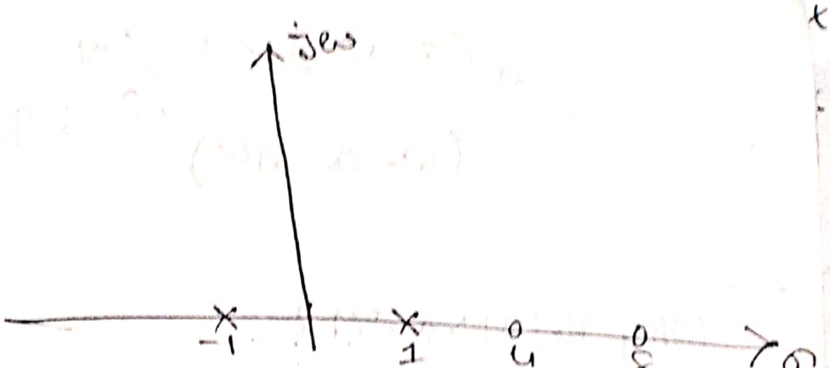
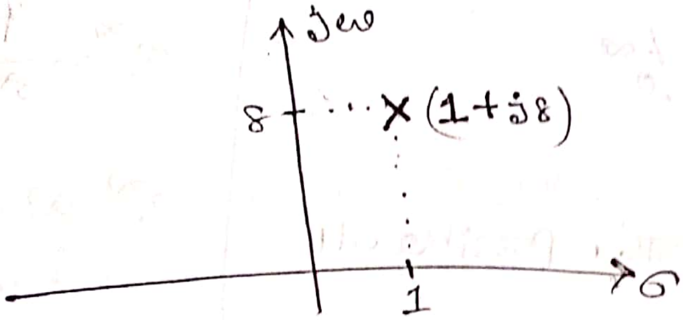
Ex: $X(s) = \dfrac{(s-8)(s-4)}{(s-1)(s+1)}$



যদি,

$s = 1 + j8$ হলে ইহা নির্দেশ এই system এর pole $(1+j8)$ এ আছে।



Ex-6.1

Determine the laplace transform of

$$x(t) = e^{at}u(t)$$

Depict the ROC and the location of poles and zeros in the s-plane. Assume that, a is real.

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

$$= \int_{-\infty}^{\infty} e^{at} \cdot u(t) e^{-st} \cdot dt$$

$$= \int_{0}^{\infty} e^{at} \cdot e^{-(\sigma + j\omega)t} dt = \int_{0}^{\infty} e^{-(\sigma - a)t} \cdot e^{-j\omega t} dt$$

$$= \frac{e^{-(\sigma-a)t}\, e^{-j\omega t}}{-(\sigma-a-j\omega)} \Big|_0^\infty \;;\; \text{ROC} \;;\; Re|s|>a$$

Why $Re|s|>a$ হবে?

যদি $\sigma<a$ হয় $e^{-(\sigma-a)t}$ হবে

$$e^{-(-1)t}\Big|_0^\infty$$

$$\Rightarrow e^{t}\Big|_0^\infty$$

$$\Rightarrow \infty-1$$ যেটা possible না।

আর; $\sigma>a$ হলে,

$e^{-(\sigma-a)t}$ এর value পাওয়া যাবে বর্ণিত উপায়ে

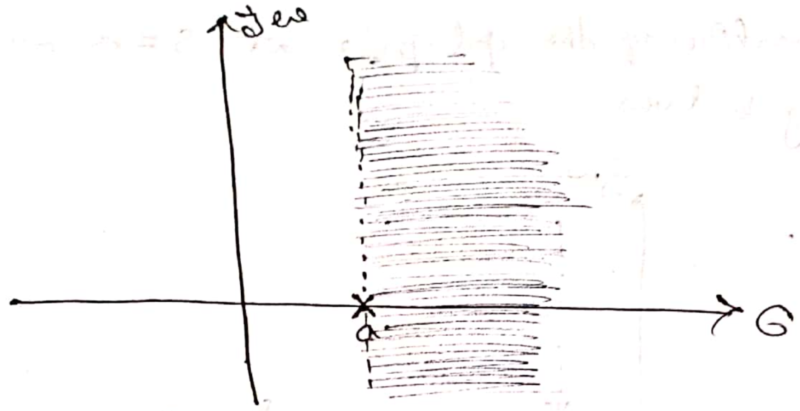
$$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

