

KHULNA UNIVERSITY OF ENGINEERING & TECHNOLOGY
B.Sc. Engineering 2nd Year 2nd Term Examination, 2019
Department of Computer Science and Engineering
CSE 2201

Algorithm Analysis and Design

TIME: 3 hours

FULL MARKS: 210

- N.B.** i) Answer **ANY THREE** questions from each section in separate scripts.
ii) Figures in the right margin indicate full marks.

SECTION A

(Answer **ANY THREE** questions from this section in Script A)

1. a) What are the differences between performance analysis and performance measurement of an algorithm. (07)
- b) Consider a 0/1 Knapsack problem. Explain the application of dynamic programming and greedy algorithm to find an optimal solution. Give the two separate repetition of problem space and compare the time complexity of the algorithm under the said paradigm. (11)
- c) "Is the minimum spanning tree generated using both Krushkal's and Prim's unique" Explain your answer if you say 'yes' or give a counter example if you say 'no'. (09)
- d) Write the control abstraction for greedy method. (08)

Question Solution - CSE 2201 (2019)

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1) If we have enough resources then to implement

a) Ans:

Performance analysis estimates space and time complexity in advance. We usually use asymptotic notations to do so.

There are mainly three asymptotic notations: Big-O notation, Omega notation and Theta notation. We consider the input value to be very large.

On the other hand, performance measurement measures the space and time taken in actual runs i.e. we count the space and time in runtime. For measuring time, a stopwatch can be used.

b) Ans:

In a greedy algorithm, we always tend to pick an object that will make our profit more than the other objects. As

this is a 0/1 knapsack problem, we can not take any fraction of an object. So in greedy method, at first we sort the objects in respect to their profit in descending order.

Then we pick the objects from the very front until the total weight of our knapsack becomes equal or less than the constraint weight.

If the value of x_{i+1} is 1 then we take the i^{th} object, otherwise i^{th} object has p_i profit and w_i weight. The capacity of knapsack is M .

In greedy method, we may find a feasible solution, but that will not be optimal solution.

In dynamic programming, we either take an object, or don't. And as we solve the same sub-problems again and again, we can store the solution to the subproblems in a 2D array $DP[n][w]$, where $n \rightarrow$ is the number of elements and w is the maximum capacity. The state $DP[i][j]$ will denote maximum value of ' j -weight' considering all values from '1 to i^{th} '.

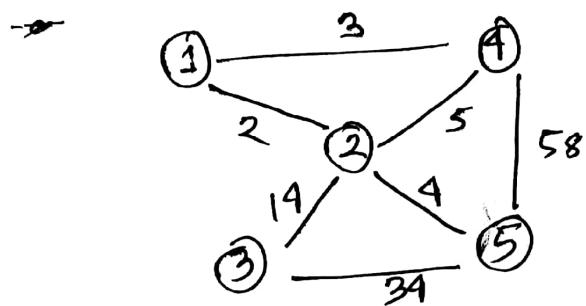
In greedy method, first we sort the array and then we take the objects. So, time complexity would be $O(n \log n)$, where n is the total objects.

In dynamic problem, we use recursion for n^{th} depth and return when we placed the maximum capacity. So, the time complexity would be $O(n \cdot w)$, where n is the number of objects and w is the capacity of the knapsack.

Q) Ans:

No, the minimum spanning tree generated using both Kruskal's and Prim's is not unique.

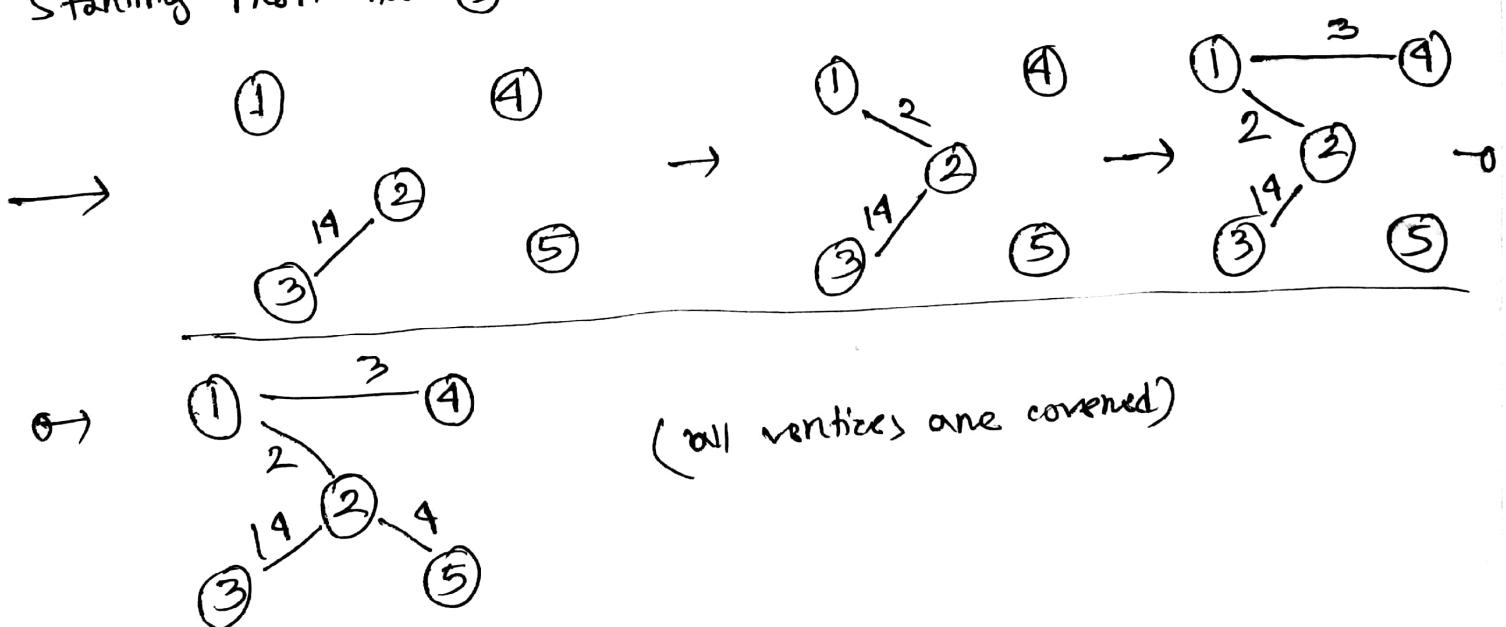
Example:



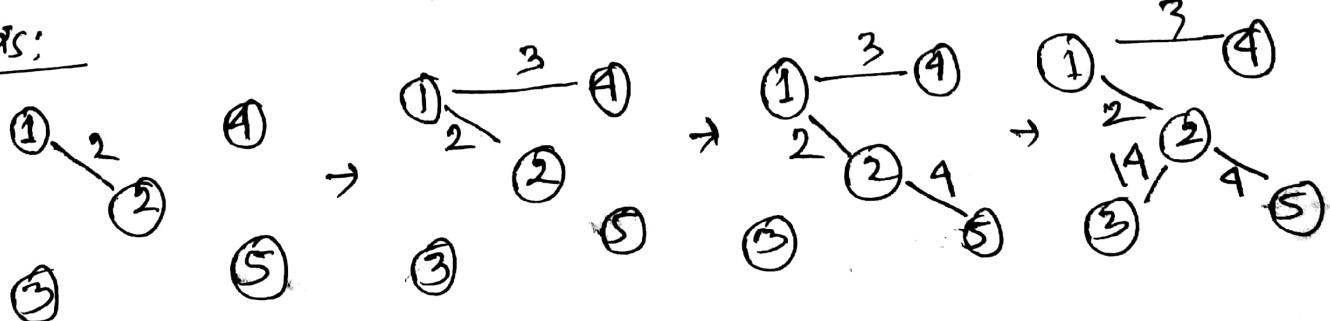
Generating the MST using Prim's and Kruskal's.

