

18-09-23 (ML)

Loss function - LMS function.

Least Mean Square

$$h(x) = \theta_0 + \theta_1 x_1$$

$\nearrow$   
Predicted.

$y \rightarrow$  Actual

$$\text{Loss} = |h(x) - y| \times \text{---}$$

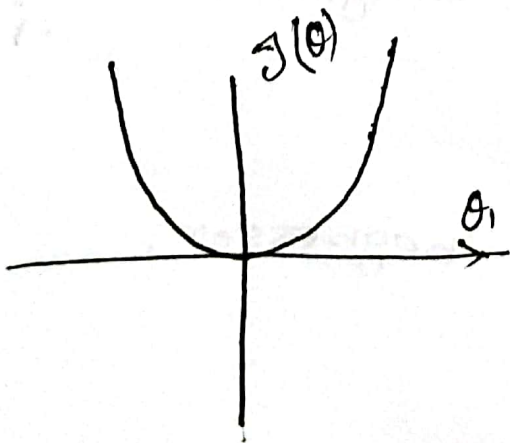
$$\text{Loss} = (h(x) - y)^2 \quad \text{---}$$

$$= (\theta_0 + \theta_1 x_1 - y)^2$$

$$y = a + bx$$

$$a = \frac{[(\sum y)(\sum x^2) - (\sum x)(\sum xy)]}{[n(\sum x^2) - (\sum x)^2]}$$

$$b = \frac{[n(\sum xy) - (\sum x)(\sum y)]}{[n(\sum x^2) - (\sum x)^2]}$$



Learning Rate.

$$\begin{aligned} \theta_1 &= \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1) \\ &= \theta_1 - \alpha (+ve) \\ &= \theta_1 + (\dots) \end{aligned}$$

Optimox

$$\theta := \theta - \alpha \frac{d}{d\theta} J(\theta_0, \theta_1)$$

Partial derivative

$$\text{For } \theta_0 \rightarrow \theta_1 := \theta_0 - \alpha \frac{1}{2} \frac{d}{d\theta_0} (h(x) - y)^2$$

$$= \theta_0 - \alpha \frac{1}{2} \frac{d}{d\theta_0} (\theta_0 + \theta_1 x - y)^2$$

$$= \theta_0 - \alpha (\theta_0 + \theta_1 x - y) \cdot 1$$

$$= \theta_0 - \alpha (h(x) - y) \cdot x$$

$$h(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \dots + \theta_j x_j$$

Repeat until Convergence {

$$\theta_j = \theta_j - \alpha (h(x) - y) x_j$$

}

ML. Linear Regression. plot diagram.

ipython

why change and update.

classification and logistic regression.

ML

25-09-2023

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$$

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \end{bmatrix}$$

$x_1, x_2, x_3, \dots$

$$h(x) = \theta^T x$$

$$h(x) = \theta_0 + \theta_1 x + \theta_2 x + \dots$$

$$J(\theta) = \frac{1}{2m} \sum (h(x) - y)^2$$

$$\approx \frac{1}{2m} (X\theta - y)^T (X\theta - y)$$

$$\frac{d}{d\theta} \left\{ (X\theta)^T X\theta - (X\theta)^T y - y^T (X\theta) + y^T y \right\}$$

