Signals and Systems

Date - 03.09.2019

Reference Book

Signals and Systems (2nd edition)

Simon Haykin

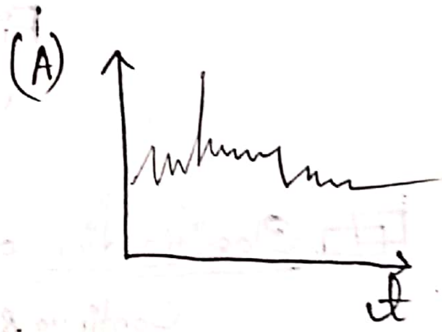
Signal:

It is defined as a function of one or more varibles that conveys information on the basis of a physical phenomenon.

→ Speech

→ Image

→ heartbeat, blood pressure, pulse.



Signal → single dimentional signal
→ multi dimentional signal

System:

It is an entity that manipulates one or more signals to accomplish a function and, thereby yeild a new signal

→ Automatic Speaker recognition.
→ Communication System.
→ Aircraft Landing System.

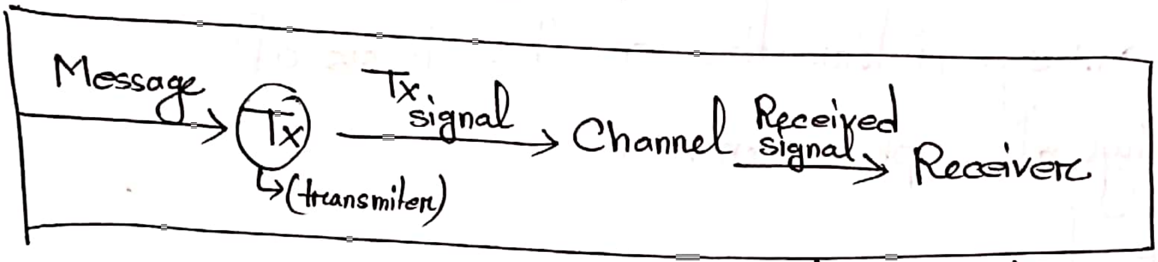


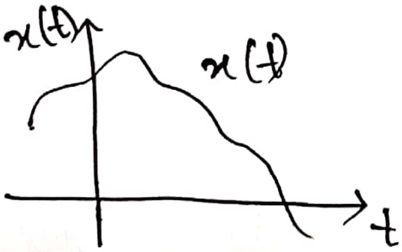
Fig: Basic Communication System

Classification of Signal:

Continuous Signal

① Its amplitude/value varies continuously with time.

[যেকোনো আকারের value হবে]

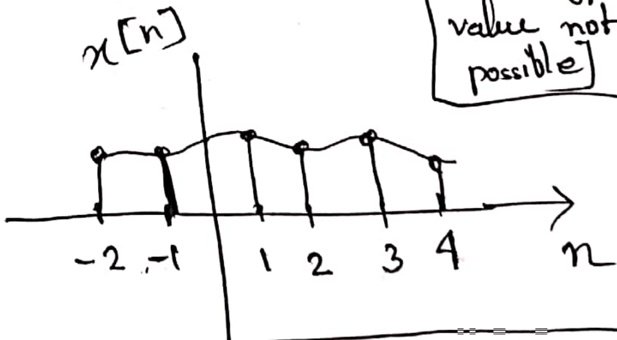


[Continuous signal থেকে value নিয়ে discrete signal তৈরি করা যাবে]

Discrete signal

① Changes in discrete instant of time.

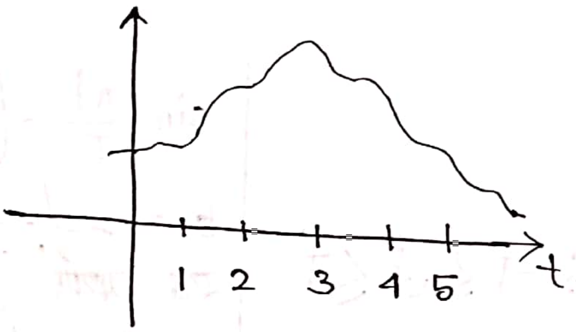
[fraction type value not possible]



