

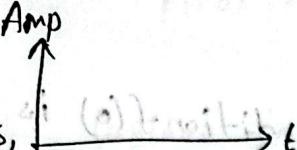
Control Systems

EEE 313

Control Systems Laboratory

EEE 314

Laplace transformation:

A signal is, 

Fourier transformable if

- In a fixed period, no. of minima and maxima has to be finite.
- " " no. of discontinuity has to be finite.
- Integral of the signal has to be finite.

$$L\{f(t)\} = F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

$$L^{-1}[F(s)] =$$

$$L\{e^{-at} u(t)\} = \frac{1}{s+a}$$

Time shifting property:

$$\int f(t) dt \leftrightarrow \frac{F(s)}{s}$$

$$\int u(t) dt \leftrightarrow \frac{1}{s^2}$$

Time differentiation:

$$\frac{d}{dt} f(t) \longrightarrow sF(s) - f(0)$$

The initial condition $f(0)$ is given.

$$\star \frac{d^2}{dt^2} f(t) \longrightarrow s[sF(s) - f(0)] - f'(0)$$

$$\longrightarrow s^2 F(s) - sf(0) - f'(0)$$

$$\star \frac{d^3}{dt^3} f(t) \longleftrightarrow s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$$

General form:

$$L\left[\frac{df(t)^n}{dt^n}\right] =$$

$$L[t] = \frac{1}{s^2}$$

$$L[t^n] = \frac{n!}{s^{n+1}}$$

$$L\{t e^{-at}\} = \frac{1}{(s+a)^2}$$

$$L\{t^4 e^{-2t}\} = \frac{4!}{(s+2)^5}$$

$$L\{t^n e^{-at}\} = \frac{n!}{(s+a)^{n+1}}$$

$$L\{\sin(\omega t)\} = \frac{\omega}{s^2 + \omega^2}$$

$$L\{\cos(\omega t)\} = \frac{s}{s^2 + \omega^2}$$

$$e^{-at} \sin(\omega t) = \frac{\omega}{(s+a)^2 + \omega^2}$$

$$e^{-at} \cos(\omega t) = \frac{s+a}{(s+a)^2 + \omega^2}$$

$$L\{u(t)\} = \frac{1}{s}$$

$$L\{e^{-at}\} = \frac{1}{s+a}$$

$$L\{t^n e^{-at}\} = \frac{n!}{(s+a)^{n+1}}$$

$$L\{t^n u(t)\} = \frac{n!}{s^{n+1}}$$

Theorem: Initial value theorem
Final value theorem.

Q. Find the Laplace transform of $i(t) = e^{-5t} \sin 3t$ (10)

Q. Find the inverse Laplace transform of $F(s) = \frac{10}{s(s+2)(s+3)^2}$

$$F(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3} + \frac{D}{(s+3)^2} = \frac{10}{s(s+2)(s+3)^2}$$

$$\Rightarrow 10 = A(s+2)(s+3)^2 + B.s(s+3)^2 + C.s(s+2)(s+3) + D.s(s+2)$$

Putting, $s=0$ | $A = \frac{5}{9}$ | $B = -5$ | $C = \frac{10}{9}$ | $D = \frac{10}{3}$

Inverse

$$(1) i = \frac{5}{9} + \frac{-5}{s+2} + \frac{\frac{10}{9}s}{(s+3)^2} = \frac{5}{9} + \frac{-5}{s+2} + \frac{10s}{9(s+3)^2} + \frac{10}{3(s+3)}$$

$$(2) t \leftarrow (1)t$$

$$(3) t \rightarrow (2)t \leftarrow \frac{(1)t}{t+3}$$

$$(4) t \rightarrow (3)t \leftarrow \frac{(1)t^2}{(t+3)^2}$$

$$(2) 9s + (3)s^2 + (1)s^2 = (2)s^2 + (2)s^2 + (2)s^2 + (2)s^2$$

$$\{s^2 + s^2 + s^2\} (2)s^2 = \{s^2 + s^2 + s^2 + s^2\} (2)s^2$$

$$\frac{s^2 + s^2 + s^2}{s^2 + s^2 + s^2 + s^2} = \frac{(2)s^2}{(2)s^2} \leftarrow$$

Mathematical Modeling of Transfer Function:

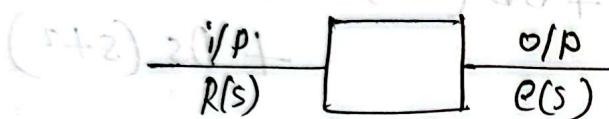
1) Transfer function

2) State space representation

* What is model?

→ is a simplified representation or abstraction of reality.

* Transfer function:



$$T.F., G(s) = \frac{\text{output}(s)}{\text{input}(s)}$$

Exercise: 2.3 find the transfer function, $G(s) = \frac{C(s)}{R(s)}$ corresponding to differential equation:

$$\frac{d^3Q}{dt^3} + 3 \cdot \frac{d^2C}{dt^2} + 7 \cdot \frac{dC}{dt} + 5C = \frac{d^2R}{dt^2} + 4 \cdot \frac{dR}{dt} + 3R. \quad \textcircled{1}$$

We know,

$$f(t) \rightarrow F(s)$$

$$\frac{df(t)}{dt} \rightarrow sF(s) - f(0)$$

$$\frac{d^2f(t)}{dt^2} \rightarrow s^2 F(s) - sf(0) - f'(0)$$

from $\textcircled{1}$

$$s^3 C(s) + 3s^2 C(s) + 7s C(s) + 5C(s) = s^2 R(s) + 4s R(s) + 3R(s)$$

$$\Rightarrow C(s) \{s^3 + 3s^2 + 7s + 5\} = R(s) \{s^2 + 4s + 3\}$$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{s^2 + 4s + 3}{s^3 + 3s^2 + 7s + 5}$$

