Statistic: The statisties is defined as the seienee of collection, organisation, presentation, analysiss and interpretation of numerical data.

Types of data:

1. Primary data 2. Secondary data 3. By product data.

Population: A population is a set of similar item or event which is interest for sone experiment.

Sample: Subset of Population.

Biased data: Bias in statisties is a professional's tendency to understimate or overstimate the value of a parameter.

Central tendency: The central tendency is defined as the number used to represent the center or middle of a set of data value.

<u>mean ($\bar{x}$)</u>: represent avarage value

→ first raw moment about origin

$$A.M = \frac{\sum f_i x_i}{\sum f_i} \qquad G.M = \text{antilog} \frac{\sum f_i \log x_i}{N}$$

$$H.M = \frac{\sum f_i}{\sum f_i\left(\frac{1}{x_i}\right)}$$

<u>Median ($\tilde{x}$)</u>: $\tilde{x} = L_{me} + \frac{N/2 - f_{p.fe}}{f_{me}} \times i$

<u>mode ($\hat{x}$)</u>: $\hat{x} = L_{mo} + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times i$

<u>Limitation of central tendency:</u>

1. If there is a data value 0 then the geometric mean will be 0.

2. The mean can not be calculated for catagorical data as the value can not be summed.

3. As the mean includes every value in the distribution the mean is influenced by outliers and skewed distribution.

* range = max − min

* Mean deviation = $\frac{\sum f_i (x_i - \bar{x})}{\sum f_i}$

* standard " = $\sqrt{\frac{\sum f_i (x - \bar{x})^2}{\sum f_i}} = \sigma$

varriance = σ^2

* co-efficient of varriation = $\frac{\sigma}{\bar{x}} \times 100\%$

skewness: skewness is the degree of asymmetry or departure from symmetry of a distribution.

$$\gamma_1 = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

< 0 (−ve) skewed $[\bar{x} - \hat{x}]$

= 0 symetric

> 0 (+ve) skewed

$\beta_1 = \gamma_1^2$

kurtosis: kurtosis is the degree of peakness of a distribution usually taken relative to a normal distribution.

$$\gamma_2 = \frac{\mu_4}{\mu_2^2}$$

< 3 − platy kurtic

= 3 − meso kurtic

> 3 − lepto kurtic

$\beta_2 = \gamma_2 - 3$

Moment: Moments are a set of statistical parameters to measure a distribution.

$$\mu_r' = \frac{\sum f_i (x_i - A)^r}{N} \qquad \mu_r = \frac{\sum f_i (x_i - \bar{x})^r}{\sum f_i}$$

$$\mu_1' = \frac{\sum f_i x_i}{N} = \text{mean}$$

$$\mu_2' = \frac{\sum f_i x_i^2}{N} \qquad \mu_3' = \frac{\sum f_i x_i^3}{N} \qquad \mu_4' = \frac{\sum f_i x_i^4}{N}$$

$$\mu_1 = 0, \quad \mu_2 = \mu_2' - \mu_1'^2$$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

Random experiment: An experiment or trial is said to be random if it has more than one possible outcomes.

Sample space: A set of all possible outcome of a random experiment is a sample space.

Event: Any subset of a sample space is called event.

Borel field: The set of all subset of a sample space or collection of all events is called Borel field.

Collectively exhasted events: If some event spans a sample space the those events is called collectively exhusted event.

Mutually exclusive event: If one event is impossible if another one event happens, then they are called mutually exclusive event.

Independent event: If the probability of B occurring is not affected by the occurrance or non occurrance of A then we say A and B are independent event.

$P(A \cap B) = P(A) \cdot P(B)$

Dependent:

$$P(A \cap B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B)$$

Axiom of Prob: A, B, C → mutually exclusive event

i) $P(S) = 1$ S = sure event

ii) $P(A) \geq 0$

iii) $P(A \cup B \cup C) = P(A) + P(B) + P(C) + \cdots$

Bayes theorem: If $B_1, B_2 \cdots B_n$ are n mutually exclusive event and also collectively exhusted, and A is any event in S.