$$2X^T X \theta - 2X^T y = 0$$

$$\theta = \theta - \alpha \frac{J}{d\theta} (h(x) - y)$$

$$\theta = (X^T X)^{-1} X^T y$$

$$h(x) = \theta^T x$$

Optimox

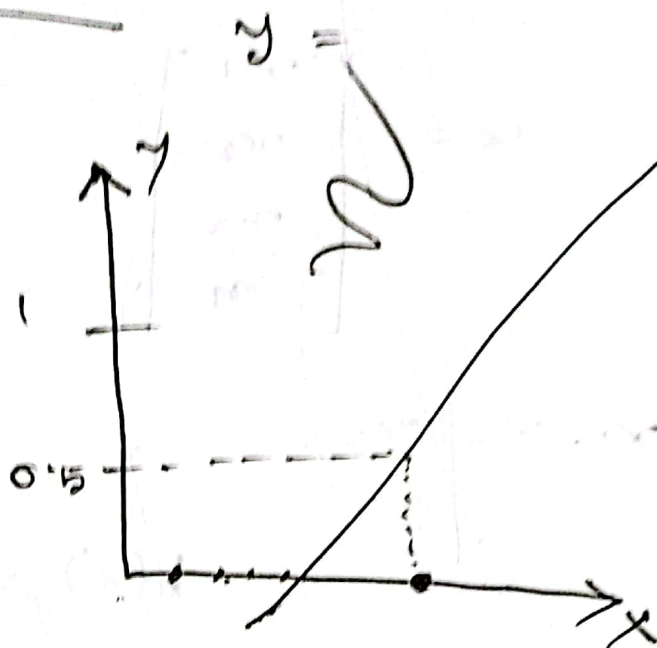
# Classification

$$y = (-\infty, \infty)$$

$$h(x) = \theta_0 + \theta_1 x$$

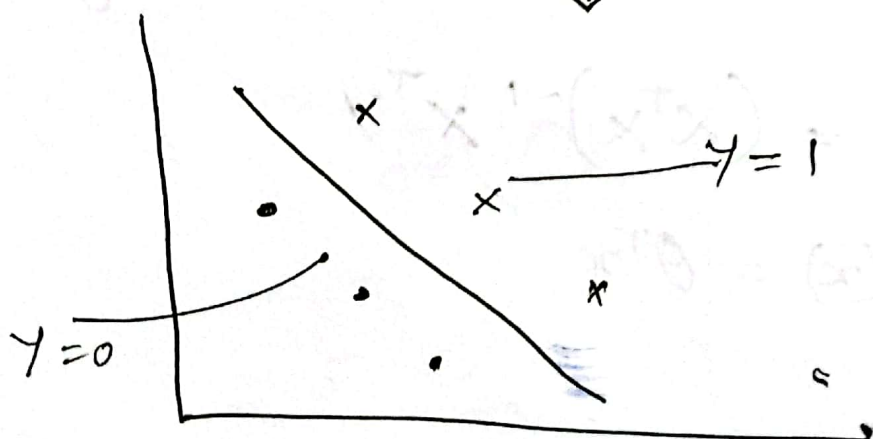
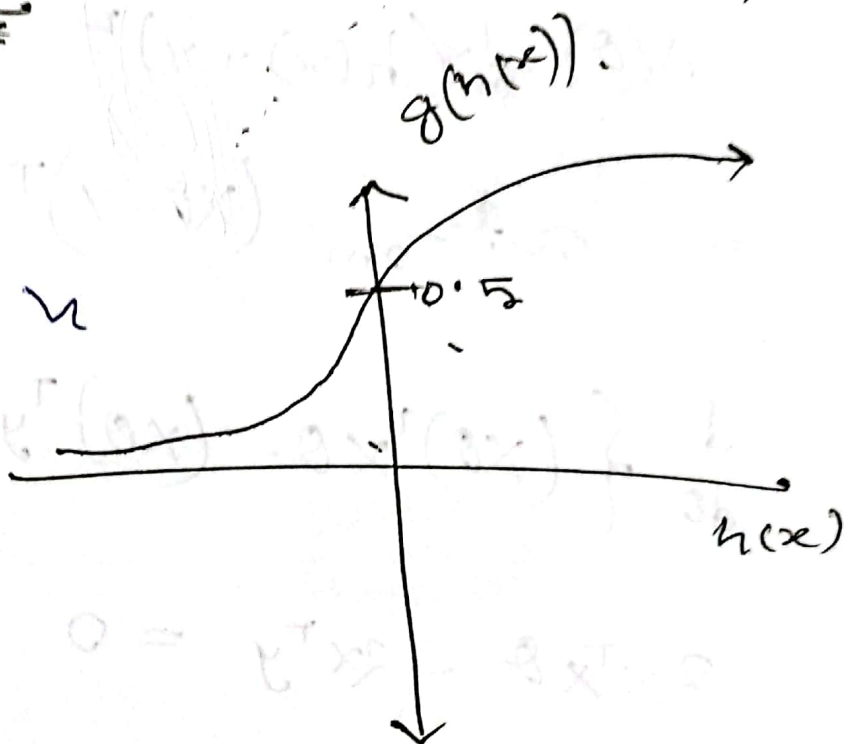
$$g(h(x)) = \frac{1}{1 + e^{-h(x)}}$$

