CSE 2207
Numerical Methods
Question 2018

1.i) True value $x = 1.25$
$y = 2.16$

Measured value $x = 2.25$
$y = 3.89$

i) $\sqrt{x^2+y^2} = \sqrt{1.25^2 + 2.16^2}$ [Using true value]
$= 2.4956$

$\sqrt{x^2+y^2} = \sqrt{2.25^2 + 3.89^2}$ [Using measured value]
$= 4.4938$

$\therefore$ Absolute error $e_a = |x_t - x_a|$

$= |2.4956 - 4.4938|$
$= 1.99824$

$\therefore$ ~~Real~~ Relative error $e_r = \dfrac{\text{absolute error}}{|\text{true value}|}$

$= \dfrac{1.99824}{|2.4956|}$

$= 0.8007$

$= 80.07\%$

(ii) $xe^y = 1.25 \times e^{2.16}$ [Using true value]

$= 10.8389$

$xe^y = 2.25 \times \cancel{3.89}\, e^{3.89}$ [Using measured value]

$= 110.04949$

$\therefore$ Absolute error $e_a = |x_t - x_a|$

$= |10.8389 - 110.04949|$

$= 99.211$

$\therefore$ Relative error $e_r = \dfrac{\text{absolute error}}{|\text{true value}|}$

$= \dfrac{99.211}{|10.8389|}$

$= 9.153$

$= 915.3\%$

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1. b) Inherent errors are those that are present in the data supplied to the model.
Inherent errors (also known as input errors) contain two components namely data errors and conversion errors

i) Data errors:

Data error (also known as empirical errors) arises when data for a problem are obtained by some experimental means and are, therefore of limited accuracy and precision. This may be due to some limitations in instruments and reading and therefore unavoidable.
A physical measurement, such as a distance, a voltage or a time period can't be exact. It is therefore, important to remember that there is

no use in performing arithmetic operations to say, four decimal places when the original data themselves are only correct to 2 decimal places.

ii)

Conversion errors:

Conversion errors (also known as representation errors) arise due to the limitations of the computer to store the data exactly.
We know that the floating point representation retains only a specified number of ~~points~~ digits. The digits that are not retained constitute the roundoff error.
As we have already seen, many numbers can't be represented exactly in a given number of decimal digits. In some cases, a decimal number can't be represented exactly in binary form. For example, the decimal number 0·1 has a non-terminating binary form like 0·00011001100110011 . . . . but the computer retains only a ~~speif~~ specified number of bits. Thus if we add 10 such numbers in a computer, the result will not be exactly 10 because of roundoff during the conversion of 0·1 to binary form.

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1.c) Iterative methods, based on the numbers of guesses they use can be grouped into 2 catego- categories:

(i) Bracketing methods (interpolation method)

(ii) Open end ~~s~~ methods (extrapolation method)

These two methods have some huge difference and they are given below:

Open end method	Bracketing method
a) In open end method, requires only a single starting value, no need to bracket a root.	a) In Bracketing method, requires two values, starting values and maximum limit. Need to bracketing a root.
b) Open methods, divergence, that means it goes from the actual root.	b) Bracketing Method is convergence. That means, it go to actual root.
c) Open methods sometimes not get the result.	c) Bracketing method gives the result ~~of~~ at any situation.
d) Open end method is faster than Bracketing method.	d) Bracketing method is slower than Open end method.

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1. d) i) ~~25~~ $25.68 \div 6.567 = 3.9105$