$$e^{\infty} = \infty$$

$$= \frac{1}{-(\sigma-a+j\omega)}(0-1) = \frac{1}{(\sigma+j\omega)-a}$$

$$= \frac{1}{s-a}$$

$$\therefore X(s) = \frac{1}{s-a} \;;\; Re|s|>a$$



Ex-6.2 An anticausal signal is zero for $t > 0$. Determine the laplace transform and ROC for the anticausal signal,

$$y(t) = -e^{at}u(-t)$$

Solution: Using $y(t) = -e^{at}u(-t)$ in place of $x(t)$ in ~~Eq. (6~~ Eq. obtain,

$$Y(s) = \int_{-\infty}^{\infty} -e^{at}u(-t)\, e^{-st}\, dt$$

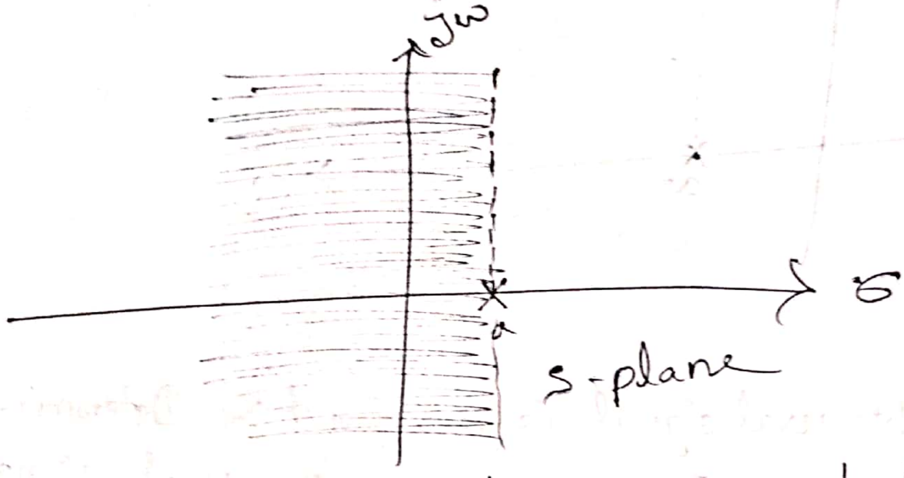
$$= -\int_{-\infty}^{0} e^{-(s-a)t}\, dt$$

$$= \frac{-1}{-(s-a)} e^{-(s-a)t}\Big|_{-\infty}^{0}$$

$$= \frac{1}{s-a} \times (a-\sigma)$$

$$= \frac{1}{s-a} \; ; \operatorname{Re}|s| < a;$$

The ROC and the location of the ~~pd~~ pole at $s=a$ are depicted in fig below.



The ROC for $y(t) = -e^{-at}u(t)$ is depicted by the shaded region. A pole is located at $s=a$.

P-6.2

(b) $x(t) = \cancel{2}\sin(3t)u(t) \rightarrow$ Do laplace transform, find ROC, ~~plo~~ poles and zeros.

$$\Rightarrow X(s) = \int_{-\infty}^{\infty} \sin(3t)\, u(t)\, e^{-st}\, dt$$

$$= \int_{0}^{\infty} \sin(3t)\, e^{-st}\, dt$$

$$= \int_{0}^{\infty} \frac{1}{2j}\left(\cancel{\sin}\, e^{j3t} - e^{-j3t}\right) e^{-st}\, dt$$

$$= \frac{1}{2j}\int_{0}^{\infty} \left\{ e^{(j3-s)t} - e^{-(j3+s)t} \right\} dt$$

$$= \frac{1}{2j}\left\{ \frac{e^{(j3-s)t}\Big/_{0}^{\infty}}{j3-s} - \frac{e^{-(j3+s)t}\Big/_{0}^{\infty}}{-(j3+s)} \right\}$$

