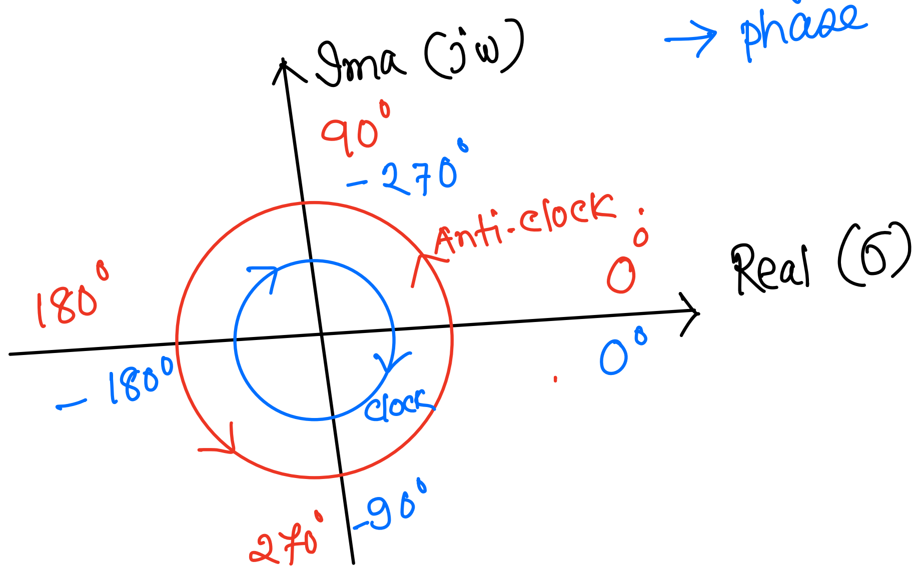
Lecture ~ 13
Polar Plot
Wise : 10

Polar Plot:

→ Used for frequency response characteristics of system
→ Plot of Mag. and phase by varying freq from 0 to ∞
→ Plotted on a real-imaginary plane with a circular pattern.

→ Magnitude = Distance w.r.to Center
→ phase = Angle w.r.to Real axis.



Procedure:

→ Plot of Mag. and phase by varying freq from $0 \to \infty$
→ Plotted using Open Loop Transfer Function G(S)

Step: 1 Determine Open loop Transfer function G(S)

Step: 2 Put $S = j\omega$,
$G(j\omega) = |G(j\omega)| \angle G(j\omega)$.

