Machine Learning

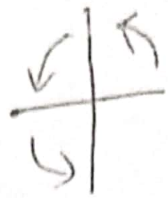
Algebra:

Date 21.8.23

- Matrix, Vector

↓

set of basis vectors



$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{\text{rotate}} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

* Linear transformation

→ vector "

to coordinate system

$\boxed{x \cdot x^T}$ → Possible

* $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

* Linear Regression → mathematical form

$$h(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$

$$= \sum_{0}^{n} \theta_i x_i = \boxed{\theta^T x}$$

$$\text{error} = \sum \{h(x) - h'(x)\}^2 \rightarrow = |h(x) - h'(x)|$$

Why square →

not square → parabola shape

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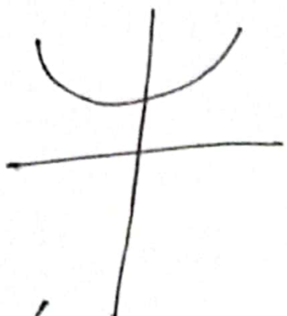
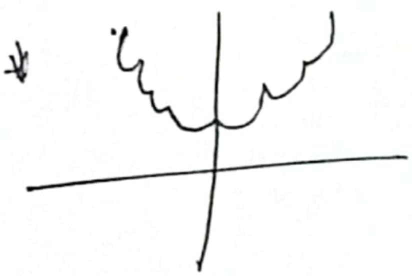
error $= \sum |h(x) - h'(x)|$



$$= \sum \{h(x) - h'(x)\}^2$$



Local min/max

* $\sum (h(x) - h'(x))^2 = 0$

$\rightarrow \sum (y - \theta_0 - \theta_1 x_1)^2 = 0$

$\rightarrow \frac{d}{d\theta_0} \sum (y - \theta_0 - \theta_1 x_1)^2 = 0$

$\rightarrow 2 \sum (y - \theta_0 - \theta_1 x_1) \cdot (-1) = 0$

$\rightarrow \sum y - \sum_{1}^{n} \theta_0 - \sum \theta_1 x_1 = 0$

$\rightarrow \sum y - n\theta_0 - \theta_1 \sum x_1 = 0$

$\therefore \theta_0 = \frac{\sum y - \theta_1 \sum x_1}{n}$

$= \boxed{\bar{y} - \theta_1 \bar{x}}$

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$$\sum (y - \theta_0 - \theta_1 x_1)^2 = 0$$

$$\frac{d}{d\theta_1} \sum (y - \theta_0 - \theta_1 x_1)^2 = 0$$

$$\rightarrow 2\sum (y - \theta_0 - \theta_1 x_1) \cdot (-x_1) = 0$$

$$\cancel{\rightarrow \sum y x_1 - \sum \theta_0 x_1 - \sum \theta_1 x_1^2 = 0}$$

$$\cancel{\rightarrow \sum y x_1 -}$$

$$\rightarrow \sum \left(\left(y - (\bar{y} - \theta_1 \bar{x}) - \theta_1 x_1 \right) x_1 \right) = 0$$

$$\rightarrow \sum \left(y x_1 - \bar{y} x_1 + \theta_1 \bar{x} x_1 - \theta_1 x_1^2 \right) = 0$$

$$\rightarrow \sum y x_1 - \sum \bar{y} x_1 + \sum \theta_1 \bar{x} x_1 - \sum \theta_1 x_1^2 = 0$$

$$\rightarrow \sum y x_1 - \sum \bar{y} x_1 + \theta_1 \left(\sum \bar{x} x_1 - \sum x_1^2 \right) = 0$$

$$\therefore \theta_1 = \frac{\sum (y x_1 - \bar{y} x_1)}{\cancel{\sum (\bar{x} x_1 - x_1^2)} \quad \sum (x_1^2 - \bar{x} x_1)}$$

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$$\therefore \theta_1 = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}$$

* Probability — distribution

H.W find the derivatives of the following formula : (chain Rule)

1) $f(z) = \log_e (1 + z)$

where $z = x^T x$, $x \in \mathbb{R}^d$

$d \rightarrow$ dimension

11) $f(z) = e^{-z/2}$

where $z = g(y)$, $g(y) = y^T S^{-1} y$

$y = h(x)$, $h(x) = x - \mu$

drive:

* Lab → prior knowledge → py library.

→ example