$$= \frac{1}{2j}\left\{ \frac{e^{(j3-\sigma-j\omega)t}\Big/_{0}^{\infty}}{j3-s} - \frac{e^{-(j3+\sigma+j\omega)t}\Big/_{0}^{\infty}}{-(j3+s)} \right\}$$

$$= \frac{1}{2j}\left\{ \frac{-1}{j3-s} - \frac{-1}{-(j3+s)} \right\}$$

$$= \frac{1}{2j}\left\{ \frac{1}{j3-s} - \frac{1}{j3+s} \right\}$$

$$= \frac{1}{2j} \cdot \frac{-2j3}{(j3)^2-(s)^2}$$

$$= \frac{-3}{-9-s^2}$$

$$= \frac{-3}{-(s^2+9)}$$

$$= \frac{3}{s^2+9}$$

$$\therefore s^2 = -9$$

$$\Rightarrow s = \pm\sqrt{-9}$$

$$= \pm\sqrt{-1}\; 3$$

$$= \pm j3$$

$s = \sigma + j\omega$

$e^{(j3-\sigma-j\omega)t}$

$\Rightarrow e^{j(3-\omega)t} \cdot e^{-\sigma t}\Big|_0^{\infty}$

where $Re|s| > 0$.

$e^{-(j3+\sigma+j\omega)t}$

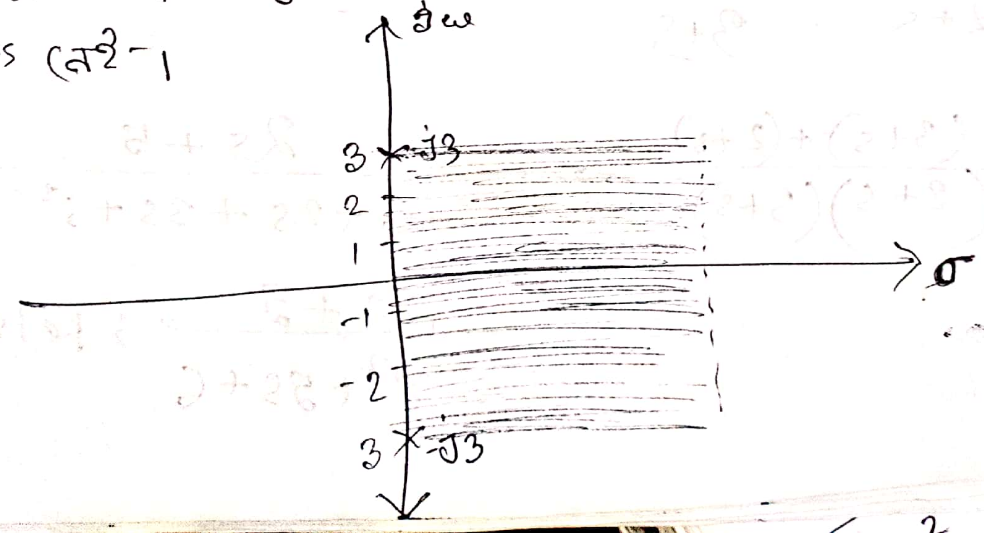
$\Rightarrow e^{-j(3+\omega)t} \cdot e^{-\sigma t}\Big|_0^{\infty}$

; $Re|s| > 0$

$j = \sqrt{-1}$

$(j)^2 = -1$

এখানে শুধুমাত্র হবে polynomial থাকবে, তাই শুধুমাত্র Poles আছে, zeros নেই।



Prob - 6.2 (c)

$$x(t) = e^{-2t}u(t) + e^{-3t}u(t)$$

$$x(s) = \int_{-\infty}^{\infty} \left\{ e^{-2t}u(t) \cancel{dt} + e^{-3t}u(t) \right\} e^{-st} \cdot dt$$

$$= \int_0^{\infty} e^{-2t} e^{-st} dt + \int_0^{\infty} e^{-3t} \cdot e^{-st} dt$$

