Salim Sir

Lecture-1

TOPIC NAME : Equation Solving Method

DAY : Monday

TIME :

DATE : 7 / 8 / 23

$27x + 6y - z = 85$

$6x + 15y + 2z = 72$

$x + y + 54z = 110$

$$D = \begin{bmatrix} 27 & 6 & -1 \\ 6 & 15 & 2 \\ 1 & 1 & 54 \end{bmatrix} = 19893$$

$$Dx = \begin{bmatrix} 85 & 6 & -1 \\ 72 & 15 & 2 \\ 110 & 1 & 54 \end{bmatrix} = 48250$$

$$Dy = \begin{bmatrix} 27 & 85 & -1 \\ 6 & 72 & 2 \\ 1 & 110 & 54 \end{bmatrix} = 71078$$

$$Dz = \begin{bmatrix} 27 & 6 & 85 \\ 6 & 15 & 72 \\ 1 & 1 & 110 \end{bmatrix} = 38313$$

$X = \dfrac{Dx}{D}$ $= 2.425$

$Y = \dfrac{Dy}{Y}$ $= 3.573$

$z = \dfrac{Dz}{z}$ $= 1.925$

TOPIC NAME: DAY: TIME: DATE: /

Gauss-Seidel iteration method:

First iteration:

$$x = \frac{1}{27}(85 - 6y + z) \quad \text{—— (i)}$$

$$y = \frac{1}{15}(72 - 6x - 2z) \quad \text{—— (ii)}$$

$$z = \frac{1}{54}(110 - x - y) \quad \text{—— (iii)}$$

put $y=0, z=0$ in (i)

$$x = \frac{85}{27} = 3.148 = x^{(1)} = \text{First approximation}$$

put $x = 3.148$, $z=0$ in (ii)

$$y = \frac{1}{15}(72 - 6 \times 3.148 - 2 \times 0) = 3.54 = y^{(1)}$$

= First approximation

put $x = 3.148, y = 3.54$ in (iii)

$$z = \frac{1}{54}(110 - 3.148 - 3.54) = 1.91 = z^{(1)} = \text{First approximation}$$

Second iteration:

$$\left.\begin{aligned}
x^{(2)} &= \frac{1}{27}\left(85 - 6y^{(1)} + z^{(1)}\right) = 2.43 \\
y^{(2)} &= \frac{1}{15}\left(72 - 6x^{(2)} - 2z^{(1)}\right) = 3.57 \\
z^{(2)} &= \frac{1}{54}\left(110 - 6y^{(2)} - z^{(2)}\right) = 1.92
\end{aligned}\right\} \text{Second approximation}$$

Third iteration:

$$x^{(3)} = \frac{1}{27}\left(85 - 6y^{(2)} + z^{(2)}\right) = 2.426$$

$$y^{(3)} = \frac{1}{15}\left(72 - 6x^{(3)} - 2z^{(2)}\right) = 3.57$$

$$z^{(3)} = \frac{1}{54}\left(110 - 6y^{(3)} - z^{(3)}\right) = 1.926$$

* Iteration বলা থাকলে ততগুলো steps, না বলা থাকলে minimum 3 steps।

* Error বের করতে দিতে পারে। 2nd ও 3rd এর মান দিয়ে error বের করলেই হবে।

* Point এর পর চার ঘর মান দিতে হবে, না বলা থাকলে।

Salim Sir

Numerical Integration

Lecture-2

TOPIC NAME : Simpson's One Third Rule

DAY: Monday

TIME:

DATE: 14/8/23

Solve integration:

$\int_4^{5.2} \log x \, dx$

$$\int \log x \, dx = \log x \int 1 \, dx - \int \left\{ \frac{d}{dx} \log x \int 1 \, dx \right\} dx$$

