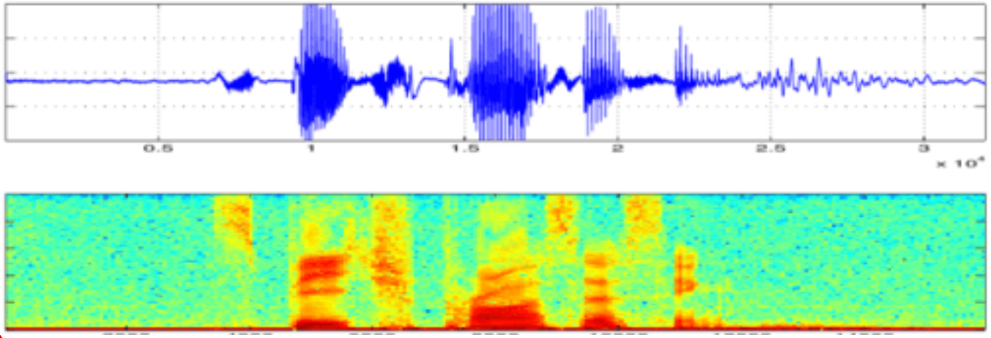
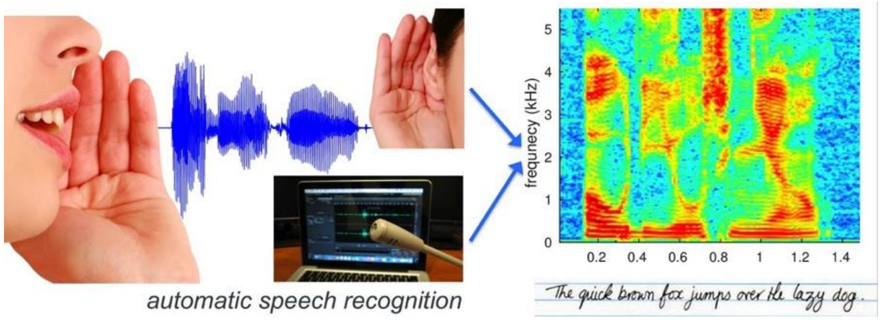
REDUCTION OF MULTIPLE SUBSYSTEM

Lecture-07

Nursadul Mamun, PhD

Center for Robust Speech Processing (CRSP)
Department of Electronics & Telecommunication Engineering
Chittagong University of Engineering & Technology





Contents

1. Formula's related to reduction of multiple subsystem.

2. Formula's related to Meson gain rule.

3. Related Examples .

Introduction

Multiple subsystems are represented by the interconnection of many subsystems.

Also known as Complicated Subsystems

Represented in two ways:

1. Block diagrams: Used for frequency domain (TF) analysis and design
2. Signal-flow graphs: Used for state-space analysis

Classifications

Block diagrams are usually used for frequency-domain analysis and design.

Signal-flow graphs for state space analysis and design

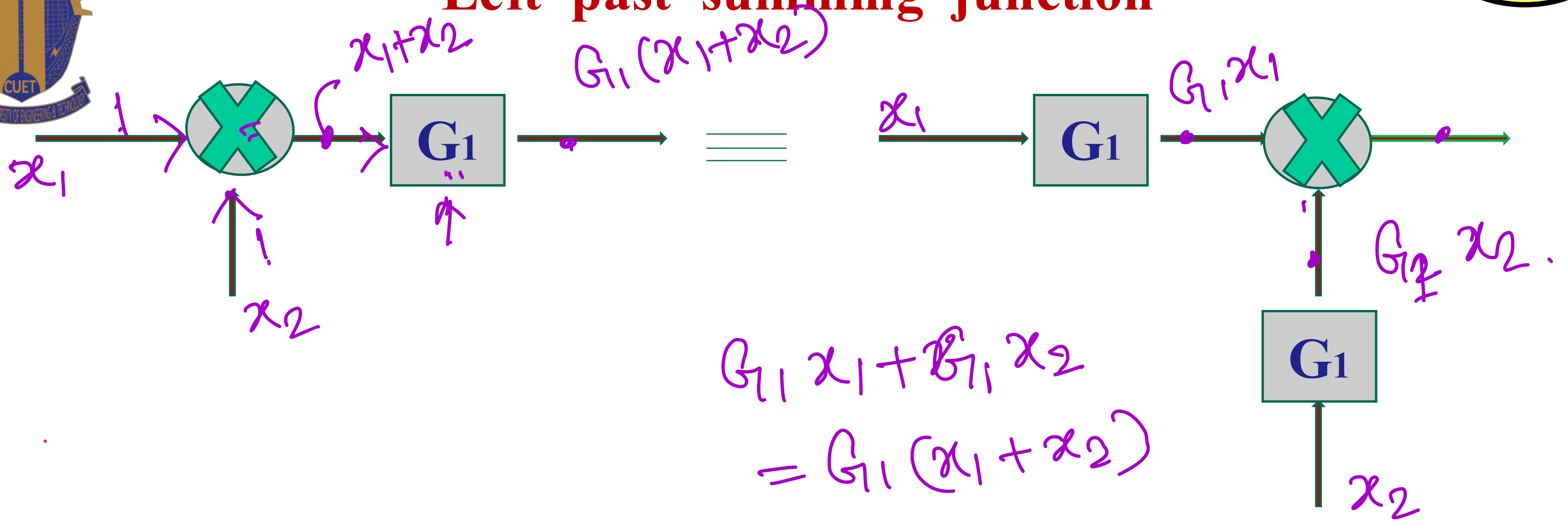
Techniques to reduce multiple subsystem to a single transfer function:-

1) Block diagram algebra –to reduce block diagrams

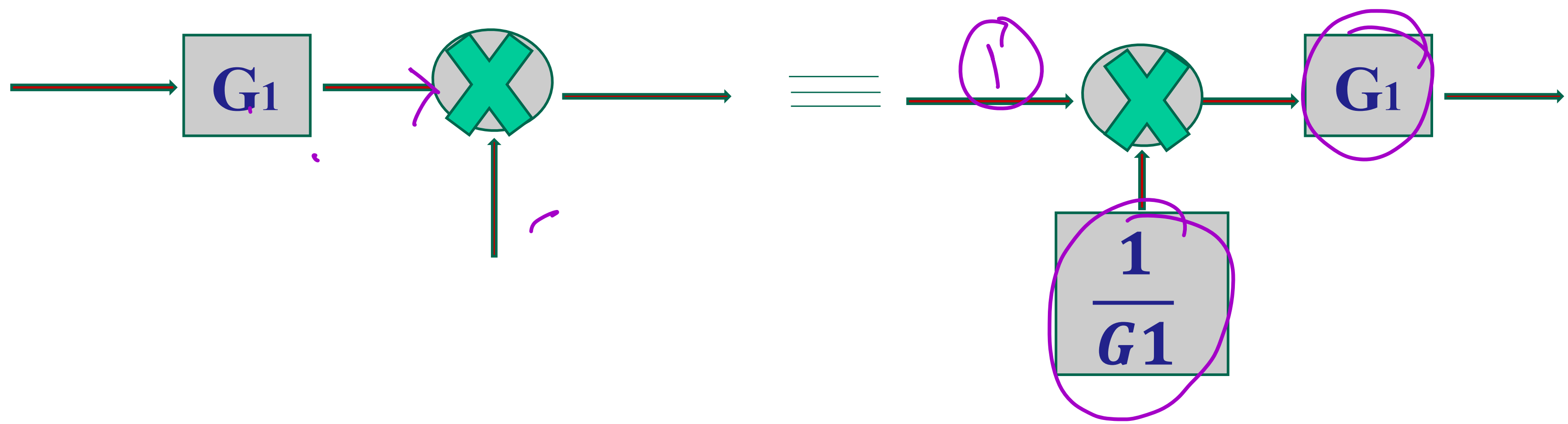
2) Manson's rule - to reduce signal-flow graphs.

