POLITECHNIKA WARSZAWKA | WARSAW UNIVERSITY OF TECHNOLOGY

FACULTY OF POWER AND AERONAUTICAL ENGINEERING

AUTOMATION AND ROBOTICS

Dynamics of Multi body system

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PROJECT REPORT

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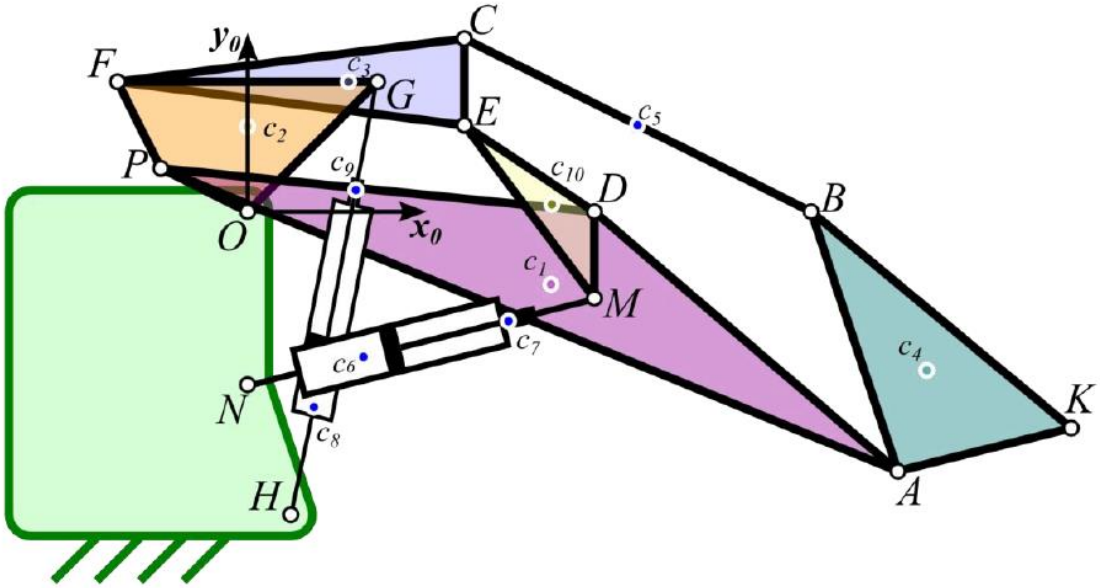

Figure 1 Kinematic scheme of the mechanism

	O	P	N	H	F	G	M	C	E	D	B	A	K
X(m)	0	-0.2	0.0	0.1	-0.3	0.3	0.8	0.5	0.5	0.8	1.3	1.5	1.9
Y(m)	0.0	0.1	-0.4	-0.7	0.3	0.3	-0.2	0.4	0.2	0.0	0.0	-0.6	-0.5

Table 1 Global coordinates of characteristics points

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
X(m)	0.70	0.0	0.20	1.55	0.9	0.20	0.60	0.15	0.25	0.7
Y(m)	-0.20	0.20	0.3	-0.35	0.20	-0.35	-0.25	0-.45	0.05	0.0

Table 2 Global coordinates of center mass

4 Theoretical Background

According to the kinematic scheme of the mechanism given above we have amount of 10 bodies fixed with ground have connected each other with different type of mechanical Joints. To analysis the kinematic pairs (mechanical joints) we have to formulate the kinematic constraint equation which is defined as follow

$$\phi^k = \phi^k(q) = 0$$

This algebraic equality of constraints is called holonomic constraints. where $\boldsymbol{q}$ denotes the vector of body-coordinates and ϕ represents a function describing the kinematic constraints.

To begin with the calculation of the positions of vector coordinates, here is some sampled 5 procedures solution how the constraint can formulate. Those are bodies from the mechanism having different mechanical joint.

5 Formulation of Kinematic constraints

5.1 Ground and Body 2

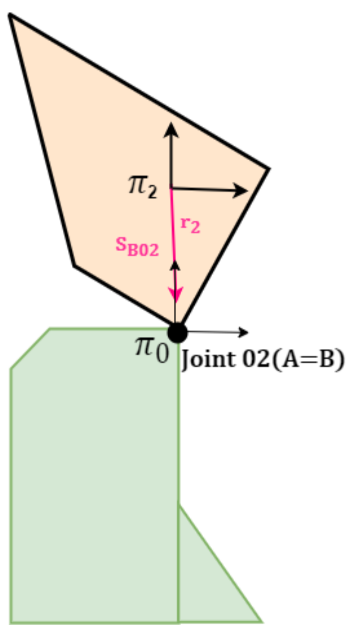

Figure 2 Revolute joint between ground and part 2

To determine the position vector of **A & B** w.r.t the GRF we have to determine the position vector **A & B** w.r.t the LRF.

From the given data for the multi body the position of center mass of body 2 ($\boldsymbol{c_2}$) is at (0.0,0.20) and the ground is (0.0,0.0).Here we consider the Global coordinates of centers of mass as LRF of that part. The LRF for part2 ($\boldsymbol{\pi_2}$) is (0.0,0.20)

Then we can calculate the position $\boldsymbol{S_{B02}}$

$$S_{B02} = (0.0,0.0) - (0.0,0.20)$$

$$= (0.0 - 0.0, 0.0 - 0.20)$$

$$\boldsymbol{S_{B02}} = [\mathbf{0.0, -0.20}]^{\boldsymbol{T}}$$

To find the position S_{A02} ,since the revolute joint and the GRF ($\boldsymbol{\pi_0}$) are at the same point then

$$\boldsymbol{S_{A02}} = [\mathbf{0.0, 0.0}]^{\boldsymbol{T}}$$

The position of LRF **($\boldsymbol{\pi_2}$)** from GRF ($\boldsymbol{\pi_0}$)

$$\boldsymbol{r_2} = (0.0,0.20) - (0.0,0.0)$$

$$= (0.0 - 0.0, 0.20 - 0.0)$$

$$\boldsymbol{r_2} = [\mathbf{0.0, 0.20}]^{\boldsymbol{T}} = -\boldsymbol{S_{B02}}$$

The constraint equation can be written in the vector form as:

$$\Phi^{k.} = r_i + R_i s_A^i - r_j - R_j s_B^j$$

$$R_2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Phi^{k.} = 0 + R_1 s_{A02}^1 - r_2 - R_2 s_{B02}^2$$

$$\Phi^{k.} = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix} - \begin{bmatrix} 0.0 \\ 0.20 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0,0 \\ 0.20 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