Prim's:

Starting from node (3)



Kruskal's:



As we can see, the MST's are the same, when the cost of edges are unique.

d) Ans:

Control Abstraction for greedy method:

Algorithm Greedy (a, n)

// $a[1:n]$ contains the n inputs

{

 solution := 0; // Initialize the solution

 for i := 1 to n do

 {

$x := \text{select}(a);$

 if Feasible (solution, x) then

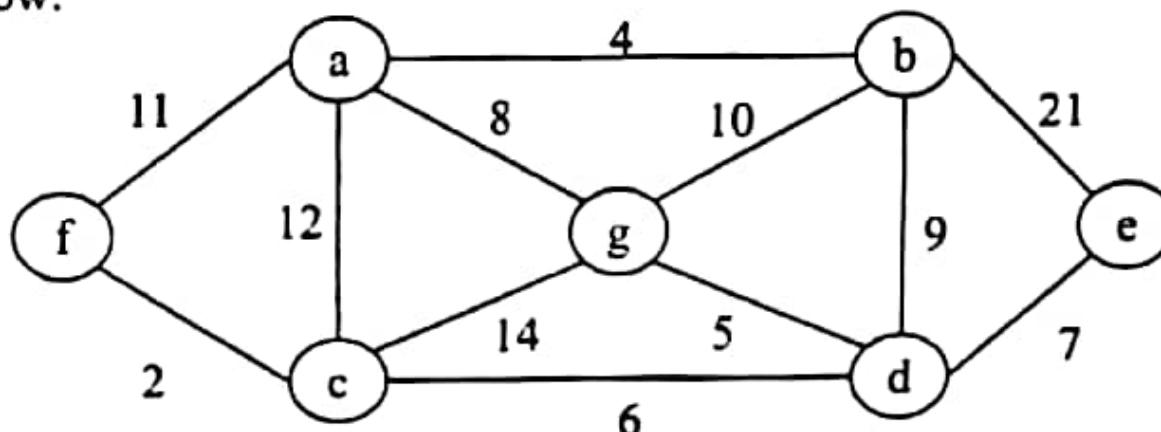
 solution := Union (solution, x);

 y

 return solution;

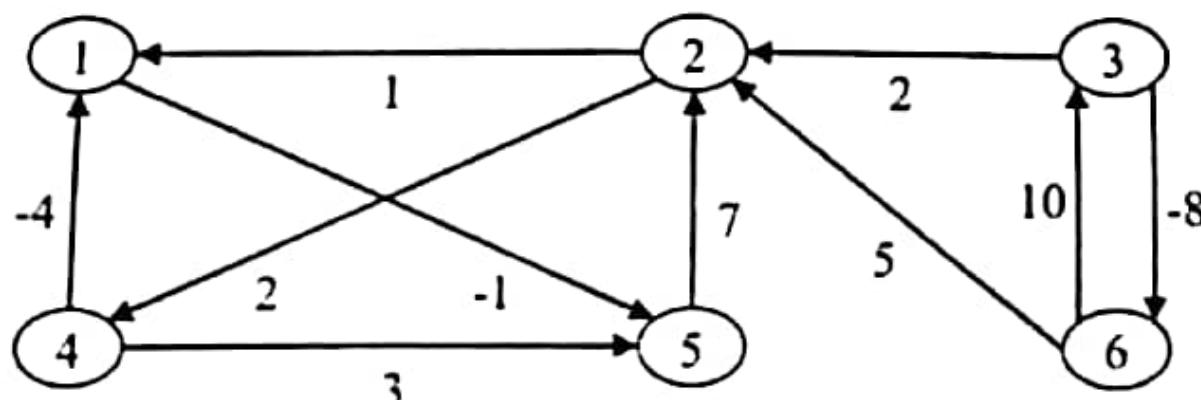
 y

2. a) Define BFS and DFS. What are the time complexity of BFS and DFS of a graph? (06)
 b) What is Spanning Tree? Use Prim's algorithm to determine the Minimum Cost Spanning Tree (10) of the graph below.



What is the total cost of the tree?

- c) Use example to distinguish between feasible solution and optimal solution for the case of (07) Knapsack problem
 d) Run the Floyd-Warshall algorithm on the weighted directed graph shown in the following (12) figure.



Show the Matrix (All pair) D^k that results for each iteration of the outer loop.

Q) What are the differences between BFS and DFS?

2



a) Ans:

BFS (Breadth First Search) is a vertex based technique for finding a shortest path in a graph. It uses a Queue data structure which follows first in first out. In BFS, one vertex is selected at a time when it is visited and then its adjacent are visited and stored in queue.

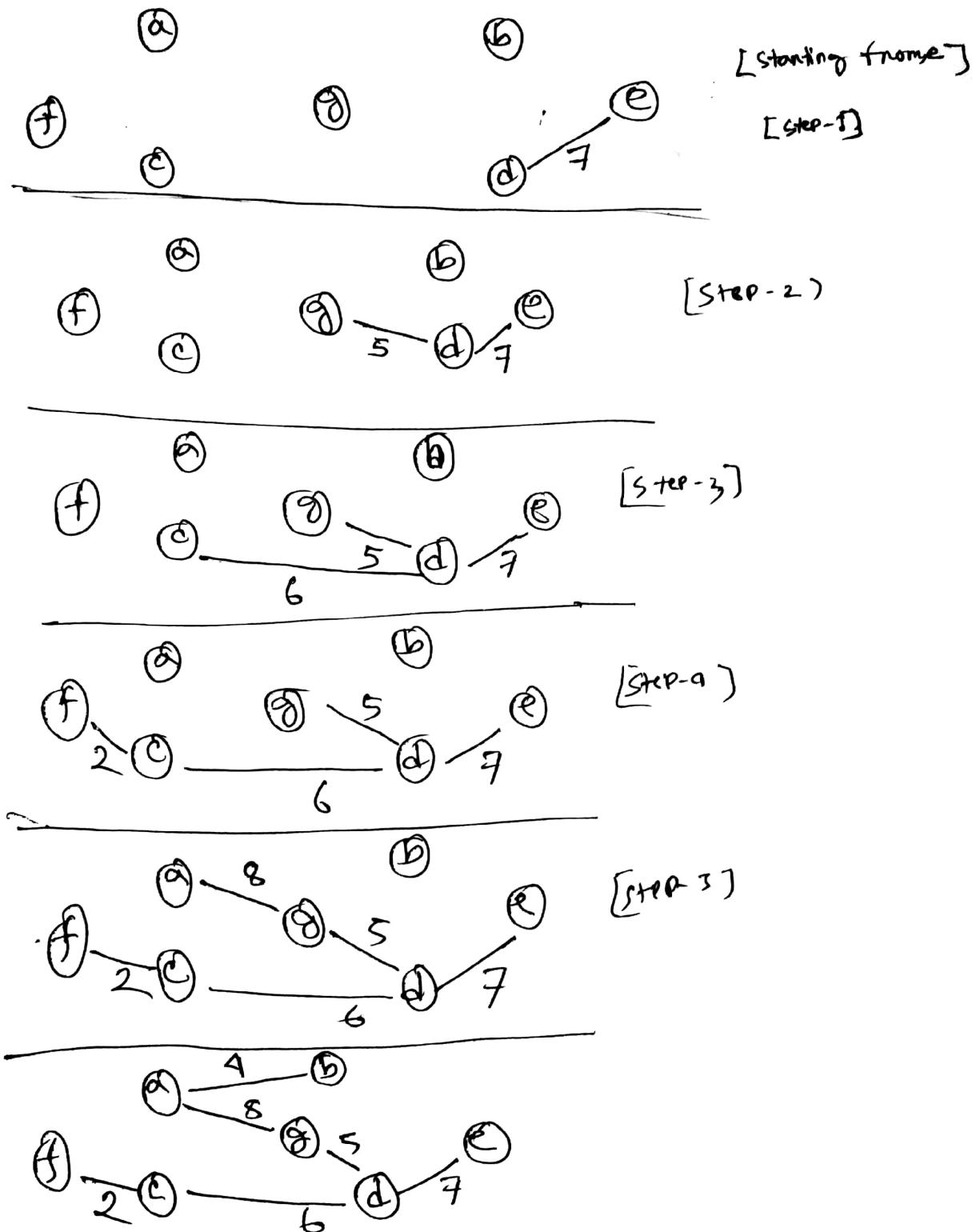
DFS (Depth First Search) is an edge based technique. It uses the Stack data structure, performs two stages, first visited vertices are pushed into stack and second if there is no vertices then visited vertices are popped.

