Question No: 1(2018)

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KHULNA UNIVERSITY OF ENGINEERING & TECHNOLOGY

B.Sc. Engineering 2nd Year 2nd Term Examination, 2018

Department of Computer Science and Engineering

MATH 2207

Complex Variables Vector Analysis and Statistics

TIME: 3 hours FULL MARKS: 210

N.B. i) Answer **ANY THREE** questions from each section in separate scripts.

ii) Figures in the right margin indicate full marks.

iii) Table, Graph (X^2 – *table*, t – *table*, z – *table*) will be supplied in the exam hall.

SECTION A

(Answer **ANY THREE** questions from this section in Script A)

1. a) The frequency distribution are given below: (17)

Class interval	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	10	15	20	5	6	4

Calculate the first four raw moment about 85 from the above table. Also find skewness, kurtosis and comment on the result.

b) Define random experiment and random variable with example. (06)

c) Find the first four moments of the following probability density function. Also draw the function; Where $f(x)=\begin{cases}\frac{1}{4} & if\ 1\le x\le 5\\ 0 & otherwise\end{cases}$ (12)

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2018
1(a)

Given that, The value of A is = 85.

~~$f_i(x_i-A)$~~ $f_i(x_i-A)$	interval	x_i	f_i	(x_i-A)	$(x_i-A)^2$	$f_i(x_i-A)^2$	$f_i(x_i-A)^3$	$f_i(x_i-A)^3$	$f_i(x_i-A)^4$	$(x_i-A)^4 \times f_i$
-400	40-50	45	10	-40	1600	16000	-64000	-640000	2560000	25600000
-450	50-60	55	15	-30	900	13500	-27000	-405000	810000	12150000
-400	60-70	65	20	-20	400	8000	-8000	-160000	160000	3200000
-50	70-80	75	5	-10	100	500	-1000	-5000	10000	50000
0	80-90	85	6	0	0	0	0	0	0	0
40	90-100	95	4	10	100	400	1000	4000	10000	40000
-1260	Total		60		~~3100~~	38400		-1206000		41040000

$\therefore$ First raw moment, $\mu_1' = \dfrac{\sum f_i(x_i-A)}{\sum f_i}$

$= \dfrac{-1260}{60} = -21$

$\therefore$ Second Raw moment, $\mu_2' = \dfrac{\sum f_i(x_i-A)^2}{\sum f_i} = \dfrac{38400}{60} = 640$

Third Raw moment, $\mu_3' = \dfrac{\sum f_i(x_i-A)^3}{\sum f_i} = -20100$

Fourth Raw moment, $\mu_4' = \dfrac{\sum f_i(x_i-A)^4}{\sum f_i} = \dfrac{41040000}{60} = 684000$

$\therefore$ ~~First~~ second central moment, $\mu_2 = \mu_2' - (\mu_1')^2 = 640 - (-21)^2 = 199$

$\therefore$ Third central moment, $\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3 = 1698$

Fourth central moment, $\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6(\mu_1')^2\mu_2' - 3(\mu_1')^4$

$= 105597$

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$$\therefore \beta_1 = \frac{\mu_3}{(\mu_2)^{3/2}} = \frac{1698}{(199)^{3/2}} = 0.6$$

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} = \frac{105597}{199^2} = 2.67$$

<u>Comment:</u> As, $\beta_1 = 0.6$ Thus, data is Positively skewed.

As $\beta_2 = 2.67 < 3$. Thus data is Platykurtic.

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(b)

Random Experiment: A Random experiment is an experiment, trial or observation that can be repeated numerous times under the same conditions. The outcome of an individual random experiment must be independent and identically distributed. It must in no way be affected by any Previous outcome and can't be Predicted with certainity.

Example: If we toss a coin, the result of the experiment is that it will either come up "tails", Symbolized by T (or 0), or "heads", symbolized by H (or 1), i.e. one of the elements of the set is $\{H, T\}$ (or, $\{0, 1\}$).

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Random Variable:

A Random variable is a variable whose value is unknown or a function that assigns values to each of an experiment's outcomes. Random variables are often designed by letters and can be classified as discrete, which are variables that have specific values or continuous, which are variables, that can have any values within a continuous range.

Example:

Suppose that, a coin is tossed twice. So that, sample space is, $S = \{HH, HT, TH, TT\}$. Let, x represents the number of heads that can come up with each sample point, we can associate a number for x as shown in table-1. Thus for example, in case of HH (i.e. 2 heads), $x=2$, while for TH (1 head), $x=1$. It follows that x is a random variable.

Sample Point	HH	HT	TH	TT
x	2	1	1	0

2(c)

Given that, $f(x) = \begin{cases} \frac{1}{4}, & \text{if } 1 \le x \le 5 \\ 0, & \text{otherwise} \end{cases}$

P.T.O.

