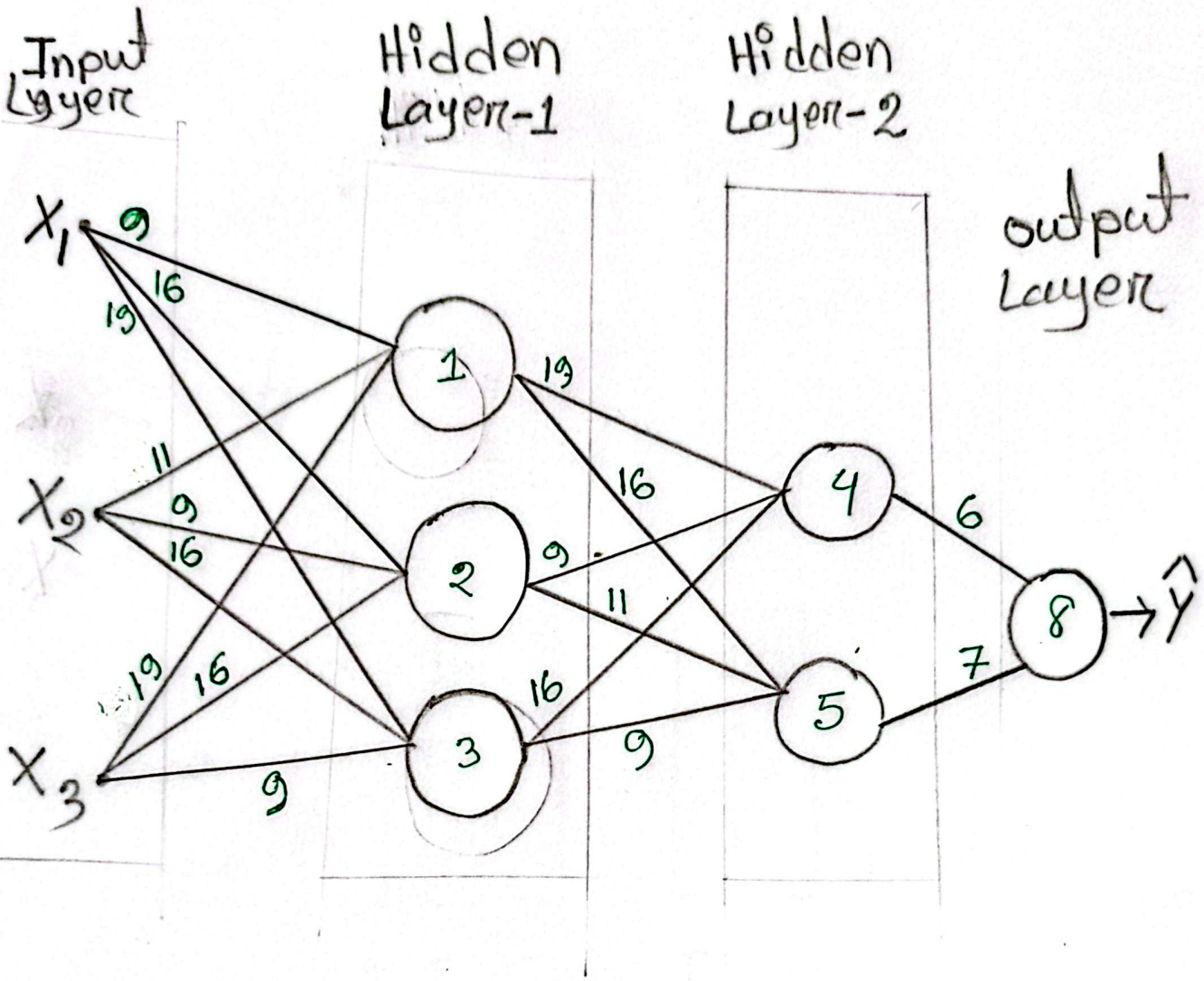
Title: Introduction to forward propagation in neural network.



Here,

Input Layer: 3 neuron (x_1, x_2, x_3)

Hidden Layer-1: 3 neuron ($h1\_1, h1\_2, h1\_3$) with ReLU Activati

Hidden Layer-2: 2 neuron ($h2\_1, h2\_2$) with ReLU Activation

Output Layer: 1 neuron ($\hat{Y}$) with Sigmoid Activation

Input Layer to hidden Layer-1

Inputs : $x_1 = 1,\ x_2 = 2,\ x_3 = 3$

weights matrix, $w_1 = \begin{bmatrix} 9 & 16 & 19 \\ 11 & 9 & 16 \\ 19 & 16 & 9 \end{bmatrix}_{(3\times 3)}$

Bias vector $b1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$$z_1 = w_1 x + b_1$$

$$= \begin{bmatrix} 9 & 16 & 19 \\ 11 & 9 & 16 \\ 19 & 16 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 11 & 19 \\ 16 & 9 & 16 \\ 19 & 16 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9+22+57 \\ 16+18+48 \\ 19+32+27 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 89 \\ 84 \\ 81 \end{bmatrix}$$

Applying ReLU Activation ($ReLU(z_1) = max(0, z_1)$)

$H_1 = [ReLU(89), ReLU(84), ReLU(81)]$

$= [max(0,89), max(0,84), max(0,81)]$

$= \begin{bmatrix} 89 \\ 84 \\ 81 \end{bmatrix}_{(3\times1)}$

Hidden Layer-1 to Hidden Layer-2

Formula: $Z_2 = H_1 \times W_2 + b_2$

Inputs $H_1 = \begin{bmatrix} 89 \\ 84 \\ 81 \end{bmatrix}$

weights matrix, $W_2 = \begin{bmatrix} 19 & 16 \\ 9 & 11 \\ 16 & 9 \end{bmatrix}_{(3\times2)}$

Bias vector $= \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$$\begin{bmatrix} 19 & 16 \\ 9 & 11 \\ 16 & 9 \end{bmatrix} \begin{bmatrix} 89 \\ 84 \\ 81 \end{bmatrix} + \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} (19\times 89) + (9\times 84) + (16\times 81) \\ (16\times 89) + (11\times 84) + (9\times 81) \end{bmatrix} + \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1691 + 756 + 1296 + 4 \\ 1424 + 924 + 729 + 5 \end{bmatrix}$$

$$= \begin{bmatrix} 3747 \\ 3082 \end{bmatrix}$$

Apply ReLU Activation

$$H_2 = [ReLU(3747), ReLU(3082)]$$

$$= [\max(0, 3747), \max(0, 3082)]$$

$$= \begin{bmatrix} 3747 \\ 3082 \end{bmatrix}_{(2\times 1)}$$

Hidden layer-2 to Output layer

Formula $z_3 = H_2 w_3 + b_3$

weight, $w_3 = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$

input, $H_2 = \begin{bmatrix} 3747 \\ 3082 \end{bmatrix}$

bias vector, $b_3 = [8]$

$$= \begin{bmatrix} 6 \\ 7 \end{bmatrix}^T \begin{bmatrix} 3747 \\ 3082 \end{bmatrix} + [8]$$

$$= \begin{bmatrix} 6 & 7 \end{bmatrix} \begin{bmatrix} 3747 \\ 3082 \end{bmatrix} + [8]$$

$$= [(6 \times 3747) + (7 \times 3082)] + [8]$$

$$= [22482 + 21574 + 8] = [44064]$$

Using Sigmoid Activation

$$z_3 = 44064$$

$$\hat{y} = \frac{1}{1+e^{-z_3}} = \frac{1}{1+e^{-(44064)}}$$

$$= 1$$