Mathematical Modeling of Transfer Function

Lecture-03

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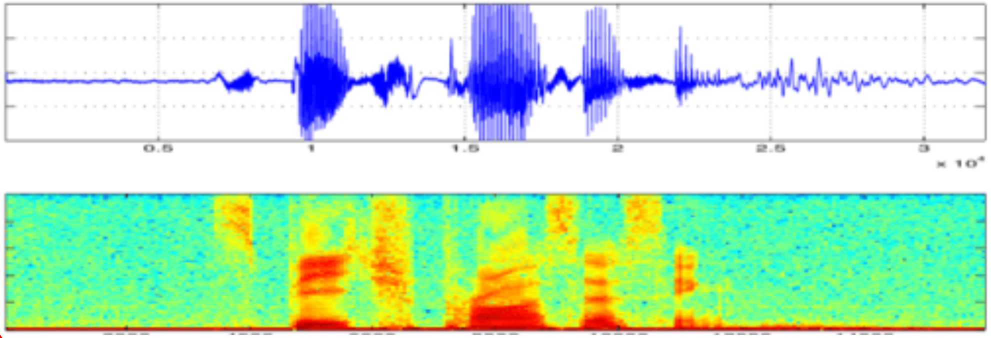
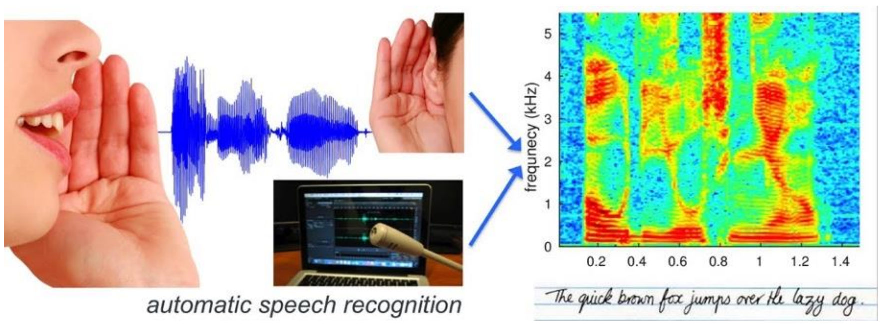
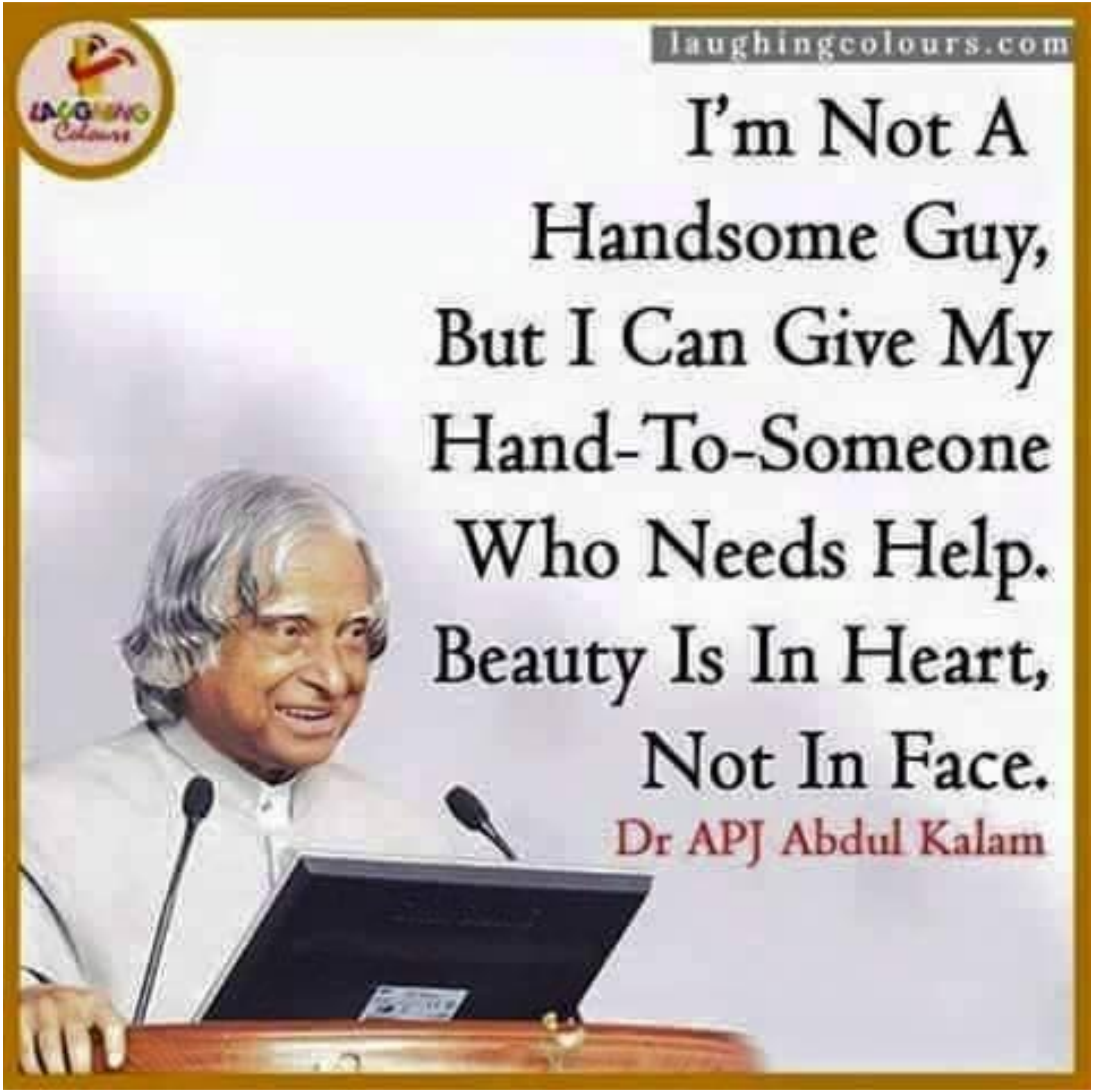





laughingcolours.com
Laughing Colours
I'm Not A
Handsome Guy,
But I Can Give My
Hand-To-Someone
Who Needs Help.
Beauty Is In Heart,
Not In Face.
Dr APJ Abdul Kalam

Introduction:

What is Model?

A *model* is a simplified representation or abstraction of reality.

What is Mathematical Model?

A set of mathematical equations (e.g., differential eqs.) that describes the input-output behavior of a system.

Why needs ?

To understand the physical behavior of a system.

