2K18:

1. a.

The frequency table is given below (Here $A = 85$)

CI	f_i	x_i	(x_i-A)	$f_i(x_i-A)$	$(x_i-A)^2$	$f_i(x_i-A)^2$	$(x_i-A)^3$	$f_i(x_i-A)^3$	$(x_i-A)^4$	$f_i(x_i-A)^4$
40-50	10	45	-40	-400	1600	16000	-64000	-640000	2560000	25600000
50-60	15	55	-30	-450	900	13500	-27000	-405000	810000	12150000
60-70	20	65	-20	-400	400	8000	-8000	-160000	160000	3200000
70-80	5	75	-10	-50	100	500	-1000	-5000	10000	50000
80-90	6	85	0	0	0	0	0	0	0	0
90-100	4	95	10	40	100	400	1000	4000	10000	40000
	$\Sigma f_i = 60$			$\Sigma f_i(x_i-A) = -1260$		$\Sigma f_i(x_i-A)^2 = 38400$		$\Sigma f_i(x_i-A)^3 = -1206000$		$\Sigma f_i(x_i-A)^4 = 41040000$

1st Raw moments

$$\mu_1' = \frac{\Sigma f_i(x_i-A)}{\Sigma f_i} = \frac{-1260}{60} = -21$$

$$\mu_2' = \frac{\Sigma f_i(x_i-A)^2}{\Sigma f_i} = \frac{38400}{60} = 640$$

$$\mu_3' = \frac{\Sigma f_i(x_i-A)^3}{\Sigma f_i} = \frac{-1206000}{60} = -20100$$

$$\mu_4' = \frac{\Sigma f_i(x_i-A)^4}{\Sigma f_i} = \frac{41040000}{60} = 684000$$

Central moments

$\mu_1 = \cancel{\bar{x}'} = \cancel{-21}\ 0$

$\mu_2 = \mu_2' - (\mu_1')^2 = 640 - (-21)^2 = 199$

$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3 = -20100 - 3\times(-21)\,640 + 2(-21)^3$

$\mu_3 = 1698$

$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6(\mu_1')^2\mu_2' - 3(\mu_1')^4$

$\mu_4 = 684000 - 4\times(-21)(-20100) + 6(-21)^2\times 640 - 3\times(-21)^4$

$\mu_4 = \cancel{1061289}\ 105597$

$\therefore \gamma_1 = \dfrac{\mu_3}{(\mu_2)^{3/2}} = \dfrac{1698}{(199)^{3/2}} = 0.6 > 0$; So it's +vely skewed.

$\therefore \gamma_2 = \dfrac{\mu_4}{(\mu_2)^2} = \dfrac{105597}{199^2} = 2.67 < 3$; so, it's platykurtic

b. Random Experiment:

A random experiment is an experiment, trial or observation that can be repeated numerous ~~time~~ ~~tries~~ under the same conditions. The outcome of an individual random experiment must be independent and identically distributed. It must ~~to~~ not be affected in any previous outcome and can't be predicted with certainty.

Example: If we toss a coin, the result of an experiment is that will either come up head or tail. Head is symbolized as "H" or 1. Tail is symbolized as T or 0. One of the elements of the set is {H,T}. (or {0,1})

Random variable:

A Random variable is a variable whose value is unknown or a function that assigns values to each of an experiment's outcomes. Random variables are often designed by letters and can be classified as discrete, which are variables that have specific values or continuous, which are variables that can have any values within a continuous range.

Example: Suppose that, a coin is tossed twice, so that, sample space is $S = \{HH, HT, TH, TT\}$. Let X represents the number of heads that can come up with each sample point we can associate a number for X as shown in table. Thus for example, in case of HH (i.e. 2 heads), $X = 2$

while for TH (1 head), $x=1$. It follows that X is a random variable.

Sample point	HH	HT	TH	TT
X	2	1	1	0

C: $f(x) = \begin{cases} 1/4 & 1 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$

Raw moments:

$$\mu_1' = E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{1} x f(x)\, dx + \int_{1}^{5} x f(x)\, dx + \int_{5}^{\infty} x f(x)\, dx$$

$$= 0 + \int_{1}^{5} x \, 1/4 \, dx + 0$$

$$= \left[\frac{x^2}{8}\right]_1^5 = \left[\frac{5^2}{8} - \frac{1^2}{8}\right] = \frac{24}{8} = 3$$

$$\mu_2' = E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_{-\infty}^{1} x^2 f(x)\, dx + \int_{1}^{5} x^2 f(x)\, dx + \int_{5}^{\infty} x^2 f(x)\, dx$$

$$= 0 + \int_1^5 x^2 \cdot 1/4 \, dx + 0$$

$$= \left[\frac{x^3}{12} \right]_1^5 = \left[\frac{125}{12} - \frac{1}{12} \right] = \frac{124}{12} = \frac{31}{3}$$

$$\mu_3' = E(x^3) = \int_{-\infty}^{\infty} x^3 f(x)\, dx = \int_{-\infty}^{1} x^3 f(x) dx + \int_1^5 x^3 \cdot 1/4 \, dx + \int_5^{\infty} x^3 f(x) dx$$

$$= 0 + \left[\frac{x^4}{16} \right]_1^5 + 0$$

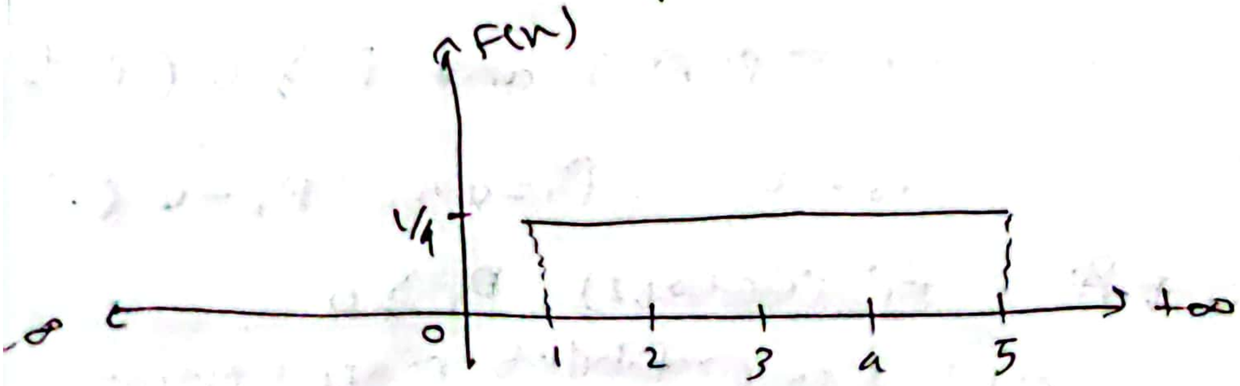
$$= \frac{5^4}{16} - \frac{1^4}{16} = \frac{624}{16} = 39$$

$$\mu_4' = E(x^4) = \int_{-\infty}^{\infty} x^4 f(x) dx = \int_{-\infty}^{1} x^4 f(x) dx + \int_1^5 x^4 f(x) dx + \int_3^{\infty} x^4 f(x) dx$$

$$= 0 + \int_1^5 x^4 \cdot \frac{1}{4} \, dx + 0 = \left[\frac{x^5}{20} \right]_1^5 = \left[\frac{5^3}{20} - \frac{1^3}{20} \right]$$

$$= \frac{781}{5}$$

$\boxed{\mu_1' = 3}$ $\boxed{\mu_2' = \frac{31}{3}}$ $\boxed{\mu_3' = 39}$ $\boxed{\mu_4' = \frac{781}{5}}$



2. a. If x is a discrete random variable $(x = x_1, x_2, x_3, \ldots\ldots, x_n)$ and $P_1, P_2, P_3 \ldots P_n$ are corresponding probability such that

(i) $\cancel{\forall}\ P_i \geq 0$ (ii) $\Sigma P_i = 1$

Then a discrete probability distribution is formed. Then $p(x)$ is called probability mass function.

(i)

x	-1	0	1	2
$p(x)$	$3k$	$-2k$	0.4	0.6

if $\Sigma p(x) = 1$, then this will be a probability mass function.

Now, $\Sigma p(x) = 1$

or, $3k - 2k + 0.4 + 0.6 = 1$

$\cancel{k + 0.1 = 0}$ $k + 1 = 1$

$k = 0$

if $k = 0$, then $\Sigma P(x) = 1$, and $P_i \geq 0$ $(i = -1, 0, 1, 2)$

$\therefore P_{-1} = 0$ $P_0 = 0$ $P_1 = 0.4$ $P_2 = 0.6$.

$\therefore$ $\cancel{a\ \forall P_i}$ $\forall i : i \in (-1, 0, 1, 2)$ $P_i \geq 0$

$\therefore p(x)$ is a probability mass function

$\therefore P(X>0) = P(X=1) + P(X=2) = 0.4 + 0.6 = 1$

$\therefore$ mean $\mu = E(x) = \mu_1' = \mu_1$

$$\mu = E(x) = \sum_{x=-1}^{2} x\,p(x)$$

$$\mu = (-1)\,P(-1) + 0\,P(0) + 1\,P(1) + 2\,P(2)$$

$$\mu = -1\times 0 + 0\times 0 + 1\times 0.4 + 2\times 0.6$$

$$\mu = 1.6.$$

(11)

x	-2	0	2	4	6	8
$p(x)$	$2k$	$2k$	k	0.5	0.3	0.2

$\therefore$ If $\sum P(x) = 1$, then this will be a probability mass function:

$$\sum P(x) = 1$$

$$2k + 2k + k + 0.5 + 0.3 + 0.2 = 1$$

$$5k + 1 = 1$$

$$5k = 0$$

$$k = 0$$

$\therefore$ if $k = 0$, then $\sum P(x) = 1$ and $p_i \geq 0$ ($i = -2, 0, 2, 4, 6, 8$)

$P_{-2} = 0$ $\quad P_0 = 0$ $\quad P_2 = 0$ $\quad P_4 = 0.5$ $\quad P_6 = 0.3$ & $P_8 = 0.2$

$P(-2) = 0$ $\quad P(0) = 0$ $\quad P(2) = 0.$

$\therefore P(x>0) = 1 - P(x \le 0)$

$= 1 - \{P(x = 0) + P(x = -2)\}$

$= 1 - 0$

$= 1$

$\therefore$ Mean $= \mu_1' = \mu = \mu_1$

$$\mu = E(x) = \sum_{x=-2}^{8} x\,p(x)$$

$$\mu = (-2)p(-2) + 0p(0) + 2p(2) + 4p(4) + 6p(6) + 8p(8)$$

$$= (-2\times 0) + (0\times 0) + (2\times 0) + (4\times 0.5) + (6\times 0.3) + (8\times 0.2)$$

$$= 5.4$$

b Normal Distribution:

Normal distribution, also known as the Gaussian distribution is a probability distribution that is symmetric about the mean showing that data near the mean are ~~Appe~~ more frequent in occurrence than data far from the mean. In graph form, normal distribution appear as a bell curve. The density function for this is given by, $f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$;

$-\infty < x < \infty$

Here, x is a continuous random variable.
$f(x; \mu, \sigma)$ is a probability density function.
μ is mean and σ is standard deviation.

mean or mode distribution:

By defn of the mean/mode we have.

$$E(x) = \int_{-\infty}^{\infty} \frac{x}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

By using Integral properties:

$$E(x) = \int_{-\infty}^{\infty} (x+\mu) \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x^2}{2\sigma^2}\right)} dx$$

$$= \int_{-\infty}^{\infty} \frac{x}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x^2}{2\sigma^2}\right)} dx + \int_{-\infty}^{\infty} \frac{\mu}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x^2}{2\sigma^2}\right)} dx$$

$$\text{---} \quad (1)$$

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For the first integral, call it I_1, we have using additivity,

$$I_1 = \int_{-\infty}^{0} \frac{x}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x^2}{2\sigma^2}\right)} dx + \int_{0}^{\infty} \frac{\mu}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x^2}{2\sigma^2}\right)} dx$$

Swapping the integration limits we have

$$I_1 = \int_{0}^{\infty} \frac{-x}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x^2}{2\sigma^2}\right)} dx + \int_{0}^{\infty} \frac{x}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x^2}{2\sigma^2}\right)} dx$$

$$I_1 = 0$$

The from eqn (I), we get that,

$$E(x) = \int_{-\infty}^{\infty} \frac{\mu}{\sigma\sqrt{2\pi}} e^{-\left(x^2/2\sigma^2\right)} dx \quad \text{--- (II)}$$

multiplying eqn (II) by $\sigma\sqrt{2}$ we have,

$$E(x) = \int_{-\infty}^{\infty} \frac{\mu}{\sqrt{\pi}} e^{-x^2} dx$$

$$E(x) = \mu \int_{0}^{\infty} \frac{2}{\sqrt{\pi}} e^{-x^2} dx$$

the last term because the integrand is an even function.

