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An. to the. Q. NO-5

Sol

Given, $f(x) = x^2 - 4x$ in $[2,4]$

~~$f'(x) \neq x^2-4x$ is exist for all $x \in [2,4]$ Hence $f(x)$ continuous in $[2,4]$~~

~~Hence, $f'(x)$ is differentiable in open interval $(-\pi, \pi)$~~

~~Hence, $f(x)$ is satisfied all condition's~~

$f(x) = x^2 - 4x$ is exist for all $x \in [2,4]$ Hence $f(x)$ continuous in $[2,4]$

$f'(x) = 2x - 4$ is exist for all $x \in [2,4]$ Hence, $f(x)$ is differentiable in open interval $(-\pi, \pi)$

Hence, $f(x)$ is satisfied all conditions

So, there exist at least one value $x = c$ such that,

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

$$\Rightarrow 2c - 4 = \frac{f(4) - f(2)}{4-2}$$

$$\Rightarrow 2c - 4 = \frac{0-(-4)}{2}$$

$$\Rightarrow 2c - 4 = \frac{4}{2}$$

$$\Rightarrow 2c - 4 = 2$$

$$\Rightarrow 2c = 2 + 4$$

$$\Rightarrow 2c = 6$$

$$\therefore c = 3 \in [2,4]$$

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$f(x) = x^2 - 4x$

$\Rightarrow f(2) = (2)^2 + 4\times 2$

$= 4 - 8$

$= -4$

$\Rightarrow f(4) = 4^2 - 4\times 4$

$= 16 - 16$

$= 0$

Hence, Lagrang's Mean value theorem is varified.

—x—

An to the Q. NO - 3

(i) ~~$y = e^{a\sin^{-1}x}$~~

Given, $y = e^{a\sin^{-1}x}$

$\Rightarrow y_1 = e^{a\sin^{-1}x} \frac{a}{\sqrt{1-x^2}}$

$\Rightarrow y_1 = \frac{ya}{\sqrt{1-x^2}}$

$\Rightarrow y_1^2 = \frac{y^2a^2}{1-x^2}$

$\Rightarrow y_1^2 (1-x^2) = y^2a^2$

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Differentiating with respect to x in bothside,

$$\frac{d}{dx}(y_1^2(1-x^2)) = \frac{d}{dx}(a^2y^2)$$

$$\Rightarrow y_1^2 \frac{d}{dx}(1-x^2) + (1-x^2)\frac{d}{dx}(y_1^2) = 2a^2yy_1$$

$$\Rightarrow y_1^2(-2x)+(1-x^2)\times 2y_1y_2 = 2a^2yy_1$$

$$\Rightarrow -2xy_1^2+2y_1y_2(1-x^2) = 2a^2yy_1$$

$$\Rightarrow -xy_1^2 + y_1y_2(1-x^2) = a^2yy_1$$

$$\Rightarrow -xy_1^2 + y_1y_2(1-x^2) - a^2yy_1 = 0$$

$$\Rightarrow y_2(1-x^2) - xy_1 - a^2y = 0$$ [Solved]

(ii) Differentinating $n$times with x using Leibnitz theorem,—

$$D^n\{(1-x^2)y_2 - xy_1 - a^2y\} = e$$

$$\Rightarrow D^n\{(1-x^2)y_2\} - D^n\{xy_1\} - D^n(a^2y) = 0$$

$$\Rightarrow [(1-x^2)D^n(y_2) + {}^nC_1D^{n-1}(y_2)D(1-x^2) + {}^nC_2D^{n-2}(y_2) D^2(1-x^2)] - [xD^n(y_1) + {}^nC_1D^{n-1}(y_1)Dx] - a^2y_n = 0$$

$$\Rightarrow [(1-x^2)y_{n+2} + ny_{n+2}(0-2x) + \frac{n(n-1)}{2!}y_n(-2)$$

$$- [xy_{n+1} + ny_n] - a^2y_n = 0$$

$\Rightarrow (1-x^2)y_{n+2} - xy_{n+1}(2n+1) - \{n^2 - n + n + a^2\} y_n = 0$

$\Rightarrow (1-x^2)y_{n+2} - xy_{n+1}(2n+1) - \{n^2+a^2\} y_n = 0$

$\Rightarrow (1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$

[showed].