Exercise 2.4 Find the differential equation corresponding to the transfer function,

$$G(s) = \frac{2s+1}{s^2+6s+2} \quad (1)$$

Given,

$$G(s) = \frac{C(s)}{R(s)} = \frac{2s+1}{s^2+6s+2}$$

$$\Rightarrow C(s)s^2 + C(s)6s + C(s)2 = R(s)2s + R(s) \quad (ii)$$

$$\Rightarrow \frac{d^2c}{dt^2} + 6\frac{dc}{dt} + 2c = 2\frac{dr}{dt} + r$$



Exercise 2.5 Find the ramp response for a system whose transfer function is $G(s)$

$$= \frac{s}{(s+4)(s+8)}$$

Therefore,

$$C(s) = G(s) \cdot R(s)$$

$$= \frac{1}{s^2} \cdot \frac{s}{(s+4)(s+8)}$$

$$= \frac{1}{s(s+4)(s+8)}$$

$$\frac{1}{s(s+4)(s+8)} = \frac{A}{s} + \frac{B}{(s+4)} + \frac{C}{(s+8)}$$

$$A = \frac{1}{32}, B = -\frac{1}{16}, C = \frac{1}{32}$$

$$G(s) = \frac{C(s)}{R(s)}$$

$$\Rightarrow C(s) = G(s) \cdot R(s).$$

$$L(f) = \frac{1}{s^2}$$

$$L\{u(f)\} = \frac{1}{s}$$

$$r(f) = t u(f)$$

$$\therefore R(s) = \frac{1}{s^2}$$

spot No V

$$C(s) = \frac{1}{32s} - \frac{1}{16(s+9)} + \frac{1}{32(s+8)}$$

$$= 32u(t) - \frac{1}{16}e^{-9t} + \frac{1}{32}e^{-8t}$$

Electrical Network Transfer functions:

- Systems
- i) \rightarrow Electrical Systems.
 - ii) \rightarrow Mechanical Systems.
 - i) Linear System
 - ii) Rotational system

Electrical Systems:

- \rightarrow Passive (doesn't store charge) element
- \rightarrow Active (can store charge) element

Resistor \rightarrow Passive

Capacitor \rightarrow Active

Resistor \rightarrow Passive

Voltage- Current relationship:

voltage across capacitor, $V_C(t) = \frac{1}{C} \int i(t) dt$

$$i(t) \quad V$$

voltage across inductor, $V_L(t) = L \frac{di(t)}{dt} + A$

$$i(t) \quad V$$

$$\frac{V}{L} = \frac{1}{R} + A \cdot \frac{1}{R^2} = A \cdot \left(\frac{1}{R} + \frac{1}{L^2} \right)$$

$$\frac{df(t)}{dt} \leftrightarrow sF(s)$$

$$\int f(t) dt \leftrightarrow \frac{1}{s} F(s)$$

Capacitor:

$$V_C(t) = \frac{1}{C} \int i(t) dt$$

$$\text{LT} \rightarrow V_C(s) = \frac{1}{C} \cdot \frac{1}{s} I(s) = \frac{1}{Cs} I(s) + \frac{(1)i_b}{s} \cdot 1 = (1)V$$

$$V_C(s) = \frac{1}{Cs} \cdot I(s)$$

$$\Rightarrow \frac{V_C(s)}{I(s)} = \frac{1}{Cs} \times \frac{1}{s} + \frac{(1)i_b}{s} + \frac{(1)i_b}{s} = (1)V$$

$$\Rightarrow Z(s) = \frac{1}{Cs} + \frac{(1)i_b}{s} + \frac{(1)i_b}{s} =$$

↳ impedance.

Inductor:

$$V_L(t) = L \frac{di(t)}{dt}$$

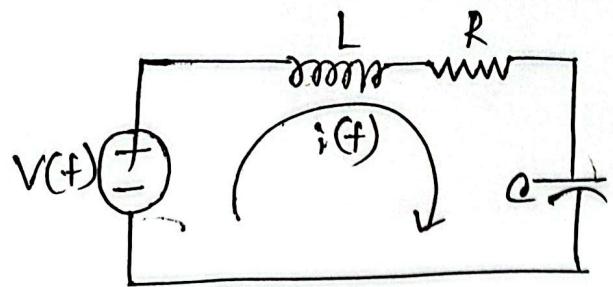
$$\Rightarrow K(s) = L \cdot s \cdot I(s)$$

$$\Rightarrow \frac{V_L(s)}{I(s)} = LS$$

Resistor:

$$\frac{V_R(s)}{I(s)} = R \quad (\text{as passive element})$$

$$\frac{1}{1 + 9A_2 + 9B_2} = \frac{(2)V}{(2)V} \Leftarrow$$



$$V(t) = L \cdot \frac{di(t)}{dt} + R \cdot i(t) + \frac{1}{C} \int i dt$$

We know,

$$i(t) = C \cdot \frac{dV_C(t)}{dt}$$

$$(2) D \cdot \frac{1}{23} = (2)_3 V$$

$$V(t) = L \frac{d^2 V_c(t)}{dt^2} + RQ \cdot \frac{dV_c(t)}{dt} + \frac{1}{C} \times \int C \cdot \frac{dV_c(t)}{dt} dt$$