BLOCK DIAGRAM REDUCTION

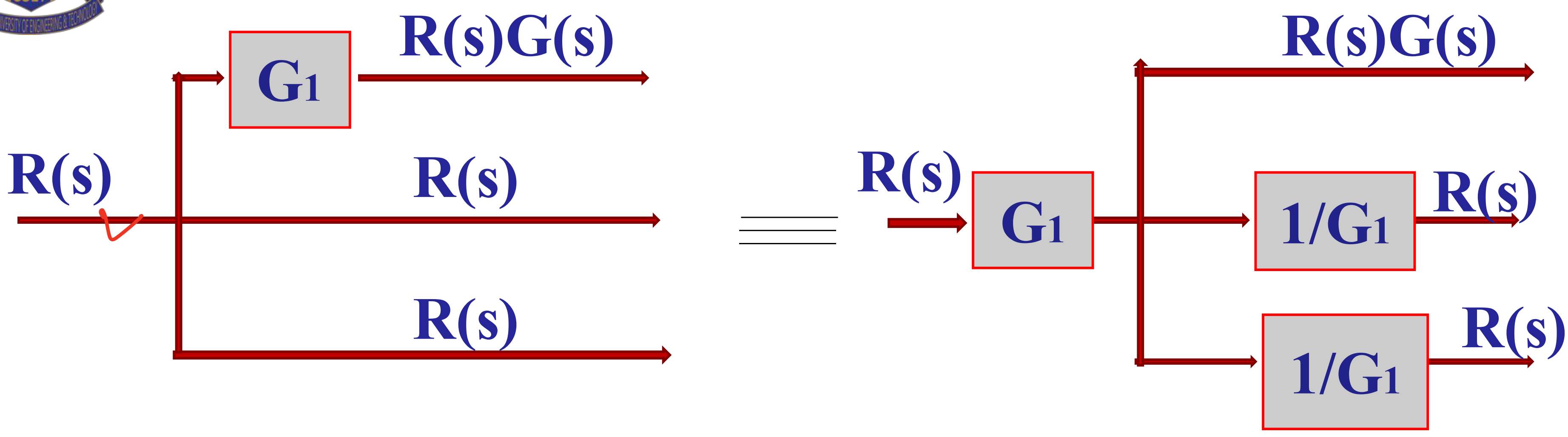
Left past summing junction



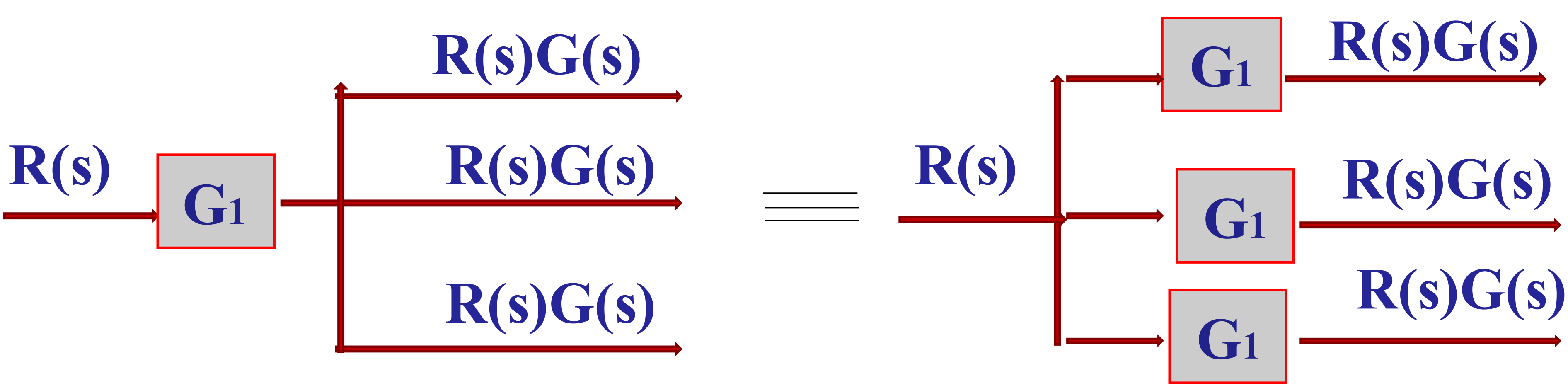
Right past summing junction



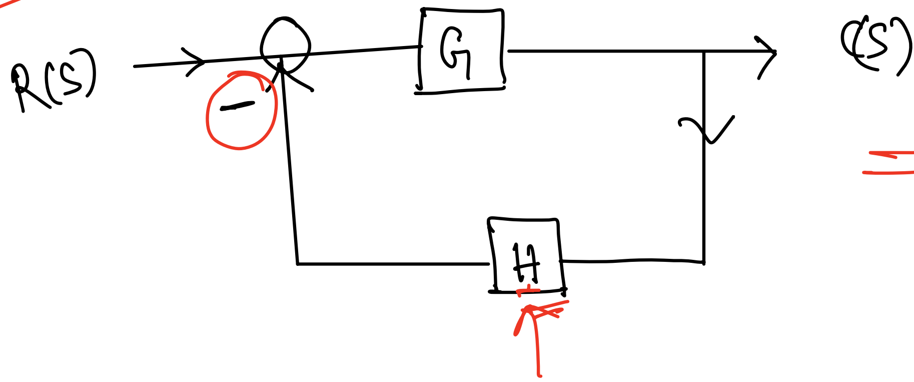
Left past pick off point



Right past pick off point



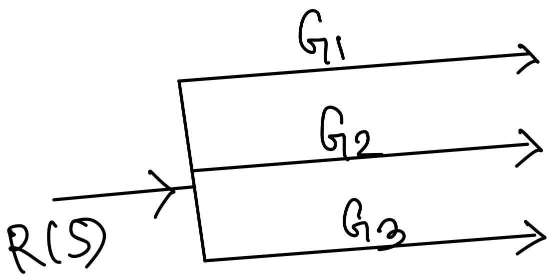
Feedback



$$\Rightarrow \quad = \frac{G}{1+GH}$$

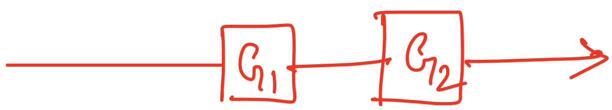
$$1 - G(-H)$$

Parallel



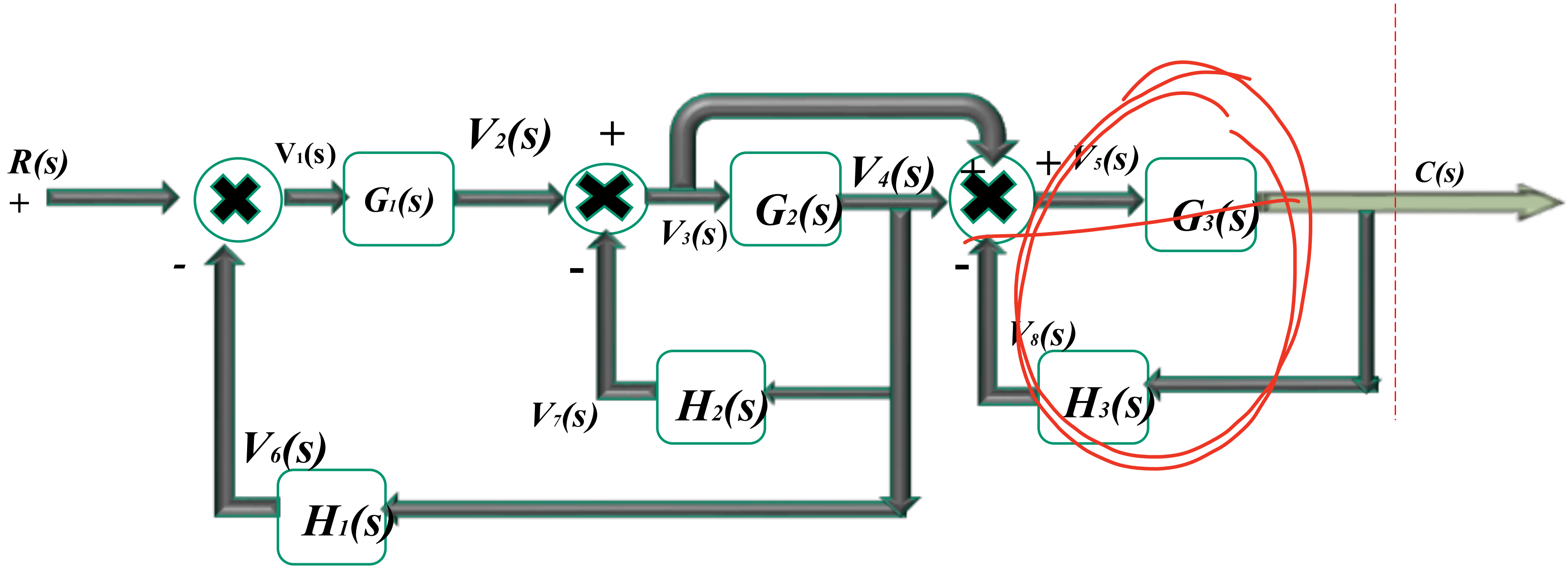
$$= G_1 + G_2 + G_3$$

Series



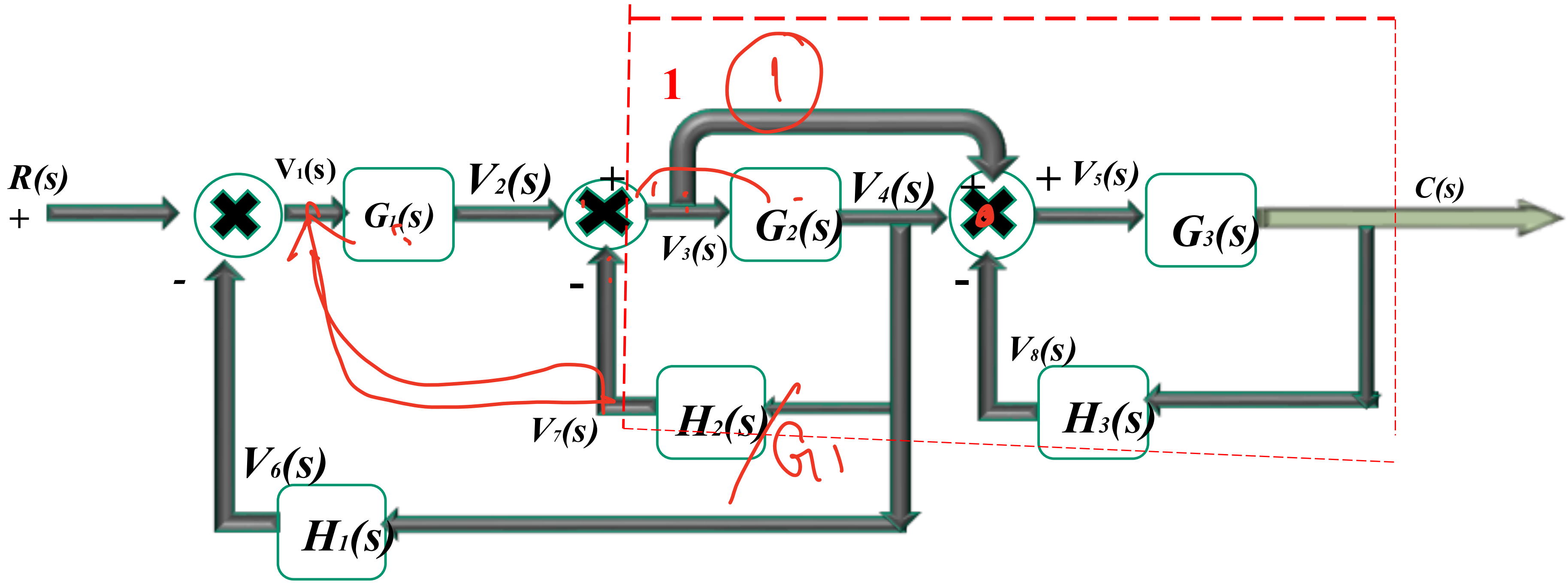
Examples

Reduce this system to a single transfer function.



