

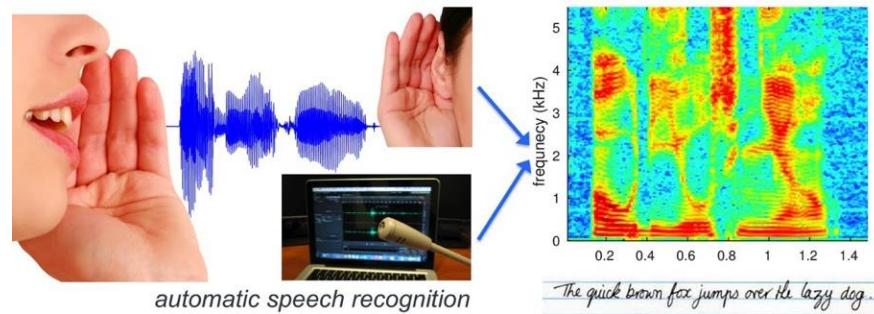
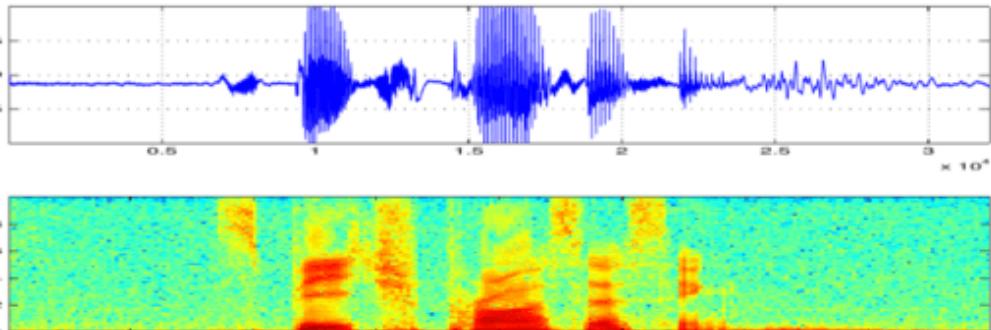


# State Space Representation

## Lecture-05

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- Definition
- Necessity
- Example
- Application
- Limitation
- Conclusion





# State Space representation:

- State Space Representation is a unified method for analyzing, modelling and designing a wide-range of system.
- A state space representation is a mathematical model of a physical system as a set of input , output & state variables related by **first order differential equation**.

Also referred to as modern system or Time domain representation .



# Necessity:

- To solve the non-linear and time-variant system.
- To model and analyse systems with multiple inputs and outputs.
- To analyse digital system.
- To analyse time variant system.

⊕ State Space System represent the future output based on current i/p & past memory. As only energy storage storage element has memory (capacitor, Inductor), so para<sup>para</sup>, cap. & Ind. will be state variable.

We can define more state variables than the minimal set; however, within this minimal set the state variables must be linearly independent. For example, if  $v_R(t)$  is chosen as a state variable, then  $i(t)$  cannot be chosen, because  $v_R(t)$  can be written as a linear combination of  $i(t)$ , namely  $v_R(t) = Ri(t)$ . Under these circumstances we say that the state variables are *linearly dependent*. State variables must be *linearly independent*; that is, no state variable can be written as a linear combination of the other state variables, or else we would not have enough information to solve for all other system variables, and we could even have trouble writing the simultaneous equations themselves.

The state and output equations can be written in vector-matrix form if the system is linear. Thus, Eq. (3.12), the state equations, can be written as

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \quad (3.15)$$

where

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{bmatrix} dq/dt \\ di/dt \end{bmatrix}; & \mathbf{A} &= \begin{bmatrix} 0 & 1 \\ -1/LC & -R/L \end{bmatrix} \\ \mathbf{x} &= \begin{bmatrix} q \\ i \end{bmatrix}; & \mathbf{B} &= \begin{bmatrix} 0 \\ 1/L \end{bmatrix}; & u &= v(t)\end{aligned}$$

Equation (3.13), the output equation, can be written as

$$y = \mathbf{Cx} + Du \quad (3.16)$$

where

$$y = v_L(t); \quad \mathbf{C} = [-1/C \quad -R]; \quad \mathbf{x} = \begin{bmatrix} q \\ i \end{bmatrix}; \quad D = 1; \quad u = v(t)$$

We call the combination of Eqs. (3.15) and (3.16) a *state-space representation* of the network of Figure 3.2. A state-space representation, therefore, consists of (1) the simultaneous, first-order differential equations from which the state variables can be solved and (2) the algebraic output equation from which all other system variables can be found. A state-space representation is not unique, since a different choice of state variables leads to a different representation of the same system.

In this section, we used two electrical networks to demonstrate some principles that are the foundation of the state-space representation. The representations developed in this section were for single-input, single-output systems, where  $y$ ,  $D$ , and  $u$  in Eqs. (3.15) and (3.16) are scalar quantities. In general, systems have multiple inputs and multiple outputs. For these cases,  $y$  and  $u$  become vector quantities, and  $D$  becomes a matrix. In Section 3.3 we will generalize the representation for multiple-input, multiple-output systems and summarize the concept of the state-space representation.

