Jamal Sir

Lecture-1

TOPIC NAME : Statistics

DAY : Sunday

TIME :

DATE : 6/8/23

We will learn elementary statistics in this course.

Marks Distribution -

30 → CT + Spot Test
(20) (10)

15 → Att + Assignment
(10) (5)

Why we need statistics?

Where uncertainty occurs and to measure the uncertainty.

Statistics : ~ may be defined as the science of collection, organization, presentation, analysis and interpretation of numerical data.

→ to apply statistics, we must convert data to numerical.

Collection : ① Raw/Primary Data ② Secondary Data

→ physically collected data

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Lecture-2

TOPIC NAME: Statistics

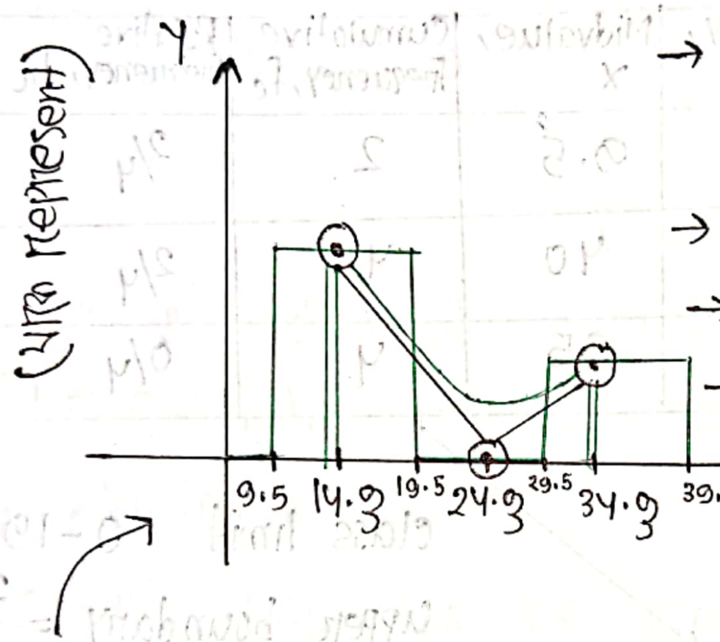
DAY: Thursday

TIME:

DATE: 10/8

Presentation:

① Graphical representation



→ Scattered Diagram
Dot "

→ Polygon — এর Dot connect করে না।

→ Curve (Green line)

→ Bar Diagram

# CI	X	f	fc
10-19	4		
20-29	0		
30-39	3		

Analysis:

① central tendency

Method to measure central tendency -

① Mean $(\bar{x})$ → ⓐ arithmetic (AM) ⓑ Geometric (GM) ⓒ Harmonic Mean (HM)

day-2
DATE: 10/8.

TOPIC NAME:
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② Median ($\tilde{x}$)

③ Mode ($\hat{x}$)

* Will depend on the nature of data.

Assignment: Submission date: 20/8/23

① Even Roll No:

⇒ Discuss about central tendency

ⓐ Formula (Group, Ungroup) → Direct method

ⓑ Use

ⓒ Merit

ⓓ Limitation with example

ⓔ Deviation
↳ for odd roll

* Mean ($\bar{x}$), Median ($\tilde{x}$), Mode ($\hat{x}$)

Mean, $\bar{x} = \frac{\sum x_i}{n}$, $\frac{\sum f_i x_i}{\sum f_i}$

19, 20, 21 ②
→ Derived

GM = $(x \cdot x)^{1/n}$

20, 21, 22, 23, 0
↓ 3 ↓ 5

10, 20, 30 (20)

TOPIC NAME : DAY : TIME : DATE :

② Deviation:

ⓐ Range

ⓑ Mean deviation

ⓒ Standard deviation (***)/Variance ⓔ

ⓓ Quartile deviation

Mean deviation:

19, 20, 21 , $\bar{x} = 20$

$$\frac{|(19-20)| + |(20-20)| + |(21-20)|}{3} \qquad \sqrt{\frac{\sum (x-\bar{x})^2}{n}} = \sigma$$

↑ Standard deviation

Standard deviation:

$$\sigma^2, SD^2 = \left(\sqrt{\frac{\sum (x-\bar{x})^2}{n}}\right)^2 \rightarrow \text{Veriance}$$

12, 24, 36

1, 2, 3

Co-efficient of variation (COV)

$$COV = \frac{SD}{\bar{x} \rightarrow \text{sample mean}} \times 100\% , \Big/ \frac{\sigma}{\mu \rightarrow \text{Population mean}} \times 100\%$$

Quartile deviation:

$f \to$ median group (20-39) $\quad f = 100$

10-19 → 45,7
20-39 → 60,10

$$\text{Median}, \tilde{x} = L + \frac{\frac{f}{2} - f_p}{f_m} \times i$$

⇒ L = Lower Boundary = 19.5

⇒ $\frac{f}{2} = \frac{100}{2} = 50$

⇒ $f_p = 45$ → median group এর আগের।

⇒ $f_m = 10$, median group এর ~~[illegible]~~ frequency.

⇒ i ⇒ 39.5 − 19.5 → যেই Group analysis করব তার class size, 20

i ↓ class size

$$\Rightarrow Q_j = i \times \frac{\frac{jf}{4} - f_{cp}}{f_j} + L$$; j = কত তম quartile

Decile, $D_j = L + \frac{\frac{jf}{10\cancel{0}\cancel{4}} + f_{cp}}{f_j} \times i$

Percentile, $P_j = L + \frac{\frac{jf}{100} + f_{cp}}{f_j} \times j$

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#

	$\bar{x}$ (Mean)	S.D	n
A	6	3	5
B	10	4	10

→ Which company has more variability?

→ S.D দিয়ে check করা যাবেনা, cov দিয়ে করা লাগবে।

$$cov(A) = \frac{S.D}{\bar{x}} \times 100 = \frac{3}{6} \times 100 = 50\%$$

$$cov(B) = \frac{4}{10} \times 100 = 40\%$$

③ Skewness: ④ Kurtosis:

* Formal definition + example জানা লাগবে।

Moment: # $\mu_r' = \frac{\sum (x-A)^r}{n}$ # $x_1, x_2 - - - - x_n$

⇒ rth moment measure from arbitrary number A.

$\mu_r' = \frac{\sum x^r}{n}$; $A = 0$

rth moment measure from origin.

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$\mu_r = \frac{\sum(x-\bar{x})^r}{n}$; $A=\bar{x}$

$\downarrow$

central moment.

For Group data:

$\mu_r' = \frac{\sum f_i (x_i - A)^r}{\sum f_i}$; f_i = ith Grp er frequency

x_i = ith Grp er mid value.

For, $A = 0$

$$\mu_r' = \frac{\sum f_i x_i^r}{\sum f_i}$$

For, $A=\bar{x} \rightarrow$ central moment

$$\mu_r = \frac{\sum f_i (x_i - \bar{x})^r}{\sum f_i}$$

First raw moment,

$$\mu_1' = \frac{\sum x_i}{n} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i} \text{ (GD)}$$

$$\mu_2' = \underset{\text{(UGD)}}{\frac{\sum x_i^2}{n}} = \frac{\sum f_i x_i^2}{\sum f_i} \text{ (GD)}$$

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$\mu_1 = \frac{\sum(x_i - \bar{x})}{n}$

$$= \frac{(x_1-\bar{x})+(x_2-\bar{x})+\cdots+(x_n-\bar{x})}{n}$$

$$= \frac{\sum x_i - \sum \bar{x}}{n}$$

$* \sum 5 = 5n$

$$= \frac{\sum x_i}{n} - \frac{\sum \bar{x}}{n}$$

$$= \frac{\sum x_i}{n} - \frac{n\bar{x}}{n}$$

$$= \bar{x} - \bar{x} = 0$$

* First central moment, $\mu_1 = 0$

GOOD LUCK™

Lecture-4 :

TOPIC NAME : Moment

DAY : Sunday

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Sir

* Second central moment, $\mu_2 = \frac{\sum(x-\bar{x})^2}{n} = \frac{\sum f(x-\bar{x})^2}{\sum f} = \sigma^2$

$= \frac{1}{n}\left[\sum(x^2 - 2x\bar{x} + \bar{x}^2)\right]$

* $SD^2 = \mu_2$

$= \frac{1}{n}\left[\sum x^2 - \sum(2x\bar{x}) + \sum \bar{x}^2\right]$

$= \frac{1}{n}\sum x^2 - 2\bar{x}\frac{\sum x}{n} + \frac{1}{n}\sum \bar{x}^2$

$= \frac{1}{n}\sum x^2 - 2\bar{x}\cdot\bar{x} + \frac{1}{\cancel{n}} \times \cancel{n}\bar{x}^2$

$= \frac{1}{n}\sum x^2 - \bar{x}^2$

$= \frac{1}{n}\sum x^2 - \left(\frac{\sum x}{n}\right)^2 = \mu_2' - \mu_1'^2$

* $\mu_3 = \frac{\sum(x-\bar{x})^3}{n}$

* $\mu_r = \frac{\sum(x-\bar{x})^r}{n}$

Skewness

* Third central moment হতে skewness পাওয়া যায়।

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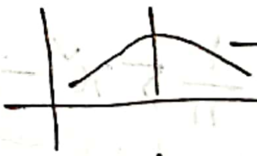
* Co-efficient of skewness $= \dfrac{\mu_3}{(\sqrt{\mu_2})^3} \Big/ \beta_1 = \dfrac{\mu_3^2}{\mu_2^3}$

$\downarrow$

কতটুকু বিচ্যুতি হয়েছে তা বুঝাতে।

$\Rightarrow$ Sign এর জন্য, $\beta_1 = \dfrac{\mu_3}{(\sqrt{\mu_2})^3} \rightarrow$ Karl Pearson's co.efficient of skewness.