Considering V is the number of vertices and E is the number of edges, the time complexity of both BFS and DFS is $O(V+E)$.

The main difference is to BFS we can visit all vertices from a single source vertex whereas in DFS we can visit all vertices from all vertices.

b) Ans:

A spanning tree is a subset of graph G₁, which has all the vertices covered with minimum possible number of edges. Hence, a spanning tree does not have cycles and it cannot be disconnected.



$$\therefore \text{the total cost of the tree} = 9 + 8 + 5 + 6 + 2 + 7 = 32$$

c) Ans?

Profit: P_1, P_2, \dots, P_n

Weight: w_1, w_2, \dots, w_n

Capacity: M

The goal is, maximize $\sum_{i=1}^n P_i x_i$, subject to $\sum_{j=1}^n w_j x_j \leq M$, $x_i = 0$ or 1 .

$$\text{or minimize } -\left(\sum_{i=1}^n P_i x_i\right) = Y$$

The upper bound can be calculated by quickly finding a feasible solution; starting from the smallest available i , scanning towards the largest i 's until M is exceeded.

Example:

$$n=6, M=34$$

i	1	2	3	4	5	6
P_i	6	10	4	5	6	4
w_i	10	19	8	10	12	8

(sorted by P_i/w_i ratio)
 $P_i/w_i \geq P_{i+1}/w_{i+1}$

A feasible solution: $x_1=1, x_2=1, x_3=0, x_4=0, x_5=0, x_6=0$

$$\therefore -(P_1 + P_2) = -16 \quad (\text{Upper bound})$$

Any solution higher than -16 can not be an optimal solution.

For fractional, $Y' = -\sum_{i=1}^n P_i x'_i$, where $0 \leq x'_i \leq 1$, and this is the optimal solution for knapsack problem.

$$\therefore Y' \leq Y \quad (\text{they are negative})$$

For fractional, $x_1=1, x_2=1, x_3=\frac{(34-10-19)}{8}=\frac{5}{8}, x_4=0, x_5=0, x_6=0$

$$\therefore -(P_1 + P_2 + \frac{5}{8} P_3) = -18.5 \quad (\text{Lower bound}) \approx -18 \quad (\text{only consider integer})$$

∴ So, the optimal solution will be between 16 and 18 .

d) Ans:

The corresponding matrix:

• bear with me guys

Node	1	2	3	4	5	6
1	0	∞	∞	∞	-1	∞
2	1	0	∞	2	∞	∞
3	∞	2	0	∞	∞	-8
4	-4	∞	∞	0	3	∞
5	∞	7	∞	∞	0	∞
6	∞	5	10	∞	∞	0

Outer loop will iterate for 6 times. Let k be current node of outer loop.

After $k=1$ complete,

Node	1	2	3	4	5	6
1	0	∞	∞	∞	-1	∞
2	1	0	∞	2	0	∞
3	∞	2	0	∞	∞	-8
4	-4	∞	∞	0	-5	∞
5	∞	7	∞	∞	0	∞
6	∞	5	10	∞	∞	0

After $k=2$ completion,

Node	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
1	0	∞	∞	∞	-1	∞	2	3	4	5	6	∞	2	3	4	5	6	
2	1	0	∞	2	0	∞	3	4	5	6	∞	2	3	4	5	6		
3	3	2	0	4	2	∞	1	2	3	4	5	3	2	3	4	5	6	
4	-4	∞	∞	0	-5	∞	0	1	2	3	4	5	19	0	1	2	3	
5	8	7	∞	9	0	∞	0	1	2	3	4	5	3	2	3	4	5	
6	6	5	10	7	3	0	0	1	2	3	4	5	2	3	4	5	6	

After $k=3$ completion,

Node	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
1	0	∞	2	∞	-1	∞	1	2	3	4	5	3	1	2	3	4	5	
2	1	0	∞	2	0	∞	2	3	4	5	6	∞	3	4	5	6	∞	
3	3	2	0	4	2	∞	1	2	3	4	5	3	2	3	4	5	6	
4	-4	2	2	0	-5	∞	0	1	2	3	4	5	19	0	1	2	3	
5	8	7	∞	9	0	∞	0	1	2	3	4	5	3	2	3	4	5	
6	6	2	10	7	5	0	0	1	2	3	4	5	2	3	4	5	6	

After $k=4$ completion,

Node	1	2	3	4	5	6
1	0	∞	∞	-1	∞	
2	-2	0	∞	2	3	∞
3	-1	2	0	4	-1	∞
4	-4	∞	∞	0	-5	∞
5	5	7	∞	9	0	∞
6	3	2	10	7	2	0

After $k=5$ convolution,

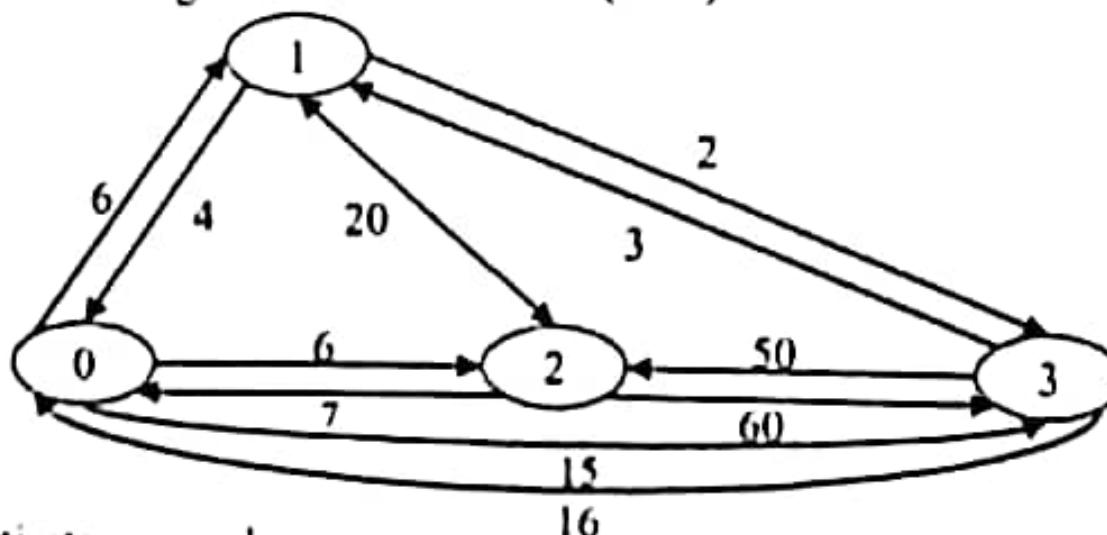
Node	1	2	3	4	5	6
1	0	6	∞	8	-1	∞
2	-2	0	∞	2	-3	∞
3	0	2	0	4	-1	-8
4	1	2	∞	0	-5	∞
5	5	7	∞	9	0	∞
6	3	2	10	7	2	0

After $k=6$ convolution,

Node	1	2	3	4	5	6
1	0	6	∞	8	-1	∞
2	-2	0	∞	2	-3	∞
3	-5	-6	0	-1	-6	-8
4	-4	2	∞	0	-5	∞
5	5	7	∞	9	0	∞
6	3	2	10	7	2	0

This is the final Matrix (All pair).