True $x = 0.39105 \times 10$

$= (0.3910 + 0.00005) \times 10$

$= (0.3910 + 0.5 \times 10^{-4}) \times 10$

$= 0.3910 \times 10 + \underbrace{0.5}_{g_x} \times 10^{-3}$

~~Appro~~ Chopping Method: Approximate $x = 0.3910 \times 10$

Error $= 0.5 \times 10^{-3}$

Symmetric Rounding: Error $= (g_x - 1) \times 10^{-3}$

$= (0.5 - 1) \times 10^{-3}$

$= -0.5 \times 10^{-3}$

Approximate $x = f_x \times 10^E + 10^{E-d}$

$= 0.3910 \times 10^1 + 10^{1-4}$

$= 0.3911 \times 10$

ii) $87.26 + 31.42 = 118.68$

True $x = 0.11868 \times 10^3$

$= (0.1186 + 0.00008) \times 10^3$

$= (0.1186 + 0.8 \times 10^{-4}) \times 10^3$

$= 0.1186 \times 10^3 + 0.8 \times 10^{-1}$

Chopping Method: Approximate $x = 0.1186 \times 10^3$

Error $= 0.8 \times 10^{-1}$

Symmetric Rounding: Error $= (g_x - 1) \times 10^{-1}$

$= (0.8 - 1) \times 10^{-1}$

$= -0.2 \times 10^{-1}$

Approximate $x = f_x \times 10^E + 10^{E-d}$

$= 0.1186 \times 10^3 + 10^{3-4}$

$= 0.1187 \times 10^3$

iii) $752.6835 \div 2.913 = 258.3877$

True $x = \cancel{0.39}\ 0.2583877 \times 10^3$

$= (0.2583 + 0.0000877) \times 10^3$

$= (0.2583 + 0.877 \times 10^{-4}) \times 10^3$

$= 0.2583 \times 10^3 + 0.877 \times 10^{-1}$

Chopping Method: Approximate $x = 0.2583 \times 10^3$

Error $= 0.877 \times 10^{-1}$

Symmetric Rounding: Error $= (g_x - 1) \times 10^{-1}$

$= (0.877 - 1) \times 10^{-1}$

$= -0.123 \times 10^{-1}$

Approximate $x = f_x \times 10^E + 10^{E-d}$

$= 0.2583 \times 10^3 + 10^{3-4}$

~~$= 0.2584 \times 10^4$~~

$= 0.2584 \times 10^3$

2018

2.(a) For a polynomial represented by:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

The largest possible root is given by,

$$x_1^* = -\frac{a_{n-1}}{a_n}$$

This value is taken as the initial approximation when no other value is suggested by the ~~knoaod~~ knowledge of the problem at hand.

Search Bracket:

Another relationship that might be useful for determining the search intervals that contain the real roots of a polynomial is,

$$|x^*| = \sqrt{\left[\frac{a_{n-1}}{a_n}\right]^2 - 2\left(\frac{a_{n-2}}{a_n}\right)}$$

where x is the root of the polynomial. Then, the ~~maxim~~ maximum absolute value of the root is,

$$|x^*_{max}| = \sqrt{\left(\frac{a_{n-1}}{a_n}\right)^2 - 2\left(\frac{a_{n-2}}{a_n}\right)}$$

This means that no root exceeds X_{max} in absolute magnitude and thus, all real roots lie within the interval $(-|X^*_{max}|, |X^*_{max}|)$.

There is yet another relationship that suggests an interval for roots: All real roots x satisfy the inequality

$$|x^*| \le 1 + \frac{1}{|a_n|} \max \{|a_0|, |a_1|, |a_2|, \ldots\ldots, |a_{n-1}|\}$$

where ~~max~~ the "max" denotes the maximum of the absolute values $|a_0|, |a_1|, \ldots\ldots, |a_{n-1}|$.

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2. b) $x^2 - 5x + 6 = 0$ $\quad x_1 = 4, \; x_2 = 5$

$$x_{i+1} = x_i - f(x_i)\,\frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})}$$

This is the secant formula.