* $\beta_1 = 0$ হলে symmetric



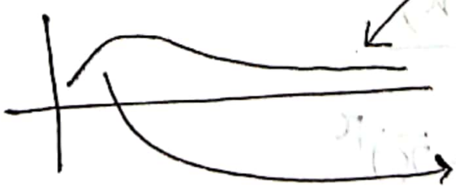
$\rightarrow$ mode থেকে সমানভাবে ছড়ানো

<u>Physical meaning</u> –

* $\beta_1 < 0$ "

Data ছড়ানো বেশি, frequency কম।

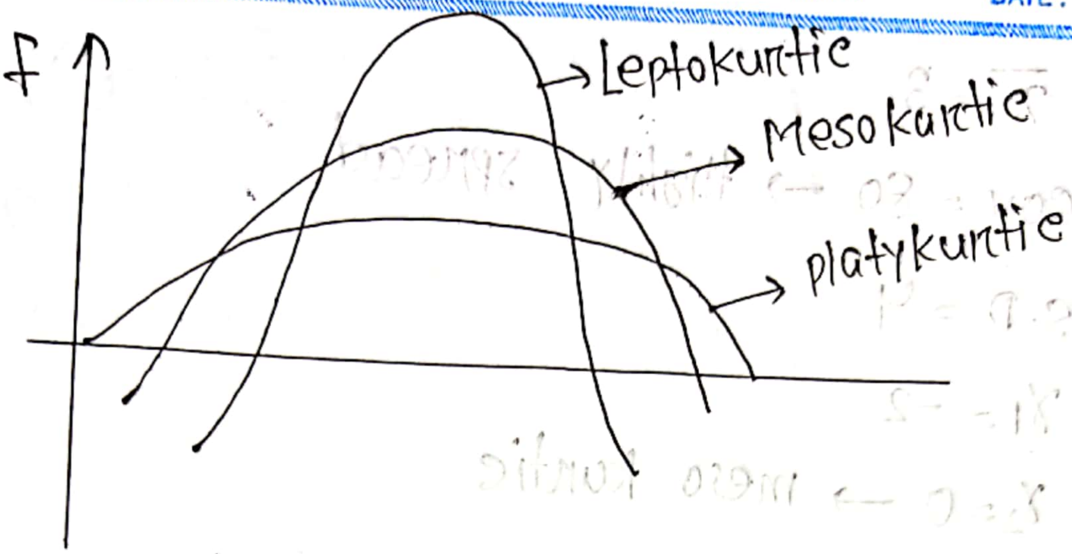
* $\beta_1 > 0$ "



টানা

<u>Kurtosis</u>

* Peakness of the frequency.
* Fourth moment use করা হয়।



$\beta_2 = \frac{\mu_4}{\mu_2^2} = 3$ (mesokurtic) * $\gamma_2 = \beta_2 - 3$

co-efficient of Kurtosis $= 0 \rightarrow$ mesto

$\gamma_2 < 0 \rightarrow$ platy

$3 < \beta_2 \rightarrow$ Leptokurtic $\gamma_2 > 0 \rightarrow$ Lepto

$3 > \beta_2 \rightarrow$ platy kurtic * $fx^3 \rightarrow$ skewness

$fx^2 \rightarrow$

#

	CI	F	x	x^2	x^3	x^4
	1-2	1	1.5			
	2-4	3	3			
	4-6	2	5			

fx	fx^2	fx^3	fx^4
1.5	2.5	3.36	
9	2.7	249	
10	50	250	

Find first fourth raw moment measured from 2ৰ ওপৰত, Hence find first central moment. Then find SD, veriance, skewness etc

TOPIC NAME : DAY : TIME : DATE :

$\bar{x} = 3$

cov = 80 → Highly spread

S.D = 4

$\gamma_1 = -2$

$\gamma_2 = 0$ → meso kurtic

* $\mu_1' = \frac{\sum fx}{\sum f} = \frac{20.5}{6} = \bar{x} = 3.42$

$\mu_2' = \frac{\sum fx^2}{\sum f} = \frac{79.5}{6} = 13.25$

$\mu_2 = \mu_2' - {\mu_1'}^2 = 13.25 - (3.42)^2 =$ Variance

$S.D = \sqrt{\mu_2}$

Lecture-5

TOPIC NAME : Data tendency

DAY: Thursday

TIME:

DATE: 24/8/23

Moment Relation:

* $\mu_1 = 0$
* $\mu_2 = \mu_2' - \mu_1'^2$

* $\mu_r = \frac{\Sigma (x-\bar{x})^r}{\Sigma f} = \frac{\Sigma f (x-\bar{x})^r}{\Sigma f}$

* $\mu_3 =$
* $\mu_4 =$

Prove করা লাগবে এই ৭টা, Exam এ আসতে পারে।
For both group data and ungrouped data.

কোন এক class test এর marks থেকে -

$n = 20$ জন

$\bar{x} = 15$ → average ভালো করছে

$\sigma = 1.2$ → $\bar{x}$ থেকে diff কম, dense

$\gamma_1 = 3$

$\gamma_2 = 4$ → leptokurtic

*** এই type analysis করতে দিবে (Both group and ungroup data।)

TOPIC NAME :

Theory of Probability

Deletermenestic Exp: input same ও output always same আসবে

Probabilistic Exp/Random/stochestic: same input এর জন্য output different আসবে।

A = {HH, HT, TH, TT}

Sample space: S → Possible all outcome, A

Sub set: Universal set এর কয়েকটা element নিয়ে যে set.

Event: Random experiment এর output

A = {H}, B = {T}, C = {H, H}

Mutually Exclusive event:

A, B, C ------

একটা হলে (A), আবার (C) হবে না → A, C mutually exclusive event.

TOPIC NAME : DAY : TIME : DATE : / /

* $A \cap B = \emptyset \Rightarrow P(A \cap B) = P(\emptyset) = 0$
* $A \cup B = A + B \Rightarrow P(A \cup B) = P(A) + P(B)$

} important for theory

Power Set: 2^n

Borel Field: Sample space's all element including $\emptyset$.

$\cap, \cup$

Mutually

Exhustive Event: $S = \{H, T\}$ (W)

$A = \{H\}$ $B = \{T\}$, $C = \{H, T\}$

$A \cup B = S$

Dependent and independent event: (W)

$A \xrightarrow{\times} B \longrightarrow$ independent

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$

$\boxed{5R, 8B}$ $P(R) = \frac{5}{13}$

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TOPIC NAME : DAY : TIME : DATE :

* $P(A\cap B) = \underbrace{P(A)\cdot P(B/A)}_{\text{Dependent}} = \underbrace{P(A)\cdot P(B)}_{\text{independent}}$

Conditional Probability: $P(B/A) = \dfrac{P(A\cap B)}{P(A)}$

$\Rightarrow P(A) \neq 0$

Lecture-6

27/8/23

Sunday

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Theory of Probability

classify mutuolly exclusive independent event

* Elementary event : {H, T}

① $A\cap B = \emptyset$

$P(A\cap B) = P(A)\cdot P(B)$ inde

② $A\cup B = A + B$

③ $P(A\cap B) = 0$

(***) Discuss about theory of probability.

Soln: 3 way to define.

GOOD LUCK

TOPIC NAME : DAY : TIME : DATE : / /

① Classical Approach :

Condition –

এই দুইটাই limitation ←

① Every element equally likely

② Possible outcome must be known.

$P = \frac{h}{n}$; h = favorable event of possible outcome

n = Possible outcome.

coin toss, $P = \frac{1}{2}$

$P = \frac{\text{either head or tail}}{2}$

② Frequency Approach :

Condition :

① Possible outcome is very large ($n \to \infty$ / $n >> 1$) lim

→ large আর large, এইটা limitation.

$P = \frac{F}{n}$; f = favorable outcome

$P = \frac{500}{100000}$; Bias | Unbias কোনটা জানি না।

TOPIC NAME : DAY : TIME : DATE :

③ Exiomatic Approach :

Condition :

① S → Sample space

A , B → event

ⓐ $0 \leq P(A) \leq 1$ → Non negative

② $P(S) = 1$ → Total probability

③ A, B mutually exclusive event

$$P(A \cup B \cup ----) = P(A) + P(B) + -----$$

Theorm of classical Approach :

For possible outcome -

① Multiple law :

A → n_1, B → n_2 ----

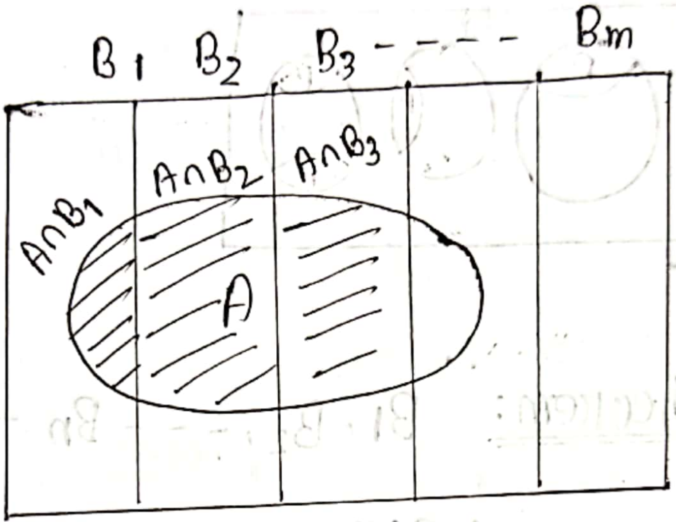
$n_1 \times n_2 \times$ ----

R → 3   C → 5   D → 2

3×5×2

(ii) Permutation law : $60P_3$, ^nP_r → order আছে ।

(iii) Combination law: $^{n}C_{r} \rightarrow$ কোন order নাই।



B_1, B_2, B_3 --- এরা mutually exclusive event.

Theory of

(*) Total probability of a event A, $P(A) = \sum_{i=1}^{m} P(B_i)\, P(A/B_i)$

$$A = A\cap B_1 + A\cap B_2 + \cdots + A\cap B_n$$

$$\Rightarrow P(A) = P(A\cap B_1 + A\cap B_2 + \cdots + A\cap B_n)$$

$$\Rightarrow P(A) = (A\cap B_1) \cup A(\cap B_2) \cup \cdots$$

$$\Rightarrow P(A) = P\left[(A\cap B_1) \cup (A\cap B_2) \cup \cdots\right]$$

$$= P(A\cap B_1) + P(A\cap B_2) + \cdots + P(A\cap B_n)$$

$$= P(B_1)\times P(A/B_1) + P(B_2)\times P(A/B_2) + \cdots + P(B_n)\times P(A/B_n)$$

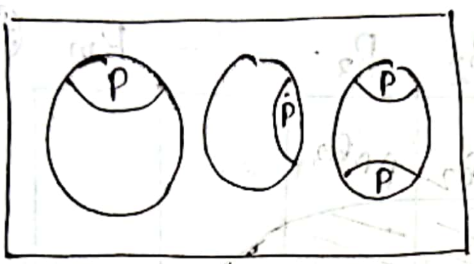
$A\cap B = \emptyset$

$A\cup B = A+B$

$A(A\cup B) = P(A) + P(B)$

TOPIC NAME : DAY : TIME : DATE :

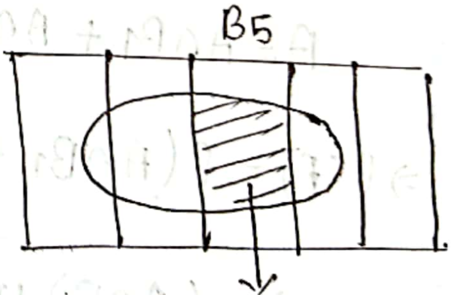
$$\therefore P(A) = \sum P(B_i)\, P(A/B_i)$$

\# 

<u>Baye's Theorem:</u> $B_1, B_2, \ldots B_n \rightarrow$ mutually exclusive event

$A, P(A) \neq 0$ $\rightarrow$ mutuall exhaustive event.

$$P(B_i/A) = \frac{P(B_i)\, P(A/B_i)}{\sum_K P(B_K)\, P(A/B_K)}$$



এইটা বের করব। (B_i)

Lecture-7

TOPIC NAME: The Theory of Probability

DAY: Sunday

TIME:

DATE: 3 / 3 / 23

Out of 100 Jobs of a computing center, 50 are class A, 30 of class B and 20 are of class C. A Sample of 30 Jobs is taken with replacement.

