02/10/19

(2)

(1) Discuss the point of discontinuity and draw the graph of the function.

$$f(x) = -x \; ; \; x \le 0$$
$$= x \; ; \; 0 < x \le 1$$
$$= 2-x \; ; \; 1 < x \le 2$$
$$= 1 \; ; \; x > 2$$

Q1
P1

$\Rightarrow$ Here we shall consider three points, $x = 0, 1$ and 2

At $x = 0$

$f(0) = 0$

$\therefore f(x) = -x$ for f $x \le 0$

L.H.L.

$$f(x) = \lim_{x \to 0-0} f(x) = \lim_{x \to 0-0} (-x)$$

$$= \lim_{h \to 0} [-(0-h)]$$

$$= \lim_{h \to 0} 0$$

1

R.H.L.

(2) R.H.L.

$$\lim_{x\to 0+0} f(x) = \lim_{x\to 0+0} (x)$$

P2

$$= \lim_{h\to 0} (0+h)$$

$$= \lim_{h\to 0} 0$$

the funtion $f(x)$ is continuous at $x=0$

At $x=1$

$f(1) = 1 \quad \therefore f(x) = x$ for $0 < x \leq 1$

R.H.L

$$\lim_{x\to 1+0} f(x) = \lim_{x\to 1+0} (2-x)$$

$$= \lim_{h\to 0} 2-(1+h) \quad \left[\begin{array}{l}\text{putting } x=1+h.\\ \quad h\to 0 \text{ as } x\to 1\end{array}\right]$$

$$= \lim_{h\to 0} (1-h)$$

$$= \lim_{h\to 0} 1$$

L.H.L (2)

$$\lim_{x\to 1-0} f(x) = \lim_{x\to 1-0} (x) \quad \therefore f(x) = x \text{ for } 0 < x \le 1$$

$$= \lim_{h\to 0} (1-h)$$

P 3

$$= 1$$

the function is continuous at $x=1$

At $x=2$

$$f(2) = 2-2$$

$$= 0 \qquad \therefore f(x) = 2-x \text{ for } 1 < x \le 2$$

R.H.L

$$\lim_{x\to 2+0} f(x) = \lim_{x\to 2+0} (1)$$

$$= 1$$

L.H.L

$$\lim_{x\to 2-0} f(x) = \lim_{x\to 2-0} (2-x) \quad \because f(x) = 2-x \text{ for } 1 < x \le 2$$