Transfer Function:

The transfer function of a linear system is the ratio of the Laplace Transform of the output to the Laplace Transform of the input variable.

$$\boxed{G(s) = \frac{Output(s)}{Input(s)}}$$

$$\mathcal{L}(f(t)) \longrightarrow F(s)$$

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} \longleftrightarrow sF(s) \; \ldots$$

$$\mathcal{L}\left\{\frac{d^2 f(t)}{dt^2}\right\} \longleftrightarrow s^2 F(s)$$

Exercise 2.3: Find the transfer function, $G(S) = \frac{C(S)}{R(S)}$ corresponding to differential eq^n:

$$\frac{d^3c}{dt^3} + 3\frac{d^2c}{dt^2} + 7\frac{dc}{dt} + 5c = \frac{d^2r}{dt^2} + 4\frac{dr}{dt} + 3r$$

Sol^n: We know that,

$$\mathcal{L}\left[\frac{d^n f(t)}{dt^n}\right] = S^n F(S) - \sum S^{n-K} \cdot f^{K-1}(0^+)$$

Here, $f(0^+) = 0$

So, $$\frac{d^3c}{dt^3} + 3\frac{d^2c}{dt^2} + 7\frac{dc}{dt} + 5c = \frac{d^2r}{dt^2} + 4\frac{dr}{dt} + 3r$$

Using L.T, where $f(0^+) = 0$

$$S^3C(S) + 3S^2C(S) + 7SC(S) + 5C(S) = S^2R(S) + 4SR(S) + 3R(S)$$

or, $$(S^3 + 3S^2 + 7S + 5)C(S) = (S^2 + 4S + 3)R(S)$$

$$\therefore \frac{C(S)}{R(S)} = \frac{S^2 + 4S + 3}{S^3 + 3S^2 + 7S + 5}$$

Exercise: 2.4: Find the differential eq^n corresponding to the transfer function, $G(S) = \dfrac{2S+1}{S^2+6S+2}$

Soln:

$$G(S) = \frac{C(S)}{R(S)} = \frac{2S+1}{S^2+6S+2}$$

$$\text{or, } C(S)\left(S^2+6S+2\right) = (2S+1)\,R(S)$$

Using inverse Laplace,

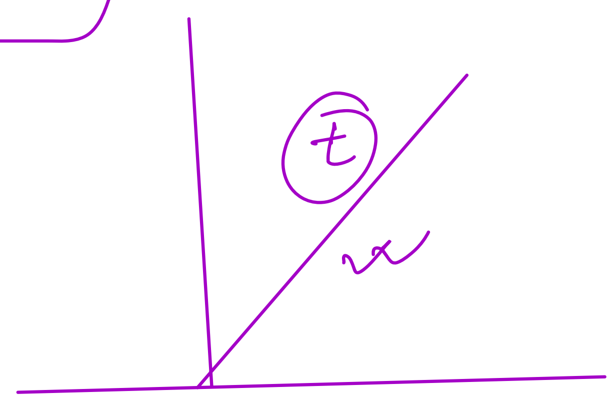
$$\frac{d^2c(t)}{dt^2} + 6\frac{dc(t)}{dt} + 2c(t) = 2\frac{dr(t)}{dt} + r(t)$$

Exercise: 2.4: Find the differential eq^n corresponding to the transfer function, $G(S) = \dfrac{2S+1}{S^2+6S+2}$.

$$\boxed{C(S) = G(S) \cdot R(S)}$$

$\mathcal{L}(t) \simeq \frac{1}{S^2}$ ✗

$\mathcal{L}\{u(t)\} \simeq \frac{1}{S}$



$r(t) = t$

$R(S) \simeq \mathcal{L}\{r(t)\} = \frac{1}{S^2}$

$C(S) = G(S) \cdot \frac{1}{S^2}$

$\boxed{\text{Exercise: 2.5}}$ Find the ramp response for a system whose transfer function is $G(s) = \dfrac{s}{(s+4)(s+8)}$.

Soln:

Here, ~~p~~ input, ~~R(s)~~ $r(t) = t\,u(t)$.

$R(s) \rightarrow \dfrac{1}{s^2}$

$$\therefore \quad C(s) = R(s)\,G(s)$$

$$= \frac{1}{s^2} \cdot \frac{s}{(s+4)(s+8)} = \frac{1}{s(s+4)(s+8)}$$

$$\therefore \quad C(s) = \frac{1}{s(s+4)(s+8)}$$

Now, $$\frac{1}{s(s+4)(s+8)} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+8}$$

or, $1 = A(s+4)(s+8) + sB(s+8) + Cs(s+4)$

$$A = \frac{1}{32}, \quad B = -\frac{1}{16}, \quad C = \frac{1}{32}$$

$$\therefore \quad C(s) = \frac{1}{32s} - \frac{1}{16(s+4)} + \frac{1}{32(s+8)}.$$

Now, Using inverse Laplace,

$$c(t) = \frac{1}{32} - \frac{1}{16}e^{-4t} + \frac{1}{32}e^{-8t}.$$

$$\mathcal{L}[e^{at}] \leftrightarrow \frac{1}{s+a}$$

Att: Section A 16/7

Pres : 2, 3, 4, 5, 8 – 12, 15, 17, 20 – 23, 26, 27,
29, 32, 34 – 36, 38, 39, :
2257, 2250, 2247, 2234, 1819, 2255, 2127
2115, 1084

Sect: C

1139, 1105, 1081, 1696, 770331020151, 2280, 2266
93, 94, 97 – 104, 106, 108, 110, 111, 114, 117 ~ 119,
121 – 124, 31, 33, 36, 38, 30, 32, 28

Sec: B

Pres: 47, 50, 51, 53 – 55, 56, 59, 71 ~ 74, 82, 87 – 89,
92,
1880,

Electrical Network Transfer Functions

The transfer function is applied to the mathematical modeling of electric circuits including passive networks and operational amplifier circuits.

Equivalent circuits for the electric networks consist of three passive linear components:

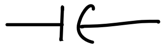
1. RESISTOR
2. CAPACITOR
3. INDUCTOR

TABLE 2.3 Voltage-current, voltage-charge, and impedance relationships for capacitors, resistors, and inductors

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
Capacitor	$v(t) = \frac{1}{C}\int_0^1 i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^1 v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ – V (volts), $i(t)$ – A (amps), $q(t)$ – Q (coulombs), C – F (farads), R – Ω (ohms), G – Ω (mhos), L – H (henries).