3. a) Define Implicit and Explicit constraints. Write Implicit and Explicit constraints for n-queens (06) and Sum of Subset problems.
b) Consider the following Travelling Salesman Problem (TSP): (14)



- i) Convert Multi-stage graph.
ii) Find the minimum cost path in the Multi-stage graph (from (i)). Do this using the forward reasoning approach.
- c) Apply backtracking technique to solve the following instance of Subset Sum problem: (10)
 $S = \{1, 3, 4, 5, 8\}$ and $d = 16$.
- d) What are the differences between Branch-and-Bound and back tracking paradigm? (05)

3

a) Ans.

Implicit Constraints: The implicit constraints are rules that determine which of the tuples in the solution space of I satisfy the criterion function. Thus implicit constraints describe the way in which the x_i must relate to each other.

Explicit Constraints: Explicit constraints are rules that restrict each x_i to take on values from a given set

N-Queens:

Let us number the rows and columns of the chessboard 1 through N . The queens can also be numbered 1 through N . Since each queen must be on a different row, we can, without loss of generality, assume queen i to be placed on row i . All solutions to the N -queens problem can therefore be represented as N -tuples (x_1, x_2, \dots, x_N) , where x_i is the column on which queen i is placed.

The explicit constraints using this formulation are $x_i \in \{1, 2, 3, \dots, N\}$, $1 \leq i \leq N$.

The implicit constraints for this problem are that no two x_i 's can be the same (i.e. all queens must be on different columns) and no two queens can be on the same diagonal.

Sum of subset:

Given positive numbers m_i , $1 \leq i \leq n$, and m , which is the sum of subset.

The explicit constraints require $x_i \in \{j \mid j \text{ is an integer and } 1 \leq j \leq n\}$.

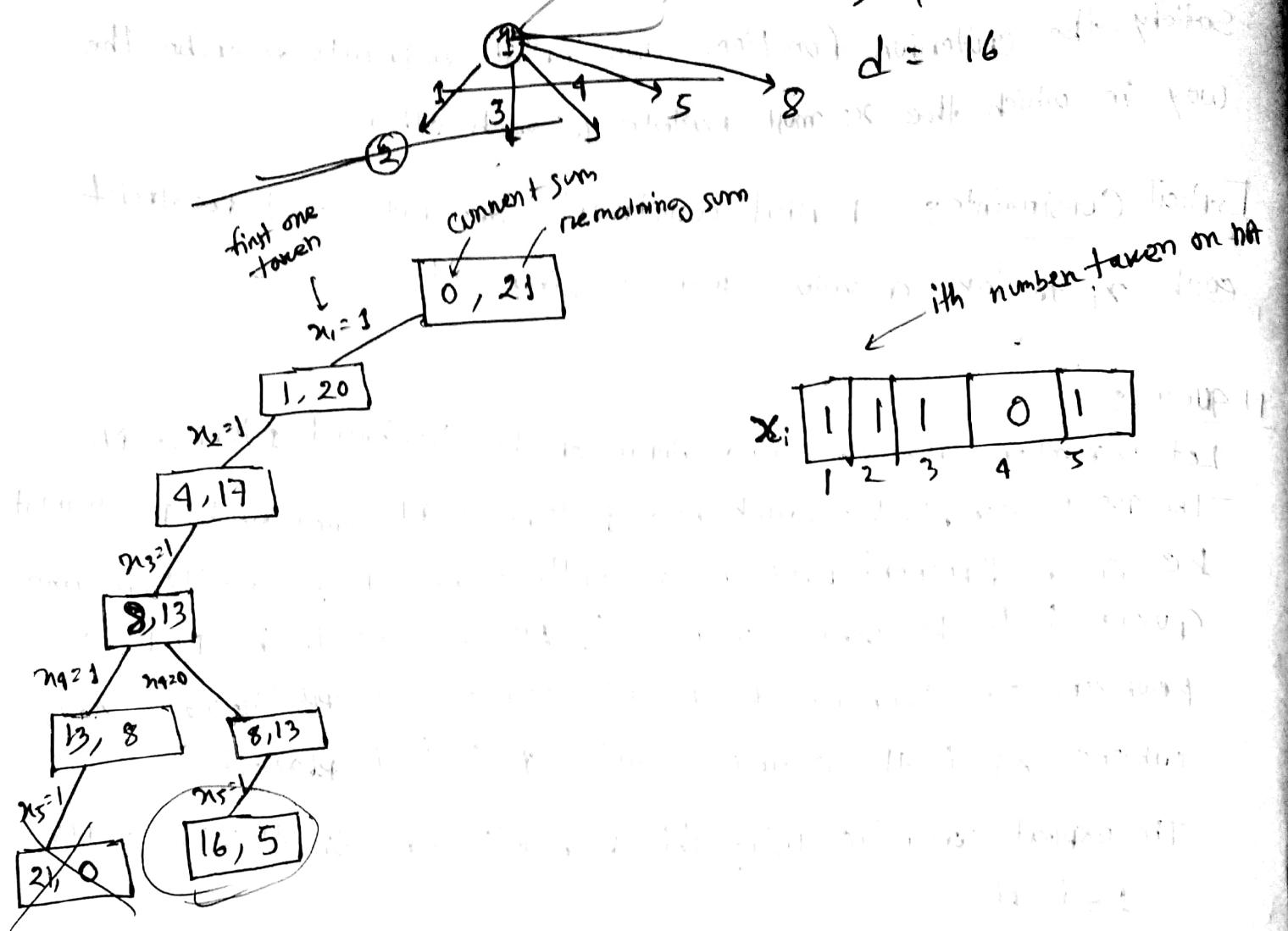
The implicit constraints require that no two be the same and that the sum of the corresponding m_i 's be m . Another implicit constraint, $x_i < x_{i+1}$, $1 \leq i < k$ where x_i is the indice.

b) Ans:

* সাধা চুম্বি কুঁজে Solution পাইনা *

c) Ans:

function ~~call~~ $S \geq \{1, 3, 4, 5, 8\}$



stems to the soft bottom, or have a little more sand on them so they don't
float. The best way to do this is to lay them down on the ground and
then roll them over and over until the sand adheres uniformly. This
way each stem has sand on it and it will not blow away.

d) Ans:

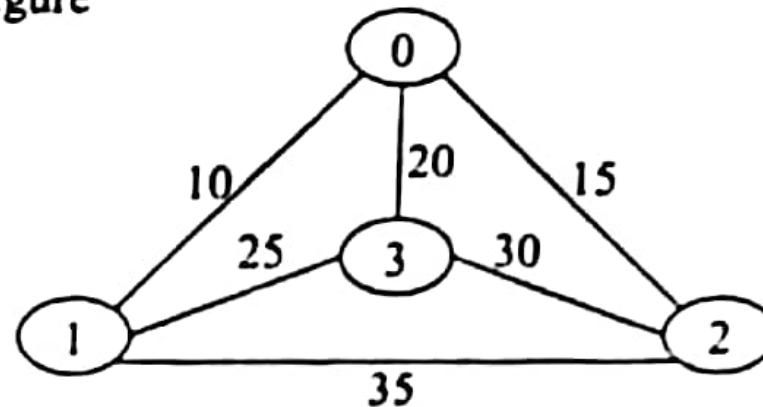
Branch-and-Bound is used to solve optimization problems. When it realises that it already has a better optimal solution than the pre-solution leads to, it abandons that pre-solution. It completely searches the state space tree to get optimal solution. It traverses the tree in any manner, DFS or BFS.

Backtracking is used for solving Decision Problem. It is used to find all possible solutions available to a problem. When it realises that it made a bad choice, it undoes the last choice by backing it up. It searches the state space tree until it has found a solution for the problem. It uses DFS to traverse the state space tree.

4. a) Draw the State-space tree for 4-queen problem. How is the solution reduced from 4^4 to optimal solution? (09)
b) Consider the following table: (13)

I	1	2	3	4	5
P _I	0.24	0.22	0.23	0.3	0.01

- Construct OBST for keys with the given probabilities.
c) Consider the following figure (13)



Find the optimal solution for TSP using Branch-and-Bound technique.