$$g(z) = \frac{1}{1 + e^{-z}}$$



For  $y = \{0, 1\}$

$x_1$	$x_2$
0.5	1.5
1	1
1.5	0.5
3	0.5
2	2
1	2.5





# \* Logistic Regression

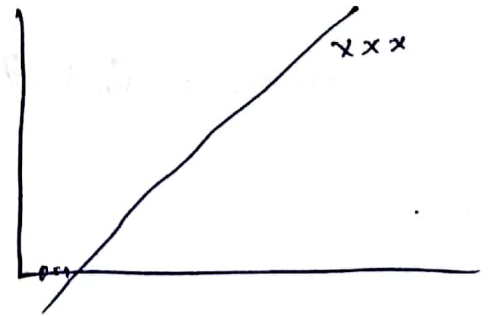
$$y = h(x) = \theta^T x = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \dots$$

$$y = \{0, 1\}$$

# Logistic function.

\* Sigmoid function.

$$\theta(z) = \frac{1}{1 + e^{-z}}$$



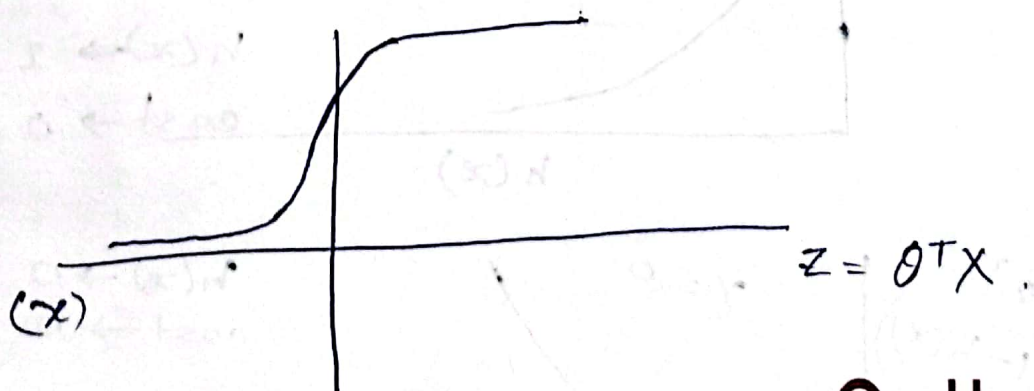
$$h(x) = \theta_0 + \theta_1 x$$

→ predicted hypothesis

$$y = \{0, 1\}$$

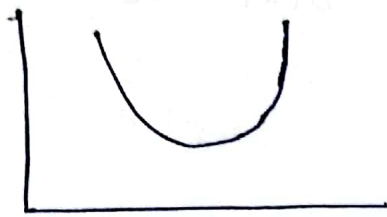
$$0 \leq h(x) \leq 1$$

$$h(x) = \frac{1}{1 + e^{-\theta^T x}}$$



Optimox

$$J(\theta) = \frac{1}{2m} \sum \left( h(x) - y \right)^2$$



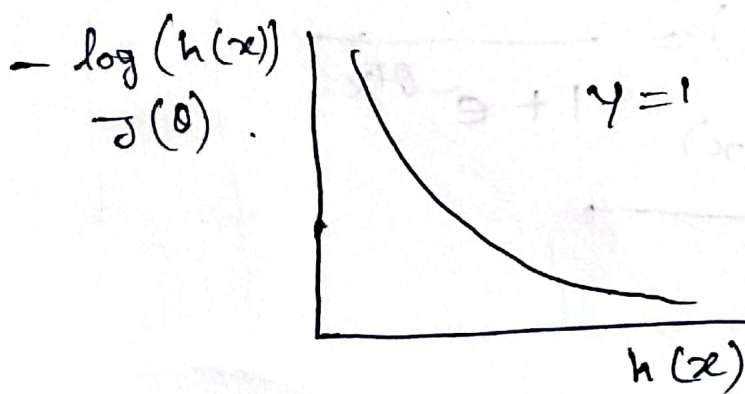
$$h(x) = \theta_0 + \theta_1 x$$



$$\log_e e^x$$

$$J(\theta) = \begin{cases} -\log(h(x)) & ; y = 1 \\ -\log(1-h(x)) & ; y = 0 \end{cases}$$

cost function.

