

## Exercise Sheet 3

### Exercise 7

A coin is thrown 3 times and one knows that head happens at least 2 times.

- (a) What is the (conditional) probability for tail with the 3rd throw?
- (b) Write a python program that simulates the above situation and estimates the conditional probability by a well-chosen relative frequency.

### Exercise 8

Simulate a fair dice  $X_i \sim U(1, 2, 3, 4, 5, 6)$  and generate a frequency distribution (histogram) and the empirical CDFs of a reasonable realisation of the random variable

$$S_n := \sum_{i=1}^n \frac{X_i - E[X_i]}{\sqrt{n \cdot V[X_i]}}, \quad n = 1, 2, 3, 5, 10, 20, 50, 100$$

Hereby  $E[X_i] = 3.5$  and  $V[X_i] = 2.9167$  the expectation and variance for one roll of the dice. The distribution of  $S_n$  approximates more and more a normal distribution  $\mathcal{N}$ . Through standardization (subtract the expectation value, divide by the square root of  $n$  times the variance)  $S_n$  equalizes to  $\mathcal{N}(0, 1)$  (plot  $\mathcal{N}(0, 1)$  as an overlay into the histograms). What fundamental mathematical theorem is behind all this?

### Exercise 9

A random variable  $X$  is distributed as follows:

k	1	2	3
$P[X=k]$	0.25	0.15	0.60

Calculate expectation and variance through

- (a) analytical calculation according to the definitions and
- (b) estimation using a simulation of the random variable  $X$  with  $n = 2000$  trials.

### Exercise 10

In a group of persons each person chooses randomly and independent from each other 2 friends (a friend can of course have multiple other friends).  $X$  denotes the number of persons who were not chosen. Estimate the expectation of  $X$  using an appropriate average of events in a simulation.

**Exercise 11**

A biometric system for access control has these benchmark data:

- The system recognises an authorised person in 95 of 100 cases correctly.
- In 1 of 100 cases yet an unauthorised person is granted access.

The system is used to grant access to a server room for only 3 persons in a company with 500 employees. Find the probability that an access granted person is indeed an authorised person

- (a) by an analytic calculation and
- (b) by estimating it by a relative frequency in a simulation.