### 3.3 The General State-Space Representation

Now that we have represented a physical network in state space and have a good idea of the terminology and the concept, let us summarize and generalize the representation for linear differential equations. First, we formalize some of the definitions that we came across in the last section.

*Linear combination.* A linear combination of  $n$  variables,  $x_i$ , for  $i = 1$  to  $n$ , is given by the following sum,  $S$ :

$$S = K_n x_n + K_{n-1} x_{n-1} + \cdots + K_1 x_1 \quad (3.17)$$

where each  $K_i$  is a constant.

*Linear independence.* A set of variables is said to be linearly independent if none of the variables can be written as a linear combination of the others. For example, given  $x_1$ ,  $x_2$ , and  $x_3$ , if  $x_2 = 5x_1 + 6x_3$ , then the variables are not linearly independent, since one of them can be written as a linear combination of the other two. Now, what must be true so that one variable cannot be written as a linear combination of the other variables? Consider the example  $K_2x_2 = K_1x_1 + K_3x_3$ . If no  $x_i = 0$ , then any  $x_i$  can be written as a linear combination of other variables, unless all  $K_i = 0$ . Formally, then, variables  $x_i$ , for  $i = 1$  to  $n$ , are said to be linearly independent if their linear combination,  $S$ , equals zero *only* if every  $K_i = 0$  and *no*  $x_i = 0$  for all  $t \geq 0$ .

*System variable.* Any variable that responds to an input or initial conditions in a system.

*State variables.* The smallest set of linearly independent system variables such that the values of the members of the set at time  $t_0$  along with known forcing functions completely determine the value of all system variables for all  $t \geq t_0$ .

*State vector.* A vector whose elements are the state variables.

*State space.* The  $n$ -dimensional space whose axes are the state variables. This is a new term and is illustrated in Figure 3.3, where the state variables are assumed to be a resistor voltage,  $v_R$ , and a capacitor voltage,  $v_C$ . These variables form the axes of the state space. A trajectory can be thought of as being mapped out by the state vector,  $\mathbf{x}(t)$ , for a range of  $t$ . Also shown is the state vector at the particular time  $t = 4$ .

*State equations.* A set of  $n$  simultaneous, first-order differential equations with  $n$  variables, where the  $n$  variables to be solved are the state variables.

*Output equation.* The algebraic equation that expresses the output variables of a system as linear combinations of the state variables and the inputs.

Now that the definitions have been formally stated, we define the state-space representation of a system. A system is represented in state space by the following equations:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \quad (3.18)$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du} \quad (3.19)$$

for  $t \geq t_0$  and initial conditions,  $\mathbf{x}(t_0)$ , where

$\mathbf{x}$  = state vector

$\dot{\mathbf{x}}$  = derivative of the state vector with respect to time

$\mathbf{y}$  = output vector

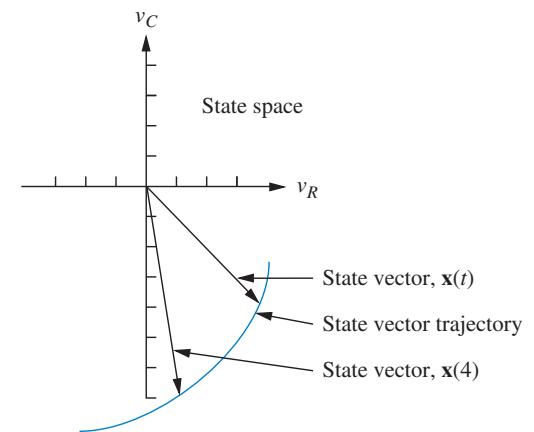
$\mathbf{u}$  = input or control vector

$\mathbf{A}$  = system matrix

$\mathbf{B}$  = input matrix

$\mathbf{C}$  = output matrix

$\mathbf{D}$  = feedforward matrix



**FIGURE 3.3** Graphic representation of state space and a state vector

(3.18)

(3.19)

Equation (3.18) is called the *state equation*, and the vector  $\mathbf{x}$ , the *state vector*, contains the state variables. Equation (3.18) can be solved for the state variables, which we demonstrate in Chapter 4. Equation (3.19) is called the *output equation*. This equation is used to calculate any other system variables. This representation of a system provides complete knowledge of all variables of the system at any  $t \geq t_0$ .

As an example, for a linear, time-invariant, second-order system with a single input  $v(t)$ , the state equations could take on the following form:

$$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + b_1v(t) \quad (3.20a)$$

$$\frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + b_2v(t) \quad (3.20b)$$

where  $x_1$  and  $x_2$  are the state variables. If there is a single output, the output equation could take on the following form:

$$y = c_1x_1 + c_2x_2 + d_1v(t) \quad (3.21)$$

The choice of state variables for a given system is not unique. The requirement in choosing the state variables is that they be linearly independent and that a minimum number of them be chosen.

## 3.4 Applying the State-Space Representation

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In this section, we apply the state-space formulation to the representation of more complicated physical systems. The first step in representing a system is to select the state vector, which must be chosen according to the following considerations:

1. A minimum number of state variables must be selected as components of the state vector. This minimum number of state variables is sufficient to describe completely the state of the system.
2. The components of the state vector (that is, this minimum number of state variables) must be linearly independent.

Let us review and clarify these statements.