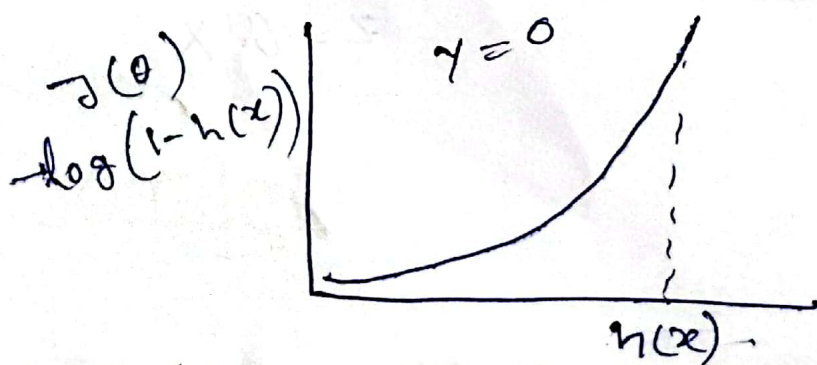


$$h(x) \rightarrow 0$$

$$\text{cost} \rightarrow \infty$$

$$h(x) \rightarrow 1$$

$$\text{cost} \rightarrow 0$$



$$h(x) \rightarrow 0$$

$$\text{cost} \rightarrow 0$$

$$h(x) \rightarrow 1$$

$$\text{cost} \rightarrow \infty$$

Merge two equation:

$$J(\theta) = -y \log(h(x)) - (1-y) \log(1-h(x))$$

$$y = 1; \quad J(\theta) = -\log(h(x))$$

$$y = 0; \quad J(\theta) = -\log(1-h(x))$$

⇒ skewed graph; ⇒

⇒ dataset observe:

⇒ threshold may be not always 0.5

⇒ May 0.95

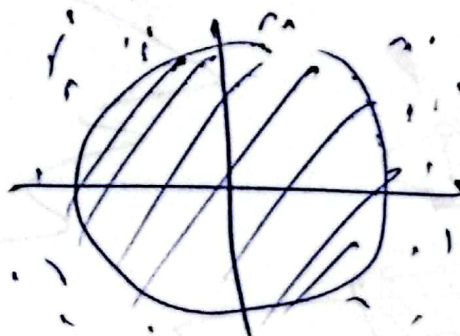
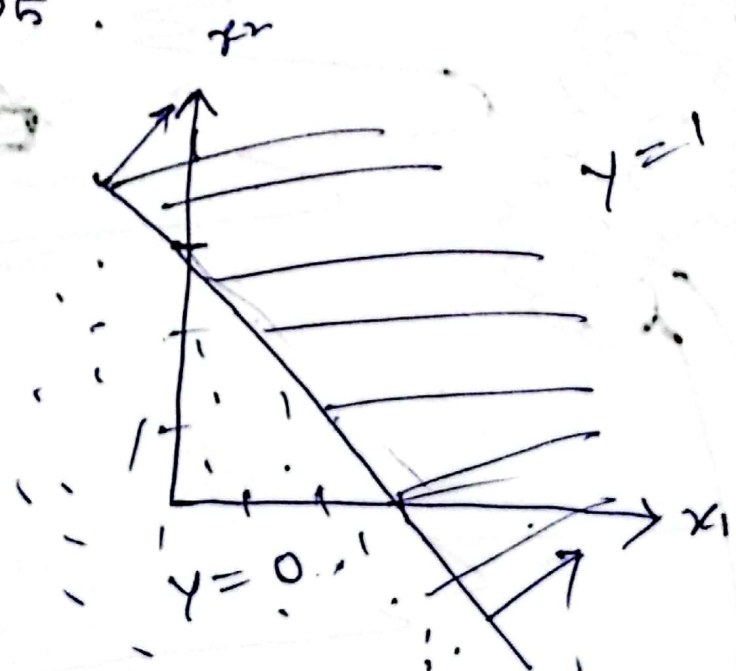
$$0 \leq h(x) \leq 1$$

