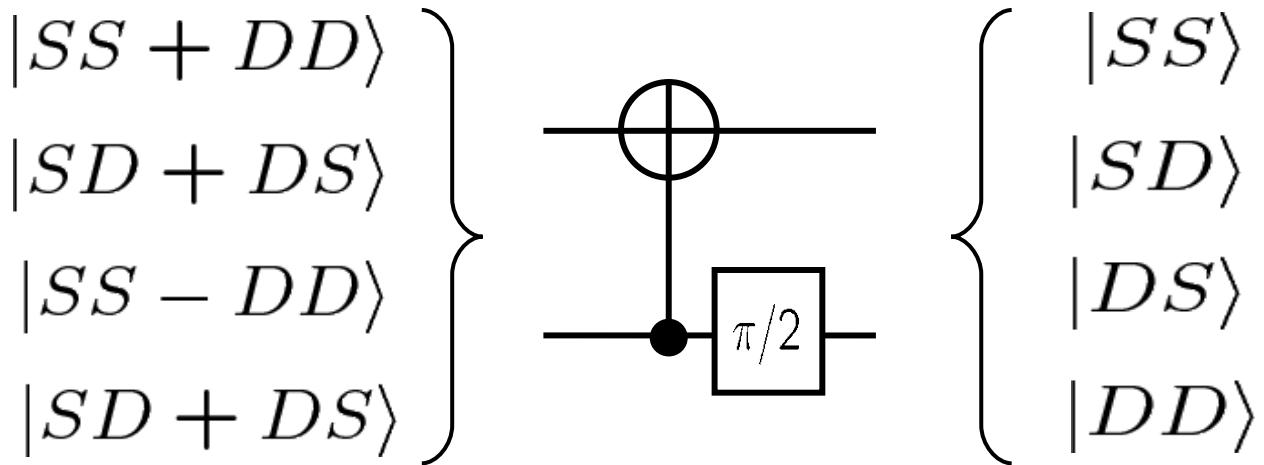


And now backwards



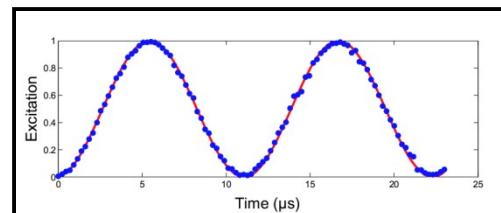
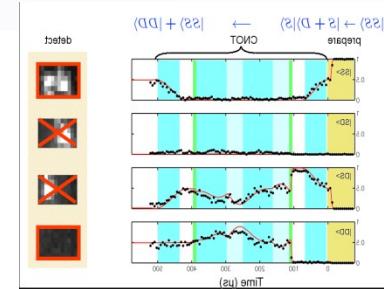
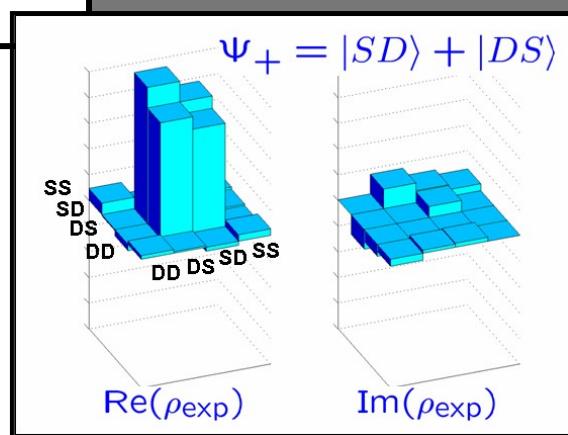
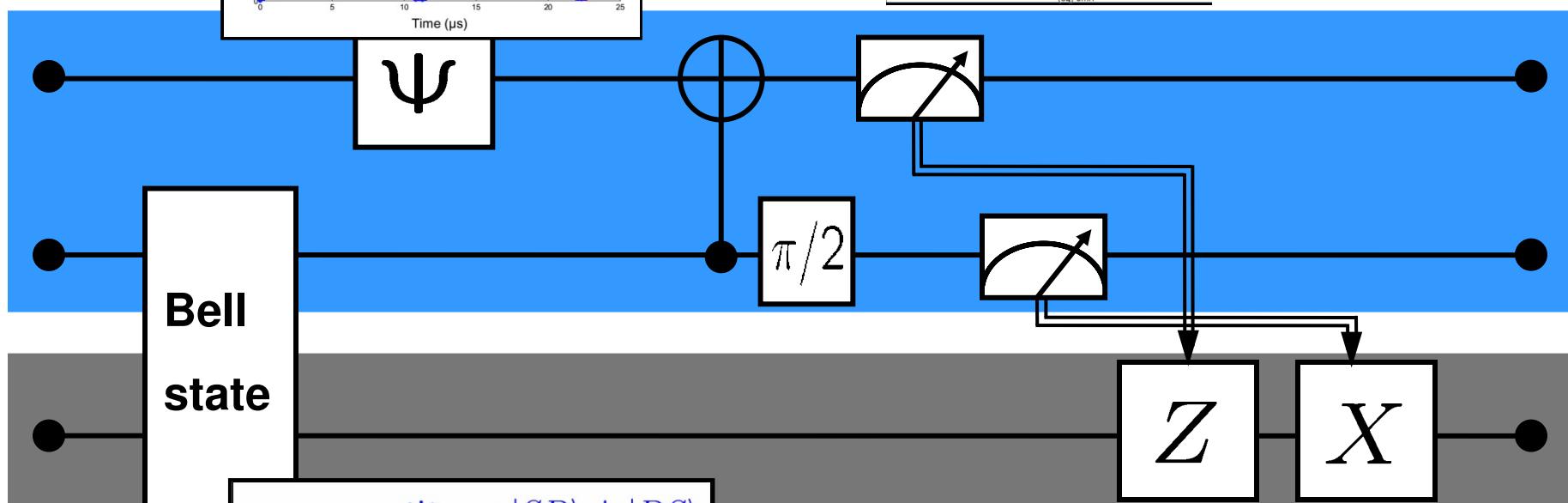


