Artificial Intelligence

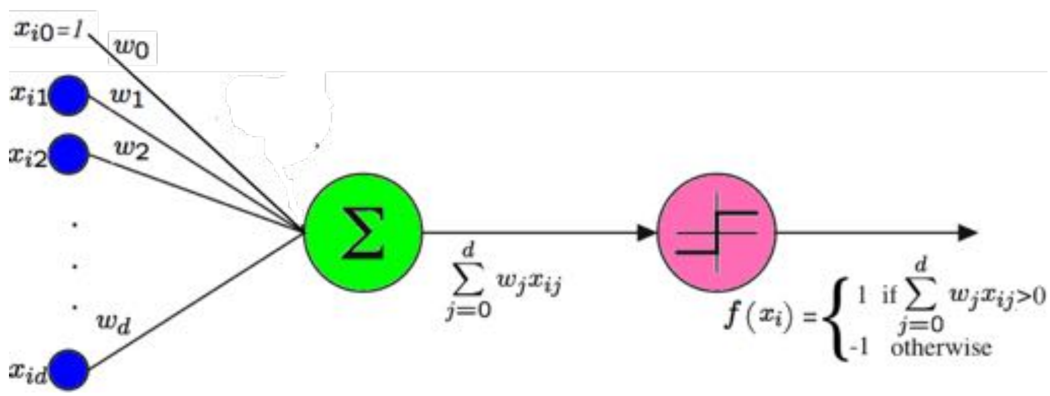
Machine Learning

Neural Networks

Neural Networks

- Algorithms that try to mimic how the brain functions.
- Worked extremely well to recognize:
  1. handwritten characters (LeCun et a. 1989),
  2. spoken words (Lang et al. 1990),
  3. faces (Cottrel 1990)
- Extensively studied in the 1990's with a moderate success.
- Now back with lots of success with deep learning thanks to the algorithmic and computational progress.
- The first algorithm used was the Perceptron (Resenblatt 1959).

Perceptron



Given n examples and d features.

$$f(x_i) = sign(\sum_{j=0}^{d} w_j x_{ij})$$

Perceptron expressiveness

- Consider the perceptron with the step function.
- Idea: Iterative method that starts with a random hyperplane and adjust it using your training data.
- It can represent Boolean functions such as AND, OR, NOT but not the XOR function.
- It produces a linear separator in the input space.

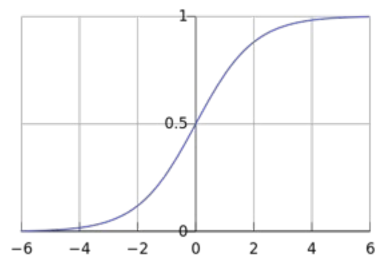
From perceptron to MLP

- The perceptron works perfectly if data is linearly separable. If not, it will not converge.
- Neural networks use the ability of the perceptrons to represent elementary functions and combine them in a network of layers of elementary questions.
- However, a cascade of linear functions is still linear,
- and we want networks that represent highly non-linear functions.