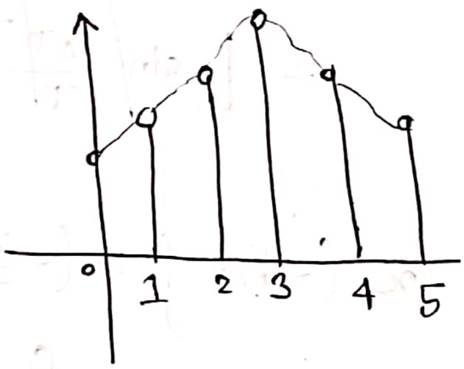
$$\boxed{x[n] = x(nT_s)}$$

→ $T_s = 2s$ (sampling period of time)/time interv

* T_s হচ্ছে simpling ~~of time~~ period. যা দ্বারা time interval নির্দেশ করে, অর্থাৎ continuous system থেকে discrete system তৈরি করার সময় ঠিক কত সময় পর পর value নিব তাই নির্দেশ করে।



$T_s = 1s$

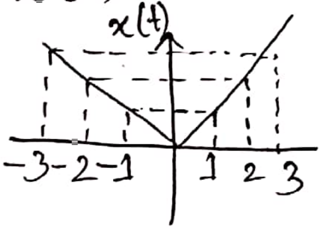


$x[n] = x(n)T_s$

$n = 0, 1, 2, 3, 4.$

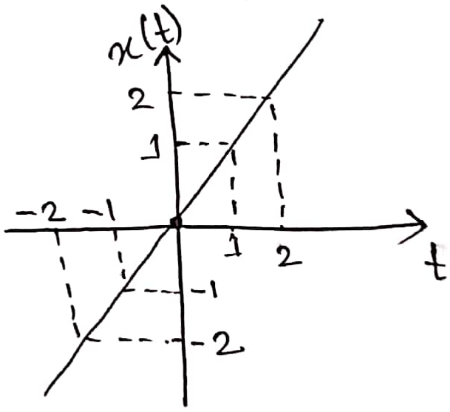
Even Signal: (Symmetric signal)

$x(-t) = x(t)$



Odd signal (inverse signal)

$x(-t) = -x(t)$



Ex: 1.1

$x(t) = x(-t)$

$x(t) = -x(-t)$

$$x(t) = \begin{cases} \sin\left(\frac{\pi t}{T}\right), & -T \leq t \leq T \\ 0, & \text{otherwise} \end{cases}$$

$$= \int_{-T}^{T} \sin\left(\frac{\pi T}{T}\right)$$

is it even or odd?

$$= \left[\sin \frac{\pi T}{T}\right]_{-T}^{T}$$

$\sin \frac{\pi t}{T} = -\left(\sin\left(\frac{\pi t}{T}\right)\right)$

because $(- -) = +$ হয়, অর্থাৎ একটি অন্যটি inverse হবে।

$$x(-t) = \begin{cases} \sin\left(-\frac{\pi t}{T}\right) & ; -T \leq t \leq T \\ 0, & \text{ot.} \end{cases}$$

$$= \begin{cases} -\sin\left(\frac{\pi t}{T}\right), & -T \leq t \leq T \\ 0, & \text{ot.} \end{cases}$$

it is odd.

Decomposing Signal in even and odd signals:

$$x(t) = \underbrace{x_e(t)}_{\text{even}} + \underbrace{x_o(t)}_{\text{odd}}$$

$$x(-t) = x_e(-t) + x_o(-t)$$

$$= \underbrace{x_e(t)}_{\text{even}} - \underbrace{x_o(t)}_{\text{odd}}$$

Thereby,

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

even value,

$\rightarrow 2x_e(t) = [x(t) + x(-t)]$

$x_e(t) = \frac{1}{2}[x(t) - x(-t)]$

$\rightarrow x_o(t) - (-x_o(t)) = [x(t) - x(-t)]$

$\rightarrow x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

Ex: 1.2

$x(t) = e^{-2t}\cos t \rightarrow$ Find the even and odd components of this signal.

$$x(t) = e^{-2t}\cos t$$

$$x(-t) = e^{-2(-t)}\cos(-t)$$

$$= e^{2t}\cos t$$

$$x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$$

$$= \frac{(e^{-2t} + e^{2t})\cos t}{2}$$

$$x_o(t) = \frac{(e^{-2t} - e^{2t})\cos t}{2}$$

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Problem 1.1

$$x(t) = (1+t^3)\cos^3(10t)$$

Find the even and odd component of this signal.