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we know, $E(x) = \int_{-\infty}^{+\infty} x f(x) dx$

$$= \int_{-\infty}^{1} x f(x) dx + \int_{1}^{5} x f(x) dx + \int_{5}^{\infty} x f(x) dx$$

$$= 0 + f(x)\left[\frac{x^2}{2}\right]_1^5 + 0$$

$$= \frac{1}{4\times 2}\left(5^2 - 1^2\right) = 3$$

$\therefore$ First raw moment $= \mu_1' = 3$

Again,

$$E(x^2) = \int_{-\infty}^{+\infty} x^2 f(x) dx$$

$$= \int_{-\infty}^{1} x^2 f(x) dx + \int_{1}^{5} x^2 f(x) dx + \int_{5}^{\infty} x^2 f(x) dx$$

$$= 0 + \frac{1}{4}\left[\frac{x^3}{3}\right]_1^5 + 0$$

$$= \frac{1}{4\times 3}\left(5^3 - 1^3\right)$$

$$= \frac{31}{3}$$

$\therefore$ Second Raw moment $= \mu_2' = \frac{31}{3}$

Again,

$$E(x^3) = \int_{-\infty}^{+\infty} x^3 f(x) dx$$

$$= \int_{-\infty}^{1} x^3 f(x) dx + \int_{1}^{5} x^3 f(x) dx + \int_{5}^{\infty} x^3 f(x) dx$$

$$= 0 + \frac{1}{4}\left[\frac{x^4}{4}\right]_1^5 + 0$$

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or, $E(x^3) = \frac{1}{4}(5^4 - 1) \times \frac{1}{4}$

$= \frac{625}{16}$

$\therefore$ Third Raw moment, $\mu_3' = \frac{625}{16}$

Again,

$$E(x^4) = \int_{-\infty}^{+\infty} x^4 f(x)\,dx$$

$$= \int_{-\infty}^{1} x^4 f(x)\,dx + \int_{1}^{5} x^4 f(x)\,dx + \int_{5}^{\infty} x^4 f(x)\,dx$$

$$= 0 + \frac{1}{4}\int_{1}^{5} x^4 \cancel{f}\,dx + \int_{5}^{\infty} (x^4 \times 0)\,dx$$

$$= \frac{1}{4} \times \frac{1}{5}\left[x^5\right]_1^5$$

$$= \frac{1}{20}(5^4 - 1)$$

$$= \frac{1}{20}(5^5 - 1) = \frac{3125}{20}$$

$\therefore$ Fourth Raw moment, $\mu_4' = \frac{3125}{20} = \frac{625}{4}$

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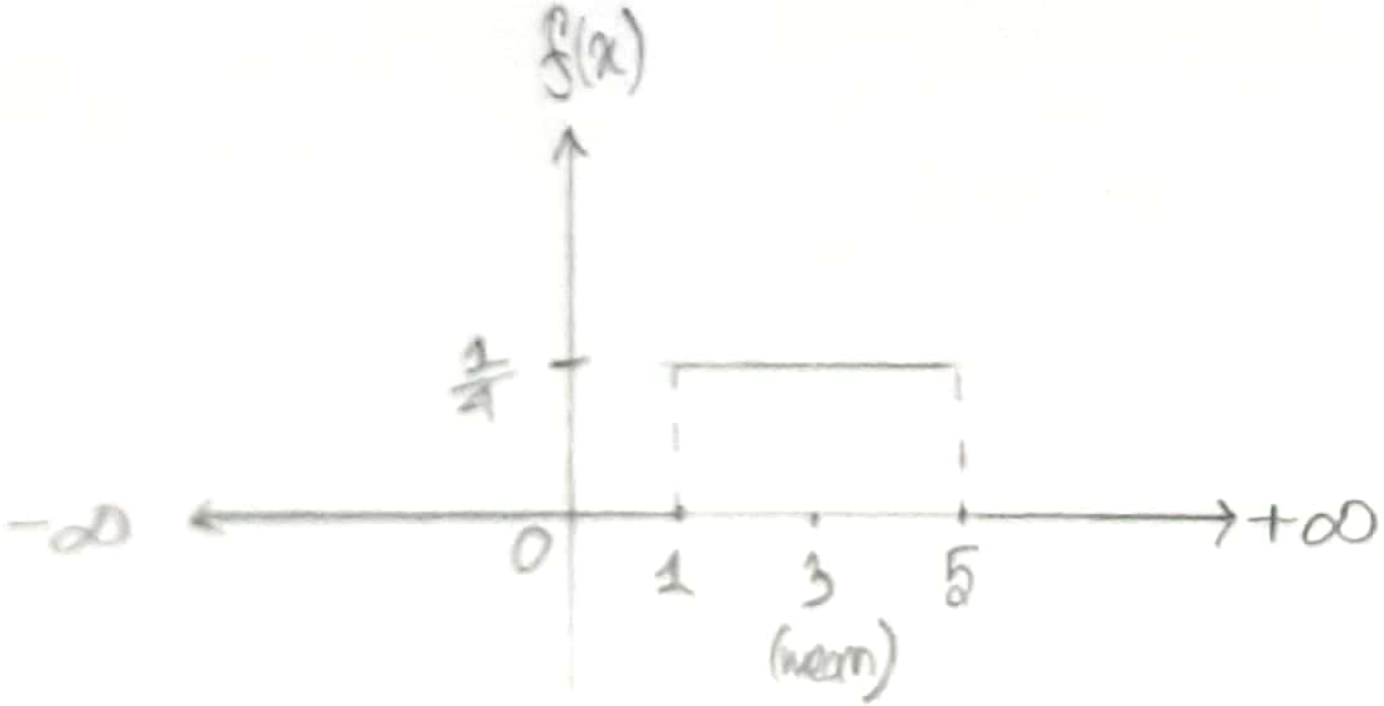
Graphical Representation :-