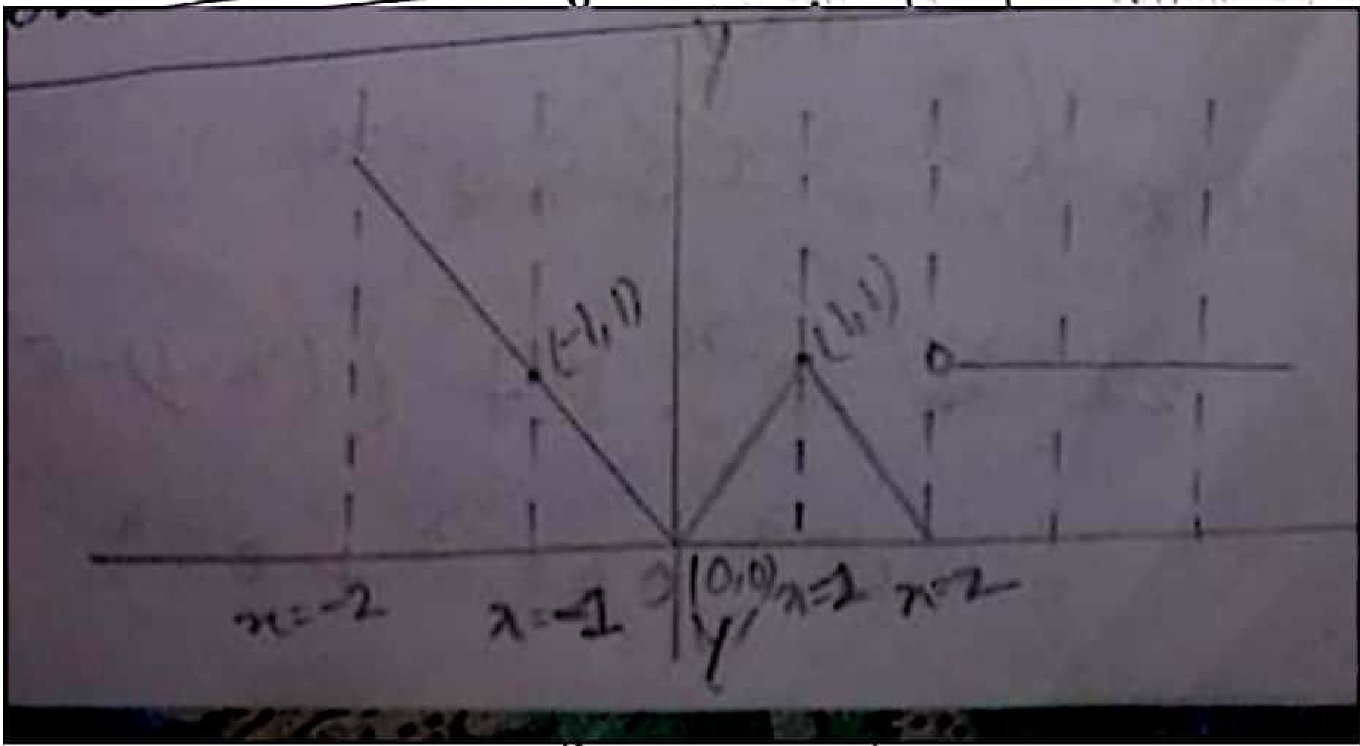
$$= \lim_{h \to 0} \left[2-(2-h)\right]$$

$$= \lim_{h \to 0} h \; 0-h$$

$$= 0$$

P 4

Graph of function $f(x)$:



Let $y = f(x)$. Then the graph of $f(x)$ will be as shown by thick lines in Fig. I. above. Here note that $f(x)$ is discontinious at $x = 2$ and has been shown by a hollow circle and the graph of the function for $x > 2$ is the line $y = 1$ as we are given $f(x) = 1$ for $x > 2$.

Q2: Find an open interval on which the following functions are increasing & decreasing.

(i) $f(x) = xe^{-x}$

(ii) $f(x) = x^3$

Solve: (i) $f(x) = xe^{-x}$

$$f'(x) = x \cdot \frac{d}{dx}(e^{-x}) + e^{-x}\frac{d}{dx}(x)$$

$$= x \cdot e^{-x}\frac{d}{dx}(-x) + e^{-x} \cdot 1$$

$$= -xe^{-x} + e^{-x}$$

$$= e^{-x} - xe^{-x}$$

$$= e^{-x}(1-x)$$

(a) For increasing,

$$f'(x) > 0$$

$$\Rightarrow e^{-x}(1-x) > 0$$

$$\Rightarrow 1-x > 0$$

$$\Rightarrow -x > -1$$

$$\Rightarrow x < 1$$

[অসমতায় (−) দিয়ে গুন করলে অসমতা চিহ্ন পাল্টে যায়]

