Notebook_nettoyage_des_données.1

August 25, 2025

ANALYSE DETECTION DE FAUX BILLETS ONCF

[91]:
```
!pip install missingno #missingno est un module très pratique pour visualiser
↪les valeurs manquantes dans un DataFrame pandas
import math
import matplotlib.pyplot as plt
#import missingno as msno
import numpy as np
import pandas as pd
import scipy.stats as st
import seaborn as sns
import statsmodels.api as sm
import statsmodels.formula.api as smf
from sklearn.linear_model import LinearRegression, RidgeCV, LassoCV
from sklearn.metrics import mean_squared_error
from sklearn.preprocessing import StandardScaler
from statsmodels.stats.outliers_influence import variance_inflation_factor,
↪OLSInfluence
from statsmodels.stats.diagnostic import normal_ad, kstest_fit,
↪het_breuschpagan, het_white

from functions import *

col = findColor('blue.png')
col = 'steelblue'  # ou n'importe quelle couleur matplotlib : 'blue', 'teal',
↪'royalblue', etc.
```

```
ERROR: Invalid requirement: '#missingno': Expected package name at the start of
dependency specifier
    #missingno
    ^
```

```
---------------------------------------------------------------------------
ModuleNotFoundError                       Traceback (most recent call last)
Cell In[91], line 17
     14 from statsmodels.stats.outliers_influence import
↪variance_inflation_factor, OLSInfluence
```

```
     15 from statsmodels.stats.diagnostic import normal_ad, kstest_fit,␣
 ↪het_breuschpagan, het_white
---> 17 from functions import *
     19 col = findColor('blue.png')
     20 col = 'steelblue'

ModuleNotFoundError: No module named 'functions'
```

1 - Import des donnees & verifications

[94]:
```
# Import data
billets = pd.read_csv('billets.csv', sep=';', encoding='latin_1')
billets.head()
```

[94]:
```
   is_genuine  diagonal  height_left  height_right  margin_low  margin_up  \
0        True    171.81       104.86        104.95        4.52       2.89
1        True    171.46       103.36        103.66        3.77       2.99
2        True    172.69       104.48        103.50        4.40       2.94
3        True    171.36       103.91        103.94        3.62       3.01
4        True    171.73       104.28        103.46        4.04       3.48

   length
0  112.83
1  113.09
2  113.16
3  113.51
4  112.54
```

La variable is_genuine est booleenne, les 6 autres variables sont quantitatives continues. Les billets ne sont pas identifies par un numero de serie ou autre identifiant, au besoin on pourra utiliser leur index dans le dataframe comme cle primaire artificielle.

[97]:
```
billets.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 1500 entries, 0 to 1499
Data columns (total 7 columns):
 #   Column        Non-Null Count  Dtype
---  ------        --------------  -----
 0   is_genuine    1500 non-null   bool
 1   diagonal      1500 non-null   float64
 2   height_left   1500 non-null   float64
 3   height_right  1500 non-null   float64
 4   margin_low    1463 non-null   float64
 5   margin_up     1500 non-null   float64
 6   length        1500 non-null   float64
dtypes: bool(1), float64(6)
memory usage: 71.9 KB
```

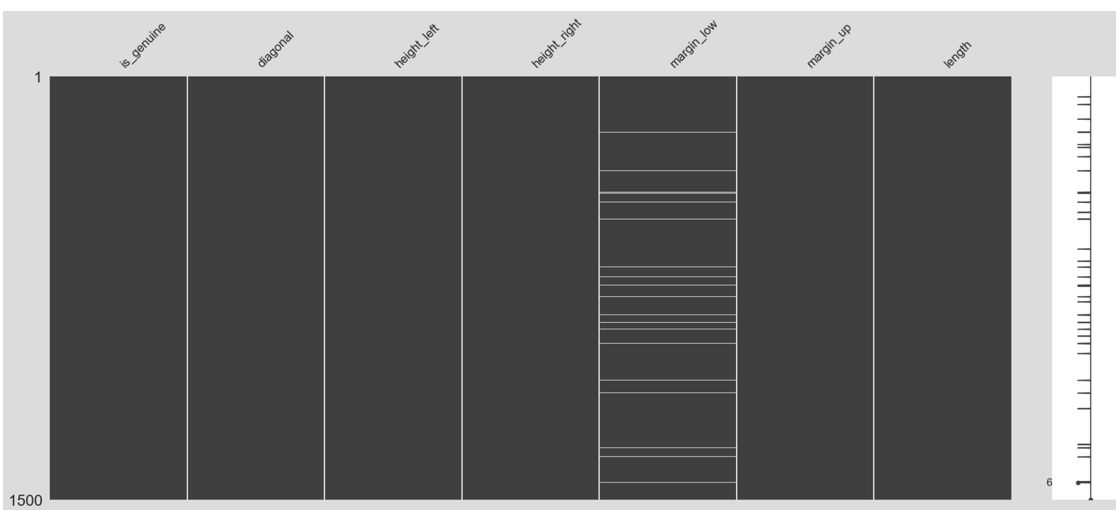
[99]:
```
billets.describe()
```

[99]:
```
          diagonal  height_left  height_right   margin_low   margin_up  \
count  1500.000000  1500.000000   1500.000000  1463.000000  1500.000000
mean    171.958440   104.029533    103.920307     4.485967     3.151473
std       0.305195     0.299462      0.325627     0.663813     0.231813
min     171.040000   103.140000    102.820000     2.980000     2.270000
25%     171.750000   103.820000    103.710000     4.015000     2.990000
50%     171.960000   104.040000    103.920000     4.310000     3.140000
75%     172.170000   104.230000    104.150000     4.870000     3.310000
max     173.010000   104.880000    104.950000     6.900000     3.910000

           length
count  1500.00000
mean    112.67850
std       0.87273
min     109.49000
25%     112.03000
50%     112.96000
75%     113.34000
max     114.44000
```

Les valeurs minimum de toutes les variables sont positives. Les maxima sont relativement proches des moyennes, dont a priori il s'agit de valeurs atypiques et non aberrantes. L'influence de ces valeurs sera analysee plus loin.

[102]:
```
msno.matrix(billets)
```

[102]: <Axes: >



[103]:
```python
# identidy lines where margin_low data is missing
missing_data = billets.loc[billets['margin_low'].isna() == True]
missing_data.head(missing_data.shape[0])
```

[103]:
```
      is_genuine  diagonal  height_left  height_right  margin_low  margin_up  \
72          True    171.94       103.89        103.45         NaN       3.25
99          True    171.93       104.07        104.18         NaN       3.14
151         True    172.07       103.80        104.38         NaN       3.02
197         True    171.45       103.66        103.80         NaN       3.62
241         True    171.83       104.14        104.06         NaN       3.02
251         True    171.80       103.26        102.82         NaN       2.95
284         True    171.92       103.83        103.76         NaN       3.23
334         True    171.85       103.70        103.96         NaN       3.00
410         True    172.56       103.72        103.51         NaN       3.12
413         True    172.30       103.66        103.50         NaN       3.16
445         True    172.34       104.42        103.22         NaN       3.01
481         True    171.81       103.53        103.96         NaN       2.71
505         True    172.01       103.97        104.05         NaN       2.98
611         True    171.80       103.68        103.49         NaN       3.30
654         True    171.97       103.69        103.54         NaN       2.70
675         True    171.60       103.85        103.91         NaN       2.56
710         True    172.03       103.97        103.86         NaN       3.07
739         True    172.07       103.74        103.76         NaN       3.09
742         True    172.14       104.06        103.96         NaN       3.24
780         True    172.41       103.95        103.79         NaN       3.13
798         True    171.96       103.84        103.62         NaN       3.01
844         True    171.62       104.14        104.49         NaN       2.99
845         True    172.02       104.21        104.05         NaN       2.90
871         True    171.37       104.07        103.75         NaN       3.07
895         True    171.81       103.68        103.80         NaN       2.98
919         True    171.92       103.68        103.45         NaN       2.58
945         True    172.09       103.74        103.52         NaN       3.02
946         True    171.63       103.87        104.66         NaN       3.27
981         True    172.02       104.23        103.72         NaN       2.99
1076       False    171.57       104.27        104.44         NaN       3.21
1121       False    171.40       104.38        104.19         NaN       3.17
1176       False    171.59       104.05        103.94         NaN       3.02
1303       False    172.17       104.49        103.76         NaN       2.93
1315       False    172.08       104.15        104.17         NaN       3.40
1347       False    171.72       104.46        104.12         NaN       3.61
1435       False    172.66       104.33        104.41         NaN       3.56
1438       False    171.90       104.28        104.29         NaN       3.24

      length
72    112.79
99    113.08
151   112.93
```

```
197     113.27
241     112.36
251     113.22
284     113.29
334     113.36
410     112.95
413     112.95
445     112.97
481     113.99
505     113.65
611     112.84
654     112.79
675     113.27
710     112.65
739     112.41
742     113.07
780     113.41
798     114.44
844     113.35
845     113.62
871     113.27
895     113.82
919     113.68
945     112.78
946     112.68
981     113.37
1076    111.87
1121    112.39
1176    111.29
1303    111.21
1315    112.29
1347    110.31
1435    111.47
1438    111.49
```

[104]:
```python
missing_data['is_genuine'].value_counts()
```

[104]:
```
is_genuine
True     29
False     8
Name: count, dtype: int64
```

La colonne margin_low comporte 37 valeurs manquantes (pour 29 vrais et 8 faux billets) qu'il faudra completer.

[106]:
```python
# check for duplicates
billets.duplicated().sum()
```

[106]: 0

Le fichier ne comporte pas de lignes dupliquees.

2 - Regression lineaire multiple 2.1 - Regression lineaire simple

[113]:
```
# create dataframe without missing data
billets_trim = billets.loc[billets['margin_low'].isna() == False]
```

2.1.1 - Regression

[116]:
```
# perform linear regression to complete margin_low column
y_reglin = billets_trim['margin_low']
x_reglin = billets_trim[['diagonal', 'height_left', 'height_right',␣
↪'margin_up', 'length']]
x_reglin = sm.add_constant(x_reglin, prepend=True, has_constant='skip')
reg_multi_reglin = sm.OLS(endog=y_reglin, exog=x_reglin).fit()
print(reg_multi_reglin.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:             margin_low   R-squared:                       0.477
Model:                            OLS   Adj. R-squared:                  0.476
Method:                 Least Squares   F-statistic:                     266.1
Date:                Fri, 27 Jun 2025   Prob (F-statistic):          2.60e-202
Time:                        18:41:36   Log-Likelihood:                -1001.3
No. Observations:                1463   AIC:                             2015.
Df Residuals:                    1457   BIC:                             2046.
Df Model:                           5
Covariance Type:            nonrobust
================================================================================
                   coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
const           22.9948      9.656      2.382      0.017       4.055      41.935
diagonal        -0.1111      0.041     -2.680      0.007      -0.192      -0.030
height_left      0.1841      0.045      4.113      0.000       0.096       0.272
height_right     0.2571      0.043      5.978      0.000       0.173       0.342
margin_up        0.2562      0.064      3.980      0.000       0.130       0.382
length          -0.4091      0.018    -22.627      0.000      -0.445      -0.374
==============================================================================
Omnibus:                       73.627   Durbin-Watson:                   1.893
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               95.862
Skew:                           0.482   Prob(JB):                     1.53e-21
Kurtosis:                       3.801   Cond. No.                     1.94e+05
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

```
[2] The condition number is large, 1.94e+05. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Toutes les variables sont significatives (p-values inferieures a 5%). La probabilite de la F-statistic est tres proche de zero, ce qui indique que notre modele est significatif (i.e. on peut rejeter l'hypothese nulle H0 que tous les coefficients sont egaux a zero puisque la probabilite de la F-statistic est inferieure au seuil alpha = 5%). Le R2 et le R2 ajuste sont faibles (< 0.5), ce qui indique cependant une qualite de prevision mediocre. Le Cond.No. indique un probleme de colinearite potentiel.

[119]:
```python
# calculate Variance Inflation Factor each variable
vif_reglin = pd.DataFrame()
variables_reglin = reg_multi_reglin.model.exog
vif_reglin["variable"] = ['diagonal', 'height_left', 'height_right',␣
 ↪'margin_up', 'length']
vif_reglin["vi_factor"] = [variance_inflation_factor(variables_reglin, i) for i␣
 ↪in np.arange(1, variables_reglin.shape[1])]
vif_reglin.to_csv('vif_reglin.csv', index=False, encoding='latin_1')
vif_reglin
```

[119]:

	variable	vi_factor
0	diagonal	1.013613
1	height_left	1.138261
2	height_right	1.230115
3	margin_up	1.404404
4	length	1.576950

Tous les VIFs sont inferieurs a 10, on peut donc exclure un probleme de multi-colinearite des variables.