$$= \int_0^{\infty} e^{-(2+s)t} dt + \int_0^{\infty} e^{-(3+s)t} dt$$

$$= \frac{e^{-(2+s)t} \Big/_0^{\infty}}{-(2+s)} + \frac{e^{-(3+s)} \Big/_0^{\infty}}{-(3+s)}$$

$$= \frac{0-1}{-(2+s)} - \frac{0-1}{3+s}$$

$$= \frac{1}{2+s} + \frac{1}{3+s}$$

$$= \frac{(3+s)+(2+s)}{(2+s)(3+s)} = \frac{2s+5}{6+2s+3s+s^2}$$

$$= \frac{2s+5}{s^2+5s+6} \; ; \; Re\{s\} > -3$$

There is a zeros at $s = -\frac{5}{2}$ and poles at

$s = -2$ and $s = -3$ Ans

Example-6.3:

Find the unilatoral laplace transforom of

$$x(t) = \left(-e^{3t}u(t) * (tu(t))\right).$$

Now,

$$x(t) = -e^{3t}u(t) * tu(t)$$

for,

$$X(s) = \int_{-\infty}^{\infty} -e^{3t}u(t)\, e^{-st}\,dt$$

$$= \int_{0}^{\infty} -e^{3t}e^{-st}\,dt$$

$$= -\int_{0}^{\infty} e^{(3-s)t}\,dt$$

$$= -\frac{e^{(3-s)t}\Big|_0^\infty}{3-s}$$

$$= -\frac{0-1}{3-s}$$

$$= -\frac{(-1)}{-(s-3)}$$

$$= \frac{-1}{s-3}$$

for, $tu(t)$: | Differentiation property:

$$-tx(t) \xrightarrow{L} \frac{d}{ds} X(s)$$

$$x(t) \xrightarrow{L} X(s)$$

Now,

$$\cancel{X} U(s) = \int_{-\infty}^{\infty} u(t)\, e^{-st}\, dt$$

$$= \int_{0}^{\infty} e^{-st}\, dt$$

$$= \left[\frac{-e^{-st}}{s} \right]_0^{\infty}$$

$$= \frac{1}{s}$$

$$\therefore u(t) \xrightarrow{L} \frac{1}{s}$$

$$tu(t) \xrightarrow{L} -\left(\frac{d}{ds}\left(\frac{1}{s}\right)\right)$$

$$= -\left(-\frac{1}{s^2}\right)$$

$$= \frac{1}{s^2}$$

$$\therefore X(s) = \frac{-1}{s-3} \times \frac{1}{s^2}$$

$$= \frac{-1}{s^2(s-3)}$$

Problem-6.4

(b) $x(t) = t^2 e^{-2t} u(t)$

Now,

$$X(s) = \int_{-\infty}^{\infty} e^{-2t} u(t)\, e^{-st}\, dt$$

$$= \int_0^{\infty} e^{-2t} \cdot e^{-st}\, dt$$

$$= \int_0^{\infty} e^{(-2-s)t}\, dt$$

$$= \int_0^{\infty} e^{-(s+2)t}\, dt$$

$$= \frac{e^{-(s+2)t} \Big|_0^{\infty}}{-(s+2)}$$

$$= \frac{-1}{-(s+2)} = \frac{1}{s+2}$$

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$\therefore \; t e^{-2t} u(t) \xrightarrow{\mathcal{L}u} -\frac{d}{ds}\left(\frac{1}{s+2}\right)$

$$= \frac{1}{(s+2)^2}$$

After that,

$t^2 e^{-2t} u(t)$

$$= t \cdot \left(t e^{-2t} u(t)\right) \xrightarrow{\mathcal{L}u} -\frac{d}{ds}\left(\frac{1}{(s+2)^2}\right)$$

$$= \frac{2}{(s+2)^3}$$

Ans.