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Question No: 2(2018)

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2. a) Test whether the following functions are probability mass function or not. If exist then find mean and $p(x > 0)$ for each function: (13)

(i)

x	-1	0	1	2
p(x)	3k	-2k	0.4	0.6

(ii)

x	-2	0	2	4	6	8
p(x)	2k	2k	k	0.5	0.3	0.2

b) Define normal distribution. Hence derive its mean or mode. (12)

c) Define uniform distribution. Show that function defined by you (for uniform distribution) be a probability density function. (10)

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2018
2(a)

If x is a discrete random variable $(x = x_1, x_2, x_3, \ldots\ldots, x_n)$ and $P_1, P_2, P_3, \ldots\ldots, P_n$ are corresponding Probablity such that,

(i) $P_i \geq 0$

(ii) total P_i, $\sum P_i = 1$

Then a discrete Probablity distribution is formed. Then, $P(x)$ is called Probablity mass function.

Ans: (i)

x	-1	0	1	2
$P(x)$	$+3k$	$-2k$	0.4	0.6

if, $\sum P(x) = 1$, then, this will be a Probablity mass function.

Now, $\sum P(x) = 1$

or, $3k - 2k + 0.4 + 0.6 = 1$

or, $k + 1 = 1$

$\therefore k = 0$

if, $k = 0$ then, $\sum P(x) = 1$ and $P_i \geq 0$ $(i = -1, 0, 1, 2)$ and $x \in \mathbb{R}$.

$\therefore$ for $k = 0$,

$P(x)$ is a Probablity mass function.

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$\therefore P(x>0) = P(x=1) + P(x=2)$

$= 0.4 + 0.6$

$= 1$

$\therefore$ Mean, $\mu = \sum_{x=-1}^{+2} x P(x)$

$= (-1)P(0) + 0 \times P(0) + 1 \times P(1) + 2 \times P(2)$

$= (-1 \times 0) + (0 \times 0) + (1 \times 0.4) + (2 \times 0.6)$

$= 0.4 + 1.2$

$= 1.6$

Ans: mean $= 1.6$.

Ans: (ii)

x	-2	0	2	4	6	8
$P(x)$	$2k$	$2k$	k	0.5	0.3	0.2

$\because \sum P(x) = 1$, then, this will be a Probablity mass function.

$\therefore \sum P(x) = 1$

or, $2k + 2k + k + 0.5 + 0.3 + 0.2 = 1$

or, $5k + 1 = 1$

$\therefore k = 0$

if, $k=0$, then this will be a Probablity mass function.

$\therefore$ Then, $P(-2) = 0$, $P(2) = 0$
$P(0) = 0$,

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2

$\therefore P(x>0) = 1 - P(x \leq 0)$

$= 1 - \{P(x=0) + P(x=-2)\}$

$= 1 - 0$

$= 1$

$\therefore$ Mean $= \mu$

$= \sum_{x=-2}^{8} x P(x)$

$= (-2)P(-2) + (0)P(0) + (2)P(2) + (4)P(4) + (6)P(6) + (8)P(8)$

$= (-2 \times 0) + (0 \times 0) + (0 \times 0) + (4 \times 0.5) + (6 \times 0.3) + (8 \times 0.2)$

$= 5.4$

<u>Ans</u>: Mean $= 5.4$.