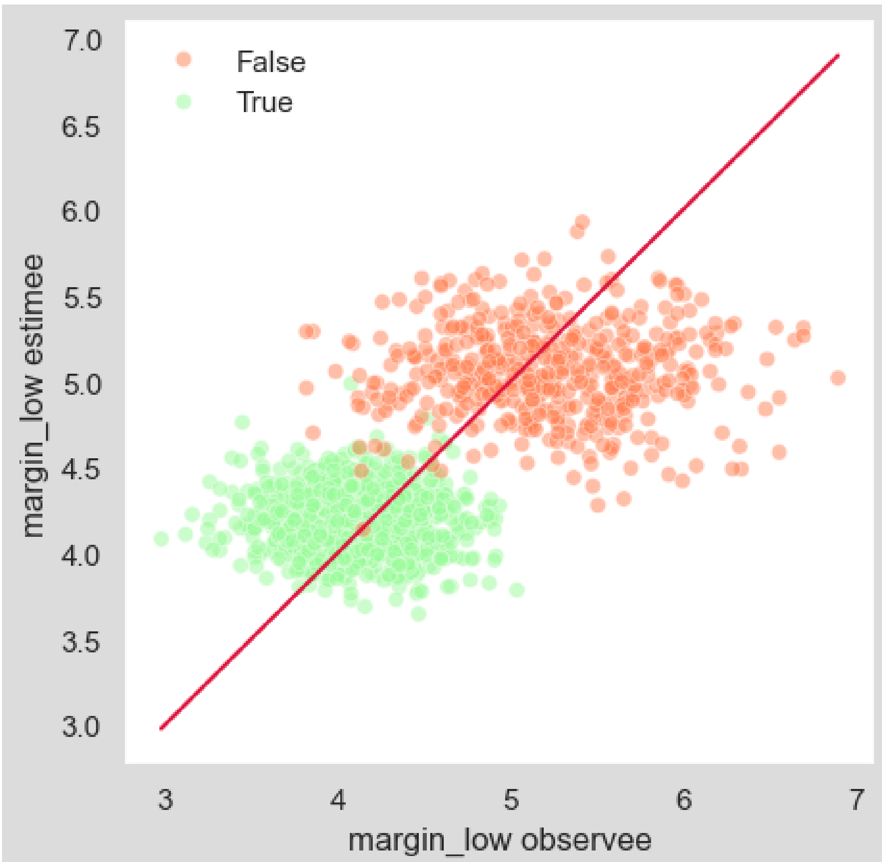
[122]:
```python
billets_trim_reg = billets_trim.copy()
billets_trim_reg['margin_low_reglin'] = reg_multi_reglin.predict()
billets_trim_reg['residus_reglin'] = reg_multi_reglin.resid
billets_trim_reg.head()
```

[122]:

	is_genuine	diagonal	height_left	height_right	margin_low	margin_up	\
0	True	171.81	104.86	104.95	4.52	2.89	
1	True	171.46	103.36	103.66	3.77	2.99	
2	True	172.69	104.48	103.50	4.40	2.94	
3	True	171.36	103.91	103.94	3.62	3.01	
4	True	171.73	104.28	103.46	4.04	3.48	

	length	margin_low_reglin	residus_reglin
0	112.83	4.788676	-0.268676
1	113.09	4.138908	-0.368908
2	113.16	4.125933	0.274067
3	113.51	4.156580	-0.536580
4	112.54	4.577425	-0.537425

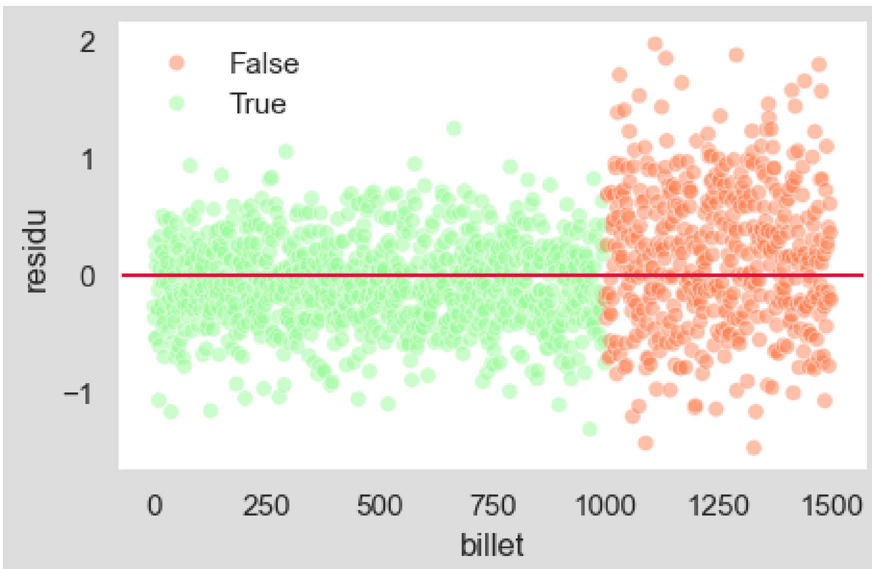
[123]:
```
# plot adjusted values against observed values
sns.set(rc={'figure.figsize':(5,5),'axes.facecolor':'white', 'figure.facecolor':
 ↪'gainsboro'})
X_plot = [billets_trim_reg['margin_low'].min(), billets_trim_reg['margin_low'].
 ↪max()]

ax = sns.scatterplot(x="margin_low", y="margin_low_reglin",␣
 ↪data=billets_trim_reg, hue='is_genuine', alpha=0.5,
                     palette=['coral','palegreen'])
ax.set(xlabel='margin_low observee', ylabel='margin_low estimee')
plt.plot(X_plot, X_plot, color='crimson')
plt.legend(title='', frameon=False)
plt.savefig('margin_low_e_reglin.png')
plt.show()
```



La droite rouge est la première bissectrice. Si la regression etait parfaite, les valeurs ajustees et observees seraient egales (residus nuls) et tous les points seraient alignes sur la premiere bissectrice d'equation y=x. On note une dispersion apparemment plus grande des faux billets autour de la premiere bissectrice, indiquant des residus plus grands pour ces billets.

2.1.2 - Analyse des residus

[127]:
```
# scatterplot residues
sns.set(rc={'figure.figsize':(5,3),'axes.facecolor':'white', 'figure.facecolor':
 ↪'gainsboro'})
ax = sns.scatterplot(x=billets_trim_reg.index,␣
 ↪y=billets_trim_reg['residus_reglin'],
                     data=billets_trim_reg, hue='is_genuine', palette=['coral',␣
 ↪'palegreen'], alpha=0.5)
ax.set(xlabel='billet', ylabel='residu')
plt.legend(title='', frameon=False)
plt.axhline(0, color='crimson')
plt.savefig('scatter_residus_reglin.png')
plt.show()
```



On n'observe pas vraiment de "cone", meme si les residus semblent s'eloigner davantage de 0 vers la droite du graphe ou sont representes les faux billets.

2.1.2.1 - Moyenne

[131]:
```
billets_trim_reg['residus_reglin'].mean()
```

[131]: 3.000501040004032e-14

La moyenne des residus est tres proche de 0.

2.1.2.2 - Homoscedasticite

[134]:
```
# Breusch-Pagan test
lm_bp, lm_pval_bp, f_val_bp, f_pval_bp = het_breuschpagan(reg_multi_reglin.
 ↪resid, variables_reglin)
print('Lagrange multiplier statistic:', lm_bp)
```

```
print('p-value Lagrange multiplier statistic:', lm_pval_bp)
```

```
Lagrange multiplier statistic: 80.16261280175073
p-value Lagrange multiplier statistic: 7.759535216194288e-16
```

La p-valeur de la statistique de test du multiplicateur de Lagrange est inferieure à 5%, on rejette l'hypothese H0 du test de Breusch-Pagan selon laquelle les variances des residus sont constantes, nous avons suffisamment d'information pour conclure a l'heteroscedasticite des residus.

[139]:
```
# White test
lm_w, lm_pval_w, f_val_w, f_pval_w = het_white(reg_multi_reglin.resid,
    variables_reglin)
print('Lagrange multiplier statistic:', lm_w)
print('p-value Lagrange multiplier statistic:', lm_pval_w)
```

```
Lagrange multiplier statistic: 109.860151968939
p-value Lagrange multiplier statistic: 2.0828244987371702e-14
```

La p-valeur de la statistique de test du multiplicateur de Lagrange est inferieure à 5%, on rejette l'hypothese H0 du test de White selon laquelle les variances des residus sont constantes, nous avons suffisamment d'information pour conclure a l'heteroscedasticite des residus.

2.1.2.3 - Normalite

[142]:
```
st.shapiro(reg_multi_reglin.resid)
```

[142]:
```
ShapiroResult(statistic=0.9857882577573781, pvalue=8.540407842391533e-11)
```

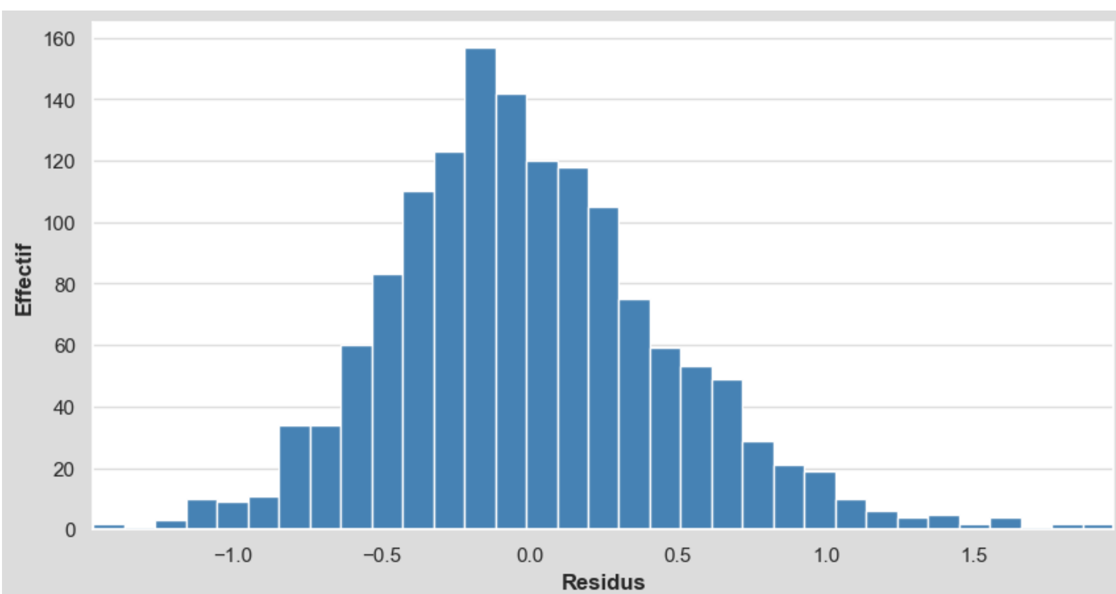
La p-value est inferieure a 5%, donc on rejette H0, les residus ne sont pas normaux, mais leur observation, le fait que leur moyenne soit tres proche de zero et que l'echantillon soit de taille suffisante (superieure a 30) permettent de dire que les resultats obtenus par le modele lineaire ne sont pas absurdes, meme si les residus ne sont pas normaux.

[145]:
```
n = billets_trim_reg.shape[0]
print("Nombre d'observations:", n)

# calculate first quartile
Q1 = np.percentile(billets_trim_reg['residus_reglin'], 25)
print("Premier quartile:", Q1)

# calculate third quartile
Q3 = np.percentile(billets_trim_reg['residus_reglin'], 75)
print("Troisieme quartile:", Q3)

# calculate interquartile range
IQ = Q3 - Q1
print("Interquartile range:", IQ)

# calculate bin width for histogram (Freedman-Diaconis rule)
bin_width = 2*((IQ)/np.cbrt(n))
```

```
print("Largeur des bins:", bin_width)

# calculate number of bins for histogram
maxi = billets_trim_reg['residus_reglin'].max()
mini = billets_trim_reg['residus_reglin'].min()
nb_bins = ((maxi - mini) / bin_width).astype(int)
print("Nombre de bins:", nb_bins)
```

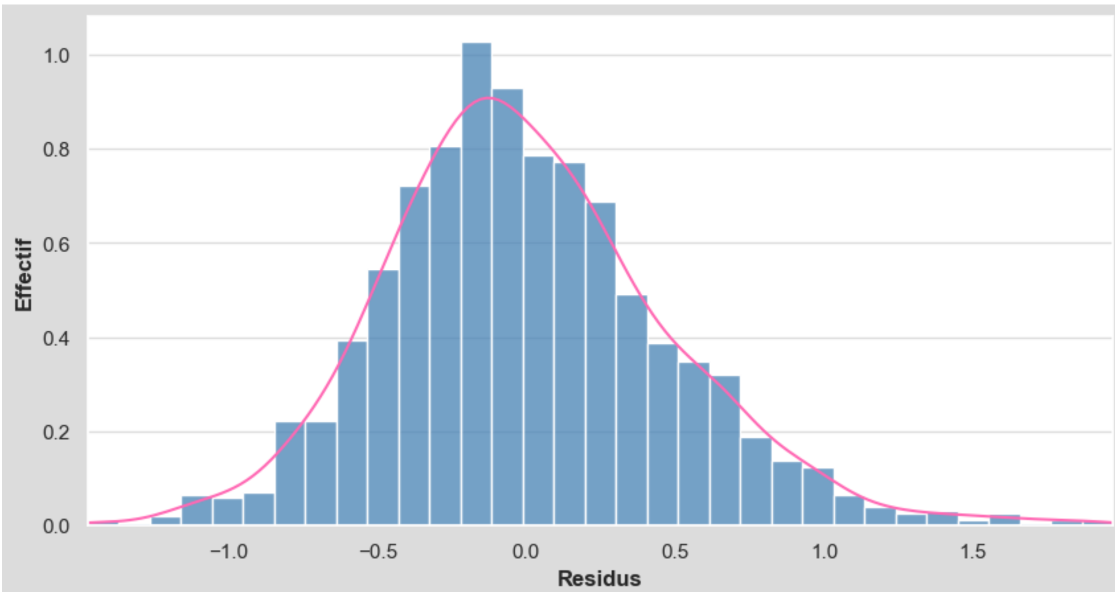
```
Nombre d'observations: 1463
Premier quartile: -0.3170737908560284
Troisieme quartile: 0.273528969656994
Interquartile range: 0.5906027605130224
Largeur des bins: 0.10405046518377252
Nombre de bins: 33
```

[147]:
```
col = 'steelblue'  # ou n'importe quelle couleur matplotlib : 'blue', 'teal',␣
↪'royalblue', etc.

# analyse residues with histogram
sns.set(rc={'figure.figsize':(10, 5), 'axes.facecolor':'white', 'figure.
↪facecolor':'gainsboro'})
graph3 = plt.hist(billets_trim_reg['residus_reglin'], bins=nb_bins, color=col)
plt.xlim([mini, maxi])
plt.xlabel('Residus', fontweight='bold', fontsize='12')
plt.ylabel('Effectif', fontweight='bold', fontsize='12')
plt.grid(axis='y', color='gainsboro')
plt.savefig("hist_reglin.png", bbox_inches='tight')
```



