Hasanuzzaman Sir

Lecture-1

TOPIC NAME : Complex Number

DAY: Tuesday

TIME:

DATE: 8 / 8 / 23

MATH 2207 : Complex Variables & Vectors

$1 \rightarrow$ integer

\# $x^2 - 4 = 0$
$\Rightarrow x = \pm 2$

\# $x^2 - 5 = 0$
$\Rightarrow x = \pm\sqrt{5}$
$= \pm 2.----$
$\downarrow$
Real Number

\# $x^2 + 4 = 0$
$\Rightarrow x = \pm 2i$
$\downarrow$
complex Number

\# $1 \rightarrow$ integer
$\downarrow$
$1.0 \rightarrow$ Real Number ; Decimal দিলেই Real
$\downarrow$
$1+0i \rightarrow$ complex Number

\# integer
$\uparrow$
$1.2 \rightarrow$ Real number
$\downarrow$
$1.2 + 0i \rightarrow$ complex

Real
$\uparrow$
\# $1+2i \rightarrow$ complex
$\updownarrow$
integer

* প্রত্যেক Number এর complex form থাকবে, কিন্তু প্রকৃত complex শুধু complex ই থাকবে।

$\oplus$ $I \subseteq \mathbb{R} \subseteq \mathbb{C}$

TOPIC NAME : DAY : TIME : DATE : / /

Complex Number: The number of the form $a+ib$ $(i=\sqrt{-1})$ is called the complex number. It is usually denoted by c. $c=a+ib$. Here a is called the real part of complex number c. $a =$ Real Part of $c = Re(c)$.

b is called the imaginary part of c. $b =$ Imaginary of $c = Im(c)$.

Complex Variable: Any pair of the variables x and y such as $(x,y) = z = x+iy$ is called the complex variable.

$$x = Re(z),\ y = Im(z)$$

Conjugate of the complex Number:

$$ax^2+bx+c=0$$

$$b^2-4ac>0$$

$b^2-4ac<0 \longrightarrow$ অবাস্তব

TOPIC NAME : DAY : TIME : DATE : / /

Let, $z = x+iy$ —— ①

Conjugate of $z = \bar{z} = \overline{(x+iy)} = x-iy$ —— ②

① + ⑪ :

$z+\bar{z} = x+iy+x-iy = 2x$

$\Rightarrow x = \dfrac{z+\bar{z}}{2}$

$\Rightarrow Re(z) = \frac{1}{2}(z+\bar{z})$

① − ⑪ :

$Im(z) = \frac{1}{2i}(z-\bar{z})$

Absolute of z } same জিনিস
Modulus of z }

$z = x+iy$

Absolute value of z, $|z| = \sqrt{x^2+y^2} = |\bar{z}|$

$\bar{z} = x-iy$, $|\bar{z}| = \sqrt{x^2+(-y)^2}$

$10 = \sqrt{7+3}$

$10 > \sqrt{7}$

$|z| \geq \sqrt{x^2}$

$\Rightarrow |z| \geq x$

$\Rightarrow |z| \geq Re(z)$

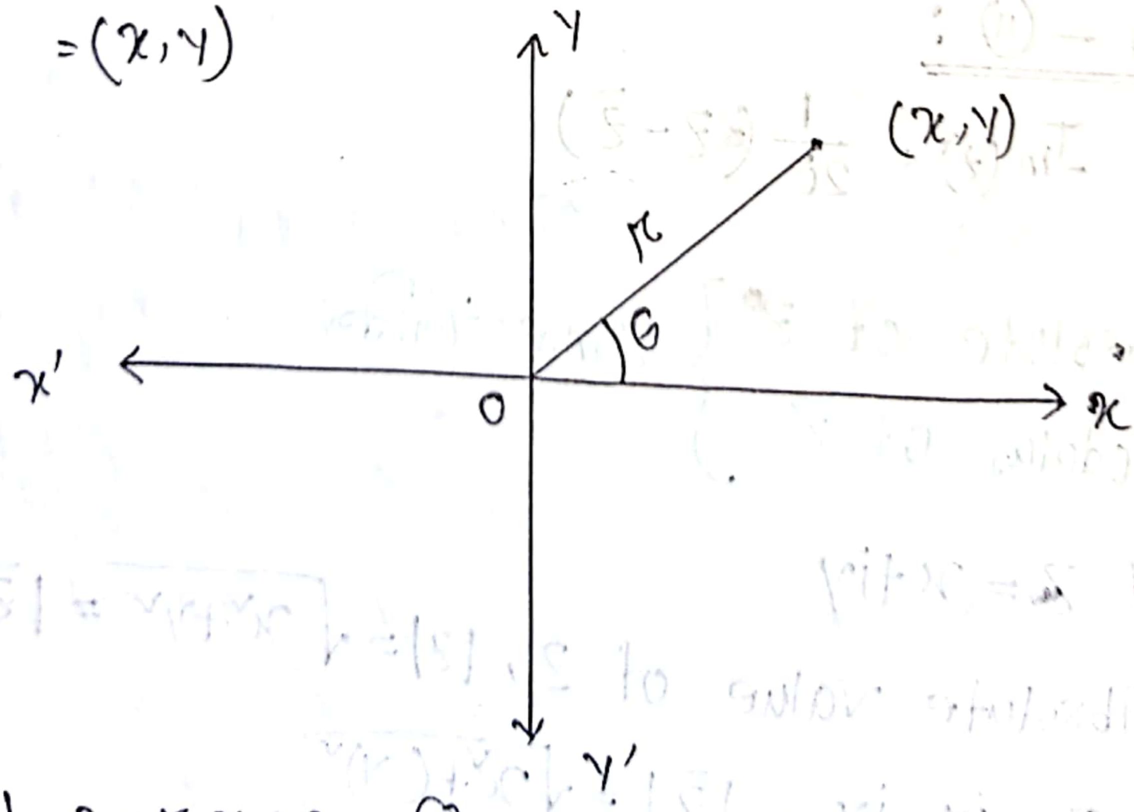
$\Rightarrow Re(z) \leq |z|$ # $Re(\bar{z}) \leq |\bar{z}|$

$Im(z) \leq |z|$, $Im(\bar{z}) \leq |\bar{z}|$

Argument:

$z = x+iy$ — ① be a complex Number

$= (x, y)$



Let, $x = r\cos\theta$ — ②

$y = r\sin\theta$ — ③

TOPIC NAME : DAY : TIME : DATE : / /

$② ^2 + ③ ^2$:

$$x^2+y^2 = r^2\sin^2\theta + r^2\cos^2\theta = r^2(\sin^2\theta+\cos^2\theta) = r^2$$

$$\Rightarrow r = \sqrt{x^2+y^2} = |z|$$

③ ÷ ② : $\tan\theta = \frac{y}{x}$

$$\Rightarrow \theta = \tan^{-1}(y/x)$$

$$① \Rightarrow z = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

$$= \sqrt{x^2+y^2}\, e^{i\tan^{-1}(y/x)}$$

Find the Polar form of $1+i$

Soln: let, $z = 1+i$ — ①

let, $1 = r\cos\theta$ — ②

$1 = r\sin\theta$ — ③

$② ^2 + ③ ^2$:

$$1^2+1^2 = r^2(\sin^2\theta+\cos^2\theta)$$

$$\Rightarrow r = \sqrt{2}$$

③ ÷ ② :

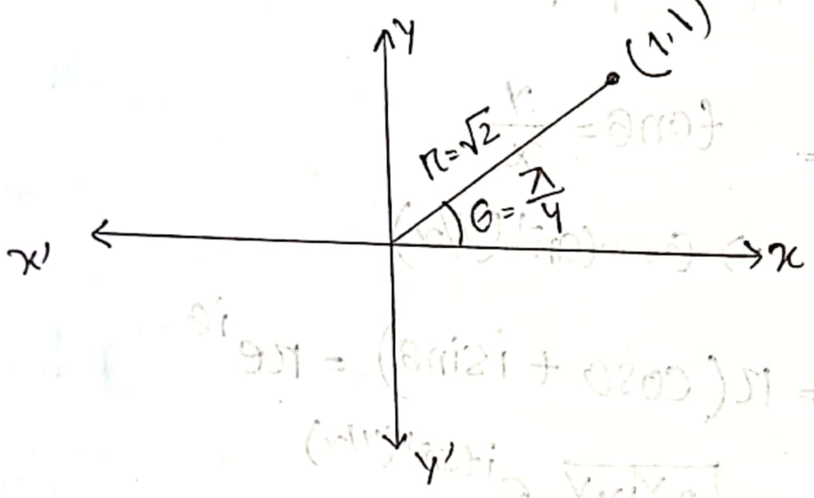
$$\theta = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$$

TOPIC NAME : DAY : TIME : DATE :

Putting the value in ①

$z = re^{i\theta}$

$= \sqrt{2}\, e^{i\pi/4}$



Lecture-2

17/8/23

Thursday

Complex Variable

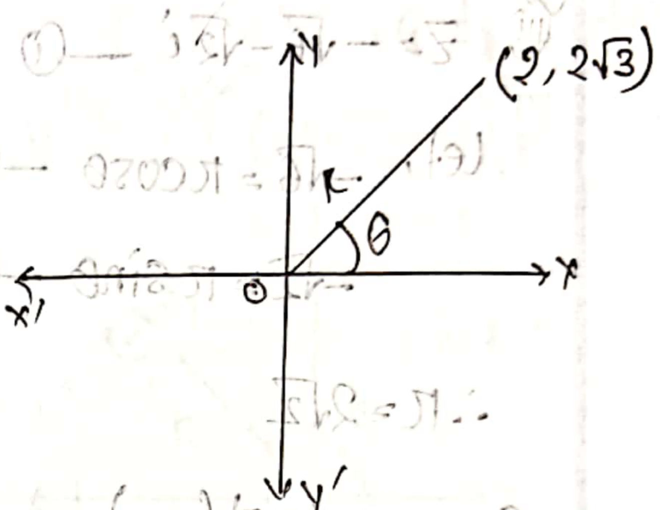
Hasanuzzaman Sir

① Express each of the following complex number in polar form (i) $2+2\sqrt{3}i$ (ii) $-5+5i$ (iii) $-6-\sqrt{2}i$, (iv) $-3i$

