Exercise Sheet 4

Exercise 12

A car rental station offers 3 different types of cars and observes, that customers choose those with the following probabilities (we assume that cars are always rent for exactly one day):

Type k of car	1	2	3
P[X=k]	0.25	0.35	0.40

The station has 5 cars of type 1, 6 of type 2 and another 6 of type 3. The number of customers per day is approximately normal distributed with mean 16 and standard deviation 2. Estimate the probability that all customers' requests can be fulfilled by implementing a suitable simulation.

Exercise 13

Generate a random sample X_i of size $n \in \{100, 1000, 10000\}$ from Poisson P_2 distribution. These samples hold the model assumptions given in the lecture (all X_i are identically distributed and independent).

(a) Compute the point estimator (see Script Chapter 3.3.1) for the parameter λ of a Poisson and compare the values with the theoretical one (2 in the above example).

(b) Show, that the point estimator for λ unbiased and consistent.

Exercise 14

Given a discrete uniform distribution with the range of $W_X = \{1, 2, 3, \ldots, m\}$, it holds: $P[X = k] = {}^1\!/\!_m$ for all $k \in W_X$. Hereafter, m is assumed to be unknown. A sample $x_1, x_2, \ldots, x_n$ in the size of n is drawn.

(a) Show that $\hat{m} = 2 \cdot \bar{x} - 1$ is an unbiased estimator for m.

(b) Illustrate the result from (a) through a simulation.

(c) At an event tickets with numbers $1, 2, 3, \ldots$ were sold. A tombola took place where from all tickets the numbers $34, 56, 17, 22, 23, 88$ were drawn. How many visitors were approximately at the event? Why is the usage of this estimator not entirely correct?

Exercise 15

We used the empirical variance to fit normal distributions to data. But is this formula a good estimator of the variance?

(a) Show that the sample variance is an unbiased estimator.

$$\hat{\sigma}^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2$$

(b) What changes if $1/(n-1)$ is replaced by $1/n$ in the above formula.