Capacitor:



$$V_C(t) = \frac{1}{C}\int i(t)\,dt$$

$$V_C(S) = \frac{1}{C}\cdot\frac{1}{S}\,I(S) = \frac{1}{CS}\,I(S)$$

$$\therefore\quad V_C(S) = \frac{1}{CS}\,I(S)$$

$$\Rightarrow\quad \frac{V_C(S)}{I(S)} = \frac{1}{CS}$$

$$\Rightarrow\quad Z_C(S) = \frac{1}{CS}$$

$$\boxed{\begin{array}{l} \dfrac{d\,f(t)}{dt} \longleftrightarrow S\,F(S) \\ \int f(t)\,dt \longleftrightarrow \dfrac{1}{S}F(S). \end{array}}$$

Inductor:

$$V_L(t) = L\,\frac{di(t)}{dt}$$

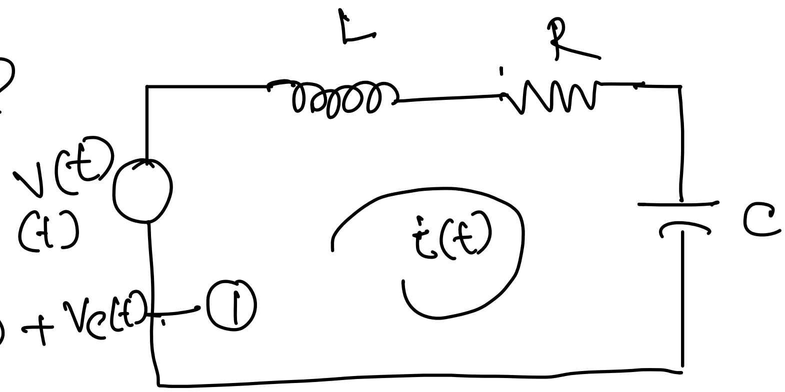
$$\xrightarrow{L.T}\quad V_L(S) = L\cdot S\;I(S)$$

$$\Rightarrow\quad \frac{V_L(S)}{I(S)} = LS$$

$$\Rightarrow\quad Z_L(S) = LS$$

Examp

$$\frac{V_c(s)}{V(s)} = ?$$



① $V(t) = V_L(t) + V_R(t) + V_c(t)$

$$= L\frac{di(t)}{dt} + Ri(t) + V_c(t) \quad ①$$

$$i(t) = \frac{dq(t)}{dt}, \quad q(t) = C \cdot V_c(t)$$

$$= \frac{d}{dt} \cdot (C V_c(t)) = C \cdot \frac{dV_c(t)}{dt}.$$

Put value of i(t) in eqn ①

$$V(t) = L \cdot \frac{d}{dt} \cdot C \cdot \frac{dV_c(t)}{dt} + RC\frac{dV_c(t)}{dt} + V_c(t)$$

$$V(t) = LC\frac{d^2V_c(t)}{dt^2} + RC\frac{dV_c(t)}{dt} + V_c(t)$$

Apply L.T.

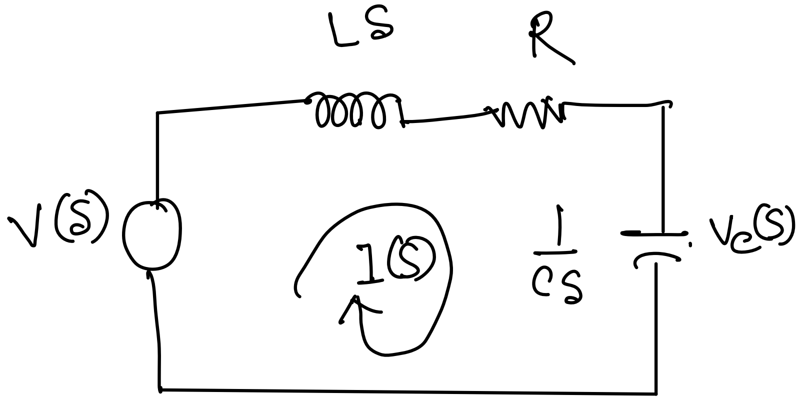
$$V(s) = LCs^2 V_c(s) + RCs\, V_c(s) + V_c(s)$$

$$= (LCs^2 + RCs + 1)V_c(s)$$

$$\therefore \boxed{\frac{V_c(s)}{V(s)} = \frac{1}{1 + RCs + LCs^2}}$$

11 ## Mesh Analysis:

Redraw the ckt with L.T.



Apply KVL:

$$V(S) = \left[LS + R + \frac{1}{CS}\right] I(S)$$

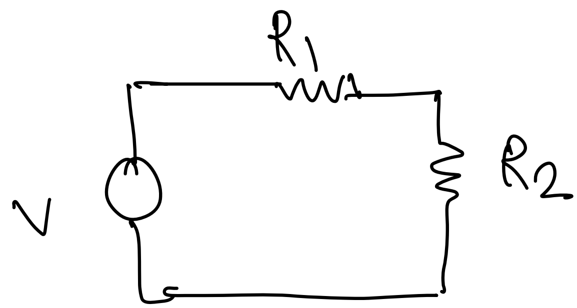
$$\frac{V_C(S)}{I(S)} = \frac{1}{CS}, \quad I(S) = CS \cdot V_C(S)$$

$$\therefore \; V(S) = \left[LS + R + \frac{1}{CS}\right] \cdot CS\, V_C(S)$$

$$\text{or} \quad \frac{V(S)}{V_C(S)} = \frac{LCS^2 + RCS + 1}{\cancel{CS}} \times \cancel{CS}$$

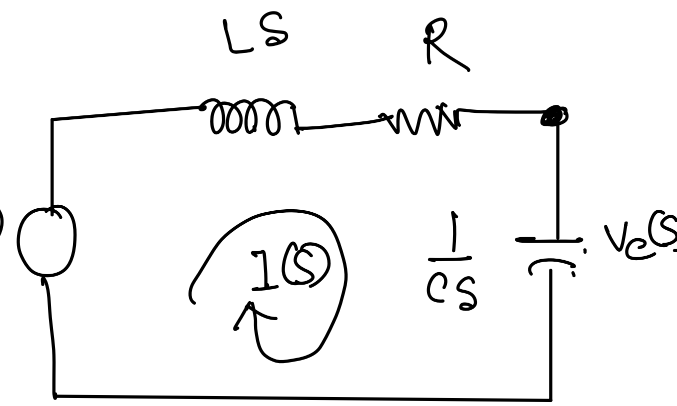
$$\therefore \; \boxed{\frac{V_C(S)}{V(S)} = \frac{1}{S^2LC + SRC + 1}}$$

(iii) Voltage divider:



$$\frac{V_c(S)}{V(S)}$$

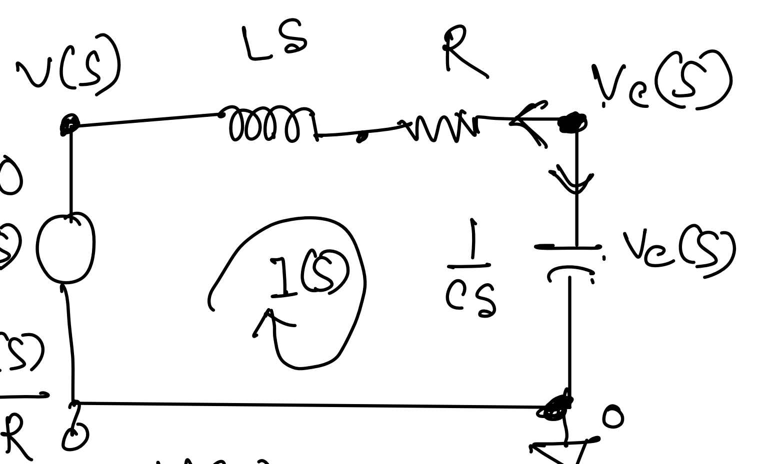
$$V_c(S) = V(S) \cdot \frac{1/CS}{LS + R + 1/CS}$$



