Lecture: PDE IV $\quad u = temp. = u(x,t)$ (P-1)

~~THE~~ LAPLACE'S EQUATION:

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→ The Laplace's equation in 2-D is:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$ and in 3-D is:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

which are also called steady-state heat equations as because it is exactly the two or three-dimensional respective ~~equa~~ equation of heat conduction when $\frac{\partial u}{\partial t} = 0$ (that is, when u is constant with time).

→ but, Laplace's equation occurs in many other contexts; for example, in potential theory, a function satisfying Laplace's equation is called harmonic function. In this context, one usually sees a vector-related notation $\nabla^2 u$ for $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in 2-D or $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ in 3-D where $\nabla^2 u$ is read as; "del squared u" and ∇^2 is called the Laplacian operator; and in this

1/A, O.R. Nizam Road, Prabortak Circle, Chittagong
Phone : 88-031-656917, Fax : 88-031-656917
PREMIER UNIVERSITY COMPUTER SOCIETY

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notation, the Laplace's ~~eq~~ equation beco...

$$\nabla^2 u = 0$$

→ Two (following) types of problems ~~are~~ commonly arise involving Laplace's equation in ~~two~~ 2-D or 3-D:

(1) A Dirichlet Problem: it is consists of finding a function satisfying $\nabla^2 u = 0$ in a given region, assuming specified values $u = f$ on the boundary of the region. For example, in 3-D we might want to satisfy $\nabla^2 u = 0$ inside a sphere, with $u(x,y,z) = f(x,y,z)$ given on the surface of the sphere.

(2) A Newmann Problem: it is consists of finding a function satisfying $\nabla^2 u = 0$ with given normal derivative $\frac{\partial u}{\partial n} = g$ on the boundary

Note: (a) Dirichlet problem has unique ~~solutio~~ solutions, with certain assumption ~~on the~~ ~~one~~ on the region and data function f.

(b) Neumann problems have solutions uniquely specified with an arbitrary added constant.

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Poisson's Equation:

* Poisson's equation has the form : $\nabla^2 u = f$ where f is given. when f is zero, Poisson's equation reduces to Laplace's equation.

Notations:

~~Equations~~ ~~Coordinate~~ ~~Standard~~ ~~Subscript~~

Equations	Coordinates	Standard form	Subscript form
Wave Eqn.	1-D	$\frac{\partial^2 y}{dt^2} = a^2 \frac{\partial^2 y}{\partial x^2}$	$y_{tt} = a^2 y_{xx}$
Heat Eqn.	1-D	$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$	$u_t = a^2 u_{xx}$
Laplace's	2-D	$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$	$u_{xx} + u_{yy} = 0$
Laplace's	3-D	$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$	$u_{xx} + u_{yy} + u_{zz} = 0$
Laplace's	Cylindrical	$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r} u\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$	$\frac{1}{r}(r u_r)_r + \frac{1}{r^2} u_{\theta\theta} + u_{zz} = 0$

Eqn.	coordinates	Standard form	Subscipt form
place's	Spherical	$\frac{1}{r}\frac{\partial^2}{\partial r^2}(ru) + \frac{1}{r^2\sin^2(\phi)}\frac{\partial^2 u}{\partial \theta^2} + \frac{1}{r^2\sin(\phi)} \times \left[\frac{\partial}{\partial \phi}\left\{\left(\frac{\partial}{\partial \phi}u\right)\sin\phi\right\}\right] = 0$	$\frac{1}{r}(ru)_{rr} + \frac{1}{r^2\sin^2\phi}u_{\theta\theta} + \frac{1}{r^2\sin\phi}\left[u_\phi \sin\phi\right]_\phi = 0$

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Φ ?