5.2 Ground and Body 8

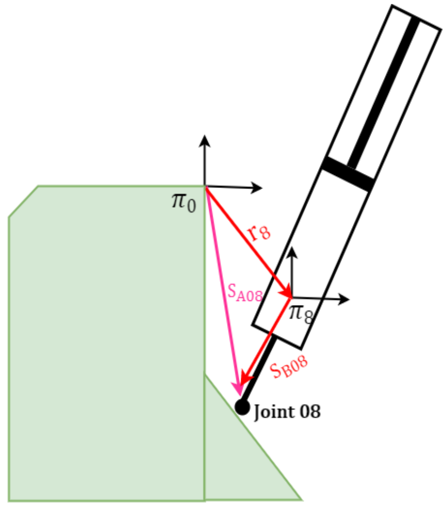

Figure 3 Revolute joints between ground and part 8

Body 8 is also connected with the ground using revolute joint at **joint 08 (A=B)**. Then we can calculate the position vectors as following by having the coordinate value

$$\pi_0(0.0)\ ,\ \pi_8(0.15, -0.45)\ , \text{Joint08}(0.1, -0.7)$$

The position vector S_{B08} from LRF

$$S_{B08} = (0.1, -0.7) - (0.15, -0.45)$$

$$= (0.1 - 0.15, -0.7 + 0.45)$$

$$\boldsymbol{S_{B08}} = [\mathbf{-0.05, -0.25}]^T$$

The position vector S_{A08} from LRF

$$S_{A08} = (0.1, -0.7) - (0.0, 0.0)$$

$$= (0.1 - 0.0, -0.7 - 0.0)$$

$$S_{A08} = [\mathbf{0.1, -0.7}]^T$$

The position vector $\boldsymbol{r_8}$ from GRF

$$r_8 = (0.15, -0.45) - (0.0, 0.0)$$

$$= (0.15 - 0.0, -0.45 - 0.0)$$

$$\boldsymbol{r_2} = [\mathbf{0.15, -0.45}]^T$$

5.3 Ground and Body 6

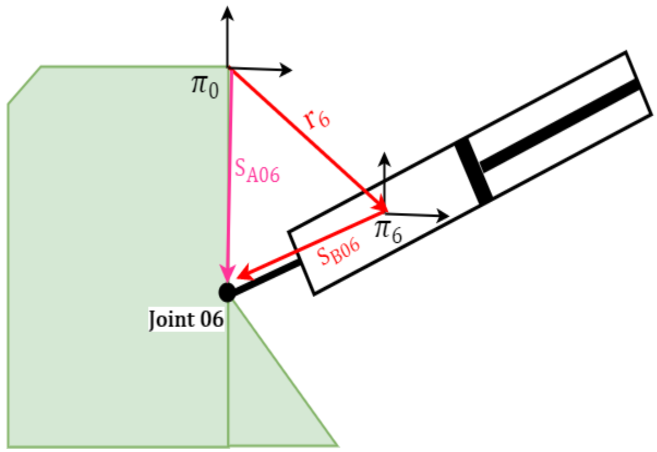

Figure 4 Revolute joint between ground and part6

$\pi_0(0.0)$, $\pi_6(0.20, -0.35)$, Joint06$(0.0, -0.4)$

$$\boldsymbol{S_{B06}} = [-\mathbf{0.20}, -\mathbf{0.05}]^T$$

$$\boldsymbol{S_{A06}} = [\mathbf{0.0}, -\mathbf{0.4}]^T$$

$$r_6 = [\mathbf{0.20}, -\mathbf{0.35}]^T$$

$$\phi^{k.} = R_6 s_{A06}^1 - r_6 - R_6 s_{B06}^6$$

5.4 Body 2 and 1

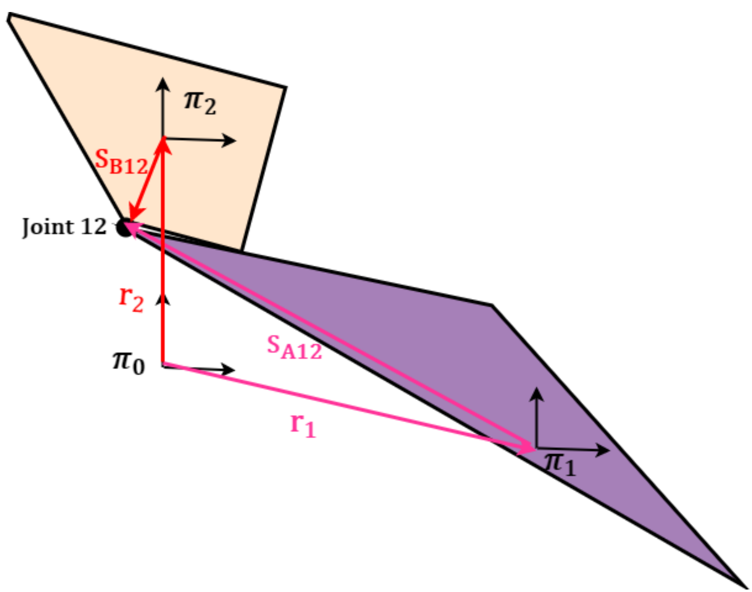

Figure 5 Revolute joint Between Part 1 &2

The mechanical joint which connects body 1 and body 2 is revolute joint. The data from the kinematic multi body system are

$\pi_0(0.0)$, $\pi_2(0.0,0.20)$, $\pi_1(0.70,-0.20)$ Joint12(-0.2,0.1)

To find the position vector $\boldsymbol{S_{A12}}$ from LRF

$$S_{A12} = [\,-0.2{,}0.1]^T - [0.70,-0.20]^T$$

$$= [-0.9\,,-0.3]^T$$

To find the position vector $\boldsymbol{S_{B12}}$ from LRF

$$S_{B12} = [\,-0.2{,}0.1]^T - [0.0{,}0.20]^T$$

$$= [-0.2,-0.1]^T$$

The position vector for LRF $\boldsymbol{\pi_1}$ from GRF $\boldsymbol{\pi_0}$

$$r_1 = [0.70,-0.20]^T$$

The position vector for LRF $\boldsymbol{\pi_2}$ from GRF $\boldsymbol{\pi_0}$

$$r_2 = [0.0{,}0.20]^T$$

The constraint equation can be written in the vector form as:

$$\Phi^{k.} = r_i + R_i s_A^i - r_j - R_j s_B^j$$

$$\Phi^{k.} = r_1 + R_1 s_{A12}^1 - r_2 - R_2 s_{B12}^2$$

$$\Phi^{k.} = \begin{bmatrix} 0.7 \\ -0.20 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.05 \end{bmatrix} - \begin{bmatrix} 0.0 \\ 0.20 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -0.2 \\ -0.1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

5.5 Body 7 and 6

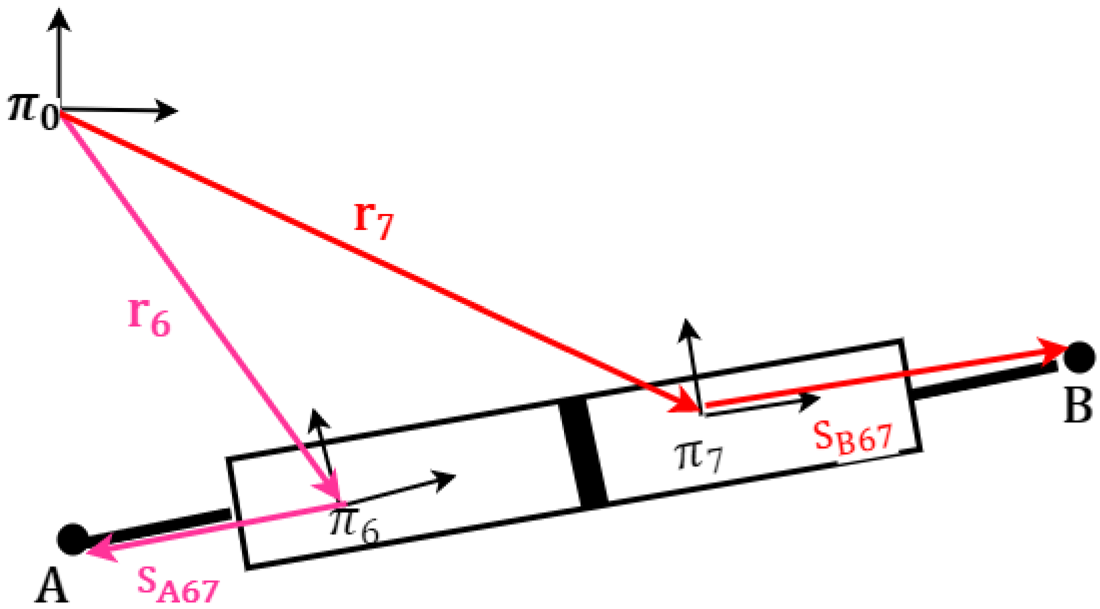

Figure 6 Translational Joint between Body 6 & 7

The mechanical joint which connects body 6 and body 7 is translational joint.

