Upper Bound for Number Of Divisors

forthright48 on July 14, 2015

Given a number N, we can find its number of divisors using prime factorization. But when the number gets too big, it gets difficult to factorize, thus harder to find the number of divisors. So given a number N, can we estimate at most how many divisors N can have? That is, we want to find the upper bound for NOD.

NOD Upper Bounds

We proceed from loose to tighter bounds.

$$NOD(N) \leq N$$

A number which is greater than N cannot divide N. For N>2, we can further say NOD(N)< N, but the improvement is negligible.

Can we make the limit tighter?

$$NOD(N) \leq rac{N}{2} + 1$$

A divisor D, such that $\frac{N}{2} < D < N$ cannot divide N, since the result would be less than 2 and greater than 1, a fraction. So possible divisors are $D \leq \frac{N}{2}$ and N.

Better than before by half, but its possible to make it tighter.

$$NOD(N) \leq 2 imes \sqrt{N}$$

If we write $N=A\times B$, where $A\leq B$, then $A\leq \sqrt{N}$. Each of A,B forms a divisor pair for N. A can take any value from $1\to \sqrt{N}$, so it is possible to form only \sqrt{N} pairs of divisors. Thus, there can be at most $2\times \sqrt{N}$ divisors for N.

$$NOD(N) pprox 2 imes \sqrt[3]{N}$$

I didn't know about this approximation until I read a <u>blog post</u> on Codeforces. From there I found out that in practice we can safely use $\sqrt[3]{N}$ as an upper bound, though I failed to understand the proof. Apparently, this approximation has been tested for $N \leq 10^{18}$, which is large enough to be used in programming contests.

Using Highly Composite Numbers

Sometimes we need upper bound for small values of N. For example, in a problem you might need to find an upper bound of NOD for $N \leq 10^9$. For such small values of N we could use NOD of largest Highly Composite Number (HCN), which is less than or equal to N, as an upper bound.

Read more about HCN here.

For programming contest, we could memorize values of HCN that comes frequently. Mainly 1344 for $N \leq 10^9$ and 103,680 for $N \leq 10^{18}$.

Application

The upper bound for NOD is needed for complexity analysis.

Reference

- 1. Codeforces Upper bound for number of divisors: http://codeforces.com/blog/entry/14463
- 2. forthright48 Highly Composite Numbers: https://forthright48.com/2015/07/highly-composite-numbers.html

Next:

Add Anycomment to your site

Post Views: 332

Category: CPPS, Number Theory

Previous:

Archives Categories Recent Comments

May 2019 (1) CPPS (45) My Shopee Interview – Shadman Protik on My Interview Experience with Shapee /

May 2019 (1) My Interview Experience with Shopee / Combinatorics (4) April 2019 (1) Garena / Sea Group Data Structure (1) March 2019 (1) Istiad Hossain Akib on SPOJ LCMSUM - Number Theory (36) LCM Sum December 2018 (2) Meta (1) Rifat Chowdhury on MyStory#02 -November 2018 (4) Deciding Where to Study CS Misc (4) **September 2018 (2)** Salman Farsi on Leading Digits of February 2018 (1) Factorial January 2018 (1) Learning Notes on Multiplicative Functions, Dirichlet Convolution, Mobius November 2017 (2) Inversion, etc – RobeZH's Blog on SPOJ **September 2015 (7)** LCMSUM – LCM Sum August 2015 (13)

July 2015 (15)