The Bad and the Ugly: decoherence in ion trap quantum computing

Hartmut Häffner

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- Fault tolerant quantum computing
- Ion trap quantum computing
- Quantum memory
- Operational errors
- Conclusions

Tel-Aviv, Apr 27th 2010

Motivation

Goal: run arbitrary (long) quantum algorithms with any desired accuracy

Wish list

1. Infinite coherence time

2. Perfect control of the qubits

Wish list

1. Infinite coherence time

Decouple qubits from the environment

2. Perfect control of the qubits

Have the qubits interact with us in a well defined way

Need to switch the environment on and off

Wish list

1. Infinite coherence time

Decouple qubits from the environment

Decoherence free subspaces

2. Perfect control of the qubits

Have the qubits interact with us in a well defined way

Wish list

1. Infinite coherence time

Decouple qubits from the environment

Decoherence free subspaces and quantum error correction

2. Perfect control of the qubits

Have the qubits interact with us in a well defined way

Undo errors with quantum error correction

Motivation

Goal: run arbitrary (long) quantum algorithms with any desired accuracy

Need for quantum error correction

Classical error correction:
- encode in a larger space
- detect errors
- correct errors

Motivation

Goal: run arbitrary (long) quantum algorithms with any desired accuracy

Need for quantum error correction

Problem: quantum operations appear to be continuous

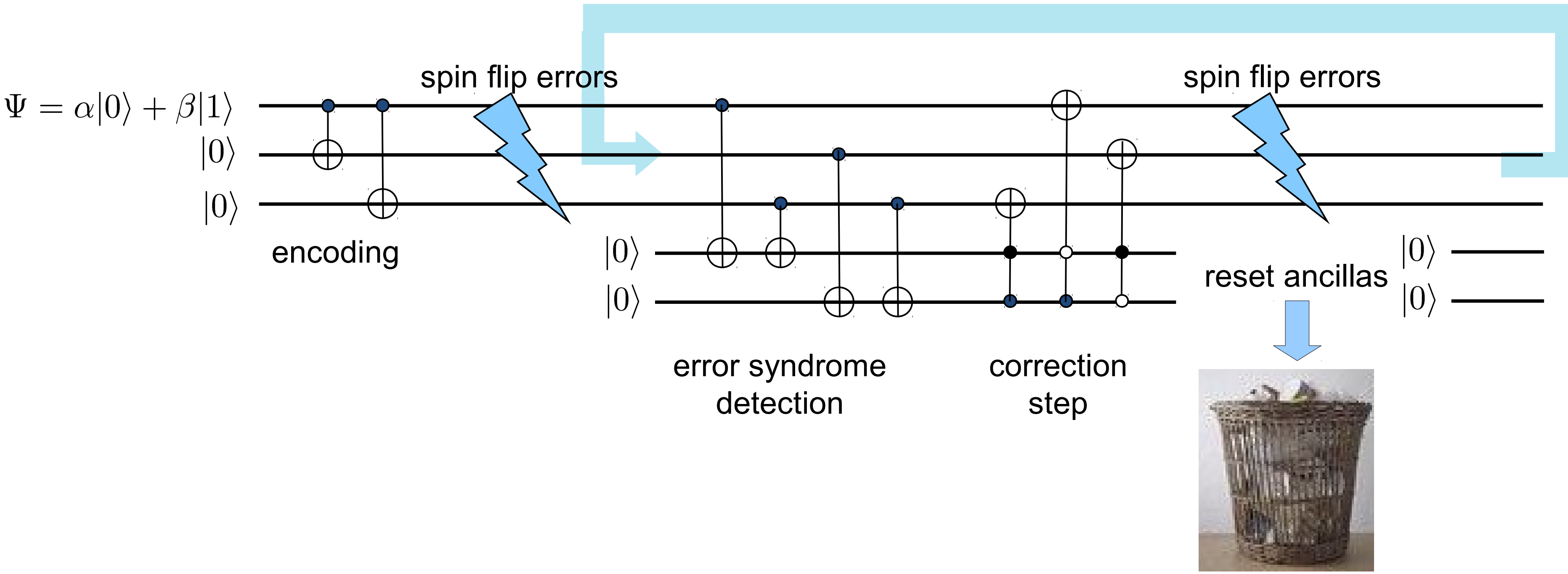
The threshold theorem for QEC

Theorem: Any quantum computation can be carried with a desired precision with polynominal overhead, provided sufficient weak decoherence and precise control

Keeping a qubit alive

Repeatable quantum error correction (bit flips only)

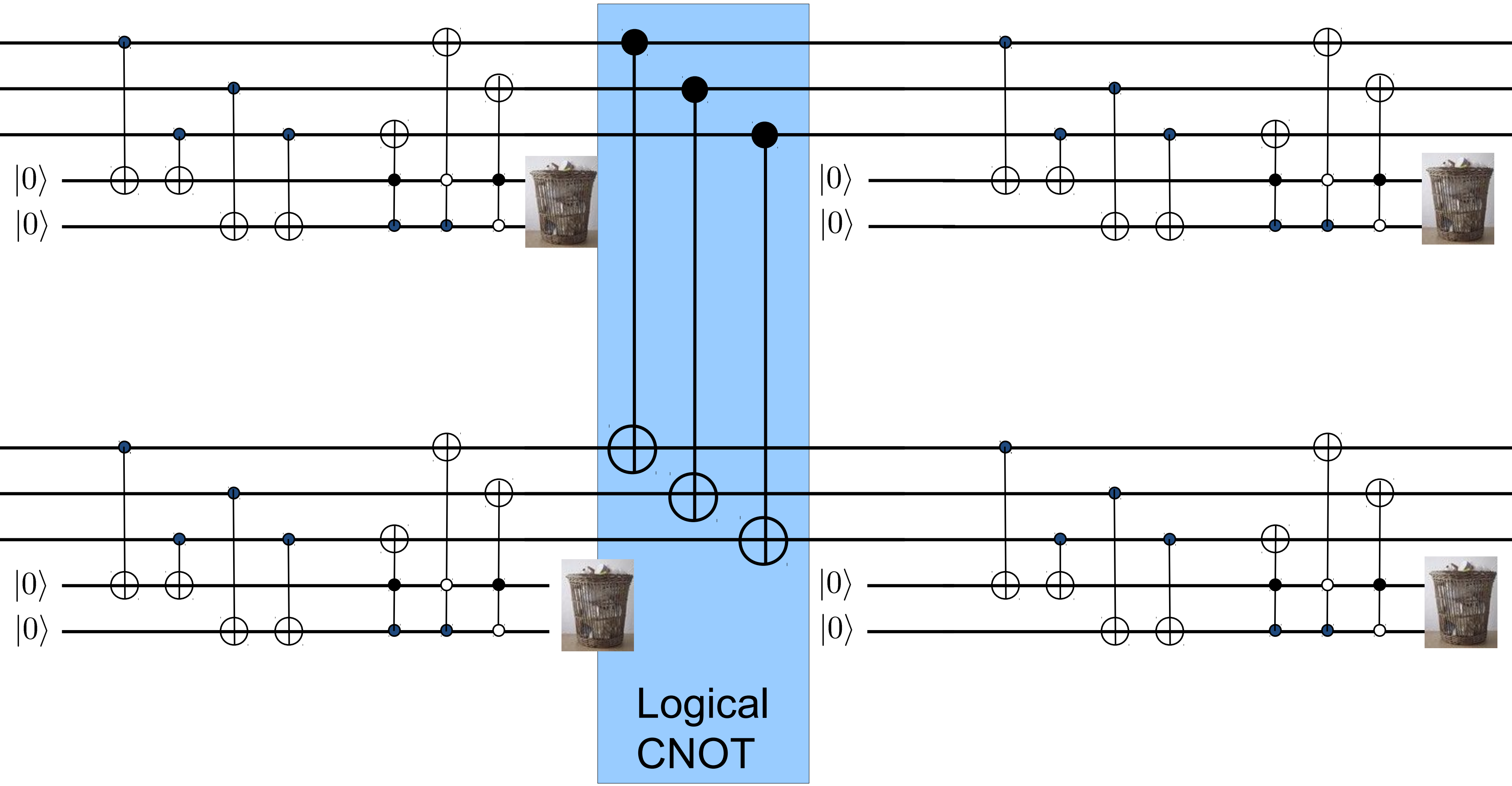
$$\alpha\,|S\rangle + \beta\,|D\rangle \rightarrow \alpha\,|SSS\rangle + \beta\,|DDD\rangle$$



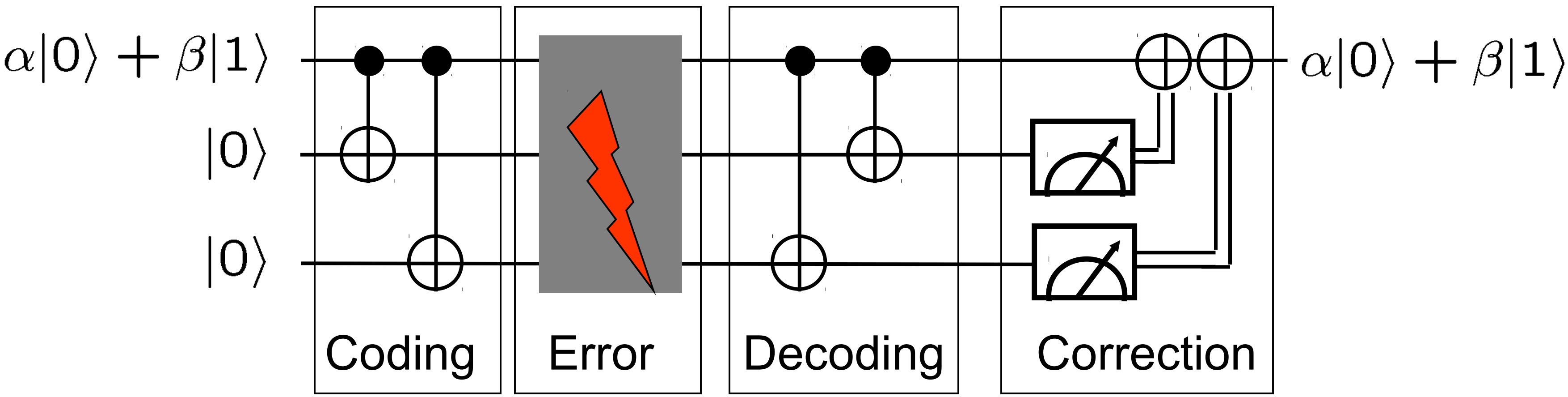
Measure only error, not data

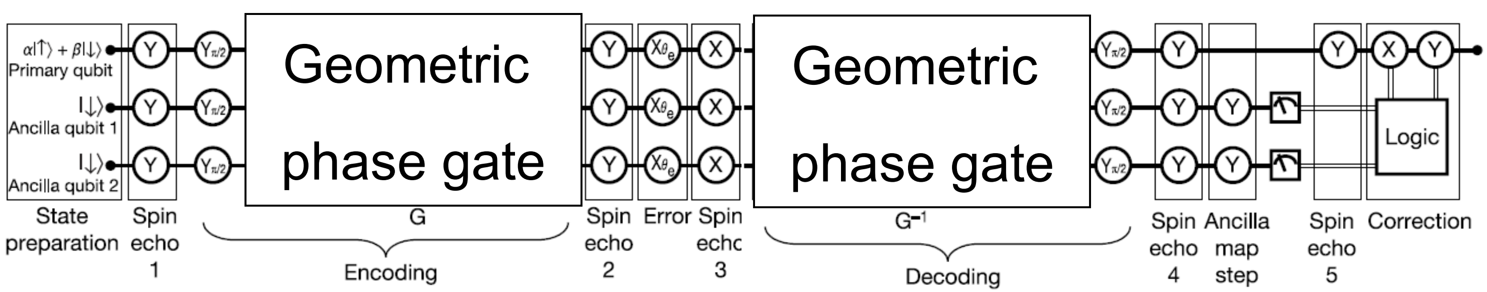
Fault tolerant quantum gates

"Simple" fault tolerant CNOT (bit flips only):



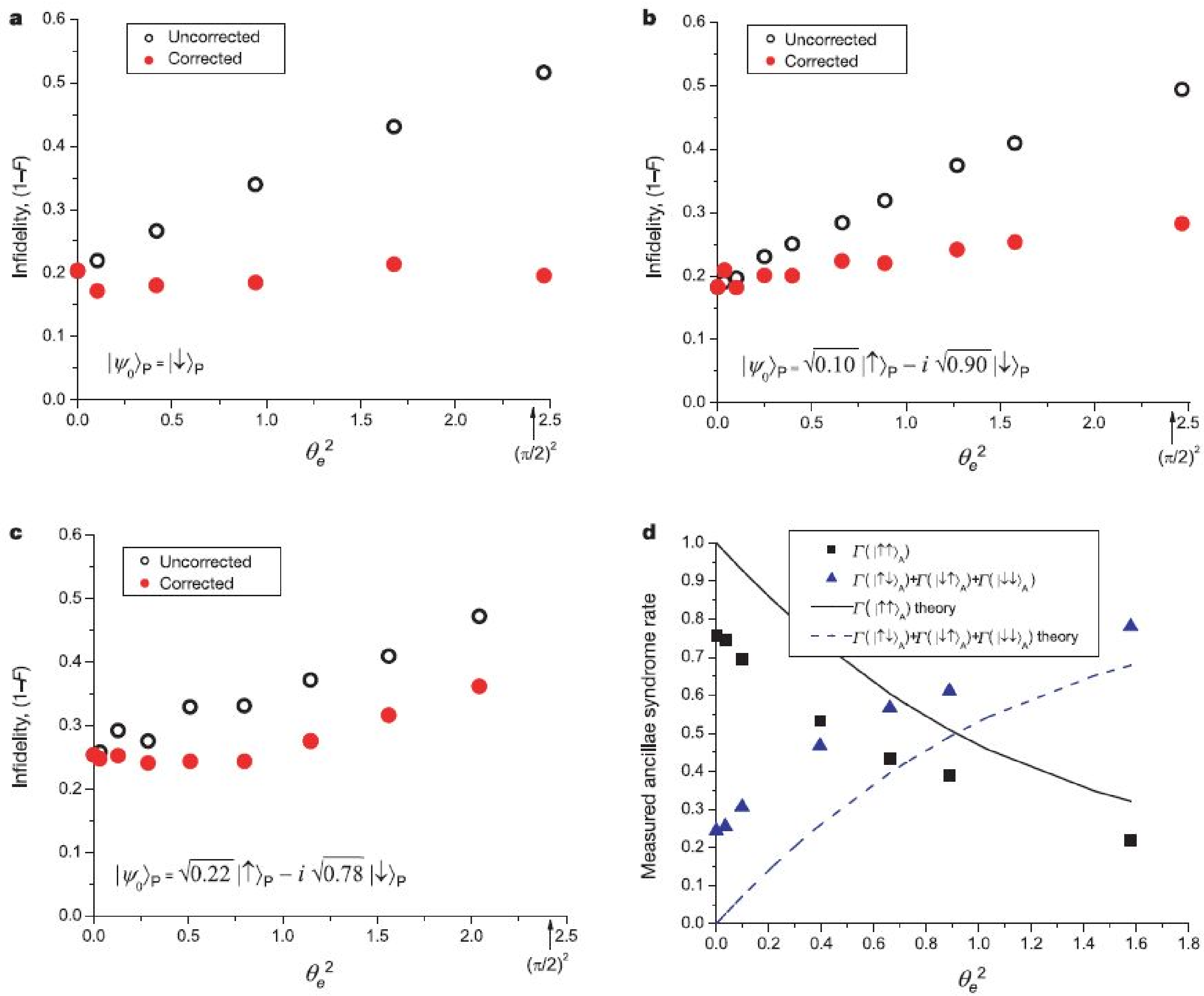
A “simple” quantum error correction





From: J. Chiaverini *et al*., Nature **432** 602 (2004), NIST

A "simple" quantum error correction



From: J. Chiaverini *et al.*, Nature **432** 602 (2004), NIST

The threshold theorem for QEC

Theorem: There is no Threshold -Theorem

(Dave Bacon, 2004)

Threshold depends strongly on:

- accuracy of control
- measurement and ancilla preparation fidelities
- level of decoherence
- correlations of errors

Realistic wish list

1. Coherence times much longer than gate times

by a factor of 10^4

2. Control the qubits

with an accuracy of 10^{-4}

Additionally: very good initialization

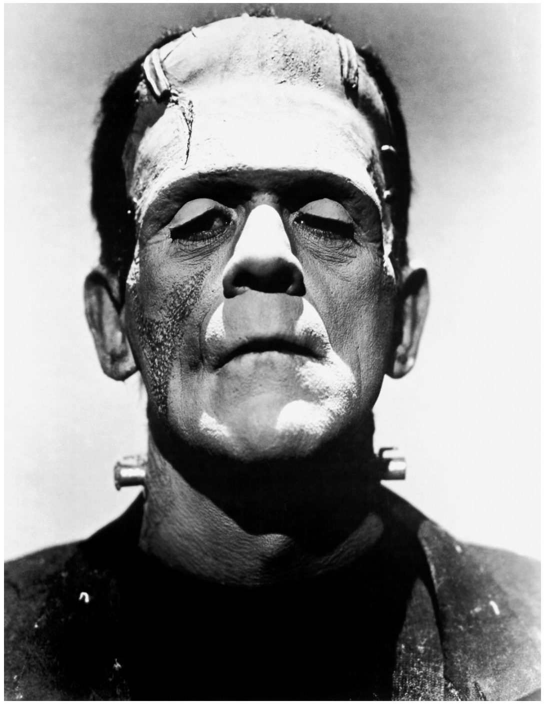
efficient and fast selective read out

parallelization of all processes (scalability)

⇒ Overhead: ~ 100 (both in qubits and time)

Steane, quant-ph/0412165

The Bad: Decoherence



The Ugly: Operational Errors

The Bad: Decoherence

The Ugly: Operational Errors

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- Quantum memory
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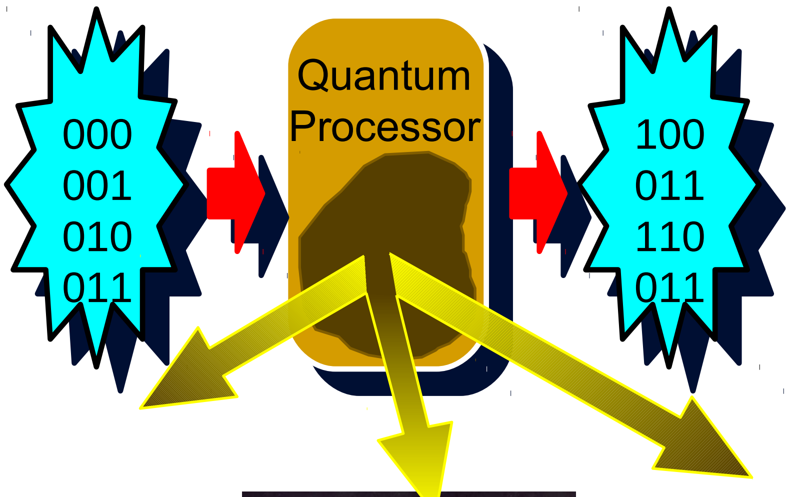
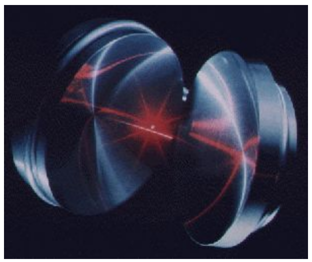
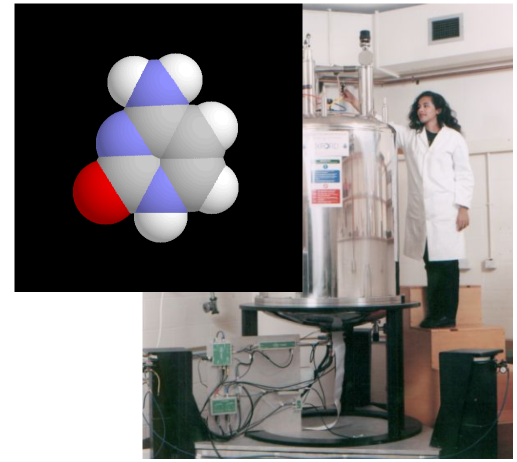
Which technology

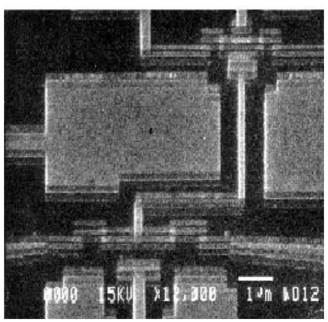




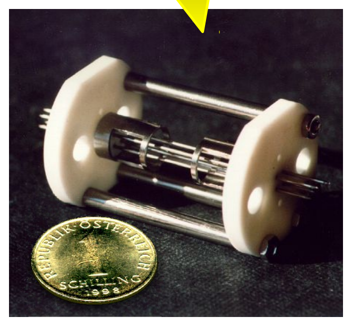
Cavity QED

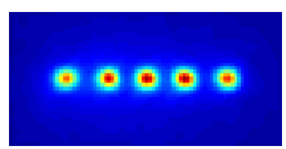


NMR

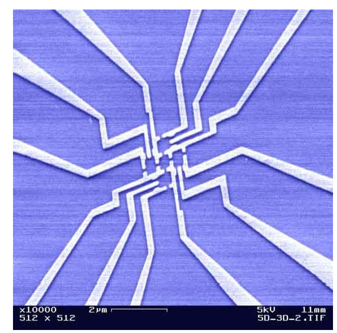


superconductors

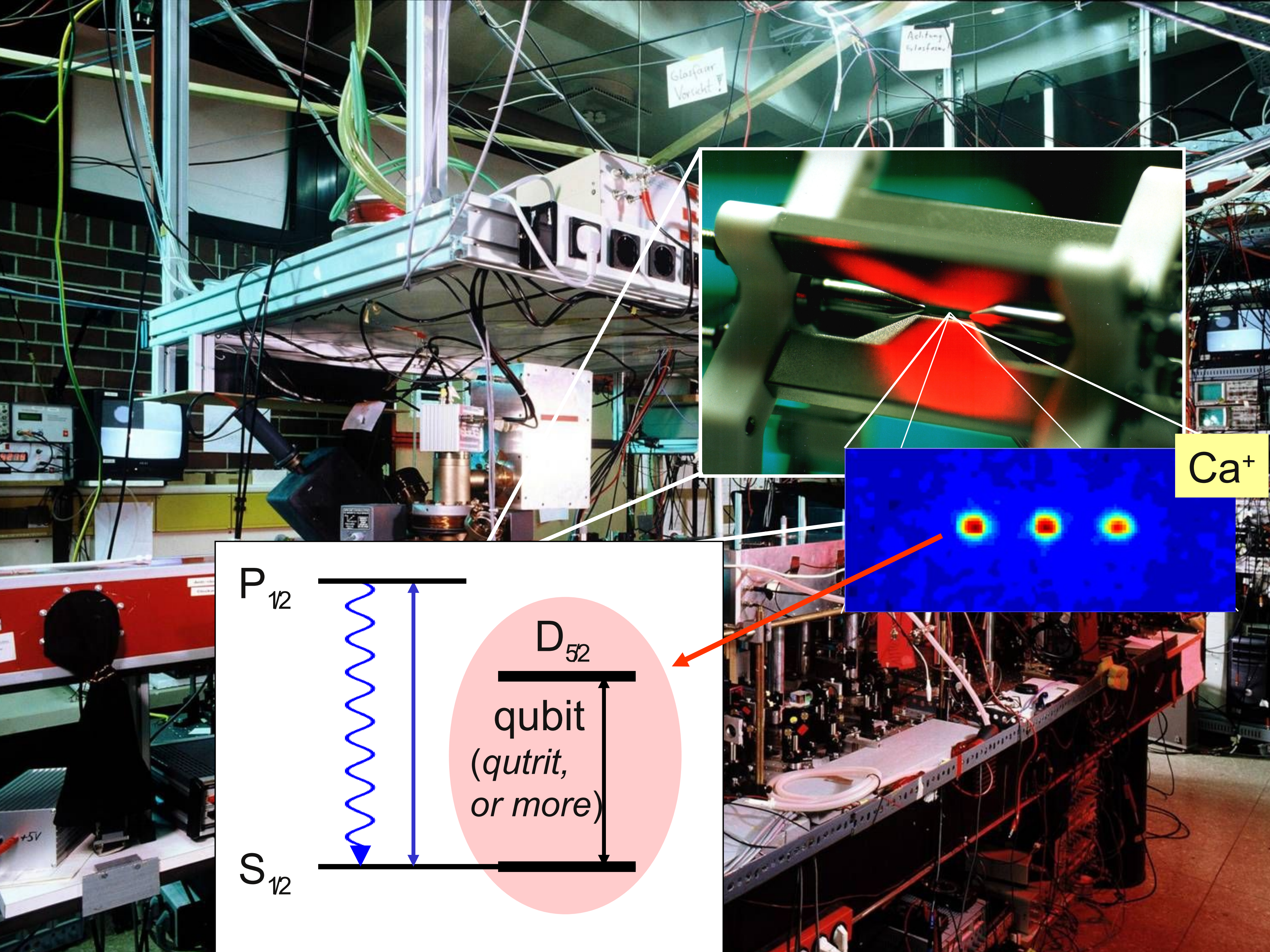




trapped ions

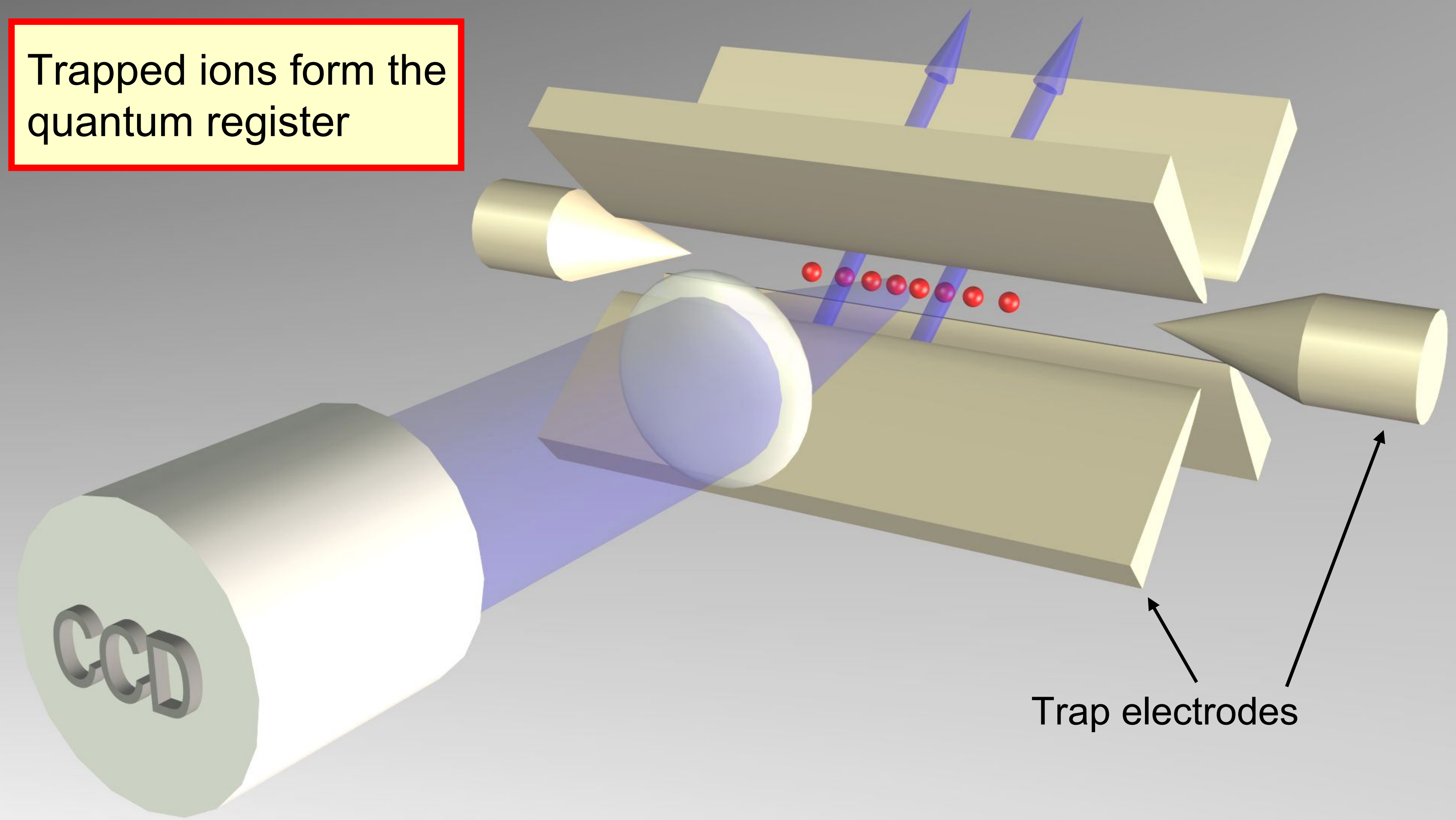


quantum dots



Ca+
P1/2
D5/2
qubit
(qutrit,
or more)
S1/2

Ion trap quantum computing



Di Vincenzo criteria

I. Scalable physical system, well characterized qubits

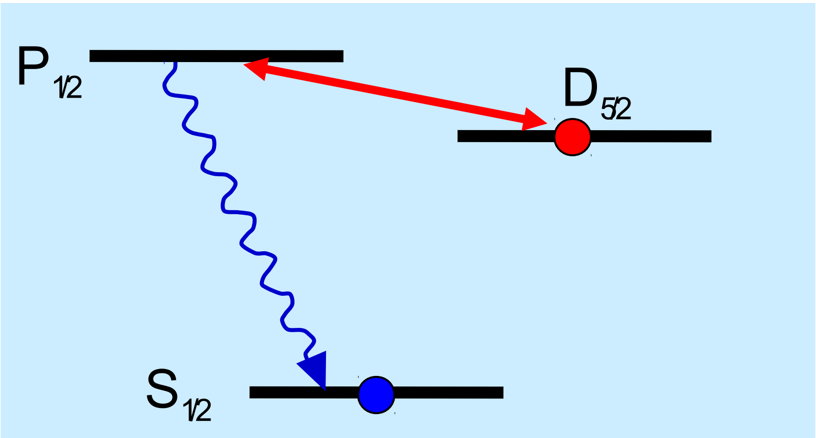
II. Ability to initialize the state of the qubits

III. Long relevant coherence times, much longer than gate operation time

IV. “Universal” set of quantum gates

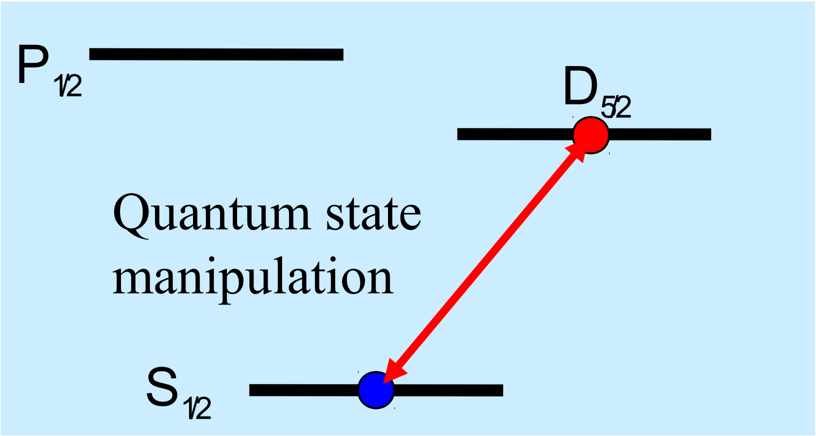
V. Qubit-specific measurement capability

