Dola Ma'am

TOPIC NAME : Course Overview

Lecture-1

DAY : Thursday

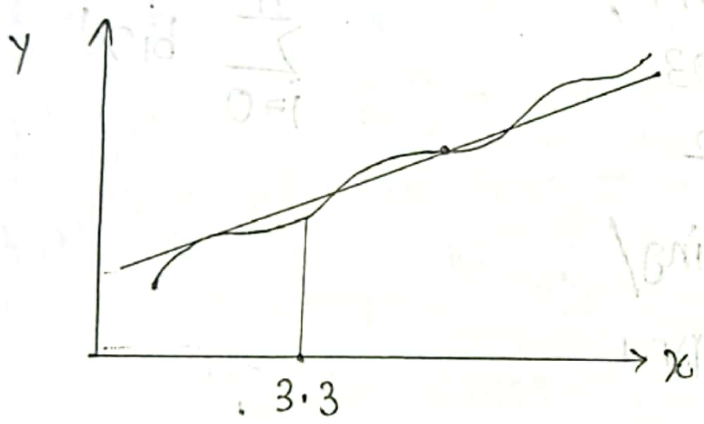
TIME :

DATE : 10 / 8 / 23

* Next হই Sunday 8:50 এ extra class.

$y = f(x)$

x	1	2	3	3.3
y	4.2	3.6	4.8	?



Method :

① Regression → close value

② Interpolation → exact value

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Lecture-2

Approximation & Errors in computing

13/8/23

Sunday

Significant digit, $\pi = 3.14159$

Significant digit = 5

* নিজেদের এর ৭/৫ হবেই দিব ।

TOPIC NAME : DAY :

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Numerical Error

Roundoff errors | Truncation errors

→ chopping, $A = 42.7293$

$= 42.72$

→ Symmetric roundoff,

$A = 42.7293$

$= 42.72$

* ওরাওরি বা, ceiling/ floor করে নিলাম।

$$\sum_{i=0}^{\infty} b_i x^i$$

↓

$$\sum_{i=0}^{n} b_i x^i$$

Error

Absolute error | Relative

$E_a = |x_r - x_a|$

যা আসল কথা

calculation এর পর যা আসছে।

Ex: Rice.

$E_r = \left| \dfrac{x_r - x_a}{x_r} \right|$

→ সূক্ষ্ম বা value অনেক বড়।

→ Unit এর value এর উপর depend করে।

Ex: Gold

TOPIC NAME : DAY : TIME : DATE : / /

Error Propagation:

→ step যত বাড়বে, error তত বাড়বে। (→ truncation error)

→ Fig slide

→ truncation error কমবে, যদি iteration বাড়ে।

i=0 ; i<n → iteration

a+b, b+e } step size

Error estimation:

① Forward error analysis

→ exact define করা যায় না।

→ Bound করে দিতে হবে।

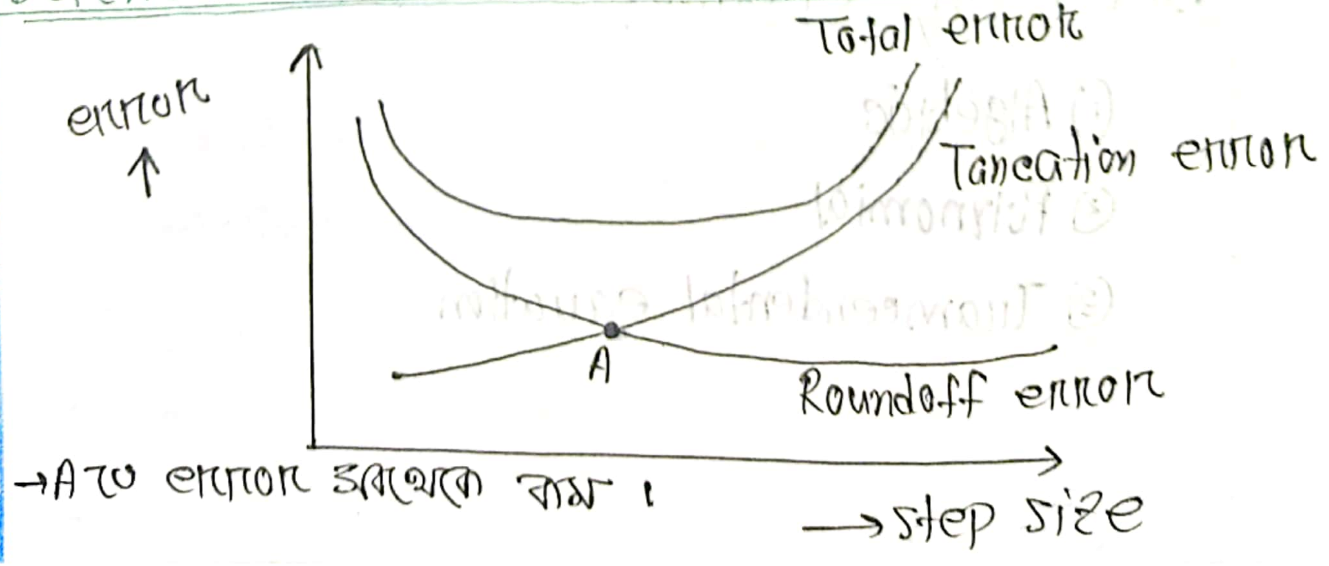
② Backward error analysis

→ Absolute error যেইটা

③ Experiment " "

→

Dependence of error on step size:



→ A তে error সবথেকে কম।

Minimize the total error :

→ initial value তোমাদের choose করা লাগবে।

→ slide.

