ID: 1390

An. to the Q. NO - 2

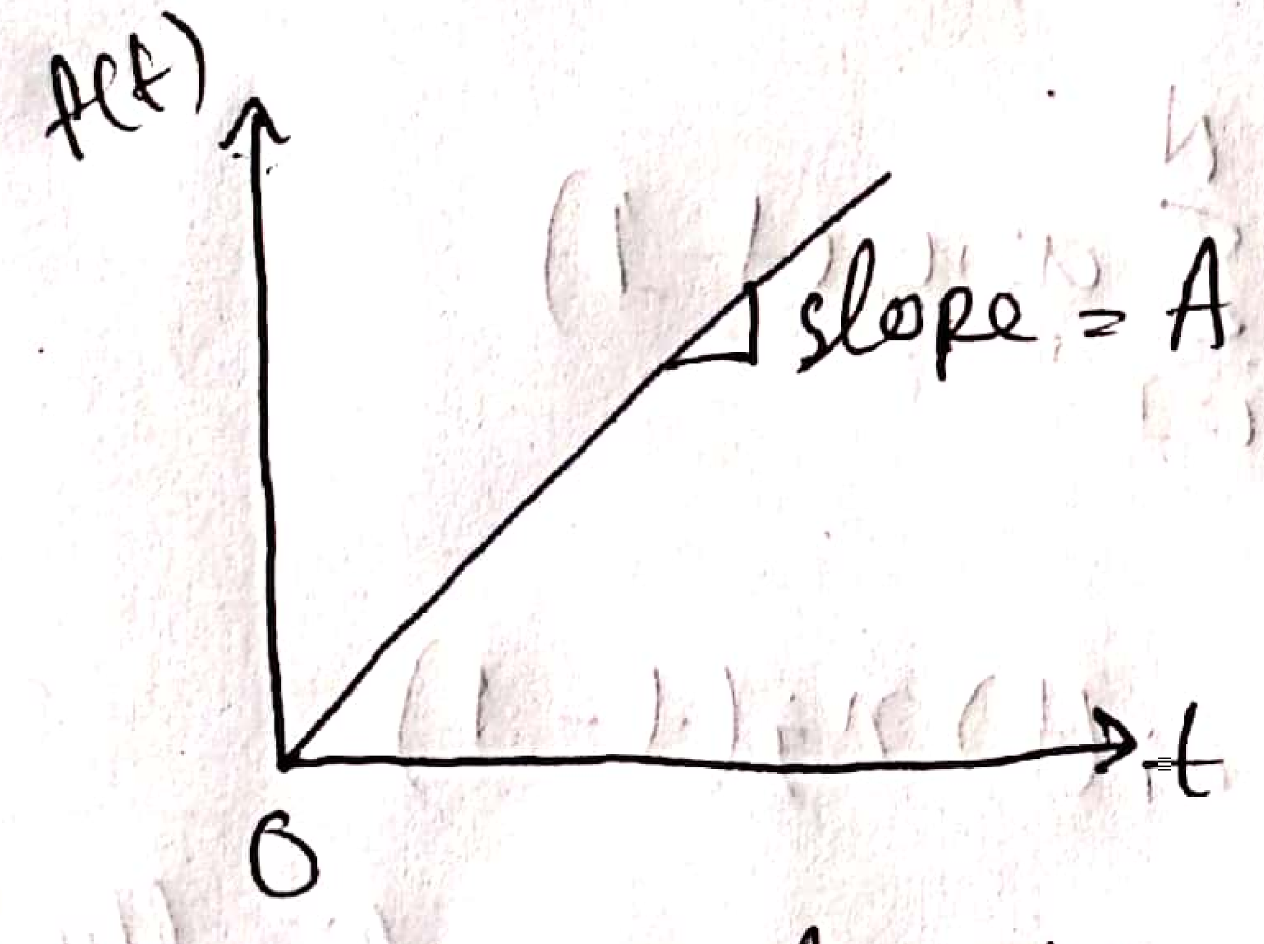
a

① Impulse function

In signal processing, the impulse response, or impulse response function (IRF), of a dynamic system is its output when presented with a brief input signal called an impulse

ⓘⓘ Ramp function:

The ramp function is unary real function, whose graph is shaped like a ramp. It can be expressed by numerous definitions.



Ramp function.

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b

~~Causal~~

Causal System:

A system is said to be ~~causa~~ causal if it does not respond before the input is applied.

$$y(t) = x(t)\,x(t-1)$$

Let the input signal $x(t)$ be expressed as the weighted sum,

$$x(t) = \sum_{i=1}^{N} a_i x_i(t).$$

Correspondingly, the output signal of the system is given by the double summation:

$$y(t) = \sum_{i=1}^{N} a_i x_i(t) \sum_{i=1}^{N} a_j x_j(t-1)$$

$$= \sum_{i=1}^{N} \sum_{i=1}^{N} a_i a_j x_i(t)\, x_j(t-1)$$

The form of this equation is radically different from that describing the input signal $x(t)$. That is here we can't write $y(t) = \sum_{i=1}^{N} a_i y_i(t)$.

Thus, the system violates the principle of superposition and is therefore nonlinear.