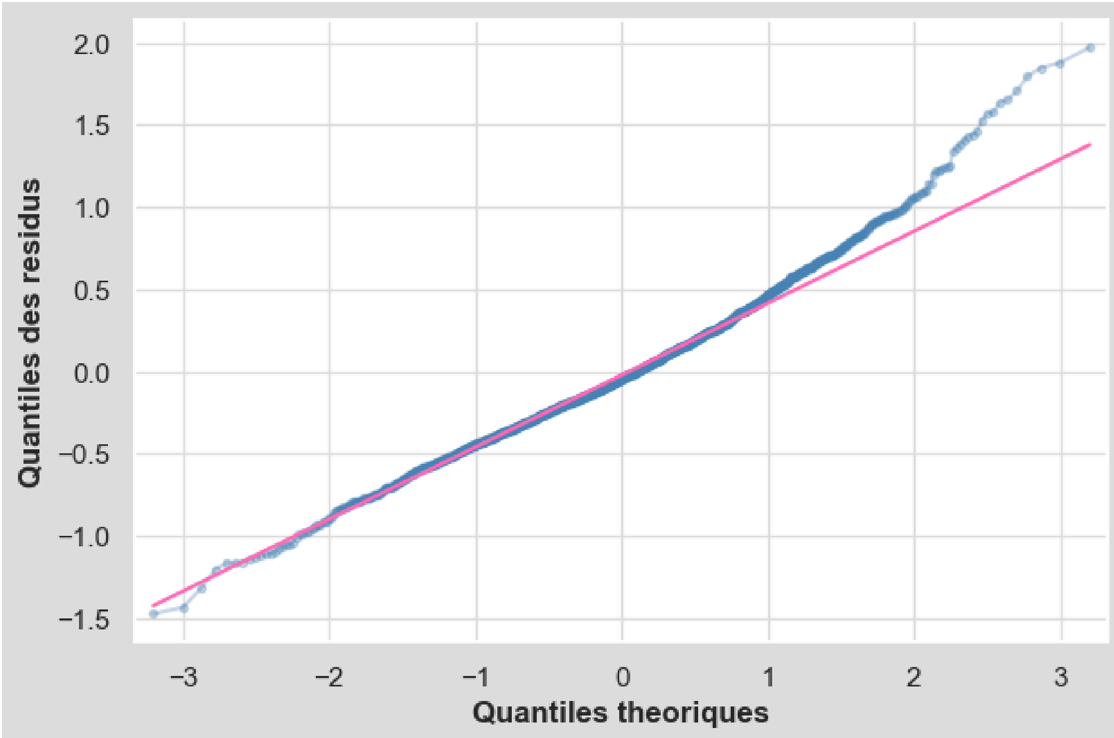
[148]:

```
sns.set(rc={'figure.figsize':(10, 5), 'axes.facecolor':'white', 'figure.
 ↪facecolor':'gainsboro'})
graph3b = sns.histplot(billets_trim_reg['residus_reglin'], bins=nb_bins,␣
 ↪kde=False, color=col, stat='density')
sns.kdeplot(data=billets_trim_reg, x='residus_reglin', color='hotpink')
plt.xlim([mini, maxi])
plt.xlabel('Residus', fontweight='bold', fontsize='12')
plt.ylabel('Effectif', fontweight='bold', fontsize='12')
plt.grid(axis='y', color='gainsboro')
plt.savefig("histplot_kde_reglin.png", bbox_inches='tight')
```

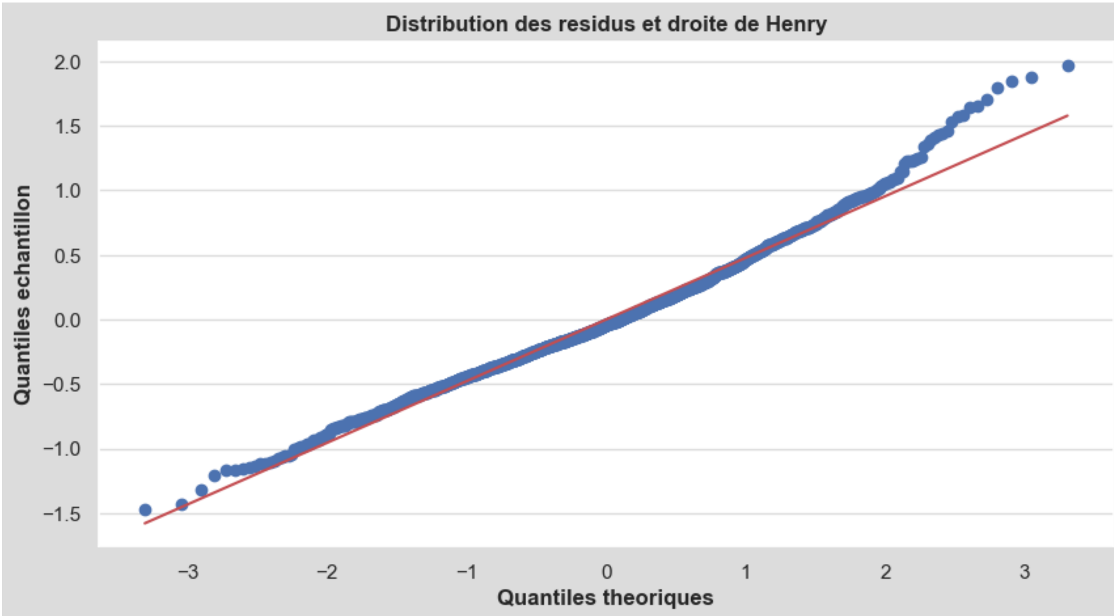


[149]:

```
# analyse residues with a QQ plot
fig, ax = plt.subplots(1, figsize=(7, 4.5))
graph4 = sm.qqplot(billets_trim_reg['residus_reglin'], marker='.',␣
 ↪markerfacecolor=col,
                   markeredgecolor=col, alpha=0.3, line="q", linestyle='solid',␣
 ↪ax=ax)
plt.xlabel("Quantiles theoriques", fontweight='bold', fontsize=12)
plt.ylabel("Quantiles des residus", fontweight='bold', fontsize=12)
ax.get_lines()[1].set_color("hotpink")
plt.grid(axis='both', color='gainsboro')
plt.savefig("qq_reglin.png", bbox_inches='tight')
plt.show()
```



[150]:
```python
# droite de Henry
fig, ax = plt.subplots()
graph5 = st.probplot(billets_trim_reg['residus_reglin'], dist="norm", plot=ax)
ax.set_title("Distribution des residus et droite de Henry", fontname='Arial',␣
 ↪fontsize=12, fontweight='bold')
plt.xlabel('Quantiles theoriques', fontweight='bold', fontsize='12')
plt.ylabel('Quantiles echantillon', fontweight='bold', fontsize='12')
plt.grid(axis='y', color='gainsboro')
plt.savefig("henry_reglin.png", bbox_inches='tight')
plt.show()
```



Sur le QQ plot comme la droite de Henry, on note un "decollement" des queues de distribution des residus, particulierement a droite.

[156]:
```
# Anderson-Darling test on residues
print(st.anderson(billets_trim_reg['residus_reglin'], dist="norm"))
```

```
AndersonResult(statistic=4.85502724639673, critical_values=array([0.574, 0.654,
0.785, 0.916, 1.089]), significance_level=array([15. , 10. ,  5. ,  2.5,  1. ]),
fit_result=  params: FitParams(loc=3.000501040004032e-14,
scale=0.47990636678802423)
 success: True
 message: '`anderson` successfully fit the distribution to the data.')
```

AndersonResult(statistic=4.85502724639673, critical_values=array([0.574, 0.654, 0.785, 0.916, 1.089]), significance_level=array([15. , 10. , 5. , 2.5, 1.]))

[159]:
```
stat, p = normal_ad(billets_trim_reg['residus_reglin'], axis=0)
print("Anderson-Darling statistic:", stat)
print("p-value:", p)
```

```
Anderson-Darling statistic: 4.85502724639673
p-value: 5.114137678791534e-12
```

Les hypothèses du test d'Anderson-Darling sont les suivantes : H0 : les données suivent une distribution spécifiée H1 : les données ne suivent pas une distribution spécifiée Utilisez la valeur de p correspondante (si disponible) pour vérifier si les données proviennent de la distribution choisie. Si la valeur de p est inférieure à alpha (généralement 0,05 ou 0,10), rejetez l'hypothèse nulle qui suppose que les données proviennent de cette distribution. source: https://support.minitab.com/fr-fr/minitab/20/help-and-how-to/statistics/basic-statistics/supporting-topics/normality/the-anderson-darling-statistic/ Ici encore, on rejette donc l'hypothese H0 de normalite des residus.

[162]:
```
# Lilliefors (Kolmogorov-Smirnov) normality test for mu and sigma unknown
stat_reglin, p_reglin = kstest_fit(billets_trim_reg['residus_reglin'],␣
 ↪dist='norm', pvalmethod='table')
print("Kolmogorov-Smirnov statistic:", stat_reglin)
print("p-value:", p_reglin)
```

```
Kolmogorov-Smirnov statistic: 0.042780969975706074
p-value: 0.0009999999999998899
```

Ici encore on rejette H0, les residus ne sont pas normalement distribues. Aucun des tests effectues ne permet de conclure a la normalite des residus de la regression lineaire simple, qui en outre met en evidence un eventuel probleme de multicollinearite (Cond. No. tres eleve). Une regression ridge semble indiquee dans ce cas. Par souci de completude, nous etudierons egalement les resultats de la regressions lasso et eventuellement elastic net en fonction des resultats.

2.1.3 - Analyse des outliers

2.1.3.1 - Individus atypiques et influents

[168]:
```
billets_trim_reg.head()
```

[168]:
```
   is_genuine  diagonal  height_left  height_right  margin_low  margin_up  \
0        True    171.81       104.86        104.95        4.52       2.89
1        True    171.46       103.36        103.66        3.77       2.99
2        True    172.69       104.48        103.50        4.40       2.94
3        True    171.36       103.91        103.94        3.62       3.01
4        True    171.73       104.28        103.46        4.04       3.48

   length  margin_low_reglin  residus_reglin
0  112.83           4.788676       -0.268676
1  113.09           4.138908       -0.368908
2  113.16           4.125933        0.274067
3  113.51           4.156580       -0.536580
4  112.54           4.577425       -0.537425
```

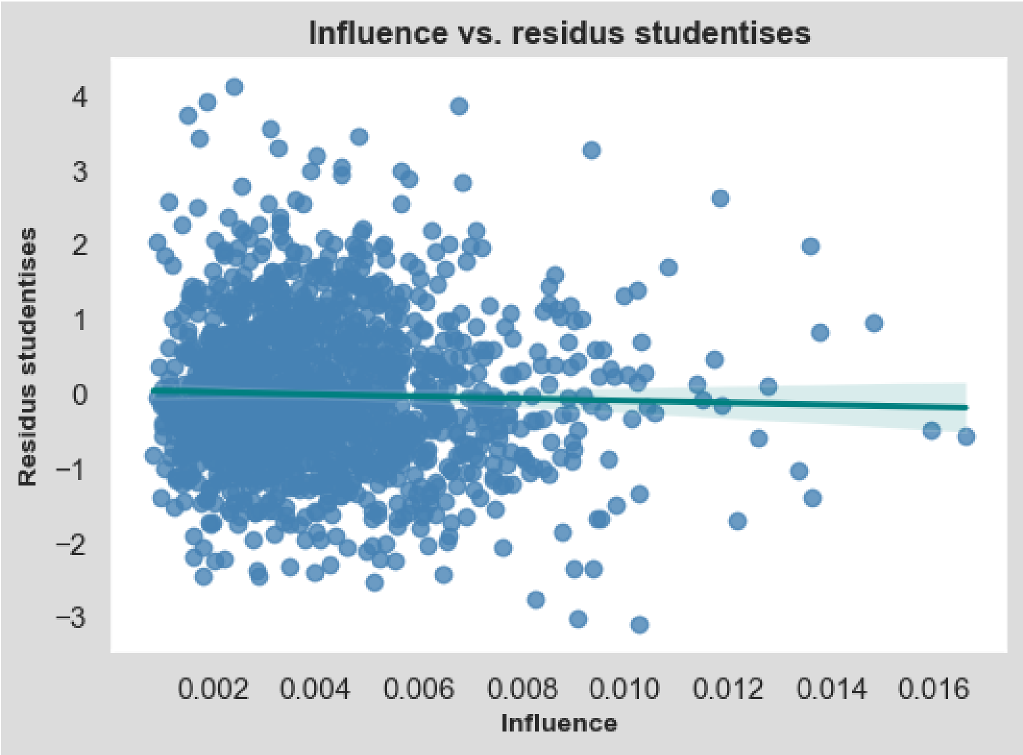
[170]:
```
# create data
stud_res = reg_multi_reglin.get_influence().resid_studentized_external  #␣
 ↪calculate studentized residues
dist_cook, pval_cook = reg_multi_reglin.get_influence().cooks_distance  #␣
 ↪calculate Cook distance
dffits, pval_dffits = reg_multi_reglin.get_influence().dffits  # calculate␣
 ↪influence of observations
lvg = reg_multi_reglin.get_influence().hat_matrix_diag  # diagonal of the hat␣
 ↪matrix for ordinary least squares method

# store data in dataframe
```

```
billets_trim_reg_2 = pd.concat([billets_trim_reg[['is_genuine', 'diagonal',␣
↪'height_left', 'height_right', 'margin_low',
                                 'margin_up', 'length', 'margin_low_reglin',␣
↪'residus_reglin']],
                                 reg_multi_reglin.get_influence().
↪summary_frame()], axis=1)
billets_trim_reg_2.head()
```

