2015

Q.1:

(a) $\int_4^{10} (x^5+2)/(x^2-4)$

$\therefore \frac{x^5+2}{x^2-4} =$

$$x^2-4 \,\big|\, \begin{array}{l} x^5+2 \\ x^5-4x^3 \\ \hline 4x^3 \\ 4x^3-16x \\ \hline 16x+2 \end{array} \,\big|\, x^3+4x$$

$$\Rightarrow \frac{x^5+2}{x^2-4} = x^3+4x+\frac{16x+2}{x^2-4}$$

Now,

$$\frac{16x+2}{x^2-4} = \frac{A}{(x+2)} + \frac{B}{(x-2)}$$

$$\Rightarrow 16x+2 = A(x-2)+B(x+2)$$

$$\Rightarrow 16x+2 = Ax+Bx-2A+2B$$

$$\Rightarrow 16x+2 = (A+B)x + (-2A+2B)$$

$2A+2B=32$

$$\therefore A+B=16 \quad \text{——— (i)}$$

$$-2A+2B=2 \quad \text{——— (ii)}$$

$$\{(i)\times 2\} + (ii) \Rightarrow$$

$$4B = 34$$

$$\therefore B = 9$$

$B=9$ using in (i) $\Rightarrow$

$$A = 16-9 = 7$$

$$\therefore \frac{x^5+2}{x^2-4} = x^3 + 4x + \frac{7}{x+2} + \frac{9}{x-2}$$

$$\therefore \int_4^{10} \frac{x^5+2}{x^2-4} = \int_4^{10} x^3 + \int_4^{10} 4x + \int_4^{10} \frac{7}{x+2} + \int_4^{10} \frac{9}{x-2}$$

$$= \left[\frac{x^4}{4}\right]_4^{10} + 4\left[\frac{x^2}{2}\right]_4^{10} + 7\left[\ln(x+2)\right]_4^{10}$$

$$+ 9\left[\ln(x-2)\right]_4^{10}$$

$$= \frac{(10)^4}{4} - \frac{4^4}{4} + \frac{4\times(10)^2}{2} - \frac{4\times 4^2}{2} + 7\ln(12)$$
$$- 7\ln(6) + 9\ln 8 - 9\ln 2$$

$\square$

Q: 2

(b)

$$\int e^{-2x}\sin(2x)\,dx = \sin 2x\int e^{-2x}dx - \int\left[\frac{d}{dx}(\sin 2x)\int e^{-2x}dx\right]dx$$

$$= \frac{-\sin(2x)e^{-2x}}{2} - \int\left(\frac{-2\cos 2x\times e^{-2x}}{2}\right)dx$$

$$= \frac{-\sin(2x)e^{-2x}}{2} + \int\left[e^{-2x}\cos(2x)\right]dx$$

$$= \frac{-\sin(2x)e^{-2x}}{2} + \cos 2x\int e^{-2x}dx$$
$$- \int\left[\frac{d}{dx}\cos(2x)\int e^{-2x}dx\right]dx$$

$$= \frac{-\sin(2x)e^{-2x}}{2} - \frac{\cos(2x)e^{-2x}}{2}$$

$$- \int \left[\frac{2\sin 2x - (e^{-2x})}{2} \right] dx$$

$$= \frac{-\sin(2x)e^{-2x}}{2} - \frac{\cos(2x)e^{-2x}}{2}$$

$$+ \int \left(\frac{\sin 2x\, e^{-2x}}{2} \right) dx$$

Q: 2 :

(b)

$$\int e^{-2x} \sin(2x)\,dx = e^{-2x} \int \sin(2x)\,dx - \int \left[\frac{d}{dx}(e^{-2x}) \int \sin(2x)\,dx \right] dx$$

$$= \frac{-\cos(2x)e^{-2x}}{2} - \int \left[\frac{(-e^{-2x})(-\cos 2x)}{2 \times 2} \right] dx$$

$$= \frac{-\cos(2x)e^{-2x}}{2} - \frac{1}{4} \int e^{-2x} \cos 2x\,dx$$

$$= \frac{-\cos(2x)e^{-2x}}{2} - \frac{1}{4} \left[e^{-2x} \int \cos 2x\,dx \right]$$

$$= \frac{\cos(2x)e^{-2x}}{2} - \frac{1}{4}\Big[e^{-2x}\int \cos(2x)\,dx$$

$$- \int\Big\{\frac{d}{dx}e^{-2x}\int \cos(2x)\,dx\Big\}dx\Big]$$

$$= \frac{-\cos(2x)e^{-2x}}{2} - \frac{e^{-2x}\sin(2x)}{2\times 4}$$

$$+ \int\Big[\frac{-e^{-2x}\sin(2x)}{4}\Big]dx$$

$$= \frac{-\cos(2x)e^{-2x}}{2} - \frac{e^{-2x}\sin(2x)}{8}$$

$$- \int\Big[\frac{e^{-2x}\sin(2x)}{4}\Big]dx$$

$$\int\Big[e^{-2x}\sin(2x)\Big]dx + \frac{1}{4}\int e^{-2x}\sin(2x)\,dx$$

$$= \frac{\cancel{\cos}\ -4\cos(2x)e^{-2x} - e^{-2x}\sin(2x)}{8}$$

$$\Rightarrow \frac{5}{4}\int\Big[e^{-2x}\sin(2x)\Big]dx = \frac{e^{-2x}\big[-4\cos(2x) - \sin(2x)\big]}{8}$$

$$\therefore \int\Big[e^{-2x}\sin(2x)\Big]dx = \frac{4e^{-2x}\big[-4\cos(2x) - \sin(2x)\big]}{5\times 8}$$

□

Q:4

$$x^3 \frac{dy}{dx} = x^2 y - 2y^3$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - 2\frac{y^3}{x^3}$$

Q:6 $\frac{dy}{dx} + \frac{1}{x}y = 3x^2$

$$\Rightarrow y' + \frac{1}{x}y = 3x^2$$

This is a linear first order o.d.e of the form $y' + P(x)y = q(x)$ with $P(x) = \frac{1}{x}$ & $q(x) = 3x^2$

$$\therefore \int P\,dx = \int \frac{1}{x}\,dx = \ln x$$

$$\therefore y = e^{-\int P dx} \int q\, e^{\int P dx}\,dx + c e^{-\int P dx}$$

$$= e^{-\ln x} \int 3x^2 e^{\ln x}\,dx + c e^{-\ln x}$$

$$= -x \int 3x^2 \times x\,dx + c(-x)$$

$$y = -x\int 3x^3\,dx - ax$$

$$y = -x\left(\frac{3x^4}{4}\right) - ax$$

$$y = -\frac{3x^5}{4} - ax \quad \square$$