Bell measurement



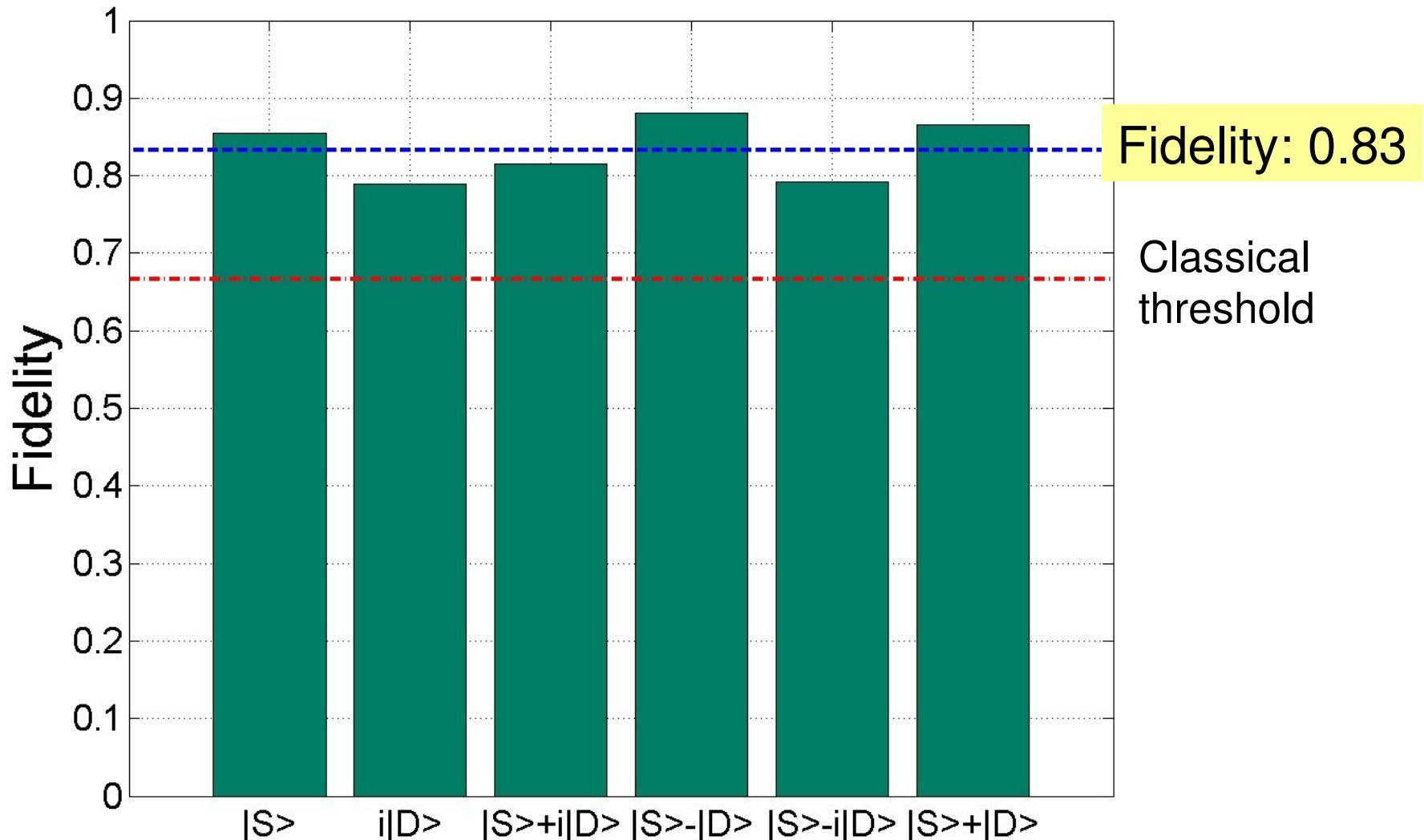


Teleportation





Deterministic teleportation

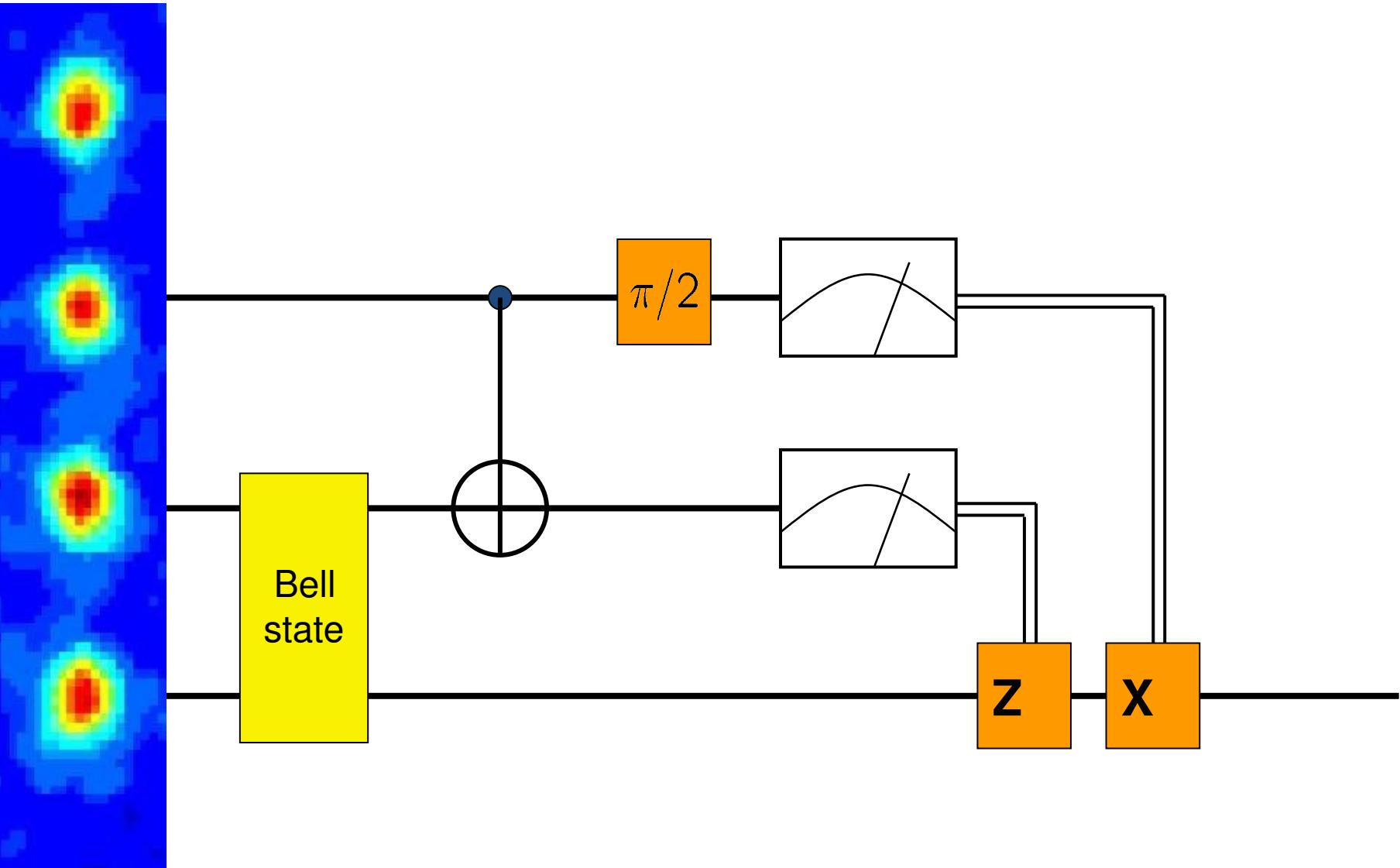


“Deterministic teleportation with atoms”

Barrett et al., Nature **429**, 737 (2004) and Riebe et al., Nature **429**, 734 (2004)

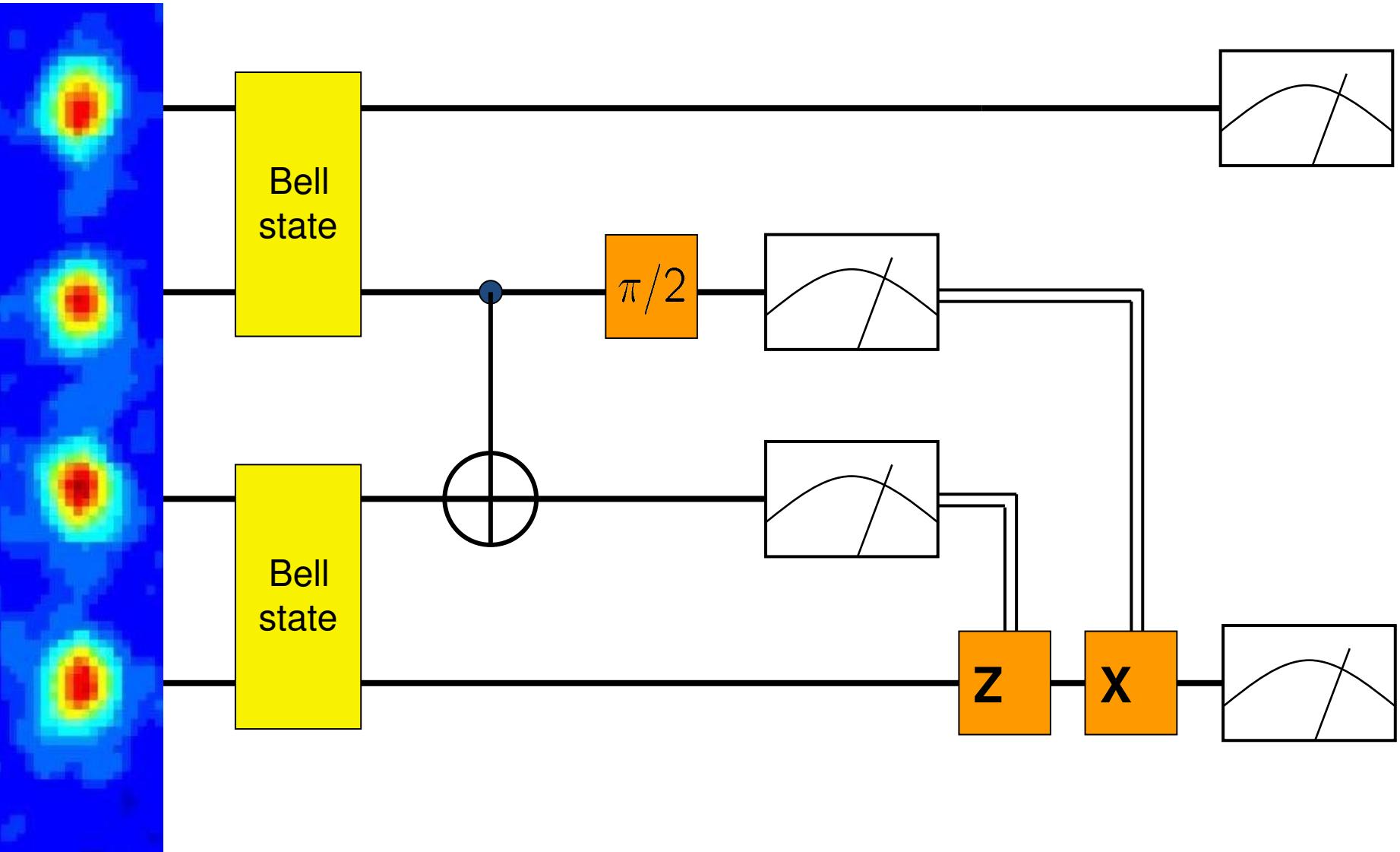


Teleporting entanglement



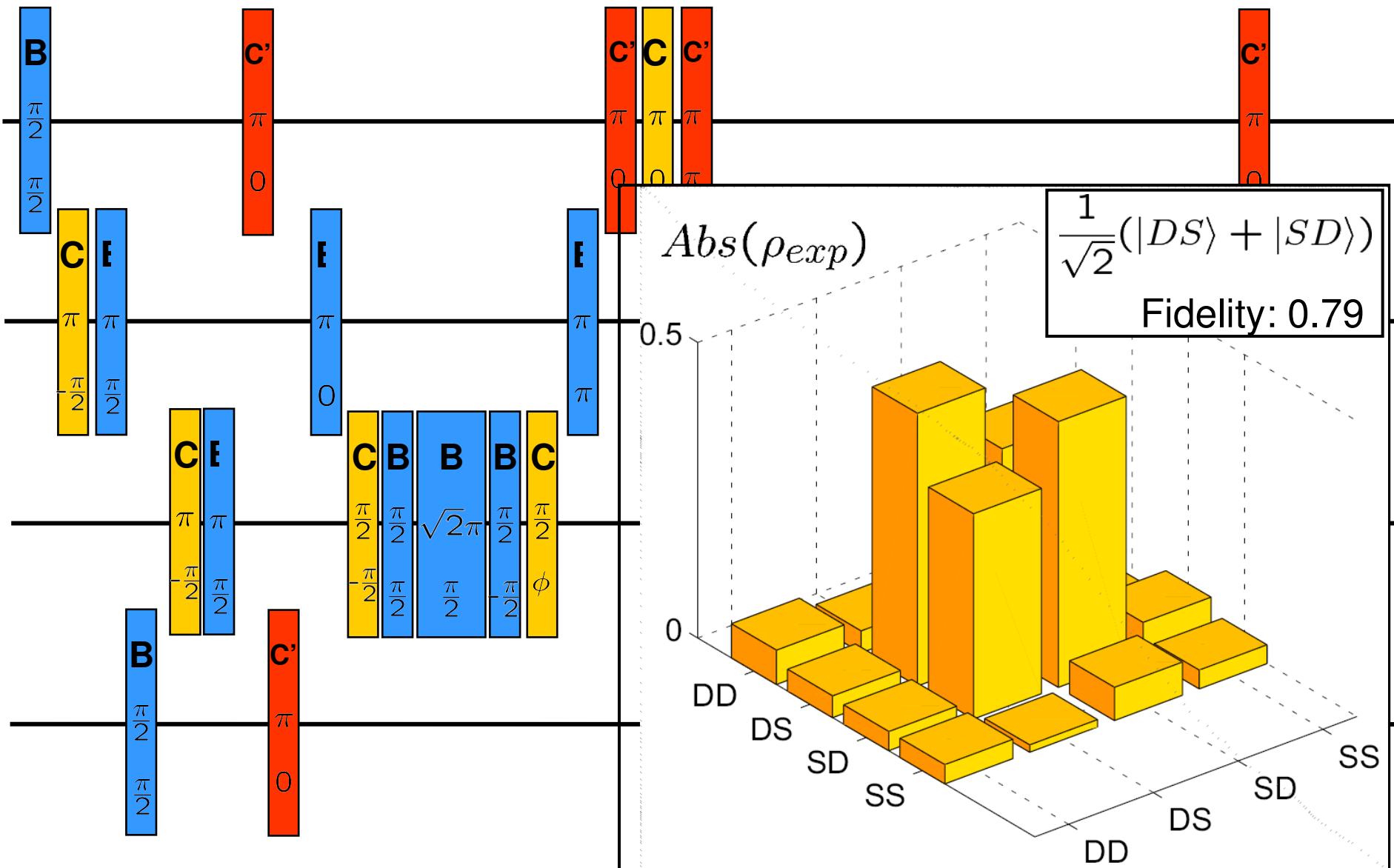


Teleporting entanglement



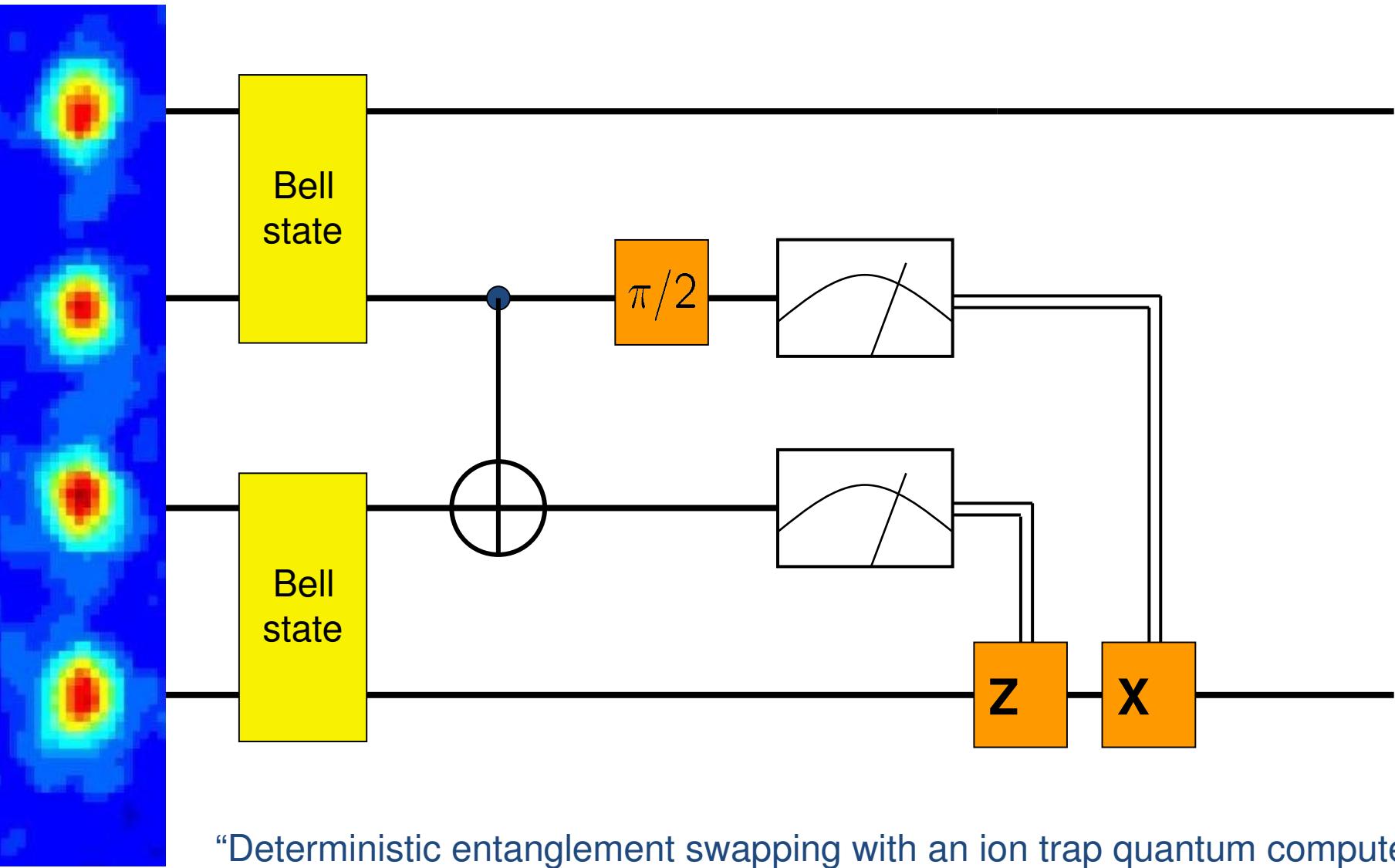


Teleporting entanglement





Entanglement swapping



“Deterministic entanglement swapping with an ion trap quantum computer”