From perceptron to MLP

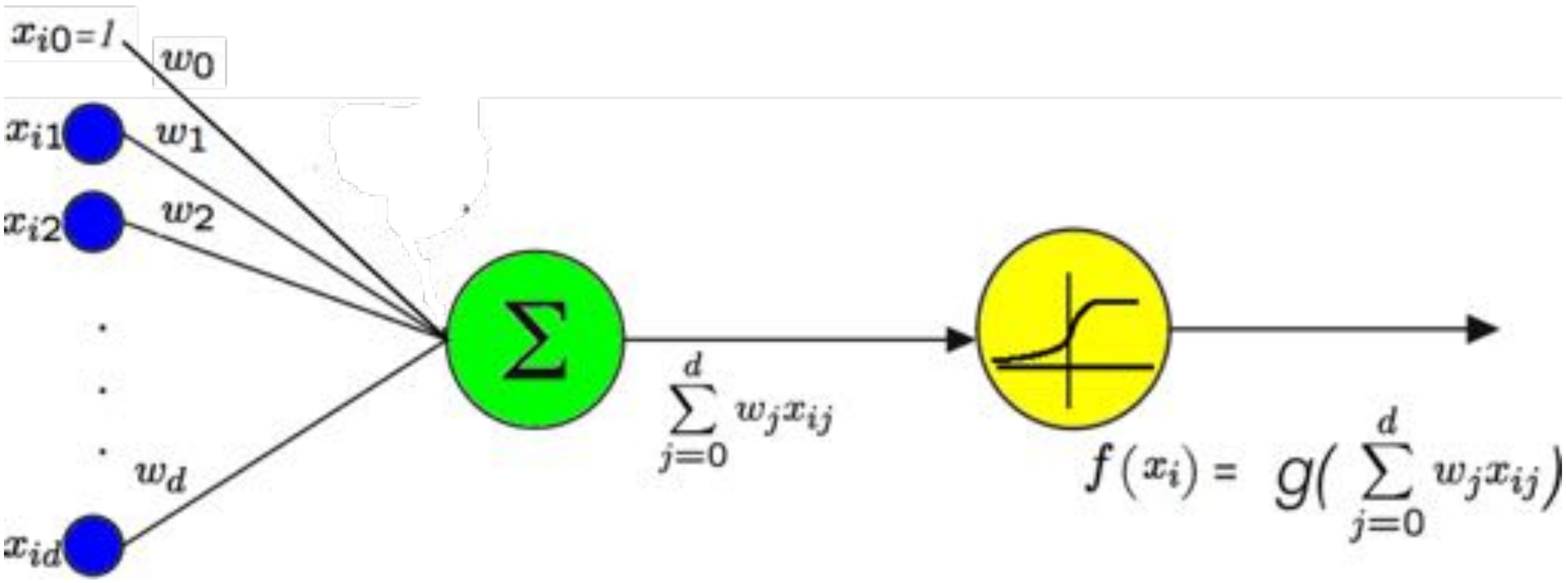
- Also, perceptron used a **threshold function**, which is undifferentiable and not suitable for gradient descent in case data is not linearly separable.
- We want a function whose output is a linear function of the inputs.
- One possibility is to use the sigmoid function:

$$g(z) = \frac{e^z}{1+e^z} = \frac{1}{1+e^{-z}}$$



$g(z) \to 1$ when $z \to +\infty$ $\quad$ $g(z) \to 0$ when $z \to -\infty$

Perceptron with Sigmoid



Given n examples and d features.

For an example x_i (the i^{th} line in the matrix of examples)

$$f(x_i) = \frac{1}{1 + e^{-\sum_{j=0}^{d} w_j x_{ij}}}$$

The XOR example

Let's try to create a MLP for the XOR function using elementary perceptrons.

The XOR example

Let's try to create a NN for the XOR function using elementary perceptrons.

First observe:

$$g(z) = \frac{1}{1 + e^{-z}}$$

$$g(10) = 0.99995$$

$$g(-10) = 0.00004$$

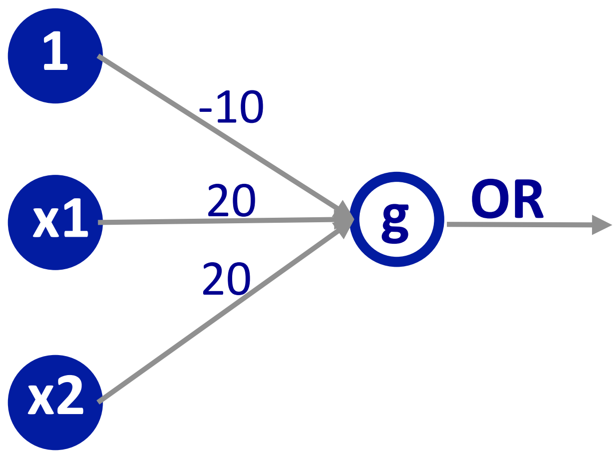
Let's consider that: For $z \geq 10$, $g(z) \rightarrow 1$. For $z \leq -10$, $g(z) \rightarrow 0$.

The XOR example

First what is the perceptron of the OR?