Experimental procedure



1. Initialization in a pure quantum state

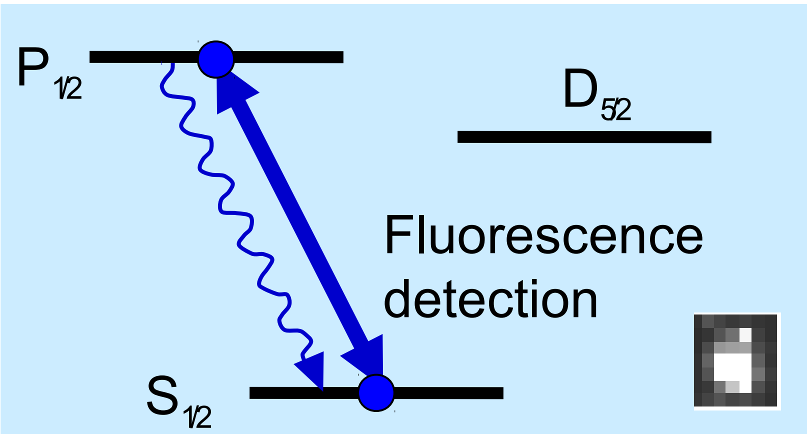
Experimental procedure



1. Initialization in a pure quantum state

2. Quantum state manipulation on $S_{1/2} - D_{5/2}$ transition

Experimental procedure

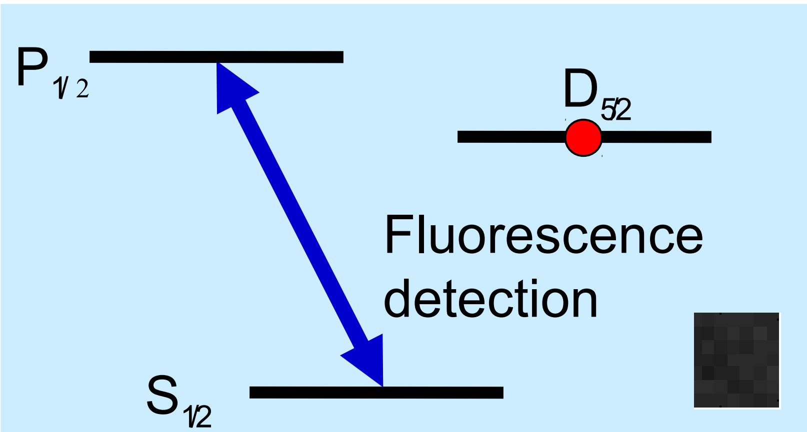


1. Initialization in a pure quantum state

2. Quantum state manipulation on $S_{1/2} - D_{5/2}$ transition

3. Quantum state measurement by fluorescence detection

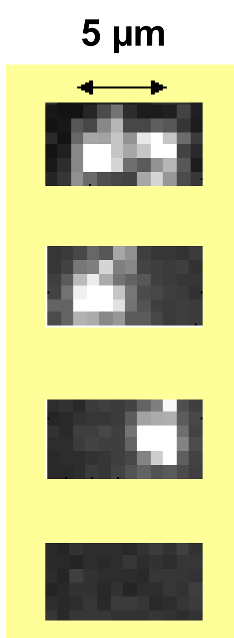
Experimental procedure



1. Initialization in a pure quantum state
2. Quantum state manipulation on $S_{1/2} - D_{5/2}$ transition
3. Quantum state measurement by fluorescence detection

Two ions:

Spatially resolved detection with CCD camera



$|SS\rangle$

$|SD\rangle$

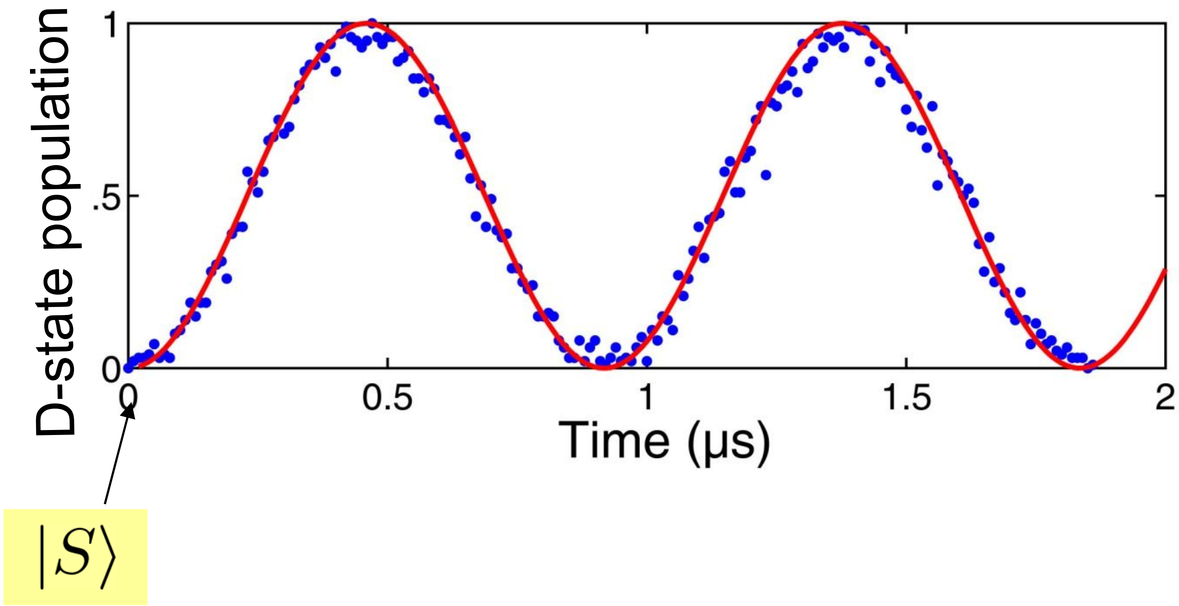
$|DS\rangle$

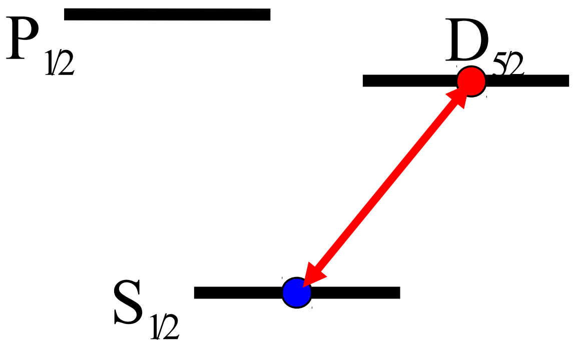
$|DD\rangle$

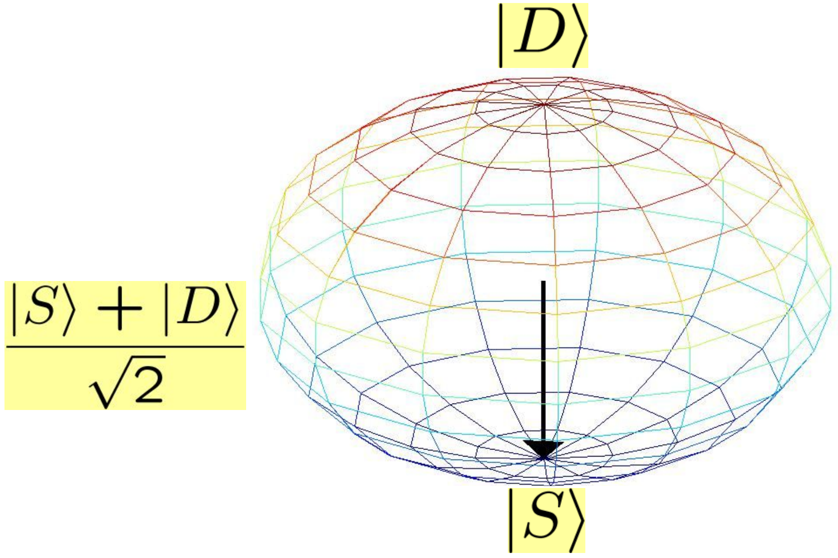
50 experiments / s

Repeat experiments 100-200 times

Rabi oscillations

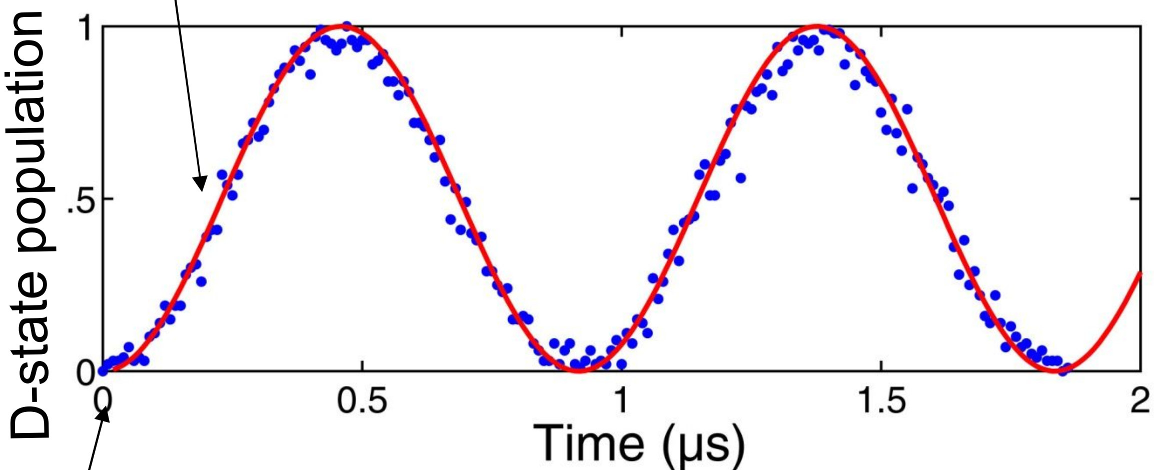




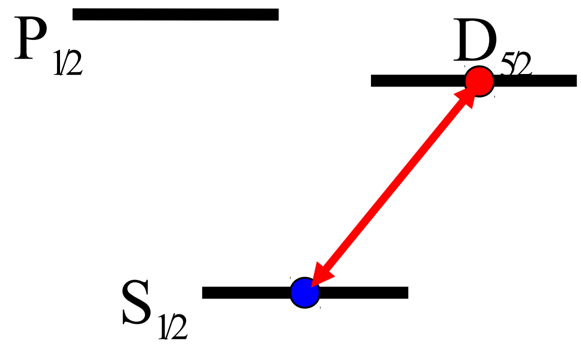


Rabi oscillations

$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$

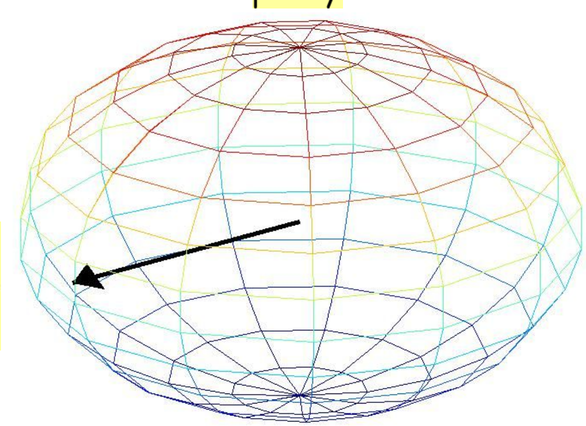


$|S\rangle$



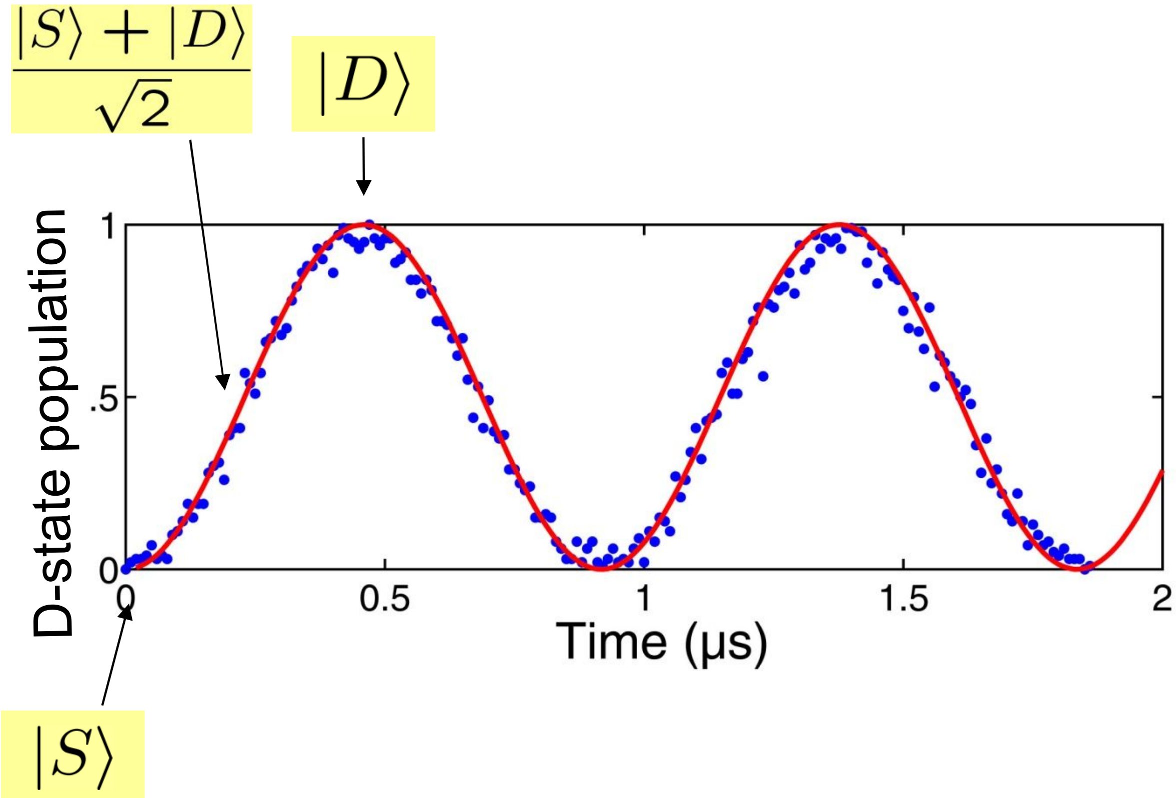
$|D\rangle$

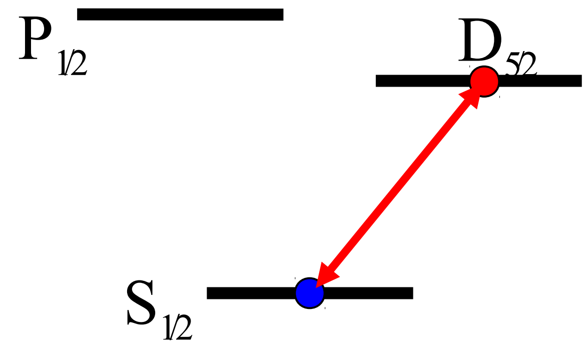
$$\frac{|S\rangle + |D\rangle}{\sqrt{2}}$$

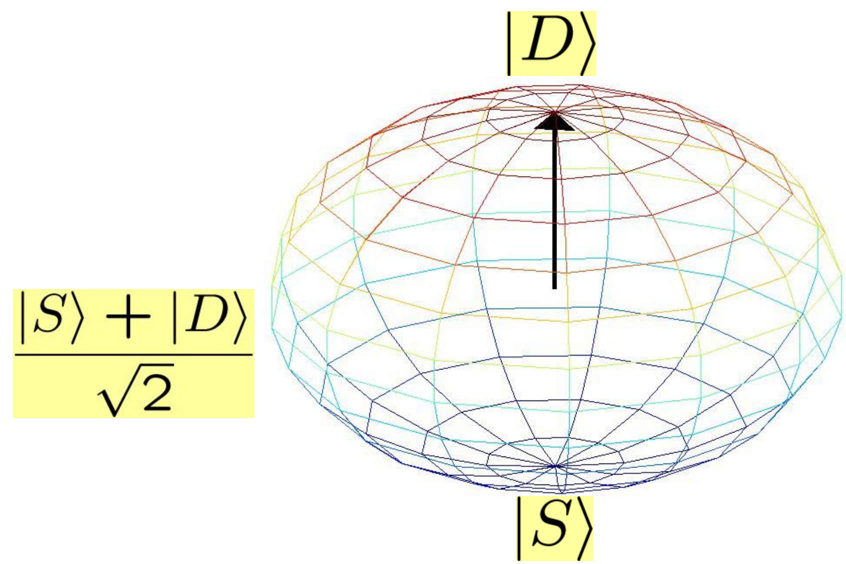


$|S\rangle$

Rabi oscillations

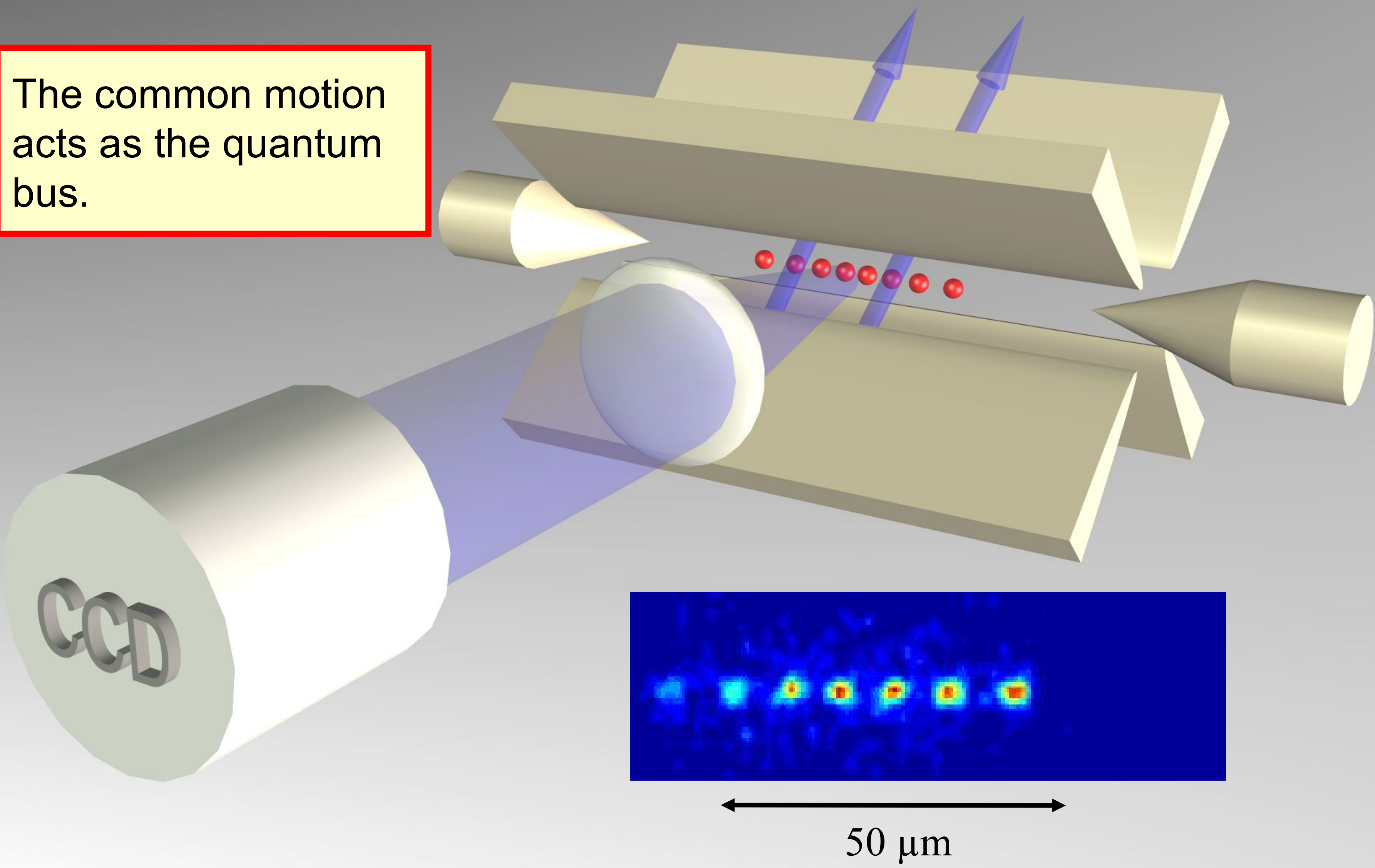




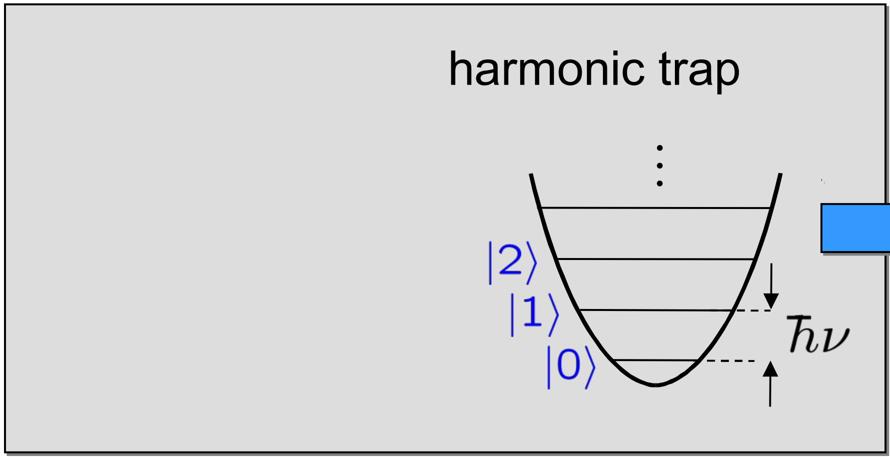


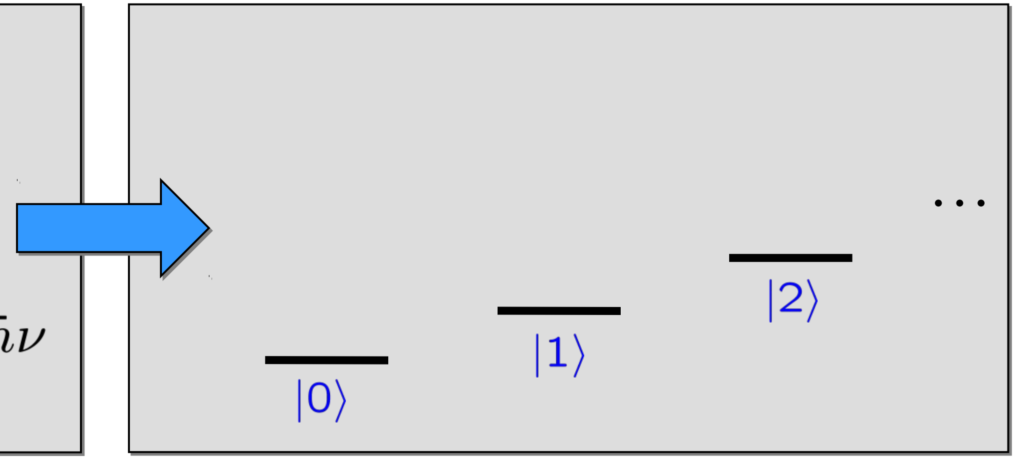
Having the qubits interact

The common motion acts as the quantum bus.

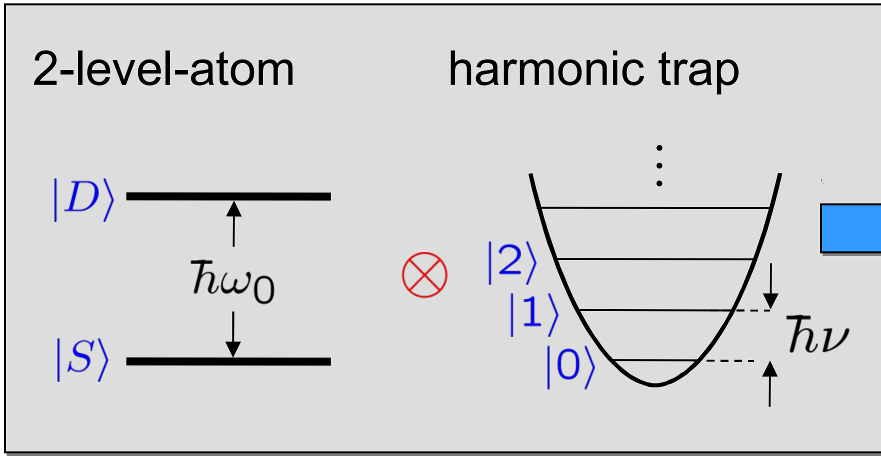


Ion motion

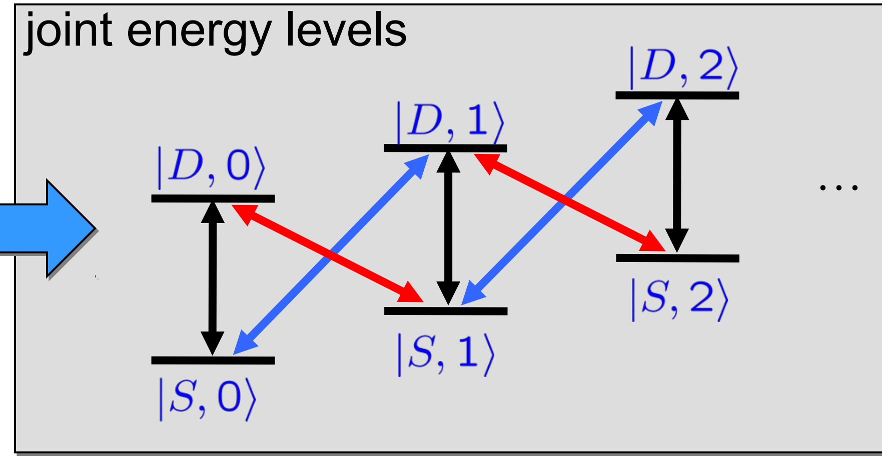

harmonic trap
$|2\rangle$
$|1\rangle$
$|0\rangle$
$\hbar\nu$


$|0\rangle$
$|1\rangle$
$|2\rangle$
...

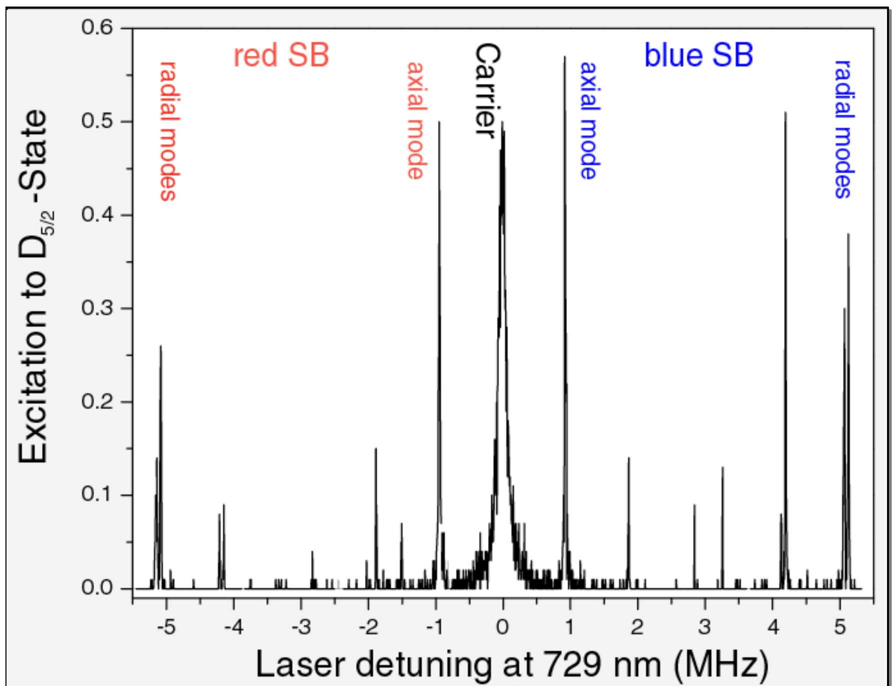
Ion motion



2-level-atom
harmonic trap
$|D\rangle$
$|S\rangle$
$\hbar\omega_0$
$\otimes$
$|2\rangle$
$|1\rangle$
$|0\rangle$
$\hbar\nu$

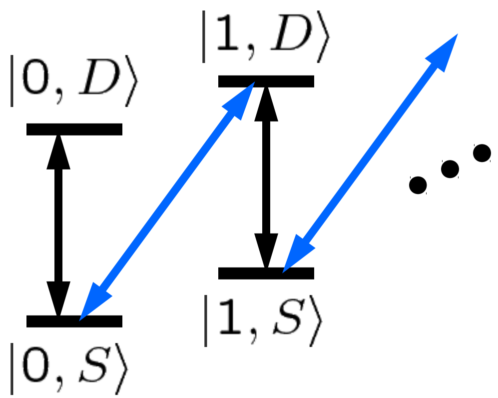


joint energy levels
$|D,0\rangle$
$|D,1\rangle$
$|D,2\rangle$
$|S,0\rangle$
$|S,1\rangle$
$|S,2\rangle$
...