An. to the Q.NO - 1

a

Given,

$$x[n] = \begin{cases} 2, & n = 1, 2 \\ -2, & n = -1, -2, -3 \\ 0, & n = 0,\ n > 2 \text{ or } n < -3 \end{cases}$$

$\therefore y[n] = x[3n+2]$

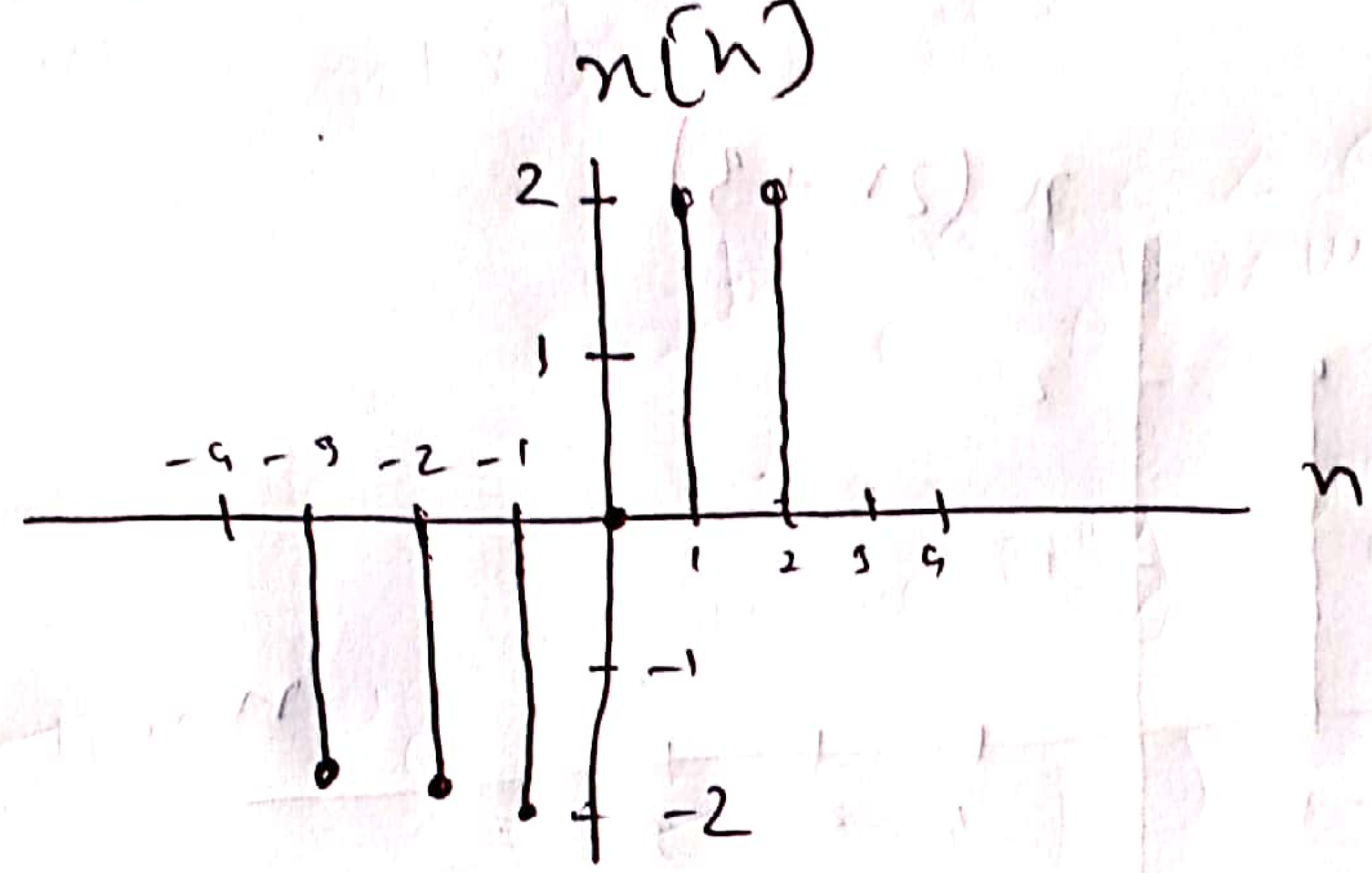

fig (a)

