Ans. to the. Q-No - 5

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{3-10}{3} = -2.33$$

$$\& \; z_2 = \frac{x_2 - \mu}{\sigma} = \frac{12-10}{3} = 0.67$$

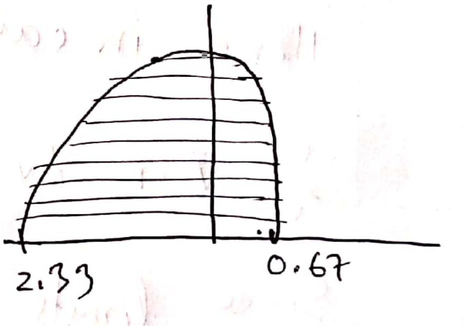
$P(x_1 < x < x_2)$

$= P(3 < x < 12)$

$= P(z_1 < z < z_2)$

$\cancel{= P(-z} = P(-2.33 < z < 0.67)$

$\therefore P(-\alpha < z < 0.67) - P(-\alpha < z < -2.33, 0)$



$\cancel{= \phi(0.67) - \phi(-2.33, 0)}$

$\cancel{= \phi(0.67) - \{1 - \phi(-2.33)} \}$

$= \phi(0.67 - \phi(-2.33)$

$= \phi(0.67) - \{1 - \phi(2.33)\}$

$= 0.7486 - \{1 - 0.0001\}$

$= 0.7387$

An. to the Q-NO - 4

Gauss's Divergence formula is:

$$\iint_S \vec{F} = \int_V \iint \vec{\nabla} \cdot \vec{F}\, dv$$

first, let us compute the

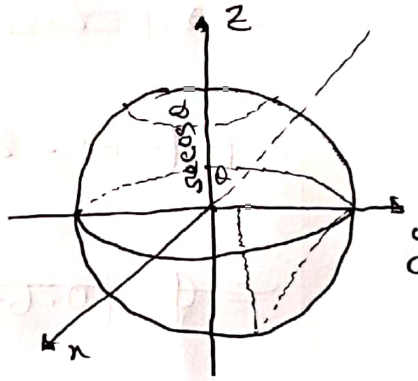
R.H.S.

$$\vec{\nabla}\vec{F} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot 3yz\hat{j}$$

$$= \frac{\partial}{\partial y}(3yz)$$

$$= 3z$$

Hence in cartesian coordinates:



$$\iiint_V \vec{\nabla} \cdot \vec{F} \cdot dv = \int_V \iint 3z\, dx\, dy\, dz$$

Since, limits are given in spherical coordinates on S enclosing the sphere with radius r, we transform to spherical coordinates as: $3z = 3r\cos\phi$, $dx\,dy\,dz = r^2 \sin\phi \, dr\, d\theta\, d\phi$ with $0 \le r \le r$, $0 \le \theta \le 2\pi$ and $0 \le \phi \le \pi$; and we get

$$\iiint_V 3z \, dx\,dy\,dz = \int_0^{\pi}\int_0^{2\pi}\int_0^{\pi} (3r\cos\theta)\, r^2 \sin\theta \, dr\,d\theta\,d\phi$$

$$= \int_0^{\pi}\int_0^{2\pi} \cos\theta \sin\theta \left[r^3\right]_0^{\pi} d\theta\, d\phi$$

$$= r^3 \int_0^{\pi}\int_0^{2\pi} \cos\theta \sin\theta \, d\phi\, d\theta$$

$$= r^3 \int_0^{\pi} \cos\theta \sin\theta \left[\theta\right]_0^{2\pi} d\theta$$