① Find the probability that the sample will contain 10 Jobs of each class.

② Find the probability that there will be exactly 12 Jobs of class C.

Soln: Here total possibility = $^{100}C_{30}$

① Favourable possibility evenets $= A_n \times B_n \times C_n$

$= {}^{50}C_{10} \times {}^{30}C_{10} \times {}^{20}C_{10}$

$\therefore$ Probability of $P(1) = \dfrac{{}^{50}C_{10} \times {}^{30}C_{10} \times {}^{20}C_{10}}{{}^{100}C_{30}}$

② Favourable possibility events $= {}^{20}C_{12} \times {}^{100-20}C_{18}$

$\therefore$ Probability of $P(2) = \dfrac{{}^{20}C_{12} \times {}^{80}C_{18}}{{}^{100}C_{3}}$

TOPIC NAME : DAY : TIME : DATE :

Four components are inspected and three events are A, B, C defined as follows -

A = All four components are found to be defective.

B = Exactly two components are found " "

C = At least three " " " " " .

Soln: ① Define above events by the element of set.

② Interpret the following events -

ⓐ $B \cup C$ ⓒ $A \cup B$

ⓑ $B \cap C$

Soln: Let, 0, 1 are indicated as defective and non defective/proper element.

So, sample space { (0,0,0,0)

(1,0,0,0)

⋮

}

TOPIC NAME : DAY : TIME : DATE : / /

0,0,0

A = (i) 0 → $^4C_4 = 1$

(i) 0 → $^4C_1 = 4$

B = (ii) 0,0 → $^4C_2 = 6$

C = (iii) 0,0,0 → $^4C_3 = 4$

P (iv) 1,1,1,1 → $^4C_0 = 1$

Sample space = { (0,0,0,0)
(1,0,0,0), (0,1,0,0) - - -
(1,1,0,0), (1,0,1,0) - - -
(0,0,0,1) - - -
(1,1,1,1)
}

A = {(0,0,0,0)}

B =

C = A ∪ P

(a) B ∪ C = {Exactly 2 deft} OR {Atleast 3 deft}
= At least two defective.

(b) B ∩ C = { " } And { " }
= Impossible event.

(c) A ∪ B = {All four event deft} OR {Exactly 2 deft}
= Either all are deft or exactly 2 element to be deft.

TOPIC NAME : Disjoint

DAY :

TIME : DATE :

$$P(A\cup B) = \frac{Poss(A\cup B)}{Poss(S)} = \frac{Pos(A)+Pos(B)}{Pos(S)}$$

$$= \frac{1+6}{16}$$

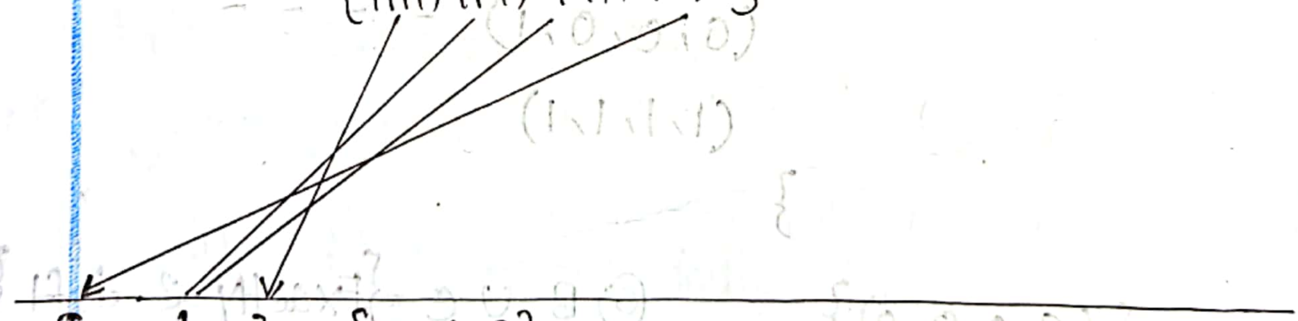
Disjoint না

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

Probability Distribution

Random variable : Formal Definition শিখা লাগবে।

$\{HH, HT, TH, TT\}$



0 1 2 $\{0,1,2\}$

$x=1 = \{HT, TH\}$

$x=2 = \{HH\}$

$x = 3 \times \text{no of } H - 5$

$x_2 = 3\times 2 - 5 = 1$

$x_1 = 3\times 1 - 5 = -2$

TOPIC NAME :   DAY :

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{HH, HT, TH, TT}

$x = \{0, 1, 2\}$ → Sample space

(P)

$1/4$ $1/2$ $1/4$

$P(x=0) = 1/4$

$P(1) = 2/4 = 1/2$

Lecture-8

7/9/23

Thursday

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Probability Distribution

1, 2, 3, 4, 5, 6 : S

$x = |1, |4, \backslash 7$

$1/6, 1/6, 1/6$ ---- $1/6$

$x = 3S - 2$

$x = S$

Random variable

* RV change হয় but probability change হয় না।

↓

Discrete variable

TOPIC NAME : DAY : TIME : DATE :

Discrete Probability Distribution :

d.r.v : x : x_1 x_2 - - - - - - x_n

P : P_1 P_2 . P_n

(i) $P \geq 0 \rightarrow$ non negative

(ii) $\sum P_i = \sum P(x_i) = 1 \rightarrow$ Total Probability

$P \rightarrow$ Probability Mass Function $\rightarrow$ Pmf

$\downarrow$

Probability or Frequency

Distribution function $\rightarrow$ cumilitive ~~fun~~ frequency : $P(x) \leq a$

TOPIC NAME : DAY : TIME : DATE : / /

x −2 0 3
P .5 .2 (.2K) .3

* দুইটা law ই পূরণ করা লাগবে।

① $P_i \geq 0 \rightarrow$ individual check

② $\sum P = .5 + .2 + .3 = 1$

So, table is discrete probability distribution.

For .2K

$\sum P_i = 1$

$\Rightarrow .5 + .2K + .3 = 1$

$\Rightarrow K = 1$ → K এর মান 1 এর জন্যই distribution.

x −2 0 3
P .5 .6 −.1

$P(x \leq 5)$ law ই মানে না

So, বের করা possible না।

TOPIC NAME : DAY : TIME : DATE :

Continuous Probability Distribution:

C.R.V x:

$f(x)$:

① $f(x) \geq 0$

② $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$\Rightarrow f(x)$ ①, ② পূরণ করলে $f(x)$ হবে continuous probability distribution

আর $f(x)$ হবে prob. dense function (PDF)

$\therefore f(x \leq a) \rightarrow$ Distribution function

$f(x) = \frac{1}{2}$; $1 \leq x \leq 3$

0 ; otherwise

① x যাই হোক মান 0 থেকে বড় / সমান।

② $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx + \int_{3}^{\infty} f(x)\,dx$

TOPIC NAME : DAY : TIME : DATE : / /

$$= 0 + \int_1^3 \tfrac{1}{2}\, dx + 0$$

$$= 1$$

$P(0 \le x \le \frac{3}{2})$

$$\int_1^{3/2} \tfrac{1}{2}\, dx$$

⊛ Pdf x integration → distribution function

distribution function x diff → Pdf

$f(x) = \tfrac{1}{2}$; $1 < x \le 2$

$= 0$; otherwise

$P(x \le a)$

$$\int_{-\infty}^{a} f(x)\, dx$$

GOOD LUCK™

TOPIC NAME : DAY : TIME : DATE :

Mathematical Expectation :

Mean, variance, Kurtosis বের করা যায়।

$x \to d.r.v$

$P(x) \to pmf$

$E(x) = \sum x P(x)$ | $\sum g(x) = \sum g(x) P(x)$

$E[g(x)]$

Expectation of x

$x \to c.r.v$

$f(x) \to pdf$

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$E[g(x)] = \int_{-\infty}^{\infty} g(x) f(x)\, dx$$

$g(x) = 3x^2 + x - 5$

Discrete:

$$E[g(x)] = E(3x^2 + x - 5)$$
$$= E(3x^2) + E(x) + E(-5)$$
$$= 3E(x^2) + E(x) + E(-5)$$

*** $E(c) = \sum c P(x)$
$= c \sum P(x)$
$= c$

TOPIC NAME : DAY : TIME : DATE : / /

$E(x^v) = \sum x^v p(x)$

$= (-2)^v (\cdot 5) + 0^v \times (\cdot 2) + (3)^v \times 0.3$

Lecture-9
Sunday
10/9/23

Jamali Sir

Mathematical Expectations

$F(x) \to$ Distribution Function

x	1	2	3	4
P	P_1	P_2	P_3	P_4

$P(2) = P_2$

$\Rightarrow F(2) = P(x \le 2) = P_1 + P_2$

$\Rightarrow F(4) = P(x \le 4) = P_1 + P_2 + P_3 + P_4$

$\Rightarrow P(2 \le x \le 4) = F(4) - F(2)$

$= (P_1 + P_2 + P_3 + P_4) - (P_1 + P_2)$

Discrete

$= P_3 + P_2$

$f(x) = 1/2 \;;\; 1 \le x \le 3$

$= 0 \;;\;$ otherwise

TOPIC NAME : DAY : TIME : DATE :

$\Rightarrow F(x) = P(\leq x)$

$f(x) = \text{pdf}$

$\Rightarrow F(x) = \int_{-\infty}^{x} f(x)\,dx$

$\Rightarrow \boxed{F'(x) = f(x)}$

$\Rightarrow P(a \leq x \leq b) = \int_{a}^{b} f(x)\,dx$

continuous

$\Rightarrow F(a) = P(x \leq a)$

$= \int_{-\infty}^{a} f(x)\,dx$

$\Rightarrow F(b) = \int_{-\infty}^{b} f(x)\,dx$

$\Rightarrow P(x \leq b) - P(x \leq a)$

$= F(b) - F(a)$

$F(x) = 2x$

$\Rightarrow F'(x) = 2 = f(x)$

$F(x) = \frac{x}{2}$

$F'(x) = f(x) = \frac{1}{2}$

0 ; otherwise

GOOD LUCK™

TOPIC NAME : DAY : TIME : DATE : / /

$\Rightarrow E(x) = \sum x\,p(x) = \mu_1'$

$= \int_{-\infty}^{\infty} x\,f(x)\,dx = \mu$ (mean)

$\Rightarrow E(x^2) = \mu_2'$

$$\left.\begin{aligned} E(x^r) &= \sum x^r\,p(x) \\ &= \int_{-\infty}^{\infty} x^r f(x)\,dx \end{aligned}\right\} \mu_r'$$

$\Rightarrow$ Mean, $\mu = \mu_1' = \sum x\,p(x)\,dx$

	TT	TH	HT	HH
H	0	1	1	2
x	-2	3	[illegible]	8
P	1/4	2/4	[illegible]	1/4

$x = 5H - 2$

$\Rightarrow \mu = E(x) = \mu_1' = \sum x\,p(x)$

$= -2\times 1/4 + 3\times 2/4 + 8\times 1/4$

$= 3$

$\Rightarrow \mu_2 = \mu_2' - (\mu_1')^2$

σ^2 $= E(x^2) - (E(x))^2$

$= 21.5 - 9$

$= 12.5$

$\Rightarrow E(x^2) = \sum x^2 p(x)$

$= (-2)^2\times 1/4 + (3)^2\times \frac{2}{4} + (8)^2\times 1/4$

$= 4\times\frac{1}{4} + 9\times\frac{2}{4} + 64\times\frac{1}{4}$

$= 1 + \frac{18}{4} + \frac{64}{4} = \frac{43}{2}$

$= 21.5$

GOOD LUCK™

TOPIC NAME : continuous

DAY :

TIME : DATE :

Test weather the following function be distribution function or not then if possible find μ, σ, cov, γ_1, γ_2, $E(3x^2+2x+5)$ $P(2 \leq x \leq 7)$

$P(2 \leq x \leq 9) \Rightarrow P(3)+P(8) = \frac{2}{4}+\frac{1}{4}$

$E(3x^2+2x+5)$

$= 3E(x^2) + 2E(x) + \underbrace{E(5)}_{5}$

Discrete হবে

where, $F(x) = \frac{1}{2}$; $5 \leq x \leq 7$

$F(x) = x/2$

$= 0$; otherwise

Soln: Here $F'(x) = 0 = f(x)$

$\int_{-\infty}^{\infty} F(x) \neq 1$

so, it is not distribution fn so, we can not find any of these.

