numerical_methods_midterm_paper

May 12, 2022

1 Numerical Methods Mid term Paper

1.1 Total Marks : 25

2 Question #1

Convert the following algorithms into python by using functions and then solve the following problem

Find the roots of the following equation

$2x^2 - 5x + 3 = 0$ analytical roots are $x = 1.5, 1$

2.1 Algorithm 1 (Secant Method)

1. Input two values for x_1 and x_2
2. Calculate $x_{new} = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2)$
3. If $|x_{new} - x_2| < tolerance$, output x_{new} the root and stop
4. If number of iterations reaches the maximum, stop.
5. Else let $x_1 = x_2$ and $x_2 = x_{new}$
6. Repeat the steps from 2

2.2 Algorithm 2 (Newton's Methd)

1. Find $f'(x)$ and define newton's raphson equation
2. Guess an initial value of x for the first iteration
3. substitute x in the NR equation and calculate x^*
4. if $|x^* - x| < tolerance$, stop iterations and output the root: x^*
5. if the number of interations reaches an assumed maximum, stop.
6. else let $x = x^*$ and repeat the steps from 3 until a condition in 4 or 5 is satisfied.

3 Question # 2

Plot the first derivative of the following function over $[-1, 1]$ by using the forward, backward and central differences.

Compare the plots with the theoretical one at $h = 0.1, 0.01$ and 0.001

$f(x) = 0.1x^5 - 0.2x^3 + 0.1x - 0.2$

Theoretical :

$f'(x) = 0.5x^4 - 0.6x^2 + 0.1$

Formulas are given for reference

FORWARD FINITE DIFFERENCES

$f'(x) = \frac{f(x_{i+1}) - f(x_i)}{h}$

$f''(x) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2}$

BACKWARD FINITE DIFFERENCES

$f'(x) = \frac{f(x_i - f(x_i))}{h}$

$f''(x) = \frac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^2}$

CENTRAL FINITE DIFFERENCES

$f'(x) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$

$f''(x) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$

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