$$= 2\pi r^3 \int_0^{\pi} \cos\theta \sin\theta \, d\theta$$

$$= 2\pi r^3 \int_0^{\pi} \frac{1}{2} \sin(2\theta)\, \theta$$

$$= \pi r^3 \int_0^{\pi} \sin(2\theta)\, d\theta$$

$$= -\pi r^3 \left(\frac{1}{2}\right)\left[\cos(2\theta)\right]_0^{\pi}$$

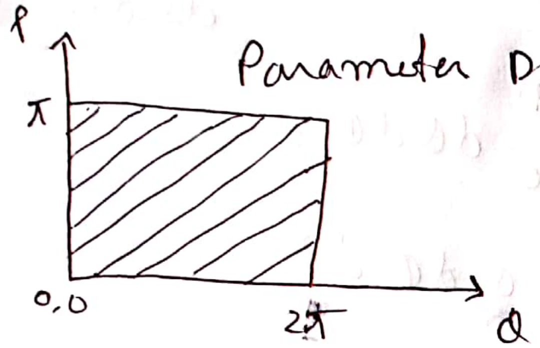
$$= -\frac{\pi}{2} r^3 [1-1]$$

$$= 0$$

Now,
we compute L.H.S. :

$\iint_S \vec{F} = (x, (\theta, \phi), y(\theta, \phi), z(\theta, \phi))$.

$\vec{N}(\theta, \phi]\, d\theta\, d\phi$ where D is the parameter domain, $D = \{(\theta, \phi) : 0 \le \theta \le 2\pi,\ 0 \le \phi \le \pi\}$



here, $\vec{F}(x(\theta,\phi), y(\theta,\phi), z(\theta,\phi)) = 3yz\,\hat{j}$

$= 3r \sin\theta \sin\phi \cos\theta\,\hat{j}$

$= 3r^2 \sin\theta \sin\phi \cos\theta\,\hat{j}$

Now, $\vec{N}(\theta,\phi) = \dfrac{\partial(y,z)}{\partial(\theta,\phi)}\hat{i} + \dfrac{\partial(z,x)}{\partial(\theta,\phi)}\hat{j} + \dfrac{\partial(x,y)}{\partial(\theta,\phi)}\hat{k}$

[in Jacobian ~~not~~ notation]

$= \left(\dfrac{\partial y}{\partial z}\dfrac{\partial z}{\partial \theta} - \dfrac{\partial z}{\partial \theta}\dfrac{\partial y}{\partial \theta}\right)\hat{i} + \left(\dfrac{\partial z}{\partial \theta}\dfrac{\partial x}{\partial \phi} - \dfrac{\partial x}{\partial \theta}\dfrac{\partial z}{\partial \phi}\right)\hat{j}$

$+ \left(\dfrac{\partial x}{\partial \theta}\dfrac{\partial y}{\partial \phi} - \dfrac{\partial y}{\partial \theta}\dfrac{\partial x}{\partial \phi}\right)\hat{k}$; where, we get:

$\dfrac{\partial x}{\partial \theta} = \dfrac{\partial}{\partial \theta}(r \sin\phi \cos\theta) = -r \sin\theta \sin\phi$, $\dfrac{\partial x}{\partial \phi} = \dfrac{\partial}{\partial \phi}$

$(r \sin\phi \cos\theta) = r \cos\phi \cos\theta$

$\dfrac{\partial y}{\partial \theta} = \dfrac{\partial}{\partial \theta}(r \sin\phi \sin\theta) = r \sin\phi \cos\theta$; $\dfrac{\partial y}{\partial \theta} = \dfrac{\partial}{\partial \phi}$

$(r \sin\theta \sin\theta) = r \cos\phi \sin\theta$

$$\frac{\partial z}{\partial \theta} = \frac{\partial}{\partial \theta}(r\cos\phi) = 0, \quad \frac{\partial z}{\partial \theta} = \frac{\partial}{\partial \phi}(r\cos\theta) = -r\sin\theta$$