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2(b)

Normal distribution :-

Normal distribution, also known as the Gaussian distribution is a probablity distribution that is symmetric about the mean showing that data near the mean are more frequent in occurance than data far from the mean. In graph form, normal distribution will appear as a bell curve. The density function for this distribution is given by,

$$f(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < +\infty$$

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Here, x is a continuous random variable, $f(x;\mu,\sigma)$ is a Probablity density function, μ is mean and σ is standard deviation.

Mean on Mode derivation :-

By defn of the mean/mode we have,

$$E(x)=\int_{-\infty}^{+\infty}\frac{x}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}dx$$

By using integral Properties,

$$E(x)=\int_{-\infty}^{+\infty}(x+\mu)\frac{1}{\sigma\sqrt{2\pi}}e^{-\left(\frac{x^2}{2\sigma^2}\right)}dx$$

$$=\int_{-\infty}^{+\infty}\frac{x}{\sigma\sqrt{2\pi}}e^{-\left(\frac{x^2}{2\sigma^2}\right)}dx+\int_{-\infty}^{+\infty}\frac{\mu}{\sigma\sqrt{2\pi}}e^{-\left(\frac{x^2}{2\sigma^2}\right)}dx \quad \ldots (i)$$

For the first integral, call it I_1, we have using additivity,

$$I_1=\int_{-\infty}^{0}\frac{x}{\sigma\sqrt{2\pi}}e^{-\left(\frac{x^2}{2\sigma^2}\right)}dx+\int_{0}^{\infty}\frac{x}{\sigma\sqrt{2\pi}}e^{-\left(\frac{x^2}{2\sigma^2}\right)}dx$$

Swapping the integration limits we have,

$$I_1=\int_{0}^{\infty}\frac{(-x)}{\sigma\sqrt{2\pi}}e^{-\left(\frac{x^2}{2\sigma^2}\right)}dx+\int_{0}^{\infty}\frac{x}{\sigma\sqrt{2\pi}}e^{-\left(\frac{x^2}{2\sigma^2}\right)}dx$$

$$=0$$

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Then, from eqn(i) we get that,

$$E(x) = \int_{-\infty}^{+\infty} \frac{\mu}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x^2}{2\sigma^2}\right)} dx \quad \cdots\cdots (ii)$$

Multiplying eqn(ii) by $\sigma\sqrt{2}$ we have,

$$E(x) = \int_{-\infty}^{+\infty} \frac{\mu}{\sqrt{\pi}} e^{-x^2} dx$$

$$= \mu \int_{0}^{\infty} \frac{2}{\sqrt{\pi}} e^{-x^2} dx$$

$\because$ the last term because the integrand is an even function

Now, $$\frac{2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-x^2} dx = \lim_{t \to \infty} \frac{2}{\sqrt{\pi}} \int_{0}^{t} e^{-x^2} dx$$

$$= \lim_{t \to \infty} \text{erf}(t)$$

$$= 1$$

where, "erf" is the error function. So, we end up with,

$$E(x) = \mu$$

$\therefore$ The mean or mode of the distribution is $= \mu$

(Ans:)

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2(c)

Uniform distribution:-

A random variable x is said to be uniformly distributed in $a \le x \le b$ if it's density function is,

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

and the distribution is called a uniform distribution.

The distribution function is given by,

$$F(x) = P(X \le x) = \begin{cases} 0, & x < a \\ (x-a)/(b-a), & a \le x \le b \\ 1, & x \geq b \end{cases}$$

The function defined for uniform distribution is,

$$f(x) = \begin{cases} \frac{1}{(b-a)}, & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

Prove: Here, x is a continuous random variable.

Condition - 01

here, $f(x) \geq 0$

as, $f(x)$ is $\left(\frac{1}{b-a}\right)$ where, $a \le x \le b$ and 0, otherwise. So, this fulfilled the condition.

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condition-02

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1$$

here, $\int_{-\infty}^{+\infty} f(x)\,dx$

$$= \int_{-\infty}^{a} f(x)\,dx + \int_{a}^{b} f(x)\,dx + \int_{b}^{\infty} f(x)\,dx$$

$$= 0 + \int_{a}^{b} \left(\frac{1}{b-a}\right) dx + 0$$

$$= \left(\frac{1}{b-a}\right) \times [x]_a^b$$

$$= \frac{(b-a)}{(b-a)}$$

$$= 1$$

So, $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

As, here,

$f(x) \geq 0$ and $\int_{-\infty}^{+\infty} f(x)\,dx = 1$.

Thus,

it can be said that, $f(x)$ is a Probablity density function.

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Question No: 7(2017)

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a) What are the necessary assumptions you need regarding Binomial distribution? In a small town there are 500 families. Each family consists of four children (boys/girls) and their parents. For a single family find the probability that there is (10)

(i) No any boy.