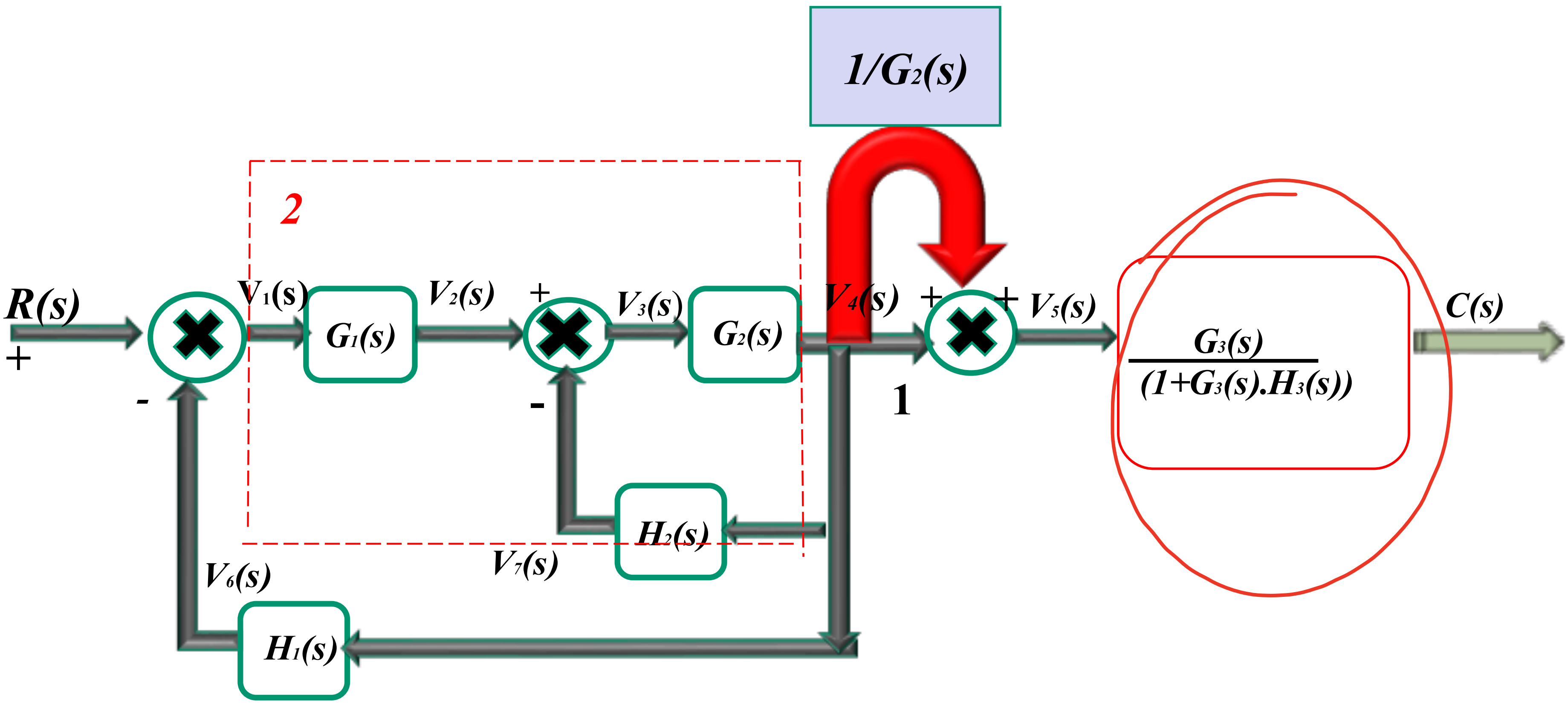
Examples

Reduce this system to a single transfer function.



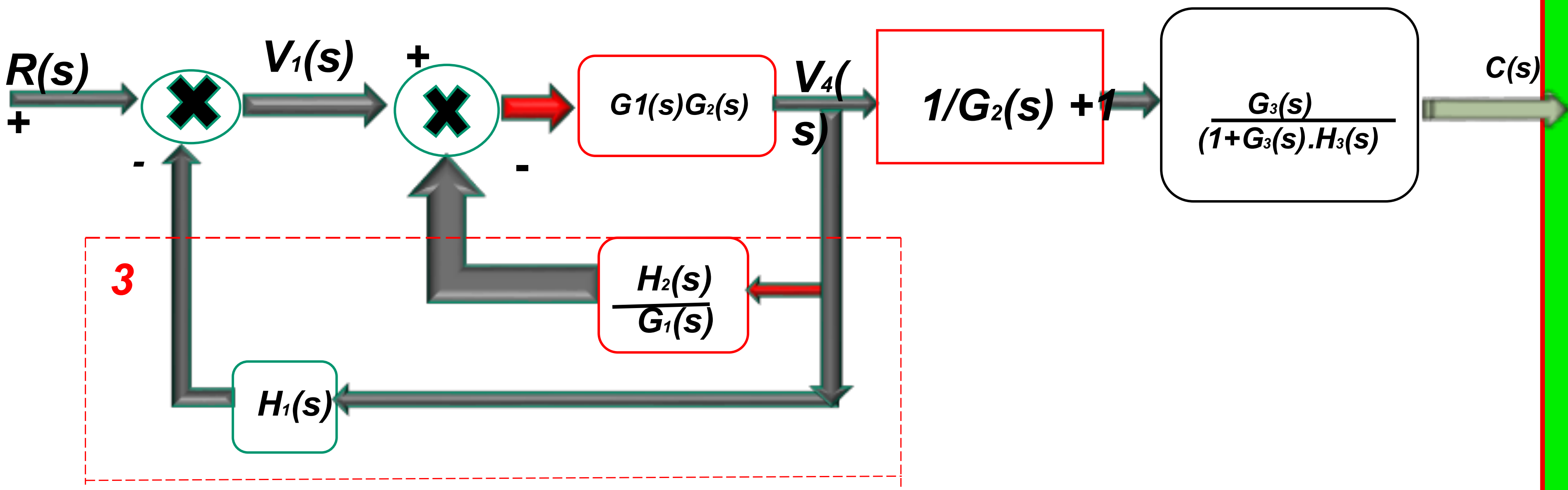
Continued…

Step:1



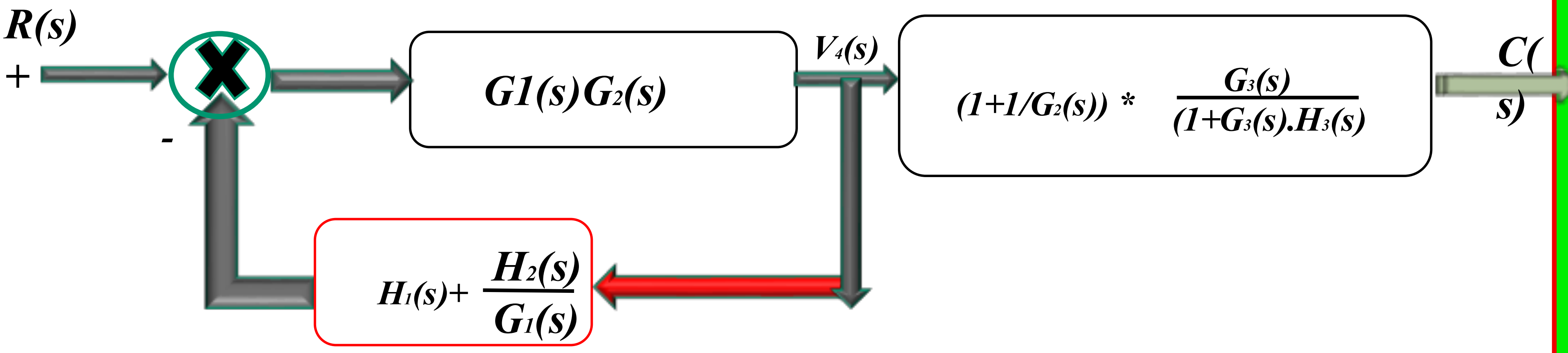
Continued…

Step:2



Continued…

Step:3



Continued…

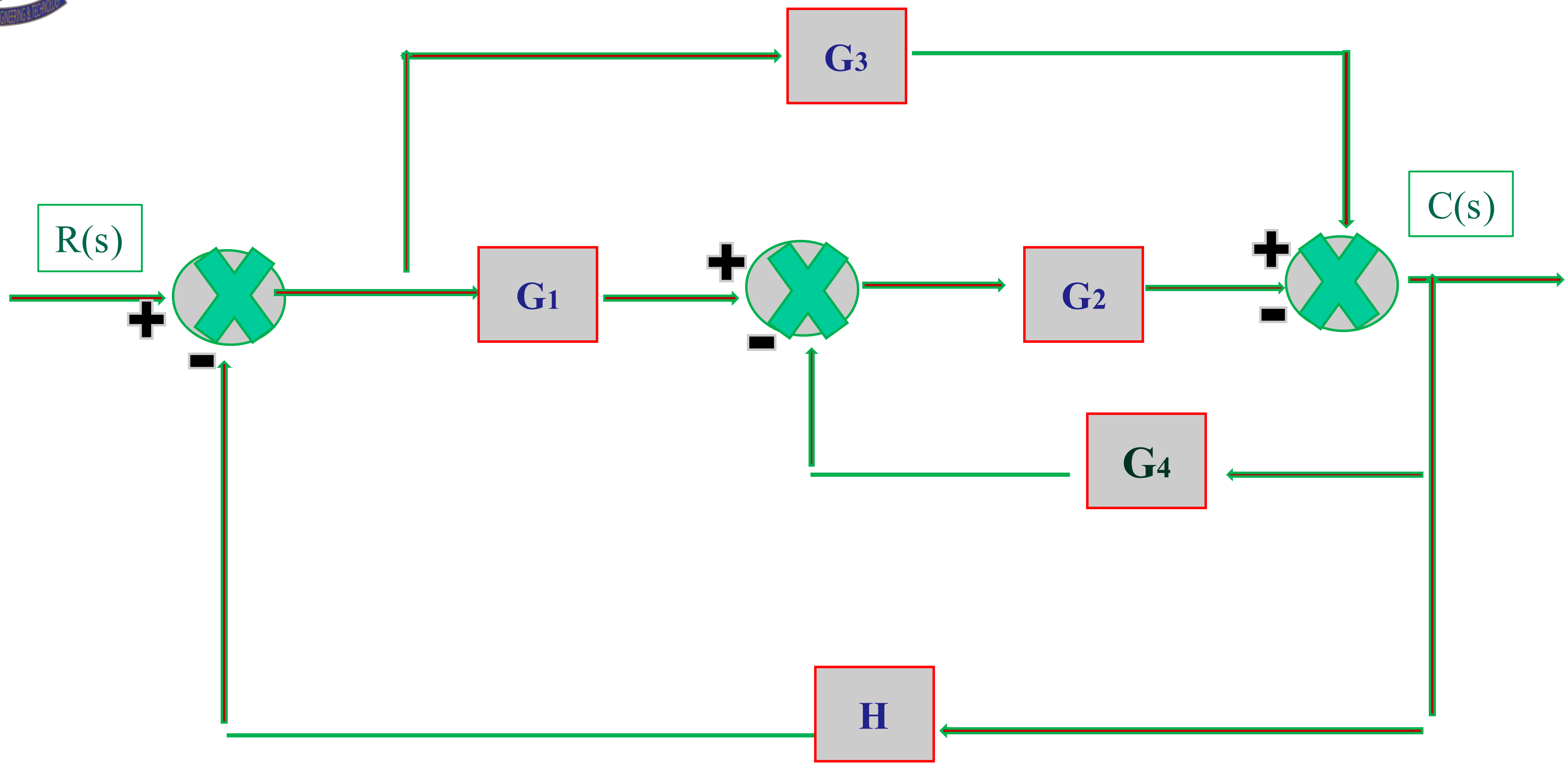
Step :4

$R(s) \rightarrow \dfrac{G1(s)G_2(s)}{1+G2(s)H2(s)+G1(s)G2(s)H1(s)} \xrightarrow{V_4(s)} (1+1/G_2(s)) * \dfrac{G_3(s)}{(1+G_3(s).H_3(s)} \rightarrow C(s)$