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Lecture-3
Thursday
17/8/23

Roots of Non linear equation

Linear vs Non Linear :

$y = 3x^2 - 4$ →

$\Rightarrow y = f(x)$

Types of equation :

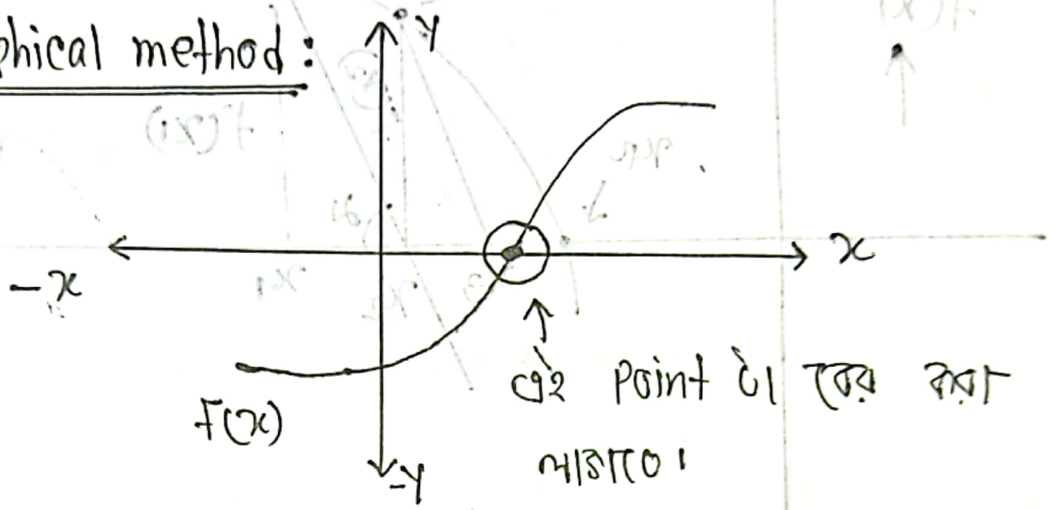
1. Algebraic
2. Polynomial
3. Transcendental equation

TOPIC NAME : DAY : TIME : DATE : / /

Methods of finding roots of nonlinear equations:

① **Direct analytical method** - $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

② **Graphical method:**



এই point টা বের করা লাগবে।

③ **Trial and Error:** x এর একটা set of value থাকবে। set এর যেই মানের জন্য 0 হবে।

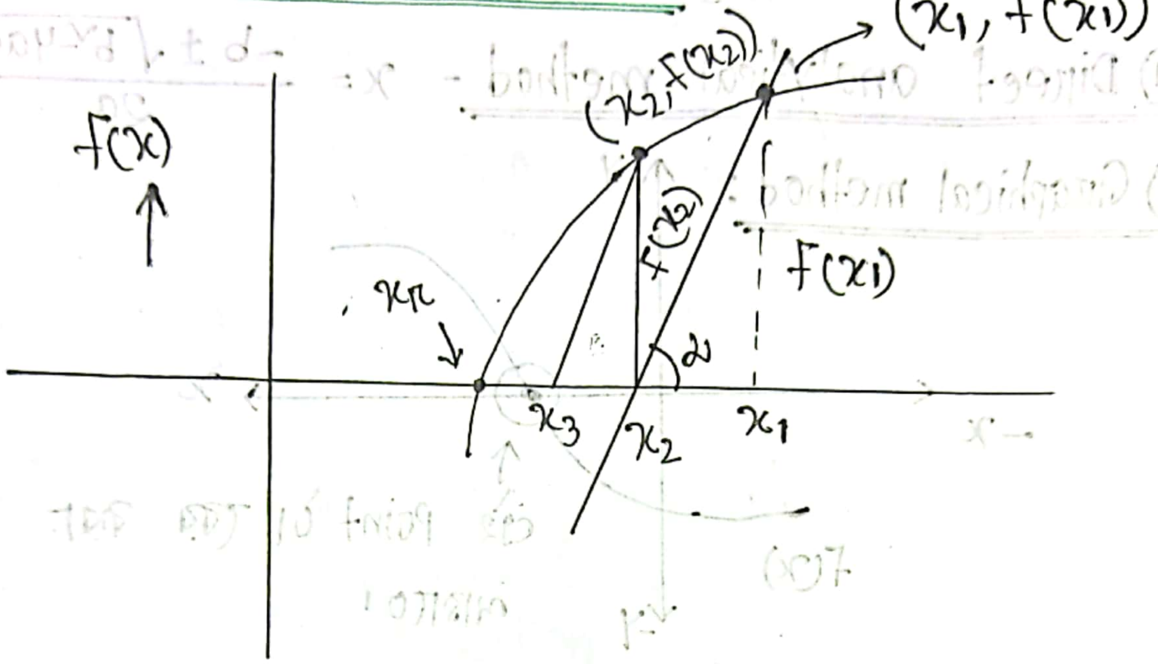
④ **Iterative method:**

Iterative Methods

① Open end methods
→ Newton-Raphson (সবচেয়ে Fast)
→ Secant

② Bracketing methods
→ Bisection method
→ False Position

iv) Newton Raphson method :



$$\tan\alpha = \frac{f(x_1)}{x_1 - x_2} = f'(x_1)$$

$$\Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

The next approximation –

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

⇒ x_2 / x_3 এর মান equation এ বসালে 0 হলে root, না হলে আবার approximation এ যাওয়া লাগবে।

TOPIC NAME : DAY : TIME : DATE : / /

→ দুইটা Successive iteration এর gap check করা লাগবে। → open end method.