Then $$P(B_i/A) = \frac{P(B_i) \cdot P(A/B_i)}{\sum P(B_i) P(A/B_i)}$$

total Prob. $P(A) = \sum P(B_i) P(A/B_i)$

Random variable: If a variable x can take a discrete or continious set of values with given probabilities then x is called a random variable.

Random function: A function that is associated with random experiments is called a random function.

Discrete random variable: A random variable that takes on a finite or countably infinite number of values is called discrete random variable.

Discrete Prob. distribution: If a variable x can assume a discrete set of values i.e. $x_1, x_2, x_3 \ldots x_n$ with respect to probabilities $p_1, p_2, p_3, \ldots p_n$ where,

i) $p_i \geq 0$

ii) $\sum p_i = 1$

Prob. mass function: Let x be a D.R.V such that $P(x) = x = P_j$ the P_j is said to be Probability mass function if it satisfy following condition-

i) $P_i \geq 0$ ii) $\sum P_j = 1$

Continious Prob. distribution: Let x is continious random variable and $f(x)$ is a Probability density function such that $f(x) \geq 0$ and $f(x) = \int_{-\alpha}^{\alpha} f(x)\,dx = 1$ then it is said that continious Prob. distribution is defined.

Prob. density function: A function $f(x)$ is Prob. if

i) $f(x) \geq 0$, $-\alpha < x < \alpha$

ii) $\int_{-\alpha}^{\alpha} f(x)\,dx = 1$

where x is c.r.v.

Mathemetical enpectation: let n be any random varriable and $g(n)$ be any function then enpection of $g(n)$ is ~~defined~~ denoted by $E\{g(n)\}$ and is defined by

$$E\{g(n)\} = \sum g(n) P(n)$$ $P(n) \rightarrow$ ~~discr~~ Prob. mass func

$$E\{g(n)\} = \int_{-\alpha}^{\alpha} g(n) f(n) \, dn$$ $f(n) \rightarrow$ " density "

$$E(n) = \mu_1' = \text{mean}$$

$$\int_0^{\alpha} n^{n-1} \cdot e^{-an} \, dn = \frac{\Gamma n}{a^n}$$

$$E(n^r) = \mu_r'$$

$$\Gamma n = (n-1)!$$

Binomial distribution: The binomial distribution is a probability distribution that summarizes the likelihood that a value will take one of two independent values under a given set of parameters or assumption

There are mainly two assumptions needed for Binomial distribution.

1. **Bernoulli trial**: Bernoulli trial is a random experiment which has to satisfy the below condition-

a) The trial must ~~have~~ have only two output.

b) Every trial has to be independent.

c) Probability of success in each trial is fixed.

2. Total number of trial is fixed and finite.

$P(x) \rightarrow$ n সংখ্যক trial এ এর x সংখ্যক success এর prob.

$P(x) = {}^nC_x \, p^x \, q^{n-x}$

Expected value $= x \times P(x)$

Properties: i) Discrete distribution

ii) two parameter, n, p

iii) Mean $\mu = np$

iv) variance $\sigma^2 = npq$

Poisson distribution: If in burnoulli distribution n is very large ($n \to \infty$) and probability for a single trial is very small ($p \to 0$) then the binomial distribution is converted into Poisson distribution.

$np < 5$

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$ $\lambda =$ mean $= np$

Properties of Poisson distribution:

1. Discrete distribution
2. One parameter, λ
3. mean $= \lambda = np$
4. variance $= \lambda = np$
5. standard deviation $\sigma = \sqrt{np}$
6. co-ef of skewness $= \frac{1}{\sqrt{\lambda}} = \frac{1}{\sigma}$
7. " " kurtosis $= 3 + \frac{1}{\lambda}$

Normal Distribution: (Gaussian) is a Probability distribution that is symmetric about the mean, showing that data near the mean are more frequent.