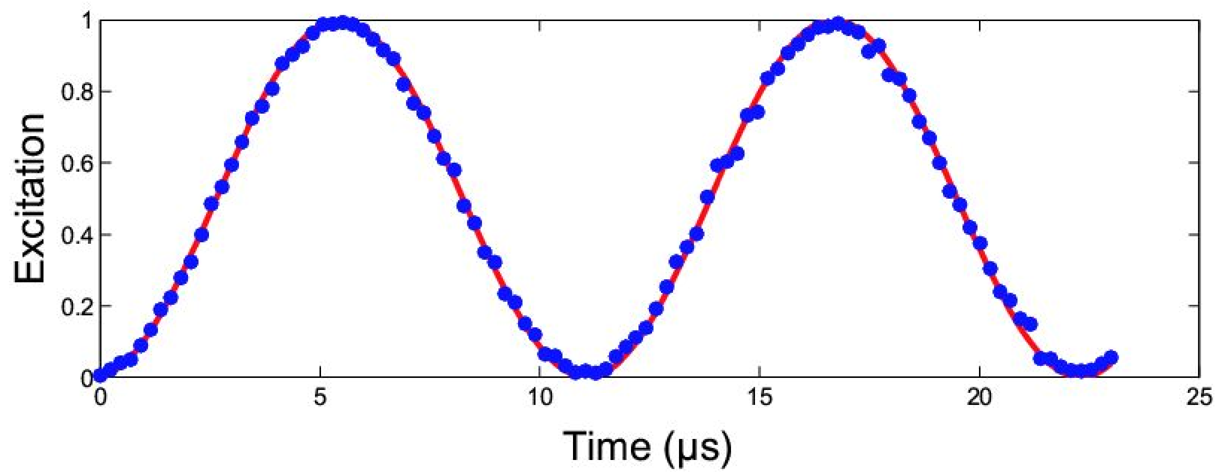


red SB
blue SB
radial modes
axial mode
Carrier
axial mode
radial modes
Excitation to $D_{5/2}$ -State
Laser detuning at 729 nm (MHz)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
-5
-4
-3
-2
-1
0
1
2
3
4
5

Ion motion

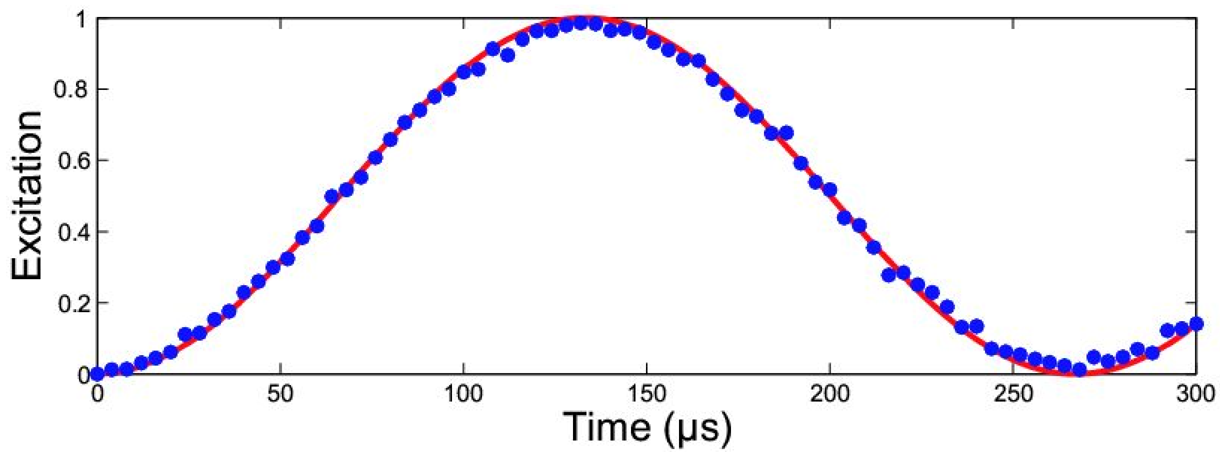


carrier



carrier and sideband
Rabi oscillations
with Rabi frequencies

$$\Omega, \quad \eta\Omega$$



$\eta = kx_0$ Lamb-Dicke parameter

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Realistic wish list

1. Coherence times much longer than gate times

   by a factor of 10^4

2. Control the qubits

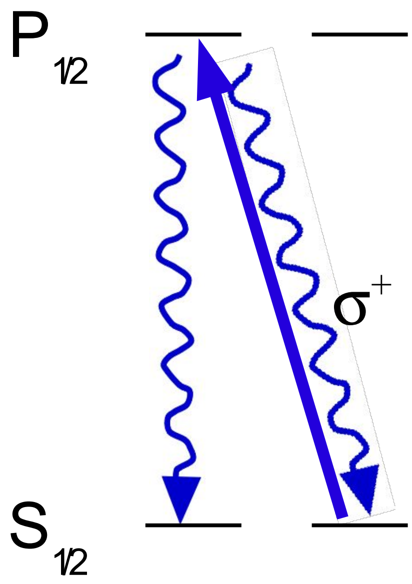
   with an accuracy of 10^{-4}

Additionally: very good initialization

efficient and fast selective read out

parallelization of all processes (scalability)

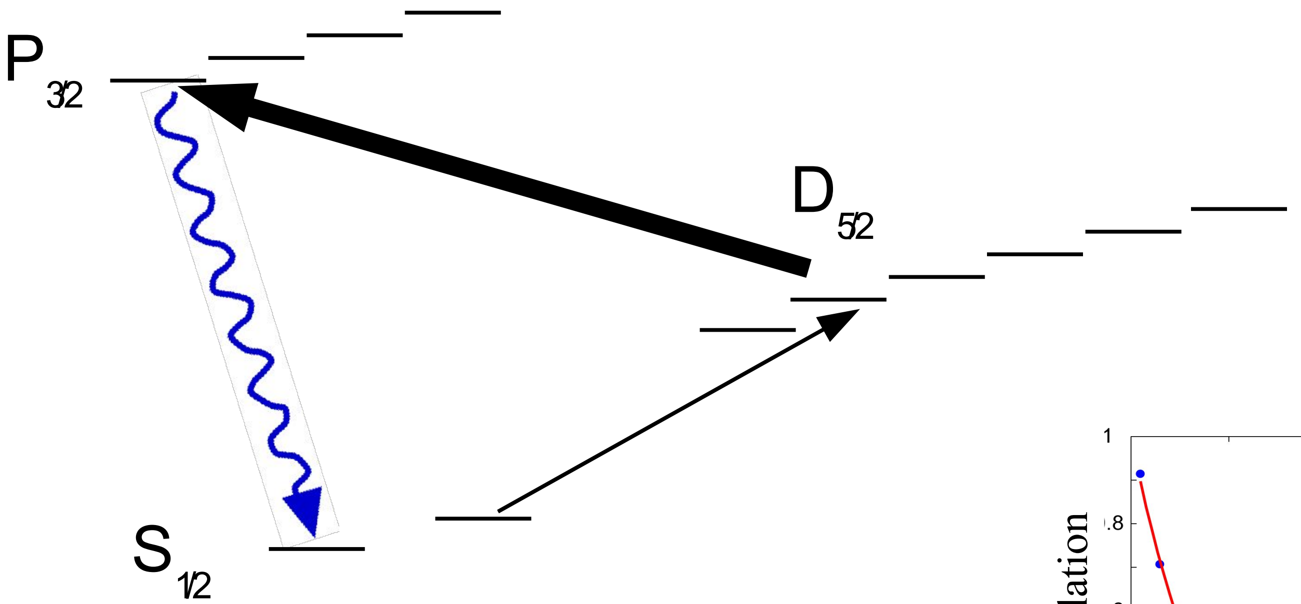
Qubit initialization

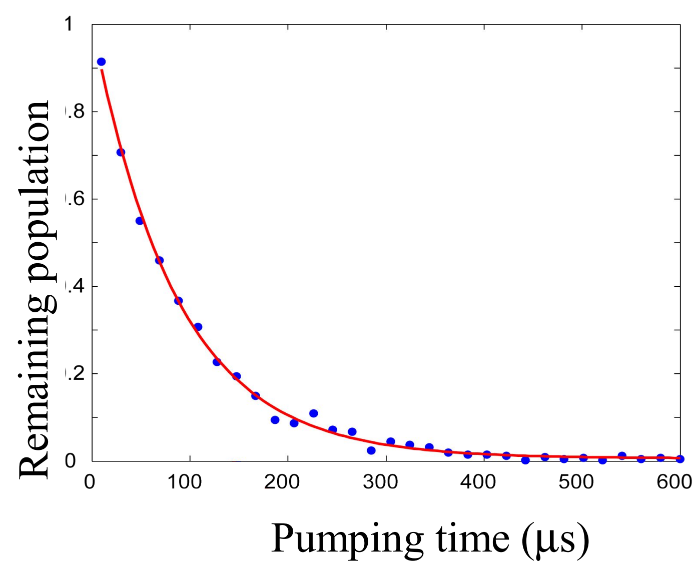


Typically optical pumping, limited often by polarization.

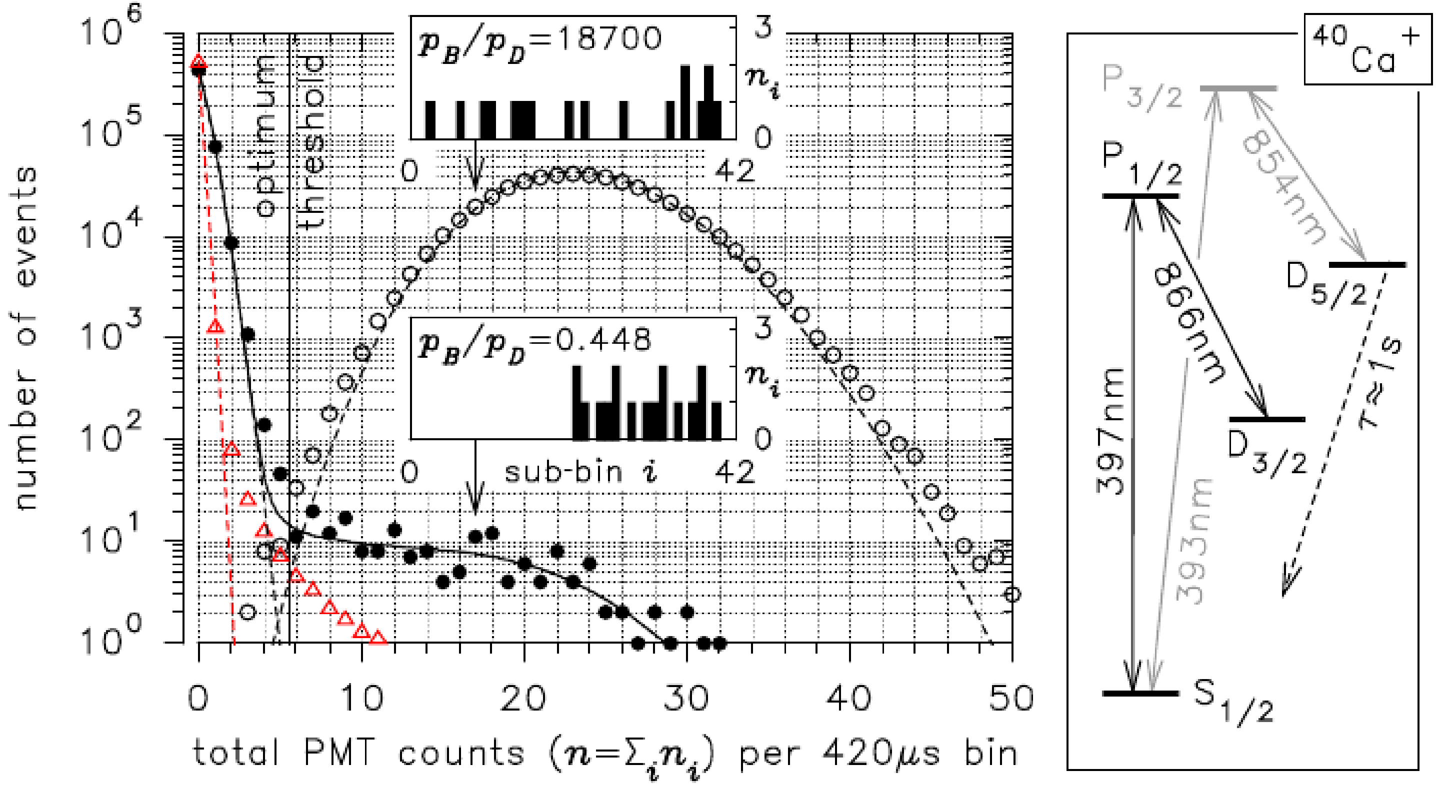
Qubit initialization

Better fidelities can be reached with frequency selectivity.





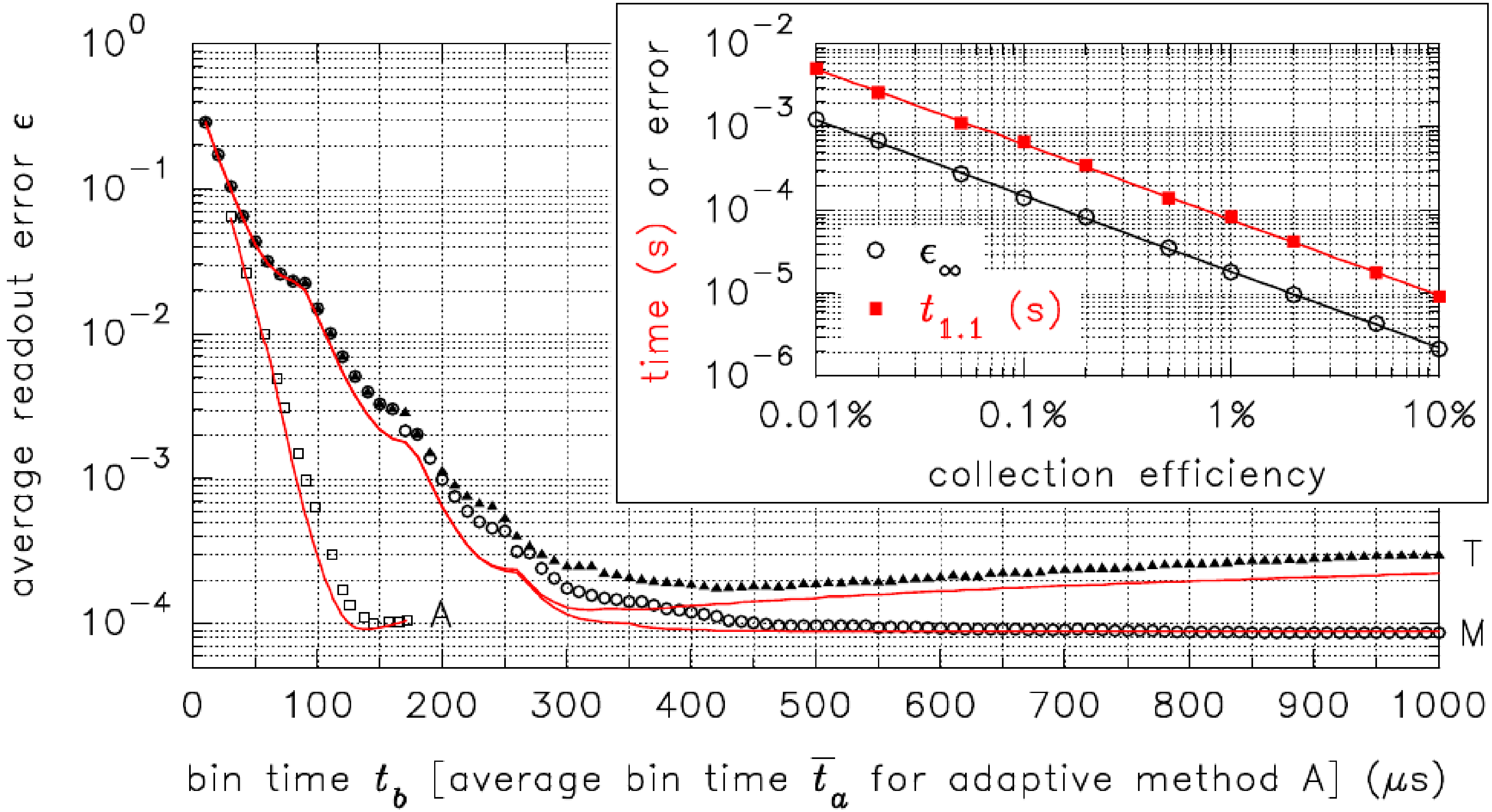
Qubit read-out



red: photon dark counts
solid circles: ion dark
open circles: ion bright

from: Myerson et al, arXiv:0802.1684v2 [quant-ph]

Qubit read-out



Fidelity limited by spontaneous decay during detection

from: Myerson et al, arXiv:0802.1684v2 [quant-ph]

Qubit read-out

Further improvements by using quantum logic:

- CNOT
  (Schätz, *et al.*, Phys. Rev. Lett. **94**, 010501 (2005))

- Repeated mapping
  (Hume *et al.*, Phys. Rev. Lett. **99**, 120502 (2007))

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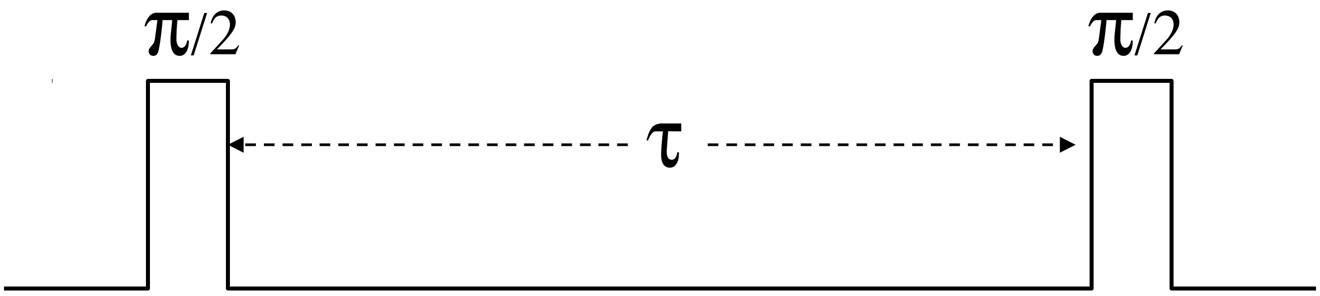
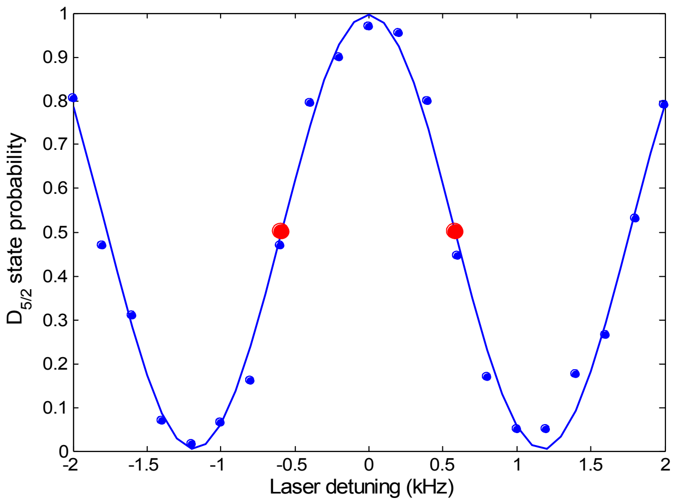
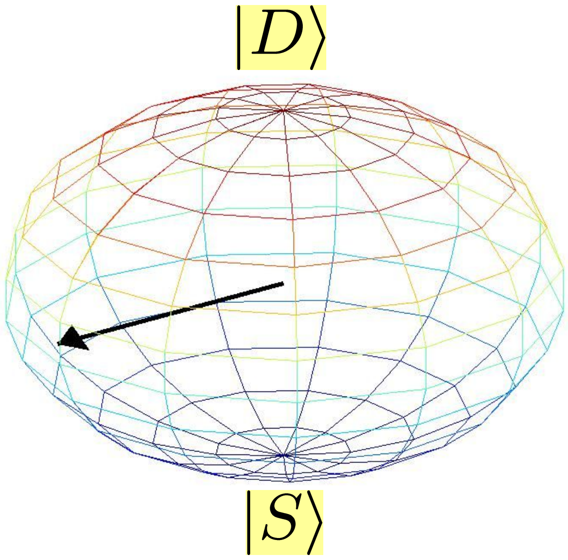
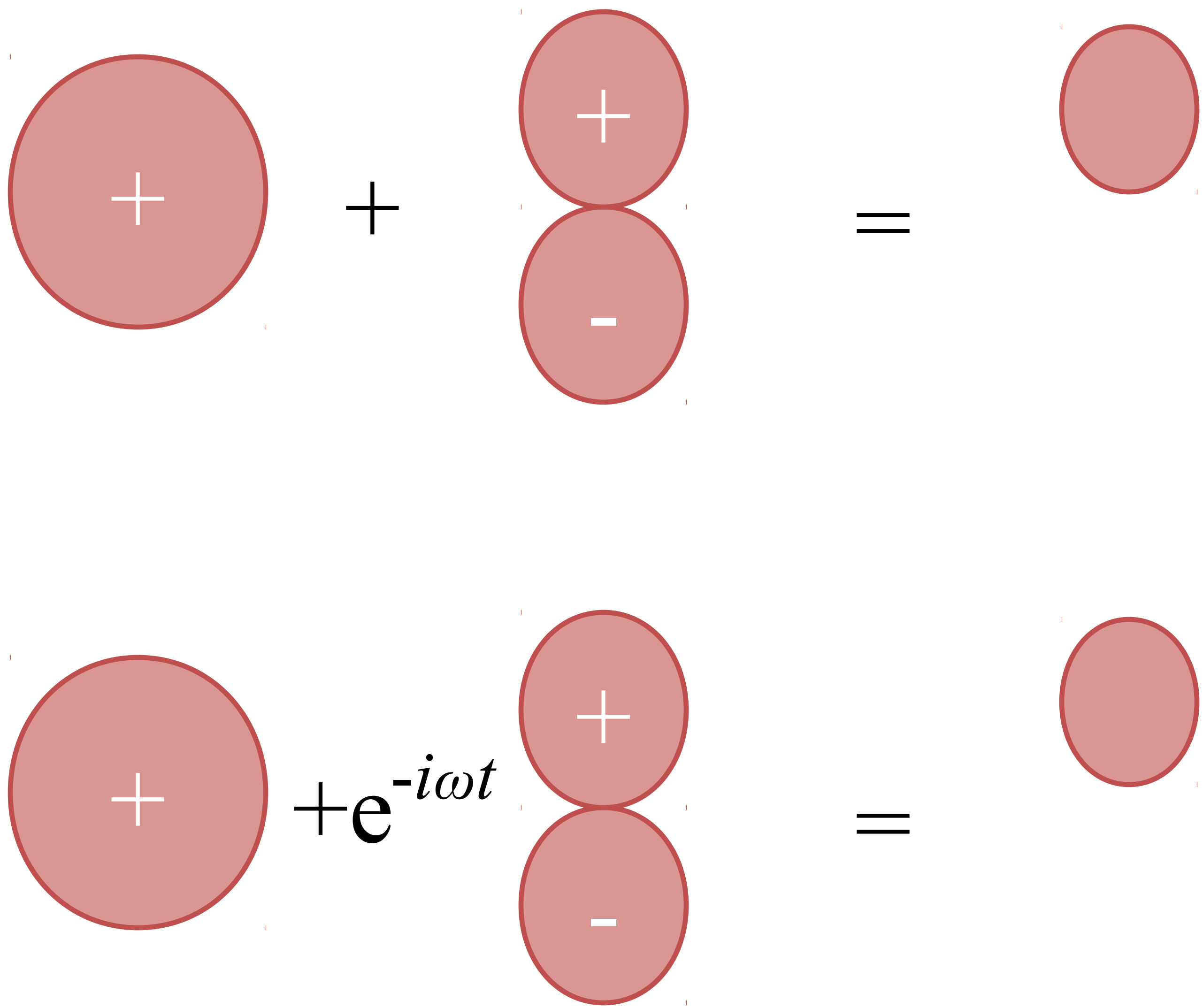
Qubit memory time



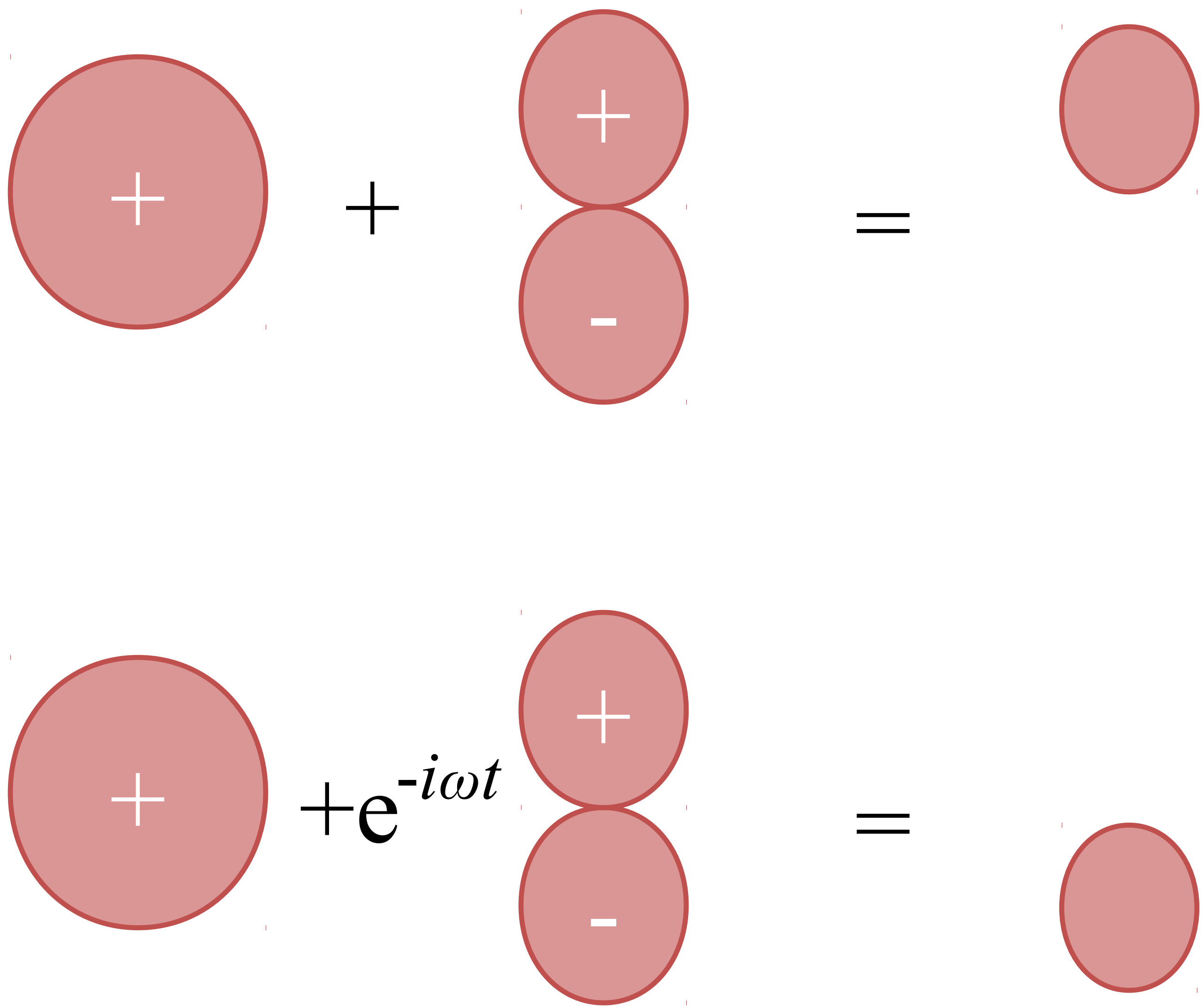




The phase ...

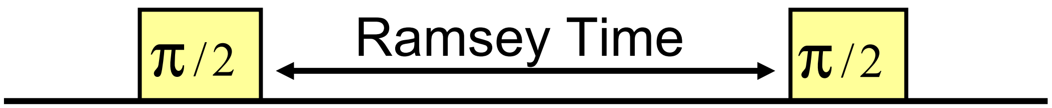


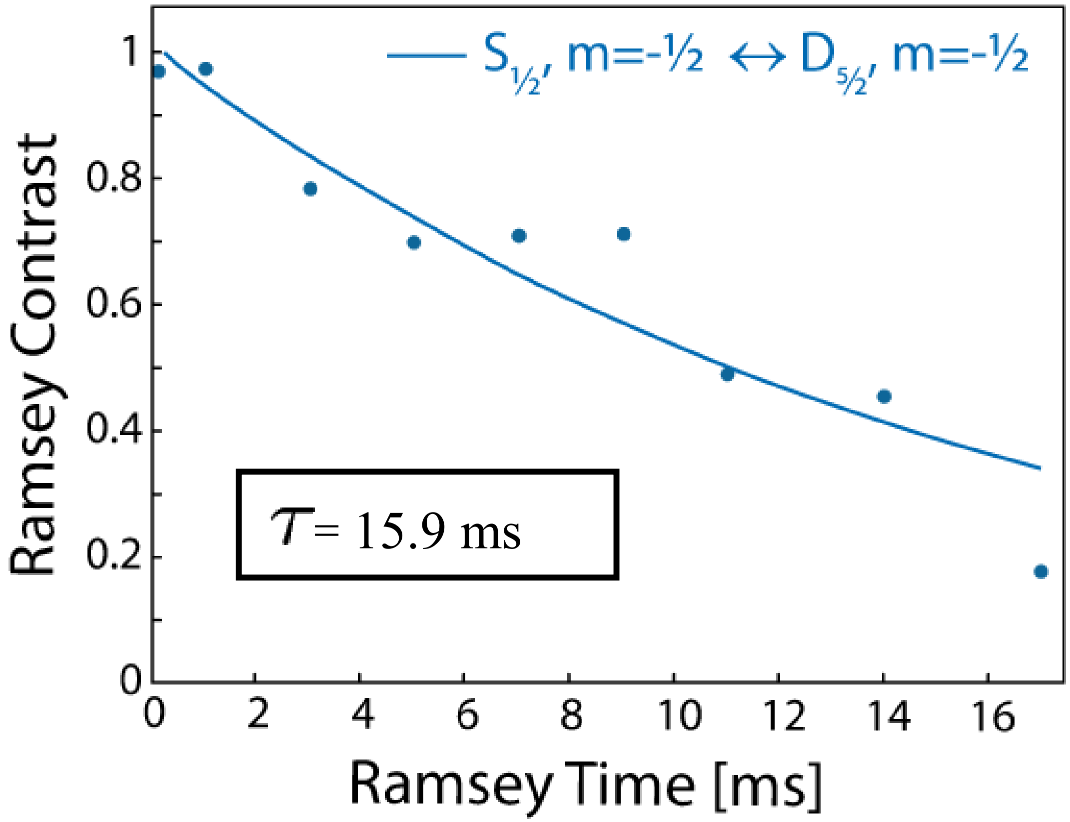
The phase ...



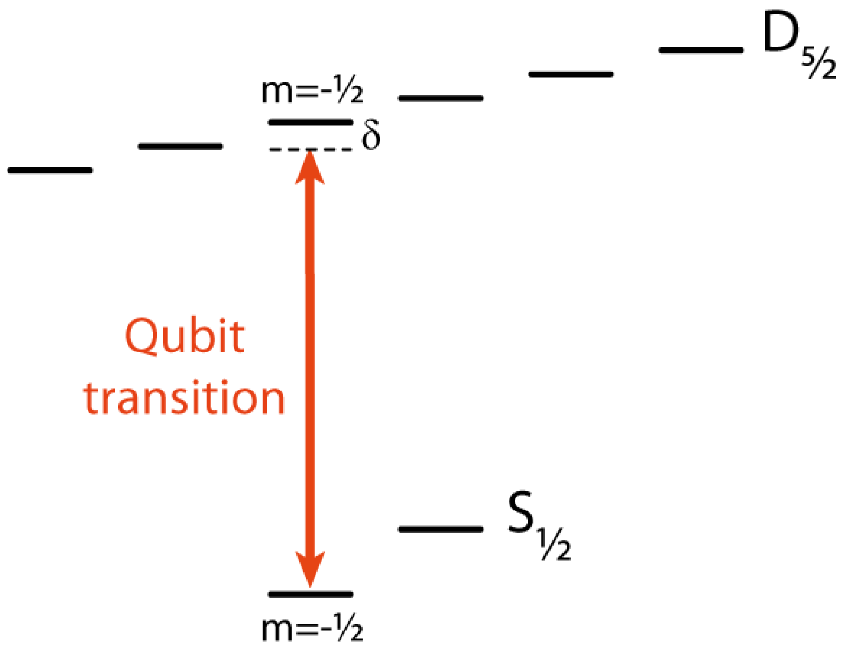
Phase coherence

Ramsey Experiment





Zeeman shift



$\Rightarrow$ Detuning causes a linear drift of the phase ϕ with $e^{i\phi} = e^{i\delta t}$

Qubits with trapped ions

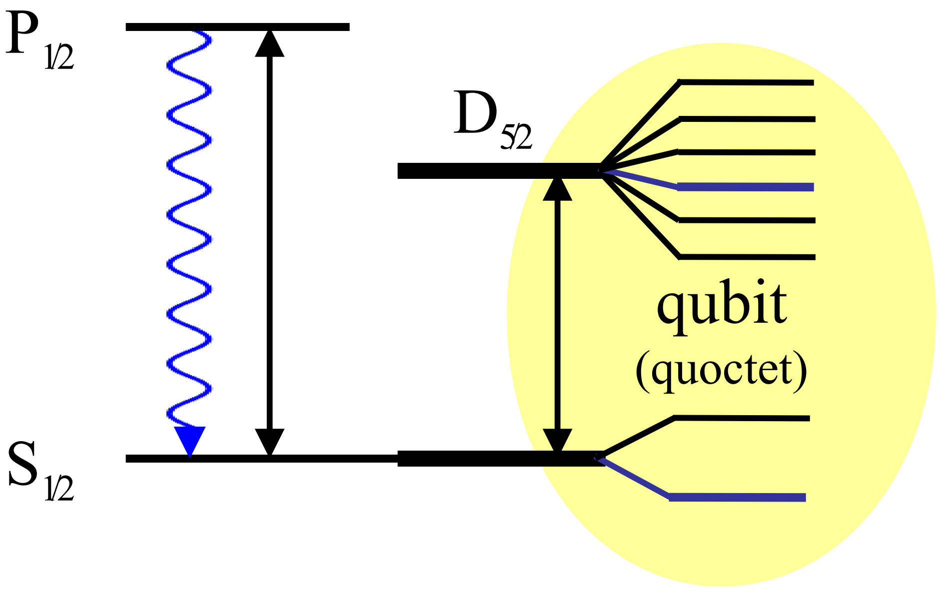
Encoding of quantum information requires long-lived atomic states:

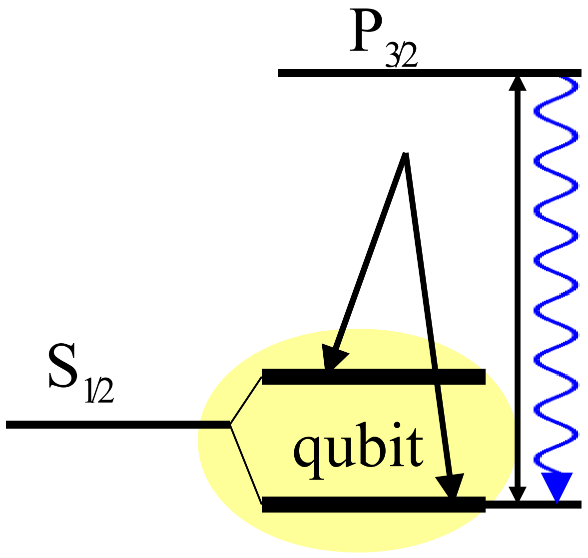
- optical transitions

  Ca^+, Sr^+, Ba^+, Ra^+, Yb^+, Hg^+ etc.