Riebe et al., Nature Physics 4, 839 (2008)



Requirements for quantum computing



Classical computer

- Initialization
- 1-bit operations (NOT)
- 2-bit gates (e.g. NAND)

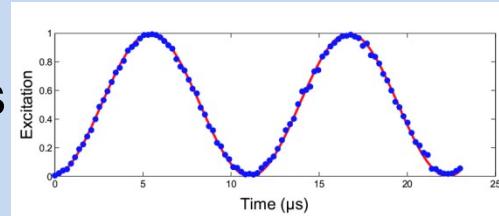
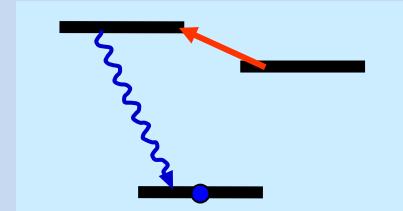
Computational space:

00
01
10
11

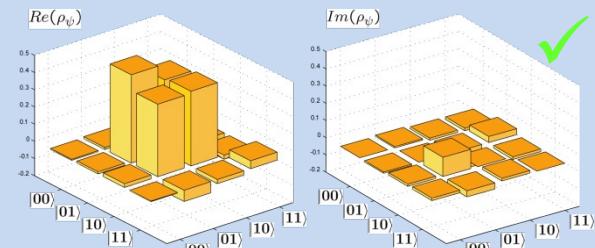
- Read out
→ result

Quantum computer

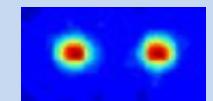
- Initialization
- 1-qubit rotations
→ superpositions
- 2-qubit gates (CNOT gate)
→ entanglement



Computational space:



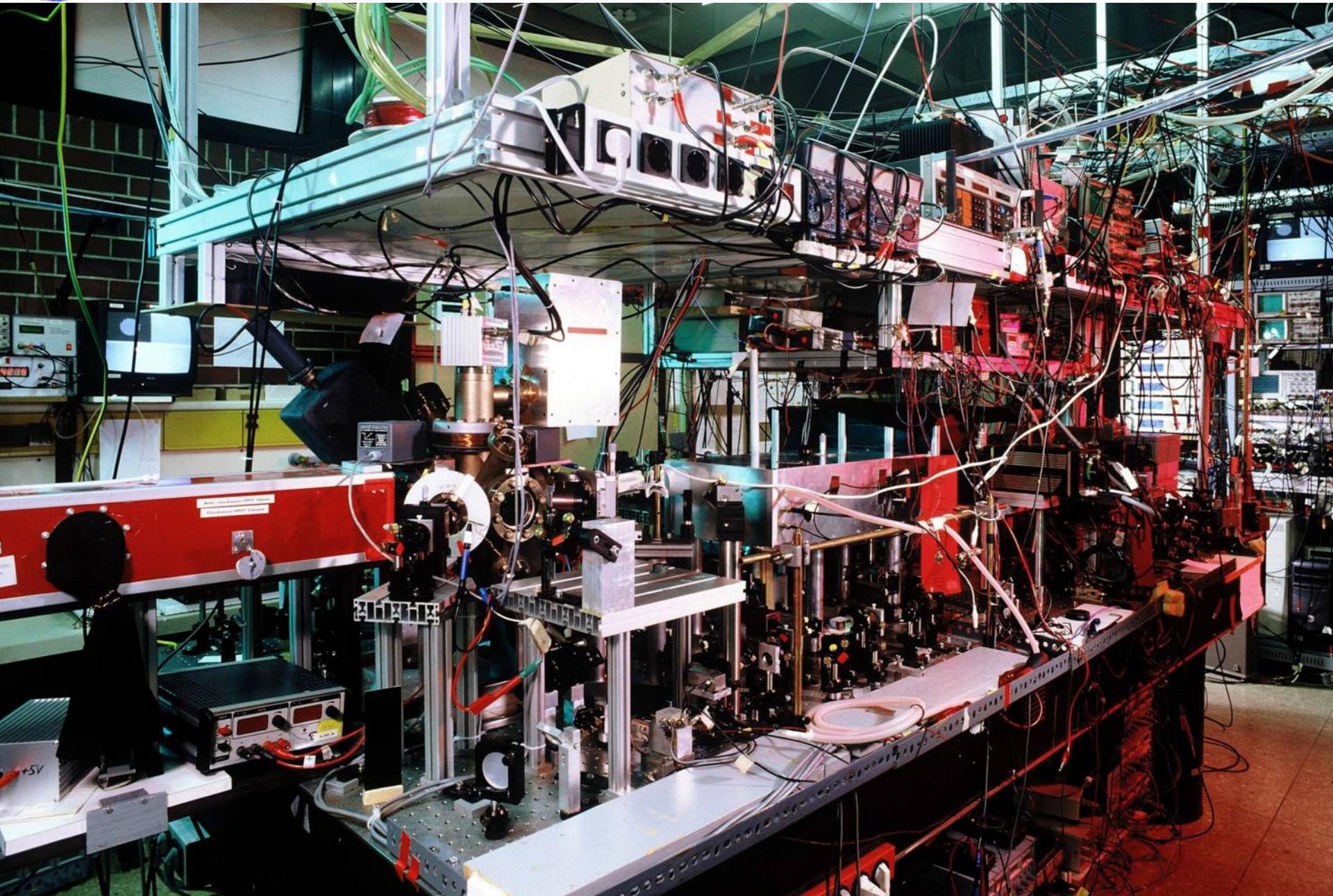
- Read out of qubits
→ gain of classical information



- Introduction to quantum information
- Ion trap quantum computing
- Teleportation and more
- Scaling of ion trap quantum computers
- Wiring up trapped ions

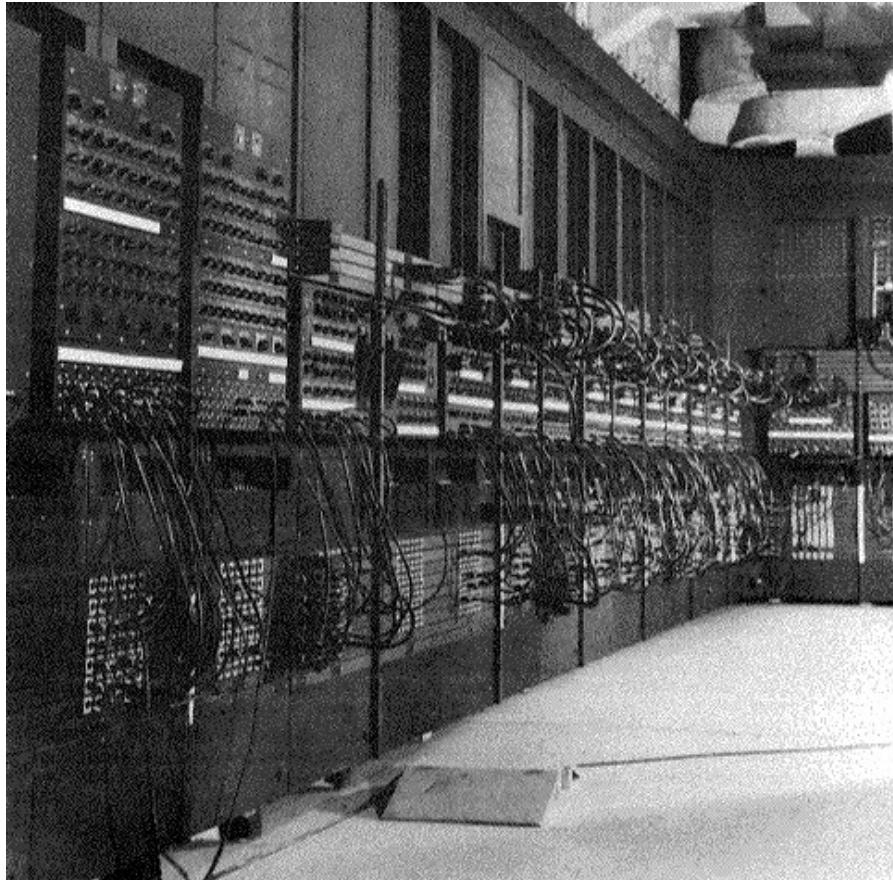


The hardware





The fate of visionaries



20th century
about the ENIAC:

„Where a calculator on the ENIAC is equipped with 18000 vacuum tubes and weighs 30 tons, computers in the future may have only 1000 tubes and weigh only 1 ½ tons.“

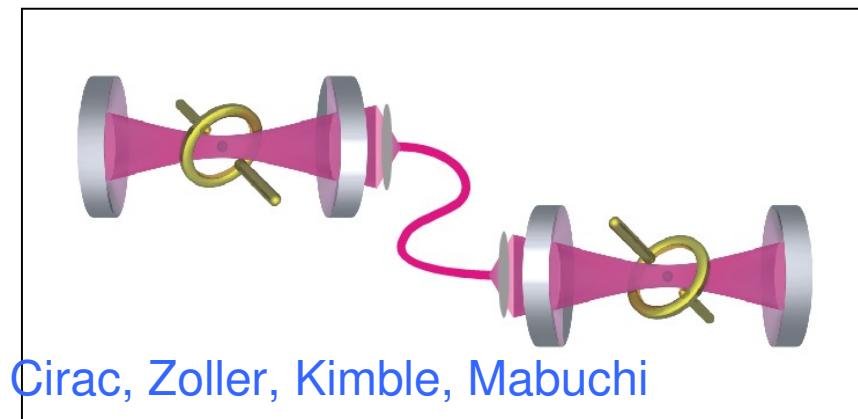
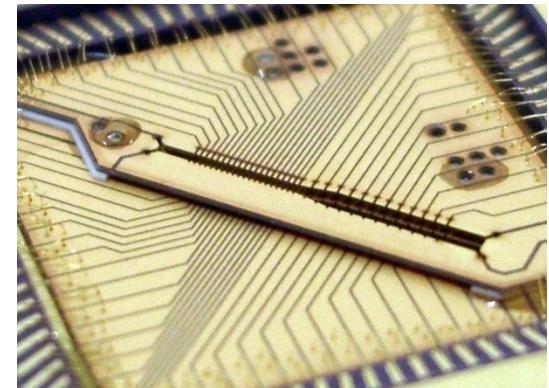
Popular Mechanics, March 1949