[170]:
```
   is_genuine  diagonal  height_left  height_right  margin_low  margin_up  \
0        True    171.81       104.86        104.95        4.52       2.89
1        True    171.46       103.36        103.66        3.77       2.99
2        True    172.69       104.48        103.50        4.40       2.94
3        True    171.36       103.91        103.94        3.62       3.01
4        True    171.73       104.28        103.46        4.04       3.48

   length  margin_low_reglin  residus_reglin  dfb_const  …  dfb_height_left  \
0  112.83           4.788676       -0.268676   0.038028  …        -0.040676
1  113.09           4.138908       -0.368908  -0.047734  …         0.040802
2  113.16           4.125933        0.274067  -0.028752  …         0.027975
3  113.51           4.156580       -0.536580  -0.031981  …         0.001406
4  112.54           4.577425       -0.537425  -0.025618  …        -0.027128

   dfb_height_right  dfb_margin_up  dfb_length   cooks_d  standard_resid  \
0         -0.050650       0.028339   -0.018016  0.000894       -0.563593
1          0.006819       0.006051    0.006096  0.000597       -0.769710
2         -0.020340      -0.010975   -0.001295  0.000457        0.572488
3         -0.016400       0.006425   -0.029579  0.000936       -1.118681
4          0.056039      -0.049448   -0.009670  0.001171       -1.121058

   hat_diag  dffits_internal  student_resid    dffits
0  0.016606        -0.073239      -0.563461 -0.073221
1  0.006015        -0.059874      -0.769603 -0.059866
2  0.008298         0.052367       0.572356  0.052355
3  0.004467        -0.074934      -1.118778 -0.074940
4  0.005558        -0.083813      -1.121157 -0.083821

[5 rows x 21 columns]
```

[174]:
```
# plot influence vs studentised residues
sns.set(rc={'figure.figsize':(6, 4), 'axes.facecolor':'white', 'figure.
↪facecolor':'gainsboro'})
graph5 = sns.regplot(x='hat_diag', y='student_resid', data=billets_trim_reg_2,␣
↪scatter_kws={"color":col},
                     line_kws={"color":"teal"})
plt.xlabel('Influence', fontweight='bold', fontsize=10)
plt.ylabel('Residus studentises', fontweight='bold', fontsize=10)
plt.title('Influence vs. residus studentises', fontweight='bold', fontsize=12)
```

```
plt.savefig('stud_res_influence.png')
plt.show()
```



[176]:
```
# overview of Studentised residues
print ("Residus studentises\n", billets_trim_reg_2['student_resid'].describe())
```

```
Residus studentises
 count    1463.000000
mean        0.000133
std         1.001419
min        -3.087577
25%        -0.660578
50%        -0.086814
75%         0.570403
max         4.127099
Name: student_resid, dtype: float64
```

[178]:
```
# identification of outliers
outliers = billets_trim_reg_2.loc[abs(billets_trim_reg_2['student_resid']) > 2]

# check if outliers influence is above threshold using leverage cutoff
# threshold = (2k+2)/n where k is the number of variables and n is the number␣
 ↪of observations
```

```
threshold = ((2 * (len(x_reglin.columns) - 1) + 2) / billets_trim_reg_2.
  ↪shape[0])
influents_2 = billets_trim_reg_2.loc[billets_trim_reg_2['hat_diag'] > threshold]

# cross outliers and influent banknotes
influent_outliers_2 =pd.merge(outliers, influents_2, left_index=True,␣
  ↪right_index=True)
```

[180]:
```
print("Il y a", outliers.shape[0], "billets atypiques dans le dataset.")
print("Il y a ", influents_2.shape[0], "billets influents dans le dataset.")
print("Il y a ", influent_outliers_2.shape[0], "billets atypiques et influents␣
  ↪dans le dataset.")
```

```
Il y a 72 billets atypiques dans le dataset.
Il y a  75 billets influents dans le dataset.
Il y a  7 billets atypiques et influents dans le dataset.
```

2.1.3.2 - Regression lineaire sans les individus atypiques et influents

[183]:
```
billets_trim_3 = billets_trim.drop(influent_outliers_2.index)
billets_trim_3.shape
```

[183]:
```
(1456, 7)
```

[185]:
```
# perform linear regression to complete margin_low column
y_reglin_2 = billets_trim_3['margin_low']
x_reglin_2 = billets_trim_3[['diagonal', 'height_left', 'height_right',␣
  ↪'margin_up', 'length']]
x_reglin_2 = sm.add_constant(x_reglin_2, prepend=True, has_constant='skip')
reg_multi_reglin_2 = sm.OLS(endog=y_reglin_2, exog=x_reglin_2).fit()
print(reg_multi_reglin_2.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:             margin_low   R-squared:                       0.493
Model:                            OLS   Adj. R-squared:                  0.491
Method:                 Least Squares   F-statistic:                     282.2
Date:                Fri, 27 Jun 2025   Prob (F-statistic):          5.39e-211
Time:                        18:51:19   Log-Likelihood:                -972.21
No. Observations:                1456   AIC:                             1956.
Df Residuals:                    1450   BIC:                             1988.
Df Model:                           5
Covariance Type:            nonrobust
================================================================================
                   coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
const           24.2866      9.516      2.552      0.011       5.621      42.952
diagonal        -0.1204      0.041     -2.938      0.003      -0.201      -0.040
height_left      0.1967      0.044      4.452      0.000       0.110       0.283
```

```
height_right     0.2537      0.042      5.974      0.000       0.170       0.337
margin_up        0.2728      0.064      4.258      0.000       0.147       0.398
length          -0.4151      0.018    -23.203      0.000      -0.450      -0.380
==============================================================================
Omnibus:                       74.826   Durbin-Watson:                   1.906
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               93.636
Skew:                           0.506   Prob(JB):                     4.65e-21
Kurtosis:                       3.720   Cond. No.                     1.94e+05
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 1.94e+05. This might indicate that there are
strong multicollinearity or other numerical problems.
```

2.1.3.4 - Analyse des residus de la regression sans individus atypiques influents

[188]:
```python
# calculate residues
margin_low_ai = reg_multi_reglin_2.predict(x_reglin_2)
billets_trim_3['margin_low_ai'] = margin_low_ai
billets_trim_3['residus_ai'] = billets_trim_3['margin_low_ai'] -
 ↪billets_trim_3['margin_low']
billets_trim_3.head()
```

[188]:
```
   is_genuine  diagonal  height_left  height_right  margin_low  margin_up  \
0        True    171.81       104.86        104.95        4.52       2.89
1        True    171.46       103.36        103.66        3.77       2.99
2        True    172.69       104.48        103.50        4.40       2.94
3        True    171.36       103.91        103.94        3.62       3.01
4        True    171.73       104.28        103.46        4.04       3.48

   length  margin_low_ai  residus_ai
0  112.83       4.794173    0.274173
1  113.09       4.133400    0.363400
2  113.16       4.122230   -0.277770
3  113.51       4.155756    0.535756
4  112.54       4.593061    0.553061
```

[190]:
```python
# mean residue
billets_trim_3['residus_ai'].mean()
```

[190]: -6.927303663540262e-14

[192]:
```python
# Breusch-Pagan test for homoscedasticity
lm_bp_ai, lm_pval_bp_ai, f_val_bp_ai, f_pval_bp_ai =
 ↪het_breuschpagan(billets_trim_3['residus_ai'], x_reglin_2)
print('Lagrange multiplier statistic:', lm_bp_ai)
```

```
print('p-value Lagrange multiplier statistic:', lm_pval_bp_ai)
```

```
Lagrange multiplier statistic: 74.98436229043278
p-value Lagrange multiplier statistic: 9.37312023328503e-15
```

La p-valeur de la statistique de test du multiplicateur de Lagrange est inferieure à 5%, on rejette l'hypothese H0 du test de Breusch-Pagan selon laquelle les variances des residus sont constantes, nous avons suffisamment d'information pour conclure a l'heteroscedasticite des residus.

[195]:
```
# Shapiro test for normality
st.shapiro(billets_trim_3['residus_ai'])
```

[195]:
```
ShapiroResult(statistic=0.9853682593487141, pvalue=5.72061553124996e-11)
```

La p-value est inferieure a 5%, donc on rejette H0, les residus ne sont pas normaux, mais leur observation, le fait que leur moyenne soit tres proche de zero et que l'echantillon soit de taille suffisante (superieure a 30) permettent de dire que les resultats obtenus apres elimination des valeurs atypiques et influentes ne sont pas absurdes, meme si les residus ne sont pas normaux. Cependant, ils ne permettent pas d'ameliorer significativment les resultats de la regression lineaire multiple. Le R2 varie peu apres elimination des valeurs atypiques et influentes (0.493 contre 0.477) et les coefficients de regression varient peu. Les residus ne sont toujorus ni Gaussiens, ni homoscedastiques. En outre, le nombre de billets atypiques et influents est faible (7 sur 1463) et on peut donc en conclure que leur influence sur les resultats de la regression lineaire est suffisemment faible pour que leur elimination ne constitue pas une amelioration significative du modele, ce qui valide statistiquement notre intuition initiale de les conserver dans l'analyse.

2.2 - Regression ridge

2.2.1 - Regression

[203]:
```
# select data
y_ridge_cv = billets_trim['margin_low']
x_ridge_cv = billets_trim[['diagonal', 'height_left', 'height_right',␣
 ↪'margin_up', 'length']]
alphas = [0.000001, 0.00001, 0.0001, 0.001, 0.01, 0.1, 0.5, 1]
```

[207]:
```
# perform ridge regression
ridge_cv = RidgeCV(alphas=alphas, fit_intercept=True, scoring=None, cv=None,␣
 ↪gcv_mode='auto', store_cv_values=True,
                    alpha_per_target=False)
ridge_reg = ridge_cv.fit(x_ridge_cv, y_ridge_cv)

# print results
print("cv values:", ridge_reg.cv_values_)
print("mean cv values:", np.mean(ridge_cv.cv_values_, axis=0))
print("alpha:", ridge_cv.alpha_)
print("const:", ridge_cv.intercept_)
print("weight vector:", ridge_cv.coef_)
print("nb endog:", ridge_cv.n_features_in_)
print("base estimator score:", ridge_cv.best_score_)
```

```
print("names endog:", ridge_cv.feature_names_in_)
```

```
cv values: [[0.07464567 0.07464566 0.0746456  … 0.07457285 0.07428156
0.07391749]
 [0.13774488 0.13774488 0.13774494 … 0.13780821 0.13806082 0.13837501]
 [0.07637518 0.07637518 0.07637512 … 0.07630915 0.07604665 0.07572212]
 …
 [0.37298655 0.37298656 0.37298664 … 0.37307482 0.37342644 0.37386276]
 [0.13121926 0.13121927 0.13121934 … 0.13129933 0.13161788 0.13201223]
 [0.04398647 0.04398646 0.04398643 … 0.04395177 0.04381389 0.04364352]]
mean cv values: [0.23215509 0.23215509 0.23215509 0.23215507 0.23215494
0.23215358
 0.23214773 0.23214087]
alpha: 1.0
const: 23.12448438252585
weight vector: [-0.1102964   0.18326792  0.25596266  0.25309142 -0.40945886]
nb endog: 5
base estimator score: -0.2321408704284659
names endog: ['diagonal' 'height_left' 'height_right' 'margin_up' 'length']
```

```
C:\Users\FAMILLE\anaconda3\Lib\site-
packages\sklearn\linear_model\_ridge.py:2341: FutureWarning: 'store_cv_values'
is deprecated in version 1.5 and will be removed in 1.7. Use 'store_cv_results'
instead.
  warnings.warn(
C:\Users\FAMILLE\anaconda3\Lib\site-packages\sklearn\utils\deprecation.py:102:
FutureWarning: Attribute `cv_values_` is deprecated in version 1.5 and will be
removed in 1.7. Use `cv_results_` instead.
  warnings.warn(msg, category=FutureWarning)
```

Rappel des resultats de la regression lineaire multiple:

Plus l'alpha de la regression ridge est important, plus la penalisation est importante et plus le resultat de la regression (poids affectes aux differentes variables) differe de celui d'une regression lineaire multiple. Ici, alpha vaut 1 et les poids des affectes aux differentes variables sont egaux a ceux de la regression lineaire multiple jusqu'a la 2eme decimale (3eme decimale pour la longueur).

[210]:
```
margin_low_ridge = ridge_reg.predict(x_ridge_cv)
score = ridge_reg.score(x_ridge_cv, y_ridge_cv)
mse = mean_squared_error(y_ridge_cv, margin_low_ridge)
print("R2:{0:.3f}, Mean Squared Error (MSE):{1:.2f}, Mean Squared Error Sqrt␣
 ↪(RMSE):{2:.2f}"
      .format(score, mse, np.sqrt(mse)))
```

```
R2:0.477, Mean Squared Error (MSE):0.23, Mean Squared Error Sqrt (RMSE):0.48
```

Le R2 n'est pas significativement plus eleve que celui de la regression lineaire multiple.