$\pi_0(0.0)$, $\pi_6(0.20, -0.35)$, $\pi_7(0.6, -0.25)$

To find the position vector $\boldsymbol{S_{A67}}$ from LRF

$$S_{A67} = [0.2\,, -0.35]^T - [0.0\,, -0.4]^T$$
$$= [0.2\,, 0.05]^T$$

To find the position vector $\boldsymbol{S_{B67}}$ from LRF

$$S_{B67} = [0.8\,, -0.2]^T - [0.6\,, -0.25]^T$$
$$= [0.2\,, 0.05]^T$$

The position vector for LRF $\boldsymbol{\pi_7}$ from GRF $\boldsymbol{\pi_0}$

$$r_7 = [0.6\,, -0.25]^T$$

The position vector for LRF $\boldsymbol{\pi_6}$ from GRF $\boldsymbol{\pi_0}$

$$r_6 = [0.20, -0.35]^T$$

Then to find the vector **V,** let consider the following figure which describes the straight-line ***l*** which connects point A and B in between body 6 and body 7.

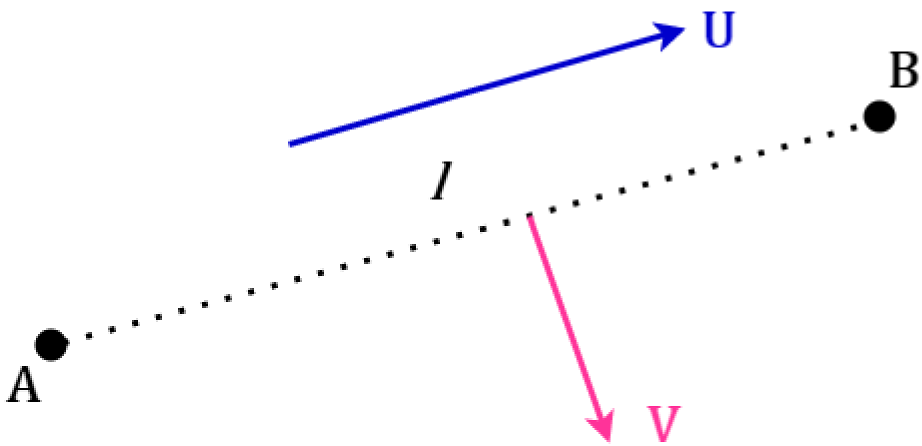

Figure 7 Calculation V and U vectors

Then the vector which is perpendicular for the line which connects the points $\pi_6(0.20, -0.35)$, $\pi_7(0.6, -0.25)$ is

$$\frac{-0.25 + 0.35}{0.6 - 0.2} = \frac{y}{x}$$

$$y = \frac{1}{4}x$$

Then to find the vector **V**, let pick one point which makes the dot product 0

Which is (-1,4).

$$V^{67} = [-1,4]^T$$

Since the joint is translational joint, then the general constraint equation is defined as follow

$$\Phi^{k\uparrow} = [R_j V^j]^T [r_j + R_j s_B^j - r_i - R_i s_A^i]$$

$$\Phi^{k<} = \varphi_i - \varphi_j - \varphi_0$$

Accordingly, the constraint equation for translational joint between body 6 and body 7 will be:

$$\Phi^{k\uparrow} = [R_7 V^{67}]^T [r_7 + R_7 s_{B67}^7 - r_6 - R_6 s_{A67}^6]$$

The rotational angle between the references is 0.

$$R_7 = R_6 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Phi^{k\uparrow} = \left[\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}-1\\4\end{bmatrix}\right]^T \left[\begin{bmatrix}0.6\\-0.25\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}0.2\\0.05\end{bmatrix} - \begin{bmatrix}0.2\\-0.35\end{bmatrix} - \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}0.2\\0.05\end{bmatrix}\right]$$

$$\Phi^{k<} = \varphi_6 - \varphi_7 - \varphi_0 = 0$$

6 Driving constraints in translational joint

$$\Phi^{\boldsymbol{D}\uparrow} = [\boldsymbol{R_j U^j}]^{\boldsymbol{T}}\left[\boldsymbol{r_j} + \boldsymbol{R_j s_B^j} - \boldsymbol{r_i} - \boldsymbol{R_i s_A^i}\right] - \boldsymbol{f_{AB}(t)}$$

First, we have to find vector **U,** since it is parallel to the straight line **l** ,

$$[0, -0.4] + [0.8, -0.2] = [0.8, -0.6]$$

$$\mathbf{U} = \frac{[0.8, -0.6]}{\sqrt{\mathbf{0.8}^2 + (-\mathbf{0.6})^2}} = [\mathbf{0.9701}, \mathbf{0.2425}]$$

Then we have also motion function for the translational joint between body 6 and 7

$$\boldsymbol{f_{67}(t)} = -0.1 * \sin(1 * t)$$

Then the driving constraint will be substituting the above calculated to the following equation.

$$\Phi^{\boldsymbol{D}\uparrow} = [\boldsymbol{R_7 U^7}]^{\boldsymbol{T}}\left[\boldsymbol{r_7} + \boldsymbol{R_7 s_{B67}^7} - \boldsymbol{r_6} - \boldsymbol{R_6 s_{A67}^6}\right] - \boldsymbol{f_{67}(t)}$$

$$\Phi^{\boldsymbol{D}\uparrow} = \left[\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}0.9701\\0.2425\end{bmatrix}\right]^T \left[\begin{bmatrix}0.6\\-0.25\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}0.2\\0.05\end{bmatrix} - \begin{bmatrix}0.2\\-0.35\end{bmatrix} - \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}0.2\\0.05\end{bmatrix}\right]$$

Note: The Above sample solution shows how can we formulate the kinematic constraint equation for few joints. The first-time derivative $\Phi^k(q)$ of yields the velocity constraints that provide relations between the velocity variables of a system. The velocity constraints can be expressed as

$$\frac{\mathbf{d}\phi(q)^{k}}{\mathbf{dt}} = \boldsymbol{J}v = 0$$

In which $\boldsymbol{J}$ is the Jacobian matrix, and $\boldsymbol{v}$ is velocity.