$$\Rightarrow x(-t) = (1+(-t)^3)\cos^3(10(-t))$$

$$= (1-t^3)\cos(10t)$$

$$\therefore x_e(t) = \frac{1}{2}\left((1+t^3)\cos^3(10t) + (1-t^3)\cos^3(10t)\right)$$

$$= \frac{1}{2}\left\{\cos^3(10t)\left[(1+\cancel{t^3}+1-\cancel{t^3})\right]\right\}$$

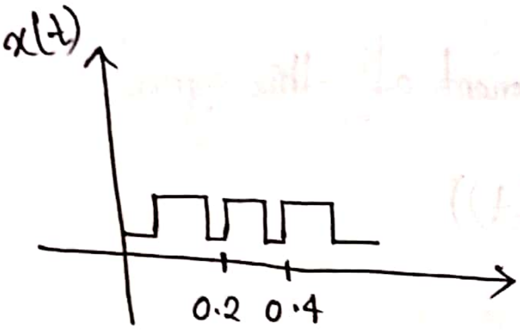
$$= \cos^3(10t)$$

$$\therefore x_o(t) = \frac{1}{2}\cos^3(10t)\left[1+t^3-1+t^3\right]$$

$$= t^3\cos^3(10t).$$

Periodic Signal:

$x(t) = x(t+T)$ ..... (i)

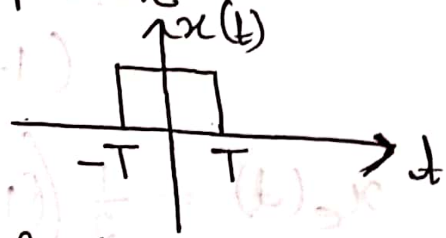


(T) period পর পর repeat হবে,

$$f = \frac{1}{T} = \frac{1}{0.2} = 20\,Hz.$$

Non periodic signal

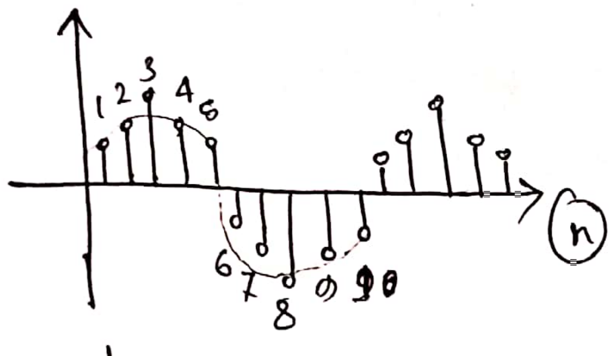
Any signal that does not satisfy (i) is non-periodic or aperiodic



নির্দিষ্ট সময় পর আর কোনো value দিবে না।

Periodic discrete signal এর শর্ত condition:

$x[n] = x[n+N]$

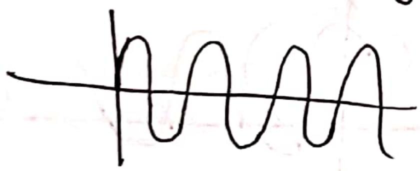


period $(N) = 10$

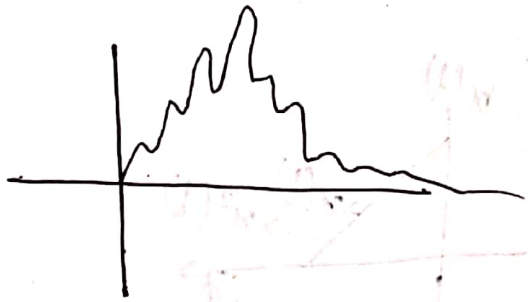
Plot(t, x(t))

stem(n, x(n))

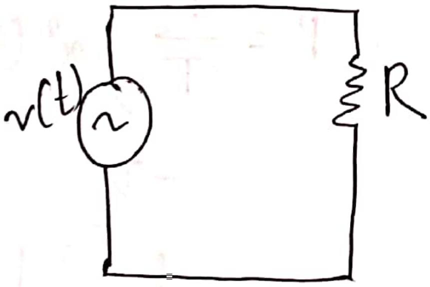
Deterministic	Random
No uncertainty with respect to it value at any time.	There is uncertainly before it occurs.