TOPIC NAME : DAY : TIME : DATE : / /

Soln:

(i) Let. $z = 2 + 2\sqrt{3}i$



Let, $2 = r\cos\theta$

$2\sqrt{3} = r\sin\theta$

$\therefore r = 4$

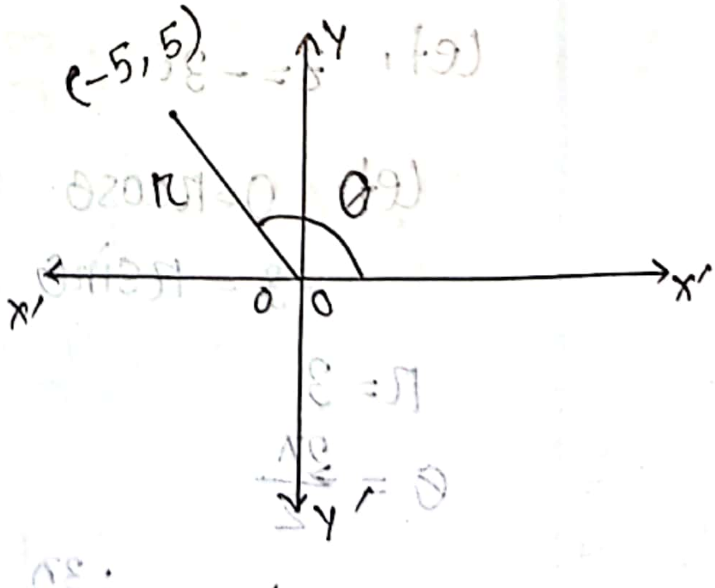
$\therefore \theta = \tan^{-1}(\ \ ) = \frac{\pi}{3}$

(1) $z = re^{i\theta} = 4e^{i\frac{\pi}{3}}$

(ii) $z = -5 + 5i$ — (1)

Let, $-5 = r\cos\theta$ — (ii)

$5 = r\sin\theta$ — (iii)



$r = 5\sqrt{2}$

$\theta = \tan^{-1}(y/x)$

$\pi -$

$= \pi - \tan^{-1}(5/5) = \frac{3\pi}{4}$

(1) $\Rightarrow z = re^{i\theta} = 5\sqrt{2}e^{i\frac{3\pi}{4}}$

TOPIC NAME :

DAY :

TIME : DATE : /

(iii) $z = -\sqrt{6} - \sqrt{2}\,i$ — (i)

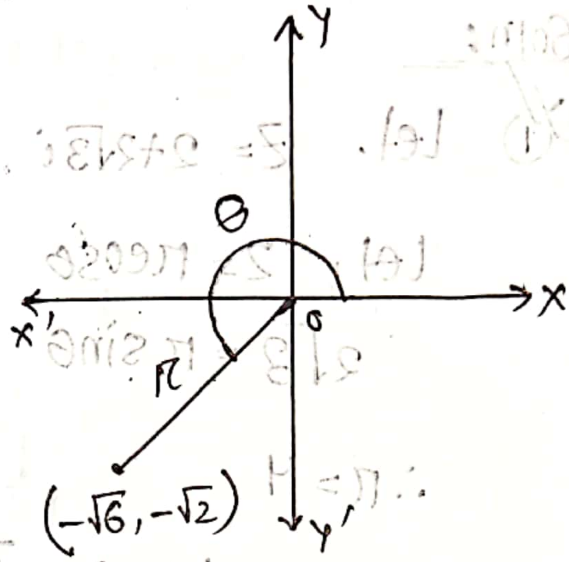
Let, $-\sqrt{6} = r\cos\theta$ — (ii)

$-\sqrt{2} = r\sin\theta$ — (iii)

$\therefore r = 2\sqrt{2}$

$\theta = \pi + \tan^{-1}(\quad)$

$=$



(iv) $z = -3i$ — (i)

Let, $z = -3i$

Let, $0 = r\cos\theta$

$-3 = r\sin\theta$

$r = 3$

$\theta = \frac{3\pi}{2}$

$\therefore z = re^{i\theta} = 3e^{i\frac{3\pi}{2}}$

4th:

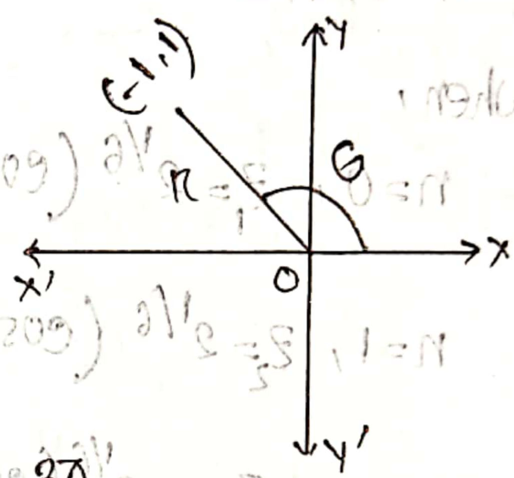
$\theta = 2\pi - \tan^{-1}(y/x)$

exam এ আসবে।

TOPIC NAME : DAY : TIME : DATE : / /

Find each of the indicated roots and locate them graphically (i) $(-1+i)^{1/3}$ (ii) $(-2\sqrt{3}-2i)^{1/4}$

Soln: let, $z=-1+i$



let, $-1=r\cos\theta$

$1=r\sin\theta$

$\therefore r=\sqrt{2}$

$\therefore \theta=\pi-\tan^{-1}(y/x)=\frac{3\pi}{4}$

$\therefore z=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$

$\Rightarrow z=-1+i=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$

$\Rightarrow z^{1/3}=(-1+i)^{1/3}=\left\{\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)\right\}^{1/3}$

$(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$

$=2^{1/6}\left\{\cos\left(\underbrace{2n\pi}_{\downarrow}+\frac{3\pi}{4}\right)+i\sin\left(2n\pi+\frac{3\pi}{4}\right)\right\}^{1/3}$

0/even হওয়া লাগবে।

গুরুত্ব step

TOPIC NAME : DAY : TIME : DATE : / /

$$= 2^{1/6}\left\{\cos\frac{2n\pi+\frac{3\pi}{4}}{3} + i\sin\frac{2n\pi+\frac{3\pi}{4}}{3}\right\}$$

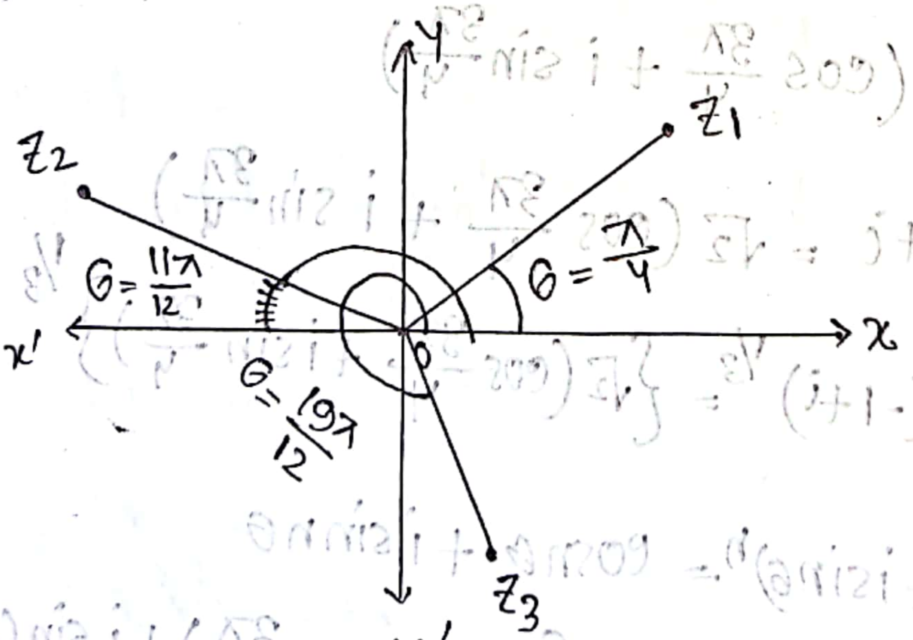
$n = 0, 1, 2$

When,

$n=0$, $z_1 = 2^{1/6}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$n=1$, $z_2 = 2^{1/6}\left(\cos\frac{11}{12}\pi + i\sin\frac{11}{12}\pi\right)$

$n=2$, $z_3 = 2^{1/6}\left(\cos\frac{19}{12}\pi + i\sin\frac{19}{12}\pi\right)$



TOPIC NAME : DAY : TIME : DATE : / /

Find the equation of circle centre at $(-5, 2)$ and radius 7.