Now, $\frac{2}{\sqrt{\pi}} \int_0^\infty e^{-u^2} du = \lim_{t \to \infty} \frac{2}{\sqrt{\pi}} \int_0^t e^{-u^2} du$

$= \lim_{t \to \infty} \operatorname{erf}(t)$

$= 1$

where "erf" is the error function. so, we end up

with, $E(x) = \mu$

$\therefore$ The mean or mode of the distribution is μ

Uniform Distribution:

random variable X is said to be uniformly distributed in $a \le x \le b$ if its density function is

$$f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

and the distribution is called a uniform distribution

This distribution function is given by

$$F(x) = P(X \le x) = \begin{cases} 0 & x < 0 \\ (x-a)/(b-a) & a < x < b \\ 1 & x \ge b \end{cases}$$

the mean and variance are, $\mu = \frac{1}{2}(a+b)$

$\sigma^2 = \frac{1}{12}(b-a)^2$

Proof
Uniform function is a probability density function.

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise.} \end{cases}$$

Here, $f(x) \geq 0$.

As $f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$; so and $f(x) = 0$; otherwise. so it's fullfilled the codition.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1.$$

Here, $\int_{-\infty}^{\infty} f(x)\,dx$

$$= \int_{-\infty}^{a} f(x)\,dx + \int_{a}^{b} f(x)\,dx + \int_{b}^{\infty} f(x)\,dx.$$

$$= 0 + \int_{a}^{b} \frac{1}{b-a}\,dx + 0$$

$$= \frac{1}{b-a} [x]_a^b = \frac{1}{b-a} [b-a]$$

$$= 1.$$

$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1.$

So, $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Thus it can be said that, $f(x)$ is a probability density function.

The binomial distribution is a probability distribution that summarizes the likelihood that a value will take one of two independent values under a given set of parameters or assumption.

There are mainly two assumptions needed regarding Binomial distribution:

i. Bernoulli Trial: Bernoulli trial is a random experiment which has to satisfy the below conditions

1. The trial must have only two o/p.
   for example: Tossing a coin. for a coin the o/p would be either Head or Tail.
2. If a family gets a new born baby. their are two possibilities, either the child is boy or a girl.

2. Every trial has to be independent.

For example, suppose, when we toss a coin for the first time we got H. When we'll toss the same coin again we don't have any idea about O/P. It can be either head (H) or tail (T), which means this O/P doesn't depend on the previous O/P.

3. Probability of such is each trial is fixed

For example: After tossing a coin, the probability of getting head & tail is 1/2 and 1/2. Whe we toss same coin the probabilities will remain same, their probability is fixed.

When a trial satisfies all these conditions it would be a bernoulli trial.

ii. Total number of trials are fixed and finite.

For example: When we say, we are tossing a coin 10 times, this means there will be 10 trial. This trial number has to be fixed. Also it can't be infinite.

poisson distribution.

If in bernoulli distribution n is very large ($n \to \infty$) and probability for a single trial is very small ($P \to 0$), then the binomial distribution is converten into poission distribution.

a properties of poission distribution.

1. poisson distribution is a discrete distribution

2. poisson distribution is given by $P(X) = \frac{\lambda^x e^{-\lambda}}{x!}$

so, it's visible that it has only one parameter λ

3. Mean of poisson distribution, $\lambda = np$

4. The mean and variance of poisson distribution are equal, $\lambda = np = \sigma^2$

Normal distribution,

Normal distribution, also known as Gaussian distribution, it's a probability distribution that is symmetric mean, showing that data near the mean are more frequent in occurance that the data far away from the mean.

a properties of Normal distribution

1. "A normal distribution comes with a perfect symmetrical shape. This means that the distribution curve can be divided in the middle to produce two equal halves

2. The mean ($\bar{x}$), mode ($\hat{x}$) and median ($\tilde{x}$) are equal.

3. The skewness of A normal distribution is zero

4. The normal distribution has a kurtosis of 3, that's why it's mesokurtic.

b. Here the Problem can be model of poisson process.

We know poisson process $P_x(t_1, t_2) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$

$\lambda = 8$ viruses per min $= 8/60$ /sec.

$\lambda t = 8/60 \times 30 \quad \cancel{= 4}$

Probability of virus attack below of the traffic

load, $P_k = \sum_{k=0}^{3} \left[\frac{e^{-\lambda t} (\lambda t)^k}{k!} \right]$

$= \sum_{k=0}^{3} \left[\frac{e^{-4} (4)^k}{k!} \right]$

$= \frac{e^{-4} 4^0}{0!} + \frac{e^{-4} 4^1}{1!} + \frac{e^{-4} 4^2}{2!} + \frac{e^{-4} 4^3}{3!}$

$=$

$= np = \bar{x} = \mu = \frac{\sum f_i x_i}{\sum f_i}$

$= \frac{0\times5+1\times10+2\times5+3\times4+4\times2+5\times1}{5+10+5+4+2+1}$

$= \frac{45}{27} = 1.67$

So the estimated $\lambda t (\mu t)$ in 1 min is $= \frac{\lambda}{30} \times 60$

$= \frac{1.67}{30} \times 60$

$= 3.33$

So the desired probability

$$P(X(0,t) < 8) = P_0 + P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7$$

$$= \frac{(4.16)^0 e^{-4.16}}{0!} + \frac{(4.16)^1 e^{-4.16}}{1!} + \frac{(4.16)^2 e^{-4.16}}{2!}$$

$$+ \frac{(4.16)^3 e^{-4.16}}{3!} + \frac{(4.16)^4 e^{-4.16}}{4!} + \frac{(4.16)^5 e^{-4.16}}{5!}$$

$$+ \frac{(4.16)^6 e^{-4.16}}{6!} + \frac{(4.16)^7 e^{-4.16}}{7!}$$

$= 0.937$

ii. $P(X(0,t) = 8) = \frac{(4.16)^8 e^{-4.16}}{8!}$

$= 0.0341.$

c. Here, $\mu = 13$, $\sigma = 4$.

$$\therefore z = \frac{x-\mu}{\sigma} = \frac{x-13}{4}.$$

f_i / O_i	x_i	O_i^2	$Z_i = \frac{x_i - \mu}{\sigma}$	$P(Z_i) = \frac{1}{\sqrt{2\pi}} \int_{-\infty} e^{-z^2/2} dz$	$\Delta P(Z) = P(Z_{i+1}) - P(Z_i)$	$E_i = N \Delta P(Z)$	$\frac{O_i^2}{E_i}$
0	$-\infty$	0	$-\infty$	0	0.0003	0	
3	-0.5	9	-3.38	0.0003	0.0297	1	9
8	5.5	64	-1.88	0.03	0.196	6	10.67
13	10	169	-0.75	0.226	0.509	15	11.27
6	15.5	36	0.63	0.735	0.234	7	5.14
0	20.5	0	1.88	0.969			

$N = 30$

$\sum O_i^2 / E_i = 36.08$

Here, $r = m - k - 1 = 4 - 0 - 1 = 3$, $\alpha = 0.05$

$\therefore \chi^2_{(r,\alpha)} = \chi^2_{(3, 0.05)}$

$\chi^2_{cal} = \sum \frac{O_i^2}{E_i} - N = 36.08 - 30 = 6.08$

$\chi^2_{tab} = \chi^2_{3, 0.05} = 0.352$

$\chi^2_{cal} > \chi^2_{tab}$; so fit is not good.

4:a $\mu = 10$ $\sigma^2 = 49$ $\sigma = 7$

$$z = \frac{x - \mu}{\sigma}$$

This problem can be modelle as normal distribution

$$: N(x : \mu, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2};$$

$$-\infty \le x \le \infty$$

$$\cancel{= \frac{1}{\sqrt{2\pi}}}$$

$$z = \frac{x - 10}{7}.$$

(1). $x = 25.5$

$$z = \frac{25.5 - 40}{7} = -2.07$$

$\therefore P(x < 25) = P(z < 2.07)$

~~= P (0 <~~ = ~~area~~ = area right to 0

$\cancel{= P(0 < z}$ $- P(0 \le z \le 2.07)$

$= 0.5 - 0.4807$

$= 0.019.$

Not qualified $= 11000 \times 0.019 = 211$

(ii) $P(25 < x < 60)$

~~$z = $~~

For, $x = 24.5$ $\qquad$ $x = 60.3$

$z = \frac{24.5 - 40}{7} = -2.21$ $\qquad$ $z = \frac{60.3 - 40}{7}$

$z = 2.92$

$P(-2.21 \leq z \leq 2.92) = P(0 \leq z \leq 2.12) +$

$P(0 \leq z \leq 2.92)$

~~$= 0.48294$~~

$= 0.9812$

$\therefore$ qualified But no eligible $= 11000 \times 0.9812$
$= 10793$

(iii) $P(x > 60)$

$x = 60.3.$

$z = \frac{60.5 - 40}{7} = 2.89.$

$$P(Z > 2.86) = 0.5 - P(0 \le Z \le 2.89)$$

~~$= 0.5 =$~~

$$= 0.00192$$

$\therefore$ Qualified and eligible $= 11000 \times 0.00192$

$$= 21$$

2k17.