$$h(x) \geq 0.5$$

$$\theta^T x \geq 0$$

$$-3 + x_1 + x_2 \geq 0$$

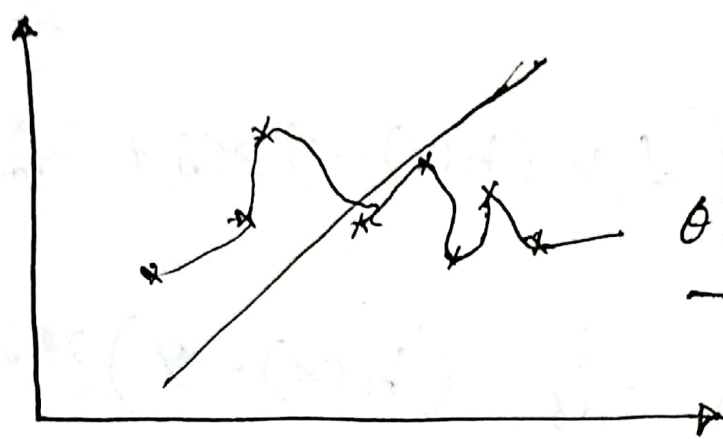
$$x_1 + x_2 \geq 3$$



not only linear.

**Optimox**



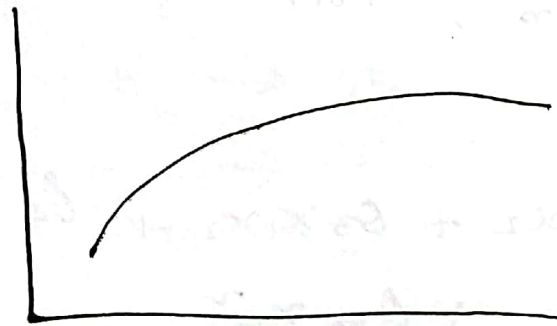


Underfit  
High bias

$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3$$

Overfit

High variance...



$\theta_0 + \theta_1 x + \theta_2 x^2$   
Just Perfect  
Low variance  
Less bias

- ⇒ Regularization
- ⇒ Boosting
- ⇒ Bagging

$$\sum (h(x) - y)^2 + 1000 \theta_3^2$$

Penalty add

Regularization term.

$$\sum (h(x) - y)^2 + \lambda \sum \theta_j^2$$

$$\theta_j := \theta_j - \alpha \frac{1}{2m} \sum (h(x) - y)^2 x_j$$

Updated cost function



Regularization

$$\theta_0 + \theta_1 x + \theta_2 x + \theta_3 x$$

$$\theta_j = \theta_j - \alpha \left[ \frac{1}{m} \sum (h(x) - y) x_j + \frac{\alpha}{m} \theta_j \right]$$

$$\theta_j = \theta_j - \alpha \frac{1}{2m} \frac{d}{d\theta_j} \left( \sum (h(x) - y)^2 \right)$$

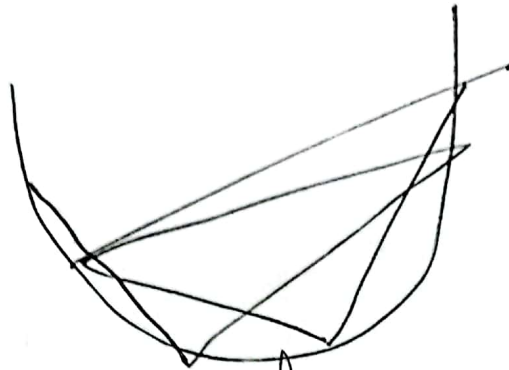
$$\theta_j = \theta_j \left( 1 - \alpha \frac{\lambda}{m} \right) - \alpha \frac{1}{m} \sum (h(x) - y) x_j$$

Non-linear regression, Polynomial regression

$$y = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1 x_2 + \theta_4 x_1^2 + \theta_5 x_2^2$$

$$x = 1.0 \text{ and } y = 3.0$$

ML | 11-10-23



$L \Rightarrow$  learning rate.

