Self Assessment Solutions

Linear Economic Models

1. **Demand and supply in a market are described by the equations**

$$Qd = 66-3P$$
$$Qs = -4+2P$$

(i) Solve algebraically to find equilibrium P and Q

In equilibrium Qd = Qs

$66-3P = -4+2P$
$-3P-2P = -4-66$
$-5P = -70$
$5P = 70$
$P^* = 14$

$Qd = Qs = 66-3P = 66-3(14) = 66-42 = 24 = Q^*$

(ii) How would a per unit sales tax t affect this equilibrium and comment on how the tax is shared between producers and consumers

Sales tax reduces suppliers price by t (P-t)
Supply curve becomes: $Qs = -4+2(P-t)$
In equilibrium Qd = Qs

tax

$66-3P = -4+2(P-t)$
$66-3P = -4+2P-2t$
$-3P-2P = -4-2t-66$
$-5P = -70-2t$
$5P = 70+2t$
$P = 14+{}^2/_5t$

$Qd = Qs = 66-3P = 66-3(14+{}^2/_5t) = 66-42-{}^6/_5t = 24-{}^6/_5t$

Equilibrium price increases by ${}^2/_5$ of the tax. This implies that the supplier absorbs ${}^3/_5$ of the tax and receives a price $P-{}^3/_5t$ for its goods. The consumer pays ${}^2/_5$ of the tax. Equilibrium quantity falls by ${}^6/_5t$.

(iii) What is the equilibrium P and Q if the per unit tax is t=5

$t = 5$, $Qs = -4+2(P-5) = -4+2P-10 = -14+2P$
In equilibrium Qd = Qs

$66-3P = -14+2P$
$-5P = -14-66$
$-5P = -80$
$5P = 80$
$P = 16$ (i.e. $14+{}^2/_5t$)

$Qd = Qs = 66-3P = 66-3(16) = 18$ (i.e. $24-{}^6/_5t$)

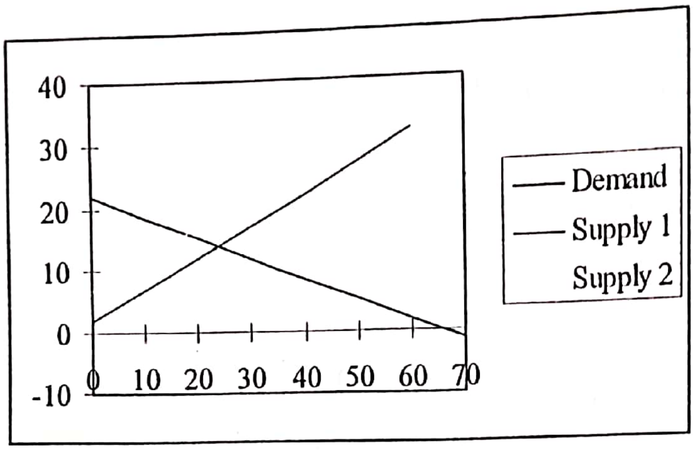
(iv) Illustrate the pre-tax equilibrium and the post-tax equilibrium on a graph

$Qd = 66-3P$ $\qquad$ $Qs = -4+2P$

Let P = 0
Qd = 66
P = 22-Qd/3 (Inverse Demand)
Let Qd = 0
P = 22

Let P = 22
Qs = -4+2(22) = -4+44 = 40
P = 2+Qs/2 (Inverse Supply)
Let Qs = 0
P = 2

Qs = -14+2P
Let P = 22
Qs = -14+2(22) = -14+44 = 30
P = 7+Qs/2
Let Qs = 0
P = 7



Fill in equilibrium before tax, equilibrium after tax, amount paid by consumer, amount paid by producer.

2. The demand and supply functions of a good are given by

Qd = 110-5P

Qs = 6P

where P, Qd and Qs denote price, quantity demanded and quantity supplied respectively.