5.a. As the given data is grouped data so we know,

$$\mu_r' = \frac{\sum f_i (x_i - A)^r}{N}$$

As the observations have to be from origin so $A = 0$.

$$\therefore \mu_r' = \frac{\sum f_i x_i^r}{N}$$

Class interval	f_i	x_i	x_i^2	x_i^3	x_i^4	$f_i x_i$	$f_i x_i^2$	$f_i x_i^3$	$f_i x_i^4$
5–7	2	6	36	216	1296	12	72	432	2592
8–10	5	9	81	729	6561	45	405	3645	32805
11–13	10	12	144	1728	20736	120	1440	17280	207360
14–16	3	15	225	3375	50625	45	675	10125	151875
	$\Sigma f_i = 20$					$\Sigma f_i x_i = 222$	$\Sigma f_i x_i^2 = 2592$	$\Sigma f_i x_i^3 = 31482$	$\Sigma f_i x_i^4 = 394632$

Here

$$\therefore \mu_1' = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{222}{20} = 11.1 = \mu = \bar{x}$$

$$\therefore \mu_2' = \frac{\Sigma f_i x_i^2}{\Sigma f_i} = \frac{2592}{20} = 129.6$$

$$\therefore \mu_3' = \frac{\Sigma f_i x_i^3}{\Sigma f_i} = \frac{31482}{20} = 1574.1$$

$$\therefore \mu_4' = \frac{\Sigma f_i x_i^4}{\Sigma f_i} = \frac{394632}{20} = 19731.6$$

So, these are the four moments.

We know, central moment $\mu_r = \dfrac{\Sigma f_i (x_i - \bar{x})^r}{\Sigma f_i}$

Here, $\mu_1 = 0$ or, $\mu_1 = \dfrac{\Sigma f_i (x_i - \bar{x})}{\Sigma f_i} = 0$

$$\mu_2 = \mu_2' - (\mu_1')^2 = 129.6 - (11.1)^2 = 6.39.$$

$$\mu_2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i} = 6.39$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3 = 1574.1 - 3 \times 129.6 \times 11.1 + 2(11.1)^3$$

$\mu_3 = -6.318$

$$\mu_3 = \frac{\Sigma f_i (x_i - \bar{x})^3}{\Sigma f_i} = -6.318$$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$\mu_4 = 19731.6 - 4 \times 1574.1 \times 11.1 + 6 \times 129.6 \times 11.1^2 - 3 \times 11.1^4$

$\mu_4 = 107.5437$

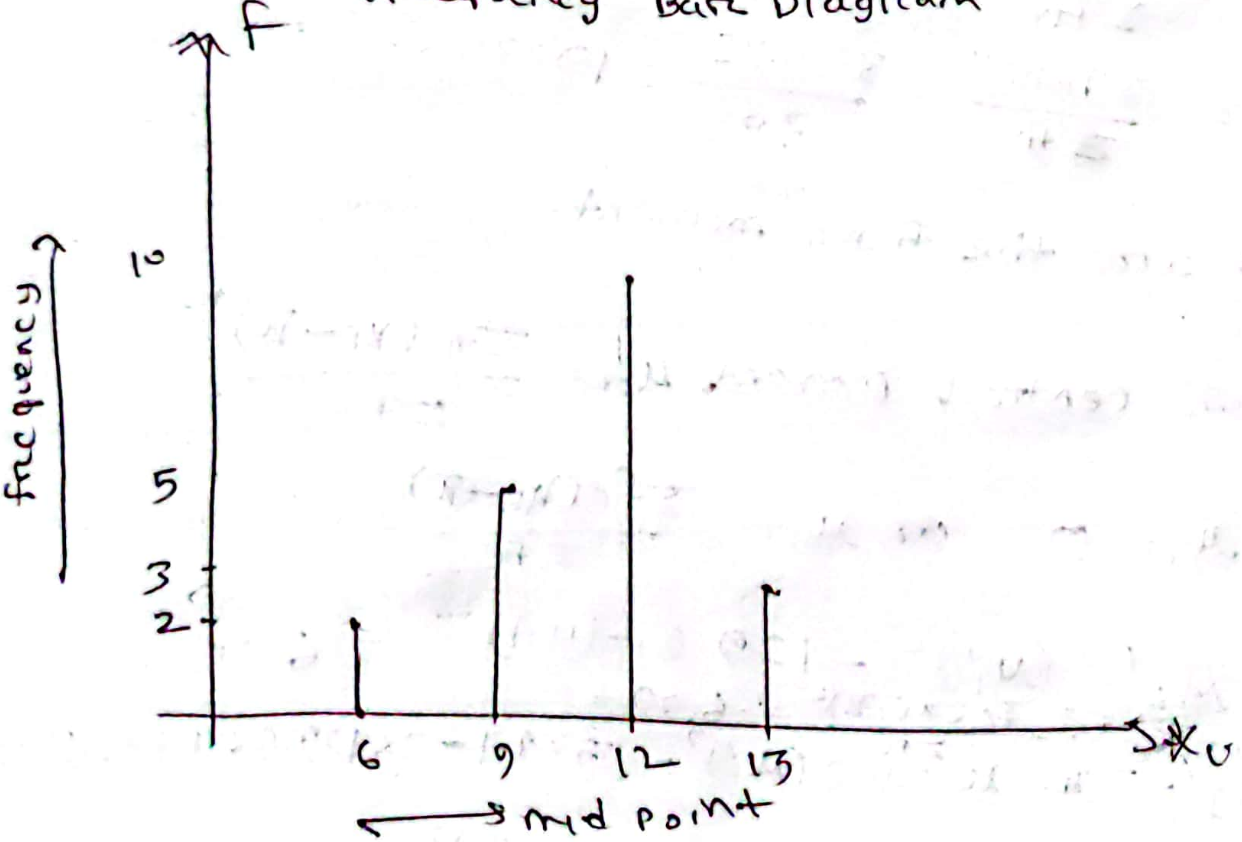
$\mu_4 = \frac{\Sigma f_i (x_i - \bar{x})^4}{\Sigma f_i} = 107.5437$

$\therefore \mu_0 = 11.1$ & $\sigma^2 = 6.39$

$\beta.\ \gamma_1 = \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{-6.318}{\sqrt{6.39^3}} = -0.39$; so negatively skewed.

$\gamma_2 = \frac{\mu_4}{\mu_2^2} = \frac{107.5437}{6.39^2} = 2.63 < 3$; platykurtic

Frequency Bar Diagram



b. Here the highest no of cars, n=4.

Since p is not given, we have to estimate p

we know, $\overline{x} = \hat{\mu} = n\hat{p}$

$$\frac{\sum f_i x_i}{\sum f_i} = n\hat{p}$$

$$\frac{0\times10 + 1\times15 + 2\times20 + 3\times12 + 4\times5}{10+15+20+12+5} = 4\times\hat{p}$$

$$\hat{p} = \frac{111}{62\times4} = 0.448$$

$\therefore \hat{q} = 1 - \hat{p} = 1 - 0.448 = 0.552$

So the B.D. model would be,

$$P(x) = {}^4C_x \, (0.448)^x \, (0.552)^{4-x}$$

No. of car min	O_i	$P_i(x)$	O_i^2	$E_i = N \cdot P(x)$	O_i^2/E_i
0	10	0.093	100	6	16.67
1	15	0.301	225	19	11.84
2	20	0.366	400	23	17.39
3	12	0.198	144	12	12
4	5	0.090	25	3	8.33
	$\sum O_i = 62$				$\sum O_i^2/E_i = 66.23$

$\therefore \chi^2_{cal} = \sum \frac{O_i^2}{E_i} - N = 66.23 - 62 = 4.23$

Here, $\nu = m - k - 1 = 5 - 1 - 1 = 3$

$\alpha = 0.05$

$\chi^2_{tab} = \chi^2_{(3, 0.05)} = 0.0717.$

$\chi^2_{cal} > \chi^2_{tab}$

So, the fit is not good at all.

6. a. For Probability distribution function :-

(i) $P(x_i) \geq 0$

(ii) $\sum P(x_r) = 1$ [For discrete]

For continuous. $\int_{-\infty}^{\infty} P(x)\, dx = 1$

(i) $\sum P(x_i) = 1$

$-3k + 2k + ak + 0.5 = 1$

$3k + 0.5 = 1$

$k = 1/6$

$\therefore P(-1) = -3 \cdot 1/6 = -1/2 < 0$

So, it's not probability density function.

$$E(2x+3) = \Sigma (2x+3) P(x)$$

$$= \Sigma 2x P(x) + \Sigma 3 P(x)$$

$$= 2 \Sigma x P(x) + 3 \Sigma P(x)$$

$$= 2\left((-1)(-3/6) + 0 \times 2/6 + 1 \times 4/6 + 0.3\right) +$$

$$3\left(-3 \times 1/6 + \cancel{0\times} 2/6 + 4/6 + 0.3\right)$$

$$= \frac{10}{3} + 3$$

$$= 19/3$$

$$P(-2 \leq x < 2) = P(-1) + P(0) + P(1)$$

$$= -1/2 + 1/3 + 2/3$$

$$= 1/2$$

(ii) $\Sigma P(x_i) = 1$

$$2k + 2k + k + 0.5 = 1$$

$$5k + 0.5 = 1$$

$$5k = 0.5$$

$$k = 0.1$$

$p(-2) = 2\times 0.1 = 0.2 > 0$

$p(0) = 2\times 0.1 = 0.2 > 0$

$p(2) = 0.1 > 0$

$p(4) = 0.5 > 0$

$\therefore$ so it's a probability density function

$E(2x+3) = \sum (2x+3)\, p(x)$

$= 2\sum x\,p(x) + 3\sum p(x)$

$= 2\left((-2)\times 0.2 + 0\times 0.2 + 2\times 0.1 + 4\times 0.5\right)$

$+ 3\times(0.2 + 0.2 + 0.1 + 0.5)$

$= 2(-0.4 + 0 + 0.2 + 2) + 3\times 1$

$= 6.6$.

$P(-2 \le x < 2) = p(-2) + p(0)$

$= 0.2 + 0.2$

$= 0.4$

iii. $p(x) = \begin{cases} kx & -1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$

$\therefore$ ~~p(x)~~

$$\int_{-\infty}^{\infty} p(x)\,dx = 1.$$

$$\int_{-\infty}^{-1} p(x)\,dx + \int_{-1}^{2} p(x)\,dx + \int_{2}^{\infty} p(x)\,dx = 1$$

$$\Rightarrow 0 + \int_{-1}^{2} kx\,dx + 0 = 1$$

$$\Rightarrow \left[\frac{kx^2}{2}\right]_{-1}^{2} = 1.$$

$$\Rightarrow \left[\frac{k4}{2} - \frac{k}{2}\right] = 1.$$

$$\Rightarrow 2k - k/2 = 1$$

$$\Rightarrow \frac{3k}{2} = 1$$

$$k = 2/3$$

$$P(x) = \begin{cases} \frac{2}{3}x & -1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

For, $x = -1$.

$P(x) = -\frac{2}{3} < 0$

so it's not probability distribution function

$$E(2x+3) = \int_{-1}^{2} (2x+3)\, \frac{2}{3}x\, dx$$

$$= \int_{-1}^{2} \frac{4x^2}{3} dx + \int_{-1}^{2} 2x\, dx$$

$$= \left[\frac{4x^3}{9}\right]_{-1}^{2} + \left[x^2\right]_{-1}^{2}$$

$$= \left[\frac{4 \cdot 2^3}{9} + \frac{4}{9}\right] + \left[4 - 1\right]$$

$$= \left[\frac{32}{9} + \frac{4}{9}\right] + 3$$

$$= \left[\frac{36}{9}\right] + 3 = 7.$$

$$P(-2 \le x \le 2) = \int_{-2}^{2} f(x)\, dx$$

$$= \cancel{\int_{-2}^{-1} \tfrac{2}{3}\, dx} = \int_{-1}^{2} \tfrac{2}{3} x \, dx$$

$$= \left[\frac{x^2}{3} \right]_{-1}^{2}$$

$$= \left[\frac{4}{3} - \frac{1}{3} \right]$$

$$= 1$$

b. According to normal distribution, we know,

$$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad ; -\infty \le x \le \infty$$

Here, given, $\mu = \bar{x} = 60$, $\sigma^2 = 4$; $\sigma = 2$

So, $$f(x; 60, 2) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-60}{2}\right)^2}$$

$$\left[\frac{1}{2\sqrt{2\pi}} e^{-\left(\frac{x-60}{2\sqrt{2}}\right)^2}\right]$$

Here x is a continuous random variable.