7 Jacobian Matrix

The Jacobian matrix is the coefficient matrix of the velocity and acceleration. The Jacobian matrix is large square matrix which have the same dimensions with the number of constraint equation. The following are Jacobian matrix corresponding to kinematic constraints equations used for the revolute joint in this project.

$$\phi_{\mathbf{r_i}}^{\mathbf{K}} = \mathbf{I_{2x2}}$$

$$\phi_{\phi_{\mathbf{i}}}^{\mathbf{K}} = \Omega\,\mathbf{R_{2x2}S_A^{(i)}}$$

$$\phi_{\mathbf{r_i}}^{\mathbf{K}} = -\mathbf{I_{2x2}}$$

$$\phi_{\phi_{\mathbf{i}}}^{\mathbf{K}} = -\Omega\,\mathbf{R_{2x2}S_B^{(j)}}$$

The **singularity** of the Jacobian matric can be checked, by the rank or determinant. Here I used rank value of the Jacobian using rank.in this case which is 30.

And the equation for translational joints is made as following

$$\phi_{\mathbf{r_i}}^{\mathbf{K<}} = \mathbf{0_{1x2}}$$

$$\phi_{\phi_{\mathbf{i}}}^{\mathbf{K<}} = \mathbf{1}$$

$$\phi_{\mathbf{r_j}}^{\mathbf{K<}} = \mathbf{0_{1x2}}$$

$$\phi_{\phi_{\mathbf{j}}}^{\mathbf{K<}} = -\mathbf{1}$$

$$\phi_{\mathbf{r_i}}^{\mathbf{K\uparrow}} = -(\mathbf{R_j V^{(f)}})^{\mathbf{T}}$$

$$\phi_{\mathbf{r_i}}^{\mathbf{K\uparrow}} = -(\mathbf{R_j V^{(f)}})^{\mathbf{T}}\Omega\,\mathbf{R_i S_A^{(i)}}$$

$$\phi_{\mathbf{r_j}}^{\mathbf{K\uparrow}} = (\mathbf{R_j V^{(f)}})^{\mathbf{T}}$$

$$\phi_{\mathbf{r_j}}^{\mathbf{K\uparrow}} = -(\mathbf{R_j V^{(f)}})^{\mathbf{T}}\,\Omega(\mathbf{r_j} - \mathbf{r_i} - \mathbf{R_i S_A^{(i)}})$$

8 Velocity

Velocity is the differentiate of the position. And the velocity is calculated in MATLAB for this project as:

$$d_q = -\frac{F_q}{F_t}$$

The last two rows contain the driving constraint equation with the function of time. The right-hand side equation contains zeros in the elements of the matrix except the last two rows with the driving constraint equations.

9 Acceleration

Acceleration is the second derivative of the position or first derivative of the velocity. The equation for acceleration will be calculated as follow:

$$\Gamma_r^{K\cdot} = \mathbf{R_i S_A^{(i)}} \phi_{\mathbf{i}}^{\mathbf{2}} - \mathbf{R_j S_B^{(j)}} \phi_{\mathbf{j}}^{\mathbf{2}}$$

$$\Gamma_r^{K<} = 0$$

10 ADAMS MODEL

The kinematic scheme is modeled in ADAMS, the units were setting in meter(**M**). The initial model is shown below in the figure.

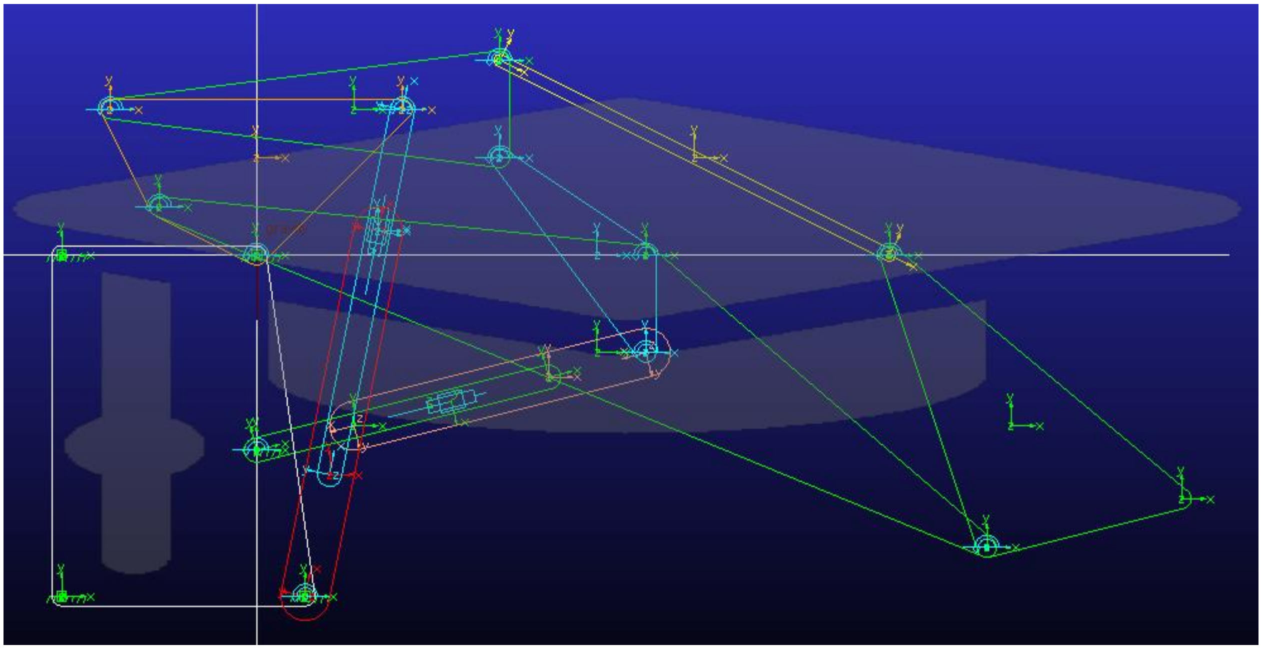

Figure 8 Initial simulation of ADAMS

After setting up the model, we must select the motion of the two actuators according to the given equation

$$x_k = l_k + a_k \sin(w_k t + \varphi_k)$$

The motion equation for **body 6** and **body 7**

$$x_1 = 0 - 0.1 * \sin(1 * t + 0)$$

The motion equation for **body 8** and **body 9**

$$x_1 = 0 + 0.05 * \sin(1 * t + 0)$$

10.1 simulation

The simulation was done for 20 seconds and 1000 steps, and the final simulation model is viewed in below:

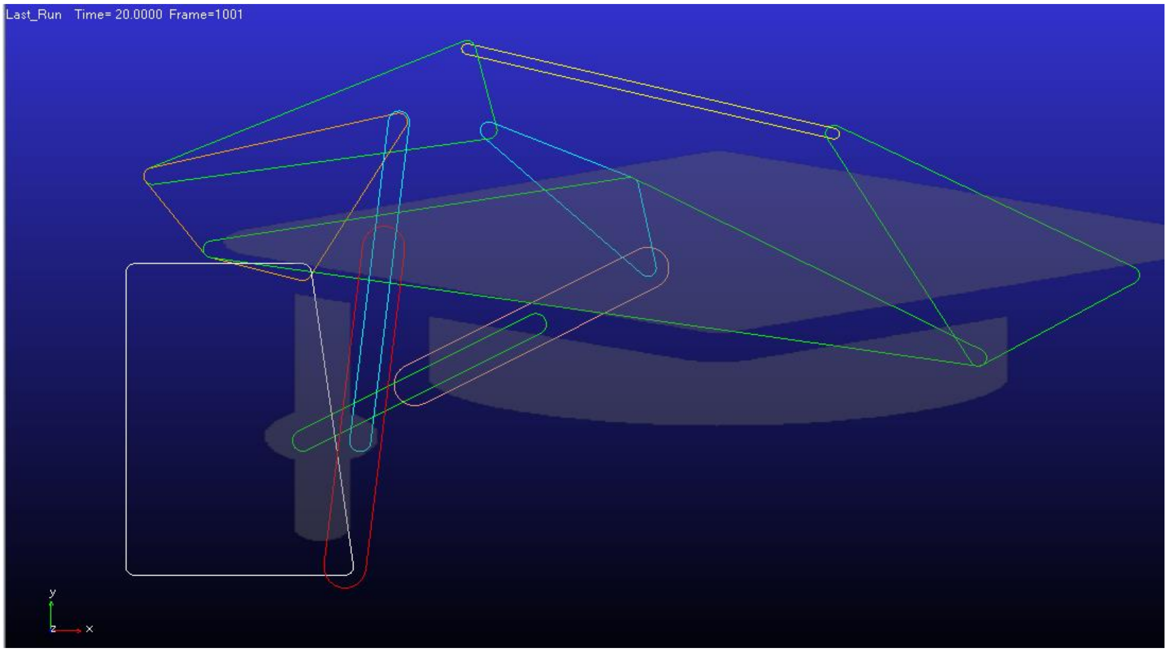

Figure 9 Final Simulation of ADAMS

10.1.1 Simulation Adams and MATLAB Result

The result was obtained for the last part **body4** $(\boldsymbol{c_2})$**.** Then the position, velocity, and acceleration of the center mass of part 4 is found as follow in the figure

10.1.2 position along X and Y in ADAMS

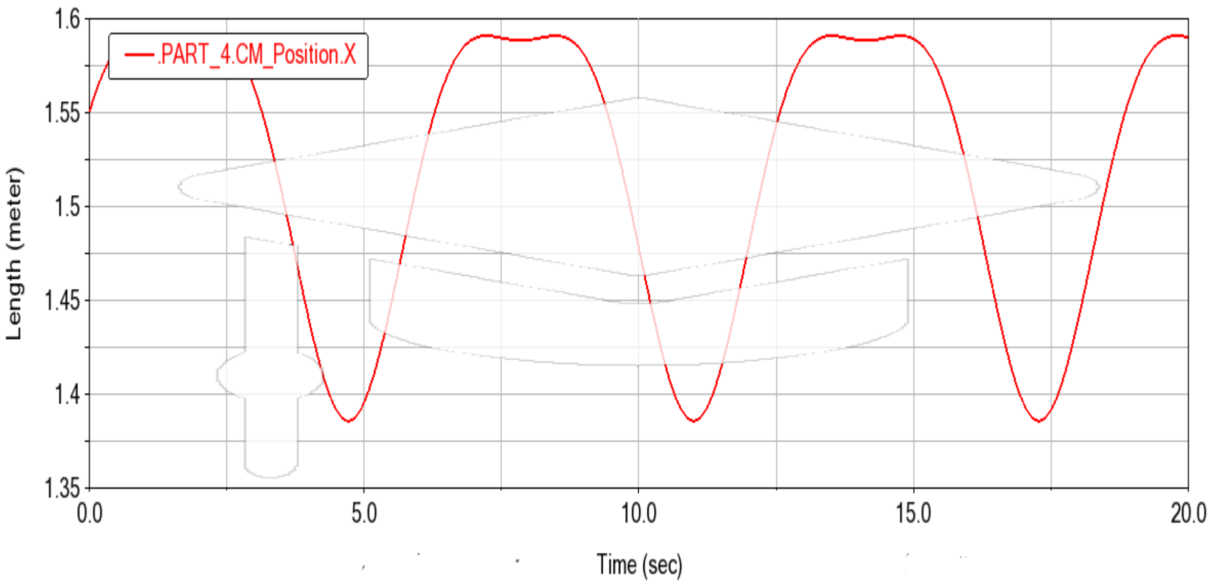

Figure 10 position of CM_part4 in x ((ADAMS result))

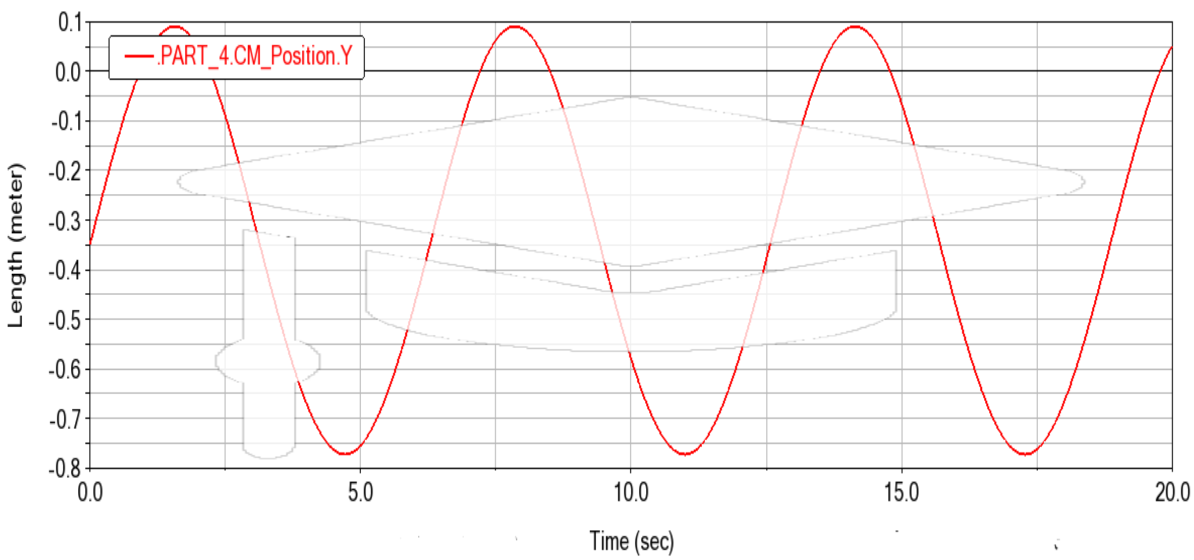

Figure 11 Position of CM_part4 in y (ADAMS result)