$x \to e, \mathbb{R}, N,$

$$f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$\therefore$ $\mu =$ mean $\quad \sigma =$ S.D

$$\therefore \quad z = \frac{x-\mu}{\sigma}$$

$$\therefore \quad f(z, 0, 1) = \int \frac{1}{\sqrt{2\pi}}\, e^{-\frac{z^2}{2}}\, dz$$

$$\text{mean } \mu = E(x) = \int_{-\alpha}^{\alpha} x f(x)\, dx = \int_{-\alpha}^{\alpha} z\, e^{-z^2/2}\, dz$$

Properties:

i) continious distribution

ii) 2 parameter (μ, σ)

iii) mean $= \mu$

iv) variance $= \sigma^2$

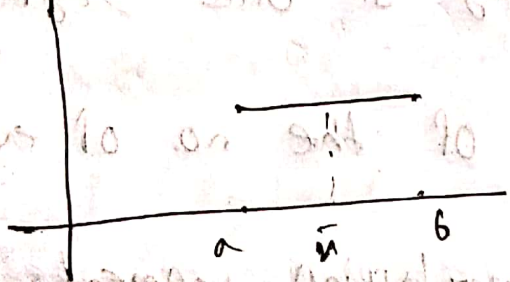
v) Standard deviation $= \sigma$

vi) co-ef of skeness, $\alpha_3 = 0$

vii) kurtosis $\alpha_4 = 3$

Uniform distribution: A continious random varriable x is said to follow a continious uniform distribution over a interval (a,b), if its p.d.f is given by,

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{when } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$



Properties:

i) continious distribution

ii) mean $= E(x) = \dfrac{a+b}{2}$

iii) varince $\sigma^2 = \dfrac{(a-b)^2}{12}$

iv) S.D $= \dfrac{a-b}{\sqrt{12}}$

v) co-effiecient of skewness $\beta_1 = 0$

Poisson Process: A Poisson Process is a series of events occurring at a mean rate over time or space.

Condition for Poisson Process:

① The no. of event occurring in one segment of time / space is independent of the no. of events in any previous non overlapping segment.

Mathematically the random variable

$x(t_1, t_2), x(t_2, t_3) - x(t_n, t_{n+1})$ $t_1 < t_2 < t_3$

are mutually independent.

② For sufficiently small Δt

$P_1(t + t+\Delta t) = \lambda \Delta t + O(\Delta t)$

where, $P_1(t, t+\Delta t)$ = Prob. of one event occur in time range $(t, t+\Delta t)$

λ = averarge density

$O(\Delta t)$ = order of Δt

$$\lim_{\Delta t \to 0} \frac{O(\Delta t)}{\Delta t} = 0$$

$\therefore P_1(t + t+\Delta t) = \alpha \lambda \Delta t$

③ for sufficient small Δt.

$$\sum_{k=2}^{\alpha} P_k(t, t+\Delta t) = O(\Delta t)$$

Prob. of occurring two or more event during a sufficiently small interval is negliable.

④ $P_k(0) = 0 \quad ; k = 0, 1, 2$

$$P(x) = \frac{u^x e^{-u}}{x!} \qquad \text{mean} = u = \lambda t$$

$$P_x(t, t+\Delta t) = \frac{(\lambda t)^x - e^{-\lambda t}}{x!}$$

Vector Quantity: The physical quantity for which both magnitude and direction are defined distictly are known as a vector quantity.

Ex: ~~tet~~ velocity of a bike is 30 ms^{-1} in a north-east direction.

Point function:

Vector field: The region in which a point function specifies physical quantity is known as field. If a vector is correspond to each point in a region by a vector point function. The region is called a vector field.

A vector field varies with time can be represented as-

$$\vec{V} = V_1(x, y, z, t)\,\hat{i} + V_2(x, y, z, t)\,\hat{j} + V_3(x, y, z, t)\,\hat{k}$$

* according to vector ~~to~~ differential operator, vector field is two types -

i) Conservative vector field:

$\vec{V}$ is a conservative vector field if and only if its curl in everywhere is zero.

$$\text{curl } \vec{V} = 0$$

ii) Solenoidal vector field:

$\vec{V}$ is defined as a solenoidal if and only if its divergence is zero.

$$\text{div } \vec{V} = 0$$

* Relation Between Binomial distribution and Poisson distribution:

The binomial distribution tends towards the Poisson distribution as: $n \to \infty$, $p \to 0$ and $q = np$ ~~stays~~ < 5

we know, Binomial:

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

here mean $\lambda = np$ $\therefore p = \frac{\lambda}{n}$

$$\therefore P(X=x) = \binom{n}{x} \left(\frac{\lambda}{n}\right)^x \left(1-\frac{\lambda}{n}\right)^{n-x}$$

As n gets larger the bionomial formula tends towards the poisson formula; that is,

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

<u>Relation between Poisson and normal distribution;</u>

we know normal distribution,

$$N(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad ; -\infty < x < \infty$$

here μ = meand σ = standand deviation.

x is a continious random variable and $f(x)$ is Probability density function

Poisson distribution is a discrete distribution with random variable $x \geq 0$.

when the mean of Poisson distribution ($\lambda = np$) is large it becomes similar to normal distribution.