(ii) At least one boy.

In that town how many families you expect that there is no any girl child. (Assume necessary data).

b) Suppose there are 12000 students in any admission test. Let the average score is 60 (in percentage) and variance is 25. Assume that scores are normally distributed. Find the expected number of students who are (12)

(i) not qualified (i.e. score is less than 40)

(ii) qualified but not eligible for choosing all departments (i.e. $40 \leq \text{score} \leq 80$)

(iii) eligible for choosing any department (i.e. score > 80).

c) What is the physical meaning of Grad ϕ and div $\overline{F}$, where ϕ is a scalar quantity and $\overline{F}$ is a vector quantity? Also classify the vector field according to the vector differential operator ∇. (13)

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1

QB-2017 (7)

7(a) Necessary assumtions regarding Binomial distribution is given ~~below~~ below:-

(a) Each trial has two Possible outcomes, in the language of reliablity called success and failure.

(b) The trials are independant. Intuitively, the outcome of one trial has no influence over the outcome of another trial.

(c) On each trial, the Probablity of success is p and the Probablity of failure is $(1-p)$ where p in $[0,1]$ is the success Parameter of the Process.

Given that,

Total no. of families in the town is, $N=500$,

Each family consists of four children and their Parents.

For a single family,

let, Probablity of a boy birth is $=\left(\frac{1}{2}\right)$

Probablity of a girl birth is $=\left(\frac{1}{2}\right)$ ~~×500~~

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(i) For a single family, Probablity of no any boy is :-

$$P(\text{no boy}) = \binom{4}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4$$

$$= \left(\frac{1}{2}\right)^4$$

$$= 0.0625 \text{ (Ans:)}$$

(ii) Probablity of at least one boy for a single family :-

$$P(\text{at least one boy}) = 1 - P(\text{no boy})$$

$$= 1 - \left(\frac{1}{2}\right)^4$$

$$= 1 - 0.0625$$

$$= 0.9375 \text{ (Ans:)}$$

(iii) Probablity of no girl child for all families :-

For a single family,

$$P(\text{no girl child}) = \binom{4}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4$$

$$= \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$\therefore$ For all families,

$$P(\text{no girl child for all families}) = N \times \left(\frac{1}{2}\right)^4 = 500 \times \frac{1}{16}$$

$$= 31.25 \approx 31$$

(Ans:)

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7(b)

Given that, total number of student is $= 12000$,

average score $=$ mean $= \mu = 60$,

standard deviation $= \sqrt{\text{variance}} = \sqrt{25} = 5$,

(i) ~~P(qualif~~

$P(\text{not qualified}) = 12000 \times P(\text{score} < 40)$ $(x < 40)$

$= 12000 \times P(-\infty \le x < 40)$ $(\text{score} = x)$

~~$= 12000 \times P($~~

$\therefore x_1 = -\infty$ $\quad \therefore z_1 = \dfrac{-\infty - 60}{5} = -\infty$

$x_2 = 40$ $\quad \therefore z_2 = \dfrac{40 - 60}{5} = -4$

$$P(-\infty \le x < 40) = P(-\infty \le z < -4) = P(-\infty \le z \le -3.99)$$

$$= \int_{-\infty}^{-3.99} N(z; 0, 1)\, dz$$

$$= \int_{-\infty}^{-3.99} \left(\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz\right)$$

$$= \frac{1}{2} + \int_{0}^{-3.99} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

$$= \frac{1}{2} - \int_{-3.99}^{0} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

$$= \frac{1}{2} - \int_{0}^{3.99} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz \neq \frac{1}{2} - 0.8000 \neq 01,$$

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(ii) $\therefore P(\text{not qualified}) = \cancel{12000 \times 0 = 0} = 12000 \times \left\{ \frac{1}{2} - \int_0^{3.99} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz \right\}$

(Ans:)

(iii) P(qualified but not eligible for ~~chossi~~ ~~chossi~~ choosing all department)

$= 12000 \times P(40 \le x \le 80)$ (score $= x$)

here, $x_1 = 40 \quad \therefore z_1 = \frac{40-60}{5} = -4$

$x_2 = 80 \quad \therefore z_2 = \frac{80-60}{5} = +4$

$\therefore P(40 \le x \le 80) = P(-4 \le z \le 4)$

$= P(-4 \le z \le 0) + P(0 \le z \le 4)$

$= P(0 \le z \le 4) + P(0 \le z \le 4)$

$= 2 \times P(0 \le z \le 4)$

$= 2 \times \int_0^4 \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$

$\cancel{= 2 \times 0.9997}$

$\therefore$ P(qualified but not eligible for choosing all dept) $= 12000 \times 2 \times \int_0^4 \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$

(Ans:)

(iii) P(eligible for choosing any dept) $= 12000 \times P(x > 80)$

$= 12000 \times [P(x < 40) + P(40 \le x \le 80)]$

$= 12000 \times \left[\frac{1}{2} - \int_0^{3.99} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz + 2 \times \int_0^4 \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz \right]$

(Ans:)

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2017
7(c)

Physical meaning of div$\vec{F}$:-

In vector calculus, the divergence is a vector operator that operates on a vector field, producing a scalar field, giving the quantity of the vector fields source at each point. More technically, the divergence represents the volume density of the outward flux of a vector field from an infinitesimal volume around a given point.