- microwave transitions

  $^{9}Be^+$, $^{25}Mg^+$, $^{43}Ca^+$, $^{87}Sr^+$, $^{137}Ba^+$, $^{111}Cd^+$, $^{171}Yb^+$

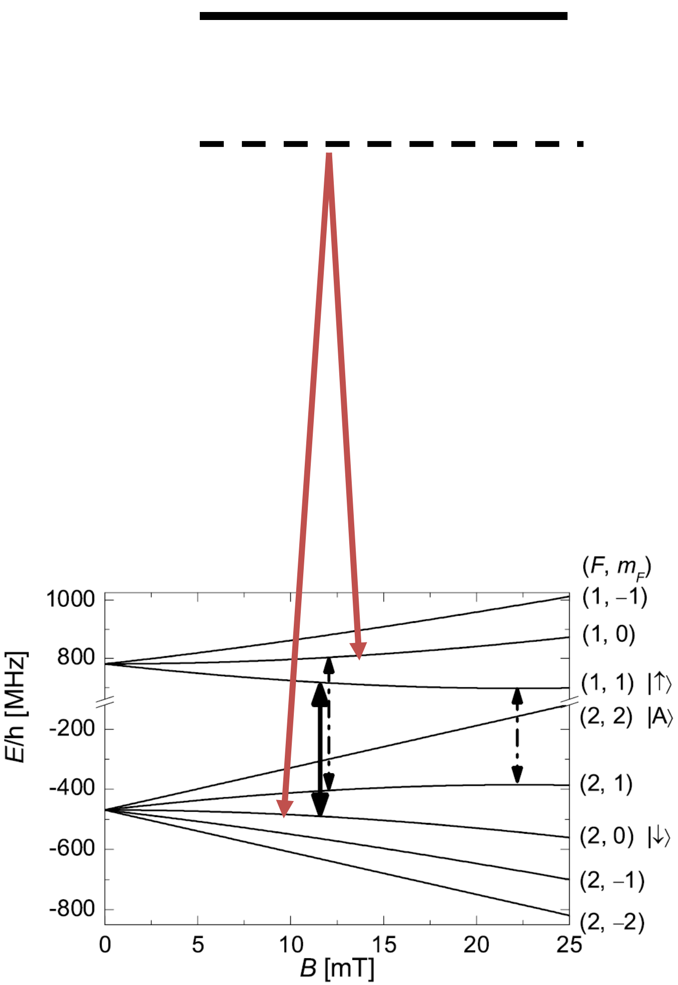




Long lived qubits

Raman transitions:

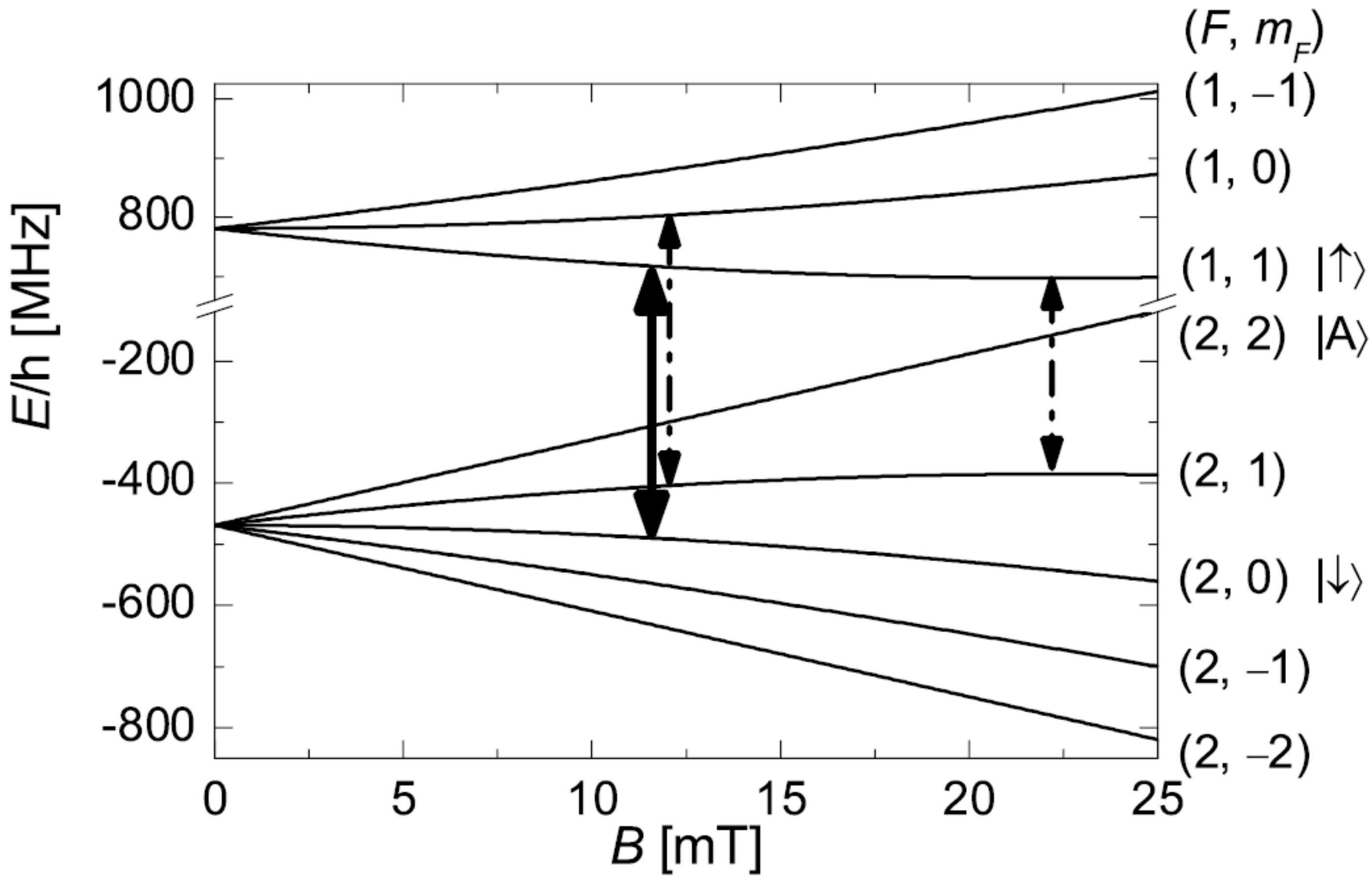
Excited state



Ground state

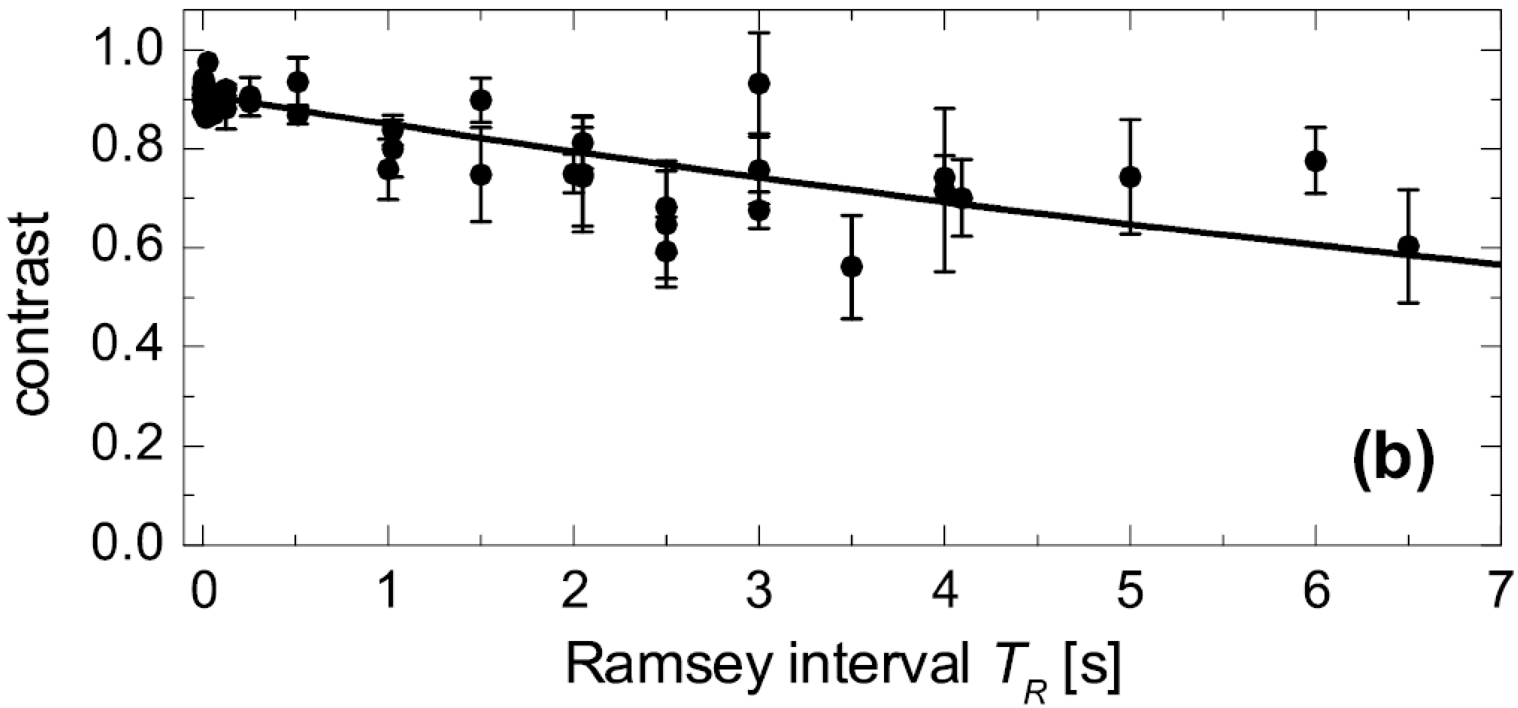
Long lived qubits

Level scheme of $^9Be^+$:



From: C. Langer *et al*., PRL **95**, 060502 (2005), NIST

Long lived qubits



From: C. Langer *et al*., PRL **95**, 060502 (2005), NIST

Time scales

Single qubit gates

10^{-6} s

Two qubit gates (Geometric phase gates)

10^{-5} s

10^{-4} s

Two qubit gates (Cirac-Zoller approach)

10^{-3} s

10^{-2} s

Single qubit coherence (magnetic field sensitive)

10^{-1} s

Coherence of the motion

10^{0} s

10^{1} s

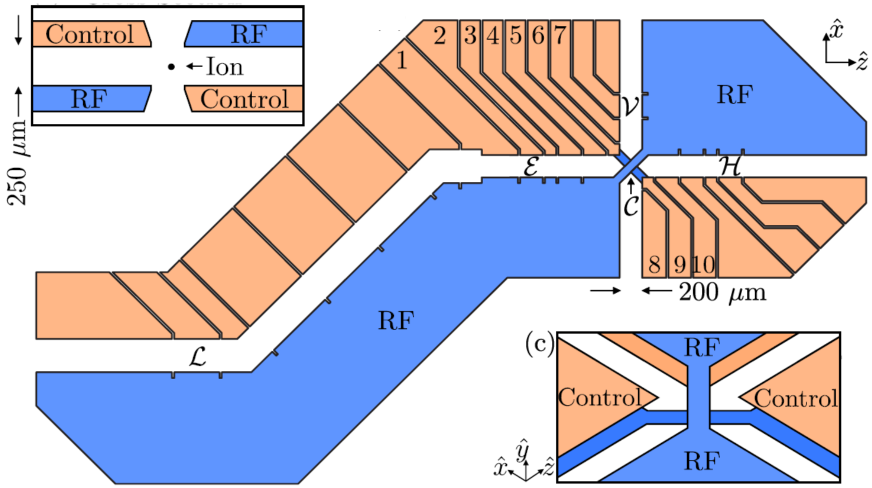
Single qubit coherence (magnetic field insensitive)

10^{2} s

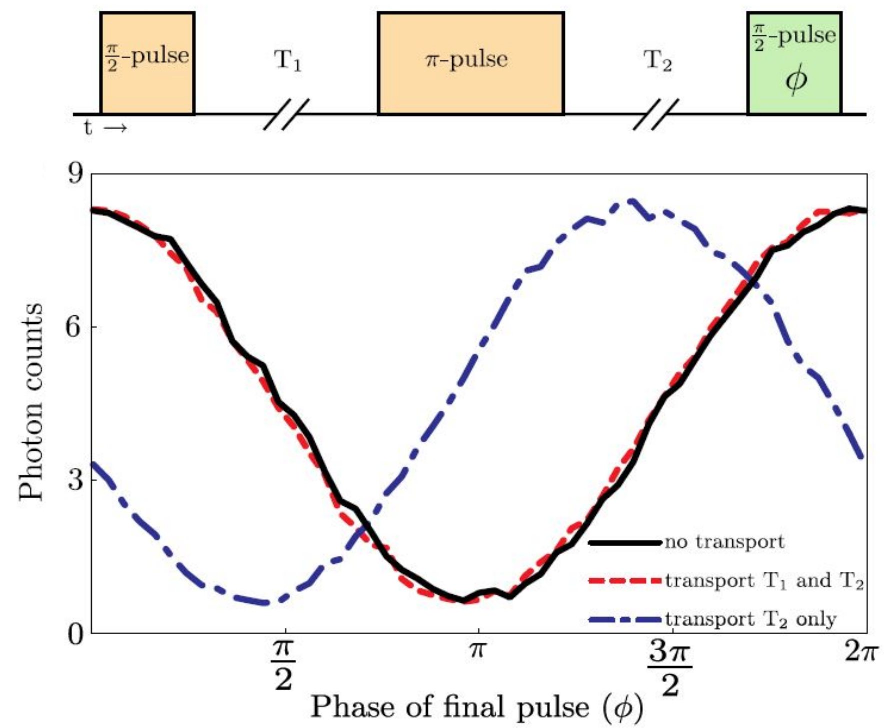
10^{3} s

Single qubit coherence (magnetic field insensitive + RF drive)

Coherent transport through a junction

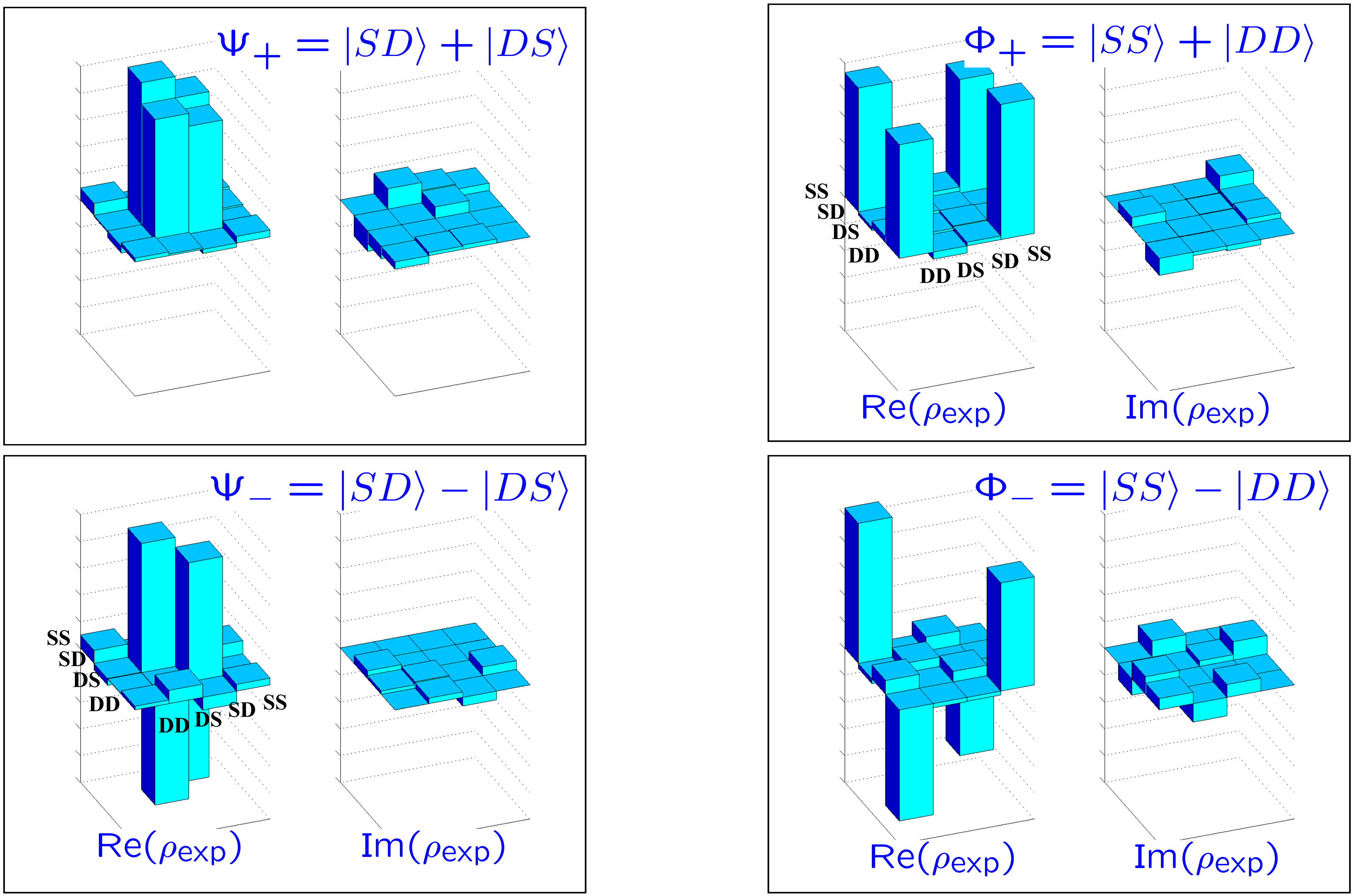


Transport		Energy Gain (recooling method) quanta/trip
$\mathcal{E}$-$\mathcal{C}$-$\mathcal{E}$	1 ion	3.2 ± 1.8
$\mathcal{E}$-$\mathcal{C}$-$\mathcal{H}$-$\mathcal{C}$-$\mathcal{E}$	1 ion	7.9 ± 1.5
$\mathcal{E}$-$\mathcal{C}$-$\mathcal{V}$-$\mathcal{C}$-$\mathcal{E}$	1 ion	14.5 ± 2.0
$\mathcal{E}$-$\mathcal{C}$-$\mathcal{E}$	2 ions	5.4 ± 1.2
$\mathcal{E}$-$\mathcal{C}$-$\mathcal{H}$-$\mathcal{C}$-$\mathcal{E}$	2 ions	16.6 ± 1.8
$\mathcal{E}$-$\mathcal{C}$-$\mathcal{V}$-$\mathcal{C}$-$\mathcal{E}$	2 ions	53.0 ± 1.2



NIST:
Blakestad, *et al.,"High fidelity transport of trapped-ion qubits through an X-junction trap array",* arXiv:0901.0533v1

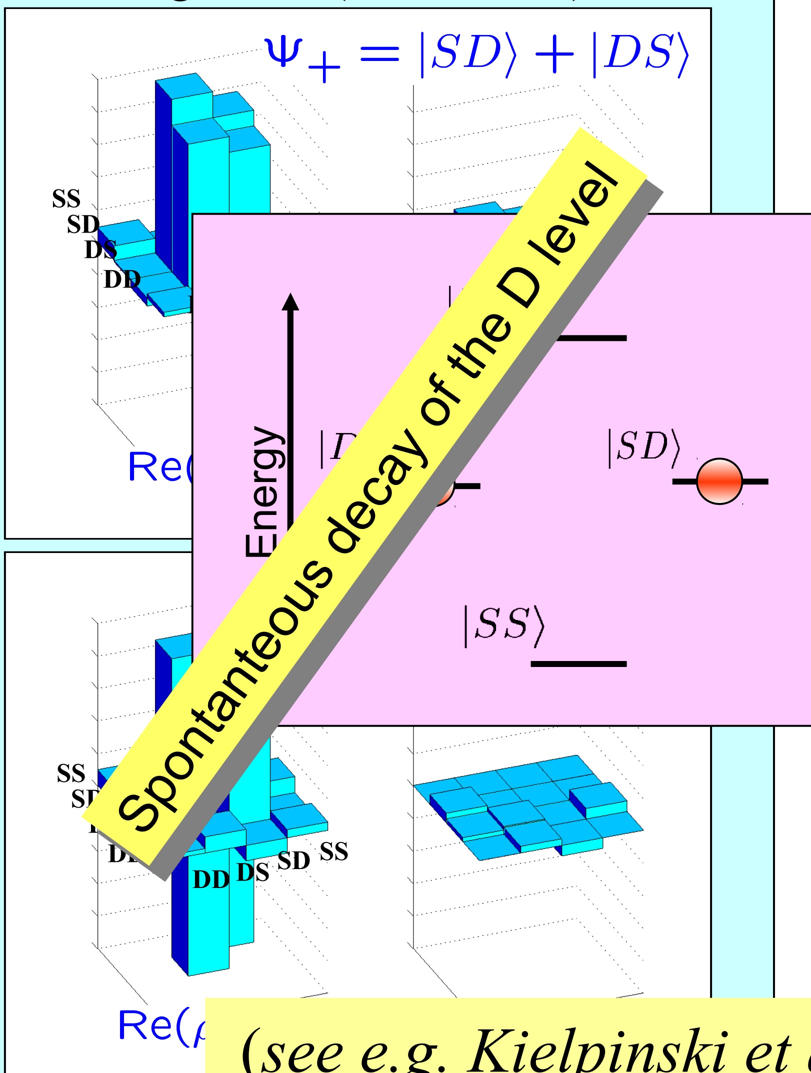
Bell states



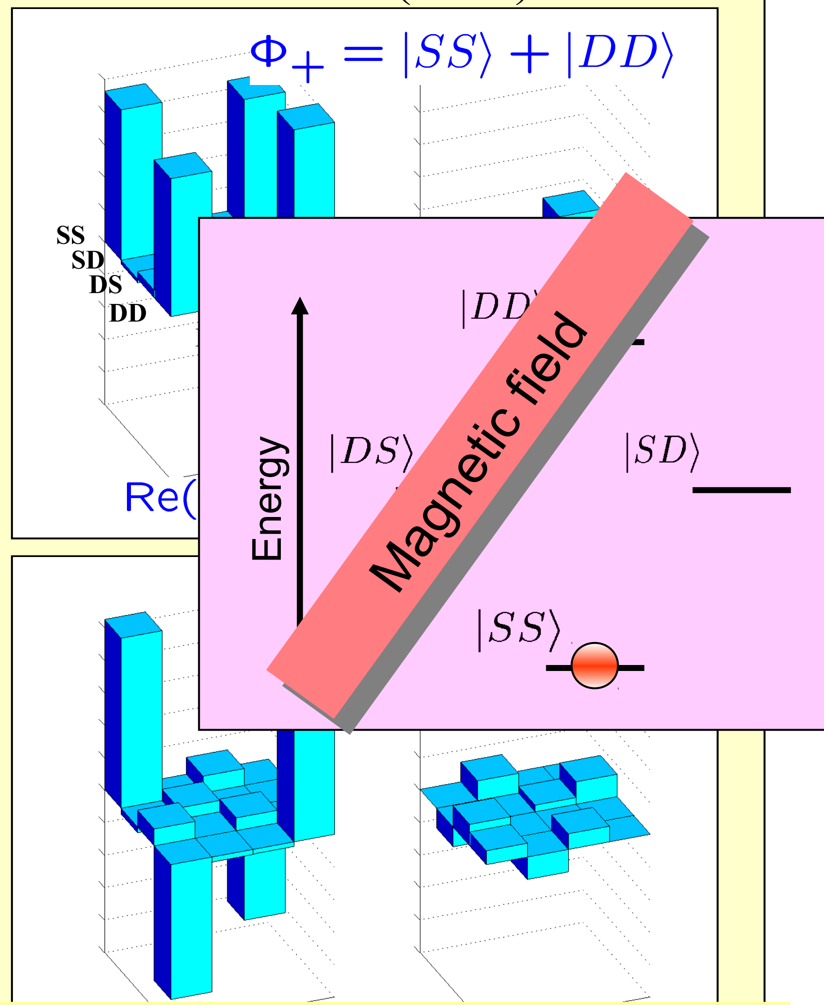
$\Psi_+ = |SD\rangle + |DS\rangle$
$\Phi_+ = |SS\rangle + |DD\rangle$
SS
SD
DS
DD
DD DS SD SS
Re(ρ_{exp})
Im(ρ_{exp})
$\Psi_- = |SD\rangle - |DS\rangle$
SS
SD
DS
DD
DD DS SD SS
Re(ρ_{exp})
Im(ρ_{exp})
$\Phi_- = |SS\rangle - |DD\rangle$
Re(ρ_{exp})
Im(ρ_{exp})

Decoherence of Bell states

long lived (~ 1000 ms)



short lived (~ ms)



(see e.g. Kielpinski et al.,Science ***291****, 1013-1015 (2001))*

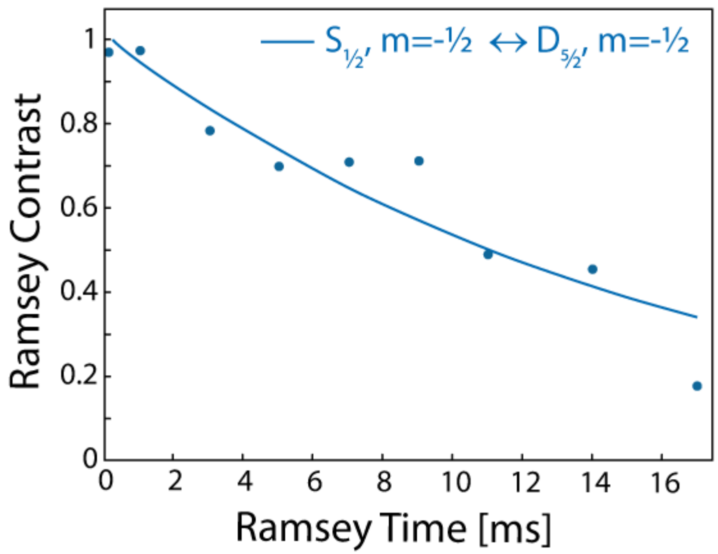
Coherence of qubits

Physical qubit: $|0\rangle + |1\rangle \longrightarrow |0\rangle + e^{i\varphi}|1\rangle$

Logical qubit: $|01\rangle + |10\rangle \longrightarrow e^{i\varphi}|01\rangle + e^{i\varphi}|10\rangle$

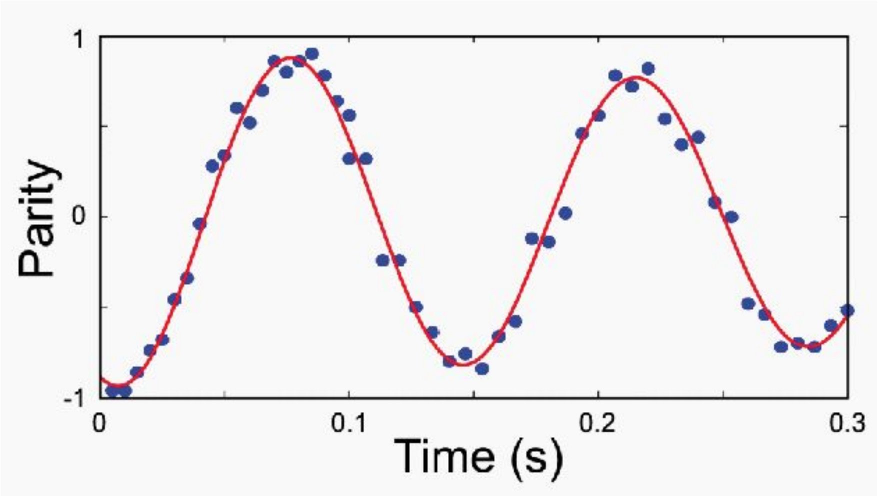
Coherence of qubits

Physical Qubit
Ramsey Experiment



τ = 16 ms

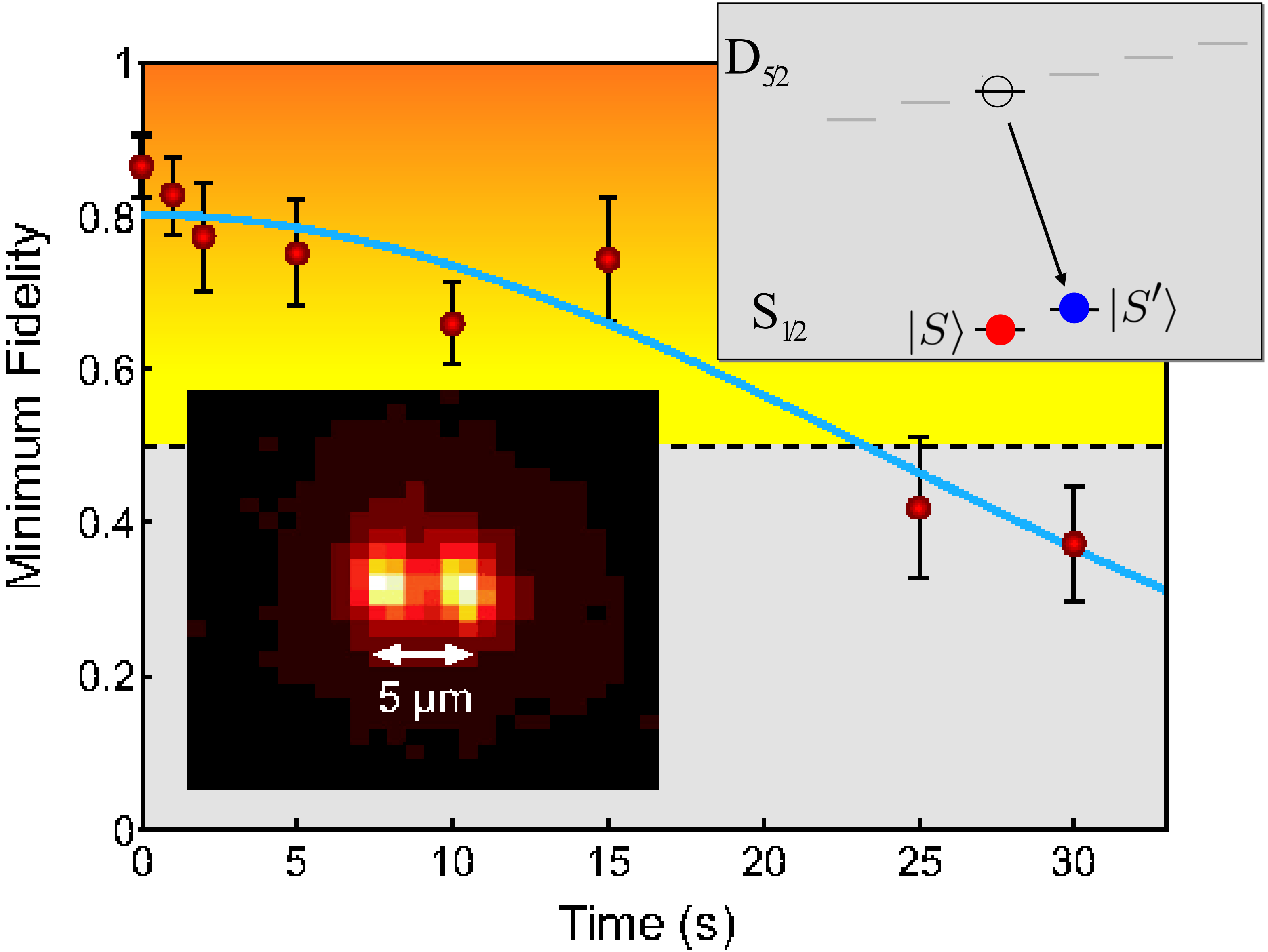
Logical Qubit
Parity Oscillations



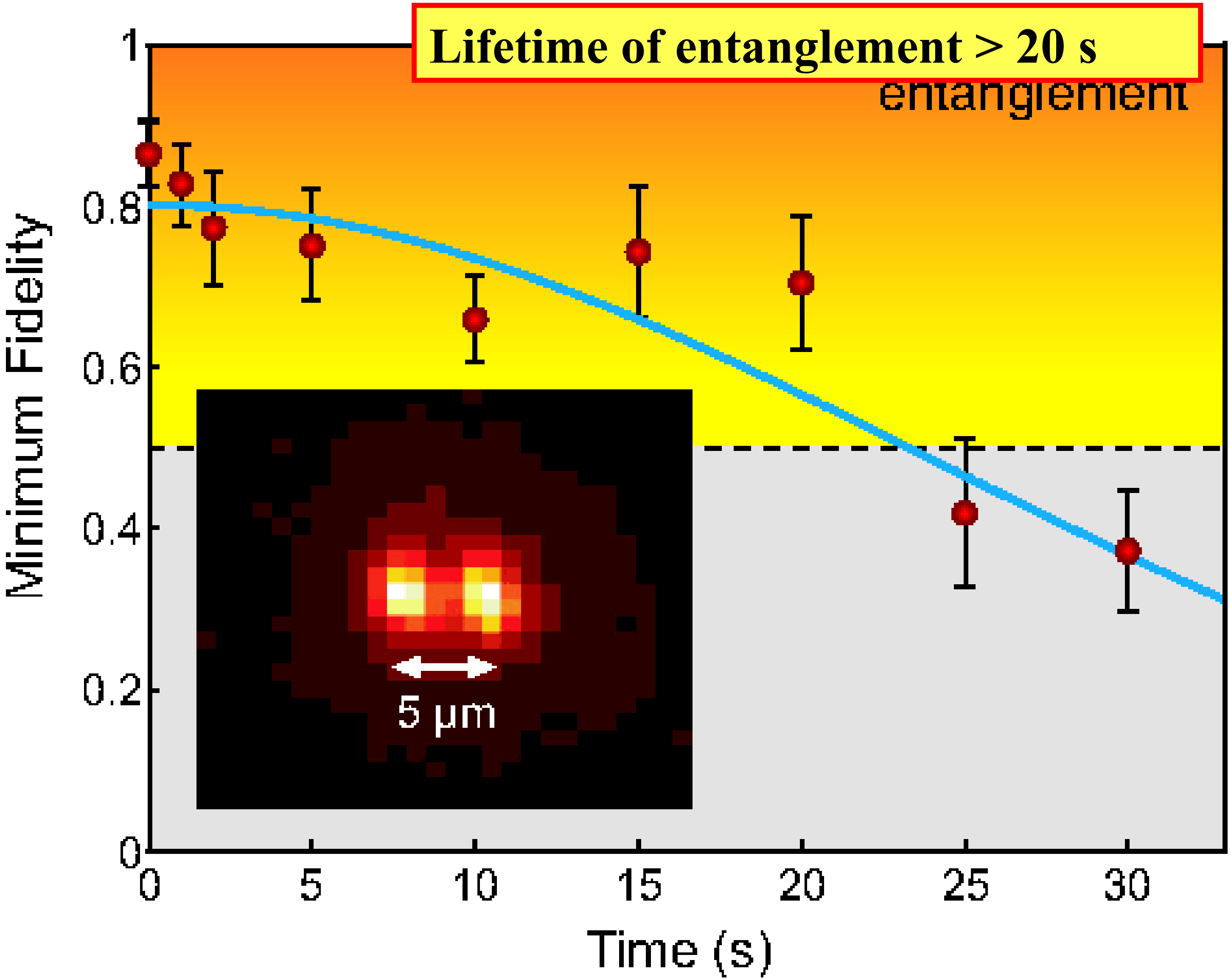
τ = 1050 ms

C.F. Roos et al., PRL 92, 220402 (2004).