$$\Rightarrow \frac{V_c(S)}{V(S)} = \frac{\cancel{1/CS}}{\frac{S^2LC + RCS + 1}{\cancel{CS}}}$$

$$\therefore \boxed{\frac{V_c(S)}{V(S)} = \frac{1}{1 + RCS + S^2LC}}$$

(iv) Nodal Analysis:



$$\frac{V_c(S) - 0}{1/CS} + \frac{V_c(S) - V(S)}{LS + R} = 0$$

$$\Rightarrow CS \cdot V_c(S) + \frac{V_c(S)}{LS + R} = \frac{V(S)}{LS + R}$$

$$\Rightarrow V_c(S)\left[CS + \frac{1}{R + LS}\right] = \frac{V(S)}{LS + R}$$

$$\Rightarrow V_c(S) \cdot \frac{S^2LC + SRC + 1}{\cancel{(R + LS)}} = \frac{V(S)}{\cancel{(LS + R)}}$$

$$\therefore \quad \boxed{\frac{V_c(S)}{V(S)} = \frac{1}{S^2LC + SRC + 1}}$$

Electrical Network Transfer Functions:

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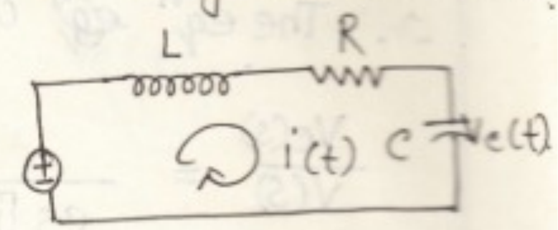
☑ $\boxed{\text{Example: } 2\cdot6, 2\cdot7, 2\cdot8, 2\cdot9}$ Find the transfer function from diffent. eqⁿ (i) relating, $V_c(S)$, $V(S)$ & (ii) Using mesh (iii) Using vol. division (iv) node.

Solⁿ:



(i) Taking, KVL,

$$V(t) = L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)\,dt \quad \cancel{V(t}$$

$$i(t) = \frac{dq(t)}{dt}.$$

$$V(t) = L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{1}{C}q(t)$$

Again, $q(t) = C\,V_c(t)$

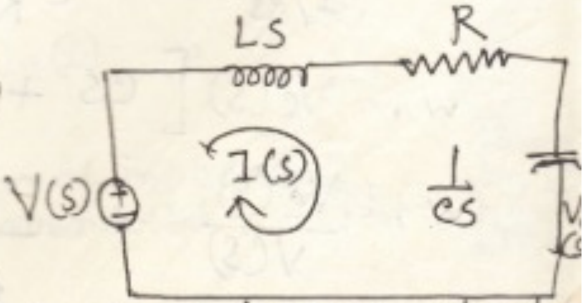
$$\therefore V(t) = LC\frac{d^2V_c(t)}{dt^2} + RC\frac{dV_c(t)}{dt} + V_c(t).$$

Using inverse laplace transformation.

$$V(S) = \left((LC)S^2 + (RC)S + 1\right)V_c(S)$$

$$\therefore \frac{V_c(S)}{V(S)} = \frac{1}{LC\left[S^2 + \frac{R}{L}S + \frac{1}{LC}\right]} = \frac{1/LC}{S^2 + \frac{RS}{L} + \frac{1}{LC}}$$

(ii) Redrawn the given ckt to Laplace transforme network.



Now, writing the mesh eqⁿ

$$\left(LS + R + \frac{1}{CS}\right)I(S) = V(S)$$

$$\therefore \frac{I(S)}{V(S)} = \frac{1}{LS + R + 1/CS}$$

Again, $V_c(S) = I(S) \cdot \frac{1}{CS}$

$\therefore I(S) = V_c(S) \cdot CS$

$\therefore$ The eq^n written as.

$$\frac{V_c(S)}{V(S)} = \frac{1}{CS\left[LS + R + 1/CS\right]} = \frac{1}{LC\left[S^2 + \frac{R}{L}S + \frac{1}{LC}\right]}$$

$$= \frac{1/LC}{S^2 + \frac{R}{L}S + \frac{1}{LC}}.$$

iii From the Laplace transformed Network.
Using vol. divider rule.

$$V_c(S) = \frac{1/CS}{LS + R + \frac{1}{CS}} V(S)$$

$$\therefore \frac{V_c(S)}{V(S)} = \frac{1}{LC\left[S^2 + \frac{R}{L}S + \frac{1}{LC}\right]}$$

iv From the Laplace transformed Network
Using nodal method.

$$\frac{V_c(S)}{1/CS} + \frac{V_c(S) - V(S)}{R + LS} = 0$$

$$\text{or,}\quad V_c(S)\left[CS + \frac{1}{R+LS}\right] = \frac{V(S)}{R+LS}$$

$$\text{or,}\quad \frac{V_c(S)}{V(S)} = \frac{1}{R+LS} \times \frac{R+LS}{1 + CS\,(R+LS)}$$

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Section A:

Present: 1-5, 7-10, 12, 15, 17-18, 20-21, 23, 26, 27, 29, 31-32, 34-36, 38, 39.

1819, 1084, 2127, 2247, 2250, 2255, 2257, 2115, 2179, 2234.

MESH ANALYSIS

CONTINUE:

To get transfer function from electrical circuit two method can be used. They are

1. MESH ANALYSIS (using KVL)
2. NODAL ANALYSIS (using KCL)

MESH ANALYSIS:

In this method KVL is being used and the following steps are followed

1. Replace passive element values with their impedances.
2. Replace all sources and time variables with their Laplace transform.
3. Assume a transform current and a current direction in each mesh.

4. Write Kirchhoffs voltage law around each mesh.

5. Solve the simultaneous equations for the output.

6. Form the transfer function

REQUIRED FORMULA:

For the capacitor,
$\boldsymbol{v(s)=I(s)/Cs}$

For the resistor,
$V\{s) = RI(s)$

For the inductor,
$V(s) = LSl(s)$

Required Formula:

Mesh:

$$\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{around Mesh 1}\end{bmatrix} I_1(s) - \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{common to the}\\ \text{two meshes}\end{bmatrix} I_2(s) = \begin{bmatrix}\text{Sum of applied}\\ \text{voltages around}\\ \text{Mesh 1}\end{bmatrix}$$

$$-\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{common to the}\\ \text{two meshes}\end{bmatrix} I_1(s) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{around Mesh 2}\end{bmatrix} I_2(s) = \begin{bmatrix}\text{Sum of applied}\\ \text{voltages around}\\ \text{Mesh 2}\end{bmatrix}$$