Limitation:

(i) $f'(x) = 0$ হতে পারে।

(ii) initial case গোলমেলে হতে হবে।

(iii) convergence loop হয়ে যেতে পারে।

Lecture-4
Sunday
20/8/23

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Secant Method

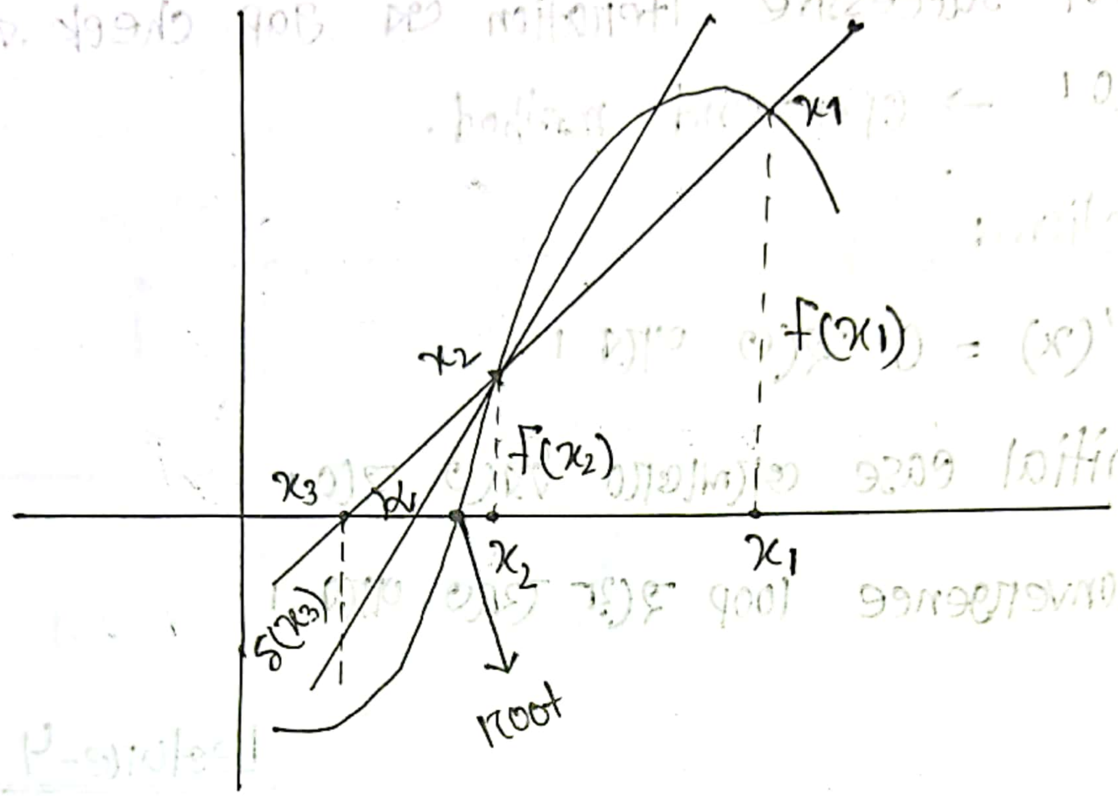
⇒ $f'(x) = 0$, limitation দূর করার জন্য secant method.

→ open end + Bracket method এর mix.

→ Mainly open end method.

GOOD LUCK™

TOPIC NAME : DAY : TIME : DATE :



$$\tan\alpha = \frac{f(x_1)}{x_1 - x_3} = \frac{f(x_2)}{x_2 - x_3}$$

জানা থাকিল -

$f(x), x_1, x_2$

$$\therefore x_3 = \frac{f(x_2)x_1 - f(x_1)x_2}{f(x_2) - f(x_1)}$$

$x_2 \Rightarrow x_1 , x_3 \Rightarrow x_2$

TOPIC NAME : DAY : TIME : DATE : / /

$f(x) = x^2 - 4x - 10 = 0$, when $x_1 = 5$, $x_2 = 3$, 4 iteration

Soln : Given,

$$f(x) = x^2 - 4x - 10$$

$$f(x_1) = f(5) = 25 - 20 - 10 = -5 \qquad x_1 = 5$$

$$f(x_2) = f(3) = 9 - 12 - 10 = -13 \qquad x_2 = 3$$

$$\cancel{x_3 = \frac{f(5)x - f(3)}{f(5) - f(3)}}$$

$$x_3 = \frac{f(x_2)x_1 - f(x_1)x_2}{f(x_2) - f(x_1)} = \frac{-13 \times 5 - (-5) \times 3}{-13 + 5} = 6.25$$

<u>2nd iteration :</u>

$$x_1 = 3, \quad x_2 = 6.25$$

$$f(x_1) = f(3) = -13$$

$$f(x_2) = f(6.25) = 4.0625$$

$$x_4 = \frac{f(x_2)x_1 - f(x_1)x_2}{f(x_2) - f(x_1)} = \frac{4.0625 \times (3) - (-13) \times 6.25}{4.0625 + 13}$$

$$= 5.4761$$

*** 21/09/23 → CT -২০ । (Time fixed
TOPIC NAME : → 18/19 তারিখের Reminder ।

3rd iteration:

$$x_1 = 6.25, \quad x_2 = 5.4761$$

$$f(x_1) = f(6.25) = 4.0625$$

$$f(x_2) = f(5.4761) = 1.9168$$

$$x_5 = \frac{4.0625 \times 6.25 - 1.9168 \times 5.4761}{1.9168 - 4.0625} = 5.7242$$

4th iteration:

$$x_1 = 5.4761, \quad x_2 = 5.7242$$

$$f(x_1) = f(5.4761) = -1.9167$$

$$f(x_2) = f(5.7242) = -0.1303$$

$$x_5 = \frac{f(x_2) \times x_1 - f(x_1) \times x_2}{f(x_2) - f(x_1)} = 5.7423$$

Lecture-5

TOPIC NAME : Bisection method

DAY : Tuesday

TIME :