$f_1 = f(x_1) = 2$. $\quad \therefore f_2 = f(x_2) = 6$

For 1st iteration,

$$x_3 = x_2 - f(x_2)\,\frac{x_2 - x_1}{f(x_2) - f(x_1)}$$

$$= 5 - 6 \times \frac{5-4}{6-2} = 3.5$$

For second iteration, $\quad f_1 = 6$

$x_1 = \cancel{3.5}\; x_2 = 5$

$x_2 = x_3 = 3.5 \quad f_2 = 0.75$

$$x_3 = x_2 - f(x_2)\,\frac{x_2 - x_1}{f(x_2) - f(x_1)} = 3.2857$$

For third iteration, $\quad f_1 = 0.75$

$x_1 = x_2 = 3.5$

$x_2 = x_3 = 3.2857 \quad f_2 = 0.3673$

$$x_3 = x_2 - f(x_2)\,\frac{x_2 - x_1}{f(x_2) - f(x_1)} = 3.08$$

for 4th iteration,

$x_1 = x_2 = 3.2857$ $\quad f_1 = 0.3673$

$x_2 = x_3 = 3.08$ $\quad f_2 = 0.0864$

$$x_3 = x_2 - f(x_2)\frac{x_2 - x_1}{f(x_2) - f(x_1)} = 3.0167$$

for 5th iteration,

$x_1 = x_2 = 3.08$ $\quad f_1 = 0.0864$

$x_2 = x_3 = 3.0167$ $\quad f_2 = 0.017017$

$$x_3 = x_2 - f(x_2)\frac{x_2 - x_1}{f(x_2) - f(x_1)} = 3.001221$$

for ~~st~~ 6th iteration,

$x_1 = x_2 = 3.0167$ $\quad f_1 = 0.017017$

$x_2 = x_3 = 3.001221$ $\quad f_2 = 0.001222$

$$x_3 = x_2 - f(x_2)\frac{x_2 - x_1}{f(x_2) - f(x_1)} = 3.00002$$

(Ans)

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2.e) Let's consider $f(x) = x^3 - 4x^2 + x + 6$

Let's assume $x_1 = 5$ (1st approximation)

$f'(x) = 3x^2 - 8x + 1$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \cancel{4} \; 5 - \frac{36}{36} = 4$$

$$\therefore x_3 = 4 - \frac{10}{17} = 3.411765$$

$$\therefore x_4 = 3.411765 - \frac{2.564624}{8.626298} = 3.114462$$

$$\therefore x_5 = 3.114462 - \frac{0.524854}{5.183921} = 3.013215$$

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$$x_6 = x_5 - \frac{f}{} \; 3.013215 - \frac{0.053736}{4.132676} = 3.000212$$

$$x_7 = 3.000212 - \frac{0.000850}{4.132676} = 3.000000$$

Iteration	Estimation	Correct Digits
1	5.000000	NIL
2	4.000000	NIL
3	3.411765	1
4	3.114462	1
5	3.013215	2
6	3.000212	4
7	3.000000	7

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2. d) $x^2 - 3x + 2 = 0$; ~~with~~ initial estimate $x_0 = 0$

$f(x) = x^2 - 3x + 2 \qquad f'(x) = 2x - 3$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{2}{-3} = \cancel{0.6666}\; 0.6667$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.6667 - \frac{0.4444}{-1.6667} = 0.9333$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.9333 - \frac{0.0711}{-1.1333} = 0.9961$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 0.9961 - \frac{0.0039}{-1.0078} = 0.9999$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 0.9999 - \frac{0.0001}{-1.0002} = 1.000$$

since $f(1.00) = 0$, the root closer to the point $x = 0$ is 1.00.

2018
3.a) A table of data may be constructed from measurements made during an experiment. In such experiments, values of the dependent variable are recorded at various values of the independent variable.

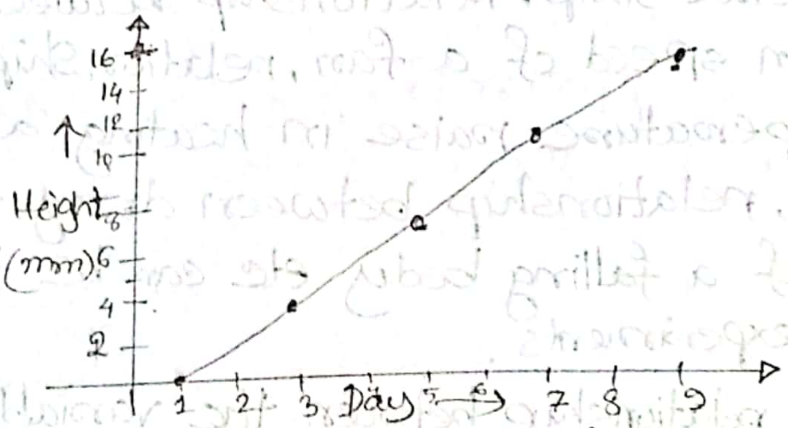
There are numerous examples of such experiments — the relationship between stress and strain on a metal strip, relationship between voltage applied on speed of a fan, relationship between time and temperature raise in heating a given volume of water, relationship between drag force and velocity of a falling body etc can be tabulated by suitable experiments.