Ultra-long coherence



Ultra-long coherence



H. Häffner et al., App. Phys. B **81** 151 (2005).

Time scales

10^{-6} s

10^{-5} s

10^{-4} s

10^{-3} s

10^{-2} s

10^{-1} s

10^{0} s

10^{1} s

10^{2} s

10^{3} s

Single qubit gates

Two qubit gates (Geometric phase gates)

Two qubit gates (Cirac-Zoller approach)

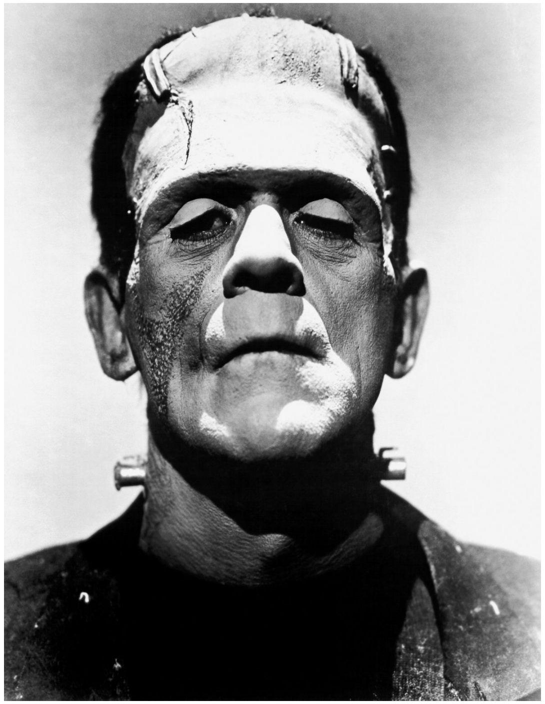
Single qubit coherence (magnetic field sensitive)

Coherence of the motion (Innsbruck)

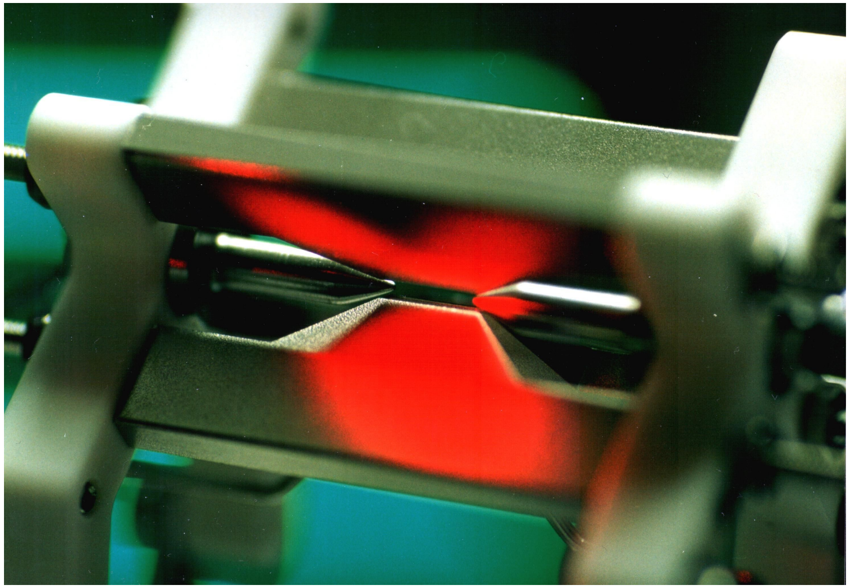
Single qubit coherence (magnetic field insensitive, Boulder)
Decoherence free subspaces

Well chosen single qubit coherence (Boulder)

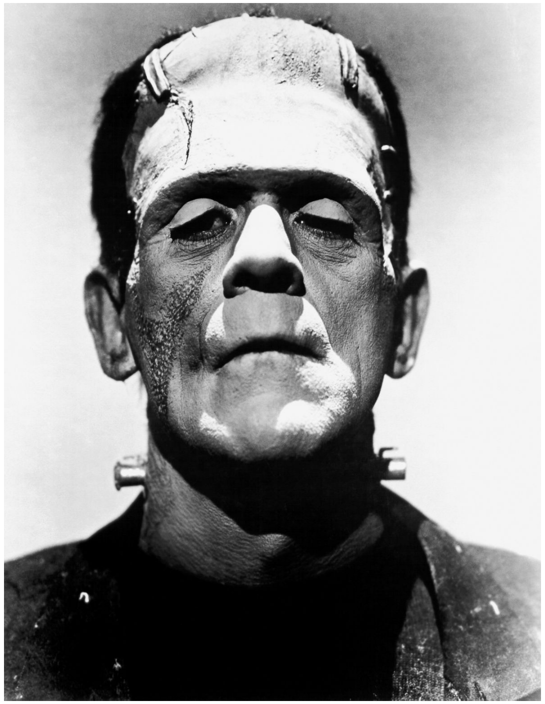
The Bad: Decoherence



The Ugly: Operational Errors



The Bad: Decoherence



The Ugly: Operational Errors

- Fault tolerant quantum computing
- Ion trap quantum computing
- Quantum memory
- Operational errors
- Conclusions

Realistic wish list

1. Coherence times much longer than gate times

by a factor of 10^4 ✓

2. Control the qubits

single qubit operations with an accuracy of 10^{-4}

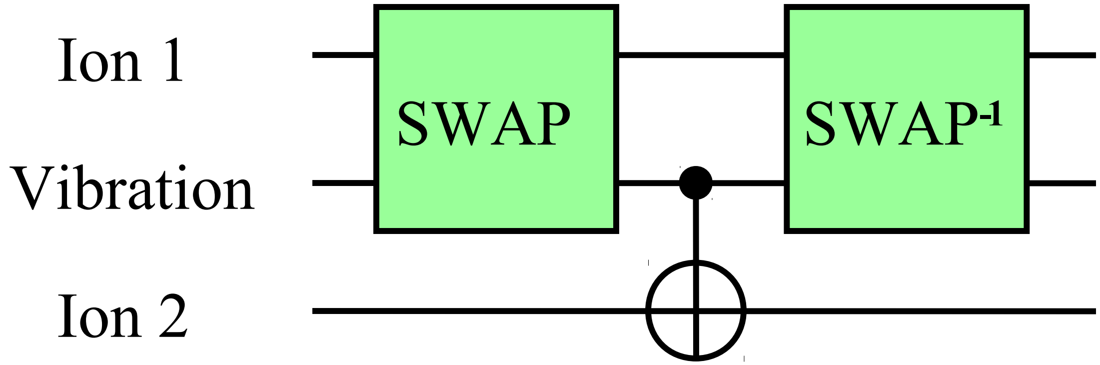
two qubit operations with an accuracy of 10^{-4}

Additionally: very good initialisation ✓

efficient and fast read out ✓

parallelisation of all processes

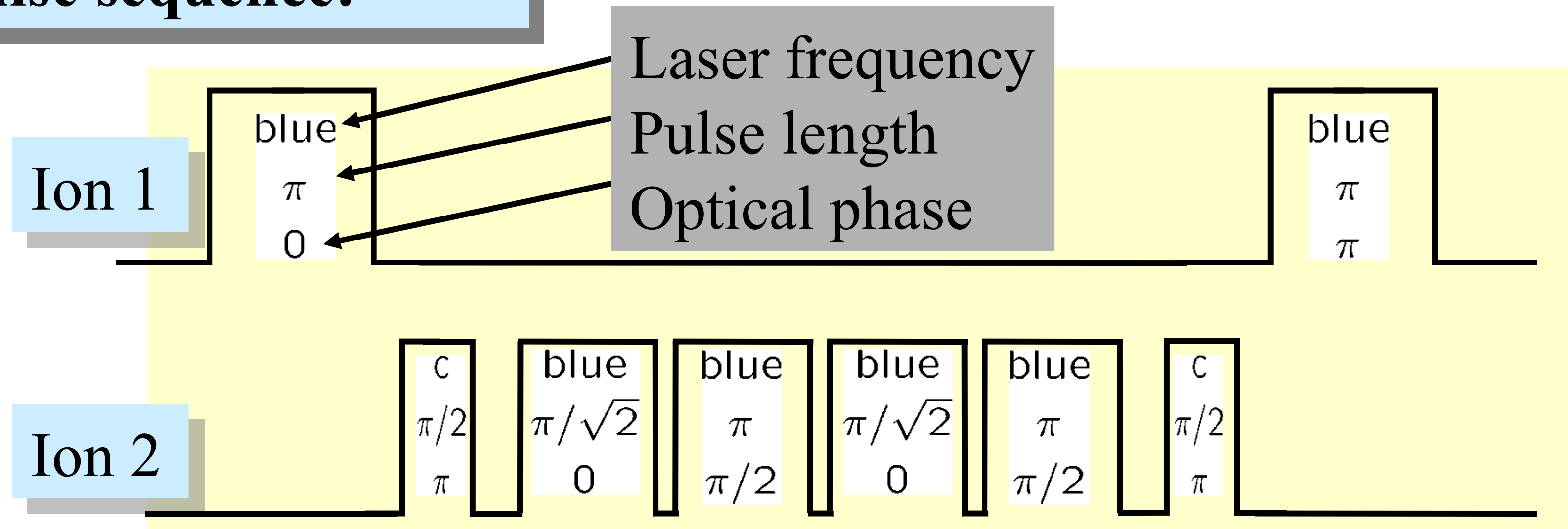
A CNOT



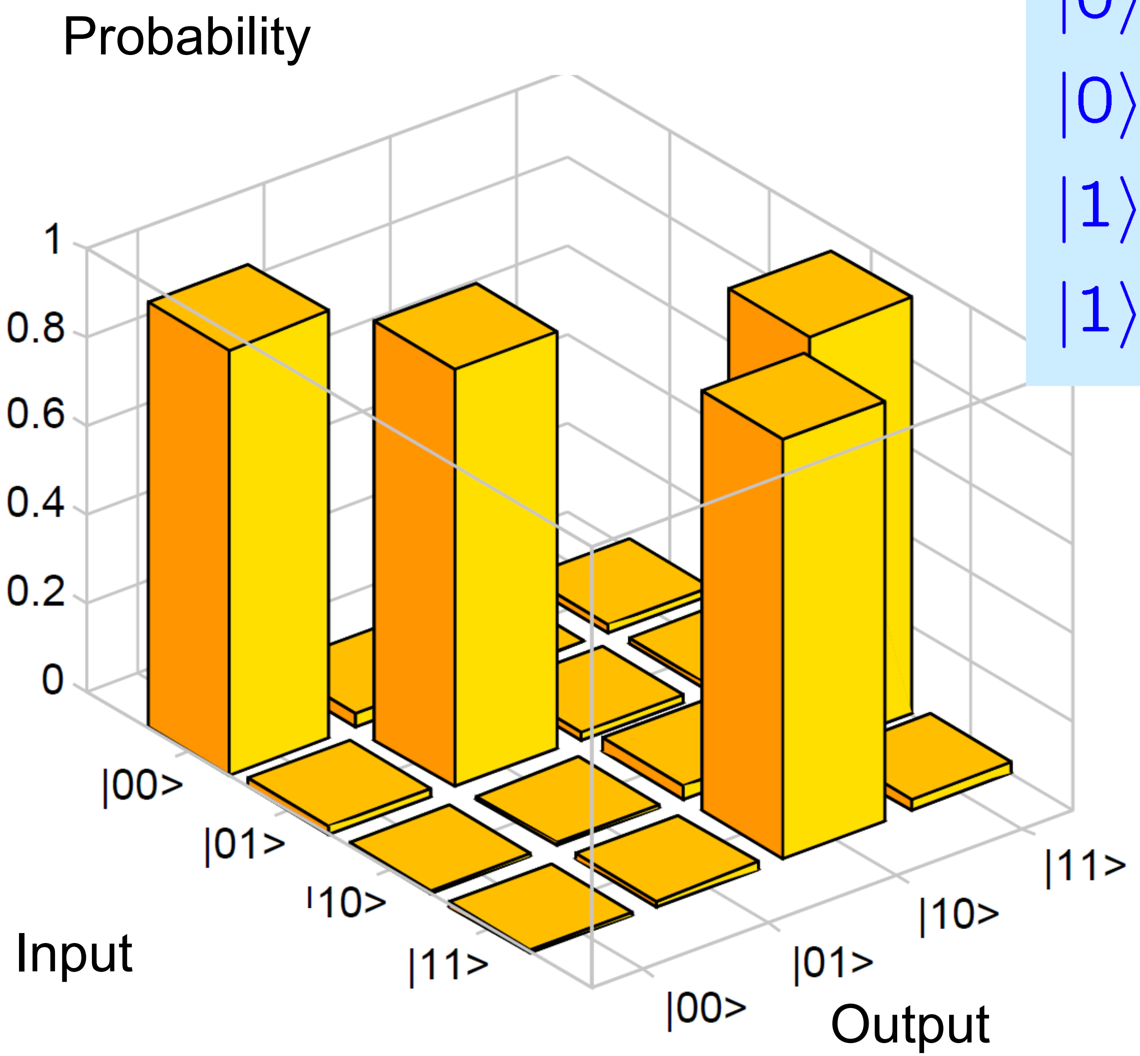
Control qubit

Target qubit

Pulse sequence:



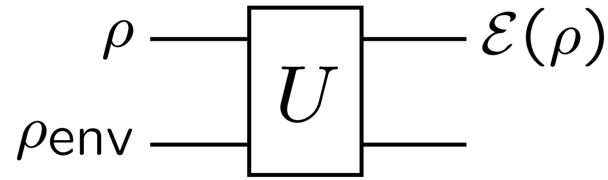
The truth table



$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

Characterizing quantum gates

How to describe action of an operation acting on system of qubits which are coupled to their noisy environment ?



Use completely positive map / quantum operations.
Poyatos, Cirac, Zoller PRL **78**, 390 (1997), Chuang & Nielsen J. Mod. Opt. **44**, 2455 (1997)

Operator sum $\mathcal{E}(\rho)$ **:**

$$\mathcal{E}(\rho) = \sum_{mn} \underbrace{\chi_{mn}}_{\text{process matrix}} \cdot A_m \rho A_n^\dagger$$

Operators A are set of 4^N basis operators for the space of matrices !

How to measure a fidelity

Recipe for quantum process tomography:

A) Choose input states for process such that corresponding density matrices form a basis set for the space of matrices.

B) For all these input states do a quantum state tomography of the output state

C) Estimate transfer matrix χ using your prefered method

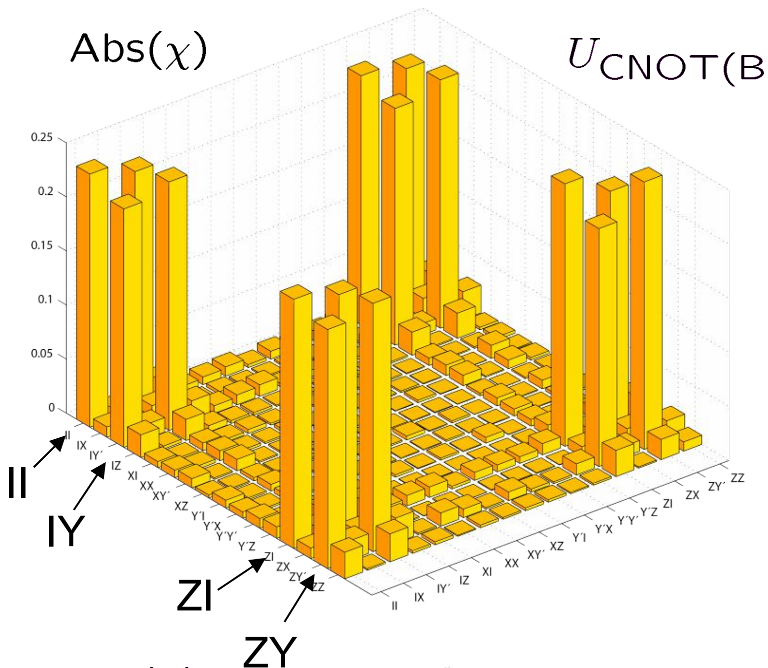
2-qubit QPT in the Ibk experimental setup:

16 input states
9 tomographic measurement settings for each state
total number of measurements is 144

Quantum process tomography takes 5- 10 min. !

Some numbers

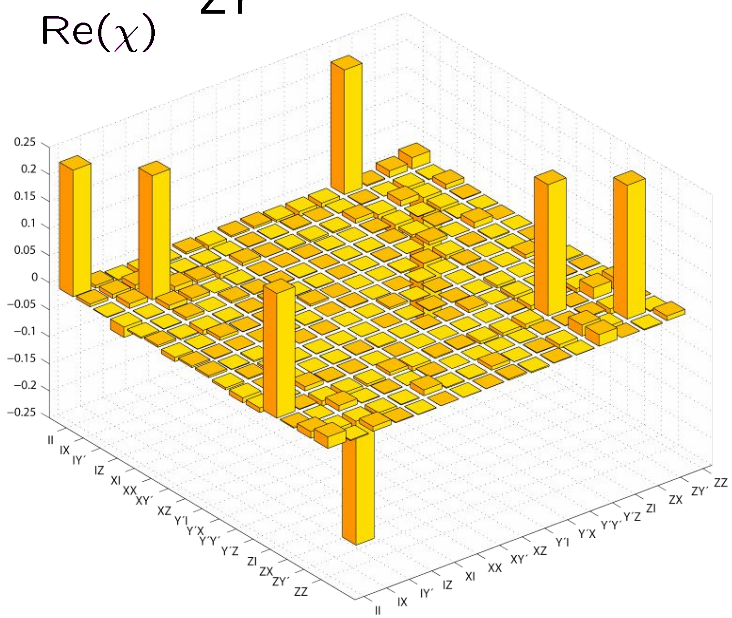
Abs(χ)



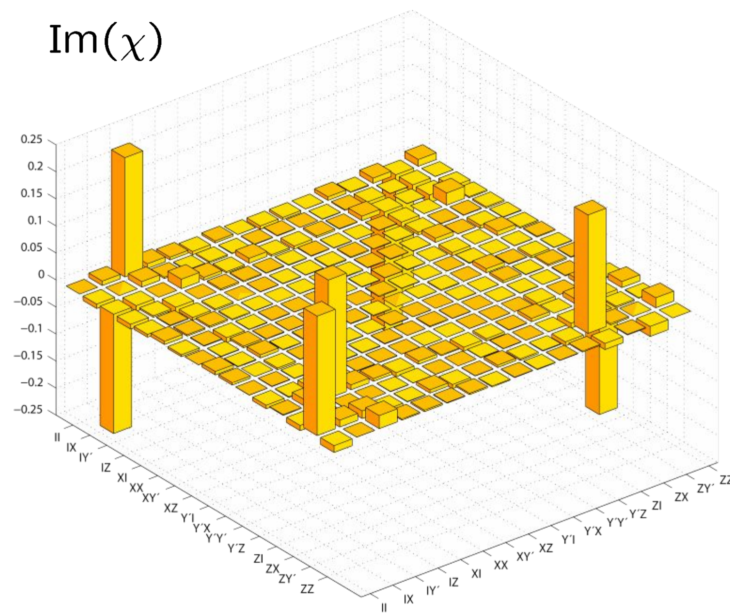
$$U_{\mathrm{CNOT(B)}} = = -\frac{1}{2}\hat{U}_Z \cdot \left(I \otimes I + Z \otimes I - iI \otimes Y' + iZ \otimes Y'\right).$$

Measure	
Process fidelity F_p	90.8(6)%
Mean fidelity $\overline{F}$	92.6(6)%

Re(χ)



Im(χ)



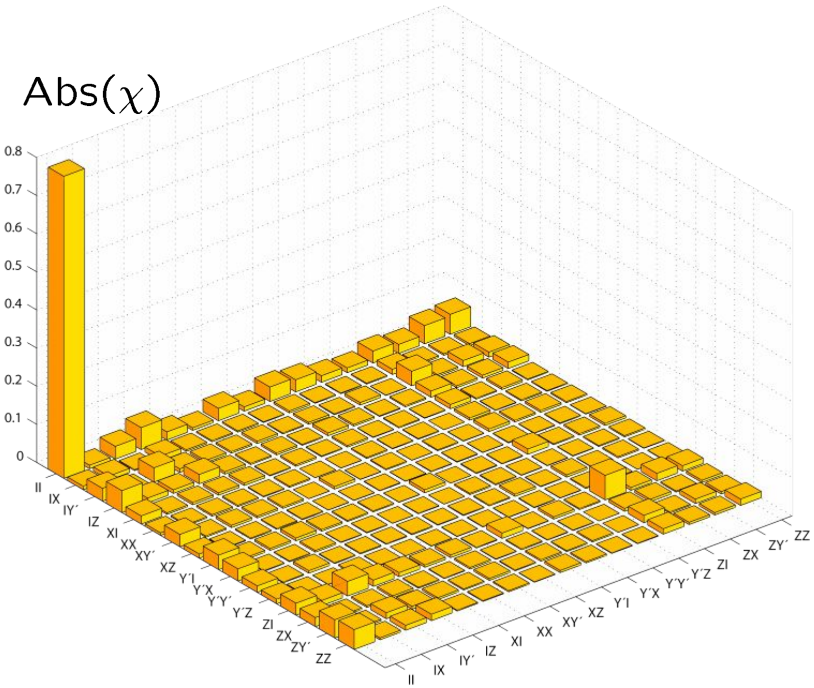
Is that all ?

Can we infer the performance of combined gate operation from a QPT of a single gate operation ?

Is that all ?

Can we infer the performance of combined gate operation from a QPT of a single gate operation ?

Experiment: QPT of two subsequently applied CNOT gates



Experimental result	**2x single gate result**
$F_p = 79(1)\%$	$F_p = 82.8\%$
$\bar{F} = 83.4(8)\%$	$\bar{F} = 86.2\%$

How useful is QPT

Is quantum process tomography practical?

How will we asses our quantum computers?

Randomized benchmarking

Rotation into the eigenbasis

Ideal $|\Psi_{\mathrm{out}}\rangle$ can be calculated

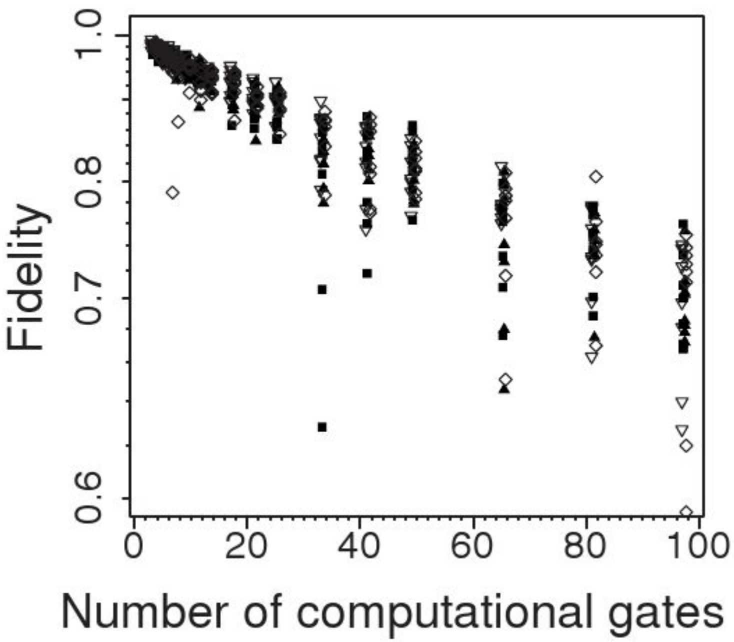
$$|\Psi_{\mathrm{out}}\rangle = RG_N P_N G_{N-1} P_{N-1} \ldots G_0 P_0 |\Psi_{\mathrm{in}}\rangle$$

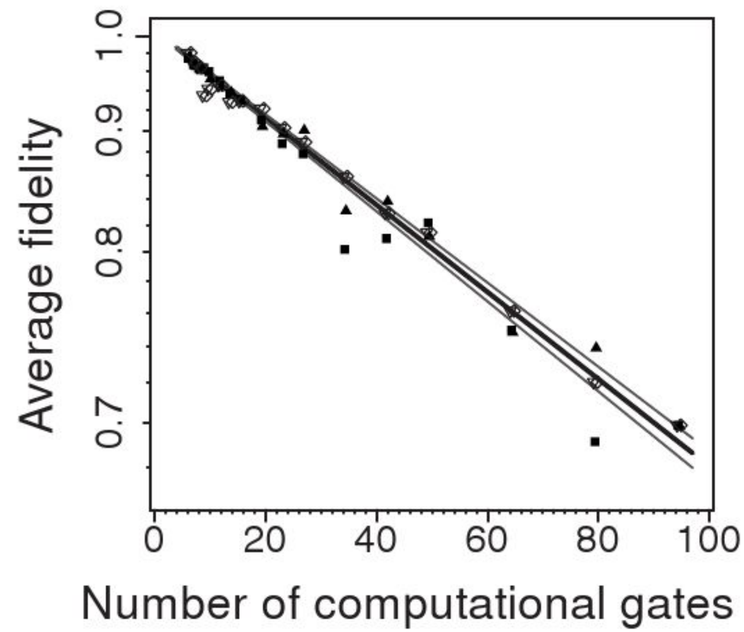
G_k : fixed random gate sequence

P_k : depolarization of noise

Randomized benchmarking

Single qubit gate fidelity: 0.995

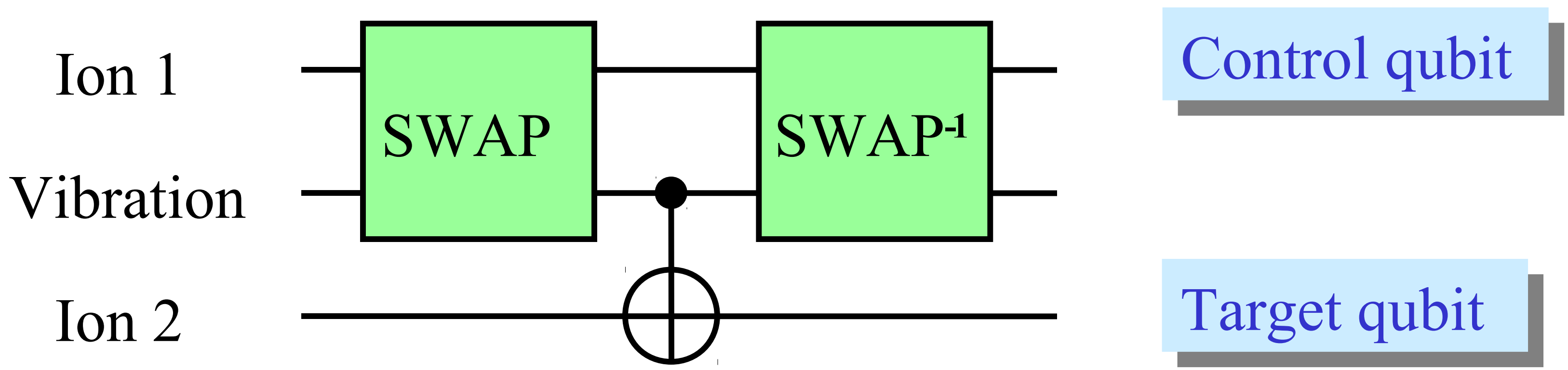




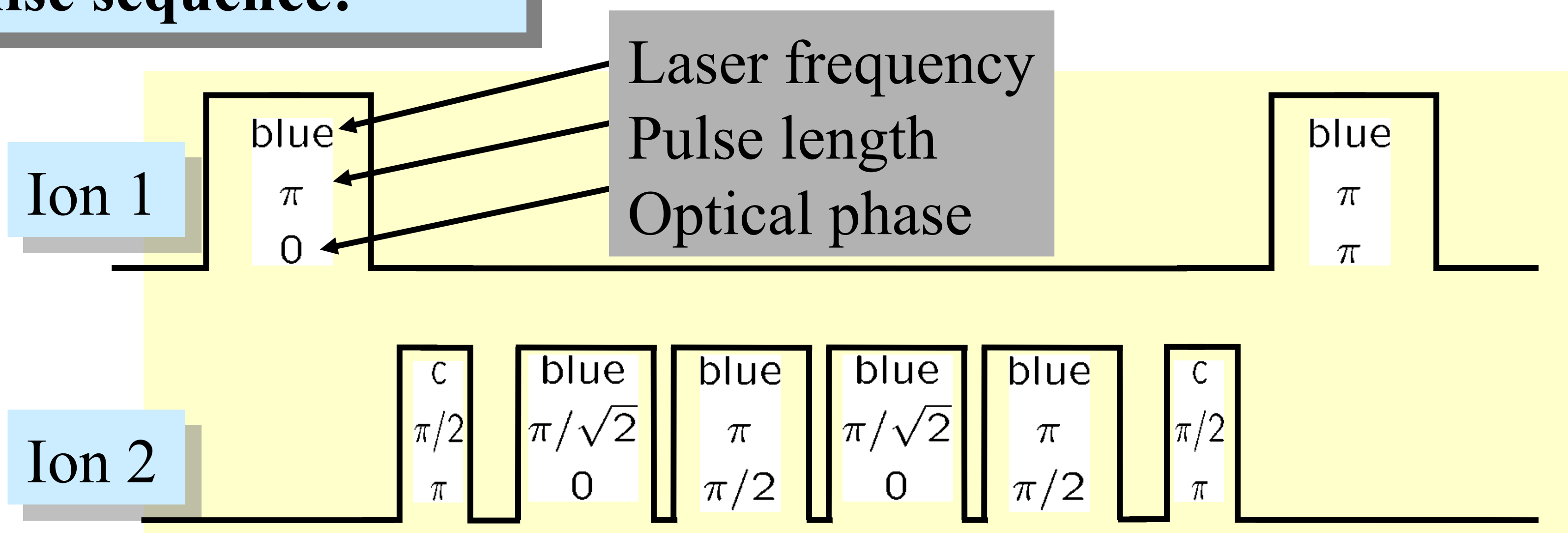
from: Knill et al., Phys. Rev A 77 012307 (2008)