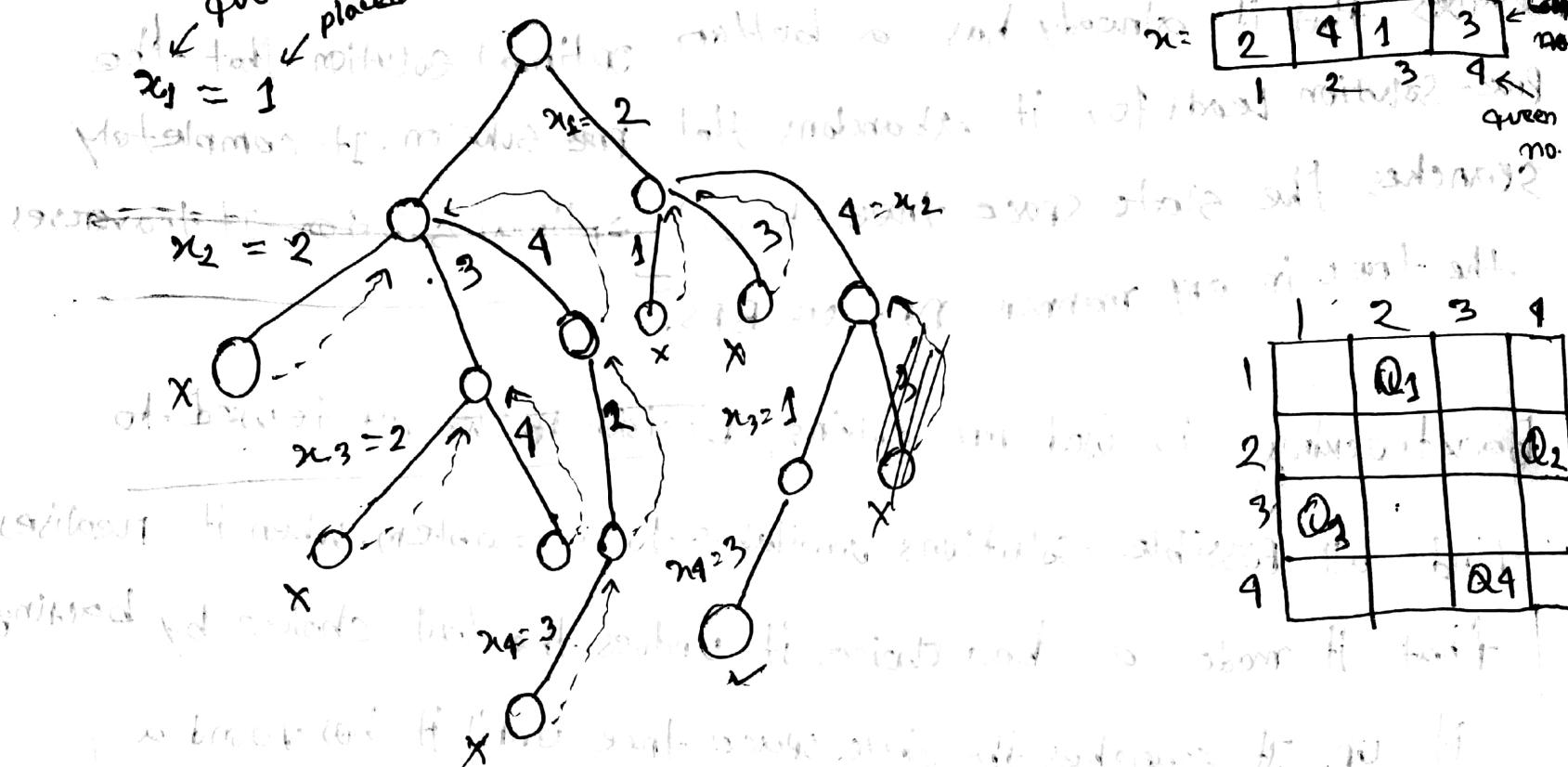
4

82m

a) Ans:

A resulting sequence of stones is shown at the bottom of board below. A green stone is placed in 1st column.

① $x_1 = 1$
tot no. of stones in 1st column



2	4	1	3
2	3	4	4
1	2	3	2
2	3	4	1

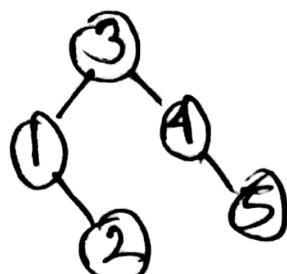
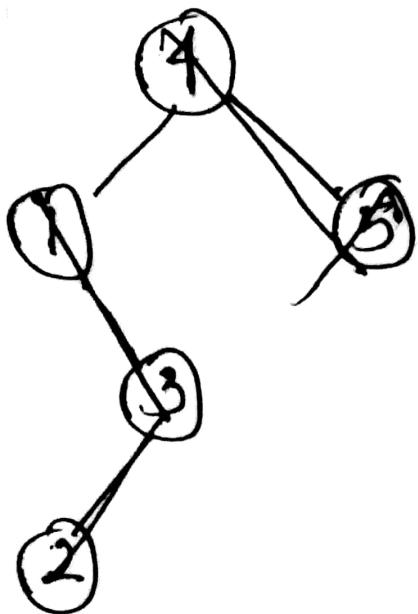
Column
no.
no.
no.
no.
green
no.

1	2	3	4
1	Q ₁		
2		Q ₂	
3	Q ₃		
4			Q ₄

Using backtracking, the solution is reduced from 4⁴ to optimal solution.

start from step 2

b) Ans:



This is the optimal binary search tree.

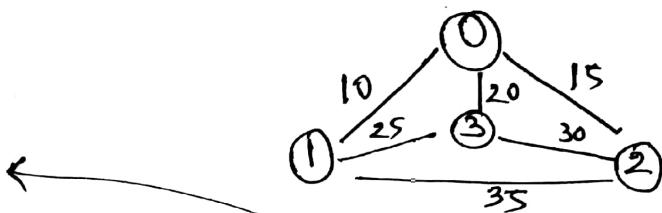
(Watch Abdul Bari's video for understanding)

i	1	2	3	4	5
p _i	0.24	0.22	0.23	0.3	0.01

j	0	1	2	3	4	5
0	0	0.24	0.48	1.36	1.9	2.88
1		0	0.22	0.67	1.42	1.99
2			0	0.23	0.76	1.1
3				0	0.3	0.31
4					0	0.01
5						0

(C) Ans.

	0	1	2	3	
0	0	5	10	∞	10
1	0	∞	25	15	10
2	0	20	∞	15	15
3	0	5	10	∞	20
					55



	0	1	2	3	
0	∞	10	15	20	10
1	10	∞	35	25	0
2	25	35	∞	30	15
3	20	25	30	∞	20

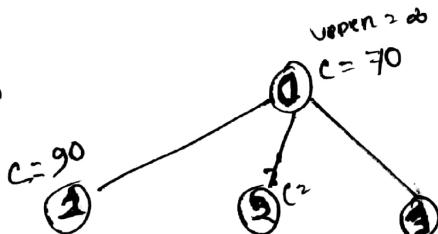
	0	1	2	3	
0	0	0	0	0	10
1	0	∞	20	5	10
2	0	20	∞	5	15
3	0	5	5	∞	20
	0	0	5	10	$15+55=70$

← This is reduced matrix

← reduced cost

← atleast the cost will be 70

12.6



$$C(0,1) + y_{1,2} \\ 10 + 70 + 10 = 90$$

[I'm sorry, I couldn't do it ~~any~~ further one]

Watch Abdul Barri's video if you want to solve it. (Travelling Salesman Problem - Branch and Bound)

(The solution may be: 0 → 1 → 3 → 2 → 0, cost = 80)

5. a) What do you mean by algorithm? Write down the basic characteristics of an algorithm. (10)
b) Define running time of an algorithm. Discuss some running time functions of an algorithm. (10)
c) Consider the code segments given in the following figure and calculate time frequency of each (15) of the segments.

i) `for (i=0; i<n; i++)
{
 for (j=0; j<n; j=j+2)
 (stmt);
}`

ii) `p = 0;
for (i=1; i<n; i=i*2)
 (p++);
for (j=1; j<p; j=j*2)
 (stmt);`

iii) `i = 1; k = 1;
while (k < n){
 stmt;
 k = k+1;
 i++;}`

5

a) Ans:

An algorithm is a set of instructions for solving a problem or accomplishing a task.