The relationship between the variables is not well defined. Accordingly we have two approaches for fitting a curve to a given set of data points.

The function is constructed such that it passes through all the data points. This method of constructing a function and estimating values at non-tabular points is called Interpolation. So Inet interpolation is a method by which related known values are used to

~~estimd~~ estimate an unknown value.
Here's an example that will illustrate the concept. A gardener kept track of his tomato plant ~~crops~~ every ~~year~~. day.

Day	Height (mm)
1	0
3	4
5	8
7	12
9	16



We can easily construct the function as this a linear pattern. $f(x) = mx$.

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3. b) Let's assume that $x_k = x_0 + kh$
The first forward difference $\Delta f_i = \Delta f_{i+1} - f_i$
The second forward difference

$$\Delta^2 f_{\cancel{i+1}} = \Delta f_{i+1} - \Delta f_i$$

In general $\Delta^j f_i = \Delta^{j-1} f_{i+1} - \Delta^{j-1} f_i$

We can now express the simple forward difference in terms of divided differences. We know that,

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f_1 - f_0}{h}$$

Therefore, ~~$f_1 =$~~ $f_1 - f_0 = hf[x_0, x_1]$

Then, $\Delta f_0 = hf[x_0, x_1]$

Similarly, $\Delta f_1 = hf[x_1, x_2]$

Now,

$$\begin{aligned}\Delta^2 f_0 &= \Delta f_1 - \Delta f_0 \\ &= hf[x_1, x_2] - hf[x_0, x_1] \\ &= h\{f[x_1, x_2] - f[x_0, x_1]\} \\ &= h\{2h\, f[x_0, x_1, x_2]\} \\ &= 2h^2 f[x_0, x_1, x_2]\end{aligned}$$

$[\because f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{2h}$

In general by induction,

$$\Delta^j f_i = j!\, h^j f[x_i, x_{i+1}, \ldots\ldots\ldots, x_{i+j}]$$

Therefore,

$$f[x_0, x_1, x_2, \ldots, x_j] = \frac{\Delta^j f_0}{j!\, h^j} \quad \text{------------ (i)}$$

We know that, Newton's divided difference interpolation polynomial is,

$$P_n(x) = \sum_{j=0}^{n} f[x_0, \ldots\ldots x_j] \prod_{i=0}^{j-1} (x - x_i) \quad \text{------------ (ii)}$$

Substituting (i) in Newton's formula (ii), we get

$$P_n(x) = \sum_{j=0}^{n} \frac{\Delta^j f_0}{j!\, h^j} \prod_{k=0}^{j-1} (x - x_k) \quad \text{.......... (iii)}$$

Let's set,

$$x = x_0 + sh \quad \text{and} \quad P_n(s) = P_n(x)$$

We assumed that, $x_k = x_0 + kh$

$$x = x_0 + sh$$
$$x_k = x_0 + kh$$
$$\overline{x - x_k = (s-k)h}$$

Substituting this eqn (iii)

$$P_n(s) = \sum_{j=0}^{n} \frac{\Delta^j f_0}{j!\, h^j} \prod_{k=0}^{j-1} (s-k)h$$

$$= \sum_{j=0}^{n} \frac{\Delta^j f_0}{j!\, h^j} \left[s(s-1)(s-2)\cdots\cdots(s-j+1) \right] h^j$$

Thus ~~P_n~~ $P_n(s) = \sum_{j=0}^{n} \frac{\Delta^j f_0}{\cancel{j!}\, h^j} \binom{s}{j} h^j$

$$= \sum_{j=0}^{n} \binom{s}{j} \Delta^j f_0$$

where $\binom{s}{j} = \dfrac{s(s-1)(s-2)\cdots\cdots(s-j+1)}{j!}$

This is known as ~~N~~ Gregory-Newton forward difference formula.