Step: 3 Find Mag $|G(j\omega)|$ and phase $\angle G(j\omega)$ at $\omega = 0$

Step: 4   Seperate real & imaginary part of $G(j\omega)$

$$G(j\omega) = \text{Real}[G(j\omega)] + j\,\text{Imag}[G(j\omega)].$$

Step: 5   If $\text{Real}[G(j\omega)] = 0$,

$\Rightarrow$ get Intersection to Imaginary axis

If $\text{Imag}[G(j\omega)] = 0$,

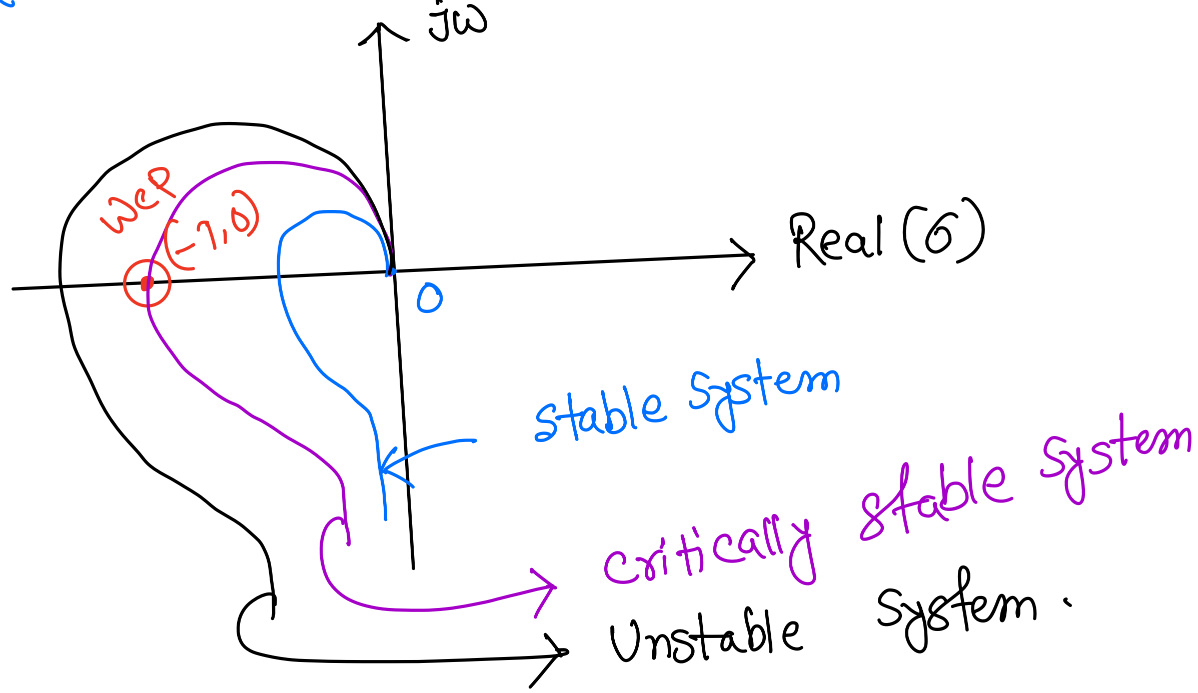
$\Rightarrow$ " " " real axis

Advantages of Polar Plot

- Single Plot plots Frequency Response Characteristics {Magnitude and Phase}.
- Graphical Study of Stability is Easy with Polar Plot {Compared to Root Locus and Bode Plot}.
- Easy to determine Gain Cross Over Frequency ω_{GC} and Phase Cross Over Frequency ω_{PC} using Polar Plot.
- Polar Plot can be plotted from the Open Loop Transfer Function {Other techniques need Closed Loop Transfer Function}.

Critical Point and stability of Polar Plot:

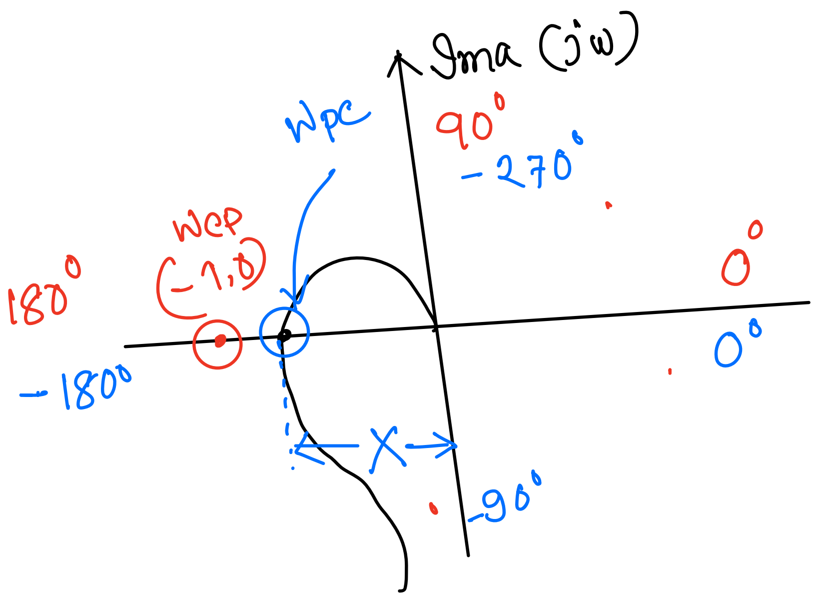
→ (−1, 0) point is Critical Point in Polar Plot.



Phase Cross-over Frequency and Gain Margin:

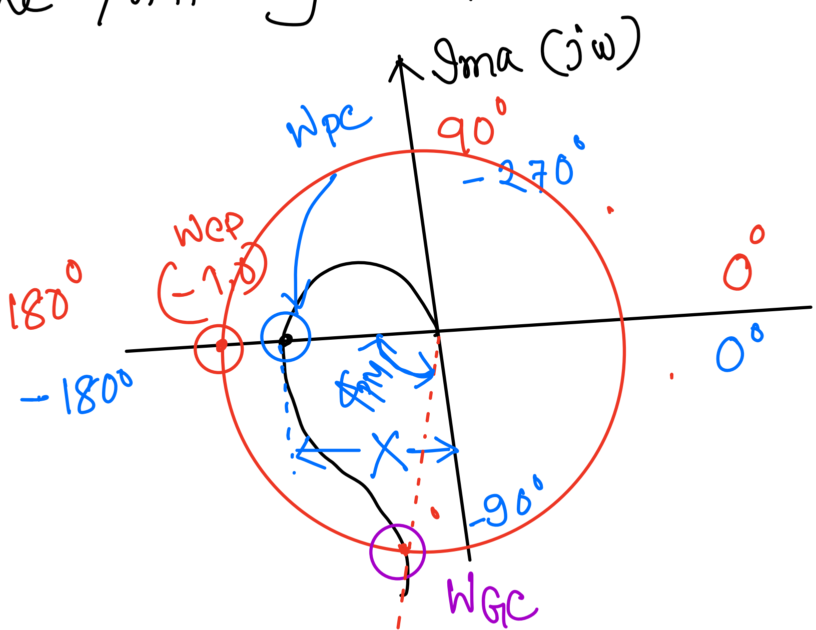
ω_{pc}: Freq at which phase is crossing $-180°$ Angle.
Let's. X = Mag. at ω_{pc} from origin.

Gain Margin $= 20 \log\left(\frac{1}{X}\right)$.