The XOR example

x_1	x_2	x_1 OR x_2	$g(z)$
0	0	0	$g(w_0 + w_1x_1 + w_2x_2) = g(-10)$
0	1	1	$g(10)$
1	0	1	$g(10)$
1	1	1	$g(10)$



The XOR example

Similarly, we obtain the perceptrons for the AND and NAND:

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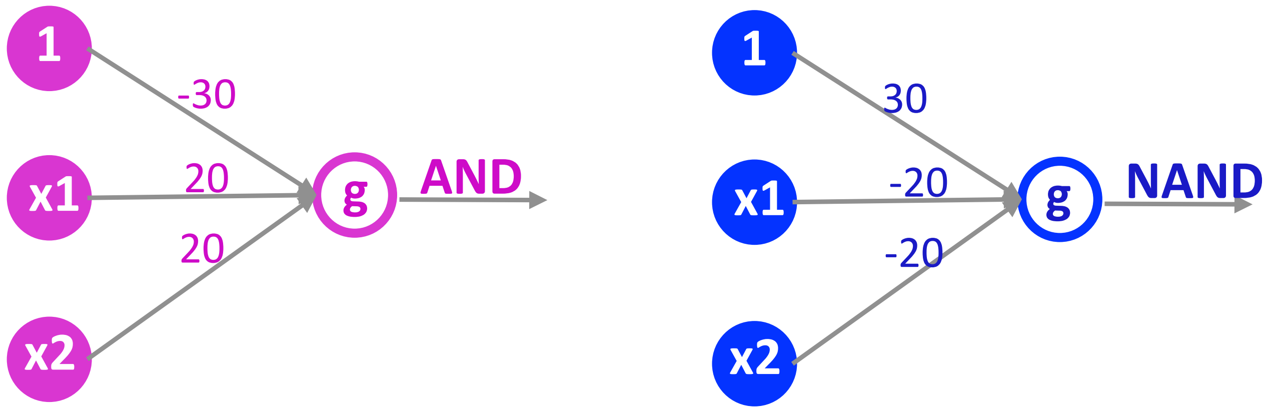

Note: how the weights in the NAND are the inverse weights of the AND.

The XOR example

Let's try to create a NN for the XOR function using elementary perceptrons.

x_1	x_2	x_1 **XOR** x_2	$(x_1$ **OR** $x_2)$ **AND** $($ x_1 **NAND** $x_2)$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	0

The XOR example

Let's put them together...