10.1.3 Position along X and Y in MATLAB

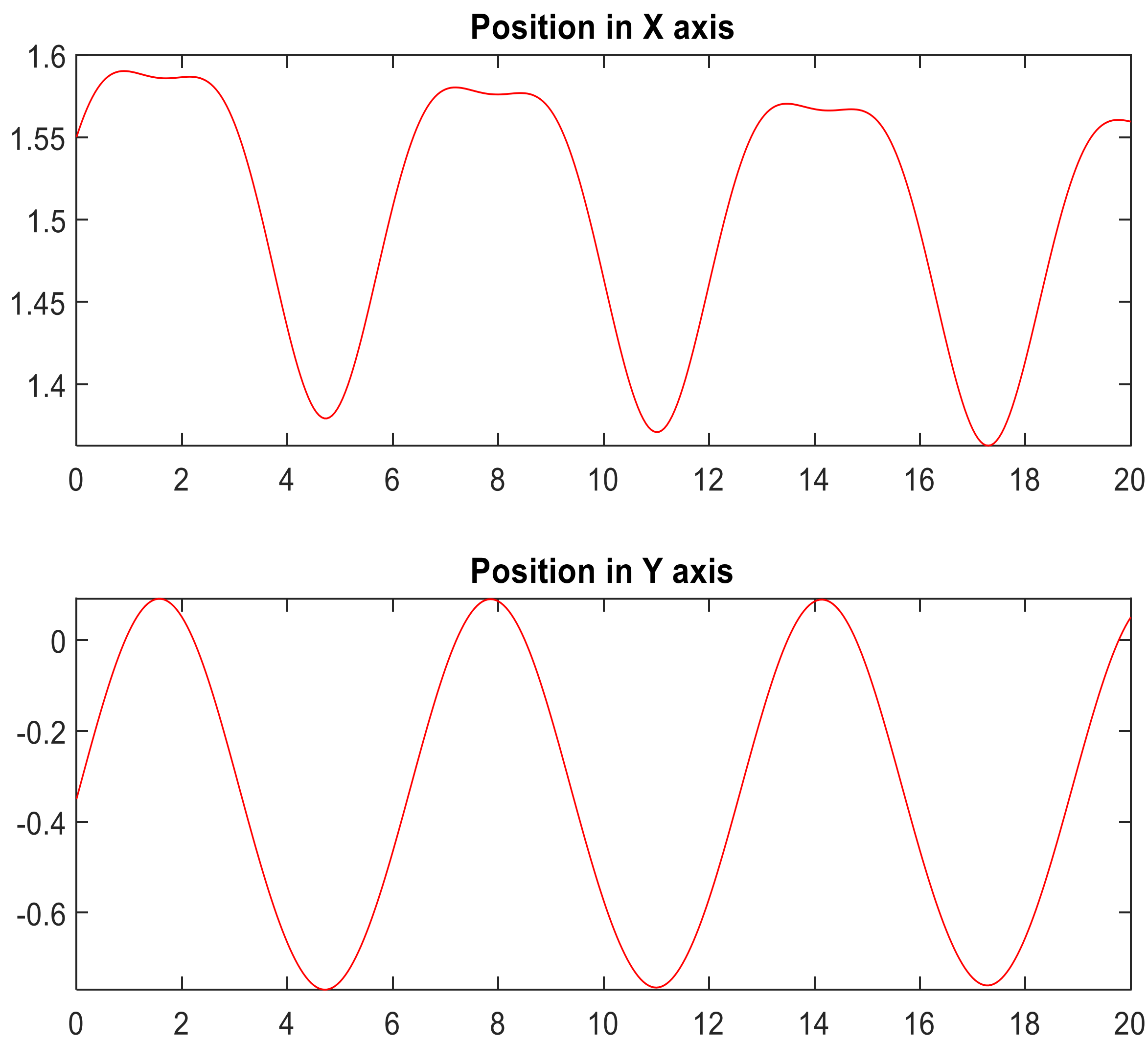

Figure 12 position of CM_part4 in x and y (MATLAB result)

10.1.4 velocity along X and Y in ADAMS

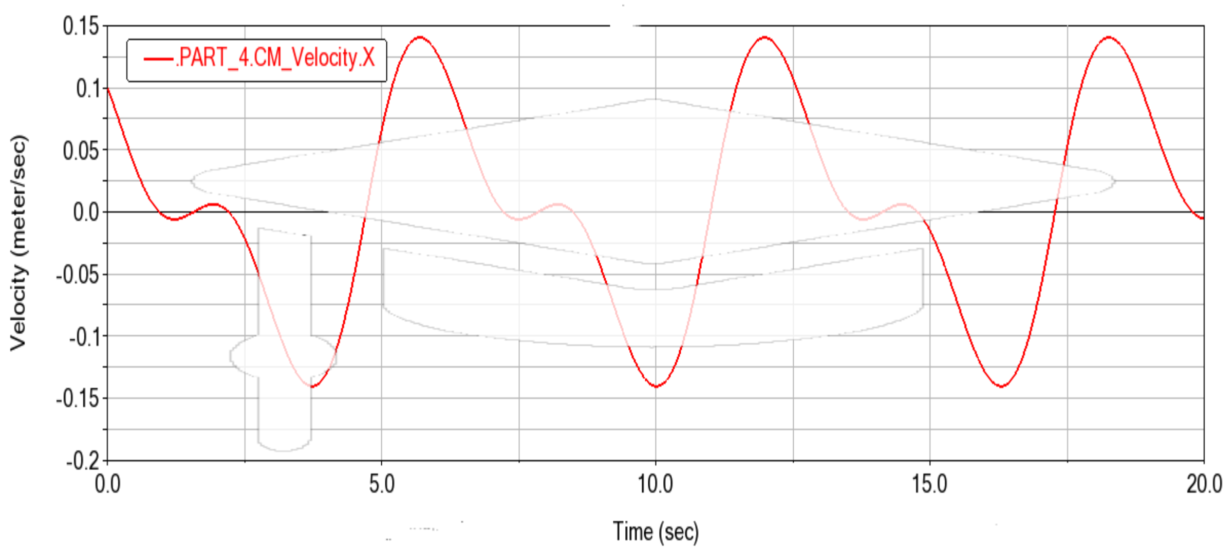

Figure 13 Velocity of CM_part4 in x(ADAMS)

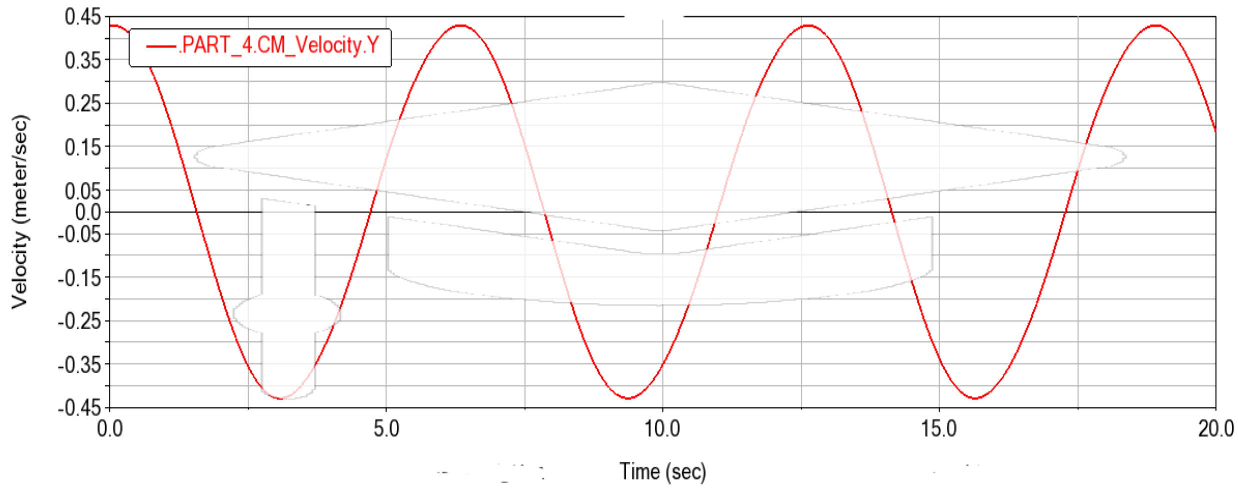

Figure 14 Velocity of CM_part4 in Y(ADAMS)