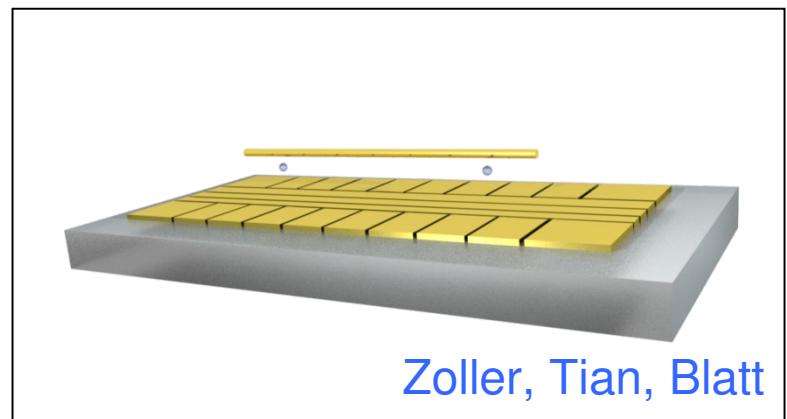
Scaling of ion trap quantum computers



Kielpinski, Monroe, Wineland



Cirac, Zoller, Kimble, Mabuchi



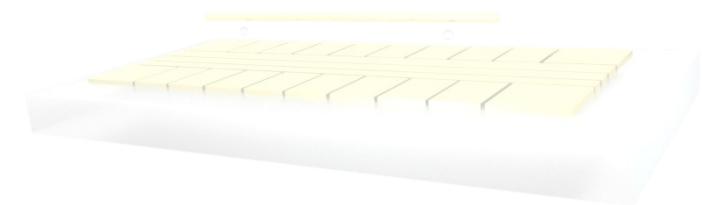
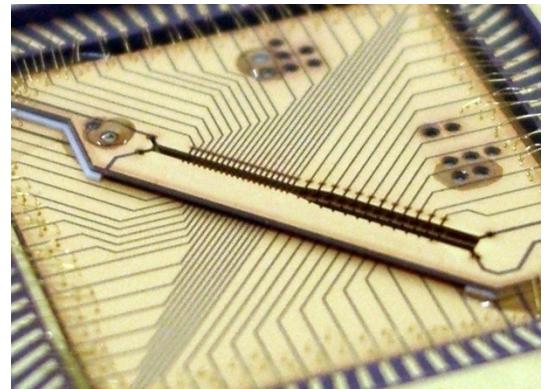
Zoller, Tian, Blatt



