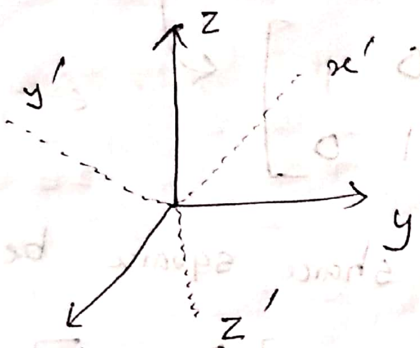
Invariance

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$



$$x' = l_{11} \cdot x + l_{12} \cdot y + l_{13} \cdot z$$
$$y' = l_{21}\, x + l_{22} \cdot y + l_{23} \cdot z$$
$$z' = l_{31} \cdot x + l_{32} \cdot y + l_{33}\, z.$$

we know for a scalar,

$$\varphi(x, y, z) = \varphi'(x', y', z').$$

imp prove

we have to show

$$\nabla \varphi(x, y, z) = \nabla \varphi'(x', y', z').$$

$$i \frac{\partial \varphi}{\partial x} + j \frac{\partial \varphi}{\partial y} + k \frac{\partial \varphi}{\partial z} = i' \frac{\partial \varphi'}{\partial x} + j' \frac{\partial \varphi'}{\partial y'} + k' \frac{\partial \varphi'}{\partial z'}$$

Gradiant of a scaler is invariant for rotation of axis.

$$\frac{\partial \varphi}{\partial x} = \frac{\partial \varphi}{\partial x'} \cdot \frac{\partial x'}{\partial x} + \frac{\partial \varphi}{\partial y'} \frac{\partial y'}{d\cancel{x}} + \frac{\partial \varphi}{\partial z'} \cdot \frac{\partial z'}{\partial \cancel{x}}$$

We know,

$$x' = l_{11} \cdot x + l_{12} \cdot y + l_{13} \cdot z.$$

$$\therefore \frac{dx'}{dx} = l_{11}.$$

$$\therefore \frac{\partial \varphi}{\partial x} = l_{11} \frac{\partial \varphi}{\partial x'} + l_{21} \frac{\partial \varphi}{\partial y'} + l_{31} \frac{\partial \varphi}{\partial z'}$$

$$\therefore \frac{\partial \varphi}{\partial x} = l_{11} \frac{\partial \varphi'}{\partial x'} + l_{21} \frac{\partial \varphi'}{\partial y'} + l_{31} \frac{\partial \varphi'}{\partial z'} \quad (\varphi = \varphi' \text{ for scalar}) \ldots (i)$$

$$\therefore \frac{\partial \varphi}{\partial y} = l_{12} \frac{\partial \varphi'}{\partial y'} + l_{22} \frac{\partial \varphi'}{\partial y'} + l_{32} \frac{\partial \varphi'}{\partial z'} \ldots\ldots (ii)$$

$$\frac{\partial \varphi}{\partial z} = l_{13} \frac{\partial \varphi'}{\partial x'} + l_{23} \frac{\partial \varphi'}{\partial y'} + l_{33} \frac{d\varphi'}{\partial z'} \ldots\ldots (iii)$$

After ~~multiplying~~ (i) $\times i$, (ii) $\times \hat{j}$, (iii) $\times \hat{k}$ we add them,

$$i \frac{d\varphi}{dx} + j \frac{d\varphi}{dy} + \hat{k} \frac{\partial \varphi}{dz} = \frac{\partial \varphi'}{\partial x'} (i l_{11} + j l_{12} + k l_{13})$$

$$+ \frac{d\varphi'}{dy'} (i l_{21} + j l_{22} + k l_{23})$$

$$+ \frac{d\varphi'}{dz'} (i l_{31} + j l_{32} + k l_{33}).$$

now, $i' = l_{11} i + l_{12} j + l_{13} k \quad (\because x' = l_{11} x + l_{12} y + l_{13} z).$

So, $i\frac{\partial \varphi}{\partial x} + j\frac{\partial \varphi}{\partial y} + k\frac{\partial \varphi}{\partial z} = i'\frac{\partial \varphi'}{\partial x'} + j'\frac{\partial \varphi'}{\partial y'} + k'\frac{\partial \varphi'}{\partial z'}$

$\therefore \nabla \varphi = \nabla \varphi'$

$\therefore$ Gradiant of a scaler is inveriant under rotation

[Proved].