Nodal:

$$\begin{bmatrix}\text{Sum of admittances}\\ \text{connected to Node 1}\end{bmatrix} V_L(s) - \begin{bmatrix}\text{Sum of admittances}\\ \text{common to the two}\\ \text{nodes}\end{bmatrix} V_C(s) = \begin{bmatrix}\text{Sum of applied}\\ \text{currents at Node 1}\end{bmatrix} \tag{2.90a}$$

$$-\begin{bmatrix}\text{Sum of admittances}\\ \text{common to the two}\\ \text{nodes}\end{bmatrix} V_L(s) + \begin{bmatrix}\text{Sum of admittances}\\ \text{connected to Node 2}\end{bmatrix} V_C(s) = \begin{bmatrix}\text{Sum of applied}\\ \text{currents at Node 2}\end{bmatrix} \tag{2.90b}$$

Related Math

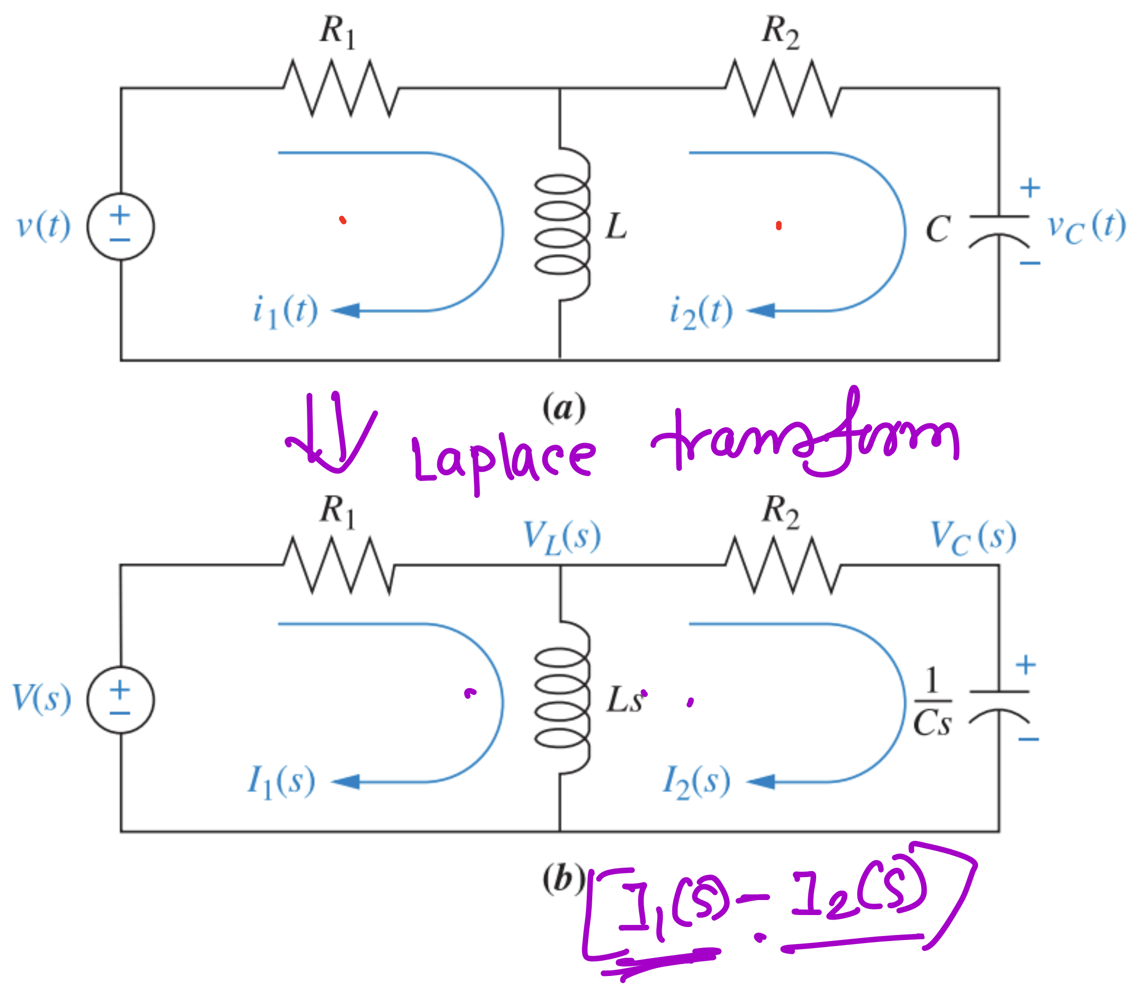
(2.10, 2.11, 2.12)

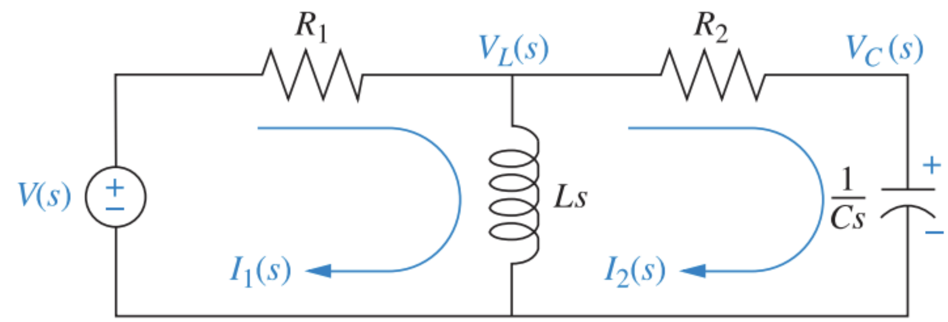
Example:2.10: Given the network of figure find the transfer function, I (s)/V(s) using mesh analysis.

Solution:

1) $I_2(s)/V(s) \rightarrow$ Mesh

2) $V_C(s)/V(s) \rightarrow$ Node





(b)

For Mesh (1):

$\sum$ voltage rise & drop $= 0$

$$R_1 I_1(S) + LS\left(I_1(S) - I_2(S)\right) = V(S)$$

$$\Rightarrow R_1 I_1(S) + LS\, I_1(S) - LS\, I_2(S) = V(S)$$

$$\Rightarrow (R_1 + LS)\, I_1(S) - LS\, I_2(S) = V(S) \quad \text{—— (i)}$$

For Mesh (2):

$$R_2 I_2(S) + \frac{1}{CS} I_2(S) + LS\left(I_2(S) - I_1(S)\right) = 0$$

$$\Rightarrow \left(R_2 + \frac{1}{CS} + LS\right) I_2(S) - LS\, I_1(S) = 0 \quad \text{—— (ii)}$$

$$(R + LS)\, I_1(S) - LS\, I_2(S) = V(S)$$

$$\begin{bmatrix}\text{sum of impedance} \\ \text{around Mesh 1}\end{bmatrix} I_1(S) - \begin{bmatrix}\text{sum of imp.} \\ \text{common around} \\ \text{Mesh (1) \& (2)}\end{bmatrix} I_2(S) = \begin{bmatrix}\text{Sum of} \\ \text{applied} \\ \text{Source} \\ \text{in} \\ \text{Mesh1}\end{bmatrix}$$

$$\left(R_2 + \frac{1}{CS} + LS\right) I_2(S) - LS\, I_1(S) = 0$$

$$\begin{bmatrix}\text{sum of impedance} \\ \text{around Mesh 2}\end{bmatrix} I_2(S) - \begin{bmatrix}\text{sum of imp.} \\ \text{common around} \\ \text{Mesh (1) \& (2)}\end{bmatrix} I_1(S) = \begin{bmatrix}\text{Sum of} \\ \text{applied} \\ \text{Source} \\ \text{in} \\ \text{Mesh2}\end{bmatrix}$$