① The basic characteristics of an algorithm:

Input: An algorithm should have 0 or more well-defined inputs.

Output: An algorithm should have at least 1 output

Definiteness: Each instruction should be clear.

Finiteness: Algorithm must terminate after a finite number of steps.

Effectiveness: Instruction must be basic to carry out principle.

b) Ans:

The running time of an algorithm is the length of time taken to run the program on some standard computer.

Θ -notation:

For a given function $g(n)$, we denote $\Theta(g(n))$ the set of functions,

$\Theta(g(n)) = \{f(n) : \text{there exist positive constants } c_1, c_2 \text{ and } n_0 \text{ such that}$

$c_1 g(n) \leq f(n) \leq c_2 g(n) \text{ for all } n \geq n_0\}$

O -notation:

For a given function $g(n)$, we denote $O(g(n))$ the set of functions,

$O(g(n)) = \{f(n) : \text{there exist a positive constants } C \text{ and } n_0 \text{ such that}$

$0 \leq f(n) \leq C g(n) \text{ for all } n \geq n_0\}$

Ω -notation:

$\Omega(g(n)) = \{f(n) : \text{there exists two constants } C \text{ and } n_0, 0 \leq c g(n) \leq f(n) \text{ for all } n \geq n_0\}$

C) Ans:

i) $\text{for } i=0; i < n; i++ \longrightarrow n+1$

$\text{for } j=0; j < n; j=j+2 \longrightarrow n \times \infty$

stmt; $\longrightarrow n \times \infty$

∴

$$T(n) = n+1 + n \times \infty + n \infty$$

ii) $P=0; \longrightarrow 1$

$\text{for } (i=1; i < n; i*2) \rightarrow \infty \text{ (infinity)}$

$\begin{cases} P++; \\ \end{cases} \rightarrow \infty$

$\text{for } (j=1; j < P; j=j*2) \rightarrow \infty$

$\begin{cases} \text{stmt}; \\ \end{cases} \rightarrow \infty$

$$T(n) = 4 \infty$$

iii) $i=1; \longrightarrow 1$

$k=1; \longrightarrow 1$

(assume $k > n$)

$\text{while } (k < n) \longrightarrow \cancel{k}$

$\begin{cases} \text{stmt}, \\ \end{cases}$

$k=k+1; \longrightarrow k$

$i++; \longrightarrow k$

↳

$\begin{array}{c} i \quad k \\ \hline 1 & 1+1=2 \\ 2 & 1+1+2=4 \\ 3 & 1+1+2+2=8 \\ \vdots & \vdots \\ p & 1+1+2+\dots+p \end{array}$

$\therefore k = 1 + \frac{p(p+1)}{2}$

$$\therefore T(n) = 4k + 2$$

$$\geq 4 \cdot \left(1 + \frac{p(p+1)}{2}\right)$$

$$\geq 4 + 2p^2 + 2p$$

$$\therefore 1 + \frac{p(p+1)}{2} > n$$

$$\therefore 1 + \frac{p^2+p}{2} > n$$

$$\therefore p^2 + p > n \Rightarrow p > \sqrt{n}$$

$$\therefore O(\sqrt{n})$$

6. a) What do you mean by recurrence? What are the methods to solve the recurrence? Solve the (10) following recurrences:

i) $T(n) = 2T(\sqrt{n}) + \log(n)$

ii) $T(n) = T(n-a) + T(a) + n$

b) Give the best Big-Oh characterization for each of the following running time estimates (where (15) n is the size of the input problem)

i) $(n+1)^3$ ii) $\sum_{i=1}^n i$ iii) $n!$ iv) $6n^3 / (\log n + 1)$ v) $6 \cdot 2^n + n^2$

c) Define algebraic simplification. Explain the principal of algebraic simplification using dense (10) polynomial representation.

a) Ans:

A recurrence is an equation or inequality that describes a function in terms of its value on smaller inputs.

There are four methods for solving recurrence:

1. Substitution Method

2. Iteration method

3. Recursion Tree Method

4. Master Method

$$i) T(n) = 2T(\sqrt{n}) + \log n \quad (1)$$

$$T(n) = 2[2T(\sqrt{\sqrt{n}}) + \log(\sqrt{n})] + \log n$$

$$T(n) = 4T(n^{1/4}) + 2\log(n) + \log n \quad (2)$$

$$T(n) = 4[2T(n^{1/8}) + \log(n^{1/4})] + 2\log(n^{1/2}) + \log n$$

$$T(n) = 8T(n^{1/8}) + 4\log(n^{1/4}) + 2\log(n^{1/2}) + \log n \quad (3)$$

$$T(n) = k \cdot T(n^{1/k}) + \log n + 2\log(n^{1/2}) + \dots + 2^{k-1} \log(n^{1/n})$$

\therefore Assume $T(n) = 1$ when $n=0$

$$n^{1/k} = 1 \Rightarrow \log n^{\frac{1}{k}} = \log 1 \Rightarrow \frac{\log n}{\log 1} = k$$

~~$$T(n) = 2^n T(n^{1/n}) + \log n + 2\log(n^{1/2}) + \dots + 2^{n-1} \log(n^{1/n})$$~~

[I don't know what to do anymore.]

b) Ans:

i) $(n+1)^3$

Definition of Big-oh suggests

$$f(n) \leq c \cdot g(n)$$

where $O(g(n))$ would be the Big-oh characterization of the function $f(n)$.

$$n^3 + 3n^2 + 3n + 1 \leq n^3 + 3n^3 + 3n^3 + 1$$

$$\Rightarrow n^3 + 3n^2 + 3n + 1 \leq n^3 (1+3+3+1)$$

$$\Rightarrow n^3 + 3n^2 + 3n + 1 \leq \frac{8n^3}{c} \cdot g(n)$$

thus, if $f(n) = O(g(n)) = O(n^3)$

ii) $\sum_{i=1}^n i = 1+2+3+\dots+(n-1)+n$

$$\text{Hence, } \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

Hence,

$$f(n) \leq c \cdot g(n)$$

$$\Rightarrow 1+2+\dots+(n-1)+n \leq n+n+\dots+n$$

$$\Rightarrow \frac{n(n+1)}{2} \leq n^2$$

$$\Rightarrow \frac{n^2}{2} + \frac{n}{2} \leq n^2 + n^2$$

$$\Rightarrow \frac{n^2}{2} + \frac{n}{2} \leq \frac{2}{c} \cdot \frac{n^2}{g(n)}$$

thus, if $f(n) = O(g(n)) = O(n^2)$

thus, we have to prove that n^2 is bounded by $c \cdot g(n)$

which can be done by induction.

iii) $n!$

iv) $6n^3 / (\log n + 1)$

v) $6 \cdot 2^n + n^2$

$$f(n) \leq c \cdot g(n)$$

Diff in complexity depends on what terms cancel

$$6 \cdot 2^n + n^2 \leq 6 \cdot 2^n + 2^n$$

$$\Rightarrow 6 \cdot 2^n + n^2 \leq 2^n(6+1)$$

$$\therefore f(n) = O(2^n)$$

c) Any:

Algebraic Simplification is the process of writing an expression in the most efficient and compact form without affecting the value of the original expression.