2

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$\therefore x < 1$

(b) For decreasing,

$f'(x) < 0$

$\Rightarrow e^{-x}(1-x) < 0$

$\Rightarrow 1 - x < 0$

$\Rightarrow -x < -1$

$\Rightarrow x > 1$

(ii) $f(x) = x^3 \quad \therefore f'(x) = 3x^2$

(a) for increasing, $f'(x) > 0$

$\Rightarrow 3x^2 > 0$

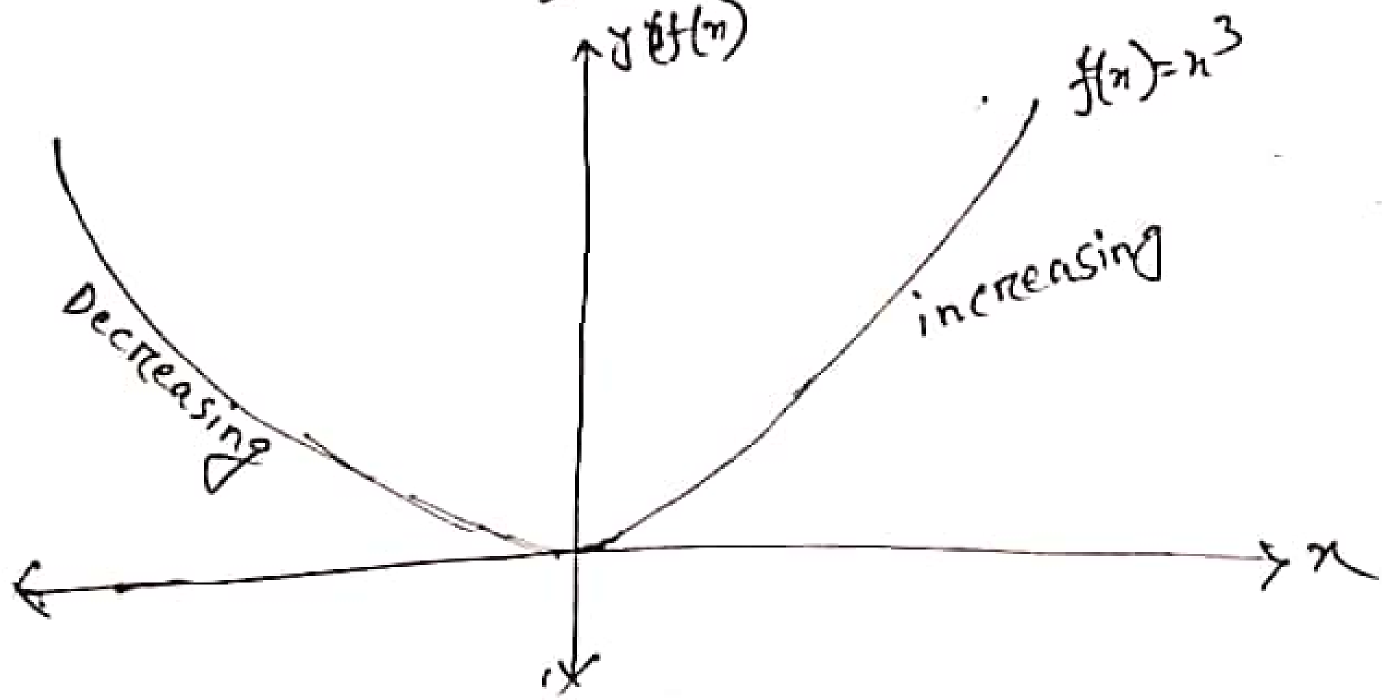
$\Rightarrow x^2 > 0$

$\Rightarrow x > 0$

(b) for decreasing, $f'(x) < 0$

$\Rightarrow 3x^2 < 0$

$\Rightarrow x < 0$



Q-3 If $y = e^{a\sin^{-1}x}$ then show that -

(i) $(1-x^2)y_2 - xy_1 - a^2y = 0$

(ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$

Soln: (i) Given,

$$y = e^{a\sin^{-1}x}$$

$$\Rightarrow y_1 = e^{a\sin^{-1}x} \frac{a}{\sqrt{1-x^2}}$$

$$\Rightarrow y_1 = \frac{ya}{\sqrt{1-x^2}}$$

$$\Rightarrow y_1^2 = \frac{y^2a^2}{1-x^2}$$

$$\Rightarrow y_1^2(1-x^2) = y^2a^2$$

Differentiating with respect to x in bothside.