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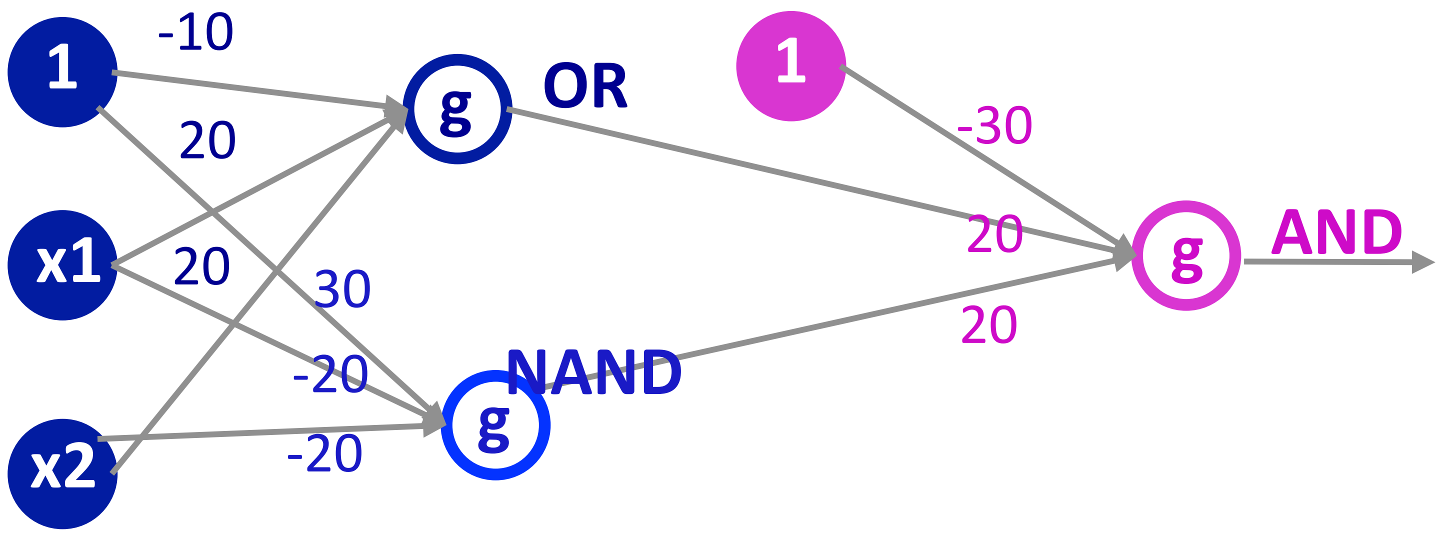
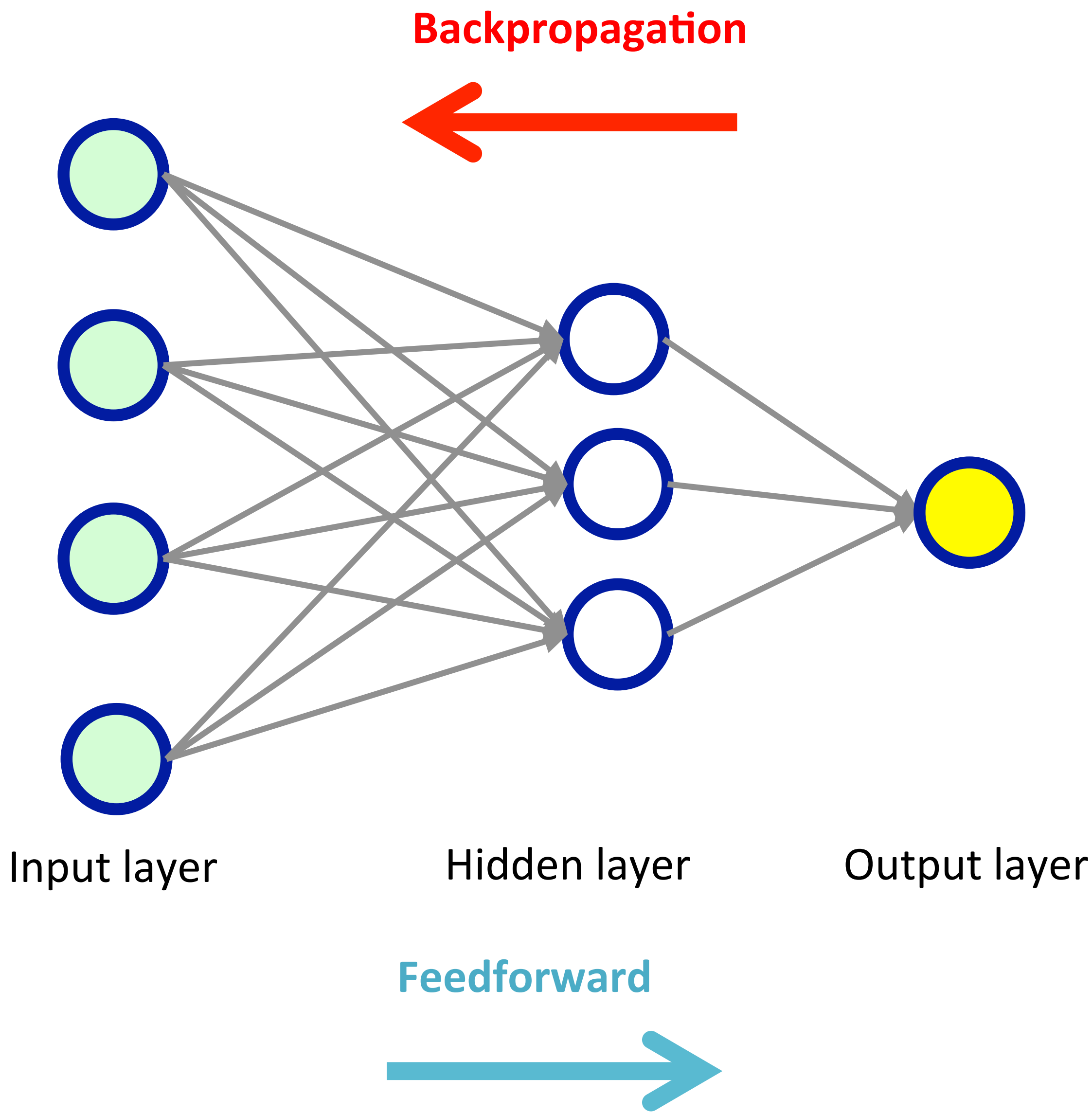


XOR as a combination of 3 basic perceptrons.