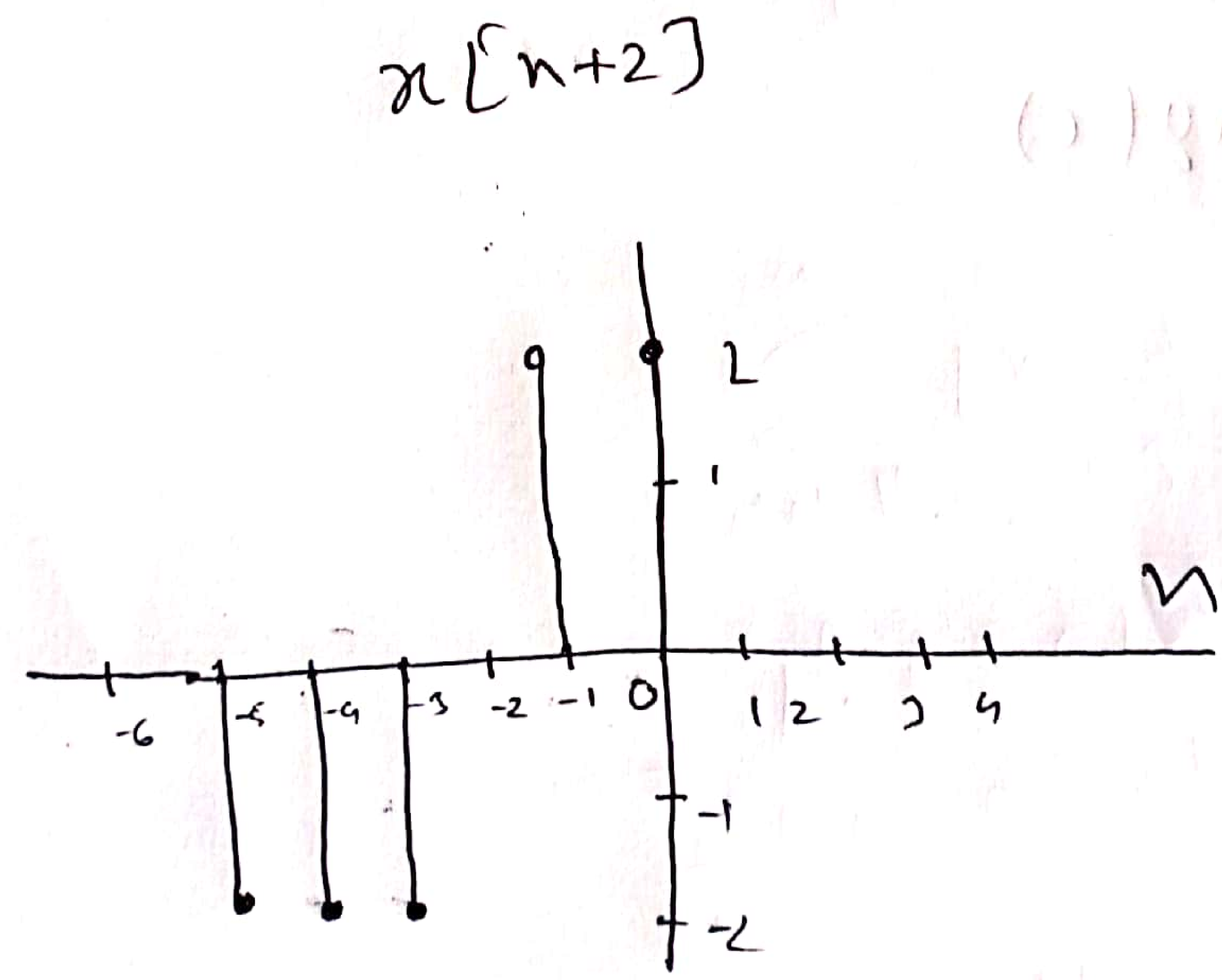

fig (b)

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Shiftting $x[n]$ to the left by 3 samples as shown in fig (b)

Now, Scaling the signal by a factor of 2.

$x(2n+3)$

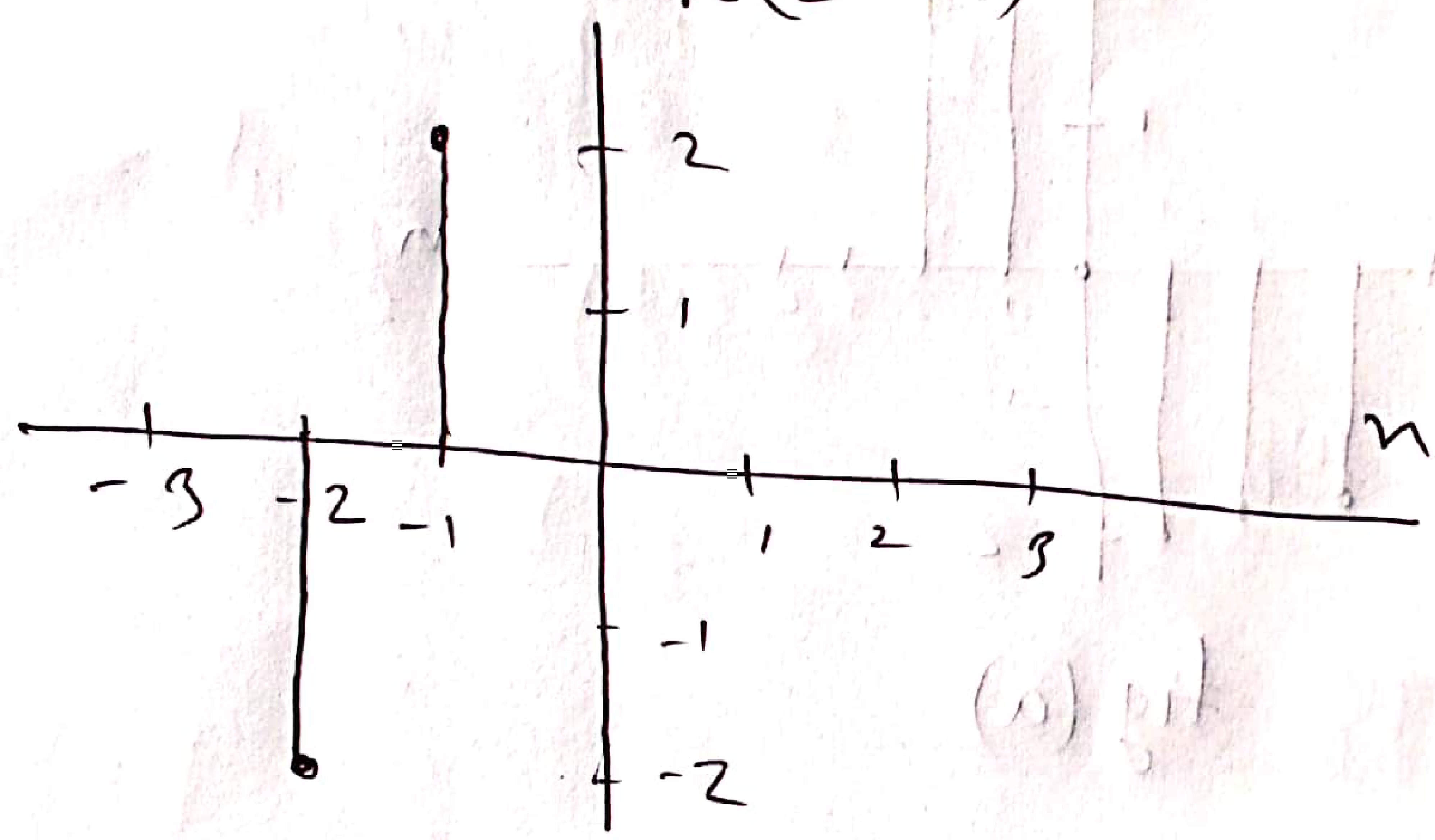

fig (e)