$$\frac{d}{dx}\left(y_1^2(1-x^2)\right) = \frac{d}{dx}(a^2y^2)$$

$$\Rightarrow y_1^2\frac{d}{dx}(1-x^2) + (1-x^2)\frac{d}{dx}(y_1^2) = 2a^2yy_1$$

$$\Rightarrow y_1^2(-2x) + (1-x^2)\times 2y_1y_2 = 2a^2yy_1$$

3

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$$\Rightarrow -2xy_1^2 + 2y_1y_2(1-x^2) = 2a^2yy_1$$

$$\cancel{\Rightarrow -xy_1^2 + 2y_1y_2}$$

$$\Rightarrow -xy_1^2 + y_1y_2(1-x^2) = a^2yy_1$$

$$\Rightarrow -xy_1^2 + y_1y_2(1-x^2) - a^2yy_1 = 0$$

$$\Rightarrow y_2(1-x^2) - xy_1 - a^2y = 0 \quad \text{[showed]}$$

(ii) Differentiating n times with respect to x using Leibnitz theorem —

$$D^n\{(1-x^2)y_2 - xy_1 - a^2y\} = 0$$

$$\Rightarrow D^n\{(1-x^2)y_2\} - D^n\{xy_1\} - D^n(a^2y) = 0$$

$$\Rightarrow \left[(1-x^2)D^n(y_2) + {}^nC_1 D^{n-1}(y_2)D(1-x^2) + {}^nC_2 D^{n-2}(y_2)D^2(1-x^2)\right] - \left[xD^n(y_1) + {}^nC_1 D^{n-1}(y_1)D(x)\right] - a^2y_n = 0$$

$$\Rightarrow \left[(1-x^2)y_{n+2} + ny_{n+2}(0-2x) + \frac{n(n-1)}{2!}y_n(-2)\right] - \left[xy_{n+1} + ny_n\right] - a^2y_n = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - xy_{n+1}(2n+1) - \{n^2 - n + n + a^2\}y_n = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - xy_{n+1}(2n+1) - \{n^2 + a^2\}y_n = 0$$

[Showed]

Bost

Q-04: Using appropriate chain rule find the $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$ for $w = 2xy$, where $x = s^2 + t^2$ and $y = s/t$ at $s = 1$, $t = 2$

Soln:

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s}$$

$$= \frac{\partial}{\partial x}(2xy)\frac{\partial}{\partial s}(s^2+t^2) + \frac{\partial}{\partial y}(2xy)\frac{\partial}{\partial s}(s/t)$$

$$= 2y\frac{\partial}{\partial x}(x)(2s+0) + 2x\frac{\partial}{\partial y}(y) \times \frac{1}{t}\frac{\partial}{\partial s}(s)$$

4

$$= 2y \times 2s + 2x \times \frac{1}{t}$$

$$= 2\frac{s}{t} \times 2s + 2(s^2+t^2) \times \frac{1}{t}$$

$$= \frac{4s^2}{t} + \frac{2(s^2+t^2)}{t} = \frac{6s^2+2t^2}{t} = 14/2 = 7$$

Ans.

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$$

$$= \frac{\partial}{\partial x}(2xy)\frac{\partial}{\partial t}(s^2+t^2) + \frac{\partial}{\partial y}(2xy)\frac{\partial}{\partial t}(s/t)$$

$$= 2y\frac{\partial}{\partial x}(x)(0+2t) + 2x\frac{\partial}{\partial y}(y)\, s\left(-\frac{1}{t^2}\right)$$

$$= 4yt + \frac{-2xs}{t^2} = 4yt - \frac{2xs}{t^2}$$

$$= \cancel{4yt - 2xy \cdot \frac{1}{t}}$$

$$= \cancel{4yt - \frac{w}{t}}$$

$$= 4s - \frac{2(s^2+t^2)s}{t^2}$$

$$= \frac{4st^2 - 2s^3 - 2st^2}{t^2}$$

$$= \frac{2st^2 - 2s^3}{t^2} = \frac{8-2}{4} = 3/2$$

Ans.