### Linearly Independent State Variables

The components of the state vector must be linearly independent. For example, following the definition of linear independence in Section 3.3, if  $x_1$ ,  $x_2$ , and  $x_3$  are chosen as state variables, but  $x_3 = 5x_1 + 4x_2$ , then  $x_3$  is not linearly independent of  $x_1$  and  $x_2$ , since knowledge of the values of  $x_1$  and  $x_2$  will yield the value of  $x_3$ . Variables and their successive derivatives are linearly independent. For example, the voltage across an inductor,  $v_L$ , is linearly independent of the current through the inductor,  $i_L$ , since  $v_L = Ldi_L/dt$ . Thus,  $v_L$  cannot be evaluated as a linear combination of the current,  $i_L$ .

### Minimum Number of State Variables

How do we know the minimum number of state variables to select? Typically, the minimum number required equals the order of the differential equation describing the system. For example, if a third-order differential equation describes the system, then three simultaneous, first-order differential equations are required along with three state variables. From the perspective of the transfer function, the order of the differential equation is the order of the denominator of the transfer function after canceling common factors in the numerator and denominator.

In most cases, another way to determine the number of state variables is to count the number of independent energy-storage elements in the system.<sup>5</sup> The number of

<sup>5</sup> Sometimes it is not apparent in a schematic how many independent energy-storage elements there are. It is possible that more than the minimum number of energy-storage elements could be selected, leading to a state vector whose components number more than the minimum required and are not linearly independent. Selecting additional dependent energy-storage elements results in a system matrix of higher order and more complexity than required for the solution of the state equations.

these energy-storage elements equals the order of the differential equation and the number of state variables. In Figure 3.2 there are two energy-storage elements, the capacitor and the inductor. Hence, two state variables and two state equations are required for the system.

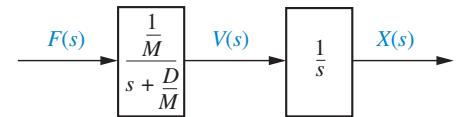
If too few state variables are selected, it may be impossible to write particular output equations, since some system variables cannot be written as a linear combination of the reduced number of state variables. In many cases, it may be impossible even to complete the writing of the state equations, since the derivatives of the state variables cannot be expressed as linear combinations of the reduced number of state variables.

If you select the minimum number of state variables but they are not linearly independent, at best you may not be able to solve for all other system variables. At worst you may not be able to complete the writing of the state equations.

Often the state vector includes more than the minimum number of state variables required. Two possible cases exist. Often state variables are chosen to be physical variables of a system, such as position and velocity in a mechanical system. Cases arise where these variables, although linearly independent, are also *decoupled*. That is, some linearly independent variables are not required in order to solve for any of the other linearly independent variables or any other dependent system variable. Consider the case of a mass and viscous damper whose differential equation is  $M \frac{dv}{dt} + Dv = f(t)$ , where  $v$  is the velocity of the mass. Since this is a first-order equation, one state equation is all that is required to define this system in state space with velocity as the state variable. Also, since there is only one energy-storage element, mass, only one state variable is required to represent this system in state space. However, the mass also has an associated position, which is linearly independent of velocity. If we want to include position in the state vector along with velocity, then we add position as a state variable that is linearly independent of the other state variable, velocity. Figure 3.4 illustrates what is happening. The first block is the transfer function equivalent to  $M\frac{dv}{dt} + Dv = f(t)$ . The second block shows that we integrate the output velocity to yield output displacement (see Table 2.2, Item 10). Thus, if we want displacement as an output, the denominator, or characteristic equation, has increased in order to 2, the product of the two transfer functions. Many times, the writing of the state equations is simplified by including additional state variables.

Another case that increases the size of the state vector arises when the added variable is not linearly independent of the other members of the state vector. This usually occurs when a variable is selected as a state variable but its dependence on the other state variables is not immediately apparent. For example, energy-storage elements may be used to select the state variables, and the dependence of the variable associated with one energy-storage element on the variables of other energy-storage elements may not be recognized. Thus, the dimension of the system matrix is increased unnecessarily, and the solution for the state vector, which we cover in Chapter 4, is more difficult. Also, adding dependent state variables affects the designer's ability to use state-space methods for design.<sup>6</sup>

We saw in Section 3.2 that the state-space representation is not unique. The following example demonstrates one technique for selecting state variables and representing a system in state space. Our approach is to write the simple derivative equation for each energy-storage element and solve for each derivative term as a linear combination of any of the system variables and the input that are present in the equation. Next we select each differentiated variable as a state variable. Then we express all other system variables in the equations in terms of the state variables and the input. Finally, we write the output variables as linear combinations of the state variables and the input.



**FIGURE 3.4** Block diagram of a mass and damper

<sup>6</sup> See Chapter 12 for state-space design techniques.