Proves

Mean measured from A :

$$E(x-A) = E(x) - E(A)$$

$$= E(x) - E(A) = E(x) - A$$

whence on, $E(x) = A + E(x-A)$

$$\bar{x} = \text{mean} = A + \frac{\Sigma(x-A)}{N}.$$

Relation between Row and Central:

~~$E(x^2$~~

Suppose we want to find 2nd central moment.

$$\therefore E(x-\bar{x})^2 = \mu_2$$

$$\therefore E(x-\bar{x})^2 = E(x^2 + \bar{x}^2 - 2x\bar{x})$$

$$= \cancel{E(\bar{x})}\ E(x^2) + E(\bar{x}^2) - 2\cdot E(x\cdot\bar{x}).$$

$$= \mu_2' + \bar{x}^2 - 2\cdot E(x)\cdot E(\bar{x})$$

$\because$ ~~constant's expectation~~

$$= \mu_2' + \bar{x}^2 - 2\cdot\bar{x}\cdot E(x)$$

$$= \mu_2' + \bar{x}^2 - 2\cdot\bar{x}\cdot\bar{x} \quad [\because E(x) = \text{mean}]$$

$$= \mu_2' - \bar{x}^2.$$

$$= \mu_2' - \{E(x)\}^2 \quad [\because \text{mean} = E(x)]$$

$$= \mu_2' - \{\mu_1'\}^2$$

Binomial Distribution

① $^nC_x \, p^x q^{n-x}$ is A P.M.F.

Proof: 1st condition: For every case ~~p(x)~~ $0 \leq P(x) \leq 1$.

Here we see p is a fraction which ~~height~~ max value is 1. And ~~if~~ q is $(1-P)$. so q's minimum value is 0.

So $P(x)$ can never be less than 0 or greater than 1

(ii) 2nd condition: Total probability $= 1$.

$$\sum_{x=0}^{n} P(x) = \sum_{x=0}^{n} {}^nC_x \, p^x \cdot q^{n-x}$$

$$= {}^nC_0 \, p^0 q^n + {}^nC_1 \, p^1 q^{n-1} + \cdots$$

$$= (p+q)^n \quad \text{[from Newton binomial theorem]}.$$

$$= 1^n = 1.$$

So it is Pmf

(II) Mean of Binomial Distribution:

$$E(x) = \sum_{x=0}^{n} x \, P(x)$$

$$= \sum_{x=0}^{n} x \cdot {}^nC_x \cdot p^x \cdot q^{n-x}$$

$$= \cancel{x}\ 0 + {}^nC_1\, p^1 \cdot q^{n-1} + 2 \cdot {}^nC_2\, p^2 q^{n-2} + {}^nC_3 \cdot 3 \cdot p^3 \cdot q^{n-3} + \cdots$$

$$\cdots\cdots + {}^nC_n \cdot n \cdot p^n \cdot q^0$$

$$= \cancel{\frac{n!}{(n-1)!\,1!}}\ np \cdot q^{n-1} + \frac{2 \cdot n!}{(n-2)! \cdot 2!} \cdot p^2 \cdot q^{n-2} + \frac{3 \cdot n!}{(n-3)!\,3!}\, p^3 \cdot q^{n-3}$$

$$\cdots\cdots + \frac{n! \cdot n}{(n-n)!\,n!} \cdot p^n \cdot q^0.$$

$$= np \left\{ q^{n-1} + \frac{2 \cdot (n-1)!}{(n-2)!\,2!}\, p^1 \cdot q^{n-2} + \frac{3 \cdot (n-1)!}{(n-3)!\,3!}\, p^2 \cdot q^{n-3} + \cdots \right\}$$

$$= np \left\{ q^{n-1} + \frac{(n-1)!}{(n-2)!\,1!} \cdot p^1 q^{n-2} + \frac{(n-1)!}{(n-3)! \cdot 2!}\, p^2 q^{n-3} + \cdots \right\}$$

$$= np \left\{ {}^{(n-1)}C_0\, p^0 \cdot q^{n-1} + {}^{(n-1)}C_1\, p^1 q^{(n-1)-1} + {}^{(n-1)}C_2\, p^2 \cdot q^{(n-1)-2} + \cdots \right\}$$

$$= np \cdot (p+q)^{n-1}$$

$$= np.$$

[Proved]

Varience/ Sandard Deviation:

$$\sigma^2 = \mu_2 = E(x^2) - \{E(x)\}^2 \text{ or } \{\mu_2'\} - \{\mu_1'\}^2$$

$$\therefore E(x^2) = \sum_{x=0}^{n} x^2 \cdot p(x)$$

$$= \sum x^2 \cdot {}^nC_x \cdot p^x q^{n-x}$$

$$= \sum x^2 \cdot \frac{n!}{(n-x)!\,x!} \cdot p^x \cdot q^{n-x}$$

$$= \sum_{x=0}^{n} x . \frac{n!}{(x-1)!\,(n-x)!} p^{x} . q^{n-x}$$

$$= \sum_{x=0}^{n} (x-1) . \frac{n!}{(x-1)!\,(n-x)!} p^{x} . q^{n-x} + \sum_{x=0}^{n} \frac{n!}{(x-1)!\,(n-x)!} . p^{x} . q^{n-x}.$$

$$= \sum_{x=0}^{n} \frac{n!}{(x-2)!\,(n-x)!} p^{x} . q^{n-x} + np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\,(n-x)!} p^{x-1} q^{n-x}.$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\{(n-2)-(x-2)\}!} p^{x-2} q^{(n-2)-(x-2)} +$$