For being continuous probability distribution it has to justify two conditions.

1. $f(x) \geq 0$: if the function $f(x; 60, 2) = e^{-x}$ can never be negative. As the S.D. is never be negative. so $f(x)$ can never be negative.

ii. ~~Assume~~ $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

At first assume, $\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$

suppose, $z = \dfrac{x-\mu}{\sqrt{2}\,\sigma} = \dfrac{x-60}{2\sqrt{2}}$

$$dz = \frac{dx}{2\sqrt{2}}$$

$$\frac{dz}{dx} = \frac{1}{2\sqrt{2}}.$$

$$dx = 2\sqrt{2}\,dz$$

So, $\int_{-\infty}^{\infty} f(x)\,dx = \dfrac{1}{2\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\left(\frac{x-60}{2\sqrt{2}}\right)^2} dx$

$$= \frac{1}{2\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-z^2} \, 2\sqrt{2} \, dz$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-z^2} dz$$

$$= \frac{1}{\sqrt{\pi}} \sqrt{\pi}$$

$$= 1$$

So, $f(x)$ is a probability density function

Now, in Normal distribution, mean, mode and median coincide, so, $\bar{x} = \hat{x} = \tilde{x} = 60$.

And coefficient of skewness $\beta_1 = 0$; as it's symmetric

7. a. Necessary assumptions regarding B.D. are given below.

i. Each trial has two possible assumptions, in the language of reliability called success and failure.

ii. The trials are independent. Intuitively, the outcome of one trial has no influence over the outcome of another trial.

iii. On each trial, the probability of success is p and the probability of failure is (1-p) where p in [0,1] is the success parameter of the process.

Given that,

total no. of families in the town is, N = 500.

Each family consists of four children and their parents.

For a single family

let, probability of a boy birth is = (1/2)

Probability of a girl birth is = (1/2)

(i) . $P(\text{no boy}) = \cancel{500 \times} \left({}^4C_0 \left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^4 \right)$

$\cancel{= 31.25}$ $= 0.0625$

(ii) $P(\text{at least one boy}) = \cancel{500 \times} (1 - P(\text{No boy}))$

$= 1 - \frac{1}{16}$

$= \frac{15}{16}$

$= 0.9375$

$P(\text{no girl for all families}) = {}^4C_0 (\tfrac{1}{2})^0 (\tfrac{1}{2})^4 \times 500$

$= 31.25$

≈ 31

b. Given that, total no. of student is $= 12000$

$\therefore$ average score = mean $= 60 = \mu$.

& variance $\sigma^2 = 25$

& $S.D = \sqrt{\sigma^2} = \sigma = \sqrt{25} = 5$

(i) $z = \dfrac{40.5 - 60}{5} = -3.9$

$P(-\infty \le x < 40.5) = P(-\infty \le z \le -3.9)$

$= P(-\infty \le z \le 0) - P(-3.9 \le z \le 0)$

$= \frac{1}{2} - P(0 \le z < 3.9)$

$= \frac{1}{2} - \dfrac{1}{\sqrt{2\pi}} \int_0^{3.9} e^{-z^2/2}\, dz$

$= 0.00004809$

$\therefore$ not qualified $= 12000 \times 0.00004809$

$= 1$

(ii) $x_1 = 39.5$ $\qquad$ $x_2 = 80.5$

$z_1 = \frac{39.5-60}{5}$ $\qquad$ $z_2 = \frac{80.5-60}{5}$

$= -4.1$ $\qquad$ $z_2 = 4.1$

$P(40 \le x \le 80) = P(-4.1 \le z \le 4.1)$

$= 2P(0 \le z \le 4.1)$

$= 2 \times \frac{1}{\sqrt{2\pi}} \int_0^{4.1} e^{-z^2/2}\, dz$

$= 0.99995$

qualified but no eligible = ~~11999~~ 12000×0.99995
$= 11999$

(iii) $x > 80.5$ $\qquad$ $P(x > 80.5) = P(z > 4.1)$

$z = \frac{80.5-60}{5}$ $\qquad$ $= P(0 \le z \le \infty) - P(0 \le z \le 4.1)$

$z = 4.1.$ $\qquad$ $= 0.5 - \frac{1}{\sqrt{2\pi}} \int_0^{4.1} e^{-z^2/2}\, dz$

$= 0.00002067.$

eligible for choosing any department. 12500 × 0.0000 ?

= 0

2K18

5. a. we have, $\lim_{z \to 0} t^{z/\bar{z}}$ where, $z = x+iy$

$\bar{z} = x-iy$

suppose, $(x,y) \to (0,0)$ along the x-axis then, $y=0$ and $x \to 0$

so the required limit

$$\lim_{x \to 0} t^{\frac{x}{x}}$$

$$= \lim_{x \to 0} t$$

$$= t$$

Suppose $(x,y) \to (0,0)$ along the y-axis, then $x \to 0$ and $y \to 0$

so the required limit

$$\lim_{y \to 0} t^{\frac{iy}{-iy}}$$

$$= \lim_{y \to 0} t^{-1}$$

$$= 1/t$$

which is different from the above

so, the required limit doesn't exist

b. $f(z) = |z|^2$

we know $z = x + iy$

$|z| = \sqrt{x^2+y^2}$

$|z|^2 = x^2+y^2$

To finid conitinuity and differentiability of ~~first~~, ~~to~~ it's limit have to be enamined.

Suppose $(x,y) \to (0,0)$ along the x anis then $y=0$ and $x \to 0$

So, the required limit

$$\lim_{x \to 0} x^2$$

$$= 0$$

Suppose $(x,y) \to (0,0)$ along the y anis, then $x=0$ and $y \to 0$

So, the required limit

$$\lim_{y \to 0} y^2$$

$$= 0$$

which is same as above

So, the required limit enists.

. functional value, at $z=0$.

$f(0) = 0^2+0^2 = 0$

$\therefore$ which is same as limiting value. so the function is continuous

This function will be differentiable if $\lim_{z \to 0} \frac{f(z)-f(0)}{z-0}$ exists

where, $f(z) = |z|^2 = x^2+y^2$

$f(0) = 0$

$z = x+iy$

$$\therefore \lim_{(x,y) \to (0,0)} \frac{x^2+y^2}{x+iy}$$

suppose $(x,y) \to (0,0)$ along the x axis, then $y=0, x \to 0$,

so, the required limit.

$$\lim_{x \to 0} \frac{x^2}{x} = \lim_{x \to 0} x = 0$$

suppose $(x,y) \to (0,0)$ along the y axis, then $x=0, y \to 0$

so the required limit

$$\lim_{y \to 0} \frac{y^2}{iy} = \lim_{y \to 0} y/i = 0$$

which is same as above. so the required limit exists so the function is differentiable.

C. $f(z) = \dfrac{xy^2(x+iy)}{x^4+y^4}$.

At first limit has to be checked

$$\lim_{z \to 0} \frac{xy^2(x+iy)}{x^4+y^4}$$

suppose $(x,y) \to (0,0)$ along the x axis so then $y \to 0$, $x \to 0$,

the required limit, $\lim_{x \to 0} \dfrac{0}{x^4} = 0$

suppose $(x,y) \to (0,0)$ along the y axis, then $x = 0$, $y \to 0$

the required limit, $\lim_{y \to 0} \dfrac{0}{y^4} = 0$

which is same as above so the required limit exists

at $z = 0$,

$f(z) = \dfrac{xy^2 \cdot z}{x^4+y^4}$; $z = x+iy$

$f(0) = 0$,

So, the function is continuous

~~For~~ To differentiable $\lim_{z \to 0} \frac{f(z) - f(0)}{z - 0}$ has to be existed.

$$\frac{f(z) - f(0)}{z - 0} = \frac{\frac{xy^2(x+iy)}{x^4+y^4} - 0}{z}$$

$$\frac{f(z) - f(0)}{z - 0} = \frac{\frac{xy^2 z}{x^4+y^4}}{z} \quad [z = x+iy]$$

$$\frac{f(z) - f(0)}{z - 0} = \frac{xy^2}{x^4+y^4}.$$

~~If~~ suppose $(x,y) \to (0,0)$ along any line $y = mx$ then

$$\lim_{x \to 0} \frac{m^2 x^3}{x^4 + m^4 x^4}$$

$$\lim_{x \to 0} \frac{m^2 x}{1 + m^4 x^2}$$

$$\lim_{x \to 0} 0$$

And, if the limit along the parabola, $x=y^2$ then

$$\lim_{y\to 0} \frac{y^4}{2y^4} = \lim_{y\to 0} \frac{1}{2} = \frac{1}{2}$$

which is different from above so the function is not differentiable at $z=0$. as if we approach origin from different directions we are getting different limits

$$f(z) = \frac{xy^2(x+iy)}{x^2+y^4}$$

$$f(z) = \frac{x^2y^2}{x^2+y^4} + i\frac{xy^3}{x^2+y^4}$$

$$f(z) = u + iv$$

$$u = \frac{x^2y^2}{x^2+y^4} \qquad v = \frac{xy^3}{x^2+y^4}$$

$$\frac{\partial u}{\partial x} = \frac{(x^2+y^4)\frac{\partial}{\partial x}(x^2y^2) - x^2y^2\frac{\partial}{\partial x}(x^2+y^4)}{(x^2+y^4)^2}$$

$$\frac{\partial u}{\partial x} = \frac{(x^2+y^4)\,2xy^2 - x^2y^2 \cdot 2x}{(x^2+y^4)^2}$$

b) $f(z) = \dfrac{xy^2(x+iy)}{x^2+y^4}$

$$f(z) = \frac{x^2y^2}{x^2+y^4} + i\,\frac{xy^3}{x^2+y^4}$$

$$f(z) = u + iv$$

$$u = \frac{x^2y^2}{x^2+y^4} \qquad v = \frac{xy^3}{x^2+y^4}$$

$$\frac{\partial u}{\partial x} = \lim_{x\to 0} \frac{u(x,0) - u(0,0)}{x}$$

$$\frac{\partial u}{\partial x} = \lim_{x\to 0} \frac{0 - 0}{x} = \lim_{x\to 0} \frac{0}{x} = 0$$

$$\frac{\partial u}{\partial y} = \lim_{y\to 0} \frac{u(0,y) - u(0,0)}{y} = \lim_{y\to 0} \frac{0}{y} = 0$$

$$\frac{\partial v}{\partial x} = \lim_{x\to 0} \frac{v(x,0) - v(0,0)}{x} = \lim_{x\to 0} \frac{0}{x} = 0$$

$$\frac{\partial v}{\partial y} = \lim_{y\to 0} \frac{v(0,y) - v(0,0)}{y} = \lim_{y\to 0} 0/y = 0$$

$\therefore \dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$ i.e. CR eq^n are satisfied at origin

6. a. Singular point:

A point at which $f(z)$ fails to be analytic is called a singular point or ~~as~~ singularity of $f(z)$.