b

Given, $x[n] = n \quad , 0 <= n < 5$

$= 20 - n \quad , 5 <= n <= 10$

$= 0 \quad$, otherwise

This signal is energy signal

$$\text{Energy} = \sum_{n=-\infty}^{\infty} x^2[n]$$

$$= \cancel{\sum_{n=0}^{9} [n]^2 + \sum_{n=5}^{10} [20-n]^2}$$

$$= \sum_{n=0}^{9} [n]^2 + \sum_{n=5}^{10} [20-n]^2$$

$$= [0 + 1^2 + 2^2 + 3^2 + 4^2] + [(20-5)^2 + (20-6)^2 + (20-7)^2 + (20-8)^2 + (20-9)^2 + (20-10)^2]$$

$$= 1 + 4 + 9 + 16 + 226 + 196 + 169 + 144 + 121 + 100$$

$$= 985$$

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PTO

Ans to the Q. NO - 5

a

Given $h(t) = RC^{-1} e^{-\frac{t}{RC}} u(t)$

$$\Rightarrow H(j\omega) = \int_{-\alpha}^{\alpha} h(\tau)\, e^{-j\omega\tau}\, d\tau$$

$$= \int_{\alpha}^{\alpha} \frac{1}{RC} e^{-\frac{\tau}{RC}} u(\tau)\, e^{-j\omega\tau}\, d\tau$$

$$= \frac{1}{RC} \int_{0}^{\alpha} e^{-(j\omega + \frac{1}{RC})\tau}\, d\tau$$

$$= \frac{1}{RC} * \frac{-1}{j\omega + \frac{1}{RC}} e^{-(j\omega + \frac{1}{RC})\tau} \Big|_{0}^{\alpha}$$

$$= \frac{1}{RC} * \frac{-1}{j\omega + \frac{1}{RC}} (0 - 1)$$

$$= 0 + \frac{\frac{1}{RC}}{j\omega + \frac{1}{RC}}$$

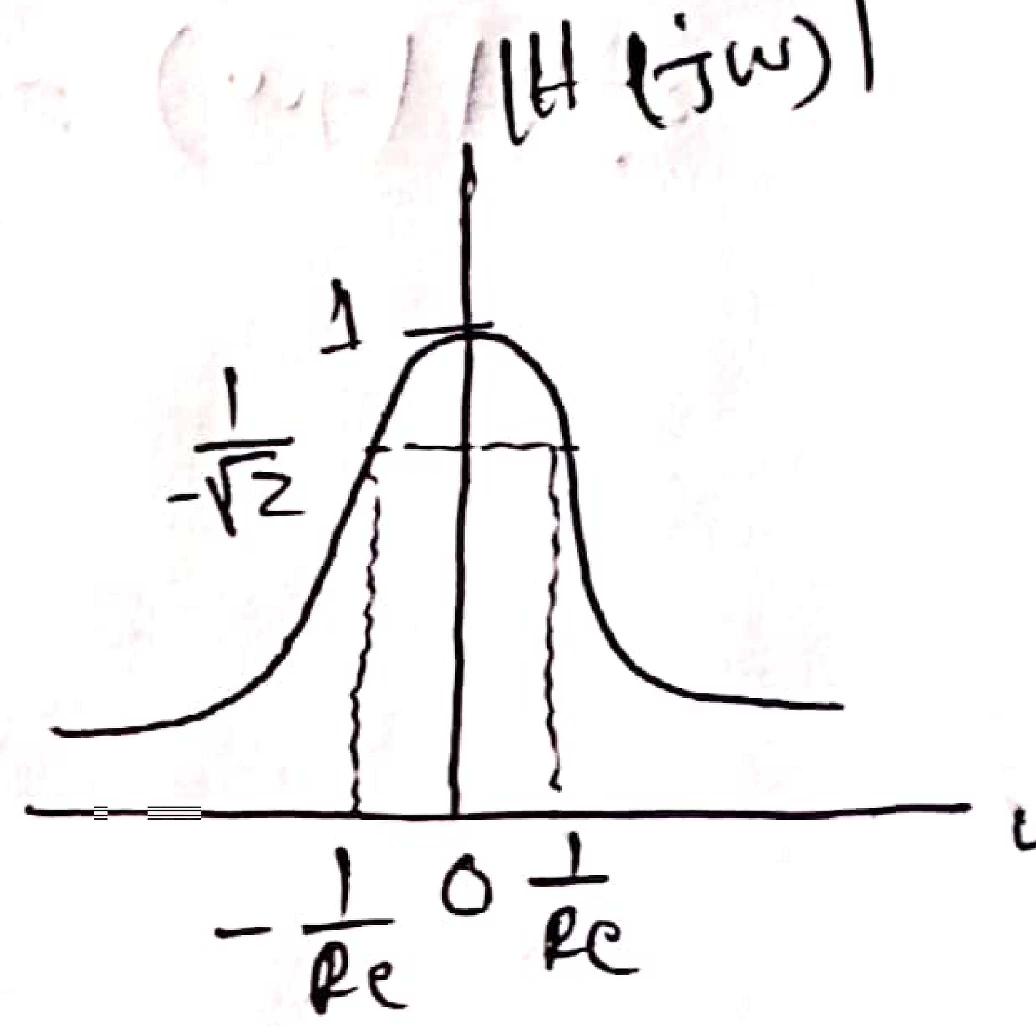
amplitude $= \left| \frac{1}{\alpha + j\beta} \right|$ $[\because \alpha + j\beta = \sqrt{\alpha^2 + \beta^2}$

$$= \frac{1}{\sqrt{\alpha^2 + \beta^2}}$$

∴ The magnitude response is

$$|H(j\omega)| = \frac{\frac{1}{RC}}{j\omega + \frac{1}{RC}} \quad \left[\begin{array}{l} \text{Let } j\omega = B \\ \frac{1}{RC} = \alpha \end{array} \right]$$

