

Premier University
Department of Computer Science & Engineering
1st Semester Special Retake Final Year Examination, August 2020
Course Code: MAT 105 **Course Title: Engineering Mathematics -I**
Time: 1 Hour 30 minutes **Full Marks: 50**

NB: Answer any of five (5) questions. Each question carries equal marks.

Q-1	Discuss the point of discontinuity and draw graph of the function given by $f(x) = \begin{cases} -x, & x \leq 0 \\ x, & 0 < x \leq 1 \\ 2-x, & 1 < x \leq 2 \\ 1, & x > 2 \end{cases}$	07
Q-2	Find an open interval on which the following functions are increasing and decreasing (i) $f(x) = xe^{-x}$ (ii) $f(x) = x^3$	07
Q-3	If $y = e^{a \sin^{-1} x}$ then show that (i) $(1-x^2)y_2 - xy_1 - a^2 y = 0$ (ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0$	07
Q-4	Using appropriate chain rule find the $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$ for $w = 2xy$, where $x = s^2 + t^2$ and $y = \frac{s}{t}$ at $s = 1, t = 2$	07
Q-5	Evaluate $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$ using L'Hospital rule. Verify Mean value theorem for $f(x) = x^2 - 4x$ in $[2, 4]$	07
Q-6	Transform the equation $11x^2 + 24xy + 4y^2 - 20x - 40y - 5 = 0$ to rectangular axes through the point $(2, -1)$ and inclined at an angle $\tan^{-1}\left(-\frac{4}{3}\right)$	07
Q-7	Test the nature of the equation $f(x, y) = 8x^2 + 4xy + 5y^2 - 16x - 14y + 13 = 0$ and also reduces to its standard form.	07

Q-8	Show that the maximum value of the function is	07
Q-9	If $u = 2(ax + by)^2 - (x^2 + y^2)$ and $a^2 + b^2 = 1$, then find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$	07
Q-10	If $y = (\sin^{-1} x)^2$ then show that (i) $(1 - x^2)y_2 - xy_1 - 2 = 0$ (ii) $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0$	07