From (i) & (ii)

$$(R + LS)\, I_1(S) - LS\, I_2(S) = V(S)$$

$$-LS\, I_1(S) + \left(R_2 + \frac{1}{CS} + LS\right) I_2(S) = 0$$

Now transfer function, $G(S)$.

$$G(S) = \frac{I_2(S)}{V(S)}$$

$$I_2(S) = \frac{\begin{vmatrix} R+LS & V(S) \\ -LS & 0 \end{vmatrix}}{\Delta} = \frac{0 + V(S)\, LS}{\Delta}$$

$$\Delta = \begin{vmatrix} R+LS & -LS \\ -LS & R_2+\frac{1}{CS}+LS \end{vmatrix}$$

$$= (R+LS)\left(R_2+\frac{1}{CS}+LS\right) - (LS)^2$$

$$= \vdots$$

$$= \frac{1}{CS}\left[S^2LC(R_1+R_2) + (R_1R_2C+L)S + R_1\right]$$

Cremer's Rule:

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\Delta} = \frac{c_1b_2 - c_2b_1}{\Delta}$$

$$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\Delta}$$

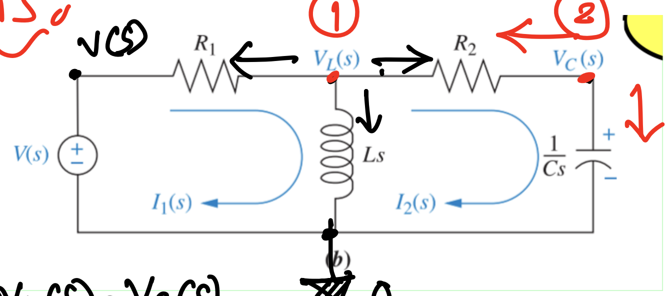
$$I_2(S) = \frac{V(S) \cdot LS}{\frac{1}{CS}\left[S^2LC(R_1+R_2) + (R_1R_2C+L)S + R_1\right]}$$

$$\frac{I_2(S)}{V(S)} = \frac{LS \times CS}{\left[S^2LC(R_1+R_2) + (R_1R_2C+L)S + R_1\right]}$$

$$= \frac{LCS^2}{\left[S^2LC(R_1+R_2) + (R_1R_2C+L)S + R_1\right]}$$

Ans

Nodal Analysis:



$$\frac{V_C(S)}{V(S)} = ?$$

① Node

$$\frac{V_L(S) - V(S)}{R_1} + \frac{V_L(S) - 0}{LS} + \frac{V_L(S) - V_C(S)}{R_2} = 0$$

$$\Rightarrow \frac{V_L(S)}{R_1} - \frac{V(S)}{R_1} + \frac{V_L(S)}{LS} + \frac{V_L(S)}{R_2} - \frac{V_C(S)}{R_2} = 0$$

$$\Rightarrow V_L(S)\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{LS}\right] + V_C(S)\left[-\frac{1}{R_2}\right] = \frac{V(S)}{R_1}$$

$$\Rightarrow V_L(S)\left[G_1 + G_2 + \frac{1}{LS}\right] - V_C(S)\, G_2 = V(S) \cdot G_1 \quad (1)$$

② Node

$$\frac{V_C(S) - V_L(S)}{R_2} + \frac{V_C(S) - 0}{1/CS} = 0$$

$$\Rightarrow V_L(S)\left[\frac{-1}{R_2}\right] + V_C(S)\left[\frac{1}{R_2} + CS\right] = 0$$

$$\Rightarrow -V_L(S)\, G_2 + V_C(S)\,(G_2 + CS) = 0 \quad (2)$$

$$\therefore V_C(S) = \frac{\begin{vmatrix} G_1 + G_2 + \frac{1}{LS} & G_1 V(S) \\ -G_2 & 0 \end{vmatrix}}{\begin{vmatrix} G_1 + G_2 + \frac{1}{LS} & -G_2 \\ -G_2 & G_2 + CS \end{vmatrix}}$$

$$V_C(S) = \frac{0 + G_1 G_2 V(S)}{\left(G_1 + G_2 + \frac{1}{LC}\right)(G_2 + CS) - G_2^2}$$

$$\frac{V_C(S)}{V(S)} = \frac{G_1 G_2}{\left(G_1 + G_2 + \frac{1}{LC}\right)(G_2 + CS) - G_2^2}$$

Ans

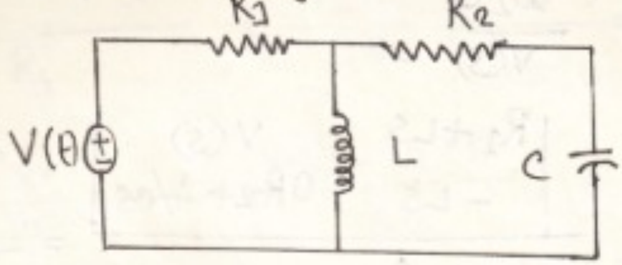
NODAL ANALYSIS:

In this method KCL is used . If there are multiple nodes then Norton theorem is used. The following steps are followed:

1. Replace passive element values with their admittances.
2. Replace all sources and time variables with their Laplace transform.
3. Replace transformed voltage sources with transformed current sources.
4. Write Kirchhoffs current law at each node.
5. Solve the simultaneous equations for the output.
6. Form the transfer function

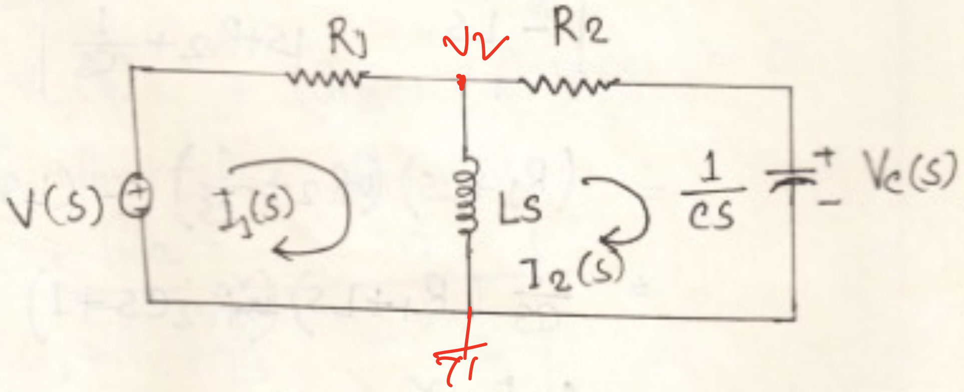
6

$\boxed{\text{Exam: 2·10, 2·19, 2·12}}$ Find the transfer function (i) $I_2(S)/V(S)$ Using Mesh / Loop analysis (ii) $V_c(S)/V(S)$, Using nodal analysis (iii) $V_c(S)/V(S)$ with current source.