Backpropagation algorithm

- Note: Feedforward NN (as opposed to recurrent networks) have no connections that loop.
- Learn the weights for a multilayer network.
- Backpropagation stands for "backward propagation of errors".
- Given a network with a fixed architecture (neurons and interconnections).
- Use Gradient descent to minimize the squared error between the network output value o and the ground truth y.
- We suppose multiple output k.
- Challenge: Search in all possible weight values for all neurons in the network.

Feedforward-Backpropagation



Backpropagation rules

- We consider k outputs
- For an example e defined by (x, y), the error on training example e, summed over all output neurons in the network is:

$$E_e(w) = \frac{1}{2} \sum_k (y_k - o_k)^2$$

- Remember, gradient descent iterates through all the training examples one at a time, descending the gradient of the error w.r.t. this example.

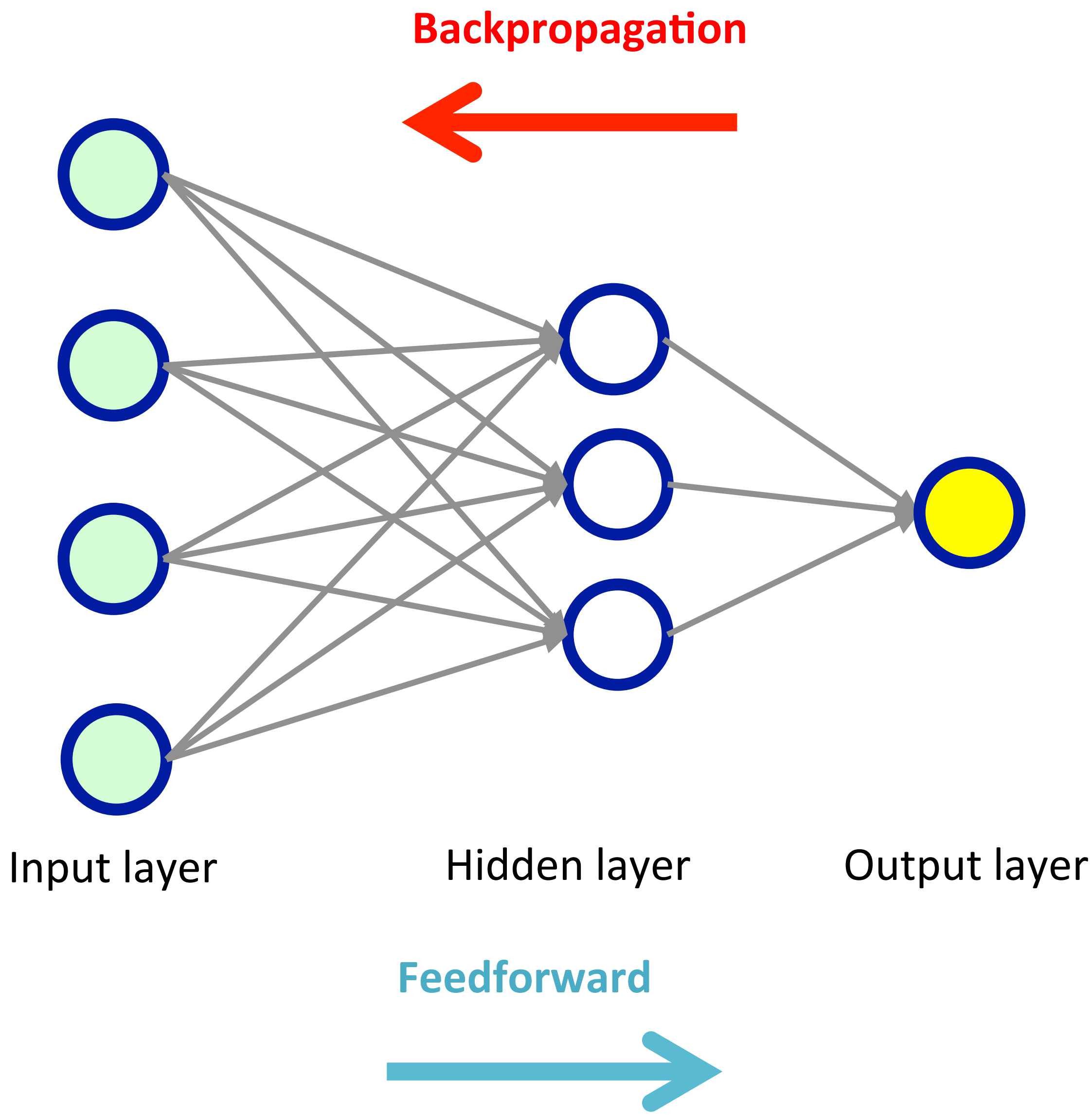
$$\Delta w_{ij} = -\alpha \, \frac{\partial E_e(w)}{\partial w_{ij}}$$