[213]:
```
# billets_trim_reg['margin_low_ridge'] = ridge_reg.predict(x_ridge_cv)
billets_trim_reg['margin_low_ridge'] = margin_low_ridge
```

```
score = ridge_reg.score(x_ridge_cv, y_ridge_cv)
mse = mean_squared_error(y_ridge_cv, margin_low_ridge)
print("R2:{0:.3f}, Mean Squared Error (MSE):{1:.2f}, Mean Squared Error Sqrt␣
↪(RMSE):{2:.2f}"
      .format(score, mse, np.sqrt(mse)))
billets_trim_reg['residus_ridge'] = billets_trim_reg['margin_low_ridge'] -␣
↪billets_trim_reg['margin_low']
billets_trim_reg.head()
```

```
R2:0.477, Mean Squared Error (MSE):0.23, Mean Squared Error Sqrt (RMSE):0.48
```

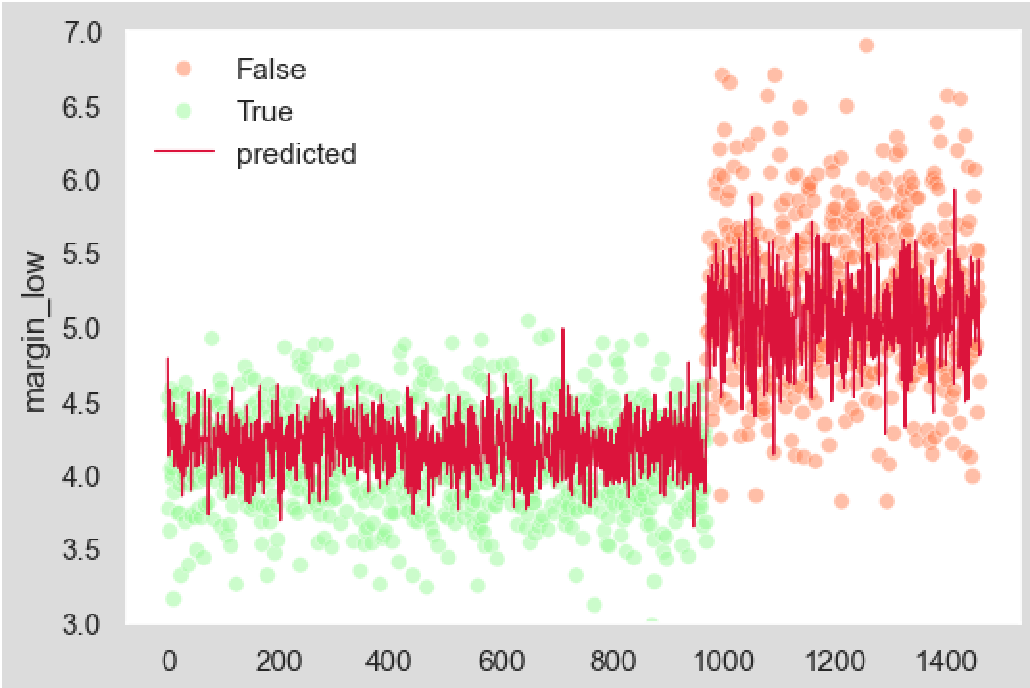
[213]:

	is_genuine	diagonal	height_left	height_right	margin_low	margin_up
0	True	171.81	104.86	104.95	4.52	2.89
1	True	171.46	103.36	103.66	3.77	2.99
2	True	172.69	104.48	103.50	4.40	2.94
3	True	171.36	103.91	103.94	3.62	3.01
4	True	171.73	104.28	103.46	4.04	3.48

	length	margin_low_reglin	residus_reglin	margin_low_ridge	residus_ridge
0	112.83	4.788676	-0.268676	4.787405	0.267405
1	113.09	4.138908	-0.368908	4.139765	0.369765
2	113.16	4.125933	0.274067	4.127090	-0.272910
3	113.51	4.156580	-0.536580	4.156350	0.536350
4	112.54	4.577425	-0.537425	4.576616	0.536616

[215]:

```
# scatterplot predicted values vs original values
sns.set(rc={'figure.figsize':(6, 4),'axes.facecolor':'white', 'figure.
↪facecolor':'gainsboro'})
graph9 = sns.scatterplot(x=range(len(x_ridge_cv)), y=y_ridge_cv, alpha=0.5,
                          hue=billets_trim_reg['is_genuine'],␣
↪palette=['coral','palegreen'])
plt.plot(range(len(x_ridge_cv)), margin_low_ridge, lw=0.8, color="crimson",␣
↪label="predicted")
plt.legend(title='', frameon=False)
ax.set(xlabel='billet', ylabel='margin_low')
plt.ylim(3, 7)
plt.savefig('margin_low_e_ridge.png')
plt.show()
```



On observe ici encore des residus plus variables sur les faux billets.

2.2.2 - Analyse des residus

[220]:
```
# mean residue
billets_trim_reg['residus_ridge'].mean()
```

[220]: 4.045673950021623e-15

La moyenne des residus est tres proche de zero.

[223]:
```
# Breusch-Pagan test for homoscedasticity
lm_bp_r, lm_pval_bp_r, f_val_bp_r, f_pval_bp_r =␣
 ↪het_breuschpagan(billets_trim_reg['residus_ridge'], variables_reglin)
print('Lagrange multiplier statistic:', lm_bp_r)
print('p-value Lagrange multiplier statistic:', lm_pval_bp_r)
```

```
Lagrange multiplier statistic: 80.16639690374598
p-value Lagrange multiplier statistic: 7.745402558978381e-16
```

La p-valeur de la statistique de test du multiplicateur de Lagrange est inferieure à 5%, on rejette l'hypothese H0 du test de Breusch-Pagan selon laquelle les variances des residus sont constantes, nous avons suffisamment d'information pour conclure a l'heteroscedasticite des residus.

[226]:
```
# Shapiro test for normality
st.shapiro(billets_trim_reg['residus_ridge'])
```

[226]: ShapiroResult(statistic=0.985719637675242, pvalue=7.890005058959495e-11)

La p-value est inferieure a 5%, donc on rejette H0, les residus ne sont pas normaux, mais leur observation, le fait que leur moyenne soit tres proche de zero et que l'echantillon soit de taille suffisante (superieure a 30) permettent de dire que les resultats obtenus par le modele ridge ne sont pas absurdes, meme si les residus ne sont pas normaux. Cependant, ils ne permettent pas d'ameliorer significativment les resultats de la regression lineaire multiple.

2.3 - Regression lasso

2.3.1 - Regression

[231]:
```
# select data
y_lasso_cv = billets_trim['margin_low']
x_lasso_cv = billets_trim[['diagonal', 'height_left', 'height_right',␣
 ↪'margin_up', 'length']]
```

[233]:
```
# perform lasso regression
lasso_cv = LassoCV(eps=0.001, n_alphas=100, alphas=alphas, fit_intercept=True,␣
 ↪precompute='auto',  max_iter=1000, tol=0.0001,
                   copy_X=True, cv=None, verbose=False, n_jobs=None,␣
 ↪positive=False, random_state=None, selection='cyclic')
lasso_reg = lasso_cv.fit(x_lasso_cv, y_lasso_cv)

# print results
print("MSE path;", lasso_reg.mse_path_)
print("alpha:", lasso_reg.alpha_)
print("dual gap:", lasso_reg.dual_gap_)
print("nb iterations:", lasso_reg.n_iter_)
print("const:", lasso_reg.intercept_)
print("weight vector:", lasso_reg.coef_)
print("nb endog:", ridge_reg.n_features_in_)
print("names endog:", ridge_reg.feature_names_in_)
```

```
MSE path; [[0.30009836 0.3257853  0.32065717 0.73249065 1.13686396]
 [0.30009836 0.3257853  0.32065717 0.73249065 1.13686396]
 [0.15823388 0.16983464 0.16970641 0.40188626 0.64157194]
 [0.1500596  0.1526748  0.14813189 0.35265839 0.459324  ]
 [0.15259126 0.15037583 0.14996131 0.34268317 0.43364512]
 [0.15321938 0.1504507  0.15052753 0.34190275 0.43077539]
 [0.15328601 0.15046161 0.15058783 0.34182953 0.43048205]
 [0.1532926  0.15046274 0.15059395 0.34182226 0.43045293]]
alpha: 1e-06
dual gap: 8.096634971317689e-07
nb iterations: 12
const: 22.99513713791545
weight vector: [-0.11104887  0.18411291  0.25713132  0.25616882 -0.40910668]
nb endog: 5
names endog: ['diagonal' 'height_left' 'height_right' 'margin_up' 'length']
```

[235]:
```
margin_low_lasso = lasso_reg.predict(x_lasso_cv)
score = lasso_reg.score(x_lasso_cv, y_lasso_cv)
mse = mean_squared_error(y_lasso_cv, margin_low_lasso)
print("R2:{0:.3f}, Mean Squared Error (MSE):{1:.2f}, Mean Squared Error Sqrt␣
↪(RMSE):{2:.2f}"
      .format(score, mse, np.sqrt(mse)))
```

```
R2:0.477, Mean Squared Error (MSE):0.23, Mean Squared Error Sqrt (RMSE):0.48
```

Ici encore, la regression Lasso n'apporte pas d'amelioration significative du R2 par rapport a la regression lineaire multiple.

[240]:
```
billets_trim_reg['margin_low_lasso'] = lasso_reg.predict(x_lasso_cv)
billets_trim_reg['residus_lasso'] = billets_trim_reg['margin_low_lasso'] -␣
↪billets_trim_reg['margin_low']
billets_trim_reg.head()
```

[240]:

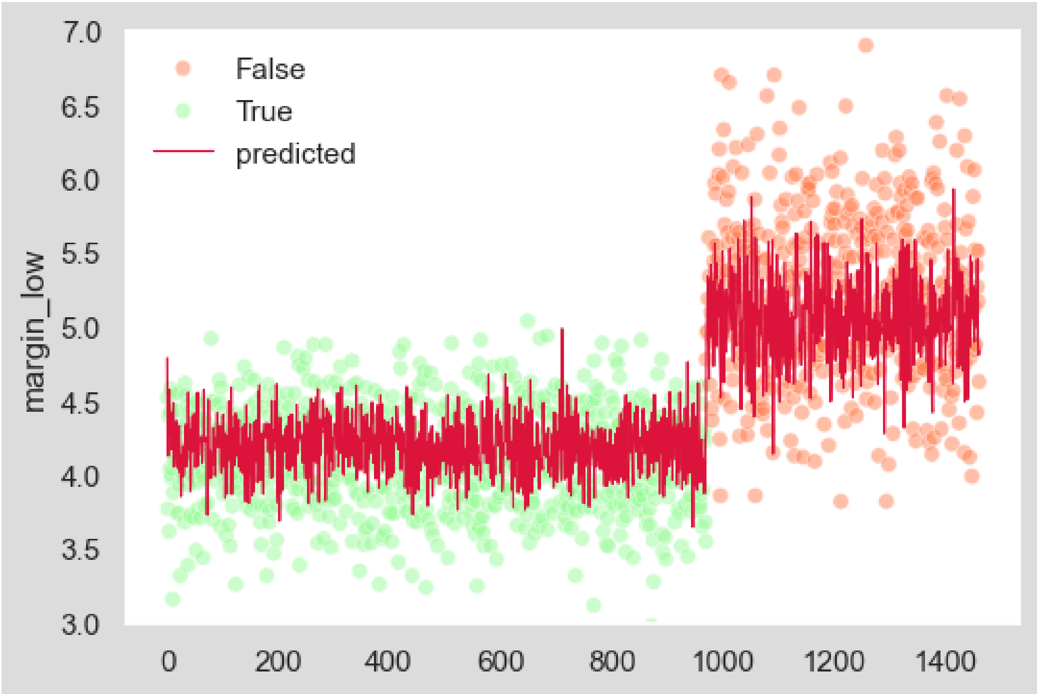
	is_genuine	diagonal	height_left	height_right	margin_low	margin_up	\
0	True	171.81	104.86	104.95	4.52	2.89	
1	True	171.46	103.36	103.66	3.77	2.99	
2	True	172.69	104.48	103.50	4.40	2.94	
3	True	171.36	103.91	103.94	3.62	3.01	
4	True	171.73	104.28	103.46	4.04	3.48	

	length	margin_low_reglin	residus_reglin	margin_low_ridge	residus_ridge	\
0	112.83	4.788676	-0.268676	4.787405	0.267405	
1	113.09	4.138908	-0.368908	4.139765	0.369765	
2	113.16	4.125933	0.274067	4.127090	-0.272910	
3	113.51	4.156580	-0.536580	4.156350	0.536350	
4	112.54	4.577425	-0.537425	4.576616	0.536616	

	margin_low_lasso	residus_lasso
0	4.788665	0.268665
1	4.138912	0.368912
2	4.125941	-0.274059
3	4.156574	0.536574
4	4.577418	0.537418

[246]:
```
# scatterplot predicted values vs original values
sns.set(rc={'figure.figsize':(6,4),'axes.facecolor':'white', 'figure.facecolor':
↪'gainsboro'})
graph9 = sns.scatterplot(x=range(len(x_lasso_cv)), y=y_lasso_cv,
                         data=billets_trim_reg, alpha=0.5, hue='is_genuine',␣
↪palette=['coral','palegreen'])
plt.plot(range(len(x_lasso_cv)), margin_low_lasso, lw=0.8, color="crimson",␣
↪label="predicted")
plt.legend(title='', frameon=False)
ax.set(xlabel='billet', ylabel='margin_low')
```

```
plt.ylim(3, 7)
plt.savefig('margin_low_e_lasso.png')
plt.show()
```



On observe ici encore des residus plus variables sur les faux billets.

2.2.2 - Analyse des residus

[251]:
```
# mean residue
billets_trim_reg['residus_ridge'].mean()
```

[251]: 4.045673950021623e-15

La moyenne des residus est tres proche de zero.

[254]:
```
# Breusch-Pagan test for homoscedasticity
lm_bp_r, lm_pval_bp_r, f_val_bp_r, f_pval_bp_r =␣
↪het_breuschpagan(billets_trim_reg['residus_ridge'], variables_reglin)
print('Lagrange multiplier statistic:', lm_bp_r)
print('p-value Lagrange multiplier statistic:', lm_pval_bp_r)
```

```
Lagrange multiplier statistic: 80.16639690374598
p-value Lagrange multiplier statistic: 7.745402558978381e-16
```

La p-valeur de la statistique de test du multiplicateur de Lagrange est inferieure à 5%, on rejette l'hypothese H0 du test de Breusch-Pagan selon laquelle les variances des residus sont constantes, nous avons suffisamment d'information pour conclure a l'heteroscedasticite des residus.

[257]:
```
# Shapiro test for normality
st.shapiro(billets_trim_reg['residus_ridge'])
```

[257]: ShapiroResult(statistic=0.985719637675242, pvalue=7.890005058959495e-11)

La p-value est inferieure a 5%, donc on rejette H0, les residus ne sont pas normaux, mais leur observation, le fait que leur moyenne soit tres proche de zero et que l'echantillon soit de taille suffisante (superieure a 30) permettent de dire que les resultats obtenus par le modele ridge ne sont pas absurdes, meme si les residus ne sont pas normaux. Cependant, ils ne permettent pas d'ameliorer significativment les resultats de la regression lineaire multiple.