Soln: The equation of a circle with center at a and radius r is $|z - a| = r$

$$\Rightarrow |(x,y) - (-3,2)| = r$$

$$\Rightarrow |x+iy-(-5+2i)| = 7$$

$$\Rightarrow (x+5)^2 + (y-2)^2 = 49$$

Represent graphically the set of values of z for which (i) $\left|\frac{z-3}{z+3}\right| = 2$ (ii) $\left|\frac{z-3}{z+3}\right| < 2$

(a) $\left|\frac{z-4}{z+4}\right| = 9 \longrightarrow PB = 9PA$

Soln: we have -

$$\left|\frac{z-3}{z+3}\right| = 2$$

$$\Rightarrow \left|\frac{z-3}{z+3}\right| = 2$$

$$\Rightarrow \left|\frac{x+iy-3}{x+iy+3}\right| = 2$$

$$\Rightarrow \left|\frac{(x-3)^2 + y^2}{(x+3)^2 + y^2}\right| = 2$$

$\Rightarrow$

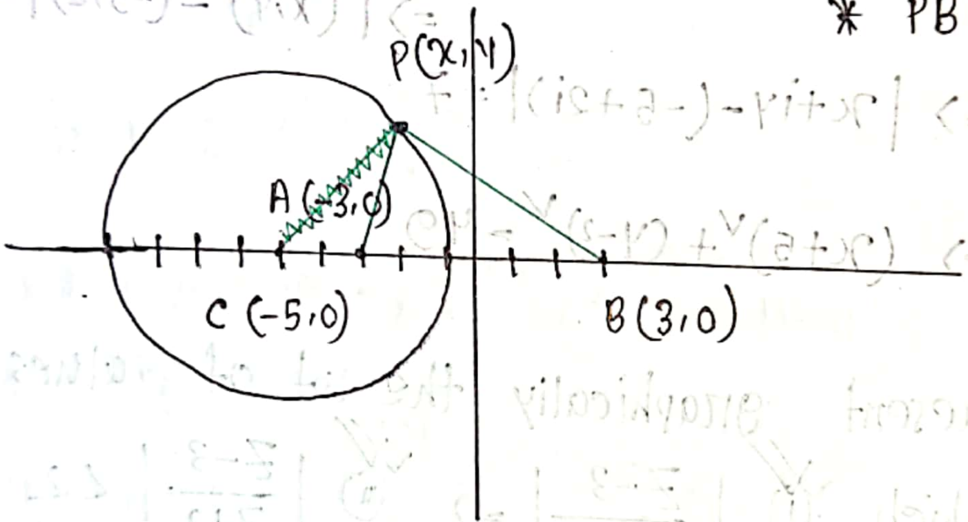
$$\Rightarrow x^2 + y^2 + 10x + 9 = 0$$

TOPIC NAME : DAY : TIME : DATE : / /

$\Rightarrow (x+5)^2+y^2=4^2$

This is a equation of circle center at $(-5,0)$ and radius is 4.

$*$ $PB = 2PA$



$PB = 2PA$, Geometrically, any Point P on this Circle is such that the distance from P to Point B(3,0) is twice the distance from P to point A(-3,0).

(ii) $(x+5)^2+y^2>16 \Rightarrow |z+5|>4$

The required set thus consists of all points external to the circle.

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Lecture-3

TOPIC NAME : Complex Variable

DAY : Tuesday

TIME :

DATE : 22/ 8 / 23

Ⓗ Find the equation of circle whose center at $(2,-5)$ and radius 9.

Soln: The equation of a circle with center at a and radius r is -

$$|z-a| = r$$

$$\Rightarrow |(x,y) - (2,-5)| = 9$$

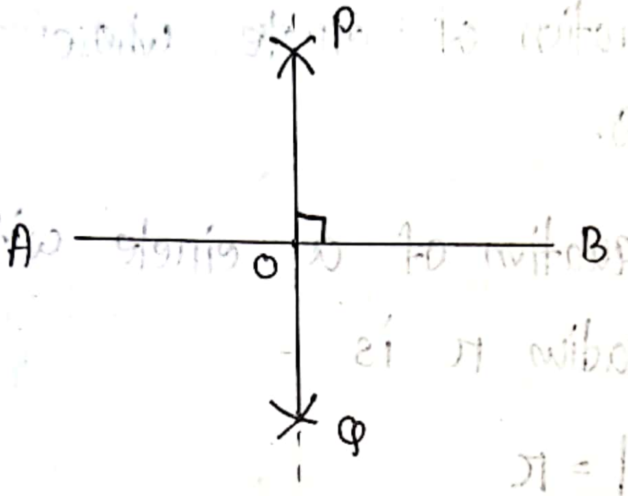
$$\Rightarrow |x+iy - (2-5i)| = 9$$

$$\Rightarrow |(x-2) + i(y+5)| = 9$$

$$\Rightarrow (x-2)^2 + (y+5)^2 = 81$$

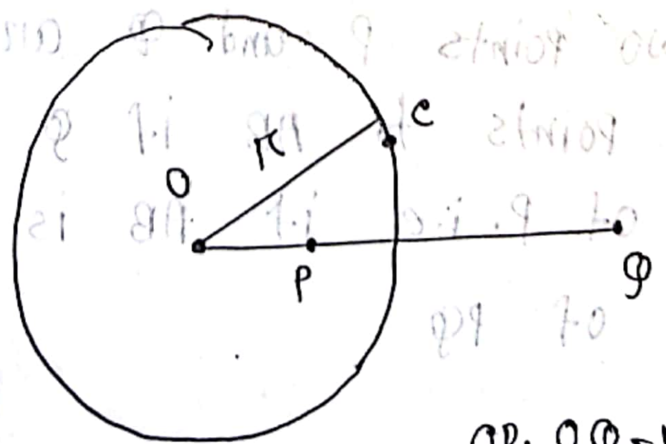
Inverse point with respect to a straight line

Two points P and Q are said to be inverse points to AB if Q is the image point of P. i.e. if AB is the right bisector of PQ



Inverse point with respect to a circle :

Two points P and Q are said to be inverse point with respect to a circle C, if they are collinear with the centre O and on the same side of it and also $OP \cdot OQ = r^2$, where r is the radius of the circle.



$OP \cdot OQ = r^2$

*** যেকোন একটা definition exam এ আসবে।

show that the inverse point of a point a with respect to a circle $|z-c| = r$ is

$$c + \frac{r^2}{\bar{a}-\bar{c}} \quad ; \quad \bar{a}, \bar{c} = \text{complex conjugate}$$

→ Assignment Submission – 19/03/23

Limit: $\lim\limits_{x\to a} f(x) = l$

$$\lim_{x\to a^{-0}} f(x) = \lim_{x\to a^{+0}} f(x) = l$$

$z = x+iy = (x,y)$

$f(z) = u(x,y) + i\,v(x,y)$ be the complex function

A function $f(z)$ is said to have a limit l to $z = z_0$ if for any $\epsilon > 0$ then there exist $\delta > 0$ such that $|f(z) - l| < \epsilon$ whenever

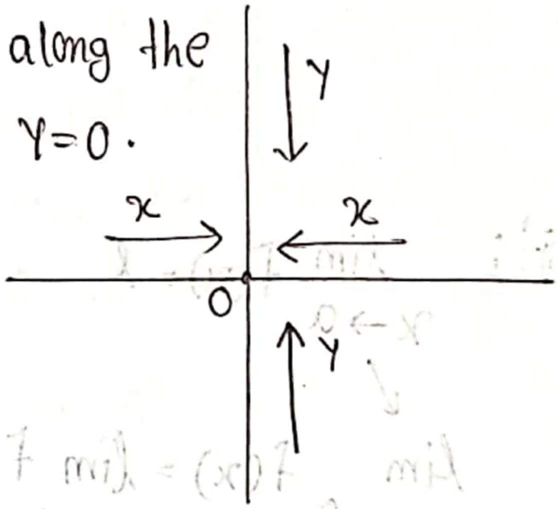
TOPIC NAME : DAY : TIME : DATE : / /

$0 < |z - z_0| < \delta$. we can write $\lim\limits_{z \to z_0} f(z) = l$

Does $\lim\limits_{z \to 0} \dfrac{\bar{z}}{z}$ exist ?

Soln: $\lim\limits_{(x,y) \to (0,0)} \dfrac{\overline{x+iy}}{x+iy} = \lim\limits_{(x,y) \to (0,0)} \dfrac{x-iy}{x+iy}$

Suppose $(x,y) \to (0,0)$ along the x-axis then $x \to 0$ and $y = 0$. So, the required limit



$\text{limit} = \lim\limits_{x \to 0} \dfrac{x}{x} = 1$

Again, suppose $(x,y) \to (0,0)$ along the y-axis then $y \to 0$ and $x = 0$. so the required

$\text{limit} = \lim\limits_{y \to 0} \dfrac{-iy}{iy} = -1$

which is different from the above.

So, $\lim_{z \to 0} \frac{\bar{z}}{z}$ does not exist.

Continuity: A function $f(z)$ is said to be continuous at $z = z_0$ if for any $\epsilon > 0$ then there exist $\delta > 0$ such that $|f(z) - f(z_0)| < \epsilon$ whenever $0 < |z - z_0| < \delta$. We can write $\lim_{z \to z_0} f(z) = f(z_0)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Differentiability:

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

TOPIC NAME : DAY : TIME : DATE : / /

Prove that a differentiable function is also continuous there but converse is not necessary true.