Soln:

Laplace transform network.



From, Mesh 1.

$$R_1 I_1(S) + LS\left(I_1(S) - I_2(S)\right) = V(S)$$

or, $$(R_1 + LS)\, I_1(S) \quad - \quad LS\, I_2(S) \quad = V(S)$$

$$\begin{bmatrix}\text{Sum of impedances} \\ \text{around Mesh 1}\end{bmatrix} I_1(S) - \begin{bmatrix}\text{Sum of impedances common} \\ \text{around Mesh 1 \& 2}\end{bmatrix} I_2(S) = \begin{bmatrix}\text{Sum of applied vol. around} \\ \text{Mesh 1}\end{bmatrix}$$

From, Mesh 2,

$$R_2 I_2(S) + \frac{1}{CS} \cdot I_2(S) + LS\left(I_2(S) - I_1(S)\right) = 0.$$

or, $$-LS \cdot I_1(S) + \left(LS + R_2 + \frac{1}{CS}\right) I_2(S) = 0$$

or $$-\begin{bmatrix}\text{Sum of impedances common} \\ \text{around Mesh 1 \& 2}\end{bmatrix} I_1(S) + \begin{bmatrix}\text{Sum of impedances} \\ \text{around Mesh 2}\end{bmatrix} I_2(S) = \begin{bmatrix}\text{Sum of applied Vol. around} \\ \text{Mesh 2}\end{bmatrix}$$

Now, Transfer function, $G(s)$.

$$G(s) = \frac{I_2(s)}{V(s)}.$$

Where,

$$I_2(s) = \frac{\begin{vmatrix} R_1+LS & V(s) \\ -LS & 0\cancel{R_2+1/cs} \end{vmatrix}}{\Delta} = \frac{V(s)\cdot LS}{\Delta}$$

$$\Delta = \begin{vmatrix} R_1+LS & -LS \\ -LS & LS+R_2+\frac{1}{cs} \end{vmatrix}$$

$$= (R_1+LS)(\cancel{LS}R_2+\frac{1}{cs}) - (LS)^2$$

$$= \frac{1}{cs}(R_1+LS)(\cancel{LS}R_2cs+1) - (LS)^2$$

$$= \frac{1}{cs}[\cancel{LCS^2R_1 + R_1R_2CS + R_1 +}$$

$$= R_1LS + R_1R_2 + \frac{R_1}{cs} + \cancel{(LS)^2} + R_2LS + \frac{L}{c} - \cancel{(LS)^2}$$

$$= R_1LS + R_1R_2 + \frac{R_1}{cs} + R_2LS + \frac{L}{c}$$

$$= \frac{1}{cs}\left[R_1LCS^2 + R_1R_2cs + R_1 + R_2LCS^2 + LS\right]$$

$$= \frac{1}{cs}\left[S^2Lc(R_1+R_2) + (R_1R_2c+L)S + R_1\right]$$

$$\therefore \frac{I_2(s)}{V(s)} = \frac{LCS^2}{S^2(R_1+R_2)LC + (R_1R_2C+L)S + R_1}.$$

(ii) Using nodal Analysis:-

$$\frac{V_L(s) - V(s)}{R_1} + \frac{V_L(s)}{Ls} + \frac{V_L(s) - V_C(s)}{R_2} = 0$$

$$\frac{V_C(s)}{1/Cs} + \frac{V_C(s) - V_L(s)}{R_2} = 0$$

$$\therefore \left(G_1 + G_2 + \frac{1}{Ls}\right) V_L(s) - (G_1 + G_2)\, V_2(s) = G_1 V(s)$$

$$- G_2 V_L(s) + (G_2 + Cs)\, V_C(s) = 0$$

$$V_C(s) = \frac{\begin{vmatrix} G_1+G_2+\frac{1}{Ls} & G_1 V(s) \\ -G_2 & 0 \end{vmatrix}}{\begin{vmatrix} G_1+G_2+\frac{1}{Ls} & -G_2 \\ -G_2 & G_2+Cs \end{vmatrix}}$$

$$= \frac{G_1 G_2 V(s)}{\left(G_1+G_2+\frac{1}{Ls}\right)(G_2+Cs) - G_2^2}$$

$$\therefore \frac{V_C(s)}{V(s)} = \frac{G_1 G_2}{\left(G_1+G_2+\frac{1}{Ls}\right)(G_2+Cs) - G_2^2}$$

* Mesh এর মত But, এখানে, associated component এর inverse এর sum হবে।

Related math:

SKILL EXERCISEMENT:2.6:

PROBLEM: Find the transfer function, $G(s) = V_L(s)/V(s)$, for the circuit given in Figure 2.14. Solve the problem two ways—mesh analysis and nodal analysis. Show that the two methods yield the same result.

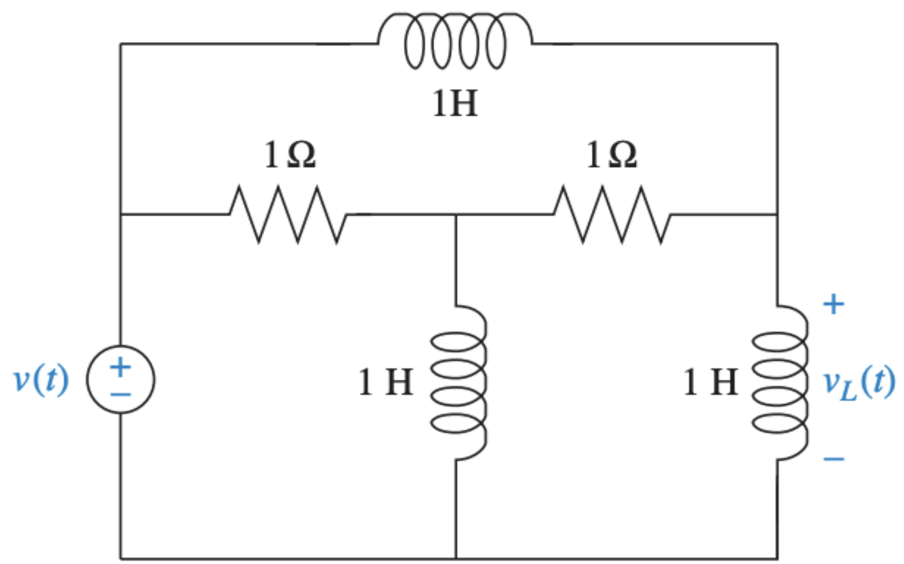
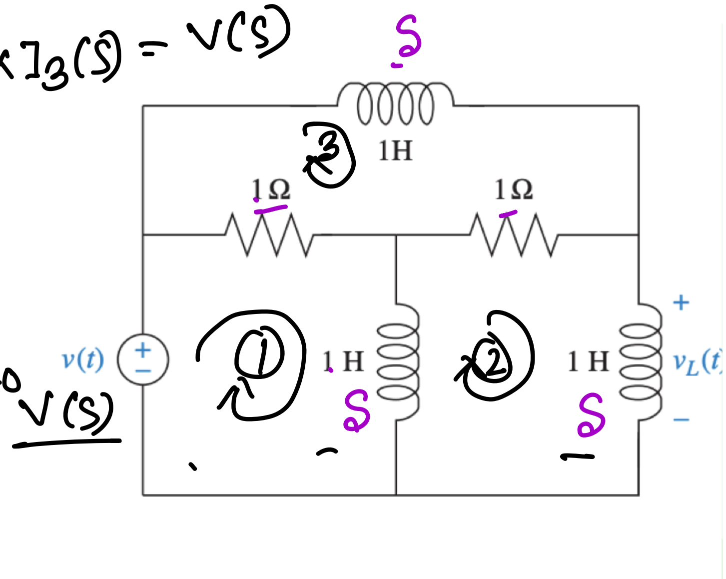