TOPIC NAME : $F(x)$ DAY : TIME : DATE : / /

$\therefore F'(x) = \frac{1}{2} \;;\; 5 \leq x \leq 7$

$= 0 \;;$ otherwise

It will be pdf if -

$\int_{-\infty}^{\infty} f(x)\, dx = 1$.

$= \int_{5}^{7} \frac{1}{2}\, dx = 1$ → This is pdf and $F(x)$ probability distribution function.

$\Rightarrow P(2 \leq x \leq 7) = F(7) - F(5)$

$= \frac{x}{2}\Big|_{7} - \frac{x}{2}\Big|_{5} = 7$

$F(7) - F(2)$

$= \frac{x}{2}\Big|_{7} - 0/_{2}$

Distribution দেখা না থাকলে $= \int_{2}^{7} f(x)\, dx$

$f(x) = \frac{1}{2}$

$F(x) = \int f(x) = \int \frac{1}{2} dx = \frac{x}{2}$

$3E(x^2) + 2E(x) + E(5)$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$

$= \int_{5}^{7} x^2 \times \frac{1}{2}\, dx$

TOPIC NAME : DAY : TIME : DATE :

$$E(x) = \int_5^7 x \times \frac{1}{2} dx$$

$$\mu_2 = \mu_2' - (\mu_1')^2$$

$$= E(x^2) - E(x)$$

Lecture-10

17/9/23

Sunday

Jamali Sir

Theory of Probability

[I] Discreet Distribution -

$x \rightarrow$ d.r

① Binomial Probalitity Distribution

Geomatric Prob. Dis — Negative "

② Bernoulli Prob. Dis.

③ Poisson Prob. Dis.

Bernoulli Trials (***)

Specification:

① Only two outcomes (Success & fail)

② Each trial is independent.

③ Probability of Success of each trial is fix.

TOPIC NAME : DAY : TIME : DATE : / /

Bernoulli Prob. Dis.

r.v, $x = 0, 1$

$P(0) = 1/2$, $P(1) = 1/2$

r.v, $x = 6$, not 6

$P(6) = 1/6$, $P(!6) = 5/6$

x : less than 2, not less than 2

Binomial Prob. Dis. (***) → Definition 72

finite and fixed ← n → trials

x → success

p → Prob. of success in each trial ↓ Bernoulli trial

$q = 1 - p$

$$\Rightarrow P(x) = {}^nC_x \times p^x \times q^{n-x}$$

Assumption :

① Must be Bernoulli trial

② n must be finit and fixed.

A family contains 4 children boy or not boy.

① What is the Prob. that the family has no any boy.

Soln: $\underbrace{n=4, p=0.49, q=0.51,}_{\text{Assumption}}$ $x = 0, 1, 2, 3, 4$

TOPIC NAME : DAY : TIME : DATE :

$P(\text{no boy}) = P(x=0) = P(0) = {}^4C_0 (0.49)^0 (0.51)^{4-0}$

② Atleast one boy

$P(\text{at least one boy}) = P(x>0) = 1 - P(x \le 0)$

Property:

(i) $P(x) > 0$

(ii) $\sum_{x=0}^{n} {}^nC_x p^x q^{n-x}$

এগুলো হলে P.d.f হবে।

① Discrete Prob. dis.

② Parameter = 2 (P, n)

③ mean = np

Variance, $\sigma^2 = np^3$

Derivation নিজে করা লাগবে।

$\Rightarrow$ COV

$E(x)$

$E(x^2) - [E(x)]^2$

Lecture-11

TOPIC NAME : Theory of probability

DAY : Thursday

TIME :

DATE : 21 / 9 / 23

Bionomial Distribution:

$\Rightarrow$ Mean $= np = \mu$

Find the mean of B.D

$$\text{mean} = E(x)/(\mu_1') = \sum x p(x)$$

$$= \sum x\, {}^{n}c_x\, p^x q^{n-x}$$

$$= 0 \cdot {}^{n}c_0\, p^0 q^{n-0} + 1 \cdot {}^{n}c_1\, p^1 q^{n-1} + 2\, {}^{n}c_2\, p^2 q^{n-2} + \cdots + n\, {}^{n}c_n\, p^n q^{n-n}$$

$$= n p q^{n-1} + 2 \cdot \frac{n(n-1)}{2!} p^2 q^{n-2} + \frac{n(n-1)(n-2)}{3!} p^3 q^{n-3} + \cdots + n \cdot p^n$$

$$= np\left[q^{n-1} + \frac{(n-1)}{1!} p q^{(n-1)-1} + \frac{(n-1)(n-2)}{2!} p^2 q^{(n-1)-2} + \cdots + p^{n-1}\right]$$

$(q+p)^{n-1}$

$$= {}^{(n-1)}c_0\, q^{n-1} p^0 + {}^{(n-1)}c_1\, q^{n-2} p^1 + {}^{(n-1)}c_2\, q^{n-3} p^2 + {}^{(n-1)}c_3\, q^{n-4} p^3 + \cdots + {}^{n-1}c_{n-1}\, q^0 p^{n-1}$$

$$= q^{n-1} + (n-1) q^{n-2} \cdot p + \frac{(n-1)(n-2)}{2!} q^{n-3} p^2 + \cdots + p^{n-1}$$

$$= np\left[q+p\right]^{n-1}$$

$$= np$$

TOPIC NAME : $\sigma^2 = \mu_2' - (\mu_1')^2$

DAY :

TIME : DATE :

$\Rightarrow$ Variance, $\sigma^2 = E(x^2) - [E(x)]^2 = npq$

$\Rightarrow$ Skewness $\Rightarrow$

$\Rightarrow$ Kurtosis $\Rightarrow$

Geometric Probability Distribution

$\Rightarrow$ 1st Success at nth trial

F, F, F, F ------- F, P

$\underbrace{\qquad\qquad\qquad}_{n-1}$ $P(F) = q$

S

P

p^n

$= q^{n-1}$

$\therefore P(x) = q^{n-1} p$

Negative B.D / Pascal P.D

$\Rightarrow$ kth success at nth trial

F F S S F S F ... S

$\underbrace{\qquad\qquad\qquad}_{k-1}$ n

$P(x) = {}^{n-1}C_{k-1} \, p^{k-1} \, q^{(n-1)-(k-1)} \, p$

$= \binom{n-1}{k-1} p^k q^{n-k}$

TOPIC NAME : DAY : TIME : DATE : / /

A carside parking facility has a capacity for 3 cars. It is estimated that six cars will pass this parking place within the time span. On the average 80% of all car will want to park there.

① What is the probability that 3 car full the park.

② " " " 6th " " "

③ Determine the probability that it will be full within the time span.

① soln: N.B.D, $P(x) = \binom{n-1}{k-1} p^k q^{n-k}$

$K = 3,\ n = 3,\ p = 0.8,\ q = 0.2,\ x = 3$

$\Rightarrow P(n,k) = P(3,3)$ $\quad P(3) = \binom{3-1}{3-1} (0.8)^3 (0.2)^0 = 1 \times (0.8)^3$

(ii) soln: $n = 6,\ K = 3,\ P = 0.8,\ q = 0.2$

$P(6,3) = \binom{6-1}{3-1} (0.8)^3 (0.2)^{6-3}$

TOPIC NAME : DAY : TIME : DATE :

(iii) Soln:

$$P(x \le 6)_{K=3} = \sum_{i=3}^{6} \binom{i-1}{K-1} p^K q^{i-K} \qquad K=3$$

$$= \binom{3-1}{3-1} p^3 q^{3-3} + \binom{4-1}{3-1} p^3 q^{4-3} + \binom{5-1}{3-1} p^3 q^{5-3} + \binom{6-1}{3-1} p^3 q^{6-3} = 0.983$$

*** $n >> 1$ and $p << 1$ and $np < 5$

Poisson Distribution

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

* $\lambda = np = \text{mean}$
* $\sigma^2 = \lambda$
*

Assignment:

(i) Derive mean (μ), SD (σ), COV, COS, COK from poisson Distribution.

Submission – mid এর আগের last thursday.

TOPIC NAME : DAY: TIME : DATE :

$P = 0.0011$

2000 individual

$n = 2000$

$P = 0.0011$

$\lambda = nP = 2$

(1) Find Proba that at least one will get the service :-

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad nP = 2$$

$$= \frac{2^x e^{-2}}{x!}$$

(1) $P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(0)$

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Lecture-12

TOPIC NAME : Theory of probability

DAY : Sunday

TIME :

DATE : 1 / 10

Several times (many times)

15 coins are tossed at a time and count the no of heads. The observation are given below —

Fit it Binomial distribution

x no of heads	f_i, O_i Frequency	$f_i x$
0	8	0
1	10	10
2	25	50
3	34	102
4	28	
5	15	75
	120	

Soln: We know B.D -

$$P(x) = {}^nC_x \, p^x \, q^{n-x}$$

$\Rightarrow$ Fair/unbiased হলে $1/2$

$n=5$

$p = ?$

* বলা না থাকলে p বের করা লাগবে।

In B.D mean, $\mu = np$

$$\mu, \bar{x} = \frac{\sum f_i x_i}{\sum f} = \frac{315}{120} = 2.625$$

exact mean না, estimate করা $\leftarrow$ $\hat{\mu} = \bar{x} = (\widehat{np}) = n\hat{p}$

$\Rightarrow 2.625 = 5\hat{p}$

$\Rightarrow \hat{p} = 0.53$

$\therefore$ Model, $P(x) = {}^5C_x (0.53)^x (0.47)^{5-x}$

Round off to nearest integer

TOPIC NAME :

DAY :