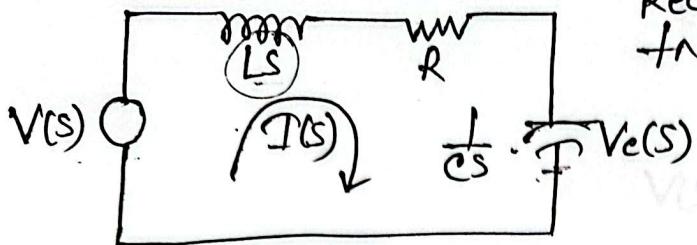
$$= LQ \cdot \frac{d^2 V_e(f)}{dt^2} + RQ \cdot \frac{dV_e(f)}{dt} + V_e(f) = (2) \leftarrow$$

initial response

Applying Laplace transformation,

$$V(s) = L C \cdot s^2 V_c(s) + R C \cdot s \cdot V_c(s) + V_c(s)$$

ii) Using mesh analysis:



Redraw circuit with Laplace transform:

$$\frac{V_c(s)}{V(s)} = \frac{R}{R + \frac{1}{Cs} + Ls}$$

Applying KVL:

$$\begin{aligned} V(s) &= Ls \times I(s) + R \times I(s) + I(s) \times \left(\frac{1}{Cs} \right) \\ &= I(s) \left[Ls + R + \frac{1}{Cs} \right] \end{aligned}$$

We know,

$$\frac{V_c(s)}{I(s)} = \frac{1}{Cs}$$

$$\Rightarrow I(s) = Cs \cdot V_c(s)$$

Therefore,

$$V(s) = Cs \cdot V_c(s) \cdot \left[Ls + R + \frac{1}{Cs} \right]$$

$$\Rightarrow \frac{V(s)}{V_c(s)} = Cs \left[Ls + R + \frac{1}{Cs} \right]$$

$$\Rightarrow \frac{V(s)}{V_c(s)} = s^2 CL + RCS + 1$$

$$\Rightarrow \frac{V_c(s)}{V(s)} = \frac{1}{1 + RCS + s^2 CL}$$

iii) Ohm's law:

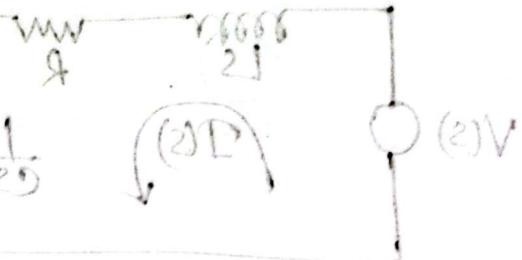
$$I \propto V$$

Voltage divider

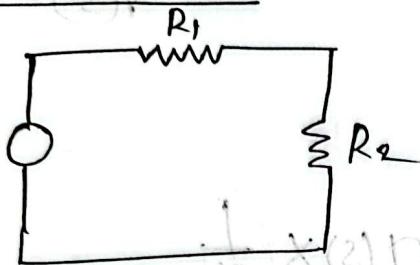
KVL
KCL

: zielone żółte pion (ii)

$$V = \frac{2}{2+1} \cdot 2V = \frac{2}{3} \cdot 2V = \frac{4}{3}V$$



Voltage divider:



$$V_2 = \frac{R_2}{R_1 + R_2} \times V$$

$$\left[\frac{1}{20} + 9 + 2 \right] (2)V =$$

$$\frac{V_c(s)}{V(s)} = ?$$

$$\frac{1}{20} = \frac{(2)V}{(2)sD}$$

$$(2)V \cdot 20 = (2)sD \quad \leftarrow$$

$$V_c(s) = V(s) \cdot \frac{1/cs}{Ls + R + 1/cs}$$

$$\Rightarrow \frac{V_c(s)}{V(s)} = \frac{\frac{1}{cs}}{\frac{s^2 L c + R s + 1}{cs}}$$

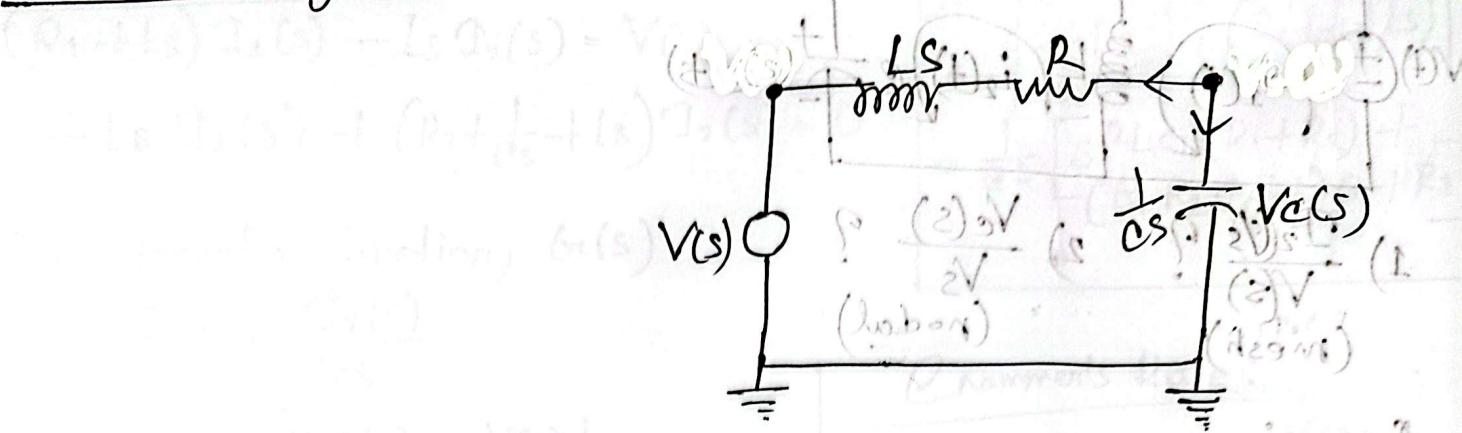
$$\left[\frac{1}{20} + 9 + 2 \right] \cdot (2)V \cdot 20 = (2)V$$