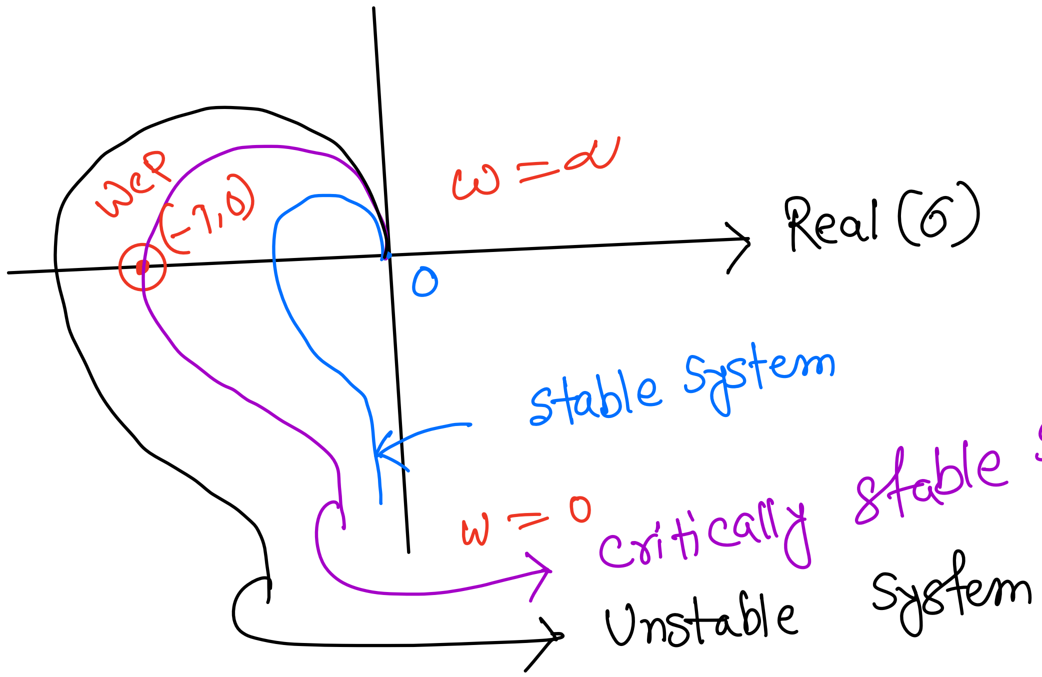
Gain Cross-over Frequency and Phase Margin:

ω_{pc}: Freq at which gain is crossing Unit Magnitude
Phase Margin ϕ_{PM} = Angle measure from $-180°$ in anticlockwise direction to the point of WGC.



Stability Analysis:

→ Polar Plot can be draw for all systems, but stability analysis is only applicable for minimum phase system.



Stable : If (-1,0) is not enclosed by polar plot
Unstable : If (-1,0) is enclosed by polar plot
Critically " : " " is crossed by " "

$\omega_{pc} > \omega_{gc}$ & $GM = +ve$ → stable
$\omega_{pc} < \omega_{gc}$ & $GM = -ve$ → unstable
$\omega_{pc} = \omega_{gc}$ & $GM = 0$ → Critically stable

Polar Plot For Type "0" System

→ Zero number of pole at origin ⇒ Type 0

Exp: $G(S) = \dfrac{K}{(1+ST_1)(1+ST_2)}$

Step 1: $G(j\omega) = \dfrac{K}{(1+j\omega T_1)(1+j\omega T_2)}$

Step 2: Write $G(j\omega)$ in polar form

$$G(j\omega) = |G(j\omega)| \angle G(j\omega)$$

$$|G(j\omega)| = \frac{K}{\sqrt{1+\omega^2 T_1^2} \cdot \sqrt{1+\omega^2 T_2^2}}$$

$$\text{phase} = \angle G(j\omega) = -\tan^{-1}\left(\frac{\omega T_1}{1}\right) - \tan^{-1}\left(\frac{\omega T_2}{1}\right)$$

Step 3 Find Mag. & phase at $\omega = 0, \omega = \infty$

→ At $\omega = 0$, Mag $= K$, phase $= 0$

→ At $\omega = \infty$ Mag $= 0$ " $= -180^0$

Step 4: Seperate real & imaginary part of $G(j\omega)$ by multiplying complex conjugate of denominator.

$$\Rightarrow G(j\omega) = \frac{K}{(1+j\omega T_1)(1+j\omega T_2)} \times \frac{(1-j\omega T_1)(1-j\omega T_2)}{(1-j\omega T_1)(1-j\omega T_2)}$$

$$= \frac{K(1-\omega^2 T_1 T_2) - Kj(\omega T_1 + \omega T_2)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}$$

$$= \underbrace{\frac{K(1-\omega^2 T_1 T_2)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}}_{\text{Real}} - j \underbrace{\frac{K(\omega T_1 + \omega T_2)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}}_{\text{img}}$$

Step 5: Intersection to Real Axis, $\text{Img}[G(j\omega)] = 0$.

$\Rightarrow \omega = 0$

Step : Intersection to Img Axis, $\text{Real}[G(j\omega)] = 0$.

