$$= 4\left[\frac{x}{2}\sqrt{1^2-x^2} + \frac{1^2}{2}\sin^{-1}\frac{x}{1}\right]_0^1$$

$$= 4\left[\frac{1}{2}\sqrt{1-1} + \frac{1}{2}\sin^{-1}\frac{1}{1}\right]$$

$$= 4\left[0 + \frac{1}{2}\cdot\frac{\pi}{2}\right] = 4\times\frac{\pi}{4} = \pi$$

Stokes Theorem verified.

— o —

Statistics

Defⁿ: The science of statistics is essentially a brach of applied mathematics dealing with the collection, presentation and analysis of Numerical Data. Satistics means quantitative data, which are affected to a market extent by multiplicity of causes which is designed to summarize or describe or describe important features of numerical Data mainly by Tables or Charts.

Frequency Distribution:

A set of classes together with the frequencies of occurance of values in each class in a given set of data, presented in a tabular form, is referred to as a frequency distribution.

Construct of tables: Health and smoking states of 50 workers.

	Health states			
Smoking states	Good	Average	Poor	total
smokers	6	10	12	28
non-smokers	3	7	12	22
Total	9	17	24	50

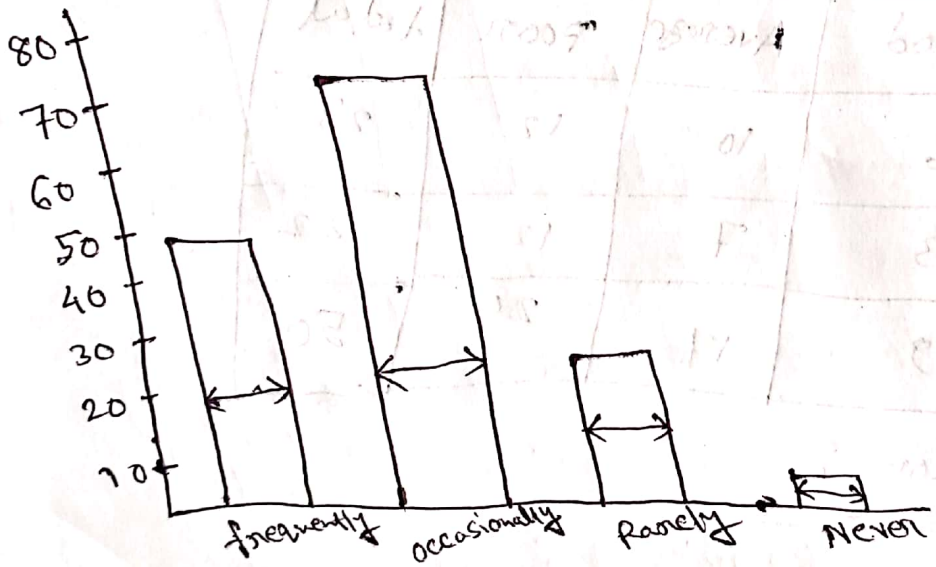
Graphical Representation:

(1) Bar diagram:

Bar diagram is also known as bar chart. Consists of horizontal or vertical bars of equal widths and lengths proportional to the magnitude they represents.

Ex: Health personnel from 150 rural health centres asked. How frequently have you visited your pear weekly?

Response	Frequently
Frequently	49
Occasionally	71
Rarely	24
Never	6
Total	150



Bar Diagram

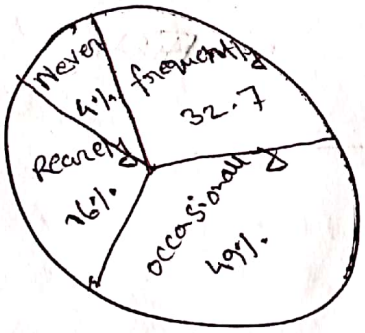
2) Pie chart : It is also known as Pie diagram is an effective way of presenting Percentage parts when the whole quantity is taken as 100.

frequently - $\frac{49}{150} \times 100\% = 32.7\%$.

Occasionally - $\frac{77}{150} \times 100\% = 49\%$.

Rarely - $\frac{24}{150} \times 100\% = 16\%$.