2.3 - Regression lasso

2.3.1 - Regression

[263]:
```
# select data
y_lasso_cv = billets_trim['margin_low']
x_lasso_cv = billets_trim[['diagonal', 'height_left', 'height_right',␣
↪'margin_up', 'length']]
```

[265]:
```
# perform lasso regression
lasso_cv = LassoCV(eps=0.001, n_alphas=100, alphas=alphas, fit_intercept=True,␣
↪precompute='auto',  max_iter=1000, tol=0.0001,
                    copy_X=True, cv=None, verbose=False, n_jobs=None,␣
↪positive=False, random_state=None, selection='cyclic')
lasso_reg = lasso_cv.fit(x_lasso_cv, y_lasso_cv)

# print results
print("MSE path;", lasso_reg.mse_path_)
print("alpha:", lasso_reg.alpha_)
print("dual gap:", lasso_reg.dual_gap_)
print("nb iterations:", lasso_reg.n_iter_)
print("const:", lasso_reg.intercept_)
print("weight vector:", lasso_reg.coef_)
print("nb endog:", ridge_reg.n_features_in_)
print("names endog:", ridge_reg.feature_names_in_)
```

```
MSE path; [[0.30009836 0.3257853  0.32065717 0.73249065 1.13686396]
 [0.30009836 0.3257853  0.32065717 0.73249065 1.13686396]
 [0.15823388 0.16983464 0.16970641 0.40188626 0.64157194]
 [0.1500596  0.1526748  0.14813189 0.35265839 0.459324  ]
 [0.15259126 0.15037583 0.14996131 0.34268317 0.43364512]
 [0.15321938 0.1504507  0.15052753 0.34190275 0.43077539]
 [0.15328601 0.15046161 0.15058783 0.34182953 0.43048205]
 [0.1532926  0.15046274 0.15059395 0.34182226 0.43045293]]
alpha: 1e-06
dual gap: 8.096634971317689e-07
nb iterations: 12
const: 22.99513713791545
```

```
weight vector: [-0.11104887  0.18411291  0.25713132  0.25616882 -0.40910668]
nb endog: 5
names endog: ['diagonal' 'height_left' 'height_right' 'margin_up' 'length']
```

[270]:
```
margin_low_lasso = lasso_reg.predict(x_lasso_cv)
score = lasso_reg.score(x_lasso_cv, y_lasso_cv)
mse = mean_squared_error(y_lasso_cv, margin_low_lasso)
print("R2:{0:.3f}, Mean Squared Error (MSE):{1:.2f}, Mean Squared Error Sqrt␣
↪(RMSE):{2:.2f}"
      .format(score, mse, np.sqrt(mse)))
```

```
R2:0.477, Mean Squared Error (MSE):0.23, Mean Squared Error Sqrt (RMSE):0.48
```

Ici encore, la regression Lasso n'apporte pas d'amelioration significative du R2 par rapport a la regression lineaire multiple.

[273]:
```
billets_trim_reg['margin_low_lasso'] = lasso_reg.predict(x_lasso_cv)
billets_trim_reg['residus_lasso'] = billets_trim_reg['margin_low_lasso'] -␣
↪billets_trim_reg['margin_low']
billets_trim_reg.head()
```

[273]:

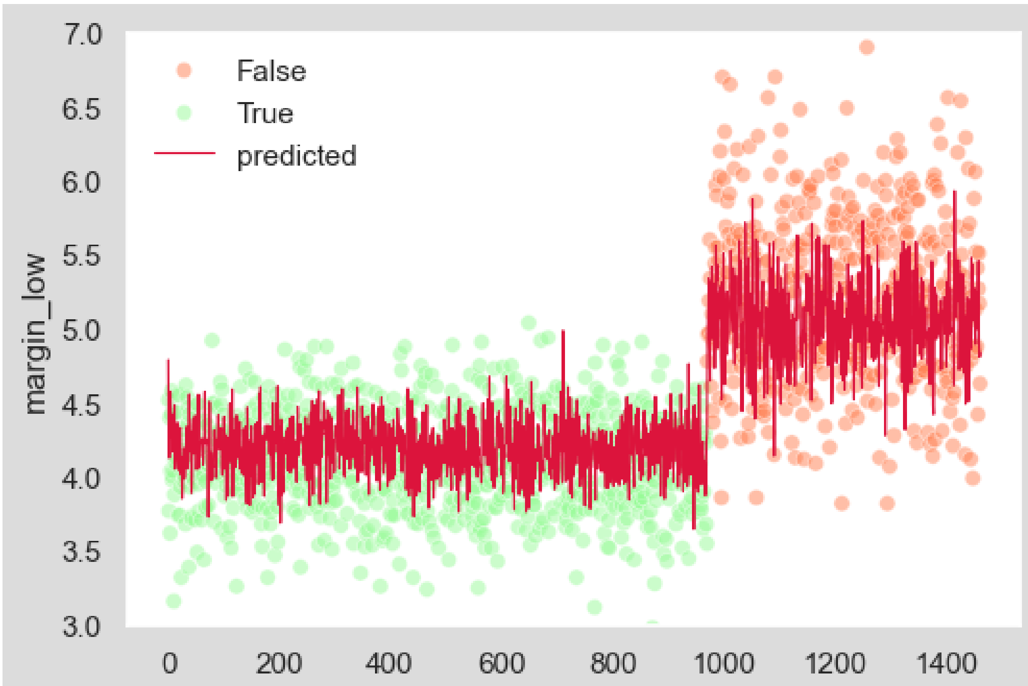
	is_genuine	diagonal	height_left	height_right	margin_low	margin_up
0	True	171.81	104.86	104.95	4.52	2.89
1	True	171.46	103.36	103.66	3.77	2.99
2	True	172.69	104.48	103.50	4.40	2.94
3	True	171.36	103.91	103.94	3.62	3.01
4	True	171.73	104.28	103.46	4.04	3.48

	length	margin_low_reglin	residus_reglin	margin_low_ridge	residus_ridge
0	112.83	4.788676	-0.268676	4.787405	0.267405
1	113.09	4.138908	-0.368908	4.139765	0.369765
2	113.16	4.125933	0.274067	4.127090	-0.272910
3	113.51	4.156580	-0.536580	4.156350	0.536350
4	112.54	4.577425	-0.537425	4.576616	0.536616

	margin_low_lasso	residus_lasso
0	4.788665	0.268665
1	4.138912	0.368912
2	4.125941	-0.274059
3	4.156574	0.536574
4	4.577418	0.537418

[275]:
```
# scatterplot predicted values vs original values
sns.set(rc={'figure.figsize':(6,4),'axes.facecolor':'white', 'figure.facecolor':
↪'gainsboro'})
graph9 = sns.scatterplot(x=range(len(x_lasso_cv)), y=y_lasso_cv,
                         data=billets_trim_reg, alpha=0.5, hue='is_genuine',␣
↪palette=['coral','palegreen'])
```

```
plt.plot(range(len(x_lasso_cv)), margin_low_lasso, lw=0.8, color="crimson",␣
 ↪label="predicted")
plt.legend(title='', frameon=False)
ax.set(xlabel='billet', ylabel='margin_low')
plt.ylim(3, 7)
plt.savefig('margin_low_e_lasso.png')
plt.show()
```



On observe ici encore des residus plus variables sur les faux billets.

2.3.2 - Analyse des residus

[279]:
```
# mean residue
billets_trim_reg['residus_lasso'].mean()
```

[279]: -1.6658657441265507e-15

La moyenne des residus est tres proche de zero.

[282]:
```
# Breusch-Pagan test for homoscedasticity
lm_bp_l, lm_pval_bp_l, f_val_bp_l, f_pval_bp_l =␣
 ↪het_breuschpagan(billets_trim_reg['residus_lasso'], variables_reglin)
print('Lagrange multiplier statistic:', lm_bp_l)
print('p-value Lagrange multiplier statistic:', lm_pval_bp_l)
```

```
Lagrange multiplier statistic: 80.16274157243085
p-value Lagrange multiplier statistic: 7.759053867341048e-16
```

La p-valeur de la statistique de test du multiplicateur de Lagrange est inferieure à 5%, on rejette l'hypothese H0 du test de Breusch-Pagan selon laquelle les variances des residus sont constantes, nous avons suffisamment d'information pour conclure a l'heteroscedasticite des residus.

[287]:
```
# Shapiro test for normality
st.shapiro(billets_trim_reg['residus_lasso'])
```

[287]:
```
ShapiroResult(statistic=0.98578785816959, pvalue=8.536463458272039e-11)
```

La p-value est inferieure a 5%, donc on rejette H0, les residus ne sont pas normaux, mais leur observation, le fait que leur moyenne soit tres proche de zero et que l'echantillon soit de taille suffisante (superieure a 30) permettent de dire que les resultats obtenus par le modele ridge ne sont pas absurdes, meme si les residus ne sont pas normaux. Cependant, ils ne permettent pas d'ameliorer significativment les resultats de la regression lineaire simple.

Le resultat des regressions ridge et lasso ne permettant pas d'ameliorer significativement celui de la regression lineaire multiple non-penalisee, nous poursuivrons l'analyse avec les resultats de celle-ci, bien que les residus soient heteroscedastiques et non-gaussiens.

4 - Creation du fichier .csv complet

[291]:
```
# add column of 1s
billets = sm.add_constant(billets, prepend=True, has_constant='skip')

# fill in missing margin_low values with regression prediction
billets.loc[(billets['margin_low'].isna() == True), 'margin_low'] =␣
↪reg_multi_reglin.predict(billets[['const','diagonal',                    ␣
↪                        'height_left',                                   ␣
↪                        'height_right',                                  ␣
↪                        'margin_up', 'length']])
# remove column of 1s
billets_final = billets.drop('const', axis=1)
billets_final.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 1500 entries, 0 to 1499
Data columns (total 7 columns):
 #   Column        Non-Null Count  Dtype
---  ------        --------------  -----
 0   is_genuine    1500 non-null   bool
 1   diagonal      1500 non-null   float64
 2   height_left   1500 non-null   float64
 3   height_right  1500 non-null   float64
 4   margin_low    1500 non-null   float64
 5   margin_up     1500 non-null   float64
 6   length        1500 non-null   float64
```

```
dtypes: bool(1), float64(6)
memory usage: 71.9 KB
```

[293]:
```python
# display basic statistics
billets_final.describe(include='all')
```

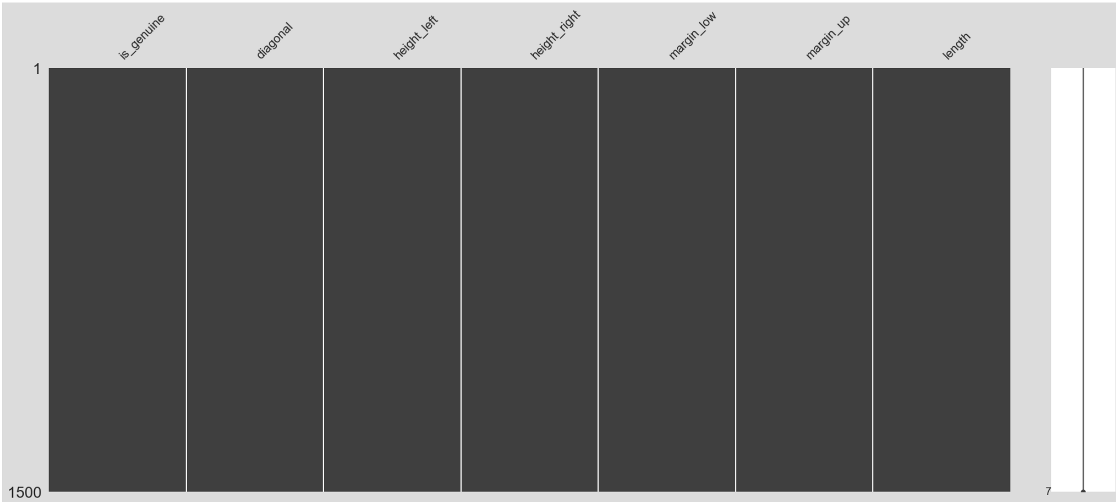
[293]:
```
       is_genuine     diagonal  height_left  height_right   margin_low  \
count        1500  1500.000000  1500.000000   1500.000000  1500.000000
unique          2          NaN          NaN           NaN          NaN
top          True          NaN          NaN           NaN          NaN
freq         1000          NaN          NaN           NaN          NaN
mean          NaN   171.958440   104.029533    103.920307     4.483475
std           NaN     0.305195     0.299462      0.325627     0.659632
min           NaN   171.040000   103.140000    102.820000     2.980000
25%           NaN   171.750000   103.820000    103.710000     4.020000
50%           NaN   171.960000   104.040000    103.920000     4.310000
75%           NaN   172.170000   104.230000    104.150000     4.870000
max           NaN   173.010000   104.880000    104.950000     6.900000

          margin_up      length
count   1500.000000  1500.00000
unique          NaN         NaN
top             NaN         NaN
freq            NaN         NaN
mean       3.151473   112.67850
std        0.231813     0.87273
min        2.270000   109.49000
25%        2.990000   112.03000
50%        3.140000   112.96000
75%        3.310000   113.34000
max        3.910000   114.44000
```

[295]:
```python
# re-check for duplicates
dup_final = billets_final.duplicated().value_counts()
dup_final
```

[295]:
```
False    1500
Name: count, dtype: int64
```

[297]:
```python
# re-check for missing values
msno.matrix(billets_final)
```

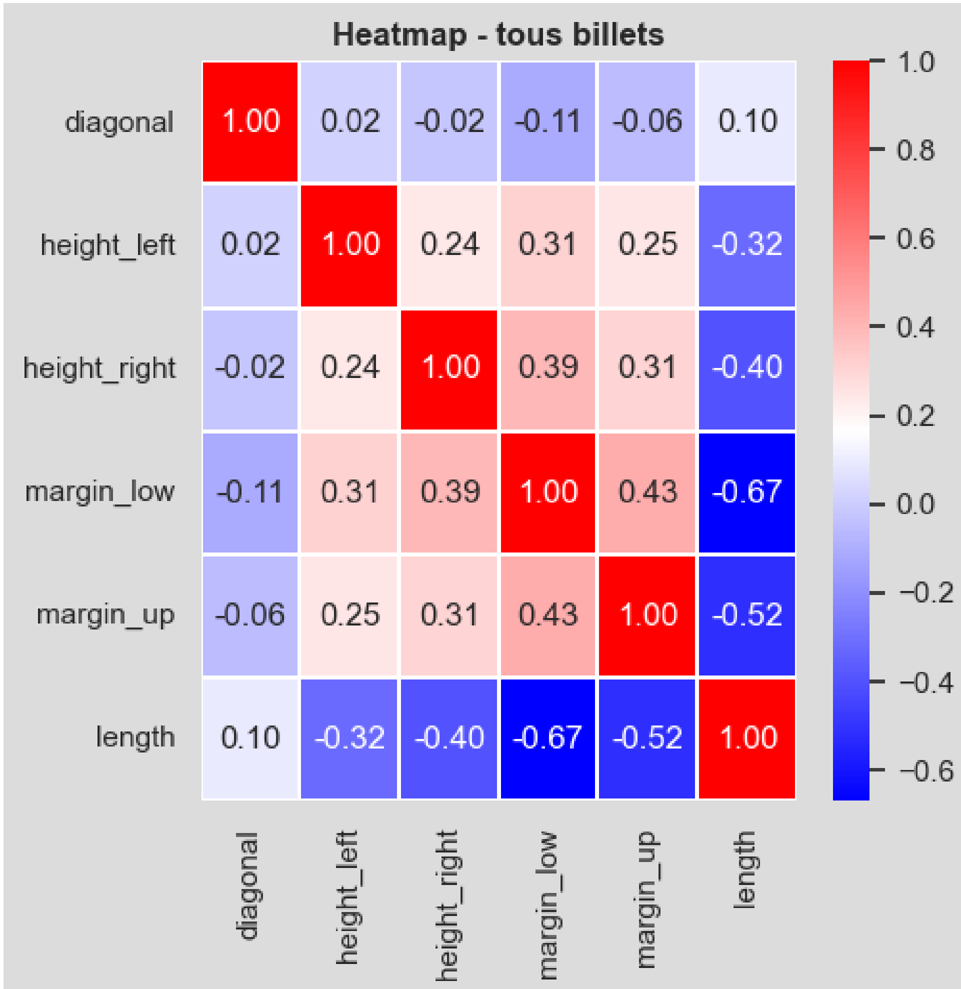
[297]:
```
<Axes: >
```



[299]:
```
billets_final.to_csv('billets_final.csv', index=False, encoding='latin_1')
```