$\Rightarrow 1 - \omega^2 T_1 T_2 = 0$

$\therefore \omega = \frac{1}{\sqrt{T_1 T_2}}$

$\therefore$ Magnitude, $|G(j\omega)| = \frac{K}{\sqrt{1+\omega^2 T_1^2} \cdot \sqrt{1+\omega^2 T_2^2}}$

Put value of ω

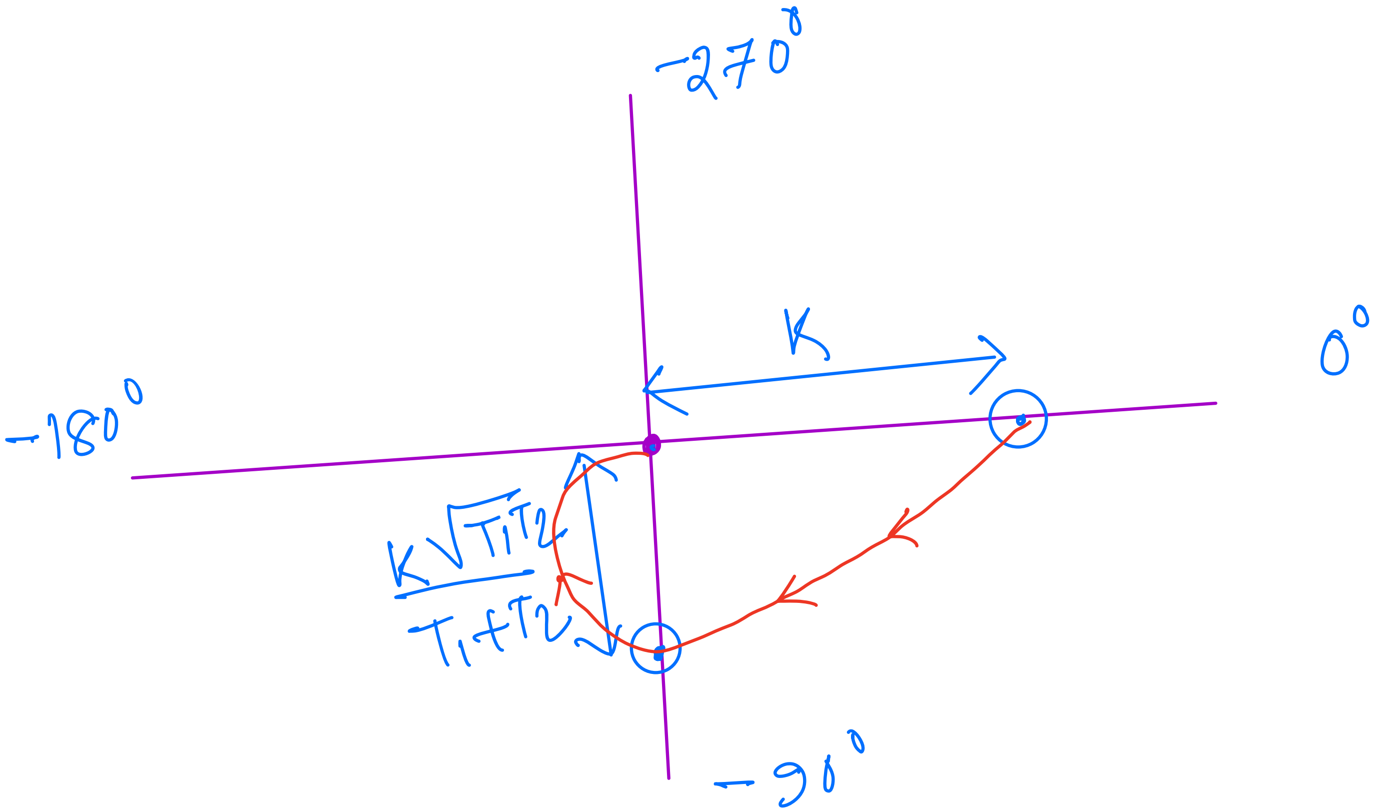
$$= \frac{K\sqrt{T_1 T_2}}{T_1 + T_2}$$

$$\text{phase} = \angle G(j\omega) = -\tan^{-1}\left(\frac{\omega T_1}{1}\right) - \tan^{-1}\left(\frac{\omega T_2}{1}\right)$$

Put value of w

$$\therefore \angle G(j\omega) = -\tan^{-1}(0) = -90^\circ$$

$$= -\tan^{-1}\tan^{-1}\left(\frac{K\sqrt{T_1T_2}}{T_1+T_2} \times T_1\right.$$



Type 1 system

one pole at origin.

$$G(S) = \frac{K}{S(1+ST_1)(1+ST_2)}$$

Step 1: $G(j\omega) = \frac{K}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$

Step 2: Write $G(j\omega)$ in polar form

$$G(j\omega) = |G(j\omega)| \angle G(j\omega)$$

$$|G(j\omega)| = \frac{K}{\omega\sqrt{1+\omega^2 T_1^2} \cdot \sqrt{1+\omega^2 T_2^2}}$$

$$\text{phase} = \angle G(j\omega) = -90^\circ - \tan^{-1}\left(\frac{\omega T_1}{1}\right) - \tan^{-1}\left(\frac{\omega T_2}{1}\right)$$

Step 3 Find Mag. & phase at $\omega = 0, \omega = \infty$

→ At $\omega = 0$, Mag $= \infty$, phase $= -90^\circ$

→ At $\omega = \infty$ Mag $= 0$ " $= -270^\circ$

Step 4: Seperate real & imaginary part of $G(j\omega)$

$$\Rightarrow G(j\omega) = \frac{K}{j\omega(1+j\omega T_1)(1+j\omega T_2)} \times \frac{(1-j\omega T_1)(1-j\omega T_2)}{(1-j\omega T_1)(1-j\omega T_2)}$$

$$= \frac{-j\,K(1-\omega^2 T_1T_2) - Kj(\omega T_1 + \omega T_2)}{\omega(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$$

$$= \frac{-K\cancel{\omega}(T_1+T_2)}{\cancel{\omega}(1+\omega^2T_1^2)(1+\omega^2T_2^2)} - j\,\frac{K(1-\omega^2 T_1 \cdot T_2)}{\omega(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$$

$$= \underbrace{\frac{-K(T_1+T_2)}{(1+\omega^2T_1^2)(1+\omega^2T_2^2)}}_{\text{Real}} - j\,\underbrace{\frac{K(1-\omega^2 T_1T_2)}{\omega(1+\omega^2T_1^2)(1+\omega^2T_2^2)}}_{\text{Img.}}$$

Step 5: Intersection to Real Axis,

$\mathrm{Img}[G(j\omega)] = 0$.