$$= \frac{1}{1 + Rcs + s^2 Lc} \cdot \left[\frac{1}{20} + 9 + 2 \right] 20 = \frac{(2)V}{(2)V} \quad \leftarrow$$

$$1 + 200 + 10^3 \Omega = \frac{(2)V}{(2)V} \quad \leftarrow$$

$$\frac{1}{10^3 + 200 + 1} = \frac{(2)V}{(2)V} \quad \leftarrow$$

iv) Nodal Analysis:



$$\frac{V_c(s) - 0}{1/s} + \frac{V_c(s) - V(s)}{Ls + R} = 0$$

$$\Rightarrow s/V_c(s) + \frac{V_c(s)}{Ls + R} = \frac{V(s)}{Ls + R}$$

$$\Rightarrow V_c(s) \left[s + \frac{1}{Ls + R} \right] = \frac{V(s)}{Ls + R}$$

$$\Rightarrow \frac{V_c(s)}{1} \cdot \left[\frac{s^2 L c + R s + 1}{Ls + R} \right] = \frac{V(s)}{Ls + R}$$

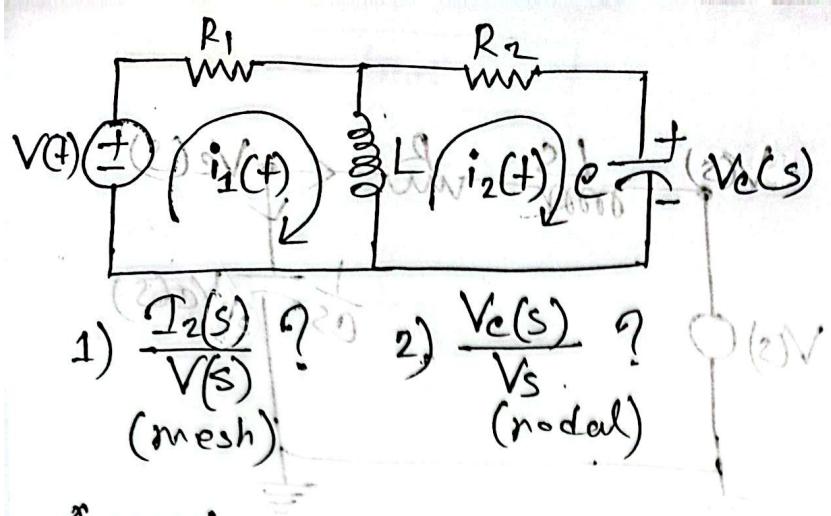
$$\Rightarrow \frac{V_c(s)}{V(s)} = \frac{1}{s^2 L c + R s + 1}$$

(2), D [consequence to node] \rightarrow (2), D [consequence to node]

new bridge to node
[draw it]

(2), D [consequence to node] \rightarrow (2), D [consequence to node]

new bridge to node
[draw it]



Leptora laboii (Vi)

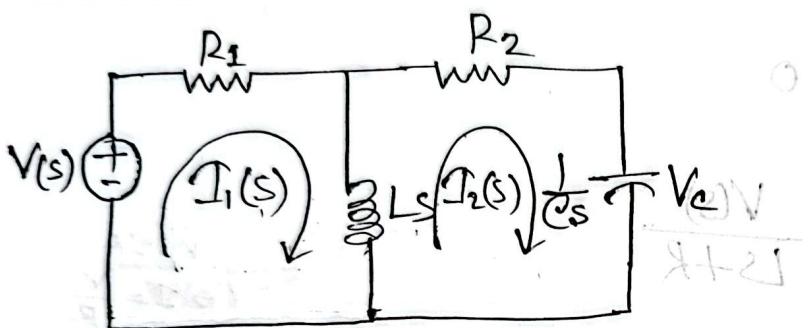
$$1) \frac{I_2(s)}{V(s)} ?$$

(mesh):

$$2) \frac{V_c(s)}{V_s} \quad ?$$

(2) V

Answer:



$$O = \frac{(2)V - (2)\delta V}{8+2} + \frac{O-(2)\delta V}{2(2)T}$$

$$I = \frac{(2) \text{ V}}{8 + 2} + (2) \text{ V } 20 \quad \leftarrow$$

$$I_2(s) = \left[\frac{1}{s+2} + 20 \right] \quad (2) \check{v} \leftarrow$$

$$\Rightarrow I_1(s)(R_1 + L_s) - L_s I_2(s) = V(s) \quad \text{--- (i)}$$

$$\Rightarrow R_1 T_1(s) + \frac{1}{Cs} T_2(s) + L_s T_2(s) - L_s T_1(s) = 0$$

$$\Rightarrow I_2(s) \left(R_1 + \frac{1}{C_s} + L_s \right) - L_s I_1(s) = 0 \quad \text{ii}$$

from (i) we can write

$$[\text{sum of impedance around mesh 1}] I_1(s) - [\text{sum of impedance around mesh ① and ②}] I_2(s) = [\text{sum of applied source in mesh 1}]$$

from (ii)

$$[\text{Sum of impedance around mesh 2}] I_2(s) - [\text{sum of impedance around mesh ① and ②}] I_1(s) = [\text{sum of applied source in mesh 2}]$$