sinωt
coswt } determinated signal



Energy Signal Vs. Power Signal:



$$P(t) = \frac{v^2(t)}{R}$$

$$= i^2(t) \cdot R$$

$$P(t) \propto x^2(t)$$

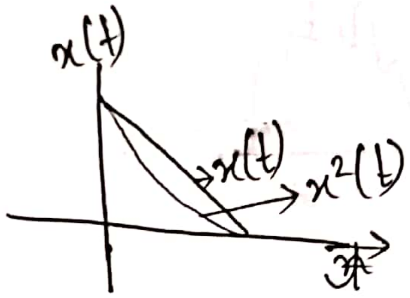
$$e(t) \propto = P(t)\, dt$$

$$\boxed{E = \int_{-\infty}^{\infty} x^2(t)\, dt}$$

* Condition:

Energy Signal $E(t)$

$$0 < E < \alpha$$

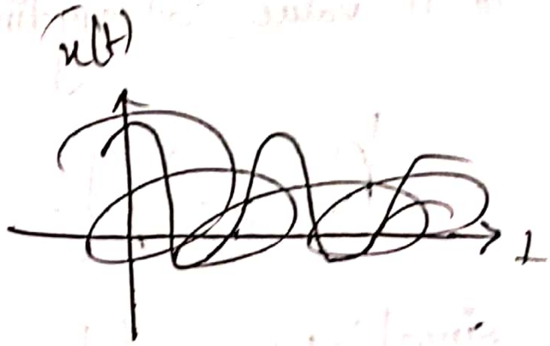


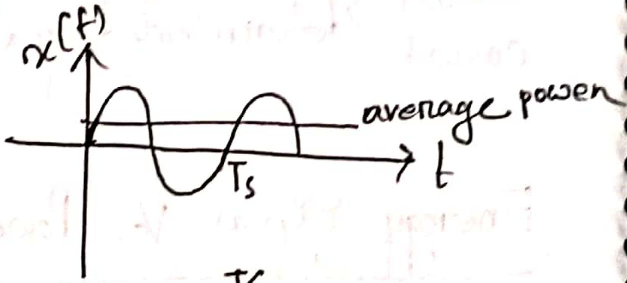
* non periodic and finite signal

$$E = \int_{-\alpha}^{\alpha} x^2(t)\, dt$$

Power Signal $P(t)$

$$0 < P < \alpha$$





$$P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt.$$

* periodical signal হবে power signal

Problem 1.9:

(a) $x(t) = \begin{cases} t & ; 0 \leq t \leq 1 \\ (2-t) & ; 1 \leq t \leq 2 \\ 0, & \text{otherwise,} \end{cases}$

is it energy or power signal? Find Energy/power of this signal?

$\Rightarrow$ ~~$E(t) = \int_0^1$~~

Soln: As it is a non-periodical signal.

So, this signal is energy signal.

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_0^1 t^2\,dt + \int_1^2 (2-t)^2\,dt$$

$$= \left[\frac{t^3}{3}\right]_0^1 + \left[\frac{(2-t)^3}{3}\right]_2^1$$

$$= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

~~$E = \int_1^2 (2-t)^2\,dt$~~

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Problem 1.9

(c) $x(t) = 5\cos(\pi t) + \sin(5\pi t)$, $-\alpha < t < \alpha$

$$P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt$$

Sinusiodalsignal

$\sin(\omega t)$

$\sin(2\pi f t)$

$\sin(2\pi (\frac{1}{2}) t)$

f

$T = 2$

$$= \frac{5}{2}\int_{-1}^{+1} \cos^2(\pi t)\,dt + \frac{1}{2/5}\int_{-1/5}^{1/5} \sin^2(5\pi t)\,dt$$

$2\cos^2\theta = 1 + \cos(2\theta)$

$2\sin^2\theta = 1 - \cos(2\theta)$

$$= \frac{5}{2}\int_{-1}^{+1} \cos^2(\pi t)\,dt + \frac{5}{2}\int_{-1/5}^{1/5} \sin^2(5\pi t)\,dt$$

$$= \frac{5}{4}\int_{-1}^{+1} 2\cos^2(\pi t)\,dt + \frac{5}{4}\int_{-1/5}^{1/5} 2\sin^2(5\pi t)\,dt$$

$$= \frac{5}{4}\left[1 + \cos(\pi t)\right]_{-1}^{+1} + \frac{5}{4}\left[1 - \cos(5\pi t)\right]_{-1/5}^{1/5}$$