Let,

$\vec{F}(x,y,z) = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$ be defined and differentiable at each point (x,y,z) in a certain region of space (i.e. $\vec{F}$ defines a differentiable vector field).

Then, the divergence of $\vec{F}$ written, $\nabla.\vec{F}$ or div$\vec{F}$ is defined by,

$$\nabla.\vec{F} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(F_1\hat{i} + F_2\hat{j} + F_3\hat{k}\right)$$

$$\therefore \nabla.\vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

Here $\nabla.\vec{F} \neq \vec{F}.\nabla$.

Classification

According to vector differential operator, vector field is of two types :-

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(i) Conservative vector field:-

$\vec{v}$ is a conservative vector field if and only if it's curl is everywhere zero.

$$\text{curl}\,\vec{v} = 0$$

(ii) Solenoidal vector field:-

$\vec{v}$ is defined as being solenoidal if and only if its divergence is everywhere zero.

$$\text{div}\,\vec{V} = 0$$

Physical meaning of Grad φ:

The gradient is a vector function which operates on a scalar function (ϕ) to produce a vector, whose scale is the max. rate of change of the function at the point of the gradient and which is pointed in that direction of that utmost rate of change. The symbol for the gradient is ∇. Let, $\phi(x, y, z)$ be defined and differentiable at each point (x, y, z) in a certain region of space (i.e. defines a differentiable scalar field). Then the gradient of φ can be,

$$\nabla\phi = \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right)\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

Note that, $\nabla\phi$ defines a vector field. The component of $\nabla\phi$ in the direction of unit vector $\hat{a}$, is given by $\nabla\phi.\hat{a}$ and is called the directional derivative of ϕ in the direction of $\hat{a}$. physically, this is the rate of change of ϕ at (x, y, z) in the direction $\hat{a}$.

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Question No: 5(2016)

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SECTION B

(Answer **ANY THREE** questions from this section in Script B)

5. a) Define and explain raw moment and central moment. (08)

b) Calculate β_1 (skewness) and β_2 (kurtosis) and coefficient of skewness based on moment from the following data. Also comment the nature of this distribution. (20)

Age below (years)	25	30	35	40	45	50	55
No. of employees	8	20	40	65	80	92	100

c) What are the difference between skew curve and normal curve? (07)

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2016
5(a)

Raw Moment:

defn: when the actual mean of a distribution is a fraction, it is tedious to calculate central moment. In such cases, moment about an arbitary origin "A" is calculated and then these moment are converted into the moment about actual mean. Such types of moments are called Raw moment.

Explanation:

(1) First raw moment is denoted by μ_1', Second raw moment is denoted by μ_2'. Thus, n'th raw moment is denoted by μ_n'

(2) n'th raw moment about origin is, $\mu_n = \frac{\Sigma(x-A)^n}{N}$

Central moment:-

defn: Moment is the arithmetic mean of various powers of the deviations of item from their assumed mean or actual mean. If the deviations of the item are taken from the arithmetic mean of distribution, it is known as central ~~mo~~ moment.

Explanation:

(1) The mean n'th power of deviations gives the n'th moment about

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mean or r'th central moment and is denoted by μ_r.

(2) r'th moment about the mean is, $\mu_r = \frac{\Sigma(x-\bar{x})^r}{N}$

(3) The first moment about the mean is always zero ($\mu_1 = 0$)

(4) The second moment about the mean is variance. ($\mu_2 = \sigma^2$)

(5) The second and third moment about the mean measures - co-efficient of skewness $\left(\beta_1 = \frac{\mu_3}{(\mu_2)^{3/2}}\right)$

(6) The fourth moment about the mean ~~we~~ measures co-efficient of kurtosis $\left(\beta_2 = \frac{\mu_4}{(\mu_2)^2}\right)$

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2016 5(b)

x_i	f_i	$x_i f_i$
25	8	200
30	20	600
35	40	1400
40	65	2600
45	80	3600
50	92	4600
55	100	5500
Total =	405	18500