$$= \frac{\frac{1}{RC}}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}}$$



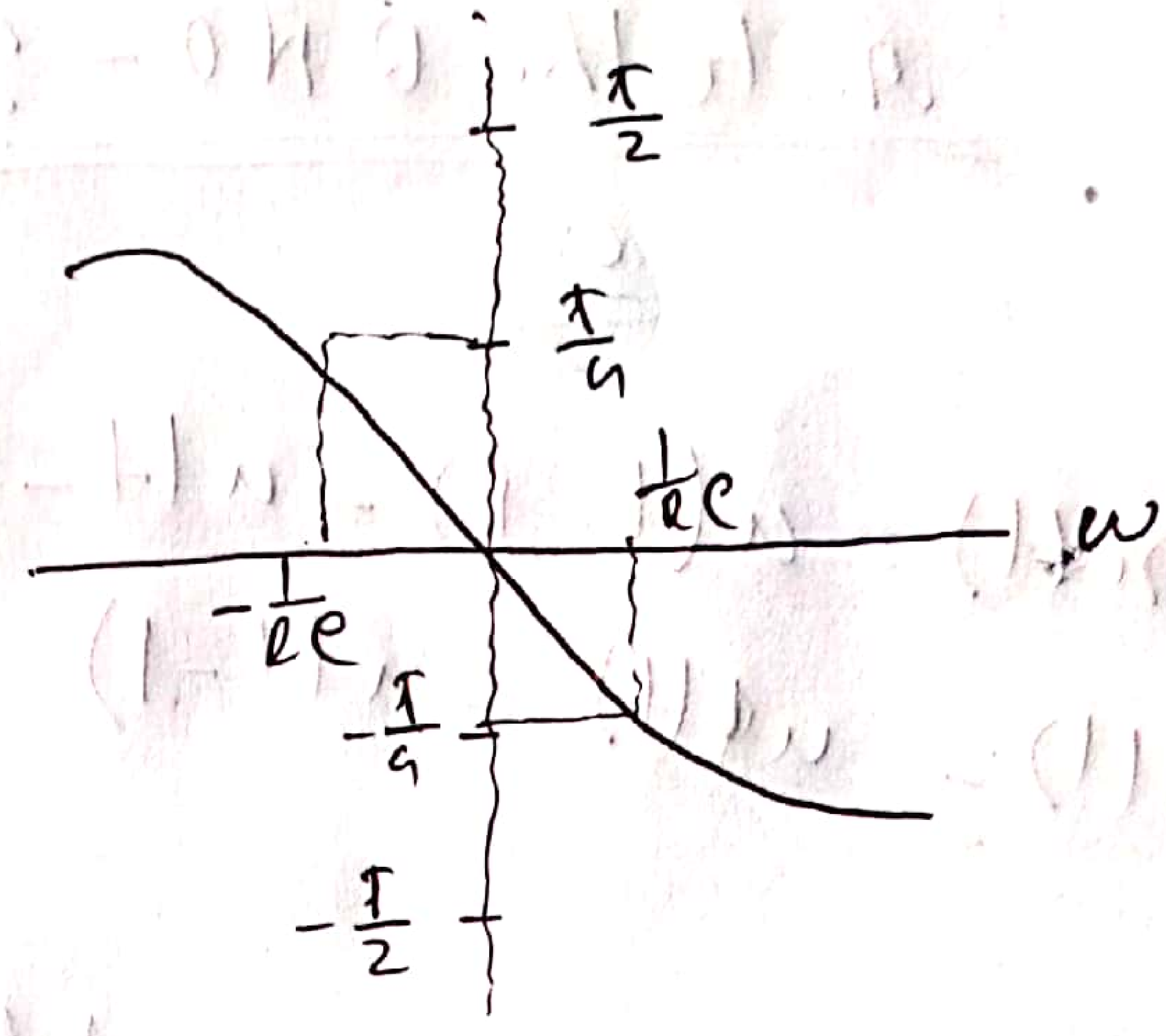
manitude response.

At $\omega = \frac{1}{RC}$,

$$|H(j\omega)| = \frac{\frac{1}{RC}}{\sqrt{\left(\frac{1}{RC}\right)^2 + \left(\frac{1}{RC}\right)^2}}$$

$$= \frac{1}{\sqrt{2}}$$

The phase response is show figh in



$\therefore$ phase response

$$\angle H(j\omega) = \tan^{-1}\left(\frac{\omega}{\frac{1}{RC}}\right)$$

$$= \tan^{-1}(\omega RC)$$ Ans

b

For, $x(t) = t$,

if input signal is bounded, that for any signal $M_x < \infty$ ~~than~~ then output signal will also be bounded. In that case it will be stable.

But for any unbounded input, the output will be unstable.

So we can say a function $x(t) = 1$ is not a stable.