$\Rightarrow 1-\omega^2T_1T_2 = 0$

$\therefore \omega = \dfrac{1}{\sqrt{T_1T_2}}$

$\therefore$ Magnitude, $|G(j\omega)| = \dfrac{K}{\omega\sqrt{1+\omega^2T_1^2}\cdot\sqrt{1+\omega^2T_2^2}}$

Put value of ω $= \dfrac{K\,T_1T_2}{T_1+T_2}$

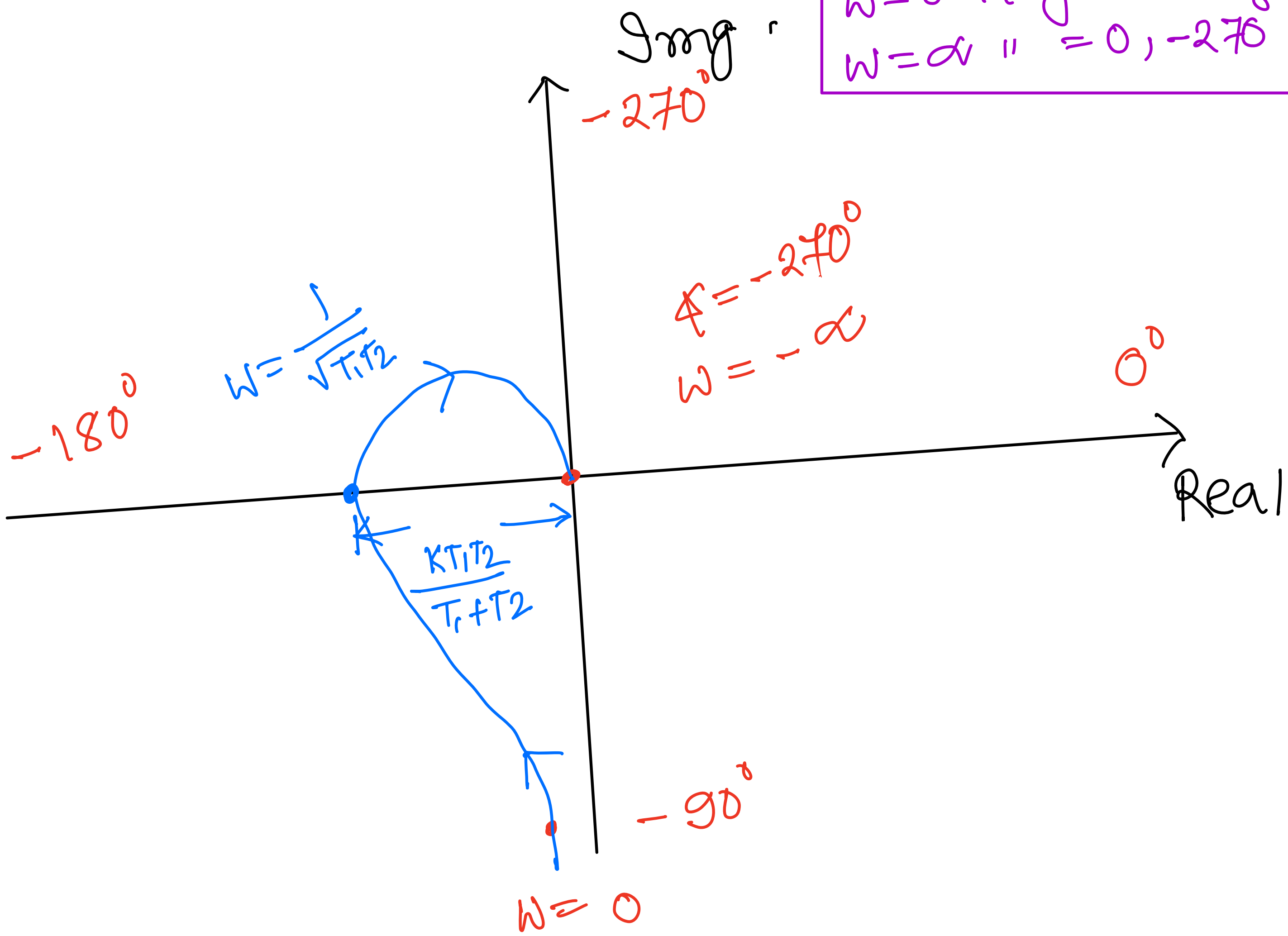
phase $= \angle G(j\omega) = -90^\circ - \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$