$$L = 0.5$$

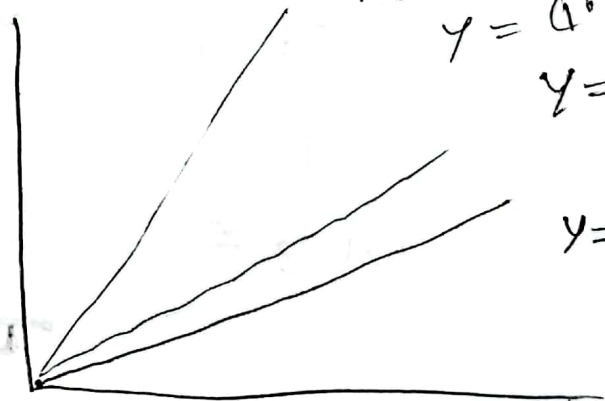
1st training data :  $x = 3.0$  and  $y = 1.0$ .

Final moderated

$$y = (0.6042)x$$

$$y = (0.3083)x$$

Initial A ;



$$y = (0.25)x$$

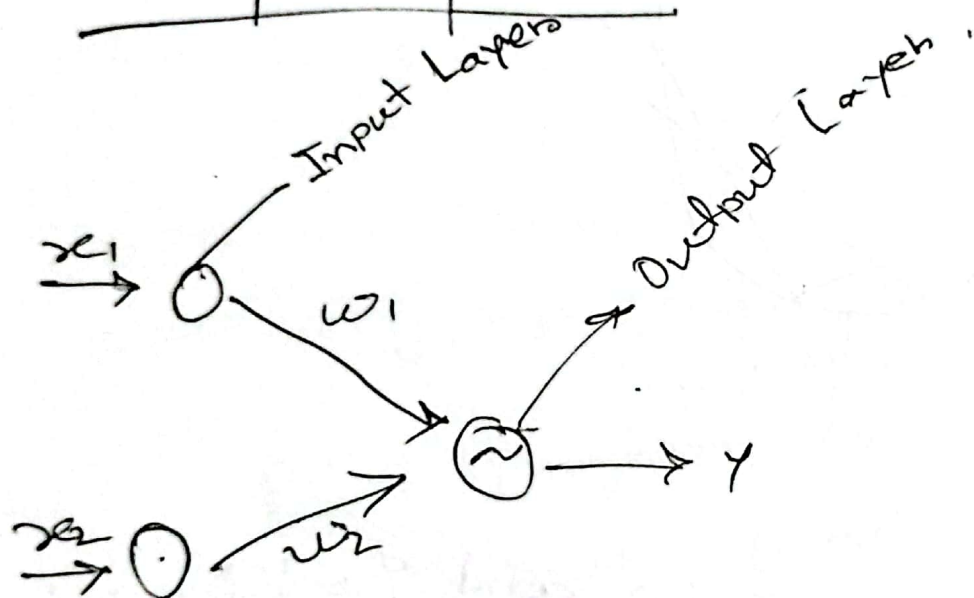
Optimox

# Neural network

Complex polynomial function.