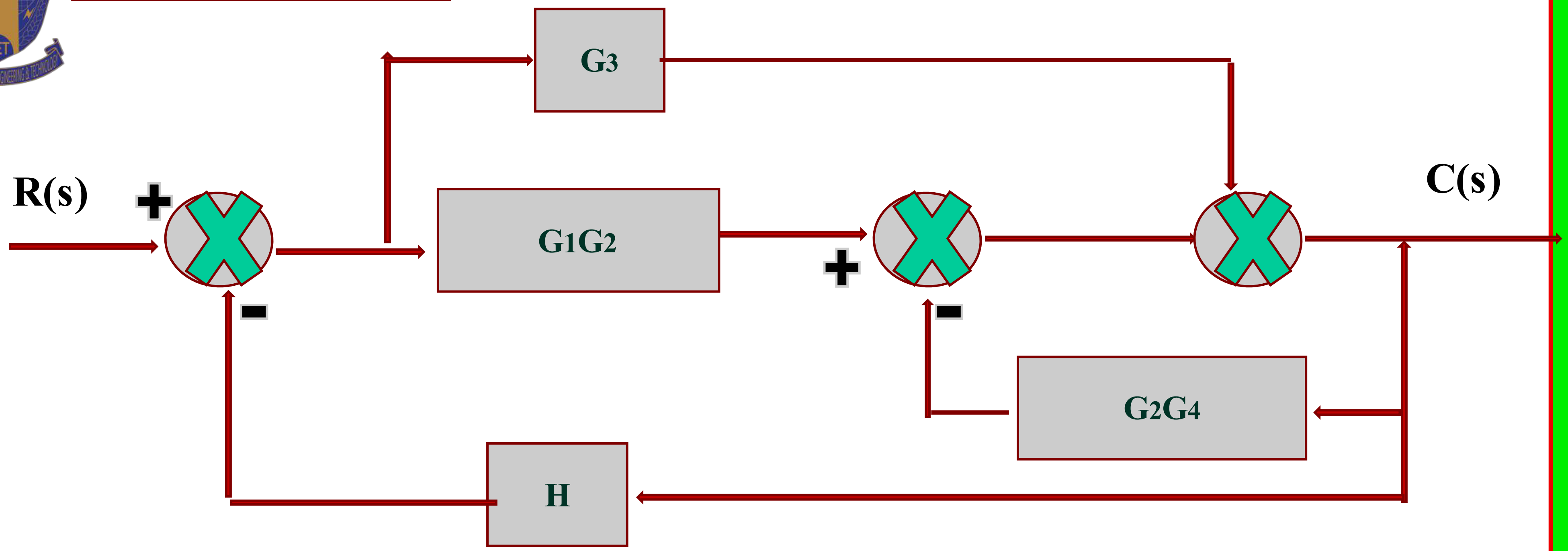
Finally,

$$R(s) \rightarrow \frac{G_1(s)G_3(s)(1+G_2(s))}{[1+G_2(s)H_2(s)+G_1(s)G_2(s)H_1(s)][1+G_3(s)H_3(s)]} \rightarrow C(s)$$

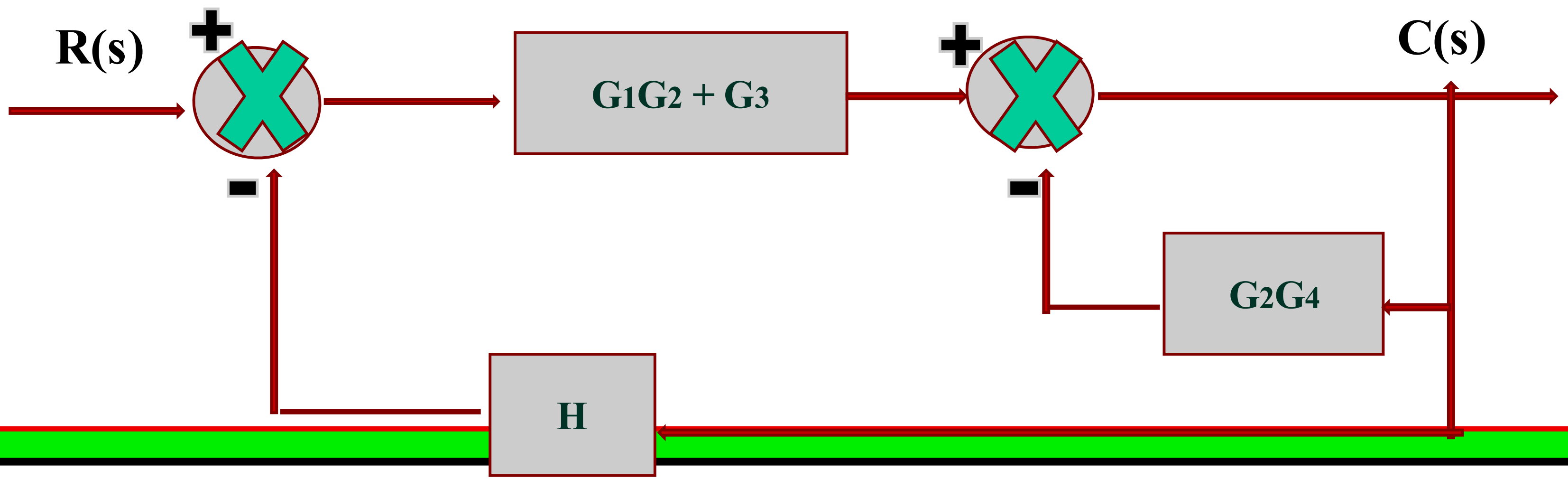
Problem 4: Reduce the system in single transfer function



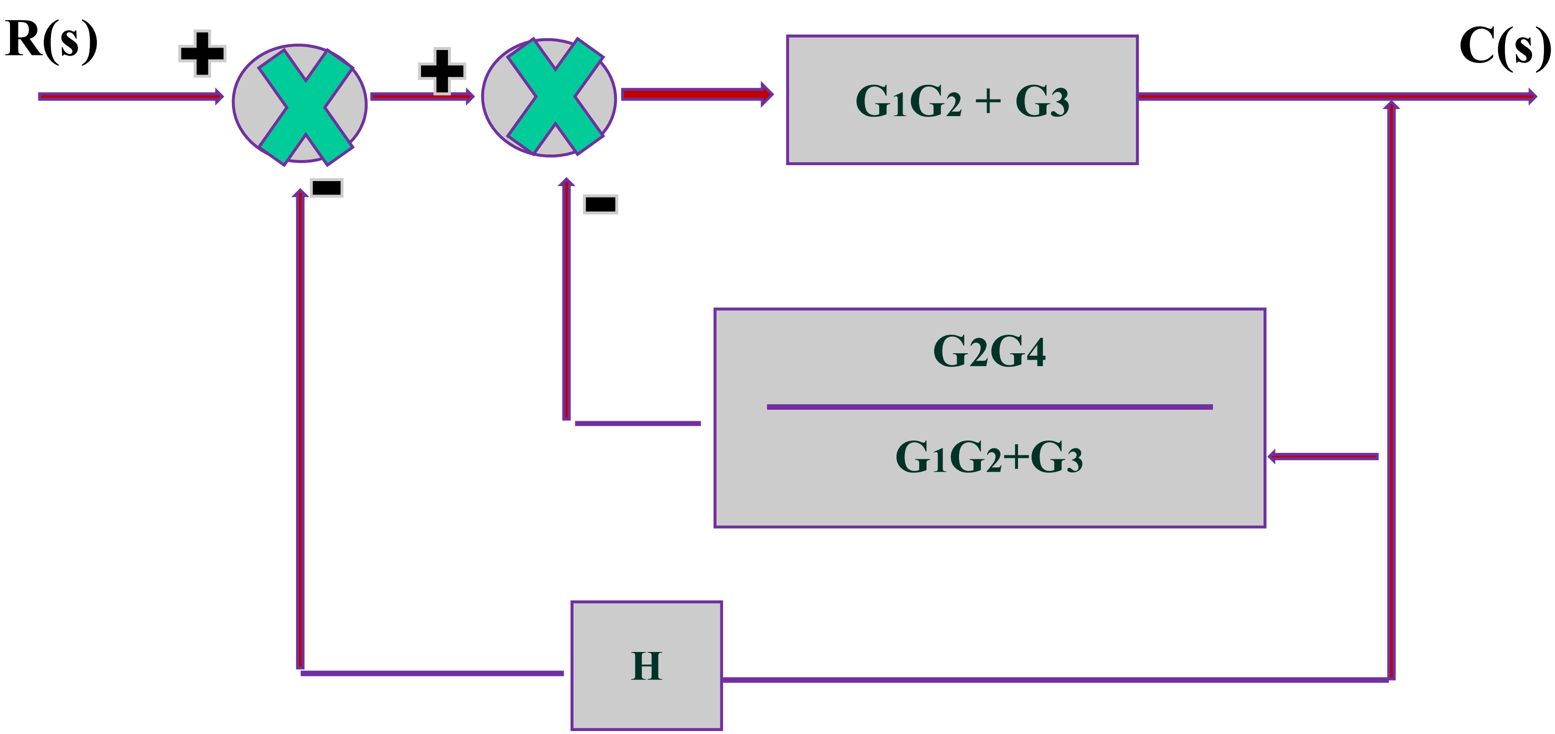
Solution:



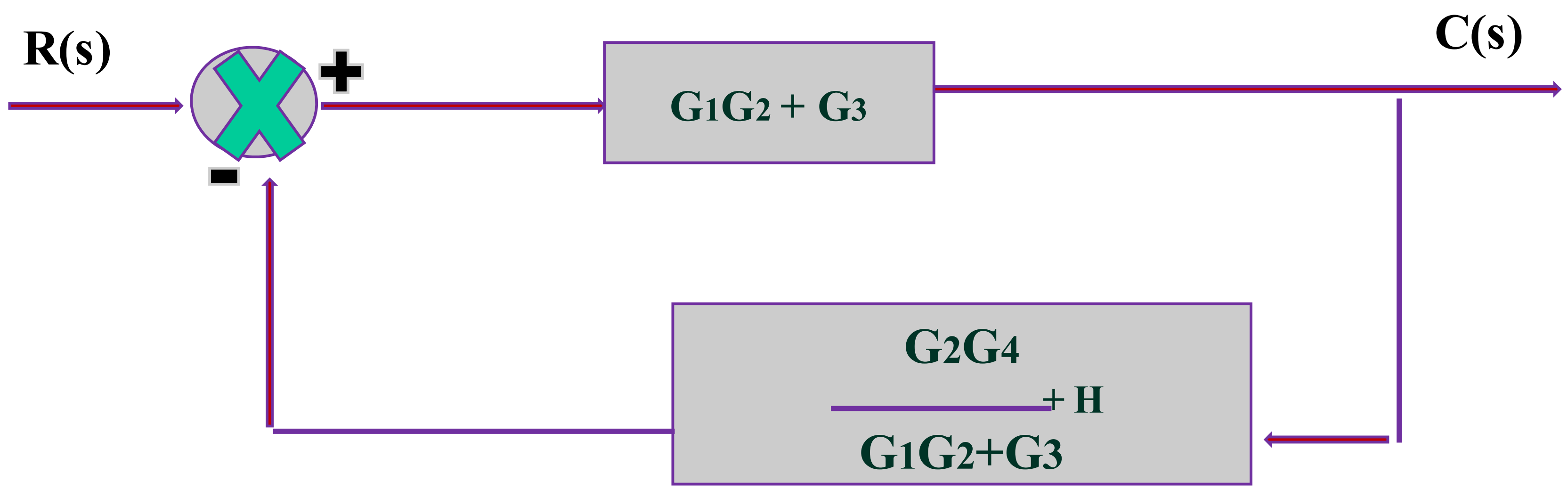
Step 2:



Step 3:



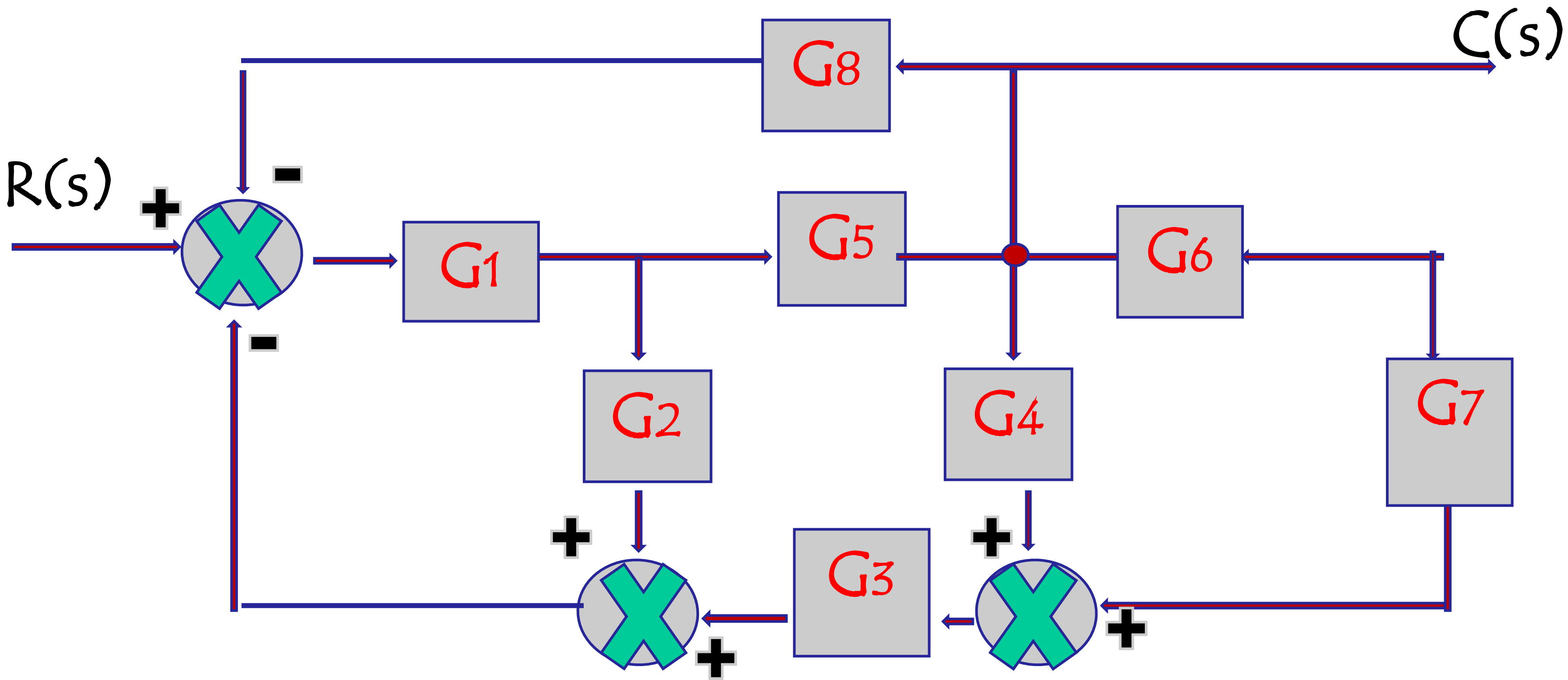
Step 4:



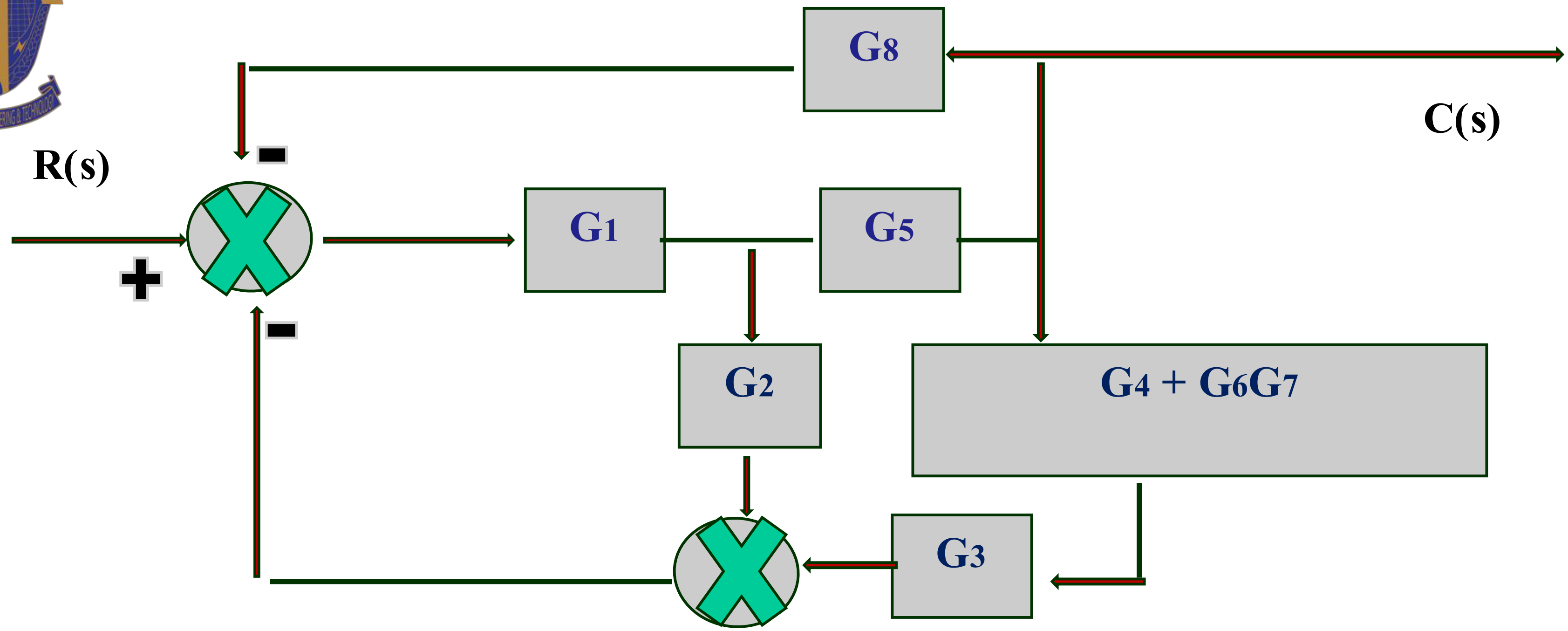
Step 5:

$$R(s) \longrightarrow \boxed{\dfrac{G_1G_2 + G_3}{1 + H(G_3 + G_1G_2) + G_2G_4}} \longrightarrow C(s)$$

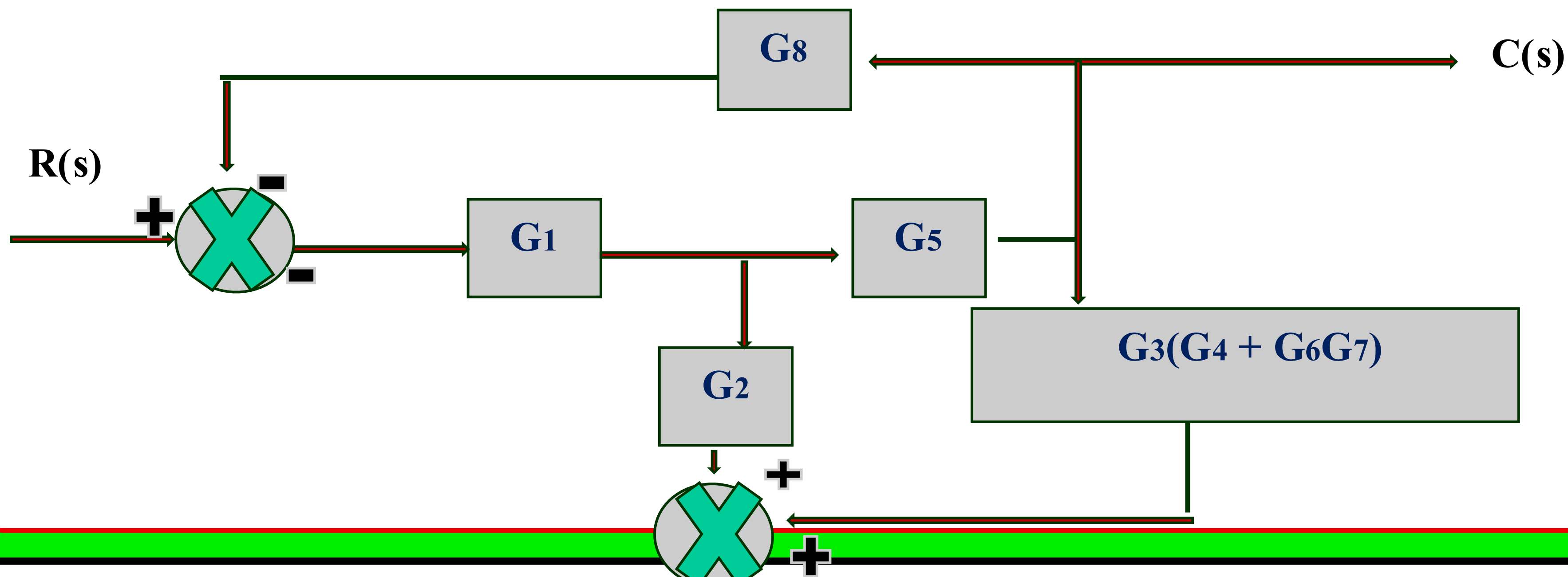
Problem 6 : Reduce the block diagram to single block T(s)= C(s)/R(s).