Matrix

Transformation: Transformation means changes in orientation, size & shape of the object. They are used to position the object, to change the shape of the object. ~~and~~

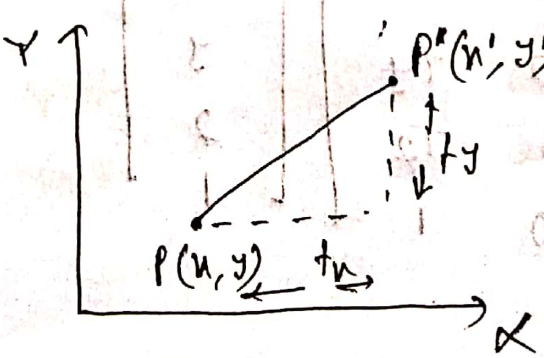
even ~~to chang~~

The basic transformations are:

1. Translation
2. Rotation
3. Scalling
4. Shearing

Translation: It is repositioning an object along straight line path from one coordinate location to another.

$(t_x, t_y) \rightarrow$ Translation vector or shift vector

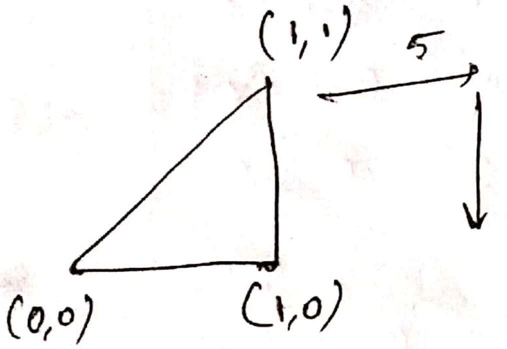


$x' = x + t_x$

$y' = y + t_y$

The matrix ~~equation~~ representation will be

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$



$$P \rightarrow \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ -6 & -6 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 6 & 6 \\ -6 & -6 & -5 \end{bmatrix}$$

another process

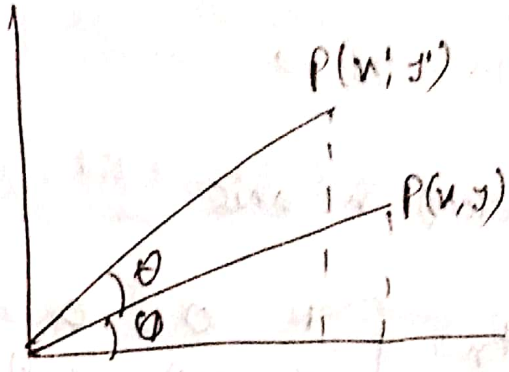
$$\rightarrow \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 6 \\ -6 & -6 & -5 \\ 1 & 1 & 1 \end{bmatrix}$$

zero row

for 3D →

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotation:



$x = r\cos\phi$

$y = r\sin\phi$

$x' = r\cos(\theta + \phi)$

$= r(\cos\theta . \cos\phi - \sin\theta . \sin\phi)$

$= r\cos\theta . \cos\phi - r\sin\theta . \sin\phi$

$$\boxed{x' = x\cos\theta - y\sin\theta}$$

$y' = r\sin(\theta + \phi)$

$= r(\sin\theta . \cos\phi + \cos\theta . \sin\phi)$

$= r\sin\theta . \cos\phi + r\cos\theta . \sin\phi$

$$\boxed{y' = x\sin\theta + y\cos\theta}$$

Matrix representation:

R = Rotation matrix

$P' = RP$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

in 3D →

along x axis

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scalling: Scalling alters the size of an object

→ Scalling of a polygon requires multiplying the co ordinate value of each vertex by the scalling factor to get the new co-ordinate.