(i) Find the inverse demand and supply functions

Qd = 110-5P
5P = 110-Qd
P = 110-Qd/5

Qs = 6P
P = Qs/6

(ii) Find the equilibrium price and quantity

Solve simultaneously:
Qd = 110-5P
Qs = 6P

At equilibrium Qd = Qs

110-5P = 6P

Collect the terms
-5P-6P = -110
11P = 110
P = 110/11
P = 10

Solve for Q*
Qd = Qs = 6P = 6(10) = 60 = Q*

3. **Demand and supply in a market are described by the equations**
Qd = 120-8P
Qs = -6+4P

a. **Solve algebraically to find equilibrium P and Q**

Qd = Qs
120-8P = -6+4P
-8P-4P = -6-120
-12P = -126
12P = 126
P* = 10.5

Qd = Qs = 120-8P = 120-8(10.5) = 120-84 = 36 = Q*

b. **How would a per unit sales tax t affect this equilibrium and comment on how the tax is shared between producers and consumers**

Supply price becomes P-t
Supply function becomes Qs = -6+4(P-t)
Solve for equilibrium
Qd = Qs
120-8P = -6+4(P-t)
120-8P = -6+4P-4t
-8P-4P = -120-6-4t
-12P = -126-4t
12P = 126+4t
P = 10.5+4t/12
P = 10.5+t/3

Qd = Qs = 120-8(10.5+t/3) = Q*
Q* = 120-84-8t/3
Q* = 36-8/3t

The impact of the tax will therefore be to increase equilibrium price by 1/3 and reduce equilibrium quantity by 8/3. Since 1/3 of tax is passed on to the consumer the supplier pays 2/3 of the tax.

c. **What is the equilibrium P and Q if the per unit tax is 4.5**

P = 10.5+t/3
P = 10.5+4.5/3
P = 10.5+1.5
P = 12

Supplier gets 10.5-2/3t = 10.5-3 = 7.5

Q = 36-8/3t
Q = 36-8/3(4.5)
Q = 36-12
Q = 24

4. **At a price of €15, and an average income of €40, the demand for CDs was 36. When the price increased to €20, with income remaining unchanged at €40, the demand for CDs fell to 21. When income rose to €60, at the original price €15, demand rose to 40.**

i) Find the linear function which describes this demand behaviour
General Form: Qd = a+bP+cY

P = 15, Qd = 36, Y = 40
P = 15, Qd = 40, Y = 60
P = 20, Qd = 21, Y = 40

Eq1 36 = a+15b+40c
Eq2 40 = a+15b+60c
Eq3 21 = a+20b+40c
Solve Simultaneously

Eq1 36 = a+15b+40c
Eq2 40 = a+15b+60c
STEP 1
a = 36-15b-40c
a = 40-15b-60c
STEP 2
36-15b-40c = 40-15b-60c
STEP 3
-15b+15b-40c+60c = 40-36
20c = 4
c = 4/20 = 1/5
STEP 4
Eq1 36 = a+15b+40(1/5)
36 = a+15b+8
36-8 = a+15b
28 = a+15b

Eq3 21=a+20b+40(1/5)
21-8 = a+20b

$13 = a+20b$

STEP 1

Eq1' $28=a+15b$

Eq2' $13=a+20b$

$a = 28-15b$

$a = 13-20b$

STEP 2

$28-15b = 13-20b$

STEP 3

$-15b+20b = 13-28$

$5b = -15$

$b = -3$

STEP 4

$a = 28-15b$

$a = 28-15(-3)$

$a = 28+45$

$a = 73$

General Form

$Qd = a+bP+cY$

$Qd = 73-3P+1/5Y$

ii) Given the supply function Qs = -7+2P find the equations which describe fully the comparative statics of the model.

$Qd = 73-3P+1/5Y$

$Qs = -7+2P$

In equilibrium $Qd = Qs$

$73-3P+1/5Y = -7+2P$

$-3P-2P = -7-73-1/5Y$

$5P = 80+1/5Y$

$P^* = 16+1/25Y$

$Qd = Qs = -7+2P = -7+2(16+1/25Y) = -7+32+2/25Y = 25+2/25Y = Q^*$

iii) What would equilibrium price and quantity be if income was €50?

$P^* = 16+1/25Y = 16+1/25(50) = 16+2 = 18$

$Q^* = 25+2/25Y = 25+2/25(50) = 25+4 = 29$

Example 2.2

Individual A, B and C's demands are described by the following three demand functions.

$$Q_A = 20 - 2P$$

$$Q_B = 30 - 3P$$

$$Q_C = 40 - 4P$$

i. Compute market demand.
ii. Determine the price at which market demand would be zero.