$$= \frac{5}{4}\left[1 + \cos(\pi) - 1 - \cos(\pi)\right] + \frac{5}{4}\left[1 - \cos(5\pi \cdot \tfrac{1}{5}) - 1 - \cos(5\pi \cdot \tfrac{1}{5})\right]$$

$$= \frac{5}{4}\left[\cos(\pi) - \cos(\pi)\right] + \frac{5}{4}\left[\cancel{1} - \cos(\pi) - \cancel{1} - \cos(\pi)\right]$$

$$= \frac{5}{4} \times 0 + \frac{5}{\cancel{4}_2}(-\cancel{2}\cos\pi)$$

$$= 0 + \left(-\frac{5}{2}\cos\pi\right)$$

$$= -\frac{5}{2}\cos(\pi).$$

$$= \frac{1}{2}\int_{-1}^{+1} (5\cos\pi t)^2\, dt + \frac{1}{2/5}\int_{-1/5}^{1/5} (\sin 5\pi t)^2\, dt$$

$$= \frac{1}{2} \times 25 \int_{-1}^{+1} \frac{1}{2}(1+\cos 2\pi t)\, dt + \frac{5}{2}\int_{-1/5}^{1/5} \frac{1}{2}(1-\cos 10\pi t)\, dt$$

$$= \frac{25}{4}\left[\left(t - \frac{\sin 2\pi t}{2\pi}\right)\right]_{-1}^{+1} + \frac{5}{4}\left[t + \sin\frac{10\pi t}{10\pi}\right]_{-\frac{1}{5}}^{\frac{1}{5}}$$

$$= \frac{25}{4}\left[\cancel{1} \frac{\sin(+1)}{} - \cancel{1} + \sin(-1)\right] + \frac{5}{4}\left(\frac{1}{5} + \cancel{\sin\left(\frac{1}{5}\right)}\right) + \frac{1}{5} + \cancel{\sin\left(-\frac{1}{5}\right)}$$

$$= \frac{25}{4}[1+1] + \frac{5}{4}\left(\frac{1}{5} + \frac{1}{5}\right)$$

$$= \frac{25}{2} + \frac{\cancel{5}}{\cancel{4}_2}\left(\frac{\cancel{2}}{\cancel{5}}\right) = \frac{25+1}{2} = \frac{26}{2} = 13$$

Class no. 3

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$\boxed{\text{discrete signal}} \rightarrow \boxed{x[n]}$

(f) $x[n] = \begin{cases} \sin(\pi n) & ; -4 \le n \le 4 \\ 0 & ; \text{otherwise.} \end{cases}$

~~sin~~ $2\pi f t$

$x_p = \frac{1}{N} \sum_{n=0}^{n-1} x^2[n]$

Solution:

$x[n] = \{ \sin(\pi n) \quad ; -4 \le n < 4$

প্রতিটা point আলাদা করে যোগ করা,

$\sin(\omega t)$

$\sin 2\pi f t$

$\sin (2\pi$

$$\text{Energy} = \sum_{-\infty}^{\infty} x^2[n]$$

$$= \sum_{n=-4}^{4} x^2(\pi n)$$

$$= \sum_{n=-4}^{4} \sin^2(\pi n)$$

$$= \sum_{n=-4}^{4} \sin^2(\pi n)$$

$$= 0$$

$$\text{Energy} = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$\therefore$ Zero signal যার power ও নাই energy ও নাই।

⊛ Sinusoidal Signal:

$x(t) = A\cos(\omega t + \varphi)$ → continuous time signal এর জন্য

$x[n] = A\cos(\Omega n + \varphi)$ → discrete signal এর angular formula

$= A\cos(\Omega n + \Omega N + \varphi)$ ← $x[n+N]$

→ $n = [n+N]$

$\Omega N = 2\pi m$

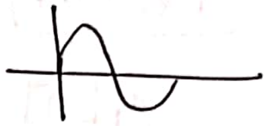
→ m এর integer value এর জন্য, Ωm এর value 2π এর multiple হবে।

$m=0, \Omega N = 20$

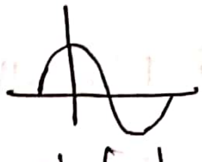
$m=1, \Omega N = 2\pi$

periodic signal হবে, নতুবা, non-periodic

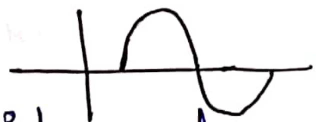
sin(ωt)



sin(ωt + 90°)



sin(ωt − 90°)



□ $x[n] = 5\sin[2n]$;

Here, $\Omega = 2$

$\boxed{\Omega N = 2\pi m}$

$N = \frac{2\pi}{\Omega} \times m$

$= \frac{2\pi}{2} \times m$

$= \pi \times m$

$= 5\sin$

non-periodic signal

* দশমিক point থাকলে non periodic.