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3. c)

	0	1	2	3	4
x	1	2	3	4	5
$f(x)$	1	1.4142	1.7321	2	2.2361

$$l_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x-x_4)}$$

$$= \frac{(x-2)(x-3)(x-4)(x-5)}{(1-2)(1-3)(1-4)(1-5)}$$

$$= \frac{(x^2-5x+6)(x^2-9x+20)}{24}$$

$$= \frac{x^4-9x^3+20x^2-5x^3+45x^2-100x+6x^2-54x+120}{24}$$

$$= \frac{x^4-\cancel{9}\,14x^3+71x^2-154x+120}{24}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}$$

$$= \frac{(x-1)(x-3)(x-4)(x-5)}{\cancel{1}(2-1)(2-3)(2-4)(2-5)}$$

$$= \frac{(x^2-4x+3)(x^2-9x+20)}{-6}$$

$$= \frac{x^4-9x^3+20x^2-4x^3+36x^2-80x+3x^2-27x+60}{-6}$$

$$= \frac{x^4-13x^3+\cancel{56x^2}\,59x^2-107x+60}{-6}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}$$

$$= \frac{(x-1)(x-2)(x-4)(x-5)}{(3-1)(3-2)(3-4)(3-5)}$$

$$= \frac{(x^2-3x+2)(x^2-9x+20)}{4}$$

$$= \frac{x^4-9x^3+20x^2-3x^3+27x^2-60x+2x^2-18x+40}{4}$$

$$= \frac{x^4-12x^3+49x^2-78x+40}{4}$$

$$l_3(x) = \frac{(x-1)(x-2)(x-3)(x-5)}{(4-1)(4-2)(4-3)(4-5)}$$

$$= \frac{(x^2-3x+2)(x^2-8x+15)}{-6}$$

$$= \frac{x^4-8x^3+15x^2-3x^3+24x^2-45x+2x^2-16x+30}{-6}$$

$$= \frac{x^4-11x^3+41x^2-61x+30}{-6}$$

$$l_4(x) = \frac{(x-1)(x-2)(x-3)(x-4)}{(5-1)(5-2)(5-3)(5-4)}$$

$$= \frac{(x^2-3x+2)(x^2-7x+12)}{4\times 3\times 2\times 1}$$

$$= \frac{x^4-7x^3+12x^2-3x^3+\cancel{7x}\,21x^2-36x+2x^2-14x+24}{24}$$

$$= \frac{x^4-10x^3+35x^2-50x+24}{24}$$

$$P_4(x) = f_0 l_0(x) + f_1 l_1(x) + f_2 l_2(x) + f_3 l_3(x) + f_4 l_4(x)$$

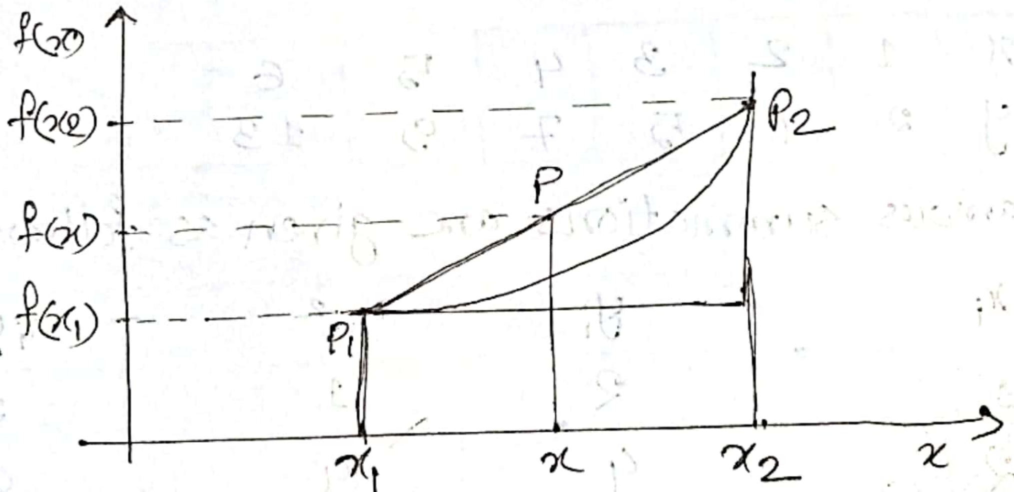
$$= 1 \times \frac{x^4 - 14x^3 + 71x^2 - 154x + 120}{24} + 1.4142 \times \frac{x^4 - 13x^3 + 59x^2 - 107x + 60}{-6} + 1.7321 \times \frac{x^4 - 12x^3 + 49x^2 - 78x + 40}{4}$$