From ① and ②

$$(R_1 + Ls) I_2(s) - Ls I_1(s) = V(s)$$

$$-Ls I_1(s) + (R_2 + \frac{1}{Cs} + Ls) I_2(s) = 0$$

Now, transfer function, $G(s)$

$$G(s) = \frac{I_2(s)}{V(s)}$$

$$I_2(s) = \frac{\begin{vmatrix} (R_1 + Ls) & V(s) \\ -Ls & 0 \end{vmatrix}}{\Delta}$$

$$\frac{(2)V}{s^2} = \left(\frac{1}{s} \right) (2)V +$$

$$2V \cdot \frac{(2)V}{s^2} = \left(\frac{1}{s} \right) (2)V +$$

$$\Delta = \begin{vmatrix} (R_1 + Ls) & -Ls \\ -Ls & (R_2 + \frac{1}{Cs} + Ls) \end{vmatrix}$$

$$= \frac{1}{Cs} \left[s^2 LC (R_1 + R_2) + (R_1 R_2 C + L) s + R_1 \right]$$

Crammer's Rule:

$$a_1 x + b_1 y = c_1 \quad \textcircled{1}$$

$$a_2 x + b_2 y = c_2 \quad \textcircled{2}$$

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\Delta}$$

$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\Delta}$$

$$\Delta = \frac{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}{\Delta} = \frac{a_1 b_2 - a_2 b_1}{\Delta}$$

$$\frac{(2)V}{s^2} = \frac{1}{s^2} (2V + 2sV)$$

$$-\frac{(2)V}{s^2} = \left(\frac{1}{s} + 2 \right) (2V + 2sV)$$

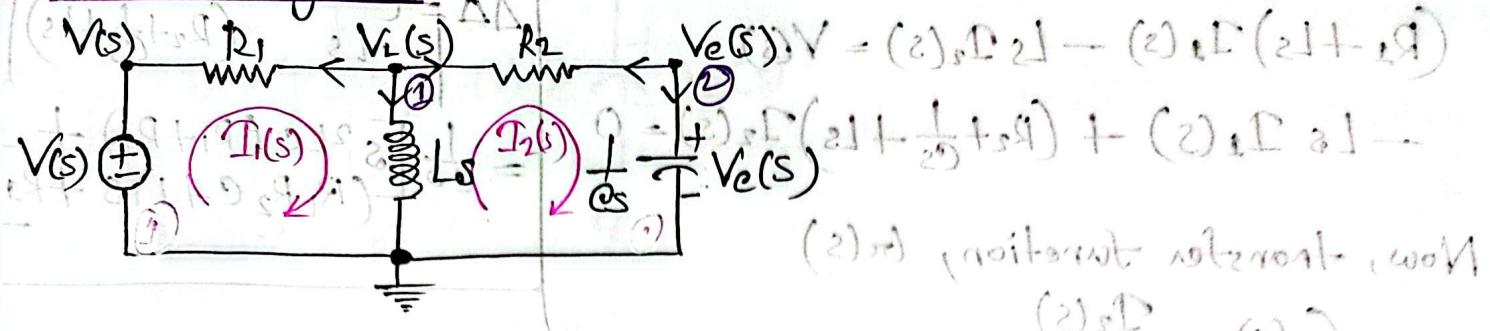
Defn.

$$\frac{I_2(s)}{V(s)} = \frac{(2)V}{s^2 LC (R_1 + R_2) + (R_1 R_2 C + L) s + R_1} = (2)V$$

$$= \frac{L C s^2}{s^2 LC (R_1 + R_2) + (R_1 R_2 C + L) s + R_1} = \frac{(2)V}{(2)V}$$

Nodal analysis:

(1) b/w ① node



For node ①

$$\frac{V_L(s) - V(s)}{R_1} + \frac{V_L(s) - 0}{Ls} + \frac{V_L(s) - V_c(s)}{R_2} = 0$$

$$\Rightarrow \frac{V_L(s)}{R_1} - \frac{V(s)}{R_1} + \frac{V_L(s)}{Ls} + \frac{V_L(s)}{R_2} - \frac{V_c(s)}{R_2} = 0$$

$$\Rightarrow V_L(s) \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{Ls} \right] + V_c(s) \left(-\frac{1}{R_2} \right) = -\frac{V(s)}{R_1}$$

$$\Rightarrow V_L(s) \left[G_{r1} + G_{r2} + \frac{1}{Ls} \right] - V_c(s) G_{r2} = V(s) \cdot G_{r1}$$

For node ③

$$\frac{V_c(s) - V_L(s)}{R_2} + \frac{V_c(s) - 0}{Cs} = 0$$

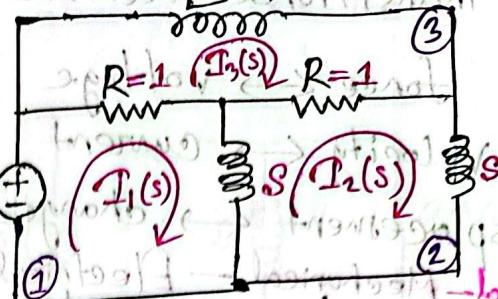
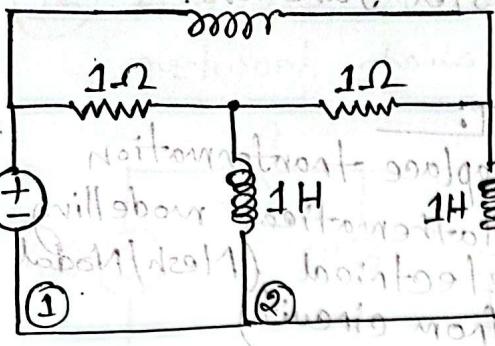
$$\Rightarrow \frac{V_c(s)}{R_2} - \frac{V_L(s)}{R_2} + \frac{V_c(s)}{Cs} = 0$$