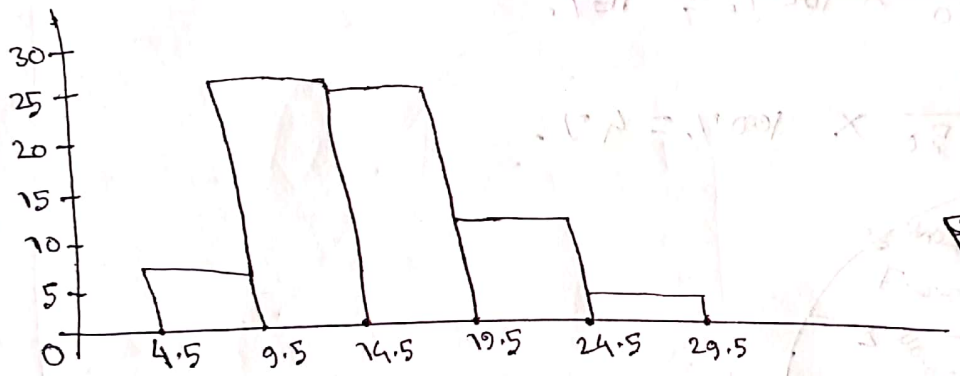
Never - $\frac{6}{150} \times 100\% = 4\%$.



③ Histogram:

The most common form of graphical presentation of a frequency distribution is Histogram.

Expenditure	class frequency	Height of rectangles	class width
4.5–9.5	8	8	5
9.5–14.5	28	28	5
14.5–19.5	27	27	5
19.5–24.5	12	12	5
24.5–29.5	4	4	5
Total	80		



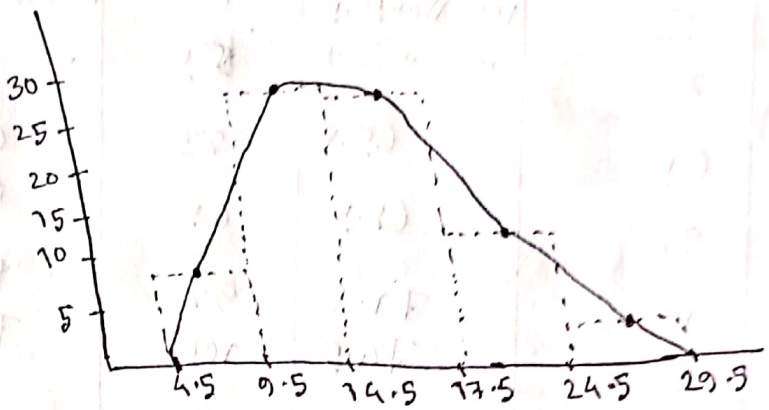
Histogram.

frequency

④ Frequency Polygon: Frequency Polygon Provideson alternati as way of graphically presenting a frequency distributio of a continuous way. The presentation involves placing the midvalues on the horizontal axis and on the vertical axis.

Previous diagram

Now,



(5.) Cumulative frequency Polygon or ogive:

An ogive is based on a cumulative frequency distribution. The frequencies are to be cumulated just by summing the class frequencies.

Two types of cumulative distribution are used "less than type" and "More than type".

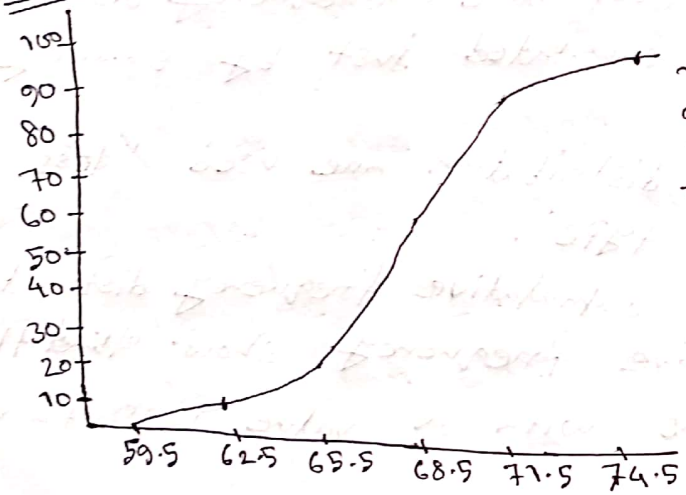
In a less than type cumulative frequency distribution for each class, the cumulative frequency show the total number of data items with a value less than the upper limit.