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An. to the Q. NO-4

Given,

$f(x,y) = 8x^2 + 4xy + 5y^2 - 16x - 14y + 13 = 0$

Now,

$f(x,y) = 8x^2 + 4xy + 5y^2 - 16x - 14y + 13 = 0$ — (i)

$= 8x^2 + 2(2)xy + (5)y^2 + 2(-8)x + 2(-7)y + (13) = 0$

here, $a = 8,$

$b = 5,$

$c = 13$

$h = 2$

$g = -8$

$f = -7$

We know,

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$= (8 \times 5 \times 13) + 2 \times (-7) \times (-8) \times 2 - 8 \times (-7)^2$

$- 5 \times (-8)^2 - 13 \times (2)^2$

$= 520 + 224 - 392 - 320 - 52$

$= -20$

$\neq 0$

Now, $ab - h^2 = (8 \times 5) - (2)^2$

$= 40 - 4$

$= 36 > 0$

So, the conic is a ellipse,

Differentiating (i) partially with respect to x and y we get,

$\frac{\partial f}{\partial x} = 16x + 4y - 16 = 0$ ——— (ii)

$\frac{\partial f}{\partial y} = 4x + 10y - 14 = 0$ ——— (iii)

Solving for (ii) & (iii) eqn

(ii) $\Rightarrow 16x + 4y - 16 = 0$

$\Rightarrow 4x + y - 4 = 0$

$\Rightarrow y = 4 - 4x$

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(iii) $4x + 10(4 - 4x) - 14 = 0$

$\Rightarrow 4x + 40 - 40x - 14 = 0$

$\Rightarrow -36x + 26 = 0$

$\Rightarrow 36x = 26$

$\therefore x = \frac{13}{18}$

& $y = 4 - 4x$

$= 4 - 4 \cdot \frac{13}{18}$

$= 4 = \frac{26}{9}$

center of conic $\left(\frac{13}{18}, \frac{26}{9}\right)$

$8x^2 + 4xy + 5y^2 + c_1 = 0$ —— (iv)

Now, $\cancel{c_1 g} = x_1 \cdot$

$c_1 = gx_1 + fy_1 + c = -8x_1 - 7y_1 + 13$

$= -8 \times \frac{13}{18} - 7 \times \frac{26}{9} + 13$

$= -\frac{52}{9} - \frac{182}{9} + 13$

$= -\frac{-117}{9}$

$= -13$

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Now from eq^n (iv)

$$8x^2+4xy+5y^2-13=0 \quad \text{—— (v)}$$

when the xy term is removed by rotation of the axis, let the reduced eq^n is,

$$a_1x^2+b_1y^2-13=0$$

$$\Rightarrow a_1x^2+b_1y^2=13 \quad \text{—— (vi)}$$

Now, $a_1+b_1=a+b=8\times5=13$ —— (vii)

$a_1\times b_1=8\times5=40$ —— (viii)

from (viii) $\Rightarrow a_1=\frac{40}{b_1}$

from (vii) $\Rightarrow a_1+b_1=13$

$$\Rightarrow \frac{40}{b_1}+b_1=13$$

$$\Rightarrow \frac{40+b_1^2}{b_1}=13$$

$$\Rightarrow b_1^2-13b_1+40=0$$

$$b_1=\frac{-(-13)\pm\sqrt{(-13)^2-4.1.40}}{2.1}$$

$$=\frac{13\pm\sqrt{169-160}}{2}$$

$$=\frac{13\pm3}{2}=8 \text{ or } 5$$

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$\therefore b_1 = 8$ or 5

$\therefore a_1 = \frac{40}{b_1} = 5$ or 8

$\therefore (a_1, b_1) = (5, 8)$ or $(8, 5)$

Now, $5x^2 + \cancel{8}8y^2 = 13$

$\Rightarrow \frac{5x^2}{13} + \frac{8y^2}{13} = 1$

$\Rightarrow \frac{x^2}{\frac{13}{5}} + \frac{y^2}{\frac{13}{8}} = 1$

$\Rightarrow \frac{x^2}{\left(\sqrt{\frac{13}{5}}\right)^2} + \frac{y^2}{\left(\sqrt{13/8}\right)^2} = 1$ Ans