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X	$P(X)$	E_i $E_i = N \times P(x)$	→ তার চিরে নেয় f_i^2/E_i
0	$P(0)=$	$120 \times P(0)$ 2	32
1	$P(1)=$	11	9.09
2	$P(2)=$	30	20.83
3	$P(3)=$	41	28.10
4	$P(4)=$	28	28
5	$P(5)=$	18	12.5
		120	$\sum f_i^2/E_i = 130.62$

Test the goodness of the fit :

(With 5% level of significance) / অথবা না থাকলে 5% level of significance দিয়ে করা লাগবে।

(" 95% confidence level)

chi /Kai square Test ← $\chi^2_{(\nu, \alpha)}$

ν = Degree of freedom $= m-1-k$
$= 6-1-1$
$= 4$

m = no of Random variable $= 6$

k = no of estimated parameter $= 1\ (\hat{p})$

*Unbiased হলে $k=0$ হবে।

α = level of significance
$= 0.05$

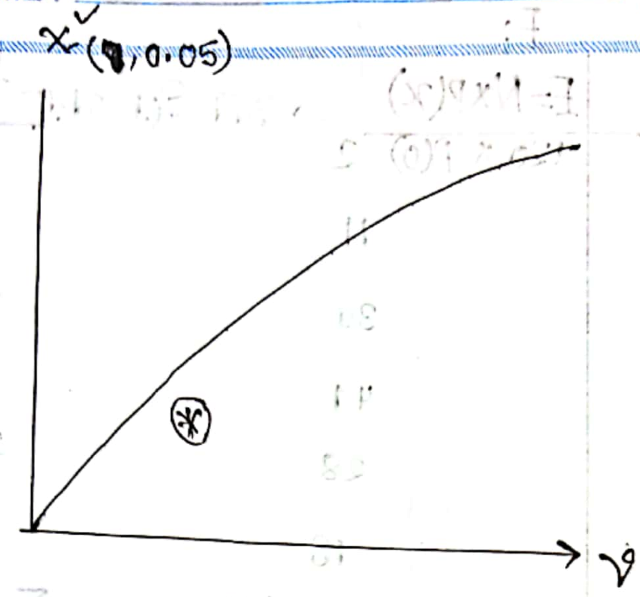
$1-\alpha = 0.95$ = confidence level

মূল্য error ← $\chi^2 = \frac{\sum (O_i - E_i)^2}{E_i} = \sum\left(\frac{O_i^2}{E_i}\right) - N = 130.62 - 120 = 10.62$

$\chi^2_{cal} = 10.62$

GOOD LUCK

TOPIC NAME : DAY : TIME : DATE :



3 condition for χ^2:

① $\chi^2_{tab} > \chi^2_{cal} \longrightarrow$ Fit is good, with 5% level of significance.

② $\chi^2_{tab} = \chi^2_{cal} \longrightarrow$ No conclusion with 5% level of significance

③ $\chi^2_{tab} < \chi^2_{cal} \longrightarrow$ Fit is not good " "

χ^2_{tab}

ν		$\chi^2_{0.95}$	
0			
1			
2			
⋮			
4		9.49	

Lecture-13

TOPIC NAME: Theory of probability

DAY: Thursday

TIME:

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☒ From previous math -

$$\chi^2_{cal} = 10.62$$

From χ^2 table -

$$\chi^2_{(\nu,\alpha)} = \chi^2_{(4,0.05)} = 0.711$$

☒ Tabular χ^2

ν		0.01	0.05	0.10
0				
1				
2				
3				
4			0.711	

tolerant / threshold

⇒ বেশি হলে = no good

⇒ 0 =

⇒ ছোট হলে = good

⇒ $\chi^2_{cal} = 10.62 > \chi^2_{(4,0.05)} = 0.711$

comment: The fit is not good at all, with respect to 5% level of significance.

⇒ Bernoulli trial এর event (yes/no) গুলোকে একসাথে Bernoulli Process বলা হয়।

Poission Process (PP)

A PP is a series of event occurring at a mean rate over time/space. All Poission Process share the following properties and differs from each other mathematically only by the level for λ (λ = average density).

Properties:

1. The number of events occurring in one segment of time/space is independent of the number of events occuring in any previous non-overlaping segment.

independent, non overlaping

Overlaping

Mathematically:

$X(t_1, t_2), X(t_2, t_3) \text{------} X(t_{n-1}, t_n)$; $t_1 < t_2 \text{---} < t_n$

are mutually independent

$\Rightarrow$ which is called memoryless Property.

TOPIC NAME : DAY : TIME : DATE : / /

2. For sufficiently small Δt

$$P_1(t, t+\Delta t) \overset{\infty}{=} \lambda \Delta t + \underline{O(\Delta t)} \rightarrow \text{Order of del } t.$$

Where - $P_1(t, t+\Delta t) \rightarrow$ Probability of one event occur in $(t+\Delta t)$

$$O(\Delta t) \rightarrow \lim_{\Delta t \to 0} \frac{O(\Delta t)}{\Delta t} = 0 \Big/ \frac{d\,o(t)}{dt} = 0$$

This assumption says that, for a sufficiently small Δt the probability of having exactly one event is directly propotional to the length of Δt.

3. For sufficiently small Δt -

$$\sum_{K=2}^{\infty} P_K(t, t+\Delta t) = O(\Delta t)$$

Poission Randon Variable (PRV) : A discrete random variable x is said to be PRV with parameter μ (average/ density) if it's range $R_x = \{0, 1, 2 ----\}$

and pmf $P_x(K)$ = $\begin{cases} \dfrac{\mu^K e^{-\mu}}{K!} & ; K \in R_x \\ 0 & ; \text{otherwise} \end{cases}$

P_x ↓ random variable

$* \mu = \lambda t$ ↳ time spell.

TOPIC NAME : ______ DAY : ______ TIME : ______ DATE : ______

\# 30/sec interval

no of vechiles	0	1	2	3	4	5	6	7	8	≥9
OBS	18	32	28	20	13	7	0	1	1	0
fixi										

let the rate of 10 vechile/min is the level of critical traffic load. Find the probability that the critical level is reached/exceeded.

Soln: avg, $\lambda = \dfrac{\sum f_i x_i}{\sum f_i} = 2.08$

$$\therefore \mu = \lambda t = \frac{2.08}{30} \times 60 = 4.16 \; / t = 1\,min$$

Assume that the system satisfies the poission process.

$$\therefore P(x) = \frac{e^{-\mu}\mu^{x}}{x!} = \left.\frac{e^{-4.16}(4.16)^{x}}{x!}\right\} \text{Model}$$

Need to find. $P(X \geq 10)$

$= 1 - P(X \leq 9) \rightarrow$ Jam না হওয়ার prob.

Next class: Poission process time interval math.

From Previous math 10.00 – 10.01 Jam থাকার Prob.:

$$P_{x\geq 10}(t\,(10.00-10.01) = P_{x\geq 0}(t=1) = 1 - P_{x<10}(t=1)$$

$$P_x(t) = \frac{e^{-\mu}\mu^x}{x!}$$

$$\therefore P_0(t=1) + P_1(t) + \text{-------------} + P_9(t=1)$$

11.30 – 11.31 → এই time এ ও Prob. same থাকলে as Poission Process memory less.

$t = 5ms$

10.00 – 10.05 → Prob of no vechile ?
= 5m

Soln:

$$\mu = \lambda t = 2.08 \times 10 = 20.8$$

$$\therefore P_0(t : 10.00 - 10.05) = \frac{e^{-20.8}(20.8)^0}{0!}$$

10.00 – 10.05 and/OR 3.00 – 3.05 এও কোন vechile থাকবে না।

Soln:

$$P_0(t : 10.00 - 10.05) \cap P_0(3.00 - 3.05)$$

$$P_0(t : 10.00 - 10.05) \cup P_0(3.00 - 3.05)$$

P_n ; n এর মান যা বেশি তা dominate করবে।

10.00 − 10.05 and 10.03 − 10.08

Soln: Overlap করতেছে, তাই Prob বের করতে পারব না।

10.00 − 10.30 এ 20টা vechile গেছে 10.30 − 10.31 Prob. কত?

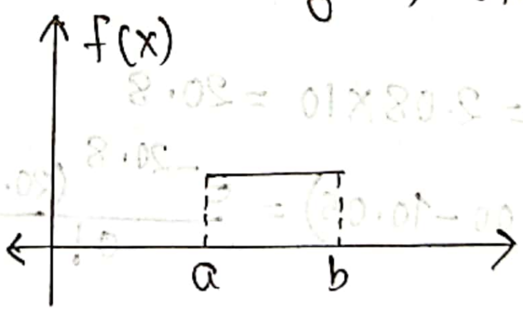
Continuous Probability Distribution

□ 3 টা method

□ **Uniform P.D**

$$f(x) = \frac{1}{b-a} \;;\; a \le x \le b$$
$$\quad\;\; 0 \;;\; \text{otherwise}$$

Property মনে রাখা লাগবে।



$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

□ **Exponential P.D / Negative E.P.D** : → memoryless Property follow করে।

$$f(x) = \lambda e^{-\lambda x} \;;\; 0 \le x \le \infty \qquad ; \lambda = \text{average density}$$
$$= 0 \;;\; \text{otherwise}$$

Some important property :

1. λ always positive.

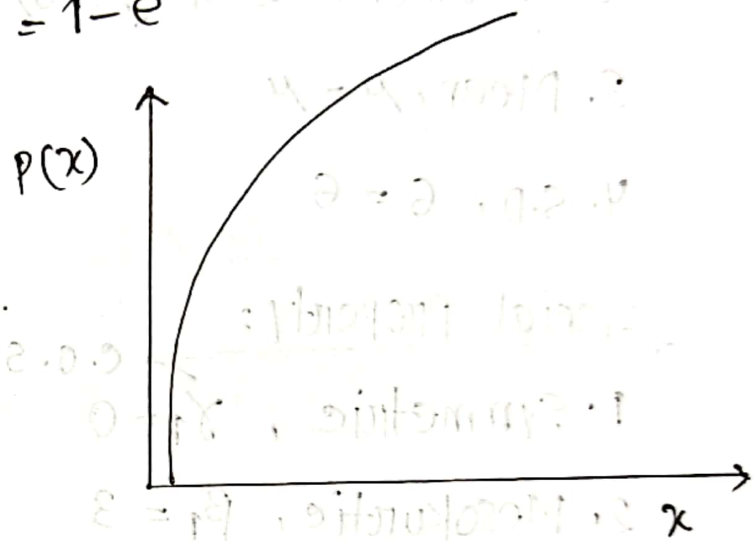
2. Cumilitive, PDF, $F(x) = 1 - e^{-\lambda x}$

3. Mean, $\mu = \frac{1}{\lambda}$

4. Mod $= 0$

5. Variance, $\sigma^2 = \frac{1}{\lambda^2}$

6. Skewness, $\gamma_1 = 2$



Normal P.D (Gaussian P.D)

$\Rightarrow$ Mean, mod, median = একই জায়গায় থাকবে ।

$\boxed{\text{ক}}$ x : c.r.v

P.d.f : $f(x : \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$; $-\infty \leq x \leq \infty$

pdf কীনা প্রমাণ করা $\rightarrow$ (i) Total integration = 1

(ii) Positive কী না

TOPIC NAME: DAY: TIME: DATE:

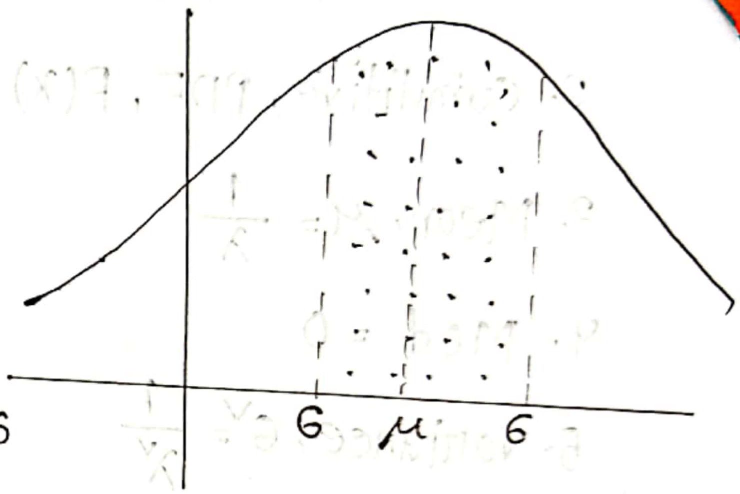
Property:

1. $x \rightarrow$ c.r.v / x = Normal variable

2. Parameter 2 টি (μ, σ)

3. Mean, $\mu = \mu$

4. S.D, $\sigma = \sigma$



Special property:

1. Symmetric, $\underbrace{\gamma_1 = 0}_{\text{c.o.s}}$

2. Mesokurtic, $\beta_1 = 3$

3. Mean, Median, Mod same

4. i) $\mu - \sigma \le x \le \mu + \sigma$ 68.27% } check করা যাবে

ii) $\mu - 2\sigma \le x \le \mu + 2\sigma$ 95.$\boxed{45}$%

iii) $\mu - 3\sigma \le x \le \mu + 3\sigma$ 99.73%

☐ standard score, $z = \dfrac{\overbrace{X - \bar{X}}^{\text{shifting}}}{\underbrace{S.D}_{\text{scalling}}}$

$$f(z:0,1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} ; -\infty \le z \le \infty$$

→ pdf standard normal distribution.

TOPIC NAME : DAY : TIME : DATE : / /

→ Mead = Mod = Median = 0

→ S.D = 1

→ $-1 \le z \le 1$ 68.27%

→ $-2 \le z \le 2$ 95.45%

→ $-3 \le z \le 3$ 99.73%

then z = Standard normal variable

when $n >> 1$ and $p \not< 1$, $p << 1$ then we normal distribution.

⇒ mean

↓

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

varianee, $\sigma^2 = E(x)^2 - [E(x)]^2$

*** SA Distribution এ mean, varianee, kurtosis, skewness বের করা শিখতে হবে।

Jamali Sir

* 5/11/23 রবিবার 12:20 Extra class

lecture-

TOPIC NAME : Gaussian Distribution

DAY : Thursday

TIME :

DATE : 2/11

The mean weight and standard deviation of 600 Students are 151 lb and 15 lb.

→ Find the no of student whose weight lie between

① 120 - 155 lb

② more than 185 lb

③ At most 120 lb

Soln: Assume that -

Students weights are normally distributed.

① Prob (120 - 155)

$\Rightarrow P(120 \le x \le 155)$

$\Rightarrow P(119.5 \le x \le 155.5)$

$$= \int_{119.5}^{155.5} f(x)\,dx = \int_{119.5}^{155.5} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

② Prob (185 - ∞)

$P(184.5 - \infty)$

$P(2.3 \le z \le \infty)$

TOPIC NAME : DAY : TIME : DATE : / /

③ $P(-\infty \le x \le 120.5) = P(-\infty \le z \le 2.03)$

$$z = \frac{x-\mu}{6}$$

z for ① $z = \dfrac{119.5 - 151}{15} = -2.10 = z_{119.5}$

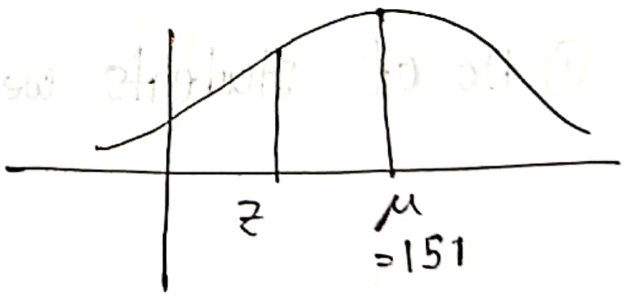
z for $z_{155.5} = \dfrac{155.5 - 151}{15} = 0.30$

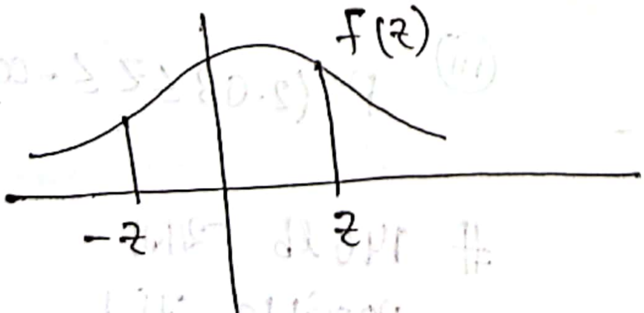
① $$\int_{-2.10}^{0.30} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

(Figure then zero)

Table :

$P(-2.10 \le z \le 0.30)$





$= P(-2.10 \le z \le 0) + P(0 \le z \le 0.30)$

$= P(0 \le z \le 2.10) + P(0 \le z \le 0.30)$

$= 0.4821 + 0.1179 = 0.6$

* z কে 2 Decimal এ নেয়া লাগবে।

TOPIC NAME : $z = 3.275$

$= 2.28$

DAY :

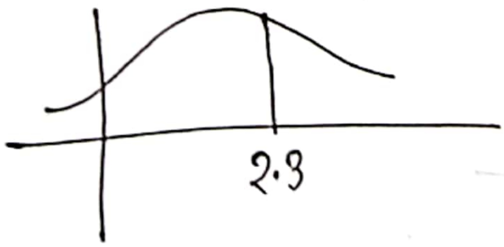
TIME : DATE :

z	0	1	2	3	4	5
0.0						
0.1						
0.3	.1179					
2.1	0.4821					

(i) No of students weight $= N \times P(x)$

$= 600 \times 0.6 = 360$ | $361.7 \rightarrow 362$

(ii)



$P(2.3 \le z \le \infty)$

$= 0.5 - P(0 \le z \le 2.30)$

(iii) $P(2.03 \le z \le -\infty)$

140 lb থাকা possible না।

*** Normal Distribution fitting → নিলে আসবে।

TOPIC NAME : DAY : TIME : DATE : / /

Poissim Process -

$\lambda = 2.08 /$

$\Rightarrow$ 10.00 – 10.05 , 11.00 – 11.05 } এর আগে একটা গাড়ি থাকার Prob

$\mu = \lambda t$

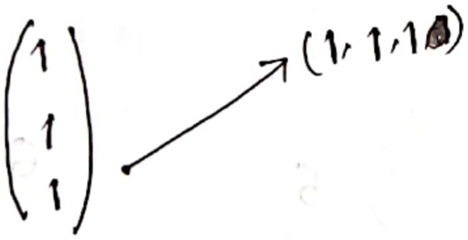
$= 2.08 \times 2 \times 5$

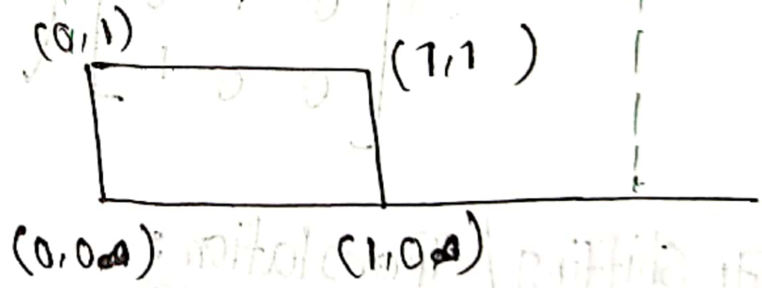
$P(10.00) + P(11.00)$

Vector

Scaling:

① Uniform ② Non-Uniform





*** $AX = H \rightarrow$ image
$\downarrow$
operator

$$5\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$AX = X$ | $AX = \lambda X \rightarrow$ Eigen value (Property change হবে, Direction change হবে না।)
$\downarrow$
I

Non Uniform Sealing

$$\begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 5 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

সবদিকে বড় হবে না, একদিকে হবে।

$$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow$$ সবদিকে বড় হবে।

Uniform

Lecture-16
Sunday
5/11/23

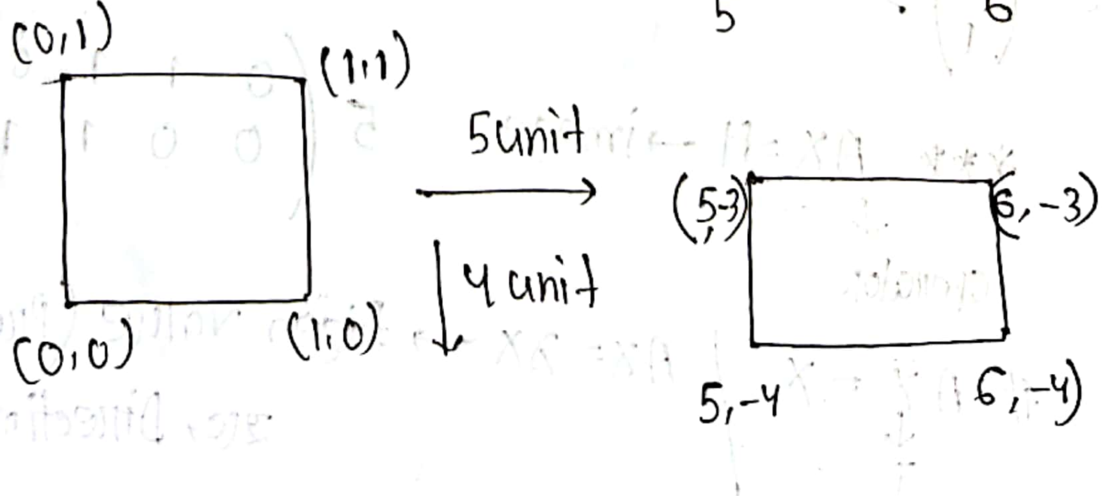
Jamali Sir

Vector

Sealing :

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Shifting / Translation :



TOPIC NAME : DAY : TIME : DATE : / /

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 5 & 5 & 5 & 5 \\ -4 & -4 & -4 & -4 \end{pmatrix} = \begin{pmatrix} \overset{5}{\cancel{0}} & 6 & 6 & \overset{5}{\cancel{0}} \\ -4 & -4 & -3 & -3 \end{pmatrix}$$