Backpropagation rules

Notations:

- x_{ij}: the i^{th} input to neuron j.
- w_{ij}: the weight associated with the i^{th} input to neuron j.
- $z_j = \sum w_{ij} x_j$, weighted sum of inputs for neuron j.
- o_j: output computed by neuron j.
- g is the sigmoid function.
- $outputs$: the set of neurons in the output layer.
- $Succ(j)$: the set of neurons whose immediate inputs include the output of neuron j.

Backpropagation notations



Backpropagation rules

$$\frac{\partial E_e}{\partial w_{ij}} = \frac{\partial E_e}{\partial z_j}\ \frac{\partial z_j}{\partial w_{ij}} = \frac{\partial \mathbf{E_e}}{\partial \mathbf{z_j}}\ x_{ij}$$

$$\Delta w_{ij} = -\alpha\ \frac{\partial \mathbf{E_e}}{\partial \mathbf{z_j}}\ x_{ij}$$

We consider two cases in calculating $\frac{\partial E_e}{\partial z_j}$ (let's abandon the index e):

- **Case 1: Neuron j is an output neuron**
- **Case 2: Neuron j is a hidden neuron**

Backpropagation rules

- **Case 1: Neuron j is an output neuron**

$$\frac{\partial E}{\partial z_j} = \frac{\partial E}{\partial o_j} \frac{\partial o_j}{\partial z_j}$$

Backpropagation rules

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$$\frac{\partial E}{\partial o_j} = \frac{\partial}{\partial o_j}\frac{1}{2}\sum_k (y_k - o_k)^2$$

$$\frac{\partial E}{\partial o_j} = \frac{\partial}{\partial o_j}\frac{1}{2}(y_j - o_j)^2$$

$$\frac{\partial E}{\partial o_j} = \frac{1}{2}\ 2\ (y_j - o_j)\frac{\partial (y_j - o_j)}{\partial o_j}$$

$$\frac{\partial E}{\partial o_j} = -(y_j - o_j)$$

Backpropagation rules

- **Case 1: Neuron j is an output neuron**

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We have: $o_j = g(z_j)$

$$\frac{\partial o_j}{\partial z_j} = \frac{\partial g(z_j)}{\partial z_j}$$

$$\frac{\partial o_j}{\partial z_j} = o_j(1 - o_j)$$

Backpropagation rules

$$\frac{\partial E}{\partial z_j} = -(y_j - o_j)o_j(1 - o_j)$$

$$\Delta w_{ij} = \alpha(y_j - o_j)o_j(1 - o_j)x_{ij}$$

We will note

$$\delta_j = -\frac{\partial E}{\partial z_j}$$

$$\Delta w_{ij} = \alpha \ \delta_j \ x_{ij}$$

Backpropagation rules

- **Case 2: Neuron j is a hidden neuron**

$$\frac{\partial E}{\partial z_j} = \sum_{k \in succ\{j\}} \frac{\partial E}{\partial z_k} \frac{\partial z_k}{\partial z_j} = \sum_{k \in succ\{j\}} -\delta_k \frac{\partial z_k}{\partial z_j}$$