3 - Etude des correlations entre les variables

[305]:
```
# plot heatmap to check correlations between variables
fig, ax = plt.subplots(figsize=(5, 5))
heatmap = sns.heatmap(billets_final[['diagonal', 'height_left', 'height_right',␣
↪'margin_low',
                      'margin_up', 'length']].corr(), annot=True, cmap="bwr",␣
↪fmt="0.2f", linewidth=0.05)
plt.title("Heatmap - tous billets", fontweight='bold', fontsize=12)
plt.savefig("heatmap_billets_final.png", bbox_inches='tight', pad_inches=0.5)
plt.show()
```



[307]:
```python
# calculate Pearson Coefficient for length and margin_low
st.pearsonr(billets_final['length'], billets_final['margin_low'])
```

[307]: PearsonRResult(statistic=-0.6709660250173901, pvalue=8.000603408726061e-197)

[309]:
```python
# calculate Pearson Coefficient for length and margin_low
st.pearsonr(billets_final['length'], billets_final['margin_up'])
```

[309]: PearsonRResult(statistic=-0.5205751349009689, pvalue=6.031895221731607e-105)

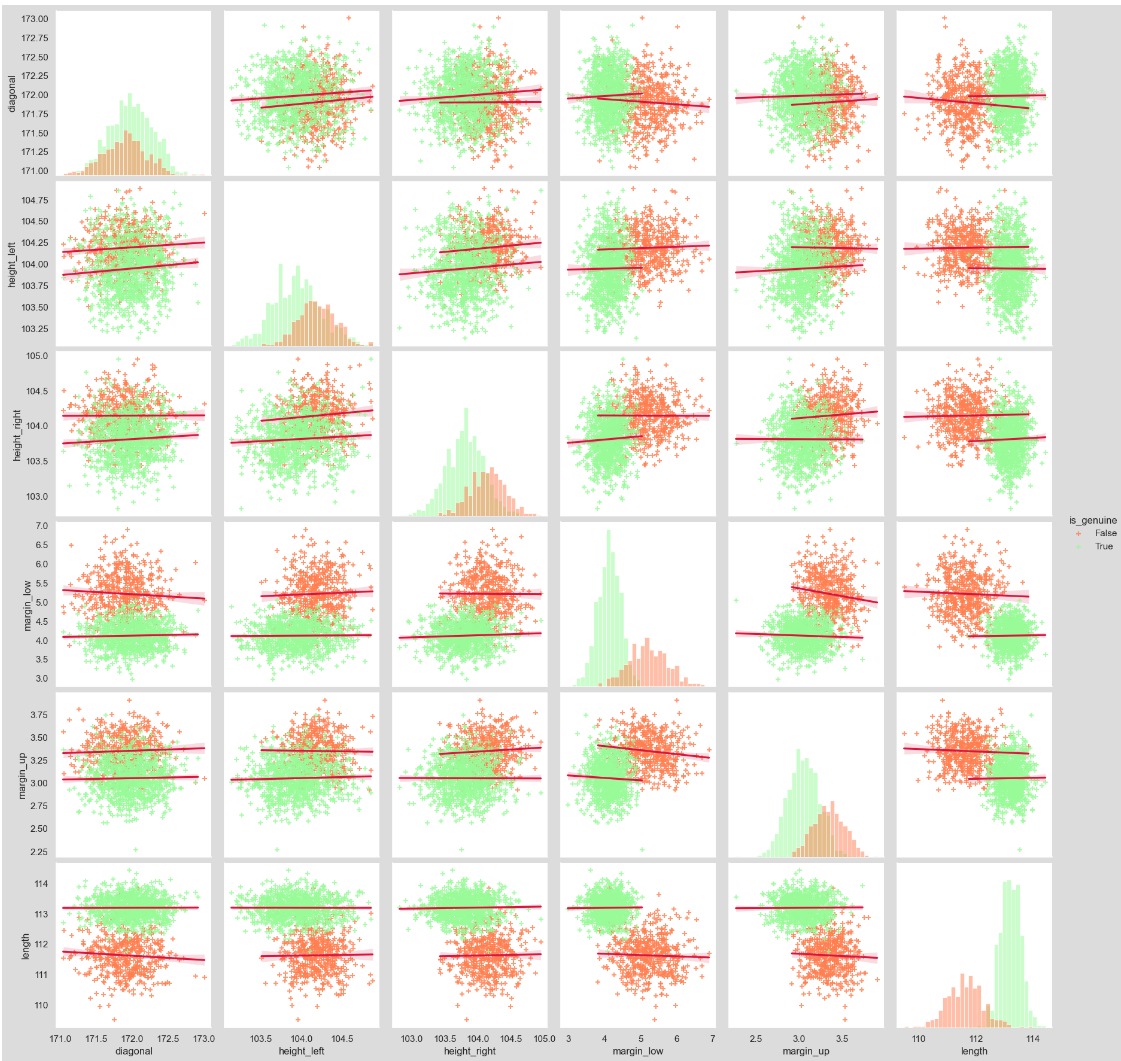
[311]:
```python
# calculate Pearson Coefficient for length and margin_low
st.pearsonr(billets_final['margin_low'], billets_final['margin_up'])
```

[311]: PearsonRResult(statistic=0.4342372129032673, pvalue=5.098188704462226e-70)

La longueur des billets apparait significativement correlee (p-value de la statistique de Pearson proche de 0) a un certain nombre d'autres dimensions, et en particulier margin_low.

[314]:

```
# visualise data correlations with a pairplot
billets_pairs = sns.pairplot(billets_final, kind='reg', height=3,␣
 ↪diag_kind='hist', markers="+", hue='is_genuine',
                             palette=['coral','palegreen'],␣
 ↪plot_kws={'line_kws':{'color':'crimson'}},
                             diag_kws={'bins':33})
plt.legend('', frameon=False)
plt.savefig("billets_final_pairplot.png")
plt.show()
```

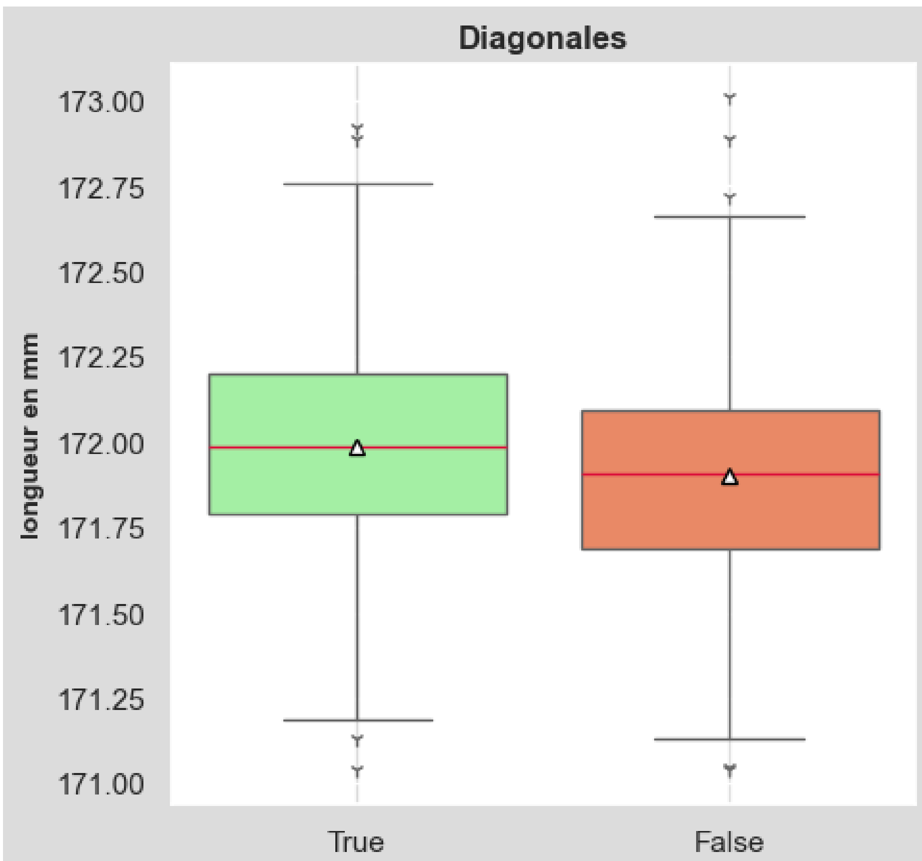


5 - Representations graphiques de la distribution des donnees

[317]:

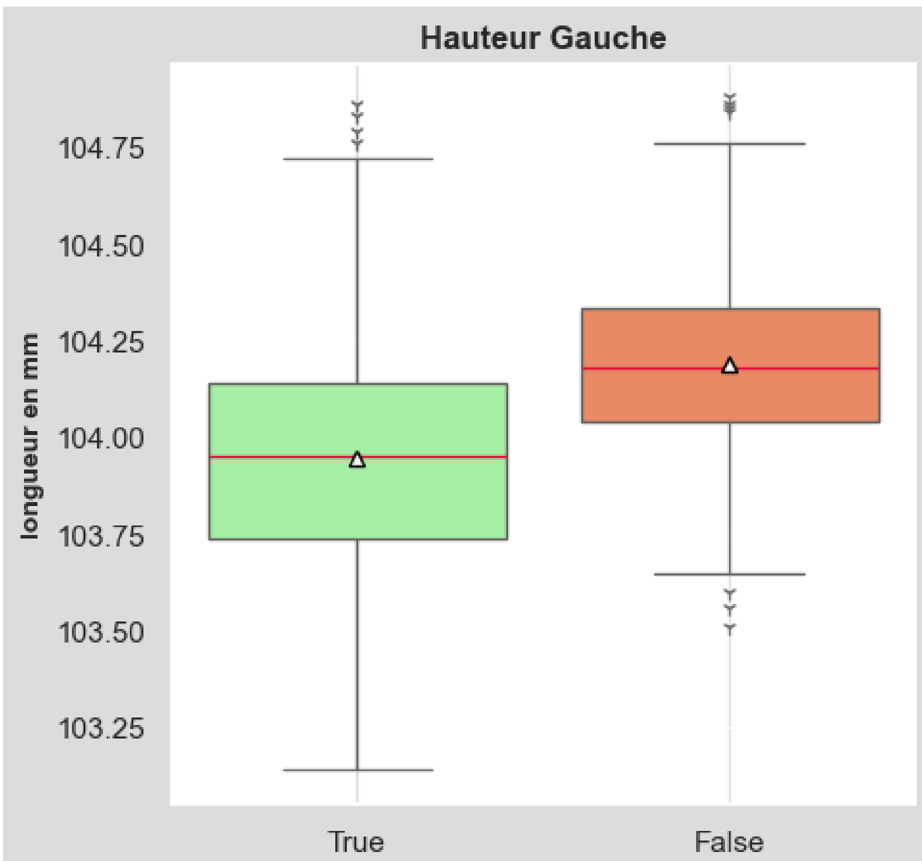
```
# analyse diagonals with a box plot
sns.set(rc={'figure.figsize':(5, 5),'axes.facecolor':'white', 'figure.
 ↪facecolor':'gainsboro'})
```

```
graph1_diag = sns.boxplot(data=billets_final, y='diagonal', x='is_genuine',␣
 ↪hue='is_genuine',
                         flierprops={"marker":"1"}, medianprops={"color":'crimson'},
                         meanprops={"markerfacecolor":"white", "markeredgecolor":
 ↪'black'},
                         orient="v", palette=['coral', 'palegreen'],␣
 ↪showmeans=True, dodge=False, order=[True, False])
plt.xlabel('')
plt.ylabel("longueur en mm", fontweight='bold', fontsize=10)
plt.title("Diagonales", fontweight='bold', fontsize=12)
plt.legend('', frameon=False)
plt.grid(axis='x', color='gainsboro')
plt.savefig("boxplot_diagonal.png", bbox_inches='tight')
plt.show()
```