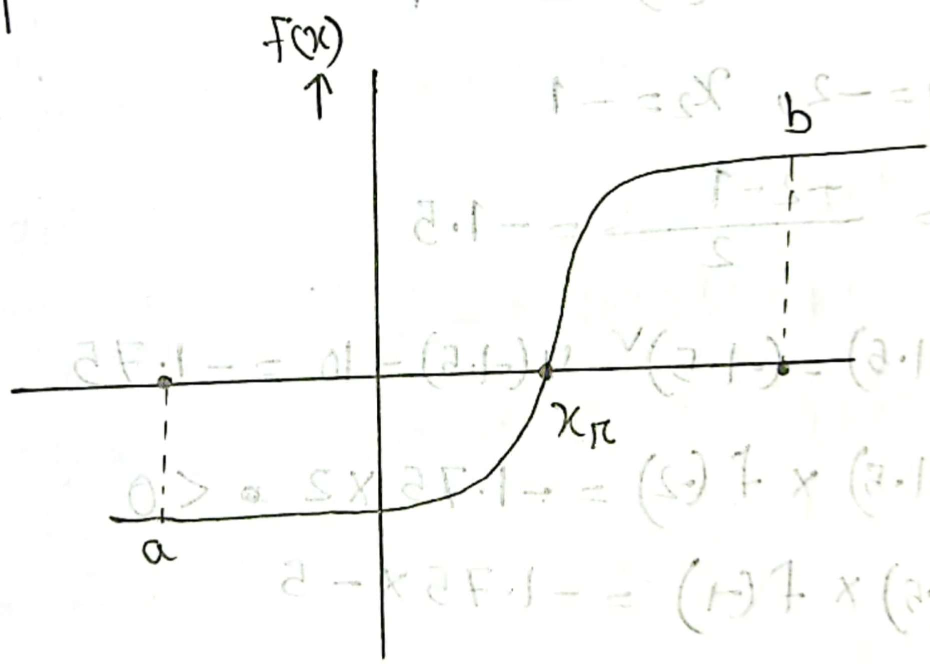
DATE : 22/8/23

Starting and stoping in iterative Process -

Search interval :

initial approximation, $x_1 = -\frac{a_{n-1}}{a_n}$

$|x_{max}|$



* $f(a) \cdot f(b) < 0$

$x_1 = a$, $x_2 = b$, Mid point, $x_0 = \frac{x_1 + x_2}{2}$

* 5 বার iteration করা লাগবে ।

* Newton Rapson, secant 7 বার iteration.

TOPIC NAME : DAY: TIME : DATE : /

$x^2 - 4x - 10 = 0$

$$|x_{max}| = \sqrt{\left(\frac{a_{n-1}}{a_n}\right)^2 - 2\left(\frac{a_{n-2}}{a_n}\right)} = 6$$

$\therefore$ so root in the interval $[-6, 6]$

$x = -2 \quad f(-2) = 2 = x_1$

$x_1 = -2, \quad x_2 = -1$

$$x_0 = \frac{-2-1}{2} = -1.5$$

$f(-1.5) = (-1.5)^2 - 4(-1.5) - 10 = -1.75$

$f(-1.5) \times f(-2) = -1.75 \times 2 < 0$

$f(-1.5) \times f(-1) = -1.75 \times -5$

$$x_0 = \frac{-2+(-1.5)}{2} = -1.75$$

$f(-1.75) = 0.0625$

$f(-1.75)\, f(-1.5) = 0.0625 \times -1.75 < 0$

$f(-1.75)\, f(-2) = 0.0625 \times 2$

TOPIC NAME :

$$x_0 = \frac{-1.75 - 1.5}{2} = -1.625$$

$f(-1.625) = -0.8593$

$f(-1.625) \times f(-1.75) = -0.8593 \times 0.0625$

$f(-1.625) \times f(-1.5) = -0.8593 \times -1.75$

$$x_0 = \frac{-1.625 - 1.75}{2} = -1.6875$$

$f(-1.6875) = -0.4023$

$f(-1.6875) \times f(-1.625) = -0.4023 \times -0.8593$

$f(-1.6875) \times f(-1.75) = -0.4023 \times 0.0625$

$$x_0 = \frac{-1.6875 - 1.75}{2} = -1.7187$$

$f(-1.7187) = -0.1712$

$f(-1.7187) \times f(-1.6875)$

$f(-1.75) = 0.0625$

$f(-1.7187) = -0.1712$

$$x_0 = \frac{-1.75 - 1.7187}{2} = -1.7343$$

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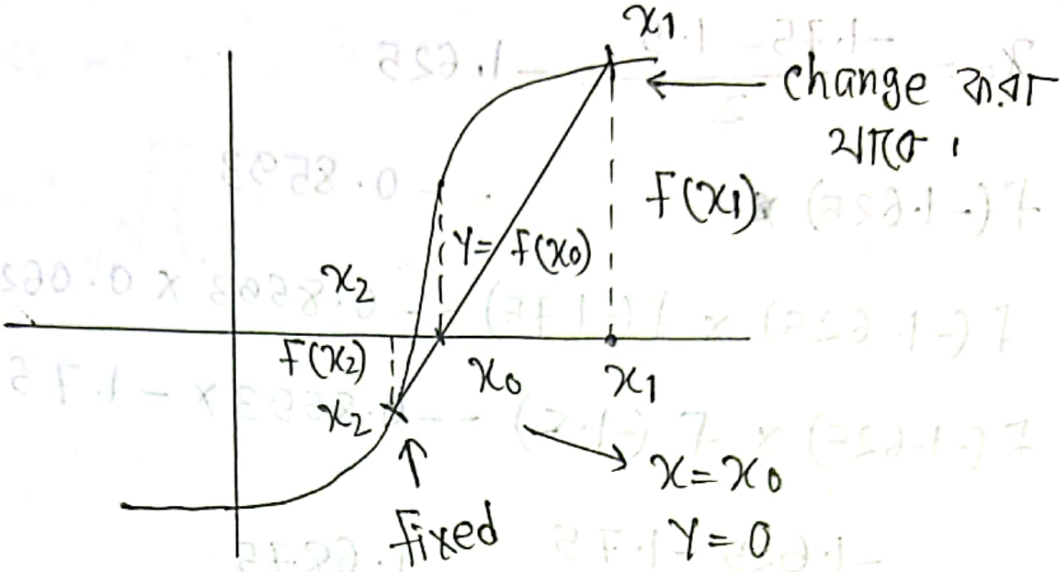
TOPIC NAME: False Position Method/ Linear interpolation