$x' = x\, S_x$ $\qquad 0 < S_x, S_y < 1 \rightarrow$ size ↓

$y' = y\, S_y.$ $\qquad S_x, S_y > 1 \rightarrow$ size ↑

$S_x = S_y \rightarrow$ uniform scalling

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

scalling = magnifying

Shearing: The shearing transformation distorts the shape of the object

2 types → i) x - shearing
ii) y - shearing

x shearing: only change x values

$P(x, y) \rightarrow P'(x + sh_x \cdot y, y)$

y shearing: y changes

$P(x, y) \rightarrow P'(x, y + sh_y \cdot x)$

x shearing:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

y ":

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Reflection:

x axis:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

y axis:

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

origin:

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Write short note on scalling?

Soln:

Scalling: Scaling is a linear transformation that enlarges or shrinks or reverse a physical quantity (object) by a scaling factor. A scaling can be represendent by a scaling matrix.

To scale an object by a vector $\bar{v} = v_x, v_y, v_z$ whose each point $P(p_x, p_y, p_z)$ would need to be multiplied with this scalling matrix

$$S_v = \begin{bmatrix} v_x & 0 & 0 \\ 0 & v_y & 0 \\ 0 & 0 & v_z \end{bmatrix}$$

so the expected result would be

$$S_v \cdot P = \begin{bmatrix} v_x & 0 & 0 \\ 0 & v_y & 0 \\ 0 & 0 & v_z \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} v_x p_x \\ v_y p_y \\ v_z p_z \end{bmatrix}$$

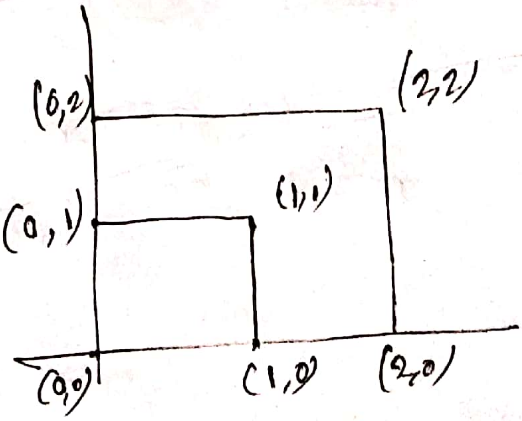
There are two types of scalling -

1. <u>Uniform scalling</u>: This scalling enlarges or shrink objects by a scalling factor that is the same in all direction.

<u>example</u>: If we want to do two times uniform scalling

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$



Non uniform: In this scalling at least one of the scalling factors is different from the others.

$$\begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

if $k_1 = 2$ and $k_2 = 3$

final matrix would be

$$= \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 3 & 3 \end{pmatrix}$$

Vector Differentiation

Point function: A point function $U = f(P)$ is a function that assigns some number or value U to each point P of some region of space.

1. Scalar point function: If to each point P in a region R, there corresponds a scalar $f(P)$ then we say f is a scalar point function.

2. Vector point function: If to each point P in a region R, there corresponds a vector $\vec{f}(P)$, then we say that f is a vector point function.

Directional derivative $=$ grad$(\phi) \cdot \hat{A}$ $A \Rightarrow$ direction vector

Some formulae:

1. Position vector $\vec{r} = xi + yj + zk$

2. $$\frac{\partial \bar{F}(x)}{\partial x} = \lim_{\Delta x \to 0} \frac{\bar{F}(x+\Delta x) - \bar{F}(x)}{\Delta x}$$

3. Tangent vector $\bar{T} = \frac{d\vec{r}}{ds} = \frac{dr}{dt} / \left|\frac{dr}{dt}\right| = \frac{\bar{V}}{|V|}$

4. $\bar{V} = \frac{d\vec{r}}{dt}$ = velocity

5. speed = $\frac{ds}{dt} = \left| \frac{d\vec{r}}{dt} \right|$

6. $\bar{V} = \left(\frac{ds}{dt}\right) \bar{T}$

7. curvature $K = \frac{d\vec{r}}{ds} = \left|\frac{d\vec{T}}{ds}\right| = \frac{d\bar{T}}{dt} / \frac{ds}{dt} = \frac{d\vec{T}}{ds}$