$$\Rightarrow V_L(s) \left[-\frac{1}{R_2} \right] + V_c(s) \left[\frac{1}{R_2} + C_s \right] = 0$$

$$A = \begin{vmatrix} G_{r1} + G_{r2} + \frac{1}{Ls} & -G_{r2} \\ -G_{r2} & G_{r2} + C_s \end{vmatrix} = (G_{r2} + C_s)(G_{r1} + G_{r2} + \frac{1}{Ls}) - G_{r2}^2$$

$$\therefore V_c(s) = \frac{\begin{vmatrix} G_{r1} + G_{r2} + \frac{1}{Ls} & G_r V(s) \\ -G_{r2} & 0 \end{vmatrix}}{(G_{r2} + C_s)(G_{r1} + G_{r2} + \frac{1}{Ls}) - G_{r2}^2} = \frac{G_{r1} G_{r2} V(s)}{(G_{r2} + C_s)(G_{r1} + G_{r2} + \frac{1}{Ls}) - G_{r2}^2}$$

$$\therefore \frac{V_c(s)}{V(s)} = \frac{G_{r1} G_{r2}}{(G_{r2} + C_s)(G_{r1} + G_{r2} + \frac{1}{Ls}) - G_{r2}^2} + \frac{(G_{r1} + G_{r2})}{(G_{r2} + C_s)(G_{r1} + G_{r2} + \frac{1}{Ls}) - G_{r2}^2}$$



For loop ①

$$(1+s)I_1(s) - sI_2(s) - I_3(s) = V(s) \quad \text{--- (i)}$$

For loop ②

$$(1+2s)I_2(s) - s \cdot I_1(s) - I_3(s) = 0 \quad \text{--- (ii)}$$

For loop ③

$$(2+s)I_3(s) - I_1(s) - I_2(s) = 0 \quad \text{--- (iii)}$$

$$\Delta = \begin{vmatrix} (1+s) & -s & -1 \\ -s & (1+2s) & -1 \\ -1 & -1 & (2+s) \end{vmatrix} = s^3 + 5s^2 + 2s$$

$$\frac{V_L(s)}{V(s)} = ?$$

$$\frac{I_3(s)}{V(s)} = ?$$

$$V_L(s) = I_3(s) \times \frac{1}{s} \Rightarrow I_3(s) = s \cdot V_L(s)$$

$$I_3(s) = \frac{\begin{vmatrix} (1+s) & -s & V(s) \\ -s & (1+2s) & 0 \\ -1 & -1 & 1 \end{vmatrix}}{s^3 + 5s^2 + 2s}$$

2.5. Transitional mechanical system transfer functions:

force \leftrightarrow voltage

velocity \leftrightarrow current

displacement \leftrightarrow charge

Mechanical \rightarrow Electrical

spring (k) \rightarrow capacitor

mass (m) \rightarrow inductor

viscous damper (f_{sr}) \rightarrow resistor

CT: $\frac{d}{dt} \rightarrow \frac{d}{ds}$

\rightarrow Laplace transformation

\rightarrow Mathematical modelling
electrical (Mesh/Nodal
from circuit.)

Representation:

Spring $\rightarrow k$

viscous damper $\rightarrow f_{sr}$

mass $\rightarrow m s^2$

force \leftrightarrow voltage

Mesh ①

$$[M_1 s^2 + (k_1 + k_2) + s(f_{v1} + f_{v3})]x_1(s) - (k_2 + f_{v3}s)x_2(s) = f(s)$$

Mesh ②

$$[M_2 s^2 + (k_2 + k_3) + s(f_{v2} + f_{v3})]x_2(s) - (k_2 + s \cdot f_{v3})x_1(s) = 0$$

Transfer function \rightarrow

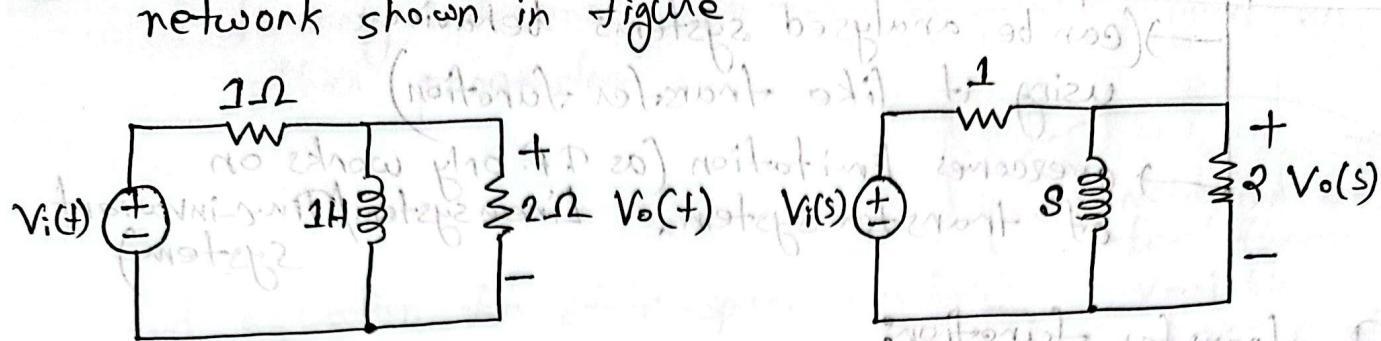
Analyses behaviour
of system

Linear system / Time-invariant
System.