Scaling of ion trap quantum computers



Kielpinski, Monroe, Wineland

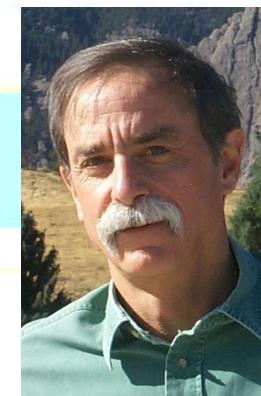




Scaling ion trap quantum computing

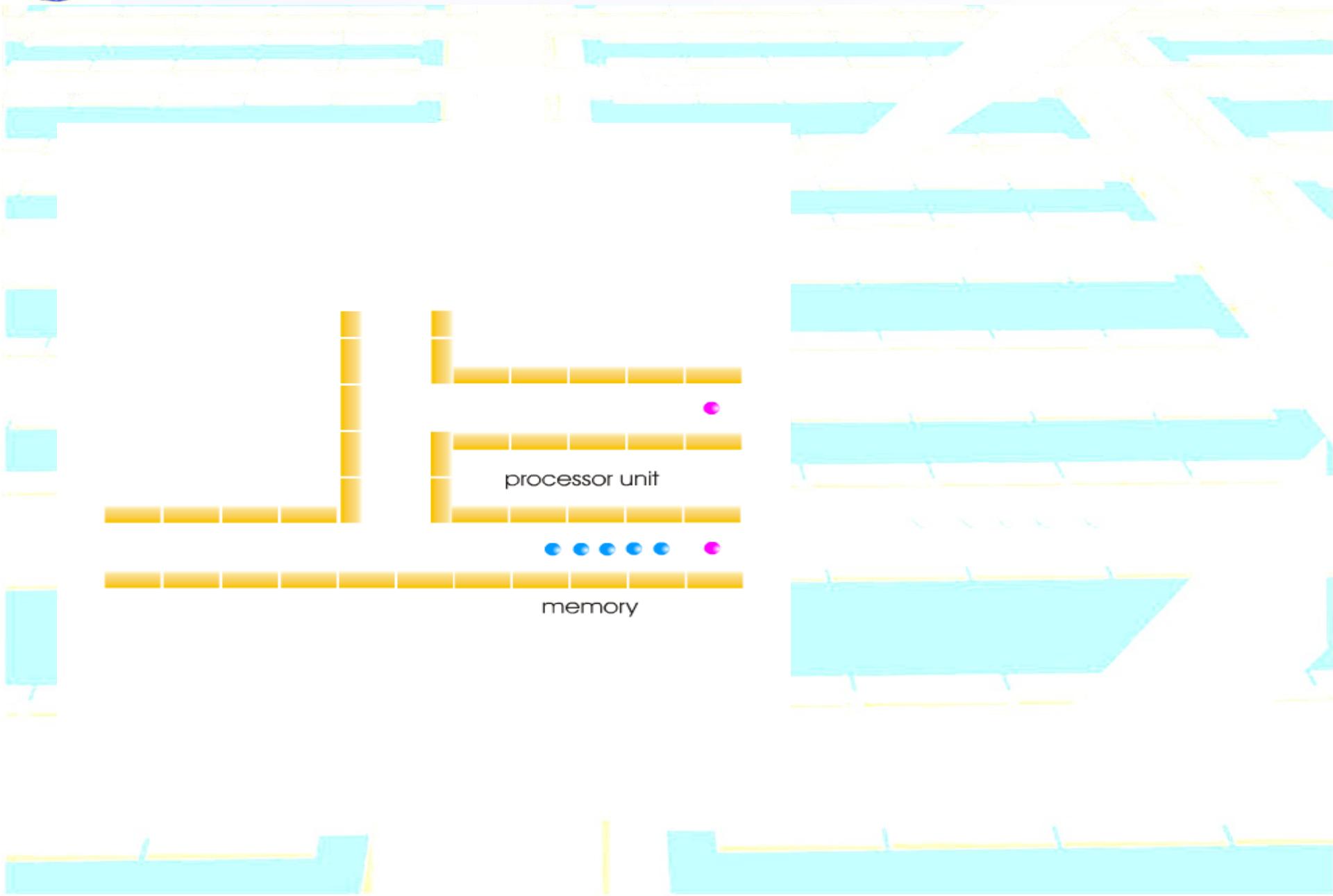


D. Leibfried, D. Wineland et al., NIST



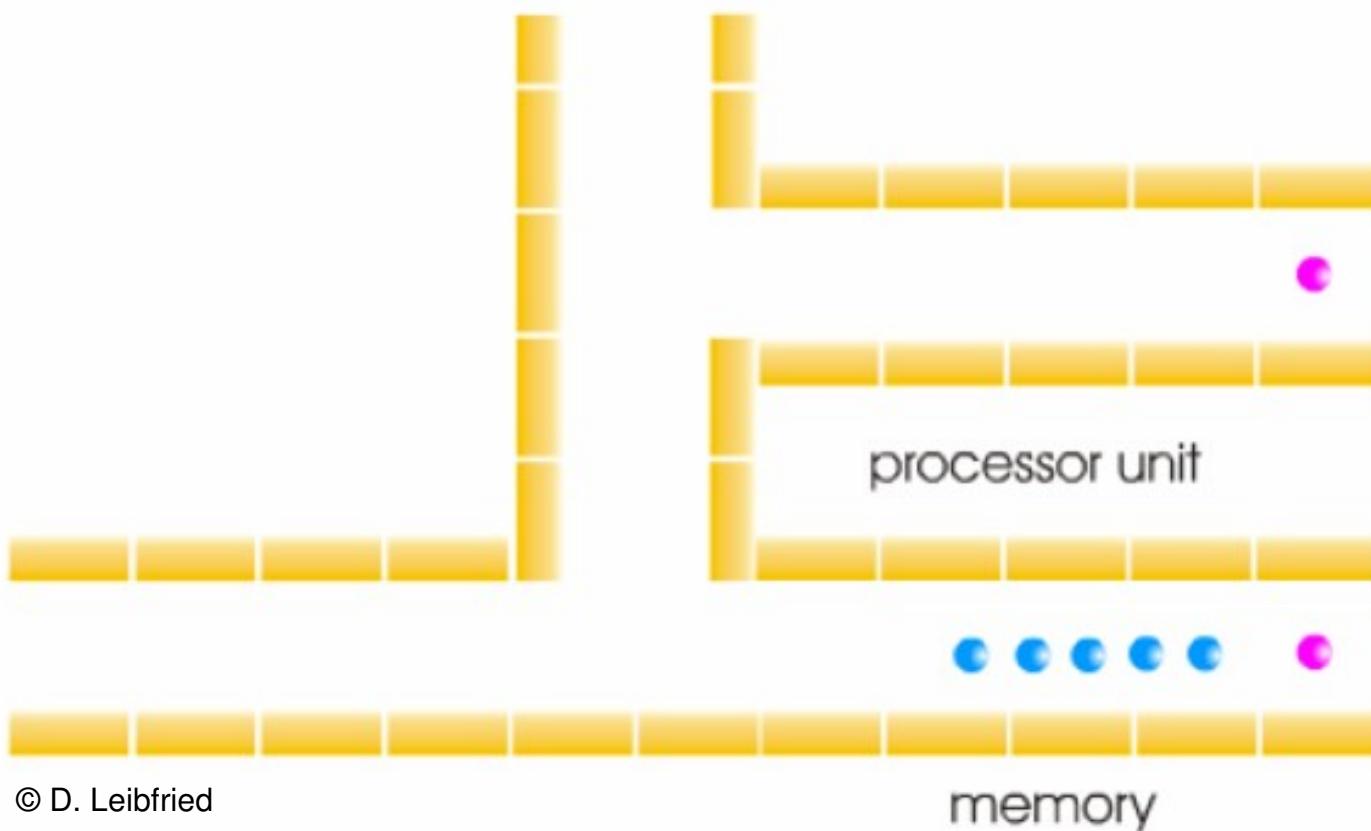


Scaling ion trap quantum computing





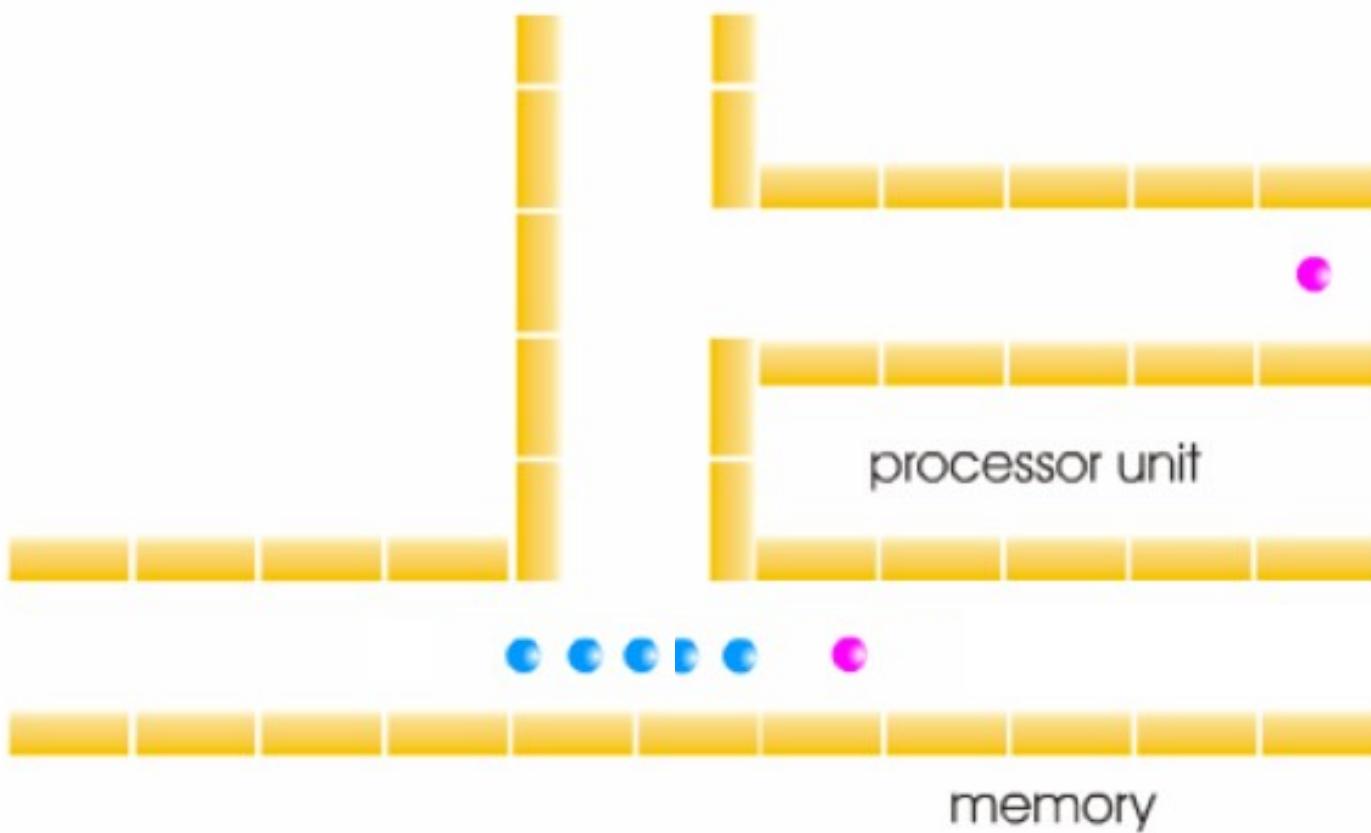
Scaling of ion trap quantum computing



© D. Leibfried

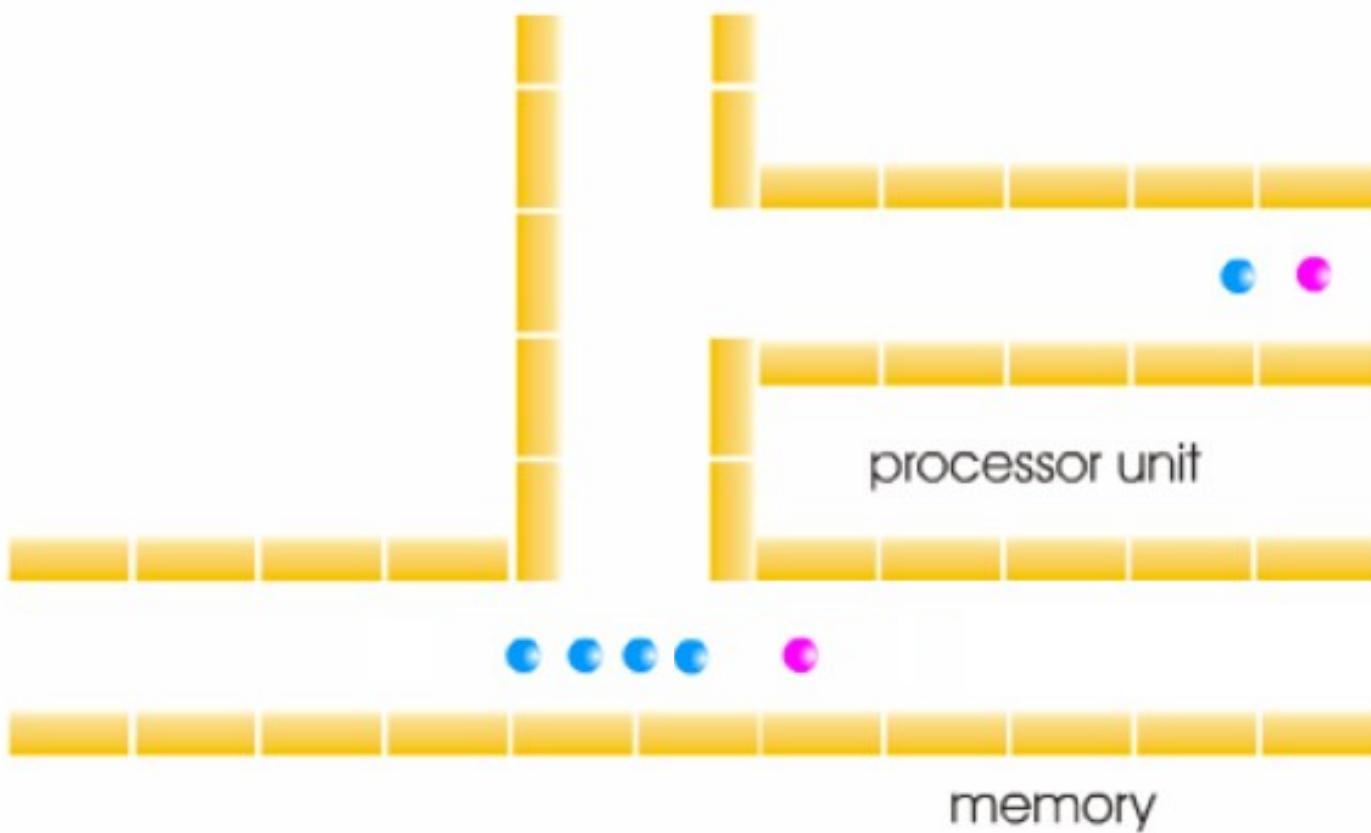


Scaling of ion trap quantum computing



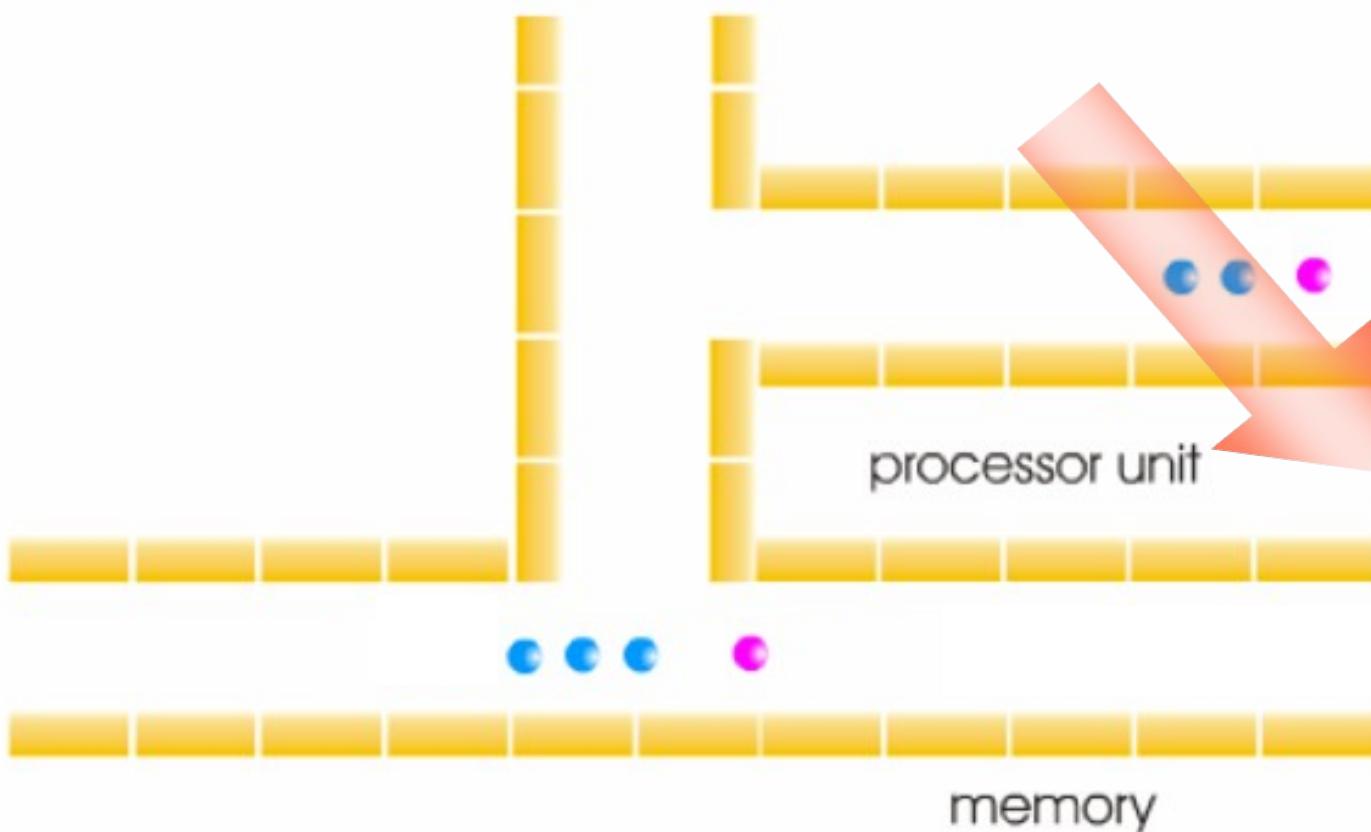


Scaling of ion trap quantum computing



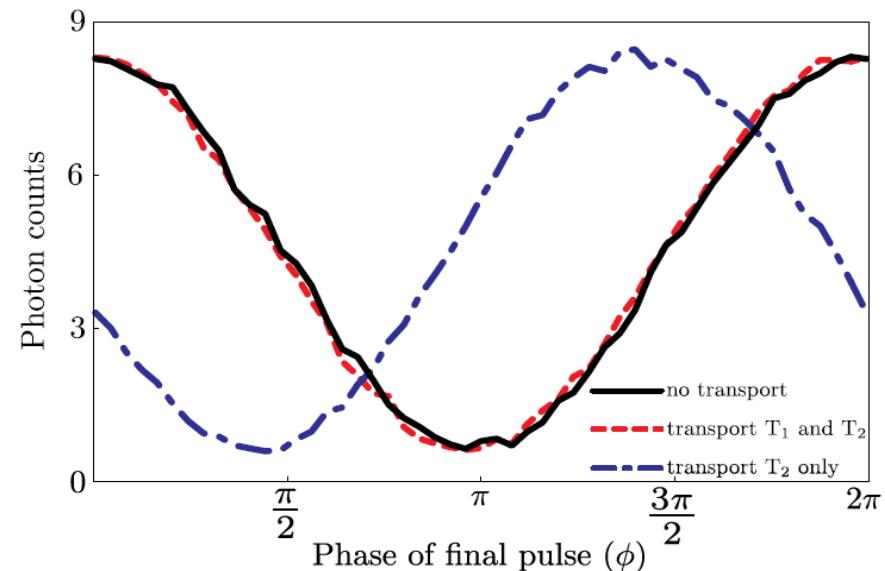
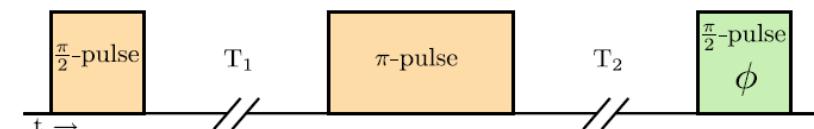
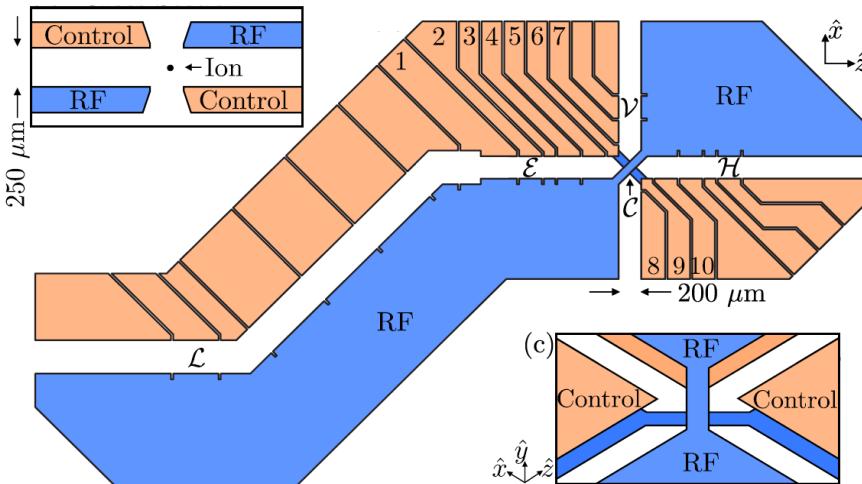