Solutions

i. Market demand

$$Q_M = Q_A + Q_B + Q_C = 20 - 2P + 30 - 3P + 40 - 4P = 90 - 9P$$

i.e, $Q_M = 90 - 9P$

ii. Set $Q_M = 0$, which follows

$$0 = 90 - 9P$$

$$\Rightarrow 9P = 90$$

$$\Rightarrow P = \frac{90}{9} = 10$$

i.e, at 10 unit price market demand would be zero.

2.4 Supply

The quantity of a good or a service that is sold or ready for sale at a certain price is called supply. Other things remaining unchanged a rise in price causes a rise in supply and vice versa- which is known as the law of supply. Because of direct relationship between price and supply, supply curve slopes upward. In addition to price of a good, input price, technology, weather and many other factors influence the supply of the good.

demand for the good. If X and Y are substitutes then $\frac{\partial Q_x}{\partial P_y} > 0$ because increase in price of Y will motivate the consumer to consume X instead of Y.

Similarly, If Z is complementary to X then $\frac{\partial Q_x}{\partial P_z} < 0$ because increase in price of Z will result in a decreased consumption of X together with Z.

$\frac{\partial Q_x}{\partial M}$ may be positive, negative or zero depending on whether good X is normal, inferior or income neutral.

Example 2.1

Demand for good A, $X_A = 100 - 2P_A + 3P_B - 4P_C + \sqrt{M}$

a) Compute demand assuming $P_A = 10, P_B = 20, P_C = 30$ & $M = 100$

b) Examine the relationship between goods A and C

c) Is good A normal? Why?

Solution

a) Demand for good A,

$$X_A = 100 - (2\times 10) + (3\times 20) - (4\times 30) + \sqrt{100} = 30$$

b) $\frac{\partial X_A}{\partial P_C} = -4 < 0$, i.e., increase in price of C reduces consumption of A, referring complementary relationship between A and C.

c) $\frac{\partial X_A}{\partial M} = \frac{1}{2}M^{-1/2} = \frac{1}{2}\frac{1}{\sqrt{M}} > 0$; This implies an increase in demand for A following an increase in income, hence good A is normal.

Example 2.2

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$$0 = 90 - 9P$$

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Equilibrium

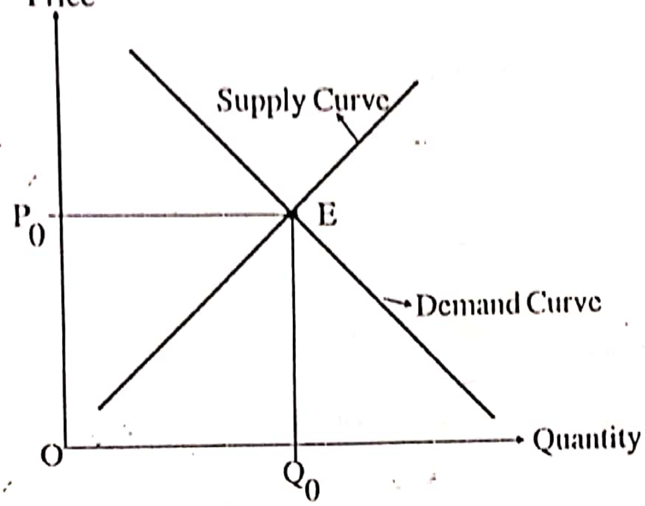

Figure- 2.8

In figure 2.8 demand curve and supply curve intersect at point E which is equilibrium point since demand and supply are equal at this point. Equilibrium price and quantity are OP_0 and OQ_0 respectively. Any price above P_0 will cause excess supply and below P_0 excess demand.

Example 2.4

Assume demand and supply functions

$$Q_d = 20 - 5P$$

$$Q_s = 5P$$

i. Compute equilibrium price and quantity
ii. Show your results in diagram
iii. Explain the nature of disequilibrium assuming separate prices above and below equilibrium price.
iv. Compute the impact of a tax at the rate of 1 dollar per unit. What is the amount of tax burden on consumer?