With the help of matrix addition

Matrix multiply operator:

$$\begin{pmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} dx & 1+dx & 1+dx & dx \\ \cancel{0}dy & \overset{dy}{\cancel{0}} & 1+dy & 1+dy \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

shifting column

$dx = 5, dy = -4$

$$= \begin{pmatrix} 5 & 6 & 6 & 5 \\ -4 & -4 & -3 & -3 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

graph এ ফর্মুলা আনতে হবে।

** যত order, তার থেকে এক বেশি নিতে হবে।

Scaling

$$\begin{pmatrix} 2 & 0 & dx \\ 0 & -1 & dy \\ 0 & 0 & 1 \end{pmatrix}$$

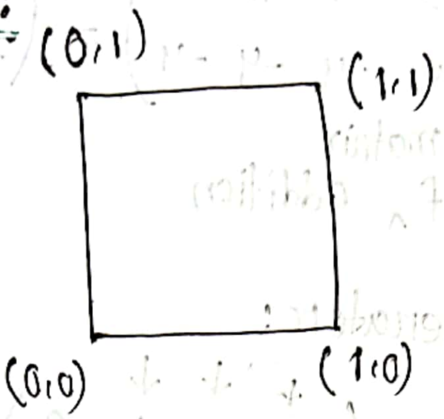
* আগে scaling হবে, then shifting হবে।

আগে shifting, পরে scaling:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow AB \neq BA$

Projection:



$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

X-axis এ Projection:

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Z- থেকে Projection ফেললে:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Reflect:

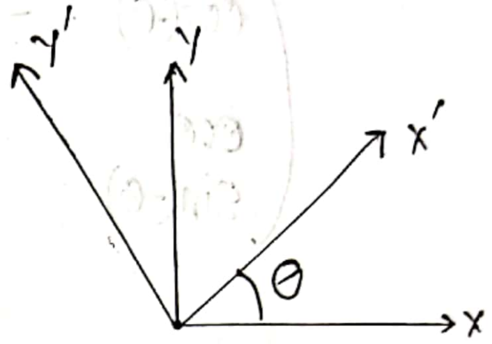
$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \rightarrow \text{Y- axis এ reflect}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} " \end{pmatrix} \rightarrow \text{X- axis এ reflect.}$$

TOPIC NAME : DAY : TIME : DATE : / /

(***) Point গুলো graph এ এঁকে দেখাতে হবে।

Rotation :



$$x' = l_{11}x + l_{12}y + l_{13}z$$

$$y' = l_{21}x + l_{22}y + l_{23}z$$

$$z' = l_{31}x + l_{32}y + l_{33}z$$

θ=0 হলে rotation হবে না।

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{pmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \end{array} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$\Rightarrow l_{11}$ = x' এর সাথে x এর direction cosine

$$X' = RX$$

↳ R = Rotational transformal matrix, Non singular matrix.

Inverse :

$$X' = R^{-1}X$$

Rotation 0 হলে $\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 2D

2D Rotation : anti clk

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \longrightarrow \text{Anti clockwise}$$

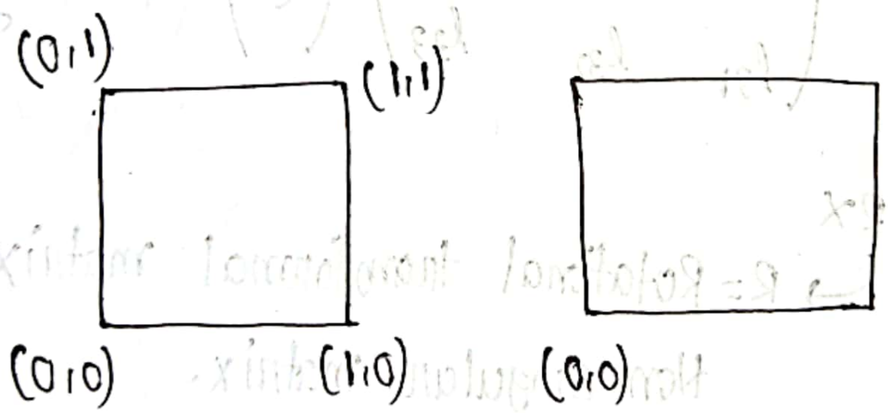
TOPIC NAME : DAY : TIME : DATE :

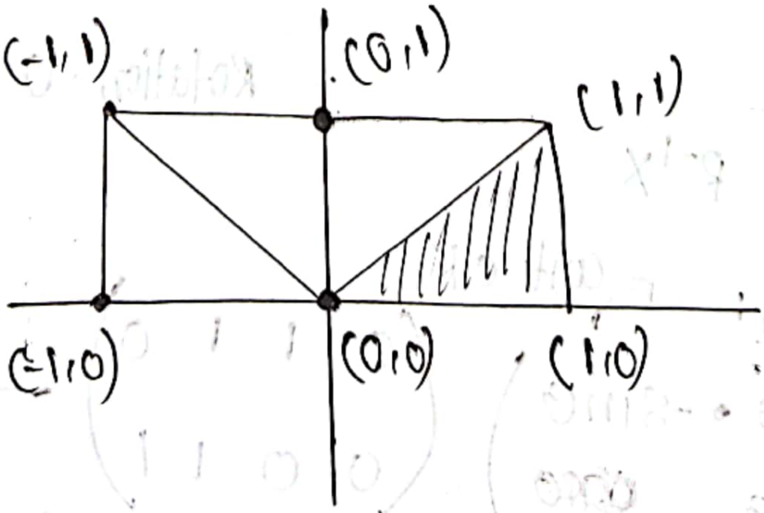
Clock wise Rotation :

$$\begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$\theta = 90^\circ$ ঘুরালো positive দিকে : anti তে $\theta = 90^\circ$ বসিয়ে

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$





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z-এর উপর Rotation

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

x-এর উপর Rotation

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

* যার উপর Rotation, তার Diagonal = 1

আগে Rotation, পরে Shifting :

$$\begin{pmatrix} \cos\theta & -\sin\theta & dx \\ \sin\theta & \cos\theta & dy \\ 0 & 0 & 1 \end{pmatrix}$$

আগে Rotation → shifting → scaling :

$$\begin{pmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dx & 0 & 0 \\ 0 & dy & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

* যেই operation matrix এর কাছে, তা আগে করা লাগবে।

GOOD LUCK™

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Lecture-17

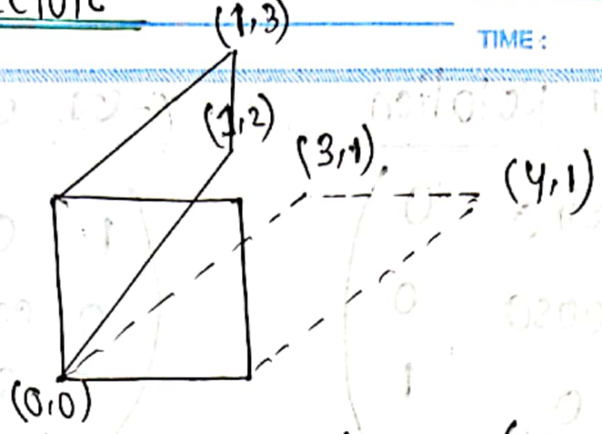
TOPIC NAME: Vector

DAY: Sunday

TIME:

DATE: 5/11

Shear:



$$\begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 2 & 3 & 1 \end{pmatrix}$$

Y কে move করছি। → along Y axis

$$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 4 & 3 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

x কে move করছি → Along x axis

$$\begin{pmatrix} 1 & S_x & 1 \\ S_y & 1 & \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow$ $R \cdot Sc \cdot S$

(***)এর একসাথে করলে কী হবে,
এক এক করে করলে কী হবে?

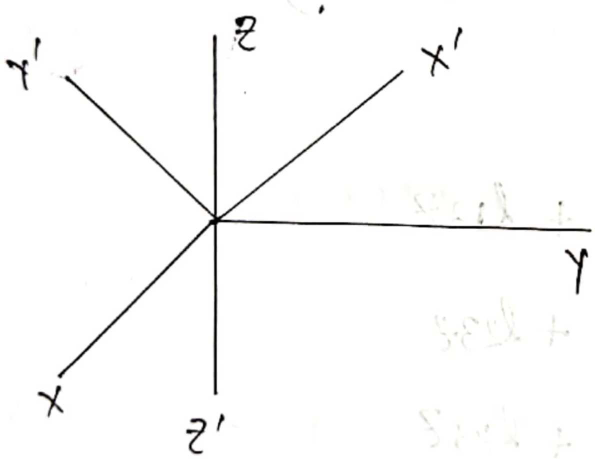
$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

$\begin{pmatrix} a & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

Matrix Image tramform:

$$A = A_1 i + A_2 j + A_3 k$$

$$A = A_1' i' + A_2' j' + A_3' k'$$



Invariant (Grad, Div, Curl এর মধ্যে):

$$\vec{r} = ix + yj + zk$$

$$r = (x'i' + y'j' + z'k')$$

$$x'i' + y'j' + z'k' = xi + yj + zk \quad \text{--- (1)}$$

$$\Rightarrow \quad A = (A \cdot i) \cdot i + (A \cdot j) \cdot j + (A \cdot k) \cdot k$$

$$i = (i\cdot i')\,i' + (i\cdot j')\,j' + (i\cdot k')\cdot k'$$

$$j = (i\cdot j')\,i' + (j\cdot j')\cdot j' + (i\cdot k')\cdot k'$$

$$k = (i, j')\,i' + (i, j)\,j' + (k\cdot k')\cdot k'$$

$$\left.\begin{aligned} i &= l_{11} i' + l_{21} j' + l_{31} k' \\ j &= l_{12} i' + l_{22} j' + l_{32} k' \\ k &= l_{13} i + l_{23} j' + l_{33} k' \end{aligned}\right\} \quad (2)$$

(2) - (1) :

$$x' = l_{11} x + l_{12} y + l_{13} z$$

$$y' = l_{21} x + l_{22} y + l_{23} z$$

$$z' = l_{31} x + l_{32} y + l_{33} z$$

$\Rightarrow \varphi(x, y, z) = \varphi'(x', y', z') \longrightarrow$ Sealar physical quantity $\longrightarrow$ বহ

$\Rightarrow$

Lecture- 18

TOPIC NAME : Vector

DAY : Thursday

TIME :

DATE : 9 / 11 / 23

Scalar function এর gradient. তার invariant হলো তার co-ordinate change হবে না। → বই থেকে দেখে নিতে হবে।

Point Function :

Every physical quantity can be expressed as a continuous function of a point. If the physical quantity is scalar point function. This function is called corresponding field scalar field. Point function / function

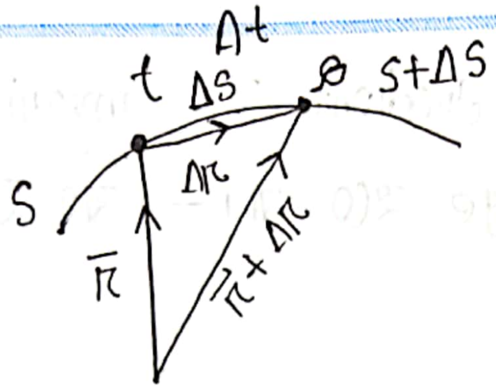
The region on which, the function is defined is called field.