8. $\bar{a} = \frac{d^2s}{dt^2} \bar{T} + \left(\frac{ds}{dt}\right)^2 K \bar{N}$, $N \to$ normal vector

9. Tangential component $a_T = \frac{d^2s}{dt^2} \bar{T}$

10. Normal component, $a_N = \left(\frac{ds}{dt}\right)^2 K$

11. $K = \frac{1}{\rho}$, ρ = radius of curvature

12. $K = \frac{d\vec{r}}{dt} / \left|\frac{d\vec{r}}{dt}\right|$

13. Bionomial vector $\bar{B} = \bar{T} \times \bar{N}$

14. For oscillating plane $(\bar{r}-\bar{r}_0)\,\bar{B}_0 = 0$ | $B = T \times N$

15. Normal plane, $(\bar{r}-\bar{r}_0)\,\bar{T}_0 = 0$ | $-\tau = \left|\frac{dB}{ds}\right|$

16. Rectifying plane, $(\bar{r}-\bar{r}_0)\,\bar{N}_0 = 0$

17. $\frac{d\bar{T}}{ds} = \kappa \hat{N}$ | $\frac{dT}{dt} = \frac{dT}{ds} \cdot \frac{ds}{dt} = vKN$

18. $\frac{d\bar{B}}{ds} = -\tau \hat{N}$; $-\tau =$ Torsion | $\frac{dT}{dt} / \frac{ds}{dt} = KN = \frac{dT}{ds}$

Q/ $\bar{r} = 2t^2 i + tj + k$ Find $\bar{V}, \bar{T}, \bar{\kappa}, \bar{N}$

Soln: $\bar{v} = \frac{d\vec{r}}{dt} = 4ti + j$

Complex number: A number of the form $a+ib$ where $i^2=-1$ is called complex number.

$c = a+ib$

$a = Re(c)$, real part of c

$b = Im(c)$, imaginary part of c

Complex variable: Any order pair of two vectors (x, y) is called the complex variable. It is usually denoted by $z(x,y) = x+iy$.

$x = Re(z)$: real part of z

$y = Im(z)$: imaginary part of z

~~coplex~~

complex conjugate: let $z = x+iy$ be a complex number the complex conjugate is $\bar{z} = x-iy$

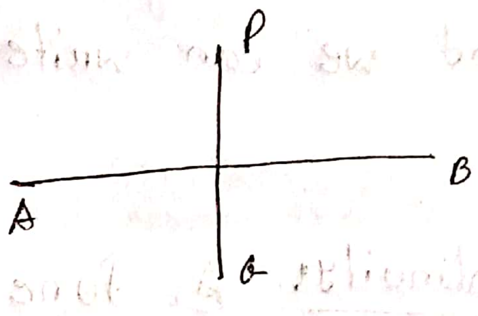
$\therefore$ $Re(z) = \frac{1}{2}(z+\bar{z})$ $\quad$ $Im(z) = \frac{1}{2i}(z-\bar{z})$

$|z| = |\bar{z}| = \sqrt{x^2+y^2} = z.\bar{z}$

$\therefore Re(z) \leq |z| \therefore Im(z) \leq |z|$

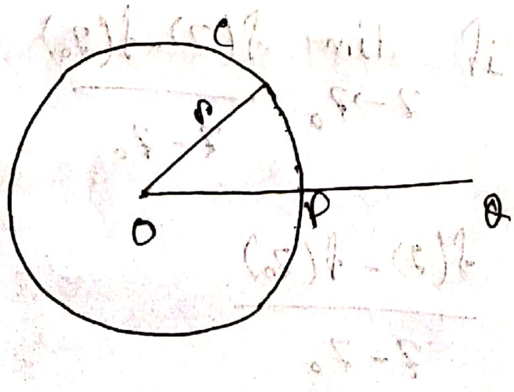
Inverse point with respect to a straight line:

Ans: Two Points P & Q said to be inverse point to AB if AB is the right bisector of PQ.