Solution

i. Equilibrium condition

$$Q_d = Q_s$$

$$\Rightarrow 20 - 5P = 5P$$

$$\Rightarrow 10P = 20$$

$\therefore \quad \overline{P} = 2$ which is equilibrium price.

Setting P=2 into demand and supply equations,

$Q_d = 20 - 5P = 20 - 5 \times 2 = 10$

$Q_s = 5P = 5 \times 2 = 10$

Thus $Q_d = Q_s = \overline{Q} = 10$ which is equilibrium quantity.

ii. Figure 2.9 describes market equilibrium

Market Equilibrium

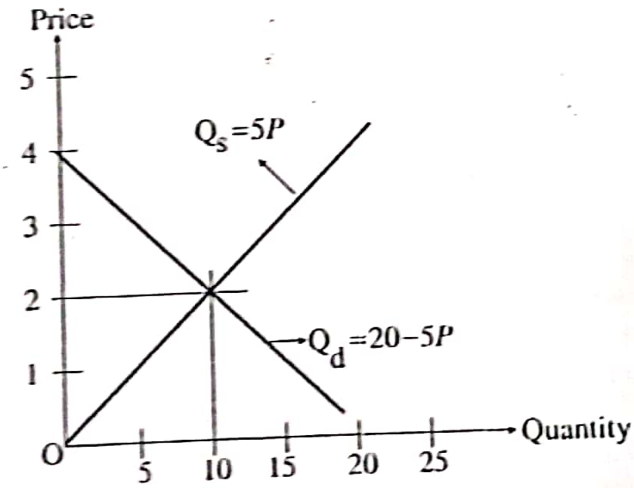

Figure- 2.9

iii. Assume a price above equilibrium

Let P=3, $Q_d = 20 - 5P = 20 - 5\times3 = 5$ and $Q_s = 5P = 5\times3 = 15$. Supply is apparently greater than demand. Amount of excess supply is 15-5=10 unit.

Assume another price P=1.5 which is below equilibrium price. In this case $Q_d = 20 - 5P = 20 - 5\times1.5 = 12.5$ and $Q_s = 5P = 5\times1.5 = 7.5$. Amount of excess demand equals to 12.5-7.5=5 unit.

iv. Imposition of tax alters supply function. Before tax, suppliers used to receive a price equal to P dollar. Upon imposition of tax at the rate of 1 dollar, they receive only (P-1) dollar. After tax supply function turns $Q_s^* = 5(P-1) = 5P - 5$.

Equilibrium condition

$$Q_d = Q_s^*$$

$$\Rightarrow 20 - 5P = 5P - 5$$

$$\Rightarrow 10P = 25$$

$$\Rightarrow \overline{P}^* = 2.5$$

Setting this new price into demand and supply equations, equilibrium quantity is computed $\overline{Q}^* = 7.5$.

Imposition of tax increases equilibrium price and decreases quantity. Earlier, consumers had to pay 2 dollars for each unit of the good but now they have to pay 2.5 dollars.

Tax burden on consumers, (2.5- 2) =0.5 dollar which is 50% of total tax per unit.

Change in Equilibrium via Larger Change in Supply

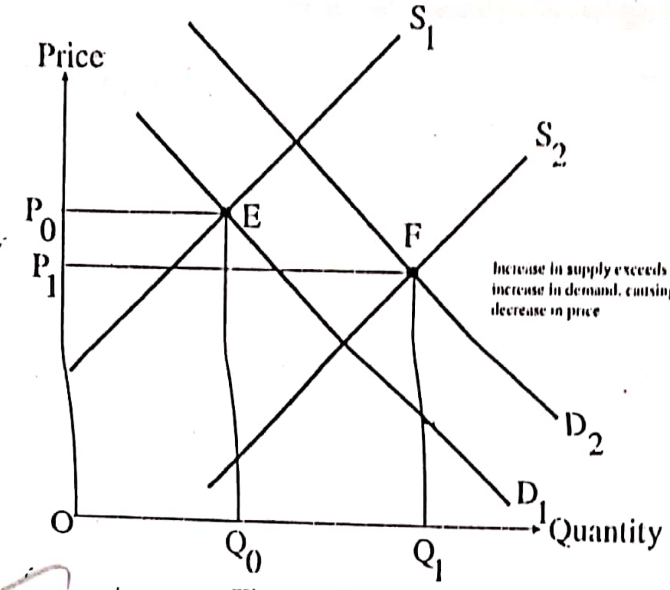

Figure- 2.14

Example 2.5

A market consists of three persons, Abiul (A), Bablu (B) and Chandrima (C), whose demand equations are as follows:

$$A: \quad P = 35 - 0.5Q_A$$

$$B: \quad P = 50 - 0.25Q_B$$

$$C: \quad P = 40 - 2Q_C$$

The industry supply equation is give by $Q_S = 40 + 3.5P$

i. Compute the equilibrium price and quantity
ii. Which individual does purchase the maximum amount?

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Soiution

i. In order to compute equilibrium price and quantity we need market demand which is the sum of individuals' demand. Simplify the demand equations given so that they can be summed.

$A: \quad 0.5Q_A = 35 - P; \Rightarrow Q_A = 70 - 2P \ldots\ldots \quad \ldots\ldots(1)$

$B: \quad 0.25Q_B = 50 - P; \Rightarrow Q_B = 200 - 4P \ldots \quad \ldots\ldots(2)$

$C: \quad 2Q_C = 40 - P; \Rightarrow Q_C = 20 - 0.5P \ldots \quad \ldots\ldots(3)$

Market demand

$$Q_d = Q_A + Q_B + Q_C = 70 - 2P + 200 - 4P + 20 - 0.5P$$

$\therefore Q_d = 290 - 6.5P$

Equilibrium condition $Q_d = Q_s$,

which follows $290 - 6.5P = 40 + 3.5P$

or, $\quad 10P = 250$

$\therefore \quad \overline{P} = 25$

Setting P=25 into market demand and supply equations

$Q_d = 290 - 6.5 \times 25 = 290 - 162.5 = 127.5$

$Q_s = 40 + 3.5 \times 25 = 40 + 87.5 = 127.5$

Setting P=25 into equations (1), (2) and (3) we compute the amounts purchased by three individuals.

Abiul's demand, $Q_A = 70 - 2P = 70 - 2 \times 25 = 20$

Bablu's demand, $Q_B = 200 - 4P = 200 - 4 \times 25 = 100$

Chandrima's demand, $Q_C = 20 - 0.5P = 20 - 0.5 \times 25 = 7.5$

Therefore Bablu purchases the maximum amount.

Exercise 2

1. Graph the demand function $Q_d = 200 - 5P$

2. Find equilibrium price and quantity assuming

   $Q_d = 20 - P^2$

   $Q_s = 5P$

3. Demand and supply functions are $Q_d = 98 - P$ and $Q_s = -2 + 4P$. Determine the impact of a subsidy at the rate of 2 Taka per unit of output.

4. Demand for good A:

   $Q_A = 400 - P_A^2 + 2P_B - 3P_C + \sqrt[3]{M}$. Compute the amount purchased at

   $P_A = 5, P_B = 10, P_c = 15$ & $M = 1000$. What happens to demand for A when price of good C rises to 20? What conclusion can you draw regarding the relationship between goods A and C?

5. a) Find market demand from three individuals' demand functions below

   $Q_1 = 70 - 2P$

   $Q_2 = 100 - P$

   $Q_3 = 20 - 2P$

   Suppose the supply equation is $Q_s = 5P$

   b) What are the amounts of equilibrium price and quantity?

   c) Compute each individual's demand in equilibrium.

T = Tastes and preferences of the individual consumer

A = Advertising expenditure made by the producers of the commodity

For many purposes in economics, it is useful to focus on the relationship between quantity demanded of a good and its own price, while keeping other determining factors such as income prices of other goods, tastes and preferences constant. With this we write the demand function of an individual in the following way:

$$Q_x = f(P_x) \qquad \text{...(2)}$$

This implies that quantity demanded of a good X is function of its own price, other determinants remaining constant. As has been explained above, there is inverse relationship between price of a commodity and its quantity demanded. Thus, when price of a commodity falls, its quantity demanded will increase and when its price rises, its quantity demanded will decrease. Therefore, when we express this relationship through a curve we get a downward-sloping demand curve of a commodity as shown in Fig. 7.1. Thus, a demand curve is a graphic representation of only a part of the demand function with price as the only independent variable.