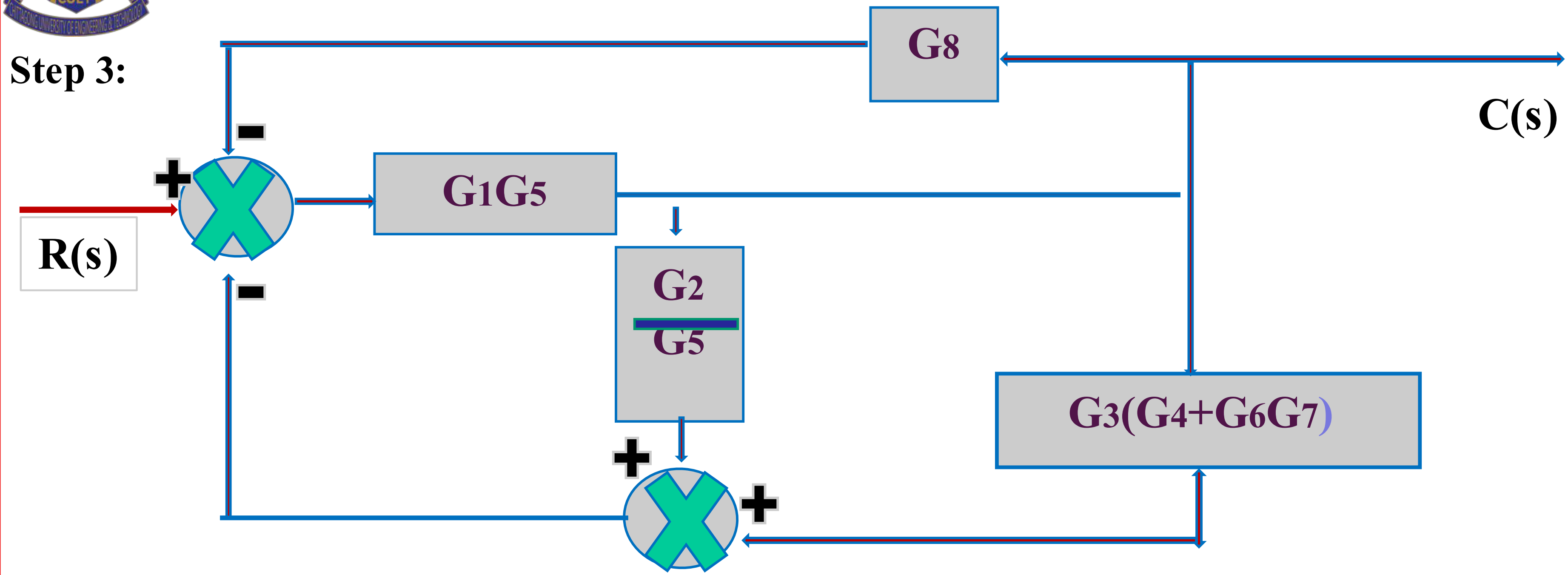
Solution : Step 1:



Step 2:



Step 3:



Final Step:

R(s)

$$\frac{G_1G_5}{1 + G_1G_5\,[(G_2/G_5) + G_3(G_4+G_6G_7) + G_8]}$$

C(s)

3/9/25

Section A

2257, 2255, 1819, 2250, 2252, 2115, 2179.

1–5, 7–13, 15, 17–18, 20–22, 26, 27, 29, 31, 34–36, 39, 68.

Section B:

2292, 1995, 1135, 1815

44, 46, 47, 49–51, 53, 55–57, 59, 62, 64, 71–76, 78, 80, 82–84, 87, 88, 92.

Sec C:

1139, 1510, 912, 1081, 2198, 1105

93–94, 97–99, 102–104, 106, 8, 11, 14, 17, 18, 19, 21–24, 28, 32, 33, 36,

Mason's rule

Skill-Assessment Exercise 5.3

PROBLEM: Convert the block diagram of Figure 5.13 to a signal-flow graph.

ANSWER: The complete solution is at www.wiley.com/college/nise.

5.5 Mason's Rule

Earlier in this chapter, we discussed how to reduce block diagrams to single transfer functions. Now we are ready to discuss a technique for reducing signal-flow graphs to single transfer functions that relate the output of a system to its input.

The block diagram reduction technique we studied in Section 5.2 requires successive application of fundamental relationships in order to arrive at the system transfer function. On the other hand, Mason's rule for reducing a signal-flow graph to a single transfer function requires the application of one formula. The formula was derived by S. J. Mason when he related the signal-flow graph to the simultaneous equations that can be written from the graph (*Mason, 1953*).

In general, it can be complicated to implement the formula without making mistakes. Specifically, the existence of what we will later call nontouching loops increases the complexity of the formula. However, many systems do not have nontouching loops. For these systems, you may find Mason's rule easier to use than block diagram reduction.

Mason's formula has several components that must be evaluated. First, we must be sure that the definitions of the components are well understood. Then we must exert care in evaluating the components. To that end, we discuss some basic definitions applicable to signal-flow graphs; then we state Mason's rule and do an example.

Definitions

Loop gain. The product of branch gains found by traversing a path that starts at a node and ends at the same node, following the direction of the signal flow, without passing through any other node more than once. For examples of loop gains, see Figure 5.20. There are four loop gains:

1. $G_2(s)H_1(s)$ (5.25a)
2. $G_4(s)H_2(s)$ (5.25b)
3. $G_4(s)G_5(s)H_3(s)$ (5.25c)
4. $G_4(s)G_6(s)H_3(s)$ (5.25d)

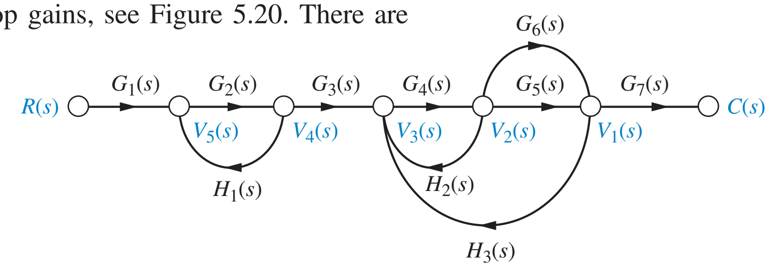


FIGURE 5.20 Signal-flow graph for demonstrating Mason's rule

Forward-path gain. The product of gains found by traversing a path from the input node to the output node of the signal-flow graph in the direction of signal flow. Examples of forward-path gains are also shown in Figure 5.20. There are two forward-path gains:

1. $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$ (5.26a)
2. $G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$ (5.26b)

Nontouching loops. Loops that do not have any nodes in common. In Figure 5.20, loop $G_2(s)H_1(s)$ does not touch loops $G_4(s)H_2(s)$, $G_4(s)G_5(s)H_3(s)$, and $G_4(s)G_6(s)H_3(s)$.

Nontouching-loop gain. The product of loop gains from nontouching loops taken two, three, four, or more at a time. In Figure 5.20 the product of loop gain $G_2(s)H_1(s)$ and loop gain

$G_4(s)H_2(s)$ is a nontouching-loop gain taken two at a time. In summary, all three of the nontouching-loop gains taken two at a time are

1. $[G_2(s)H_1(s)][G_4(s)H_2(s)]$ (5.27a)
2. $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$ (5.27b)
3. $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$ (5.27c)

The product of loop gains $[G_4(s)G_5(s)H_3(s)][G_4(s)G_6(s)H_3(s)]$ is not a nontouching-loop gain since these two loops have nodes in common. In our example there are no nontouching-loop gains taken three at a time since three nontouching loops do not exist in the example.

We are now ready to state Mason's rule.

Mason's Rule

The transfer function, $C(s)/R(s)$, of a system represented by a signal-flow graph is

$$\boxed{G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta}} \tag{5.28}$$

where

k = number of forward paths

T_k = the kth forward-path gain

Δ = $1 - \Sigma$ loop gains $+ \Sigma$ nontouching-loop gains taken two at a time $- \Sigma$ nontouching-loop gains taken three at a time $+ \Sigma$ nontouching-loop gains taken four at a time $- \ldots$

Δ_k = $\Delta - \Sigma$ loop gain terms in Δ that touch the kth forward path. In other words, Δ_k is formed by eliminating from Δ those loop gains that touch the kth forward path.

Notice the alternating signs for the components of Δ. The following example will help clarify Mason's rule.

Example 5.7

Transfer Function via Mason's Rule

PROBLEM: Find the transfer function, $C(s)/R(s)$, for the signal-flow graph in Figure 5.21.