$$\frac{\partial E}{\partial z_j} = \sum_{k \in succ\{j\}} -\delta_k \frac{\partial z_k}{\partial o_j} \frac{\partial o_j}{\partial z_j}$$

$$\frac{\partial E}{\partial z_j} = \sum_{k \in succ\{j\}} -\delta_k \; w_{jk} \frac{\partial o_j}{\partial z_j}$$

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Backpropagation algorithm

Input: training examples (x, y), learning rate α (e.g., $\alpha = 0.1$), n_i, n_h and n_o.

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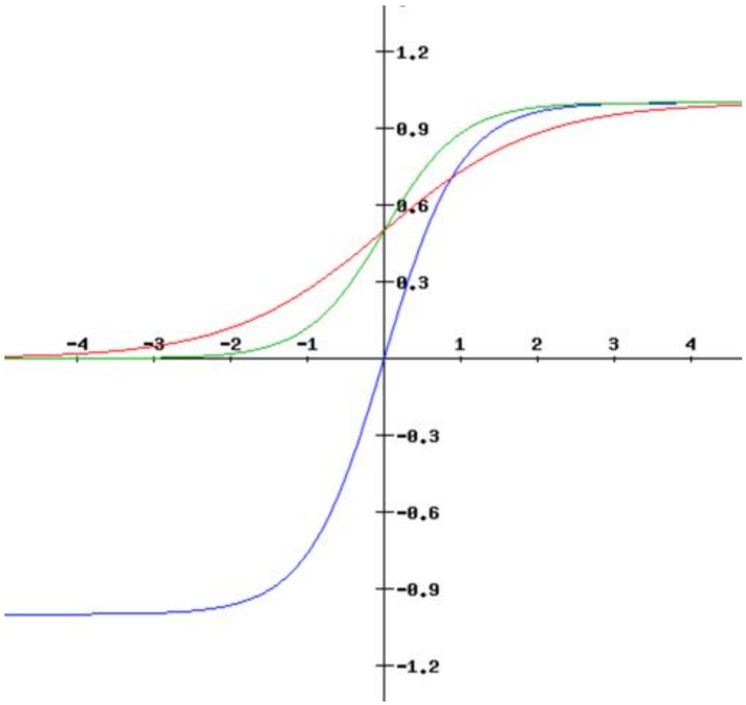
      iii. Update each weight $w_{ij} \leftarrow w_{ij} + \alpha \delta_j x_{ij}$

Observations

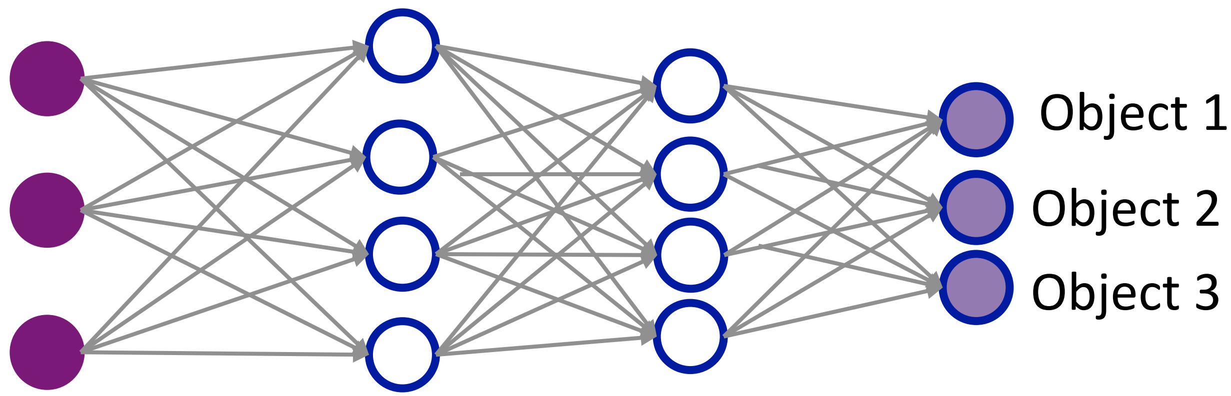
- Convergence: small changes in the weights
- There are other activation functions. Hyperbolic tangent function, is practically better for NN as its outputs range from -1 to 1.