Inverse Point w.r.t a circle:

Two Point P and Q are said to be inverse point with respect to a circle C if they are co-linear with the centre O and on the same side of it and $OP.OQ = \rho^2$ where ρ is the radius of the circle.



Limit: A function $f(z)$ is said to have a limit l at $z = z_0$ if for any $\varepsilon > 0$ their exists $\delta > 0$ such that $|f(z) - l| < \varepsilon$ whenever $0 < |z - z_0| < \delta$ and we can write $\lim_{z \to z_0} f(z) = l$

Continuity: A function $f(z)$ said to be continious at $z = z_0$ if for any $\varepsilon > 0$ there exist $\delta > 0$ such that $|f(z) - f(z_0)| < \varepsilon$ whenever $0 < |z - z_0| < \delta$ and we can write,

$$\lim_{z \to z_0} f(z) = f(z_0)$$

Differentiability: A function $f(z)$ is said to be differentiable at $z = z_0$ if $\lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$ exists and we can write,

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

Analytic function in a region: A function $f(z)$ is defined in a region R is called analytic if $f'(z)$ exist at every point of R. Also known as regular function.

Analytic function at a point: A function $f(z)$ is said to be analytic at a point $z = z_0$ if there exist a neighbourhood N of z_0 such that $f'(z)$ exist at each point of N.

Complex function: $f(x) = u(x, y) + iv(x, y)$

Harmonic function: Any function of x and y which posseses continious partial derivatives and satisfies the laplace equation is called harmonic function.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Cauchy - Reiman eqn:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Cauchy theorem:

Statement: If $f(z)$ is analytic inside and on a region R bounded by a simple closed curve C and $f'(z)$ is continious there then,

$$\oint_C f(z)\,dz = 0$$

C

$|z|=2$

$f(z) = \frac{1}{z-3}$

Consequence of cauchy's theorem:

Statement: If $f(z)$ is analytic inside and on a region bounded by simple closed curves C_1 and C_2

then, $\oint_{C} f(z)\,dz = \oint_{C_2} f(z)\,dz$

where C_1 and C_2 are both transversed in the positive sense to their interior.

Singular Point: A Point at which $f(z)$ fails to be analytic is a singular Point of $f(z)$.

$f(z) = \frac{1}{z}$, $z=0$ is a singular Point

Isolated singularity: The singular Point $z=z_0$ is called an isolated singularity of $f(z)$ if we can find $\delta > 0$ such that the circle $|z-z_0| = \delta$ enclose no singular Point than z_0.

Ex: $f(z) = \frac{1}{(z-1)(z-3)}$ $|z| = 2$

$z = 1, 3$ are singular Point but $z=1$ is isolated singularity.

Pole: A singular Point $z=z_0$ is called a Pole of order m if $\lim\limits_{z \to z_0} (z-z_0)^m f(z)$ exists; where m is any Positive integer.

Ex: $f(z) = \dfrac{z^2+2}{z^3(z-1)^5(z-3)}$

Here $z = 0, 1, 3$ are singular point

$z = 0$ is a pole of order 3

$z = 1$ " " " " " 5

$z = 3$ " " " " " 1 / simple pole

Removable singularity: The singular point $z = z_0$ is called a removable singularity if $f(z_0)$ does not exist but $\lim_{z \to z_0} f(z)$ exist.

Ex: $f(z) = \dfrac{\sin z}{z}$; $f(0) = \alpha$

$$\lim_{z \to 0} \frac{\sin z}{z} = 1$$

Essentially singularity: A singular point $z = z_0$ which is not a pole or removable singularity is called an

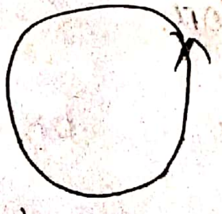
Essential singularity: A point z_0 is said to essential singular point if $f(z)$ is not analytic at $z=z_0$ and ~~there is a~~ every neighborhood of $f(z_0)$ ~~such there~~ contains infinite number of points in which f is analytic.

Singularity at infinity: Let $f(z)$ be a complex valued function then singularity of $f(z)$ at $z=\infty$ is same as singularity of $f(1/z)$ at $z=0$.