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5 (i) Evaluate $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$ using L' Hospital Rule.

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(ii) Verify Mean Value Theorem for $f(x) = x^2 - 4x$ in $[2,4]$

Soln: $\boxed{\text{* Using Lagrange's Mean Value Theorem}}$

Given, $f(x) = x^2 - 4x$ $[2,4]$

① $f(x) = x^2 - 4x$ is exist for all $x \in [2,4]$. Hence $f(x)$ continuous in $[2,4]$

② $f'(x) = 2x - 4$ is exist for all $x \in [2,4]$ Hence, $f(x)$ is differentiable in open interval $(-\pi, \pi)$

Hence, $f(x)$ is satisfied all condition's

So, there exist at least one value $x = c$ such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow 2c - 4 = \frac{f(4) - f(2)}{4 - 2}$$

$$\Rightarrow 2c - 4 = \frac{0 - (-4)}{2}$$

$$\Rightarrow 2c - 4 = \frac{4}{2}$$

$$\Rightarrow 2c - 4 = 2$$

$$\Rightarrow 2c = 6$$

$$\therefore c = 3 \in (2,4)$$

$f(x) = x^2 - 4x$

$\Rightarrow f(2) = (2)^2 - 4 \times 2$

$= 4 - 8$

$= -4$

$f(4) = 4^2 - 4 \times 4$

$= 0$

5

Hence, Lagrange's Mean Value Theorem is verified.

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① Transform the equation $11x^2+24xy+4y^2-20x-40y-5=0$ to rectangular axes through the point $(2,-1)$ and inclined at angle $\tan^{-1}\left(\frac{-4}{3}\right)$

Q 6
P 12

= eqn → $11x^2+24xy+4y^2-20x\cancel{y}-40y-5=0$ —— ①

and the point ~~to~~ which origin $(2,-1)$ and an angle $\theta = \tan^{-1}(-4/3)$

Replacing x and y $(x'+2)$ and $(y'-1)$ respectively

$\Rightarrow 11(x'+2)^2+24(x'+2)(y'-1)+4(y'-1)-20(x'+2)$

6

$-40(y'-1)-5=0$

$\Rightarrow \cancel{121x'}\ 11\left(x'^2+\cancel{2\cdot 2\cdot x'\cdot 2}+2^2\right)+(24x'+48+24y'-24)$ $\quad 2\cdot x\cdot 2\ (2)^2$

$+4y'-4-20x'-40-40y'+40-5=0$

$\Rightarrow 11x'^2+22x'y+11y^2+24x'+24y'+24+$

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$$11x'^2 + 24x'y' + 4y'^2 - 5 = 0$$

$$11x^2 + 24xy + 4y^2 - 5 = 0 \quad \text{—— (2)}$$

Rotating the axis through an angle 'θ' Replacing x and y by,

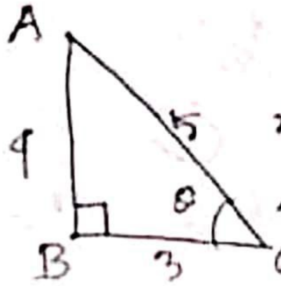
$(x'\cos\theta - y'\sin\theta)$ and $(x'\sin\theta + y'\cos\theta)$

Now,

P 13

$\tan^{-1}\left(-\frac{4}{3}\right) = \theta$

$\Rightarrow \tan\theta = -\frac{4}{3}$



$AC^2 = AB^2 + BC^2$

$AC^2 = (4)^2 + (3)^2$

$= 5$

$\therefore \cos\theta = \frac{-3}{5}$, $\sin\theta = \frac{4}{5}$

<u>eq^n – 2</u>

$$11(x'\cos\theta - y'\sin\theta)^2 + 24(x'\cos\theta - y'\sin\theta) + 4(x'\sin\theta + y'\cos\theta)^2 - 5 = 0$$

$$\Rightarrow 11\left(x'\frac{-3}{5} - y'\frac{4}{5}\right)^2 + 24\left(x'\frac{-3}{5} - y'\frac{4}{5}\right) + 4\left(x'\frac{4}{5} + y'\left(\frac{-3}{5}\right)\right)^2 - 5 = 0$$