Put value of ω

$\angle G(j\omega) = -180^\circ$

$\therefore \angle G(j\omega)$

$w = 0$, Mag $= \infty, -90^\circ$
$w = \infty$ " $= 0, -270^\circ$



Examp: Sketch polar plot for given open loop T.F.

$$G(S) = \frac{S^3}{(S+1)(S+2)}$$

step 1: Put $s = j\omega$

$$G(j\omega) = \frac{(j\omega)^3}{(1+j\omega)(2+j\omega)}$$

step 2: $G(j\omega) = |G(j\omega)| \angle G(j\omega)$

$$= \frac{\omega^{\,}}{\sqrt{1+\omega^2}\sqrt{4+\omega^2}} \angle 270^0 - \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega}{2}\right)$$

Step 3:

At $w=0$

$|G(j\omega)| = 0$

$\angle G(j\omega) = 270^0$

At $w = \infty$

$|G(j\omega)| = \infty$

$\angle G(j\omega) = 270^0 - 90^0 - 90^0$

$= 90^0$

Step 4: Seperate real & imaginary.

$$G(j\omega) = \frac{(j\omega)^3}{(1+j\omega)(2+j\omega)} \times \frac{(1-j\omega)(2-j\omega)}{(1-j\omega)(2-j\omega)}$$

$$= \underbrace{\frac{-3\omega^4}{(1+\omega^2)(4+\omega^2)}}_{\text{Real}} + \underbrace{\frac{j\,\omega^3(\omega^2-2)}{(1+\omega^2)(4+\omega^2)}}_{\text{img}}$$