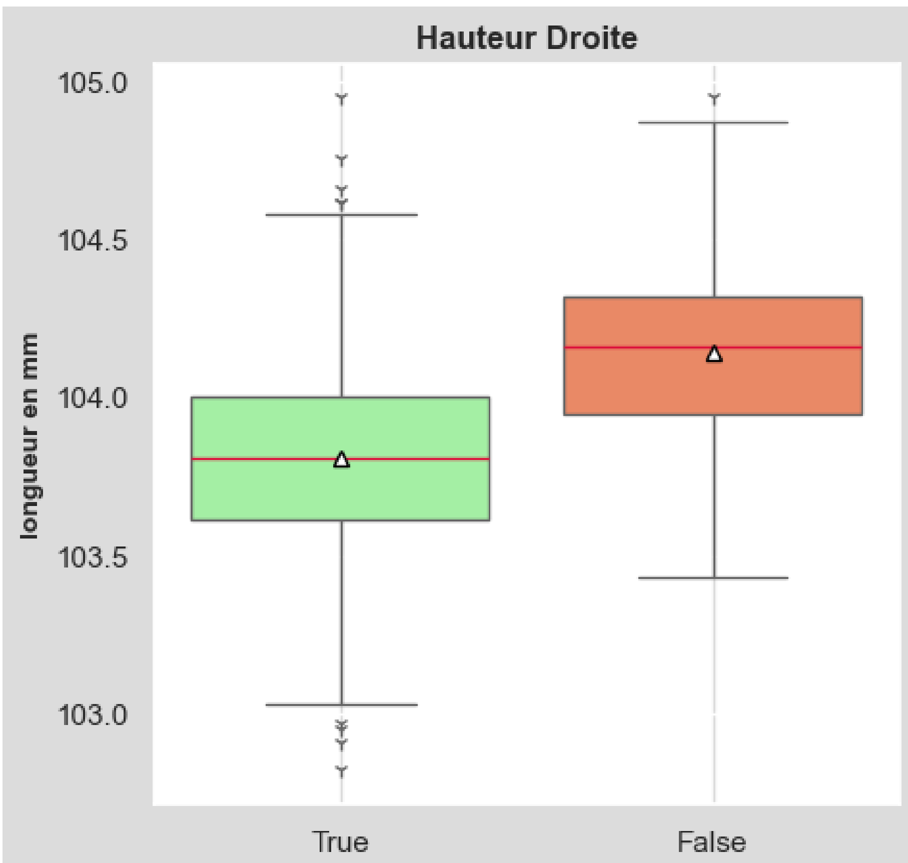
[319]:
```
# analyse height_left with a box plot
sns.set(rc={'figure.figsize':(5, 5),'axes.facecolor':'white', 'figure.
 ↪facecolor':'gainsboro'})
```

```
graph1_hl = sns.boxplot(data=billets_final, y='height_left', x='is_genuine',␣
 ↪hue='is_genuine',
                     flierprops={"marker":"1"}, medianprops={"color":'crimson'},
                     meanprops={"markerfacecolor":"white", "markeredgecolor":
 ↪'black'},
                     orient="v", palette=['coral', 'palegreen'],␣
 ↪showmeans=True, dodge=False, order=[True, False])
plt.xlabel('')
plt.ylabel("longueur en mm", fontweight='bold', fontsize=10)
plt.title("Hauteur Gauche", fontweight='bold', fontsize=12)
plt.legend('', frameon=False)
plt.grid(axis='x', color='gainsboro')
plt.savefig("boxplot_height_left.png", bbox_inches='tight')
plt.show()
```



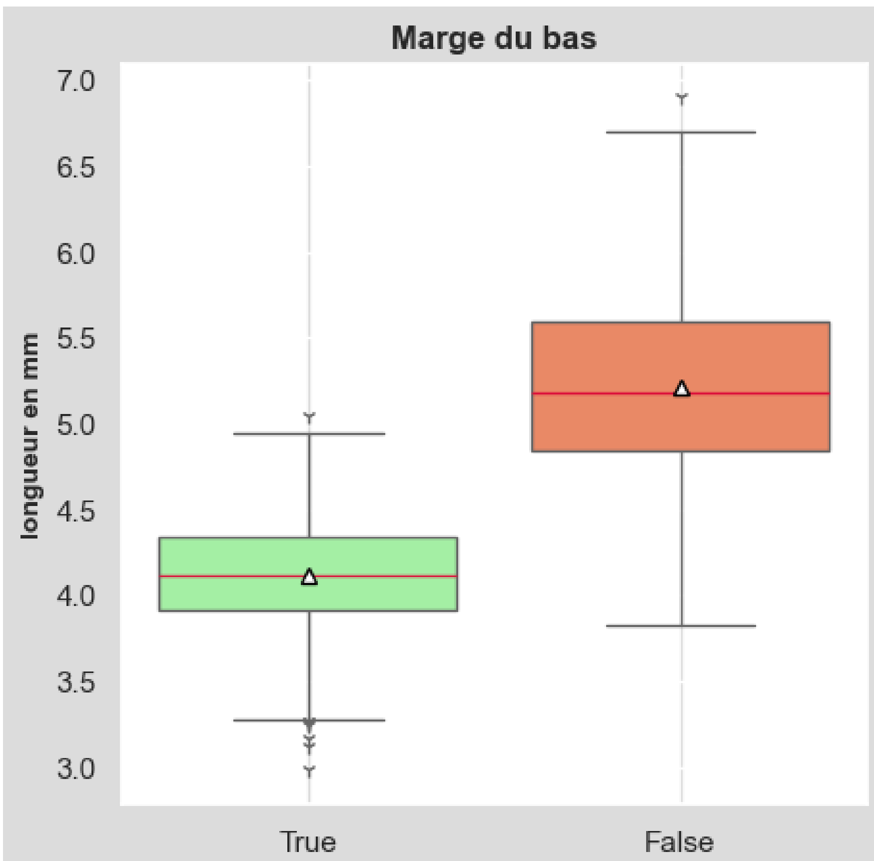
[323]:
```
# analyse height_right with a box plot
sns.set(rc={'figure.figsize':(5, 5),'axes.facecolor':'white', 'figure.
 ↪facecolor':'gainsboro'})
```

```
graph1_hr = sns.boxplot(data=billets_final, y='height_right', x='is_genuine',␣
 ↪hue='is_genuine',
                        flierprops={"marker": "1"}, medianprops={"color" :␣
 ↪'crimson'},
                        meanprops={"markerfacecolor":"white", "markeredgecolor":
 ↪'black'},
                        orient="v", palette=['coral', 'palegreen'],␣
 ↪showmeans=True, dodge=False, order=[True, False])
plt.xlabel('')
plt.ylabel("longueur en mm", fontweight='bold', fontsize=10)
plt.title("Hauteur Droite", fontweight='bold', fontsize=12)
plt.legend('', frameon=False)
plt.grid(axis='x', color='gainsboro')
plt.savefig("boxplot_height_right.png", bbox_inches='tight')
plt.show()
```



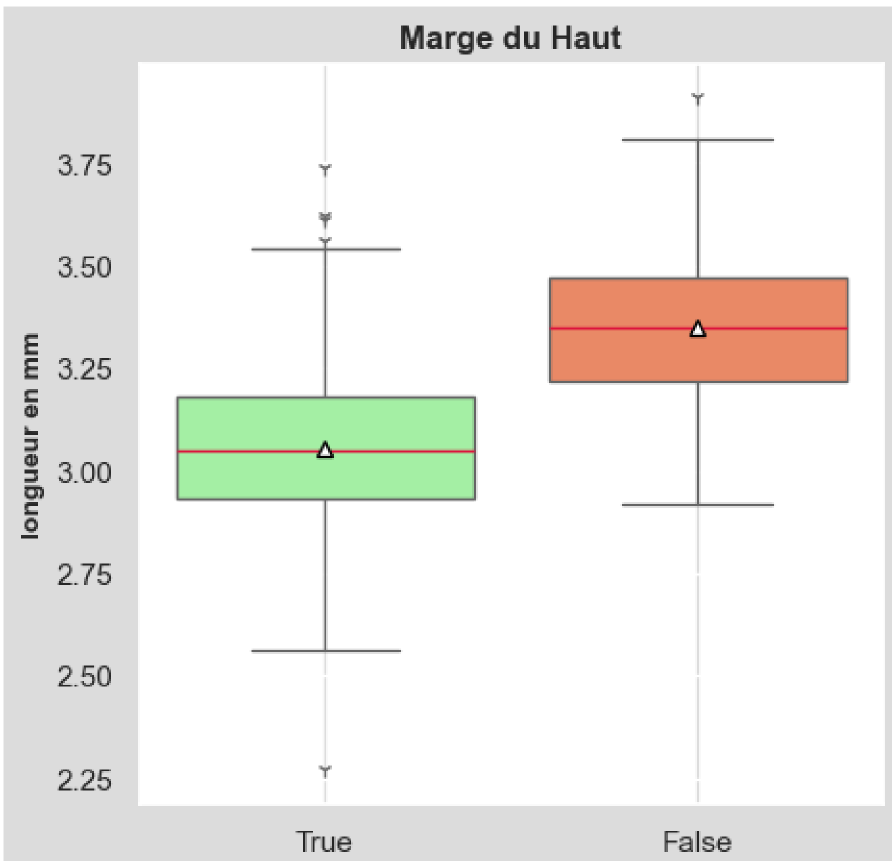
[325]:
```
# analyse margin_low with a box plot
sns.set(rc={'figure.figsize':(5, 5),'axes.facecolor':'white', 'figure.
 ↪facecolor':'gainsboro'})
```

```
graph1_ml = sns.boxplot(data=billets_final, y='margin_low', x='is_genuine',␣
 ↪hue='is_genuine',
                     flierprops={"marker":"1"}, medianprops={"color":'crimson'},
                     meanprops={"markerfacecolor":"white", "markeredgecolor":
 ↪'black'},
                     orient="v", palette=['coral', 'palegreen'],␣
 ↪showmeans=True, dodge=False, order=[True, False])
plt.xlabel('')
plt.ylabel("longueur en mm", fontweight='bold', fontsize=10)
plt.title("Marge du bas", fontweight='bold', fontsize=12)
plt.legend('', frameon=False)
plt.grid(axis='x', color='gainsboro')
plt.savefig("boxplot_margin_low.png", bbox_inches='tight')
plt.show()
```



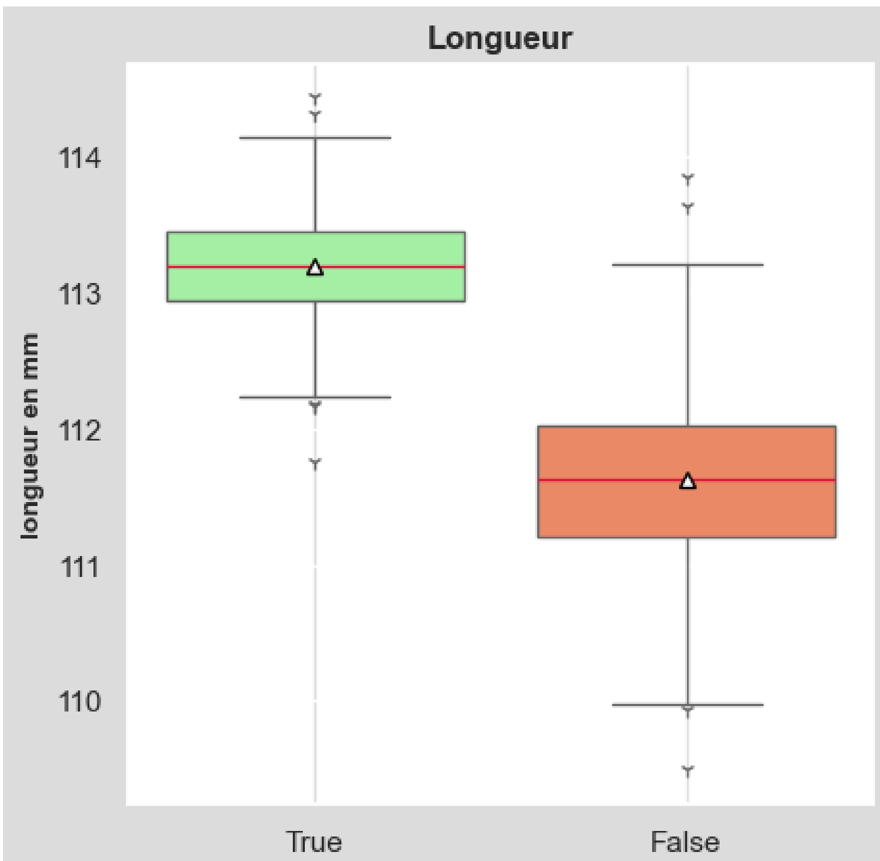
[328]:
```
# analyse margin_up with a box plot
sns.set(rc={'figure.figsize':(5, 5),'axes.facecolor':'white', 'figure.
 ↪facecolor':'gainsboro'})
```

```
graph1_mu = sns.boxplot(data=billets_final, y='margin_up', x='is_genuine',␣
↪hue='is_genuine',
                        flierprops={"marker":"1"}, medianprops={"color":'crimson'},
                        meanprops={"markerfacecolor":"white", "markeredgecolor":
↪'black'},
                        orient="v", palette=['coral', 'palegreen'],␣
↪showmeans=True, dodge=False, order=[True, False])
plt.xlabel('')
plt.ylabel("longueur en mm", fontweight='bold', fontsize=10)
plt.title("Marge du Haut", fontweight='bold', fontsize=12)
plt.legend('', frameon=False)
plt.grid(axis='x', color='gainsboro')
plt.savefig("boxplot_margin_up.png", bbox_inches='tight')
plt.show()
```



[330]:
```
# analyse length with a box plot
sns.set(rc={'figure.figsize':(5, 5),'axes.facecolor':'white', 'figure.
↪facecolor':'gainsboro'})
```

```
graph1_long = sns.boxplot(data=billets_final, y='length', x='is_genuine',␣
 ↪hue='is_genuine',
                         flierprops={"marker":"1"}, medianprops={"color":'crimson'},
                         meanprops={"markerfacecolor":"white", "markeredgecolor":
 ↪'black'},
                         orient="v", palette=['coral', 'palegreen'],␣
 ↪showmeans=True, dodge=False, order=[True, False])
plt.xlabel('')
plt.ylabel("longueur en mm", fontweight='bold', fontsize=10)
plt.title("Longueur", fontweight='bold', fontsize=12)
plt.legend('', frameon=False)
plt.grid(axis='x', color='gainsboro')
plt.savefig("boxplot_length.png", bbox_inches='tight')
plt.show()
```



Les caracteristiques des vrais et faux billets sont nettement differentes sauf pour la longueur de la diagonale, ou la difference est moins marquee. Les outliers etant relativement proches des moustaches, nous les considererons comme des valeurs atypiques (et non aberrantes) et nous les conserverons pour la suite de l'analyse.

```
[ ]:
```