$$= x \log x - \int \frac{1}{x} \times x \, dx$$

$$\int_4^{5.2} \log_e x \, dx$$

$$= x \log x - \int 1 \, dx$$

$$= x \log x - x$$

$$\int_4^{5.2} \log x = \left[x \log x - x \right]_4^{5.2}$$

$$= (5.2 \log 5.2 - 5.2) - (4 \log x - 4)$$

Step $n = \dfrac{\text{Upper limit} - \text{lower limit}}{n}$

n = ইচ্ছামত ধরে নিব।

* n যত বড় হবে, result তত accurate আসবে।

Steps: x, $y = \log_e x$

$x_0 = 4 = LL$ $\quad y_0 = 1.3862$

$x_1 = 4 + 0.2 \times 1$ $\quad y_1 = 1.4350$

$x_2 = x_0 + 0.2 \times 2$ $\quad y_2 = 1.4816$

$x_3 = x_0 + 0.2 \times 3$ $\quad y_3 = 1.5260$

$x_4 = x_0 + 0.2 \times 4$ $\quad y_4 = 1.5686$

$x_5 = x_0 + 0.2 \times 5$ $\quad x_5 = 1.6094$

$x_6 = x_0 + 0.2 \times 6$ $\quad y_6 = 1.6486$
$= UL$

[illegible] $\int_4^{5.2} \log_e x \, dx = h\left[\frac{1}{2}(y_0 + y_6) + (y_1 + y_2 + y_3 + y_4 + y_5)\right]$

$= 1.8271$

=> Slide এর তিনটা ques note + মুখস্থ করে আসতে, Next class এ লিখতে দিবে। → marks থাকবে।

=> এইভাবে তিনটা Rule এর উপর Assigment থাকবে।
+ বাকি দুইটা ques

Salim Sir

Lecture-3

TOPIC NAME : Spot Test

DAY : Monday

TIME :

DATE : 21/8/23

⇒ Spot test : Compute the general equation

→ Trapezoidal

→ Simpson's 1/3 Rule

→ Simpson's 3/8 Rule

Lecture-4

Salim Sir

Method of Factorization

22/8/23

Tuesday

From the relation $A = LU$, we get-

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \times \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

*** Find out the factors of a matrix.

TOPIC NAME :

DAY :

TIME : DATE : / /

$$\begin{array}{ccc|ccc} 1 & 0 & 0 & U_{11} & U_{12} & U_{13} \\ l_{21} & 1 & 0 & 0 & U_{22} & U_{23} \\ l_{31} & l_{32} & 1 & 0 & 0 & U_{33} \end{array}$$

$1 \times U_{11} + 0 \times 0 + 0 \times 0 = U_{11} = a_{11}$ — (1)

$1 \times U_{12} + 0 + 0 = U_{12} = a_{12}$ — (ii)

$1 \times U_{13} + 0 + 0 = a_{13}$ — (3)

$U_{11} \times l_{21} = a_{21}$

$\Rightarrow l_{21} = \dfrac{a_{21}}{a_{11}} =$ — (4)

$U_{12} \times l_{21} + U_{22} = a_{22}$ — (5)

$l_{21} \times U_{13} + U_{23} = a_{23}$

$a_{12} \times l_{21} + U_{22} = a_{22}$ — (5)

$l_{21} \times a_{13} + U_{23} = a_{23}$ — (6)

$l_{31} \times U_{11} = a_{31}$

$\Rightarrow l_{31} = \dfrac{a_{31}}{a_{11}}$ — (7)

$l_{31} \times U_{12} + l_{32} \times U_{22} = a_{32}$ — (8)

$l_{31} \times U_{13} + l_{32} \times U_{23} + U_{33} = a_{33}$ — (9)

Salim Sir

Lecture-5

TOPIC NAME : Method of Factorization

DAY : Monday

TIME :

DATE : 28 / 8 / 23

#

$2x + 3y + z = 9$

$x + 2y + 3z = 6$

$3x + y + 2z = 8$

coefficient matrix $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

From the matrix :

$U_{11} = a_{11} = 2$

$U_{12} = a_{12} = 3$

$U_{13} = a_{13} = 1$

$U_{11} \times l_{21} = a_{21}$

$\Rightarrow l_{21} = \frac{1}{2}$

$a_{12} \times l_{21} + U_{22} = a_{22}$

$\Rightarrow 3 \times l_{21} + U_{22} = 2$

$3 l_{21} + U_{22} = 2$ — (i)

$l_{21} \times a_{31} + U_{23} = a_{23}$

$\Rightarrow 3 l_{21} + U_{23} = 3$ — (ii)

$l_{31} = \frac{a_{31}}{a_{11}} = \frac{3}{2}$

$l_{31} \times U_{21} + l_{32} \times U_{22} = a_{32}$

$\Rightarrow \frac{3}{2} U_{21} + l_{32} \times U_{22} = 1$

$\Rightarrow 3U_{21} + 2 l_{32} U_{22} = 2$ — (iii)

$l_{31} \times U_{13} + l_{32} \times U_{23} + U_{33} = a_{33}$

$l_{31} + l_{32} \times U_{23} + U_{33} = 2$ — (iv)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

*** 31/8/23

=> Assignment (Factorization method) এর উপর দুইটা math.