Solⁿ: Suppose $f(z)$ is differentiable at $z = z_0$

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0} \quad \text{—— (1)}$$

Now, $f(z) - f(z_0) = \dfrac{f(z) - f(z_0)}{z - z_0} \times (z - z_0)$

$\lim_{z \to z_0} f(z) = f(z_0)$

$f(z) = f(z_0)$

$f(z) - f(z_0)$

Taking limit on both sides as $z \to z_0$, we get -

$$\lim_{z \to z_0} \left\{ f(z) - f(z_0) \right\} = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0} \times (z - z_0)$$

$$\Rightarrow \lim_{z \to z_0} f(z) - \lim_{z \to z_0} f(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0} \times \lim_{z \to z_0} (z - z_0)$$

$$\Rightarrow \lim_{z \to z_0} f(z) - f(z_0) = f'(z_0) \times 0$$

$$\Rightarrow \lim_{z \to z_0} f(z) = f(z_0)$$

Hence, $f(z)$ is continuous at $z = z_0$.

Lecture-4

29/8/23

Tuesday

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complex variables

conversely -

For ~~conversely~~, $f(z) = |z| = \sqrt{x^2 + y^2}$

Here, $f(z)$ is continuous at $z = 0$

Now, $f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$

$$\Rightarrow f'(0) = \lim_{z \to 0} \frac{f(z) - f(0)}{z - 0}$$

$$= \lim_{(x,y) \to (0,0)} \frac{\sqrt{x^2 + y^2}}{x + iy}$$

TOPIC NAME : DAY : TIME : DATE : / /

Suppose, $(x,y) \to (0,0)$ along the x-axis so $x \to 0, y=0$, then the required limit

$$\lim_{x \to 0} \frac{\sqrt{x^2}}{x} = 1$$

Again, suppose $(x,y) \to (0,0)$ along the y-axis so, $y \to 0, x=0$, then the required limit -

$$\lim_{y \to 0} \frac{\sqrt{y^2}}{iy} = -i$$

which is is different from above.

So, the limit does not exist.

Hence $f'(z)$ does not exist at $z=0$.

Analytic function in a region:

A function $f(z)$ is defined in a region R is called analytic if $f'(z)$ exists at every point of R. It is also known regular

TOPIC NAME : DAY : TIME : DATE : / /

Regular function : (*** Definition)

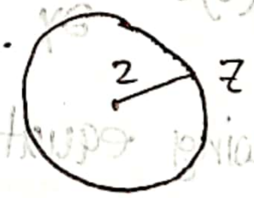
$F(z)$

$|z| = 5$

$f(z) = z^2 + 1$

$\Rightarrow f'(z) = 2z$

$f(z) = \dfrac{1}{z-1}$; $|z| = 2$



$f'(z) = \dfrac{-1}{(z-1)^2}$

Analytic function at a point: A function $f(z)$ is said to be analytic at a point $z = z_0$ if there exists a neighborhood N of z_0 such that $f'(z)$ exists at each point of N.

$f(z) = \dfrac{1}{z-1}$ $\quad f'(z) = \dfrac{-1}{(z-1)^2}$ at $z = 1$

TOPIC NAME : DAY : TIME : DATE : / /

0.99 1.0001

$\odot_N$

Cauchy - Riemann Equations:

$f(z) = u(x,y) + iv(x,y)$ be analytic function

$$\Rightarrow f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

$$\text{or } f'(z) = \frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y}$$

The following equations -

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial y}$ are called the Cauchy Riemann equations.

Laplace Equation:

$u(x,y) = \text{----}$

$v(x,y) = \text{----}$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \rightarrow \text{Laplace equation.}$$

GOOD LUCK

Hermonic Function: Any function of x and y whose have continuous partial derivative such as $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$ and satisfied the laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Then u is called the hermonic function. Here v is called the hermonic conjugate.

(*** উপরের 3টা definition এর যেকোন একটা আসবে!) .

→ আসবেই!

\# Prove that $u = e^{-x}(x\sin y - y\cos y)$ is hermonic. Find v such that $f(z) = u + iv$ is analytic, where v is the hermonic conjugate.

Soln: We have -

$$u = e^{-x}(x\sin y - y\cos y) \quad \text{—} \quad (1)$$

Diff (1) Partially w.r.t x we get -

$$u = e^{-x} x \sin y - e^{-x} y \cos y$$

TOPIC NAME : DAY : TIME : DATE : / /

$$\frac{\partial u}{\partial x} = e^{-x}\sin y - xe^{-x}\sin y + e^{-x} y\cos y \quad \text{--- (ii)}$$

$$\frac{\partial^2 u}{\partial x^2} = -e^{-x}\sin y - (e^{-x}\sin y - xe^{-x}\sin y) - e^{-x} y\cos y \quad \text{--- (iii)}$$

$$= -2e^{-x}\sin y + xe^{-x}\sin y - ye^{-x}\cos y$$

Again, Diff (i) w.r.t y we get -

$$\frac{\partial u}{\partial y} = xe^{-x}\cos y - (e^{-x}\cos y + e^{-x} y(-\sin y)) \quad \text{--- (iv)}$$

$$= xe^{-x}\cos y - e^{-x}\cos y + ye^{-x}\sin y$$

$$\frac{\partial^2 u}{\partial y^2} = -xe^{-x}\sin y + e^{-x}\sin y + e^{-x}\sin y + ye^{-x}\cos y \quad \text{--- (v)}$$

Now, (iii) + (v) =>

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

So, u is hermonic

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Lecture- 5

TOPIC NAME: Complex Variables

DAY: Thursday

TIME:

DATE: 31/ 8 / 23

2nd Part:

we know that the cauchy- Reimann equation are -

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{--- (6)}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad \text{--- (7)}$$

Since $\therefore F(z) = u(x,y) + iv(x,y)$ --- (8) is analytic.

Diff (1) Partially w.r.t x we get -

$$f'(z) = \frac{\partial u(x,y)}{\partial x} + i\frac{\partial v(x,y)}{\partial x}$$

$$= \frac{\partial u(x,y)}{\partial x} - i\frac{\partial u(x,y)}{\partial y} \quad [\text{from (7)}]$$

--- (9)

(*) Put, $x = z, y = 0$ in (9)

$$f'(z) = \frac{\partial u(z,0)}{\partial x} - i\frac{\partial u(z,0)}{\partial y} \quad \text{--- (10)}$$

Put, $x = z, y = 0$ in (2) & (4) we get -

$$\frac{\partial u(z,0)}{\partial x} = \text{---} = 0$$

TOPIC NAME : DAY : TIME : DATE : / /

$$\frac{\partial u(z,0)}{\partial y} = e^{-z}(z-1)$$

(10) $\Rightarrow f'(z) = 0 - ie^{-z}(z-1)$

$\Rightarrow f'(z) = -ize^{-z} + ie^{-z}$

Integrating -

$f(z) = -i\int ze^{-z}dz + i\int e^{-z}dz + ic$

$\Rightarrow f(z) = ize^{-z} + ic$

$\Rightarrow u+iv = i(x+iy)e^{-x-iy} + ic$

$= (ix-y)e^{-x}(\cos y - i\sin y) + ic$

$\Rightarrow u+iv = e^{-x}(ix\cos y + x\sin y - y\cos y + iy\sin y) + ic$

Equating real and imaginary parts, we get -

$u = e^{-x}(x\sin y - y\cos y)$

TOPIC NAME : DAY : TIME : DATE : / /

$v = e^{-x}(x\cos y + y\sin y) + c$

CT syllabus (20 Marks)

Date: 10/9/23 to 14/9/23

If $f(z) = u+iv$ is analytic function of z and $u-v = e^{x}(\cos y - \sin y)$, find $f(z)$ interms of z.

Soln: Since $f(z) = u+iv$ — ① is analytic

$\Rightarrow$ আগের math এর hermonic test এর মত শুরু হবে।

$\therefore if(z) = iu - v$ — ②

① + ⑪

$f(z) + if(z) = u + iu + iv - v$

$\Rightarrow (1+i)\, f(z) = u - v + i(u+v)$

$\Rightarrow F(z) = U(x,y) + iV(x,y)$ — ③

TOPIC NAME : DAY : TIME : DATE : / /

Where -

$$F(z) = (1+i)\, f(z)$$

$$U(x,y) = u - v = e^{x}(\cos y - \sin y)$$

$$V(x,y) = u + v$$

③ Partial diff w.r.t x -

$$F'(z) = \frac{\partial U(x,y)}{\partial x} + i\,\frac{\partial V(x,y)}{\partial x}$$

$$= \frac{\partial U(x,y)}{\partial x} - i\,\frac{\partial U(x,y)}{\partial y} \quad \text{— ⑤} \left[\text{by } \frac{\partial V}{\partial x} = -\frac{\partial U}{\partial y}\right]$$

Put, $x = z$, $y = 0$ in ⑤ we get -

Diff ④ Partially w.r.t x, y

$$\frac{\partial u(z,0)}{\partial x} = e^{z}$$

$$\frac{\partial U(z,0)}{\partial y} = -e^{z}$$

TOPIC NAME : DAY : TIME : DATE : / /

(6) $\Rightarrow F'(z) = (1+i)e^z$

Integrating we get -

$$F(z) = (1+i)e^z + ic$$

$$\Rightarrow (1+i)f(z) = (1+i)e^z + i.c$$

$$\therefore f(z) = e^z + \frac{i}{1+i}c$$

Lecture - 6
12/9/23
Tuesday

Hasanuzzaman Sir

Complex Variables

Singular point / singularity:

A point $z = z_0$ of a function $f(z)$ is said to be a singular point if its to analytic at $z = z_0$

* ✓ এর মান same value repeat হয়ে গেছে।

Example: $f(z) = \dfrac{1}{z(z-1)(z+3)^{✓}}$

Simple pole ← z; $(z+3)$ → pole of order 2

Here - $z = 0, 1, -3$ are the singular points of $f(z)$.