# REPRESENTATION OF A SYSTEM

A system is represented in state space by following equation-

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU}$$

$$\mathbf{y} = \mathbf{CX} + \mathbf{DU}$$

**$\dot{\mathbf{X}}$ =State vector**  
 **$\mathbf{x}$ =Derivative of  $x$  with respect to time**  
 **$\mathbf{y}$ =Output vector**  
 **$\mathbf{U}$ =Input or control vector**

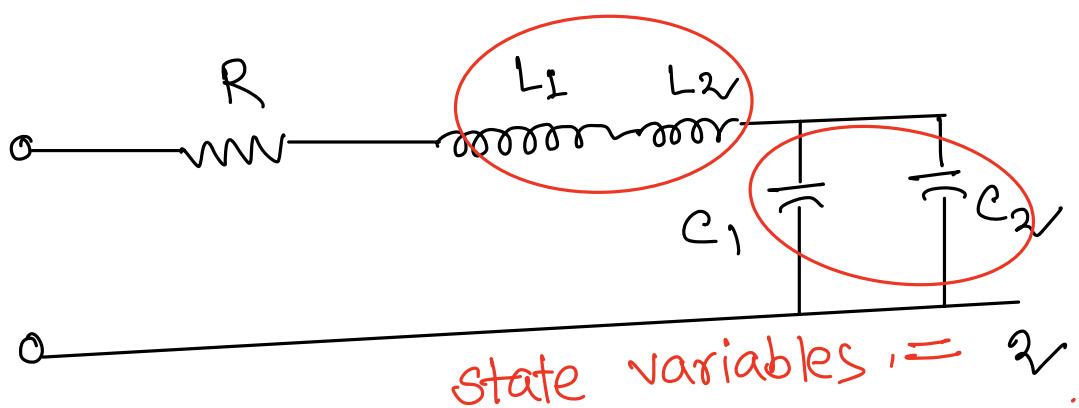
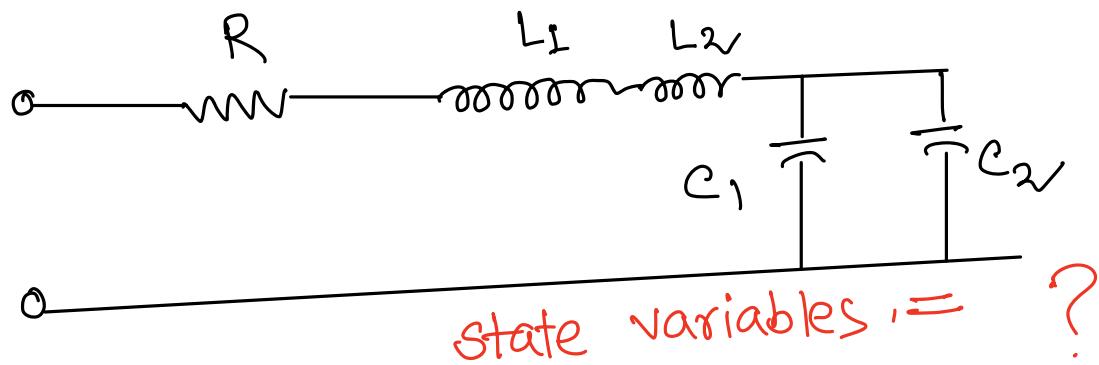
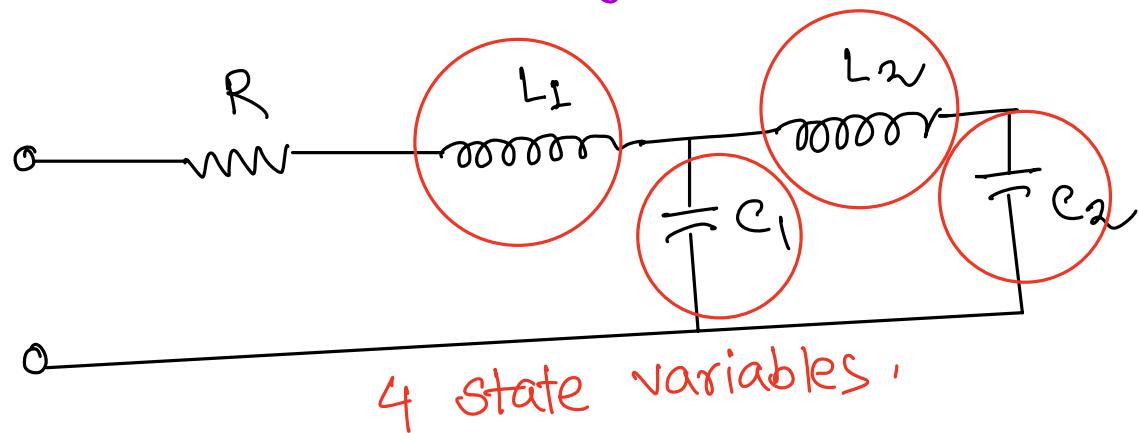
**A=System matrix**  
**B=Input matrix**  
**C=Output matrix**  
**D=Feed forward matrix**

order of the system defines the number of states variable.