Example:

$$f(z) = \frac{z+1}{z^5(z-1)}$$

put $z^5(z-1)=0$

$\therefore z=0, z=1$ are singular points.

pole: ~~If~~

If z_0 is not a singular and we can find a positive integer n such that $\lim_{z \to z_0} (z-z_0)^n f(z) = A \neq 0$, then $z=z_0$ is called a pole of order n. if $n=1$, z_0 is called a simple pole.

Example:

$$f(z) = \frac{3z-2}{(z-1)^2(z+1)(z-4)}$$

has a pole of order 2 at $z=2$ and simple poles at $z=-1$ and $z=4$

~~iii~~.

Branch point:

A branch point of an analytic function is a point in the complex plane whose complex argument can be mapped from a single point in the domain to multiple points in the range.

Example:

$f(z) = (z-3)^{1/2}$ has a branch point at $z=3$.

Removable singular point:

An isolated singular point z_0 is called a removable singular point of $f(z)$ if $\lim_{z \to z_0} f(z)$ exists. By defining $f(z_0) = \lim_{z \to z_0} f(z)$, it can be shown that $f(z)$ is not only continuous at z_0 but is also analytic at z_0.

Example:

The singular point ~~of~~ $z=0$ is a removable singularity of $f(z) = \frac{\sin z}{z}$ since $\lim_{z \to 0} \frac{\sin z}{z} = 1$

b. (i) $f(z) = \frac{e^{-z}}{(z-2)^4}$

To ~~st.~~ find pole

put $(z-2)^4 = 0$

$z-2 = 0$

$z = 2$

$z = 2$ is the singular point of the function $f(z)$ and ~~it i~~ $z=2$ is a pole of order 4

~~Ae.~~

(ii) $f(z) = \sin \frac{1}{z-1}$

To find pole, put $(z-1) = 0$

$z = 1$

$z = 1$ is the singular point of the function $f(z)$ and $z = 1$ is an essential singularity

c. $\oint_c \dfrac{z\,dz}{(9-z^2)(z+i)}$ $|z|=2$

The integrand has singularities, where $(9-z^2)(z+i)=0$

i.e. $z = \pm 3, -i$

The circle $|z|=2$ has centre at $z=0$

$\therefore f(z)=z$, is an analytic function.

$$\frac{1}{(9-z^2)(z+i)} = \frac{\frac{1}{6(i-3)}}{3+z} + \frac{\frac{1}{6(i+3)}}{3-z} + \frac{\frac{1}{10}}{z+i}$$

$$\int_c \frac{z}{(9-z^2)(z+i)} = \int_{C_1} \frac{\frac{z}{6(i-3)}}{3+z}\,dz + \int_{C_2} \frac{\frac{z}{6(i+3)}}{3-z}\,dz +$$

$$\int_{C_3} \frac{\frac{z}{10}}{z+i}\,dz$$

According to cauchy's theorem,

$$\int_c \frac{\frac{1}{6(i-3)}}{3+z}\,dz = 0 \qquad \int_c \frac{\frac{1}{6(i+3)}}{3-z}\,dz = 0$$

Because these are outside the pole

$$\int_C \frac{z}{(9-z^2)(z+i)}\,dz \Rightarrow \int_C \frac{z/10}{z+i}\,dz$$

$$\int_C \frac{z}{(9-z^2)(z+i)}\,dz = \frac{1}{10}\,2\pi i\, f(-i)$$

$$[f(z)=z]$$

$$\int_C \frac{z}{(9-z^2)(z+i)} = \frac{1}{10}\,2\pi i\,(-i)$$

$$\int_C \frac{z}{(9-z^2)(z+i)} = \frac{1}{10}\,2\pi\,(-i)^2 = \frac{1}{\cancel{10}\,5}\,\cancel{2}\pi$$

$$\int_C \frac{z}{(9-z^2)(z+i)} = \pi/5$$

2a. By the divergence theorem,

$$\oiint \vec{F}\cdot\hat{n}\, ds = \iiint_V \nabla\cdot F\, dv \qquad ; F = x^2 \underline{i} + y^2 \underline{j} + z^2 \underline{k}$$

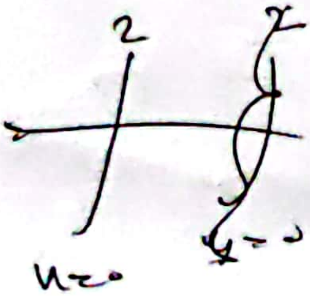
$$= \iiint_V \left(\frac{\partial}{\partial x} x^2 + \frac{\partial}{\partial y} y^2 + \frac{\partial}{\partial z} z^2\right) dv$$

$$= \iiint_V (2x+2y+2z)\, dv$$

$$= \int_{x=0}^{2}\int_{y=0}^{2}\int_{z=0}^{2} (2x+2y+2z)\, dz\, dy\, dx$$

$$= \int_{x=0}^{2}\int_{y=0}^{2} \left[2xz+2yz+z^2\right]_0^2 dy\, dx$$

$$= \int_{x=0}^{2}\int_{y=0}^{2} \left[4x+4y+4\right] dy\, dx$$



$$= \int_{x=0}^{2} \left[4xy+2y^2+4y\right]_0^2 dx$$

$$= \int_{x=0}^{2} \left[8x+8+8\right] dx$$

$$= \int_{x=0}^{2} (8x+16)\, dx$$

$$= \left[4x^2 + 16x\right]_0^2$$

$$= \left[16 + 32\right] = 48$$

b. $\overline{F} = 3xy\,\underline{i} - y^2\,\underline{j}$

we know $\overline{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$d\overline{r} = dx\,\underline{i} + dy\,\underline{j} + dz\,\underline{k}$

$$\int_c \underline{F} \cdot d\underline{r} = \int_c \left[(3xy\,\underline{i} - y^2\,\underline{j}) \cdot (dx\,\underline{i} + dy\,\underline{j} + dz\,\underline{k})\right]$$

$\cancel{F \cdot dr}$

$$\int_c \underline{F} \cdot d\underline{r} = \int_c (3xy\,dx - y^2\,dy)$$

Here, $y = 2x^2$

$$\int_c \underline{F}\, d\underline{r} = \int_{x=0}^{1} 3x \cdot 2x^2\, dx - \int_0^2 y^2\, dy$$

$$\int_c \underline{F}\, d\underline{r} = \int_{x=0}^{1} 6x^3\,dx - \int_0^2 y^2\,dy$$

$$\int_c \underline{F}\, d\underline{r} = \left[\frac{6x^4}{4}\right]_0^1 - \left[\frac{y^3}{3}\right]_0^2$$

$$\int_c \underline{F}\, d\underline{r} = \frac{6}{4} - \frac{8}{3} = \frac{18-32}{12}$$

$$\int_c \underline{F}\, d\underline{r} = \frac{-14}{12} = -\frac{7}{6}$$

C. $\frac{\overline{r}}{r}$ = We know, $\overline{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$|\overline{r}| = r = \sqrt{x^2+y^2+z^2}$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\frac{\overline{r}}{r} = \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{j} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{k}$$

$\overline{\nabla}$. Let $Q = \vec{\nabla}\cdot\frac{\overline{r}}{r}$

$$= \cancel{\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\hat{i} + \frac{y}{\sqrt{}}\right.}$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{j} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{k}\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial}{\partial y}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial}{\partial z}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

$$= \frac{\sqrt{x^2+y^2+z^2} - x\,\frac{2x}{2\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2} + \frac{\sqrt{x^2+y^2+z^2} - y\cdot\frac{2y}{2\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2} + \frac{\sqrt{x^2+y^2+z^2} - z\,\frac{2z}{2\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}$$

$$= \frac{2(x^2+y^2+z^2) - 2x^2}{2(x^2+y^2+z^2)^{3/2}} + \frac{2(x^2+y^2+z^2) - 2y^2}{2(x^2+y^2+z^2)^{3/2}} + \frac{2(x^2+y^2+z^2) - 2z^2}{2(x^2+y^2+z^2)^{3/2}}$$

$$= \frac{2y^2+2z^2+2x^2+2z^2+2x^2+2y^2}{2(x^2+y^2+z^2)^{3/2}}$$

$$= \frac{9(x^2+y^2+z^2)}{2(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}}$$

$$Q = \frac{2}{\sqrt{x^2+y^2+z^2}}$$

Let, grad $Q = \vec{\nabla} Q$

$$= \left(\underline{i}\frac{\partial}{\partial x} + \underline{j}\frac{\partial}{\partial y} + \underline{k}\frac{\partial}{\partial z}\right)\left(\frac{2}{\sqrt{x^2+y^2+z^2}}\right)$$

$$= \underline{i}\frac{\partial}{\partial x}\frac{2}{\sqrt{x^2+y^2+z^2}} + \underline{j}\frac{\partial}{\partial y}\frac{2}{\sqrt{x^2+y^2+z^2}} + \underline{k}\frac{\partial}{\partial z}\frac{2}{\sqrt{x^2+y^2+z^2}}$$

$$= \underline{i}\left(-\frac{\cancel{2}\cdot 2x}{\cancel{2}(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}}\right) + \underline{j}\left(-\frac{\cancel{2}\cdot 2y}{\cancel{2}(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}}\right) + \underline{k}\left(\frac{-\frac{\cancel{2}\cdot 2z}{\cancel{2}(x^2+y^2+z^2)}}{\sqrt{x^2+y^2+z^2}}\right)$$

$$= \cancel{\underline{i}} = -\left(\frac{2x}{(x^2+y^2+z^2)^{3/2}}\underline{i} + \frac{2y}{(x^2+y^2+z^2)^{3/2}}\underline{j} + \frac{2z}{(x^2+y^2+z^2)^{3/2}}\underline{k}\right)$$

8.a. we know, $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$

$$|\underline{r}| = \sqrt{x^2+y^2+z^2}$$

$$|\underline{r}| = r$$

$$r^2 = x^2+y^2+z^2$$

$$\overline{F} = \frac{\overline{r}}{r^2} = \frac{x}{x^2+y^2+z^2}\underline{i} + \frac{y}{x^2+y^2+z^2}\underline{j} + \frac{z}{x^2+y^2+z^2}\underline{k}$$

$$\overline{\nabla} = \underline{i}\frac{\partial}{\partial x} + \underline{j}\frac{\partial}{\partial y} + \underline{k}\frac{\partial}{\partial z}$$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{x^2+y^2+z^2} & \frac{y}{x^2+y^2+z^2} & \frac{z}{x^2+y^2+z^2} \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}\left(\frac{z}{x^2+y^2+z^2}\right) - \frac{\partial}{\partial z}\left(\frac{y}{x^2+y^2+z^2}\right)\right)$$

$$-\hat{j}\left(\frac{\partial}{\partial x}\left(\frac{z}{x^2+y^2+z^2}\right) - \frac{\partial}{\partial z}\left(\frac{x}{x^2+y^2+z^2}\right)\right)$$

$$+\hat{k}\left(\frac{\partial}{\partial x}\left(\frac{y}{x^2+y^2+z^2}\right) - \frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2+z^2}\right)\right)$$

$$= \hat{i}\left(\frac{-2yz}{(x^2+y^2+z^2)^2} + \frac{2yz}{(x^2+y^2+z^2)^2}\right) - \hat{j}$$

$$\left(\frac{-2zx}{(x^2+y^2+z^2)^2} + \frac{2zx}{(x^2+y^2+z^2)^2}\right) + \hat{k}$$

$$\left(\frac{-2xy}{(x^2+y^2+z^2)^2} + \frac{2xy}{(x^2+y^2+z^2)^2}\right)$$

$$= 0\underline{i} + 0\underline{j} + 0\underline{k}$$

$$= \underline{0}$$

So, $\underline{F}$ is not rotational.