Scaling of ion trap quantum computing

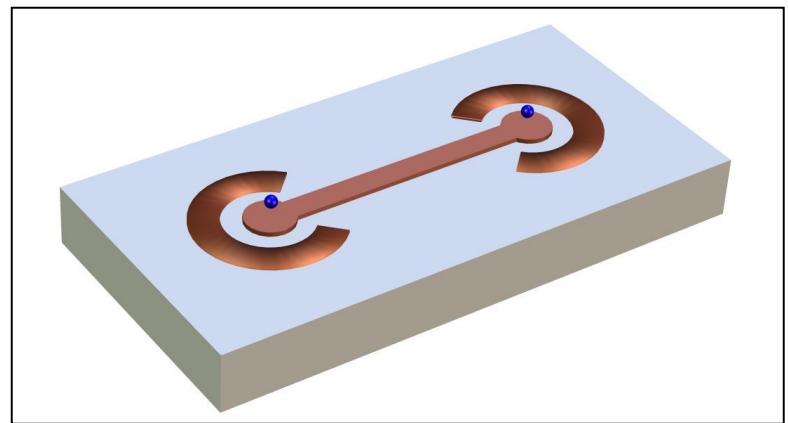


„Architecture for a large-scale ion-trap quantum computer“,
D. Kielpinski et al., Nature **417**, 709 (2002).

Coherent transport through a junction



Transport	Energy Gain (recooling method)	
	quanta/ion	quanta/trip
$\mathcal{E}-\mathcal{C}-\mathcal{E}$	1 ion	3.2 ± 1.8
$\mathcal{E}-\mathcal{C}-\mathcal{H}-\mathcal{C}-\mathcal{E}$	1 ion	7.9 ± 1.5
$\mathcal{E}-\mathcal{C}-\mathcal{V}-\mathcal{C}-\mathcal{E}$	1 ion	14.5 ± 2.0
$\mathcal{E}-\mathcal{C}-\mathcal{E}$	2 ions	5.4 ± 1.2
$\mathcal{E}-\mathcal{C}-\mathcal{H}-\mathcal{C}-\mathcal{E}$	2 ions	16.6 ± 1.8
$\mathcal{E}-\mathcal{C}-\mathcal{V}-\mathcal{C}-\mathcal{E}$	2 ions	53.0 ± 1.2



- Introduction to quantum information
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- Wiring up trapped ions



Wiring up trapped ions



Two trapped ions ...