Ans to the Q. NO - 3

a

Given, $x(t) = u(t-1) - u(t-3)$

& $h(t) = u(t) - u(t-1)$

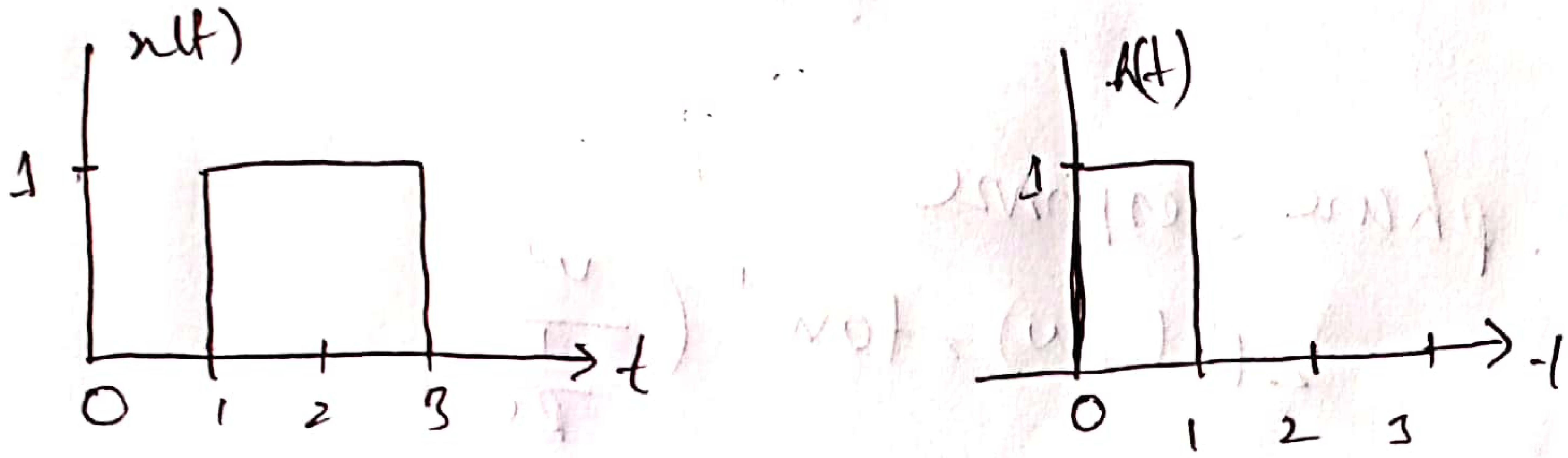


figure (a)

figure(a) represents the input signal and LTI system impulse response for given question.