$$np \sum_{x=1}^{n} {}^{n-1}C_{x-1} \, p^{x-1} q^{(n-1)-(x-1)}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} {}^{n-2}C_{x-2} \, p^{x-2} . q^{(n-2)-(x-2)} + np . 1$$

$$= n(n-1) . p^2 + np .$$

$$\therefore \mu_2 = E(x^2) - \{E(x)\}^2$$

$$= n(n-1)p^2 + np - n^2p^2$$

$$= np - np^2$$

$$\sigma^2 = np(1-p)$$

$$\therefore S.D = \sqrt{np(1-p)} .$$

Poisson

mean:

$$\bar{x} = E(x) = \sum x\,p(x)$$

$$= \sum_{x=0}^{n} x \cdot \frac{\lambda^x \cdot e^{-\lambda}}{x!}$$

$$= e^{-\lambda} \left\{ \sum_{x=0}^{n} \frac{\lambda^x}{(x-1)!} \right\}$$

$$= \lambda \cdot e^{-\lambda} \left\{ \sum_{x=1}^{n} \frac{\lambda^{x-1}}{(x-1)!} \right\}$$

$$= \lambda \cdot e^{-\lambda} \cdot \left(\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \cdots\cdots \right)$$

$$= \lambda \cdot \cancel{e^{-\lambda}} \cdot \cancel{e^{\lambda}}$$

$$= \lambda.$$

Standard Deviation:

$$\sigma^2 = \mu_2 = E(x^2) - \{E(x)\}^2$$

$$E(x^2) = \sum x^2 \frac{\lambda^x \cdot e^{-\lambda}}{x!}$$

$$= \sum x \cdot \frac{\lambda^x e^{-\lambda}}{(x-1)!}$$

$$= e^{-\lambda} \sum (x-1) \frac{\lambda^x \cdot \cancel{e}}{(x-1)!} + e^{-\lambda} \sum \frac{\lambda^x}{(x-1)!}$$

$$= e^{-\lambda} \sum \frac{\lambda^x}{(x-2)!} + e^{-\lambda} \sum \frac{\lambda^x}{(x-1)!}$$

$$= e^{-\lambda} \cdot \lambda^2 \sum_{x=2}^{n} \frac{\lambda^{x-2}}{(x-2)!} + e^{-\lambda} \cdot \lambda \sum_{x=1}^{n} \frac{\lambda^{x-1}}{(x-1)!}$$

$$= e^{-\lambda}.\lambda^2\left\{\frac{\lambda^0}{0!}+\frac{\lambda^1}{1!}+\frac{\lambda^2}{2!}+\cdots\right\} + e^{-\lambda}.\lambda\left\{\frac{\lambda^0}{0!}+\frac{\lambda^1}{1!}+\frac{\lambda^2}{2!}+\cdots\right\}$$

$$= \cancel{e^{-\lambda}}.\lambda^2.\cancel{e^{\lambda}} + \cancel{e^{-\lambda}}.\lambda.\cancel{e^{\lambda}}$$

$$= \lambda^2+\lambda.$$

$$\therefore \cancel{E}\ \mu_2 = E(x^2) - \{E(x)\}^2$$

$$= \lambda^2+\lambda - \lambda^2$$

$$= \lambda.$$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

mean of normal distribution

We know

$$\bar{x} = \mu_1' = E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

Let,

$$\frac{x-\mu}{\sigma} = z.$$

$$\frac{1}{\sigma} dx = dz.$$

$$\therefore dx = \sigma dz.$$

Again,

$$x = (\sigma z + \mu).$$

$$= \frac{1}{\sqrt{2\pi}\cancel{\sigma}} \int_{-\infty}^{\infty} (\sigma z + \mu) e^{-\frac{z^2}{2}} \cdot \cancel{\sigma}\, dz$$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mu e^{-\frac{z^2}{2}} dz + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sigma z \cdot e^{-\frac{z^2}{2}} dz$$

[But here $\int_{-\infty}^{\infty} \sigma z\, e^{-\frac{z^2}{2}} = 0$ because it is an odd function].

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mu e^{-\frac{z^2}{2}} dz$$

Let

$$z^2/2 = p.$$

$$\frac{1}{\cancel{2}} \cancel{2} z\, dz = dp$$

$$dz = \frac{dp}{\sqrt{2p}}$$

$$= \frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-p} \frac{dp}{\sqrt{2p}}$$

$$= \frac{2\mu}{\sqrt{2\pi}} \int_{0}^{\infty} \frac{e^{-p}}{\sqrt{2p}} dp \quad [\because \text{even function}].$$

$$= \frac{\cancel{2}\mu}{\cancel{\sqrt{2}}\cancel{\sqrt{2}}\sqrt{\pi}} \int_{0}^{\infty} p^{-\frac{1}{2}} \cdot e^{-p}\, dp.$$

from gamma function rule.

$$= \frac{\mu}{\cancel{\sqrt{\pi}}} \cancel{\sqrt{\pi}}$$

$$= \mu$$