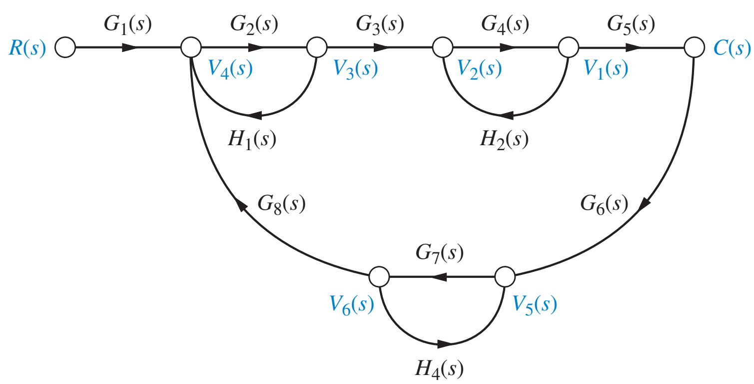


FIGURE 5.21 Signal-flow graph for Example 5.7

SOLUTION: First, identify the *forward-path gains*. In this example there is only one:

$$G_1(s)G_2(s)G_3(s)G_4(s)G_5(s) \tag{5.29}$$

Second, identify the *loop gains*. There are four, as follows:

1. $G_2(s)H_1(s)$ (5.30a)
2. $G_4(s)H_2(s)$ (5.30b)
3. $G_7(s)H_4(s)$ (5.30c)
4. $G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)$ (5.30d)

Third, identify the *nontouching loops taken two at a time*. From Eqs. (5.30) and Figure 5.21, we can see that loop 1 does not touch loop 2, loop 1 does not touch loop 3, and loop 2 does not touch loop 3. Notice that loops 1, 2, and 3 all touch loop 4. Thus, the combinations of nontouching loops taken two at a time are as follows:

$$\text{Loop 1 and loop 2:}\ G_2(s)H_1(s)G_4(s)H_2(s) \tag{5.31a}$$

$$\text{Loop 1 and loop 3:}\ G_2(s)H_1(s)G_7(s)H_4(s) \tag{5.31b}$$

$$\text{Loop 2 and loop 3:}\ G_4(s)H_2(s)G_7(s)H_4(s) \tag{5.31c}$$

Finally, the *nontouching loops taken three at a time* are as follows:

$$\text{Loops 1, 2, and 3:}\quad G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s) \tag{5.32}$$

Now, from Eq. (5.28) and its definitions, we form Δ and Δ_k. Hence,

$$\begin{aligned}\Delta = 1 - [&G_2(s)H_1(s) + G_4(s)H_2(s) + G_7(s)H_4(s) \\ &+ G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)] \\ +[&G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s) \\ &+ G_4(s)H_2(s)G_7(s)H_4(s)] \\ -[&G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]\end{aligned} \tag{5.33}$$

We form Δ_k by eliminating from Δ the loop gains that touch the kth forward path:

$$\Delta_1 = 1 - G_7(s)H_4(s) \tag{5.34}$$

Expressions (5.29), (5.33), and (5.34) are now substituted into Eq. (5.28), yielding the transfer function:

$$G(s) = \frac{T_1\Delta_1}{\Delta} = \frac{[G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)][1 - G_7(s)H_4(s)]}{\Delta} \tag{5.35}$$

Since there is only one forward path, $G(s)$ consists of only one term, rather than a sum of terms, each coming from a forward path.

Skill-Assessment Exercise 5.4

PROBLEM: Use Mason's rule to find the transfer function of the signal-flow diagram shown in Figure 5.19(*c*). Notice that this is the same system used in Example 5.2 to find the transfer function via block diagram reduction.

ANSWER:

$$T(s) = \frac{G_1(s)G_3(s)[1 + G_2(s)]}{[1 + G_2(s)H_2(s) + G_1(s)G_2(s)H_1(s)][1 + G_3(s)H_3(s)]}$$

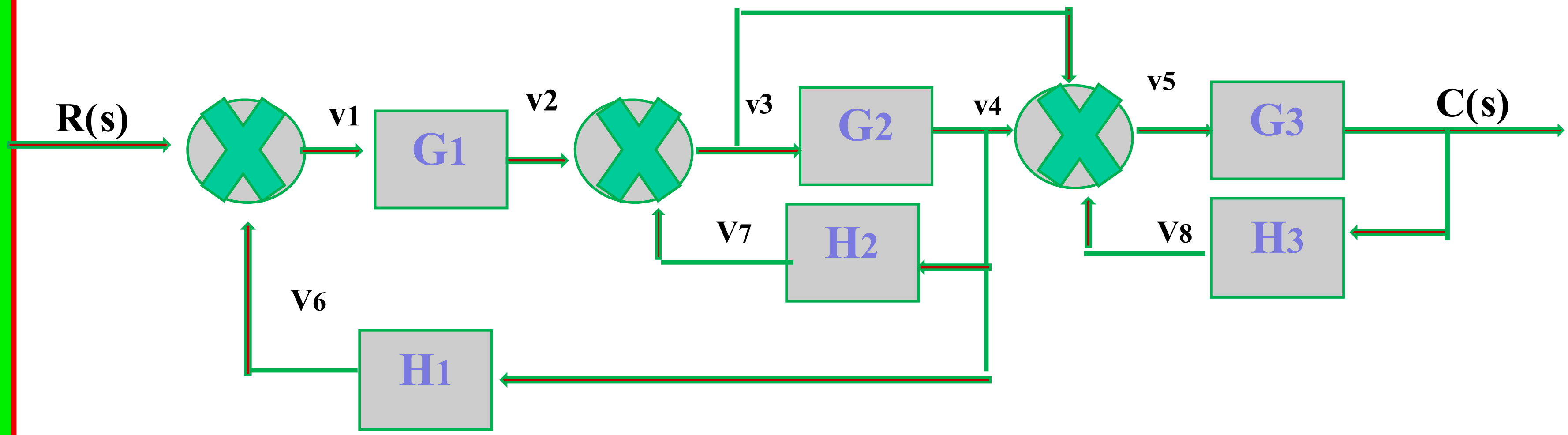
The complete solution is at www.wiley.com/college/nise.

T_k = The k^{th} forward path gain

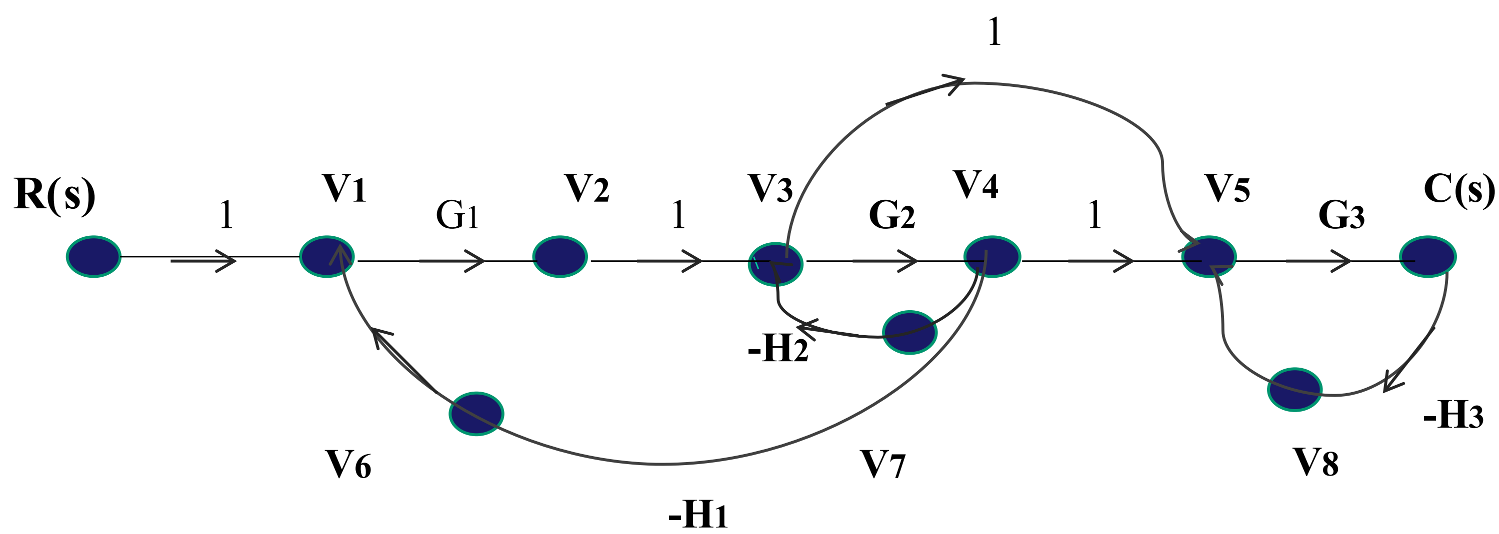
Δ = 1 - $\sum$ loop gains + $\sum$non touching loop gain taken two at a Time + $\sum$non touching loop gains taken three at a time…….

Δ_k = Δ - $\sum$ loop gains terms in Δ that touch the kth forward Path

Exercise 5.4: Use Mason's rule to find the transfer Function from the signal flow diagram.



Solution:



Forward path gains: G1G2G3 and G1G3

Loop gains: -G1G2H1, -G2H2 and -G3H3

Non-touching loops: G1G2G3H1H3 and G2G3 H2 H3

Δ = 1 + G1G2H1 + G3H2 + G3H3 + G1G2G3H1H3 + G2G3 H2 H3

Finally $\Delta_1 = 1$ and $\Delta_2 = 1$.

T(s) = C(s) / R(s) = $\sum$Tk Δk / Δ

$$= \frac{G1G3[1 + G2]}{\Delta}$$

Loop gain: starting & ending point same.

Forward path gain: The way of different reaching to output from input

Non touching loop gain taking two at a time:

" " " " " " three " "

$$G(S) = \frac{C(S)}{R(S)} = \frac{\sum T_K \Delta_K}{\Delta}$$

T_K = K^{th} no. of forward path gain.

No. of forward path.

$$G(S) = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$$

Δ = 1 − Σ loop gain + Σ Nontouching loop gain taking 2 at a time − Σ Nontouc. loop 3.

$T_1 = G_1 G_2 G_3$, $T_2 = G_1 G_3$

$$\Delta = 1 + G_1G_2H_1 + G_2H_2 + G_3H_3 + G_1G_2G_3H_1H_3 + G_2G_3H_2H_3$$

$$\Delta_1 = 1 + \cancel{G_1G_2H_1} + \cancel{G_2H_2} + \cancel{G_3H_3} + \cancel{G_1G_2G_3H_1H_3} + \cancel{G_2G_3H_2H_3}$$

$$\Delta_2 = 1 + \cancel{G_1G_2H_1} + \cancel{G_2H_2} + \cancel{G_3H_3} + \cancel{G_1G_2G_3H_1H_3} + \cancelto{0}{G_2G_3H_2H_3}$$

$$G = \frac{T_1\cancelto{1}{\Delta_1} + T_2\cancelto{1}{\Delta_2}}{\Delta}$$

$$= \frac{G_1G_2G_3 + G_1G_3}{\Delta}$$

$G_4(s)H_2(s)$ is a nontouching-loop gain taken two at a time. In summary, all three of the nontouching-loop gains taken two at a time are

1. $[G_2(s)H_1(s)][G_4(s)H_2(s)]$ (5.27a)
2. $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$ (5.27b)
3. $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$ (5.27c)

The product of loop gains $[G_4(s)G_5(s)H_3(s)][G_4(s)G_6(s)H_3(s)]$ is not a nontouching-loop gain since these two loops have nodes in common. In our example there are no nontouching-loop gains taken three at a time since three nontouching loops do not exist in the example.