Problem - 6.4:

(d) $x(t) = e^{-t} u(t) * \cos(t-2)\, u(t-2)$

let,

$y(t) = \cos(t)\, u(t)$

$\therefore \; Y(s) = \int_{-\infty}^{\infty} \cos(t)\, u(t)\, e^{-st}\, dt$

$$= \int_{0}^{\infty} \cos(t)\, e^{-st}\, dt$$

* Tim shift

$x(t-\tau) \xrightarrow{\mathcal{L}u} e^{-s\tau} X(s)$

$x(t) \xrightarrow{\mathcal{L}u} X(s)$

$$= \frac{1}{2}\int_0^{\infty} (e^{j} + e^{-j}) e^{-st}\, dt$$

$$= \frac{1}{2}\int_0^{\infty} \left(e^{(j-s)t} + e^{-(j+s)t} \right) dt$$

$$= \frac{1}{2} \left\{ \frac{e^{(j-s)t}\Big|_0^{\infty}}{j-s} + \frac{e^{-(j+s)t}\Big|_0^{\infty}}{-(j+s)} \right\}$$

$$= \frac{1}{2} \left\{ \frac{-1}{j-s} - \frac{(-1)}{j+s} \right\}$$

$$= \frac{1}{2} \cdot \frac{-j - s + j - s}{j^2 - s^2}$$

$$= \frac{1}{2} \times \frac{-2s}{j^2 - s^2}$$

$$= \frac{-s}{(\sqrt{-1})^2 - s^2}$$

$$= \frac{-s}{-(s^2+1)}$$

$$= \frac{s}{s^2+1}$$

$$y(t-2) \xrightarrow{\mathcal{L}u} e^{-2s} \cdot Y(s)$$

$$\Rightarrow e^{-2s} \cdot \frac{s}{s^2+1}$$

Again,

$$y(t) = e^{-t} u(t)$$

$$Y(s) = \int_{-\infty}^{\infty} e^{-t} u(t) \cdot e^{-st} dt$$

$$= \int_{0}^{\infty} e^{-t} \cdot e^{-st} dt$$

$$= \int_{0}^{\infty} e^{-(1+s)t} dt$$

$$= \frac{e^{-(1+s)t} \Big|_0^{\infty}}{-(1+s)}$$

$$= \frac{-1}{-(1+s)}$$

$$= \frac{1}{1+s}$$

$$\therefore e^{-t} u(t) \xrightarrow{\mathcal{L}u} \frac{1}{1+s}$$

$$e^{-t}u(t) * \cos(t-2)u(t-2) \xrightarrow{\mathcal{L}u} e^{-2s} \cdot \frac{s}{s^2+1} \times \frac{1}{1+s}$$

$$= \frac{e^{-2s} \cdot s}{(s+1)(s^2+1)}$$

Ans.

Problem
6.4 (c)

$$x(t) = tu(t) - (t-1)u(t-1) - (t-2)u(t-2) + (t-3)u(t-3)$$

$$\mathcal{L}_0(s) = \int_{-\infty}^{\infty} u(t)\, e^{-st}\, dt$$

$$= \int_0^{\infty} e^{-st}\, dt$$

$$= \frac{1}{s}$$

$$tu(t) \xrightarrow{\mathcal{L}u} -\frac{d}{ds}\left(\frac{1}{s}\right)$$

$$= \frac{1}{s^2}$$

$$\cancel{X_2(s) =} \int_0^{\infty}$$

$$y(t-1) \xrightarrow{\mathcal{L}} e^{-s}\, y(s)$$

$$= e^{-s} \frac{1}{s^2}$$

$$= \frac{e^{-s}}{s^2}$$

$$y(t-2) = e^{-2s} \cdot y(s)$$

$$= e^{-2s} \cdot \frac{1}{s^2}$$

$$= \frac{e^{-2s}}{s^2}$$

$$y(t-3) = e^{-3s} \cdot y(s)$$

$$= e^{-3s} \cdot \frac{1}{s^2}$$

$$= \frac{e^{-3s}}{s^2}$$

$$x(t) \xrightarrow{\mathcal{L}} X(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s^2} - \frac{e^{-3s}}{s^2}$$

$$= \frac{1 - e^{-s} + e^{-2s} - e^{-3s}}{s^2}$$ Ans.