Total no. of state variables  $\approx$  Max order of the system.

$$s^4 + 3s^3 + 2s^2 + 1 = 0$$

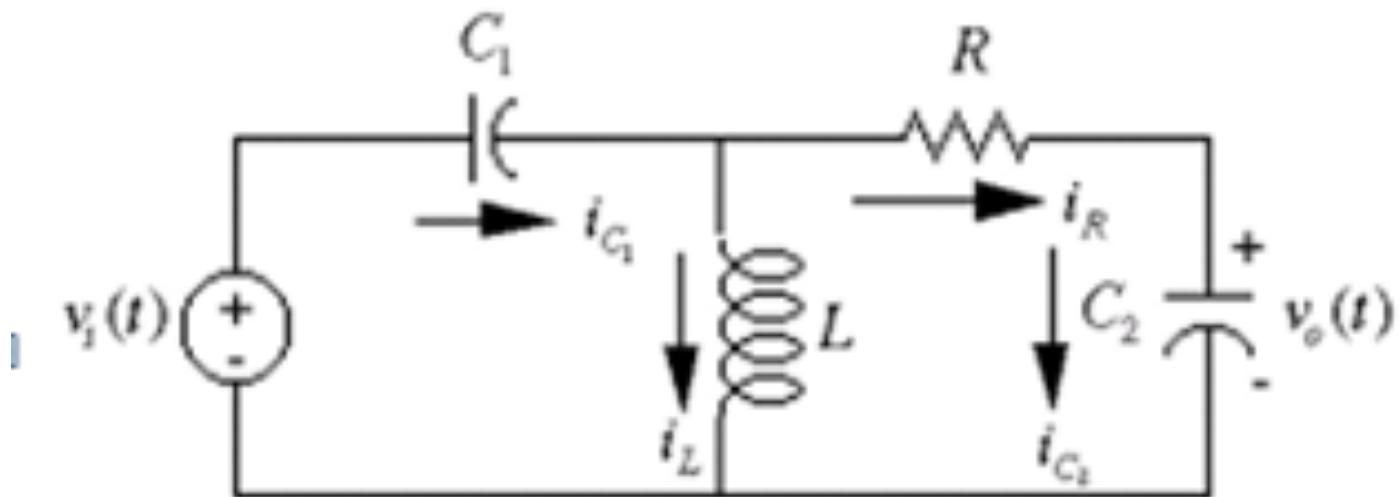
so, No. of State variables  $\approx 4$





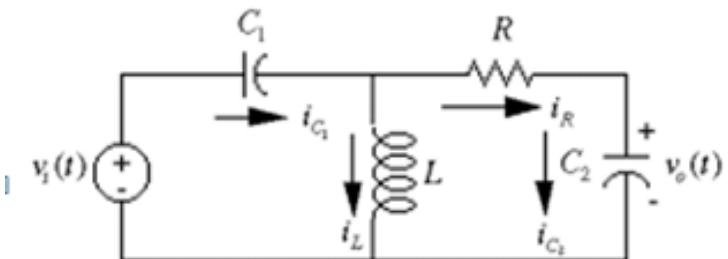
## Representing Electrical Network

Example: Find the state space representation of the Electrical network given below-





# Solution



**STEP 1:** Label all the branch current

**STEP 2:** Write the derivative equation for all energy storage element.

$$\rightarrow \frac{dV_{C1}}{dt} = \frac{i_{C1}}{C_1}$$

$$V_{C1}$$

$$\dot{X} = AX + BU$$

$$\rightarrow \frac{di_L}{dt} = \frac{V_L}{L}$$

$$i_L$$

$$V_i$$

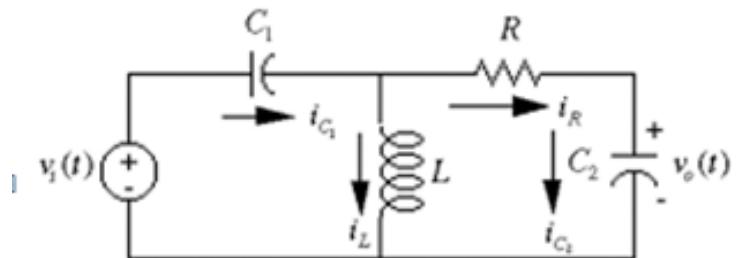
$$\rightarrow \frac{dV_{C2}}{dt} = \frac{i_{C2}}{C_2}$$

$$V_{C2}$$



**STEP 3:**

Convert  $V_L$  &  $i_C$  into state variable & input using KCL & KVL



$$\rightarrow \frac{dV_{C1}}{dt} = \frac{i_{C1}}{C1} \quad i_{C1} = i_L + \frac{(V_L - V_{C2})}{R}$$

$$\rightarrow \frac{di_L}{dt} = \frac{V_L}{L} \quad V_L = -V_{C1} + V_i$$

$$\rightarrow \frac{dV_{C2}}{dt} = \frac{i_{C2}}{C2} \quad I_{C2} = V_L - \frac{(V_L - V_{C2})}{R}$$