$x[n] = 5\sin(5\pi n)$

$N = \frac{2\pi}{5\pi} \times m$

$= \frac{2}{5} \times m$ [if $m=5$]

$= \frac{2}{\cancel{5}} \times \cancel{5}$

$= 2$ [fundamental period]

সব থেকে ছোট value.

* unit না থাকলে sample দিয়ে লিখতে হবে.

After 2 sample the signal repeat.

Ex: $\boxed{1.7}$

$x_1[n] = \sin[5\pi n]$

$x_2[n] = \sqrt{3}\cos[5\pi n]$

(a) Find the common fundamental period?

Solution:

$x_1[n] = \sin[5\pi n]$

$\cancel{N = \frac{2\pi}{\Omega} \times m} \quad \cancel{N = 2\pi \times \ldots}$

Here,

$\boxed{N = 2}$

$\cancel{= \frac{2\pi}{5\pi} \times m}$

$\cancel{= \frac{2\pi}{5\pi} \times \frac{1}{5}}$

(a) The common fundamental period 2.

(b) $\cancel{y[n] = x_1[n] + x_2[n]}$

$= \sin[5\pi n] + \sqrt{3}\cos[5\pi n]$

$\cancel{= \frac{1}{2}\left(\sin[5\pi n] + \sqrt{3}\cos[5\pi n]\right)}$

$= \cancel{\frac{1}{2}\sin[5\pi n] + \frac{\sqrt{3}}{2}}\cos[5\pi n]$

$=$

$$A\cos(\omega t + \phi_1) + B\sin(\omega t + \phi_2)$$

$$= \sqrt{(A\cos\phi_1 + B\sin\phi_2)^2 + (A\sin\phi_1 + B\cos\phi_2)^2} \times \cos\left(\omega t + \tan^{-1}\frac{A\sin\phi_1 + B\cos\phi_2}{A\cos\phi_1 + B\sin\phi_2}\right)$$

$$= \sqrt{(A\cos\phi_1 + B\sin\phi_2)^2 + (A\sin\phi_1 + B\cos\phi_2)^2} \times \boxed{\cos\left(\omega t + \tan^{-1}\frac{A\sin\phi_1 + B\cos\phi_2}{A\cos\phi_1 + B\sin\phi_2}\right)}$$

amplitude

$$\therefore\ y[n] = x_1[n] + x_2[n]$$

Operation performed on dependent variable:

$$\text{Signal} \rightarrow x(t)$$

Addition $\rightarrow y(t) = x_1(t) + x_2(t)$

Multiplication $\rightarrow y(t) = x_1(t) \times x_2(t)$

Differentiation $\rightarrow y(t) = \frac{d}{dt} x(t)$

Integration $\rightarrow y(t) = \int_{-\alpha}^{t} x(t)\, d\tau$

Operation performed on independent variable:

Time Scaling :

$$\text{Signal} \rightarrow x(t)$$

$y(t) = x(at) \rightarrow$ If $0 < a < 1 \rightarrow$ signal will be stretched

$a > 1 \rightarrow$ signal will be compressed.

* $y(t) = x(2t)$

$t = \frac{1}{2}$

$2t = 2 \times \frac{1}{4}$

t

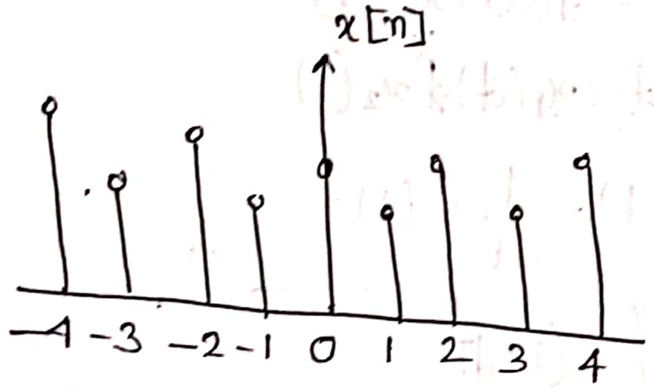
* time বাড়াই দিলে file size compressed হয়ে যায়।