$$\therefore \underline{F} = \frac{x}{x^2+y^2+z^2}\underline{i} + \frac{y}{x^2+y^2+z^2}\underline{j} + \frac{z}{x^2+y^2+z^2}\underline{k}$$

$$\underline{F} = \underline{\nabla} a$$

$$\frac{x}{x^2+y^2+z^2}\underline{i} + \frac{y}{x^2+y^2+z^2}\underline{j} + \frac{z}{x^2+y^2+z^2}\underline{k} = \frac{\partial a}{\partial x}\underline{i} + \frac{\partial a}{\partial y}\underline{j} + \frac{\partial a}{\partial z}\underline{k}$$

$$\frac{\partial Q}{\partial x} = \frac{x}{x^2+y^2+z^2} \quad \text{———} \quad (i)$$

$$\frac{\partial Q}{\partial y} = \frac{y}{x^2+y^2+z^2} \quad \text{———} \quad (ii)$$

$$\frac{\partial Q}{\partial z} = \frac{z}{x^2+y^2+z^2} \quad \text{———} \quad (iii)$$

Integrating (i) partially ~~with respec~~ w.r.t. x

$$Q = \int \frac{x\,dx}{x^2+y^2+z^2}$$

let $x^2+y^2+z^2 = t$

$x\,dx = dt/2$

$$Q = \int \frac{1}{2t}\,dt$$

$$Q = \frac{1}{2}\ln|t| + h(y,z).$$

$$Q = 1/2 \ln|x^2+y^2+z^2| + h(y,z)$$

$$Q = \ln\sqrt{x^2+y^2+z^2} + h(y,z)$$

[Integrating (ii) partially w.r.t y]

Similarly, $Q = \ln\sqrt{x^2+y^2+z^2} + v(z,x)$

[Integrating (iii) partially w.r.t. z]

$Q = \ln\sqrt{x^2+y^2+z^2} + w(x,y)$

$\therefore \varphi = \ln\sqrt{x^2+y^2+z^2} + \text{const}.$

~~[without condition]~~

But it's not possible to find ~~& find~~ the scalar potential such the scalar solution be zero at (0,0,0)

b.

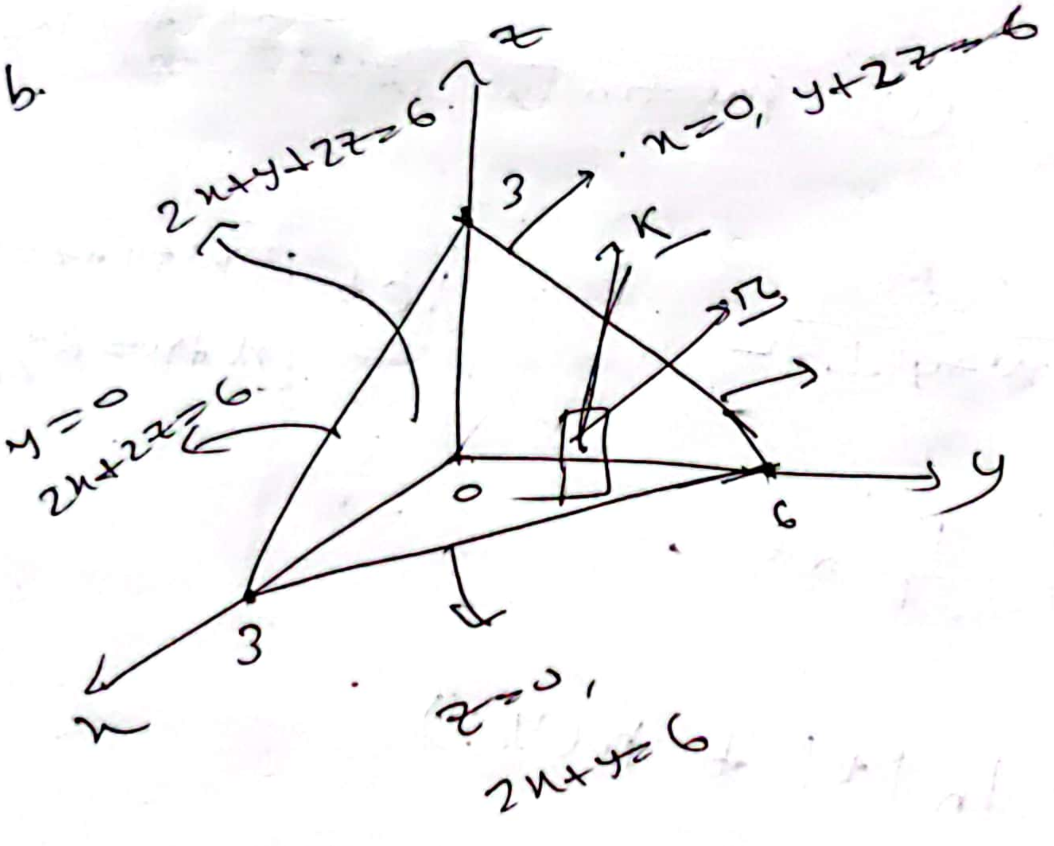


Fig: 1

The surface s and its projection R on the yz plane are shown in fig 1.

we know that, $\iint_S \underline{A}\cdot\underline{n}\, ds = \iint_R \underline{A}\cdot\underline{n}\,\dfrac{dy\,dz}{|\underline{n}\cdot\underline{i}|}$

To obtain $\underline{n}$, note that a vector perpendicular to the surface ~~2x+~~ $2x+y+2z=6$ is given by

is ~~given b~~ $\underline{\nabla}(2x+y+2z)$

$$\cancel{\nabla\phi}\ \nabla\phi = \underline{\nabla}(2x+y+2z)$$

$$= 2\underline{i} + \underline{j} + 2\underline{k}$$

$$\underline{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{2}{3}\underline{i} + \frac{1}{3}\underline{j} + \frac{2}{3}\underline{k}$$

Now, $\underline{n}\cdot\underline{k} = (2/3\,\underline{i} + 1/3\,\underline{j} + 2/3\,\underline{k})\cdot\underline{i}$

$$= 2/3$$

$$\underline{A}\cdot\underline{n} = \left((x+y^2)\underline{i} - 2x\underline{j} + 2yz\underline{k}\right)(2/3\,\underline{i} + 1/3\,\underline{j} + 2/3\,\underline{k})$$

$$= 2/3\,(x+y^2) - 2/3\,x + 4/3\,yz$$

$$= \frac{2x+2y^2-2x+4yz}{3}$$

~~$2x+y+2z=6$~~

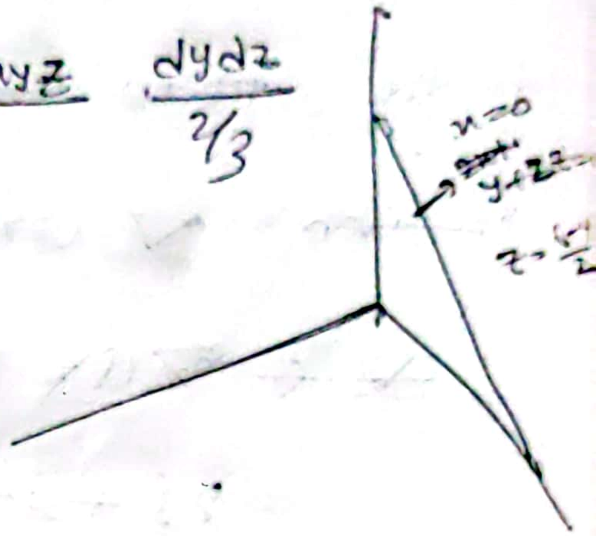
~~$y+2z=6-2x$~~

$$= \frac{2y^2+4yz}{3} = \cancel{\frac{2y(y+2z)}{3}}$$

$\cancel{\boxed{\begin{array}{l}2x+y+2z=6\\ y+2z=6-2x\end{array}}}$

$\approx$

$$\iint_S \underline{A}\cdot\underline{n}\, ds = \int_{y=0}^{y=6}\int_{z=0}^{z=\frac{6-y}{2}} \frac{2y^2+4yz}{3}\, \frac{dy\,dz}{2/3}$$



$$= \int_{y=0}^{y=6}\int_{z=0}^{z=\frac{6-y}{2}} (y^2+2yz)\,dy\,dz$$

$$= \int_{y=0}^{y=6}\left[\int_{z=0}^{z=\frac{6-y}{2}} y^2+2yz\,dz\right]dy$$

$$= \int_{y=0}^{6}\left[y^2z+yz^2\right]_0^{\frac{6-y}{2}} dy$$

$$= \int_{y=0}^{6}\left(y^2\cdot\frac{6-y}{2} + y\cdot\left(\frac{6-y}{2}\right)^2\right)dy$$

$$= \int_{y=0}^{6}\left(\frac{6-y}{2}\,y\left(y+\frac{6-y}{2}\right)\right)dy$$

$$= \int_0^6 \left[\frac{6-y}{2} \cdot y \left(\frac{2y+6-y}{2} \right) \right] dy$$

$$= \int_0^6 \frac{y \cdot (6-y) \cdot (6+y)}{4} \, dy$$

$$= \int_0^6 \frac{y(36-y^2)}{4} \, dy$$

$$= \int_0^6 \frac{36y - y^3}{4} \, dy$$

$$= \int_0^6 \left(9y - y^3/4 \right) dy$$

$$= \left[\frac{9y^2}{2} - \frac{y^4}{16} \right]_0^6$$

$$= \left[\frac{9 \cdot 6^2}{2} - \frac{6^4}{16} \right]$$

$$= 81$$

$$\boxed{\text{Thus: } \iint_S \underline{A} \cdot \underline{n} \, dS = 81}$$

c. Green Theorem is $\oint_C M dx + N dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx dy$