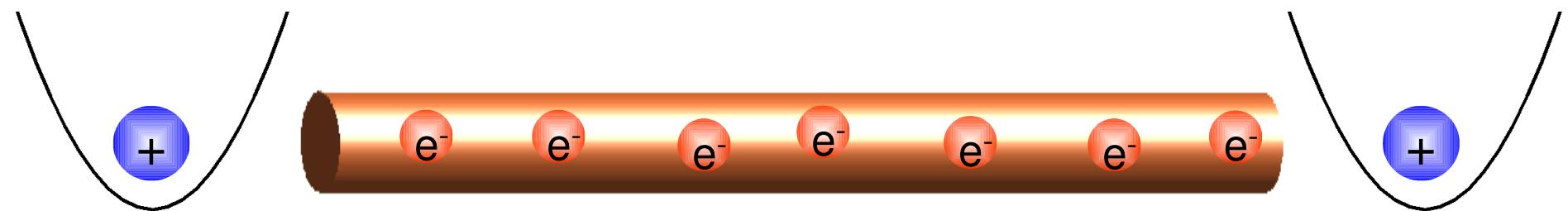




Wiring up trapped ions



Two trapped ions + a wire

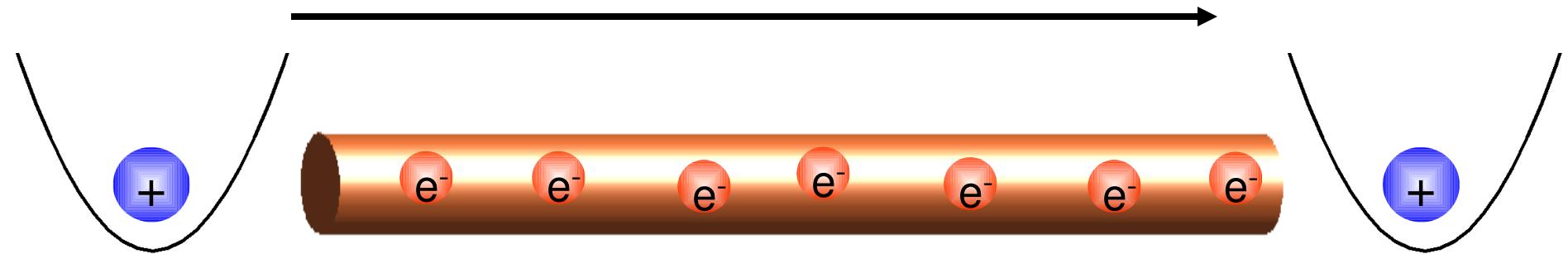




Wiring up trapped ions



Transport of quantum information

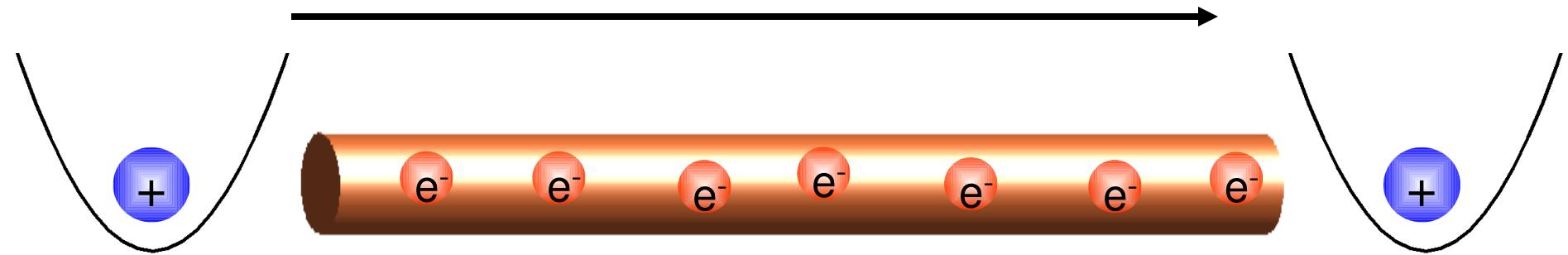




Wiring up trapped ions



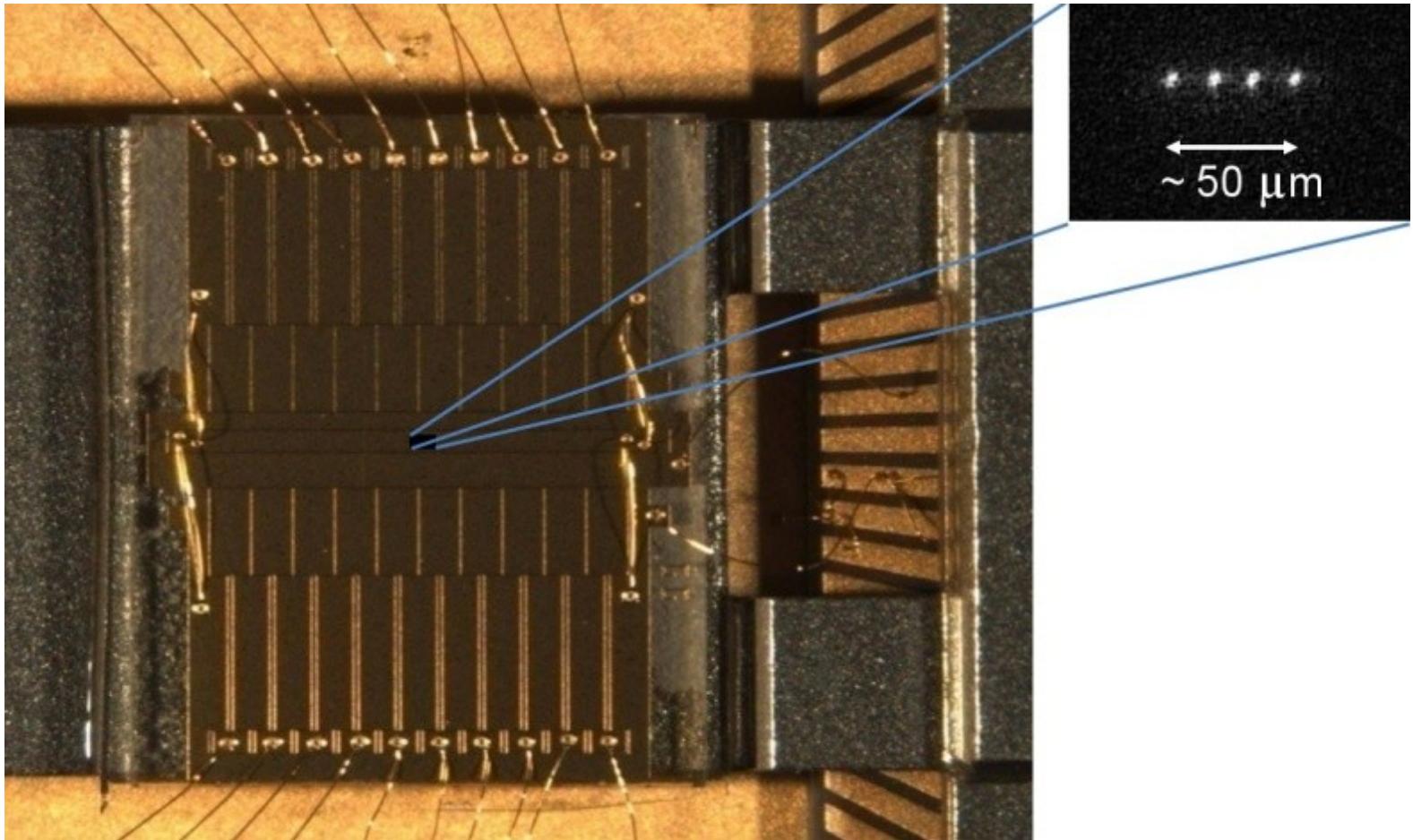
Transport of quantum information



No trace of the quantum information should remain in the wire

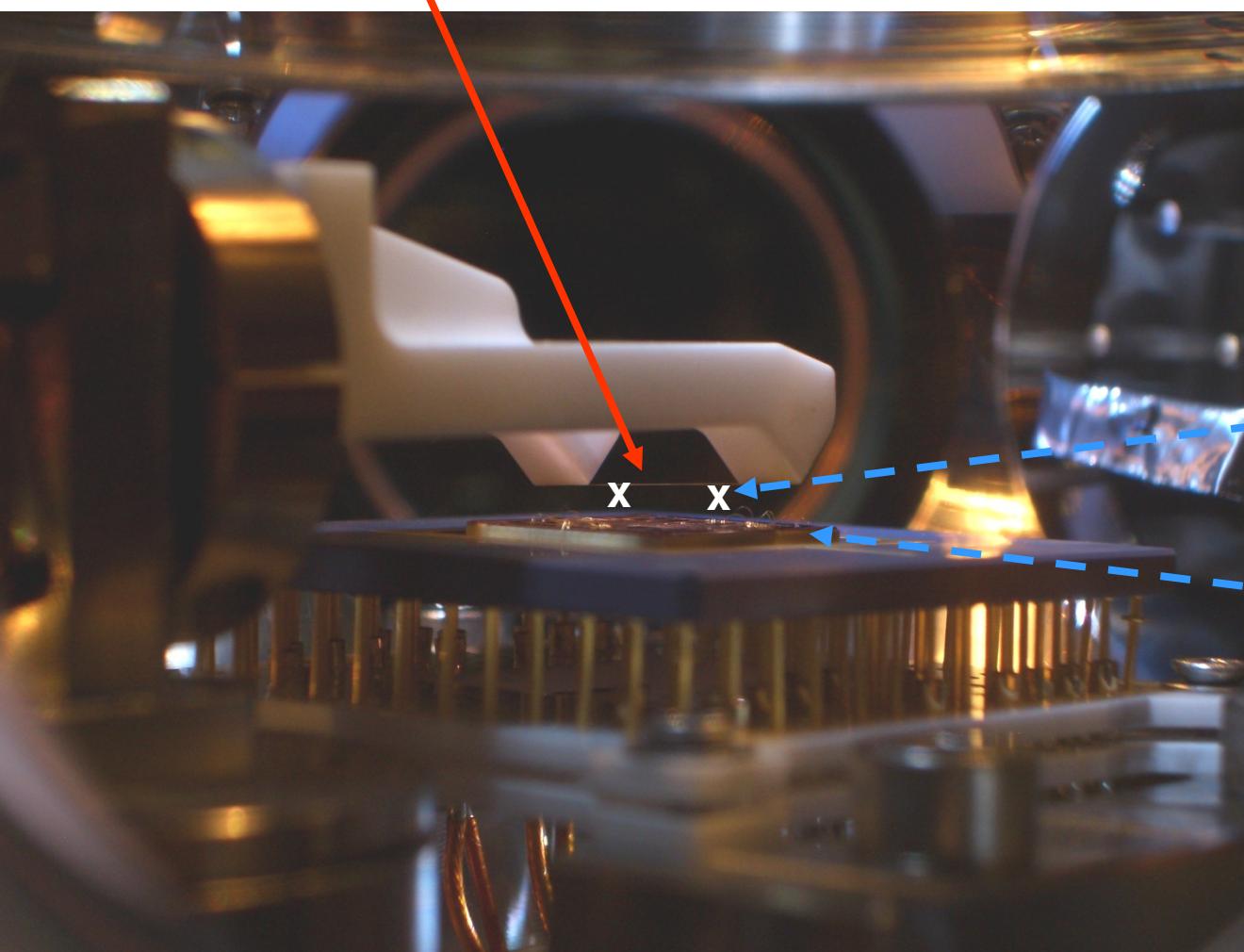
→ ~~super conducting wire~~

Experimental set-up





Experimental set-up



Separation of trapping and coupling

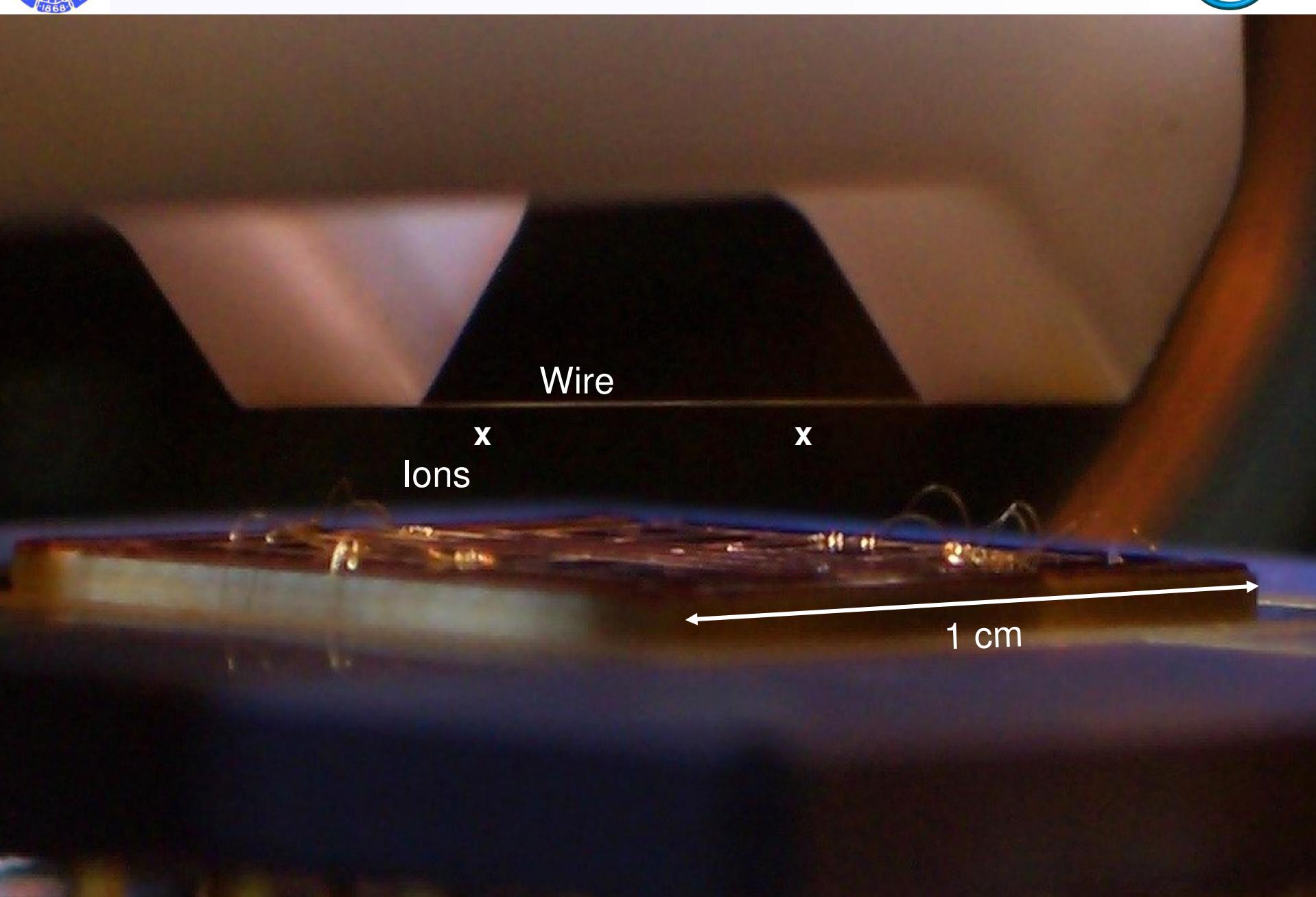
Wire

Ions

Trap electrodes

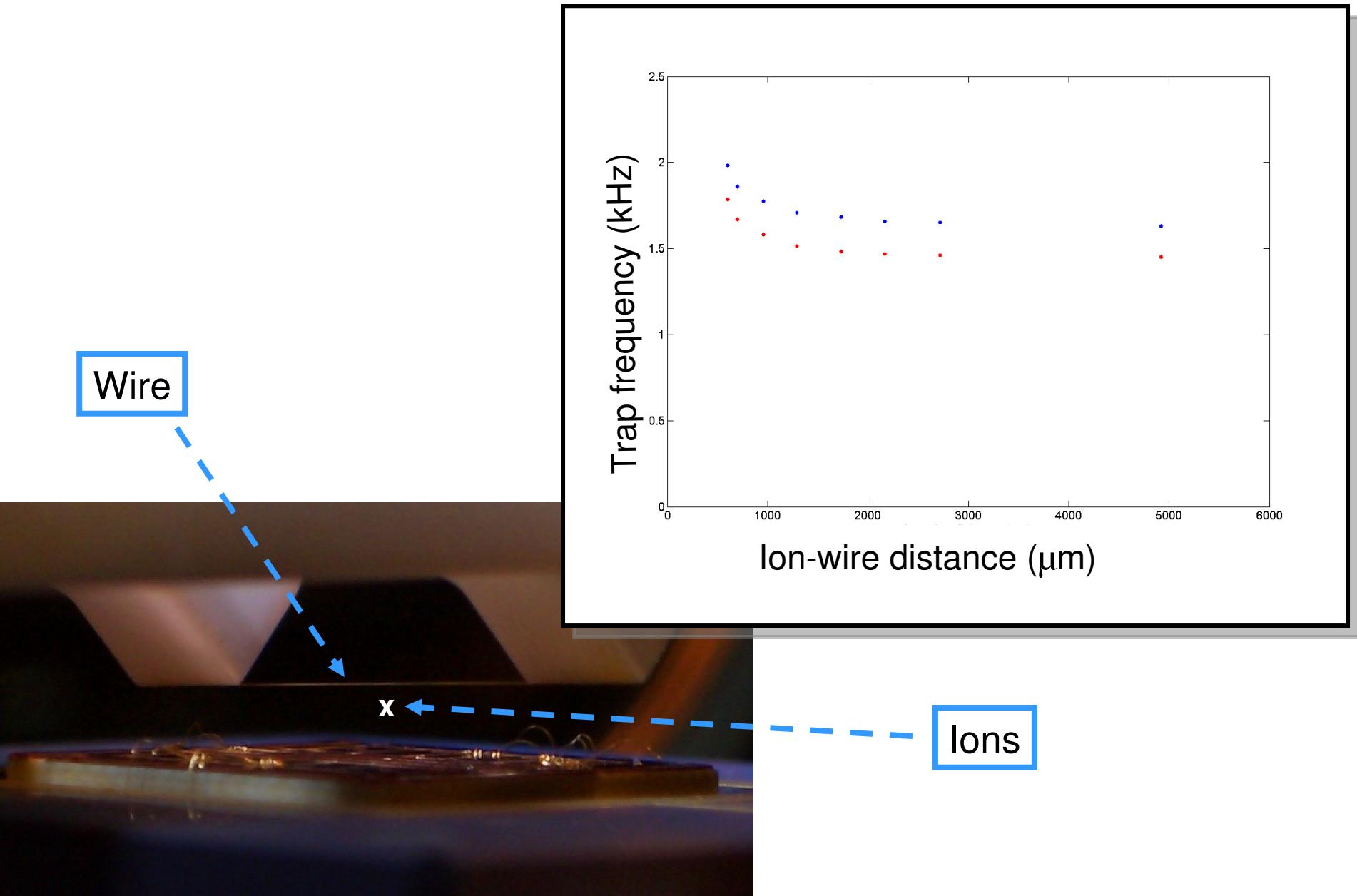


Experimental set-up





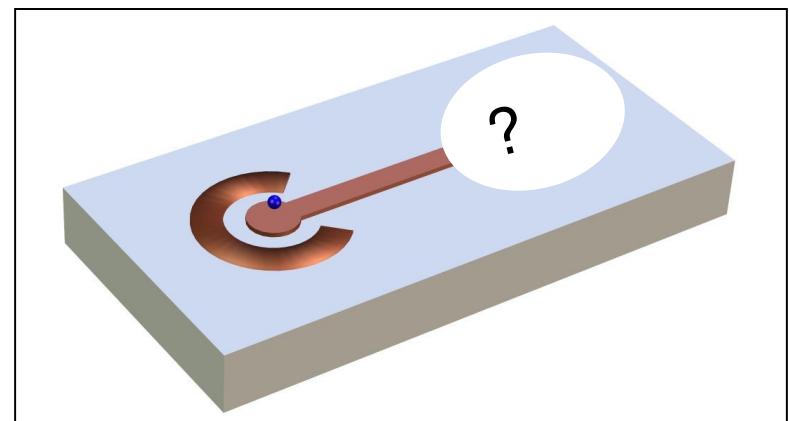
Ion-wire interaction



More physics with this set-up

Physics:

- Decoherence in charge transport
- Wire mediated laser cooling to a few μK
- Detection of tiny AC-currents (10^{-16} A)
- Cooling of LC resonators
- Hybrid quantum computing

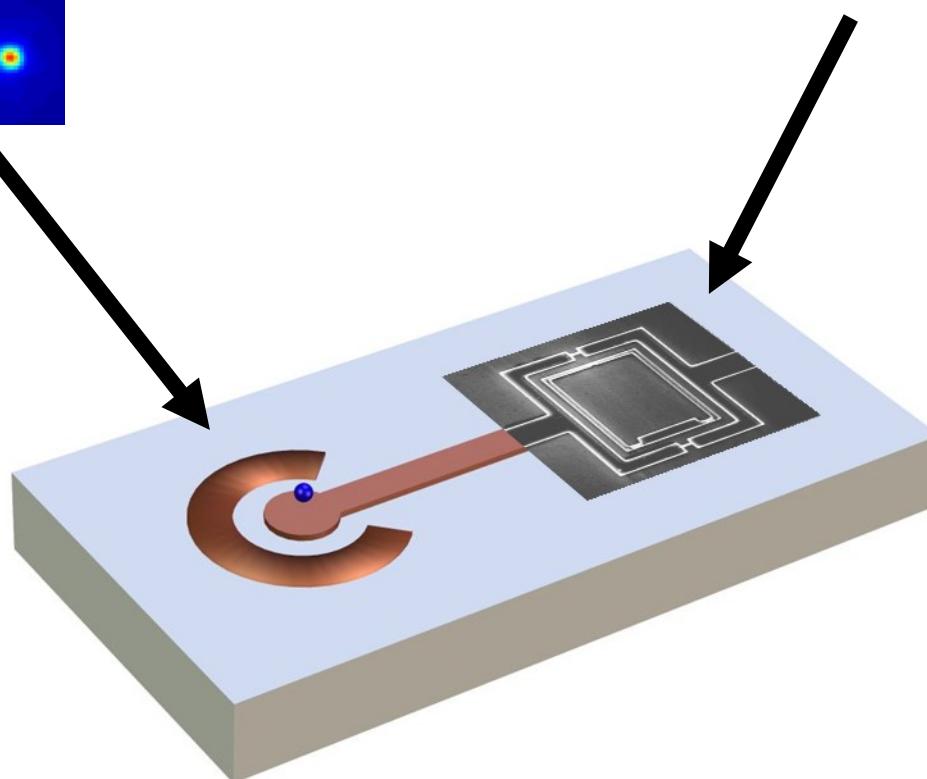
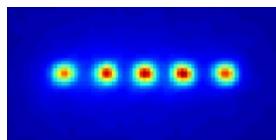




Interfacing quantum systems



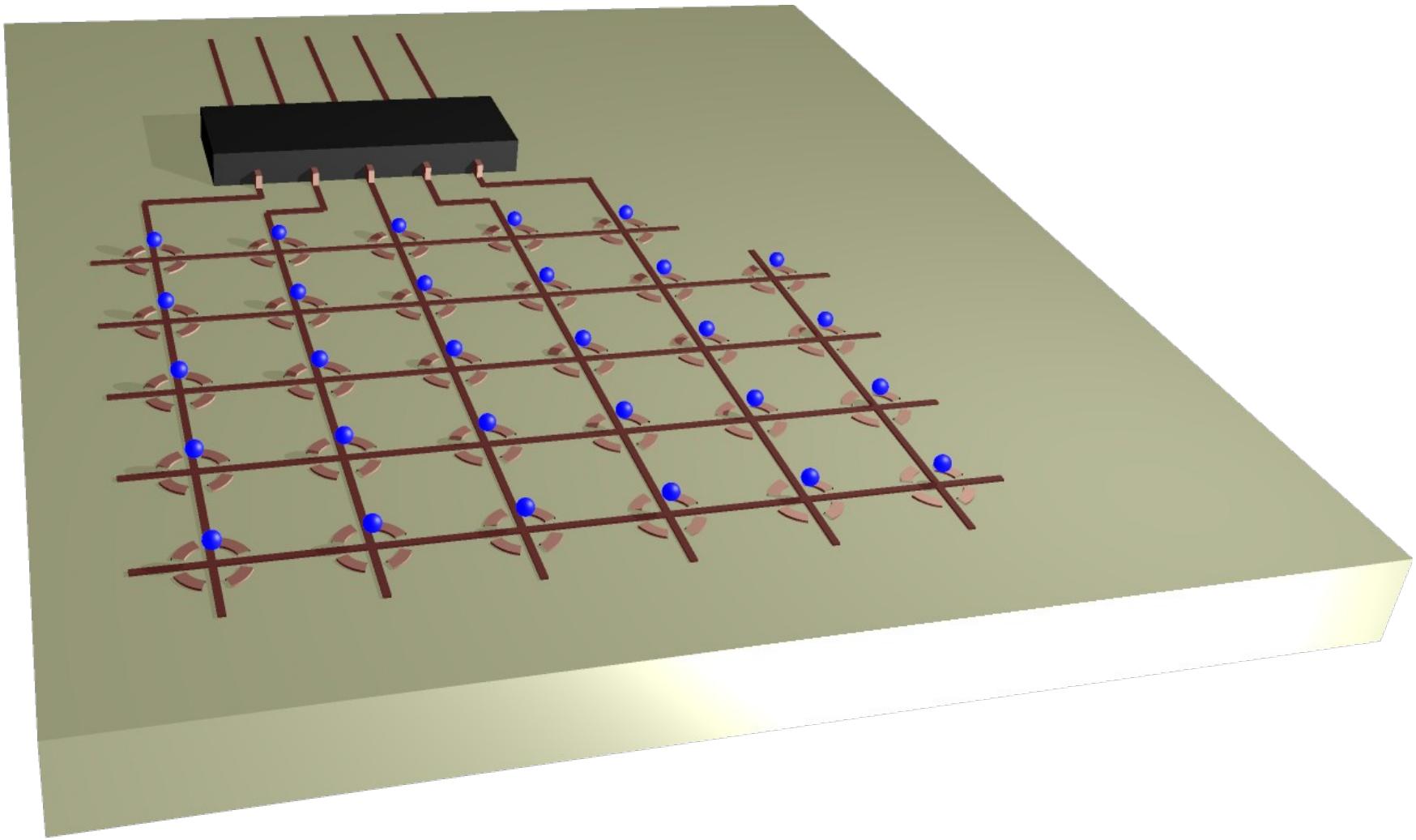
Trapped ions



Josephson qubits



A vision

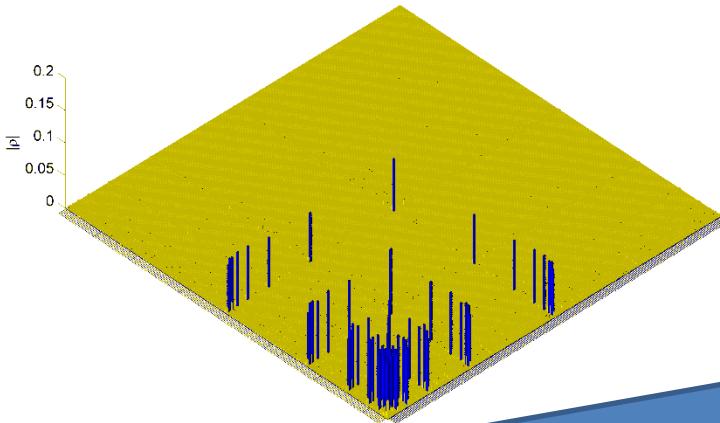




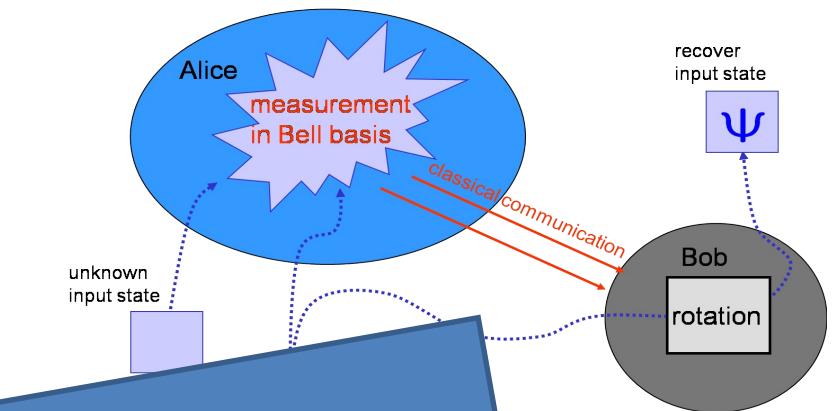
Conclusions



Entanglement

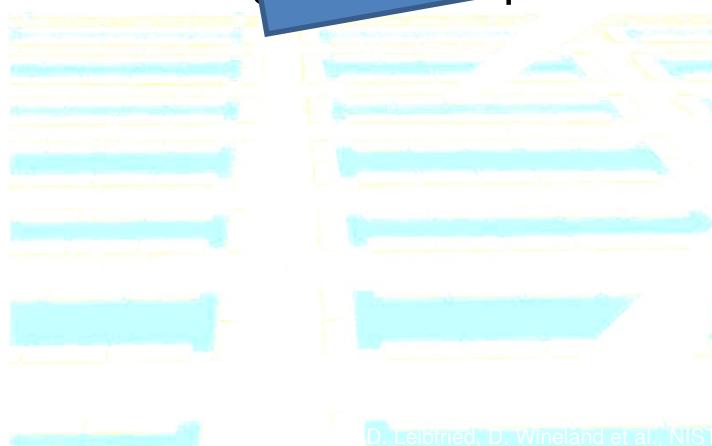


Teleportation



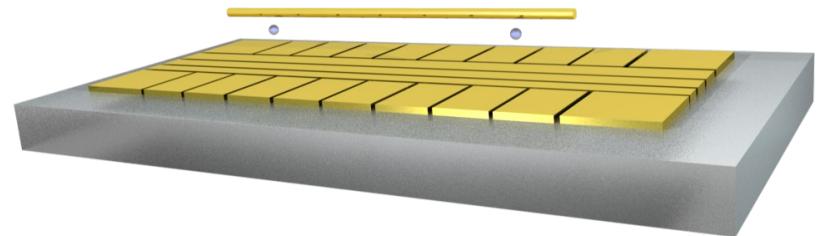
Future: small microfabricated traps
=> will run into surface science problems

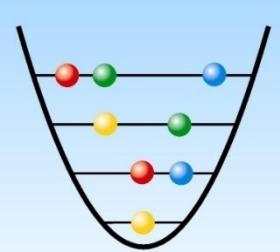
Scaling



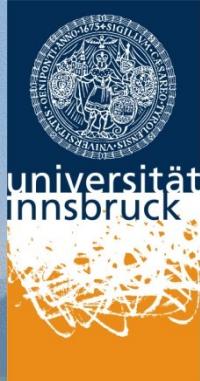
D. Leibfried, D. Wineland et al., NIST

Wiring up trapped ions





The Innsbruck ion trap group



AG Quantenoptik
und Spektroskopie



€



FWF
SFB



CONQUES
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SCALA

FWF | bm:bwk



Industrie
Tirol



IQI
GmbH

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