$y(t) = x(\frac{1}{2}t) \rightarrow x(\frac{1}{2})$ t এর value 1 দিতে হবে

Class no. 5

Tuesday

17.09.2019

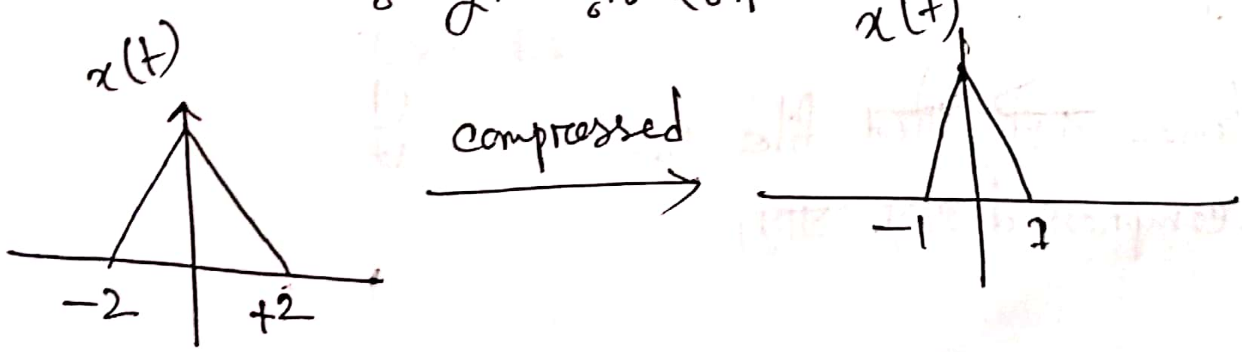


$Y[n] = x[2n]$

→ 1 এর থেকে বড় হলে ~~expand হয়ে যাবে~~। signal compress হয়ে যেতে পারে তাই time scale কে এই value দিয়ে ভাগ করতে হবে।

$x(2(t)) \rightarrow$ maximum value 2. অর্থাৎ t এর value 1 হবে।

→ যদি 1 থেকে ছোট হয় তাহলে signal expand হবে তখন time scale কে ঐ value দিয়ে গুন করতে হবে।



$x[n] = x[\frac{1}{2}x]$

$n = \frac{1}{2} n \times 2$

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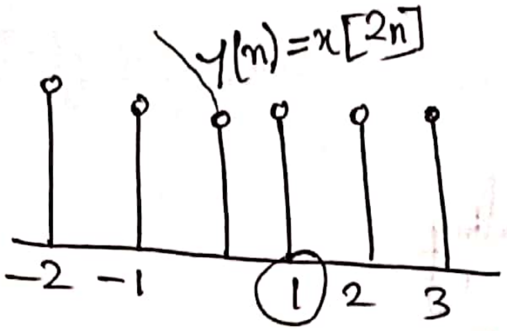


$y(0)=x[0]$

$y(1)=x[2]$ [$n=1$ হলে]

$y[2]=x[4]$

$y[3]=x[6]$

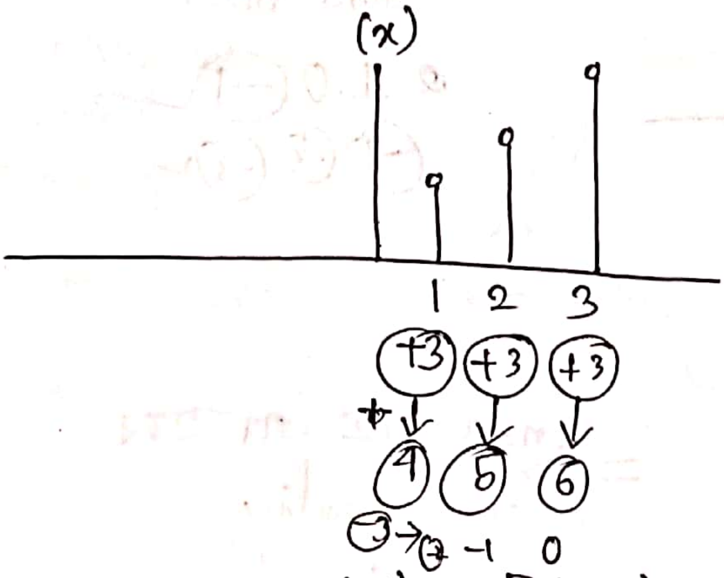
Time Shifting:

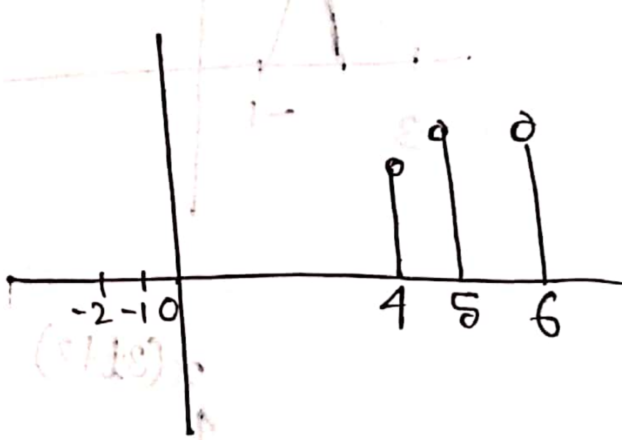
$y[n]=x[n-3]$

$y(n)=x[n+3]$

(−) operator হলে, time shifting হবে ডান দিকে

(+) operator হলে, time shifting হবে বাম দিকে



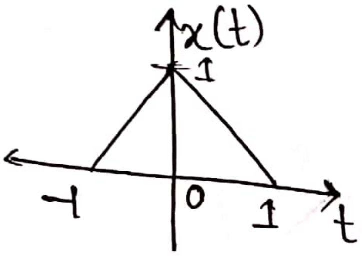


$y(4)=x[4-3]$

$=x[1]$

$y[n] = x[2n-3]$

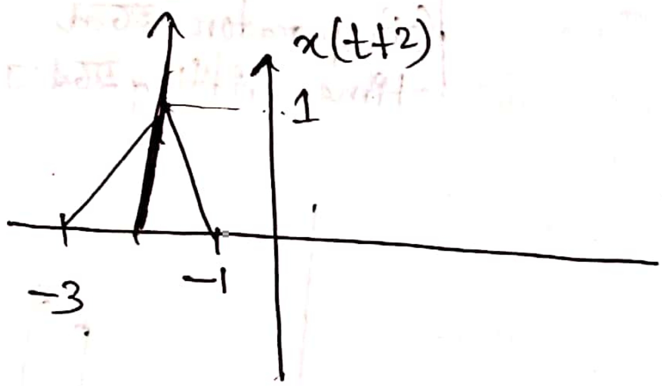
* আগে হবে time shifting তারপর হবে scaling



y(t) → থাকা মানে continuous
y(n) → ,, ,, discrete

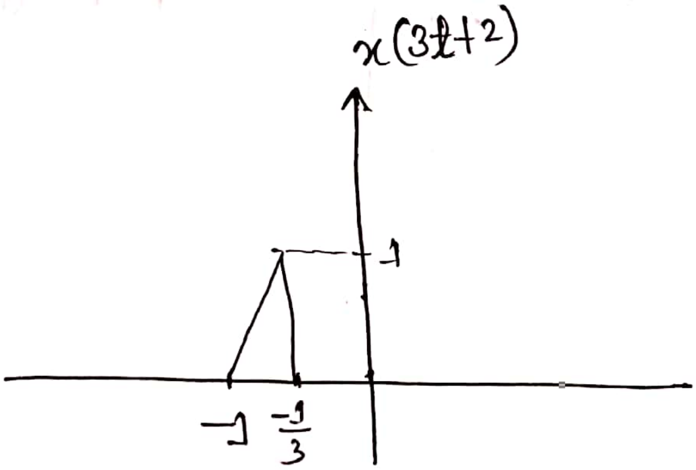
Find, $y(t) = x(3t+2)$

$= x(t+2)$



⟹ 2 ঘর বামে যাবে তাই

1, 0, −1 → −1, −2, −3



⟹ n=3, তাই ভাগ হবে time scaling

$\frac{-1}{3}, -\frac{3}{3}$

$= -1$