$$x_1 x_2 x_3^2 x_4^4 x_5^2$$

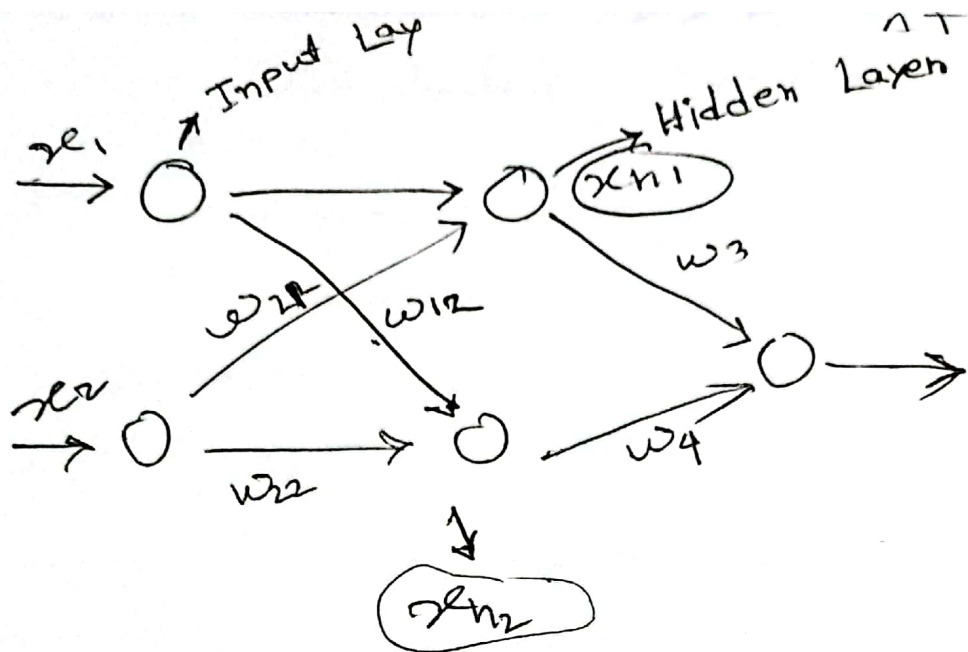
$x_1$	$x_2$	$y$
10	5	0
12	0	1
15	10	0



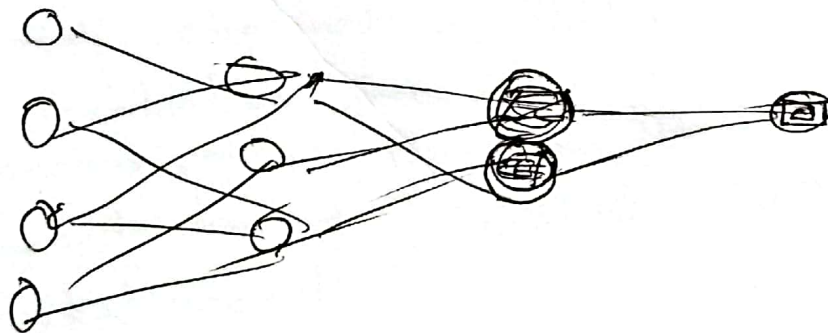
$$y = g(x_1 w_1 + x_2 w_2)$$

$$g(z) = \frac{1}{1 + e^{-z}}$$



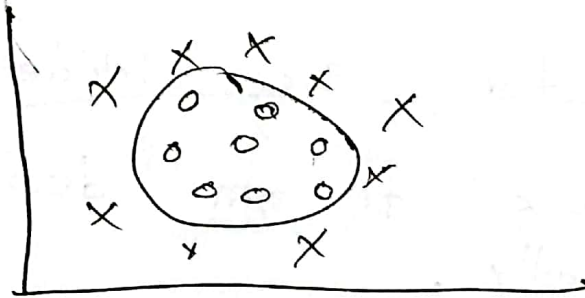
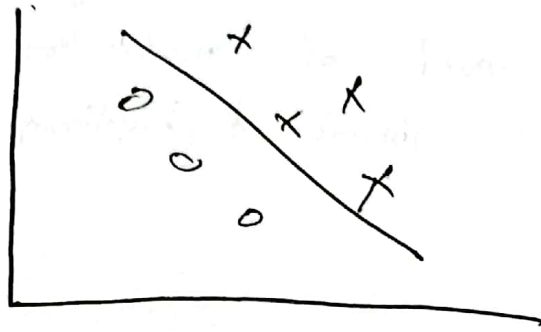