Step 5: Intersection to real axis at img $(G(j\omega))=$

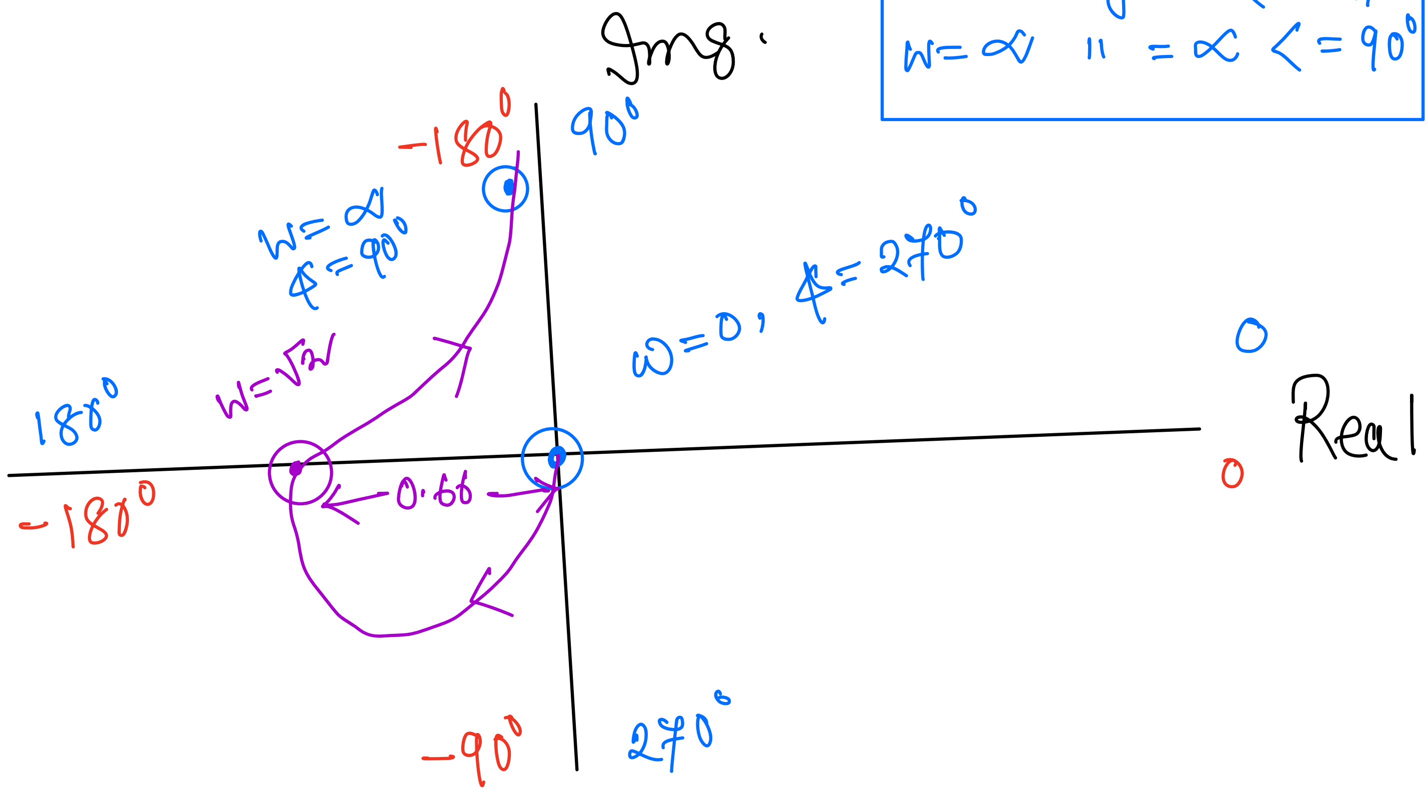
$w=0$, $w=\sqrt{2}$,

At, $w=\sqrt{2}$ $|G(j\omega)| = 0.66 \angle 180^0$

Step 6: " " img " " real $(G(j\omega))=$

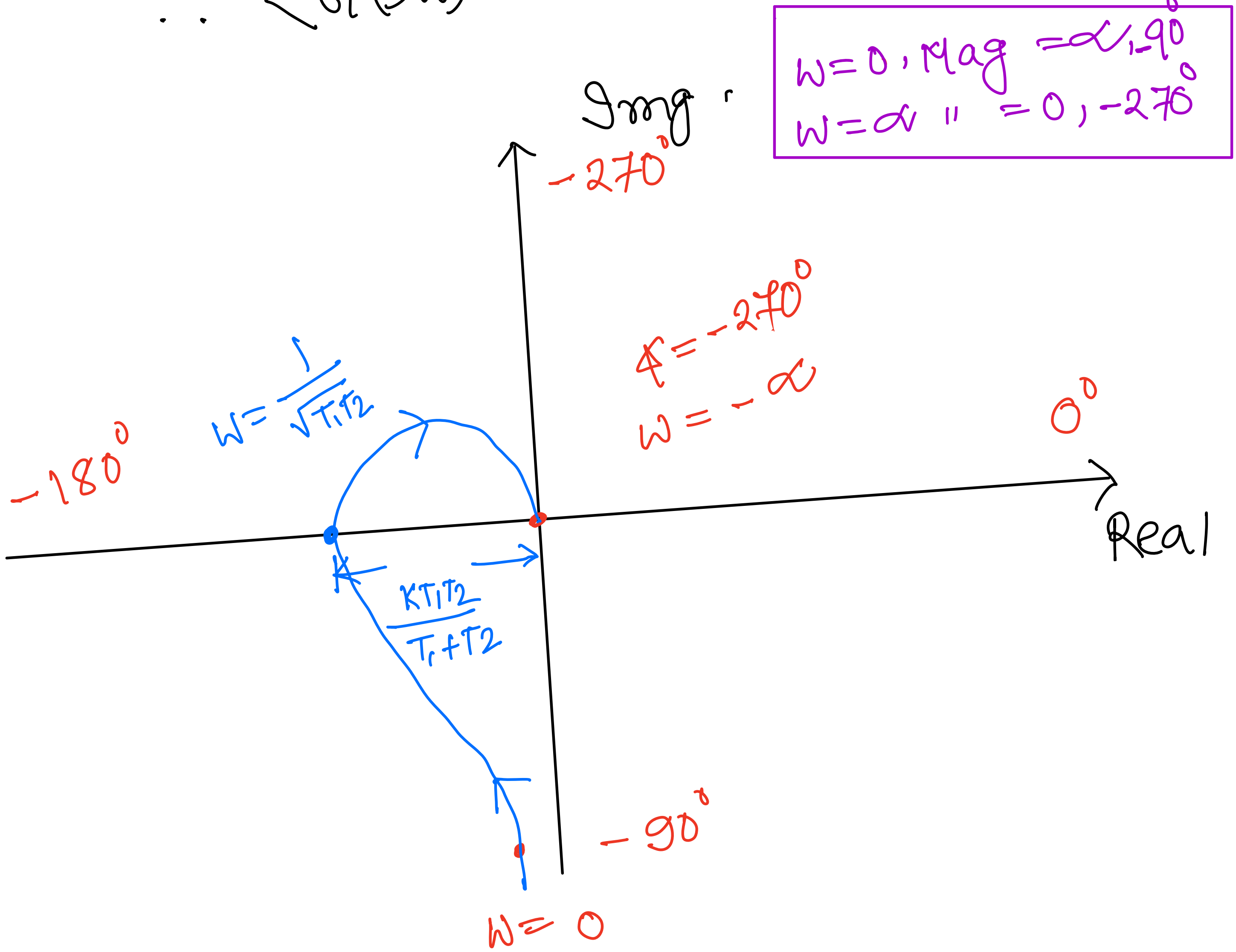
$\Rightarrow w = 0$,

$W=0,\ Mag=0,\ \angle=270^0$
$W=\infty\ \ \text{"}\ =\infty\ \ \angle=90^0$



$\angle G(jw)$

$\therefore \angle G(j\omega)$



Examp: Sketch polar plot for given open loop T.F.

$$G(S) = \frac{S^3}{(S+1)(S+2)}$$

step 1: Put $s = j\omega$

$$G(j\omega) = \frac{(j\omega)^3}{(1+j\omega)(2+j\omega)}$$

step 2: $G(j\omega) = |G(j\omega)| \angle G(j\omega)$

$$= \frac{\omega^{...}}{\sqrt{1+\omega^2}\sqrt{4+\omega^2}} \angle 270^0 - \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega}{2}\right)$$

Step 3 :

$\underline{\text{At } w=0}$

$|G(j\omega)| = 0$

$\angle G(j\omega) = 270^0$

At $w = \infty$

$|G(j\omega)| = \infty$

$\angle G(j\omega) = 270^0 - 90^0 - 90^0$

$= 90^0$

Step 4: Seperate real & imaginary.

$$G(j\omega) = \frac{(j\omega)^3}{(1+j\omega)(2+j\omega)} \times \frac{(1-j\omega)(2-j\omega)}{(1-j\omega)(2-j\omega)}$$

$$= \ldots$$

$$= \underbrace{\frac{-3\omega^4}{(1+\omega^2)(4+\omega^2)}}_{\text{Real}} + \underbrace{\frac{j\,\omega^3(\omega^2-2)}{(1+\omega^2)(4+\omega^2)}}_{\text{img}}$$

Step 5: Intersection to real axis at img $(G(j\omega))$ = ...