$$+ 2 \times \frac{x^4 - 11x^3 + 41x^2 - 61x + 30}{-6} + 2.2361 \times \frac{x^4 - 10x^3 + 35x^2 - 50x + 24}{24}$$

$$= -1.17083 \times 10^{-3} x^4 + 0.019425x^3 - 0.135429x^2 + 0.702075x + 0.4151 \text{ (Ans)}$$

$$P_4(2.5) = 1.6731 \text{ (Ans)}$$

2018 3.d)



Suppose we are given two points $(x_1, f(x_1))$ and $(x_2, f(x_2))$. These two points can be connected linearly as shown in the figure above. Using the concept of similar triangles we can show that,

$$\frac{f(x) - f(x_1)}{x - x_1} = \frac{f(x_2) - f(x_1)}{\cancel{f}\ x_2 - x_1}$$

$$\therefore f(x) = f(x_1) + (x - x_1)\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

This ~~t~~ is known as Linear interpolation formula.

2018.

4. a) Regression Analysis.

Regression Analysis is a set of statistical methods used for the estimation of relationships between a dependent variable and one or more independent variables. It can be utilized to assess the ~~streg~~ strength of the relationship between variables and for modelling the future relationship between ~~b~~ them.

x	1	2	3	4	5	6
y	2	4	5	7	9	13

The various summations are given as follows:

x_i	y_i	x_i^2	$x_i y_i$
1	2	1	2
2	4	4	8
3	5	9	15
4	7	16	28
5	9	25	45
6	13	36	78
Σ 21	40	91	176

We know;

$$b = \frac{n\Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n \Sigma x_i^2 - (\Sigma x_i)^2}$$

$$= \frac{6\cancel{5} \times 176 - 21 \times 40}{6 \times 91 - (21)^2} = 2.057$$

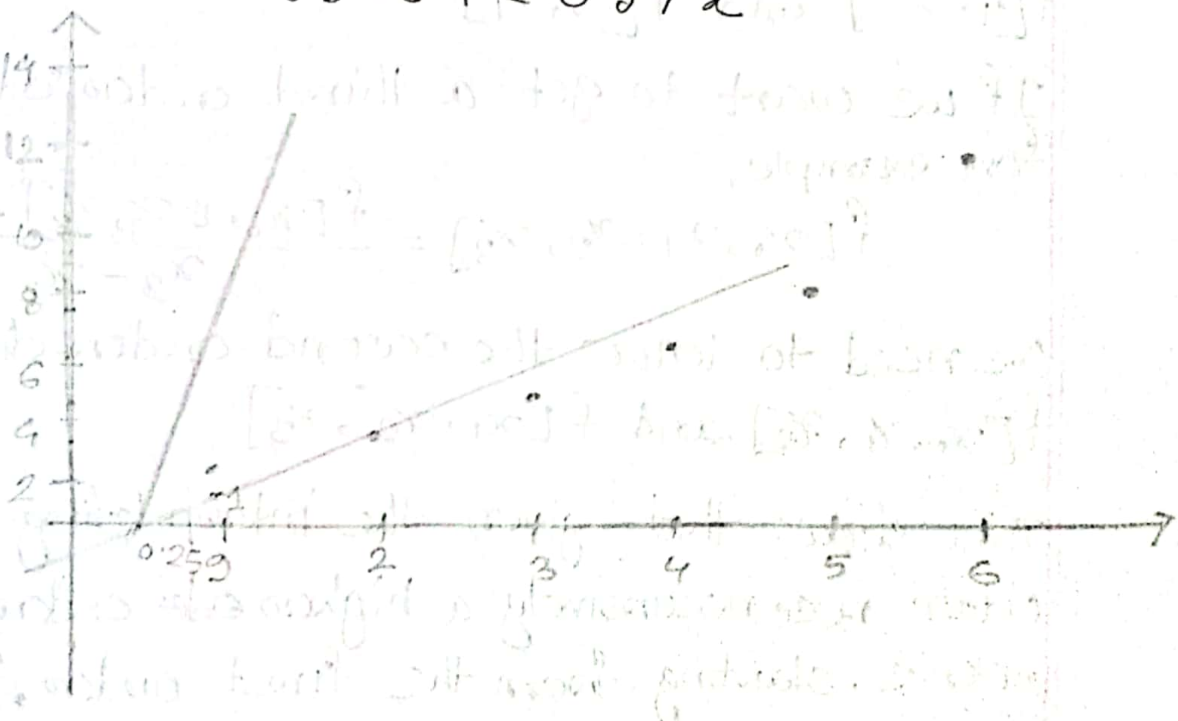
$$a = \frac{\Sigma y_i}{n} - b \frac{\Sigma x_i}{n}$$