Lecture-6

DAY: Sunday

TIME:

DATE: 27/8



$$\frac{f(x_1) - f(x_2)}{x_1 - x_2} = \frac{y - f(x_2)}{x - x_2}$$

$$\Rightarrow \frac{f(x_1) - f(x_2)}{x_1 - x_2} = \frac{-f(x_2)}{x_0 - x_2}$$

[4/5 step করা লাগবে]

$$\Rightarrow x_0 = x_1 - \frac{f(x_1)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

→ Range বলা থাকলে ভালো, তা হলে interval বের করে নিতে হবে।

TOPIC NAME : DAY : TIME : DATE : / /

Convergance Comparison

Newton Raphson < secant < False Position < Bisection

→ iteration

← convergance rate high

Lecture-7
3/10/23
Tuesday

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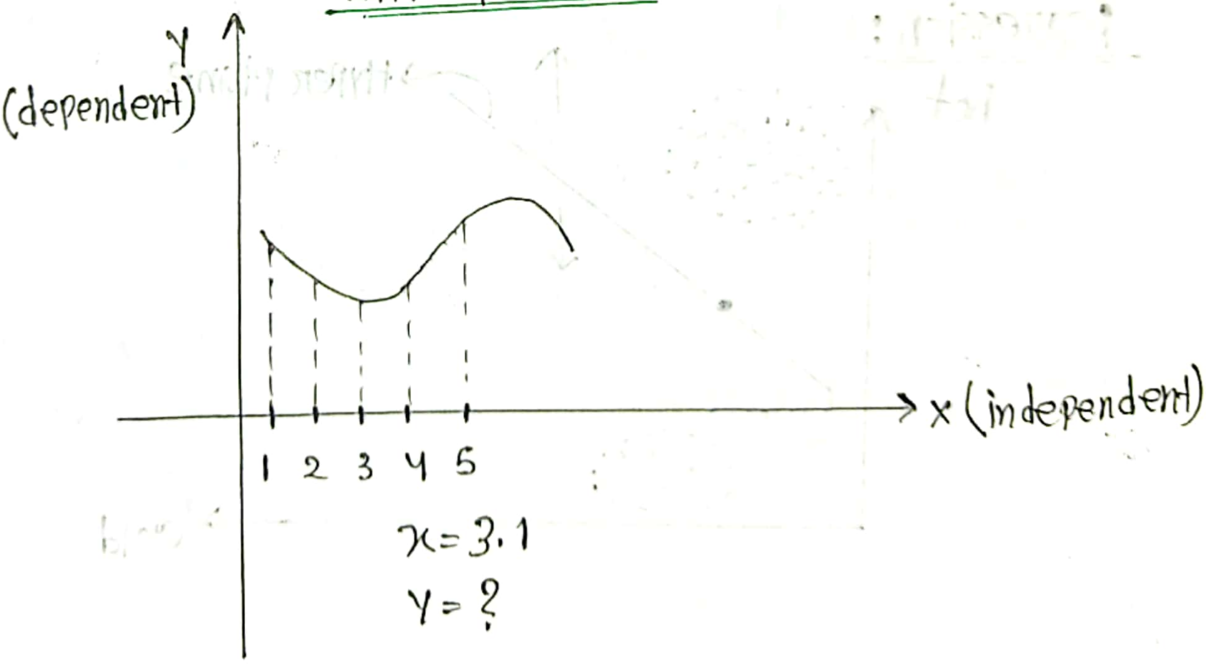
Curve Fitting:

Interpolation



$x = 3.1$

$y = ?$

The process of constructing

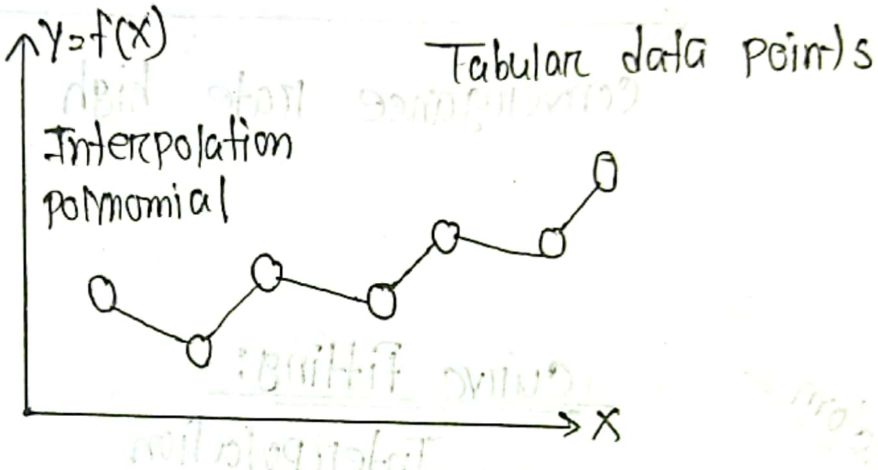
TOPIC NAME : DAY : TIME : DATE : /

Tabular data point -

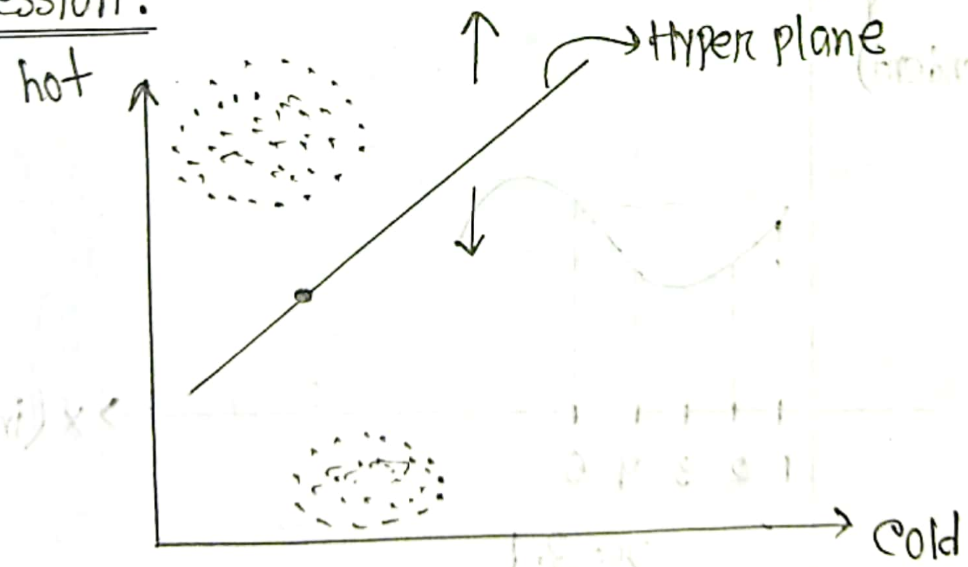
1. Table of values of well defined functions.

2. Data tabulated from experimental measurement.

Interpolation:



Regression:



Curve fitting Lecture-8

TOPIC NAME : Interpolation DAY: Thursday

TIME : DATE : 5 / 10 / 23

Different form of nth order polynomial :

=> Power form

=> Shifted power form

=> Newton form

=> another form

Linear Interpolation

=> solving first ~~1st~~ order polynomial

=> To avoid huge error we use lagrange interpolation polynomial.

Lagrange int Polynomial

2nd order polynomial of the form -

$$P_2(x) = b_0(x-x_0)(x-x_1) + b_1(x-x_1)(x-x_2) + b_2(x-x_2)(x-x_0)$$

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Curve fitting

TOPIC NAME : Interpolation

Lecture

DAY : Tuesday

TIME :

DATE : 10 / 10

Lagrange limitations :

1. Computation requires more number of operations.

2. Doesn't add another point to improve accuracy

To remove limitations

Newton interpolation

আগের step ও পরের step এর divided difference 0 হলে থামা যেতে হবে।

$\Rightarrow f_0 = f[x_0]$

$\Rightarrow f_1 = f_0 a_1 (x_1 - x_0) \Rightarrow a_1 = \frac{f_1 - f_0}{x_1 - x_0} = f[x_0, x_1]$

is called first divided difference

$\Rightarrow f_2 = a_0 + a_1(x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1)$

$\Rightarrow a_2 = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = f[x_0, x_1, x_2]$

Second divided difference

General Format

$$P_n(x) = \sum_{i=0}^{n} f[x_0 \text{-----} x_i] \prod_{j=0}^{i-1} (x - x_j)$$

TOPIC NAME : DAY : TIME : DATE : / /

Divided Difference Table

$$\# \; f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

Assignment : slide page-10 Table value find $f(4.5)$

$\rightarrow f(3.5) \rightarrow 1-4$

5 add করো then $f(4.5)$ value find করো, then comment.

Submission : 17/10/23

Lecture-10
12/10/23
Thursday

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Interpolation

Equi Distance Interpolation :

$x_0 = 0 \quad x_1 = 0 \quad x_2 = 0 \quad x_3 = 0 \quad x_4 = 0 \quad x_5 = 0$

$$x_k = x_0 + kh$$

First forward, $\Delta f_i = f_{i+1} - f_i = \Delta f_0$

" forward, $\Delta^2 f_i = \Delta f_{i+1} = \frac{\Delta^2 f_0}{2h^2}$

In General, $f[x_0, x_1, x_2 ----, x_j] = \frac{\Delta^j f_0}{j!\, h^j}$

TOPIC NAME : DAY : TIME : DATE :

Forward Difference:

We know that - $x_k = x_0 + kh$

let - $x = x_0 + sh$ and $P_n(s) = P_n(x)$

so, $x - x_k = h(s-k)$

$$P_n(s) = \sum_{j=0}^{n} \frac{\Delta^j f_0}{j!\,h^j} \prod_{k=0}^{j-1} h(s-k)$$

$$= \sum_{j=0}^{n} \frac{\Delta^j f_0}{j!\,h^j} \times h^j \left[s(s-1)(s-2)\cdots(s-j+1)\right]$$

$$= \sum_{j=0}^{n} \Delta^j f_0 \times \frac{s(s-1)(s-2)\cdots(s-j+1)}{j!}$$

$$= f_0 + \Delta f_0 s + \Delta^2 f_0 \frac{s(s-1)}{2!} + \cdots + \Delta^n f_0 \frac{s(s-1)(s-2)\cdots(s-(n-1))}{n!}$$

GOOD LUCK

lecture-11

TOPIC NAME : Interpolation

DAY : Tuesday

TIME :

DATE : 17 / 10 / 23

☐ Backward Difference Table :