### STEP 4:

Putting the value into step 2-

$$\frac{dV_{c1}}{dt} = -\frac{V_{c1}}{RC_1} + \frac{i_L}{C_1} + -\frac{V_{c2}}{RC_1} + \frac{V_i}{RC_1}$$

$$\frac{di_L}{dt} = -\frac{V_{c1}}{L} + \frac{V_i}{L}$$

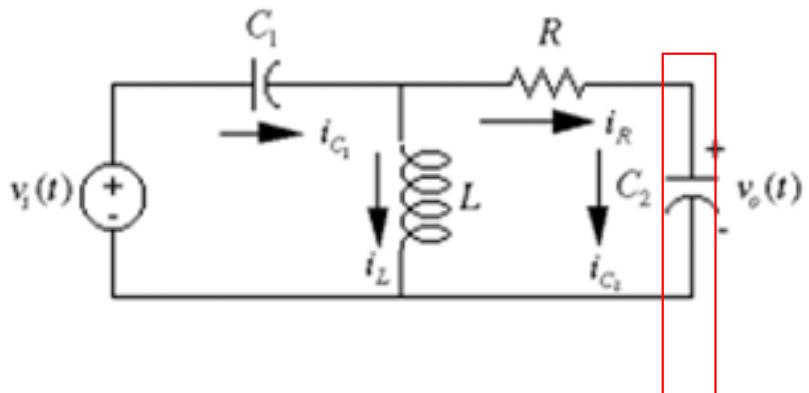
$$\frac{dV_{c2}}{dt} = -\frac{V_{c1}}{RC_2} + -\frac{V_{c2}}{RC_2} + \frac{V_i}{RC_2}$$



### STEP 5:

Find the output equation-

$$V_0 = V_{C2}$$



Vector matrix form-

$$\dot{\mathbf{X}} = \begin{bmatrix} -\frac{1}{R} & \frac{1}{C_1} & -\frac{1}{RC_1} \\ \frac{C}{L} & 0 & 0 \\ -\frac{1}{RC_2} & 0 & \frac{1}{RC_2} \end{bmatrix} \mathbf{X} + \begin{bmatrix} \frac{1}{RC_1} \\ \frac{1}{L} \\ \frac{1}{RC_2} \end{bmatrix} V_i$$

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU}$$

$$\mathbf{y} = \mathbf{CX} + \mathbf{DU}$$



# CONTINUE.....

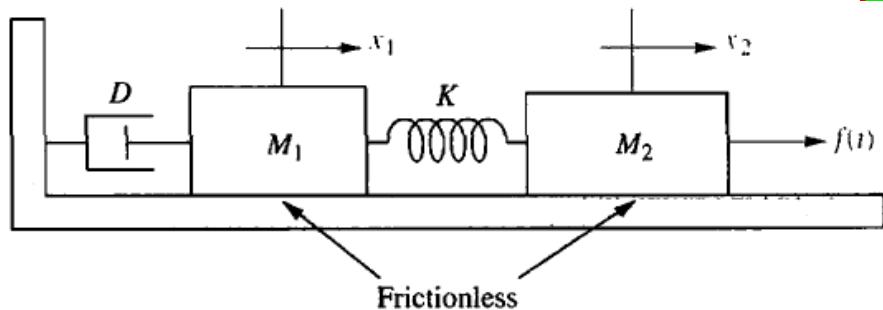
Output vector matrix

$$\mathbf{Y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} 0 \end{bmatrix}$$



# Example:

Find the state equations for the translational mechanical system shown in Figure.



$$M_1 \frac{d^2x_1}{dt^2} + D \frac{dx_1}{dt} + Kx_1 - Kx_2 = 0 \quad \text{--- 1}$$

$$-Kx_1 + M_2 \frac{d^2x_2}{dt^2} + Kx_2 = f(t) \quad \text{--- 2}$$



Now let  $d^2x_1/dt^2 = dv_1/dt$ , and  $d^2x_2/dt^2 = dv_2/dt$ ,  
 add  $dx_1/dt = v_1$  and  $dx_2/dt = v_2$

Now set of the state equations will be,

$$\frac{dx_1}{dt} = +v_1 \quad 3$$

$$\frac{dv_1}{dt} = -\frac{K}{M_1}x_1 - \frac{D}{M_1}v_1 + \frac{K}{M_1}x_2 \quad 4$$

$$\frac{dx_2}{dt} = +v_2 \quad 5$$

$$\frac{dv_2}{dt} = +\frac{K}{M_2}x - \frac{K}{M_2}x_2 + \frac{1}{M_2}f(t) \quad 6$$



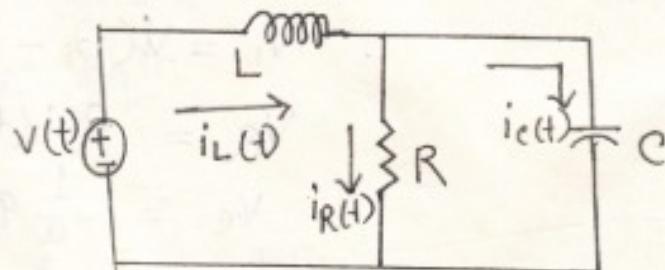
$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/M_1 & -D/M_1 & K/M_1 & 0 \\ 0 & 0 & 0 & 1 \\ K/M_2 & 0 & -K/M_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/M_2 \end{bmatrix} f(t)$$

(ii)  $\dot{x} = Ax + Bu$  = Same as before.

$$y = Cx + DU$$

$$\begin{bmatrix} V_L \\ V_R \\ V_c \end{bmatrix} = \begin{bmatrix} -1/c & -R \\ 0 & R \\ 1/c & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ i(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot v(t)$$

Example: 3.1 Find a state-space representation if the output is the current through the resistor.



Sol:

Step 1: Label all the branch current.