$$T(s) = \frac{C(s)}{X(s)}$$

$$\Rightarrow C(s) = T(s) \cdot X(s)$$

(17) Find the transfer function, $G(s) = V_o(s)/V_i(s)$, for each network shown in figure

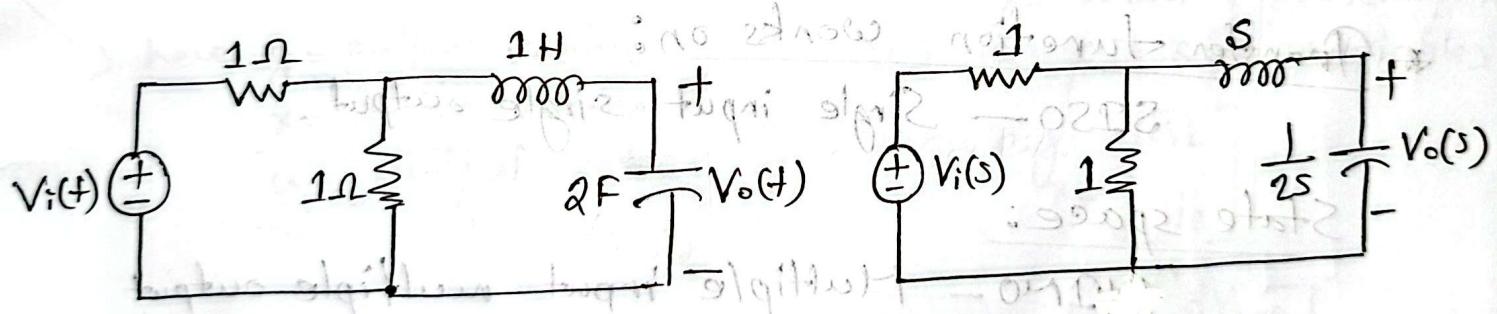


$$\text{Total impedance} = 1 + s + 2$$

$$= s + 3$$

$$V_o(s) = \frac{V_i(s)}{s+3} \times 2$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{2}{s+3}$$



$$\text{Total impedance} = 1 + s + \frac{1}{2s}$$

$$V_o(s) = \frac{1}{1+s+\frac{1}{2s}} \times V_i(s)$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{1}{2s^2 + 2s + 1}$$

State space representation:

- can be analysed system's behaviour through using it like transfer function
- overcomes limitation (as T.F. only works on linear system/time-invariant system)

* In transfer function:

output depends on current input

In state-space:

- output depends on current input as well as previous state variable
 - it can analyse wide-range of system (not only linear)
- ~~past memory~~ — capacitor and inductor are used for it.

* Transfer function works on:

SISO — Single input single output

State space:

MIMO — Multiple input multiple output

Question: Differentiate between transfer function and state space representation.
 (it is necessary over transfunc. to analyse a system)

capacitor, inductor — stores energy, works as memory

State variable: linearly independent variable, doesn't depend on other parameters.

Required equations for state-space analysis

Combination of two/three variable
not a state variable

derivative of x , $\dot{x} = \frac{dx}{dt}$

$$i) \dot{x} = Ax + Bu \rightarrow \text{state equation}$$

$$ii) y = Cx + Du \rightarrow \text{output equation}$$

$$\ddot{x} = \frac{d^2x}{dt^2}$$

$$\dddot{x} = \frac{d^3x}{dt^3}$$

here, x = state vector
 u = input vector

here, y = output vector
 x = state vector
 u = output vector.