* z এর power যত ততগুলো singular point.

Pole: A singularity of a function $f(z)$ is said to be pole of order m if there exists a positive integer m such that –

$$\lim_{z \to z_0} (z-z_0)^m f(z) \text{ exists.}$$

If $m=1$ then it is called a simple pole.

A $z=-3$

$$\lim_{z \to -3} (z+3)^2 f(z)$$

$$= \lim_{z \to 3} \cancel{(z+3)^2} \times \frac{1}{z(z-1)\cancel{(z+3)^2}}$$

$$= \frac{1}{12} \to \text{finite value}$$

TOPIC NAME : DAY : TIME : DATE : / /

Removable Singularity:

A singularity z_0 of a function $f(z)$ is said to be removable if -

$$\lim_{z \to z_0} f(z) \text{ exists.}$$

$$\lim_{z \to 0} \frac{\cos z}{z} \to \infty$$

$$\lim_{x \to 0} \frac{\tan x}{x} = 1$$

Ex: $f(z) = \frac{\sin z}{z} \Rightarrow f(0) = \frac{0}{0} \to \infty$

$$\Rightarrow \lim_{z \to 0} f(z) = \lim_{z \to 0} \frac{\sin z}{z} = 1$$

$$\Rightarrow \lim_{z \to 0} \frac{1}{z}\left(z - \frac{z^3}{3!} + \cdots\right)$$

$$= \lim_{z \to 0}\left(1 - \frac{z^2}{3!} + \frac{z^4}{5!} - \cdots\right)$$

$$= 1$$

$$\# \lim_{z \to 0} \frac{\cos z}{z} = \infty$$

$$= \lim_{z \to 0} \frac{1}{z}\left(1 - \frac{z^2}{2!} + \cdots\right) = \infty$$

Essential singularity:

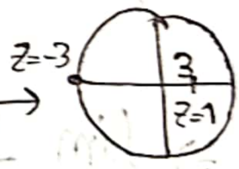
A singularity which is neither a pole nor a removable singularity is called an essential singularity.

Ex: $f(z) = e^{1/z}$, $e^{1/z-1} = g(z)$

$z=0$, $z=1$ are essential singularity.

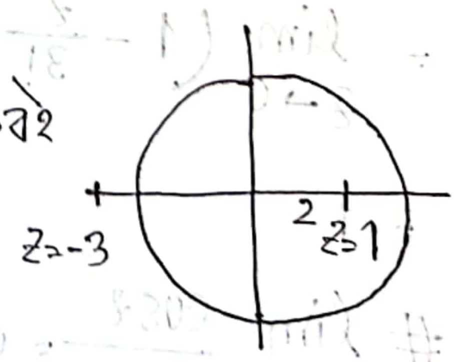
Isolated Singularity: Definition → বই

$$f(z) = \frac{1}{(z-1)(z+3)} \longrightarrow \text{(i) } |z| = 3 \rightarrow$$



(ii) $|z| = 2$

↓



Singularity at infinity: Definition → বই

$f(z) = z^2 + 1$

$f(\infty) = \infty$

TOPIC NAME : DAY: TIME: DATE: / /

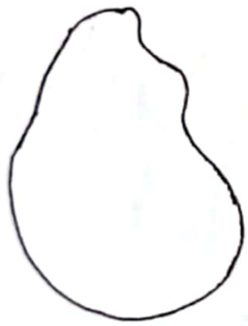
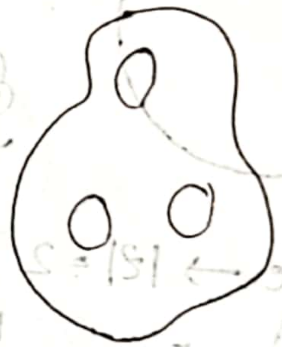
Closed curve:

① Simple close curve → no intersect point

② Not simple close curve → intersect point exists

① ②

Simple closed Region NOT simple region

Hosanuzzaman Sir

Lecture - 7

TOPIC NAME: Complex Variables

DAY: Thursday

TIME:

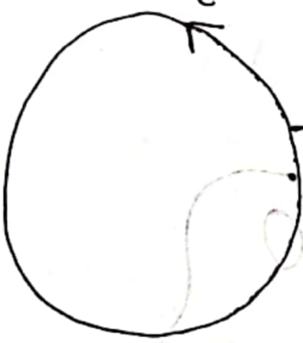
DATE: 14 / 9 /

Cauchy's theorem:

Statement: If $f(z)$ is analytic inside and on a region R bounded by a simple closed curve c and $f'(z)$ is continuous, then $\oint_c f(z)\,dz = 0$

#

$c \rightarrow |z| = 5 \rightarrow [-5, 5]$



$\rightarrow f(z) = z^2 + 2z + 3$

$\oint_c f(z)\,dz = 0$

#

$c \rightarrow |z| = 2$

$\rightarrow f(z) = \frac{1}{z-1}$ $\quad \oint_c f(z)\,dz \neq 0$

#

$c \rightarrow |z| = 4.9$

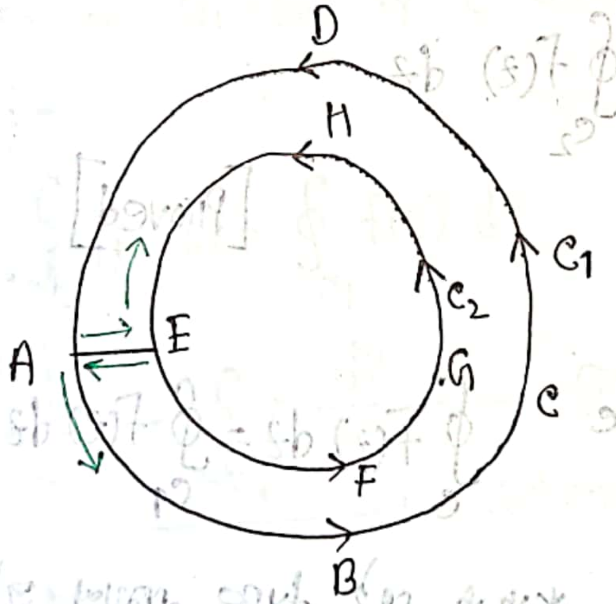
$\rightarrow f(z) = \frac{1}{z-5}$ $\quad \oint_c f(z)\,dz = 0$

TOPIC NAME : DAY : TIME : DATE : / /

*** If $f(z)$ is analytic inside and on a region R bounded by two simple closed curve c_1 and c_2, then $\oint_{c_1} f(z)\,dz = \oint_{c_2} f(z)\,dz$

where, c_1 and c_2 are both ~~co~~ ~~traaa~~ traversed in positive sense relative to their interior.

Soln :



Draw a cross section AE. Then AEHGFEABCDA is a simple closed curve and $f(z)$ is analytic. Then by cauchy's theorem -

$$\int_{AE---A} f(z)\,dz = 0$$

$\int_0^{10} f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^7 f(x)\,dx + \int_7^{10} f(x)\,dx$

TOPIC NAME : DAY : TIME : DATE : / /

$$= \int_{AE} f(z)\,dz + \int_{EHGFE} f(z)\,dz + \int_{EA} f(z)\,dz + \int_{ABCDA} f(z)\,dz = 0$$

$$\Rightarrow \int_{AE} f(z)\,dz - \oint_{C_2} f(z)\,dz - \int_{AE} f(z)\,dz + \oint_{C_1} f(z)\,dz = 0$$

$$\Rightarrow \oint_{C_1} f(z)\,dz = \oint_{C_2} f(z)\,dz$$

[Proved]

#

$$\oint_C f(z)\,dz = \oint_{C_1} f(z)\,dz + \oint_{C_2} f(z)\,dz$$

*** এই type প্রশ্ন বই থেকে পড়ে নেয়া লাগবে।

Evaluate $\oint_C \frac{dz}{z-a}$, where C is a closed curve and

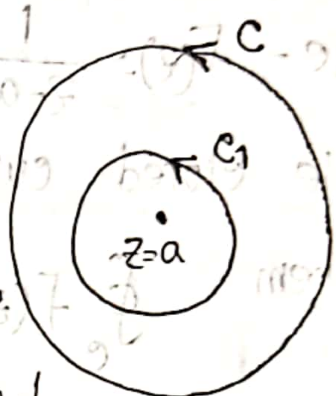
(i) $z=a$ inside C

(ii) $z=a$ is outside C.

Soln:

① Suppose $z=a$ is inside C

Let - $\frac{1}{z-a} = f(z)$



Since, $f(z)$ is analytic inside on a region bounded by C and C_1. Then -

$$\oint_C f(z)\,dz = \oint_{C_1} f(z)\,dz$$

$$\Rightarrow \oint_C \frac{dz}{z-a} = \oint_{C_1} \frac{dz}{z-a} \quad \text{—— ①}$$

$\longrightarrow$ where C_1 is a circle

On C_1:

$|z-a| = r$

$\Rightarrow z-a = re^{i\theta}$

$\therefore dz = rie^{i\theta}d\theta$

$$① \Rightarrow \oint \frac{dz}{z-a} = \int_0^{2\pi} \frac{rie^{i\theta}\,d\theta}{re^{i\theta}} = i\int_0^{2\pi} d\theta = 2\pi i$$

(Ans)

TOPIC NAME : DAY : TIME : DATE : / /

(ii) Suppose $z=a$ is outside c.