→ If the physical quantity is vector then its vector point function. And the corresponding field is vector field.

$\bar{r} = xi + yj + zk$ → Position vector of a moving Particle.

same $\left\{ \begin{array}{l} x = x(t) \\ y = y(t) \\ z = z(t) \end{array} \right.$

TOPIC NAME :
DAY :
TIME : DATE :



cord lenght = $|d\bar{r}|$

$$\lim_{Q\to P} \frac{(\bar{r}+\Delta\bar{r})-\bar{r}}{(t+\Delta t)-\Delta t} = \lim_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \vec{v}$$

$$\# \lim_{Q\to P}\frac{|d\vec{r}|}{\Delta t} = \lim_{Q\to P}\frac{\Delta s}{\Delta t}$$

$$\Rightarrow v = \left|\frac{dr}{dr}\right| = \frac{ds}{dt} \rightarrow \text{Speed}$$

$\# \ \bar{v} = v\hat{T}$; $\hat{T}$ = unit tangent vector

Another theory :

$\bar{A}$; $|\bar{A}|$ = constant

$\bar{A} = 5\cos t\, i + 5\sin t\, j$

$\bar{A}\cdot\bar{A}$ = const

$$\frac{d\bar{A}}{dt}\cdot\bar{A} + \bar{A}\cdot\frac{d\bar{A}}{dt} = 0 \qquad \therefore \bar{A}\perp\frac{d\bar{A}}{dt} = 0$$

TOPIC NAME : DAY : TIME : DATE : / /

$T \perp \frac{dT}{dt}$, $T \perp \frac{dT}{ds}$

$$\frac{dT}{ds} = \underbrace{\kappa}_{\text{curvature}} \underbrace{N}_{\text{Normal vector / Principal Normal vector}} \quad ; \quad T \perp N$$

Rate of change in direction, with respect to ~~time~~ Displacement.

$$\frac{d\bar{T}}{ds} = \frac{dT}{dt} \Big/ \frac{ds}{dt} = \frac{dT}{dt} \Big/ v = \kappa N$$

$$\left|\frac{dT}{ds}\right| = |\kappa \bar{N}| = \kappa$$

$\frac{dT}{ds}$ কে kappa দিয়ে ভাগ করলে N পাওয়া যায়।

$$\therefore \frac{dT}{dt} = \kappa v \hat{N} = \frac{v}{r} \hat{N}$$

$$\therefore \kappa = \frac{1}{\rho}$$ (ρ → radius of curvature)

$\hat{T} \times \hat{N} = \hat{B}$ → BiNormal vector

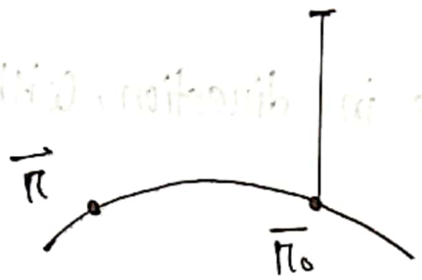
T N B of a moving particle continued a

TOPIC NAME : DAY : TIME : DATE :

moving coordinate system called trihedral/tried

$\Rightarrow N, \hat{B}$ এগুলো coordinate plane.

<u>Drawing vector plane:</u>



এখানে Point দেয়া আছে, আর যে plane এ আছে তার Normal দেয়া আছে।

$(\bar{r}-\bar{r}_0)\bar{T}_0 = 0 \rightarrow N, B$ } Normal Plane

$(\bar{r}-\bar{r}_0)\bar{N}_0 = 0 \rightarrow T, B$ } Retifying plane

$(\bar{r}-\bar{r}_0)\bar{B}_0 = 0 \rightarrow N, B$ } Osculating plane

$\frac{dB}{ds} = -\tau N$

$\left|\frac{dB}{ds}\right| = |\tau N| = \kappa$

τ = Torsion

$\sigma = \frac{1}{\tau} \rightarrow$ Radius of Torsion

TOPIC NAME : DAY: TIME: DATE: / /

$\frac{dB}{ds} = \frac{dB}{dt} \Big/ \frac{ds}{dt}$

$$\bar{a} = \frac{dv}{dt} T + v \frac{dT}{dt}$$

$$= \frac{dv}{dt} + v \cdot \frac{v}{\rho} \bar{N}$$

$$\bar{a} = \frac{d\overset{\rightarrow \text{speed}}{v}}{dt} + \frac{v^2}{\rho} \bar{N}$$

$$\Rightarrow \bar{a} = \underbrace{a_T}_{\text{Tangential component}} \bar{T} + a_N \hat{N} \rightarrow \text{Normal component}$$

⇒ Straight line এ move করলে $\rho = 0$, মানে infinity curvature.

⇒ constant speed হলে শুধু Normal compent থাকবে।

$$\left\{ \begin{array}{c} \bar{v}, v, \bar{a}, a_T, a_N \\ T, N, B, \kappa, \tau \\ \text{Specific point এ plane} \end{array} \right\}$$

Moving particle এর parametric equation -

$x = 2\cos t,$
$y = 2\sin t$
$z = 3$

Find, $\bar{v}, v, \bar{a}, |a|, a_T, a_N, T, N, B, \kappa, \tau$ and all cordinates planes, at $t=0, ----,$ also comment about the path.

Comment v must, $\left.\begin{matrix} t \to 0, \\ t \to \infty \end{matrix}\right\} \tau, \kappa$

Soln:

Position vector, $\vec{r} = 2\cos t\, i + 2\sin t\, j + 3k$

$\vec{v} = -2\sin t\, i + 2\cos t\, j$

Velocity, $|v| = \frac{ds}{dt} = v = 2$

$\therefore$ Tangent, $\bar{T} = \frac{\bar{v}}{|v|} = -\sin t\, i + \cos t\, j$

$\bar{a} = \frac{dv}{dt} = -2\cos t\, i - 2\sin t\, j \quad |a| = 2$

Direction $-i, -j$ এর দিকে

$\frac{d\bar{T}}{ds} = \frac{dT}{dt} \Big/ \frac{ds}{dt} = \frac{-\cos t\, i - \sin t\, j}{2}$ $\quad \frac{dT}{dt} = -\cos t\, i - \sin t\, j$

$\frac{ds}{dt} \to 2$

TOPIC NAME : DAY : TIME : DATE : / /

$$\left|\frac{d\bar{T}}{ds}\right| = \left|\frac{dT}{dt}\right|\left|\frac{ds}{dt}\right| = \left|\frac{-\cos t\, i - \sin t\, j}{2}\right| = \chi N.$$

$$\chi = \frac{1}{2}, \quad \rho = 2$$

$$\hat{N} = -\cos t\, i - \sin t\, j$$

$$\hat{B} = \hat{T} \times \hat{N} = \begin{vmatrix} i & j & k \\ -\sin t, & \cos t & 0 \\ -\cos t & -\sin\theta & 0 \end{vmatrix} = \hat{k}$$

$$\frac{dB}{ds} = -\tau \hat{N} = 0 \quad \therefore \tau = 0$$

$$\Rightarrow \frac{dB}{ds}(\hat{k}) = 0$$

$$\bar{a} = \frac{dv}{dt}\bar{T} + \frac{v^2}{\rho}\hat{N}$$

$$\therefore a_T = \frac{dv}{dT} = \frac{d}{dT}(2) = 0$$

$$\therefore a_N = \frac{v^2}{\rho} = \frac{2^2}{2} = 2$$

$\therefore$ Speed : v $t \to 0$, or $t \to \infty$ always constant.

$|a| = 2$ for all

ρ = curvature = constant (circle) | a_N = constant, centrifugal.

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$\kappa=0$ → মোচড় নাই।

$\hat{B} = k$, Particle xy plane এ move করতেছে।

তারপর t এর মান যেকোনো value ধরে করা যাবে।

\# $\bar{\kappa} = 2t$ → Spiral

$\kappa = 2t$ → মোচড় আস্তে, আস্তে বাড়তে।

$\Rightarrow \vec{\kappa} = \frac{2}{t}$

$\Rightarrow \kappa = \frac{2}{t}$ → আস্তে আস্তে মোচড় কমতে থাকবে।

Normal plane:

$$(r-r_0)\,\bar{T}_0 = 0$$

$$\left[(x-x_0)\,i + (y-y_0)\,j + (z-z_0)\,k\right]\cdot\left[-\sin t\, i + \cos t\, j\right] = 0$$

$$\Rightarrow (x-x_0)(-\sin t)\Big|_{t=0} + (y-y_0)\cos t\Big|_{t=0} = 0$$

When, $t=0$,

$x_0 = \cos t = 1$, $y_0 = \sin t = 0$, $z_0 = 3$

$\Rightarrow (x-1)\,0 + (y-0)\,1 = 0$

$\Rightarrow y = 0$

Tangent plane = xz plane

\# $\bar{F} = 3t^3 i + 2t^2 j + 3K$, Find velocity, Parametric equation (position vector) such that $v(0) = j$,
$\vec{r}(0) = i$

Soln:

$\bar{F} = m\bar{a}$, let, $m = 1$

$\bar{a} = 3t^3 i + 2t^2 j + 3K$

$\Rightarrow \dfrac{d\bar{v}}{dt} = 3t^3 i + 2t^2 j + 3K$

$\Rightarrow \bar{v} = \frac{3}{4} t^4 i + \frac{2}{3} t^3 + 3tK + \bar{c}$

$v(0) = 0 + 0 + 0 = \bar{c} = j$

impo math, আসবে

TOPIC NAME : Distribution

DAY :

TIME : DATE : / /

$\mu = 12,\ \sigma = 3$

$Z = \dfrac{X-\mu}{\sigma}$

$Z_{5.5} = \dfrac{5.5-12}{3}$

$= -2.17$

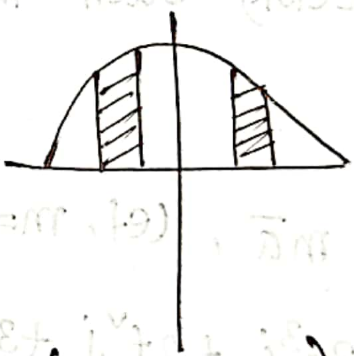
$Z_{10.5} = -0.5$

$Z_{15.5} = 1.17$

# Z	f	z	P(z)	E=NP(z)
$-\infty - 5$	0	$-\infty - (-2.17)$	.010	
6–10	3	$-2.17 - (-0.5)$	2.983	8
11–15	2	$-0.5 - 1.17$	.327	7
$16 - \infty$	0	$1.17 - \infty$	.411	2
	N=5			

$0 \le z \le z_0$

$.5 + P(0 \le z \le 2.17)$



$P(0 \le z \le 2.17) - P(0 \le z \le 1.5)$

$= 3.10 - .117 = 2.983$

$P(0 \le z \le 1.17) + P(0 \le z \le 1.5)$

$= .21 + .117 = .327$

$\chi^2 = \sum \dfrac{O_i^2}{E_i} - N$

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