$$A =$$

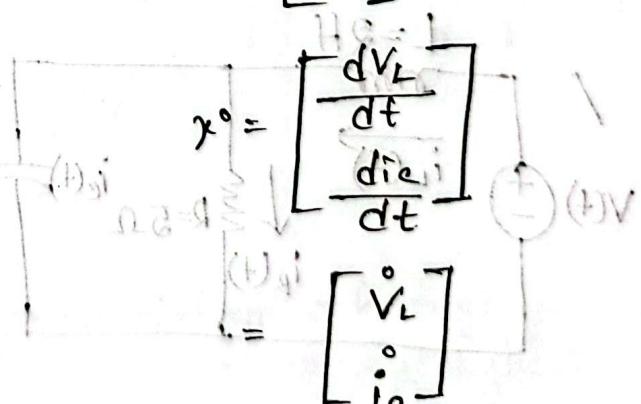
$$B =$$

$$C =$$

$$D =$$

x = vector representation
of state variables

e.g. $x = \begin{bmatrix} V_L \\ i_C \end{bmatrix}$



A system is represented in state space by following equation,

$$\dot{x} = Ax + Bu$$

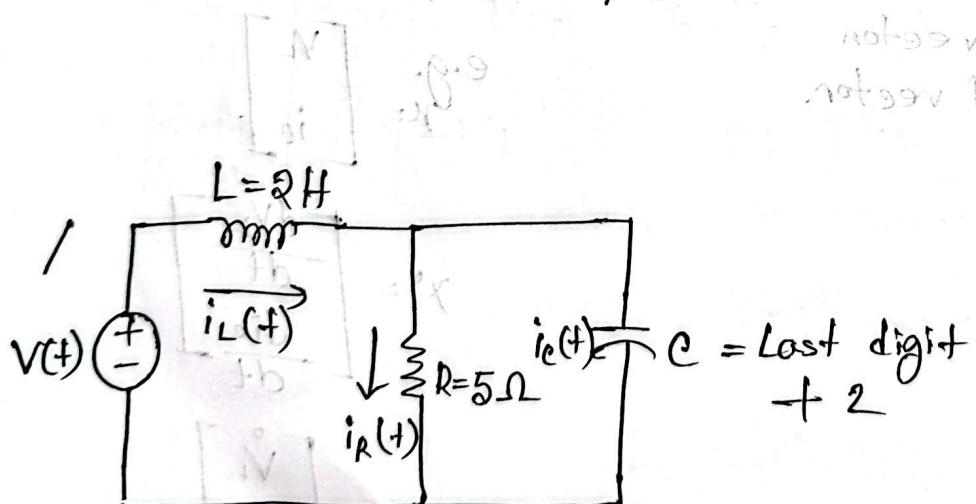
$$y = Cx + Du$$

① ~~unit state variable~~ — order 20, no. of state variable - 00,

② state variable = no. of energy element (inductor/capacitor) $\omega I + \frac{1}{\omega} A^T = \dot{x}$ (i)
Scenario. $\left\{ \begin{array}{l} \text{series - } \textcircled{1} \text{ induction} \\ \text{parallel - } \textcircled{2} \text{ capacitor} \end{array} \right\}$ considered equivalent

Example 3.1°

Find a state-space representation



Step 1: Label all branch current.

equation of state
variable out input
25% of total

Step 2: Write the derivative
of all energy equation.

Derivative

$$V_L = L \cdot \frac{di_L}{dt}$$

$$i_L = \frac{1}{L} \int V_L dt$$

$$\Rightarrow \frac{di_L}{dt} = \frac{1}{L} \cdot V_L \quad \textcircled{1}$$

$$i_C = C \cdot \frac{dV_C}{dt} \quad \textcircled{ii}$$

$$\Rightarrow \frac{dV_C}{dt} = \frac{1}{C} i_C \quad \textcircled{ii}$$

$$\Rightarrow V_C = \frac{1}{C} \int i_C dt$$

as, (energy storage element's) which variable
can be represent through derivative $(\frac{di_L}{dt}, \frac{dV_C}{dt})$
they are state variables.

now,

$$x = \begin{bmatrix} V_C \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{dV_C}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} \frac{dV_C}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} \frac{dV_C}{dt} \\ \frac{dV_L}{dt} \end{bmatrix} = \begin{bmatrix} \frac{dV_B}{dt} \\ \frac{dV_B}{dt} \end{bmatrix} = \dot{x}$$

$\text{(ex)} \quad \text{(ex)}$

Step 3: non-state variable representation through
state variable.

In eqn ①

$$\frac{di_L}{dt} = \frac{1}{L} \cdot V_L$$

$$= \frac{1}{L} (V - V_C)$$

$$= \frac{1}{L} V - \frac{1}{L} V_C$$

— iii

$$\begin{aligned} \frac{dV}{dt} &= \frac{dV_L}{dt} + \frac{dV_R}{dt} \\ V &= V_L + V_R \\ \Rightarrow V_L &= V - V_R \end{aligned}$$

$$i_L = i_R + i_C$$

$$i_C = i_L - i_R$$

$$\text{current equation} \Rightarrow i_L = \frac{V_R}{R}$$

$$= i_L - \frac{V_C}{R}$$

from eqn (ii)

$$\frac{dV_C}{dt} = \frac{1}{C} \cdot i_C$$

$$= \frac{1}{C} \left(i_L - \frac{V_C}{R} \right)$$

$$= \frac{1}{C} i_L - \frac{1}{RC} V_C$$

$$\frac{dV_C}{dt} = \frac{1}{RC} V_C + \frac{1}{C} i_L + 0 \cdot V_i \quad \text{(1)}$$

$$\frac{di_L}{dt} = -\frac{1}{L} V_C + 0 \cdot i_L + \frac{1}{L} V_i \quad \text{(2)}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1/RC & 1/C \\ -1/L & 0 \end{bmatrix} \begin{bmatrix} V_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_i$$

(2x1) — 2 equations (1) and (2)

Output equation:

$$\text{Output, } y = i_R = \frac{V_R}{R} = \frac{V_C}{R}$$

$$i_R = \frac{1}{R} V_C + 0 \cdot i_L + 0 \cdot V_i$$

$$i_R = \begin{bmatrix} 1/R & 0 \end{bmatrix} \begin{bmatrix} V_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} V_i$$

[as, V_C and i_L are equivalent for being parallel]

$$\text{①} \rightarrow N \cdot \frac{1}{L} = \frac{V_b}{Tb}$$

$$\text{②} \rightarrow \frac{V_b}{Tb} \cdot 0 = 0$$

$$Tb \cdot 0 = 0$$

$$0 = 0$$

$$0 = 0$$

$$0 = 0$$

$$0 = 0$$

$$0 = 0$$

$$0 = 0$$

$$0 = 0$$

$$0 = 0$$

$$0 = 0$$

$$0 = 0$$

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$$0 = 0$$

Mid

Lec 1: → Basics of CS

→ open system

Vs
closed system

- functional block diagram of closed loop system.
- steps of engineering CS.

Lec 2: → Laplace transform (cont. Lec 1)

— 3rd condition

→ Lap. tran. math.

→ Mathematical model
(mechanical must engg.)

E

Lec 3: State space & representation (Theory + Math)