$$\therefore \vec{N}(\theta,\phi) = [(r\sin\phi\cos\theta)(-r\sin\phi) - 0(r\cos\phi\sin\theta)]\hat{i}$$
$$+[0(r\cos\phi\sin\theta) - (-r\sin\phi\sin\theta)$$
$$(-r\sin\phi)]\hat{j} + [(-r\sin\phi\sin\theta)(r\cos\phi\sin\theta)$$
$$-r\sin\phi\cos\theta\,[r\cos\phi\cos\theta]\hat{k}$$

$$= r^2\sin^2\phi\cos\theta - r^2\sin^2\phi\sin\theta\,\hat{j} + (-r^2\sin\phi\sin^2\theta$$
$$- r^2\sin\phi\cos\phi\cos^2\theta)\hat{k})$$

$$= 0 - r^2\sin^2\phi\cos\theta\,\hat{i} - r^2\sin^2\phi\sin\theta\,\hat{j} -$$
$$r^2\sin\phi\cos\phi\,(\sin^2\theta + \cos^2\theta)\hat{k}$$

$$= -r^2\sin\phi\,[\cos\theta\sin\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} +$$
$$\cos\phi\,\hat{k}]$$

$$\therefore f(x(\theta,\phi), y(\theta,\phi), z(\theta,\phi))$$

$$\vec{N}(\theta,\phi) = [3r^2\sin\theta\sin\phi\cos\phi\,\hat{j}].$$
$$[-r^2\sin\phi\{\cos\theta\sin\theta\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\phi\,\hat{k}\}]$$

$$= (3r^2\sin\theta\sin\phi\cos\phi)(-r^2\sin\theta\sin^2\phi)$$

$$= -3r^2\sin\theta$$

$\sin^3\theta \cos\phi$

$\therefore \iint_S \vec{F} \int_0^{2\pi} \int_0^{\pi} (-4r^4 \sin^2\theta \sin^3\phi \cos\phi)$

$d\phi\, d\theta = -3r^4 \int_0^{2\pi} \sin^2\theta \left[\int_0^{\pi} \sin^3\phi \cos\phi\, d\phi \right] d\theta$

$= 0$

Thus, L.H.S. = R.H.S.

Ans. to. the. Q. No – 1

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta+2k\pi}{n}\right) + i\sin\left(\frac{\theta+2k\pi}{n}\right)\right]$$

where, $r = |z|$

$= \sqrt{x^2+y^2}$ for $z = x+iy$;

$n=3 \rightarrow k = 0, 1, \text{-----}, n-1$; $k = 0, 1, 2, 3$

θ = any angle of $\arg z$ = $\operatorname{Arg} z$ = $\arctan\left(\frac{y}{x}\right)$

Here, $z = 1-i \rightarrow r = \sqrt{(1)^2+(-1)^2}$

$= \sqrt{2}$

$= |z|$

$\theta = \arctan\left(\frac{-1}{1}\right)$

$= \arctan(-1)$

$= -\frac{\pi}{3}$

Thus, then ~~requir~~ required roots are:

(i) $k=0 \rightarrow (1-i)^{\frac{1}{3}}$

$= (\sqrt{2})^{\frac{1}{3}}\left[\cos\left(\frac{-\frac{\pi}{3}+0}{3}\right) + i\sin\left(\frac{-\frac{\pi}{4}+0}{3}\right)\right]$

$= \left(2^{\frac{1}{2}}\right)^{\frac{1}{3}} [\cos\left(-\frac{\pi}{9}\right) + \cancel{i\sin\left(-\frac{\pi}{9}\right)}\ i\sin\left(-\frac{\pi}{9}\right)$

$= 2^{\frac{1}{6}} [\cos\left(\frac{\pi}{9}\right) - i\sin\left(\frac{\pi}{9}\right)$

$= \cancel{0.9396 - i0.3420} = 0.8192 - i0.3420$

$\cancel{= 1.0555 - i0.38374}24$

(ii) $K = 1 \rightarrow (1-i)^{\frac{1}{3}}$

$= 2^{\frac{1}{6}} [\cos\left(\frac{-\frac{\pi}{3} + 2\pi}{3}\right) + i\sin\left(\frac{-\frac{\pi}{3} + 2\pi}{3}\right)]$