Here - $f(z) = \frac{1}{z-a}$ is analytic inside and on a simple closed curve c, then by cauchy's theorem

$$\oint_c f(z)\, dz = 0$$

$$\Rightarrow \oint_c \frac{dz}{z-a} = 0$$

(Ans)

Lecture-8
19/9/23
Tuesday.

Hasanuzzaman Sir

Complex Variable

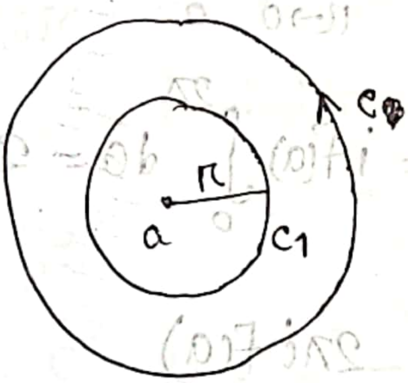
Cauchy integral formula:

If $f(z)$ is analytic inside and on region R bounded by a simple closed curve c and a is any point inside c, then

$$f(a) = \frac{1}{2\pi i} \oint_c \frac{f(z)}{z-a}\, dz$$

Soln:

Construct a circle C_1 inside C with centre at a and radius r. Then $\dfrac{f(z)}{z-a}$ is analytic inside and on the region bounded by C and C_1.



$$\therefore \oint_C \frac{f(z)}{z-a}\,dz = \oint_{C_1} \frac{f(z)}{z-a}\,dz \quad \text{—— (i)}$$

On C_1:

$$|z-a| = r$$

$$\Rightarrow z-a = re^{i\theta}$$

$$\Rightarrow z = a + re^{i\theta}$$

$$\therefore dz = rie^{i\theta}\,d\theta$$

Now - $$\oint_{C_1} \frac{f(z)}{z-a}\,dz = \int_0^{2\pi} \frac{f(a+re^{i\theta})}{re^{i\theta}}\, rie^{i\theta}\,d\theta$$

TOPIC NAME : DAY : TIME : DATE : / /

$$\Rightarrow \oint_{C_1} \frac{f(z)}{z-a}\,dz = i\int_0^{2\pi} f(a+re^{i\theta})\,d\theta$$

Taking limit $r\to 0$ on both sides we get –

$$\lim_{r\to 0}\oint_{C_1} \frac{f(z)}{z-a}\,dz = i\lim_{r\to 0}\int_0^{2\pi} f(a+re^{i\theta})\,d\theta$$

$$\Rightarrow \oint_{C_1} \frac{f(z)}{z-a}\,dz = i f(a)\int_0^{2\pi} d\theta = 2\pi i\, f(a)$$

$$(1) \Rightarrow \oint_{C} \frac{f(z)}{z-a}\,dz = 2\pi i\, f(a)$$

$$\Rightarrow f(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{z-a}\,dz$$

Cauchy integral 2nd formula :

$$f'(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{(z-a)^2}\,dz$$

$$f''(a) = \frac{2!}{2\pi i}\oint_C \frac{f(z)}{(z-a)^3}\,dz$$

$$f^n(a) = \frac{n!}{2\pi i}\oint_C \frac{f(z)}{(z-a)^{n+1}}\,dz$$

n = কয় বার diff হবে।

Evaluate $\frac{1}{2\pi i}\oint_C \frac{e^z}{z-2}\,dz$ if c is the circle

ⓐ The circle $|z|=3$

ⓑ The circle $|z|=1$

ⓐ Soln: Suppose c is the circle $|z|=3$.

Here $z=2$ is inside c.

let

$$f(z)=e^z$$

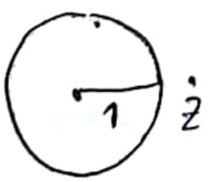
$$\therefore \frac{1}{2\pi i}\oint_C \frac{e^z}{z-2}\,dz = \frac{1}{2\pi i}\oint_C \frac{f(z)}{z-2}\,dz = f(2)$$

$$= e^2 \text{ (Ans)}.$$

ⓑ Soln: Suppose c is the circle $|z|=1$

Here $z=2$ is outside c.

Then by cauchy's theorem -



$$\oint_C \frac{e^z}{z-2}\,dz = 0$$

$$\Rightarrow \frac{1}{2\pi i}\oint_C \frac{e^z}{z-2}\,dz = 0 \text{ (Ans)}.$$

$\frac{1}{2\pi i}\oint_c \frac{e^z}{(z-2)^{10}}\,dz$

W/ $\frac{1}{2\pi i}\oint_c \frac{e^z}{(z-2)^{10}}\,dz = \frac{1}{2\pi i}\oint_c \frac{f(z)}{(z-2)^{10}}\,dz$

$= f^9(2) = e^2$ (Ans)

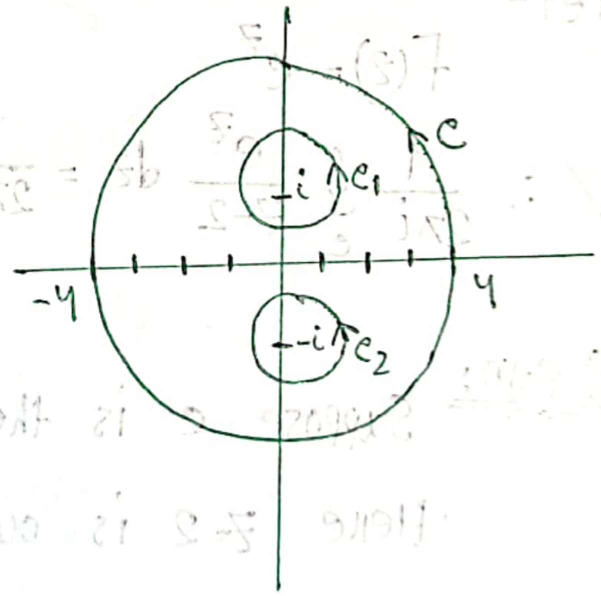
$\frac{1}{2\pi i}\oint_c \frac{e^{zt}}{(z^2+1)^2}\,dz$ $c: |z| = 4$

Put, $(z^2+1)^2 = 0$

$\Rightarrow z^2+1 = 0$

$\Rightarrow z = \pm i$

$\Rightarrow |z| = 1$



$\Rightarrow \oint_c \frac{e^{zt}}{(z+i)^2(z-i)^2}\,dz$

<u>On c_1:</u> $f(z) = \frac{e^{zt}}{(z+i)^2}$

<u>On c_2:</u> $g(z) = \frac{e^{zt}}{(z-i)^2}$

Now $\frac{1}{2\pi i}\oint_C \frac{e^{zt}}{(z^2+1)^2} = \frac{1}{2\pi i}\oint_{C_1} \frac{e^{zt}}{(z^2+1)^2}\,dz + \frac{1}{2\pi i}\oint_{C_2} \frac{e^{zt}}{(z^2+1)^2}\,dz$

$$= \frac{1}{2\pi i}\oint_{C_1} \frac{e^{zt}}{(z+i)^2(z-i)^2}\,dz + \frac{1}{2\pi i}\oint_{C_2} \frac{e^{zt}}{(z+i)^2(z-i)^2}\,dz$$

$$= \frac{1}{2\pi i}\oint_{C_1} \frac{f(z)}{(z-i)^2}\,dz + \frac{1}{2\pi i}\oint_{C_2} \frac{g(z)}{(z+i)^2}\,dz$$

$$= f'(i) + g'(-i)$$

lecture-9
Tuesday
3/10/23

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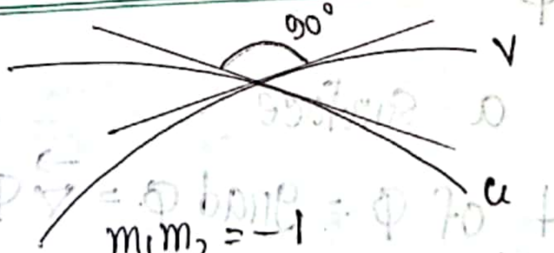
Complex Variable

Orthogonal Properties :



$m_1 m_2 = -1$

*** বই থেকে পড়ে নেয়া লাগবে orthogonal properties + related math.

TOPIC NAME : DAY : TIME : DATE: / /

Vector

⇒ Gradiant, divergence and curl.

Vector Differentiation: The vector differentiation is

$\vec{A}$

$\underrightarrow{B}$

$\overline{B}$

$\underline{B}$

$\mathbb{B}$

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

$$= \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

* $\vec{i}$ is unit vector with respect to x-axis.

* Unit vector of $\vec{i}$ $= \frac{\vec{i}}{1} = \vec{i}$

* $\vec{\nabla}$ is called the nabla operator.

* $\vec{\nabla}\cdot\vec{\nabla} = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Gradiant: $\vec{\nabla}\phi$

Let, ϕ be a surface

then gradiant of ϕ = grad $\phi = \vec{\nabla}\phi$

$$= \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)\phi(x,y,z)$$

$$= \frac{\partial\phi}{\partial x}\vec{i} + \frac{\partial\phi}{\partial y}\vec{j} + \frac{\partial\phi}{\partial z}\vec{k}$$

TOPIC NAME: DAY: TIME: DATE: / /

Directional Derivative:

$\phi(x,y,z)$ $\vec{A} = A_1\vec{i} + A_2\vec{j} + A_3\vec{k}$

grad $\phi = \vec{\nabla}\phi$

$$\vec{n} = \frac{\vec{A}}{|\vec{A}|}$$

Now, Directional derivative $= \vec{\nabla}\phi \cdot \vec{n}$ = constant

Divergence: $\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$

Scalar

$$\vec{A} = A_1\vec{i} + A_2\vec{j} + A_3\vec{k}$$

Divergence of $\vec{A}$ = div $\vec{A} = \vec{\nabla}\cdot\vec{A}$

$$= \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)(A_1\vec{i} + A_2\vec{j} + A_3\vec{k})$$

$$= \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$$

* $A\cdot A = |A||B|\cos\theta$
* $A\times B = |\vec{A}||\vec{B}|\sin\theta\ \hat{n}$

Curl: Curl $\vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$

$\Rightarrow$ $\vec{\nabla}\times\vec{A} = 0$; A is irrotational, conservative.