Can be extended to multi-qubit processes

A CNOT



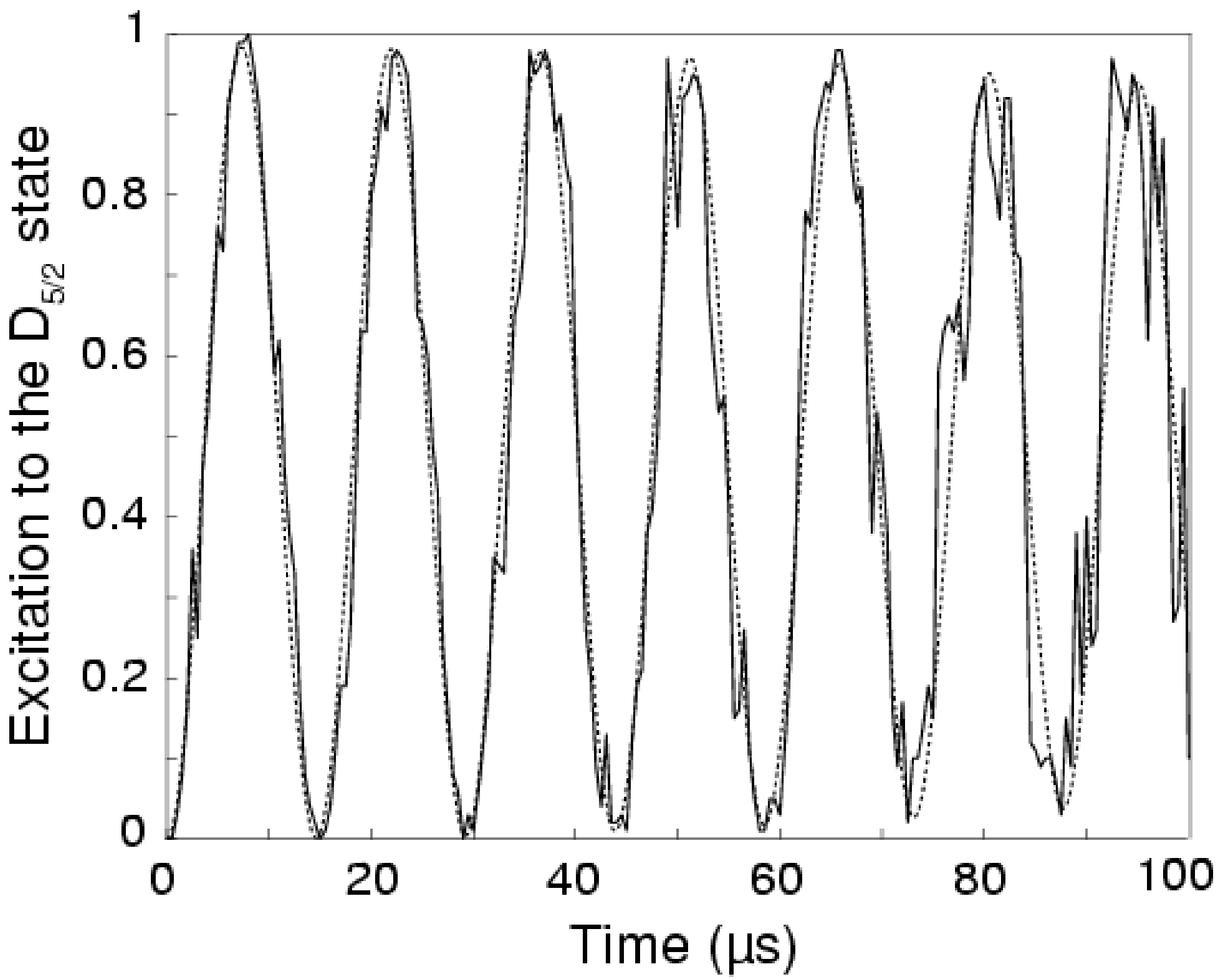
Pulse sequence:



Error budget for Cirac-Zoller CNOT

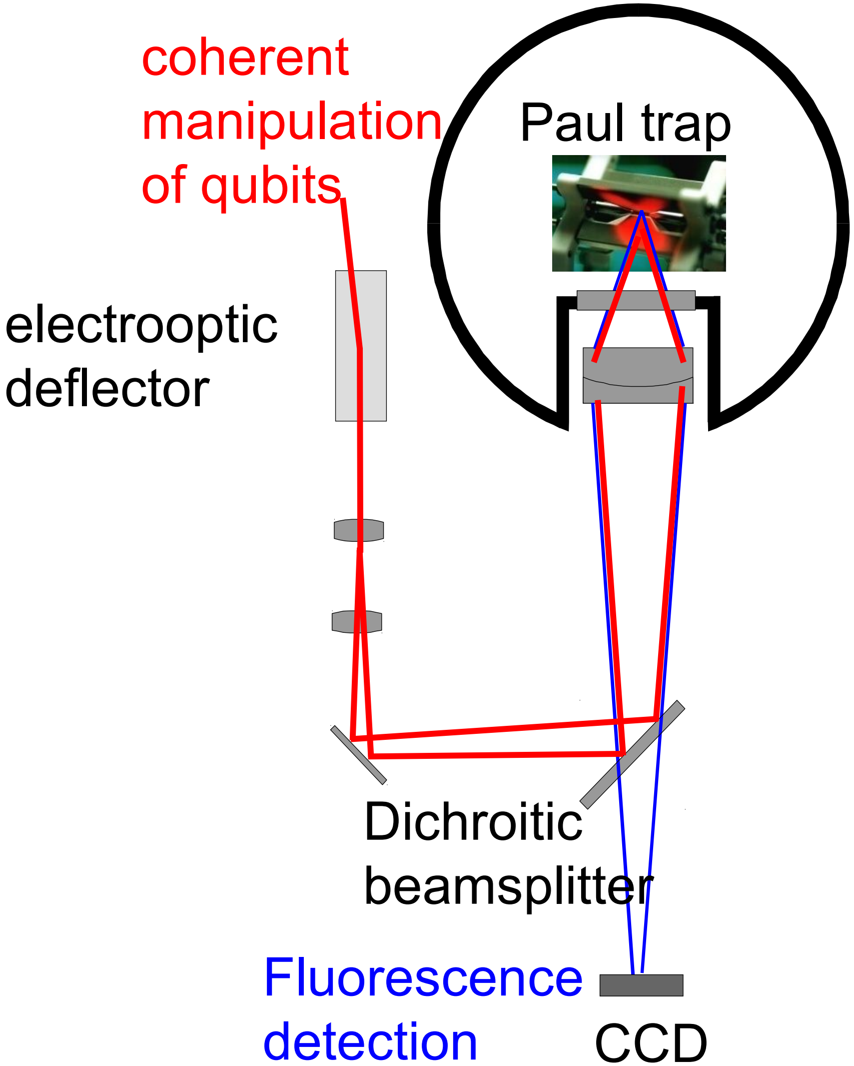
Error source	Magnitude	Fidelity loss
Frequency noise (fast)	< 160 Hz (FWHM)	< 5 %
Frequency noise (slow)	~ 160 Hz (FWHM)	~ 0.6 %
Laser intensity noise	3 % peak to peak	0.1 %
Addressing error (can be corrected for partially)	3 % in Rabi frequency (at neighbouring ion)	1 %
Off resonant excitations	for $t_{gate} = 600$ μs	- (Pulse shaping)
Residual thermal excitation	<n>bus < 0.02 <n>spec = 6	< 2 % 0.4 %
Total	February 2006	~ 9 %

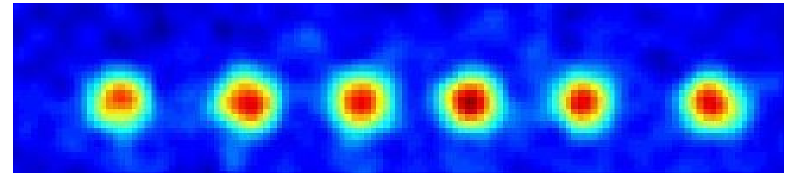
Intensity noise

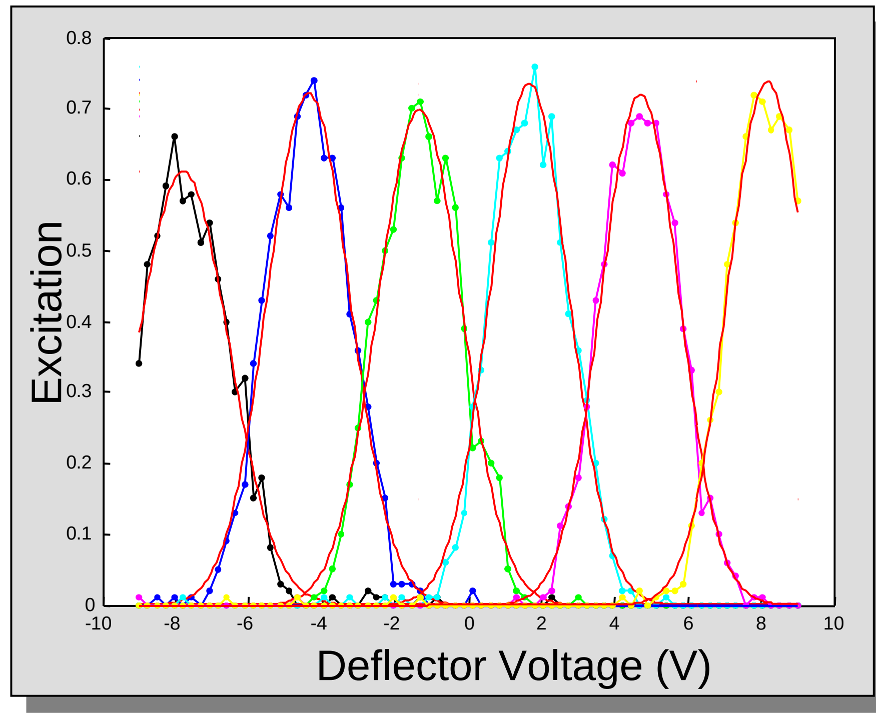


=> Laser intensity noise: 0.03

Addressing single qubits







- inter ion distance: $\sim 4\ \mu m$

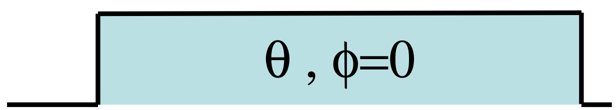
- addressing waist: $\sim 2\ \mu m$

< 0.1% intensity on neighbouring ions

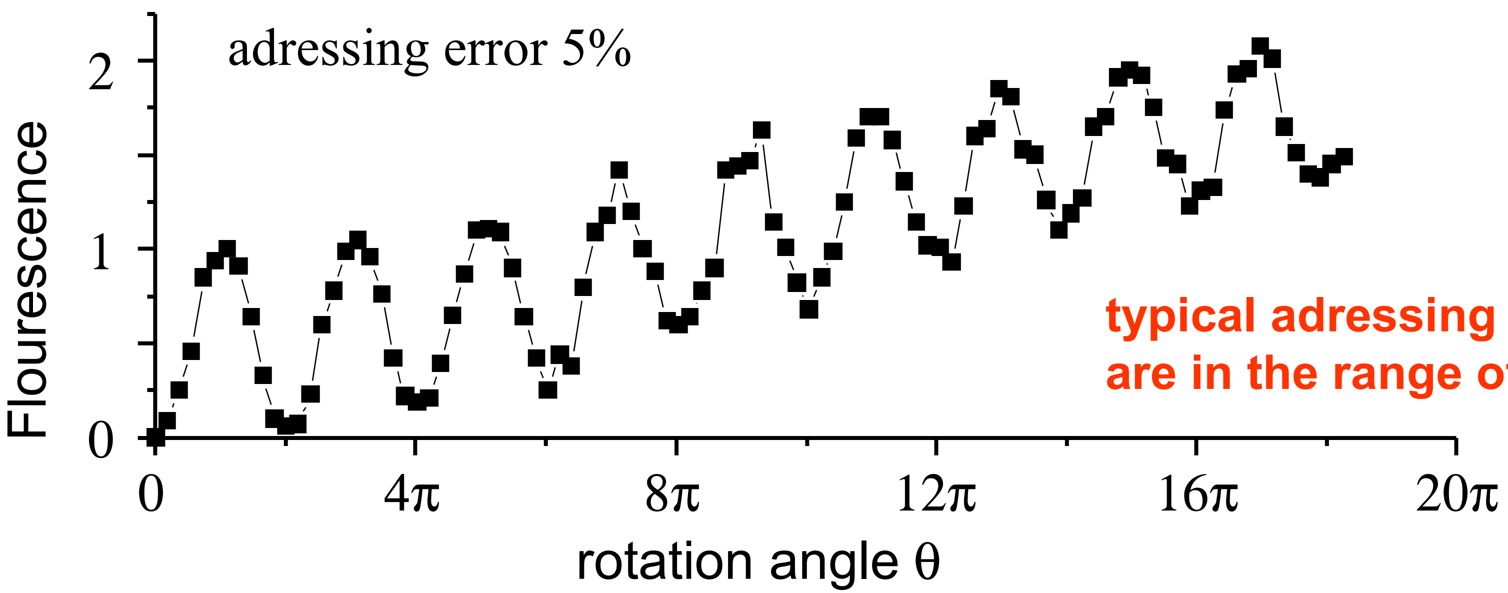
Qubit rotations with two ions

Pulse sequence:

carrier
pulse



$$\textbf{error} = \frac{\Omega_{\text{neighbour}}}{\Omega_{\text{target}}} = \sqrt{\frac{I_{\text{neighbour}}}{I_{\text{target}}}}$$



typical adressing errors are in the range of 3-5%

Error budget for Cirac-Zoller CNOT

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Frequency noise (fast)	< 160 Hz (FWHM)	< 5 %
Frequency noise (slow)	~ 160 Hz (FWHM)	~ 0.6 %
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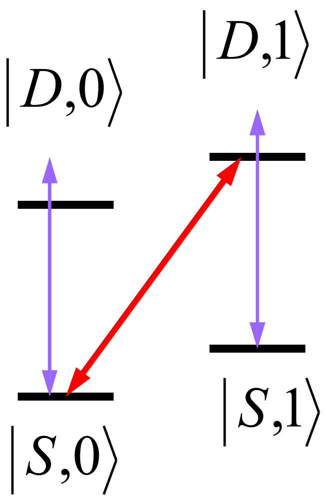
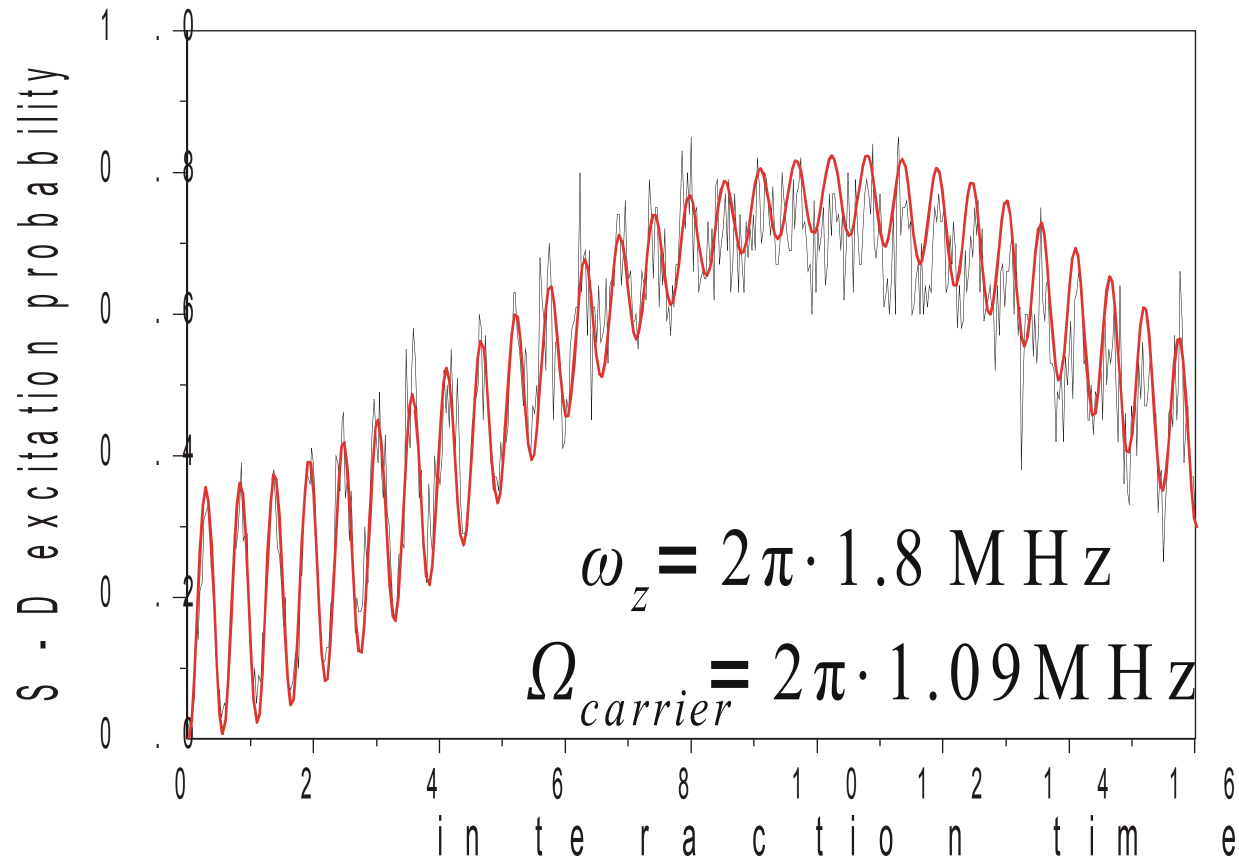
Addressing error (can be corrected for partially)	3 % in Rabi frequency (at neighbouring ion)	1 %
Off resonant excitations	for $t_{gate} = 600$ μs	- (Pulse shaping)
Residual thermal excitation	<n>bus < 0.02 <n>spec = 6	< 2 % 0.4 %
Total	February 2006	~ 9 %

Off-resonant excitations





Problem:

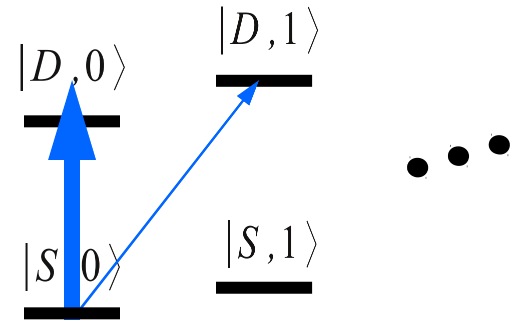
- AC Stark shifts
- off-resonant (carrier) excitation (spectator modes)

A. Steane, C. F. Roos, D. Stevens, A. Mundt, D. Leibfried, F. Schmidt-Kaler, R. Blatt, Phys. Rev. A **62**, 042305 (2000)

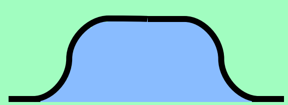
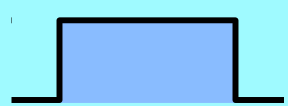
Improve gate fidelity with pulse-shaping

Off-resonant excitation:
Driving sideband transitions leads to unwanted off-resonant excitation of the much stronger carrier transition.

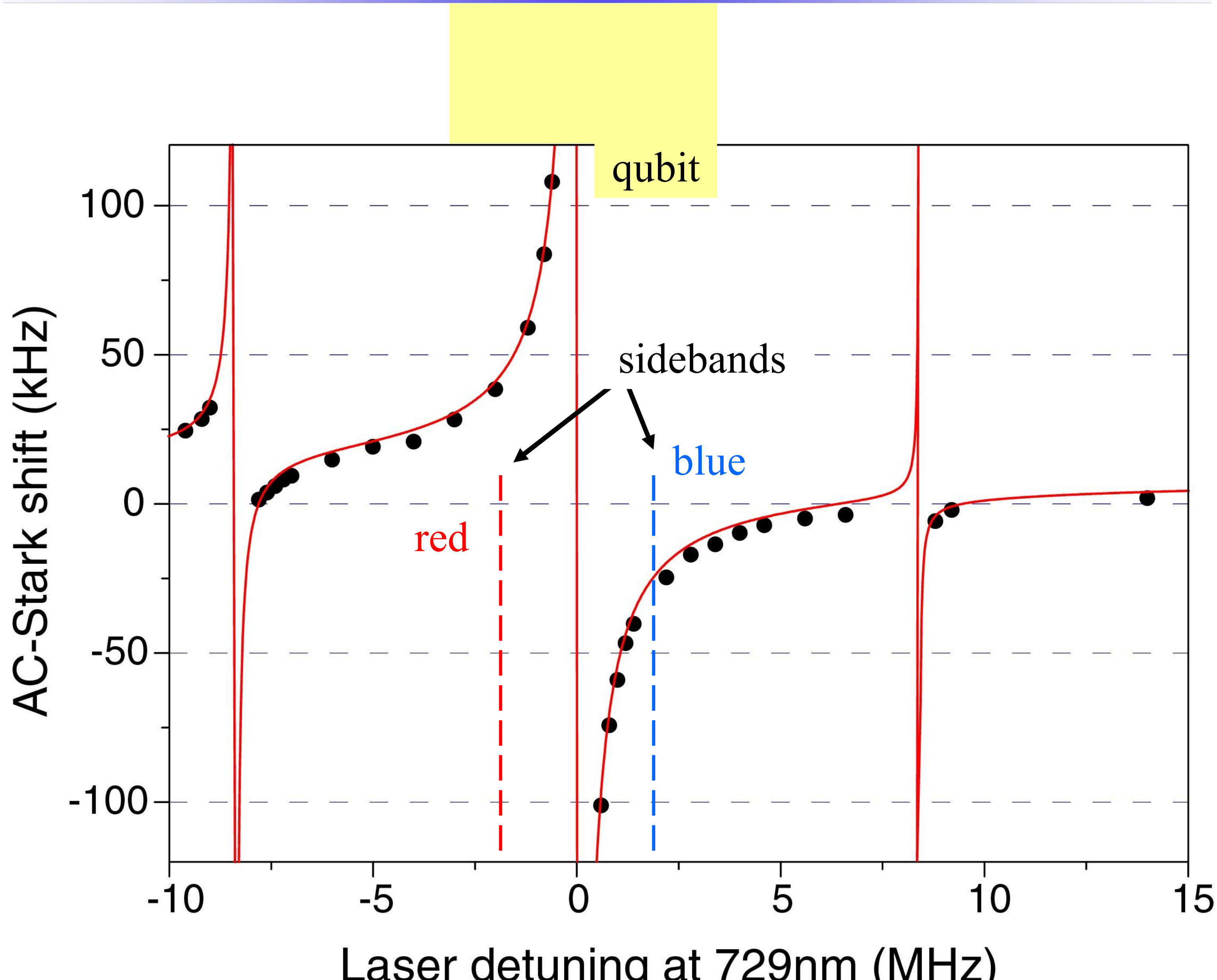
- decreases gate fidelity
- limits gate speed to about 500 μs



Solution: Adiabatically switch on/off laser pulses

Adiabatic 5 μs rise/fall time		$F_p = 87.6(7)\%$	$\bar{F} = 90.1(6)\%$
Square pulse		$F_p = 75(1)\%$	$\bar{F} = 80(1)\%$

AC – Stark shift of qubit transition



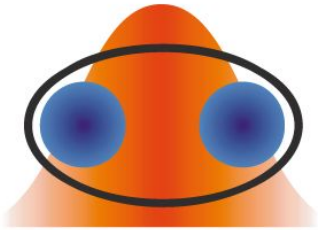
Complications of the Cirac-Zoller CNOT

- **the gate is slow (off-resonant excitations)**
  => sensitive to frequency fluctuations / deviations.

- **the gate requires addressing** => trap frequency needs to reduced
  Use other gate types which do not require addressing.

- **the gate is sensitive to motional heating**
  Not really a problem yet.

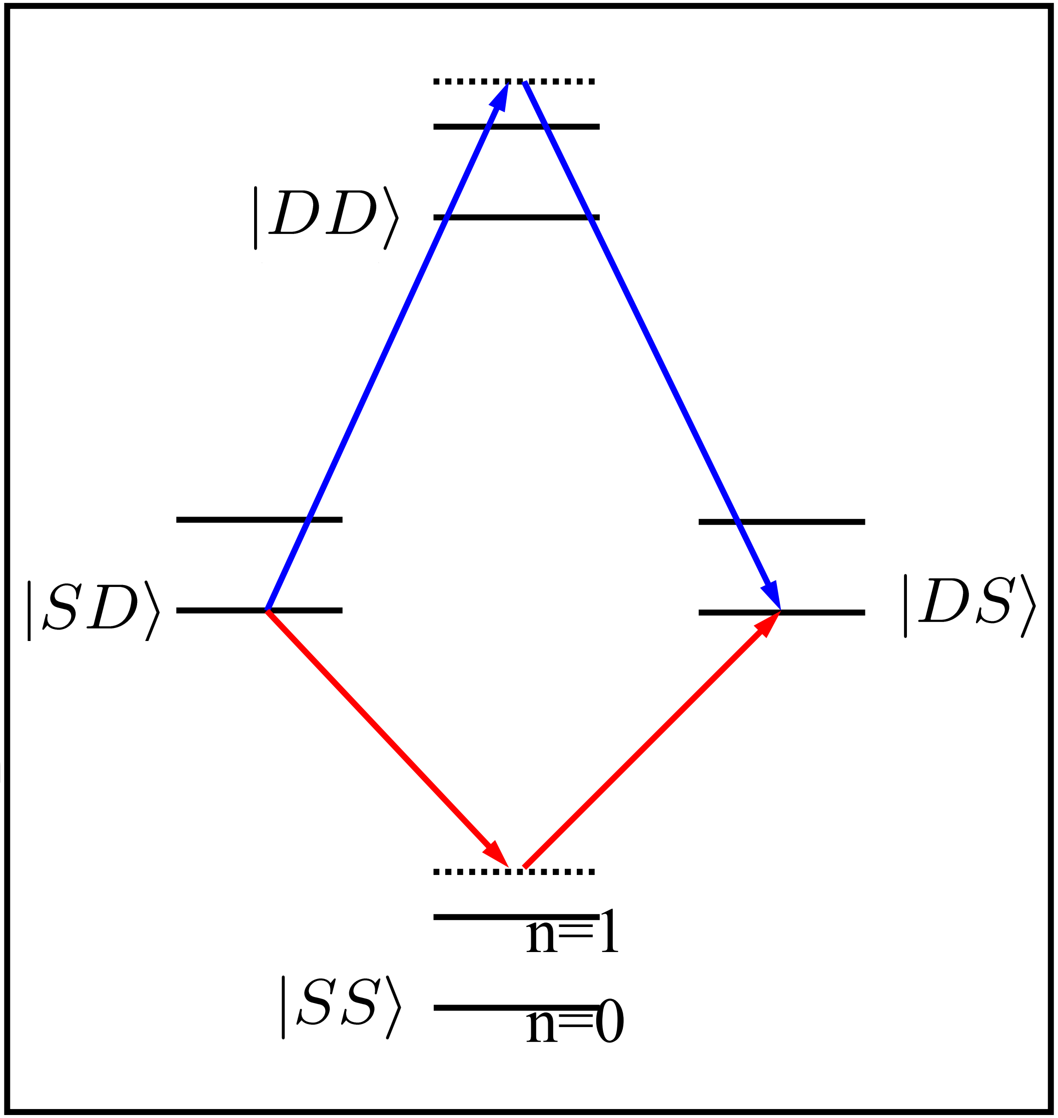
Mølmer-Sørensen gate



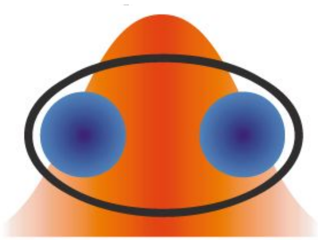
Raman transitions between

$$|SD\rangle \quad \Leftrightarrow \quad |DS\rangle$$

Interaction of two ions via common motion.



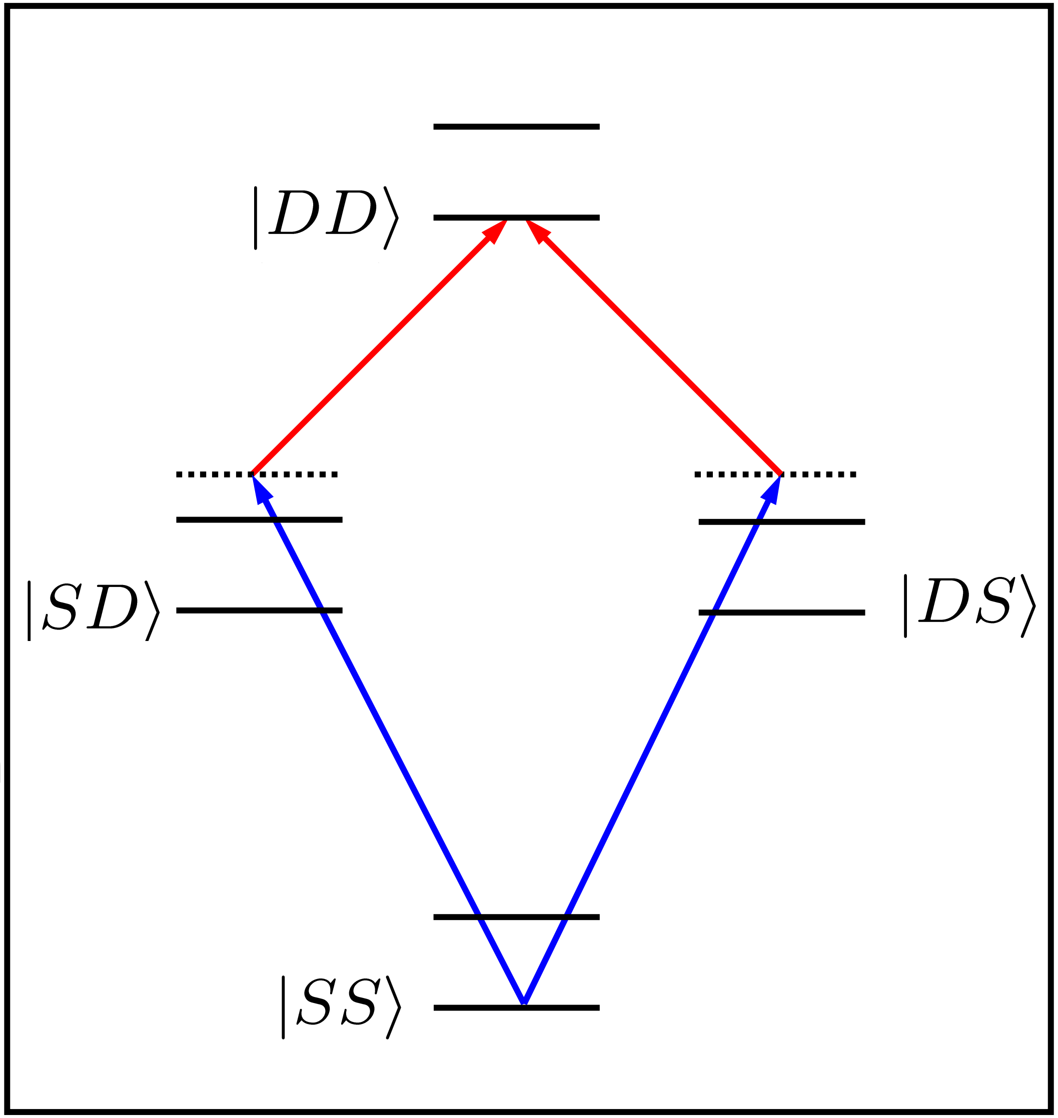
Mølmer-Sørensen gate



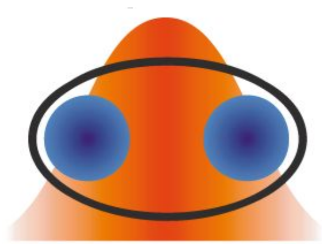
Raman transitions between

$$|SS\rangle \quad \Leftrightarrow \quad |DD\rangle$$

Interaction of two ions via common motion.