$\Rightarrow$

-> X^2-4y^2+1=0

Q.7 Given.

$$f(x,y) = 8x^2 + 4xy + 5y^2 - 16x - 14y + 13 = 0$$

Solution:-

~~Let~~. Now.

$$f(x,y) = 8x^2 + 4xy + 5y^2 - 16x - 14y + 13 = 0 \quad \text{--(i)}$$

$$= 8x^2 + 2(2)xy + (5)y^2 + 2(-8)x$$

$$+ 2(-7).y + (13) = 0$$

Here,

$$a = 8,\ b = 5,\ c = 13,\ h = 2,\ g = -8,\ f = -7$$

We know,

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

$$= (8 \times 5 \times 13) + 2 \times (-7) \times (-8) \times 2 - 8 \times (-7)^2$$

$$- 5 \times (-8)^2 - 13 \times (2)^2$$

$$= 520 + 224 - 392 - 320 - 52$$

7

$$= -20$$

$$\neq 0$$

Now, $ab - h^2 = (8 \times 5) - (2)^2 = 40 - 4 = 36 > 0$

$\therefore$ So, the conic is an ellipse.

Differentiating (i) partially with respect to x and y we get,

$$\frac{\partial f}{\partial x} = 16x + 4y - 16 = 0 \quad \cdots (ii)$$

$$\frac{\partial f}{\partial y} = 4x + 10y - 14 = 0 \quad \cdots (iii)$$

Solving for (ii) and (iii).

$$16x + 4y - 16 = \cancel{4x + 10y - 14}$$

or, $12x - 6y - 2 = 0$

$16x + 4y - 16 = 0$

or, $4x + y - 4 = 0$

or, $y = 4 - 4x$

(iii) $\Rightarrow$ $4x + 10(4 - 4x) - 14 = 0$

or, $4x + 40 - 40x - 14 = 0$

or, $-36x + 26 = 0$

or, $36x = 24$

or, $3x = 2$

$\therefore x = 2/3$ and $y = 4/3$

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$\therefore$ Center of conic ~~(2/3)~~ $\left(\frac{2}{3}, \frac{4}{3}\right)$

