Womanium Final Project: Development of Novel Quantum Algorithms

Dimitrios Kranas & Haemanth Velmurugan

Context

Project: Exponential quantum speedup in simulating coupled classical oscillators **[Ryan Babbush et al (2023)]**

Description:

- The evolution of a system of N coupled oscillators to the evolution of an 2^{2n+1} system of qubits.
- The correspondence enables us to leverage power full tools from the arsenal of quantum algorithms to implement the system's evolution (**Hamiltonian simulation**).

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Main result: Assuming access to masses and spring constant oracles, the authors' evolution algorithm outputs a state that is ϵ-close $(0 < \epsilon \ll 1)$ to a target state $|\psi(t)\rangle$.

Our implementation

(Quantum) algorithms we developed/extended

Algorithmic milestones:

1. Code development that performs the **LCU decomposition of a** $2N^2 \times 2N^2$ **Hamiltonian** into identity and Pauli strings.

2. **Encoding** the LCU decomposition of a Hamiltonian, extending Classiq functions to the case where the decomposition coefficients are negative.

3. Combining the Classiq high-level functions functions $prepare_amplitudes(amplitudes = amps.tolist($ $0.01, out = x)$ and $unitary(U_matrix.tolist(), x)$ (where the U_matrix contains 1's and "i"'s appropriately in its diagonal) to implement **state preparation** of quantum states with **complex amplitudes**.

Post-analysis & take-home messages

Resources Investigation and Optimization:

- We optimized our quantum algorithm for width and depth: min(depth)=3776, min(width)=12

  Conclusion: The qubitization method for Hamiltonian simulation of the coupled oscillators could be achieved on hardware with very few qubits as long as the number of gates is sufficient.

- For a relatively small number of oscillators (small N), the topology of connectivity is insignificant, i.e., whether the oscillators form a linear chain or they are connected all together.

  Remark: We would love to investigate the relationship between the optimal depth and the topology of the connectivity if we are given access to hardware with more resources.

Critical scale: We identified the critical scale κ_{ij}/m_i below which our quantum algorithm becomes inefficient in terms of winning time. We need to ensure $\kappa_{ij}/m_i > \mathcal{O}(10^3)$ to achieve significant speedup.

Thank you for the amazing quantum journey this summer!!!