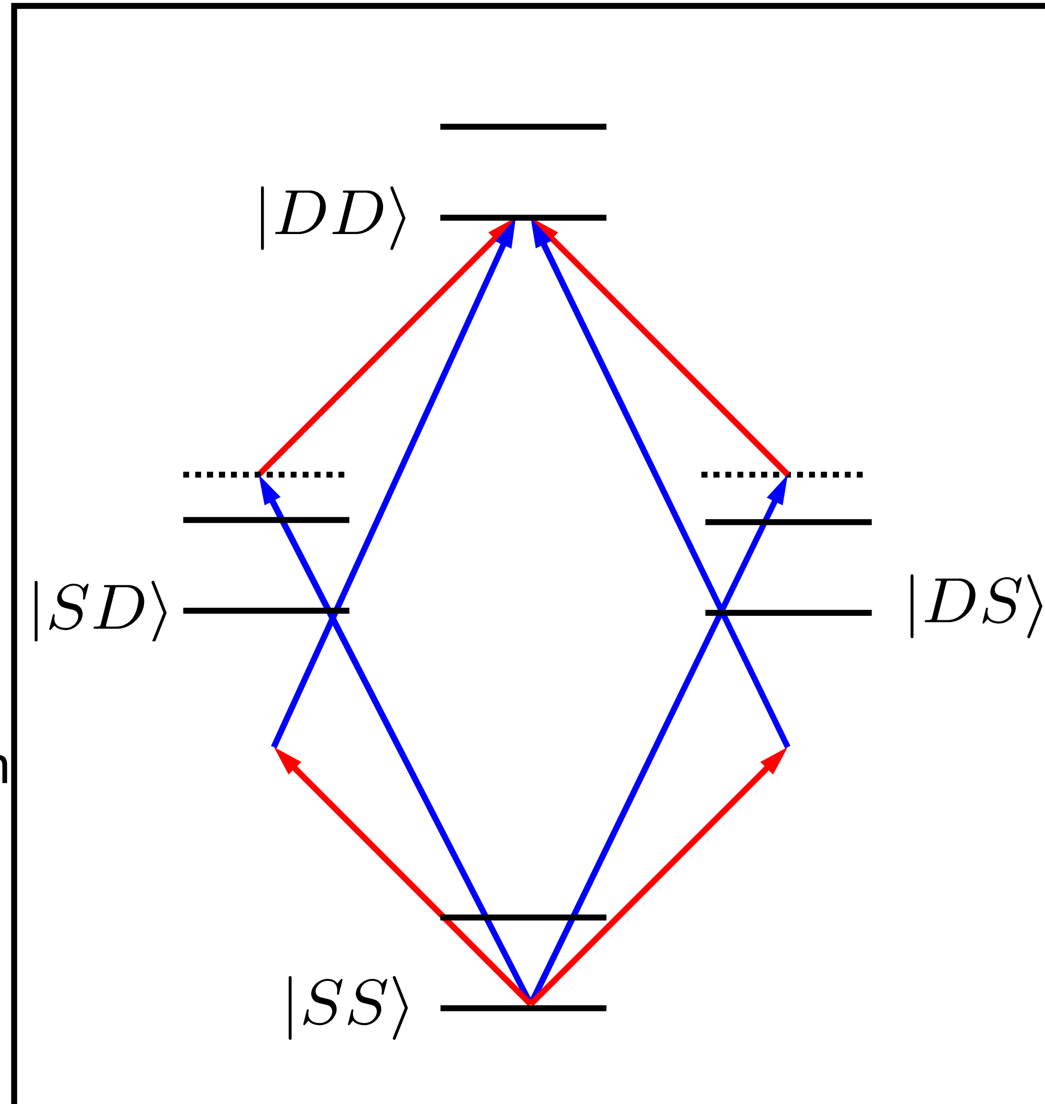
Mølmer-Sørensen gate



Raman transitions between

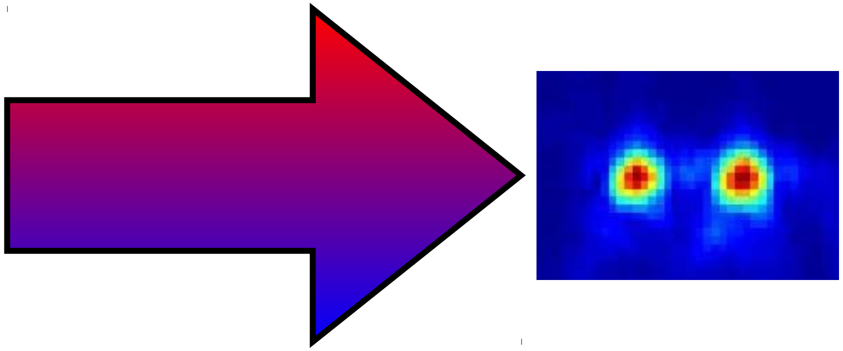
$$|SS\rangle \quad \Leftrightarrow \quad |DD\rangle$$

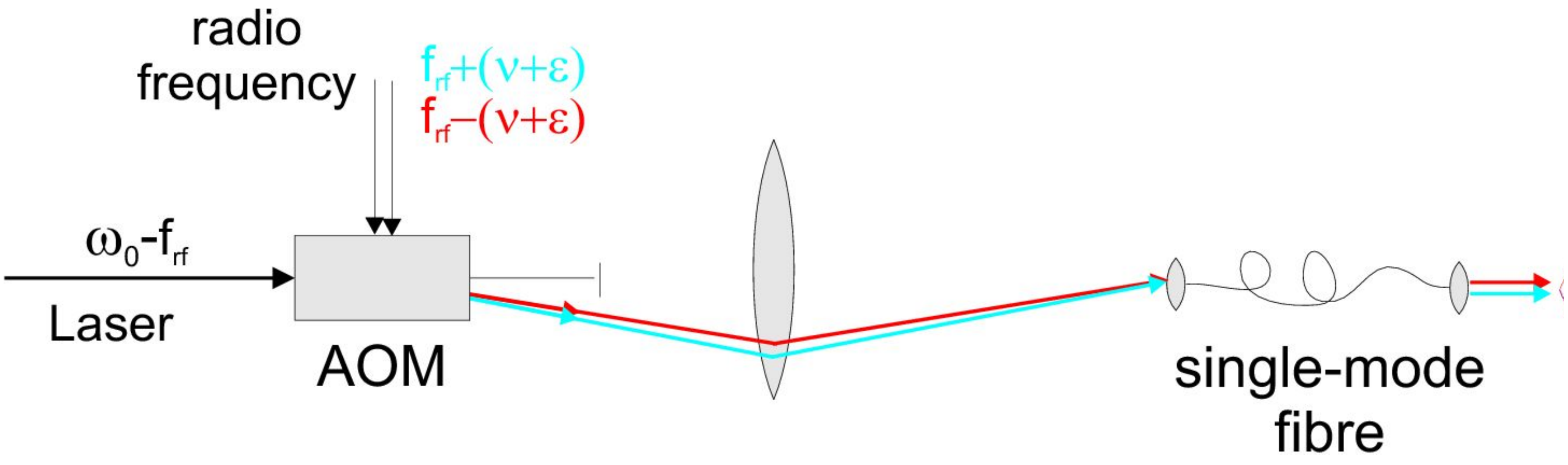
Interaction of two ions via common motion.



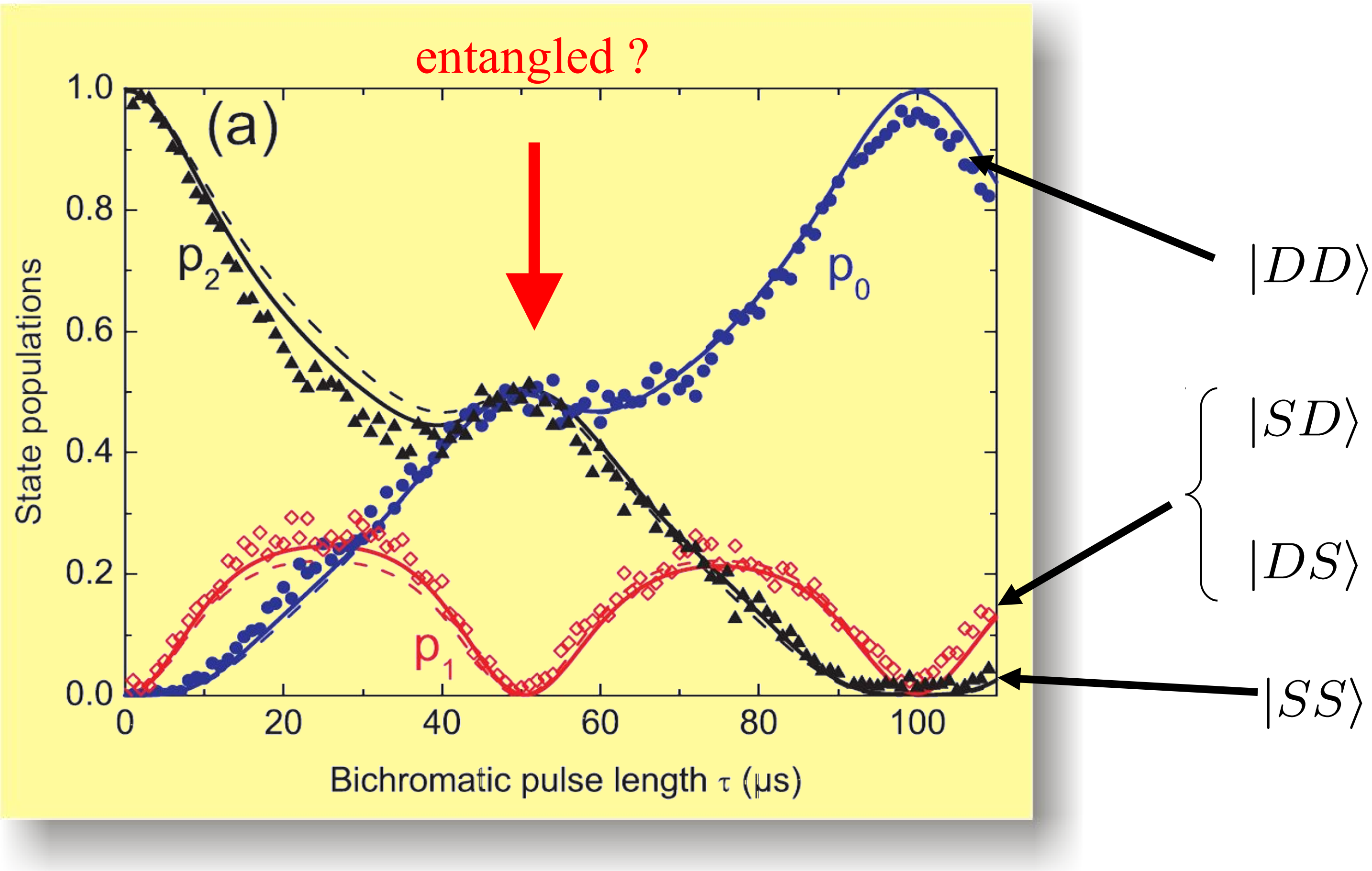
Technical realization

bicromatic beam
applied to both ions





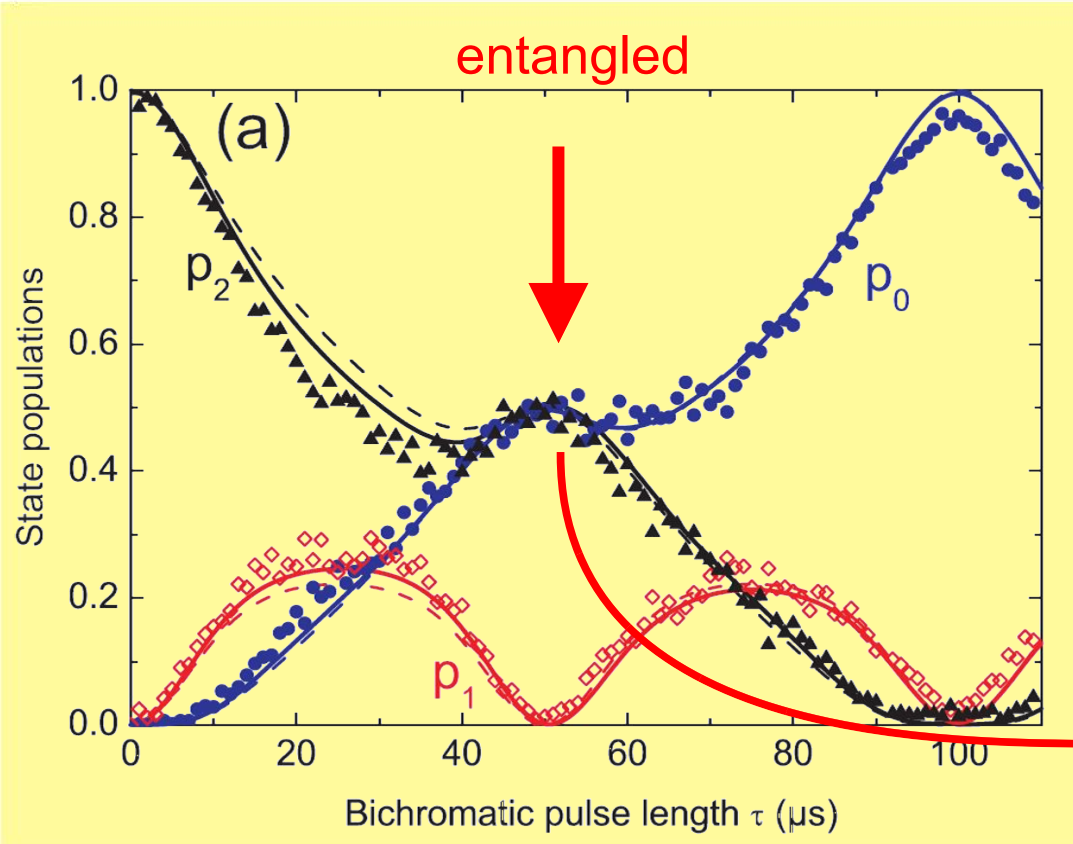
Entangling ions



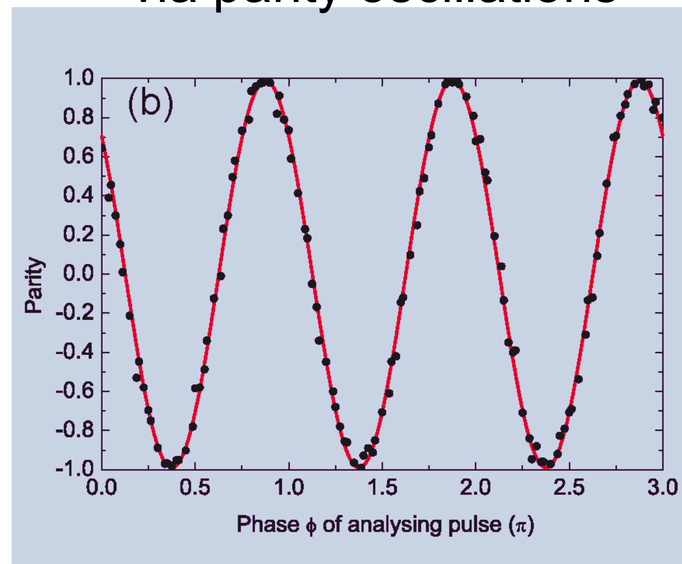
J. Benhelm et al., Nature Physics **4**, 463 (2008)

Theory: C. Roos, NJP **10,** 013002 (2008)

Entangling ions



measure entanglement
via parity oscillations



gate duration 51 μs

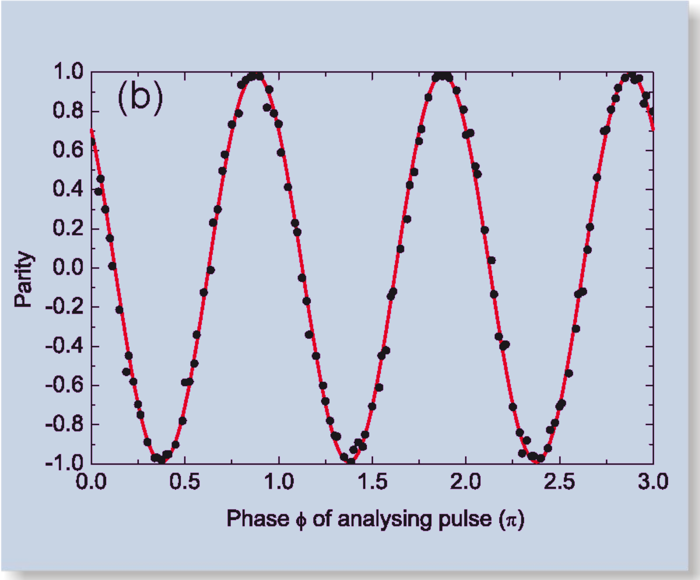
average fidelity: 99.3 (2) %

$$|00\rangle + |11\rangle \xrightarrow{R_2^C(\pi/2,\varphi),R_1^C(\pi/2,\varphi)}$$

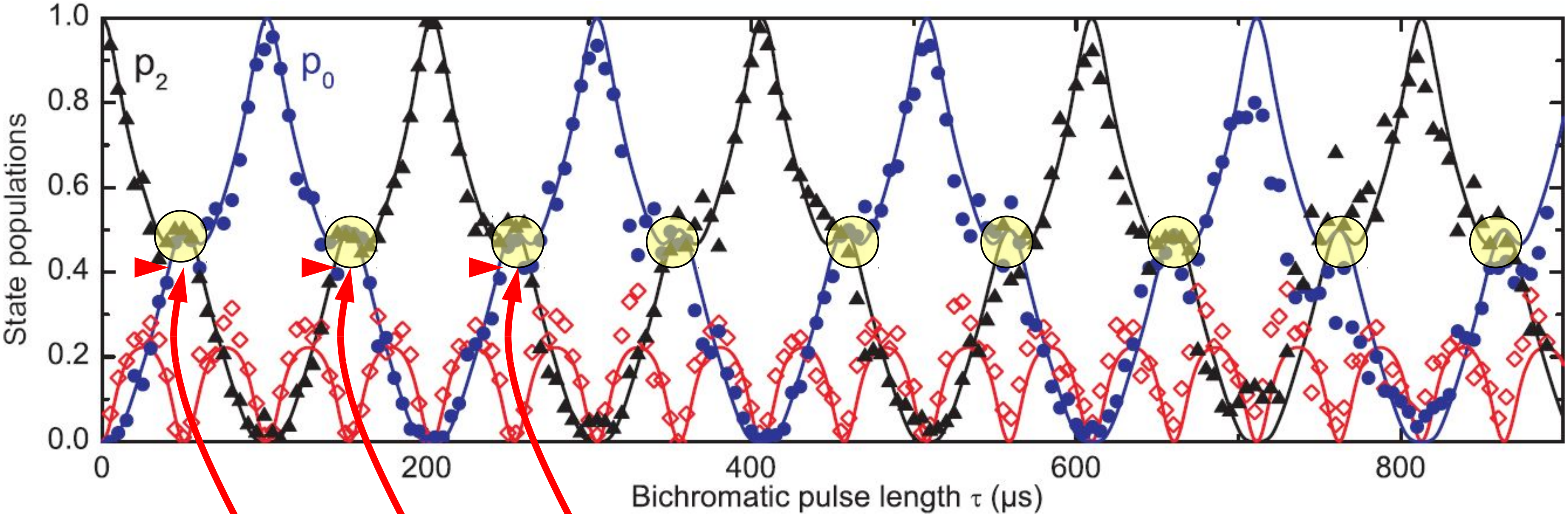
$$\left(|0\rangle + ie^{i\varphi}|1\rangle\right)\left(|0\rangle + ie^{i\varphi}|1\rangle\right) + \left(|1\rangle + ie^{-i\varphi}|0\rangle\right)\left(|1\rangle + ie^{-i\varphi}|0\rangle\right)$$

$$= (1 - e^{-2i\varphi})|00\rangle + ie^{i\varphi}(1 + e^{-2i\varphi})|01\rangle$$

$$+ ie^{i\varphi}(1 + e^{-2i\varphi})|10\rangle + (1 - e^{-2i\varphi})|11\rangle\,,$$



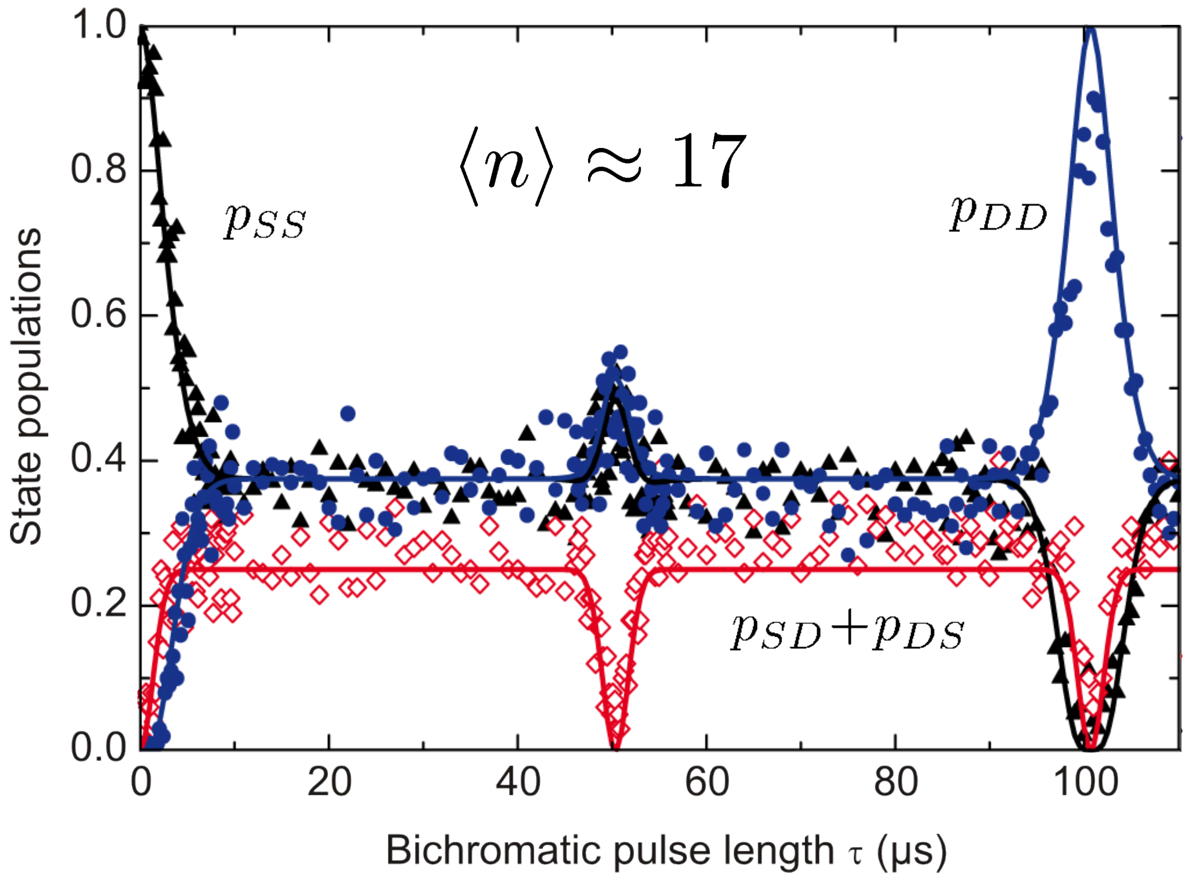
Multiple gates



maximally entangled states

Mølmer-Sørensen for thermal states

Gate operation after Doppler cooling



Bell state:

$\Psi = |SS\rangle + i|DD\rangle$

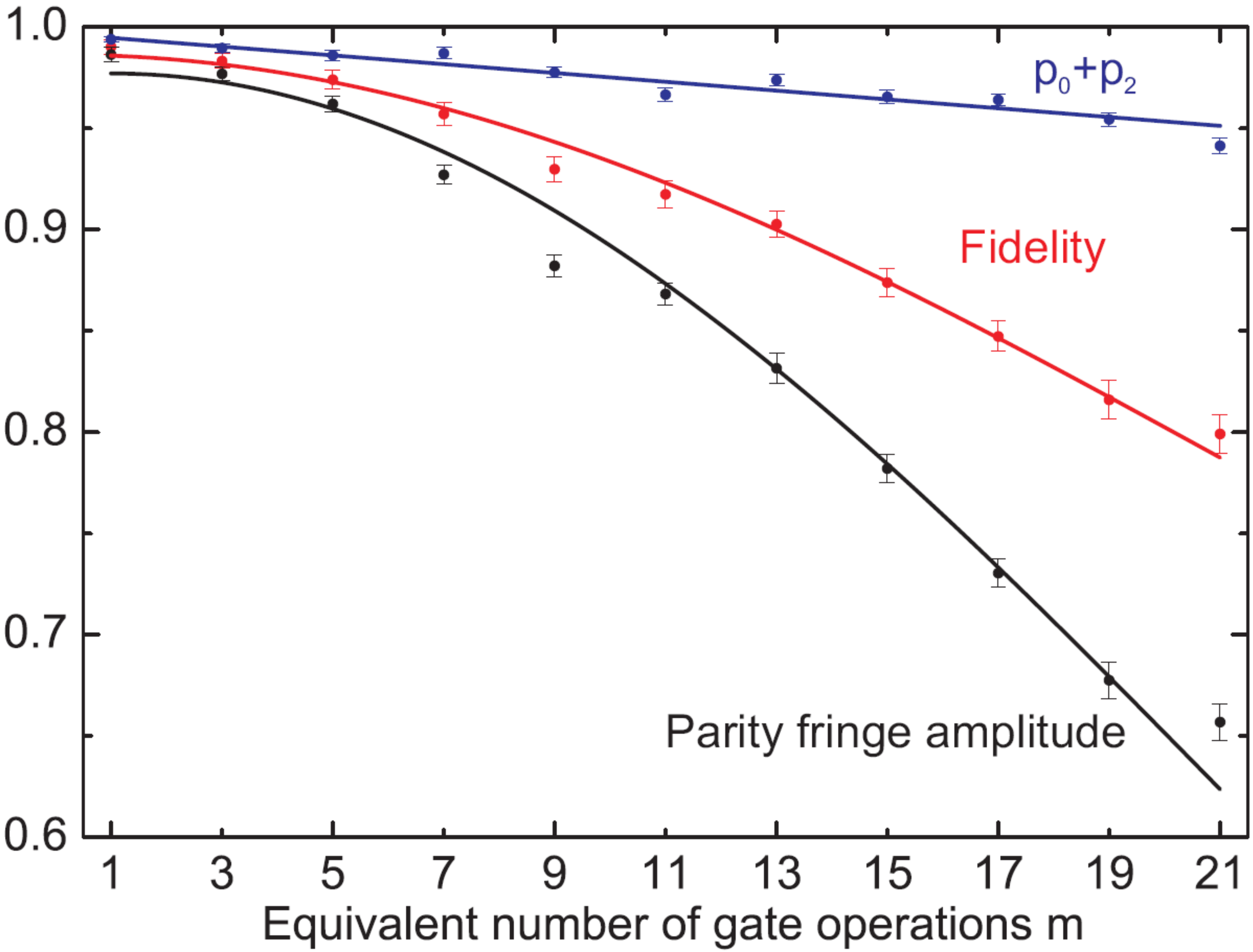
Fidelity :

F = 96.1(5) %

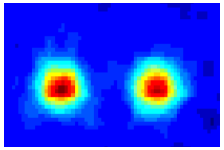
Gate operation ≈ independent of motional state !

Kirchmair et al., New. J. Phys. 11, 023002 (2009).