And, $8x^2 + 5y^2 = 13$

$\Rightarrow \frac{8x^2}{13} + \frac{5y^2}{13} = 1$

$\Rightarrow \frac{x^2}{\frac{13}{8}} + \frac{y^2}{\frac{13}{5}} = 1$

$\Rightarrow$

$$\Rightarrow \frac{x^2}{(\sqrt{13/8})^2} + \frac{y^2}{(\sqrt{13/5})^2} = 1 \quad \underline{A.}$$

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Ans to the Q. No – 1

Here, we shall test the function $f(x)$ for $x = 0, 1, \frac{1}{2}$

$f(x) = x$ for $x \leq 0$ and $x < \frac{1}{2}$

L.H.S. $f(x) = \lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} (x)$

$= \lim\limits_{h \to 0} (h)$

$= \lim\limits_{h \to 0} 0$

R.H.S: $f(x) = \lim\limits_{x \to \frac{1}{2}} f(x) = \lim\limits_{x \to \frac{1}{2}} (x)$

$= \lim\limits_{h \to 0} (h)$

$= \lim\limits_{h \to 0} 0$

The function $f(x)$ is continous at $x = 0$

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R.H.S. $\lim_{x \to \frac{1}{2}+0} f(x) = \lim_{x \to 0} 1 = 1$

R.H.S.

$\lim_{x \to \frac{1}{2}-0} f(x) = \lim_{x \to \frac{1}{2}-0} (1-x)$

$f(x) = 1-x$ for $\frac{1}{2} < x < 1$

$= \lim_{h \to 0} [1-(1-h)]$

$= \lim_{h \to 0} [0+h] = 0$

At, $x=1$. $f(x)=1$ for $x \geq 1$

R.H.S.

$\lim_{x \to 1+0} f(x) = \lim_{x \to 1+0} (1-x)$

$= \lim_{h \to 0} [1-(1-h)]$

$= \lim_{h \to 0} h$

$= 0$

putting $x = 1-h$, $h \to 0$ as $x \to 1$

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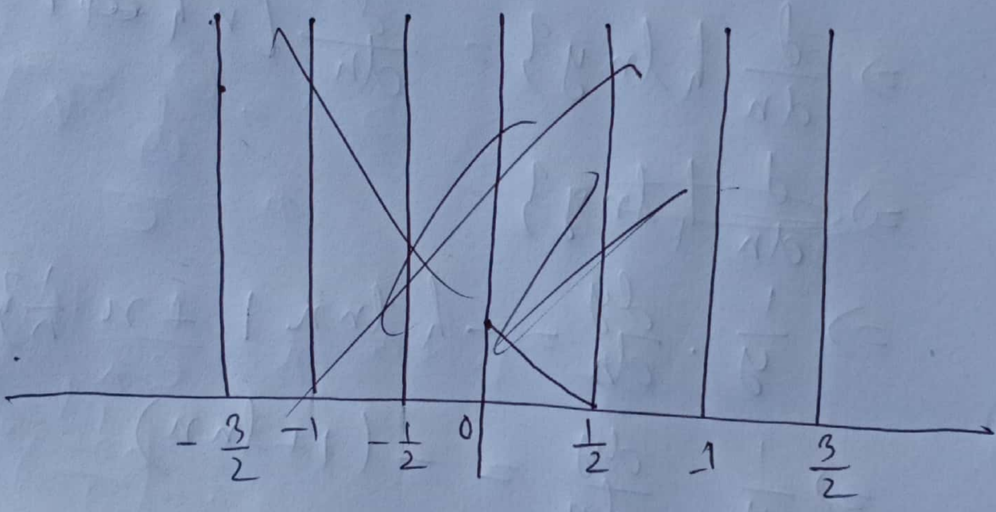
L.H.S.