10.1.5 Velocity along X and Y in MATLAB

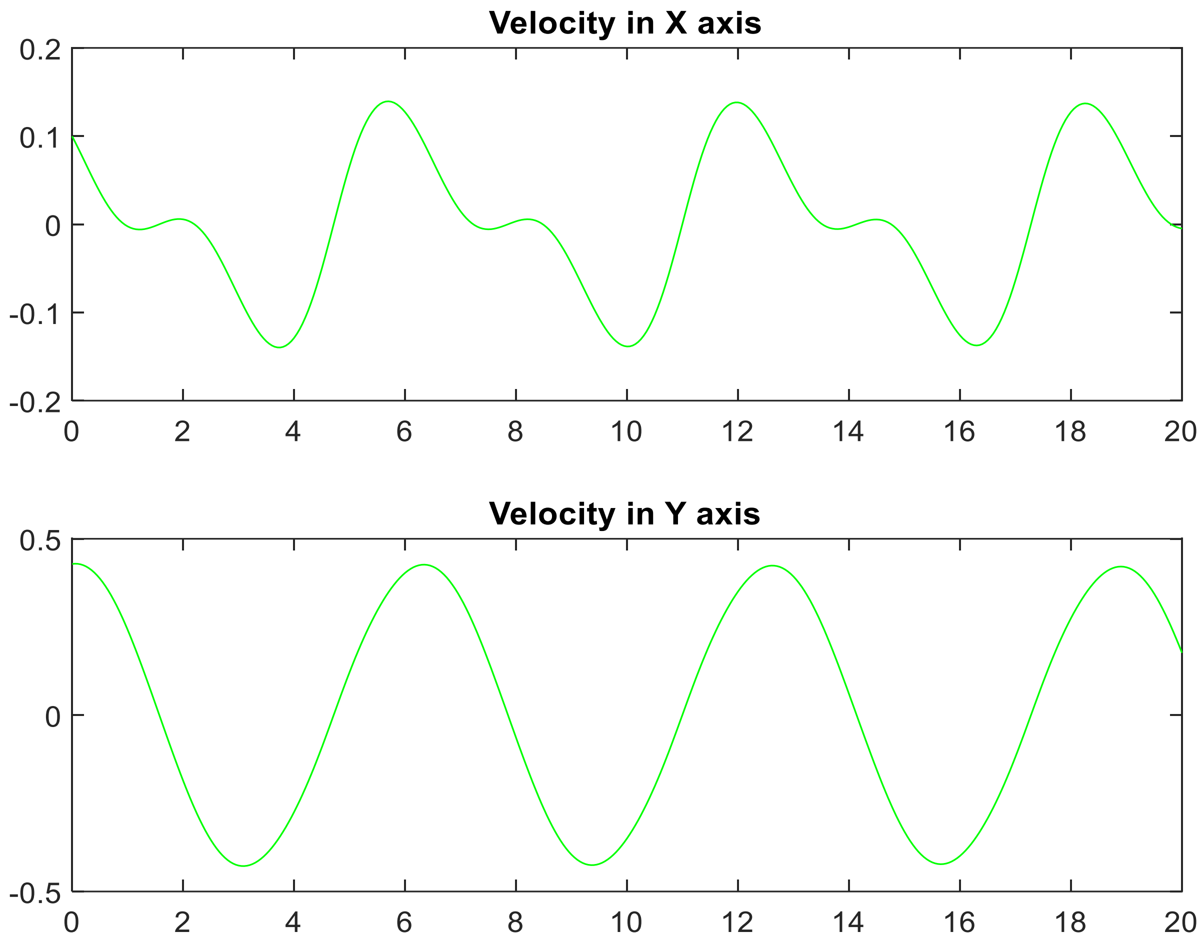

Figure 15 Velocity of CM_part4 in x and y (MATLAB result)

10.1.6 Acceleration along X and Y in ADAMS

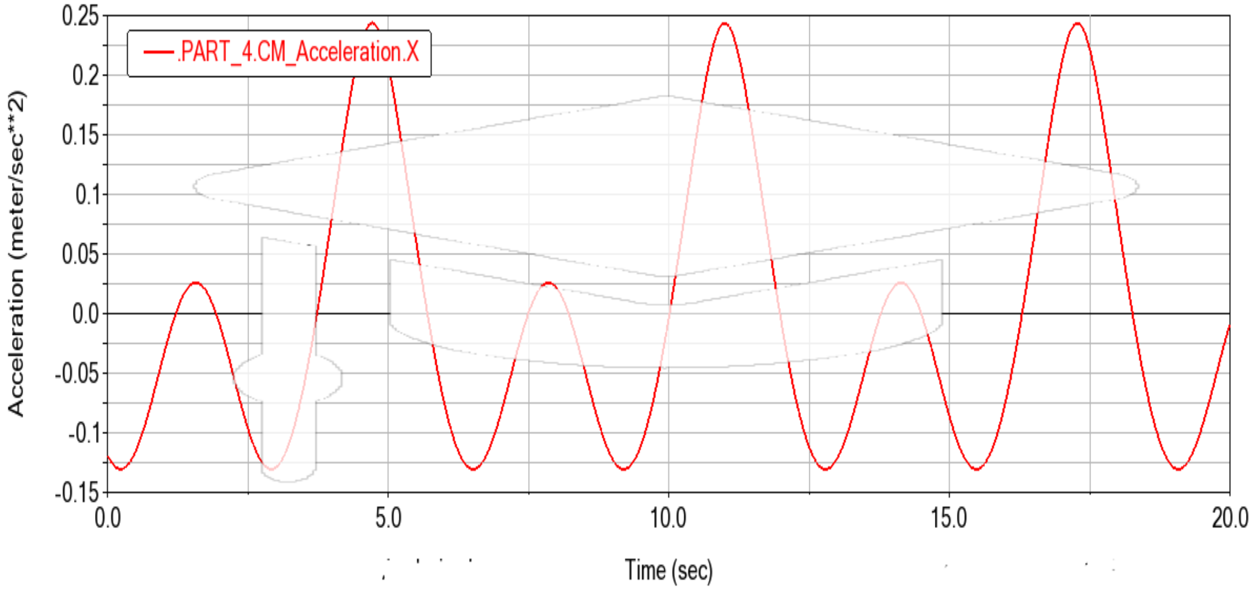

Figure 16 Acceleration of CM_part4 in X (ADAMS)