$$= \frac{40}{6} - 2.057 \times \frac{21}{6} = -0.5328$$

Therefore the linear equation is,

$$y = a + bx$$

$$= -0.5328 + 2.057x$$



2018. 4.b) We know, the first order divided differences are given by,

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$f[x_1, x_2] = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$f[x_n, x_{n+1}] = \frac{f(x_{n+1}) - f(x_n)}{x_{n+1} - x_n}$$

If we want to get a second order divided difference for example,

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

we need to know the first order divided difference $f[x_1, x_2]$ and $f[x_0, x_1]$

If we want to get a third order divided difference for example,

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_0, x_1, x_2] - f[x_1, x_2, x_3]}{x_3 - x_0}$$

we need to know the second order divided difference $f[x_0, x_1, x_2]$ and $f[x_1, x_2, x_3]$

This shows that, given the interpolating points, we can obtain ~~rece~~ recursively a higher ~~ode~~ order divided difference, starting from the first order differences.

A ~~for~~ fourth order divided Difference Table is given below: (Let's assume we have 5 data points

i	x_i	$f[x_i]$	First Difference	2nd Difference	Third Difference	Fourth Difference
0	x_0	$f[x_0]$	$f[x_0, x_1]$			
1	x_1	$f[x_1]$		$f[x_0, x_1, x_2]$	$f[x_0, x_1, x_2, x_3]$	
			$f[x_2, x_1]$			
2	x_2	$f[x_2]$		$f[x_1, x_2, x_3]$		$f[x_0, x_1, x_2, x_3, x_4]$
			$f[x_3, x_2]$		$f[x_1, x_2, x_3, x_4]$	
3	x_3	$f[x_3]$		$f[x_2, x_3, x_4]$	⋮	
			$f[x_4, x_3]$	⋮		
4	x_4	$f[x_4]$				
⋮						

2018 4.c)

i	0	1	2	3
x	1	2	3	4
y	0	0.6931	1.0986	1.3863

We know,
Newton Gregory backward difference interpolation polynomial is:

$$P_n(s) = f_n + s\nabla f_n + \frac{s(s+1)}{2!} + \cdots + \frac{s(s+1)\cdots(s+n-1)}{n!}\nabla^n f_n$$

where $s = \frac{x - x_n}{h}$

$x_n = 4 \quad x = 3.5 \quad h = 1$

$\therefore s = \frac{x - x_n}{h} = \frac{3.5 - 4}{1} = -0.5$

as $n = 3$

$$\therefore P_3(s) = f_3 + s\nabla f_3 + \frac{s(s+1)}{2!}\nabla^2 f_3 + \frac{s(s+1)(s+2)}{3!}\nabla^3 f_3$$

Let's generate backward difference table:

i	x_i	f_i	∇f	$\nabla^2 f$	$\nabla^3 f$
0	1	0.0			
			0.6931		
1	2	0.6931		-0.2876	
			0.4055		0.1698
2	3	1.0986		-0.1178	
			0.2877		
3	4	1.3863			

$$P_3(-0.5) = 1.3863 + (-0.5)\times 0.2877 + \frac{(-0.5)(-0.5+1)}{2!} \times (-0.1178) + \frac{(-0.5)\times(-0.5+1)(-0.5+2)}{3!} \times 0.1698$$

$$= \cancel{1.3863} \; 1.24656$$