A more than type is also constructed simply by cumulating the frequencies from lower limit of the first class boundary. the first cumulating shows the total frequency in the distribution.

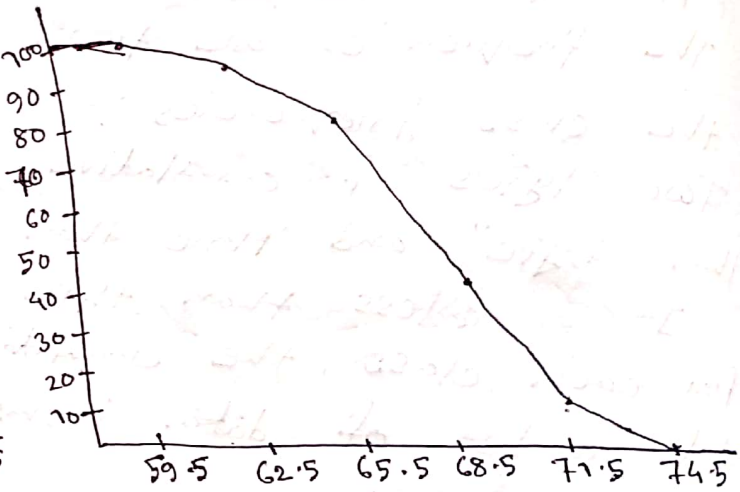
Q. Table	frequency
59.5 – 62.5	5
62.5 – 65.5	18
65.5 – 68.5	42
68.5 – 71.5	27
71.5 – 74.5	8
Total	100

Heights (inch)	frequencies	cumulative frequency	cumulative frequency Polygon			
			Less than type		More than type	
			Heights	frequencies	Heights	frequencies
59.5 – 62.5	5	5	less than 59.5	0	More than 59.5	100
62.5 – 65.5	18	23	" " 62.5	5	62.5	95
65.5 – 68.5	42	65	65.5	23	65.5	77
68.5 – 71.5	27	92	68.5	65	68.5	35
71.5 – 74.5	8	100	71.5	92	71.5	8
			74.5	100	74.5	0
Total	100					

Graph



less than ~~ogive~~ type ogive



More than type ogive

Average or Measures of central Tendency :-

1. The Arithmatic Mean.
2. The Median
3. The Mode
4. The Harmonic Mean
5. The Geometic Mean

① The Arithmetic Mean:

$$\bar{x} = \frac{\Sigma x_i}{N}$$

If $X = 1, 2, 3, 9, 7, 10$

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{1+2+3+9+7+10}{6} =$$

for grouped data,

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

Weekly wages in TK	frequency, f_i	Midvalue, x_i	$f_i x_i$
48.5 – 53.5	2	51	102
53.5 – 58.5	2	55	110
58.5 – 63.5	3	61	183
63.5 – 68.5	5	65	325
68.5 – 73.5	5	71	355
73.5 – 78.5	5	75	375
78.5 – 83.5	5	81	405
83.5 – 88.5	7	85	595
88.5 – 93.5	10	91	910
93.5 – 98.5	6	95	570
Total	50		3930

$$\therefore \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{3930}{50}$$

$$= 78.6$$

Average or Measures of central tendancy:

1. The Arithmetic Mean
2. The Median.

for grouped data.

$$\text{Median}, M_e = L_0 + \frac{n/2 - F}{f_0} \times h$$

Where,

L_0 = lower limit of the median class.

n = total number of class.

f = cumulative frequency prior the median class.

f_0 = frequency of the median class.

h = class width of the median class.

Calculating the Median Class:

1. Complete less than type cumulative frequency
2. Determine $n/2$.
3. Locate median class for which the cumulative frequency is more than $n/2$.
4. Determine the lower limit l_0 of the median class.
5. Determine the sum of frequencies of all classes prior the median class f.
6. Determine the frequency of the median class f_0.
7. Determine class width of the median class, h.