$\Rightarrow$ $\vec{\nabla}\times\vec{A} \neq 0$; A is rotational.

TOPIC NAME : DAY : TIME : DATE : / /

(#) Find a unit normal to the surface $x^2y+2xz=4$ at the $(2,-2,3)$.

Soln: Let - $\varphi = x^2y+2xz-4$ — ①

The normal to the surface ① is

$$\vec{N} = \vec{\nabla}\varphi$$

$$= \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)(x^2y+2xz)$$

$$= (2xy+2z)\vec{i} + x^2\vec{j} + 2x\vec{k}$$

At $(2,-2,3)$, $\vec{N} = -2\vec{i} + 4\vec{j} + 4\vec{k}$.

Then a unit normal to the surface $= \dfrac{\vec{N}}{|\vec{N}|}$

another " " " " --- $\dfrac{\vec{N}}{|\vec{N}|}$

GOOD LUCK™

Hasanuzzaman Sir

Lecture-10

TOPIC NAME : Vector

DAY : Thursday

TIME :

DATE : 5 / 10 / 23

\# Find the directional derivative of $\varphi = x^2yz + 4xz^2$ at $(1,-2,-1)$ in the direction $2\vec{i} - \vec{j} - 2\vec{k}$.

Soln: The gradient of $\varphi = \vec{\nabla}\varphi = \frac{\partial\varphi}{\partial x}\vec{i} + \frac{\partial\varphi}{\partial y}\vec{j} + \frac{\partial\varphi}{\partial z}\vec{k}$

At $(1,-2,-1)$, $\vec{\nabla}\varphi = 8\vec{i} - \vec{j} - 10\vec{k}$

We know, Direction derivative $= \vec{\nabla}\varphi \cdot \hat{n}$ — (i)

where $\hat{n} = \frac{2\vec{i} - \vec{j} - 2\vec{k}}{3}$ — (ii)

Putting (ii) in (i) we get -

Direction derivative $= (8\vec{i} - \vec{j} - 10\vec{k}) \cdot \frac{2\vec{i} - \vec{j} - 2\vec{k}}{3}$

dot product

$= \frac{16}{3} + \frac{1}{3} + \frac{20}{3} = \frac{37}{3}$

\# Find the angle between the surfaces $x^2+y^2+z^2 = 9$ and $z = x^2+y^2-3$, at the point $(2,-1,2)$

Soln: Let, $\varphi_1 = x^2+y^2+z^2-9$ / $x^2+y^2+z^2$

$\varphi_2 = x^2+y^2-z-3$ / x^2+y^2-z

The normal of φ_1 is $\vec{N_1} = \vec{\nabla}\varphi_1$

At $(2,-1,2)$, $\vec{N_1} = 4\vec{i} - 2\vec{j} + 4\vec{k}$

TOPIC NAME : DAY : TIME : DATE : / /

The Normal of ϕ_2, $\vec{N_2} = \vec{\nabla}\phi_2$

At $(2,-1,2)$, $\vec{N_2} = 4\vec{i} - 2\vec{j} - \vec{k}$

let θ be the angle between the surfaces.

$\therefore \vec{N_1} \cdot \vec{N_2} = |\vec{N_1}||\vec{N_2}| \cos\theta$

$\Rightarrow \theta = \cos^{-1}(\quad)$

Find an equation for the tangent plane to the surface $3xz^2 - 3xy - 4x = 7$ at the point $(1,-1,2)$.

Soln: Let $\phi = 3xz^2 - 3xy - 4x$

The normal of ϕ is $\vec{N} = \vec{\nabla}\phi$

At $(1,-1,2)$, $\vec{N} = 7\vec{i} - 3\vec{j} + 8\vec{k}$

The equation of a plane passing through a point whose position vector is $\vec{r_0} = (1,-1,2)$ and which is perpendicular to the normal $\vec{N}$ is

$(\vec{r} - \vec{r_0}) \vec{N} = 0$

TOPIC NAME : DAY : TIME : DATE : / /

$$\Rightarrow \left\{ (x-1)\vec{i} + (y+1)\vec{j} + (z-2)\vec{k} \right\} . (7\vec{i} - 3\vec{j} + 8\vec{k}) = 0$$

$$\Rightarrow 7(x-1) - 3(y+1) + 8(z-2) = 0$$

(a)

Find the values of a, b, c so that the vector $\vec{V} = (x+2y+az)\vec{i} + (bx-3y-z)\vec{j} + (4x+cy+2z)\vec{k}$ is irrotational or conservative.

(b) Show V can be expressed as the gradient of a scalar function.

(a) Soln: we know that $\text{curl}\,\vec{V} = \vec{\nabla} \times \vec{V}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+2y+az & bx-3y-z & 4x+cy+2z \end{vmatrix}$$

$$= (c+1)\vec{i} + (a-4)\vec{j} + (b-2)\vec{k}$$

Since $\vec{V}$ is irrotational then, $\vec{\nabla} \times \vec{V} = \vec{0}$

$c = -1,\ a = 4,\ b = 2$

$$\therefore \vec{V} = (x+2y+4z)\vec{i} + (2x-3y-z)\vec{j} + (4x-y+2z)\vec{k}$$

TOPIC NAME : DAY : TIME : DATE : / /

(b) Soln : Since $\vec{V}$ is irrotational then we can write -

$\vec{V} = \vec{\nabla}\phi$ where ϕ is a scalar function

$$\Rightarrow (x+2y+4z)\vec{i} + (2x-3y-z)\vec{j} + (4x-y+2z)\vec{k} = \frac{\partial\phi}{\partial x}\vec{i} + \frac{\partial\phi}{\partial y}\vec{j} + \frac{\partial\phi}{\partial z}\vec{k}$$

$$\therefore \frac{\partial\phi}{\partial x} = x+2y+4z \quad \text{--- (1)}$$

$$\frac{\partial\phi}{\partial y} = 2x-3y-z \quad \text{--- (2)}$$

$$\frac{\partial\phi}{\partial z} = 4x-y+2z \quad \text{--- (3)}$$

Integrating (1) partially w.r to x taken y and z constant, we get -

$$\phi = \frac{x^2}{2} + 2xy + 4zx + f(y,z) \quad \text{--- (4)}$$

" (2) " " y " x and z "

$$\phi = 2xy - \frac{3y^2}{2} - zy + g(z,x) \quad \text{--- (5)}$$

" (3) " " z " x and y "

TOPIC NAME: DAY: TIME: DATE: / /

$\phi = 4zx - yz + z^2 + h(x,y)$ —— ⑥

Hence the required scalar function is -

$$\phi = \frac{x^2}{2} + 2xy + 4zx - \frac{3y^2}{2} - yz + z^2 + \text{constant}$$

(Ans)

lecture-11
Tuesday
10/10/23

Hasanuzzaman Sir

Vector

Vector Integration:

If $\vec{A} = (3x^2+6y)\vec{i} - 14yz\,\vec{j} + 20xz^2\vec{k}$, evaluate $\oint_C \vec{A}\, d\vec{r}$ from $(0,0,0)$ to $(1,1,1)$ along the following Path C :

① $x = t,\ y = t^2,\ z = t^3$

② The straight line form $(0,0,0)$ to $(1,0,0)$ then to $(1,1,0)$ and then to $(1,1,1)$.

③ The straight line Joining $(0,0,0)$ and $(1,1,1)$

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TOPIC NAME : DAY : TIME : DATE : / /

Soln: Let $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ be the position vector.

$d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$

Now, $\oint_C \vec{A}\cdot d\vec{r} = \oint_C \{(3x^2+6y)dx - 14yz\,dy + 20xz^2\,dz\}$ — (i)

(1) $x = t,\ y = t^2,\ z = t^3$

$dx = dt,\ dy = 2t\,dt,\ dz = 3t^2\,dt$

When, $x=0, y=0, z=0$ then $t=0$

" $x=1, y=1, z=1$ then $t=1$

From equ (i) =>

$$\oint_C \vec{A}\cdot d\vec{r} = \int_{t=0}^{1} (\qquad\qquad\qquad) dt = 5$$

(2) Along the straight line from $(0,0,0)$ to $(1,0,0)$

So that -

$$\frac{x-1}{1-0} = \frac{y-0}{0-0} = \frac{z-0}{0-0} = t$$

x varies from 0 to 1, $y=0, z=0$ and

$\Rightarrow x-1 = t,\quad y=0,\quad z=0$

$dx = dt,\quad dy = 0,\quad dz = 0$

TOPIC NAME : DAY : TIME : DATE : / /

from equ ① $\Rightarrow \oint_c \vec{A}\cdot d\vec{r} = \int_{-1}^{0} (\quad) dt = 1$

$\therefore \int_0^1 (\quad) = 1$

$x-1=t, \quad y-1=t, \quad z=0$

$\therefore dx = dt, \quad dy = dt, \quad dz = 0$

equ ① $\Rightarrow \oint_c \vec{A}\cdot d\vec{r} = \int_{t=-1}^{0} (\quad) = 0$

$x-1=t, \quad y-1=t, \quad z-1=t$

$\therefore dx = dt, \quad dy = dt, \quad dz = dt$

equ ① $\Rightarrow \oint_c \vec{A}\cdot d\vec{r} = \int_{t=-1}^{0} (\quad) dt = \frac{20}{3}$