Spline Interpolation

Problem $\longrightarrow$ Part by Part

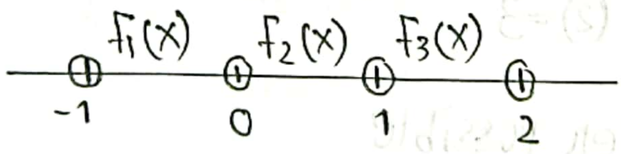
Solve

$\downarrow$

Σ Solution

☐

$$f(x) = \begin{cases} \overset{f_1(x_1)}{x+1} \; ; & -1 \le x \le 0 \\ \overset{f_2(x_2)}{2x+1} \; ; & 0 \le x \le 1 \\ \overset{f_3(x_3)}{4-x} \; ; & 1 \le x \le 2 \end{cases}$$



$f_1(0) = 1$, $f_2(0) = 1$ $\quad \therefore f_1(0) = f_2(0)$

$f_2(1) = 3$, $f_3(1) = 3$ $\quad \therefore f_2(1) = f_3(1)$

$\therefore$ linear spline

$f_1'(x) = 1$, $f_2'(x) = 2$, $f_3'(x) = -1$

$f_1'(x) \neq f_2'(x)$

$f_2'(x) \neq f_3'(x)$

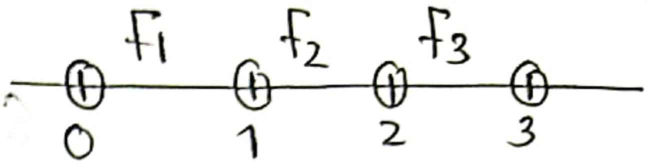
TOPIC NAME : DAY :

TIME : DATE : /

$\boxed{\text{Ex}}$ $f_1(x) = x \quad ; \quad 0 \le x \le 1$

$f_2(x) = x^2 - x + 1 \quad ; \quad 1 \le x \le 2$

$f_3(x) = 3x - 3 \quad ; \quad 2 \le x \le 3$



$f_1(1) = 1 \qquad f_2(2) = 3$

$f_2(1) = 1 \qquad f_3(2) = 3$

$\therefore$ They are linear spline

Now -

$f_1'(x) = 1, \; f_2'(x) = 2x - 1, \; f_3'(x) = 3$

$f_1'(1) = 1, \; f_2'(1) = 1$

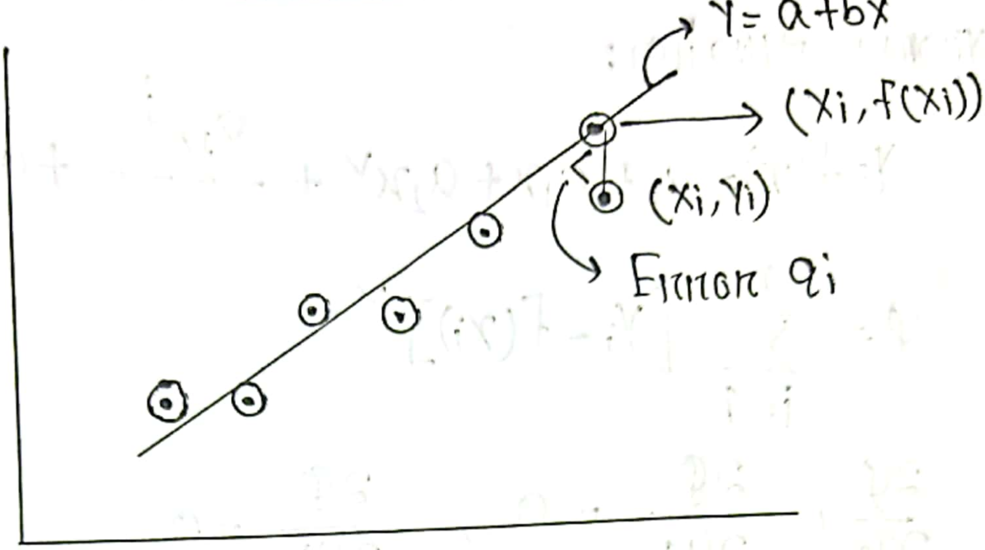
$f_2'(2) = 3, \; f_3'(2) = 3$

$\therefore$ First order possible

Lecture-12

TOPIC NAME : Regression DAY : Wednesday

TIME : DATE : 18/10/23



Objective: We have to minimize the sum of error.

Least square method -

=> linear equation, $Y = a+bx$

=> Transcendental equation, $Y = ae^{bx}$

=> Polynomial equation, $Y = a+bx^2$

=> Multi-variable, $Y = a_1 + a_2 x + a_3 z$

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Lecture -

TOPIC NAME : Regression

DAY: 1/11/23 (We

TIME :

DATE : 1 / 11

Polynomial equation :

$$Y = f(x) = a_0 + a_1 x + a_2 x^2 + \cdots a_j x^j \cdots + a_{m-1} x^{m-1}$$

$$\varphi = \sum_{i=1}^{n} [Y_i - f(x_i)]^2$$

$$\frac{\partial \varphi}{\partial a_0}, \frac{\partial \varphi}{\partial a_1} = 0, \frac{\partial \varphi}{\partial a_j} = 0$$

$$a = [0 - (m-1)]$$

\# second order → $m = 3$

$(0, 1, 2)$

$$Y = a_0 + a_1 x + a_2 x^2$$

Multiple linear equation :

$$Y = a_0 + a_1 x + a_2 z$$

Lecture- 14

TOPIC NAME : Eigen value and Eigen vector

DAY : Thursday

TIME :

DATE : 2 / 11 / 23

General equation:

$A = \lambda$; $A = (n \times n)$ matrix

$\Rightarrow A\vec{x} = \lambda \vec{x}$; λ = Eigen values of A correspond to $\vec{x}$