$f(z) = z^2+2z+3$

$f(\alpha) = \alpha$

Simple closed curve: A closed curve is called a simple closed curve if it does not intersect itself at any point.



simple closed curve

not simple closed curve

Orthogonal families:

Let $u(x,y) = \alpha$ and $v(x,y) = \beta$ when u and v are the real and imaginary part of an ~~anal~~ analytic function $f(z)$ and α, β are any constants represent two families of curve e. The two families are called orthogonal family.

$$m_1 \times m_2 = -1, \quad m_1 = -\frac{\partial u/\partial x}{\partial u/\partial y}$$

$$m_2 = -\frac{\partial v/\partial x}{\partial v/\partial y}$$

Vector Differential Operator:

$\nabla = \frac{\partial}{\partial x}\underline{i} + \frac{\partial}{\partial y}\underline{j} + \frac{\partial}{\partial z}\underline{k}$ is called vector differential operator or nabla operator.

Gradient: Let $\varphi = \varphi(x,y,z)$ be a function of x, y

and z the gradiant of $\varphi = \text{grad}\,\varphi = \nabla\varphi$

$$= \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right) \varphi(x,y,z)$$

Physical meaning:

The gradiant is a vector function which operates on a scalar function to produce a vector, It is the rate of change with respect to a variable quantity in the direction of maximum change.

Divergence: $V = V_1 i + V_2 j + V_3 k$

$$\text{div } v = \nabla V = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right) (V_1 i + V_2 j + V_3 k)$$

div $= 0$ solinoidal

div > 0 source

div < 0 sink

Physical meaning:

The divergence is a vector operator that operates on a vector field, producing a scalar field, giving the quantity of the vector field source at each point.

Directional Derivative: is the rate at which any function changes at any particular point in a fixed direction.

$\varphi = \varphi(x, y, z)$

$$\text{grad } \varphi = \nabla \varphi = \frac{\partial \varphi}{\partial x} \underline{i} + \frac{\partial \varphi}{\partial y} \underline{j} + \frac{\partial \varphi}{\partial z} \underline{k}$$

in the direction $\underline{A} = A_1 \underline{i} + A_2 \underline{j} + A_3 \underline{k}$

$$\underline{n} = \frac{\vec{A}}{|A|}$$

$$\therefore \text{D.D} = \nabla \varphi \cdot n$$

Curl: $\underline{A} = A_1 i + A_2 j + A_3 k$

$$\text{curl } \underline{A} = \nabla \times \underline{A} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

if curl $= 0 \Rightarrow$ conservative or irrotational.

Physical meaning: The curl of a vector is the amount of rotation or angular momentum of the contents of given region of space.

Cauchy Integral formula: If $f(z)$ is analytic inside and on the boundary C of a simple connected region R and 'a' is any point ~~that is~~ inside C then

$$f(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z-a}\, dz$$

$$f^n(a) = \frac{n!}{2\pi i} \oint \frac{f(z)}{(z-a)^{n+1}}\, dz$$

Green's theorem:

Statement: suppose R is a closed region in the xy plane bounded by a simple closed curve C and suppose $M(x, y)$ are continious function of x and y having continious derivatives in R, then

$$\oint_C M\,dx + N\,dy = \int_R \int \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx\,dy$$

surface integral:

$$\int_S \int \underline{A} \cdot \hat{n}\, ds = \int_R \int \underline{A} \cdot \hat{n} \frac{dx \cdot dy}{|\hat{n} \cdot \hat{k}|}$$

Branch Point: A branch Point of an analytic function is a point in the complex plane whose complex argument can be mapped from a single point in the domain to multiple points in the range.

$f(z) = z^a$

Put $z = e^{i\theta}$ and taking θ from 0 to 2π gives

$$f(e^{0i}) = 1$$

$$f(e^{2\pi i}) = e^{2\pi i a}$$

which are different.

<u>Linearly Dependent</u>: Let $V(F)$ be a vector space.

A finite set $\{\alpha_1, \alpha_2 \dots \alpha_n\}$ of vectors of V is said to be linearly dependent if there exist scalar $(a_1, a_2 \dots a_n \in F)$ not all of them 0 such that

$$a_1 \alpha_1 + a_2 \alpha_2 + a_3 \alpha_3 + \dots + a_n \alpha_n = 0$$