Weekly wages	Frequency f_i	cumulative frequency
48.5 – 53.5	2	2
53.5 – 58.5	2	4
58.5 – 63.5	3	7
63.5 – 68.5	5	12
68.5 – 73.5	5	17
73.5 – 78.5	5	22
78.5 – 83.5	(5)	27
83.5 – 88.5	7	34
88.5 – 93.5	10	44
93.5 – 98.5	6	50
Total	50	

$n/2 = \frac{50}{2} = 25$

Median class = 78.5 – 83.5

$l_0 = 78.5$

$f = 22$

$f_0 = 5$

$h = 5$

The mode:

1, 2, 3, 1, 5, 6, 7, 5, 1

for grouped data,

$$\text{Mode}, M_o = l_o + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h$$

where,

l_o = lower limit of the modal class

Δ_1 = Absolute difference of frequency between modal and pre-modal class.

Δ_2 = Absolute difference between modal and post modal class.

h = Class width of the modal class.

যদি 2টি same highest frequency হয় তাহলে- modal class 2টি frequency হবে।

Class boundary	frequency
1.45 – 1.95	2
1.95 – 2.45	1
2.45 – 2.95	4
2.95 – 3.45	10
3.45 – 3.95	10
3.95 – 4.45	5
4.45 – 4.95	3
Total	

Modal class = 2.9 – 3.95

$\Delta_1 = 20 - 5$

$\Delta_2 = 20 - 14$

$h = 1.00$

$l_o = 2.95$

$\Delta_1 = 15 - 4 = 11$

$\Delta_2 = 15 - 10 = 5$

$h = 0.50$

$$\text{Mode}, M_o = l_o + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h$$

$$= 2.95 + \frac{11}{11+5} \times 0.50$$

$=$

Ex:

Calculate AM, Median and Mode From the following data.

Age in years	Number of births
14.5 – 19.5	677
19.5 – 24.5	1908
24.5 – 29.5	1737
29.5 – 34.5	1040
34.5 – 39.5	294
39.5 – 44.5	91
44.5 – 49.5	16

(2.)

Age	No. of women
9.5 – 14.5	27
14.5 – 19.5	34
19.5 – 24.5	41
24.5 – 29.5	45
29.5 – 34.5	45
34.5 – 39.5	43
39.5 – 44.5	35
44.5 – 49.5	30

Ex: Construct histogram, frequency polygon, less than type and more than type ogive.

wages	frequency
250-260	8
260-270	10
270-280	16
280-290	15
290-300	10
300-310	5
310-320	3
320-330	3

The Standard deviation or other measures of Dispersion

1. The Range
2. The quartile deviation
3. The mean deviation
4. The variance
5. The standard deviation

The mean deviation:

for grouped data,

$$MD(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$$

where, x_i = Mid values of class boundary

$\bar{x}$ = A.M Arithmetic Mean

f_i = Total frequency

Compute M.D

Class Interval	frequency
48.5 – 53.5	2
53.5 – 58.5	2
58.5 – 63.5	3
63.5 – 68.5	5
68.5 – 73.5	5
73.5 – 78.5	5
78.5 – 83.5	5
83.5 – 88.5	7
88.5 – 93.5	10
93.5 – 98.5	6
Total	50

class interval	Mid value x_i	frequency f_i	$f_i x_i$	$\bar{x}$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
48.5 – 53.5	51	2	102		28.1	56.2
53.5 – 58.5	56	2	112		23.1	46.2
58.5 – 63.5	61	3	183		18.1	54.3
63.5 – 68.5	66	5	330	$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$	13.1	65.5
68.5 – 73.5	71	5	355		8.1	40.5
73.5 – 78.5	76	5	380	$= \frac{3955}{50}$	3.1	15.5
78.5 – 83.5	81	5	405	$= 79.1$	1.9	9.5
83.5 – 88.5	86	7	602		6.9	48.5
88.5 – 93.5	91	10	910		11.9	11.9
93.5 – 98.5	96	6	579		16.9	101.4
Total		50	3955			556.4

$$MD(\bar{x}) = \frac{\Sigma f_i |x_i - \bar{x}|}{\Sigma f_i}$$

$$= \frac{556.4}{50}$$

$$= 11.128$$

27 May
statistic

variace ,

$$\sigma^2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}$$

Standard deviation ,

$$\sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}}$$

probability

Exercise

(1) One day 20 file were presented to an income tax officer for disposal 5 files contained bogus entries. All the files were thoroughly mixed and there was no indication about bogus files. What is the probability that one file with bogus entries is selected?

(2) A bag contains 4 white and 6 red balls. A ball is drawn at random from the bag. What is the probability that it is red? white?