Table-1

$$\therefore \bar{x} = \text{mean} = \frac{\sum x_i f_i}{\sum f_i}$$

$$= \frac{18500}{405}$$

$$= 45.7$$

x_i	f_i	$(x_i-\bar{x})$	$(x_i-\bar{x})^2$	$f_i(x_i-x)^2$	$(x_i-\bar{x})^3$	$f_i(x_i-\bar{x})^3$	$(x_i-\bar{x})^4$	$f_i(x_i-\bar{x})^4$
25	8	-20.7	428.49	3427.92	-8869.74	-70957.94	183603.7	1468829.4
30	20	-15.7	246.49	4929.8	-3869.9	-77397.9	60757.32	1215146.4
35	40	-10.7	114.5	4579.6	-1225.04	-49001.72	13107.96	524318.4
40	65	-5.7	32.49	2111.85	-185.2	-12037.5	1055.6	68614
45	80	-.7	0.49	39.2	-0.343	-27.44	0.2401	~~0.2401~~ 19.2
50	92	4.3	18.49	1701.1	79.5	7314.6	341.9	31452.9
55	100	9.3	86.49	864.9	804.36	8043.6	7480.52	74805.2
Total	405			17654.37		-194064.3		3383185.2

Table-02

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The second central moment is, $\mu_2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}$

$= \frac{17654.37}{405}$

$= 43.6$

The third central moment is, $\mu_3 = \frac{\Sigma f_i (x_i - \bar{x})^3}{\Sigma f_i}$

$= \frac{-194064.3}{405}$

$= -479.17$

The fourth central moment is, $\mu_4 = \frac{\Sigma f_i (x_i - \bar{x})^4}{\Sigma f_i}$

$= \frac{3383185.3}{405}$

$= 8353.54$

$\therefore \beta_1 = \frac{\mu_3}{(\mu_2)^{3/2}} = \frac{-479.17}{(43.6)^{3/2}} = -1.73$

$\therefore \beta_2 = \frac{\mu_4}{(\mu_2)^2} = \frac{8353.54}{(43.6)^2} = 4.39$

Comment Since β_1 is -1.73. That means data is negatively skewed.

Since, β_2 is 4.39. So data are mesokurtic.

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5(c) (2016)

The differances between the skew curve and normal curve is given below :-

Skew Curve	Normal Curve
Skew Curve is not Symmetrical.	Normal curve is Symmetrical.
A skew curve is the Probablity distribution curve of a skewed random variable.	A normal curve is the Probablity distribution curve of a normal random variable.
It is a graphical representation of skewed distribution.	It is a graphical representation of normal distribution.
The "bell" is not centred in a skew curve.	The "bell" is in the middle of a normal curve.
Skew curve doesn't ~~know~~ show an even distribution around the mean.	Normal curve shows even distribution around the mean.
The mean, mode, median are not the same in a skew curve.	The values of mean, median and mode are located on the same point.
Skew curve can be negative.	Normal curve can't be negative.

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Question No: 6(2016)

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6. a) Calculate the coefficient of variation and Karl Pearson's coefficient of skewness from the following data: (12)

Marks	No. of students
Less than 30	18
" " 40	40
" " 50	70
" " 60	30
" " 80	10
" " 100	5

b) Define and explain Uniform distribution and Hyper geometric distribution. (10)

c) From a lot of 15 items, 4 are selected at random and fired. If the lot contains 3 defective items that will not fire. What is the probability that (13)

i) None will be defective.

ii) At least one will be defective.

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2016 (6(a))

Table 1

Marks	x_i	f_i	$x_i f_i$
0-29	14.5	18	261
30-39	34.5	40	1380
40-49	44.5	70	3115
50-59	54.5	30	1635
60-~~69~~ 79	69.5	10	695
80-99	89.5	5	447.5
Total		173	7533.5

$$\therefore \bar{x} = \frac{\sum x_i f_i}{\sum f_i}$$

$$= \frac{7533.5}{173}$$

$$= 43.5 = \text{mean}$$

Table-2

x_i	f_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
14.5	18	-29	841	15138
34.5	40	-9	81	3240
44.5	70	1	1	70
54.5	30	11	121	3630
69.5	10	26	676	6760
89.5	5	46	2116	10580
Total	173			39418

Table-2

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$\therefore \mu_2$ = Second central moment

$$= \frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}$$

$$= \frac{39418}{173}$$

$$= 227.85$$

$\therefore$ standard deviation $= \sigma = \sqrt{\mu_2} = \sqrt{227.85} = 15.09$

$\therefore$ Co-efficient of variation $= \frac{\sigma}{\bar{x}} \times 100\%$

$$= \frac{15.09}{43.5} \times 100\% = 34.69\%$$

(Ans:)

<u>Mode</u> :

L_1 = lower class boundary of the modal class $= \frac{39+40}{2} = 39.5$,

Δ_1 = excess of modal frequency over frequency of next lower class

$= (70 - 40) = 30$

Δ_2 = excess of modal f over f of next higher class $= (70-30) = 40$

c = Size of the modal class interval = 10.