$= 2^{\frac{1}{6}} [\cos\left(\frac{\frac{-\pi + 6\pi}{3}}{3}\right) + i\sin\left(\frac{\frac{-\pi + 6\pi}{3}}{3}\right)]$

$= 2^{\frac{1}{6}} [\cos\left(-\frac{\pi}{9} + \frac{2\pi}{3}\right) + i\sin\left(-\frac{\pi}{9} + \frac{2\pi}{3}\right)]$

$= 2^{\frac{1}{6}} [\cos\left(\frac{5\pi}{9}\right) + i\sin\left(\frac{5\pi}{9}\right)]$

$= -0.05709 + i0.0848$

(iii) $K = 2 \rightarrow (1-i)^{\frac{1}{3}}$

$= 2^{\frac{1}{6}} [\cos\left(\frac{-\frac{\pi}{3} + 4\pi}{3}\right) + i\sin\left(\frac{-\frac{\pi}{3} + 4\pi}{3}\right)]$

$= 2^{\frac{1}{6}} [\cos\left(\frac{\frac{-\pi + 12\pi}{3}}{3}\right) + i\sin\left(\frac{\frac{-\pi + 12\pi}{3}}{3}\right)]$

$= 2^{\frac{1}{6}} (\cos\left(\frac{-\pi + 12\pi}{9}\right) + i\sin\left(\frac{-\pi + 12\pi}{9}\right)]$

$= \cancel{2^{\frac{1}{6}}} [$

$= 2^{\frac{1}{6}} [\cos\left(\frac{11\pi}{9}\right) + i\sin\left(\frac{11\pi}{9}\right)]$

$= -0.2559 - i0.6424$

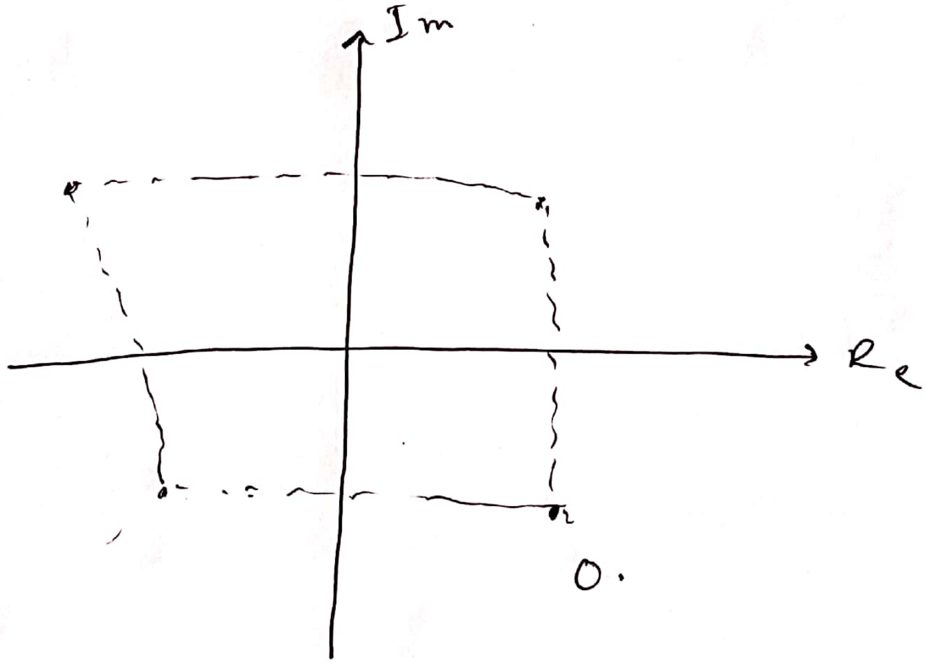
(iii) $k = 3 \rightarrow (1-i)^{\frac{1}{3}}$

