

Assignment 1, Theoretical Exercises

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Course: EEN020
Computer Vision

Exercices

Theoretical Exercise 1: Points in Homogeneous Coordinates

Given:

$$x_1 = \begin{pmatrix} -8 \\ 6 \\ 2 \end{pmatrix}, x_2 = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix}, x_3 = \begin{pmatrix} -3\lambda \\ 12\lambda \\ 6\lambda \end{pmatrix}, \lambda \neq 0$$

Convert to Cartesian:

$$(x, y) = \left(\frac{X}{W}, \frac{Y}{W} \right)$$

$$x_1 : (-4, 3), x_2 : (6, -4), x_3 : \left(\frac{-3\lambda}{6\lambda}, \frac{12\lambda}{6\lambda} \right) = (-0.5, 2)$$

Interpretation of $x_4 = (5, -4, 0)$: point at infinity in direction $(5, -4)$. $x_4 \neq x_5 = (5, -3, 0)$: different directions at infinity.

Theoretical Exercise 2: Lines and Intersections

Lines:

$$\ell_1 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \ell_2 = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$$

Intersection:

$$x = \ell_1 \times \ell_2 = \begin{pmatrix} 5 \\ 2 \\ -16 \end{pmatrix} \sim \left(-\frac{5}{16}, -\frac{1}{8} \right)$$

Second pair:

$$\ell_3 = (-5, 0, 2), \ell_4 = (2, 0, 3)$$

$$x = \ell_3 \times \ell_4 = (0, 19, 0) \text{ (point at infinity, lines are parallel)}$$

Line through $x_1 = (2, 3)$, $x_2 = (4, -2)$:

$$\ell = \tilde{x}_1 \times \tilde{x}_2$$

$$\ell = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 1 \\ 4 & -2 & 1 \end{vmatrix} = (3 \cdot 1 - 1 \cdot (-2), 1 \cdot 4 - 2 \cdot 1, 2 \cdot (-2) - 3 \cdot 4)^\top = (5, 2, -16)^\top.$$

Line equation in Cartesian form is

$$5x + 2y - 16 = 0$$

Theoretical Exercise 3: Nullspace

Matrix:

$$M = \begin{pmatrix} 4 & -2 & 1 \\ 2 & 3 & 1 \end{pmatrix}, \quad x = (5, 2, -16)$$
$$Mx = (0, 0)^\top$$

Intersection point lies in nullspace. Nullspace is 1D: all scalar multiples of $(5, 2, -16)$.

Theoretical Exercise 4: Projective Transformation

$$H = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$
$$x_1 = (1, 0, 1), \quad x_2 = (0, 1, 1)$$
$$y_1 = Hx_1 = (2, 2, 1) \sim (2, 2), \quad y_2 = Hx_2 = (0, 2, 1) \sim (0, 2)$$

Lines:

$$\ell_1 = x_1 \times x_2 = (-1, -1, 1), \quad \ell_2 = y_1 \times y_2 = (0, 1, -2)$$
$$(H^{-1})^\top \ell_1 \sim \ell_2$$

Theoretical Exercise 5: Proof

Let a line l_1 be given. A point x is on l_1 iff $l_1^\top x = 0$. Let $y \sim Hx$. Define the image line

$$l_2 \sim (H^{-1})^\top l_1.$$

Then

$$l_2^\top y \sim ((H^{-1})^\top l_1)^\top Hx = l_1^\top (H^{-1}H)x = l_1^\top x = 0.$$

So any image point y of a point $x \in l_1$ lies on l_2 . Therefore H maps the whole line l_1 to a line l_2 . Projective transforms preserve lines.

Theoretical Exercise 6: Classification

- Projective: H_1, H_2, H_3 (H_4 not, $\det=0$)
- Affine: H_2, H_3
- Similarity: H_2
- Euclidean: None
- Preserve lengths: None
- Map lines to lines: All invertible
- Preserve parallelism: H_2, H_3

Theoretical Exercise 7: Pinhole Camera

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$X_1 = (1, 1, 2, 1) : PX_1 = (1, 1, 1) \sim (1, 1)$$

$$X_2 = (3, -1, 1, 1) : PX_2 = (3, -1, 0) \text{ (point at infinity)}$$

$$X_3 = (1, -1, -1, 1) : PX_3 = (1, -1, -2) \sim \left(-\frac{1}{2}, \frac{1}{2}\right)$$

Camera center:

$$PC = 0 \implies C = (0, 0, 1, 1)$$

Principal axis: along positive z-direction.

The principal axis is the straight line that starts at the camera center and goes perpendicular to the image plane. In this setup, the camera center is at $(0, 0, 1)$ and the image plane faces the camera, so the axis is simply the line that goes straight forward from that point along the positive z direction

Theoretical Exercise 8, OPTIONAL

Calibrated cameras $P_1 = [I \ 0]$ and $P_2 = [R \ t]$. Let $x \in \mathbb{P}^2$ be the image of a 3D point $U \in \mathbb{P}^3$ in the first camera, so $x \sim P_1 U$.

From $x \sim [I \ 0] U$ it follows that the first three coordinates of U are proportional to x . Hence

$$U \sim \begin{pmatrix} x \\ s \end{pmatrix}, \quad s \in \mathbb{R}.$$

As s varies this describes the entire back-projection ray of x through the center of P_1 . With only P_1 the value of s cannot be determined, since depth is ambiguous from a single view.

Assume U lies on the plane $\Pi = (\pi, 1)$. The plane equation gives

$$\Pi^T U = 0 \implies \begin{pmatrix} \pi^T & 1 \end{pmatrix} \begin{pmatrix} x \\ s \end{pmatrix} = \pi^T x + s = 0,$$

which yields

$$s = -\pi^T x.$$

The second camera forms

$$y \sim P_2 U = [R \ t] \begin{pmatrix} x \\ s \end{pmatrix} = Rx + ts.$$

Substituting $s = -\pi^T x$ gives

$$y \sim Rx - t\pi^T x = (R - t\pi^T)x.$$

Therefore the mapping between the two images induced by the plane is a homography

$$H = R - t \pi^T, \quad y \sim Hx.$$

The set $\{U(s)\}$ is a 3D ray back-projected from x . Depth s cannot be recovered from the first camera alone. With the plane prior s equals $-\pi^T x$. The corresponding point in the second image is given by the planar homography $y \sim (R - t \pi^T)x$.