$\therefore \; w_t(\tau) = x(\tau)\, h(t-\tau)$

$\therefore \; y(t) = \int_{-\infty}^{\infty} w_t(\tau)\, d\tau$

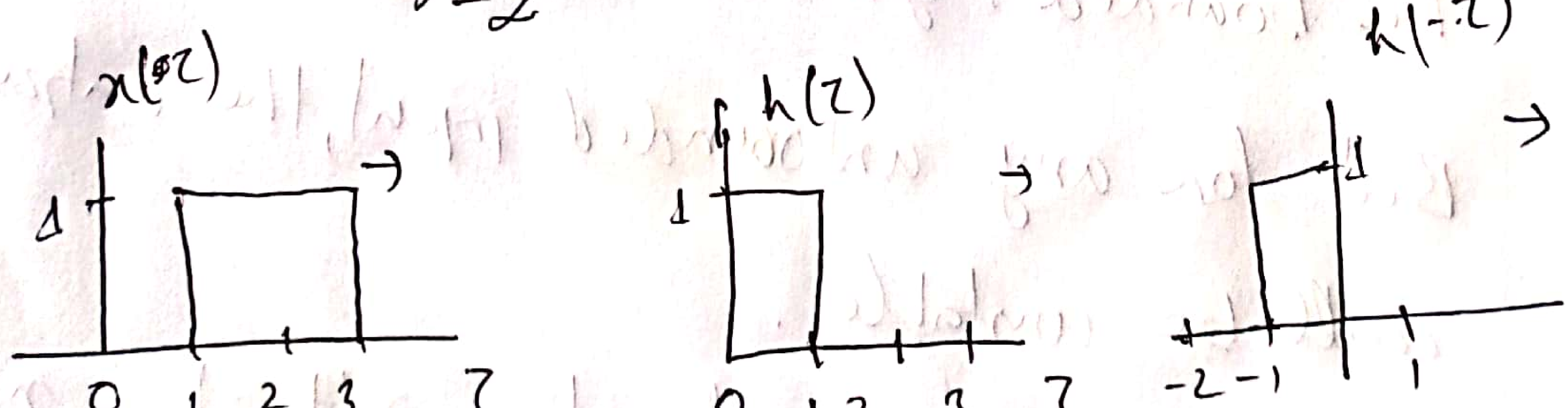
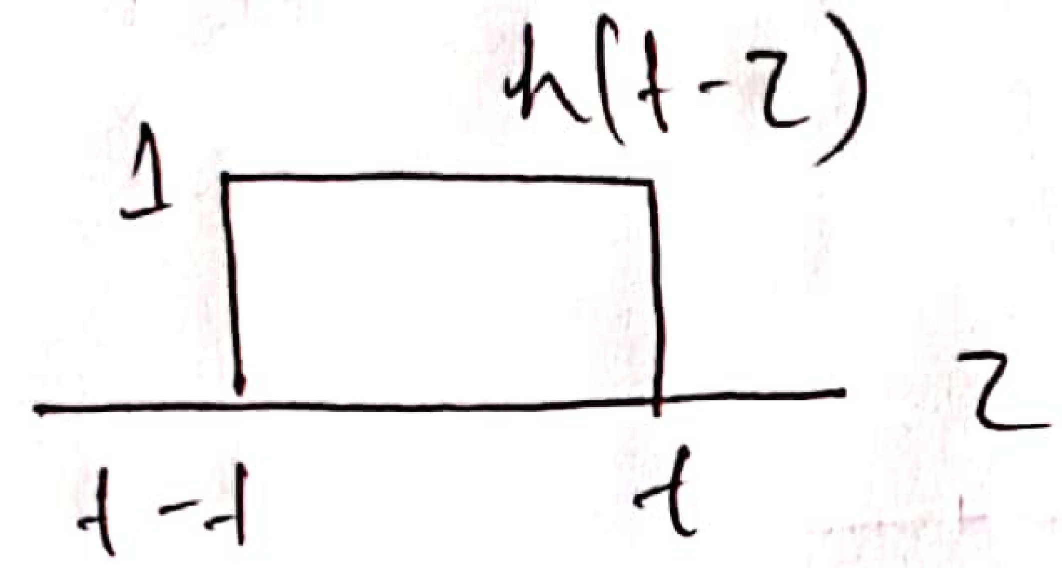
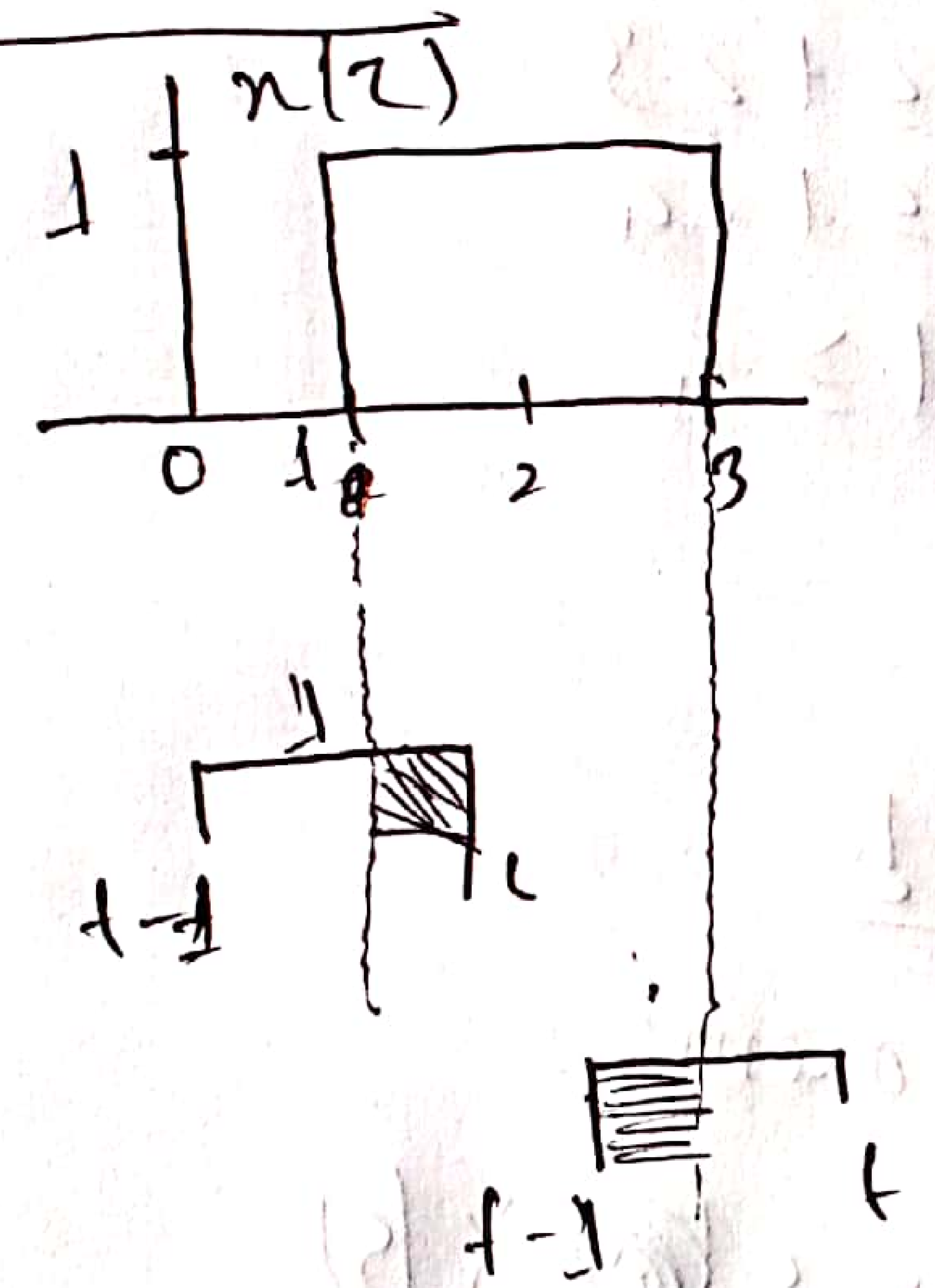