$$= 2^{\frac{1}{6}}\left[\cos\left(\frac{-\frac{\pi}{3}+6\pi}{3}\right) + i\sin\left(\frac{-\frac{\pi}{3}+6\pi}{3}\right)\right]$$

$$= 2^{\frac{1}{6}}\left[\cos\left(\frac{\frac{-\pi+18\pi}{3}}{3}\right) + i\sin\left(\frac{\frac{-\pi+18\pi}{3}}{3}\right)\right]$$

$$= 2^{\frac{1}{6}}\left[\cos\left(\frac{17\pi}{9}\right) + i\sin\left(\frac{17\pi}{9}\right)\right]$$

$$= 0.3132 - i\,0.3420$$



An. to the Q. NO - 2

Here C is a simple closed path and the domain inside the C is simple connected But $0 \in D$, and $f'(z) = \frac{-2}{z^2}$ does not exist) for $z=0$, Hence $f(z)$ is not analytic is the entire domain D.

Thus we cannot apply cauchy Intergral Theorem. But we can evaluate $\oint f(z)\, dz$

$= \oint \frac{2}{z} dz$ as follow

We can rewrite,

$c = z(\theta)$

$= x+yi$

$= r\cos\theta + ir\sin\theta$, for $0 \leq \theta \leq 2\pi$

$= \cos\theta + i\sin\theta$ [$\because$ for unit circle, $r=1$ $0 \leq \theta \leq 2\pi$]

$\Rightarrow z(\theta) = e^{i\theta}$

$\Rightarrow z'(\theta) = ie^{i\theta}$ and we can write

$\oint_C \frac{2}{z} dz$

$= \oint_C \frac{2}{z} dz$

$$= \int_0^{2\pi} f(z(\theta)) z'(\theta) d\theta$$

$$= \int_0^{2\pi} \frac{2}{z(\theta)} z'(\theta)\, d\theta$$

$$= \int_0^{2\pi} \frac{2}{e^{i\theta}} \cdot ie^{i\theta}\, d\theta$$

$$= i\int_0^{2\pi} 2\, d\theta$$

$$= i\, 2 \int_0^{2\pi} d\theta$$

$$= 2i\, [\theta]_0^{2\pi}$$

$$= 2i\, [2\pi - 0]$$

$$= 4\pi i$$

So, cuchy intergrat theorem does not apply

the above integration is not equal

to zero.

Ans. to the Q.No - 3

Given, $\vec{f} = \cancel{a}\, 2\hat{i} + \hat{j} - 3\hat{k}$

we know,

direction cosines are:

$\cos\alpha = \frac{a_1}{\|\vec{a}\|}$, $\cos\beta = \frac{a_2}{\|\vec{a}\|}$ & $\cos\gamma = \frac{a_3}{\|\vec{a}\|}$

Now, $a_1 = 2$, $a_2 = 1$ and $a_3 = -3$ gives

$$\|\vec{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$= \sqrt{2^2 + 1^2 + (-3)^2}$$

$$= \sqrt{4 + 1 + 9}$$

$$= \sqrt{14}$$

$$= \cancel{3.74}$$

$\therefore \cos\alpha = \cancel{\frac{2}{3.74}} \quad \frac{2}{\sqrt{14}}$

$\therefore \cos\beta = \cancel{\frac{1}{3.74}} \quad \frac{1}{\sqrt{14}}$

$\therefore \cos\gamma = \cancel{\frac{-3}{3.74}} \quad \frac{-3}{\sqrt{14}}$

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So, the direction angels are –

$\alpha = \text{arc}\cos\frac{2}{\sqrt{14}} \cong 1.006$ radian [~58°]

$\beta = \text{arc}\cos\frac{1}{\sqrt{14}} \cong 1.30$ radian [~75°]

$\gamma = \text{arc}\cos\frac{-3}{\sqrt{14}} \cong 2.5001$ radian [~143°]