Partial differential Equation:

(H.W) Find the non-intrivial solution of

(i) $\frac{\delta^2 u}{\delta x^2} = 0$

Solution:

$$\frac{\delta^2}{\delta x^2}(u) = \frac{\delta}{\delta x}\left(\frac{\delta}{\delta x} u\right)$$

Let, $u = x + f(y)$ then:

$$\frac{\delta u}{\delta x} = \frac{\delta}{\delta x}[x + f(y)] = 1 + 0$$

$$\therefore \frac{\delta^2 u}{\delta x^2} = \frac{\delta}{\delta x}\left(\frac{\delta u}{\delta x}\right)$$

$$= \frac{\delta}{\delta x}(1)$$

$$= 0$$

$\therefore u = x + f(y)$ is the general from of the solution function.

(ii) $\frac{\delta u}{\delta x} + \frac{\delta u}{\delta y} = 0$

Solution:

Let $u(x,y) = k(x-y)$, k is constant.

$$\frac{\delta u}{\delta x} + \frac{\delta u}{\delta y} = \frac{\delta}{\delta x}[k(x-y)] + \frac{\delta}{\delta y}[k(x-y)]$$

$$= \frac{\delta}{\delta x}(kx) - \frac{\delta}{\delta x}(ky) + \frac{\delta}{\delta y}(kx) - \frac{\delta}{\delta y}(ky)$$

$$= \cancel{k} - 0 + 0 - \cancel{k}$$

$$= 0$$

$\therefore$ $u(x,y) = k(x-y)$, k is a solution.

(3) $\frac{\delta^2 u}{\delta x \delta y} = y$

Solⁿ: Let $u(x,y) = \frac{1}{2}xy^2$ then:

$$\frac{\delta^2 u}{\delta x \delta y} = \frac{\delta}{\delta x}\left[\frac{\delta}{\delta y}\left(\frac{1}{2}xy^2\right)\right]$$

$$= \frac{\delta}{\delta x}\left[\frac{1}{\cancel{2}}x(\cancel{2}y) + y^2(0)\right]$$

$$= \frac{\delta}{\delta x}(xy)$$

$$= x\frac{\delta}{\delta x}(y) + y\frac{\delta}{\delta x}(x)$$

$$= 0 + y \cdot 1 = y$$

④ $\frac{\delta^4 u}{\delta x^4} - x = 0$

Solnⁿ: Let $u(x,y) = \frac{x^5}{120} + f(y)$, then

$$\frac{\delta u}{\delta x} = \frac{x^4}{24}, \quad \frac{\delta^2 u}{\delta x^2} = \frac{\delta}{\delta x}(x^4/24) = \frac{x^3}{6}$$

$$\frac{\delta^3 u}{\delta x^3} = \frac{\delta}{\delta x}\left(\frac{x^3}{6}\right) = \frac{x^2}{2}$$

$$\frac{\delta^4 (u)}{\delta x^4} = \frac{\delta}{\delta x}\left(\frac{x^2}{2}\right)$$

$$= \frac{1}{2}\frac{\delta}{\delta x}(x^2)$$

$$= \frac{1}{\cancel{2}} \times \cancel{2}x$$

$$= x$$

which matches the eqⁿ $\frac{\delta^4 u}{\delta x^4} - x = 0$.

⑤ $\frac{\delta^2 u}{\delta x^2} = 0$, $u(1,1) = 1$

Solⁿ: Since $u(1,1) = 1$ is only one value, we can easily solve for one unknown co-efficient.

Let $u(x,y) = A(x+y)$

then $\frac{\delta u}{\delta x} = A$; $\frac{\delta^2 u}{\delta x^2} = 0$

and,

$$1 = A(1+1) = 2A \Rightarrow A = \frac{1}{2}$$

~~and~~

$\therefore u(x,y) = \frac{1}{2}(x+y)$ will work 1.