Step 2: Write the derivative eqn for all energy storage elements.

$$i_C = C \frac{dV_c}{dt} \quad \therefore \frac{dV_c}{dt} = \frac{1}{C} i_C$$

$$V_L = L \frac{di_L}{dt} \quad \therefore \frac{di_L}{dt} = \frac{1}{L} V_L$$

so,  $V_c$  &  $i_L$  are the state variable. & input.

Step 3: convert,  $i_C$  &  $V_L$  to state variable using

KCL & KVL.

$$V_L = V(t) - i_R(t) \cdot R \quad \Rightarrow \quad V_L = V(t) - V_c(t)$$

Step 4

Putting the value in step 2.

$$\therefore \frac{dVc(t)}{dt} = \frac{1}{C} i_L(t) - \frac{1}{RC} Vc(t)$$

$$\therefore \frac{di_L}{dt} = \frac{V(t)}{L} - \frac{Vc(t)}{L}$$

Step 5: Find the output eq?

$$i_R = \frac{Vc}{R}$$

State Space equation:-

$$\dot{x} = Ax + Bu$$

$$\star \begin{bmatrix} \frac{dVc}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} Vc \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{RC} \\ \frac{1}{L} \end{bmatrix} V(t)$$

$$y = cx + du$$

$$\begin{aligned} i_R &= [1/R \ 0] \begin{bmatrix} Vc \\ i_L \end{bmatrix} + 0 \cdot V(t) \\ &= [1/R \ 0] \begin{bmatrix} Vc \\ i_L \end{bmatrix} \end{aligned}$$

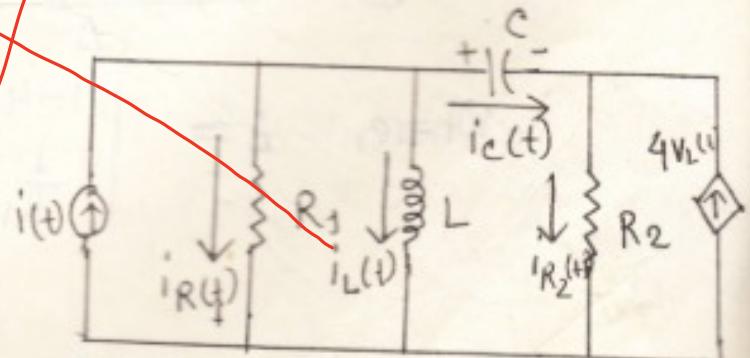
Example 3.2 Find the state & output equations for the electrical network, if the output vector  $y = [V_{R_2} \ i_{R_2}]^T$  where, T means transpose.

Soln:

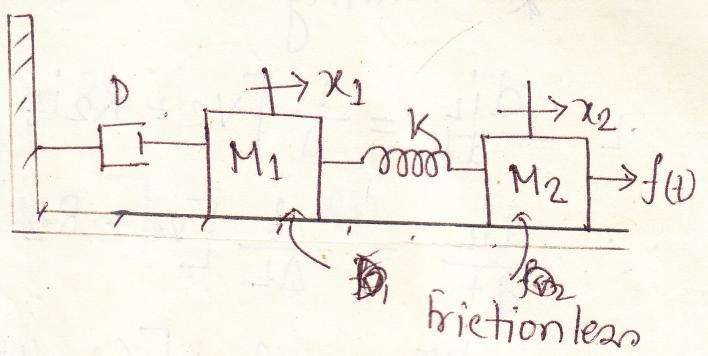
$$L \frac{di_L}{dt} = V_L$$

$$C \frac{dVc}{dt} = i_C$$

so,  $i$  &  $Vc$  are state variable



Example: 3.3 Find the state eqn's for the following network.



Sol<sup>n</sup>:

$$M_1 \frac{d^2x_1}{dt^2} + D \frac{dx_1}{dt} + Kx_1 - Kx_2 = 0$$

$$\& M_2 \frac{d^2x_2}{dt^2} - Kx_1 + M_2 \frac{d^2x_2}{dt^2} + Kx_2 = f(t)$$

or,

$$M_1 \frac{dV_1}{dt} + DV_1 + Kx_1 - Kx_2 = 0$$

$$\therefore \frac{dV_1}{dt} = -\frac{K}{M_1}x_2 - \frac{D}{M_1}V_1 - \frac{K}{M_1}x_1 \dots (i)$$

$$\frac{dx_1}{dt} = V_1 \dots (ii)$$

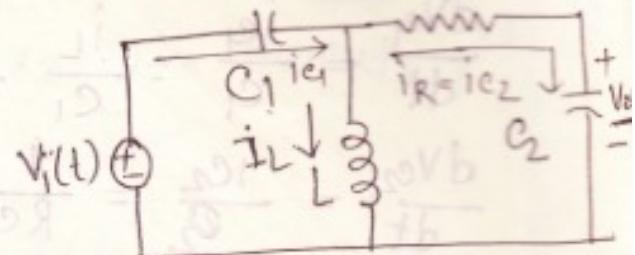
$$\frac{dV_2}{dt} = \frac{f(t)}{M_2} + \frac{K}{M_2}x_1 - \frac{K}{M_2}x_2 \dots (iii)$$

$$\frac{dx_2}{dt} = V_2 \dots (iv)$$

so, state vector eqn:

$$\begin{bmatrix} \dot{x}_1 \\ \vdots \\ V_1 \\ \dot{x}_2 \\ \vdots \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K}{M_1} & -\frac{D}{M_1} & \frac{K}{M_1} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{K}{M_2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ V_1 \\ x_2 \\ V_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1/M_2 \end{bmatrix} f(t)$$