Gate performance



Two-ion GHZ-state



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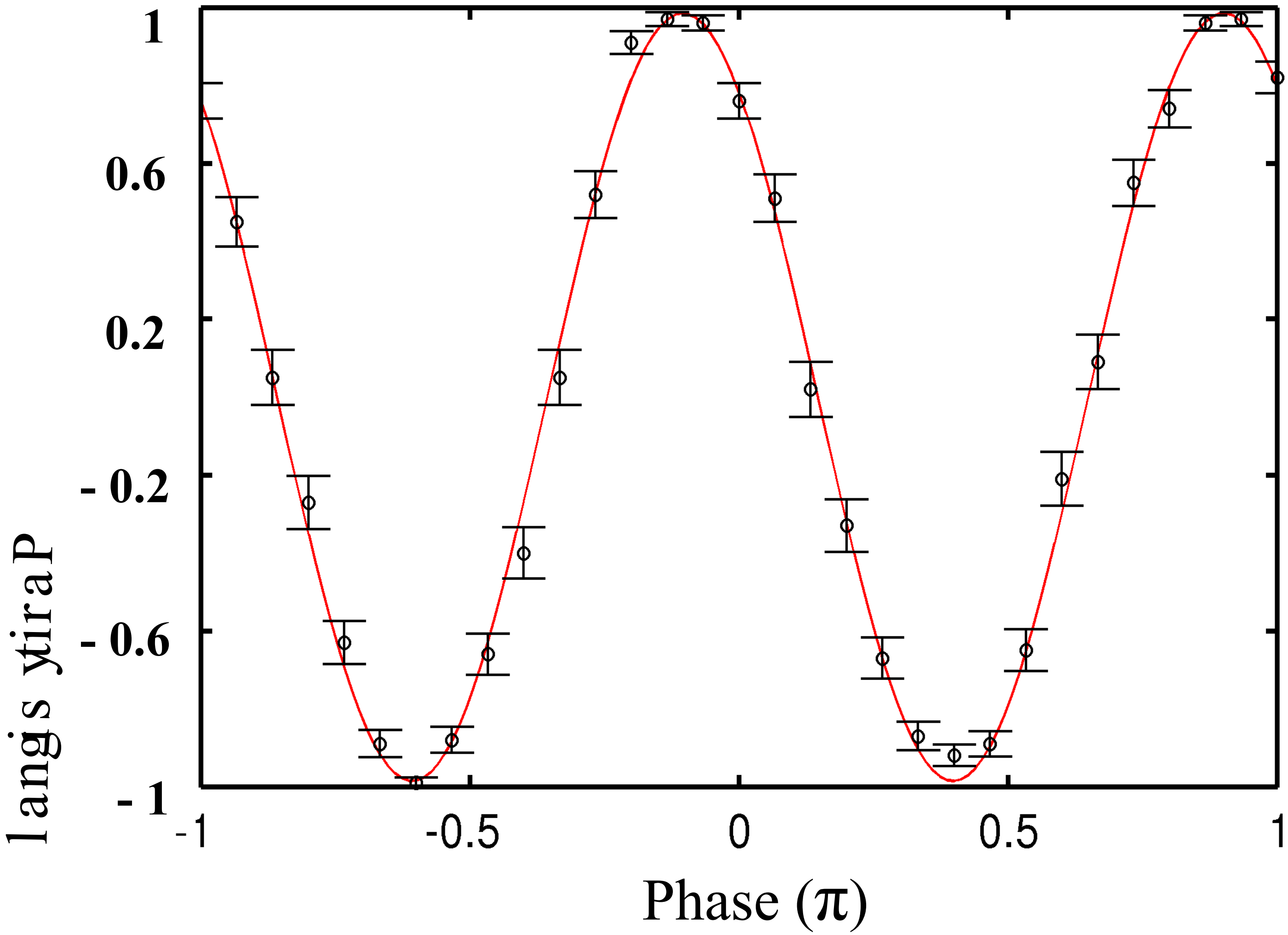
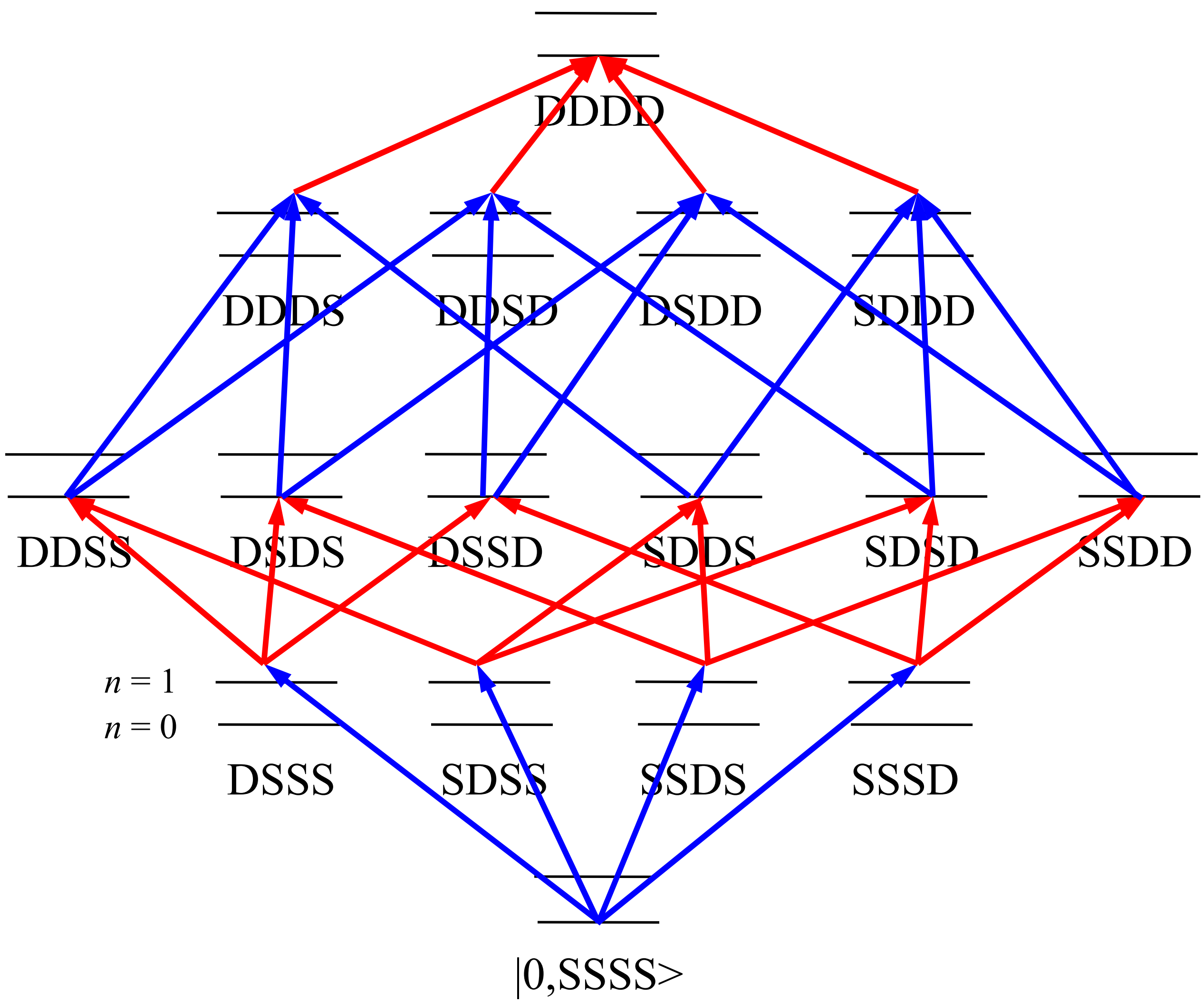
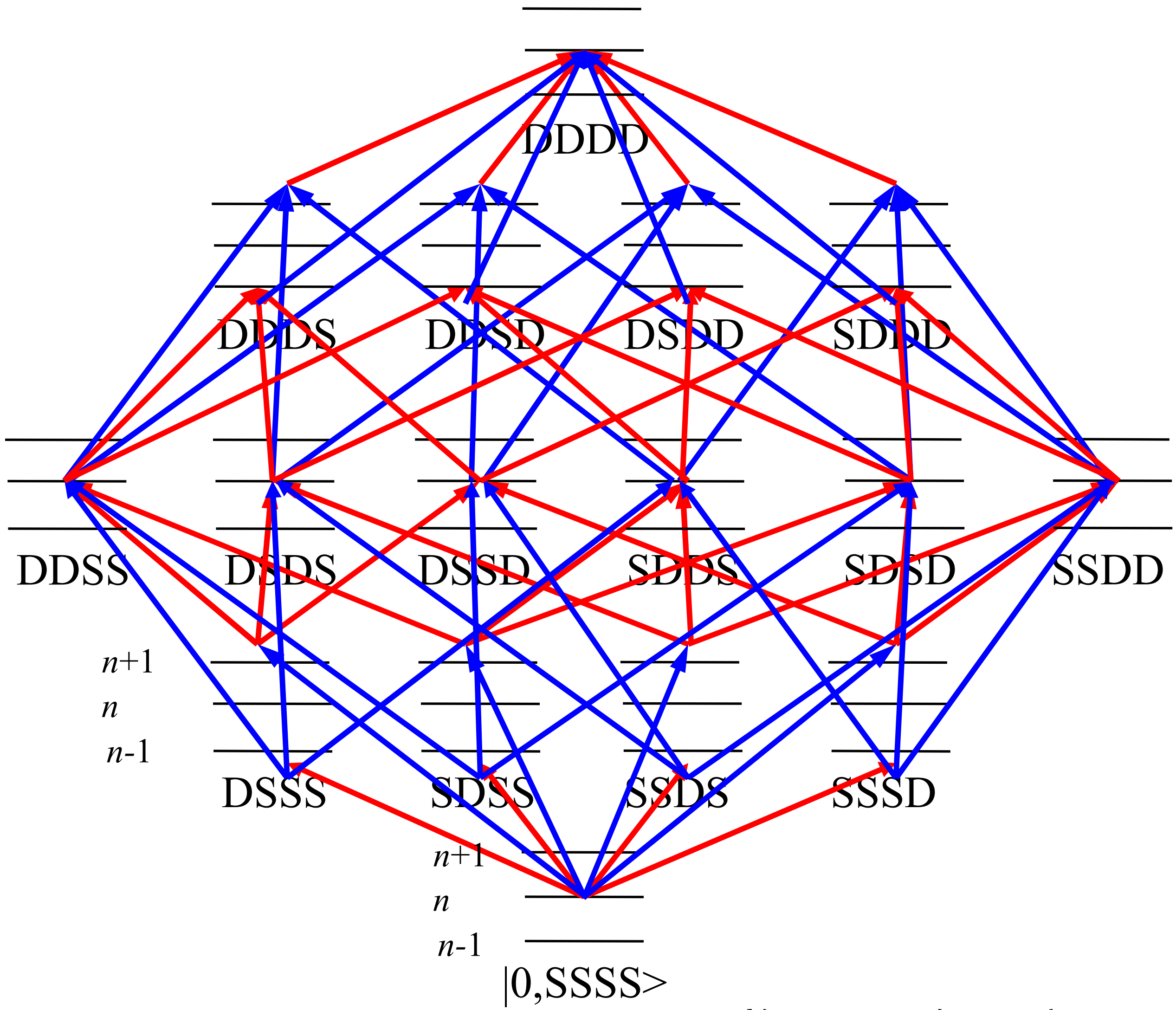


Entangling four ions



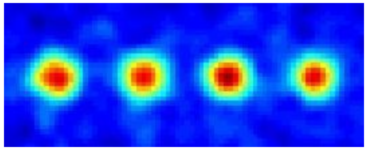
$$\Rightarrow (|SSSS\rangle + |DDDD\rangle)/\sqrt{2}$$

A complex interferometer ...



$$\Rightarrow (|SSSS\rangle + |DDDD\rangle)/\sqrt{2}$$

Four-ion GHZ state



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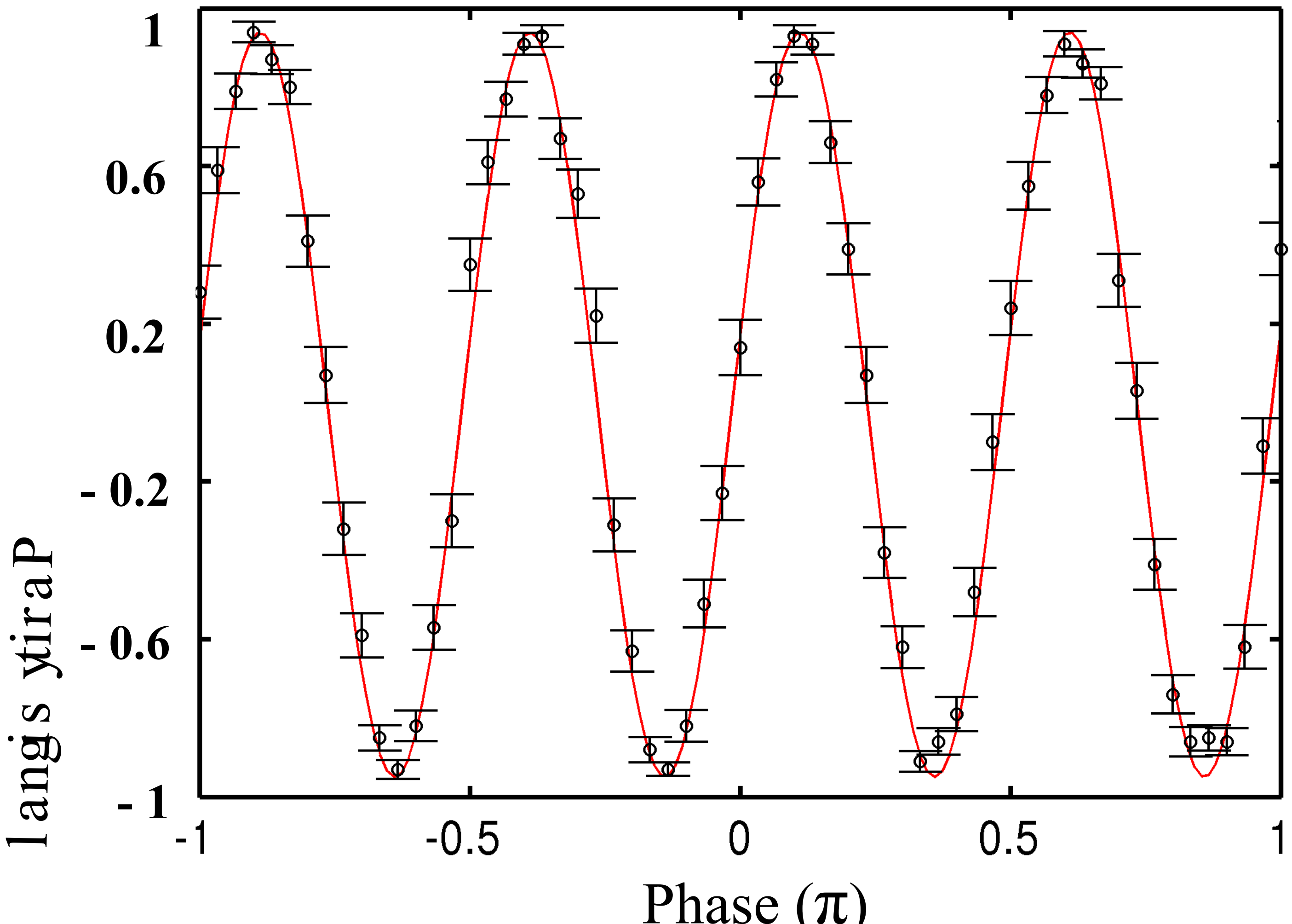
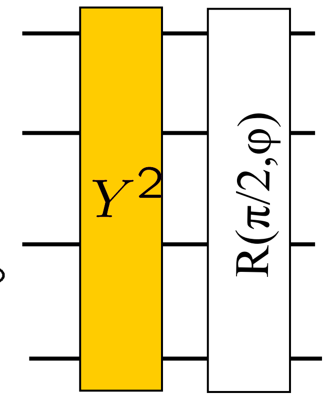
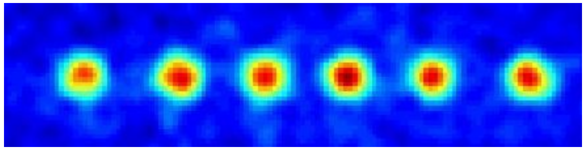




See also: Leibfried *et al.*, Nature **438**, 639 (2005)

Six-ion GHZ-state



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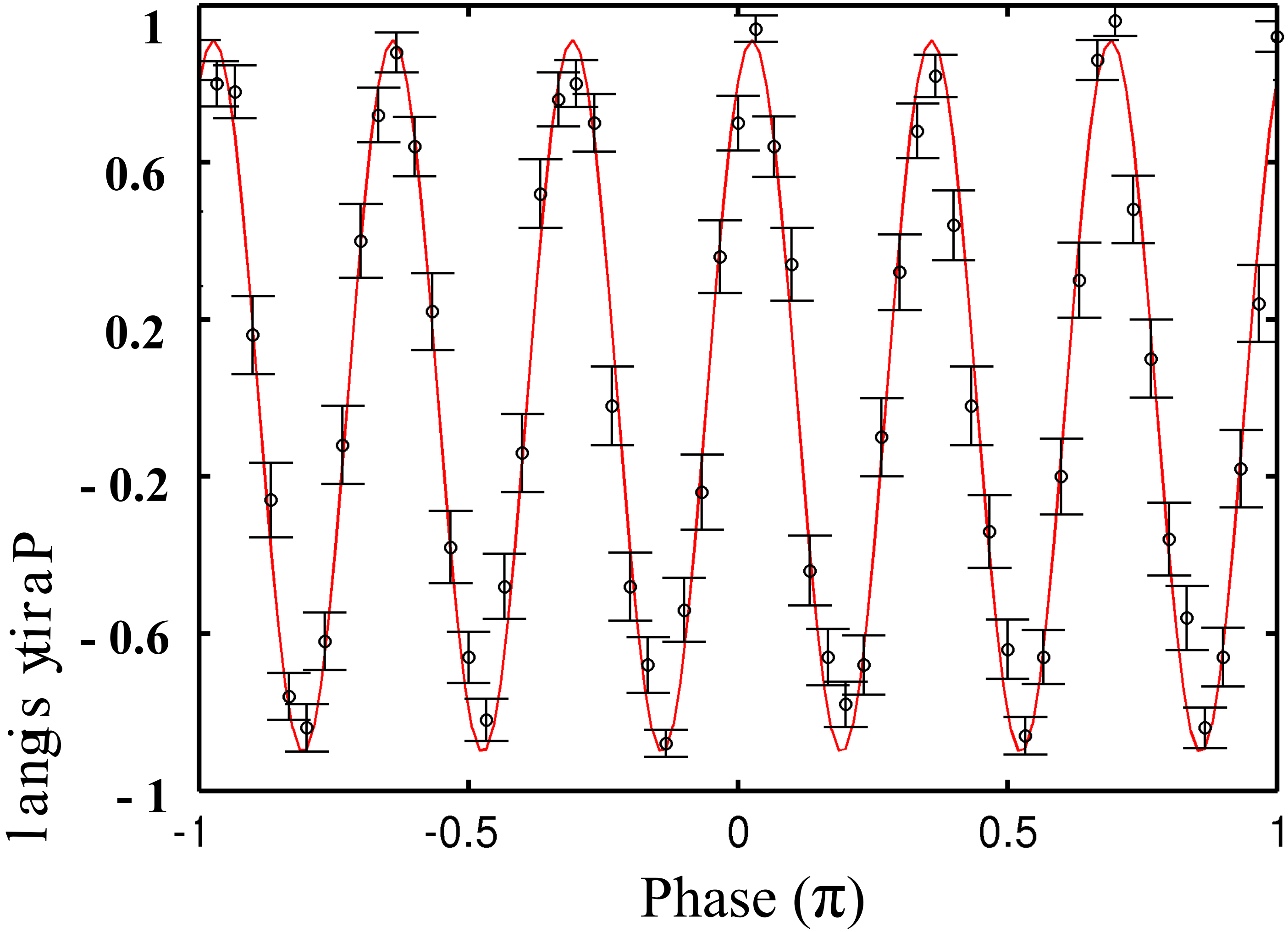
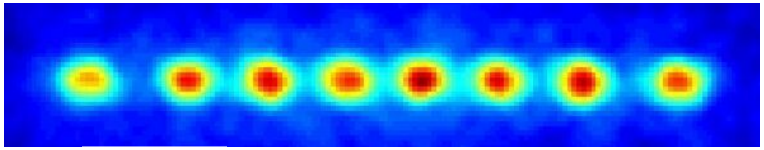


See also: Leibfried *et al.*, Nature **438**, 639 (2005)

Eight-ion GHZ state



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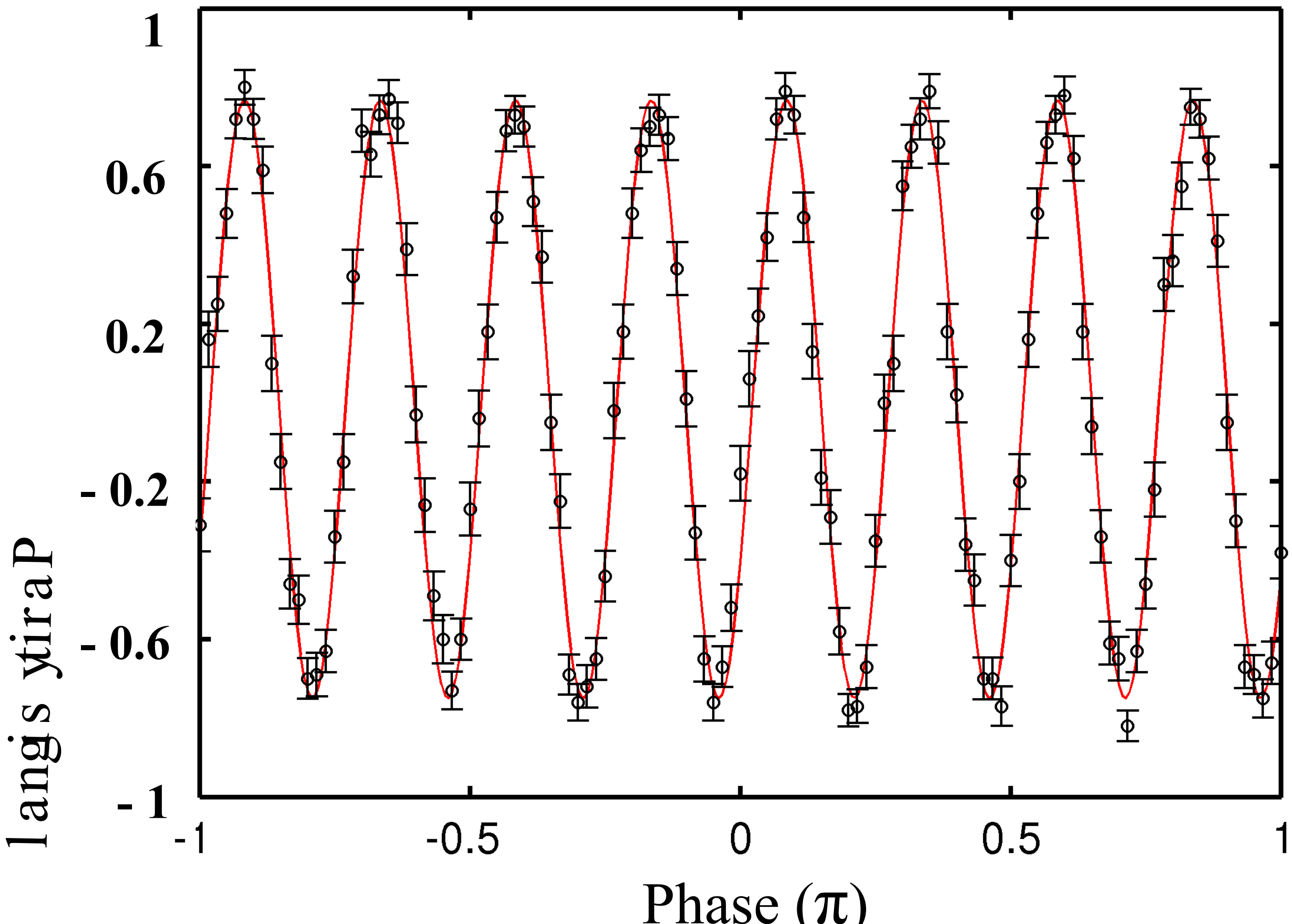


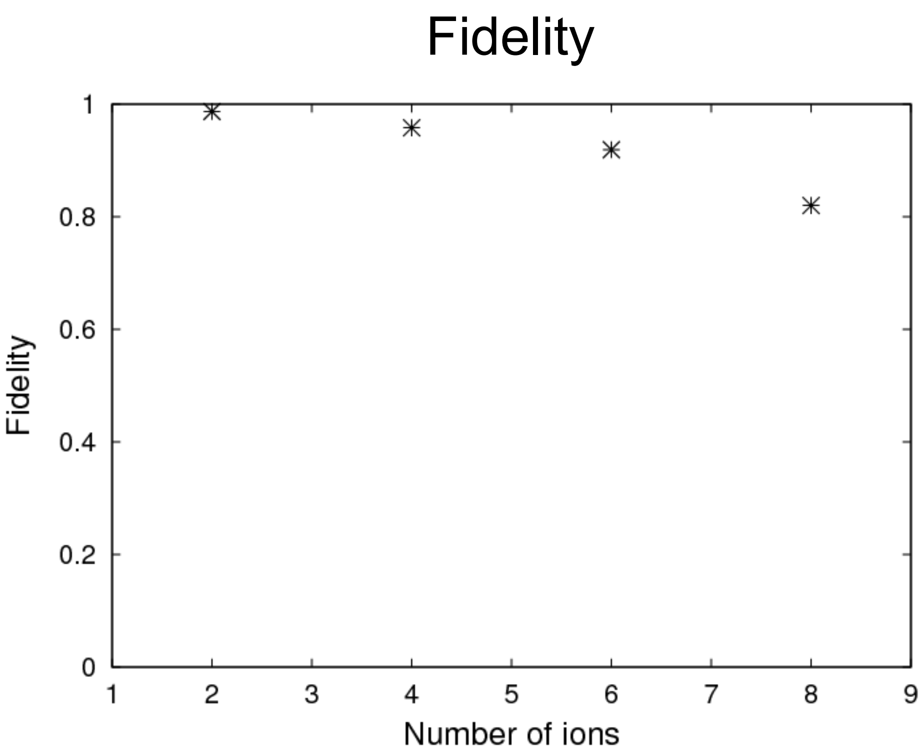
GHZ-state fidelities

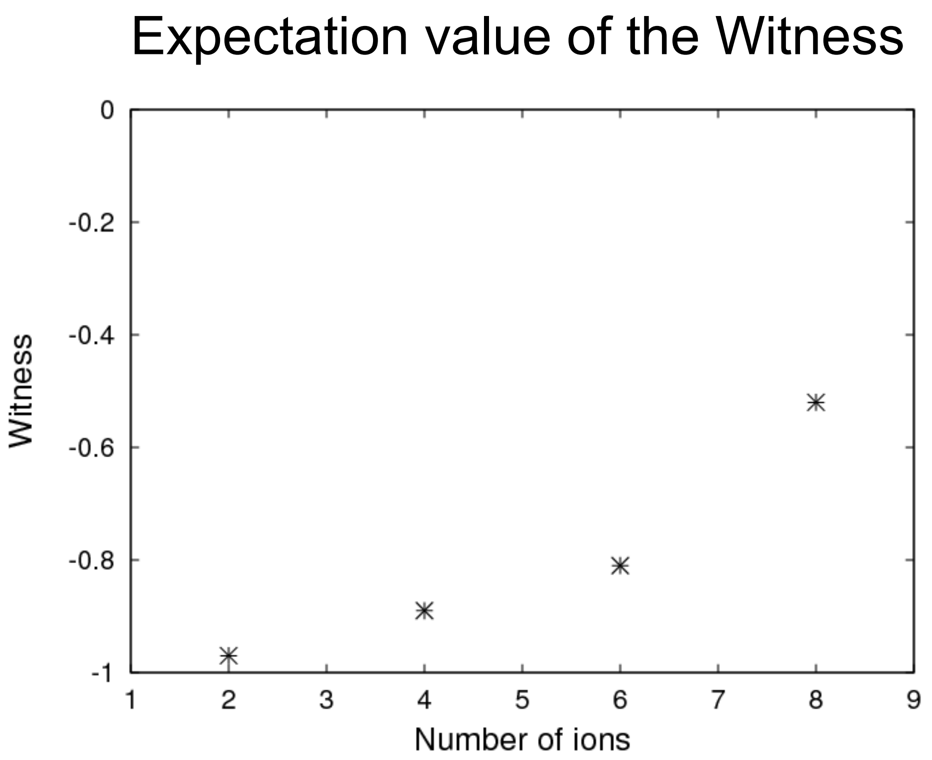
# ions	Fidelity	Witness
2	0.99 (1)	-0.97 (1)
4	0.96 (2)	-0.89 (1)
6	0.92 (3)	-0.81 (3)
8	0.82 (3)	-0.52 (3)

$$F = (P_{SS\ldots S} + P_{DD\ldots D} + 2\,\mathsf{Contrast})/2$$

$$W = 1 - 4\,\mathsf{Contrast}$$

GHZ-state fidelities





GHZ-state fidelities

Why is the fidelity loss non-linear in the ion number ?

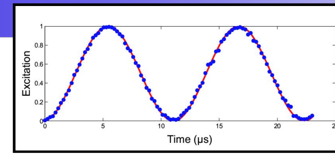
Why is the coherence time non-linear with the ion number ?

GHZ-state fidelities

Presumably can be explained by noise components at high frequencies.

Realized time scales

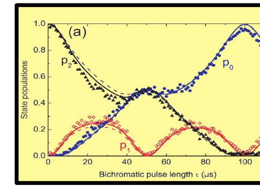
Single qubit gates



10^{-6} s

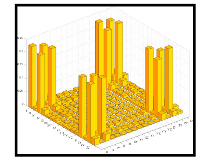
10^{-5} s

Two qubit gates (Geometric phase gates)



10^{-4} s

Two qubit gates (Cirac-Zoller approach)



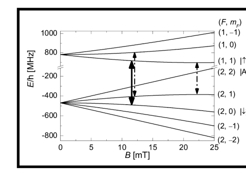
10^{-3} s

10^{-2} s

Single qubit coherence (magnetic field sensitive)

Coherence of the motion (Innsbruck)

10^{-1} s

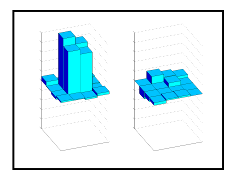


10^{0} s

Single qubit coherence (magnetic field insensitive, NIST)

10^{1} s

Decoherence free subspaces



10^{2} s

10^{3} s

Well chosen single qubit coherence (NIST)

Status

1. Coherence times much longer than gate times

by a factor of 10^4 ✓

2. Control the qubits

single qubit operations with an accuracy of 10^{-3} (✓)

two qubit operations with an accuracy of 10^{-2}

Additionally: very good initialization ✓

efficient and fast read out ✓

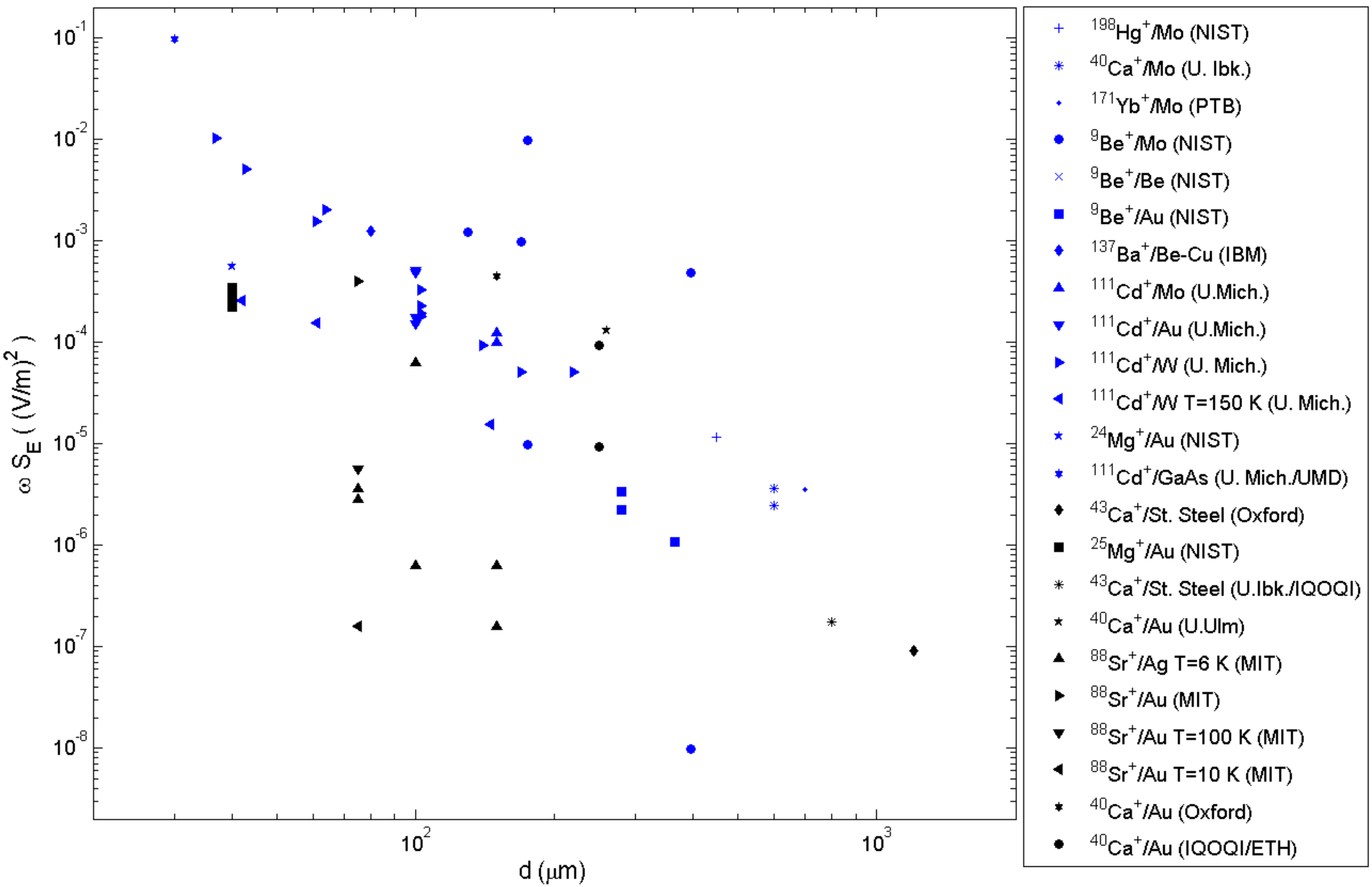
parallelization of all processes

Fundamental road blocks

Combine all high-fidelity operations in a single scalable apparatus/network

Major challlenge: scalable trap architecture with small enough heating rates

Fundamental road blocks



The Innsbruck ion trap group

AG Quantenoptik
und Spektroskopie

IQI

universität
innsbruck

€ FWF SFB

CONQUEST
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