Hence, the required value $= 1 + 0 + \frac{20}{3} = \frac{23}{3}$ (Am)

③ The straight line joining (0,0,0) and (1,1,1) so that -

$$\frac{x-0}{0-1} = \frac{y-0}{0-1} = \frac{z-0}{0-1} = -t$$

TOPIC NAME : DAY : TIME : DATE : / /

$\therefore x = t, \; y = t, \; z = t$

$\therefore dx = dt, \; dy = dt, \; dz = dt$

From equ ① $\Rightarrow \oint_C \vec{A} \cdot \vec{dr} = \int_0^1 (\qquad) dt = \frac{13}{3}$

Must WRITE

Find the work done in moving a particle once around a circle C in the xy plane. If the circle has centre at origin and radius 3, and if the force field is given by -

$$\vec{F} = (2x - y + z)\vec{i} + (x + y - z^3)\vec{j} + (3x - 2y + 4z)\vec{K}$$

Soln: We know that the work done -

$$W = \oint_C \vec{F} \cdot \vec{dr} \text{ —— ①}$$

let $\vec{r} = x\vec{i} + y\vec{j} + z\vec{K}$ be the position vector of a particle.

$$\therefore \vec{dr} = dx\vec{i} + dy\vec{j} + dz\vec{K}.$$

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TOPIC NAME : DAY : TIME : DATE : / /

Putting the values in (1) $\Rightarrow$

$$W = \oint_C \{(2x-y+z)\vec{i} + (x+y-z^2)\,dy + (3x-2y+4z)\,dz\} \quad \text{--- (2)}$$

Since c is in xy-plane, so $z=0$, $dz=0$

$$(2) \Rightarrow W = \oint_C \{(2x-y)\,dx + (x+y)\,dy\} \quad \text{--- (3)}$$

Circle equation:

$$x^2+y^2=3^2$$

let - $x = 3\cos\theta$, $y = 3\sin\theta$

$\therefore dx = -3\sin\theta\, d\theta$, $dy = 3\cos\theta\, d\theta$

$$(3) \Rightarrow W = \int_0^{2\pi} (\qquad) = 18\pi$$

Hasanuzzaman Sir

Lecture-12

TOPIC NAME : Vector

DAY : Tuesday

TIME :

DATE : 17 / 10 / 23

Green theorem :

Statement : If R be a closed region of the xy-plane bounded by a simple curve c and if M and N are continuous function of x and y having continuous derivatives in R, then -

$$\oint_c M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

where c is traversed in the positive direction.

Type math exam এ আসবে।

\# +বই থেকে আগে দেখে যাবে।

\# Verify Green's theorem in the plane for $\oint_c (xy+y^2)\,dx + x^2\,dy$ where c is the closed curve of the region bounded by $y=x$ and $y=x^2$.

Soln: Here, $y=x$ and $y=x^2$ intersect at $(0,0)$ and $(1,1)$

The positive direction in traversing c is shown as in - Fig 1.

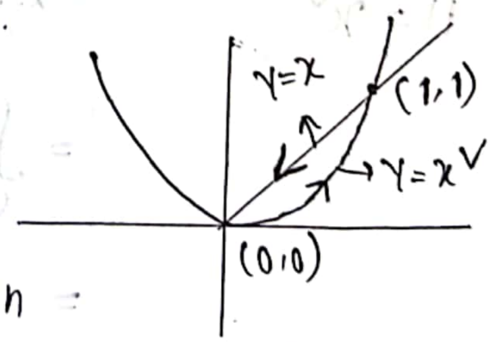


Fig-1

TOPIC NAME : DAY : TIME : DATE : / /

Along $y=x^2$, then integral become $=\int_0^1 (x^3+x^4)\,dx + x^2 2x\,dx$

$dy = 2x\,dx$

$= \frac{19}{20}$

Along $y=x$ then the integral become $= \int_1^0 (x^2+x^2+x^2)\,dx$

$dy = dx$

$= -1$

Thus the required integral $= \frac{19}{20} - 1 = -\frac{1}{20}$

According to Green's theorem we know that –

$$\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

$M = xy + y^2$

$N = x^2$

$$= \int_{x=0}^{1}\int_{y=x}^{x^2} (2x - x - 2y)\,dx\,dy$$

$$= \int_0^1 \left\{ \left| xy - y^2 \right|_{y=x}^{y=x^2} \right\} dx$$

$=$

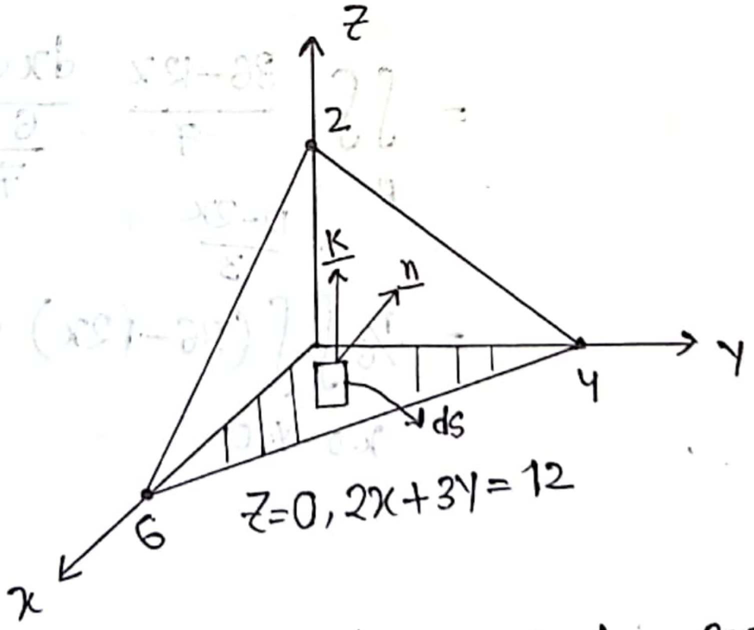
$= -\frac{1}{20}$

4] Surface Integration:

$$\iint_S \underline{A}\cdot\underline{n}\, ds = \iint_S \underline{A}\cdot \underline{ds} = \iint_R \underline{A}\cdot\underline{n}\,\frac{dx\,dy}{|\underline{n}\cdot\underline{K}|} \quad \text{--- (1)}$$

\# Evaluate $\iint_S \underline{A}\cdot\underline{n}\, ds$, where $\underline{A} = 18z\underline{i} - 12\underline{j} + 3y\underline{k}$ and S is the part of the plane $2x+3y+6z=12$, which is located in the first octant.

Soln: The surface S and its projection R on the xy-plane are shown in Fig-1-



To obtain $\underline{n}$ note that a vector perpendicular to the surface $2x+3y+6z=12$ is given by

$$\vec{N} = \nabla(2x+3y+6z) = 2\underline{i} + 3\underline{j} + 6\underline{K}$$

TOPIC NAME : DAY : TIME : DATE : / /

$$\therefore \underline{n} = \frac{\underline{N}}{|\underline{N}|} = \frac{2}{7}\underline{i} + \frac{3}{7}\underline{j} + \frac{6}{7}\underline{k}$$

$$\underline{n}\cdot\underline{k} = \left(\frac{2}{7}\underline{i} + \frac{3}{7}\underline{j} + \frac{6}{7}\underline{k}\right)\cdot$$

$$= \frac{6}{7}$$

$$\underline{A}\cdot\underline{n} = \frac{36z - 36 + 18y}{7} = \qquad = \frac{36-12x}{7}$$

$$① \Rightarrow \iint_S \underline{A}\cdot\underline{n}\, ds = \iint_R \underline{A}\cdot\underline{n}\frac{dx\,dy}{|\underline{n}\cdot\underline{k}|}$$

$$= \iint_R \frac{36-12x}{7}\,\frac{dx\,dy}{\frac{6}{7}}$$

$$= \frac{1}{6}\int_{x=0}^{6}\int_{y=0}^{\frac{12-2x}{3}} (36-12x)\,dy\,dx \quad \Bigg| \quad \frac{1}{6}\int_{y=0}^{4}\int_{x=0}^{\frac{12-3y}{2}} (36-12x)\,dx\,dy$$

$$= 24$$

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TOPIC NAME : Surface Integration

Lecture-14

DAY : Tuesday

TIME :

DATE : 7 / 11 / 23

Find the volume of the region common to the intersecting cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$.

Soln:

$$\text{volume} = \iiint dv \qquad dv = dx\,dy\,dz$$

$$= \int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}}\int_{z=0}^{\sqrt{a^2-x^2}} dz\,dy\,dx$$

$$= \int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}} |z|_0^{\sqrt{a^2-x^2}}\,dy\,dx$$

$$= \int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2}\,dy\,dx$$

$$= \int_{x=0}^{a} |y|_0^{\sqrt{a^2-x^2}}\left(\sqrt{a^2-x^2}\right)dx$$

$$= \int_{x=0}^{a} (a^2-x^2)\,dx$$

$$\vdots$$

$$= \frac{2a^3}{3} \qquad \therefore \text{Total volume} = 8\times\frac{2a^3}{3}$$

GOOD LUCK

Some Theorem:

1. Divergence theorem
2. Stoke's theorem

Hasanuzzaman sir que করাত না, Jamali sir করতে পারে, তাই easy math গুলা করা লাগতে, time পারলে।

#littleGiant_34