We are now ready to state Mason's rule.

Mason's Rule

The transfer function, $C(s)/R(s)$, of a system represented by a signal-flow graph is

$$\boxed{G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta}} \tag{5.28}$$

where

k = number of forward paths

T_k = the kth forward-path gain

Δ = 1 − Σ loop gains + Σ nontouching-loop gains taken two at a time − Σ nontouching-loop gains taken three at a time + Σ nontouching-loop gains taken four at a time − . . .

Δ_k = Δ − Σ loop gain terms in Δ that touch the kth forward path. In other words, Δ_k is formed by eliminating from Δ those loop gains that touch the kth forward path.

Notice the alternating signs for the components of Δ. The following example will help clarify Mason's rule.

Example 5.7

Transfer Function via Mason's Rule

PROBLEM: Find the transfer function, $C(s)/R(s)$, for the signal-flow graph in Figure 5.21.

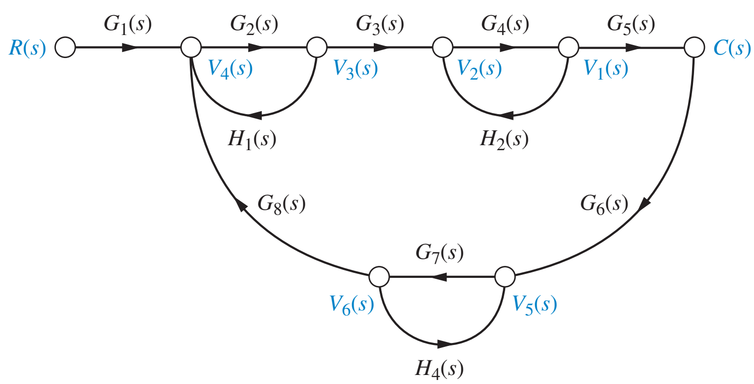


FIGURE 5.21 Signal-flow graph for Example 5.7

SOLUTION: First, identify the *forward-path gains*. In this example there is only one:

$$G_1(s)G_2(s)G_3(s)G_4(s)G_5(s) \tag{5.29}$$

Second, identify the *loop gains*. There are four, as follows:

1. $G_2(s)H_1(s)$ (5.30a)
2. $G_4(s)H_2(s)$ (5.30b)
3. $G_7(s)H_4(s)$ (5.30c)
4. $G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)$ (5.30d)

Third, identify the *nontouching loops taken two at a time*. From Eqs. (5.30) and Figure 5.21, we can see that loop 1 does not touch loop 2, loop 1 does not touch loop 3, and loop 2 does not touch loop 3. Notice that loops 1, 2, and 3 all touch loop 4. Thus, the combinations of nontouching loops taken two at a time are as follows:

$$\text{Loop 1 and loop 2:}\; G_2(s)H_1(s)G_4(s)H_2(s) \tag{5.31a}$$

$$\text{Loop 1 and loop 3:}\; G_2(s)H_1(s)G_7(s)H_4(s) \tag{5.31b}$$

$$\text{Loop 2 and loop 3:}\; G_4(s)H_2(s)G_7(s)H_4(s) \tag{5.31c}$$

Finally, the *nontouching loops taken three at a time* are as follows:

$$\text{Loops 1, 2, and 3:}\quad G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s) \tag{5.32}$$

Now, from Eq. (5.28) and its definitions, we form Δ and Δ_k. Hence,

$$\begin{aligned}\Delta = 1 &-[G_2(s)H_1(s) + G_4(s)H_2(s) + G_7(s)H_4(s)\\ &\qquad + G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)]\\ &+[G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s)\\ &\qquad + G_4(s)H_2(s)G_7(s)H_4(s)]\\ &-[G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]\end{aligned} \tag{5.33}$$

We form Δ_k by eliminating from Δ the loop gains that touch the kth forward path:

$$\Delta_1 = 1 - G_7(s)H_4(s) \tag{5.34}$$

Expressions (5.29), (5.33), and (5.34) are now substituted into Eq. (5.28), yielding the transfer function:

$$G(s) = \frac{T_1\Delta_1}{\Delta} = \frac{[G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)][1 - G_7(s)H_4(s)]}{\Delta} \tag{5.35}$$

Since there is only one forward path, $G(s)$ consists of only one term, rather than a sum of terms, each coming from a forward path.

Skill-Assessment Exercise 5.4

PROBLEM: Use Mason's rule to find the transfer function of the signal-flow diagram shown in Figure 5.19(c). Notice that this is the same system used in Example 5.2 to find the transfer function via block diagram reduction.

ANSWER:

$$T(s) = \frac{G_1(s)G_3(s)[1 + G_2(s)]}{[1 + G_2(s)H_2(s) + G_1(s)G_2(s)H_1(s)][1 + G_3(s)H_3(s)]}$$

The complete solution is at www.wiley.com/college/nise.

25. Repeat Problem 24 for

State Space **SS**

$$G(s) = \frac{20}{s(s-2)(s+5)(s+8)}$$

[Section: 5.7]

26. Using Mason's rule, find the transfer function, $T(s) = C(s)/R(s)$, for the system represented in Figure P5.22. [Section: 5.5]

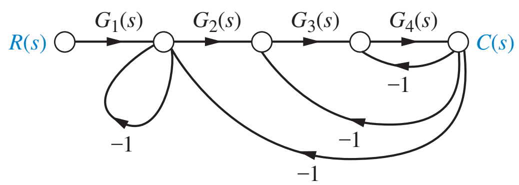


FIGURE P5.22

27. Using Mason's rule, find the transfer function, $T(s) = C(s)/R(s)$, for the system represented by Figure P5.23. [Section: 5.5]

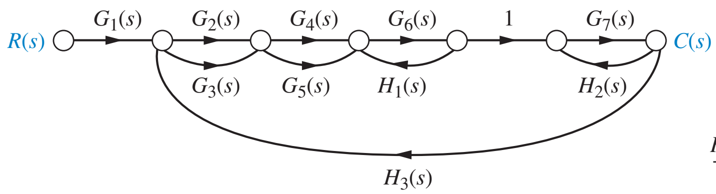


FIGURE P5.23

28. Use Mason's rule to find the transfer function of Figure 5.13 in the text. [Section: 5.5]

29. Use block diagram reduction to find the transfer function of Figure 5.21 in the text, and compare your answer with that obtained by Mason's rule. [Section: 5.5]

30. Represent the following systems in state space in Jordan canonical form. Draw the signal-flow graphs. [Section: 5.7]

State Space **SS**

a. $G(s) = \dfrac{(s+1)(s+2)}{(s+3)^2(s+4)}$

b. $G(s) = \dfrac{(s+2)}{(s+5)^2(s+7)^2}$

c. $G(s) = \dfrac{(s+4)}{(s+2)^2(s+5)(s+6)}$

31. Represent the systems below in state space in phase-variable form. Draw the signal-flow graphs. [Section: 5.7]

State Space **SS**

a. $G(s) = \dfrac{s+3}{s^2+2s+7}$

b. $G(s) = \dfrac{s^2+2s+6}{s^3+5s^2+2s+1}$

c. $G(s) = \dfrac{s^3+2s^2+7s+1}{s^4+3s^3+5s^2+6s+4}$

32. Repeat Problem 31 and represent each system in controller canonical and observer canonical forms. [Section: 5.7]

State Space **SS**

33. Represent the feedback control systems shown in Figure P5.24 in state space. When possible, represent the open-loop transfer functions separately in cascade and complete the feedback loop with the signal path from output to input. Draw your signal-flow graph to be in one-to-one correspondence to the block diagrams (as close as possible). [Section: 5.7]

State Space **SS**

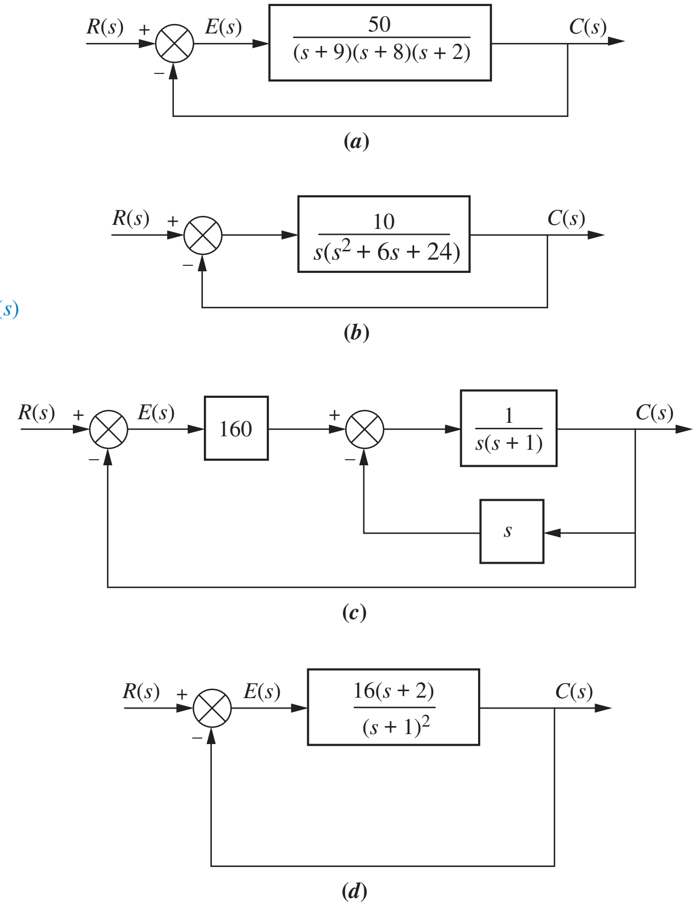


FIGURE P5.24

34. You are given the system shown in Figure P5.25. [Section: 5.7]

State Space **SS**

a. Represent the system in state space in phase-variable form.

b. Represent the system in state space in any other form besides phase-variable.

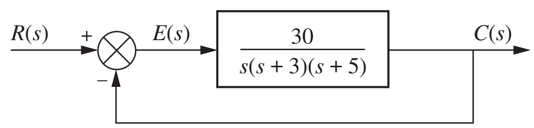


FIGURE P5.25

Assignment

Example 5.1 , 5.2, 5.7,

Skill assessment

5.1,5.4

Problem

1,2,3,4,5,6,8,9,10

Reference
Control System Engineering by Norman-Nise

Thank you!
Question ?