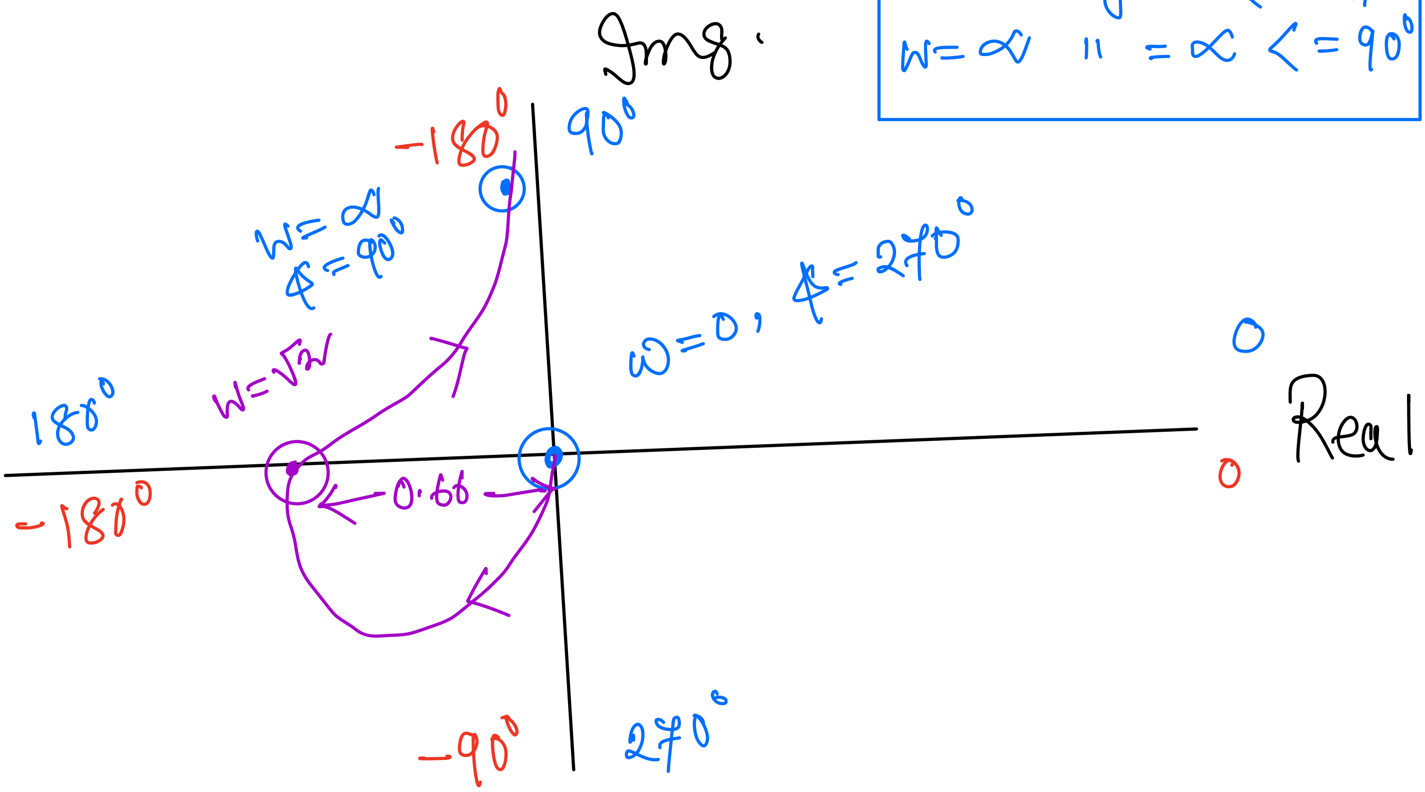
$w = 0, \quad w = \sqrt{2},$

At, $w = \sqrt{2}$ $|G(j\omega)| = 0.66 \angle 180^0$

Step 6: " " img " " real $(G(j\omega))$ = ...

$\Rightarrow w = 0,$

$W=0$, Mag $=0$, $\angle = 270^0$
$W=\infty$ " $=\infty$ $\angle = 90^0$



Section A

Pres

ID: 2257, 2234, 2115, 2179, 2127

1–3, 5, 9, 10–13, 15, 20, 17, 18, 21, 22, 27, 29,
31, 32, 34, 36, 68.

Section B

1815, 1995, 1880, 2292, 1135

44, 46, 47, 49, 53, 55, 56, 57, 59, 60, 62, 64,
71, 72, 73, 74, 78, 76, 77, 82–84, 86–88,
92,

Sec. C

2280, 1510, 1105, 1081.

93, 94, 97 ~ 99, 101 ~ 4, 108, 11, 14, 17, 18, 19,
21, 22, 23, 24, 28, 33, 36, 38