It should be noted that when there is a change in the other determining factors which are held constant such as income, tastes, prices of related commodities, the whole demand curve will shift. For example, if income increases, the whole demand curve will shift to the right and, on the contrary, if income decreases, then the whole demand curve shifts to the left. Similarly, changes in other determining factors such as tastes, prices of related commodities, advertising cause shift in the demand curve and are therefore called *shift factors*.

The individual's demand function in (2) above is a *general* functional form and does not show how much quantity demanded of a consumer will change following a unit change in price (P_x). For the purpose of actually estimating demand for a commodity we need a specific form of the demand function. Generally, demand function is considered to be of a *linear* form. The specific demand function of a linear form is written as

$$Q_x = a - bP_x \qquad \text{...(3)}$$

where a is a constant intercept term on the X-axis and b is the coefficient showing the slope of the demand curve. If on estimating the demand function (3) from the information about monthly quantities demanded of sugar at its various prices by an individual consumer, we find the constant a to be equal to 12 and the constant b to be equal to 2, we can write individual's demand function as

$$Q_x = 12 - 2P_x$$

This is interpreted as *one rupee fall in price of sugar will cause its quantity demanded to increase by 2 units of sugar.*

Market Demand Function

A market consists of several individuals. Market demand function is obtained by summing up the demand functions of the individuals constituting the market.

Example 1

A market for a commodity consists of three individuals A, B and C whose demand functions for the commodity are given below. Find out the market demand function.

$$Q_A = 40 - 2P$$

$$Q_B = 25.5 - 0.75P$$

$$Q_C = 36.5 - 1.25P$$

When individual demand functions are expressed as *'quantity as a function of price'* as is the case in our problem stated above, market demand function can be obtained by summing up the individual demand functions. Thus, market demand function is

$$Q_M = Q_A + Q_B + Q_C$$

$$= (40 - 2P) + (25.5 - 0.75P) + (36.5 - 1.25P)$$

$$= (40 + 25.5 + 36.5) - (2 + 0.75 + 1.25)P$$

$$= 102 - 4P$$

However, note that when individual demand functions are expressed as *"price as function of quantity"*, then in order to obtain the market demand, they have to be first converted into 'quantity as function of price'.

Example 2

Suppose a market consists of three consumers, A, B and C whose individual demand functions are given below:

(A) : $P = 35 - 0.5Q_A$

(B) : $P = 50 - 0.25Q_B$

(C) : $P = 40 - 2.00Q_C$

(i) *Find out the market demand function for the commodity.*

(ii) *If the market supply function is given by $Q_S = 40 + 3.5P$, determine the equilibrium price and quantity*

Since the individual demand functions are expressed as 'price as function of quantity, that is, we are given *"inverse demand functions"* we have first to transform them into 'quantity demanded as function of price'. Transforming them yields the following demand functions:

$$Q_A = 70 - 2P$$

$$Q_B = 200 - 4P$$

$$Q_C = 20 - 0.5P$$

Market demand function :

$$Q_D = (70 - 2P) + (200 - 4P) + (20 - 0.5$$

$$Q_D = 70 + 200 + 20 - (2 + 4 + 0.5)P$$

$$= 290 - 6.5P$$

Market supply function $(Q_S) = 40 + 3.5P$

In equilibrium,

$$Q_D = Q_S$$

$$290 - 6.5P = 40 + 3.5P$$

$$10P = 250$$

$$P = \frac{250}{10} = 25$$

Substituting the equilibrium value of price in the demand function equation, we have

$$Q = 290 - 6.5(25)$$

$$= 290 - 162.5$$

$$= 127.5$$

Thus, the equilibrium price is Rs. 25 and equilibrium quantity is 127.5 units.

QUESTIONS FOR REVIEW

1. Define demand for a commodity. Explain the various factors which determine demand for a commodity.
2. Demand for a commodity refers to :
   (*a*) desire for a commodity (*b*) need for a commodity
   (*c*) desire for a commodity backed by ability to pay for it.
   (*d*) ability to pay for a commodity.
3. What is meant by *Ceteris Paribus* ? What factors are covered under *Ceteris Paribus* condition in relation to demand for a commodity ?
4. You are given the following demand function for a commodity :