$\therefore$ Mode $= L_1 + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) \times c = 40 + \left(\frac{30}{70} \times 10\right) = 44.28$.

$\therefore$ Pearson's first co-efficient of skewness $= \frac{\bar{x} - \text{mode}}{\sigma} = \frac{43.5 - 44.28}{15.09}$

$$= -0.05$$

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Median

As the marks of the table are continuously distributed. Thus,

The median is $\left(\frac{\text{Total frequency}}{2} = \frac{173}{2} = 86.5\right)$ that mark for which 86.5 lies half above it and half lies below it. Since the sum of first two and ~~fir~~ first three class frequencies are $(18+40) = 58$ and $(18+40+70) = 128$. Therefore, median lies in the third class. which is therefore the median class. Then,

L_1 = lower class boundary of the median class $= \frac{39+40}{2} = 39.5$,

N = no. of items in the data $= 173$,

$(\Sigma f)_1$ = sum of f of all ~~lower~~ classes lower than the median class $= 58$,

f_{median} = frequency of the median class $= 70$,

C = size of the median class interval $= 10$.

$$\therefore \text{Median} = L_1 + \left(\frac{N/2 - (\Sigma f)_1}{f_{median}}\right) \times c = 39.5 + \left(\frac{173/2 - 58}{70}\right) \times 10$$

$$= 43.571$$

$\therefore$ Pearson's 2nd co-efficient of skewness,

$$= \frac{3(\text{mean} - \text{median})}{S.D.} = \frac{3(43.5 - 43.571)}{15.09} = -0.014$$

(Ans)

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2016
6(b)

The Uniform distribution :-

A random variable X is said to be uniformly distributed in $a \le x \le b$ if it's density function is,

$$f(x) = \begin{cases} 1/(b-a) & , a \le x \le b \\ 0 & , \text{otherwise} \end{cases}$$

and, the distribution is called uniform distribution.

The distribution function is given by,

$$F(x) = P(X \le x) = \begin{cases} 0 & , x < a \\ (x-a) \div (b-a) & , a \le x \le b \\ 1 & , x \geq b \end{cases}$$

The mean and variance are respectively,

$$\text{mean} = \mu = \frac{1}{2}(a+b)$$

$$\text{variance}, \sigma^2 = \frac{1}{12}(b-a)^2$$

and, standard deviation $= \sigma = \dfrac{(b-a)}{2\sqrt{3}}$

The Hypergeometric distribution :

The hypergeometric distribution is a discrete Probablity distribution that describes the Probablity of k successes (random draws for which the ~~obje~~

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object drawn has a specified feature) in n draws without replacement, from a finite population of size N that contains exactly k objects with that feature, wherein each draw is either a success or a failure.

Suppose that,

a box contains b blue marbles and r red marbles. Let us perform n trials of an experiment in which a marble is chosen at random, its color is observed and then the marble is put back in the box.

In such ~~case~~ cases, if x is the random variable denoting the number of blue marbles chosen (successes) in n trials, then using the binomial distribution, we see that, the probablity of exactly x successes is,

$$P(X=x) = \binom{n}{x} \frac{b^x r^{n-x}}{(b+r)^n}$$

Since, $p = \frac{b}{b+r}$, $q = 1-p = \frac{r}{b+r}$

If we modify the above so that sampling without replacement, i.e., the marbles are not replaced after being chosen, then,

$$P(X=x) = \frac{\binom{b}{x}\binom{r}{n-x}}{\binom{b+r}{n}}, \quad x = \max(0, n-r) \ldots \min(n, b)$$

This is the hypergeometric distribution. The mean and variance are,

$$\mu = \frac{nb}{b+r}, \quad \sigma^2 = \frac{nbr(b+r-n)}{(b+r)^2(b+r-1)}$$

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2016

6(c)

Here, The Probablity of defective items $= p = \frac{\text{No. of items that are defective}}{\text{Total items}}$

or, $p = \frac{3}{15}$

$\therefore p = 0.2$

$\therefore$ The Probablity of non defective items, $q = (1-p) = (1-0.2)$
$= 0.8$

Let, the random variable x be the no. of defective items.

(i) None will be defective:

$$P(x=0) = \binom{4}{0}(0.2)^0(0.8)^4$$

$= 0.4096$

(Ans:)

(ii) At least one will be defective:-

$$P(x \geq 1) = 1 - P(x=0)$$

$= 1 - 0.4096$

$= 0.5904$

(Ans:)

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