=> class এ math করতে হবে এর উপরে আগে equation বানাতে হবে, then solve.

$Y_1 = 9$

$\frac{Y_1}{2} + Y_2 = 6$

$\frac{3Y_1}{2} - 7Y_2 + Y_3 = 8$

$Y_1 = 9$

$Y_2 = \frac{3}{2}$

$Y_3 = 5$

$$\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3/2 \\ 5 \end{bmatrix}$$

$2x + 3y + z = 9$

$\frac{y}{2} + \frac{5z}{2} = \frac{3}{2}$

$\Rightarrow y + 5z = 3$

$18z = 5$

$\Rightarrow z = \frac{5}{18}$

$\Rightarrow y = \frac{29}{18}$

$x = \frac{35}{18}$

$y_{n+1} \approx y_n + hf(x_n, y_n)$

Given, $\frac{dy}{dx} = \frac{y-x}{y+x}$, with $y=1$ for, $x=0$ in five steps. Find y approximation for $x=0.1$ by Euler's method (in five steps).

Soln: Here we want the value at $x=0.1$ from $x=0$ in five steps. So, we breakup the interval 0 to 0.1 into five subintervals by introducing the points x_1, x_2, x_3, x_4, x_5. Let $h=0.02$. We shall find the values of y at $x=0.02, 0.04, 0.06, 0.08$ and 0.1 Successively.

$x_0 = 0, y_0 = 1, h = 0.02$

$f(x_0, y_0) = 1 = 1.02$

$$y_1 = y_0 + hf(x_0, y_0)$$
$$= 1.02$$

$$y_2 = y_1 + hf(x_1, y_1) = 1.02 + 0.02 \times \frac{1.02 - 0.02}{1.02 + 0.02}$$
$$= 1.03923$$

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$$Y_3 = Y_2 + hf(x_2, Y_2)$$

$$= 1.03923 + 0.02 \times \frac{1.03923 - 2\times0.02}{1.0392 - 2\times0.02} \simeq 1.05751$$

$$Y_4 = Y_3 + hf(x_3, Y_3)$$

$$= 1.05751 + 0.02 \times \frac{1.05751 - 3\times0.02}{1.05751 - 3\times0.02} = 1.0748$$

$$Y_5 = Y_4 + hf(x_4, Y_4)$$

$$= 1.0748 + 0.02 \times \frac{1.0748 - 4\times0.02}{1.0748 + 4\times0.02} = 1.0922$$

Hence $Y = 1.0928$ where $x = 0.1$

=> Modified Euler's method.

Spot Test : 7/9/23 → Modified Euler's method derivation.

Salim Sir

Lecture-7

TOPIC NAME : Runge Kutta Method

DAY : Monday
TIME :
DATE : 4/9/23

$K_1 = hf(x_0, y_0)$

$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$

$K_4 = hf\left(x_0 + h, y_0 + K_3\right)$

$\Delta y = \frac{1}{6}\left(K_1 + 2K_2 + 2K_3 + K_4\right)$

$x_1 = x_0 + h$

and

$y_1 = y_0 + \Delta y$

Given, $\frac{dy}{dx} = x + y$ (when $x=0$ then $y=1$)

approximate y, when $x = 0.1$ and $x = 0.2$

Soln: Let us take $h = 0.1$ Here $f(x,y) = x+y$

Now, $K_1 = ? = hf(x_0, y_0) = 0.1(0+1) = 0.1$

$K_2, K_3, K_4 = ?$

$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= 0.1 f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}\right)$

$= 0.1(0.05, 1.05)$

$= 0.1(0.05 + 1.05)$

$= 0.11$

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$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$

$= 0.1\,(0.05, 1.055)$

$= 0.1105$

$K_4 = hf\,(x_0 + h, y_0 + K_3)$

$= 0.1\,(0.1, 1.1105)$

$= 0.12105$

$\Delta y = \frac{1}{6} \times 0.66205$

$= 0.110341$

$x_1 = x_0 + h = 0 + 0.1 = 0.1$

$y_1 = y_0 + \Delta y = 1 + 0.110341$
$= 1.110341$

এই step বারা লাগবে।

2nd iteration:

$x_0 = 0.1, \; y_0 = 1.110341$

$K_1 = hf\,(x_0, y_0)$

$= 0.1\,(0.1 + 1.110341)$

$= 0.121034$

$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= 0.1\,(0.15, 1.17085)$

$= 0.132085$

GOOD LUCK™

TOPIC NAME : DAY : TIME : DATE : / /

$K_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2})$

$= 0.1(0.15, 1.1763)$

$= 0.13263$

$K_4 = hf(x_0 + h, y_0 + K_3)$

$= 0.1(0.2, 2.28664)$

$= 0.248664$

$\Delta Y = \frac{1}{6} \times 0.899128 = 0.14985$

$x_2 = x_0 + h = 0.1 + 0.1 = 0.2$

$Y_2 = Y_0 + \Delta Y = 1.110341 + 0.14985 = 1.26326$

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Lecture-8

TOPIC NAME : Inverse Matrix

DAY : Tuesday

TIME :

DATE : 5/9/23

Find the inverse, $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

$|A| = -2$

$A_{11} = -1 \qquad A_{12} = 8 \qquad A_{13} = -5$

$A_{21} = 1 \qquad A_{22} = -6 \qquad A_{23} = 3$

$A_{31} = -1 \qquad A_{32} = 2 \qquad A_{33} = -1$

$\cancel{A^{-1}} = \cancel{\frac{1}{-2}} \cancel{\begin{vmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{vmatrix}} = \cancel{\begin{vmatrix} 1/2 & -1/2 & 1/2 \\ -4 & & \\ & & \end{vmatrix}}$

$A^T = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix} \qquad A^{-1} = \begin{vmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{vmatrix}$

TOPIC NAME : DAY : TIME : DATE : / /

Find inverse Matrix :

① Gauss Jordan method

② Choleski method

③ Escalator method

Solve : 5 (from slide)

Lecture-9
Monday
11/9/23

Salim Sir

Curve Fitting

Fit a second degree Parabola to the data -

x :	0	1	2	3	4
y :	1	5	10	22	38

Soln : let the parabola to be fitted to the given be - $y = a + bx + cx^2$. Then the normal equations -

$$\Sigma y = ma + \Sigma bx + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$

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$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4$

$\sum y = ma + b\sum x$ and $\sum xy = a\sum x + b\sum x^2$

straight line.

$\sum x = 10 \quad \sum y = 76$

$\sum x^2 = 0^2 + 1^2 + 2^2 + 3$

x	y	x^2	x^3	x^4	xy	x^2y
0	1	0	0	0	0	0
1	5	1	1	1	5	5
2	10	4	8	16	20	40
3	22	9	27	81	66	198
4	38	16	64	256	152	608
$\sum x = 10$	$\sum y = 76$	$\sum x^2 = 30$	$\sum x^3 = 100$	$\sum x^4 = 354$	$\sum xy = 243$	$\sum x^2 y = 851$

Here, $m = 5$

$\sum y = ma + b\sum x$

$\Rightarrow 76 = 5a + 10b$

$\sum xy = a\sum x + b\sum x^2$

$\Rightarrow 243 = 10a + 30b$

$a = -3,\ b = \frac{91}{10}$

$\sum y = ma + b\sum x + c\sum x^2$

$\Rightarrow 76 = 5\times(-3) + \frac{91}{10}\times 10 + c\times 30$

$\Rightarrow 76 = -15 + 91 + c\times 30$

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$76 = 5a + 10b + 30c$

$243 = 10a + 30b + 100c$

$851 = 30a + 100b + 354c$

Solving we get -

$a = 10/7,\ b = 17/70,\ c = 31/14$

x : 0 1 2 3 4

y : 1 1.8 3.3 4.5 6.3

$\Sigma y = ma + b\Sigma x,\quad \Sigma xy = a\Sigma x + b\Sigma x^2$

x	y	x^2	xy	
0	1	0	0	$16.9 = 5a + 10b$
1	1.8	1	1.8	$47.1 = 10a + 30b$
2	3.3	4	6.6	$a = 18/25,\ b = 133/100$
3	4.5	9	13.5	
4	6.3	16	25.2	$y = \frac{133}{100}x + \frac{18}{25}$
$\Sigma x = 10$	$\Sigma y = 16.9$	$\Sigma x^2 = 30$	$\Sigma xy = 47.1$	