**Exercise 3.1** Find the state-space representation of the electrical network. The output of which is across  $C_2$ .



Soln:

$$i_{C_1} = C_1 \frac{dV_{C_1}}{dt} \quad \text{--- (I)}$$

$$i_{C_2} = C_2 \frac{dV_{C_2}}{dt} \quad \text{--- (II)}$$

$$V_L = L \frac{di_L}{dt} \quad \text{--- (III)}$$

Now, convert  $i_{C_1}$ ,  $i_{C_2}$  &  $V_L$  into,  $V_{C_1}$ ,  $V_{C_2}$ ,  $i_L$  & inputs

$$i_{C_1} = i_L + i_{C_2}$$

$$\therefore i_{C_1} - i_{C_2} = i_L$$

$$\therefore i_{C_1} = i_L + i_R \quad [i_{C_2} = i_R]$$

$$= i_L + \frac{VR}{R}$$

$$= i_L + \frac{V_L - V_{C_2}}{R}$$

$$= i_L + \frac{1}{R} [V_i(t) - V_{C_1} - V_{C_2}]$$

$$\therefore i_{C_1} = i_L + \frac{1}{R} [V_i(t) - V_{C_1} - V_{C_2}]$$

$$i_{C_2} = i_R = \frac{1}{R} [V_i(t) - V_{C_1} - V_{C_2}]$$

$$V_L = V_i(t) - V_{C_1}$$

Now, Putting the value in eq<sup>n</sup> ①, ii & ③

$$\frac{dV_{C_1}}{dt} = \frac{i_C}{C_1} = \frac{i_L}{C_1} + \frac{1}{RC_1} [V_i(t) - V_{C_1} - V_{C_2}]$$

$$\frac{dV_{C_2}}{dt} = \frac{i_{C_2}}{C_2} = \frac{1}{RC_2} [V_i(t) - V_{C_1} - V_{C_2}]$$

$$\frac{di_L}{dt} = \frac{1}{L} V_i(t) - \frac{1}{L} V_{C_1}$$

$$\text{Output eq}^n: V_o = V_{C_2},$$

The static eq<sup>n</sup>:

$$\begin{bmatrix} \dot{V}_{C_1} \\ i_L \\ \dot{V}_{C_2} \end{bmatrix} = \begin{bmatrix} \frac{1}{RC_1} & \frac{1}{C_1} & -\frac{1}{RC_1} \\ -\frac{1}{L} & 0 & 0 \\ -\frac{1}{RC_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} V_{C_1} \\ i_L \\ V_{C_2} \end{bmatrix} + \begin{bmatrix} \frac{1}{RC_1} \\ \frac{1}{L} \\ \frac{1}{RC_2} \end{bmatrix} V_i(t)$$

$$\dot{x} = Ax + Bu$$

$$V_{C_2} = [0 \ 0 \ 1] \begin{bmatrix} V_{C_1} \\ i_L \\ V_{C_2} \end{bmatrix} + 0$$

$$y = cx + du$$



## Application:

- Can be used to represent a non-linear system that have backlash , saturation and dead zone.
- Can handle initial non-zero condition.
- Can represent time varying system.
- Availability of software packages.

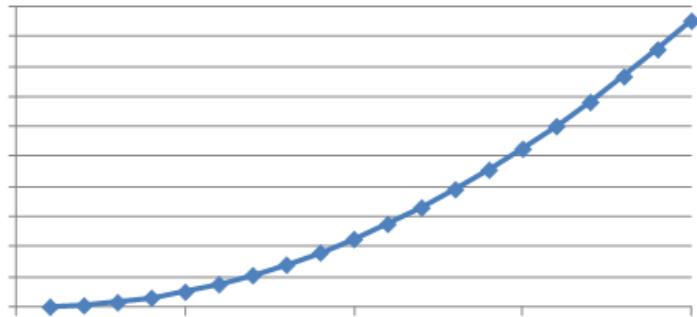


Fig 1: non-linear system.





## Limitations:

- Becomes difficult for systems of high order.
- Complex to analyse and takes more time .
- Provide less stability and transient response.





# Conclusion:

In this section , we have learned how to convert a transfer function representation to a state-space representation.

For linear, time-invariant systems, the state space representation a simply another way of mathematically modeling them .





# Assignment

Example; 3.1, 3.2, 3.3

Exercise; 3.1, 3.2

Problem; 1, 2, 3



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Section A

Present: 2257, 2250, 2234, 2179, 2115, 1819  
1, 2, 3, 5, 9, 10, 11, 15, 18, 20, 26, 27, 21, 35, 34,  
36, 39,

Section B

Present: 2292, 1880,  
49, 53, 56, 59, 60, 62, 71, 73, 87, 92.

Section C

912, 1510, 1025, 1081, 1105, 2266,  
93, 94, 97, 103, 102, 110, 108, 111, 114, 117, 119,  
123, 124, 128, 132, 133, 131, 136, 139,



Thank you!  
Question ?