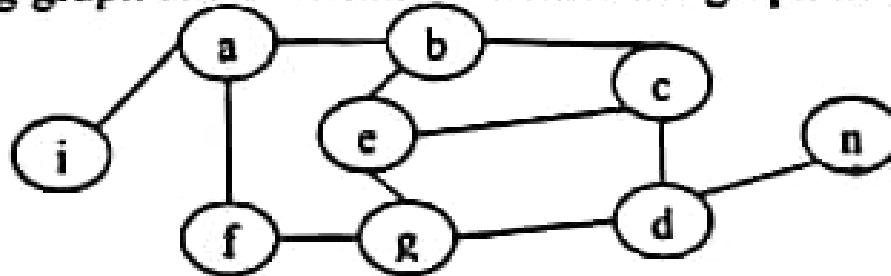
Let's consider a polynomial. A univariate polynomial is generally written as

$$A(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

Where x is an indeterminate and the a_i may be integers, floating point numbers, etc. When considering the representation of a polynomial by its coefficient, there are at least two alternatives. The first calls for storing the degree followed by degree's coefficient; (Degree begin n , $a_n \neq 0$)
 $(n, a_n, a_{n-1}, \dots, a_1, a_0)$

This is termed dense representation because it implicitly stores all coefficient whether or not they are zero.

7. a) Let $C(x) = A(x) \times B(x)$, where $A(x) = 3x^2 + 4x + 1$ and $B(x) = x^2 + 2x + 5$. Now, find the (10) resultant polynomial $C(x)$ using algebraic transformation and evaluation.
- b) "Any algorithm that computes the largest and smallest elements of a set of n ordered elements (05) requires $(\lceil 3n/2 \rceil - 2)$ comparison." – Prove the statement.
- c) What is tight lower bound? Show that the lower bound of an insertion sort is tight. (10)
- d) Consider the following graph and determine whether the graph has a cycle using DFS. (10)



71

a) Ans:

[Didn't find solution anywhere]

b) Ans:

[sorry]

c) Ans:

[sorry]

d) Ans:

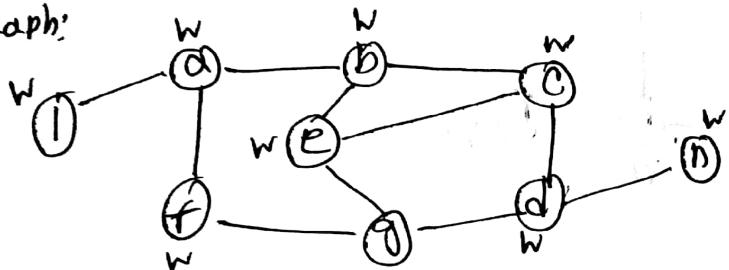
Let's consider something. If a node is

(W) white = not found/visited yet

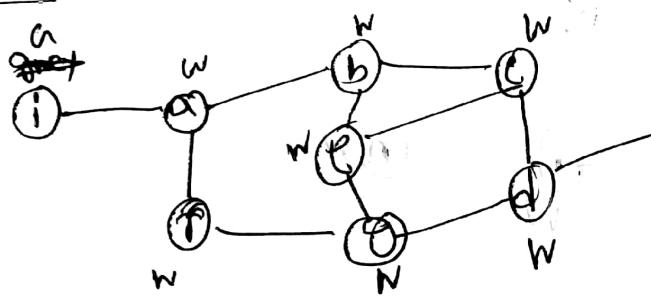
(G) grey = found but hasn't finished visiting its nodes yet

(B) black = finished working / visited.

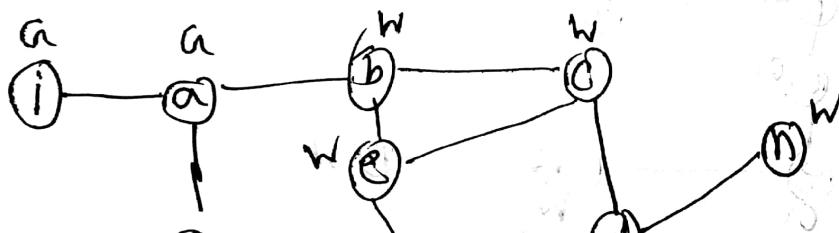
the graph:



first like run:



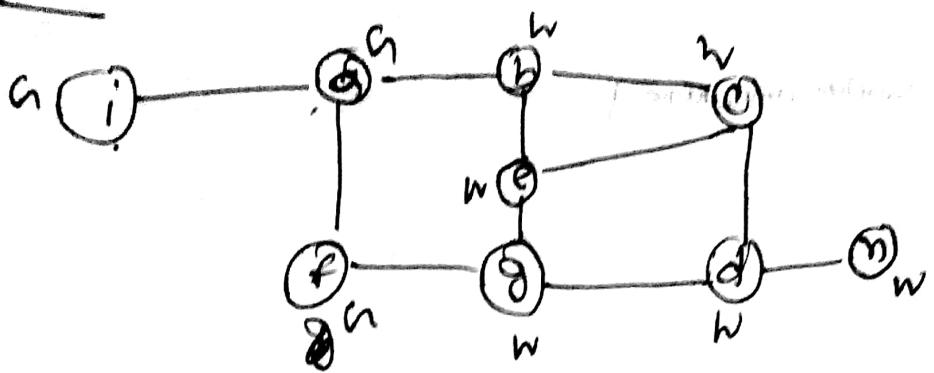
2nd



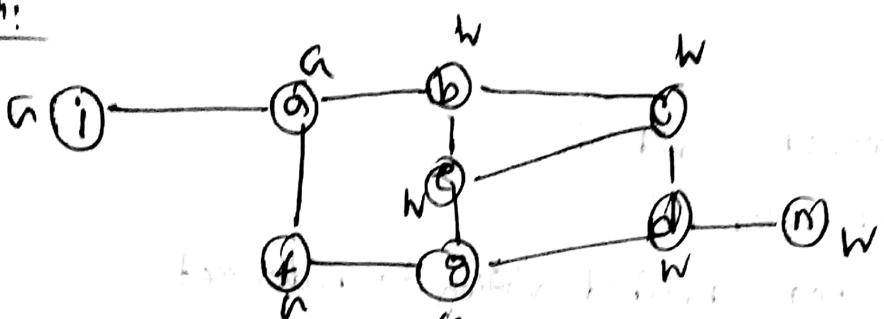
It's not hard to see that after stage N of N of visitments we will
either have the answer or it won't, which will take a lot of time.

So, if we do this, it will take a lot of time.

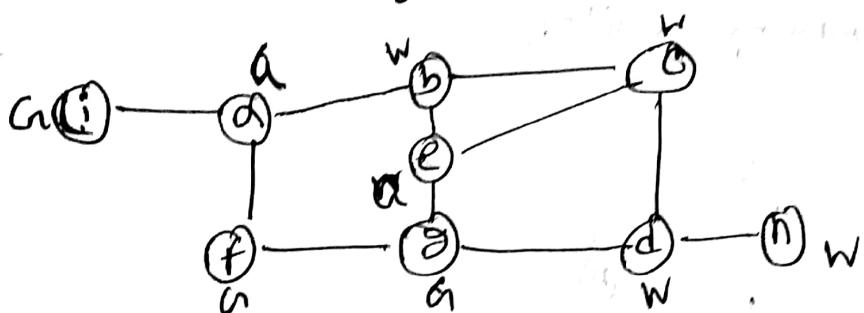
3rd:



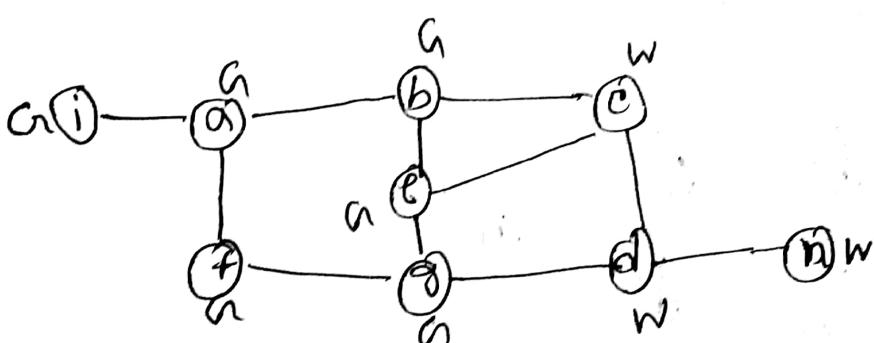
4th:



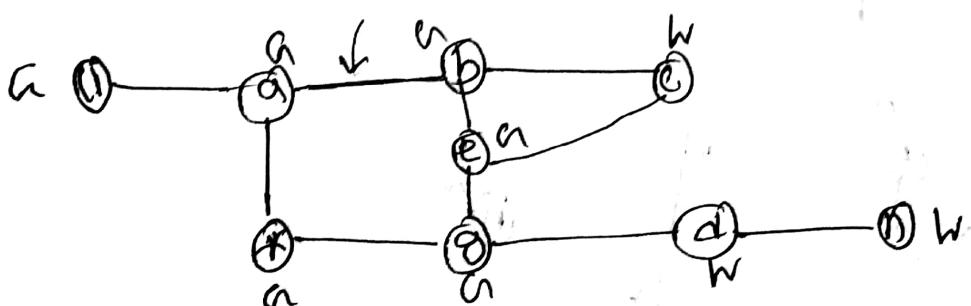
5th:



6th:

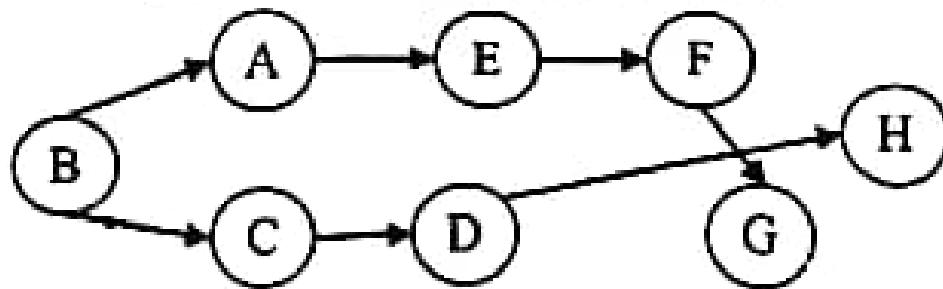


7th:



As we are attempting to go to a grey node (still not finished working) from a grey node (still working), there is a cycle at least one cycle and that is $a \rightarrow f \rightarrow g \rightarrow e \rightarrow b \rightarrow a$.

8. a) Why do we need non-deterministic algorithm? Convert a deterministic search into a non- (10)
deterministic search.
- b) Justify the statement – “the Halting Problem is a NP-hard problem that is not NP.” (06)
- c) Discuss the basic paradigm to find space complexity of a recursive algorithm. (10)
- d) Apply Topological sort in Lexicographical order of the graph mentioned in the following (09)
figure. If the graph has multiple answer, then mention all of them.



8

a) Ans:

We need non-deterministic algorithm for finding approximate solutions, when an exact solution is difficult or expensive to derive using a deterministic algorithm.

A nondeterministic search algorithm:

```

①  $i := \text{Choice}(s, n);$ 
    if  $A[i] = x$  then  $\{\text{write}(i); \text{success}();\}$ 
     $\text{writeln}(); \text{Failure}();$ 
  
```

b) Ans:

A problem will be NP if it can be verified by deterministic turing machine in polynomial time. As halting problem is undecidable. So, Halting problem is not NP.

To show that Halting problem is NP-hard, we show that satisfiability is at halting problem. For this let us construct an algorithm A whose input is a propositional formula X. Suppose X has n variables.

Algorithm A tries out all 2^n possible truth assignments and verifies if X is satisfiable. If it is satisfied then A stops. If X is not satisfiable, then A enters an infinite loop. Hence A halts on input if X is satisfiable. If we had a polynomial time algorithm for the

Halting problem, then we could solve the satisfiability problem in polynomial time using A and X as input to the algorithm for the halting problem.

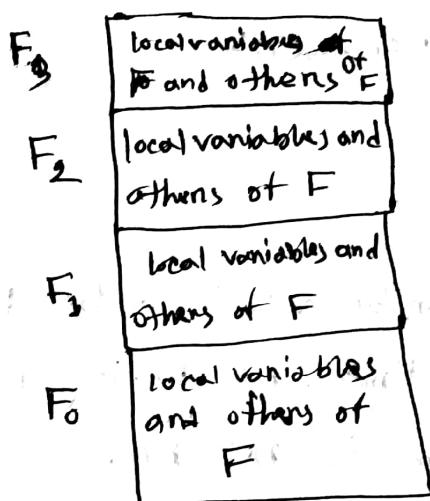
Hence, the halting problem is an NP-hard problem which is not NP.

C) Ans:

To find space complexity of a recursive algorithm, we have to consider the depth of the recursive call. And we have

to calculate how much space it is taking in each call.

If a function F is recursive, and its depth is N, then the stack looks like this,



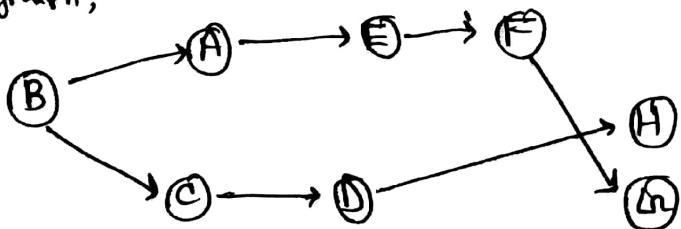
if N is 4, that is the depth of the recursive tree, then and in each call, if F takes S(F) space, then the space complexity will be mostly $N \times S(F)$. As the F₃ returns, the F₃ will get vanished from the recursive tree.

Now we shall discuss about the space complexity of the recursive algorithm. It is mainly to calculate the maximum size of the

frame or stack which is required to store the local variables and other information of the function. In this case, if the function is not recursive, then the space complexity will be O(1), but if the function is recursive, then the space complexity will be O(N), where N is the depth of the recursive tree.

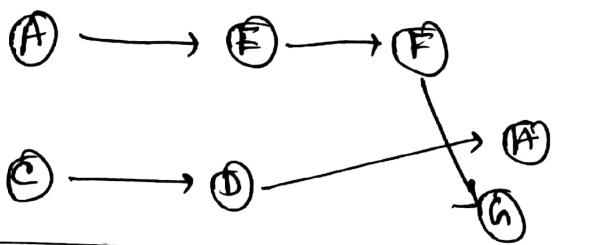
(d) Ans:

The graph,



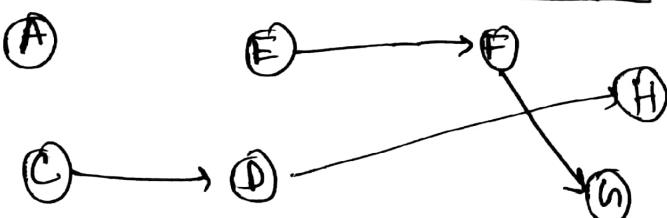
Applying Topological sort in Lexicographical order,

(B)



[Finished removing B]
indegree of

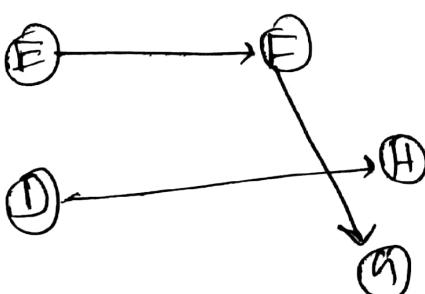
(B)



[A comes before C]

(B)

(A)

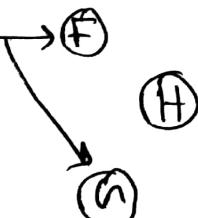


[C, as its indegree is 0]

(B)

(A)

(C)



[D]

(B)

(A)

(E)

(F)

(H)

(C)

(D)

(G)

The order is : B → A → C → D → E → F → G → H