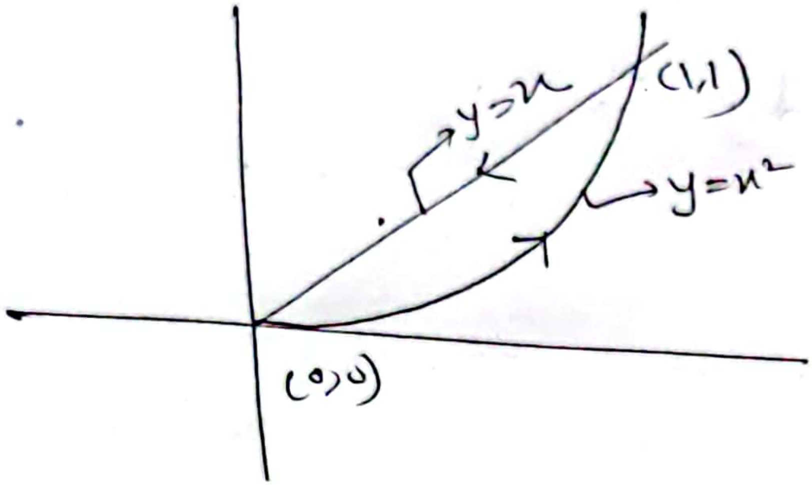


fig 1

$y = x$

$y = x^2$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, 1$

$y = 0, 1$

In fig 1, $y=x$ and $y=x^2$ intersect at $(0,0)$ and $(1,1)$

~~Along~~ Here, $\oint_C (xy+y^2)dx + x^2 dy$

Along $y=x^2$, then the line integral =
$dy = 2x dx$

$$\int_0^1 \left\{(x\cdot x^2 + (x^2)^2) dx + x^2 \cdot 2x dx\right\}$$

$$= \int_0^1 (x^3 + x^4) dx + \int_0^1 2x^3 dx$$

$$= \int_0^1 x^3 dx + \int_0^1 x^4 dx + \int_0^1 2x^3 dx$$

$$= \left[\frac{x^4}{4}\right]_0^1 + \left[\frac{x^5}{5}\right]_0^1 + \left[\frac{x^4}{2}\right]_0^1$$

$$= 1/4 + 1/5 + 1/2$$

$$= \frac{5+4+10}{20}$$

$$= 19/20$$

Along $y = x$, then the line integral

$dy = dx$

$$\int_1^0 \{(x \cdot x + x^2) dx + x^2 dx\}$$

$$= \int_1^0 3x^2 dx$$

$$= [x^3]_1^0$$

$$= -1$$

Then the required line integral $= \frac{19}{20} - 1$

$= -1/20$

Here, $M = xy + y^2$ $\qquad \frac{\partial M}{\partial y} = x + 2y$

$N = x^2$ $\qquad \frac{\partial N}{\partial x} = 2x$

Then by greens theorem.

$$\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

$$= \int_{x=0}^{1} \int_{y=x^2}^{x} (2x - x - 2y)\,dx\,dy$$

$$= \int_{x=0}^{1} \left[\int_{y=x^2}^{x} (x - 2y)\,dy \right] dx$$

$$= \int_0^1 \left[\left[xy - y^2 \right]_{x^2}^{x} \right] dx$$

$$\int_0^1 [(x^2 - x^2) - (x^3 - x^4)]\,dx$$

$$= \int_0^1 (x^4 - x^3)\,dx$$

$$= \left[\frac{x^3}{5} - \frac{x^4}{4}\right]_0^1$$

$$= \frac{1}{5} - \frac{1}{4}$$

$$= \frac{4-5}{20}$$

$$= -\frac{1}{20}$$

$$\therefore \oint_C (M\,dx + N\,dy) = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

Hence green's theorem is verified.

2K17

1\. a (i) $\lim_{z \to (1+i)} (z^2 - 5z + 10)$

$= \lim_{z\to(1+i)} z^2 - \lim_{z\to(1+i)} 5z + \lim_{z\to(1+i)} 10$

$= (1+i)^2 - 5(1+i) + 10$

$= 1+2i+i^2 - 5 - 5i + 10$

$= 1+2i-1-5-3i+10$

$= -3i+5$

$= 5-3i$

(ii) $\lim_{z\to 0} \frac{z}{\bar{z}}$

we know $z = x+iy$

$\bar{z} = x - iy$

$\lim_{(x,y)\to(0,0)} \frac{x+iy}{x-iy}$

suppose, $(x,y) \to (0,0)$ along the x axis, then $y=0, x\to 0$

So, the required limit.

$\lim_{x\to 0} \frac{x}{x}$

$= 1$

suppose $(x,y)\to(0,0)$ along the y axis so then

$x=0$, $y\to0$ so the required limit

$$\lim_{y\to0}\frac{iy}{-iy}$$

$= -1$

which is not same as above. so the limit

doesn't exist

b. $f(z)=(x^3-3xy^2)+i(y^3-3x^2y)$

~~limit~~ for checking continuity at first

limit has to be checked

$$\therefore \lim_{z\to0}(x^3-3xy^2)+i(y^3-3x^2y)$$

Here, $z=x+iy$.

$z\to0$ means $(x,y)\to(0,0)$

suppose $(x,y)\to(0,0)$ along the x axis. then

$y=0$ and $x\to0$, so the required limit.

$$\lim_{x\to0}x^3 = 0$$

suppose, $(x,y) \to (0,0)$ along the y axis, then $x \to$ and $y \to 0$ so the required limit.

$$\lim_{x\to 0} \lim_{y\to 0} i y^3$$

$= 0$

which is same as above.

Along the, $y = mx$ line

$$\lim_{x\to 0} (x^3 - 3x m^2 x^2) + i(m^3 x^3 - 3x^2 mx)$$

$$= \lim_{x\to 0} (x^3 - 3m^2 x^3) + i(m^3 x^3 - 3x^3 m)$$

$= 0$

which is also same as above two.

so the limit exists

at, $z = 0$, $f(0) = 0$.

which is also same as limiting value

so the function is continuous.

Differentiability

$$\lim_{z\to 0} \frac{f(z)-f(0)}{z-0}$$

~~f(0)=~~

$$\lim_{z\to 0} \frac{(x^3-3xy^2)+i(y^3-3x^2y) - 0}{x+iy}$$

~~suppose $(x,y)\to(0,0)$ along the x axis, then~~ $y=0$

~~$x\to 0$, so the required~~ limit

$$\cancel{\lim_{x\to 0} \frac{x^3}{x}}$$

$$\cancel{=\lim_{x\to 0} x^2}$$

$\cancel{=0}$

suppose ~~to $(x,y)\to(0,0)$ along the~~

$$\lim_{z\to 0} \frac{(x-iy)^3}{z}$$

$\Big[\ z=x+iy$

$\bar{z}=x-iy$

$(x-iy)^3 = x^3-3x^2iy+3x(iy)^2-(iy)^3$

$(x-iy)^3 = (x^3-3xy^2)+i(y^3-3x^2y)\ \Big]$

$$\lim_{z\to 0} \frac{(\bar{z})^3}{z};$$

$= \frac{(\bar{z})^3}{0}$

$=$ infinity; so at $z=0$; the function is not differentiable

$f(z) = (x^3 - 3xy^2) + i\,(y^3 - 3x^2y)$

$f(z) = u + iv$

$u = x^3 - 3xy^2$ $\qquad$ $v = y^3 - 3x^2y$

$\frac{\partial u}{\partial x} = 3x^2 - 3y^2$ $\qquad$ $\frac{\partial u}{\partial y} = \cancel{-6xy}\ -6xy$

at $(0,0)$ $\frac{\partial u}{\partial x} = 0$ $\qquad$ at $(0,0)$ $\frac{\partial u}{\partial y} = 0$

$\frac{\partial v}{\partial x} = -6xy$ $\qquad$ $\frac{\partial v}{\partial y} = 3y^2 - 3x^2$

at $(0,0)$ $\frac{\partial u}{\partial x} = 0$ $\qquad$ at $(0,0)$ $\frac{\partial u}{\partial y} = 0$

According to C-R equation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\& \ \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

So, C-R equation is satisfied.

c. Let, $u = 3x^2y + 2x^2 - y^3 - 2y^2$

$\frac{\partial u}{\partial x} = 6xy + 4x$ $\quad \ldots\ldots$ (1)

$\frac{\partial^2 u}{\partial x^2} = 6y + 4$

$\frac{\partial u}{\partial y} = 3x^2 - 3y^2 - 4y$ $\quad \ldots\ldots$ (2)

$\frac{\partial^2 u}{\partial y^2} = -6y - 4$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6y+4-6y-4=0$$

so it satisfies laplace equation. so the function is harmonic

Let v be the harmonic conjugate of u

According to C-R equation.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{———} \quad (1)$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad \text{———} \quad (ii)$$

$$f(z) = u(x,y) + i v(x,y) \quad \text{———} \quad (iii)$$

Differentiating (iii) partially w.r.t x, we get

$$f'(z) = \frac{\partial u(x,y)}{\partial x} + i \frac{\partial v(x,y)}{\partial x}$$

$$f'(z) = \frac{\partial u(x,y)}{\partial x} - i \frac{\partial u(x,y)}{\partial y}$$

$$f'(x+iy) = U_1(x,y) - i U_2(x,y) \quad --- \quad (iv)$$

According to millons theorem

put $x=z$, $y=0$ in (iv), we get.

$$f(z) = U_1(z,0) - i U_2(z,0) \quad --- \quad (v)$$

put $x=z$, and $y=0$ in ① we get

$$\frac{\partial u}{\partial x} = U_1(z,0) = 4z$$

put $x=z$ and $y=0$ in ② we get

$$\frac{\partial u}{\partial y} = U_2(z,0) = 3z^2$$

From ⑤ we get

$$f'(z) = 4z - i3z^2$$

Now integrating we get.

$$f(z) = \cancel{\frac{4z^2}{2}}\ 2z^2 - iz^3 + ic$$

$$f(x+iy) = 2(x+iy)^2 - i(x+iy)^3 + ic$$

~~f(x+iy)~~

$$f(z) = 2(x^2+2xiy-y^2) - i(x^3+3x^2iy+3xi^2y^2+i^3y^3)+ic$$

$$f(z) = 2x^2 + 4xy.i - 2y^2 - x^3i + 3x^2y + 3xy^2i - y^3 + ic$$

$$f(z) = (3x^2y + 2x^2 - 2y^2 - y^3) + i(4xy - x^3 + 3xy^2 + c)$$

$$u+iv = (3x^2y + 2x^2 - 2y^2 - y^3) + i(4xy - x^3 + 3xy^2 + c)$$

equating the real and imaginary part we get

$$u = 3x^2y + 2x^2 - 2y^2 - y^3$$

$$v = 4xy - x^3 + 3xy^2 + c$$

2 ~~a i, ii, iii, iv~~

2. a. v. Essential singularity

An isolated singularity that is not a pole or removable singularity is called an essential singularity

~~f(z~~ Example:

$f(z) = e^{1/z-2}$; has an essential singularity at $z=2$

~~If a function has an isolated singularity, the~~

b. $\oint_C \frac{e^{3z}}{z-\pi i}\, dz$ ~~with~~

again. $|z-2|+|z+2|=6$

$|x+iy-2|+|x+iy+2|=6$

$\sqrt{(x-2)^2+y^2}+\sqrt{(x+2)^2+y^2}=6$

$\sqrt{(x-2)^2+y^2} = 6-\sqrt{(x+2)^2+y^2}$

$(x-2)^2+\cancel{y^2} = 36-12\sqrt{(x+2)^2+y^2}+(x+2)^2+\cancel{y^2}$

$\cancel{(x-2)^2}\ 12\sqrt{(x+2)^2+y^2}-36=(x+2)^2-(x-2)^2$

$12\sqrt{(x+2)^2+y^2}-36=8x$

$8x+36 = 12\sqrt{(x+2)^2+y^2}$

$2x+9 = 3\sqrt{(x+2)^2+y^2}$

$4x^2+36x+81 = 9(x+2)^2+9y^2$

$4x^2+\cancel{36x}+81 = 9x^2+\cancel{36x}+36+9y^2$

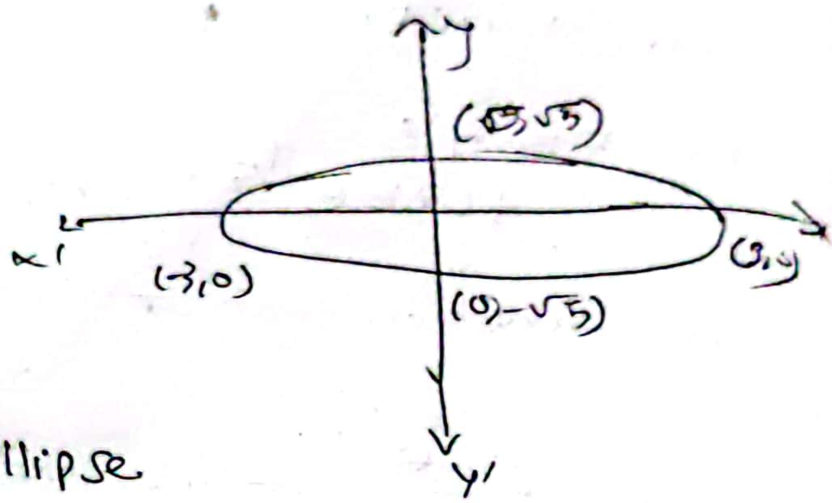
$5x^2+9y^2 = 81-36$

$5x^2+9y^2 = 45$

$\frac{x^2}{9}+\frac{y^2}{5} = 1$

$\frac{x^2}{3^2}+\frac{y^2}{(\sqrt{5})^2} = 1$

It's the eqn of ellipse



put

~~Here, $f(z) = e^{3z}$~~ $z-\pi i = 0$,

$z = \pi i$

~~$\therefore z_0 = \pi i$ Here, $x=0$, $y=\pi$~~ $(x=0, y=\pi$

Here, $\pi > \sqrt{5}$. which is outside the circle

So, According to cauchys theorem,

~~$\oint_c \frac{f(z)}{z-a}\,dz$~~ $\oint_c f(z)\,dz = 0$

c. $\oint_C \frac{e^{iz}}{z^3} dz \qquad |z|=2$

At $z^3=0$

$z=0$ is the singularity.

$$\oint_C \frac{e^{iz}}{z^3} dz = \oint_{C_1} \frac{e^{iz}}{z^3} dz$$

$$= \oint_{C_1} \frac{f(z)}{z^3} dz$$

where, $f(z) = e^{iz}$

$$= \oint_{C_1} \frac{f(z)}{(z-0)^3} dz$$

here, $a=0$

$\cancel{\oint_C \frac{e^{iz}}{z^3} = \frac{2\pi i}{2!} f}$

According to Cauchy's Integral formula

$$\boxed{f^n(a) = \frac{n!}{2\pi i} \oint_C \frac{f(z)}{(z-a)^{n+1}} dz}$$

$$= \frac{2\pi i \, f''(0)}{2!}$$

$$= \pi i \, f''(0)$$

$$= \pi i \cdot (-1)$$

$$= -\pi i$$

$f(z) = e^{iz}$

$f'(z) = i e^{iz}$

$f''(z) = i^2 e^{iz}$

$f''(0) = -e^{i0}$

$f''(0) = -1$

3.a. Linearly dependent:

Let $V(F)$ be a vector space a finite set $\{\alpha_1, \alpha_2, \ldots \alpha_n\}$ of vectors of V is said to be linearly dependent if there exists scalar $\{a_1, a_2, \ldots a_n \in F\}$ not all of them 0 (some of them may be 0) such that $a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 + \cdots + a_n\alpha_n = 0$.

Linearly independent:

Let $V(F)$ be a vector space. a finite set $\{\alpha_1, \alpha_2, \ldots \alpha_n\}$ of vectors of V is said to be linearly independent if every relation of the form

$a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 + \cdots + a_n\alpha_n = 0$

$a_i \in F$, $1 \leq i \leq n \Rightarrow a_i = 0$ for each $1 \leq i \leq n$

$\underline{A} = 2\underline{i} + \underline{j} - 3\underline{k}$

$\underline{B} = \underline{i} - 4\underline{k}$

$\underline{C} = 4\underline{i} + 3\underline{j} - \underline{k}$

Taking ~~vectors in matrix form~~

Determinant

$$\begin{vmatrix} 2 & 1 & -3 \\ 1 & 0 & -4 \\ 4 & 3 & -1 \end{vmatrix}$$

$= 2(0+12) - 1(-1+16) - 3(3-0)$

$= 24 + 15 - 9$

$= 30 \neq 0$

so, vectors are linearly independent.

b $\varphi(x,y,z) = z - x^2 - y^2$ ———— (1)

The Normal to the surface (1) is

$\underline{N} =$ grad φ

$= \underline{\nabla}\varphi$

$= \frac{\partial \varphi}{\partial x}\underline{i} + \frac{\partial \varphi}{\partial y}\underline{j} + \frac{\partial \varphi}{\partial z}\underline{k}$

$= -2x\hat{i} - 2y\underline{j} + \underline{k}$

at $(2,-1,5)$. $\underline{N} = \nabla \phi = -4\underline{i} + 2\underline{j} + \underline{k}$.

The eqn of the tangent plane passing through a point ~~whose~~ $(2,-1,5)$ whose position vector is $\underline{r}_0 = 2\underline{i} - \underline{j} + 5\underline{k}$ and perpendicular to the normal $\underline{N}$ is $(\underline{r} - \underline{r}_0) \cdot \underline{N} = 0$

$$\left\{ (x\underline{i} + y\underline{j} + z\underline{k}) - (2\underline{i} - \underline{j} + 5\underline{k}) \right\} (-4\underline{i} + 2\underline{j} + \underline{k}) = 0$$

$$\left\{ (x-2)\underline{i} + (y+1)\underline{j} + (z-5)\underline{k} \right\} (-4\underline{i} + 2\underline{j} + \underline{k}) = 0$$

$$-4(x-2) + 2(y+1) + (z-5) = 0$$

~~The eq~~ $\frac{\partial \phi}{\partial x} = -2x \qquad \frac{\partial \phi}{\partial y} = -2y \qquad \frac{\partial \phi}{\partial z} = 1$

~~$\partial \phi$~~ at $(2,-1,5)$

$$\frac{\partial \phi}{\partial x} = -4 \qquad \frac{\partial \phi}{\partial y} = 2 \qquad \frac{\partial \phi}{\partial z} = 1.$$

$\therefore$ Normal line

$$\frac{x-2}{-4} = \frac{y+1}{2} = \frac{z-5}{1}$$

Let assume, $\bar{r} = xi + yj + zk$

Then $f(r)\,\underline{r} = f(r)(x\underline{i} + y\underline{j} + z\underline{k})$

$= f(r)\,x\,\underline{i} + f(r)\,y\,\underline{j} + f(r)\,z\,\underline{k}$

As, $f(r)\,\underline{r}$ is solenoid, then.

$\operatorname{div}[f(r).\underline{r}] = 0$

Therefore $\operatorname{div} f(r)\,r = \frac{\partial}{\partial x}[f(r).x] + \frac{\partial}{\partial y}[f(r)\,y] + \frac{\partial}{\partial z}[f(r)\,z]$

$\frac{\partial}{\partial x}[f(r)\,x] = f(r) + x f'(r).\frac{x}{r} = f(r) + \frac{x^2}{r} f'(r)$

$\frac{\partial}{\partial y}[f(r)\,y] = f(r) + y f'(r).\frac{y}{r} = f(r) + \frac{y^2}{r} f'(r)$

$\frac{\partial}{\partial z}[f(r)\,z] = f(r) + z f'(r).\,z/r = f(r) + \frac{z^2}{r} f'(r)$

put these values in above expression.

$\operatorname{div}[f(r).r] = 3f(r) + \frac{f'(r)}{r}(x^2+y^2+z^2)$

$= 3f(r) + \frac{f'(r)}{r}\,r^2$

$= 3f(r) + f'(r).r.$

So, $f(r)$ is solenoid,

$\operatorname{div}[f(r)\,r] = 0$

$3f(r) + f'(r).r = 0$

$\therefore f(r) = -\frac{r}{3} f'(r)$

$3f(r) = -r f'(r)$

$\cancel{3f = -rf'}$

$3f(r) = -r \cdot \frac{d f(r)}{dr}$

$-\frac{3 dr}{r} = \frac{df(r)}{f(r)}$

$\cancel{-3 \ln|r| = \ln|f(r)| + \ln|C|}$

$\ln|f(r)| = \cancel{\ln} -3\ln|r| + \ln|C|$

$\ln|f(r)| = \ln|r|^{-3} + \ln|C|$

$\ln|f(r)| = \ln|Cr^{-3}|$

$f(r) = \cancel{Cr} \; Cr^{-3}$

$f(r) = C/r^3$

where 'C' is an arbitrary const.

4. a. $\int_C^{\cancel{}} \bar{F} \, d\cdot\bar{r}$

We know $\bar{r} = x\underline{i} + y\underline{j} + z\underline{k}$

$d\bar{r} = dx\underline{i} + dy\underline{j} + dz\underline{k}$

again, $y = x^3$

$dy = 3x^2 dx$

$$\int_C F\,d\cdot r = \int_C \{(5xy-6x^2)\,\underline{i} + (2y-4x)\,\underline{j}\}\cdot(dx\,\underline{i} + dy\,\underline{j} + dz\,\underline{k})$$

$$= \int_C \left[(5xy-6x^2)\,dx + (2y-4x)\,dy\right]$$

$$= \int_1^2 (5x\cdot x^3 - 6x^2)\,dx \cancel{+ \int_1^8 (2y - 4\sqrt[3]{y})\,dy}$$

$$+ \int_1^2 (2x^3 - 4x)\cdot 3x^2\,dx$$

$$= \int_1^2 (5x^4 - 6x^2)\,dx + \int_1^2 (6x^5 - 12x^3)\,dx$$

$$= \cancel{5x}$$

$$= \left[x^5 - 2x^3\right]_1^2 + \left[x^6 - 3x^4\right]_1^2$$

$$= \left[2^5 - 2\cdot 2^3 + 2^6 - 3\cdot 2^4 - 1^5 + 2\cdot 1^3 - 1^6 + 3\cdot 1^4\right]$$

$$= 35$$

b. $\underline{A} = 2x^2y\,\underline{i} - y^2\,\underline{j} + 4xz^2\,\underline{k}$

$x^2 + y^2 + z^2 = 3^2$ and $x = 2$

$$\underline{\nabla} \cdot \underline{A} = \frac{\partial}{\partial x}(2x^2y) + \frac{\partial}{\partial y}(-y^2) + \frac{\partial}{\partial z}(4xz^2)$$

$$= 4xy - 2y + 8xz.$$

$r = 3.$

$$\int_{z=-3}^{3} \int_{y=-\sqrt{9-z^2}}^{\sqrt{9-z^2}} \int_{x=0}^{2} (4xy - 2y + 8xz)\, dx\, dy\, dz$$