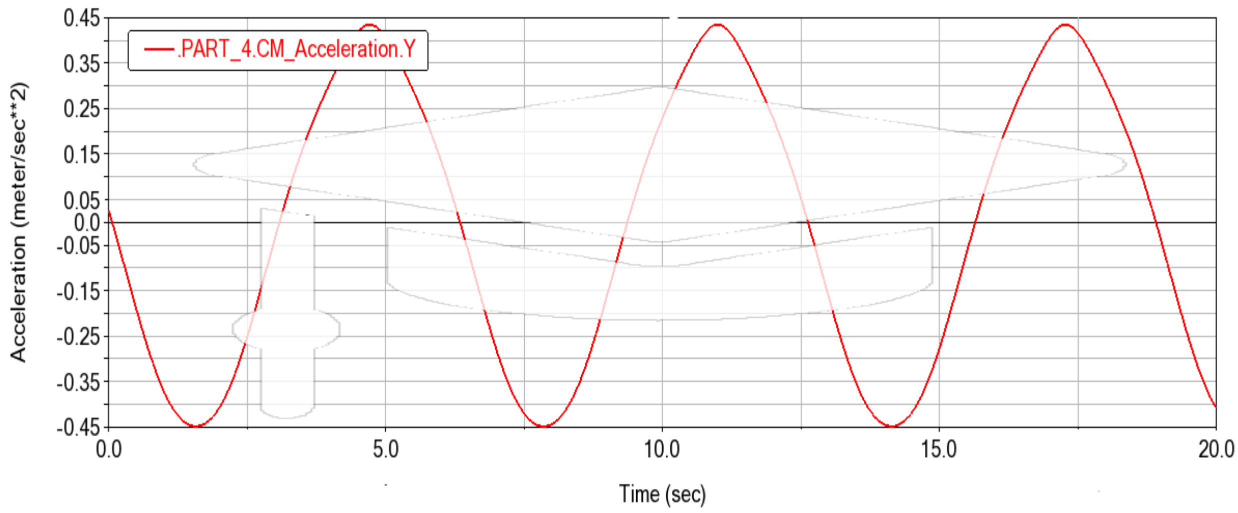

Figure 17 Acceleration of CM_part4 in Y (ADAMS)

10.1.7 Acceleration along X and Y in MATLAB

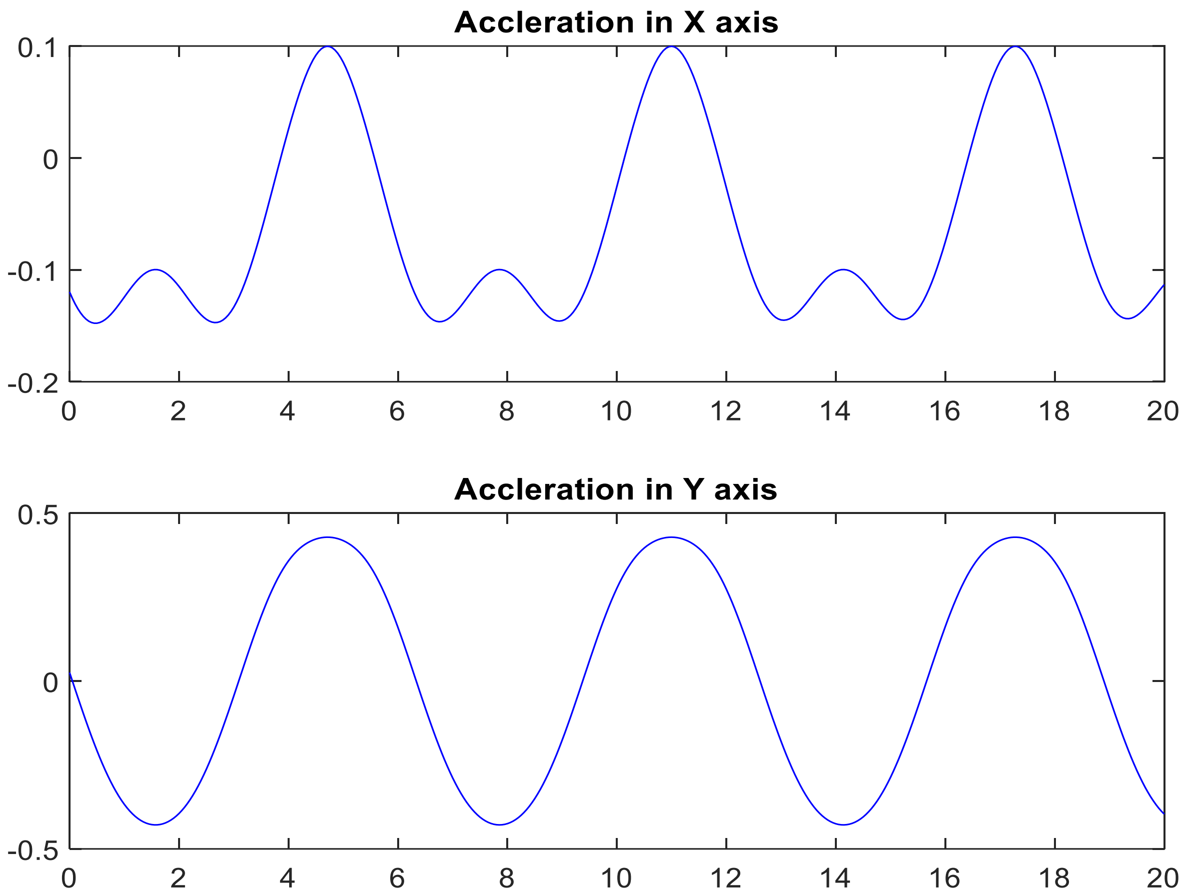

Figure 18 Acceleration of CM_part4 in X and Y (MATLAB result)

11 Comparison of results

Here I have put the comparison of some sample points from the ADAMS and MATLAB simulation result for the position, velocity and accelerations along X and Y for the selected CM part4.

11.1 Comparison of positions in MATLAB and ADAMS

Time (s)	Position in X(m)			Position in Y(m)		
	MATALAB	**ADAMS**	**Error**	**MATLAB**	**ADAMS**	**Error**
0.4	1.5796	1.5797	0.0001	-0.1811	--0.1811	0.0000
7.8	1.5763	1.5840	0.0077	0.08915	0.0901	0.00095
11	1.3709	1.3855	0.0146	-0.7650	-0.7729	0.0079

Table 3 position comparison

11.2 Comparison of Velocity in MATLAB and ADAMS

Time (s)	Position in X(m)			Position in Y(m)		
	MATALAB	**ADAMS**	**Error**	**MATLAB**	**ADAMS**	**Error**
1.2	-0.0059	-0.0060	0.0001	0.1624	0.1626	0.0002
8	0.0034	0.0035	0.0001	-0.0643	-0.0653	0.0010
16.2	-0.1362	-0.1392	0.0030	-0.3655	0.3697	0.0042

Table 4 Velocity comparison

11.3 Comparison of Acceleration in MATLAB and ADAMS

Time (s)	Position in X(m)			Position in Y(m)		
	MATALAB	ADAMS	Error	MATLAB	ADAMS	Error
2.2	-0.1281	-0.45	0.3219	-0.3538	-0.3574	0.0036
10.8	0.0930	0.2263	0.1333	0.4237	0.4210	0.0027
14	-0.1014	0.0216	0.1230	-0.4252	-0.4448	0.0196

Table 5 Acceleration comparison

12 Conclusion

As summary, we can see that from the MATLAB and ADAMS simulation results are the same. The procedures that I use to do this project are, primarily I calculate the absolute coordinate positions for the characteristic's points and center mass of the parts. By having those vectors, we must formulate the kinematic constraints according to their corresponding way of formulation and formulation of the driving constraints. The Jacobian matrix is helpful on calculating the velocity and acceleration and checking of the singularity, in order to check the singularity of the Jacobian matrix the rank of the matrix has to be equal to the number of scalar equations, in this case the rank is 30. For the newton Raphson method, the chosen accuracy 10^{-10} and the number of iterations is 25.

Generally, this project helps on Analysis of kinematic for multi body system connected with different types mechanical joints.