$g(x) = sigmoid(x) = \frac{e^{kx}}{1+e^{kx}}$ for $k = 1$, $k = 2$, etc.

$g(x) = tanh(x) = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$ (It is a rescaling of the logistic sigmoid function!).

Multi class case etc.



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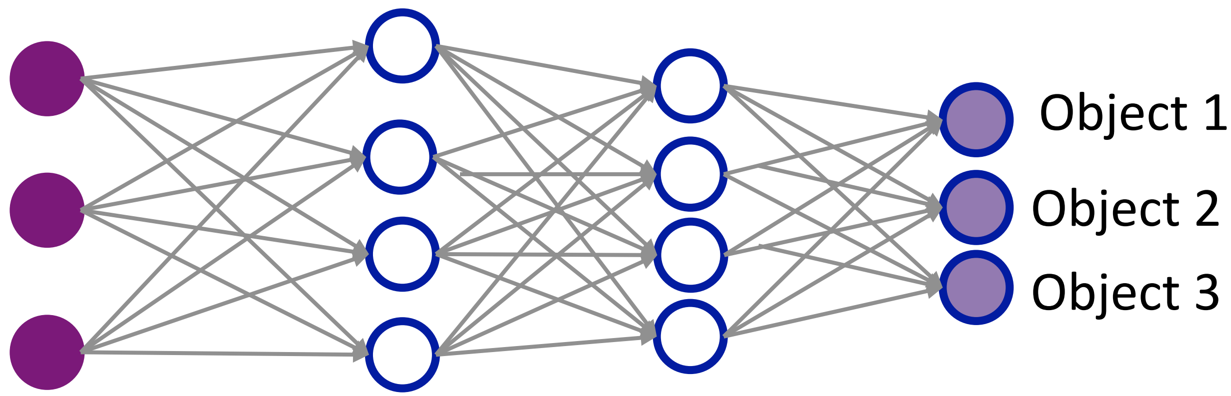
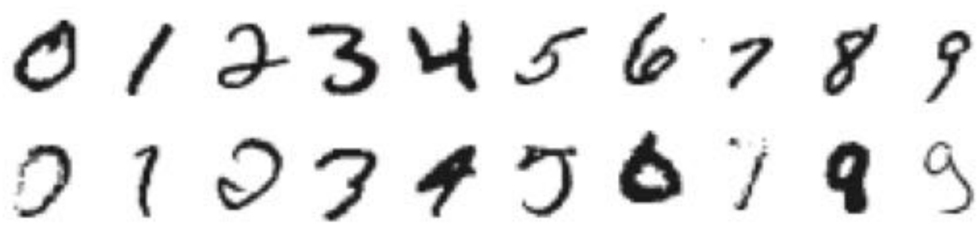


- Nowadays, networks with more than two layers, *a.k.a.* deep networks, have proven to be very effective in many domains.
- Examples of deep networks: restricted Boltzman machines, convolutional NN, auto encoders, etc.

MNIST database

http://yann.lecun.com/exdb/mnist/

- The MNIST database of handwritten digits
- Training set of 60,000 examples, test set of 10,000 examples
- Vectors in $\mathbb{R}^{784}$ (28x28 images)
- Labels are the digits they represent
- Various methods have been tested with this training set and test set



- Linear models: 7% - 12% error
- KNN: 0.5%- 5% error
- Neural networks: 0.35% - 4.7% error
- Convolutional NN: 0.23% - 1.7% error

Demo: Tensorflow

http://playground.tensorflow.org/

- Open source software to play with neural networks in your browser.
- The dots are colored orange or blue for positive and negative examples.
- It's possible to choose the activation function, architecture, rate etc.
- Very well done! Let's check it out!

Credit

- Machine Learning 1997. T. Mitchell.
- Andrew Ng's lecture notes.
- http://playground.tensorflow.org/