figure (b)



from figure (b) The input $x(\tau)$ depicted above the ~~scaled~~ reflected and time-shifted impulse response $h(t-\tau)$, depicted as a function of τ

To find limit:



for $1 \le \tau < t$

$$y(t) = \int_1^t 1 \cdot d\tau$$

$= t - 1$ where $1 \le t \le 3$

for $t-1 \le \tau \le 3$

$$y(t) = \int_{t-1}^{3} 1 \cdot d\tau = 4 - t \quad \text{where, } 3 \le t \le 4$$

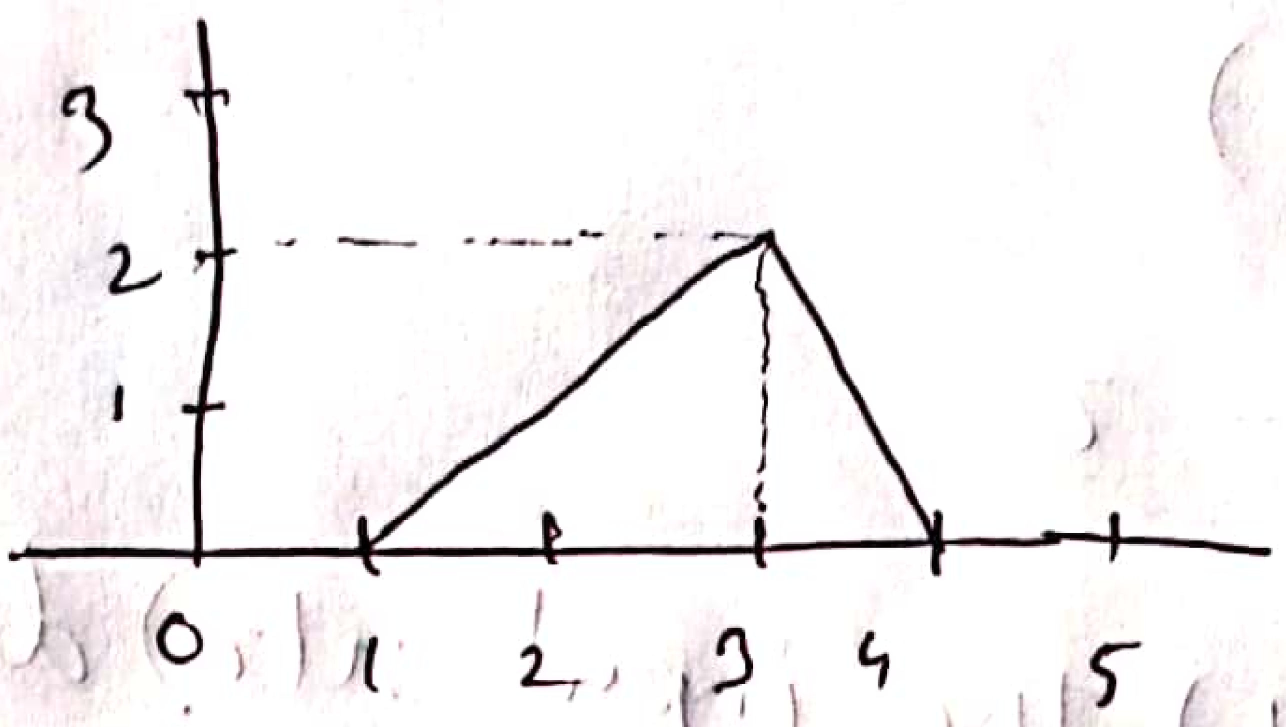


figure (e) the system output of $y(t)$

Combining the soluation for ~~one~~ each interval of time sifting given the output as shown in figure (e).

$$y(t) = \begin{cases} 0 & t < 1 \\ t-1 & 1 \le t < 3 \\ 4-t & 3 \le t < 4 \\ 0 & t \ge 4 \end{cases}$$

b

① Initial value theorem;

$$x(0) = \lim_{s \to \infty} s X(s)$$

② final value theorem,

$$x(\infty) = \lim_{s \to \infty} s X(s)$$

Ans. to the Q. No - 4

a

$x(t) = \sin(4t)\, u t$

$x(s) = \int_{-\alpha}^{\alpha} x(t)\, e^{-st}\, dt$

$= \int_{0}^{\alpha} \frac{(e^{4jt} - e^{-4jt})\, e^{-st}\, dt}{2j}$

$= \frac{1}{2j} \left\{ \frac{e^{(j4-s)t} \Big/_0^{\alpha}}{j4-s} - \frac{e^{-(j4+s)t} \Big/_0^{\alpha}}{-(j4+s)} \right\}$

$= \frac{1}{2j} \left\{ \frac{(0-1)}{j4-s} - \frac{(0-1)}{-(j4+s)} \right\}$

$= \frac{1}{2j} \left\{ \frac{1}{4s-s} - \frac{1}{j4+s} \right\}$

$= \frac{1}{2j} \left\{ \frac{-j4-s-j4+s}{(j4)^2-(s)^2} \right\}$

$= \frac{1}{\cancel{2j}} \times \frac{-\cancel{2}\times\cancel{j}\times 4}{(j4)^2-(s)^2}$

$= \frac{-4}{(-1\times 4)-s^2}$

$= \frac{-4}{-4-s^2}$

$= \frac{-4}{-(4+s^2)}$

$$= \frac{4}{4+s^2}$$

$$\therefore s^2 = \sqrt{-4}$$

$$\therefore s = \sqrt{-(2)^2}$$

$$= \pm j2$$

There are ~~po~~ poles at $j = \pm j2$

b

$$y(s) = \int_{-\alpha}^{\alpha} \sin(t)\, u(t)\, e^{-st}\, dt$$

$$= \int_{0}^{\alpha} \frac{e^{jt} - e^{-jt}}{2j} - e^{-(j-s)t}) dt$$

$$\cancel{= \frac{1}{2j} \int_{0}^{\alpha} \frac{e^{(j-s)t}}{j-s} - e^{-(j-s)}\Big|_0^{\alpha}}$$

$$= \frac{1}{2j} \int_{0}^{\alpha} (e^{(j-s)t} - e^{-(j-s)t}) dt$$

$$= \frac{1}{2j} \left\{ \frac{e^{(j-s)}\Big|_0^{\alpha}}{j-s} - \frac{e^{-(j-s)}\Big|_0^{\alpha}}{-(j+s)} \right\}$$

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$$= \frac{1}{2j}\left\{\frac{-1}{j-s} - \frac{-1}{-(j+s)}\right\}$$

$$= \frac{1}{2j}\left\{-\frac{1}{j-s} - \frac{1}{j+s}\right\}$$

$$= \frac{1}{2j}\left(\frac{-j-s-j+s}{(j)^2-(s)^2}\right.$$

$$= \frac{1}{2j} \times \frac{-2j}{j^2-s^2}$$

$$= \frac{-1}{(\sqrt{-1})^2-s^2}$$

$$= \frac{-1}{-1-s^2}$$

$$= \frac{-1}{-(1+s^2)}$$

$$= \frac{1}{1+s^2}$$

$$\therefore y(t-3) \xrightarrow{lu} e^{-3s}\,y(s)$$

$$= e^{-3s} \times \frac{1}{1+s^2}$$

Let, $t = e^{-l}u(t)$

$$= \int_{-\alpha}^{\alpha} e^{-l}u(t)\,e^{-st}dt$$

$$= \int_0^{\infty} e^{-(1+s)t}\, dt$$

$$= \frac{1}{1+s}$$

$$\therefore e^{-t} u(t) \xrightarrow{Lu} \frac{1}{1+s}$$

$$\therefore e^{-t} u(t) * \sin(t-3)\, u(t-3)$$

$$= e^{-3s} \times \frac{1}{1+s^2} \times \frac{1}{1+s}$$

$$= \frac{e^{-3s}}{(1+s^2)(1+s)}$$