Assignment 1, Theoretical Exercises

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Course: EEN020
Computer Vision

Excercises

Theoretical Exercise 1: Points in Homogeneous Coordinates

Given:

$$x_1 = \begin{pmatrix} -8 \\ 6 \\ 2 \end{pmatrix},\ x_2 = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix},\ x_3 = \begin{pmatrix} -3\lambda \\ 12\lambda \\ 6\lambda \end{pmatrix},\ \lambda \neq 0$$

Convert to Cartesian:

$$(x, y) = \left(\frac{X}{W}, \frac{Y}{W}\right)$$

$$x_1 : (-4, 3),\ x_2 : (6, -4),\ x_3 : \left(\frac{-3\lambda}{6\lambda}, \frac{12\lambda}{6\lambda}\right) = (-0.5, 2)$$

Interpretation of $x_4 = (5, -4, 0)$: point at infinity in direction $(5, -4)$. $x_4 \neq x_5 = (5, -3, 0)$: different directions at infinity.

Theoretical Exercise 2: Lines and Intersections

Lines:

$$\ell_1 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix},\ \ell_2 = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$$

Intersection:

$$x = \ell_1 \times \ell_2 = \begin{pmatrix} 5 \\ 2 \\ -16 \end{pmatrix} \sim (-\frac{5}{16}, -\frac{1}{8})$$

Second pair:

$$\ell_3 = (-5, 0, 2),\ \ell_4 = (2, 0, 3)$$

$$x = \ell_3 \times \ell_4 = (0, 19, 0) \text{ (point at infinity, lines are parallel)}$$

Line through $x_1 = (2, 3)$, $x_2 = (4, -2)$:

$\ell = \tilde{x}_1 \times \tilde{x}_2$

$$\ell = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 1 \\ 4 & -2 & 1 \end{vmatrix} = \big(3 \cdot 1 - 1 \cdot (-2),\ 1 \cdot 4 - 2 \cdot 1,\ 2 \cdot (-2) - 3 \cdot 4\big)^\top = (5,\ 2,\ -16)^\top.$$

Line equation in Cartesian form is

$$5x + 2y - 16 = 0$$

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Theoretical Exercise 3: Nullspace

Matrix:

$$M = \begin{pmatrix} 4 & -2 & 1 \\ 2 & 3 & 1 \end{pmatrix}, \ x = (5, 2, -16)$$

$$Mx = (0, 0)^\top$$

Intersection point lies in nullspace. Nullspace is 1D: all scalar multiples of $(5, 2, -16)$.

Theoretical Exercise 4: Projective Transformation

$$H = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$x_1 = (1, 0, 1), \ x_2 = (0, 1, 1)$$

$$y_1 = Hx_1 = (2, 2, 1) \sim (2, 2), \ y_2 = Hx_2 = (0, 2, 1) \sim (0, 2)$$

Lines:

$$\ell_1 = x_1 \times x_2 = (-1, -1, 1), \ \ell_2 = y_1 \times y_2 = (0, 1, -2)$$

$$(H^{-1})^\top \ell_1 \sim \ell_2$$

Theoretical Exercise 5: Proof

Let a line l_1 be given. A point x is on l_1 iff $l_1^\top x = 0$. Let $y \sim Hx$. Define the image line

$$l_2 \sim (H^{-1})^\top l_1.$$

Then

$$l_2^\top y \sim \left((H^{-1})^\top l_1\right)^\top Hx = l_1^\top (H^{-1}H)x = l_1^\top x = 0.$$

So any image point y of a point $x \in l_1$ lies on l_2. Therefore H maps the whole line l_1 to a line l_2. Projective transforms preserve lines.

Theoretical Exercise 6: Classification

- Projective: H_1, H_2, H_3 (H_4 not, det=0)
- Affine: H_2, H_3
- Similarity: H_2
- Euclidean: None
- Preserve lengths: None
- Map lines to lines: All invertible
- Preserve parallelism: H_2, H_3

Theoretical Exercise 7: Pinhole Camera

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$X_1 = (1,1,2,1) : PX_1 = (1,1,1) \sim (1,1)$$

$$X_2 = (3,-1,1,1) : PX_2 = (3,-1,0) \text{ (point at infinity)}$$

$$X_3 = (1,-1,-1,1) : PX_3 = (1,-1,-2) \sim (-\frac{1}{2}, \frac{1}{2})$$

Camera center:

$$PC = 0 \implies C = (0,0,1,1)$$

Principal axis: along positive z-direction.
The principal axis is the straight line that starts at the camera center and goes perpendicular to the image plane. In this setup, the camera center is at (0, 0, 1) and the image plane faces the camera, so the axis is simply the line that goes straight forward from that point along the positive z direction

Theoretical Exercise 8, OPTIONAL

Calibrated cameras $P_1 = [I\ 0]$ and $P_2 = [R\ t]$. Let $x \in \mathbb{P}^2$ be the image of a 3D point $U \in \mathbb{P}^3$ in the first camera, so $x \sim P_1 U$.

From $x \sim [I\ 0]\, U$ it follows that the first three coordinates of U are proportional to x. Hence

$$U \sim \begin{pmatrix} x \\ s \end{pmatrix}, \qquad s \in \mathbb{R}.$$

As s varies this describes the entire back–projection ray of x through the center of P_1. With only P_1 the value of s cannot be determined, since depth is ambiguous from a single view.

Assume U lies on the plane $\Pi = (\pi, 1)$. The plane equation gives

$$\Pi^T U = 0 \implies \begin{pmatrix} \pi^T & 1 \end{pmatrix} \begin{pmatrix} x \\ s \end{pmatrix} = \pi^T x + s = 0,$$

which yields

$$s = -\,\pi^T x.$$

The second camera forms

$$y \sim P_2 U = [R\ t] \begin{pmatrix} x \\ s \end{pmatrix} = Rx + ts.$$

Substituting $s = -\pi^T x$ gives

$$y \sim Rx - t\,\pi^T x = (R - t\,\pi^T)\,x.$$

Therefore the mapping between the two images induced by the plane is a homography

$$H = R - t\,\pi^T, \qquad y \sim Hx.$$

The set $\{U(s)\}$ is a 3D ray back–projected from x. Depth s cannot be recovered from the first camera alone. With the plane prior s equals $-\pi^T x$. The corresponding point in the second image is given by the planar homography $y \sim (R - t\,\pi^T)x$.