Tranferring the origin to ~~(1,0)~~ $\left(\frac{2}{3}, \frac{4}{3}\right)$

$$8x^2 + 4xy + 5y^2 + C_1 = 0 \quad \cdots \text{(iv)}$$

Now,

$$C_1 = gx_1 + fy_1 + c = -8x_1 - 7y_1 + 13$$

$$= -8 \cdot \frac{2}{3} - 7 \cdot \frac{4}{3} + 13$$

$$= -\frac{5}{3}$$

Now from eq^n(iv),

$$8x^2 + 4xy + 5y^2 - \frac{5}{3} = 0 \quad \cdots \text{(v)}$$

When the xy term is removed by rotation of the axis, Let, the reduced eq^n is,

$$a_1x^2 + b_1y^2 - \frac{5}{3} = 0$$

$$\text{or, } a_1x^2 + b_1y^2 = \frac{5}{3} \quad \cdots \text{(vi)}$$

Now,

$$a_1 + b_1 = a + b = 8 + 5 = 13 \quad \cdots \text{(vii)}$$

$$a_1b_1 = 8 \times 5 = 40 \quad \cdots \text{(viii)}$$

From (viii) $\Rightarrow$ $a_1 = \frac{40}{b_1}$

From (vii) $\Rightarrow$ $a_1 + b_1 = 13$

or, $\frac{40}{b_1} + b_1 = 13$

or, $\frac{40 + b_1^2}{b_1} = 13$

or, $b_1^2 - 13b_1 + 40 = 0$

$\therefore$ $b_1 = \frac{-(-13) \pm \sqrt{(-13)^2 - 4.1.40}}{2.1}$

$= \frac{13 \pm \sqrt{169 - 160}}{2}$

$= \frac{13 \pm 3}{2}$

$= 8, 5$

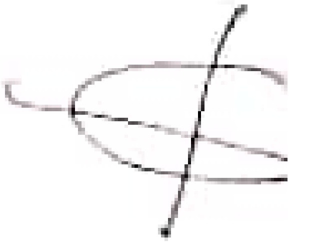
$\therefore$ $b_1 = 8$ and 5

$\therefore$ $a_1 = \frac{40}{b_1} = 5, 8$

$\therefore$ $(a_1, b_1) = (5, 8) \& (8, 5)$

Now,

$$5x^2 + 8y^2 = 5/3$$



or, $$\frac{5x^2}{5/3} + \frac{8y^2}{5/3} = 1$$

or, $$\frac{x^2}{1/3} + \frac{y^2}{5/24} = 1$$

or, $$\frac{x^2}{(\sqrt{1/3})^2} + \frac{y^2}{(5/24)^2} = 1 \text{ — (Ans)}$$

And,

$$8x^2 + 5y^2 = 5/3$$

or, $$\frac{8x^2}{5/3} + \frac{5y^2}{5/3} = 1$$

or, $$\frac{x^2}{5/24} + \frac{y^2}{1/3} = 1$$

or, $$\frac{x^2}{(\sqrt{5/24})^2} + \frac{y^2}{(\sqrt{1/3})^2} = 1 \text{ (Ans)}$$

Sub:

Example: show that the maximum value of $\left(\frac{1}{x}\right)^x$ in $e^{\frac{1}{e}}$.

Solution: $y = \left(\frac{1}{x}\right)^x$

$$\ln y = \ln\left(\frac{1}{x}\right)^x$$

$$\ln y = x \ln\left(\frac{1}{x}\right)$$

$$\Rightarrow \frac{d}{dx}\{\ln y\} = \frac{d}{dx}\left[x\{\ln 1 - \ln x\}\right].$$

$$\Rightarrow \frac{d}{dx}\{\ln y\} = \frac{d}{dx}\left[x\{-\ln x\}\right]$$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = -\left\{\ln x \cdot 1 + x\frac{1}{x}\right\}$$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = -(1+\ln x)$$

$$\Rightarrow \frac{dy}{dx} = -y(1+\ln x) \longrightarrow (i)$$

$$\therefore \frac{d^2y}{dx^2} = -\left[y\frac{d}{dx}(1+\ln x) + (\ln x + 1)\frac{dy}{dx}\right]$$

$$= -\left[y \times \frac{1}{x} - (1+\ln x) \times y(1+\ln x)\right]$$

[From i]

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Sub:

$$= -y\left[\frac{1}{x} - (1+\ln x)^2\right]$$

$$= -\left(\frac{1}{x}\right)^x\left[\frac{1}{x} - (1+\ln x)^2\right]$$

The necessary and sufficient condition for maximum and minimum value is:

$$\frac{dy}{dx} = 0$$

$$\Rightarrow -y(1+\ln x) = 0$$

$$\Rightarrow 1+\ln x = 0$$

$$\Rightarrow \ln x = -1$$

$$\Rightarrow \ln e^x = -1$$

$$\therefore x = e^{-1} = \frac{1}{e}$$

When $x = \frac{1}{e}$ then $\frac{d^2y}{dx^2}$

$$= -\left(\frac{1}{\frac{1}{e}}\right)^{1/e}\left[\frac{1}{\frac{1}{e}} - \left(1+\ln\frac{1}{e}\right)^2\right]$$

$$= -(e)^{1/e}\left[e - \left(1+\ln\frac{1}{e}\right)^2\right] \quad \left[\because \ln\frac{1}{e} = \ln e^{-1} = 1\right]$$

$$= -e^{\frac{1}{e}}\left[e - (1-1)^2\right]$$

$$= -e^{\frac{1}{e}}.e < 0$$

hence $f(x)$ is maximum at the point $x = \frac{1}{e}$

maximum value $= \left(\frac{1}{\frac{1}{e}}\right)^{\frac{1}{e}}$

$$= e^{\frac{1}{e}}$$

Q9 if $u = 2(ax+by)^2 - (x^2+y^2)$ and $a^2+b^2=1$, then find the value of $\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2}$