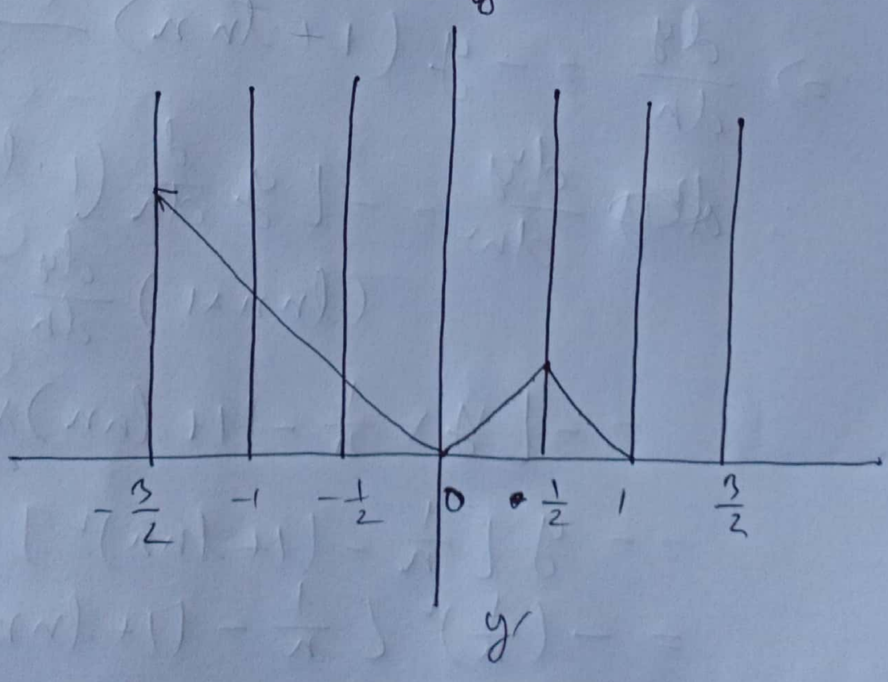
$$\lim_{x \to 1-0} f(x) = \lim_{x \to 1-0} (x)$$

$$= \lim_{h \to 0} (h)$$

$$= \lim_{h \to 0} 0$$

$$= 0$$





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An. to the Q. NO - 2

The maximum value of the function $\left(\frac{1}{x}\right)^x$

$y = \left(\frac{1}{x}\right)^x$

$\Rightarrow \ln y = \ln\left(\frac{1}{x}\right)^x$

$\Rightarrow \ln y = x\ln\left(\frac{1}{x}\right)$

$\Rightarrow \frac{d}{dx}\{\ln y\} = \frac{d}{dx}[x\{\ln 1 - \ln x\}]$

$\Rightarrow \frac{d}{dx}\{\ln y\} = \frac{d}{dx}[x\{1 - \ln x\}]$

~~$\Rightarrow \frac{d}{dx}\{\ln y\}$~~

$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = -\{\ln x \cdot 1 + x \frac{1}{x}\}$

$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = -(1 + \ln x)$

$\Rightarrow \frac{dy}{dx} = -y(1 + \ln x)$ —— (i)

$\therefore \frac{d^2y}{dx^2} = -[y \frac{d}{dx}(1 + \ln x) +$
$(\ln x + 1)\frac{dy}{dx}]$

$= -[y \times \frac{1}{x} - (1 + \ln x) \times y(1 + \ln x)]$

$= -y[\frac{1}{x} - (1 + \ln x)^2]$

$= -\left(\frac{1}{x}\right)^x [\frac{1}{x} - (1 + \ln x)^2]$

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The neccessary and sufficient condition for maximum and minimum value is

$\frac{dy}{dx} = 0$

$\Rightarrow -y(1+\ln x) = 0$

$\Rightarrow 1+\ln x = 0$

$\Rightarrow \ln x = -1$

$\Rightarrow \ln e^x = -1$

$\Rightarrow x = e^{-1}$

$\therefore x = \frac{1}{e}$

when $x = \frac{1}{e}$ then $\frac{d^2y}{dz^2}$

$= -\left(\frac{1}{\frac{1}{e}}\right)^{ve} \left[\frac{1}{\frac{1}{e}} - (1+\ln \frac{1}{e})^2 \right]$

$= (-e)^{ve} \left[e - (1+\ln \frac{1}{e})^2 \right]$

$= -e^{\frac{1}{e}} \left[e - (1-1)^2 \right]$ $[\because \ln \frac{1}{e} = \ln e^{-1} = 1]$

$= -e^{\frac{1}{e}} . e < 0$

if $f(x)$ is maximum at the point of

$x = \frac{1}{e}$ ~~max~~ maximum value $= \frac{1}{e}$

and the maximum value $= \left(\frac{1}{\frac{1}{e}}\right)^{\frac{1}{e}}$

$= e^{\frac{1}{e}}$.