$$y = g(w_3 x_{h1} + w_4 x_{h2})$$

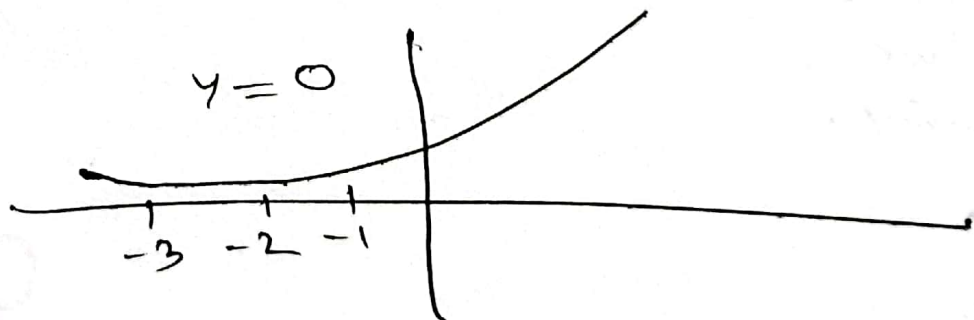
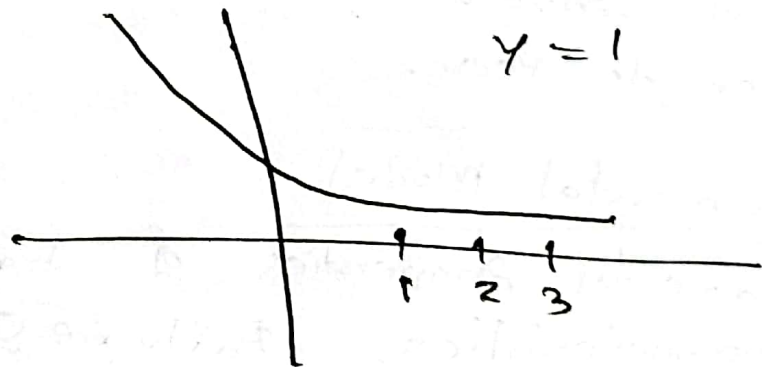


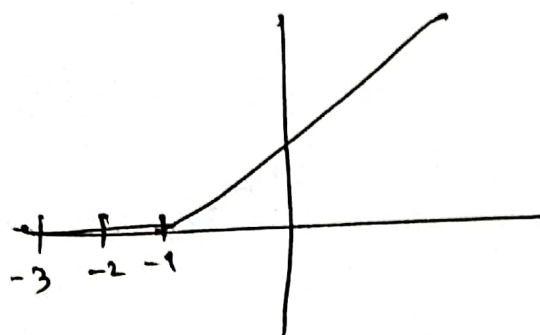
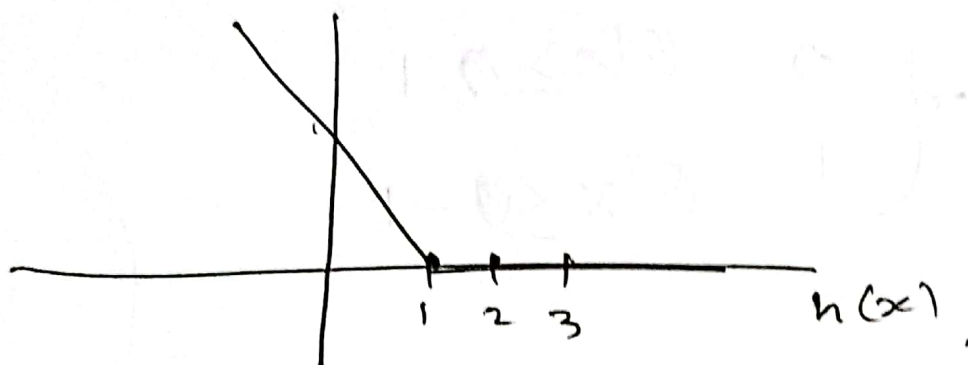
The ~~structure~~

# Support Vector Machine (SVM)



$$\text{cost} = -y \log h(x) - (1-y) \log (1-h(x))$$





$$-\min \frac{1}{n} \left[ \sum y \log h(x) + (1-y) \log (1-h(x)) \right] + \frac{\lambda}{2n} Q_y^2$$

$$\min (u-5)^2 + 1 \quad \text{where } u = 5.$$

$$\min \{ (u-5)^2 \} \rightarrow \text{too large}$$

$$A + \lambda B$$

$$c = \frac{1}{\lambda}$$

$$cA + B \rightarrow \text{too small}$$

Optimox



$$h(x) = \begin{cases} 0 & \theta^T x \geq 0 \\ 1 & \theta^T x \leq -1 \end{cases}$$

$$\frac{-1}{1 + e^{\theta^T x}}$$