$\Rightarrow A\vec{x} - \lambda\vec{x} = 0$ — (i) ; $\vec{x}$ Eigen vector of A correspond to λ

$\Rightarrow A\vec{x} = \lambda I \vec{x}$ — (ii) $\vec{x} \neq 0$

$\Rightarrow A\vec{x} - \lambda I \vec{x} = 0$

$\Rightarrow \vec{x}(A - \lambda I) = 0$ — (iii)

$\det (A-\lambda I)\vec{x} = 0$

$A = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$

$\lambda, \vec{x} = ?$

$$A - \lambda I = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & 1 \\ 4 & 5-\lambda \end{bmatrix}$$

$$\det (A - \lambda I) = [(2-\lambda)(5-\lambda)] - 4$$

$$= 10 - 7\lambda + \lambda^2 - 4$$

$\Rightarrow \lambda^2 - 7\lambda + 6$

$\Rightarrow (\lambda - 1)(\lambda - 6)$

$\therefore \lambda = 1, 6$

TOPIC NAME : DAY : TIME : DATE :

when $\lambda = 1$

$$(A-\lambda I)\vec{x} = 0$$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & 1 \\ 4 & 5-\lambda \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 4 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$\Rightarrow$ $R_2 \to R_2 - 4R_1$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$\Rightarrow x_1 + x_2 = 0$

$\Rightarrow x_1 = -x_2$

let, $x_2 = 1$

$x_1 = -1$

$$\vec{x} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

when $\lambda = 6$

$$(A-\lambda I)\vec{x} = 0$$

$$A - \lambda I = \begin{bmatrix} -4 & 1 \\ 4 & -1 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$$\begin{bmatrix} -4 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$-4x_1 + x_2 = 0$

$\Rightarrow 4x_1 = x_2$

$\Rightarrow x_1 = \frac{x_2}{4}$

$x_2 = 1,$ $\quad \vec{x} = \begin{bmatrix} 1/4 \\ 1 \end{bmatrix}$

$x_1 = 1/4$

TOPIC NAME : Polynomial method/ Fadeev Leverrier Method

DAY : 9/11/23/Thursday
TIME :
DATE : 9 / 11 / 23

$A = n \times n$

$\lambda^n - P_1\lambda^{n-1} - P_2\lambda^{n-2} - \cdots - P_n = 0$

For $i = 1$,

$A_1 = A$ and $P_1 = \text{tr } A_1$ → trace of A_1 (Summation of Diagonal values)

For $i = 2, \ldots, n$

$A_i = A(A_{i-1} - P_{i-1}I)$ and $P_i = \frac{\text{tr } A_i}{i}$

$A_2 = A(A_1 - P_1 I)$

$(-1-\lambda)x_1 = 0$

$x_1 + (-2-\lambda)x_2 + 3x_3 = 0$

$2x_2 + (-3-\lambda)x_3 = 0$

$Ax - \lambda I x = 0$

$\Rightarrow (A - \lambda I)\bar{x} = 0$

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 3 \\ 0 & 2 & -3 \end{bmatrix} = A_1$$

$\lambda^3 - P_1\lambda^2 - P_2\lambda - P_3 = 0$

$P_1 = A_1 \text{ tr}$

$= A_1 \times (-6)$

$$A_2 = A(A_1 - P_1 I)$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 3 \\ 0 & 2 & -3 \end{bmatrix} \left(\begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 3 \\ 0 & 0 & -3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right. \begin{bmatrix} 6 & 0 & 0 \\ -6 & 12 & -18 \\ 0 & 0 & 18 \end{bmatrix}$$

$$P_2 = \frac{-6 \cancel{\begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 3 \\ 0 & 2 & -3 \end{bmatrix}} A_2}{2} = -5$$

$$\Rightarrow \lambda^3 + 6\lambda^2 + 5\lambda = 0$$

$$\therefore \lambda = 0, -1, -5$$

For, $\lambda = 0$

$$-x_1 = 0$$

$$x_1 - 2x_2 + 3x_3 = 0$$

$$2x_2 - 3x_3 = 0$$

$\therefore x_1 = 0$,

$$2x_2 = 3x_3$$

if $x_2 = 1$, $x_3 = \frac{2}{3} = 0.667$

$$\therefore X = \begin{bmatrix} 0 \\ 1 \\ 0.667 \end{bmatrix}$$

For, $\lambda = -1$

$$x_1 = 0$$

$$x_1 - x_2 + 3x_3 = 0$$

$$2x_2 - 2x_3 = 0$$

$$\Rightarrow x_2 - x_3 = 0$$

$$\Rightarrow x_2 = x_3$$

$$3x_3 = x_2$$

Power Method

(large one)

$\lambda = 0, -1, 5$

* বড় value নিয়ে কাজ করলে Power method, ছোট value নিয়ে কাজ করলে inverse Power method.

$$X = \begin{bmatrix} \quad \end{bmatrix}$$

$$Y = A \cdot X$$

$$X = \frac{1}{K} Y \quad [K = \text{max value of } Y]$$

X: eigen vector

K: eigen value

$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$$Y = AX$$

$$= A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

; $x_1, x_2, x_3 \neq 0$ at the same time.

Let, $X = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$Y = AX = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

<u>1st it:</u> $\therefore X = \frac{1}{K} Y = \frac{1}{2} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \\ 0 \end{bmatrix} \rightarrow X$

<u>2nd it:</u>

$Y = AX = [\quad] \begin{bmatrix} 1 \\ 0.5 \\ 0 \end{bmatrix}$

$X = \frac{1}{2.5} \begin{bmatrix} 2 \\ 2.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 1 \\ 0 \end{bmatrix}$

$\therefore$ Continue till $X_{n-1} = X_n$

6 it: $X = \underbrace{\frac{1}{3}}_{K} \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}_{\text{vector}}$