Soln:

$$\frac{\delta^2 u}{\delta x^2} = \frac{\delta}{\delta x}\left(\frac{\delta u}{\delta x}\right)$$

$$\frac{\delta u}{\delta x} = 2\cdot 2(ax+by)\frac{\delta}{\delta x}(ax+by) - 2x$$
$$= 4(ax+by)\cdot a - 2x$$
$$= 4a(ax+by) - 2x$$
$$= 4a^2x + 4aby - 2x$$

$$\frac{\delta^2 u}{\delta x^2} = 4a^2 + 0 - 2 = 4a^2 - 2$$

$$\frac{\delta u}{\delta y} = 2\cdot 2(ax+by)\frac{\delta}{\delta y}(ax+by) - 2y$$
$$= 4(ax+by)\cdot b - 2y$$
$$= 4b(ax+by) - 2y$$
$$= 4abx + 4b^2y - 2y$$

$$\frac{\delta^2 u}{\delta y^2} = 0 + 4b^2 - 2 = 4b^2 - 2$$

$$\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2}$$
$$= 4a^2 - 2 + 4b^2 - 2$$
$$= 4a^2 + 4b^2 - 4$$
$$= 4(a^2+b^2) - 4$$
$$= (4\times 1) - 4 \qquad [\because a^2+b^2=1]$$
$$= 0$$

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Q-10: If $y = (\sin^{-1} x)^2$ then show that

(i) $(1-x^2)y_2 - xy_1 - 2 = 0$

(ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$

Soln.

(i) $y = (\sin^{-1}x)^2$

$\Rightarrow y_1 = 2(\sin^{-1}x)\dfrac{1}{\sqrt{1-x^2}}$

$\Rightarrow (y_1)^2 = \dfrac{4(\sin^{-1}x)^2}{(1-x^2)}$

10

$\Rightarrow (y_1)^2 = \dfrac{4y}{1-x^2}$

$\Rightarrow (1-x^2)\cdot y_1^2 - 4y = 0$

$\Rightarrow (1-x^2)\,2y_1y_2 + y_1^2(-2x) - 4y_1 = 0$

$\Rightarrow (1-x^2)y_2 - xy_1 - 2 = 0$

[showed]

(11) Now Differentiating n times with respect to x using Leibnitz theorem -

$$D^n\{(1-x^2)y_2\} - D^n(xy_1) - D^n(2) = 0$$

$$\Rightarrow [D^n\{(1-x^2)y_2\}] - D^n(xy_1) - 0 = 0$$

$$\Rightarrow [(1-x^2)D^n y_2 + {}^nC_1 D^{n-1}(y_2)D(1-x^2) + {}^nC_2 D^{n-2}y_2 D^2(1-x^2)]$$

$$- [xD^n y_1 + \cancel{ne}.\,{}^nC_1 D^{n-1}y_1 D(x)] = 0$$

$$\Rightarrow (1-x^2)y_{n+2} + ny_{n+1}(-2x) + \frac{n(n-1)}{2}y_n(-2)] - [xy_{n+1} + ny_n] = 0$$

$$\Rightarrow \cancel{a}(1-x^2)y_{n+2} - 2xny_{n+1} - (n^2 - n)y_n - xy_{n+1} + ny_n = 0$$

$$\cancel{\Rightarrow (1-x^2)y_{n+2} \quad 2xny}.$$

$$\Rightarrow (1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0 \quad \boxed{\text{showed}}$$