FIGURE 2.14 Electric circuit for Skill-Assessment Exercise 2.6

Loop 1

$$(1+S)I_1(S) - S\,I_2(S) - 1\times I_3(S) = V(S)$$



Loop 2

$$(1+2S)I_2(S) - S\,I_1(S) - 1\times I_3(S) = 0$$

Loop 3

$$(2+S)I_3(S) - 1\times I_1(S) - 1\times I_2(S) = 0$$

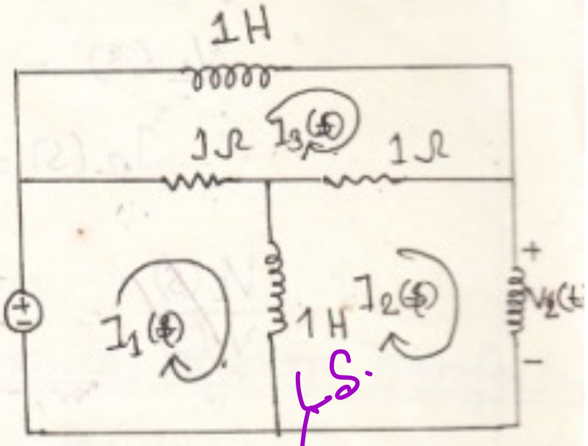
Class Test – I

→ Laplace transformation

→ TF of electrical Networks.

Date: 4/8/25

Exercise: 2.6 Find. $G(s) = \dfrac{V_L(s)}{V(s)}$ Using mesh analysis & nodal analysis.



Ls.

L = 1×S

Soln:

Mesh Analysis:

$$(1+s)\, I_1(s) - s\, I_2(s) - I_3(s) = V(s)$$

$$-\, s\, I_1(s) + (2s+1)\, I_2(s) - I_3(s) = 0$$

$$-\, I_1(s) - I_2(s) + (2+s)\, I_3(s) = 0$$

Now,
$$I_2(s) = \frac{\begin{vmatrix} 1+s & V(s) & -1 \\ -s & 0 & -1 \\ -1 & 0 & 2+s \end{vmatrix}}{\begin{vmatrix} 1+s & -s & -1 \\ -s & 2s+1 & -1 \\ -1 & -1 & 2+s \end{vmatrix}}$$

$$= \frac{-V(s)\left[(2+s)(-s) - 1\right]}{(-1)\left[s+2s+1\right) + (-1-s-s) + (2+s)\left[(s+1)(2s+1) - s^2\right]}$$

$$\frac{I_2(s)}{V(s)} = \frac{+(2s+s^2+1)}{-3s-1-2s-1+(2+s)(2s^2+3s+1-s^2)}$$

$$= \frac{s^2+2s+1}{-5s-2+(2+s)(3s+1+s^2)}$$

$$= \frac{s^2+2s+1}{-5s-2+6s+2+2s^2+3s^2+s+s^3}$$

$$\boxed{\frac{s^2+2s+1}{4s-2+\cancel{2s^2}+2+2s^2+s}}$$

s^2+2s+1 $\quad$ s^2+2s+1

Now, $LSI_2(S) = V_L(S)$

$\therefore V_L(S) = S I_2(S) \quad [L=1]$

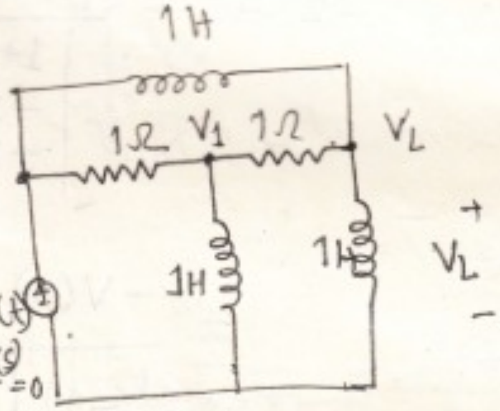
$\therefore I_2(S) = \frac{V_L(S)}{S}$. $\therefore V$

$\therefore \cancel{\frac{V_L(S)}{V(S)}} \quad I_2(S) = \frac{(S^2+2S+1)\,V(S)}{S(S^2+5S+2)}$

or, $\frac{V_L(S)}{S} = \frac{(S^2+2S+1)\,V(S)}{S(S^2+5S+2)}$

$\therefore \frac{V_L(S)}{V(S)} = \frac{S^2+2S+1}{S^2+5S+12}$.

Using Nodal Analysis:



$\frac{V_1(S)-V(S)}{1} + \frac{V_1(S)}{S} + \frac{V_1-V_L}{1} = 0$

$\frac{V_L(S)-V_1(S)}{1} + \frac{V_L(S)-}{S} + \frac{V_L-V(S)}{S} = 0$

~~V(S)~~

or, $SV_1(S) - SV(S) + V_1(S) + SV_1(S) - SV_L(S) = 0$

or, $(2S+1)V_1(S) - SV_L(S) = S\,V(S)$.

& $-S\,V_1(S) + (2+S)V_L(S) = V(S)$

Now, $V_L(S) = \frac{\begin{vmatrix} 2S+1 & SV(S) \\ -S & V(S) \end{vmatrix}}{\begin{vmatrix} 2S+1 & -S \\ -S & 2+S \end{vmatrix}} =$

$$= \frac{V(s)\left[2s+1+s^2\right]}{\left[(2s+1)(s+2)-s^2\right]} = \frac{V(s)\left[s^2+2s+1\right]}{\left[2s^2+5s+2-s^2\right]}$$

$$\therefore \quad \frac{V_L(s)}{V(s)} = \frac{s^2+2s+1}{s^2+5s+2}.$$

Application:

In automobile suspension

an electrical system be developed for mechanical system

Electric motor

Electrical energy required to rotate the motor

Electric Generator

mechanical energy is required to generates electricity

Conclusion:

From this discussion topic we can conclude that , an electrical network can be interfaced to a mechanical system by cascading their transfer functions, provided that one system is not loaded by the other.

Thank you!
Question ?